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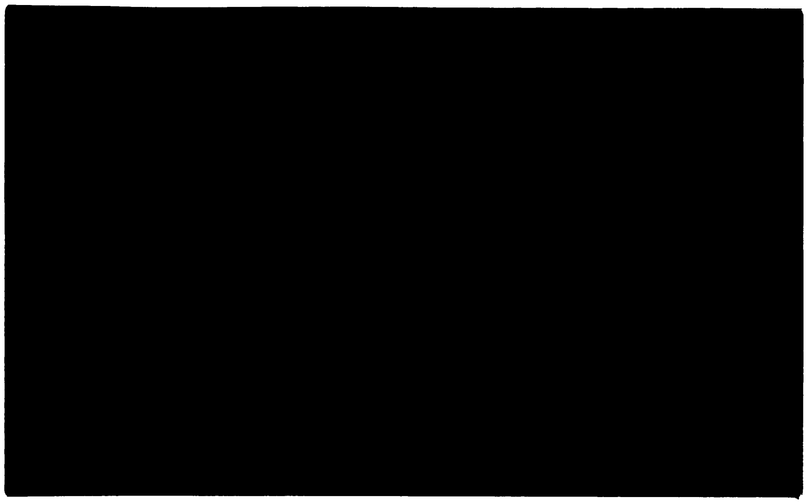

Oligopoly Behavior When Conjectural Variations Are Rational

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Abstract: This paper considers the following question: if each firm in an n-firm oligopoly has "rational conjectural variations," if n is exogenously given, and if we rule out increasing returns to scale, must aggregative industry output and its market price converge to competitive levels as n diverges to infinity? By rational conjectural variations we mean that at equilibrium, the responses from its rivals which each firm believes will follow a change in its output must correspond to reactions the rivals themselves perceive to be profit maximizing. In this framework we show that the answer to our question is "no" -- market outcomes near the cartel level and near the competitive level can both result no matter how large n is.

Oligopoly Behavior When Conjectural Variations Are Rational

The purpose of this paper is to consider the following question: if each firm in an n-firm oligopoly has "rational conjectural variations," and if we rule out increasing returns to scale and explicit collusion, must the aggregate industry output and its market price converge to competitive levels as n diverges to infinity? The latter type of convergence does generally emerge in the context of the Cournot model. As is well known, however, although the Cournot behavioral assumption requires each firm to believe that a change in its output will elicit no quantity adjustments from its rivals, normally the rivals will, in fact, want to react (given the overall Cournot framework). With "rational conjectural variations," on the other hand, the responses that a firm anticipates from its rivals must correspond to changes which the rivals themselves desire.[1/] Laitner [1980] shows that for a fairly broad class of duopoly problems, many industry outputs can be rational conjectural equilibria. The present paper shows that as we shift from duopoly to an n-firm oligopoly, a similar result carries over: given an arbitrary n = 2,3,..., any aggregative industry quantity of output strictly between monopoly and perfect competition solutions can be incorporated into a rational conjectural outcome. Thus, the answer to our basic question is "no" -- with rational conjectural variations, as n approaches infinity, an oligopoly's output and price need not converge to competitive levels.

All our firms are alike, we examine only symmetric outcomes, the single good which is produced is perfectly homogeneous, each firm has constant or increasing costs, and, in particular, the number of firms is exogenously given. Within this framework, our analysis implies that an antitrust action which converts, say, a duopoly into a ten firm oligopoly might cause industry output to rise toward the competitive level, might leave it unchanged, or

might cause it to decline toward the monopoly level. There is no *a priori* reason to believe one result is more likely than the others -- in the context presented. (This does not imply, however, that real world data should show no correlation between concentration and monopolistic pricing -- in practice, low concentration may often accompany low entry barriers, for example, and the latter may lead to low prices.)

Sections 1 and 2 establish our result for n-firm oligopolies. Although the arguments are constructive, the rational conjectural variation functions which we derive are somewhat complicated. Section 3, however, shows that the functions can easily be approximated. Section 4 uses the approximations in contrasting the Cournot and Bresnahan [1981] models with our own.

1. Assumptions and Definitions

We begin with a list of our assumptions and a precise definition of rational conjectural variations.

The assumptions are:

(A1) There are $n \geq 2$ firms producing for the market in which we are interested. All firms are alike. The number n is exogenously given.

(A2) The production costs of any firm producing X units of output are given by C(X). $C(\cdot)$ is twice continuously differentiable. $C'(X) \geq 0$ and $C''(X) \geq 0$ for $X \geq 0$. $C(0) = 0$.[2/]

(A3) Let Z be aggregative industry output. Then P(Z) gives the market price per unit of output. $P(\cdot)$ is continuous and nonincreasing for all $Z \geq 0$. $P(0) > C(0)$. For all $Z > 0$ with $P(Z) > 0$, we have $P(\cdot)$ twice continuously differentiable, $P'(Z) < 0$, and $P''(Z) \leq 0$. $P(0) > C'(0)$.

(A4) If firm i has output X_i and if the aggregative output of the remaining n - 1 firms is Y_i, let $F(X_i,Y_i)$ give the rate at which firm i anticipates that Y_i will change in response to an infinitesimal change in X_i -- in other words, let firm i believe that $dY_i/dX_i = F(X_i,Y_i)$.[3/] Then all firms use the same function (in each case the given firm's output and Z minus its output being the appropriate arguments of $F(\cdot)$). Furthermore, in the same notation, firm i believes that all of its rivals will react in the same way to a change in X_i -- in other words, if firm j, $j \neq i$, produces output X_j, firm i believes that $dX_j/dX_i = F(X_i,Y_i)/(n - 1)$.

We call $F(\cdot)$ the "conjectural variation function" of each firm. We say that $\phi: R^3 \to R$ "solves" $F(\cdot)$ if $\phi(X,Y,X) = Y$ and $\partial\phi(X,Y,X^*)/\partial X^* = F(X^*,\phi(X,Y,X^*))$ all $X,Y,X^* \geq 0$. Notice that if $\phi(\cdot)$ is the unique solution for $F(\cdot)$, and if (X_i,Y_i) characterizes the present outputs of firm i and all other firms, respectively, then firm i believes that if it changes its output from X_i to X_i^*, Y_i will change to $Y_i^* = \phi(X_i, Y_i, X_i^*)$ in response.

Given an output X_i for firm i and an aggregative output Y_i for its n - 1 rivals, the profits of firm i will be

$$H(X_i, Y_i) = P(X_i + Y_i) \cdot X_i - C(X_i). \tag{1}$$

Suppose the conjectural variation function $F(\cdot)$ used by all firms has a unique solution $\phi(\cdot)$. Then we define the "stable set" for firm i, $K_i(F(\cdot))$, as

$$K_i(F(\cdot)) = \{(\overline{X}_i, \overline{Y}_i) \geq 0: \ H(\overline{X}_i, \overline{Y}_i) > H(X, \phi(\overline{X}_i, \overline{Y}_i, X_i)) \quad \text{all } X_i \geq 0 \text{ and } X_i \neq \overline{X}_i\}. \tag{2}$$

$K_i(F(\cdot))$ is a set of "stable points" for firm i in the sense that if $(\overline{X}_i, \overline{Y}_i) \varepsilon K_i(F(\cdot))$ and firm i's output is $\overline{X}_i$, then the firm anticipates lower profits subsequent to any change in X_i on its part -- in other words, firm i will want to avoid initiating such a change.

For any given Y_i there may be more than one X_i with $(X_i, Y_i) \varepsilon K_i(F(\cdot))$. Suppose, for example, that (X_i, Y_i), $(\overline{X}_i, Y_i) \varepsilon K_i(F(\cdot))$. Then one of the two points may yield higher profits to firm i than the other (although if, for example, $H(X_i, Y_i) > H(\overline{X}_i, Y_i)$ and $\phi(\cdot)$ solves $F(\cdot)$, we cannot have $\phi(\overline{X}_i, Y_i, X_i) = Y_i$). Thus, if $F(\cdot)$ has a unique solution $\phi(\cdot)$, we define a "preferred stable set" for firm i as:

$$K_i^*(F(\cdot)) = \{(X_i, Y_i) \varepsilon K_i(F(\cdot)): \ H(X_i, Y_i) \geq H(\overline{X}_i, Y_i) \text{ all } (\overline{X}_i, Y_i) \varepsilon K_i(F(\cdot))\}. \tag{3}$$

We have

$$K_i^*(F(\cdot)) \subset K_i(F(\cdot)), \tag{4}$$

although both sets might be empty.[4/]

We are interested in symmetric outcomes. Suppose (A1)-(A4) hold, $\phi(\cdot)$ is the unique solution of $F(\cdot)$, all firms have output $X \geq 0$, and $Y = (n-1)X$. Suppose further that all firms are satisfied --

$$(X,Y) \in K_i(F(\cdot)) \text{ all } i. \tag{5}$$

Then the tuple $(X,F(\cdot))$ potentially defines an equilibrium for our model. In fact, we will pose the slightly more demanding requirement that

$$(X,Y) \in K_i^*(F(\cdot)) \text{ all } i. \tag{6}$$

And, we want conjectural variations to be "rational" rather than arbitrary. Suppose firm j contemplates modifying its output from $X_j = X$ to $X_j = X + \delta$. Given (A4), firm j believes that if it does so, $Y_j = \sum_{i \neq j} X_i$ will change from $Y_j = Y$ to $Y_j = \phi(X,Y,X+\delta)$ -- the output for each firm, ℓ, $\ell \neq j$, changing from $X_\ell = X$ to $X_\ell = \phi(X,Y,X+\delta)/(n-1)$. The latter adjustment will be sensible from the point of view of firm ℓ if

$$(X_\ell, Y_\ell) = (\phi(X,Y,X+\delta)/(n-1), \\ [(n-2)/(n-1)] \cdot \phi(X,Y,X+\delta) + X + \delta) \in K_\ell^*(F(\cdot)). \tag{7}$$

Our "rationality" concept is a "sensibility" requirement on anticipations along these lines:

<u>Definition</u>: <u>Let</u> <u>(A1)</u>-<u>(A4)</u> <u>hold</u>. <u>Let</u> $X^* > 0$ <u>and suppose</u> $\phi(\cdot)$ <u>is the unique solution for</u> $F^*(\cdot) \equiv F(\cdot)$. <u>Then</u> $(X^*,F^*(\cdot))$ <u>defines a "symmetric rational conjectural equilibrium"</u> (<u>SRCE</u>) <u>for our model if</u>

(i) $(X^*,Y^*) = (X^*,(n-1)X^*) \in K_i^*(F^*(\cdot))$ <u>all</u> $i = 1,\dots,n$, <u>and</u>

(ii) <u>there exists an</u> $\varepsilon > 0$ <u>such that</u> $|\delta| < \varepsilon$ <u>implies</u> $(\phi(X^*,Y^*,X^*+\delta)/(n-1),$
$[(n-2)/(n-1)] \cdot \phi(X^*,Y^*,X^*+\delta) + X^* + \delta) \in K_i^*(F^*(\cdot))$ <u>all</u> $i = 1,\dots,n$.

Our definition is analogous to Laitner's [1980] and Bresnahan's [1981]. It requires that firms' conjectural variations be "sensible" in the immediate vicinity of a given equilibrium point (X*,Y*). In contrast to Bresnahan, we allow F*( ) to have two arguments and we do not require that it be a polynomial.[5/]

2. Symmetric Rational Conjectural Equilibria

We now show that as we choose larger and larger values of n, the set of aggregate output and price realizations which can be consistent with rational conjectural variations does not shrink to the perfectly competitive outcome alone.

We use the following notation. As in Section 1, Z stands for total market output. For any given n, we call $Z^m(n)$ the "monopoly" value of Z if

$$P(Z^m(n)) + Z^m(n) \cdot P'(Z^m(n)) - C'(Z^m(n)/n) = 0. \tag{8}$$

Given assumptions (A1)-(A4), $Z^m(n)$ corresponds to market output when (given n) joint profits are maximized in a regime of full, explicit cooperation. We call $Z^c(n)$ the "competitive" value for Z if it is such that

$$P(Z^c(n)) - C'(Z^c(n)/n) = 0. \tag{9}$$

Thus, $Z^c(n)$ gives the value of Z if each firm seeks to maximize its own profits and believes that the market price of output is totally independent of its behavior. (In this case, each firm's profits will be at least zero, and they may be positive.) Our assumptions imply that

$$0 < Z^m(n) < Z^m(n+1) \text{ and } Z^m(n) < Z^c(n) < Z^c(n+1) \quad \text{all } n \geq 2. \tag{10}$$

Let

$$S(n) = \{Z: Z^m(n) < Z < Z^c(n)\}. \tag{11}$$

Note that if

$$s(n) = Z^c(n) - Z^m(n), \tag{12}$$

our restrictions (A2)-(A3) guarantee

$$\lim_{n\to\infty} \inf_{n^*\geq n}\{s(n^*)\} > 0. \tag{13}$$

Our basic result is

Proposition 1: Suppose assumptions (A1)-(A4) hold. Fix any $n \geq 2$. Let Z be aggregative industry output. Let S(n) be as in line (11). Then for any $Z^* \varepsilon S(n)$, if $X^* = Z^*/n$, we can construct a conjectural variation function $F^*(\cdot) = F(\cdot)$ such that $F^*(\cdot)$ has a unique solution $\phi(\cdot)$ and such that $(X^*, F^*(\cdot))$ defines an SRCE.

A proof is given in the appendix to this paper.

The conclusions of Proposition 1 can be restated in slightly different terms as follows:

Corollary 1: Let the suppositions and notations of Proposition 1 hold. Let $S^*(n) = \{Z: Z = nX^* \text{ with } (X^*, F^*(\cdot)) \text{ an SRCE}\}$. Then $S(n) \subset S^*(n)$ all $n \geq 2$.

Thus, while Cournot behavior in our framework would imply $Z(n) \to Z^c(n)$ as $n \to \infty$ for all $Z(n) \varepsilon S^*(n)$, such convergence is not implied if conjectural variations are rational.

Economists sometimes argue that when n is large, if firm i changes its output, the effect on the profits of each individual rival will be inconsequential and will therefore be ignored. Actually, let $(X^*, F^*(\cdot))$ be an SRCE generated in the proof of Proposition 1, let X_i give the output of firm i, and let X_j give the output of firm j with $j \neq i$. Then in the proof of Proposition 1 the only *a priori* restrictions that we have on firm i's anticipations about dX_j/dX_i are (assuming $Y^* = (n - 1)X^*$)

$$dX_j/dX_i \Big|_{(X^*,Y^*)} = F^*(X^*,Y^*)/(n - 1) \; \varepsilon \; (-1/(n - 1),1), \qquad (14)$$

given rationality. In fact, the proof shows that for any $x \; \varepsilon \; (-1/(n - 1),1)$ we can find a $Z^* \varepsilon S(n)$ with $X^* = Z^*/n$, $Y^* = (n - 1)X^*$, $(x^*, F^*(\cdot))$ an SRCE, and $F^*(X^*,Y^*)/(n - 1) = x$. Hence, dX_j/dX_i in line (14) need not converge to 0 as $n \to \infty$.

If we look at the first-order condition for profit maximization on the part of firm i, the effects of the "rationally" anticipated reactions of each individual rival do converge to 0 as $n \to \infty$. The output choice of firm i depends on the sum of such effects taken over $n - 1$ rivals, however, and the sum does not necessarily become infinitesimal for large values of n.

A second corollary which follows directly from the proof of Proposition 1 is

Corollary 2: *Given assumptions (A1)-(A4), let $(X^*, F^*(\cdot))$ be an SRCE constructed in Proposition 1. Let $Y^* = (n - 1) \cdot X^*$. Then for some open set W with $(X^*, Y^*) \in W$, we have $F^*(X,Y)$ continuously differentiable all $(X,Y) \in W$.*

This corollary provides the basis for the analysis in the next section.

3. Approximations

The conjectural variation function $F^*(\cdot)$ generated in the proof of Proposition 1 is somewhat complicated. Nevertheless, since, as Corollary 2 notes, $F^*(\cdot)$ is continuously differentiable in the vicinity of the equilibrium point (X^*,Y^*), at each of our SRCE tuples $(X^*,F^*(\cdot))$ (with $Y^* = (n-1)X^*$) we can be sure there is a linear function

$$G(X,Y;(X^*,F^*(\cdot))) = \alpha X + \beta Y + \gamma \tag{15}$$

such that

$$G(X^*,Y^*;(X^*,F^*(\cdot))) = F^*(X^*,Y^*) \tag{16}$$

and such that for any $\varepsilon > 0$ we can find $\tau > 0$ with

$$|(X,Y) - (X^*,Y^*)| < \tau$$

implying

$$F^*(X,Y) - G(X,Y;(X^*,F^*(\cdot)))| < \varepsilon.$$

$G(\cdot)$ will be the first-order Taylor series approximation of $F^*(\cdot)$ about (X^*,Y^*). We now derive α, β, and γ, and briefly examine $G(\cdot)$'s properties.

Line (16) gives one linear constraint for α, β, and γ. Step 4 in the proof of Proposition 1 implies two others. On the one hand, given the construction of $F^*(\cdot)$, for an open set U containing (X^*,Y^*) we have

$$F^*(X,Y) = \psi(X,Y) \text{ all } (X,Y) \in U \cap K \tag{17}$$

(where $\psi(\cdot)$ is defined in Step 1 of the proof of Proposition 1). Thus,

$$\begin{aligned} dG(X^*,Y^*;\cdot)/dY\Big|_K &\equiv dG(X^*,Y^*;\cdot)dY\Big|_{dX = B^{-1}dY} = \\ &\alpha B^{-1} + \beta = dF^*(X^*,Y^*)/dY\Big|_K = \\ &d\psi(X^*,Y^*)/dY\Big|_K = \\ &\Delta/[X^* \cdot P'(X^* + Y^*)]^2 \end{aligned} \tag{18}$$

where B is defined in the proof of Proposition 1 and

$$\Delta \equiv X^* \cdot P'(X^* + Y^*) \cdot [C''(X^*) \cdot B^{-1} -$$
$$P'(X^* + Y^*) \cdot (1 + B^{-1}) - P'(X^* + Y^*) \cdot B^{-1} -$$
$$X^* \cdot P''(X^* + Y^*) \cdot (1 + B^{-1})] -$$
$$[C'(X^*) - P(X^* + Y^*) - X^* \cdot P'(X^* + Y^*)] \cdot$$
$$[P'(X^* + Y^*) \cdot B^{-1} + X^* \cdot P''(X^* + Y^*)(1 + B^{-1})].$$

On the other hand, Step 4 (in the proof of Proposition 1) also shows that $F^*(\cdot)$ is constant along a line L. Thus, if b is as defined in the appendix,

$$dG(X^*,Y^*;\cdot)/dX\Big|_L \equiv dG(X^*,Y^*;\cdot)/dX\Big|_{dY = (n-1)bdX} =$$
$$\alpha + (n-1)b\beta = dF^*(X^*,Y^*)/dX\Big|_L = 0. \tag{19}$$

Lines (18)-(19) show that

$$\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = N^{-1}v$$

where

$$N = \begin{pmatrix} B^{-1} & 1 \\ 1 & (n-1)b \end{pmatrix},$$

$$v = \begin{pmatrix} \Delta/[X^* \cdot P'(X^* + Y^*)]^2 \\ 0 \end{pmatrix}.$$

The matrix N is invertible because

$$\det(N) = 0 \quad \text{iff} \quad B = (n-1)b$$

and Step 2 in the proof of Proposition 1 shows $B \neq (n-1)b$. Given α and β, line (16) determines γ.

For a specified X^* with $nX^* \in S(n)$, Section 2 does not prove that there is a unique conjectural variation function $F(\cdot)$ making $(X^*,F^*(\cdot))$ an SRCE. However, the first-order condition for each firm does imply that if $(X^*,F(\cdot))$ and $(X^*,F^*(\cdot))$ are SRCE's and $Y^* \equiv (n-1)X^*$, then

$$F^*(X^*,Y^*) = F(X^*,Y^*) = \psi(X^*,Y^*). \tag{20}$$

Furthermore, if $F^*(\cdot)$ and $F(\cdot)$ are differentiable, line (18) must hold for both. We can understand this as follows. If firm j changes its output by dX_j, for firm j's expectations to be borne out and for symmetry among its rivals to be preserved, firm i, $i \neq j$, must react by changing its output by

$$dX_i = F^*(X^*,Y^*)dX_j/(n - 1) = F(X^*,Y^*)dX_j/(n - 1) \equiv bdX_j \tag{21}$$

(see line (20) and the definition of b in the proof of Proposition 1). Firm i will only desire the latter reaction if its marginal profits are still zero (as at (X^*,Y^*)) when we reach

$$(X^* + dX_i, Y^* + dX_j + [(n - 2)/(n - 1)] F^*(X^*,Y^*)dX_j). \tag{22}$$

Thus, using lines (21) and (22), marginal profits for firm i must be unchanged in the direction

$$dY_i/dX_i = [1 + (n - 2)b]dX_j/dX_i = [1 + (n - 2)b]b^{-1} \equiv B \tag{23}$$

(where B is as defined in the proof of Proposition 1). For marginal profits to remain equal to zero, $F(X,Y)$ and $F^*(X,Y)$ must remain equal to $\psi(X,Y)$ as we move away from (X^*,Y^*) in this direction -- which is what line (18) requires. In contrast, our decision to make the $F^*(\cdot)$ in the proof of Proposition 1 constant along L was arbitrary. Line (19) need not, therefore, hold for every $F(\cdot)$ with $(X^*,F(\cdot))$ an SRCE.

Finally, notice that if $(X^*,F^*(\cdot))$ is an SRCE and $G(\cdot;(X^*,F^*(\cdot)))$ approximates $F^*(\cdot)$ near (X^*,Y^*), we cannot conclude that $(X^*,G(\cdot;(X^*,F^*(\cdot)))$ itself is an SRCE -- that usually will not be the case. For, while the definition of an SRCE requires $F^*(\cdot)$ to have exact properties for a range of values $X^* + \delta$ where $|\delta| < \varepsilon$ some $\varepsilon > 0$, a Taylor series approximation such as $G(\cdot)$ will normally only be precise at one point (in our case, (X^*,Y^*)).

4. Examples

We now study several specific examples.

Example 1: Cournot's Solution.

Suppose $(\hat{X},\hat{Y})$ with $\hat{Y} = (n - 1)\hat{X}$ constitutes a Cournot equilibrium. The Cournot conjectural variation function is, of course, $\hat{F}(X,Y) = 0$ all (X,Y). Let $(\hat{X},F^*(\cdot))$ be the SRCE constructed in the proof of Proposition 1, and let $G(\cdot) = G(\cdot;(\hat{X},F^*(\cdot)))$. We can then compare $G(\cdot)$, $F^*(\cdot)$, and $\hat{F}(\cdot)$.

Line (18) shows that

$$dF^*(\hat{X},\hat{Y})/dY\Big|_K = dF^*(\hat{X},\hat{Y})/dY\Big|_{dX = 0} = dG(\hat{X},\hat{Y};\ )/dX\Big|_{dX = 0} =$$

$$\beta = -[P'(\hat{X} + \hat{Y}) + \hat{X} \cdot P''(\hat{X} + \hat{Y}]/$$

$$[\hat{X} \cdot P'(\hat{X} + \hat{Y})] < 0. \tag{24}$$

Line (19) gives

$$dF^*(\hat{X},\hat{Y})/dX\Big|_L = dF^*(\hat{X},\hat{Y})/dX\Big|_{dY = 0} =$$

$$dG(\hat{X},\hat{Y};\cdot)/dX\Big|_{dY = 0} = \alpha = 0. \tag{25}$$

Thus,

$$G(X,Y;(\hat{X},F^*(\cdot))) = \beta Y - \beta\hat{Y},\ \beta < 0. \tag{26}$$

Line (25) shows that

$$\hat{F}(X,\hat{Y}) = G(X,\hat{Y};\cdot) = F^*(X,\hat{Y}) \quad \text{all} \quad X \geq 0. \tag{27}$$

However, line (24) implies that $\hat{F}(X,Y)$ does not equal $G(X,Y;(\hat{X},F^*(\cdot)))$ or $F^*(X,Y)$ any $(X,Y) \neq (X,\hat{Y})$ with (X,Y) near $(\hat{X},\hat{Y})$. Since Section 3 establishes that for any SRCE $(\hat{X},F^{**}(\cdot))$ with $F^{**}(\cdot)$ differentiable, $G(\cdot;(\hat{X},F^{**}(\cdot)))$ must satisfy equation (18) -- and, hence, line (24) -- we see that $(\hat{X},\hat{F}(\cdot))$ is not an SRCE and that $\hat{F}(\cdot)$ is not a linear Taylor series approximation at $(\hat{X},\hat{Y})$ for any differentiable $F^{**}(\cdot)$ with $(\hat{X},F^{**}(\cdot))$ an SRCE.

Example 2: *Bresnahan's Solution*.

Bresnahan [1981] develops a model in which

$$C(X) = cX + (\bar{c}/2)X^2; \; c,\bar{c} > 0, \tag{28}$$

$$P(Z) = \text{Max}\{0, d_0 - d \cdot Z\}; \; d_0, d > 0. \tag{29}$$

His most important restriction, however, is the fact that he limits his attention to SRCE tuples (X*,F*(•)) in which F*(X,Y) is a polynomial (of finite degree) in X. Within that class, Bresnahan shows that (given lines (28)-(29)) there is only a single SRCE tuple, (X**,F**(•)), and that F**(X,Y) = f, a constant, all (X,Y). We now compare Bresnahan's solution with our own.

We have

Proposition 2: *Suppose lines (28)-(29) hold. Let* $P(0) - C'(0) = d_0 - c > 0$. *Fix any integer* $n \geq 2$. *Let* R(n) = {(X*,F*(•)): X* = Z*/n *with* Z* ε S(n), (X*,F*(•)) *is the SRCE generated in the proof of Proposition 1, and for some open set* V *with* (X*,Y*) = (X*,(n - 1)X*) ε V *we have* G(X,Y;(X*,F*(•))) = F*(X,Y) *all* (X,Y) ε V}. *Then* R(n) *has one and only one element. The element is Bresnahan's solution. If* (X*(n),F**(•)) ε R(n) *and* Z*(n) = n • X*(n), *then* $\lim_{n\to\infty} Z^*(n) = Z^c(n)$.

A proof is provided in the appendix.

Proposition 2 shows that if (X*,F*(•)) is one of our SRCE's from Proposition 1, and if we demand that F*(•) and G(•;(X*,F*(•))) equal one another one some open set containing (X*,Y*) = (X*,(n - 1)X*), then X* will be unique. Requiring that G(•) and F*(•) exactly coincide on an open set is a very restrictive behavioral assumption, however. Yet, the significance of the unique value of X* emerging in this example seems to depend on the realism of that assumption.[6/]

5. Conclusion

We have examined a partial equilibrium model of oligopoly behavior in one market. The good sold in the market is homogeneous. There are $n \geq 2$ firms in the oligopoly, and the number is exogenously given. Explicit collusion is not allowed. We find that under specified demand and cost conditions, any aggregative output (strictly) between monopoly and perfectly competitive levels can emerge and be consistent with all firms having rational conjectural variation functions. Because this outcome is shown to be independent of n, we find that a large n alone is no guarantee that market behavior will closely mimic the perfect competition paradigm.

Appendix

1. Proof of Proposition 1:

Fix X^* as stated. Let $Y^* = (n - 1)X^*$. Let

$$\psi(X,Y) = [C'(X) - P(X + Y) - X \cdot P'(X + Y)] / [X \cdot P'(X + Y)].$$

Notice that $|\psi(X^*,Y^*)| < \infty$.

Step 1: Let

$$b = \psi(X^*,Y^*)/(n - 1),$$

$$a = Y^* \cdot (1 - b).$$

Let

$$B = b^{-1} + (n - 2),$$

$$A = B^{-1} \cdot Y^* - X^*.$$

Define

$$K = \{(X,Y) \geq 0: \quad X = B^{-1}Y - A\},$$

$$L = \{(X,Y) \geq 0: \quad Y = a + (n - 1)bX\}.$$

We want a subset of K to serve as $K_i^*(F^*(\cdot))$ any i, and L to give firm i's anticipation of the aggregate reaction of all other firms if it changes its output X when the initial point is (X^*,Y^*). Notice that by construction,

$$(X^*,Y^*) \in K \cap L.$$

Step 2: We digress to show that $|B^{-1}| < \infty$ and that K and L are not coincident.

Let

$$\lambda(X) = \psi(X,(n - 1)X).$$

Then

$$\lambda'(X) < 0.$$

So, $Z^* \varepsilon S(n)$ implies

$$\lambda(Z^c(n)/n) = -1 < \lambda(Z^*/n) = (n-1) \cdot b < \lambda(Z^m(n)/n) = n - 1.$$

Thus,

$$-1/(n - 1) < b < 1.$$

So,

$$B > n - 1 \quad \text{or} \quad B < -1$$

Hence,

$$|B^{-1}| < \infty.$$

Suppose

$(n - 1)b = B$ --

in other words, that K and L coincide. The preceding paragraph shows

$$-1 < (n - 1)b < (n - 1)$$

and

$$B > n - 1 \quad \text{or} \quad B < -1.$$

Thus, we have a contradiction. So,

$$(n - 1)b \neq B.$$

Step 3: We now define $F^*(\cdot)$ on L in such a way that $(X^*,Y^*) \varepsilon K_i^*(F^*(\cdot))$.

Define

$$F^*(X,Y) = \psi(X^*,Y^*) \quad \text{all} \quad (X,Y) \varepsilon L \text{ with } Y > 0.$$

Define

$$F^*(X,0) = 0 \quad \text{all} \quad X \geq 0.$$

Let $\phi(\cdot)$ solve $F^*(\cdot)$ starting at (X^*,Y^*). Then for any $X \geq 0$,

$$(X,\phi(X^*,Y^*,X)) \varepsilon L \quad \text{or} \quad \phi(X^*,Y^*,X) = 0.$$

Let $(X,Y) = (X,\phi(X^*,Y^*,X))$ some $X \geq 0$. If $Y = 0$,

$$1 + F^*(X,Y) = 1 > 0.$$

If $Y > 0$, using the function $\lambda(\cdot)$ defined in Step 2,

$$1 + F^*(X,Y) = 1 + \psi(X^*,Y^*) = 1 + \lambda(Z^*/n) > 1 + \lambda(Z^c(n)/n) > 0.$$

Let (X,Y) be as above. If X is firm i's output, the firm perceives its marginal profit at (X,Y) to be

$$M(X,Y) = P(X + Y) + X \cdot P'(X + Y) \cdot (1 + F^*(X,Y)) - C'(X).$$

If $Y > 0$,

$$dM(X,Y)/dX\Big|_{dY = F^*(X,Y)dX} =$$

$$2 \cdot P'(X + Y) \cdot (1 + F^*(X,Y)) +$$

$$X \cdot P''(X + Y) \cdot (1 + F^*(X,Y))^2 -$$

$$C''(X) < 0.$$

By construction,

$$M(X^*,Y^*) = 0.$$

Thus, as we move along L to larger values of X with $X \geq X^*$, $M(X,Y)$ is negative. If Y becomes 0, $M(X,Y)$ drops to an even lower value and remains negative.[7/] If we move away from $X = X^*$ in the other direction, $M(X,Y)$ is positive. If Y becomes 0, $M(X,Y)$ remains positive.[8/] Thus, $(X^*,Y^*) \in K_i(F^*(\cdot))$ any i.

Step 4: We now define $F^*(\cdot)$ for points not in $L \cup \{(X,0): X \geq 0\}$.

Step 2 shows that if $B > 0$, then $B > |(n - 1)b|$. The definition of B shows that if $B < 0$, then $b < 0$. In the latter case, Step 2 shows that $|B| > |(n - 1)b|$. Thus, in all circumstances, the absolute value of the slope of $K = |B| > |(n - 1)b|$ = the absolute value of the slope of L. So,

$$|B| > |(n - 1)b| = |\psi(X^*,Y^*)|.$$

We also have $X^* > 0$, $Y^* > 0$, $\psi(X^*,Y^*) + 1 > 0$, and $\psi(\cdot)$ continuous. So, there exists $\eta \in (0,\infty)$ such that $X^* - \eta > 0$, $Y^* - \eta > 0$,

$$U = (X^* - \eta,\ X^* + \eta) \times (Y^* - \eta,\ Y^* + \eta)$$

implies

$$\psi(X,Y) + 1 > 0 \quad \text{all} \quad (X,Y) \in Cl(U),$$

and

$$|\psi(X,Y)| < |B| \quad \text{all} \quad (X,Y) \in Cl(U).$$

Let

$$\overline{L}(X,Y) = \{(X,Y)\} + \{L - \{(X^*,Y^*)\}\}.$$

Let Y_L and Y_U be such that

$$Y_L = \inf\{Y\colon (X,Y) \in U \cap K\},$$

$$Y_U = \sup\{Y\colon (X,Y) \in U \cap K\}.$$

Then

$$0 < Y_L \leq Y_U < \infty.$$

In fact, since Step 2 shows $0 \neq B$ = the slope of K,

$$Y_L < Y_U.$$

Let X_L and X_U be such that

$$(X_L,Y_L),\ (X_U,Y_U) \in K,$$

and let

$$\hat{L} = \{(X,Y) \geq 0\colon (X,Y) \in \overline{L}(X_L,Y_L) \cup \overline{L}(X_U,Y_U)\}.$$

Let

$$M^*(X,Y) = P(X + Y) + X \cdot P'(X+Y) \cdot (1 + F^*(X^*,Y^*)) - C'(X).$$

Since

$$1 + F^*(X^*,Y^*) > 0,$$

for some $\hat{Z} \in (X^* + Y^*,\infty)$ we have

$$M^*(X,Y) < 0 \quad \text{all} \quad (X,Y) \in V = \{(\overline{X},\overline{Y}) \geq 0\colon \overline{X} > X^* \text{ and } \overline{X} + \overline{Y} > \hat{Z}\}.$$

Let $V\#$ be

$$V\# = \{(\overline{X},\overline{Y}) \geq 0\colon (\overline{X},\overline{Y}) \notin V\}.$$

Then adjusting our choice of η (above) downward if necessary, Step 3 shows that if $\eta^* = X^*/2$, $X \geq X^* + \eta^*$ and $(X,Y) \in \hat{L} \cap V\#$ imply $M^*(X,Y) < 0$, and $X \leq X^* - \eta^*$ and $(X,Y) \in \hat{L}$ imply $M^*(X,Y) > 0$. Thus, $M^*(X,Y) < 0$ for $X \geq X^* + \eta^*$ and $(X,Y) \in \hat{L}$, and $M^*(X,Y) > 0$ for $X \leq X^* - \eta^*$ and $(X,Y) \in \hat{L}$.

Let

$$F^*(X,Y) = \psi(X,Y) \quad \text{all} \quad (X,Y) \in K \cap U.$$

For $(\overline{X},\overline{Y}) \in \overline{L}(X,Y)$ some $(X,Y) \in K \cap U$, $\overline{X} \geq 0$, and $\overline{Y} > 0$, let

$$F^*(\overline{X},\overline{Y}) = F^*(X,Y).$$

Let

$$F^*(X,Y) = \psi(X^*,Y^*) \text{ all } (X,Y) \in \hat{L} \text{ with } X \geq 0 \text{ and } Y > 0.$$

For all other $(\overline{X},\overline{Y})$ with $\overline{Y} > 0$, let

$$F^*(\overline{X},\overline{Y}) = -1.$$

Let

$$T = \{(\overline{X},\overline{Y}) \geqq 0\colon \overline{Y} > 0 \text{ and } (\overline{X},\overline{Y}) \in \overline{L}(X,Y) \text{ some } (X,Y) \in K \cap U\} \cup \{(\overline{X},\overline{Y}) \geqq 0\colon (\overline{X},\overline{Y}) \in \hat{L} \text{ and } \overline{X} \geq X^* + \eta^* \text{ or } \overline{X} \leq X^* - \eta^*\} \cup \{(\overline{X},\overline{Y}) \geq 0\colon \overline{Y} = 0\}.$$

Let $\phi(\cdot)$ solve $F^*(\cdot)$ everywhere. Then the continuity of $\psi(\cdot)$ implies that there exists an open interval I with $Y^* \in I$ such that if $Y \in I$ and $(X,Y) \in K$, we have $\phi(X,Y,\tilde{X}) \in T$ all $\tilde{X} \geq 0$.

Note that

$$dF^*(X,Y)/dX\big|_{dY = F^*(X^*,Y^*)dX} = 0 \quad \text{all} \quad (X,Y) \in T.$$

Thus,

$$dM(X,Y)/dX\big|_{dY = F^*(X^*,Y^*)dX} =$$

$$2 \cdot P'(X + Y) \cdot (1 + F^*(X,Y)) +$$

$$X \cdot P''(X + Y) \cdot (1 + F^*(X,Y))^2 -$$

$$C''(X) < 0 \text{ all } (X,Y) \in T.$$

We have

$$M(X,Y) = 0 \quad \text{all} \quad (X,Y) \in K \cap U.$$

Suppose $(\overline{X},\overline{Y}) \in K \cap (R^1 \times I)$, $\tilde{X} \geq 0$, and $(\hat{X},\hat{Y}) = (\tilde{X},\phi(\overline{X},\overline{Y},\tilde{X}))$. Then

$$|\psi(X,Y)| < |B| \quad \text{all} \quad (X,Y) \in Cl(U),$$

our condition for dM(X,Y)/dX above, and our discussion of $M^*(\cdot)$ on $\hat{L}$ imply 9/

$$M(\hat{X},\hat{Y}) > (<)\ 0 \text{ for } \tilde{X} < (>)\ \overline{X}.$$

Hence,

$$K \cap (R^1 \times I) \subset K_i(F^*(\cdot)) \text{ all } i.$$

Step 5: Let T# be the complement of T. Let $(X,Y) \in T\# \cap (R^1 \times I)$. Then $F^*(X,Y) = -1$. So, $M(X,Y) = P(X + Y) - C'(X) > 0$, or $H(X,Y) < H(\overline{X},Y)$ with $(\overline{X},Y) \in K$. For $(X,Y) \in (T \cap (R^1 \times I)) - K$, $M(X,Y) \neq 0$. Thus,

$$K \cap (R^1 \times I) \subset K_i^*(F^*(\cdot)) \text{ all } i.$$

Step 6: We have

$$(X^*,Y^*) = (X^*,(n - 1)X^*) \in K \cap (R^1 \times I) \subset K_i^*(F^*(\cdot)) \text{ all } i.$$

We now show that $(X^*,F^*(\cdot))$ satisfies the rest of the definition of an SRCE.

Let $\phi(\cdot)$ solve $F^*(\cdot)$. There exists an $\varepsilon > 0$ such that $|\delta| < \varepsilon$ implies

$$[(n-2)/(n-1)] \cdot \phi(X^*,Y^*,X^*+\delta) + X^* + \delta \in I.$$

So, $|\delta| < \varepsilon$ implies

$$\begin{aligned}
&(\phi(X^*,Y^*,X^*+\delta)/(n-1),[(n-2)/(n-1)] \cdot \\
&\quad \phi(X^*,Y^*,X^*+\delta) + X^* + \delta) = \\
&\quad ([Y^* + \delta \cdot b \cdot (n-1)]/(n-1),[(n-2)/(n-1)] \cdot \\
&\quad [Y^* + \delta \cdot b \cdot (n-1)] + X^* + \delta) = \\
&\quad (X^* + \delta \cdot b,(n-2)(X^* + \delta \cdot b) + X^* + \delta) = \\
&\quad (X^* + \delta \circ b, Y^* + \delta \circ b \cdot (n-2) + \delta) = \\
&\quad (X^* + \delta \cdot b, Y^* + B \cdot \delta \cdot b) = \\
&\quad (X^* + B^{-1}B\delta b, Y^* + B\delta \cdot b) \in K \cap (R^1 \times I) \subset \\
&\quad K_i^*(F^*(\cdot)) \quad \text{all } i
\end{aligned}$$

(where the second-to-the-last equality follows from the definition of B in Step 1, and the first inclusion follows from the definition of K and our restrictions on the size of δ). Thus, $(X^*,F^*(\cdot))$ is an SRCE. //

2. Proof of Proposition 2:

Step 1: Step 4 in Proposition 1 implies that for $F^*(\cdot)$ and $G(\cdot)$ to be equal on some open set W containing (X^*,Y^*), $G(\cdot)$ must equal $\psi(\cdot)$ on $K \cap U \cap W$. Line (18) shows

$$dG(X,Y;\cdot)/dY\Big|_K = \alpha B^{-1} + \beta.$$

Also,

$$d\psi(X,Y)/dY\Big|_K = \Delta/[X \cdot P'(X+Y)]^2$$

where, given lines (28) - (29),

$$\Delta = -Xd[\bar{c}B^{-1} + d + dB^{-1} + dB^{-1}] + [c + \bar{c}X - d_0 + dX + dY + dX]dB^{-1} = - Xd^2 + cdB^{-1} - d_0dB^{-1} + d^2YB^{-1}.$$

But, along K,

$$YB^{-1} = A + X.$$

So,

$$\Delta = -Xd^2 + cdB^{-1} - d_0dB^{-1} + d^2A + d^2X = [c - d_0 + dAB]dB^{-1}.$$

Thus,

$$dG(X,Y;\cdot)/dY\Big|_{K\cap V} = \alpha B^{-1} + \beta$$

implies

$$A = (d_0 - c)/(dB)$$

and

$$\alpha B^{-1} + \beta = 0.$$

<u>Step 2</u>: Using Step 1 of the proof of Proposition 1,

$$A = [(n - 1)B^{-1} - 1]X^*.$$

So,

$$(d_0 - c)/d = (n - 1 - B)X^*.$$

Also, Step 1 shows

$$B = b^{-1} + n - 2.$$

So,

$$(d_0 - c)/d = (n - 1 - b^{-1} - n + 2)X^* = (1 - b^{-1})X^*.$$

From Proposition 1 we also have that

$$(n-1)b = \psi(X^*,Y^*) =$$

$$[c + \bar{c}X^* - d_0 + dX^* + dY^* + dX^*]/[-X^*d] =$$

$$[(c - d_0)/(-dX^*)] - [(\bar{c} + nd + d)/d].$$

Thus, substituting from the preceding paragraph,

$$(n-1)b = 1 - b^{-1} - \bar{c}d^{-1} - (n+1) = -b^{-1} - \bar{c}d^{-1} - n.$$

So,

$$(n-1)b^2 + (n + \bar{c}d^{-1})b + 1 = 0.$$

Therefore,

$$b = \{-(n + \bar{c}d^{-1}) \pm \sqrt{(n + \bar{c}d^{-1})^2 - 4(n-1)}\}/$$
$$(2 \cdot (n-1)).$$

Step 3: For b to be real, we need

$$(n + \bar{c}d^{-1})^2 \geqq 4 \cdot (n-1).$$

That is true iff

$$n^2 + 2\bar{c}d^{-1}n + (\bar{c}d^{-1})^2 \geqq 4n - 4 \quad \text{iff}$$
$$\theta(n) = n^2 + (2\bar{c}d^{-1} - 4)n +$$
$$((\bar{c}d^{-1})^2 + 4) \geqq 0.$$

We have

$$\theta(2) > 0 \quad \text{and} \quad \theta'(2) > 0.$$

So,

$$\theta(n) \geqq 0 \quad \text{all} \quad n \geqq 2.$$

Step 4: Step 2 in the proof of Proposition 1 shows

$$-1/(n-1) < b < 1.$$

Thus, for $n \geqq 2$, only

$$b = \{-(n + \bar{c}d^{-1}) + \sqrt{(n + \bar{c}d^{-1})^2 - 4(n-1)}\}/$$
$$(2 \cdot (n-1))$$

is feasible. Since this $b < 0$, we must have $b > -1/(n-1)$.

$$b > -1/(n-1) \quad \text{iff} \quad -2 <$$

$$-(n + \bar{c}d^{-1}) + \sqrt{(n + \bar{c}d^{-1})^2 - 4(n-1)} \quad \text{iff}$$

$$n + \bar{c}d^{-1} - 2 < \sqrt{(n + \bar{c}d^{-1})^2 - 4(n-1)} \quad \text{iff}$$

$$(n + \bar{c}d^{-1})^2 - 4(n + \bar{c}d^{-1}) + 4 <$$

$$(n + \bar{c}d^{-1})^2 - 4(n-1) \quad \text{iff} \quad -4\bar{c}d^{-1} < 0,$$

which is true.

<u>Step 5</u>: Returning to Step 2,

$$X^* = (d_0 - c)/[d \cdot (1 - b^{-1})].$$

We need

$$P(nX^*) = d_0 - dnX^* > 0.$$

But,

$$b > -1/(n-1) \quad \text{iff} \quad -b < 1/(n-1)$$

$$\text{iff} \quad -b^{-1} > n-1 \quad \text{iff} \quad 1 - b^{-1} > n$$

$$\text{iff} \quad d_0 - dnX^* > d_0 - d_0 + c =$$

$$c > 0.$$

<u>Step 6</u>: Let $n = 2$. Then

$$b = -1 - [\bar{c}/(2d)] + [\bar{c} \cdot (\sqrt{1 + (4d/\bar{c})})/(2d)].$$

Our b corresponds to Bresnahan's r. Thus, our solution corresponds to Bresnahan's.

<u>Step 7</u>: Let $b = b(n)$ and

$$X^*(n) = (d_0 - c)/[d \cdot (1 - (b(n))^{-1})].$$

Let $Z^*(n) = n \cdot X^*(n)$. Suppose "lim" stands for "$\lim_{n\to\infty}$". Then

$$\lim Z^*(n) = [(d_0 - c)/d] \cdot \lim n/[1 - (b(n))^{-1}].$$

We have

$$\lim n/[1 - (b(n))^{-1}] =$$

$$\lim n/\{1 + 2(n-1)/[n + \bar{c}d^{-1} - \sqrt{(n + \bar{c}d^{-1})^2 - 4(n-1)}]\} =$$

$$\lim\{2/(n + \bar{c}d^{-1} - \sqrt{(n + \bar{c}d^{-1})^2 - 4(n-1)})\}^{-1} =$$

$$.5 \cdot \lim\{n + \bar{c}d^{-1} - \sqrt{(n + \bar{c}d^{-1})^2 - 4(n-1)}\} =$$

$$.5 \cdot \lim\{n + \bar{c}d^{-1} - \sqrt{[n + (\bar{c}d^{-1} - 2)]^2 + 4\bar{c}d^{-1}}\} =$$

$$.5 \cdot \lim\{n + \bar{c}d^{-1} - [n + \bar{c}d^{-1} - 2]\} = 1.$$

Therefore,

$$\lim Z^*(n) = (d_0 - c)/d.$$

So, $d_0 - d \cdot \lim Z^*(n) = c = \lim C'(X^*(n))$. //

Notes

1. See Laitner [1980] and Bresnahan [1981] for detailed discussions of rational conjectural variations. See also Marschak and Selten [1974].

2. The last restriction here is not strictly necessary for most of our results. It simplifies our proofs, however, by ruling out some types of "corner solutions."

3. Note that even more general specifications -- $dY_i/dX_i = \Omega(X_1,...,X_n)$ -- are possible. We avoid such generalizations in the interests of simplicity. If we did not restrict our attention to symmetric equilibria, we would have to use $\Omega(\cdot)$.

4. The distinction between $K_i^*(F(\cdot))$ and $K_i(F(\cdot))$ is a detail which does not play an essential role in our analysis.

5. See Section 4. Note also that Bresnahan [1981] (and Laitner [1980]) sets $n = 2$.

6. As stated, Bresnahan [1981] allowed $F^*(\cdot)$ to be a polynomial in X, but then proved that only the constant polynomial was relevant. Note also that he allowed a constant term in line (28) -- see note 2 above. Finally, recall that the definition of an SRCE involves exact properties on an open set. For a less restrictive concept in the same vein, see page 644 of Laitner [1980].

7. Note that for Y to drop to 0 as X increases we must have $F^*(X,Y) < 0$ along L. Also, along the X-axis, $dM(X,Y)/dX = 2 \cdot P'(X) + X \cdot P''(X) - C''(X) < 0$.

8. In order for Y to drop to 0 as X decreases, we must have $F^*(X,Y) > 0$ along L. See note 7.

9. See also note 7.

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