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## STRATEGIC TRADE POLICY AND DIRECT FOREIGN INVESTMENT: TARIFFS VERSUS QUOTAS

by

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**Abstract.** This paper investigates the equivalence of tariffs and quotas when the market in question is imperfectly competitive and open to direct foreign investment. The absence of a foreign supply response under a quota, so critical to the analysis of the differential impact of tariffs and quotas under imperfect competition, is called into question by the potential occurrence of direct foreign investment. The paper proves the equivalence of optimal tariffs and quotas when the markets are oligopolistic and open to direct foreign investment.

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# Strategic Trade Policy and Direct Foreign Investment: Tariffs versus Quotas

James Levinsohn

## 1. Introduction

The equivalence of tariffs and quotas in a perfectly competitive world is well understood. It is by now equally well known that the presence of a monopoly may destroy the tariff-quota equivalence. While these are informative limiting cases, much international trade is conducted in markets that lie somewhere between the extremes of pure competition and monopoly. This trade is often characterized by large, frequently multinational, players who recognize their influence on markets and hence act strategically. Recent developments in trade theory have explored how trade policy works in this setting. This work is nicely surveyed in Grossman and Richardson (1985).

Most of the recent work on trade policy with imperfect competition has looked at various tariff-subsidy schemes. By concentrating on prices, as opposed to quantities, as policy instruments, researchers have made an important choice. While under perfect competition there will be an equivalent quota associated with any tariff, under imperfect competition this need not be the case. Bhagwati (1965) first noted this point. He showed that a tariff might dominate a quota or vice versa depending on the basis for comparison (same imports or same total consumption) and on where the monopoly was located. While the existence of a competitive fringe is an essential element of Bhagwati's model,<sup>1</sup> the asymmetric foreign supply response to a tariff and a quota is at the heart of the nonequivalence result. Under a tariff, the foreign producer can still increase output while under a binding quota, it cannot.

Krishna (1983) was among the first to investigate quotas in an oligopoly setting. Her analysis provided yet another example of nonequivalence between tariffs and quotas. Using a game theoretic framework in which firms played strategically against other firms but took government actions as given, Krishna showed that a quota might serve as a facilitating device while a tariff would not. She demonstrates that in a Bertrand duopoly setting, each firm would like to raise its price if it

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This paper is based on my Ph.D. dissertation at Princeton University. I have benefitted from the insightful comments and suggestions of Avinash Dixit and Gene Grossman to whom I am grateful.

<sup>1</sup> If a monopolist does not face a fringe of competitive suppliers, tariffs and quotas will be equivalent. That is, if the monopolist is a world, as opposed to domestic, monopolist, Bhagwati's result does not hold.

were sure that its rival would do likewise. Under a quota, it is irrational for a firm to lower its price in order to gain market share since quotas would bind. Quotas, then, allow firms to credibly precommit to higher prices. Like Bhagwati's results, Krishna's exploit the lack of a foreign supply response imposed by a quota but not by a tariff.<sup>2</sup>

In Krishna's introduction, she writes:

Most of the literature has dealt with the two polar cases of monopoly and perfect competition, neglecting the strategic interaction crucial to the analysis of oligopoly. Such interaction between firms is a dominant feature of many markets, especially in some international markets in which large multinationals operate.

Her work, though, like much of the analysis of tax based policies, ignores the possible multinational aspects of the game firms play. In this paper, I consider the tariff quota equivalence question in a setting which explicitly accounts for the possibility of direct foreign investment (dfi).

The absence of a foreign supply response under a quota, so critical to the analysis of the differential impact of tariffs and quotas under imperfect competition is called into question by the potential occurrence of dfi. The credible precommitment permitted by a government imposed quota is crucial to Krishna's analysis. Yet when the possibility of dfi exists, the key assumption of no foreign supply response to a quota may no longer hold. While a foreign producer may not be able to increase local exportable output under a binding quota, the firm may produce the additional output in the home country. In this case, a quota is no longer a facilitating device, and many of the strategic interactions that differentiate a quota from a tariff disappear.

The rest of this paper is organized as follows. Section 2 presents a diagrammatic analysis of tariff-quota comparisons in the presence of dfi. This section provides a useful taxonomy for the general model of Section 3. Section 2 also provides some intuition about the results of Section 3. In that section, a general oligopoly model with dfi is developed. The generality extends to the number of firms and their mode of conduct. In this setting, the tariff-quota nonequivalence of Bhagwati and Krishna is reconsidered. In particular, the optimal profit-shifting tariff is compared to the optimal profit-shifting quota. Section 4 entertains the notion that trade policy might be motivated by considerations of domestic employment. Allowing this to be the case, equivalence results are re-examined. Section 5 relaxes assumptions about firms' cost functions and reconsiders the results of Section 3. A brief summary is given in Section 6.

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<sup>2</sup> Sweeney (1985) recently studied tariffs and quotas in a conjectural variations oligopolistic setting. He noted that quotas in effect change a firm's conjectures in a way that tariffs do not. This is because the rational firm's conjectures will be conditioned on the type of policies its competitors face. While there are problems in trying to model a sequential and hence dynamic process using conjectural variations, the underlying idea of the CV approach is the same as Bhagwati's and Krishna's. That is, there is no foreign supply response to a binding quota while such a response exists for a tariff.

## 2. A Diagrammatic Analysis of Tariff Quota Comparisons.

Consider a market in which a foreign monopolist is the sole supplier of a good in the home-country market.<sup>3</sup> Figure 1 represents the market for the foreign good in the usual space of own price and quantity. The foreign monopolist faces the demand schedule  $D$  and the corresponding marginal revenue schedule  $MR$ .

In Figure 1, it is well known that the optimal tariff is positive, as the home country can improve its welfare by extracting monopoly rents from the foreign country in excess of the lost consumer surplus.<sup>4</sup> This result requires only that  $MR$  be more steeply sloped than the linear  $D$ . It is noteworthy that some positive tariffs are welfare improving. Since it is rational to impose such a tariff, (non)equivalence results are meaningful in a way they might not be if the imposition of a tariff was itself economically irrational. For example, the equivalence of tariffs and quotas in the perfectly competitive paradigm begs the question of why the tariff or quota is present in the first place.

Initially, the foreign firm produces only in the foreign country (this will be referred to as domestic production) at constant marginal cost  $MC^{dom}$ . The assumption of constant marginal cost is not incidental. While it simplifies the analysis, it is also the logical choice. If marginal cost were increasing, one might ask why additional plants, foreign or domestic, do not already exist. (This analysis, like its predecessors, glosses over the implications of large fixed costs.) If marginal costs were decreasing, a new set of issues for strategic trade policy arises.<sup>5</sup>

Now let us introduce the possibility of dfi by the foreign firm in the home country. Dfi production is assumed to be a perfect substitute for non-dfi production. For example, the consumer is assumed to be indifferent between a Honda Accord built in Ohio and the same model manufactured in Japan. The marginal cost of production under dfi,  $MC^{dfi}$ , also is assumed to be constant.  $MC^{dfi}$  must be at least as high as  $MC^{dom}$ . If this were not the case, cost minimization would preclude any domestic production. The optimal rent-extracting tariff may be either at least as large as or smaller than the difference in marginal costs. I compare tariffs and quotas for each case.

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<sup>3</sup> As stated in note 1, the optimal tariff and optimal quota are equivalent in this case when dfi is not possible, since there is no competitive fringe and the monopolist is a global, as opposed to domestic, monopolist. Nevertheless, this framework provides a very useful taxonomy for the more general oligopoly model of Section 3.

<sup>4</sup> This result and related others have been demonstrated in a series of papers by Brander and Spencer.

<sup>5</sup> Krugman(1984) has shown that with increasing returns to scale, an advantage given to a firm in one market via trade or industrial policy may spill over into advantages in another market. Introducing direct foreign investment into this scenario is an interesting problem, but it obscures the more basic issues this paper addresses.

With inverse demand represented by  $p = a - bx$  and the constant marginal cost by  $c$ , the optimal rent seeking tariff absent the possibility of dfi is given by  $\frac{a-c}{3}$ . The calculations for this linear case are given in Appendix A. This tariff and the difference between  $MC^{dfi}$  and  $MC^{dom}$  are exogenous to the policymaker. Whether or not the optimal tariff is greater than the difference in marginal costs is a function only of tastes and technologies.<sup>6</sup>

Case 1: *The optimal tariff, neglecting the possibility of dfi is at least as large as the difference in marginal costs.* This corresponds to a quota of at most  $Q^*$ . This case is given in the figure by a tariff equal to EH. In the absence of dfi, the quota yielding the same level of imports is given by  $\bar{Q}$ . Now introduce the possibility of dfi at  $MC^{dfi}$ . Dfi constrains the optimal tariff to a level of AP.<sup>7</sup> Any larger tariff induces dfi and no revenue is raised. Some semantic clarification is useful. A tariff of EH which was optimal before the possibility of dfi is termed the no-dfi optimal tariff. The tariff which is optimal after dfi is introduced is termed the cum-dfi optimal tariff.

There is no cum-dfi tariff which will ensure imports of  $\bar{Q}$  while a quota does just that. While this is an obvious nonequivalence, it is not a very meaningful one, since a quota of  $\bar{Q}$  in the presence of dfi is not optimal. At a quota of  $\bar{Q}$ , the foreign producer faces a kinked MC schedule given by AEKLS. Quota licence revenue is given by APKE. While imports equal  $\bar{Q}$ , the amount KL is produced as dfi in the home country. Total consumption of the good is  $Q^*$ . Home welfare is given by consumer surplus and licence revenue. This is area MNR plus PKEA. A quota of  $Q^*$ , which corresponds to the cum-dfi optimal tariff, yields the same consumer surplus but gives strictly greater licence revenue by amount KLRE. The cum-dfi optimal tariff is equivalent to the cum-dfi optimal quota.

Simple jumping of a quota such as  $\bar{Q}$  leads to dfi concurrent with home production even though marginal costs are constant. Without quantity restrictions, such behavior is incompatible with standard cost minimization. Seen from a different angle, the very presence of dfi as quota jumping is evidence that the quota is set at a suboptimal level.

With imperfect competition, dfi may be welfare worsening, as it may undermine optimal rent extracting trade policies.<sup>8</sup> This may be seen in figure 1. Without dfi and an optimal tariff equal to EH, welfare is given by tariff revenue ZHEA plus consumer surplus RGF. With dfi, the optimal

<sup>6</sup> As section 3 will prove, the linearity of this example is not necessary to the argument presented here. Even in a general model, the optimal policies will be functions of only tastes and technology.

<sup>7</sup> Actually, the optimal tariff is constrained to a level of AP minus epsilon. Throughout the analysis, the open set aspect of the optimal tariff will be ignored.

<sup>8</sup> A similar point has been made in a perfectly competitive model by Grossman(1984).

tariff yields revenue PLRA and consumer surplus RNM. If area ZHKP is greater than areas KLRE plus GFMN, dfi has been welfare worsening. For a quota set at  $\bar{Q}$ , the relevant comparison is GFMN versus ZHKP. If the former is smaller, dfi is again welfare worsening. Although it is not obvious from the geometry, a revealed-preference argument shows that dfi is welfare worsening since the cum- dfi policies were feasible but not chosen when selecting the no-dfi optimal policies.

Case 2: *The optimal tariff is less than the difference in marginal costs.*

For quantity setting, this corresponds to an optimal quota of between  $Q$  and  $Q^*$ . In this case the usual tariff-quota nonequivalence results hold except that the marginal cost of dfi imposes an upper bound beyond which a quota will provoke a supply response in the form of dfi. If the cost function of dfi is similar to the cost function of domestic production, the difference in marginal costs will be small and it becomes less likely that case 2 will obtain.

### **3. A More General Analysis of Tariff-Quota Comparisons.**

The previous section demonstrated that in a fairly specific setting, if the no-dfi optimal tariff exceeded the difference in local and dfi marginal costs, then the introduction of dfi made the optimal rent-extracting tariff and the optimal rent-extracting quota equivalent policies. In section 2, there was a foreign monopolist facing a linear demand schedule. In this section, I generalize the diagrammatic results by considering a differentiated-product oligopoly in which firms face general demand schedules and market conduct is not restricted. This latter generalization is of some significance, as Eaton and Grossman (1986) have shown that choice of market conduct is often key to characterizing optimal policies.

I consider tariff-quota equivalence in the presence of dfi when firms compete only in the home market. The restrictive assumption that is in part carried over from Section 2 is that marginal costs must be constant in the neighborhood where production actually occurs. The reasons are the same as before. With upward sloping marginal costs, one must consider why another plant does not already exist. (The ramifications of upward sloping marginal costs are discussed in Section 5.) With downward sloping marginal costs, an entirely new set of issues arises and these may obscure the original intent of the analysis.

The more general setting described above is of more than just theoretical interest as it describes the U.S./Japanese automobile market fairly accurately. U.S. and Japanese cars are certainly differentiated products. They are produced with constant marginal costs by a small number of very large firms in each country. These firms compete almost solely in the U.S. market. Furthermore, the market has been subject to tariffs and, more recently, quotas. Completing the picture, dfi in the U.S. by Japanese firms is a rapidly growing phenomenon.

*The Set Up:*

It is useful to establish some notation at the outset. Let:

$n_1$  = number of domestic firms.

$n_2$  = number of foreign firms.

I consider a symmetric market structure in which firms within a country are identical. Dfi production represents an increase in output but it is not an increase in the number of firms.

Output is given by:

$q_1$  = output of a firm in industry 1.

$q_2$  = output of a firm in industry 2.

$Q_2 = q_{2d} + q_{2f}$  where  $q_{2d}$  is home production by foreign firms and  $q_{2f}$  is dfi production.

Let:

$$Q_1 = n_1 q_1$$

$$Q_{2f} = n_2 q_{2f}$$

$$Q_{2d} = n_2 q_{2d} \text{ and } Q_{2f} + Q_{2d} = Q_2$$

Cost functions are:

$$C^1 = c(q_1)$$

$$C^2 = c(q_{2d}, q_{2f})$$

*Assumption 1:* Marginal costs, denoted  $c_1, c_{2f}$ , and  $c_{2d}$ , are constant and  $c_{2f} > c_{2d}$ .

Let  $v_{ij}$  be firm  $j$ 's conjecture of firm  $i$ 's response to a change in its own quantity ( $i, j = 1, 2$ ). The conjectural variations (CV) parameter is used in this context as a flexible parameter which can represent myriad market conducts. It is no more than a convenient parameterization.

The home country's utility function is given by:  $U = U(Q_1, Q_2)$  Inverse demands are:

$$\frac{\partial U}{\partial Q_1} = P^1(Q_1, Q_2) \quad \text{and} \quad \frac{\partial U}{\partial Q_2} = P^2(Q_1, Q_2).$$

Subscripts on  $P^1$  and  $P^2$  will denote partial derivatives.

Before dfi is introduced, the home country is able to freely set a specific tariff  $t$  on imports and a specific subsidy  $s$  on home production. The optimal policies for this standard no-dfi case are derived in Appendix B. Finally, I make the following two assumptions.

*Assumption 2:* The no-dfi optimal tariff is profitably jumped. (This corresponds to Case 1 of the diagrammatic analysis.)

*Assumption 3:* Home welfare is continuous and strictly concave in quantities.



*Proof of Tariff Quota Equivalence:*

I prove that the optimal tariff and quota are equivalent in two steps. In step 1, I characterize the optimal tariff and subsidy combination. It is important to consider subsidy schemes. Because the no-dfi optimal tariff is, by assumption 2, profitably jumped, the no-dfi optimal tariff raises no revenue. In this sense, dfi constrains tariff setting to a level no greater than the difference in foreign marginal costs. With two quantities involved,  $Q_1$  and  $Q_2$ , a first best solution will usually require two unconstrained policy tools. As the cum-dfi optimal tariff is exogenously constrained, the subsidy may capture some of the foreign oligopoly rents that an unconstrained optimal tariff would have captured. The approach here follows Dixit (1983 and 1984).

In step 2, I characterize the optimal quota and subsidy combination. Dfi does not constrain quota setting as it constrains the optimal tariff. Hence the home country has an unconstrained policy tool for each quantity involved. I prove that the ability to set a binding quota has no value in terms of home country welfare. Furthermore, the solution to the optimal tariff/subsidy scheme will always be the solution to the optimal quota/subsidy scheme. This equivalence will be shown to be independent of the number of firms on either side of the market and independent of the mode of market conduct.

Appendix C extends the equivalence by proving that the optimal tariff and quota are equivalent when no subsidy is available. This result requires an assumption slightly stronger than Assumption 3.

*Step 1.* Here I characterize the optimal tariff subsidy scheme. Firms choose outputs strategically but take government policies as given. The government maximizes national welfare conditional on firms' profit maximization.

I first consider firms' profit maximizing behavior. By assumption 2, the cum-dfi optimal tariff must be no greater than  $c_{2f} - c_{2d}$ . (Were it greater, no revenue would be raised.) Because marginal costs are constant by assumption 1, cost minimization precludes dfi concurrent with domestic production. Thus  $q_2 = q_{2d}$ .

Firms' profit functions are given by:

$$\pi_1 = P^1(Q_1, Q_2)q_1 - C^1(q_1) + sq_1 \quad (1)$$

$$\pi_2 = (P^2(Q_1, Q_2) - t)q_2 - C^2(q_2) \quad (2)$$

An interior solution to an individual firm's profit maximization implies:

$$\mu^1 = \frac{\delta \pi_1}{\delta q_1} = 0 = P^1(Q_1, Q_2) + s - c_1 + q_1(P_1^1(Q_1, Q_2)g_0 + P_2^1(Q_1, Q_2)g_1) \quad (3)$$

$$\mu^2 = \frac{\delta\pi_2}{\delta q_2} = 0 = P^2(Q_1, Q_2) - t - c_{2d} + q_2(P_1^2(Q_1, Q_2)h_1 + P_2^2(Q_1, Q_2)h_0) \quad (4)$$

$$\text{where: } g_0 = [1 + (n_1 - 1)v_{11}]$$

$$g_1 = n_2v_{21}$$

$$h_0 = [1 + (n_2 - 1)v_{22}]$$

$$h_1 = n_1v_{12}$$

These  $g$  and  $h$  terms reflect the mode of market conduct and the number of firms. They are treated as constants although making them functions of quantities does not affect any of the equivalence results.

Some comparative statics analysis on firms' profit maximization will be useful for the later welfare analysis. It will be helpful to introduce some new notation. Arguments of partial derivatives of the inverse demand functions will be omitted for brevity. Let:

$$\begin{aligned} \Omega_1(Q_1, Q_2) &= \frac{P_{11}^2 h_1 + P_{12}^2 h_0}{n_2} & \Omega_3(Q_1, Q_2) &= \frac{P_{12}^1 g_0 + P_{22}^1 g_1}{n_1} \\ \Omega_2(Q_1, Q_2) &= \frac{P_{12}^2 h_1 + P_{22}^2 h_0}{n_2} & \Omega_4(Q_1, Q_2) &= \frac{P_{11}^1 g_0 + P_{12}^1 g_1}{n_1} \end{aligned}$$

This set of terms is related to the degree of concavity or convexity of the inverse demand functions. With linear inverse demands, these terms all become zero. A second set of terms deals with conjectures aggregated to an industry level. These terms are:

$$R_1(Q_1, Q_2) = \frac{P_1^1 g_0 + P_2^1 g_1}{n_1} \quad \text{and} \quad R_2(Q_1, Q_2) = \frac{P_1^2 h_1 + P_2^2 h_0}{n_2}$$

Using this notation, firms' first order conditions simplify to:

$$P^1 + Q_1 R_1 - c_1 = -s \quad (3')$$

$$P^2 + Q_2 R_2 - c_{2d} = t \quad (4')$$

Conditional on assumption 2, equations (3') and (4') implicitly define a one to one mapping between policy tools  $s$  and  $t$  and quantities  $Q_1$  and  $Q_2$ . As policy tools change, firms respond by adjusting quantities. This relationship is given by totally differentiating (3') and (4') to give the below system:

$$\begin{pmatrix} P_1^1 + R_1 + Q_1 \Omega_4 & P_2^1 + Q_1 \Omega_3 \\ P_1^2 + Q_2 \Omega_1 & P_2^2 + R_2 + Q_2 \Omega_2 \end{pmatrix} \begin{pmatrix} dQ_1 \\ dQ_2 \end{pmatrix} = \begin{pmatrix} -ds \\ dt \end{pmatrix} \quad (5)$$

Conditional on assumptions 1 through 3 and firms' profit maximization, the home country sets  $s$  and  $t$  to maximize home welfare. Home welfare is given by consumer surplus, domestic profits, and net trade taxes. Hence:

$$\begin{aligned} W &= U(Q_1, Q_2) - P^1 Q_1 - P^2 Q_2 + n_1 \pi_1 + t Q_2 - s Q_1 \\ &= U(Q_1, Q_2) - P^2(Q_1, Q_2) Q_2 - c_1 Q_1 + t Q_2 \end{aligned} \quad (6)$$

This implicitly defines welfare as a function of  $s$  and  $t$ . That is:

$$W = f(Q_1(s, t), Q_2(s, t), t) \quad (7)$$

I proceed by characterizing the optimal policy pair  $(s^*, t^*)$ . The home production subsidy is not constrained by the potential of dfi. Hence,  $s$  is set such that :

$$\frac{\delta W}{\delta s} = 0 \quad \text{for any tariff } t \quad (8)$$

The tariff, though, is constrained in that any tariff greater than the difference in marginal costs induces dfi and hence raises no revenue.

*Lemma 1: The optimal cum-dfi tariff is the largest tariff that does not induce dfi. Hence  $t^* = c_{2f} - c_{2d}$ .*

*Proof:* See Appendix D.

Equation (8) implicitly defines the optimal subsidy coupled with the cum- dfi optimal tariff. Using  $t = c_{2f} - c_{2d}$  and substituting (3') into (8) yields:

$$\frac{\delta W}{\delta s} = 0 = [-s - Q_1 R_1 - P_1^2 Q_2] \frac{\delta Q_1}{\delta s} + [c_{2f} - c_{2d} - P_1^2 Q_2] \frac{\delta Q_2}{\delta s}.$$

Using system (5) to solve for  $\frac{\delta Q_1}{\delta s}$  and  $\frac{\delta Q_2}{\delta s}$  and solving for  $s$  gives:

$$s^* = -Q_1 R_1 - Q_2 P_1^2 - \frac{[c_{2f} - c_{2d} - Q_2 P_2^2][P_1^2 + Q_2 \Omega_1]}{P_2^2 + R_2 + Q_2 \Omega_2} \quad (9)$$

The optimal subsidy is a function of market conduct, number of firms, tastes, and technology. It is not in general possible to sign the expression. For the special case of Cournot competition and linear inverse demands,

$$\frac{\delta s}{\delta(c_{2f} - c_{2d})} = \frac{P_1^2}{P_2^2 + R_2} < 0.$$

This accords with intuition. As the difference in marginal costs contracts, the cum-dfi optimal tariff becomes more constrained. Since the profit shifting ability of the tariff is more constrained, the home production subsidy acts to capture some of the foreign oligopoly rents that the tariff no longer can.

*Step 2.* I next characterize the optimal quota/subsidy scheme and show that it is equivalent to the optimal tariff/subsidy scheme of step 1.

The home government may set two policy tools—a quota on  $Q_{2d}$  and a subsidy on home production. As in the tariff/subsidy scheme, I assume  $c_{2f} - c_{2d}$  is greater than the no-dfi optimal tariff. The dual of this this assumption is that dfi, if it existed, would occur at the no-dfi optimal quota. Unlike the case of the tariff, the difference in foreign marginal costs does not constrain the choice of the quota. This corresponds to the diagrammatic case in which no tariff could ensure imports of  $\bar{Q}$  while a quota did just that. The cost differential,  $c_{2f} - c_{2d}$ , now sets the maximum price a foreign firm would be willing to pay for a quota licence. So that comparison with the tariff/subsidy scenario is valid, I assume quotas are auctioned to foreign producers.

As in step 1, I first consider firms' profit maximization and then characterize optimal policy conditional on firms' optimizing behavior. Since foreign firms pay  $c_{2f} - c_{2d}$  for the quota licence on non-dfi production, a foreign firm's profit function is now given by:<sup>9</sup>

$$\pi_2 = [P^2(Q_1, Q_2) - (c_{2f} - c_{2d})]q_{2d} + P^2(Q_1, Q_2)q_{2f} - C^2(q_{2d}, q_{2f}) \quad (10)$$

A home firm's profit function is unchanged from (1) as is its first order condition—(3) or (3').

The foreign firms maximize profits only with respect to  $q_{2f}$ . They take  $q_{2d}$  as given since the quota binds and is exogenously set. Foreign profit maximization, then, implies:

$$\frac{\delta \pi_2}{\delta q_{2f}} = 0 = P^2 - c_{2f} + q_2 [P_1^2 h_0 + P_2^2 h_0] \quad \text{and} \quad q_2 = q_{2d} + q_{2f} \quad (11)$$

$$= P^2 - c_{2f} + (Q_{2f} + Q_{2d})R_2 \quad (11')$$

Equations (3') and (11') implicitly define a mapping between policy tools  $s$  and  $Q_{2d}$  and free quantities  $Q_1$  and  $Q_{2f}$ . As the government changes policies, quantities adjust. For foreign firms, this relationship is represented by totally differentiating (11'). This gives:

$$[P_1^2 + (Q_{2f} + Q_{2d})\Omega_1]dQ_1 + [P_2^2 + (Q_{2f} + Q_{2d})\Omega_2 + R_2][dQ_{2f} + dQ_{2d}] = 0 \quad (12)$$

As quantities change,  $P^2$  changes. Since  $dP^2(Q_1, Q_2) = p_1^2 dQ_1 + P_2^2 dQ_2$ , (12) can be written:

$$dP^2 = (-Q_2\Omega_1)dQ_1 - (Q_2\Omega_2 + R_2)dQ_2 \quad (13)$$

<sup>9</sup> It is well known that an equilibrium in pure strategies may not exist for Bertrand behavior in the presence of a quota. Krishna then shows an equilibrium will exist in mixed strategies. Non-existence of equilibria in pure strategies is not a problem when dfi occurs. To understand why, it helps to understand why a pure strategies equilibrium in the presence of a quota might fail to exist in the first place. A binding quota acts like a binding capacity constraint. Firms playing Nash in prices may cycle endlessly around the capacity constraint or quota. Introducing dfi is analogous to removing the capacity constraint that is causing the cycling behavior. An equilibrium in pure strategies will exist when the the quota is jumped via dfi.

Finally, (12) can be manipulated to show how free quantities change as the quota is adjusted.

This gives:

$$dQ_{2d} = -\frac{P_1^2 + Q_2\Omega_1}{P_2^2 + Q_2\Omega_2 + R_2}dQ_1 - dQ_{2f} \quad (14)$$

Equations (13) and (14) will be useful for the home welfare maximization to which I now turn.

Home welfare is given by consumer surplus, domestic profits, and licence revenues.

$$W = U(Q_1, Q_2) - P^1Q_1 - P^2Q_2 + n_1\pi_1 + (c_{2f} - c_{2d})Q_{2d} - sQ_1 \quad (15)$$

Unlike the tariff case, revenue is possible in the presence of dfi, hence  $Q_{2f}$  need not equal zero. An incremental change in welfare is given by:

$$dW = (P^1 - c_1)dQ_1 - (Q_{2f} + Q_{2d})dP^2 + (c_{2f} - c_{2d})dQ_{2d} \quad (16)$$

Equation (16) implies a possible first best situation since there is an unconstrained policy tool for each of the free quantities— $Q_1$  and  $Q_{2f}$ .

Substituting (3'), (13) and (14) into the welfare maximand, (16) and simplifying gives:

$$dW = (-Q_1R_1 - s + (Q_2)^2\Omega_1 - [(Q_2)^2\Omega_2 + Q_2R_2 + c_{2f} - c_{2d}][\frac{P_1^2 + Q_2\Omega_1}{P_2^2 + Q_2\Omega_2 + R_2}])dQ_1 \\ + (c_{2d} - c_{2f})dQ_{2f} \quad (17)$$

The second term in (17) is very informative. Since  $c_{2f} > c_{2d}$ , dfi enters negatively in home welfare. The ability to set a strictly binding quota which provokes dfi, then, is of no benefit to domestic welfare. At a welfare optimum,  $Q_{2f}$  equals 0. The quota on  $Q_{2d}$  is set such that revenue  $Q_{2d}(c_{2f} - c_{2d})$  is collected but no dfi is provoked.<sup>10</sup> This is exactly what the cum-dfi optimal tariff accomplishes.

The first term in (17) implicitly defines the optimal subsidy that is coupled with the optimal quota. Setting the term multiplied by  $dQ_1$  equal to zero and solving for  $s$  gives the optimal subsidy. Straightforward calculations show that this optimal subsidy is identical to the optimal subsidy in the tariff/subsidy scheme of step 1. Again, the optimal subsidy depends on the number of domestic and foreign firms and their mode of market conduct.

The tariff/subsidy scheme is completely equivalent to the quota/subsidy scheme. This result is independent of the number of firms, their market conduct, and the inverse demand system they face.

<sup>10</sup> Proof that this assignment does indeed achieve the optimum is analogous to Lemma 1. We know dfi enters welfare negatively, yet licence revenue, like the constrained tariff revenue, enters welfare positively. Hence, the optimum is at the knife-edge described in the text. It is not possible to set  $\frac{\delta W}{\delta s}$  and  $\frac{\delta W}{\delta t} = 0$  to explicitly solve for the optimal quota because price is a general function of quantities.

#### 4. Domestic Employment Considerations and Tariff-Quota Equivalence.

Section 3 showed that the existence of dfl in a constant marginal cost imperfectly competitive industry is evidence of suboptimal trade policy. Dfl, though, is often praised for the increased employment associated with it. I ask in this section how the equivalence results of section 3 are affected by domestic employment concerns.

I assume the employment associated with production in the home country ( $Q_1 + Q_{2f}$ ) enters national welfare but does not directly enter firms' profit functions.<sup>11</sup> The approach used in this section is intentionally agnostic as to why this might be the case.<sup>12</sup>

Let  $\phi = \phi(Q_1 + Q_{2f})$  be a flexible function that measures how society values a dollar of wages relative to a dollar of trade tax revenue or consumer surplus.  $\phi$  is normalized such that  $\phi = 1$  implies society is indifferent between a dollar paid in wages and a dollar of consumer surplus.  $\phi = 0$  implies wages do not enter national welfare. This was the implicit assumption throughout section 3. This might be a logical choice if there is no involuntary unemployment in the home economy.  $\phi$  may be constant ( $\phi' = 0$ ). In this case, the first dollar of wages is just as important as the nth. Similarly,  $\phi$  may be concave or convex in employment. One might well imagine that the home country derives decreasing marginal benefits to increasing employment.

Employment enters national welfare in the following way.<sup>13</sup>

$$\begin{aligned} \text{benefits to home country employment} &= \phi(Q_1 + Q_{2f})\alpha w[Q_1 + Q_{2f}] \\ \text{where: } \alpha &= \text{the assumed constant input coefficient on labor.} \\ w &= \text{the assumed constant wage.} \end{aligned} \tag{18}$$

Firms' first order conditions and comparative statics on them are unchanged since employment considerations do not enter profit functions. A tariff/subsidy scheme, then, still imposes a bang-bang solution to the firms' cost minimization problem. While a quota/subsidy scheme does not

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<sup>11</sup> Employment associated with the production of  $Q_{2f}$  enters national welfare in the same way as the employment associated with the production of  $Q_1$ . It is unclear why this might not be the case. Nonetheless, the (non-)equivalence results of this section are unchanged by letting only employment associated with dfl production enter welfare considerations via (18).

<sup>12</sup> This approach is in the spirit of Atkinson's work on modelling attitudes concerning inequality in optimal tax problems.

<sup>13</sup> The modeling approach does not address the possible endogeneity of the employment externality. That is, I do not account for the idea that the existence of protection may lead to a divergence between the actual and the shadow wage. Resulting unemployment in this case is endogenously determined with policy. Neglecting this endogeneity may result in misguided policymaking. See Rodrik (1986) on this issue.

force this behavior, the optimal quota/subsidy scheme of Section 3 induces it. I next investigate whether introducing employment concerns into the home welfare function alters this equivalence.

With a quota/subsidy scheme, home welfare is now:

$$W = U(Q_1, Q_2) - P^1 Q_1 - P^2 Q_2 + n_1 \pi_1 + (c_{2f} - c_{2d}) Q_{2d} - s Q_1 + \phi(Q_1 + Q_{2f}) \alpha w [Q_1 + Q_{2f}] \quad (19)$$

An incremental change in welfare is given by:

$$dW = [P^1 - c_1 + \phi \alpha w + Q_1 \alpha w \phi'] dQ_1 - Q_2 dP^2 + (c_{2f} - c_{2d}) dQ_{2d} + [\phi \alpha w + Q_{2f} \alpha w \phi'] dQ_{2f} \quad (20)$$

Using the same substitutions used in step 2 of section 3 yields:

$$dW = [-Q_1 R_1 - s + (Q_2)^2 \Omega_1 + \phi \alpha w + Q_1 \phi' \alpha w - [(Q_2)^2 \Omega_2 + Q_2 R_2 + c_{2f} - c_{2d}] \left[ \frac{P_1^2 + Q_2 \Omega_1}{P_2^2 + Q_2 \Omega_2 + R_2} \right]] dQ_1 + (c_{2d} - c_{2f} + \phi \alpha w + Q_{2f} \phi' \alpha w) dQ_{2f} \quad (21)$$

Tariff- quota equivalence now depends on whether the welfare benefits to employment are constant over the relevant range of output.

*Case 1:*  $\phi' = 0$ . In this case, tariff/subsidy and quota/subsidy schemes are equivalent. Due to constant marginal costs, firms either completely jump a tariff or engage in no dfi. Tariff revenue per unit of  $Q_{2d}$  is  $c_{2f} - c_{2d}$ . The welfare benefits to dfi are  $\phi \alpha w$  per unit  $Q_{2f}$ . If  $\phi \alpha w > c_{2f} - c_{2d}$ , the home country prefers to collect no tariff revenue and instead induce dfi and its attendant employment benefits. If  $\phi \alpha w < c_{2f} - c_{2d}$ , the home country prefers no dfi and instead collects tariff revenue  $(c_{2f} - c_{2d}) Q_{2d}$ . The bang- bang solution hinges on  $c_{2f} - c_{2d} [ > \text{ or } < ] \phi \alpha w$ .

The optimal quota hinges on exactly the same inequality. This is evident by examining the second term in (21). At  $\phi' = 0$ , if  $\phi \alpha w > c_{2f} - c_{2d}$ ,  $Q_{2f}$  enters welfare positively and the more dfi the better. The optimal quota will set  $Q_{2d}$  at zero and induce only dfi while raising no licence revenue. If  $\phi \alpha w < c_{2f} - c_{2d}$ , dfi enters welfare negatively and it will be optimal to collect licence revenue but provoke no dfi.

*Case 2:*  $\phi' \neq 0$ . In this case, a non-trivial binding quota may be optimal and tariff-quota equivalence fails to hold. At a welfare optimum,

$$\phi \alpha w - c_{2f} + c_{2d} + Q_{2f} \phi' \alpha w = 0. \quad (22)$$

From (11), we know:

$$Q_{2f} = \frac{c_{2f} - P^2}{R_2} - Q_{2d}$$

Substituting this into (35) and solving for  $Q_{2d}$  yields the optimal quota.<sup>14</sup>

$$Q_{2d}^* = \frac{\phi}{\phi'} + \frac{c_{2f} - P^2(Q_1, Q_2)}{R_2} - \frac{c_{2f} - c_{2d}}{\phi' \alpha w} \quad (23)$$

For some specifications of  $\phi$ , the optimal quota will involve dfi concurrent with foreign production. This is intuitive when  $\phi$  is very concave in employment. The welfare benefits of the first few hours of employment are very large but diminish quickly as domestically produced output expands. No dfi in this case may be suboptimal since the foregone welfare benefits for the first bit of extra employment will be very large relative to the licence revenue collected. All dfi, on the other hand, may also be suboptimal as the increase in domestic employment will be so large that the marginal benefits of such employment are negligible. A quota allows the home country to ensure dfi at a level somewhere between the all or nothing levels imposed by the tariff/subsidy scheme. In this case, the home welfare associated with an optimal quota will never be less than that associated with the optimal tariff.

### 5. Increasing Marginal Costs and Tariff-Quota Nonequivalence: An Example.

Throughout this paper, marginal costs have been assumed to be constant. In this section I show that this assumption is essential to the general equivalence results. This is done by the use of a numerical counterexample. I consider a simple Cournot duopoly. Let firms' cost functions and marginal costs be given by:

$$\begin{aligned} TC_1 &= 10 + \frac{1}{2}(Q_1)^2 & c_1 &= Q_1 \\ TC_{2d} &= 10 + \frac{1}{2}(Q_{2d})^2 & c_{2d} &= Q_{2d} \\ TC_{2f} &= 10 + 3Q_{2f} + \frac{1}{2}(Q_{2f})^2 & c_{2f} &= 3 + Q_{2f} \end{aligned} \quad (38)$$

As in Section 3, the marginal cost schedule of dfi lies strictly above the marginal cost schedule of foreign local production. Inverse demands are given by:

$$\begin{aligned} P^1 &= 10 - .1Q_1 - .05Q_2 \\ P^2 &= 10 - .05Q_1 - .1Q_2 \end{aligned} \quad (39)$$

It is no longer useful to use the taxonomy of the constant marginal cost case when calculating optimal policies. Even if the tariff is greater than  $(c_{2f} - c_{2d})$ , dfi and foreign local production

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<sup>14</sup> One should note that for some modes of market conduct, the optimal quota may not be well defined. This is because the optimal quota may be negative. While it is clear that a negative tax is a subsidy, a negative quota is somewhat trickier.



might coexist. Another difference relative to the constant marginal cost case is that the value of a quota licence will depend on the quantities being produced. In particular, a licence will be worth the difference in marginal costs at the equilibrium quantities. With these points in mind, it is straightforward to calculate optimal policy schemes using the same methodology explained in Section III. Using the cost functions and inverse demands from (38) and (39), resulting equilibria are given in Table 1.

Table 1 illustrates several points. First, the optimal tariff/subsidy scheme is not equivalent to the optimal quota/subsidy scheme. This is due to the very different ways a tariff and a quota affect the foreign firm's profit function. In free trade, the profit maximizing foreign firm will produce where  $c_{2f}(Q_{2f}) = c_{2d}(Q_{2d})$  at an interior solution. This implies  $Q_{2d} > Q_{2f}$  since  $c_{2f}(\cdot) > c_{2d}(\cdot)$ . A tariff shifts  $c_{2d}$  upward but the foreign firm continues to produce where  $c_{2f} = c_{2d} + t$ . As output expands past the point at which dfi becomes profitable, the foreign firm minimizes cost by dividing production between dfi and foreign local production. When a binding quota is in place, the foreign firm cannot divide output between its two plants. Instead, the firm must on the margin produce only via dfi. This results in marginal costs that rise more quickly per unit of output. Whereas with a tariff the foreign firm could in effect spread the increased marginal costs between two plants, now only one plant may produce output beyond the amount of the quota. This is why in Table 1 the foreign good price is higher and foreign output and profits are lower with the quota than with the tariff. The lower  $Q_2$  associated with a quota yields a larger  $P^1$ . While this increases domestic profits, this does not offset the loss in consumer surplus and trade tax revenue relative to the tariff scheme. Net home welfare is lower with the optimal quota/subsidy than with the optimal tariff/subsidy.

Second, Table 1 dramatically illustrates the concept of profit shifting trade policy. Foreign profits fall from 29.2 with free trade to 6.95 with a tariff and 2.35 with a quota. Home welfare in turn rises from 41.31 to 64.52 with a tariff and to 55.42 with a quota.

The results of Table 1 are a specific example of tariff-quota nonequivalence. They are not a general comparison of tariffs and quotas with increasing marginal costs. The results of Table 1 are sensitive to the mode of market conduct and the functional forms used.

## 6. Summary.

Recent work on tariff-quota nonequivalence under imperfect competition has ignored the possibility that the firms concerned might be multinational. Introducing the possibility of dfi, which is often associated with multinational firms, greatly simplifies the tariff quota comparison. Many of the

strategic interactions that are difficult to model in a general way disappear when dfi is introduced into the model.

Section 2 analyzed the equivalence issue in a linear foreign monopoly model. Using a diagrammatic analysis, optimal tariffs and quotas were shown to be equivalent when the cum-dfi optimal tariff exceeded the difference between the marginal cost of dfi production and the marginal cost of foreign production.

Section 3 extended the equivalence to a general demand system, general market structure, and general mode of market conduct. The restrictive assumption maintained was that of constant marginal costs.

Section 4 introduced domestic employment considerations into the general model of section 3. It was shown that tariff quota equivalence depends on exactly how employment enters the home country's welfare function.

Section 5 relaxed the assumption of constant marginal costs. A numerical counterexample proved the equivalence of section 3 breaks down under increasing marginal costs.

There are at least two broad areas for extending the analysis presented in this paper. Recent empirical work on strategic trade policy by Dixit (1986) and Baldwin and Krugman (1986) has been restricted to the investigation of price based policies. The results of this paper should facilitate investigation of quantity based policies when the restrictive assumptions of Section 3 apply. The results of this paper are applicable to an empirical analysis of the current United States quota on Japanese automobiles. This is the subject of current research. Finally, this model does not consider dynamic effects of dfi. In an uncertain world, dfi might pre-empt future protectionist trade policy.

## Appendix A

The optimal tariff in this situation is derived as follows:

$$\text{Inverse demand is given by: } p = a - bx \quad (1A)$$

$$\text{Marginal revenue is: } a - 2bx \quad (2A)$$

$$\text{Marginal cost plus the tariff is: } c + t \quad (3A)$$

Marginal cost including the tariff set

$$\text{equal to marginal revenue implies: } x = \frac{a - c - t}{2b} \quad (4A)$$

$$\text{Substitution of 4A into 1A yields } a - p = \frac{a - c - t}{2} \quad (5A)$$

$$\text{Consumer surplus is given by: } \frac{(a - c - t)^2}{8b} \quad (6A)$$

$$\text{Tariff revenue is: } t \left[ \frac{a - c - t}{2b} \right] \quad (7A)$$

Welfare is consumer surplus plus tariff revenue. Setting  $\frac{\delta W}{\delta t} = 0$  and solving for the optimal tariff  $t^*$  gives:

$$t^* = \frac{a - c}{3} \quad (8A)$$

## Appendix B

Standard tariff/subsidy scheme without dfi

Profit functions are given by:

$$\pi_1 = P^1(Q_1, Q_2)q_1 - C(q_1) + sq_1 \quad (1B)$$

$$\pi_2 = [P^2(Q_1, Q_2) - t]q_2 - C(q_2) \quad (2B)$$

Welfare is :

$$W = U(Q_1, Q_2) - P^1Q_1 - P^2Q_2 + n_1\pi_1 + tQ_2 - sQ_1 \quad (3B)$$

Incremental changes in welfare are:

$$\begin{aligned} dW &= (P^1 - c_1)dQ_1 - Q_2dP^2 + d(tQ_2) \\ &= (P^1 - c_1)dQ_1 - Q_2d(P^2 - t) + tdQ_2 \end{aligned} \quad (4B)$$

Profit function first order conditions simplify to:

$$\begin{aligned} P^2(Q_1, Q_2) &= -R_2Q_2 + c_2 + t \\ P^1(Q_1, Q_2) &= -R_1Q_1 + c_1 - 2 \end{aligned} \quad (5B)$$

As foreign producers change quantities in response to policy changes,

$$d(P^2 - t) = [-R_2Q_2 - Q_2\Omega_2]dQ_2 - [Q_2\Omega_1]dQ_1.$$

Substitution into (4B) gives:

$$dW = (P^1 - c_1 + (Q_2)^2\Omega_1)dQ_1 + (Q_2R_2 + (Q_2)^2\Omega_2 + t)dQ_2 \quad (6B)$$

At an optimum,  $P^1 = c_1 - (Q_2)^2\Omega_1$  which implies  $s^* = -R_1Q_1 + (Q_2)^2\Omega_1$  (from (5B)). Similarly,  $t^* = -R_2Q_2 - (Q_2)^2\Omega_2$ . The equilibrium quantities  $Q_1$  and  $Q_2$  are implicitly defined by the profit function first order conditions.

## Appendix C

For some modes of market conduct, the optimal production subsidy associated with the optimal cum-dfi tariff or quota will be positive. Subsidizing domestic oligopolists, though, may often be politically infeasible. In this appendix, I amend Assumption 3. I then prove that restricting the home production subsidy to zero does not affect the equivalence of the optimal tariff and optimal quota, as demonstrated in section 3.

Due to the generality of the set-up described in Section 3, Part A, it is not possible to prove that  $\frac{\delta W}{\delta t} > 0$  for all dfi-constrained tariffs. When the production subsidy is restricted to zero, there is no longer a one to one mapping between policy tools and quantities. Hence, Lemma 1 no longer applies. In order to impose some structure on the problem, I amend Assumption 3 as follows.

*Assumption 3C:* Home welfare is strictly concave in policy tools. It follows immediately that  $\frac{\delta W}{\delta t} > 0$  for any dfi-constrained tariff. The cum-dfi optimal tariff, then, is  $c_{2f} - c_{2d}$ . I next show that the optimal quota is equivalent to this optimal tariff.

*Proposition:* The ability to set a binding quota which provokes dfi provides no benefit to home welfare.

*Proof:* Foreign firms' profit functions and their respective first order conditions are unchanged by restricting a home production subsidy to zero. Hence, (11'), (12), and (13) still obtain.

A home firm's profit function is now:

$$\pi_1 = P^1(Q_1, Q_2)q_1 - C(q_1) \quad (1C)$$

Its first order condition at an interior solution is given by:

$$P^1 - c_1 + Q_1 R_1 = 0 \quad (2C)$$

Home welfare is still given by (15) and an incremental change in it by (16). Substituting (2C), (13), and (14) into (16) now yields:

$$\begin{aligned} dW = & [-Q_1 R_1 + (Q_2)^2 \Omega_1 - [(Q_2)^2 \Omega_2 + Q_2 R_2 + c_{2f} - c_{2d}] \left[ \frac{P_1^2 + Q_2 \Omega_1}{P_2^2 + Q_2 \Omega_2 + R_2} \right]] dQ_1 \\ & + (c_{2d} - c_{2f}) dQ_2 \end{aligned} \quad (3C)$$

Since  $c_{2d} < c_{2f}$ , dfi enters negatively into home welfare even when the subsidy is constrained to zero. The ability to set a binding quota that provokes dfi confers no welfare benefit to the home country. Q.E.D.

As was the case in step 2 of Section 3, the quota on  $Q_{2d}$  is set such that  $Q_{2d}(c_{2f} - c_{2d})$  is collected as licence revenue but no dfi is provoked. This is equivalent to the dfi constrained optimal tariff scenario.

## Appendix D

### Proof of Lemma 1

Proof: A sufficient condition for the proposition to hold is  $\frac{\delta W}{\delta t} > 0$  for all  $t$  such that dfi does not occur.

I proceed by first characterizing the optimal no-dfi tariff. Stability results ensure that the optimal tariff yields a local welfare maximum, not minimum. At a tariff set epsilon less than the optimal no-dfi tariff, then,  $\frac{\delta W}{\delta t} > 0$ . I next suppose there exists a tariff less than the optimal no-dfi tariff at which  $\frac{\delta W}{\delta t} < 0$ . I show such a tariff violates assumption 3.

It will be useful to define the following terms:

$$\mu_1^1 = a_1 = P_1^1 + R_1 + Q_1 \Omega_4 \quad (1D)$$

$$\mu_2^1 = b_1 = P_2^1 + Q_1 \Omega_3 \quad (2D)$$

Similarly,

$$\mu_2^2 = a_2 = P_2^2 + R_2 + Q_2 \Omega_2 \quad (3D)$$

$$\mu_1^2 = b_2 = P_1^2 + Q_2 \Omega_1 \quad (4D)$$

Following Dixit (1984), a natural myopic adjustment process is one in which each firm increases its output starting at a given point if it perceives positive marginal profit from doing so. Hence:

$$\dot{q}_i = \sigma_i \mu^i(Q_1, Q_2) \quad (5D)$$

where  $\sigma_i > 0$  are adjustment speeds. I have assumed a symmetric industry structure. This symmetry is also assumed to extend to firms' adjustment paths to equilibrium quantities when perturbed away from such quantities. In this case, it is sufficient to examine a representative firm from each industry.

Taking linear approximations around equilibrium quantities  $(q_1^*, q_2^*)$  yields,

$$\begin{pmatrix} \dot{q}_1 \\ \dot{q}_2 \end{pmatrix} = \begin{pmatrix} s_1 a_1 & s_1 b_1 \\ s_2 b_2 & s_2 a_2 \end{pmatrix} \begin{pmatrix} q_1 - q_1^* \\ q_2 - q_1^* \end{pmatrix} \quad (6D)$$

Given the symmetric industry structure assumed above, a sufficient condition for stability is that the coefficient matrix in (6D) have eigenvalues with negative real parts. This implies the coefficient matrix must have a negative trace and a positive determinant. If this is to hold for any adjustment speeds,

$$a_1, a_2 < 0, \quad \text{and} \quad a_1 a_2 - b_1 b_2 > 0. \quad (7D)$$

These are the stability conditions. Using them, system (5) can be rewritten as:

$$\begin{pmatrix} a_1 & b_1 \\ b_2 & a_2 \end{pmatrix} \begin{pmatrix} dQ_1 \\ dQ_2 \end{pmatrix} = \begin{pmatrix} -ds \\ dt \end{pmatrix} \quad (8D)$$

Implicitly differentiating (7) with respect to  $t$  yields:

$$\frac{\delta W}{\delta t} = \frac{\delta W}{\delta Q_1} \frac{\delta Q_1}{\delta t} + \frac{\delta W}{\delta Q_2} \frac{\delta Q_2}{\delta t} + Q_2 \quad (9D)$$

From (6),

$$\frac{\delta W}{\delta Q_1} = P^1 - c_1 - P_1^2 Q_2 \quad \text{and} \quad \frac{\delta W}{\delta Q_2} = t - P_2^2 Q_2.$$

While from (8D),

$$\frac{\delta Q_1}{\delta t} = -\frac{b_1}{a_1 a_2 - b_1 b_2} \quad \text{and} \quad \frac{\delta Q_2}{\delta t} = \frac{a_1}{a_1 a_2 - b_1 b_2}.$$

Substitution of the above into (9D) yields:

$$\frac{\delta W}{\delta t} = \frac{1}{D} [-(P^1 - c_1 - P_1^2 Q_2)b_1 + (t - P_2^2 Q_2)a_2] + Q_2 \quad (10D)$$

where  $D = a_1 a_2 - b_1 b_2$ .

While the tariff is constrained by potential dfi, the subsidy is not. Hence,  $\frac{\delta W}{\delta s} = 0$  for all tariffs.

This yields:

$$\frac{\delta W}{\delta s} = \frac{1}{D} [-(P^1 - c_1 - P_1^2 Q_2)a_2 + (t - P_2^2 Q_2)b_2] = 0.$$

This implies:

$$P^1 - c_1 - P_1^2 Q_2 = t - P_2^2 Q_2. \quad (11D)$$

Substitution of (11D) into (10D) and some manipulation yields:

$$\frac{\delta W}{\delta t} = \frac{1}{a_2} [t + Q_2 R_2 + (Q_2)^2 \Omega_2] \quad (12D)$$

The optimal no-dfi tariff is given by:

$$T^{* \text{ no-dfi}} = -Q_2 R_2 - (Q_2)^2 \Omega_2.$$

The second order condition of (12D) is:

$$\frac{\delta^2 W}{\delta t^2} = \frac{1}{a_2} < 0.$$

This ensures that welfare is increasing at a tariff just less than  $t^{* \text{ no-dfi}}$ . In figure 2, this is a point epsilon to the left of point A.

Suppose that at some tariff less than  $t^*$  *not*  $t$ ,  $\frac{\delta W}{\delta t} < 0$ . This is a point such as B in figure 2. Because welfare is continuous in quantities and there is a one to one mapping between quantities and policy tools, welfare must also be continuous in policy tools. By the continuity of the welfare function, there must exist a point such as C at which  $\frac{\delta W}{\delta t} = 0$ . At all  $t$  in figure 2, the subsidy is optimally set. Hence, at point C,  $\frac{\delta W}{\delta s}$  and  $\frac{\delta W}{\delta t} = 0$ . But at point A,  $\frac{\delta W}{\delta s}$  and  $\frac{\delta W}{\delta t} = 0$ . Because welfare is, by assumption 3, strictly concave in quantities and there exists a one to one mapping between quantities and policy tools, there can be only one point at which  $\frac{\delta W}{\delta s}$  and  $\frac{\delta W}{\delta t}$  are jointly equal to 0. Hence, it must be the case that for all  $t$  such that *dfi* does not take place,  $\frac{\delta W}{\delta t} > 0$ . Q.E.D.



## References

- Atkinson, Anthony B. and Joseph Stiglitz (1980) *Lectures in Public Economics*. New York: McGraw Hill.
- Baldwin, Richard, and Paul Krugman (1986) "Market Access and International Competition: A Simulation Study of 16K Random Access Memories," NBER Working Paper no. 1936.
- Bhagwati, Jagdish (1965) "On the Equivalence of Tariffs and Quotas," in *Trade, Growth, and the Balance of Payments*, ed. R. Baldwin. Chicago: Rand McNally.
- Bhagwati, J. and R. Brecher (1980) "National Welfare in an Open Economy in the Presence of Foreign-Owned Factors of Production," *Journal of International Economics*, **10**, 103-115.
- Brander, J. and P. Krugman (1983) "A 'Reciprocal Dumping' Model of International Trade," *Journal of International Economics*, **15**, 313-321.
- Brander, J. and B. Spencer (1984) "Tariff Protection and Imperfect Competition," in *Monopolistic Competition in International Trade*, ed. H. Keirzkowski. New York: Oxford.
- Brander, J. (1985) "Export Subsidies and International Market Share Rivalry," *Journal of International Economics*, **18**, 83-100.
- Brecher, R. and C Diaz-Alejandro (1977) "Tariffs, Foreign Capital and Immiserizing Growth," *Journal of International Economics*, **7**, 317-322.
- Buffie, E. (1985) "Quantitative Restrictions and the Welfare Effects of Capital Inflows," *Journal of International Economics*, **19**, 291-303.
- Dei, F. (1985) "Voluntary Export Restraints and Foreign Investment," *Journal of International Economics*, **19**, 305-312.
- Dixit, A. (1984) "International Trade Policy for Oligopolistic Industries," *Economic Journal*, **94**, 305-312.
- Dixit, A. (1986) "Comparative Statics for Oligopoly," *International Economic Review*, **27**, 107-122.
- Dixit, A. (forthcoming) "Optimal Trade and Industrial Policies for the U.S. Automobile Industry," in *Empirical Methods for International Trade*, ed. R. Feenstra. Cambridge: MIT Press.
- Eaton, J. and G. Grossman (1986) "Optimal Trade and Industrial Policy Under Oligopoly," *Quarterly Journal of Economics*, **101**, 383-406.
- Fishelson, G. and F. Flatters (1975) "The (Non)Equivalence of Optimal Tariffs and Quotas under Uncertainty," *International Economic Review*, **5**, 385-393.
- Grossman, G. and J. D. Richardson (1985) "Strategic Trade Policy: A Survey of Issues and Early Analysis," International Finance Section, Department of Economics, Princeton University, Special Papers in International Economics, No. 15.
- Katz, M. and H. Rosen (1983) "Tax Analysis in an Oligopoly Model," NBER Working Paper No. 1088.
- Krishna, K. (1983) "Trade Restrictions as Facilitating Practices," Discussion Papers in Economics No. 55, Princeton University, Woodrow Wilson School.
- McCulloch, R. (1973) "When are a Tariff and a Quota Equivalent?," *Canadian Journal of Economics*, **6**, 503-511.

- Rodrik, D. (1986) "Policy Targeting with Endogenous Distortions: The Theory of the Optimum Subsidy Revisited," JFK School of Government, Harvard University, Discussion Paper No. 146D.
- Sweeny, G. (1985) "Import Quotas and Oligopolistic Interactions," Vanderbilt University Department of Economics Working Paper No. 84-W17.

Table 1  
 Optimal Tariff/Subsidy and Optimal Quota/Subsidy Schemes  
 with Increasing Marginal Costs: An Example.

|                               | Free Trade | Tariff/Subsidy | Quota/Subsidy            |
|-------------------------------|------------|----------------|--------------------------|
| Optimal Subsidy               | —          | 1.127          | 1.061                    |
| Optimal Tariff                | —          | 3.687          | (value of licence) 4.229 |
| Optimal Quota                 | —          | —              | 2.115                    |
| Dfi ( $Q_{2f}$ )              | 4.291      | 4.806          | 3.344                    |
| Total Imports ( $Q_2$ )       | 11.582     | 8.745          | 5.549                    |
| Domestic Production ( $Q_1$ ) | 7.851      | 8.909          | 8.990                    |
| $P^1$                         | 8.636      | 8.671          | 8.828                    |
| $P^2$                         | 8.450      | 8.680          | 9.005                    |
| $\pi_1$                       | 26.980     | 37.619         | 38.494                   |
| $\pi_2$                       | 29.200     | 6.952          | 2.351                    |
| Consumer Surplus              | 14.335     | 11.687         | 7.984                    |
| Home Welfare                  | 41.316     | 64.524         | 55.422                   |

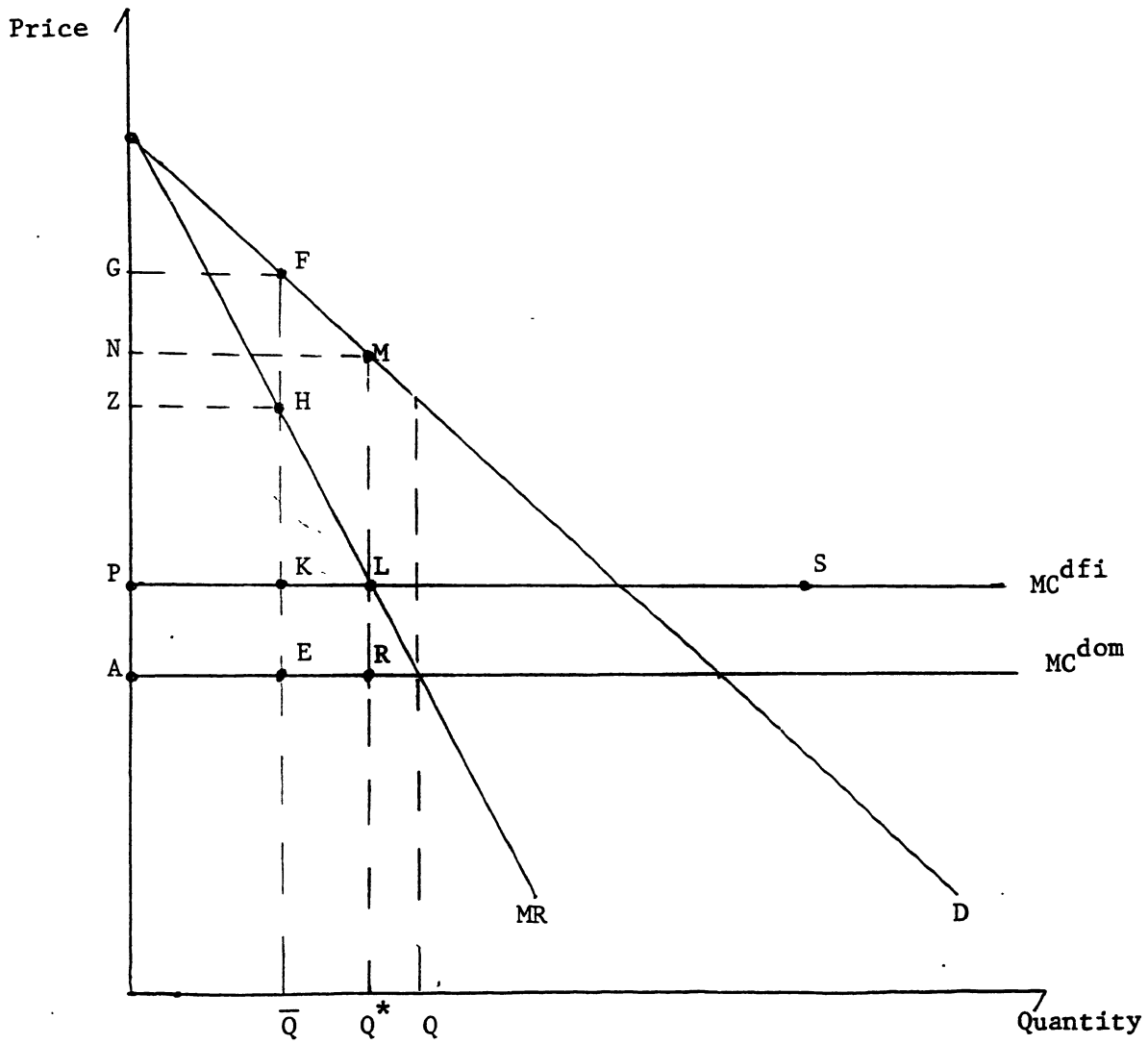


FIGURE 1



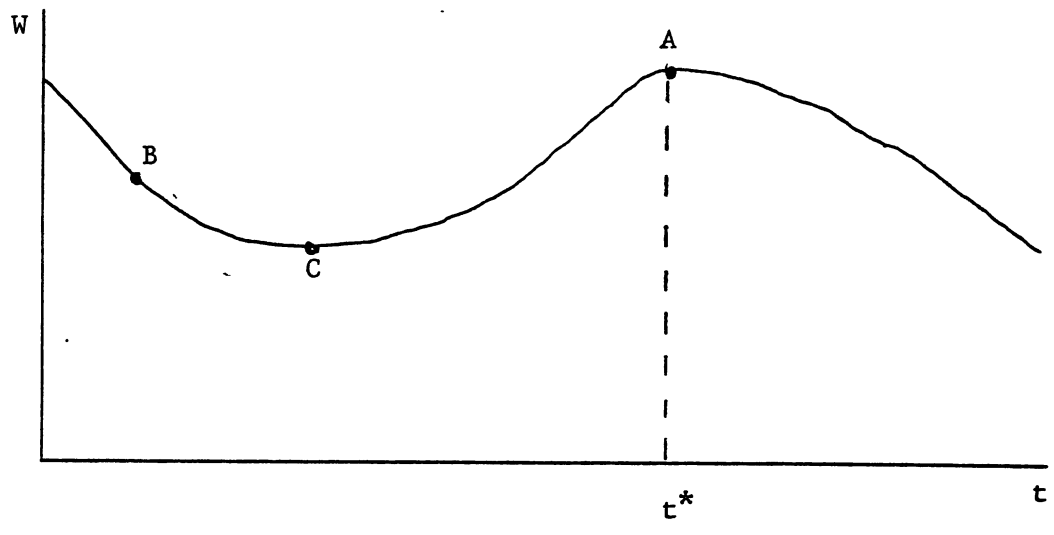


FIGURE 2

