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Abstract. Many recent papers have estimated components of the disturbance term in the "market model" of equity returns. In particular, several studies of regulatory changes and other policy events have decomposed the event effects in order to allow for heterogeneity across firms. In this paper we demonstrate that the econometric method applied in some papers yields biased and inconsistent estimates of the model parameters. We demonstrate the consistency of a simple and easily-implemented alternative method.

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1. Introduction

Empirical researchers in industrial organization, international trade, and macroeconomics have recently found imaginative ways to exploit the abundance of financial market data. Their methods typically focus on decomposing abnormal equity returns. In the field of industrial organization, for example, Nancy Rose [1985] asks whether there are identifiable firm characteristics which might help explain the effect of deregulation on rents in the trucking industry. Rodney Smith, Michael Bradley, and Greg Jarrell [1986] apply a similar method to investigate the effects of oil price regulation on firms in the oil industry. These papers and others are creative attempts to use available stock market data to analyze interesting policy-related questions.

It is well understood that stock returns are generated by highly efficient, forward-looking markets. This has important economic and econometric implications. Some of the implications have been ignored in recent attempts to decompose abnormal returns. The result has been that several researchers have inadvertently employed econometric methods that yield biased and inconsistent estimates of the model parameters of interest. Other researchers using the same type of data and also decomposing abnormal returns have correctly estimated their models. In all cases, the choice of econometric technique appears to have been haphazard. Our goal in this paper is to set the record straight. There is a right way and a wrong way to econometrically decompose the disturbance term in financial models. Fortunately, the right way is simple and straightforward.

In Section 2, we consider estimating components of a well-behaved disturbance term in a general context. In the usual case components of the disturbance that are observable up

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to an unknown parameter vector would—indeed should—be included in the specification of the model's explanatory variables and estimated directly. However in an apparently peculiar circumstance it turns out that the conventional approach yields biased and inconsistent parameter estimates. That is, it may be better to leave some explanatory variables in the disturbance. Of course, the econometrician may want to estimate the effects of these disturbance components; we propose a simple and consistent method for doing so.

It may seem that our peculiar case is of little practical interest. In fact the circumstance can arise in any rational expectations model. In particular, the situation occurs quite naturally in applications of the Capital Asset Pricing Model (CAPM). Thus, in Section 3 we explain how the *economics* underlying the efficient markets hypothesis imposes constraints on the choice of *econometric* technique. We also briefly discuss several recently published papers employing econometric techniques that may yield biased and inconsistent estimates of the model parameters. We conclude by summarizing a consistent and easily-implemented estimation method and comment on its wide applicability.

2. General Treatment of the Problem.

Consider a standard linear model,

$$y_t = X_t\beta + \epsilon_t, \qquad t = 1, \ldots, T, \tag{1}$$

for which the classical assumptions hold: $\mathrm{E}[\epsilon|X] = 0$, $\mathrm{E}[\epsilon\epsilon'|X] = \sigma^2 I_T$, and plim $X'X/T$ is a positive definite matrix, with X_t a $1 \times K_1$ vector of explanatory variables. Then the least squares estimates of the parameter vector are unbiased, and consistent in large samples.

Suppose the econometrician believes that some components of the disturbance, ϵ_t, are in fact observable, and models the disturbance as

$$\epsilon_t = Z_t\gamma + \omega_t \tag{2}$$

with Z_t a $1 \times K_2$ vector of observable variables, but still believes the orthogonality restriction on the X_t, that $\mathrm{E}[\epsilon|X] = 0$. If the parameters γ are of no economic interest, the decomposition of the disturbance term can be ignored and the parameters β can be estimated by applying least squares to equation (1). Typically, however, one is interested in estimates of γ as well as in estimates of β.

The obvious approach would seem to be the following. Combine equations (1) and (2) to obtain

$$y_t = X_t\beta + Z_t\gamma + \omega_t. \tag{3}$$

The econometrician then might jointly estimate the parameters (β, γ) using least squares. Would such estimates be consistent? The requisite conditions are that:

$$\frac{1}{T}X'\omega \xrightarrow{p} 0 \tag{4}$$

$$\frac{1}{T}Z'\omega \xrightarrow{p} 0 \tag{5}$$

(where the notation $\xrightarrow{p}$ means "converges in probability to"); that is, the structural variables X and observable disturbance components Z must both be asymptotically uncorrelated with the unobserved disturbance components ω. For simplicity, we assume throughout this note that the Z are asymptotically uncorrelated with the ω.

Consider equation (4). Since we have assumed that X is uncorrelated with the total disturbance, ϵ, we might expect that X is also uncorrelated with each of ϵ's components, ω and $Z\gamma$. In fact, in many interesting cases, this will not be the case. If X is correlated with any of the observable disturbance components, Z, then X must also be correlated with ω, and least squares estimates of the parameters of equation (3) will be biased and inconsistent.[1]

Proposition 1. *In the model given by (1) and (2), if the classical assumptions hold for (1), and if any columns of X are asymptotically correlated with $Z\gamma$, then least squares will yield biased and inconsistent estimates of γ and β in equation (3).*

The premise of Proposition 1 may seem to have little practical relevance. After all, if X is uncorrelated with ϵ, but correlated with $Z\gamma$, then X must be correlated with ω by precisely the right amount, since

$$\text{corr}(X, \epsilon) = \text{corr}(X, Z\gamma + \epsilon) \;=\; \text{corr}(X, Z\gamma) + \text{corr}(X, \omega).$$

---

[1] Proofs of the propositions are given in the appendix.

We shall show in Section 3 that such a peculiar happenstance arises quite naturally in rational expectations models, and describe a number of CAPM applications as examples. We shall also discuss the sensitivity of our results to this apparently strong assumption.

First, we provide a solution to the estimation problem. There is a simple alternative to one-stage estimation which yields consistent estimates of both the original structural parameters, β, and the disturbance components parameters, γ. The consistent method has been used in some empirical papers in the literature, but no proof of its consistency has appeared.

The consistent method estimates the original model (1) using least squares. Under the classical assumptions, the estimated residuals from this regression are consistent for the true disturbances. The econometrician then estimates the model in (2), substituting the estimated disturbances from the first-stage regression for the true disturbances ϵ:

$$\hat{\epsilon}_t = Z\gamma_t + \xi_t. \tag{6}$$

Letting $P = X(X'X)^{-1}X'$ be the projection matrix of X, we show in the appendix that the disturbance vector ξ in (6) is given by

$$\xi = \omega - P\epsilon. \tag{7}$$

It might appear from (7) that least squares applied to (6) will yield inconsistent estimates, since ϵ appears in the disturbance, and we know from the model (2) that Z is correlated with ϵ. However, since the disturbances are estimated consistently in the first-stage regression, the measurement error term $P\epsilon$ vanishes as the sample size gets large, and thus Z is asymptotically uncorrelated with the disturbance in (6).

Proposition 2. *Under the assumptions of Proposition 1, the least squares estimates of β in equation (1) are best linear unbiased, and the coefficients on the disturbance components (γ) in (6) are consistent.*

We also present the asymptotic variance-covariance matrix for the parameter estimates, $\hat{\gamma}$. As the sample size T approaches infinity, $\sqrt{T}(\hat{\gamma}-\gamma)$ has a limiting multivariate normal distribution with mean vector zero, and covariance matrix

$$\Omega = \sigma_\omega^2 Q_{zz}^{-1}(I - Q_{zx}Q_{xx}^{-1}Q_{xz}Q_{zz}^{-1}) + \sigma_\nu^2 Q_{zz}^{-1}Q_{zx}Q_{xx}^{-1}Q_{xz}Q_{zz}^{-1}$$

where $Q_{zx} = \text{plim } Z'X/T$ and so forth, $\nu \equiv Z\gamma$, and $\sigma_\nu^2 \equiv \text{plim } (1/T)(Z\gamma\gamma' Z')$.

We have presented our method for the simple case in which the classical assumptions hold for the model of equation (1). It is, however, straightforward to extend the results to the cases of heteroskedastic disturbances and, in a panel data context to a random effects model. The main point remains: if a disturbance term which is orthogonal to the explanatory variables has some observable components which are not orthogonal to the regressors, then the unobservable components will also not be orthogonal. Including the observable disturbance components in a one-stage regression will yield biased and inconsistent parameter estimates. The two-stage method described above is always appropriate and easy to implement.

3. Application to "Market Model" Studies.

Several papers have applied the standard Capital Asset Pricing Model (CAPM) of Sharpe-Lintner-Mossin to analyze the effect of various factors on a firm's equity value. In this section, we show how the above results may be used to ensure consistent and efficient estimation of the CAPM in several common applications.

Notation.

We employ the following notation:

r_{it}	realized return on security i in period t
r_{mt}	realized return on "market" portfolio in period t
r_f	risk-free return, assumed constant
Z_{it}	vector of effects, not necessarily firm-specific

The purpose and interpretation of the effects Z are explained below. Expectations are taken to be conditional on all information publicly available in the prior period.

"The Market Model."

We briefly derive the empirical implementation of the CAPM, and demonstrate that it satisfies the orthogonality condition for unbiased and consistent estimation. This result is well-known, but is repeated here for comparison to cases in which the parameters are estimated inconsistently.

Let:

$$r_{it} = \mathrm{E}[r_{it}] + w_{it} \quad \text{and} \quad r_{mt} = \mathrm{E}[r_{mt}] + w_{mt} \tag{8}$$

where the ω's are i.i.d. "white noise" forecasting errors, according to the efficient markets hypothesis.

Assuming that the joint distribution of one-period percentage returns on assets is multivariate normal, it can be shown that the CAPM implies:[2]

$$\mathrm{E}[r_{it}] = (1-\beta_i)r_f + \beta_i \mathrm{E}[r_{mt}] \quad \text{where} \quad \beta_i \equiv \mathrm{cov}(r_{it}, r_{mt})/\,\mathrm{var}(r_{mt}) = \sigma_{im}/\sigma_m^2 \tag{9}$$

Substitution yields:

$$\begin{aligned} r_{it} &= \alpha_i + \beta_i r_{mt} + \epsilon_{it} \\ \epsilon_{it} &= w_{it} - \beta_i w_{mt} \end{aligned} \tag{10}$$

where $\alpha_i \equiv (1-\beta_i)r_f$.

From (8) it is clear that $\mathrm{cov}(r_{mt}, w_{mt}) \neq 0$, and thus it may appear that the market model (10) has a simultaneity bias preventing consistent estimation by least squares. However, the orthogonality required is that $r_m' \epsilon_i / T \xrightarrow{p} 0$, *i.e.*, that the asymptotic correlation between the market return and the *total* disturbance be zero. But

$$\begin{aligned} \mathrm{cov}(r_{mt}, \epsilon_{it}) &= \mathrm{cov}(r_{mt}, r_{it} - \alpha_i - \beta_i r_{mt}) \\ &= \mathrm{cov}(r_{mt}, r_{it}) - \frac{\mathrm{cov}(r_{mt}, r_{it})}{\mathrm{var}(r_{mt})}\,\mathrm{var}(r_{mt}) \\ &= 0 \end{aligned} \tag{11}$$

where the first equality follows from substitution of (10) and the second uses the definition of β_i under the maintained efficient markets hypothesis. Thus, the orthogonality condition is met, and least squares estimation is consistent.

It may be helpful to suggest an intuition for this apparently fortuitous result. The CAPM follows from the efficient markets hypothesis, which is equivalent in this setting to the assumption of rational expectations. Thus, $\alpha_i + \beta_i \mathrm{E}[r_{mt}]$ is the best predictor of the return on security i. Any deviations from this prediction must be independent and

[2] See the appendix in Jensen [1972] for a derivation of this result. Multivariate normality is sufficient, but not necessary. The same result holds if expected utility is quadratic, or if trading is continuous and security prices follow a Wiener diffusion process.

wholly unpredictable given prior information. If the disturbance *were* correlated with the market return, it would be possible to exploit the covariance to produce a lower variance predictor of the return on security i. This lower variance, unbiased predictor would provide an arbitrage opportunity, which cannot persist in equilibrium under the efficient markets hypothesis. Thus, the zero covariance between r_{mt} and the disturbance ϵ_{it} is guaranteed by the assumed absence of arbitrage opportunities. A similar condition holds in many rational expectations models.

Decomposing the Disturbance in the Market Model.

Suppose that the econometrician believes that

$$\epsilon_{it} = \sum_{k=1}^{K} Z_{kit}\gamma_{ki} + \nu_{it}, \tag{12}$$

where the Z_k are observable effects (but need not necessarily vary across firms). We propose a taxonomy of cases when such a specification arises:

(1) *"Standard Event Study."* Let the disturbance components be

$$Z_{kit} = \begin{cases} 1, & \text{if the } k-\text{th event occurs on day } t \\ 0, & \text{otherwise.} \end{cases} \tag{13}$$

Then, γ_{ki} estimates the abnormal return for firm i on the day of the k-th event.[3]

(2) *"Heterogeneous-Effect Event Study."* It has recently become standard practice to investigate whether events might have predictably different effects on different firms, particularly in studies of the effects of regulatory changes on firm value.[4] Heterogeneity is modeled by decomposing abnormal returns into components which depend in part on observable firm characteristics. In general, let

$$Z_{kit} = \delta_{kit}\tilde{Z}_{kit}$$
$$\delta_{kit} = \begin{cases} 1, & \text{if the } k-\text{th event occurs on day } t \\ 0, & \text{otherwise.} \end{cases} \tag{14}$$

[3] This particular method of estimating abnormal returns presumes that the parameters of the market model for firm i, α_i and β_i, are not affected by the event; thus, the entire sample including the post-event period is employed to efficiently estimate those parameters.

[4] See, *e.g.*, Rose [1985]; Smith, Bradley, and Jarrell [1986]; Borenstein and Zimmerman [1987]; Mitchell and Maloney [1988].

The $\tilde{Z}_{kit}$, then, are variables affecting the impact of the event on the firm's return; they may vary across firm or time, or both. Examples include the leverage ratio and elements of the firm's production function (such as average haul length for trucks; see Rose [1985]).

(3) *"Error Components Study."* Some studies do not focus on particular events, but are interested in more generally decomposing the disturbance in security returns into various sources of unexpected shocks. Typically, the econometrician will model various "news" or "innovations" variables.[5] Such a study might propose various sources of the prediction errors which correspond specifically to firms (w_{it}), and other shocks which correspond to the return on the market (w_{mt}).

As shown in Section 1, the consistency of a one-stage estimation procedure in the above examples will depend on whether the market return is correlated with the observable disturbance components. First consider the "Standard Event Study". In this case it is usually reasonable to assume that the days on which events occur are not correlated with market returns, at least for events which affect a small fraction of all firms in the market. Thus, estimating the abnormal event-day returns in one-stage will typically yield consistent results.[6]

The outlook is not so sanguine for one-stage estimation of either the "Heterogeneous-Effect" or "Error Components" models. It is reasonable to believe that aggregate demand shocks (Z_{it}) will affect various firms differently. Since in general aggregate demand shocks will be correlated with realized market returns, $\text{cov}(r_{mt}, Z_{it}) \neq 0$ and one-stage estimates will be inconsistent. Also, attempts to account for firm heterogeneity in event-day effects usually correct for variations in leverage, which translate the effect of an event on firm value into an effect on equity value (Rose [1985]; Smith, Bradley and Jarrell [1986]). If a firm's

5 See, *e.g.*, Grossman and Levinsohn [1989]; Pearce and Roley [1988].

6 In fact, most event studies estimate the market model on pre-event data, and then use prediction errors as measures of abnormal returns, in order to avoid the problems that arise if the events change the model parameters.

choice of leverage is correlated with returns in the stock market,[7] then one-stage estimation which incorporates leverage will yield inconsistent results. Other firm characteristics might also be expected to vary with market returns.

It is instructive to consider formally the sources of inconsistency in the one-stage method. Suppose that both sources of error in the market model are decomposed into observables and unobservables as follows:

$$\begin{aligned} w_{it} &= Z_{it}\gamma_i + \omega_{it} \\ w_{mt} &= Z_{mt}\gamma_m + \omega_{mt}. \end{aligned} \tag{15}$$

where Z_i is a $T\times K_i$ matrix, and Z_m is a $T\times K_m$ matrix, both of observables, and γ_i is $K_i\times 1$ and γ_m is $K_m \times 1$, both vectors of unknown parameters. Thus, the Z_{it} represent *ex post* observable components of innovations in firm returns, and the Z_{mt} represent observable components of innovations in the market return. Substitution of (15) into the market model (10) yields:

$$\omega_{it} - \beta_i\omega_{mt} = r_{it} - \alpha_i - \beta_i r_{mt} - Z_{it}\gamma_i + \beta_i Z_{mt}\gamma_m \tag{16}$$

Recall that consistency of the one-stage method is determined by the correlation between the market return and the unobservable components of the disturbance.[8] We can write this as

$$\text{cov}(r_{mt}, \omega_{it}-\beta_i\omega_{mt}) = [\text{cov}(r_{mt}, r_{it}) - \beta_i \text{cov}(r_{mt}, r_{mt})]-[\text{cov}(r_{mt}, Z_{it}\gamma_i) - \beta_i \text{cov}(r_{mt}, Z_{mt}\gamma_m)] \tag{17}$$

by substituting (16) into the covariance expression on the left.

As shown in (11), the first square-bracketed term equals zero under the efficient markets hypothesis. Thus,

$$\sigma_{r_m\epsilon} \equiv \text{cov}(r_{mt}, \omega_{it} - \beta_i\omega_{mt}) = -\left[\text{cov}(r_{mt}, Z_{it}\gamma_i) - \frac{\text{cov}(r_{mt}, r_{it})}{\text{var}(r_{mt})}\text{cov}(r_{mt}, Z_{mt}\gamma_m)\right] \tag{18}$$

7 As it typically will be since the market value of a firm's debt and equity will move differently as market returns move. As an empirical matter, it is well known that firms are more likely to issue new equity during market rises than falls; see, *e.g.*, Marsh [1982]; MacKie-Mason [1988].

8 We are assuming that $\text{cov}(Z_{it}\gamma_i, \omega_{it}) = \text{cov}(Z_{it}\gamma_i, \omega_{mt}) = \text{cov}(Z_{mt}\gamma_m, \omega_{it}) = \text{cov}(Z_{mt}\gamma_m, \omega_{mt}) = 0$. Since the Z's are regressors in the second stage of the two-stage method, these are sufficient (although somewhat stronger than necessary) assumptions for consistent estimation in the second stage of the two-stage method. They are not necessary for consistent estimation in the first stage.

Typically, the covariance in (18) will be nonzero. For example, suppose there are no observable components of the shock to the market return, $Z_m = 0$. Then $\sigma_{r_m \epsilon} = -\operatorname{cov}(r_{mt}, Z_{it}\gamma_i)$. The Z_i might include factors such as leverage which will tend to be correlated with the market return (since real leverage depends on the market value of the firm's equity). Or, if the firm is a mineral producer, an exogenous commodity price shock might be an observable Z_i, and if the mineral is important enough (*e.g.*, oil), the market return might also move with such a price shock. Here, the price shock is also a Z_m, and both terms in the brackets of (18) are nonzero. The sum, then, will in general be non-zero.

The conclusion is that the one-stage estimation method will typically be inconsistent in market model applications. Nonetheless, several papers have used such an estimator when decomposing shocks to security returns into (*ex post*) observable components. These include the previously mentioned studies by Rose and by Smith, Bradley, and Jerrell. Other papers include Thomas Gilligan [1986] and Douglas Pearce and Vance Roley [1988].

A plausible criticism of our approach is that to obtain the consistency of our two-stage method we need the very restrictive condition that the components of the disturbance term are correlated with the explanatory variables such that the correlations precisely cancel out. Even though we have shown that this result follows from the efficient markets hypothesis or rational expectations, it could be argued that those conditions will never hold exactly in reality, thus rendering the two stage method inconsistent as well.

The point is well-taken. However, our assumptions are exactly those *which virtually every CAPM-based regression analysis maintains.* If the efficient markets assumption doesn't hold or very nearly so, then any estimation of the market model will yield inconsistent parameter estimates even if disturbance components are ignored altogether, because of the measurement error from that results replacing expected market returns with realized returns. Numerous event studies and other uses of the CAPM rely on the efficient markets hypothesis for consistent estimation of the market model parameters. If one is relying on this hypothesis, it seems reasonable to apply our two-stage method for estimating disturbance components, since it rests on exactly the same assumptions. If the two-stage method yields inconsistent estimates then the market model is flawed to begin with and should not be estimated using least squares in the first place.

In some cases, one's intuition predicts that the correlation between the observable component of the disturbance and the market return will be small. This might be because the disturbance component enters the model only on event days. [9] In other cases we might expect the correlation to be small based on our economic intuition about what does and does not co-vary with the market return. Finally, there will be those cases in which the correlation between the disturbance component and the market return may be quite substantial. Some such examples are news about money supply, interest rates, and inflation. Another variable likely to be correlated with the market return and frequently used in studies which use firm characteristics is the firm's leverage choice. [10]

We are sympathetic to the notion that it is not always worth employing complicated econometric techniques if the economics of the problem suggest that the added econometric complexity is unlikely to substantially improve the quality of the results. On the other hand, if it is very simple to do the estimation correctly there is little reason not to. Our procedure is very simple and straightfoward. Hence, even in cases in which the results may not change much one ought to estimate the parameters on disturbance components correctly.

4. Conclusion.

Empirical researchers frequently confront a paucity of reliable data. One source of plentiful data is stock market returns. Researchers in several fields of economics have turned to this data source. By estimating components of shocks to security returns, they have creatively used stock return data to investigate a wide variety of applied economic questions.

We have shown that an estimation procedure commonly employed in these studies will usually yield biased and inconsistent estimates of the model parameters. An alternative, straightforward two-stage estimation method is suggested. The econometrician should run

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[9] When the disturbance components enter the model only on event days, they are interacted with event dummies. Hence, they take on non-zero values on only a few days of the sample. Thus, even if the parts of the disturbance components are highly correlated with the market return, the correlation of the entire disturbance component may not be large. An example of this is in Rose.

[10] This is almost guaranteed if leverage is correctly measured using market values.

the simple CAPM, save the residuals, and use these residuals as dependent variables in a second regression. We provide a simple proof of consistency and intuition for why the one-stage estimator is inconsistent. We also note the correct covariance matrix for the second-stage coefficient estimates.

Our results are not limited to market model analyses. Any model which has the following characteristics is subject to the same problems: (a) the model without specification of the disturbance components satisfies the classical assumptions; (b) the observable components of the disturbance have a nonzero correlation with the other regressors in the model. For example, many rational expectations models satisfy these conditions. In any such case, the two-stage method should be used to estimate both the structural parameters and the parameters of the disturbance decomposition.

Appendix

1. ***Proof of Proposition 1.*** The assumptions of equation (1) and (2) include:

$$\frac{1}{T}X(Z\gamma + \omega) \xrightarrow{p} 0. \qquad (A.1)$$

Consistent estimation of β using least squares in (3) requires that $\frac{1}{T}X\omega \xrightarrow{p} 0$. (A.1) implies that plim $\frac{1}{T}X\omega = -$plim $\frac{1}{T}XZ\gamma$. By assumption, $\frac{1}{T}XZ\gamma$ has non-zero probability limit. ▮

2. ***Proof of Proposition 2.*** Applying least squares to equation (1) yields $\hat{y} = Py$, where $P = X(X'X)^{-1}X'$ the projection matrix of X, and the estimated disturbances:

$$\hat{\epsilon} \;=\; y - \hat{y} \;=\; (I - P)y \;=\; M\epsilon \qquad (A.2)$$

for $M = I - P$, where the last equality follows by substituting $X\beta + \epsilon$ for y. Substitute $\epsilon = \hat{\epsilon} + P\epsilon$, into (2) to obtain

$$\hat{\epsilon} = Z\gamma + [\omega - P\epsilon]. \qquad (A.3)$$

It is clear from equation (2) that Z is correlated with ϵ, suggesting that least squares estimation of (A.3) might yield inconsistent results. However, since the estimated disturbances from the first-stage regression are consistent estimates of the true disturbances, the measurement error term $(P\epsilon)$ in (A.3) vanishes as the sample size increases. Formally,

$$\frac{1}{T}Z'(P\epsilon) = \frac{Z'X}{T}\left(\frac{X'X}{T}\right)^{-1}\frac{X'\epsilon}{T}.$$

To invoke consistent estimation in the first-stage we assumed that $T^{-1}(X'X)$ is stochastically bounded and that $\frac{1}{T}X'\epsilon \xrightarrow{p} 0$. Thus, as long as the limiting covariances between Z and X are finite, $\frac{1}{T}Z'P\epsilon \xrightarrow{p} 0$, and least squares on the second-stage model yields consistent estimates of γ. The model of equation (1) satisfies the Gauss-Markov conditions. and thus the least squares estimates of β are best linear unbiased. ▮

3. ***Covariance Matrix for Second-Stage Regression.***

Write the second-stage estimated coefficients as:

$$
\begin{aligned}
\hat{\gamma} &= (Z'Z)^{-1}Z'\hat{\epsilon} \\
&= \gamma + (Z'Z)^{-1}Z'\omega - (Z'Z)^{-1}Z'P\epsilon
\end{aligned}
$$

by substitution of (A.2) to obtain the second equality. Then,

$$
\sqrt{T}(\hat{\gamma} - \gamma) = \left(\frac{Z'Z}{T}\right)^{-1} \frac{Z'\omega}{\sqrt{T}} - \left(\frac{Z'Z}{T}\right)^{-1} \frac{Z'X}{T} \left(\frac{X'X}{T}\right)^{-1} \frac{X'\epsilon}{\sqrt{T}} \qquad (A.4)
$$

Letting plim $Z'Z/T = Q_{zz}$, plim $Z'X/T = Q_{zx}$, and plim $X'X/T = Q_{xx}$, each positive definite, and assuming the other classical assumptions given in the text, we can apply the Lindeberg-Lévy central limit theorem, and the properties of multivariate normal vectors to yield a limiting normal distribution for the estimated parameter vector,

$$
\sqrt{T}(\hat{\gamma} - \gamma) \xrightarrow{d} \mathcal{N}[0, \sigma_\omega^2 Q_{zz}^{-1} + \sigma_\epsilon^2 Q_{zz}^{-1} Q_{zx} Q_{xx}^{-1} Q_{xz} Q_{zz}^{-1} - 2\sigma_{\omega\epsilon} Q_{zz}^{-1} Q_{zx} Q_{xx}^{-1} Q_{xz} Q_{zz}^{-1}]. \quad (A.5)
$$

Since $\epsilon = Z\gamma + \omega$, with $Z\gamma$ and ω assumed to be orthogonal, we have $\sigma_{\omega\epsilon} = \sigma_\omega^2$, and $\sigma_\epsilon^2 = \sigma_\nu^2 + \sigma_\omega^2$ where $\sigma_\nu^2 I_T = \text{plim } (1/T)(Z\gamma\gamma' Z')$, thus.

$$
\sqrt{T}(\hat{\gamma} - \gamma) \xrightarrow{d} \mathcal{N}[0, \sigma_\omega^2 Q_{zz}^{-1}(I - Q_{zx} Q_{xx}^{-1} Q_{xz} Q_{zz}^{-1}) \sigma_\nu^2 Q_{zz}^{-1} Q_{zx} Q_{xx}^{-1} Q_{xz} Q_{zz}^{-1}]. \qquad (A.6)
$$

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