Monopoly and Long-Run Capital Accumulation*

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Abstract

This paper constructs a decentralized growth model with two production sectors, one having competitive firms and the other monopolies. Since capitalized pure profits for the latter sector constitute an asset which household savings must finance, we show that imperfect competition can reduce steady-state national output through both a "static effect" on allocative efficiency and a "dynamic effect" on aggregative capital accumulation. After presenting a theoretical analysis, we generate several numerical examples. The latter suggest the "dynamic effect" of monopoly may be ten times or more as large as the "static effect" in practice.
Monopoly and Long-Run Capital Accumulation

Induced discrepancies between social marginal rates of transformation and household marginal rates of substitution are the focus of conventional general equilibrium studies of the consequences of monopoly. Using such an approach, Harberger (1954) estimated that allocative-efficiency losses to the U.S. economy from imperfect competition amount to about .1% of national income (see also Schwartzman (1960, 1961) and Worcester (1973)). Recent work -- see Kamerschen (1966), Bergson (1973), and Cowling and Mueller (1978) -- implies losses of 1 to 5% (or more) are conceivable if greater interproduct elasticities of substitution, monopoly markups of price over marginal cost, and degrees of disaggregation than Harberger employed are appropriate. In each case the basic framework of analysis is the same, however, and it is static.

The purpose of this paper, in contrast, is to consider one set of possible dynamic implications of the existence of monopolies in an economy. To carry out the investigation we construct a decentralized long-run growth model (in other words, a simple general equilibrium model) having a competitive sector and a sector with monopolies. We posit the existence of a function which aggregates all value added into a single index, "gross national product." The function plays a role roughly analogous, for example, to Bergson's (1973) method of calculating "real income" or to a rule for deducing total consumer surplus in Harberger's (1954) sense. Thus we can calculate a "static loss" in aggregate output due to monopoly corresponding to the losses computed in the works listed above. A second channel through which monopoly can adversely affect national output also emerges, however.

The new channel operates as follows. If all economic agents have perfect foresight and there is no uncertainty, the market value of a competitive firm should coincide with the replacement value of its undepreciated physical assets.
whereas -- assuming all profits accrue to stockholders -- the market value of a firm which is a monopoly will include both the value of tangible assets and the capitalized value of present and future (after-tax) pure profits. Now the household sector will desire to hold a certain amount of wealth to meet life-cycle needs. If there are no monopolies (and no government activities), all such wealth will be available to finance physical capital. On the other hand, if there are monopolies in the economy, part of the wealth will be diverted to finance capitalized monopoly profits -- and only the remainder will be left for tangible capital. The existence of monopolies, therefore, may tend to reduce the overall physical capital stock which competitive household-sector behavior can support, lowering the steady-state gross national product.

Our "dynamic effect" of monopoly on gross national product does not freely translate into a welfare loss the way the "static effect" does: permanently destroying monopoly profits would reduce the present consumption possibilities of the owners of the former profit flows, although the consumption possibilities of future generations would presumably be increased. The "dynamic effect" of imperfect competition on steady-state aggregate output is interesting in its own right, nevertheless, and it can be compared in magnitude with its "static effect" counterpart.

This paper has four sections. Section I presents our model and proves the existence of a unique steady-state growth path. Section II separates and examines the "static" and "dynamic" components of the effect of monopolies on the gross national product. Section III then attempts to make the basic model more realistic (using a framework resembling that in Tobin (1967)) and to assess the possible empirical importance of "dynamic" monopoly consequences. Section IV concludes the analysis.
I. The Model

This section presents a decentralized growth model incorporating both monopolies and competitive firms. To preserve a degree of simplicity and make unambiguous comparative-static results possible (in Section II), we employ stringent specializing assumptions about functional forms (Cobb-Douglas production and Bergson utility functions) and about the feasibility of forming various aggregates. After setting up our model, we prove the existence and uniqueness of a steady state.

Production

We begin with a description of the model's production framework and of the implications for factor allocation of cost-minimizing behavior on the part of firms. There will be one sector of competitive businesses and a second of monopolies.3/

Consider the competitive sector first. Suppose, for the moment, that at time \( t \) along with a labor input, \( LC_{it} \), each competitive firm \( i \) uses a single type of capital stock, measured as \( KC_{it} \), to produce a quantity of output \( QC_{it} \), given in terms of value added. Let each such firm have a production function

\[
QC_{it} = FC_i(LC_{it}, KC_{it}) \cdot (1 + \lambda)^t,
\]

where \( FC_i(\cdot) \) is linearly homogeneous.4/ We assume the conditions of Green's (1964) Corollary to Theorem 10 are satisfied -- all \( FC_i(\cdot) \) functions have parallel expansion paths -- so that there exists a linearly homogeneous function \( FC(\cdot) \) and constants \( a^{\cdot}_i \) such that

\[
LC_t = \sum_i LC_{it} \quad \text{and} \quad KC_t = \sum_i KC_{it}
\]

imply
\[ QC_t = \Sigma a_i Q_{it} = FC(LC_t, KC_t) \cdot (1 + \lambda)^t. \]

Given the aggregation above, if \( dLC_t = dKC_t = 0 = dQC_k \) for \( k \neq i \) or \( j \),
\[
dQC_{it}/dQC_{jt} = -a_j/a_i.
\]

On the other hand, if \( PC_{it} \) is the competitive price of \( QC_{it} \), profit-maximizing behavior leads to
\[
dQC_{it}/dQC_{jt} = -PC_{jt}/PC_{it}.
\]

So, the \( a_i \) coefficients must be proportional to the prices \( PC_{it} \) for any \( t \).
Because the coefficients are independent of \( t \), therefore, relative prices in the competitive sector must remain unchanged over time. In other words, the production possibilities frontier for competitive-sector value added is linear, it shifts in a parallel fashion over time, and its slope reflects relative competitive prices.

Repeating the same analysis for the sector of monopolies (or, more generally, of "imperfect competitors"),
\[ QM_t = \Sigma a_i Q_{Mi} = FM(LM_t, KM_t) \cdot (1 + \lambda)^t \]

with
\[ LM_t = \Sigma_i LM_{it}, \]
\[ KM_t = \Sigma_i KM_{it}. \]

We assume there is a constant \( \eta \in (1, \infty) \)

such that if \( MC_{it} \) is the marginal cost of monopoly-sector firm \( i \) at time \( t \),
\[ MC_{it} = (1 - \frac{1}{\eta}) \cdot PM_{it} \quad \text{all } i, t. \]

If all firms in the sector are pure monopolies facing no entry threats, we can think of \( \eta \) as an average price elasticity for the demand curves the individual
firms perceive that they face. In the case of oligopoly, monopoly with a threat of entry, or monopolistic competition, on the other hand, \( \eta \) might represent a multiple of the average (product) elasticity. Given the dependence of our interpretation on the particular specification of the nature of noncompetitive behavior, we will treat \( \eta \) as a parameter — rather than attempting to derive the connection between \( \text{MC}_{it} \) and \( \text{PM}_{it} \) from postulates of profit-maximizing firm behavior.\(^5\) Assuming \( \eta \) is given,

\[
\frac{a_{mj}}{a_{mi}} = \frac{\text{PM}_{jt}}{\text{PM}_{it}} \quad \text{all } i,j,t
\]

as in the last paragraph. Again, relative prices must remain unchanged over time.

The variables \( QC_t \) and \( QM_t \) are Hicksian aggregates by construction. Let

\[
PC_t = PC_{it} / ac_i \quad \text{all } t,
\]

\[
PM_t = PM_{it} / am_i \quad \text{all } t.
\]

Then we can treat \( QC_t \) and \( QM_t \) as "quantities" having "prices" \( PC_t \) and \( PM_t \) (see page 28 of Green (1964)). Aggregate expenditures on competitive-sector output will be

\[
PC_t \cdot QC_t = PC_t \cdot \sum_i ac_i \text{QC}_{it} =
\]

\[
PC_t \cdot ac_i \cdot \sum_i (ac_i / ac_i) \cdot QC_{it} =
\]

\[
PC_{it} \cdot \sum_i (PC_{it} / PC_{it}) \cdot QC_{it} =
\]

\[
\sum_i PC_{it} \cdot QC_{it}.
\]

Similarly,

\[
PM_t \cdot QM_t = \sum_i PM_{it} \cdot QM_{it}.
\]

All economic agents can determine their expenditures for competitive and monopoly-sector goods bundles simply on the basis of \( PC_t \), \( PM_t \), and a total budget.
We want to aggregate a step further, however. Suppose all physical capital depreciates at the same constant rate \( \mu \kappa (0,1) \) (which is independent of usage). Then a firm's capital stock can be summarized with a two-dimensional vector \( (K_{1t}, K_{2t}) \) where \( K_{1t} \) measures undepreciated units of \( QC_t \) and \( K_{2t} \) does the same for \( QM_t \). We assume that there exists a linearly homogeneous function \( G: R^2 \rightarrow R \) such that the firm's (value added) output can be determined from its labor input and \( G(K_{1t}, K_{2t}) \) alone and such that the same function \( G(\cdot) \) works for all firms. Let

\[
Q_t = G(QC_t, QM_t).
\]

Then each firm's output at time \( t \) depends only on its labor input and a single-number capital-stock input, measured in terms of units of \( Q_t \). This provides an interpretation for our \( KC_{1t}, KM_{1t}, KC_t, \) and \( KM_t \) variables above.

We also assume each household can make intertemporal budget allocations with a direct utility function having one (consumption) argument per period of life and with that argument constructed using \( G(\cdot) \). We call \( Q_t \) the "gross national product" at time \( t \). There is no money in our model, so we establish all relative prices by normalizing the current price per unit of \( Q_t \) to 1 in all periods.

To interpret \( Q_t \), we can think of all households and firms as being interested (and interested in the same manner) in a collection of abstract characteristics of their consumption (and investment) bundles. A mixture of competitive and monopoly-sector production can create bundles with given attributes. Reducing the contribution of one sector does not preclude an equally desirable result.

A great inbalance between sector outputs may make the resource cost of providing a specific result enormous, however. (Note that some of the characteristics embodied in any particular final output good may have been fixed at an early stage of production, and our analysis allows monopoly (competitive) products to enter the fabrication of competitive-sector (monopoly-sector) final outputs as intermediate goods.)
As stated, we specialize to Cobb-Douglas functional forms. Thus, we assume

\[ QC_t = (LC_t)^{1-\alpha}(KC_t)^{\alpha}(1 + \lambda)^{\tau}, \alpha \in (0,1), \lambda \geq 0, \]
\[ QM_t = (LM_t)^{1-\alpha}(KM_t)^{\alpha}(1 + \lambda)^{\tau}, \]
\[ Q_t = (QC_t)^{\beta}(QM_t)^{1-\beta}, \beta \in (0,1). \]

For the sake of simplicity (and because we have no specific predisposition to the contrary) we use the same \( \alpha \) parameter in lines (1) and (2).\(^6\) Because we do not add multiplicative scaling constants in lines (1) – (3), only steady-state ratios in the case of quantity variables will be of interest in our simulations of Section III.

Notice that line (3) implies there will be a unitary elasticity of substitution between \( QC_t \) and \( QM_t \). This seems roughly consistent with Harberger's (1954) analysis cited in the introduction of this paper.\(^7\) Kamerschen (1966), Bergson (1973), and Cowling and Mueller (1978), however, obtain more dramatic welfare losses from monopoly than Harberger in part because they allow larger substitution elasticities. From this standpoint our results in Section III will be conservative.

On the other hand, Carson (1975) and Worcester (1975) argue that the parameter we call \( \eta \) cannot be too small relative to the elasticity of substitution (see also Bergson's (1975) comments). By adopting a unitary elasticity in line (3), we free ourselves to use any value of \( \eta > 1 \) without fear of internal inconsistency in this respect.

Cost-minimizing behavior on the part of firms implies the following relationships among \( LC_t, LM_t, KC_t, \) and \( KM_t \). Let \( L_t \) be the total labor supply at time \( t \). Then provided the wage rate, \( w_t \), adjusts to clear the labor market at all times,

\[ LC_t + LM_t = L_t. \]

Let \( K_t \) be the aggregate (physical) capital stock (measured in the same units as \( Q_t \)). Then assuming full employment again,
The chain rule of differentiation shows
\[ \frac{dQ_t}{dL_{C_t}} = \left( \frac{\partial Q_t}{\partialQC_t} \right) \left( \frac{dQC_t}{dL_{C_t}} \right) = \left( \frac{\partial Q_t}{\partialQC_t} \right) \cdot \sum_i ac_i \left( \frac{\partial QC_{it}}{\partial L_{C_{it}}} \right) \]
\[ \left( \frac{dL_{C_{it}}}{dL_{C_t}} \right) = \left( \frac{\partial QC_t}{\partial QC_{it}} \right) \cdot \sum_i ac_i \left( \frac{w_t}{PC_{it}} \right) \left( \frac{dL_{C_{it}}}{dL_{C_t}} \right). \]

Given a price of 1 for units of \( Q_t \), \( \frac{\partial Q_t}{\partial QC_{it}} = \frac{3(1 - Q_t)}{3QC_{it}} \) equals \( PC_{it} \), the amount by which a marginal unit of \( QC_{it} \) enhances the value of the gross national product. So,
\[ PC_{it} = \frac{\partial Q_t}{\partial QC_{it}} = \left( \frac{\partial Q_t}{\partial QC_t} \right) \left( \frac{\partial QC_t}{\partial QC_{it}} \right) = \left( \frac{\partial Q_t}{\partial QC_t} \right) \cdot ac_i. \]

Combining the last two strings of equalities,
\[ \frac{dQ_t}{dL_{C_t}} = w_t \cdot \sum_i \left( \frac{dL_{C_{it}}}{dL_{C_t}} \right) = w_t. \]

The same analysis for \( LM_t \) yields
\[ \frac{dQ_t}{dL_{M_t}} = \left( \frac{\partial Q_t}{\partial QM_t} \right) \cdot \sum_i am_i \left( \frac{w_t}{PM_{it}} \cdot (1 - \frac{1}{\eta}) \right) \left( \frac{dL_{M_{it}}}{dL_{M_t}} \right) \]
\[ = w_t / (1 - \frac{1}{\eta}). \]

Thus,
\[ \left( \frac{\partial Q_t}{\partial QC_t} \right) \left( \frac{dQC_t}{dL_{C_t}} \right) = \left( 1 - \frac{1}{\eta} \right) \left( \frac{\partial Q_t}{\partial QM_t} \right) \left( \frac{dQ_t}{dL_{M_t}} \right). \]

If
\[ \theta \equiv (1 - \frac{1}{\eta}) \cdot (1 - \beta)/\beta, \]
the last line above yields
\[ \text{LM}_t = \theta \cdot \text{LC}_t. \]

Line (4) then implies

(6) \[ \text{LC}_t = \frac{1}{1 + \theta} \cdot L_t, \]

(7) \[ \text{LM}_t = \frac{\theta}{1 + \theta} \cdot L_t. \]

Repeating the arguments for capital,

(8) \[ \text{KC}_t = \frac{1}{1 + \theta} \cdot K_t, \]

(9) \[ \text{KM}_t = \frac{\theta}{1 + \theta} \cdot K_t. \]

**Consumption**

We assume (until Section III) that there are overlapping cohorts of households in the economy and that each household lives two periods. Every household inelastically supplies one unit of labor in its first period of life and retires in its second. All households are identical other than their initiation dates. The number of first-period-of-life families at time \( t \) will be \( (1 + n)^t, \ n \geq 0 \). Thus, in terms of the notation of the preceding subsection, we have

(10) \[ L_t = (1 + n)^t \text{ all } t. \]

We now examine each family's saving behavior.\(^3\)

As stated, we assume the utility function used by all families is a member of the Bergson class. Our discussion of aggregation indicated that we will think of the function as having a single consumption argument, measured in units of current gross national product, for each period of life. For a household born at time \( t \) and consuming \( c_t \) units of \( Q_t \) in the first period of life and \( c^*_{t+1} \) units of \( Q_{t+1} \) in the second, lifetime utility will be

\[ u(c_t, c^*_{t+1}) = \left( c_t^{\gamma}/\gamma \right) + \gamma \cdot \left( c^*_{t+1}^{\gamma}/\gamma \right) \]
with \( r > 0 \) and \( \gamma < 1 \). If \( \gamma = 0 \), this becomes

\[
\frac{u(c_t, c^*_t)}{c_t} = \ln(c_t) + \Gamma \cdot \ln(c^*_t) .
\]

The household will determine its first-period-of-life saving, \( s_t \), from

\[
\begin{align*}
\max \quad & u(c_t, c^*_t) \\
\text{subject to:} & \quad c_t + c^*_t/(1 + r_{t+1}) = w_t
\end{align*}
\]

where \( r_{t+1} \) is the interest rate accruing at time \( t + 1 \) on period-\( t \) savings. Solving,

\[
s_t = w_t \cdot \phi(r_{t+1})
\]

with

\[
\phi(r) \equiv [1 + r^{-1/(1-\gamma)}(1 + r)^{-\gamma/(1-\gamma)}]^{-1} .
\]

For future reference, note that

\[
-\gamma/(1 - \gamma) < 1
\]

and

\[
\phi(r) \in (0,1) \text{ if } r > 0 .
\]

Section III modifies household budget constraints to include taxes. At no point will we add bequests and inheritances, however. Bequests would, of course, affect family saving patterns in general. On the other hand, bequest behavior would seem to have no special interaction with monopolies given our assumption that all assets pay the same rate of return (relative to their market values).2/ Although at the inception of each existing monopoly presumably a windfall capital gain accrued to stockholders, we limit our attention to steady states in this paper, and the proof of Proposition IX in Laitner (1979) shows that in a steady state current families will not inherit any part of such original gains.
Steady-state growth

This subsection combines our production and saving functions to form a complete decentralized growth model. We then prove that the model has one and only one steady state.

We need two variables in addition to those already defined. One, \( m_t \), gives total current (pure) monopoly profits at time \( t \): if \( TR_t \) equals total revenues in the QM-sector, \( TC_t \) gives total costs (net of depreciation), and the rate of depreciation is \( \mu \),

\[
m_t = TR_t - TC_t - \mu \cdot KM_t.
\]

Total revenues are

\[
TR_t = P_i \cdot PM_{it} \cdot QM_{it}.
\]

Our analysis of production shows

\[
PM_{it} = \left( \frac{\partial Q_t}{\partial QM_{it}} \right) \cdot a_t.
\]

So,

\[
TR_t = \left( \frac{\partial Q_t}{\partial QM_t} \right) \cdot P_i \cdot a_t \cdot QM_{it} = \left( \frac{\partial Q_t}{\partial QM_t} \right) \cdot QM_t = (1 - \beta) \cdot Q_t.
\]

Total costs will be

\[
TC_t = \omega_t \cdot LM_t + r_t \cdot KM_t.
\]

The analysis of production shows

\[
\omega_t = (1 - \frac{1}{\eta}) \cdot \left( \frac{\partial Q_t}{\partial QM_t} \right) \cdot \left( \frac{dQ_t}{dLM_t} \right) = (1 - \frac{1}{\eta})(1 - \beta)(1 - \alpha)(Q_t/LM_t),
\]

\[
r_t = (1 - \frac{1}{\eta}) \cdot (1 - \beta) \cdot \alpha \cdot (Q_t/KM_t) - \mu.
\]
So,

\[ TC_t = (1 - \frac{1}{n}) \cdot (1 - \beta) \cdot Q_t - \mu \cdot KM_t. \]

Thus,

\[ m_t = (1 - \beta) \cdot (1/\eta) \cdot Q_t \quad \text{all } t. \]

The second new variable, \( M_t \), measures the present value of current and future (pure) monopoly profits -- the capitalized value of pure profits. Using the variable \( m_t \),

\[ M_t = m_t + \sum_{s=t+1}^{\infty} [m_s / (1 + r_s)] \cdot (1 + r_t^s). \]

We are now ready to present equations for our complete growth model. We have

\[ K_{t+1} + M_{t+1} / (1 + r_{t+1}) = \beta \cdot (1 - \alpha) \cdot (1 + \theta) \cdot Q_t \cdot \phi(r_{t+1}), \]

\[ Q_t = h(\theta) \cdot (1 + \lambda)^t \cdot (L_t)^{1-\alpha} (K_t) \alpha \quad \text{where } h(\theta) = \theta^{1-\beta} / (1 + \theta), \]

\[ M_t = (1/\eta) \cdot (1 - \beta) \cdot Q_t + M_{t+1} / (1 + r_{t+1}), \]

\[ r_t = [\beta \cdot (1 - \theta) \cdot Q_t / K_t] - \mu. \]

Equation (14) is an accounting identity for wealth: Our household life-cycle analysis implies the total wealth carried from period \( t \) to \( t + 1 \) will equal the aggregative saving of all young families born at time \( t \), \( L_t - s_t \). Using lines (6) and (11) and the relationship

\[ w_t = (\partial Q_t / \partial QC_t) \cdot (dQC_t / dLC_t) \]

derived in the production subsection, this explains the right-hand side of (14). The left-hand side sums physical capital for time \( t + 1 \) and the present value of capitalized future monopoly profits -- the two assets which must be financed at time \( t \). Line (15) combines lines (1) - (3) and (6) - (9). Line (16) is a recursive
representation of (13) — given (12) — provided

\[ M_t < \infty \] all \( t \) and

\[
\lim_{s \to t} \frac{M_s}{\Pi (1 + r_u)} = 0
\]

Line (17) follows from the production subsection and line (8). Throughout this paper we assume there is no uncertainty and foresight on the part of all economic agents is perfect.

We will say \( \sigma > 0, \delta > 0, K_0 > 0, M_0 > 0, \) and \( Q_0 > 0 \) define a steady state if

\[
K_t = \sigma^t K_0 \quad \text{all} \ t, \\
Q_t = \sigma^t Q_0 \quad \text{all} \ t, \ \text{and} \\
M_t = \delta^t M_0 \quad \text{all} \ t
\]

provide a solution for lines (14) - (18). Notice that

\[
r_t = r \quad \text{all} \ t
\]

along a steady-state path. We have

**Proposition 1:** There exists one and only one steady state for our model.

**Proof:** Let \( \sigma > 0, \delta > 0, K_0 > 0, M_0 > 0, \) and \( Q_0 > 0 \) constitute a prospective steady-state solution.

**Step 1:** Lines (10) and (15) imply

\[
\sigma = \sigma (1 + n) (1 + \lambda)
\]

if the prospective solution is to be feasible. Thus, we must have

\[
\sigma = (1 + n) (1 + \lambda) \frac{1}{(1 - \alpha)} \geq 1 > 0
\]

**Step 2:** If \( r_t = r \) all \( t \) and \( M_{t+1} = \delta \cdot M_t \), line (16) shows

\[
M_t \cdot \left[ (1 + r - \delta)/(1 + r) \right] = (1/\eta) \cdot (1 - \beta) \cdot Q_t
\]
Thus, the steady-state growth rates of $M_t$ and $Q_t$ must be the same. So, we must have

(20) \[ \delta = \sigma . \]

**Step 3:** Step 2 shows

\[ M_t = \left( \frac{1}{n} \right) \cdot (1 - \beta) \cdot \left[ \frac{(1 + r)/(1 + r - \sigma)}{1} \right] \cdot Q_t . \]

Line (17) shows $r > 0$ if $Q_0 > 0$ and $K_0 > 0$. Thus, for $M_0 > 0$ and $Q_0 > 0$ we must have

(21) \[ r > \sigma - 1 . \]

Using our expression for $M_t$, line (14) shows

\[ \sigma \cdot \frac{K_t}{Q_t} + \sigma \cdot \left( \frac{1}{n} \right) \cdot (1 - \beta) \cdot \frac{1/(1 + r - \sigma)}{1} \cdot Q_t = \]

\[ \beta \cdot (1 - \alpha) \cdot (1 + \theta) \cdot \phi(r) \cdot Q_t . \]

Dividing through by $Q_t$ and using line (17),
\[ \frac{\sigma \alpha(1 + \theta)/(r + \mu)}{[\sigma(1/\eta)(1 - \theta)/(1 + r - \sigma)] = \beta(1 - \alpha)(1 + \theta) \phi(r). \]

Dividing through by \( \phi(r) \) and recalling that \(-\gamma/(1 - \gamma) < 1 \) (see equation (11)), a simple graph shows (22) has one and only one root \( r > \sigma - 1 \geq 0 \). Fix this \( r \).

**Step 4:** Given \( r \), we can solve equations (15) and (17) simultaneously to determine \( K_0 \) and \( Q_0 \):

\[ K_0 = \frac{\alpha \beta 1 - \beta/(r + \mu)}{(1 - \alpha)} \]

\[ Q_0 = h(\theta) \cdot (K_0)^\alpha \]

where \( h(\cdot) \) is defined in line (15). Step 2 then yields \( M_0 > 0 \).

**Step 5:** Steps 1 - 4 show there is at most one steady state for our model. To prove there is one we need to verify that line (18) holds for the prospective solution of lines (19) - (24).

For that solution,

\[ M_t = \sigma^t M_0. \]

So,

\[ M_t < \infty \text{ all } t \]

Since \( 1 + r > \sigma \) (see line (21)), however,

\[ \lim_{s \to \infty} \frac{M_s}{\prod_{u=t}^{s} (1 + r)} = \lim_{s \to \infty} M_s^{(s-t)}(1 + r)^{s-t+1} = 0. \]

Having established the existence and uniqueness of a steady state, we turn next to comparative-static (or "comparative-steady-state") analyses.
II. The Markup of the Monopoly Sector

We now investigate the steady state of Section I in detail and, in particular, derive a number of comparative-static results. We accomplish the latter by differentiating various steady-state variables with respect to $\eta$. Recall that the magnitude of $\eta$ reveals the percentage difference between price and marginal cost in the QM-sector: as $\eta \to 1$, the difference diverges to $\infty$; as $\eta \to \infty$, the difference converges to 0. In other words, we can think of the QM-sector as becoming more and more "competitive" for larger and larger choices of $\eta$. The effects of variations in $\eta$ are of special interest because antitrust legislation presumably can influence monopoly markups.

**Comparative-static results**

We first calculate the signs of the derivatives of the steady-state variables of Section I with respect to $\eta$. The steps in the proof of Proposition I provide the basis for the analysis. Note that in a steady-state, for all $t$

$$r_t = r,$$

$$K_t = \sigma^t K_0,$$

$$Q_t = \sigma^t Q_0$$

where $r$ is defined by lines (21) and (22), $K_0$ by line (23), and $Q_0$ by line (24). We also have

$$w_t = w_0^{\xi^t}$$

where

$$w_0 \equiv \beta \cdot (1 - \alpha) \cdot (1 + \theta) \cdot Q_0,$$

$$\xi \equiv \sigma/(1 + n).$$

If

$$k_t \equiv K_t/L_t,$$
in the steady state
\[ k_t = K_0 \cdot \zeta^t. \]

Changes in \( \eta \) have no effect on the steady-state growth rates of \( Q_t, K_t, w_t, \) and \( k_t \): Line (19) shows
\[ \frac{d\sigma}{d\eta} = 0. \]

So the definition of \( \zeta \) establishes that
\[ \frac{d\zeta}{d\eta} = 0. \]

Line (20) shows the growth rate of \( M_t, \delta - 1, \) is also independent of \( \eta \).

Results are less trivial for the steady-state interest rate. Letting
\[ \phi(r) \equiv l/\phi(r), \]
we can rewrite line (22) as
\[
[\sigma \cdot \alpha \cdot \beta \cdot \phi(r)/(r + \mu)] + [\sigma \cdot (1/\eta) \cdot (1 - \beta) \cdot \phi(r)/
(1 + \theta) \cdot (1 + r - \sigma)] = \beta \cdot (1 - \alpha). \tag{25}
\]

The definition of \( \theta \) (where \( \theta \equiv (1 - \frac{1}{\eta})(1 - \beta)/\beta \)) shows
\[ \frac{d\theta}{d\eta} > 0. \tag{26} \]

The steady-state interest rate is the value of \( r \) solving (25) and satisfying \( r > 0 \) and \( r > \sigma - 1 \). If we graph the left-hand side of line (25) (against \( r \) on the abscissa), the root \( r \) with \( r > 0 \) and \( r > \sigma - 1 \) will occur with the graph falling as it cuts a horizontal line of height \( \beta \cdot (1 - \alpha) > 0 \). Line (26) shows an increase in \( \eta \) will lower the downward sloping graph. Thus,
\[ \frac{dr}{d\eta} < 0. \tag{27} \]

In other words, a decrease in the markup of price over marginal cost in the monopoly sector will lead to a lower steady-state interest rate.

All of the steady-state variables of interest in this section can be written
as functions of $\eta$ and $K_0$. Thus, for any such variable $x$ we have

$$x = x(\eta, K_0).$$

For example, lines (15) and (17) show

$$(28) \quad r = r(\eta, K_0) \equiv \beta \cdot \alpha \cdot \theta^{1-\beta} \cdot (K_0)^{\alpha-1} - \nu$$

where $\theta$ depends directly on $\eta$. Static analyses such as Harberger's and Bergson's (see the introductory section of this paper) consider only the direct effect of changes in $\eta$ on $x$ -- $\partial x / \partial \eta$ or, for noninfinitesimal changes, perhaps

$$\int_\eta^\infty (\partial x(z, K_0) / \partial z) dz.$$

We will therefore say $\partial x / \partial \eta$ registers the "static effect" of changes in $\eta$ on $x$. The "total effect" contains a second component as well:

$$\frac{dx}{d\eta} = \left( \frac{\partial x}{\partial \eta} \right) + \left( \frac{\partial x}{\partial K_0} \right) \left( \frac{dK_0}{d\eta} \right).$$

Since the second component operates through changes in the steady-state value of $K_0$ -- a variable which is fixed in static analyses -- we call $\left( \frac{\partial x}{\partial K_0} \right) \left( \frac{dK_0}{d\eta} \right)$ the "dynamic effect" on $x$ of a change in $\eta$.

Returning to the variable $r$, line (28) shows

$$\frac{dr}{d\eta} = r_1 + r_2 \cdot \left( \frac{dK_0}{d\eta} \right)$$

with

$$r_1 \equiv \frac{\partial r}{\partial \eta} = \beta \cdot \alpha \cdot \theta^{-\beta} \cdot (K_0)^{\alpha-1} \cdot$$

$$\left( \frac{d\theta}{d\eta} \right),$$

$$r_2 \equiv \frac{\partial r}{\partial K_0} = \beta \cdot \alpha \cdot \theta^{1-\beta} \cdot (\alpha - 1) \cdot (K_0)^{\alpha-2}.$$
on $r$ of a change in $\eta$ must be negative (and larger in absolute terms than the "static effect"). Since $\alpha \in (0,1)$,

$$r^2 < 0 .$$

So we must have

$$dK_0/d\eta > 0$$

(29) to obtain $dr/d\eta < 0$. The sign of the "static effect" is not too surprising: an increase in $\eta$ leads to an improvement in static allocative efficiency, which allows the marginal revenue product of capital to rise. The "dynamic effect" is negative, however, because an increase in $\eta$ somehow raises the steady-state growth trajectory of the physical capital stock (see line (29)).

Appealing to the accounting identity in line (14), we cannot tell whether the result of line (29) occurs because a rise in $\eta$ increases household wealth accumulations or because it reduces the steady-state value of capitalized pure profits. However, let wealth carried by the household sector from period $t$ to $t+1$ be $W_t$. Then $W_t$ equals the right-hand side of line (14). Dividing line (14) through by $W_t$,

$$[K_{t+1}/W_t] + [M_{t+1}/W_t \cdot (1 + r)] = 1 .$$

(30) This is only trivially different from equation (25). Looking at lines (25) and (27) we can see, therefore, that

$$d(K_{t+1}/W_t)/d\eta > 0$$

(assuming $K_{t+1}$ and $W_t$ are steady-state values). So, if $M_{t+1}$ and $W_t$ are steady-state variables,

$$d[M_{t+1}/W_t \cdot (1 + r)]/d\eta < 0$$

by line (30). Thus, an increase in $\eta$ causes a reduction in the percentage of steady-state life-cycle wealth that must be devoted to financing intangible assets. In this sense, a rise in $\eta$ creates more "room" in household portfolios for the financing
of physical capital. The result is \( \frac{dK_0}{dn} > 0 \), which makes \( \frac{dr}{dn} < 0 \) possible.

Line (29) already shows the effect of an increase in \( n \) on \( K_0 \). There is only a "dynamic effect" in this case by definition. Since \( k_0 = K_0 \), we also have

\[
\frac{dk_0}{dn} > 0 .
\]

For \( Q_0 \) there are again two effects. Using the function \( h(\cdot) \) defined in line (15),

\[
\frac{dQ_0}{dn} = Q_1 + Q_2
\]

where

\[
Q_1 = h'(\theta) \cdot (K_0^\alpha \cdot (d\theta/dn) ,
\]
\[
Q_2 = h(\theta) \cdot a \cdot (K_0^{\alpha-1} \cdot (dK_0/dn)) .
\]

Differentiating \( h(\cdot) \),

\[
h'(\theta) = (1 - \beta)(1/\eta)/[\beta^2(1 + \theta)^2] > 0 .
\]

Given line (26), this implies

\[
Q_1 \equiv \frac{\partial Q_0}{\partial \eta} > 0 .
\]

Q1 and its integrated version are the direct analogues for our model of the national output losses from monopoly that Harberger, Bergson, and others mentioned above have tried to measure. On the other hand, line (29) shows

\[
Q_2 \equiv (\partial Q_0/\partial K_0)(dK_0/d\eta) > 0 .
\]

In our model, therefore, an increase in \( n \) leads to a rise in the steady-state growth path of \( Q_t \) through both static and dynamic channels:

\[
(31) \quad \frac{dQ_0}{dn} = Q_1 + Q_2 > Q_1 > 0 .
\]

In comparing the model of this paper and the static analyses referenced in the introduction we would like to know the magnitude of \( Q_2 \), our "dynamic effect," relative to \( Q_1 \). Because
\(e = 0 \text{ when } n = 1,\)
\(h(0) = 0, \text{ and}\)
\(h'(0) = \infty,\)

it appears that
\[\lim_{n \to 1} Q_2/Q_1 = 0. \text{ 10/}\]

For values of \(n\) bounded away from 1 we unfortunately have no such guidance. However, Section III derives a number of numerical values of \(Q_2/(Q_1 + Q_2)\) which suggest that \(Q_2\) may be large relative to \(Q_1\) in practice.

Each household's ultimate source of funds is its first-period-of-life labor income. Differentiating the expression for \(w_0\) given at the beginning of this section, lines (26) and (31) show
\[\frac{dw_0}{d\eta} > 0.\]

As in the case of \(Q_0\), "static" and "dynamic effects" operate in the same direction.

**Consumption**

Despite its importance, however, the level of the time path of \(w_t\) is an imperfect indicator of steady-state family well-being: the lifetime utility of a family started at time \(t\) depends positively on both \(w_t\) and \(r_{t+1}\), yet an increase in \(n\) lowers \(r\) while raising \(w_0\). Let \(C_t\), on the other hand, be aggregate consumption at time \(t\) (or \(L_t + L_{t-1}\) times average consumption per family). Then
\[C_t = Q_t - (K_{t+1} - K_t) - \mu \cdot K_t.\]

So, in the steady state
\[C_t = \sigma^t C_0.\]

If \(\sigma\) is fixed, \(C_0\) is a better variable for steady-state welfare judgments than \(w_0\) -- comparing steady states with \(C_0 = A\) and \(C_0 = B > A\), for example, for any feasible distribution of consumption goods between young and old families along the
first growth path we can find a distribution along the \( C_0 = B \) path which makes all families better off. Using \( C_0 \) as a measure of welfare (see note 11) in this sense, we now study \( dC_0/d\eta \).

Before investigating changes in \( \eta \) though, consider the golden-rule value of \( K_0 \) -- the value of \( K_0 \) yielding the highest steady-state \( C_0 \). We have

\[
C_0 = Q_0 - (\sigma - 1) \cdot K_0 - \mu K_0
\]

for balanced-growth paths. Differentiating,

\[
\frac{dC_0}{dK_0} = \alpha \cdot h(\theta) \cdot (K_0)^{\alpha-1} - (\sigma - 1) - \mu,
\]

\[
\frac{dC_0}{dK_0} < 0.
\]

A unique golden-rule \( K_0 \), say, \( K^* \), exists therefore, and

\[
\frac{dC_0(K^*)}{dK_0} = 0 = \frac{dQ_0(K^*)}{dK_0} - (\sigma - 1) - \mu.
\]

If we have balanced growth with \( K_0 > K^* \), there is Pareto inefficiency; if \( K_0 < K^* \), more steady-state consumption is possible, although the transition to a "better" path may not be socially desirable.

If we exclude monopolies from our model, steady-state growth with \( K_0 > K^* \) will be possible. An interesting consequence of including monopolies is that such an outcome is ruled out. We can see this mathematically as follows. Section I shows

\[ r > \sigma - 1 \]

in the steady state. Using lines (15) and (17),

\[
\frac{dQ_0}{dK_0} = \alpha \cdot h(\theta) \cdot (K_0)^{\alpha-1} > r + \mu
\]

iff \( \alpha \cdot h(\theta) \cdot (K_0)^{\alpha-1} > \alpha \cdot \beta \cdot (1 + \theta) \cdot Q_0/K_0 = \)

\[
\alpha \cdot \beta \cdot (1 + \theta) \cdot h(\theta) \cdot (K_0)^{\alpha-1}
\]

iff \( 1 > \beta \cdot (1 + \theta) = \beta \cdot [1 + (1/\beta) - 1 - (1/\eta \cdot \beta) + (1/\eta)] = [1 - (1/\eta) + (\beta/\eta)] \) iff \( 1 > \beta \).
Therefore, since \( \beta \in (0,1) \),

\[
\frac{\partial Q_0}{\partial K_0} - \mu > r > \sigma - 1
\]

in the steady state. So, \( K_0 < K^*_0 \). In words, line (13) shows the capitalized value of pure profits will not be finite in the steady state unless \( r > \sigma - 1 \).

However, \( r \) is less than \( \frac{\partial Q_0}{\partial K_0} - \mu \) because if we add an extra unit of physical capital to our decentralized economy, part of the increment to national output will be siphoned into monopoly profits. Thus, \( \frac{\partial Q_0}{\partial K_0} - \mu > \sigma - 1 \).

Turning to changes in \( n \), we have

\[
dC_0/\partial n = C1 + C2
\]

where

\[
C1 \equiv \frac{\partial C_0}{\partial n} = \frac{\partial Q_0}{\partial n},
\]

\[
C2 \equiv \left( \frac{\partial C_0}{\partial K_0} \right) \cdot \left( \frac{\partial K_0}{\partial n} \right).
\]

The first term is familiar from the preceding subsection:

\[
C1 = Q1 > 0.
\]

This registers the "static effect" of a change in \( n \). The paragraph above, on the other hand, shows \( K_0 \) must fall short of \( K^*_0 \), so

\[
\frac{\partial C_0}{\partial K_0} > 0.
\]

Thus, line (29) shows the "dynamic effect" is also positive:

\[
C2 > 0.
\]

Combining the two positive effects,

\[
dC_0/\partial n > 0.
\]

Steady-state consumption will be augmented through two conduits, therefore, if markups in the monopoly sector are moderated.
Summary

Recapitulating,

**Proposition II:** Assuming the economy has reached a steady state at time $t = 0$,

i) $\frac{d\sigma}{d\eta} = \frac{d\delta}{d\eta} = \frac{d\zeta}{d\eta} = 0$

ii) $\frac{dx}{d\eta} < 0$, $\frac{dK_0}{d\eta} > 0$

iii) $\frac{dQ_0}{d\eta} > 0$, $\frac{dC_0}{d\eta} > 0$; and,

iv) $K_0 < K^*_0$. 
III. Sample Computations

In this section we return to our original model and attempt to alter it in the direction of greater realism: We allow taxes on labor and property incomes; government debt; and, for households, multi-period life spans, life-cycle changes in family composition, and empirically-based lifetime labor supply profiles. We then present computer simulations for credible values of n. Examining the sizes of Q1, Q2, C1, and C2 (see Section II), we find that in our examples the "dynamic effects" associated with imperfect competition considerably exceed the "static effects" in magnitude.

Modifications to our model

We modify the household model of Section I along the lines of Tobin's (1968) life-cycle framework. We then choose our production-sector parameters using Tobin (1968), Scherer (1971), and other sources.

Our basic household-sector changes are as follows: we expand our model to allow (family) life spans of up to 80 years (although we continue to use discrete time); we adopt Tobin's (see page 251) index of "equivalent adults" to reflect life-cycle changes in family composition; using standard survival probabilities for U.S. males and females, we incorporate Tobin's actuarially-based life insurance and annuities; and, assuming each family begins with an 18 year old male and female, we correct 1973 earnings data from Table 53 of the Current Population Report (1973) for participation rates, and then use it to deduce a "representative" household's relative labor supply at different ages. Because we use Tobin's logarithmic Bergson utility function for all households, the distribution of labor earnings within each cohort is irrelevant. Although the Population Report income data includes property income, the errors introduced by using it to estimate labor earnings hopefully are not too great since (like Tobin) we use medians for different age-sex categories rather than means. The figures include social security, welfare,
and private pension fund payments. Although they do not cover employer contributions for retirement programs or fringe benefits, as long as such items constitute a fixed fraction of measured income, they will not affect our results.

The male and female income data terminate with 65-and-over age categories. We extrapolate the figures to age 68 (in other words, to a family age of 50) and assume retirement follows. Although our labor supply profiles incorporate worker contributions to private pensions and, as stated, can be interpreted as including employer contributions as well, we will deduct all tax payments below in deriving household budget constraints. Thus, using figures from Table 67 of the Social Security Bulletin Annual Statistical Supplement, 1973, we assign social security benefit payments to surviving males and females of ages 68 to 98. We assume aggregate benefits remain a constant fraction of the gross national product over time.

Following Tobin, we set our labor parameter $\alpha$ in lines (1) and (2) equal to .33; the rate of population growth, $n$, equal to .0168; and $A$ where

$$1 + A = (1 + \lambda)^{1/(1-\alpha)}$$

equal to .03. Assuming the manufacturing-industries percentages of Scherer's (1971) Table 3.4 can also be applied to the economy as a whole, we try two values for $1 - \beta$, the monopoly sector's share of the gross national product (see line (3)), .5 and .2. (Bergson (1973) uses .5 in his two-sector model.) We set $\mu = .025$.\(^{12}\)

The simulations below use three values of $n$: 5, 10, and 20. These yield monopoly-sector markups of price over marginal cost (as a percentage of the latter) of 25.0%, 11.1%, and 5.3%, respectively. While the last seems the most consistent with Harberger's (1954) empirical data (provided manufacturing-sector figures can be applied for the whole economy),\(^{13}\) Stigler (1956), Bergson (1973), and Cowling and Mueller (1978) argue on theoretical grounds that Harberger's numbers need to be adjusted upward. Bergson's simulations use markups of 10%, 20% and 30%.

For the sake of making comparisons, Table I presents steady-state information for different choices of $n$ and $1 - \beta$ on pure profits divided by all property income,
on the rate of return per unit of physical capital in the monopoly sector divided by the rate for the competitive sector, and on pure profits divided by net national product. The first two ratios can be interpreted in either net or gross-of-tax terms (see our assumptions about taxes below). Harberger (1954) thought the ratio in row 1 would surely be less than .33 for the U.S. economy, and that is the case for all of our parameter selections. Bain (1951) (see also Scherer (1971)) presented data showing a figure of about 1.75 would be appropriate in row 2 if the monopoly sector includes only industries with 8 firm concentration ratios of 70% or more. Looking at Scherer's Table 3.4 again, Bain's ratio would seem to be relevant for 1 - β = .2 at least. Scherer (1971) remarks (see page 409) that the ratio of row 3 may be in the neighborhood of .02-.03 (or more) empirically. None of the comparisons, therefore, seem to rule out parameter combinations of 1 - β = .5 and η = 20 or 1 - β = .2 and η = 10 or 20.

Our new model also includes a government sector. Data for 1973 from Pechman (1977) implies a ratio of the sum of local, state, and federal debt in private hands to gross national product of .35. Our simulations assume the ratio remains fixed over time. Using aggregative tax data from the same source, we assume a 35% proportional tax on all income at all times. Tax revenues not required for debt service and transfer payments are assumed to be spent on current government consumption—a there is no government (physical) capital stock.14/

Results

With Cobb-Douglas production functions and Tobin's (1968) logarithmic utility function for households, Proposition I remains valid, as do our sign calculations of Section II.15/ Given our assumptions about parameters, we now present illustrative numerical results.

Table II presents steady-state outcomes for different values of η and 1 - β. As stated in Section I, only r and ratios of other variables are of interest. Given
Table I

Steady-State Profit and Rate-of-Return Ratios

<table>
<thead>
<tr>
<th></th>
<th>$1 - \beta = .5$</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\eta = 5$</td>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>$m_0/(m_0 + rK_0)$</td>
<td>.29</td>
<td>.17</td>
<td>.09</td>
</tr>
<tr>
<td>$(r + m_0/KM_0)/r$</td>
<td>1.93</td>
<td>1.42</td>
<td>1.20</td>
</tr>
<tr>
<td>$m_0/(q_0 - \mu K_0)$</td>
<td>.11</td>
<td>.05</td>
<td>.03</td>
</tr>
<tr>
<td>Notes:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a) The first two ratios can be interpreted in either net or gross-of-tax terms -- see the text.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table II

Steady-State Variables

<table>
<thead>
<tr>
<th></th>
<th>$1 - \beta = .5$</th>
<th></th>
<th>$1 - \beta = .2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$n = 5$</td>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>$r^b$</td>
<td>.07</td>
<td>.06</td>
<td>.06</td>
</tr>
<tr>
<td>$K_0/(Q_0 - uK_0)$</td>
<td>2.34</td>
<td>2.68</td>
<td>2.88</td>
</tr>
<tr>
<td>$K_1/W_0$</td>
<td>.41</td>
<td>.54</td>
<td>.65</td>
</tr>
<tr>
<td>$[M_1/(1+r)]/W_0^c$</td>
<td>.52</td>
<td>.39</td>
<td>.27</td>
</tr>
<tr>
<td>$D_1/W_0^d$</td>
<td>.07</td>
<td>.07</td>
<td>.08</td>
</tr>
</tbody>
</table>

Notes:

a) Actual computations used $n = 100,000$ in columns 4 and 8.

b) All steady-state variables are given in net-of-tax terms.

c) See line (14).

d) $D_t$ is total government debt at time $t$. 
our choices of \( \eta \) and \( \lambda \), the steady-state growth rate of \( Q_t \), \( \sigma - 1 \), is 4.8%.

Tobin's (1968) best simulation-based estimate for \( K_0/(Q_0 - \mu K_0) \) (using 1964 data on earnings) was 5.4. Although Tobin's model had no monopolies, our ratio with \( \eta = \infty \) is 3.20. Tobin's model, however, also lacked a government sector and any discussion of depreciation. If we run our model without taxes, government debt, and depreciation, the steady-state capital-to-net national product ratio climbs to 5.68 (when \( \eta = 100,000 \)). Tobin cites an empirical estimate of 4 for the ratio.

The most interesting feature of Table II is the enormous role capitalized monopoly profits play as an asset in household portfolios. In no case with \( \eta < 20 \) are such capitalized pure profits less than 15% of total wealth holdings, and with \( 1 - \beta = .2 \) and \( \eta = 10 \) or \( 1 - \beta = .5 \) and \( \eta = 20 \) they constitute 24% or more of all assets.

Table III presents numerical versions of the comparative-static results of Section II. Because of the meaninglessness of our units, all derivatives are presented in elasticity form.\(^{16/}\) Row 1 is of particular interest -- the ratio \( Q_1/(Q_1 + Q_2) \) shows the relative importance of our "dynamic" and "static effects" for the sample parameter values. The numbers show a "dynamic effect" (for \( Q_0 \)) from 7 to 50 times as large as the accompanying "static effect" in all cases. The relative importance of the "dynamic effect" rises with \( \eta \) in Table III, although the magnitude of the "total effect" (in terms of elasticities) falls.

Table IV presents integrated versions of the "total effects" given in point elasticity form by Table III. We saw in Section I that our assumption about the elasticity of substitution between \( QM_t \) and \( QC_t \) was somewhat analogous to Harberger's (1954). Harberger's calculation of the "static effect" for completely eliminating monopoly was approximately .1% of national income. The numbers of row 1 in Table III indicate that our "total effect" of removing monopolies could be 10 to 50 times (or more) as high. As stated, \( \eta = 20 \) gives markups roughly in the range of Harberger's. Table IV shows for \( 1 - \beta = .5 \) and \( \eta = 20 \) a total gain of 5.0% in the steady-state net national product following a rise in \( \eta \) to \( \infty \). Thus we have an
### Table III

Comparative-Static Results

<table>
<thead>
<tr>
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<th>$1 - \beta = .5$</th>
<th>$1 - \beta = .2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\eta = 5$</td>
<td>10</td>
</tr>
<tr>
<td>$Q_2/(Q_1 + Q_2)$</td>
<td>.89</td>
<td>.96</td>
</tr>
<tr>
<td>$dln(Q_0)/dln(\eta)$</td>
<td>.13</td>
<td>.07</td>
</tr>
<tr>
<td>$C_2/(C_1 + C_2)$</td>
<td>.87</td>
<td>.95</td>
</tr>
<tr>
<td>$dln(C_0)/dln(\eta)$</td>
<td>.12</td>
<td>.06</td>
</tr>
<tr>
<td>$dln(K_0)/dln(\eta)$</td>
<td>.34</td>
<td>.19</td>
</tr>
<tr>
<td>$dln(w_0)/dln(\eta)$</td>
<td>.24</td>
<td>.12</td>
</tr>
</tbody>
</table>
Table IV
Percentage Steady-State Changes for Complete Elimination of Monopoly

<table>
<thead>
<tr>
<th></th>
<th>$1 - \beta = .5$</th>
<th></th>
<th>$1 - \beta = .2$</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\eta = 5$</td>
<td>10</td>
<td>20</td>
<td>10</td>
</tr>
<tr>
<td>$Q_0$</td>
<td>16.2</td>
<td>8.8</td>
<td>5.0</td>
<td>7.8</td>
</tr>
<tr>
<td>$C_0$</td>
<td>14.2</td>
<td>7.7</td>
<td>4.3</td>
<td>6.8</td>
</tr>
<tr>
<td>$W_0$</td>
<td>-28.6</td>
<td>-23.0</td>
<td>-16.6</td>
<td>-20.6</td>
</tr>
<tr>
<td>$K_0$</td>
<td>54.6</td>
<td>28.8</td>
<td>15.8</td>
<td>24.2</td>
</tr>
<tr>
<td>$w_0$</td>
<td>29.1</td>
<td>14.6</td>
<td>7.7</td>
<td>12.3</td>
</tr>
<tr>
<td>$r$</td>
<td>-20.3</td>
<td>-13.8</td>
<td>-8.9</td>
<td>-12.0</td>
</tr>
</tbody>
</table>

Notes:

a) In other words, we change from $\eta = 5, 10, \text{ or } 20$ to $\eta = 100,000$. 
integrated "total effect" roughly 50 times as large as Harberger's "static effect"
alone. For \( \eta = 20 \) and \( 1 - \beta = .2 \) Table IV shows a "total effect" of 2.4% of gross
national product.

Overall, the numbers in Table IV are much more dramatic than the early results
cited in the introduction. We obtain (steady-state) output losses in the same
range as Kamerschen (1966), Bergson (1973), and Cowling and Mueller (1978), but
we do so without adopting a large elasticity of substitution between competitive
and monopoly-sector products. Row 4 of Table IV reveals the basis of our results:
eliminating monopoly has a substantial positive effect on the steady-state capital
stock in all of our examples. Interestingly enough, row 3 shows that in every
case the increases in \( K_0 \) occur despite reductions in total wealth accumulations.17/
IV. Conclusion.

We have constructed several versions of a long-run growth model in which imperfect competition can affect steady-state aggregate output in two ways: through reductions in static allocative efficiency, and through the crowding of asset instruments which finance physical capital out of household-sector portfolios. We call the former channel the "static effect" of monopolies, and the latter the "dynamic effect." Our sample simulations in Section IV show that capitalized pure profits can easily make up a substantial fraction of all assets (see Table II) and that the "dynamic effect" of monopoly may in practice outweigh the "static effect" by a factor of 10 or more (see Table III). As a consequence of the "dynamic effect," in our simulation examples ridding the economy of monopoly leads to steady-state increases of 2.4% or more in physical capital accumulation, gross national product, and wage rates (see Table IV). This occurs despite our assumption of a unitary overall elasticity of substitution between monopoly and competitive-sector Hicksian aggregates.

As stated, potential gains in steady-state national output and consumption may be obtainable only after transition intervals during which one or more cohorts suffer standard-of-living reductions. In particular, this is true for steady-state gains from excising the obstructive "dynamic effect" of monopolies on capital accumulation. To determine the most socially advantageous degree of government intervention in the monopoly sector, we might, therefore, use an aggregative welfare function which allows intergenerational comparisons -- perhaps a function of the form

$$V(C_t, C_{t+1}, \ldots) = \sum_{i=t}^{\infty} \eta^i \cdot v(C_i)$$

where $\eta < 1$ is a social discount factor and $C_i$ is aggregate consumption at time $i$. Maximizing $V(\cdot)$ with respect to $\eta$, we could determine an optimal national "anti-trust policy" (for period $t$ and beyond). A large $\eta < 1$ would presumably imply a
large optimizing $\eta$ -- in other words, a strict degree of control over monopoly price markups.
Notes

1. See, however, Leibenstein (1966) and chapter 17 of Scherer (1971).

2. We are ignoring the capitalized values of advertising and "good will" established in the past. We are also ignoring possible downward valuations in specialized equipment from surprise changes in consumer demand.

3. The word "monopolies" here can be interpreted to encompass oligopoly, monopolistic competition, monopoly with entry threats or partial regulation, or unadulterated monopoly. We do not, however, deal with "monopoly" in the supply of labor.

4. This, unfortunately, rules out increasing returns to scale as a reason for imperfect competition in this paper.

5. Below we do, however, mention a link between $\eta$ and an overall elasticity of substitution in demand.

6. Note that lines (1) and (2) imply a linear production possibility frontier for $QC_t$ and $QM_t$. Bergson (1973) has a similar framework. (Of course, the ratio of $PC_t$ to $PM_t$ is not determined solely by the slope of the production possibility frontier unless $\eta = \infty$.)

7. Given our production assumptions, we do not need any restrictions on demand elasticities of substitution for products within the QC and QM sectors.

8. At this point our model of household behavior is a replica of Diamond's (1965).

9. If monopolies paid abnormally high rates of return to stockholders and only high income families had the know-how to obtain such securities, further complications might arise.

10. The word "appears" is appropriate because we do not rigorously investigate $\lim_{n \to 1}(dK_0/d\eta)$.

11. "Steady-state household well-being" refers to comparisons of alternative steady states which ignore transition stages. See Section IV.

13. Schwartzman (1961) empirically derives a markup figure of 8.3%.

14. Government transfer payments (other than social security retirement benefits) are treated as a component of labor income in our model of household behavior.

15. In other words, we can still write the steady-state version of $s_t$ -- see line (11) -- as $s_t = w_t \cdot \phi(r_{t+1})$. We can easily show that $\phi(r) \rightarrow \infty$ as $r \rightarrow \infty$ and $\phi'(r) > 0$ all $r > 0$. Thus, because $\phi(r) \leq 0$ can never yield a steady-state equilibrium, Step 3 in the proof of Proposition I remains valid, as does the analysis of Section II. Since our goal here is to present numerical examples, we will not go into further details.

16. Although the elasticities are labeled in terms of $Q_0$, $C_0$, $w_0$, etc., the same figures would apply for $Q_t$, $C_t$, $w_t$, ... any $t$. The ratios $Q_1/(Q_1 + Q_2)$ and $C_1/(C_1 + C_2)$ are also independent of time, as are the results in Table IV.

17. In a life-cycle model which allows families to have labor incomes in many consecutive periods, a reduction in the interest rate can reduce saving and wealth accumulation quite appreciably even when households have logarithmic utility functions.

18. Compare the figures of Table IV with $1 - \delta = .5$ to those of row 2 in Bergson's (1973) Table 1, for example.
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