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ABSTRACT: This paper analyzes marriage market equilibria when the gains from marriage result from joint consumption of household public goods. Assuming a class of utility functions which guarantee transferable utility within marriage, the paper proves that marriage markets will be characterized by positive assortative mating on income. A tendency for positive assortative mating on wages is also demonstrated, contrasting with Becker's predictions for marriages based on gains from specialization. The implications of the results for empirical analysis of household composition decisions are explored. An econometric technique is developed to deal with a wide class of problems in which the behavior of two agents depends on the dispersion in some characteristic.

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Introduction

Economists have increasingly recognized that decisions about household living arrangements, such as when and whom individuals marry, the age at which children leave home, and the sharing of living quarters by children and their elderly parents, should be explicitly modeled as choice variables in the analysis of many economic and demographic issues. Household composition may influence individual labor supply and consumption decisions because of direct effects on tax rates or eligibility for government transfer programs, or because of indirect effects on relative shadow prices due to joint consumption economies or returns to specialization. Analysts of government welfare programs are especially aware of the importance of household composition, and many have modeled household living arrangement decisions as one part of a complete set of joint decisions which simultaneously determine labor supply, program participation, and living arrangements.

As economists move in the direction of explicitly modeling joint decisions about household formation and internal household allocations, a number of difficult theoretical and empirical issues arise. This paper focuses attention on a relatively unexplored set of issues resulting from the role of household public goods in the determination of optimal living arrangements and consumption allocations. The economies in household size estimated in the household equivalence scale literature[1] suggest that some form of joint consumption economies are an important component of household consumption. This paper explores the implications of household public goods for the extent and direction of assortative mating in joint household formation. Although the theoretical results are developed for the specific case of marriage markets, the results are shown to have implications for all living arrangements decisions. The paper exploits a class of utility functions which allow transferable utility in the presence of a public good, making it possible to characterize the joint payoffs from all possible combinations of partners without making any assumptions about the intra–household division of utility or about the nature of within household bargaining. The properties of marriage assignments which are consistent with a competitive marriage market are derived and used to characterize the extent and direction of assortative mating when there exist household public goods. The results both extend and contrast with Becker's (1981) predictions about assortative mating, with the most important difference being a prediction of positive, rather than negative, assortative mating on wages.

An important implication of the paper's theoretical results is that dispersion in the characteristics of potential housemates will be an important determinant of the probability of marriage or the formation of other joint living arrangements. Specification of a simple empirical model demonstrates that serious econometric problems result from the role of dispersion in characteristics. The effects of dispersion will in general be difficult to estimate or control for in empirical work whenever the characteristics of potential housemates are unobserved for individuals who do not form joint households. The paper demonstrates that estimation techniques such as conventional multi–stage solutions to selectivity bias cannot be applied to cases in which the dispersion of characteristics are important. A general econometric solution to these problems is developed which explicitly accounts

[1] See, for example, Lazear and Michael (1980), Muellbauer (1977), and Pollak and Wales (1981).

for the unknown sample separation problem caused by dependence of behavior on the dispersion in unobserved characteristics.

The likelihood function for the general problem is derived, providing a simple demonstration that the parameters cannot be estimated using multi-stage single equation procedures, and demonstrating the potentially misleading nature of estimates based on linear parameterizations that do not recognize dispersion of characteristics. The procedure allows maximum likelihood estimation of structural parameters which have straightforward interpretations in a theoretical model of households formation based on public goods. The estimation strategy applies to to a wide class of problems in which behavior depends on the dispersion in characteristics among two agents, one of whom is observed only for particular values of some structural equation.

1. Household Formation and Marriage Market Equilibrium

Researchers on a wide variety of topics compare the economic behavior of married versus single young adults, intact elderly couples versus widows, or grown children living at home versus grown children living alone. While economists working in these areas have recognized the importance of making household living arrangements endogenous, there has been little theoretical guidance on appropriate empirical specifications. The empirical implications of endogenous household composition cannot be understood in the absence of theoretical guidance on the choice of household living arrangements.

Becker's (1973, 1974, 1981) seminal work on the economics of marriage and the family provides an important foundation for analysis of marriage markets and household formation. Becker's results include characterizations of the properties of marriage market equilibria, including predictions about assortative mating on spouses' traits. One of the limitations of Becker's approach to the economics of marriage is the inability to incorporate joint consumption economies into his analysis. Becker was able to derive simple characterizations of equilibrium assignments in a competitive marriage market by assuming a single composite commodity, a restriction which guarantees transferable utility between spouses. Gains to marriage in Becker's model result from specialization in home production and market work, with all inputs used to produce a single private good which can be divided in any way between spouses. Joint consumption is likely to be pervasive in households, however, and to be a critical determinant of the gains to marriage and the formation of joint households. The introduction of household public goods in any interesting way requires the existence of at least two goods, a modification which immediately complicates Becker's proofs of the properties of marriage market equilibria.[2]

Although public goods cannot be readily incorporated into Becker's analysis, some of the basic foundations of Becker's model of marriage markets based on comparative advantage are also useful for the analysis of a model based on household public goods. One central result draws directly on Koopmans and Beckmann's (1957) analysis of the problem of assigning firms to locations, which was

[2] Examples of the problems caused by introducing multiple goods in Becker's model of the household can be found in Bergstrom (1986) and Bernheim, Shleifer, and Summers (1985), both papers demonstrating problems in extending Becker's Rotten Kid Theorem to the case of multiple goods.

insightfully borrowed by Becker (1981) for analysis of the marriage market. A useful reformulation of this result is the following:

Proposition 1. *If there exists a set of strictly increasing weighting functions g_i such that $g_i(U_i)+g_j(U_j)$ equals some constant g_{ij} for all divisions of utility between potential partners i and j, then any assignment of partners which is in the core must maximize $\sum_{i\in S} g_i(U_i)$, where S is the set of all potential partners.*

Proof. Denote a permutation of males and females which maximizes $\sum_i g_i(U_i)$ by P^*, and arrange the spouses in P^* such that the ith male is married to the ith female, with corresponding utilities denoted by U_i^m and U_i^f, and with "weighting functions" g_i^m and g_i^f. Consider an alternative assignment $\hat{P}$ in which individuals receive utilities $\hat{U}_i^m$ and $\hat{U}_i^f$, with

$$\sum_{i\in M} g_i^m(U_i^m) + \sum_{i\in F} g_i^f(U_i^f) > \sum_{i\in M} g_i^m(\hat{U}_i^m) + \sum_{i\in F} g_i^f(\hat{U}_i^f), \tag{1}$$

where M and F are the sets of all males and females respectively. The alternative assignment $\hat{P}$ will be blocked by any male and female who would both receive higher utility married to each other in P^* than they receive in their marriages in $\hat{P}$, i.e. if

$$U_j^m > \hat{U}_j^m \quad \text{and} \quad U_j^f > \hat{U}_j^f \quad \text{for some} \quad j. \tag{2}$$

Given condition (1), it must be the case that there exists some j such that

$$g_j^m(U_j^m) + g_j^f(U_j^f) > g_j^m(\hat{U}_j^m) + g_j^f(\hat{U}_j^f). \tag{3}$$

If $g_j^m(U_j^m) + g_j^f(U_j^f)$ is constant for couple (j,j), implying that *weighted* utility is transferable one to one between spouses by transfers of money, it follows that if (3) is true, then there exists some division of utility such that

$$g_j^m(U_j^m) > g_j^m(\hat{U}_j^m) \quad \text{and} \quad g_j^f(U_j^f) > g_j^f(\hat{U}_j^f). \tag{4}$$

But since g_j is assumed to be increasing in U_j, it follows from (4) that (2) holds, and therefore that permutation $\hat{P}$ cannot be in the core. This establishes Proposition 1. The usefulness of the result will be seen below, where it will be shown that a particular class of utility functions will produce the conditions necessary for the proposition to hold.

In Becker's model there exists a single aggregate commodity Z_{ij} which is produced jointly by spouses i and j according to a household production function. Given no restrictions on the division of output withing marriage, the condition for optimal assignments in Becker's model is simply that aggregate output be maximized, a straightforward reformulation of the Koopmans and Beckmann assignment problem. Using this condition, Becker proves that positive assortative mating will characterize the efficient assignment of partners if

$$\frac{\partial^2 F(A_m, A_f)}{\partial A_m \partial A_f} > 0, \tag{5}$$

where $F(\cdot)$ is a household production function and A_m and A_f are the values for some quantitative trait for the male and female partners.[3]

2. Assortative Mating on Income in Public Good Households

If the gains from forming combined households are entirely the result of capturing pure public good economies, it is instructive to consider whether the optimal assignment of partners will be characterized by positive assortative mating on income or other characteristics. Becker (1981: 81) briefly mentions the implications of joint consumption in marriage assignments, suggesting that there should be positive assortative mating on preferences in such a case. The role of income *per se* is not discussed by Becker, however, and no formal analysis is provided of the role of joint consumption.

Extending Becker's analysis of marriage market equilibrium and assortative mating to the case of household public goods is difficult in the general case in which transferable utility is not imposed. Given any arbitrary utility functions, there will in general be no simple characterization of the payoff of a potential marriage that is independent of the distribution of utility within the marriage. If payoffs cannot be assigned to potential marriages independent of bargaining outcomes over the surplus generated by the marriage then the convenient properties of the Koopmans–Beckmann assignment problem cannot be exploited, and little can be said about marriage market equilibria. Some structure can be regained by imposing specific bargaining solutions, as in Rochford's (1984) proof of the existence of equilibrium under any symmetric pairwise bargaining solution. An alternative approach is taken in the analysis which follows. A class of utility functions is adopted which make it possible to recover transferable utility in the presence of public goods. As proven below, it is then possible to derive theoretical predictions about the nature of assortative mating in marriage markets in the presence of household public goods. The results provide important extensions to Becker's results and in some important cases provide contradictory theoretical predictions.

The fundamental assumption is that individuals have utility functions of the form shown by Bergstrom and Cornes (1981, 1983) to be necessary and sufficient to make the efficient allocation of public goods independent of the distribution of income. Specifically, individual i's utility is given by

$$U_i(Y, X_i) = A(Y)X_i + B_i(Y), \tag{6}$$

where Y is the level of a household public good which can be jointly consumed, and X_i is individual

[3] Proofs are given in Becker (1981: 70-72) and Sattinger (1975). Schirm (1986) demonstrates that the Becker result based on a single trait need not hold in general when there are multiple traits.

i's consumption of a private good.[4] Analysis of (6) reveals that the sum of the marginal rates of substitution of a group of individuals with this form of utility function depends only on the level of the public good Y and the aggregate quantity of the private good X. For a group of individuals who face the same price for the private good, it follows that the level of Y which satisfies the Samuelson condition for the efficient level of the public good depends only on total income and prices and is independent of the distribution of income. Utility of form (6) also implies that the sum of the utilities of any group of individuals sharing the public good Y is independent of the distribution of the private good. Utility is transferable across individuals at a one for one rate through transfers of the private good.

Suppose individuals i and j, with incomes I_i and I_j, combine into a single household, denoting aggregate household quantities by $I = I_i + I_j$ and $X = X_i + X_j$. Since utility can be transferred one to one between the two by allocations of the private good, it is efficient for the couple to maximize joint utility $U_i + U_j$. If the two face a budget constraint $pY + wX = I$, where p and w are the market prices of the public and private goods respectively, then substituting the budget constraint into the sum $U_i + U_j$, their joint utility can be represented as

$$Z_{ij}(Y, p, w, I) = U_i + U_j = A(Y)\left[\frac{I - pY}{w}\right] + B_i(Y) + B_j(Y). \tag{7}$$

The independence of the efficient level of public goods from the distribution of income (or utility) is demonstrated by the fact that income appears in (7) only in terms of the total quantity $I = I_i + I_j$. The household's equilibrium consumption of both the public and private good can be determined without resolving any bargaining problems over the distribution of utility.

Taking advantage of the transferable utility implied by (6), it follows from Proposition 1 that if utility is divided between spouses as the outcome of a competitive marriage market,[5] then all sets of marriage assignments which are in the core must have the property that they maximize $\sum_i U_i$ over all individuals i. The power of assuming utility of form (6) is that Becker's assumption of a single composite commodity can be relaxed and household public goods can be introduced without losing the transferable utility which makes the assignment problem tractable.[6]

In order to analyze the pure public good household, the household production function can simply be thought of as the joint utility function in (7). The traits to be considered are the two prospective partners' incomes, I_i and I_j. Since one partner's income is a perfect substitute for the other's, the restatement of Becker's condition (5) for the case of positive assortative mating on incomes is

$$\frac{\partial^2 V(I, p)}{\partial I^2} > 0, \tag{8}$$

[4] While utility function (6) is obviously restrictive, it does not impose that preferences are identical or homothetic and it allows quite general income and price elasticities which may differ greatly across individuals.

[5] Requiring that in equilibrium there cannot be two individuals who would both get higher utility married to each other than they receive from their assigned spouses.

[6] See Bergstrom (1986) for applications of this utility function to extensions of Becker's Rotten Kid Theorem.

where $V(\cdot)$ is the indirect utility function implied by (7), i.e. $V = \max_Y Z_{ij}(Y)$ subject to $wX + pY \leq I$. Using condition (8), the following proposition can be established.

Proposition 2. *If the efficient level of household public goods is independent of the distribution of income between spouses, then marriage market equilibrium will be characterized by positive assortative mating on spouses' incomes.*

Proof. To see whether condition (8) holds, note that (7) has been written to imply an unconstrained maximum in Y, with first order condition

$$\frac{\partial Z_{ij}(Y,p,w,I)}{\partial Y} = A'\left[\frac{I-pY}{w}\right] - \frac{p}{w}A(Y) + B_i' + B_j' = 0, \tag{9}$$

and second order condition

$$\frac{\partial^2 Z_{ij}}{\partial Y^2} = A''X + B_i'' + B_j'' - \frac{2p}{w}A' < 0, \tag{10}$$

where $A' = \frac{\partial A}{\partial Y}$, $A'' = \frac{\partial^2 A}{\partial Y^2}$, etc. Denote the left hand side of the inequality in (10) by D, and note that (9) implies Y as a function of I. By the implicit function theorem, the comparative static result for the effect of income on household demand for the public good is

$$\frac{\partial Y}{\partial I} = \frac{-A'}{D}. \tag{11}$$

Since $D < 0$ by (10), $\partial Y/\partial I$ and A' must always be of the same sign.

Differentiating the household indirect utility function with respect to income, the first derivative is

$$\frac{\partial V}{\partial I} = \frac{\partial Z_{ij}}{\partial Y}\frac{\partial Y}{\partial I} + \frac{\partial Z_{ij}}{\partial I} = A. \tag{12}$$

The first term in (12) disappears by first order condition (9), a standard implication of the envelope theorem. The second derivative, then, is simply

$$\frac{\partial^2 V(I,p)}{\partial I^2} = \frac{\partial Y}{\partial I}\frac{\partial A}{\partial Y} \tag{13}$$

By (11), the two terms in (13) will always be of the same sign, implying that $\partial^2 V/\partial I^2$ is unambiguously positive as long as the income elasticity of demand for the public good is not zero. Following Becker's theorem, then, since the indirect utility function is convex in income, positive assortative mating on income will always be optimal when the gains from combining result strictly from capturing public goods economies, given independence of the level of public goods from the distribution of household income.

3. Assortative Mating on Wages

The results above assume potential spouses bring some exogenous income endowment to the marriage and that they pay the same price for the private good. It is more interesting to consider the case in which spouses earn income at potentially different wages. In this case leisure can be defined as the private good and preferences can still be characterized by utility function (6). The analysis is now complicated, however, by the fact that the two spouses face different prices for the private good. The budget constraint is now

$$I_i + I_j + w_i(T - X_i) + w_j(T - X_j) = pY, \tag{14}$$

where w_i is the ith person's wage, T is the maximum potential labor supply, and X_i is the ith person's consumption of leisure. Unlike the case in which the private good can be purchased at a single price, utility under budget constraint (14) cannot be transferred between spouses one to one, since an hour of leisure for the lower wage spouse can only be converted into less than an hour of leisure for the other spouse. It is no longer the case, then, that joint utility for the couple is independent of the distribution of utility. Substituting budget constraint (14) into utility function (6), joint utility can now be written as

$$Z_{ij} = A(Y)\left[\frac{I - pY}{w_i} - X_i(\frac{w_i - w_j}{w_j})\right] + B_i(Y) + B_j(Y), \tag{15}$$

where I is the couple's full income, $I = I_i + I_j + T(w_i + w_j)$. Comparing (15) with (7), note the dependence of joint utility in (15) on the distribution of leisure between spouses. For any given level of utility, it is clear from (15) that joint utility is maximized by giving all leisure to the lower wage spouse. It is no longer true, then, that each party views maximization of joint utility as the desirable outcome.

It remains true, however, that the efficient level of the public good is independent of the distribution of income. Choosing Y to maximize one spouse's utility subject to a given level of utility for the other gives first order condition

$$A'[w_iX_i + w_jX_j] - pA(Y) + w_iB_i' + w_jB_j' = 0, \tag{16}$$

which can be shown to be the Samuelson efficiency condition when the private good is defined as money rather than as leisure.[7] Note that $w_iX_i + w_jX_j = I - pY$ is simply the total dollar value of leisure consumption in the marriage. It follows that (16), like the efficiency condition for the pure endowment case in (9), can be solved to give the efficient level of the public good independent of the distribution of the private good between the spouses.

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[7] Note that the marginal rate of transformation of money for the public good is the same for both spouses, even though the marginal rate of transformation of leisure for the public good differs between spouses.

The second order condition for the efficient level of Y is

$$A''(I - pY) + w_i B_i'' + w_j B_j'' - 2pA' < 0, \tag{17}$$

directly analogous to condition (10).

When individuals face different wages and view leisure as the private good in (6), the sum of utilities of person i and person j sharing a public good will depend on the distribution of leisure. The key to making further progress is to note that the *wage–weighted* sum of utilities is independent of the distribution of leisure, however. The wage–weighted sum of utilities can be written as

$$w_i U_i + w_j U_j = A(Y)[w_i X_i + w_j X_j] + w_i B_i(Y) + w_j B_j(Y). \tag{18}$$

Substituting from the budget constraint, the weighted sum reduces to

$$Z_{ij}^w = A(Y)[I - pY] + w_i B_i(Y) + w_j B_j(Y). \tag{19}$$

These results make it possible to prove the following proposition:

Proposition 3. *If utility is of form (6) and the benefits of marrying result entirely from sharing household public goods, then when wages differ across the population, marriage market equilibrium will be characterized by positive assortative mating on spouses' wages.*

Proof. Since the wage–weighted sum of utilities is constant for any i and j who marry, it follows from Proposition 1 that marriage assignments which are in the core must maximize the *weighted* sum of utilities, $\sum_{i \in S} w_i U_i$, where the weights are individual wages and S is the set of all potential spouses.[8]

The first order condition (16) for the efficient level of Y in a marriage in which spouses face different wages implies Y as an implicit function of w_i and w_j, with

$$\frac{\partial Y}{\partial w_i} = \frac{-(A'T + B_i')}{D_2}, \tag{20}$$

where A' and B_i' are defined as before, and where D_2 is the left hand side of (17) and is strictly negative by second order conditions. It follows, then, that $\partial Y / \partial w_i$ must be of the same sign as $A'T + B_i'$.

Since we have shown that the optimal assignment of spouses must maximize $\sum_i w_i U_i$, the condition which determines whether there will be positive or negative assortative mating on wage

[8] In general, when individuals face different prices of the private good, the analogous result is to weight utility functions by the individual prices of the private good. The previous results for the case of exogenous income endowments are a special case, since the price of the private good is the same for all partners and hence the equilibrium assignments maximize the unweighted sum of utilities.

rates depends on the cross–partial derivatives of $w_iU_i + w_jU_j$. Let $V_2 = \max_Y Z^w_{ij}(Y, p, w_i, w_j, I)$ subject to the budget constraint (14). The appropriate extension of Becker's theorem is that there will be positive assortative mating on wage rates if

$$\frac{\partial^2 V_2}{\partial w_i \partial w_j} > 0. \tag{21}$$

Differentiating V_2 with respect to w_i,

$$\frac{\partial V_2}{\partial w_i} = \frac{\partial Z^w_{ij}}{\partial Y}\frac{\partial Y}{\partial w_i} + \frac{\partial Z^w_{ij}}{\partial w_i} = AT + B_i, \tag{22}$$

using the fact that $\partial Z_{ij}/\partial Y = 0$ by first order conditions. The cross–partial derivative is

$$\frac{\partial^2 V_2}{\partial w_i \partial w_j} = \frac{\partial Y}{\partial w_i}[A'T + B'_i] \tag{23}$$

From (20), the two terms on the right hand side of (23) will always be of the same sign. This implies that $\partial^2 V_2/\partial w_i \partial w_j$ is unambiguously positive as long as the elasticity of demand for the public good with respect to wages is not zero. Following Becker's theorem, then, Proposition 3 is established. Positive assortative mating on wages will always be observed in equilibrium when the gains from combining result strictly from capturing public goods economies, given independence of the level of public goods from the distribution of household income.

Becker (1981) concluded from his analysis that negative assortative mating on wages should be observed in marriage market equilibrium.[9] The model developed here based on household public goods, however, predicts positive assortative mating on wages. The difference results from the different source of gains from marriage. When the gains result from specialization and comparative advantage, as in Becker, it will often be optimal to match a spouse who has high market wages with a mate who has low market wages and can specialize in home production. When the gains result from joint consumption of goods which are purchased in the market, it is always optimal to match spouses who have similar full incomes and therefore similar demands for the public good. Although the latter effect has been hypothesized in the literature on both marriage markets and local public goods, no formal proofs have been provided and no general conditions for the result established. This paper proves the existence of a kind of "Tiebout equilibrium" (Tiebout, 1956) in marriage markets by assuming a particular class of utility functions but without making any assumptions about intra–household distributions of utility or the form of within marriage bargaining.

[9] His theoretical results are actually ambiguous on this prediction, but imply negative assortative mating if the elasticity of labor supply with respect to spouse's wage is negative, a condition Becker argues will generally hold empirically.

4. Incorporating the Role of Comparative Advantage

The effects of household public goods can be combined with the effects of comparative advantage by making household full income a more general function of spouses' traits. Suppose that the household budget constraint is

$$F(a_i, a_j) = pY + wX, \tag{24}$$

where we return to the case in which there is a single price for the private good but make no assumptions about the relationship between traits and household income. Assuming that utility is still of form (6), budget constraint (24) implies transferable utility of the same form as the original model with exogenous income, since utility is linear in the private good. By the same logic as above, then, any two spouses assigned to each other will maximize joint utility and the equilibrium assignment of spouses will maximize $\sum_i U_i$. Replacing I with $F(a_i, a_j)$ in the expression for joint utility (7) and the first order condition (9), and noting that the second order condition (10) is unchanged, the effect of a_i on demand for Y is

$$\frac{\partial Y}{\partial I} = \frac{-A'}{D}\frac{\partial F}{\partial a_i}, \tag{25}$$

which, as above, implies that $\frac{\partial Y}{\partial I}$ and $A'\partial F/\partial a_i$ have the same sign.

Defining $V_3 = \max_Y Z_{ij}(Y)$ subject to (24),

$$\frac{\partial V_3}{\partial a_i} = \frac{\partial Z_{ij}}{\partial Y}\frac{\partial Y}{\partial a_i} + \frac{\partial Z_{ij}}{\partial a_i} = A\frac{\partial F}{\partial a_i}. \tag{26}$$

The cross–partial derivative is

$$\frac{\partial^2 V_3}{\partial a_i \partial a_j} = A'\frac{\partial Y}{\partial a_i} + A\frac{\partial^2 F}{\partial a_i \partial a_j} \tag{27}$$

The first term on the right hand side of (27) is unambiguously non–negative as proven above for the two previous cases. The term implies that for any trait which influences household full income, there will always be a force working toward positive assortative mating on that trait as long as there are household public goods. If the trait also has the property that the higher values of the trait lower the marginal product of the trait for the other member, then there may also be a force working in the direction of negative assortative mating, the case originally analyzed by Becker. This effect is captured in the second term in (27), which now has the possibility of being negative, capturing Becker's notion of the role of comparative advantage in household production. If the second term is positive, suggesting that the husband's and wife's traits are complementary, then the household production effect reinforces the tendency to match persons with similar full incomes caused by the presence of public goods.

5. Implications of Disparities in Traits for Empirical Analysis

In addition to their direct application to assortative mating in marriage markets, the above results have implications for the decision of whether or not to combine by any two individuals or households, independent of whether there is a complete "market" for household members. Suppose we are interested, for example, in the effect of individual full incomes on the decision by an elderly parent and a grown child about whether to form a joint household. Returning to the case in which individuals differ only in exogenous incomes and all goods can be purchased in the market, consider the effect of holding the total income of the parent and child constant while increasing the dispersion in their incomes. If individual preferences can be expressed as form (6), then total household utility when the parent and child combine is independent of the distribution of income. It follows that the gross benefit to combining is unaffected by an increase in the dispersion. But it follows directly from the convexity of the indirect utility function proven in (13) that the combined utility of the parent and child when living separately must *increase* as the dispersion in incomes increases. Holding total income of the potential members of a joint household constant, the net benefits of combining into a joint household are a strictly decreasing function of the absolute difference in their incomes, as long as the income elasticity of demand for the public good is not zero. The magnitude of the decline in the benefits of combining caused by an increase in the dispersion of incomes increases as the absolute value of the income elasticity of demand for the public good increases.

These theoretical results shed light on the appropriate specification of empirical models of marriage and joint household formation. In particular, they clarify the role of the incomes of potential household members and the importance of the distribution of income among these members. Let Z_i^* represent the net benefits for family i of forming a joint household with a given potential match. To take a concrete example for illustrative purposes, assume that family i refers to parents and that Z_i^* is the benefits from combining with a family headed by one of their grown children. For simplicitiy the analysis will consider the role of family incomes, but the model could be recast in terms of wages or other characteristics. Represent the benefits of combining as

$$Z_i^* = \beta_1' C_{1i} + \gamma_1(I_{1i} + I_{2i}) + \gamma_2|I_{1i} - I_{2i}| + u_i, \tag{28}$$

where I_{1i} is the natural log of the parents' family income, I_{2i} is the natural log of the child's family income, C_{1i} is a vector of characteristics of the parents, and u_i is a normally distributed disturbance term assumed to be uncorrelated with the other right hand side variables. The use of log income is a functional form assumption which will be seen below to simplify the solution to the inherent unobservables problem in (28). Rather than observing Z_i^* directly we observe the indicator variable Z_i,

$$Z_i = 1 \quad \text{if} \quad Z_i^* > 0$$

$$Z_i = 0 \quad \text{if} \quad Z_i^* \leq 0.$$

If both I_{1i} and I_{2i} are observed for a sample of families independent of their living arrangements, (28) could be estimated by a conventional probit. Estimates of γ_1 indicate the income effect on demand for privacy, with $\gamma_1 > 0$ if privacy is a normal good. The parameter γ_2 captures the effect of income disparities between the families on the benefits of extending. In the case of efficient households with utility functions of type (6), γ_2 will be negative if the gains to sharing a household result from joint consumption economies.

A standard problem of unobservables plagues direct estimation of (28). The income of the potential joint family is typically only observed for those families who choose to combine, a problem complicated by the non–linearity of (28). If I_{1i} and I_{2i} entered linearly into (28) the problem could be handled with a straightforward modification of Heckman's (1974, 1976) solution to the problem of unobserved wages for women who do not work. What is needed is a second structural equation describing I_{2i} as a function of characteristics of family i, say

$$I_{2i} = \beta_2' C_{2i} + v_i, \tag{29}$$

where C_{2i} is a vector of characteristics of family i and u and v are assumed to be bivariate normal with zero means and covariance matrix

$$\Sigma = \begin{bmatrix} \sigma_u^2 & \sigma_{uv} \\ \sigma_{uv} & \sigma_v^2 \end{bmatrix}.$$

If (28) could be reparameterized as

$$Z_i^* = \beta_1' C_{1i} + \alpha_1 I_{1i} + \alpha_2 I_{2i} + u_i, \tag{30}$$

and assuming variables exist to insure identification, consistent estimates could be obtained by full information maximum likelihood or by a three step procedure in which a reduced form probit is estimated for (28), a selection bias corrected OLS regression is estimated for (29), and predicted values of I_{2i} are used in a third stage probit for (28) (Heckman, 1976, Nelson and Olsen, 1978).

Structural equation (28) could be rewritten as (30) if the rankings of the two incomes were the same for all observations. If $I_{1i} > I_{2i}\ \forall i$, then $\alpha_1 = \gamma_1 + \gamma_2$ and $\alpha_2 = \gamma_1 - \gamma_2$. If $I_{1i} < I_{2i}\ \forall i$, then $\alpha_1 = \gamma_1 - \gamma_2$ and $\alpha_2 = \gamma_1 + \gamma_2$. It is worth noting that even if I_{1i} and I_{2i} were observed for all households, making probit estimation of (30) straightforward, the interpretation of the parameters α_1 and α_2 could be misleading. Consider a case, for example, in which $I_{1i} < I_{2i}\ \forall i$, and $\gamma_2 < \gamma_1 < 0$. This implies that $\alpha_1 > 0$ in (30), a result which might be interpreted as implying a positive income effect on the propensity to form joint households. The positive α_1 would actually reflect the fact that if $I_{1i} < I_{2i}$, holding I_{2i} constant and raising I_{1i} raises total income but also reduces the dispersion of incomes. If the effect of reduced dispersion is strong enough, the effect of raising I_{1i} could be positive, even though privacy is a normal good as indicated by $\gamma_1 < 0$.

In general, it will be inappropriate to enter the characteristics of individuals as separate linear variables if behavior depends on the dispersion in the values of the variables. The specific nature of the mis–specification caused by adopting this commonly used functional form can be seen in the likelihood function, which will be derived below. The problem with trying to rewrite (28) as (30) is that it will generally be unreasonable to assume that the ranking of I_{1i} and I_{2i} is the same for all households. If (30) is estimated on either a complete sample or by using parameter estimates from (29) to predict I_{2i} for non–joint families, there is no clear interpretation of α_1 and α_2 in (30) if the rankings of I_{1i} and I_{2i} vary across households. The estimation problem becomes more complicated since we now require not simply a prediction of the mean of I_{2i} but also a prediction of its ranking relative to I_{1i}. Since the ranking depends on the stochastic component of I_{2i}, a multistage procedure using reduced forms will not be sufficient to provide consistent estimates of the parameters.

The problem can be interpreted as unknown sample separation in the ranking of the two incomes for non–joint families. Let the indicator function δ_i denote the ranking of I_{1i} and I_{2i}, with $\delta_i = 1$ if $I_{1i} > I_{2i}$ and $\delta_i = 0$ if $I_{1i} < I_{2i}$. Rewrite (28) as

$$Z_i^* = \beta_1' C_{1i} + \alpha_1 I_{1i} + \alpha_2 I_{2i} + u_i \qquad \text{if} \quad \delta_i = 1 \tag{31}$$

$$Z_i^* = \beta_1' C_{1i} + \alpha_2 I_{1i} + \alpha_1 I_{2i} + u_i \qquad \text{if} \quad \delta_i = 0, \tag{32}$$

where $\alpha_1 = \gamma_1 + \gamma_2$ and $\alpha_2 = \gamma_1 - \gamma_2$. The likelihood function can be thought of as separating into four parts corresponding to the four combinations of Z_i and δ_i. Sample separation on Z_i is known for all observations and sample separation on δ_i is known conditional on $Z_i = 1$. If $Z_i = 0$, however, the value of δ_i is unknown, implying that it is unknown whether (31) or (32) is the correct linear structural equation for a given non–joint family. The problem is directly analogous to problems of unknown sample separation in market disequilibrium models and the solution takes a similar form.[10]

To see the form of the likelihood function, note first that for those with $Z_i = 1$, the probability that $Z_i^* > 0$ can be conditioned on the value of I_{2i} and therefore on the value of δ_i:

$$\begin{aligned} P[Z_i^* > 0 | \delta_i = 1] &= P[\beta_1' C_{1i} + \alpha_1 I_{1i} + \alpha_2 I_{2i} > -u_i | I_{2i}] \\ &= \frac{1}{\sigma_v} \phi \left[\frac{I_{2i} - \beta_2' C_{2i}}{\sigma_v} \right] \Phi \left[\frac{\beta_1' C_{1i} + \alpha_1 I_{1i} + \alpha_2 I_{2i} + \sigma_{uv}(I_{2i} - \beta_2' C_{2i})/\sigma_v^2}{s_{12}} \right] \end{aligned}$$

$$\begin{aligned} P[Z_i^* > 0 | \delta_i = 0] &= P[\beta_1' C_{1i} + \alpha_2 I_{1i} + \alpha_1 I_{2i} > -u_i | I_{2i}] \\ &= \frac{1}{\sigma_v} \phi \left[\frac{I_{2i} - \beta_2' C_{2i}}{\sigma_v} \right] \Phi \left[\frac{\beta_1' C_{1i} + \alpha_2 I_{1i} + \alpha_1 I_{2i} + \sigma_{uv}(I_{2i} - \beta_2' C_{2i})/\sigma_v^2}{s_{12}} \right], \end{aligned}$$

where ϕ and Φ are the density and cumulative respectively of the standardized normal distribution, and $s_{12} = [\sigma_u^2 - (\sigma_{uv}/\sigma_v^2)]^{1/2}$.

For observations for which $Z_i = 0$ we do not know whether the benefit of extending is described by regime (31) or (32). The reason that a standard multistage estimation strategy will not work is that it is not sufficient to simply predict an expected value for the unobservable I_{2i}. Estimation requires knowing whether $I_{2i} > I_{1i}$ for each observation, a condition which depends on the stochastic disturbance term v_i. For observations for which $Z_i = 0$, the value of I_{2i} is not observed, so

[10] See, for example, Goldfeld and Quandt (1975) and Maddala and Nelson (1974).

the likelihood function must be written as a function only of observables C_{1i}, C_{2i}, and I_{1i}, the parameters, and the stochastic disturbances u_i and v_i:

$$P[Z_i^* < 0] = P[\beta_1' C_{1i} + \alpha_1 I_{1i} + \alpha_2 \beta_2' C_{2i} < -(u_i + \alpha_1 v_i) \quad \text{and} \quad I_{1i} - \beta_2' C_{2i} > v_i]$$
$$+ P[\beta_1' C_{1i} + \alpha_2 I_{1i} + \alpha_1 \beta_2' C_{2i} < -(u_i + \alpha_2 v_i) \quad \text{and} \quad I_{1i} - \beta_2' C_{2i} < v_i].$$

Given the distributional assumption on the disturbances, this can be rewritten as a function of a standardized bivariate normal distribution, recognizing that v and $u + \alpha_1 v$ are jointly normal with means zero and covariance matrix Σ_1 and that v and $u + \alpha_2 v$ are jointly normal with means zero and covariance matrix Σ_2, where

$$\Sigma_j = \begin{bmatrix} \sigma_v^2 & \sigma_{uv} + \alpha_j \sigma_v^2 \\ \sigma_{uv} + \alpha_j \sigma_v^2 & \sigma_{3j}^2 \end{bmatrix} \quad (j = 1, 2),$$

with $\sigma_{3j}^2 = \sigma_u^2 + \alpha_j^2 \sigma_v^2 + 2\alpha_j \sigma_{uv}$. Transforming u and $v + \alpha_j v$ $(j = 1, 2)$ into standardized normal variables, the probability of observing non–joint families is

$$P[Z = 0] = \Phi(c_{2i}, d_{2i}, \rho_2) + \Phi(c_{1i}, d_{1i}, \rho_1),$$

where Φ is the standardized bivariate normal cumulative distribution. The first two arguments in $\Phi(\cdot,\cdot,\cdot)$ are upper limits of integration, the lower limits of integration are $-\infty$, and the third argument is the correlation between v and $u + \alpha_j v$, where $\rho_j = (\sigma_{uv} + \alpha_j \sigma_v^2)/(\sigma_v \sigma_{3j})$. The upper limits of integration are

$$\begin{aligned} c_{1i} &= \frac{-[\beta_1' C_{1i} + \alpha_2 I_{1i} + \alpha_1 \beta_2' C_{2i}]}{\sigma_{31}} \\ c_{2i} &= \frac{-[\beta_1' C_{1i} + \alpha_1 I_{1i} + \alpha_2 \beta_2' C_{2i}]}{\sigma_{32}} \end{aligned} \tag{33}$$

and

$$\begin{aligned} d_{1i} &= I_{1i} - \beta_2' C_{2i} \\ d_{2i} &= \beta_2' C_{2i} - I_{1i}. \end{aligned} \tag{34}$$

The log likelihood function for the entire sample of joint and non–joint families, then, is

$$
\begin{aligned}
&\log L = \\
&\sum_{\substack{Z=1\\ \delta=1}} \left[\log \phi \left(\frac{I_{2i} - \beta_2' C_{2i}}{\sigma_v} \right) + \log \Phi \left(\frac{\beta_1' C_{1i} + \alpha_1 I_{1i} + \alpha_2 I_{2i} + \sigma_{uv}(I_{2i} - \beta_2' C_{2i})/\sigma_v^2}{s_{12}} \right) - \log \sigma_v \right] \\
&+ \sum_{\substack{Z=1\\ \delta=0}} \left[\log \phi \left(\frac{I_{2i} - \beta_2' C_{2i}}{\sigma_v} \right) + \log \Phi \left(\frac{\beta_1' C_{1i} + \alpha_2 I_{1i} + \alpha_1 I_{2i} + \sigma_{uv}(I_{2i} - \beta_2' C_{2i})/\sigma_v^2}{s_{12}} \right) - \log \sigma_v \right] \\
&+ \sum_{Z=0} \log \left[\Phi(c_{2i}, d_{2i}, \rho_2) + \Phi(c_{1i}, d_{1i}, \rho_1) \right].
\end{aligned} \tag{35}
$$

The likelihood function (35) characterizes the information contributed by the four possible combinations of Z and δ, taking explicit account of the fact that the value of δ will be unobserved when $Z = 0$. As is customary, only the ratios β_1/σ_u, γ_1/σ_u, and γ_2/σ_u can be recovered, but these make it possible to test whether privacy is a normal good and the direction of the effect of dispersion in characteristics.

The empirical model described by (35) is applicable to a wide class of problems in which behavior depends on the dispersion in characteristics among two parties, one of whom is observed only for particular values of some structural equation. As the theoretical results of this paper demonstrate, dispersion in characteristics is likely to be an important determinant of household composition decisions. Researchers who compare the economic behavior of married versus single young adults, intact elderly couples versus widows, or grown children living at home versus grown children living alone must consider the role of endogenous household composition choices. While many researchers have recognized the importance of making household living arrangements endogenous, theoretical guidance on appropriate empirical specifications has been limited.

When dispersion in unobservable characteristics is important, as the theoretical results from this paper suggest will often be the case, econometric modeling becomes even more difficult than conventional problems with unobservables. The results derived here provide a general solution to these problems, permitting estimation of structural parameters which have straightforward interpretations in a theoretical model of households formed to capture public good economies.

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