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A Simple, Consistent Estimate for Disturbance Components in Financial Models by

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Abstract. Many recent papers have estimated components of the disturbance term in the "market model" of equity returns. In particular, several studies of regulatory changes and other policy events have decomposed the event effects in order to allow for heterogeneity across firms. In this paper we demonstrate that the econometric method applied in some papers yields biased and inconsistent estimates of the model parameters. We demonstrate the consistency of a simple and easily-implemented alternative method.

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Empirical researchers in industrial organization, international trade, and macroeconomics have recently found imaginative ways to exploit the abundance of financial market data. Their methods typically focus on decomposing abnormal equity returns. In the field of industrial organization, for example, Rose [1985] asks whether there are identifiable firm characteristics which might help explain the effect of deregulation on rents in the trucking industry. Smith, Bradley, and Jarrell [1986] apply a similar method to investigate the effects of oil price regulation on firms in the oil industry. These papers and others are creative attempts to use available stock market data to analyze interesting policy-related questions.

It is well understood that stock returns are generated by highly efficient, forward-looking markets. This has important economic and econometric implications. Some of the implications have been ignored in recent attempts to decompose abnormal returns. The result has been that several researchers have inadvertently employed econometric methods that yield biased and inconsistent estimates of the model parameters of interest. Other researchers using the same type of data and also decomposing abnormal returns have correctly estimated their models. In all cases, the choice of econometric technique appears to have been haphazard. Our goal in this paper is to set the record straight. There is a right way and a wrong way to econometrically decompose the disturbance term in financial models. Fortunately, the right way is simple and straightforward.

In Section 2, we consider estimating components of a well-behaved disturbance term in a general context. In the usual case components of the disturbance that are observable up to an unknown parameter vector would—indeed should—be included in the specification of the model's explanatory variables and estimated directly. However in an important class

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of problems it turns out that the conventional approach yields biased and inconsistent parameter estimates. That is, it may be better to leave some explanatory variables in the disturbance. Of course, the econometrician may want to estimate the effects of these disturbance components; we propose a simple and consistent method for doing so.

It may seem that our finding is of little practical interest. In fact the circumstance can arise in any rational expectations model. In particular, the situation occurs quite naturally in applications of the Capital Asset Pricing Model (CAPM). Thus, in Section 3 we explain how the economics underlying the efficient markets hypothesis imposes constraints on the choice of econometric technique. We also briefly discuss several recently published papers employing econometric techniques that may yield biased and inconsistent estimates of the model parameters.

1. General Treatment.

Consider a standard linear model,

\[ y_t = X_t \beta + \epsilon_t, \quad t = 1, \ldots, T, \]

for which the classical assumptions hold: \( E[\epsilon | X] = 0, E[\epsilon \epsilon' | X] = \sigma^2 I_T \), and \( \text{plim} \ X'X/T \) is finite, with \( X_t \) a \( 1 \times K_1 \) vector of explanatory variables. Then the least squares estimates of the vector \( \beta \) are unbiased, and consistent in large samples.

Suppose the econometrician believes that some components of the disturbance, \( \epsilon_t \), are in fact observable, and models the disturbance as

\[ \epsilon_t = Z_t \gamma + \omega_t, \]

with \( Z_t \) a \( 1 \times K_2 \) vector of observable variables, but still believes the orthogonality restriction on the \( X_t \), that \( E[\epsilon | X] = 0 \). Combine equations (1) and (2) to obtain

\[ y_t = X_t \beta + Z_t \gamma + \omega_t. \]

If both \( \beta \) and \( \gamma \) are of interest, the econometrician then might jointly estimate these parameters using least squares. Would such estimates be consistent? The requisite orthogonality conditions are that:

\[ \text{plim} \ X' \omega/T = 0 \quad \text{and} \quad \text{plim} \ Z' \omega/T = 0. \]
For simplicity, we assume throughout this note that the $Z$ are asymptotically uncorrelated with the $\omega$.

Since we have assumed that $X$ is uncorrelated with the total disturbance, $\epsilon$, we might expect that $X$ is also uncorrelated with each of $\epsilon$'s components, $\omega$ and $Z\gamma$. In fact this will not be true in many interesting cases. If $X$ is correlated with any of the observable disturbance components, $Z$, then $X$ must also be correlated with $\omega$, and least squares estimates of the parameters of equation (3) will be biased and inconsistent.\(^1\)

**Proposition 1.** *In the model given by (1) and (2), if the classical assumptions hold for (1), and if any columns of $X$ are asymptotically correlated with $Z\gamma$, then least squares will yield biased and inconsistent estimates of $\gamma$ and $\beta$ in equation (3).*

There is a simple alternative to one-stage estimation that yields consistent estimates of both the original structural parameters, $\beta$, and the disturbance components parameters, $\gamma$. Estimate the original model (1) using least squares. The estimated residuals from this regression are consistent for the true disturbances. Then estimate the model in (2), substituting the estimated disturbances from the first-stage regression for the true disturbances $\epsilon$:

$$\tilde{\epsilon}_t = Z\gamma + \xi_t. \tag{4}$$

Letting $P = X(X'X)^{-1}X'$ be the projection matrix of $X$, we show in the appendix that the disturbance vector $\xi$ in (4) is given by

$$\xi = \omega - P\epsilon. \tag{5}$$

Although $Z$ is correlated with $\epsilon$, the disturbances are estimated consistently in the first-stage regression and thus the measurement error term $P\epsilon$ vanishes as the sample size gets large. Thus $Z$ is asymptotically uncorrelated with the disturbance in (4) when, as we've assumed, $Z$ is uncorrelated with $\omega$.

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\(^1\) Proofs of the propositions are given in the appendix.
Proposition 2. Under the assumptions of Proposition 1, and if \( \text{plim} \ X'Z/T \) is finite and \( \text{plim} \ Z'\omega/T = 0 \), then the least squares estimates of \( \beta \) in equation (1) are best linear unbiased, and the estimates of \( (\gamma) \) in (4) are consistent.

The vector \( \sqrt{T}(\hat{\gamma} - \gamma) \) has a limiting multivariate normal distribution with mean vector zero, and covariance matrix

\[
\Omega = \sigma^2_\nu Q^{-1} (I - Q_{zz} Q^{-1} Q_{zz} Q^{-1}) + \sigma^2_\nu Q^{-1} Q_{zz} Q^{-1} Q_{zz} Q^{-1}
\]

where \( Q_{zz} = \text{plim} \ Z'X/T \) and so forth, \( \nu \equiv Z\gamma \), and \( \sigma^2_\nu \equiv \text{plim} \ (1/T)(Z\gamma'Z') \). It is straightforward to extend the results for heteroskedastic disturbances and, in a panel data context to a random effects model.\(^2\)


Several papers have applied the standard Capital Asset Pricing Model (CAPM) of Sharpe-Lintner-Mossin to analyze the effect of various factors on a firm's equity value. In this section, we show how the above results may be used to ensure consistent and efficient estimation of the CAPM in several common applications.

**Notation.** We employ the following notation:

- \( r_{it} \) realized return on security i in period t
- \( r_{mt} \) realized return on "market" portfolio in period t
- \( r_f \) risk-free return, assumed constant\(^3\)
- \( Z_{it} \) vector of effects, not necessarily firm-specific

The purpose and interpretation of the effects \( Z \) are explained below. Expectations are taken to be conditional on all information publicly available in the prior period.

"The Market Model." We briefly derive the empirical implementation of the CAPM, and demonstrate that it satisfies the orthogonality condition for unbiased and consistent

\(^2\) For instance, we show in the appendix how to use White's [1980] method to estimate the covariance matrix if there is heteroskedasticity of an unknown form. It should also be noted that since this is a problem of statistical endogeneity, instrumental variables methods could also be used to consistently estimate the parameters if appropriate instruments are available. A generalized method-of-moments estimator could be derived that made use of all of the statistical information and thus obtained relatively more efficient estimates than the two-stage estimates without additional instrumental information.
estimation. This result is well-known, but is repeated here for comparison to cases in which
the parameters are estimated inconsistently.

Equilibrium expected rates of return under the Capital Asset Pricing Model (CAPM) are:4

\[ E[r_{it}] = (1 - \beta_i) r_f + \beta_i E[r_{mt}] \quad \text{where} \quad \beta_i \equiv \frac{\text{cov}(r_{it}, r_{mt})}{\text{var}(r_{mt})} = \frac{\rho_{im}}{\sigma_m^2} \tag{6} \]

To obtain the empirically testable "market model", assume:

\[ r_{it} = E[r_{it}] + \varepsilon_{it} \quad \text{and} \quad r_{mt} = E[r_{mt}] + \varepsilon_{mt} \tag{7} \]

where the \( \omega \)'s are uncorrelated forecasting errors, according to the efficient markets hy-
pothesis. Substitution yields the market model:

\[ r_{it} = \alpha_i + \beta_i r_{mt} + \varepsilon_{it}, \quad \text{with} \quad \varepsilon_{it} = \varepsilon_{it} - \beta_i \varepsilon_{mt} \tag{8} \]

where \( \alpha_i \equiv (1 - \beta_i) r_f \).

From (7) it is clear that \( \text{cov}(r_{mt}, \varepsilon_{mt}) \neq 0 \), and thus it may appear that the mar-
ket model (8) has a simultaneity bias preventing consistent estimation by least squares.
However, the orthogonality required is that \( \text{plim} \frac{r_{mt}^t \varepsilon_{i}}{T} = 0 \), i.e., that the asymptotic
correlation between the market return and the total disturbance be zero. But

\[ \text{cov}(r_{mt}, \varepsilon_{it}) = \frac{\text{cov}(r_{mt}, r_{it}) - \text{cov}(r_{mt}, r_{it})}{\text{var}(r_{mt})} \var(r_{mt}) \tag{9} \]

where the first equality follows from substitution of (8) and the second uses the definition
of \( \beta_i \). Thus, the orthogonality condition is met, and least squares estimation is consistent.
The result follows from the assumption of efficient markets. If the disturbance were corre-
lated with the market return, it would be possible to exploit the covariance to produce an

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4 See Jensen [1972] for a derivation of this result. The CAPM requires only that all investors have
identical subjective beliefs. We invoke rational expectations below to derive the "market model" from the
CAPM.
arbitrage opportunity with the same expected return but lower risk. Such arbitrages are ruled out in equilibrium.

**Decomposing the Disturbance in the Market Model.** Suppose the econometrician believes that

\[ \epsilon_{it} = \sum_{k=1}^{K} Z_{kit} \gamma_{ki} + \omega_{it}, \]  

(10)

where the \( Z_k \) are observable effects (but need not necessarily vary across firms). We propose a taxonomy of cases when such a specification arises:

(1) **"Standard Event Study."** Let the disturbance components be

\[ Z_{kit} = \begin{cases} 1, & \text{if the } k - \text{th event occurs on day } t \\ 0, & \text{otherwise.} \end{cases} \]  

(11)

Then, \( \gamma_{ki} \) estimates the abnormal return for firm \( i \) on the day of the \( k \)-th event.\(^5\)

(2) **"Heterogeneous-Effect Event Study."** It has recently become standard practice to investigate whether events might have predictably different effects on different firms, particularly in studies of the effects of regulatory changes on firm value.\(^6\) Heterogeneity is modeled by decomposing abnormal returns into components which depend in part on observable firm characteristics. In general, let

\[ Z_{kit} = \delta_{kit} \tilde{Z}_{kit} \]

\[ \delta_{kit} = \begin{cases} 1, & \text{if the } k - \text{th event occurs on day } t \\ 0, & \text{otherwise.} \end{cases} \]  

(12)

The \( \tilde{Z}_{kit} \) are variables affecting the impact of the event on the firm's return; they may vary across firm or time, or both. Examples include the leverage ratio and elements of the firm's production function (such as average haul length for trucks; see Rose [1985]).

\(^5\) We need not be concerned if the *ex ante* mean of the event effects is non-zero: \( E[Z_{it} \gamma_{i}] \neq 0 \). As usual in a regression model, the mean can be absorbed into the intercept without affecting any of the results. The same remark holds for the other cases in our taxonomy as well.

\(^6\) See, e.g., Rose [1985]; Smith, Bradley, and Jarrell [1986]; Borenstein and Zimmerman [1987]; Mitchell and Maloney [1988].
(3) "Error Components Study." Some studies do not focus on particular events, but are interested in more generally decomposing the disturbance in security returns into various sources of unexpected shocks. Typically, the econometrician will model various "news" or "innovations" variables.\(^7\)

As shown in Section 1, the consistency of a one-stage estimation procedure in the above examples will depend on whether the market return is correlated with the observable disturbance components. First consider the "Standard Event Study". In this case it is usually reasonable to assume that the days on which events occur are not correlated with market returns, at least for events which affect a small fraction of all firms in the market. Thus, estimating the abnormal event-day returns in one-stage will typically yield consistent results.\(^8\)

The outlook is not so sanguine for one-stage estimation of either the "Heterogeneous-Effect" or "Error Components" models. For example, aggregate demand shocks \((Z_t)\) may affect various firms differently. Since in general aggregate demand shocks will be correlated with realized market returns, \(\text{cov}(r_{mt}, Z_{it}) \neq 0\) and one-stage estimates will be inconsistent.

It is instructive to consider formally the sources of inconsistency in the one-stage method. Suppose that both sources of error in the market model are decomposed into observables and unobservables as follows:

\[ e_{it} = Z_{it} \gamma_i + \omega_{it} \quad \text{and} \quad e_{mt} = Z_{mt} \gamma_m + \omega_{mt} \]  

(13)

where \(Z_i\) is a \(T \times K_i\) matrix, and \(Z_m\) is a \(T \times K_m\) matrix, both of observables, and \(\gamma_i\) is \(K_i \times 1\) and \(\gamma_m\) is \(K_m \times 1\), both vectors of unknown parameters. Thus, the \(Z_{it}\) represent ex post observable components of innovations in firm returns, and the \(Z_{mt}\) represent observable

\(^7\) See, e.g., Grossman and Levinsohn [1989]; Pearce and Roley [1988]; French, Ruback and Schwert [1983].

\(^8\) In fact, most event studies estimate the market model on pre-event data, and then use prediction errors as measures of abnormal returns, in order to avoid the problems that arise if the events change the model parameters.
components of innovations in the market return. Substitution of (13) into the market model (8) yields:

\[ \omega_{it} - \beta_i \omega_{mt} = \tau_{it} - \alpha_i - \beta_i r_{mt} - Z_{it} \gamma_i + \beta_i Z_{mt} \gamma_m \]  

Recall that consistency of the one-stage method is determined by the correlation between the market return and the unobservable components of the disturbance.\(^9\) We can write this as

\[
\text{cov}(r_{mt}, \omega_{it} - \beta_i \omega_{mt}) = \text{cov}(r_{mt}, \tau_{it}) - \beta_i \text{cov}(r_{mt}, r_{mt}) - \beta_i \text{cov}(r_{mt}, Z_{it} \gamma_i) - \beta_i \text{cov}(r_{mt}, Z_{mt} \gamma_m)
\]

(15)

by substituting (14) into the covariance expression on the left.

The first square-bracketed term in (15) equals zero by the definition of \( \beta_i \). Thus,

\[
\sigma_{r_{mt}} \equiv \text{cov}(r_{mt}, \omega_{it} - \beta_i \omega_{mt}) = - \left[ \text{cov}(r_{mt}, Z_{it} \gamma_i) \right] - \text{cov}(r_{mt}, Z_{it} \gamma_i)
\]

(16)

Typically, the covariance in (16) will be nonzero. For example, suppose there are no observable components of the shock to the market return, \( Z_m = 0 \). Then \( \sigma_{r_{mt}} = - \text{cov}(r_{mt}, Z_{it} \gamma_i) \). The \( Z_i \) might include factors such as leverage which will tend to be correlated with the market return.\(^10\) Or, if the firm is a mineral producer, an exogenous commodity price shock might be an observable \( Z_i \), and if the mineral is important enough (e.g., oil), the market return might also move with such a price shock. Here, the price shock is also a \( Z_m \), and both terms in the brackets of (16) are non-zero. The sum, then, will in general be non-zero.

The conclusion is that the one-stage estimation method will typically be inconsistent in market model applications. Nonetheless, several papers have used such an estimator when decomposing shocks to security returns into (ex post) observable components. These

\(^9\) We are assuming that \( \text{cov}(Z_{it} \gamma_i, \omega_{it}) = \text{cov}(Z_{it} \gamma_i, \omega_{mt}) = \text{cov}(Z_{mt} \gamma_m, \omega_{it}) = \text{cov}(Z_{mt} \gamma_m, \omega_{mt}) = 0 \). Since the \( Z_i \)'s are regressors in the second stage of the two-stage method, these are sufficient (although somewhat stronger than necessary) assumptions for consistent estimation in the second stage of the two-stage method. They are not necessary for consistent estimation in the first stage.

\(^10\) Real leverage depends on the market value of the firm's equity. Also, firms are more likely to issue equity during market rises than falls; MacKie-Mason [1988].
include the previously mentioned studies by Rose and by Smith, Bradley, and Jarrell. Other papers include Gilligan [1986], and Pearce and Roley [1988], and French, Ruback and Schwert [1983].

In some cases, one's intuition predicts that the correlation between the observable component of the disturbance and the market return will be small. This might be because the disturbance component enters the model only on event days.\(^{11}\) In other cases we might expect the correlation to be small based on our economic intuition about what does and does not co-vary with the market return.\(^{12}\) Finally, there will be those cases in which the correlation between the disturbance component and the market return may be quite substantial. Such examples might include news about money supply and interest rates.

We are sympathetic to the notion that it is not always worth employing complicated econometric techniques if the economics of the problem suggest that the added econometric complexity is unlikely to substantially improve the quality of the results. On the other hand, if it is very simple to do the estimation correctly there is little reason not to. Our procedure is very simple and straightforward. Hence, even in cases in which the results may not change much one ought to estimate the parameters on disturbance components consistently.

3. Conclusion.

Empirical researchers frequently confront a paucity of reliable data. One source of plentiful data is stock market returns. Researchers in several fields of economics have turned to this data source. By estimating components of shocks to security returns, they have creatively used stock return data to investigate a wide variety of applied economic questions.

We have shown that an estimation procedure commonly employed in these studies will usually yield biased and inconsistent estimates of the model parameters. An alternative,

\(^{11}\) When the disturbance components enter the model only on event days, they are interacted with event dummies. Hence, they take on non-zero values on only a few days of the sample. Thus, even if the parts of the disturbance components are highly correlated with the market return, the correlation of the entire disturbance component may not be large.

\(^{12}\) Pearce and Roley [1988] found a corrected $R^2$ of only 0.06 in a regression of the unanticipated inflation change on the market return.
straightforward two-stage estimation method is suggested. The econometrician should run the simple CAPM, save the residuals, and use these residuals as dependent variables in a second regression. We provide a simple proof of consistency and intuition for why the one-stage estimator is inconsistent. We also note the correct covariance matrix for the second-stage coefficient estimates.
Appendix

1. **Proof of Proposition 1.** Consistent estimation of $\beta$ using least squares in (3) requires that $\text{plim} \ X'(\omega)/T = 0$. The assumptions of equation (1) include $\text{plim} \ X'(Z\gamma + \omega)/T = 0$, which implies that $\text{plim} \ X'(\omega)/T = -\text{plim} \ X'Z\gamma/T$. By assumption, $\text{plim} \ X'Z\gamma/T$ has non-zero probability limit.

2. **Proof of Proposition 2.** Least squares applied to equation (1) yields $\hat{\epsilon} = (I - P)\epsilon$, where $P = X(X'X)^{-1}X'$. Substitution into (2) gives $\hat{\epsilon} = Z\gamma + [w - P\epsilon]$. By assumption, $Z'(\omega)/T = 0$. Plim $Z'(P\epsilon)/T = \text{plim} \ (Z'X/T)(X'X/T)^{-1}(X'\epsilon/T) = 0$ because the first two terms have finite plim and the last term has zero plim by assumption, so the estimates of $\gamma$ are consistent. The model of equation (1) satisfies the Gauss-Markov conditions, and thus the least squares estimates of $\beta$ are best linear unbiased.

3. **Covariance Matrix for Second-Stage Regression.** Write the second-stage estimated coefficients as:

$$\hat{\gamma} = (Z'Z)^{-1}Z'\hat{\epsilon} = \gamma + (Z'Z)^{-1}Z'\omega - (Z'Z)^{-1}Z'P\epsilon$$

by substitution for $\hat{\epsilon}$ to obtain the second equality. Then,

$$\sqrt{T}(\hat{\gamma} - \gamma) = \left( Z'Z \right)^{-1}Z'\omega \sqrt{T} - \left( Z'Z \right)^{-1}Z'X \left( X'X \right)^{-1}(X'\epsilon/T) \sqrt{T}$$

Let plim $Z'/T = Q_{zz}$, plim $X'/T = Q_{xx}$, and plim $Z'/X = Q_{zx}$ and apply the Lindeberg-Lévy central limit theorem to get

$$\sqrt{T}(\hat{\gamma} - \gamma) \xrightarrow{d} \mathcal{N}[0, \sigma_{\omega}^2 Q_{zz}^{-1} + \sigma_{\epsilon}^2 Q_{zx}^{-1} Q_{zz} Q_{xx} Q_{zz}^{-1} - 2\sigma_{\omega}\sigma_{\epsilon} Q_{zz}^{-1} Q_{xx} Q_{zx}^{-1} Q_{xx}^{-1}].$$

Since $\epsilon = Z\gamma + \omega$, with $Z\gamma$ and $\omega$ assumed to be orthogonal, we have $\sigma_{\omega}\epsilon = \sigma_{\omega}^2$, and $\sigma_{\epsilon}^2 = \sigma_{\epsilon}^2 + \sigma_{\omega}^2$ where $\sigma_{\omega}^2 = \text{plim} \ (1/T)(Z\gamma'(Z\gamma'))$, so we can write

$$\sqrt{T}(\hat{\gamma} - \gamma) \xrightarrow{d} \mathcal{N}[0, \sigma_{\omega}^2 Q_{zz}^{-1}(I - Q_{xx} Q_{xx}^{-1} Q_{zx} Q_{zx}^{-1}) + \sigma_{\epsilon}^2 Q_{zz}^{-1} Q_{xx} Q_{zx}^{-1} Q_{xx}^{-1}].$$
Suppose the disturbances exhibit heteroskedasticity of unknown form: $E[\epsilon'X] = \Omega_\epsilon$, $E[\omega'|X,Z] = \Omega_\omega$, $\Omega_\epsilon$ and $\Omega_\omega$ diagonal. Then the covariance matrix is

$$Q_{zz}^{-1} \left( \text{plim} \frac{Z'\omega\omega'Z}{T} \right) + Q_{zz}^{-1} Q_{xx}^{-1} \left( \text{plim} \frac{X'\epsilon\epsilon'X}{T} \right) Q_{xx}^{-1} Q_{zz}^{-1} Q_{zz}^{-1}$$

$$- 2Q_{zz}^{-1} \left( \text{plim} \frac{Z'\omega\epsilon'X}{T} \right) Q_{xx}^{-1} Q_{zz}^{-1}$$

which can be consistently estimated using $\hat{\epsilon}$ and $\hat{\omega}$ from the two regressions, following the method of White [1980].
References


