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THE NATURAL RATE OF UNEMPLOYMENT*

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THE NATURAL RATE OF UNEMPLOYMENT

The purpose of this paper is to derive comparative-static results for short and long-run determinants of one component of the natural rate of unemployment. The component arises in the models of Stiglitz (1974) and Salop (1979), for example, with the following basis:[1/] If the labor market becomes "tight," individual firms may decide to offer wage premiums to reduce the quit rates they experience. If firms are fundamentally similar, however, all will reach roughly the same decision about premiums, and they will reach it simultaneously. Although all will then be frustrated to discover they have not succeeded in outbidding their competitors, the resulting rise in average wage rates will increase unemployment, making the labor market less tight. If the latter development reduces each firm's incentives to offer premiums, the market may settle into a permanent state in which no firm desires to change its wage offer yet a positive amount of unemployment persists.[2/] Changes affecting the magnitude of that unemployment are the topic of this paper. The theories pioneered in Phelps et al. (1970) are the foundation of the analysis: we assume that the nonpecuniary aspects of employment at different firms, as well as the tastes of different laborers for such characteristics, are heterogeneous; that labor turnover is a quantitatively important phenomenon; and, that data acquisition costs lead to "imperfect information" on the part of labor market participants (workers in particular).

We first develop a short-run profit function for individual firms. The key feature of the function is that the efficiency of each firm's labor input varies positively with the difference between the wage rate offered by the firm, w, and the average rate offered in the remainder of the economy, W, and with the overall rate of unemployment, U. Section I attempts to motivate these assumptions by demonstrating that our specification in terms of efficiency can subsume several existing theories of quit-rate costs and supervision difficulties.

Section II then examines the functioning of the labor market in the short run. After using the correspondence principle to check stability, we show that decreases in the overall supply of labor, technological improvements, or increases in firms' capital stocks may well increase the permanent values of both U and W. These surprising results about the unemployment rate can be understood as follows. Suppose initially the labor market clears, each firm has adjusted its wage so that the marginal value of an increment to its wage premium just equals the marginal cost of the increment, and all chosen premiums are zero. Then let the marginal productivity of labor (measured in "effective" units) rise -- perhaps because of a sudden addition to every firm's capital stock. The change will cause W to be bid up, as is normally the case. If the supply of natural labor units is fixed so that firms cannot increase their hiring, however, the marginal cost to each firm of incrementing its wage premium will not have changed (the marginal cost will equal the magnitude of the firm's natural labor input), yet the marginal value of increasing the efficiency of existing workers will have risen.[3/] After the productivity change, therefore, firms will want to offer positive wage premiums to reap the benefits of the resulting efficiency gains. As all firms attempt to do so, W will be bid up above its market clearing level, causing U to rise.[4/]

Section III considers long-run developments. The model predicts that an increase in the average propensity to save or a decrease in the population growth rate will increase the steady-state values of both W and U. Section IV concludes the paper with an analysis of why our comparative-static results differ from those in other articles.

I. Labor Efficiency

We define the short-run profit function of each firm with the following expression:

$$\pi = f(k, n \cdot \Theta(w - W, U)) - n \cdot w \tag{1}$$

where f is the firm's production function, k is its (physical) capital stock, n is the number of natural units of labor input, $\Theta > 0$ gives the number of "effective" units of labor input per natural unit employed, w is the firm's wage rate per natural unit of labor, W is the average wage per natural unit of labor elsewhere in the economy, and U is the aggregate unemployment rate. We assume that k is fixed in the "short run," that the firm takes U and W as given, and that the production function displays constant returns to scale and is twice continuously differentiable, increasing in each argument, and strictly concave. For the sake of simplicity, we assume all firms produce the same output good, the good's price is normalized to 1, and all firms have the same production function, capital stock, and efficiency function Θ.[5]

We also assume that Θ is twice continuously differentiable; that Θ is an increasing function of a firm's proffered wage premium and the overall rate of unemployment,

$$\Theta_i(w - W, U) > 0 \text{ for } i = 1, 2; \tag{2}$$

and, that efficiency gains from increments to (w - W) and U taper off as the magnitude of either variable increases,

$$\Theta_{ij}(w - W, U) < 0 \text{ for } i, j = 1, 2. \qquad (3)$$

This section is devoted to motivating our use of Θ and the inequalities of lines (2) and (3). We use two alternative approaches to do that: a human capital model and a labor supervision model.

Human Capital Approach

Consider first the case of what Becker (1964) calls "specific human capital." We deal with the polar case of specific capital which is useful at a single firm. Although Becker shows that in practice both a firm and its employees will invest in such capital, we study only the investments of firms.[6/] To streamline the analysis, we use discrete time (here and in Section II) and assume that (specific) human capital investments take place at the beginning of each period, that the investment process is instantaneous, that human capital entirely depreciates at the end of a single period, and that a firm experiences a period's quits at the start of the period, immediately after human capital investments have been made.

Suppose the quit rate for each firm's labor force is $q(w - W, U) \varepsilon [0, 1]$ with q nonstochastic, twice continuously differentiable, and

$$q_i(w - W, U) < 0 \text{ for } i = 1, 2.$$

The empirical importance of quit rates is well known (see Holt 1970). Our derivative conditions imply that the firm can reduce its flow of quits by raising its wage offer and that quit rates will be high in a tight labor market -- when new jobs are presumably easily located. Because of imperfect information and worker and job heterogeneity, slight changes in w need not lead to enormous changes in q. This type of quit rate model is familiar from the work of Mortensen (1970), Phelps (1970), Stiglitz (1974), Calvo (1979), and Salop (1973, 1979).[7/]

We also let

$$q_{ij}(w - W, U) > 0 \text{ for } i, j = 1, 2.$$

These inequalities mean successive increments to a firm's wage premium are decreasingly effective at stemming quits, such increments are most effective in a tight labor market, and successive increments to the rate of unemployment have less and less impact on quit rates. The inequalities are consistent with the properties of the quit rate functions of Calvo (1979) and Salop (1979).[8/]

Suppose each firm's specific human capital investment per worker is an exogenously determined constant h -- we could think of h as hiring costs.[9/] Let the investment process consume only labor, and let h be measured in the same units as n. Then the firm's profit function is

$$\pi = f(k, n - n \cdot h/(1 - q)) - n \cdot w.$$

Setting

$$\Theta \equiv 1 - h/(1 - q),$$

we have line (1). Differentiating,

$$\Theta_i = -h \cdot q_i/(1 - q)^2 > 0 \text{ for } i = 1, 2$$

because of our assumptions about q_i. Thus, we can justify line (2). Taking second derivatives,

$$\Theta_{ij} = -\{(1 - q)^2 \cdot h \cdot q_{ij} + h \cdot q_i \cdot 2 \cdot (1 - q) \cdot q_j\}/$$
$$(1 - q)^4 \text{ for } i, j = 1, 2.$$

Since $q_{ij} > 0$ and $q_i < 0$ for i, j = 1, 2,

$$\Theta_{ij} < 0 \text{ for } i, j = 1, 2,$$

which is consistent with line (3).

A more sophisticated model makes worker productivity a function of firms' expenditures for training t. Suppose the training is firm specific in each case and that the magnitude of t is endogenous. We might write an individual firm's profit function as

$$\pi = f(k, n \cdot \alpha(t) - n \cdot t/(1 - q)) - n \cdot w$$

where

$$\alpha_1(t) > 0 \text{ and } \alpha_{11}(t) < 0,$$

so that increments of training increase worker productivity, but by diminishing amounts.

The new profit function is concave in t. If

$$t^* = t^*(w - W, U)$$

is the profit-maximizing choice of t, t* satisfies the first-order condition

$$\alpha_1(t^*) = 1/(1 - q).$$

Define

$$\Theta \equiv \alpha(t^*) - t^*/(1 - q)$$

in this case. Then we again have a profit function given by line (1). Differentiating Θ and using the first-order condition for t to simplify,

$$\Theta_i = \alpha_1 \cdot t^*_i - t_i^*/(1 - q) -$$

$$t^* \cdot q_i/(1 - q)^2 =$$

$$-t^* \cdot q_i/(1 - q)^2 > 0 \text{ for } i = 1, 2.$$

So, we have another justification for line (2).

Suppose $t^* > 0$ and $e(t) \equiv d\ \ln(\alpha_1)/d\ \ln(t) \leq -1/2$. (The latter will be true, for example, if $\alpha(t) = t^a$ with $a \leq 1/2$.) Then we can verify that line (3) holds as follows. Differentiating the first-order condition for t^*,

$$t^*_i = q_i/[\alpha_{11} \cdot (1 - q)^2] > 0 \text{ for } i = 1, 2.$$

Differentiating Θ_i,

$$\Theta_{ij} = -\{(1 - q)^2 \cdot (t^*_j \cdot q_i + t^* \cdot q_{ij}) +$$

$$t^* \cdot q_i \cdot 2 \cdot (1 - q) \cdot q_j\}/(1 - q)^4 =$$

$$-\{t^* \cdot q_{ij}/(1 - q)^2\} - \{q_i \cdot t_j^* \cdot$$

$$(1 + t^* \cdot 2 \cdot \alpha_{11} \cdot (1 - q))/(1 - q)^2\} =$$

$$-\{t^* \cdot q_{ij}/(1 - q)^2\} - q_i \cdot t_j^* \cdot$$

$$\{(1 + 2 \cdot e(t^*))/(1 - q)^2\} <$$

$$-t^* \cdot q_{ij}/(1 - q)^2 < 0 \text{ for } i, j = 1, 2.$$

Supervision Model

Using the ideas of Calvo (1979) (and Calvo and Wellisz (1978)) we can develop a different argument for lines (2) and (3): Suppose every employee's disutility per hour spent at work depends positively on the degree of effort he or she expends, and let each firm's production processes require supervised teamwork rather than piecework on an individual basis. Then if supervision occurs at random intervals (continuous supervision being prohibitively expensive), the amount of employee shirking will depend on the frequency of supervision, the degree of risk aversion of workers, and the severity of the punishment of those caught shirking. Suppose firing is one punishment. The level of U (measuring the difficulty of finding a new job) and the degree to which a firm's wage rate exceeds W may be the important determinants of the discomfort of getting fired. Then if

$$\gamma = \gamma(w - W, U, s)$$

(with s giving the ratio of supervision time to work time) registers labor efficiency, we expect (see Calvo (1979))

$$\gamma_i(w - W, U, s) > 0 \text{ for } i = 1, 2, 3.$$

For Calvo's model we also have

$$\gamma_{ij}(w - W, U, s) < 0 \text{ for } i, j = 1, 2.$$

With the new approach, each firm's profit function is

$$\pi = f(k, n \cdot \gamma - n \cdot s) - n \cdot w.$$

Provided supervision yields diminishing benefits -- $\gamma_{33} < 0$ -- the profit function is concave in s. Optimal supervision requires

$$\gamma_3 = 1.$$

Let

$$\Theta \equiv \gamma(w - W, U, s^*) - s^*$$

where $s^* = s^*(w - W, U)$ is the optimizing level of s. Then we again obtain lines (2) and (3):

$$\Theta_i = (\gamma_3 - 1) \cdot s_i^* + \gamma_i = \gamma_i > 0 \text{ for } i = 1, 2,$$

$$\Theta_{ij} = \gamma_{ij} < \text{ for } i, j = 1, 2.$$

The second general approach depends on having firing being one penalty for apprehended shirkers and on there being a need for supervision in the first place -- employees must desire to shirk, and piecework contracts must be infeasible. As in the specific human capital case, we find that lines (1) - (3) provide a convenient summary formulation.[10/]

II. Short-Run Equilibrium

We now turn our attention to the overall labor market. We assume (physical) capital stocks are fixed and each firm adjusts its natural labor input and wage rate to maximize its profit function (see line (1)). Because there are many firms in the economy, none worries about the effects of its decisions on its competitors' behavior. Although the nonpecuniary attributes of employment differ among firms, we assume worker tastes for such attributes differ symmetrically, so that all firms behave alike and none need offer compensating wage differentials (see Salop (1979, p. 121)). We will say the economy reaches an "equilibrium" when the wage rate each firm wishes to pay equals the average amount offered elsewhere (W) and when the amount of (natural) labor input each firm desires to hire equals $(1 - U) \cdot N$, where N is the amount each must hire if there is to be full employment.[11/] (Note that our concept of an "equilibrium" is nonstandard: our "equilibria" correspond to "rest" states for the economy, but not states in which unemployment is necessarily zero -- see the introduction to this paper.) In this section we examine the stability and determinants of such equilibria.

Firm Behavior

Each firm's goal is to solve

$$\underset{\{n,\ w\}}{\text{Max}} \{f(k,\ n \cdot \Theta(w - W,\ U)) - n \cdot w\} =$$

$$\underset{\{n,\ w\}}{\text{Max}} \{\pi(n,\ w)\}.$$

Maximizing with respect to n alone yields a necessary and sufficient condition:

$$f_2(k,\ n \cdot \Theta(w - W,\ U)) \cdot \Theta(w - W,\ U) = w. \tag{4}$$

If $n^* = n^*(w)$ satisfies (4), let $\pi^*(w) = \pi(n^*(w),\ w)$. Then

$$d\pi^*/dw = f_2 \cdot [n_1^* \cdot \Theta + n^* \cdot \Theta_1] -$$

$$[n_1^* \cdot w + n^*] = f_2 \cdot n^* \cdot \Theta_1 - n^*. \tag{5}$$

Let ε be the elasticity of worker efficiency with respect to w for each firm:

$$\varepsilon(w,\ W,\ U) = w \cdot \Theta_1(w - W,\ U)/\Theta(w - W,\ U).$$

Section I shows ε should be positive. Substituting from (4), line (5) yields

$$d\pi^*/dw = [\varepsilon(w,\ W,\ U) - 1] \cdot n^*(w). \tag{6}$$

Assuming there is a unique, "interior solution" (with $n^* > 0$), π^* will have a maximum at

$$w^* = w^*(W,\ U) > 0 \text{ such that}$$

$$\varepsilon(w^*(W,\ U),\ W,\ U) = 1. \tag{7}$$

For such a solution we also must have

$$\varepsilon_1(w^*(W,\ U),\ W,\ U) < 0. \tag{8}$$

We will assume line (8) holds from this point forward.

Notice the intuitive plausibility of condition (7); each firm will raise its wage offer w until the realized percentage gain in efficiency per worker exactly equals the percentage increase in cost per worker. Thus, each firm will adjust w to minimize the cost it pays per effective unit of labor.

The function w* gives the desired wage offer of an individual firm for each average wage W and aggregate unemployment rate U. To understand the shape of the graph of w*, define a new function

$$E(W, U) = \varepsilon(W, W, U).$$

E gives the elasticity of labor efficiency for the firm if it pays the average wage rate. Since $E(W, U) = (\Theta_1(0, U)/\Theta(0, U)) \cdot W$, E is linear in W, as shown in Diagram 1. Lines (2) and (3) show $U^* > U$ implies $E(W, U^*) < E(W, U)$ all $W > 0$.

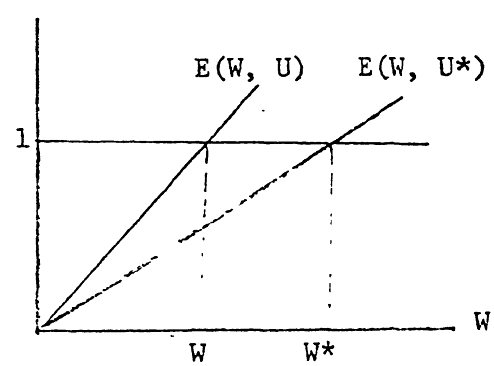

Diagram 1: Graphs of the function E if U* > U

Diagram 1 and line (8) enable us to draw the graph of w^*. At the value of W such that $E(W, U) = 1$, the graph of w^* must cross the 45-degree line shown in Diagram 2. Diagram 1 and line (8) show that to the right (left) of W the graph of $w^*(W, U)$ must lie above (below) the diagonal. Lines (2), (3), and (8) show the graph of $w^*(W, U)$ must lie above that of

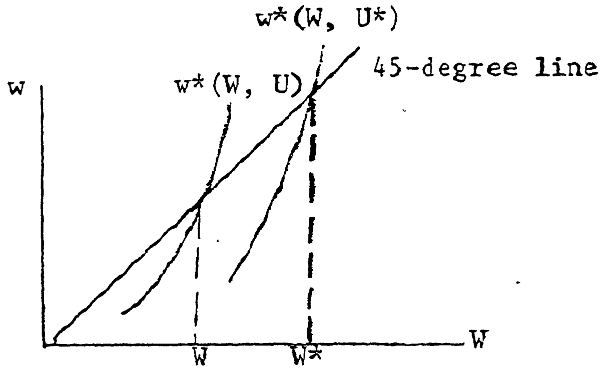

Diagram 2: The graph of w^* -- where $U^* > U$ and W and W^* refer to Diagram 1

$w^*(W, U^*)$ if $U^* > U$.

Equilibrium

In Diagram 2 the equilibrium wage rate associated with unemployment U is W: If the average wage is W, (given U) each firm will offer $w = W$, so that W will persist. Leaving aside stability conditions for the time being, we can derive sufficient conditions for the existence of a short-run equilibrium pair (W^E, U^E) -- a pair such that

$$w^*(W^E, U^E) = W^E, \tag{9}$$

$$n^*(W^E) = N \cdot (1 - U^E) \text{ --} \tag{10}$$

as follows.

One condition needed for $(W, U) = (W^E, U^E)$ is

$$E(W, U) = 1. \tag{11}$$

This follows from lines (7) and (9). Lines (4), (9), and (10) give

$$f_2(k, \Theta(0, U) \cdot N \cdot (1 - U)) \cdot \Theta(0, U) = W. \tag{12}$$

Combining lines (11) and (12), a necessary and sufficient condition for the existence of a pair $(W, U) = (W^E, U^E)$ is that there be a root $U = U^E \, \varepsilon [0, 1]$ for

$\Psi(U) = 1$ where $\Psi(U) \equiv$

$$f_2(k, \Theta(0, U) \cdot N \cdot (1 - U)) \cdot \Theta_1(0, U). \tag{13}$$

After solving for $U = U^E$, we can determine $W = W^E$ from line (11) (see Diagram 1).

None of our assumptions guarantee that equation (13) will have a root. If $\lim_{x \downarrow 0} f_2(k, x) = \infty$, Ψ will have a vertical asymptote at $U = 1$, as illustrated in Diagram 3. The graph of Ψ may or may not drop as low as 1 for some $U \varepsilon [0, 1]$; but, if it does, there will be at least one short-run equilibrium pair (W^E, U^E).

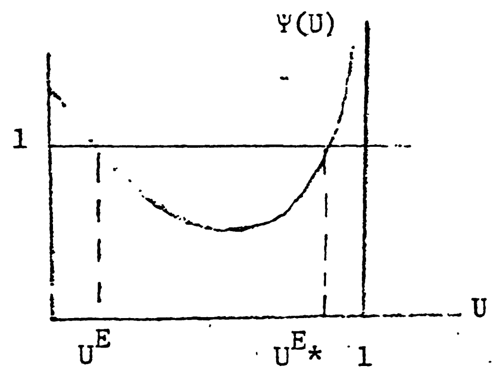

Diagram 3: The graph of Ψ

Stability

We now digress to investigate the stability of the two types of equilibria (U^E and U^E*) pictured in Diagram 3. Let W^E be the equilibrium wage rate that equation (11) associates with unemployment rate U^E, and let W^E* be the wage corresponding to U^E*.

Suppose that based on past experience firms set their wage offers at time t in such a way that W_t is the average wage rate. Assuming each firm sets $w_t = W_t$, we can attempt to solve line (12) for the aggregate unemployment rate, U_t. Suppose (12) associates a unique U_t, say

$$U_t = \sigma(W_t),$$

with each value of W_t in the immediate vicinity of W^E and W^E*. Wherever a single-valued σ exists, it is differentiable. At time t, $U_t = \sigma(W_t)$ gives the unemployment rate consistent with profit-maximizing hiring decisions on the part of all firms.

Given W_t and U_t, firms would all like to offer a wage rate $w*(W_t, U_t)$. Because wages at time t are already established, we assume[12/]

$$W_{t+1} = w*(W_t, U_t).$$

Thus we have a dynamic model of the functioning of the labor market: W_t determines the excess supply of labor at time t; given the magnitude of the excess supply, measured by U_t, if profitable price changes are possible, all firms make them at time t + 1. The difference equation

$$W_{t+1} = \Sigma(W_t) \equiv w*(W_t, \sigma(W_t)) \tag{14}$$

determines the evolution of W_t -- at least for the neighborhoods of W^E and W^E* in which σ is defined. An equilibrium value of W, W = W*, will be locally stable if

$$-1 < \partial\Sigma(W^*)/\partial W < 1. \qquad (15)$$

The dynamic model shows equilibria such as U^{E*} in Diagram 3 (ie, equilibria at which the graph of Ψ cuts the unit-high horizontal line while rising) are unstable. To see why, let

$$\Gamma(W) \equiv E(W, \sigma(W)).$$

Then lines (12) and (13) show

$$\Gamma(W) = \Psi(\sigma(W)). \qquad (16)$$

So,

$$\Gamma(W^{E*}) = \Psi(\sigma(W^{E*})) = \Psi(U^{E*}) = 1.$$

Let

$$\phi(U) \equiv \Theta(0, U) \cdot N \cdot (1 - U).$$

Computing the derivative of Ψ, we can see the only way that the graph in Diagram 3 can be rising at U^{E*} is if

$$\phi_1(U^{E*}) < 0.$$

Totally differentiating line (12), we find σ must be an increasing function if $\phi_1 < 0$. Thus, σ is an increasing function in the vicinity of W^{E*}. Hence, line (16) shows the graph of Γ must look as shown in Diagram 4.

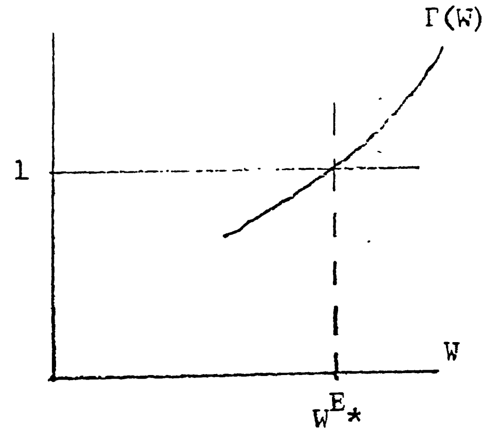

Diagram 4: The graph of Γ in the vicinity of W^{E*}

Suppose W_t is slightly larger (smaller) than W^{E*}. Let $U_t = \sigma(W_t)$. Then Diagram 4 shows $\Gamma(W_t) = \varepsilon(W_t, W_t, U_t) > (<) 1$. So, $W_{t+1} = w^*(W_t, U_t) > (<) W_t$ (see lines (7) and (8)). The adjustment process, therefore, will (perpetually) mov_ us away from W^{E*}. Hence, (U^{E*}, W^{E*}) is an unstable equilibrium. The opposite is true for (U^E, W^E) if inequality (15) holds.

Comparative-Static Analysis

Con_ider an increase in k, a decrease in N, or an improvement in technology that raises f_2 directly. The definition of ψ (see line (13)) shows that in any of these cases the Ψ-curve in Diagram 3 will shift upward. Confining our attention to a stable equilibrium U^E, we can see that the equilibrium rate of unemployment will increase. Condition (11) and Diagram 1 show that as U^E increases, W^E will increase as well. In comparing two countries (with closed economies), for instance, this analysis suggests the most capital-intensive economy may have both the highest short-run equilibrium average wage rate and the highest natural rate of unemployment. The reason is summarized in the introduction to this paper; Suppose we increase the function f_2 in one country. After the change, a unit increment to w will increase each firm's costs in the country by the same amount as before, but the (expected) efficiency gain stemming from a unit wage premium will now translate into a larger output increase. Thus, a higher f_2 will cause firms to bid up W^E to the point of increasing U^E.

III. A Simple Growth Model

We now add a linear saving function to our model and begin keeping track of capital accumulation. We assume that physical capital does not depreciate and that the natural labor force grows at a constant rate $M > 0$. For the sake of simplicity we will assume there is no technological change.

Steady States

Let $S \varepsilon (0, 1)$ be the average propensity to save for the economy as a whole. Since we will be interested only in sequences of short-run equilibrium states, let $\Theta_t = \Theta(0, U_t)$ all t. Because all firms have the same production function, output, capital stock, and labor input, we have the following equations:

$$\dot{k}_t = S \cdot f(k_t, \Theta_t \cdot N_t \cdot (1 - U_t)),$$

$$\dot{N}_t = M \cdot N_t.$$

Let

$$z_t = k_t/(\Theta_t \cdot N_t \cdot (1 - U_t)).$$

Then because f has constant returns to scale, we have a steady-state equilibrium for the economy if

$$S \cdot f(z^E, 1) = M \cdot z^E, \tag{17}$$

$$f_2(z^E, 1) \cdot \Theta(0, U^E) = W^E, \tag{18}$$

$$f_2(z^E, 1) \cdot \Theta_1(0, U^E) = 1 \tag{19}$$

where z^E, U^E, and W^E are steady-state values.

Because the only endogenous variable in line (17) is z^E, we can determine the steady-state capital-to-effective labor ratio from Diagram 5. We assume

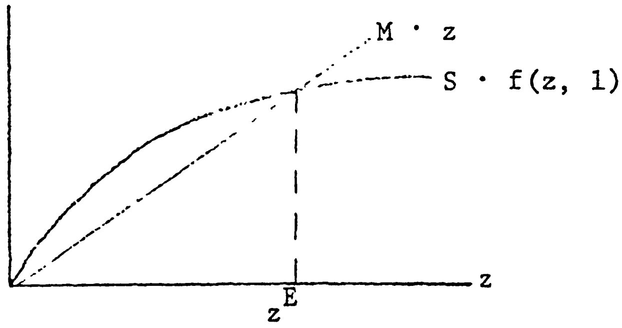

Diagram 5: The determination of z^E

$$\lim_{z \downarrow 0} f_1(z, 1) = \infty,$$

$$\lim_{z \uparrow \infty} f_1(z, 1) < M/S \tag{20}$$

for all values of S under consideration. Condition (20) and the concavity of f insure the existence of a unique z^E. After determining z^E, we can use line (19) to calculate U^E. As Diagram 6 illustrates, U^E will be unique, if it exists, because of line (3). A sufficient condition for the existence of a $U^E \varepsilon (0, 1)$ is

$$\Theta_1(0, 0) > 1/f_2(z^E, 1) > \Theta_1(0, 1). \tag{21}$$

Given z^E and U^E, line (18) defines W^E.

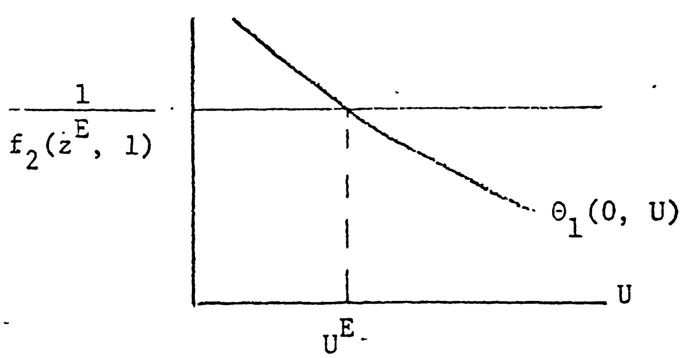

Diagram 6: The determination of U^E given z^E

Deriving stability conditions is an easy task if we assume that k_t changes slowly relative to W_t and U_t. For, provided the local stability condition of Section II (see line (15)) holds, the model will converge to short-run equilibrium values of W and U for each k_t -- in other words, short-tun adjustments will make equations (18) and (19) valid for each value of k_t. The equation for $\dot{k}_t$ and Diagram 5 show z_t will converge directly to z^E in the long run. Thus, the outcome (U^E, W^E, z^E) will be locally stable if line (15) holds.

Comparative-Static Results

Suppose we raise S. Then Diagram 5 shows z^E will rise. Since $f_{12} > 0$ (because of constant returns to scale and concavity), the horizontal line in Diagram 6 will fall, causing U^E to rise. Line (18) shows that W^E will rise as well. If we raise M, the same reasoning shows that z^E, U^E, and W^E will fall.

We can explain both comparative-static results at once: An increase in S or a decrease in M will raise z^E just as such changes would raise the steady-state capital-to-labor ratio in Solow's (1956) growth model. That, in turn, will raise the marginal product of labor for each firm. As shown in Section II, an increase in the marginal productivity of labor will stimulate firms to bid up the average wage rate. The outcome, as in Section II, will be increases in both U^E and W^E.

IV. Conclusion

We have constructed a model in which individual firms can increase the efficiency of their labor forces by offering wage premiums. We have argued that rises in the overall rate of unemployment will also increase the efficiency of those remaining on the job. Section I suggested justifications for the model based on "specific human capital" and supervision costs.

Section II showed that sudden improvements in technology or increases in the average capital-to-labor ratio would tend to raise the short-run equilibrium average wage and unemployment rates. Section III showed that an increase in society's average propensity to save or a decrease in the growth rate of the labor force would increase the steady-state values of W and U. Our results apply only for locally stable equilibria, and that limitation is particularly important in the case of our short-run outcomes.

We close by contrasting our analysis to existing papers. The first two sections of Stiglitz (1974) provide, perhaps the most illuminating comparison. Stiglitz bases his profit function for each firm on assumptions similar to those of our first human capital model in Section I. There is an important difference, however: Stiglitz's profit function is

$$\pi = f(k, n) - \lambda \cdot n, \tag{22}$$

$$\lambda \equiv w + h \cdot q. \tag{23}$$

In our model human capital is valued in labor units, so that the "cost" of a quit depends on f_2 and, hence, through first-order conditions, on w. Our approach is correct if human capital is constructed mainly from labor inputs, either in the form of personnel-staff time or lost production time on the part of workers being processed or trained. Stiglitz's model, on the other hand, makes human capital investment costs, h in line (23), entirely independent of wage rates. The consequences of this different formulation are pronounced.

Assuming the economy has reached an equilibrium (see lines (9) and (10)), the first-order conditions for each of Stiglitz's firms are

$$f_2(k,\ N \cdot (1 - U)) = W + h \cdot q(1,\ U), \tag{24}$$

$$W + h \cdot q_1(1,\ U) = 0 \tag{25}$$

where we use Stiglitz's quit function $q = q(w/W, U)$ in place of $q = q(w - W, U)$. Assuming the sign conventions of Section I remain valid, the locus of (U, W) points satisfying line (24) will be an upward sloping curve as shown by graph A in Diagram 7. Similarly, the locus of points satisfying line (25) will have the shape illustrated by graph B. The economy can be in equilibrium only when W and U lie at the intersection of the two curves.

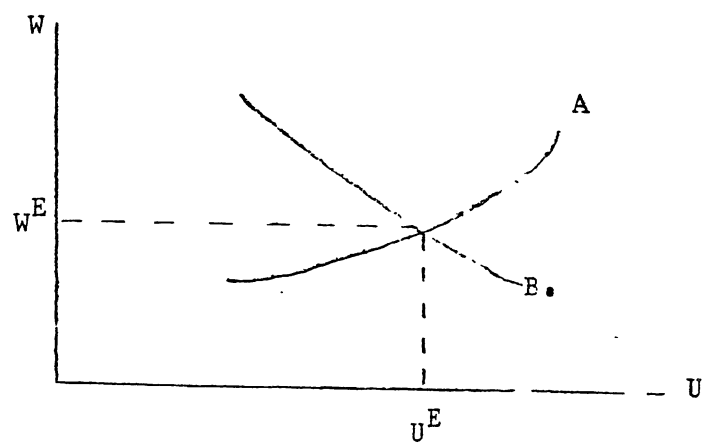

Diagram 7: Loci of points satisfying the first-order conditions of the Stiglitz model.

We cannot make a correspondence-principle argument ruling out a priori some categories of equilibria for the model of lines (22) - (25 ~~26~~) -- Diagram 7 shows only one type of equilibria exists. It is not diffucult to see that the comparative-static results for U of the Stiglitz model are opposites to ours: for example, if we raise k for each firm, the A-curve in Diagram 7 will shift upward and the B-curve will remain the same, so U^E will fall.

To illustrate the root of the difference in outcomes, change line (23) to

$$\lambda = w + w \cdot h \cdot q. \tag{26}$$

In line (26) the cost of human capital depends on the wage rate, as it does in our model. The first-order conditions for the model of lines (22) and (26) can be written

$$f_2(k, N \cdot (1 - U))/(1 + h \cdot q(0, U)) = W \tag{27}$$

$$1 + h \cdot q(0, U) + W \cdot h \cdot q_1(0, U) = 0 \tag{28}$$

where we have changed back to $q = q(w - W, U)$. For the new analogue of Diagram 7, line (27) will still produce an upward sloping curve such as A. The new B-curve, however, may assume any shape. The latter fact makes our comparative-static results possible. In Section II our analysis, in effect, shows only equilibria in Diagram 7 at which the B-curve crosses the A-curve with a steeper slope can meet necessary conditions for stability.

Salop (1979) uses a variation of the Stiglitz model in which h depends positively on n, but not w. Salop's comparative-static results resemble Stiglitz's, therefore, although they are somewhat more ambiguous.

Mortensen (1970) (see also Phelps (1970)) considers long-run equilibria and finds the natural rate of unemployment will vary positively with the rate of population growth. This contrasts with our result in Section III that $dU^E/dM < 0$. While Mortensen does not consider (physical) capital accumulation, however, our result stems from the effects of changes in M on the steady-state capital-to-labor ratio. Note, on the other hand, that the signs of the comparative-static results in Section III do not depend on our correspondence-principle arguments of Section II.

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FOOTNOTES

1. We do not discuss other components of the natural rate, such as unemployment due to job-search time -- see Salop (1979). We also do not cover unemployment stemming from insufficient aggregate demand.

2. Our outcomes are analogous, therefore, to those of the monopolistic competition model -- see Stiglitz's (1974) comments on this subject.

3. Although we assume the natural labor supply is inelastic within each period, our analysis would not be fundamentally changed if the total natural labor supply varied positively with W.

4. The underlying cause of this unemployment emerges in Section I: in practice firms and their employees cannot establish binding contracts specifying labor time, wages, and the quality of each laborer's effort. The problems associated with "specific human capital" investments in this regard are well-known, for instance. (See also Salop's (1979, p. 121) discussion of "quantity rationing as the clearing device in some markets....")

5. The importance of interfirm "nonpecuniary" differences becomes apparent below. Further comments about the differences appear at the beginning of Section II.

6. We could think of n as incorporating all efficiency gains from workers' human capital investments, although a more sophisticated model would make such investments endogenous.

7. See also Tobin's (1972) well-known comments on the inverse relationship of quit rates and U. Bardham (1979) uses a model similar to ours too, although his analysis incorporates assumptions specific to underdeveloped economies; our analysis is intended to deal mainly with developed economies.

8. They are also roughly consistent with the properties of Stiglitz's (1974) quit rate function.

9. Note that to avoid the complexities of dynamic maximization problems (see, for instance, Salop (1973)) -- complexities which seem to add little insight to the models in Sections II and III of this paper -- we should think of all labor being freshly hired each period if h stands for hiring costs.

10. Stiglitz (1975) suggests a specific model in which wage payments that are partly time rates and partly piece rates emerge: Suppose the output per time unit of each worker depends partly on his or her effort and partly on random factors; then if workers are risk averse, optimal employment contracts may include time-rate as well as piece-rate pay. In

such a situation, Stiglitz goes on to suggest that firms may have incentives to raise their time-rate wages above market-clearing levels in order to attract high quality workers. Thinking of w and W as time-rate wages, allowing quality differences among laborers, and assuming recruiting diffuculties are inversely related to U, we could motivate lines (1) - (3) from such a model as well as Calvo's.

11. We can set the units on both n and N to be manyears/year. Then U will stand for the percentage of unemployed people in the aggregate labor force.

12. Note that the length of the period between times t and t + 1 here need not be the same as the "period" used in Section I. Notice also that this subsection's step-by-step wage adjustments accompanied by instantaneous (intraperiod) quantity adjustments correspond to the stage-by-stage price adjustments of conventional Walrasian stability models.