Public Ownership as a Sufficient Condition for the Soft Budget Constraint

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Abstract

The soft budget constraint refers to the phenomenon that a firm expects to survive in the future when it is financially insolvent. This is widely recognized as one of the important reasons for the inefficiency and, eventually, the collapse of contemporary socialist economies. The purpose of this paper is to explore the cause of the soft budget constraint in a formal way. Using a comparative setting which includes both the socialist system and the market system, I show that public ownership of capital is a sufficient condition for the soft budget constraint. The intuition is that under public ownership of capital, the firm (which consists of labor and management) retains its control right regarding the disposition of the capital. Thus, the ex post termination decision of the firm’s investment project is socially efficient, which means that so long as the financial loss is not as big as the potential loss of human capital due to the termination of the firm, the firm should survive. This gives rise to the soft budget constraint. Similarly, the “hard” budget constraint occurs under private ownership, where the firm can be excluded from the termination decision. Next, I show that ex ante, the soft budget constraint is inefficient while the “hard” budget constraint can be efficient, since given limited liability, the only way to discourage the firm from proposing bad projects is by threatening to incur losses to the human capital when the project indeed is bad. Furthermore, it is shown that public ownership of capital causes excessive (relative to the socially optimal level) demand for investment in socialist economies, while private ownership of capital results in under-investment in market economies. Such phenomena have been widely recognized in comparative economics.

JEL Classification Code: P31, P51

Key Words: Public Ownership, Soft Budget Constraint, Investment Hunger
1. Introduction

The purpose of this paper is to explore the cause and consequences of the phenomenon of the soft budget constraint in contemporary socialist economies. It is argued that public ownership of capital is a sufficient condition for the soft budget constraint. For the purpose of comparison, the case of private ownership of capital in market economies is also studied.

The concept of the soft budget constraint was first introduced by Janos Kornai (see Kornai (1980)). He argued that in socialist economies the financial constraint on the enterprise is *ex ante* "soft". In other words, when making economic decisions the socialist firm is not concerned with negative profit in the future, since it can expect to get financial subsidies in the future in case of economic failure. Therefore, the behavior of the socialist enterprise can never be the same as that of its counterparts in market economies.

The phenomenon of the soft budget constraint (SBC) is regarded as one of the most fundamental causes of the collapse the contemporary socialist economies. The soft budget constraint gives rise to investment hunger, chronical shortage and inflation. Moreover, the market socialist reforms in 1970's and 1980's did not help "harden" the budget constraint at all. For example, in China through the reform years from 1986 to 1988, a state-owned enterprise can expect to get 0.97RMB subsidy for every 1.00RMB of financial loss and these numbers hardly changed through the years [Li (1990)].

While the importance of the soft budget constraint is widely recognized, there are not many formal economic explanations for the cause of the soft budget constraint. The difficulty in finding such explanations seems to be that the cause of the soft budget constraint must be peculiar to the socialist economy, since in general the soft budget constraint is not a problem in market economies. Moreover, such explanations should answer the question why the market socialist experiment failed to solve the problem of the soft budget constraint.

This paper explores one cause of the soft budget constraint in the socialist economy, that is, the public ownership of capital. It will be argued that public ownership entitles the socialist enterprise in an adverse economic environment to relatively better treatment than their counterparts in the market economies. Therefore, *ex ante* firms in the socialist economies will face less economic pressure and will demand more investment than firms in market economies.

The basic idea can be explained in the following scenario. To begin with, the manager of a firm proposes a project whose profitability is random. The project can last either one period or two periods. The profitability becomes known when the state is realized at the end of the first period and it will be decided whether the project may last for one more period. The owner of the capital has the sole decision right at this time, since it is impossible to sign a contract that links this decision and various possible states which are very complicated. Financially, management and labor (together they will be referred to as "the firm") have only limited liability. That is, if the profit level is negative, no one can force the firm to absorb part of this loss. However, if the project is terminated, management as well as labor will suffer a human capital loss. For example, management and labor may have to search for new jobs and the search is costly.

Given the above set-up, ownership will be a factor in the investment decision. At the end of period 1, if the creditor is the sole owner, then he will decide to terminate the project whenever the profit is negative, since management and labor cannot help absorb the financial loss of the firm. At the same time, if the creditor,
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At the end of period 1, if the creditor is the sole owner, then he will decide to
terminate the project whenever the profit is negative, since management and labor
cannot help absorb the financial loss of the firm. At the same time, if the creditor,
the management and the labor jointly own the capital, then the project cannot
be terminated unless the combined welfare is maximized by such a decision. That
means that the termination decision is socially efficient at time 1. This explains
the phenomenon of the soft budget constraint. It seems that a soft budget con-
straint is more efficient than a "hard" budget constraint. However, this is not true.
The correct argument is that a hard budget constraint is good because it exposes
management to the risk of human capital loss when the profit is negative. In fact
this is the only way to discipline management so that it can avoid proposing bad
investment projects. Moreover, a profit sharing rule can serve as a complement to
such a "hard" budget constraint, since management will be compensated when the
profit is positive. Quite the opposite, a soft budget constraint fails to discipline
management and a profit sharing rule cannot fill the role, since management and
labor have limited financial liability and thus cannot be punished in the case of
financial losses. To summarize, a "hard" budget constraint is caused by private
ownership of capital by the creditor while a soft budget constraint is caused by
public ownership of capital. The former is ex post inefficient, the latter ex post
efficient. However, it will be shown that, in general, the former is more efficient ex
ante than the latter.

Dewatripont and Maskin (1989) have a different explanation for the soft budget
constraint. Their starting point is that a fundamental difference between the so-
cialist economy and the market economy is the degree of concentration of financial
institutions, such as banks. They observe that banks in market economies are of
limited size. Therefore if a project turns out to be a "bad" one, then the bank has
to terminate the project due to the bank's own liquidity constraint, even though it
is sequentially efficient to finance the "bad" project for one more period. Therefore,
the size of the bank serves as commitment not to re-finance "bad" projects in the
market economy. To the contrary, in the socialist economy there is only one bank
and such a commitment device does not exist. Since public ownership of capital and a high concentration of financial institutions go hand in hand in reality, their arguments and those in this paper are complementary.

Central to the arguments in this paper is an assumption that under public ownership in socialist economies employees (management and labor) of the firm have an inalienable control right in the disposition of physical capital of the firm. The official socialist ideology claims that the socialist system is superior to the capitalist system. It criticizes harshly the capitalist system by claiming that labor is alienated from production materials (capital) and that for the first time in human history, labor is not separated from production materials. In the constitutions of all socialist countries, it is stated that to work is both the right and obligation of all citizens. It can be argued that citizens in socialist countries treated (although involuntarily) their personal political rights for this inalienable right to have a job. This is perhaps the fundamental reason for Kornai's observation that in socialist countries the employees of the firm and the bureaucrats have a paternalist relationship — the formal can always rely on the latter for protection (Kornai 1980). The most prominent manifestation of this inalienable right came after the demise of the socialist system. In the process of privatization in the former socialist countries, one of the biggest problem is that employees of the state-owned enterprise have strong demand for significant amount of shares of the privatized firm. Shleifer and Vishny (1992) described this situation in Russia.

2. The Model

Consider the life cycle of an enterprise. There are two time periods and three players: the creditor, the management and labor. The creditor becomes the bureaucrat under the socialist system. At the beginning of period 1, management has a blueprint of a project. A firm may be set up for this project. The profitability of this project is uncertain to management, although management is better informed about it than other parties. The profit will be known to all at the end of period 1. Then the firm can last one more period if so decided. Management knows about the exact distribution of the profitability of the project, but the creditor only has some rough knowledge about the parameter of the distribution. By the end of period 2, the firm will be liquidated.

In order to start up the firm to do the project, both physical capital and hu-
human capital will be needed. Management and labor provide the human capital. Management will propose the project to the creditor to obtain a unit of physical capital.

The physical capital is sunk in the project once it is invested. As long as the firm is in operation, its management enjoys a private benefit of $B$, which is not necessarily monetarized. If the project is terminated at the end of period 1, management will lose this control benefit for the whole period 2. In other words, management has to wait until the end of period 2 to find a new position. Finally, management can always pursue a riskless project, which gives 0 profit all the time and therefore management would stay in control and enjoys $B$ all the time. Under public ownership, the control benefit is shared by management and labor, since the difference between management and labor is blurred in this kind of economies. From now on, I will treat management and labor as one integrated entity under public ownership.

The utility function of management is the sum of the control benefit $B$ and the monetary incentive associated with profit. The creditor maximizes the expected total profit.

At the beginning of period 1, management and the creditor can sign any contract contingent upon the profitability of the project. However, the contract cannot refer to the underlying state, since as argued above the state can be very complicated. Theoretically, there is a distinction between the state and the profitability. At the end of period 1, it is the realized state, rather than the realized profit, that determines whether the project should last for one more period. For example, let the state be $t$ and $e(t)$ be the fundamental profit and $\epsilon_t$ be random noise for period $i$ with 0 expectation. Then the profit for period 1 is $\pi_1 = e(t) + \epsilon_1$ and the profitability at the end of period 2 is $\pi_2 = e(t) + \epsilon_2$. At the end of period 1 when the state $t$ is realized, the expected $\pi_2$ is $e(t)$. Therefore, it is $e(t)$ or the state $t$ rather than $\pi_1$ that determines whether the project should last for one more period. Since the contract is not contingent upon $t$, the decision of the project's fate is left unspecified in the initial contract. The owner(s) of the capital is left with this decision right.

However, purely for the purpose of technical ease, the discussion will be confined to noise-less cases, i.e. $\epsilon_t = 0$. A more general situation with noise should yield similar results. Moreover, assume that there are only three states, the profit corresponding to each state is $\theta_1$, $\theta_2$ and $\theta_3$ respectively. Again, this is merely a simplifying assumption, as a larger number of states should not change the conclusion at all. The following assumptions are made on the profit level.

A1. $\theta_1 > 0$
A2. $\theta_2 < 0 < \theta_3 + B$
A3. $\theta_2 < \theta_3 < 0$ and $\theta_3 + B < 0$

In simple English, the above assumptions are that state 1 is profitable; state 2 is unprofitable but socially efficient; state 3 is socially undesirable.

The information structure is the following. Management knows the probability of the occurrence of each state: $q_1$, $q_2$ and $q_3$. As a simplifying assumption, $q_2 = \gamma(1 - q_1)$, where $0 \leq \gamma \leq 1$ and is a constant. The creditor only knows that $q_1$ follow a uniform distribution in the range of $[0,1]$. To summarize:

A4. $q_2 = \gamma(1 - q_1), 0 \leq \gamma \leq 1$
A5. $q_1 \sim U(0,1)$

The payoffs to management and to the creditor are dependent on a profit sharing contract signed at the beginning of period 1. As a result, management will get $s$ share of the profit at the end of period 1 and 2. However, due to limited liability,
management cannot honor such a contract when the profit is negative. Thus the creditor will absorb all the negative profit.

With the above set up, once the project is approved by the creditor, the expected payoff of a manager with a project \((q_1, q_2, q_3)\), is

\[
Y_M = s\theta_1 q_1 + B + (B + s\theta_1)P(c)
\]

where \(P(c)\) is the probability that the project continues at the end of period 1. As an alternative, management can always propose a risk-free project which has 0 profit but guarantees the management's control benefit. The payoff to management with a risk-free project is \(2B\). Management proposes a risky project only when the project brings a higher payoff for him than a risk-free one.

The payoff to the creditor is more complicated to analyze than that of management. Suppose that the approved project is characterized by the parameters \((q_1, q_2, q_3)\). At the end of period 1, if the profit is positive (i.e. when state 1 is realized), then the incentive contract is honored and the creditor gets \((1 - s)\) share of the profit. If the other two states are realized, the profit is negative. Due to limited financial liability, the creditor carries all the negative profit. In the second period, if the project continues, the same situation arises. In short, the expected payoff to the creditor is

\[
Y_M(q_1, q_2, q_3) = (1 - s)\theta_1 q_1 + \theta_2 q_2 + \theta_3 q_3 + P(c)(1 - s)\theta_1 q_1 + \theta_2 + \theta_3
\]

The creditor approves the proposed project only when her expected payoff from all the projects proposed exceeds 0, which is the expected payoff from a risk-free project. That is,

\[
Y_M = E[y_M(q_1, q_2) \mid (q_1, q_2, q_3) \text{ is proposed}] \geq 0
\]

The creditor’s control variable is the profit sharing rule \(s\), other than the decision whether to approved a project. In general, a high \(s\) will encourage management to propose many investment projects.

3. The Cause of the Soft Budget Constraint

At the end of period 1, uncertainties about the profitability of the firm disappear. A soft budget constraint means that the firm continues to operate even when the observed profit is negative, while a hard budget constraint says that the firm will be terminated when the observed profit is positive. In the following it will be argued that the hard budget constraint arises under private ownership and the soft budget constraint arises under public ownership.

3.1. The Hard Budget Constraint and Private Ownership

When the creditor is the sole owner of capital, she has the decision to terminate the firm at the end of period 2. If state 1 is realized, the profit is \(\theta_1\) which is positive. It is in the interest of the creditor to let the firm continue for one more period. When either state 2 or state 3 are the true state, the profit is either \(\theta_2\) or \(\theta_3\) and is negative. It is not in the interest of the creditor to see the firm to survive, unless management can compensate the creditor for losses caused by the continuation of the firm. However, this is not possible since management has limited wealth. Therefore, under private ownership the firm will be terminated in state 2 and state 3 at the end of period 1. To summarize, here is lemma 1.
Lemma 1: (The hard budget constraint) Under private ownership, the firm will survive beyond period 1 only if state 1 is realized, which implies that the profit is positive.

3.2. The Soft Budget Constraint and Public Ownership

Under public ownership of capital, both the creditor and management/labor are owners of capital. Management and labor in particular have inalienable rights regarding decisions unspecified in the contract signed at the beginning of period 1. The creditor, management and labor bargain about whether and how the firm should be terminated.

Without losing generality, I will use the generalized Nash bargaining solution to model the outcome, since the solution to any extensive form bargaining game is equivalent to that of a generalized Nash bargaining game. The outside option of the bargaining is that the firm will continue to exist, since the capital is sunk in the form of tangible assets. In addition, I will assume that the bargaining power \( \alpha \) of management and labor is 0. This seems to be a very realistic assumption, since the bureaucrat is at a higher level of hierarchy than management/labor. In other words, the best that management/labor can achieve in terms of utility level is that of the outside option.

The outcome of the Nash bargaining game is efficient, i.e. the outcome should maximize the combined payoff of both parties. By assumptions A1, A2 and A3, the firm will continue in states 1 and 2 and both parties should agree to terminate the project in state 3 after some transfer arrangements. In state 1, the firm will continue. Assuming that the profit sharing contract will still be honored, the payoff to the bureaucrat and that to the management are

\[
((1 - s)\theta_1, s\theta_1 + B).
\]

In state 2, the firm will continue, since this is the efficient outcome. To put it another way, although it is costly for the bureaucrat to let the firm survive, bribing management to agree to close the firm is more costly. Clearly, in this case the payoff is \((\theta_2, B)\), since management will not be able to sustain any profit loss due to the limited liability condition.

Finally, in state 3, the profit is negative. The bureaucrat would like the firm to be terminated as soon as possible. In order to persuade management/labor to agree, the bureaucrat has to compensate their human capital loss. Let \( t \) be the compensation. In order for management/labor to agree to the termination of the firm, \( t \) has to be at least as big as \( B \). The generalized Nash bargaining outcome says that both parties divide the efficiency gain according to their bargaining power. For the bureaucrat, the payoff is

\[
\theta_3 + (1 - \alpha)(0 - \theta_3 - B) = -B
\]

where \( \alpha = 0 \) is the bargaining power of the firm. For management the payoff is

\[
B + \alpha(0 - \theta_3 - B) = B
\]

The following lemma summarizes the condition under which the firm will be terminated.

Lemma 2: (Soft Budget Constraint) Under public ownership, the firm will survive if the realized state is 1 or 2. The firm will be terminated at the end of period 1 only when the realized state is state 3. The termination decision for the project under rule becomes \( s_2 \), the same set of results can still be reached. What actually matters is the sum of \( s_1 \) and \( s_2 \).
4. Investment Decisions under Private and Public Ownership

As was argued above, after the uncertainty is resolved and the state is realized at the end of period 1, the private ownership system terminates the project more often than the public ownership system. Moreover, ex post, a hard budget constraint under private ownership is socially inefficient, while a soft budget constraint under public ownership is always socially efficient. In this section, I will look at the whole picture and compare the overall efficiency of investment decisions in these systems.

4.1. The Case of Private Ownership

When the creditor is the sole owner of capital, the termination decision is very harsh to management/labor. The project will be terminated as soon as the realized profit is negative. Knowing that, management is careful in proposing investment projects. What kind of investment project will then be proposed by management?

Let \((q_1, q_2, q_3)\) be parameter of an investment project. The expected payoff to management is easy to calculate. If the state is 1, then the project will last for two periods and the total payoff to management is \(2B + 2s\theta_1\). If the state is 2 or 3, then the project will be terminated after 1 period and the payoff to the management is \(B\), since management cannot be held responsible for negative profit. Therefore management can expect to get the payoff of

\[ Y_M(q_1, q_2, q_3) = q_1(2B + 2s\theta_1) + q_2B + q_3B \]

Management is only interested in projects which bring higher payoff than the risk free project. Therefore, if \((q_1, q_2, q_3)\) is proposed, it must be true that

\[ Y_M(q_1, q_2, q_3) = q_1(2s\theta_1 + B + q_1) \geq 2B \]

That is

\[ q_1 \geq \frac{B}{2s\theta_1 + B} \]

Lemma 3: (Project Selection Criterion (PSC1)) Under private ownership of capital, management will propose a project such that

\[ q_1 \geq q = \frac{B}{2s\theta_1 + B} \]

From this project selection criterion, a high profit share \(s\) will set a relatively low minimum project quality level \(q_0\). A high control benefit will increase this minimum level. These relationships are intuitively easy to understand, because a high \(s\) is equivalent to a high lump sum transfer in expected utility to management team that proposes an investment project at the beginning of period 1.

The creditor anticipates this project selection criterion by management and will set the profit sharing rule accordingly in order to maximize the expected profit. Supposing that a project \((q_1, q_2, q_3)\) is approved the creditor’s expected profit is calculated in the following way. If the state turns out to be 1, the project will last for two periods and the creditor will get \(2(1 - s)\theta_1\) from these two periods. If the state is 2 (or 3), the project will be canceled at the end of period 1 and the creditor has to sustain the entire loss from the first period, which is \(\theta_2\) (or \(\theta_3\)). Therefore
the expected payoff to the creditor from such a specific project is

\[ y_C(q_1, q_2, q_3) = 2\theta_1(1 - s)q_1 + \theta_2 q_2 + \theta_3 q_3 \]

By assumption A4, \( y_C \) becomes

\[ y_C = 2\theta_1(1 - s)q_1 + \theta_2 q_2(1 - \gamma)(1 - q_1) \]

\[ = 2\theta_1(1 - s)q_1 + \gamma \theta_2 (1 - \gamma) q_2(1 - q_1) \]

Due to information asymmetry, the creditor does not know the exact value of \((q_1, q_2, q_3)\) when the projected is approved. Therefore \( y_C \) is still a random variable to the creditor at time 0. However, the creditor is able to calculate the distribution of \((q_1, q_2, q_3)\) for the proposed projects according to management's project selection criterion (PSC1). The creditor will maximize his expected payoff conditional on PSC1. The creditor sets the appropriate level of the profit sharing rule \( s \) so that PSC1 maximizes the expected payoff to the creditor.

**Proposition 1:** Under private ownership of capital, the optimal profit sharing rule \( s \) is strictly between 0 and 1 and is increasing with \( B \).

**Proof:** The fact that \( s \) is strictly between 0 and 1 is very easy to establish. \( s = 1 \) is not optimal since all the positive profit will be given out to management. By slightly reducing \( s \), the creditor can do better. \( s = 0 \) is not optimal either, since no project will be proposed by management. Management will lose control benefit \( B \) in states 2 and 3, it only gets \( B \) in state 1. Thus, management will be better off by staying with risk-free projects.

To prove that \( s \) increases with \( B \), one needs to utilize that first order condition

of the creditor's maximization problem. The expected profit of the creditor is

\[ Y_C = E(y_C | PSC1) = 2\theta_1(1 - s) \int q_1 d q_1 + \theta_2 + (1 - \gamma)\theta_3 \int (1 - q_1) d q_1 \]

since \( q_1 \) is assumed to follow a uniform distribution. Computing the integral and substituting the expression of \( q \), \( Y_C \) becomes:

\[ Y_C = \theta_1(1 - s)(1 - q_2^2) + \frac{1}{2}\theta_2(1 - \gamma)(1 - q_2)^2 \]

\[ = \theta_1(1 - s)[1 - (\frac{B}{2s \theta_1 + B}) \frac{2}{2s \theta_1 + B}] + \frac{1}{2}\theta_2(1 - \gamma)(\frac{2s \theta_1}{2s \theta_1 + B})^2 \]

Since it is already argued that the optimal \( s \) is strictly within \((0, 1)\), the first order condition must be a necessary condition. Therefore, one has

\[-\theta_1(\frac{B}{2s \theta_1 + B})^2 + \theta_1(1 - s)\frac{4B \theta_1^2}{(2s \theta_1 + B)^3} + \frac{1}{2}\theta_2(1 - \gamma)(\frac{2s \theta_1}{2s \theta_1 + B})^3(\frac{2s \theta_1}{2s \theta_1 + B} - 4s \theta_1^2) = 0 \]

Notice that \( 2s \theta_1 + B > 0 \), therefore the FOC can be simplified as

\[-\theta_1(2s \theta_1 + B)^3 + \theta_1 B^2(2s \theta_1 + B) + 4B \theta_2^2(1 - s) + 2\theta_2 B[\gamma \theta_2 + (1 - \gamma) \theta_3] = 0 \]

i.e.

\[-(2s \theta_1 + B)^3 + [4\theta_1 B(\gamma \theta_2 + (1 - \gamma) \theta_3) - 2B^2 \theta_1]s + B^3 + 4B \theta_1 = 0 \]

The first order condition is in the form of a cubic algebraic equation. In general, there is at least one real root and if it either has no imaginary roots or it will have two conjugate imaginary ones. It will be proved in the appendix that there is only one positive real root, which must be the optimal \( s \). In the following, I will establish the fact that \( s \) increases with \( B \).
The unique root $t^*$ to the first order condition corresponds to a unique profit share $s^*$, which is function of $B$. Taking derivative with respect to $B$ on the first order condition, remembering that $s^*$ is a function of $B$, we have

$$-3(2s^\theta_1 + B)^2(2s^\theta_1 s^\gamma + 1) + [4\theta_1 B(\gamma \theta_2 + (1 - \gamma) \theta_3) - 2B^2 \theta_1]s^\gamma$$

$$+ [4\theta_1 (\gamma \theta_2 + (1 - \gamma) \theta_3) - 4D\theta_1]s + 3D^2 + 8D\theta_1 = 0$$

which gives

$$s^\gamma = \frac{-3D^2 - 8D\theta_1 - 4\theta_1 (\gamma \theta_2 + (1 - \gamma) \theta_3) - 4D\theta_1]s}{-6\theta_1 (s\theta_1 + B)^2 + 4\theta_1 B(\gamma \theta_2 + (1 - \gamma) \theta_3) - 2B^2 \theta_1}$$

The denominator is clearly negative. It remains to be shown that the numerator is also negative. Substituting the first order condition into the third term, the numerator becomes:

$$-3D^2 - 8D\theta_1 - \frac{1}{B}(2s\theta_1 + B)^3 + B^2 + 4B\theta_1 + 2B\theta_1 s$$

$$= -2D^2 - 4D\theta_1 - \frac{1}{B}(8s^3 \theta_1^3 + 8s^3 \theta_1 B + 4s\theta_1 B^2 + 3B^3) + 2B\theta_1 s$$

$$= -2D^2 - 4D\theta_1 - \frac{1}{B}(8s^3 \theta_1^3 + 8s^3 \theta_1 B + 4s\theta_1 B^2 + 3B^3) - 2B\theta_1 s < 0$$

Therefore $s^\gamma > 0$

Q.E.D.

It is not difficult to understand this proposition, intuitively. The profit sharing rule $s$ is needed, because otherwise the only possible benefit in each period to the management is $B$ and in some states management cannot get it. Thus, without a profit sharing rule, management will not be willing to take risky investment projects. However, $s$ cannot be too big, since this may encourage management to take too risky projects. When $B$ increases with other things being equal, a larger $s$ is needed to compensate management for the potential loss of return to its human capital.

4.2. The Case of Public Ownership

Under public ownership, management and labor anticipate that the project will not be terminated unless it is in state 3 when the profit is really low. Moreover, they know that even if the project is terminated, they can expect to get compensation no worse than that associated with a risk-free project. Given this, management and labor will propose whatever investment opportunity available. This implies that the profit sharing scheme $s$ will not serve its purpose of inducing management/labor to choose projects which are desirable to the bureaucrat. Therefore, if the bureaucrat maximizes his own profit, he might as well set $s$ as low as possible, i.e. 0.

Proposition 2: Under public ownership of capital, management and labor propose unselectively all investment projects. For a bureaucrat who maximizes profit, the optimal profit sharing rule $s^\gamma$ is 0.

4.3. Comparisons between Private and Public Ownership

It is interesting to first compare the investment decisions under these two alternative systems with the first best outcome. The first best outcome is achieved if there is no information asymmetry between the creditor and the management and if a social planner makes both decisions of project selection and projection termination. It will be demonstrated that compared with the first best outcome, fewer projects will be proposed under private ownership and more projects will be proposed under public ownership.
In the first best situation, the social planner knows perfectly the parameter of the project \((q_1, q_2, q_3)\). Furthermore, at the end of the first period, the social planner’s decision should be efficient, that is, he will let the project continue for one more period if state 1 or 2 are realized; otherwise the project will be aborted. Thus, the expected total social welfare from such a project is

\[
sw_0(q_1, q_2, q_3) = 2(q_1q_1 + q_2q_2) + q_3q_3 + B + (q_1 + q_2)
\]

If this project is indeed socially desirable, then the social welfare \(sw_0\) should be at least as large as that from a risk-free project, i.e.

\[
(2\theta_1 + B)q_1 + (2\theta_2 + B)\gamma(1 - q_1) + \theta_3(1 - \gamma)(1 - q_1) + B \geq 2B
\]

which is

\[
[2\theta_1 + B - \gamma(2\theta_2 + B) - \theta_3(1 - \gamma)]q_1 + [(2\theta_2 + B)\gamma + \theta_3(1 - \gamma) - B] \geq 0
\]

Therefore the first best choice of projects is like the following.

**Lemma 4:** *(First Best Project Selection Criterion (PSCO))* The project selection criteria in the first best outcome is: a project is invested if and only if

\[
q_1 \geq \frac{[2\gamma\theta_2 + (1 - \gamma)\theta_3] + (1 - \gamma)B}{2\theta_1 - [2\gamma\theta_3 + (1 - \gamma)\theta_3] + (1 - \gamma)B}
\]

Next, let us compare the three alternative project selection criteria of private ownership, public ownership and first best outcome. It turns out that relative to the first best outcome, the private ownership system selects too few projects while the public ownership system has bigger demand for investment in risky projects.

**Proposition 3:** *Strictly fewer projects are proposed under private ownership than in the first best outcome while strictly more projects are proposed under public ownership.*

**Proof:** To prove the first assertion, notice that the project selection criterion under private ownership is

\[
q_1 \geq 1 = \frac{B}{2\theta_1 + B}
\]

while the equivalent criterion for the first best outcome can be re-written as (\(s\) being the optimal profit sharing rule under private ownership):

\[
q_1 \geq \frac{s[-2\gamma\theta_2 + (1 - \gamma)\theta_3] + (1 - \gamma)B}{2s\theta_1 - s[(2\gamma\theta_2 + (1 - \gamma)\theta_3] + (1 - \gamma)B}
\]

In order to prove that the private ownership system selects fewer projects, one has to show:

\[
\frac{B}{2s\theta_1 + B} > \frac{s[-2\gamma\theta_2 + (1 - \gamma)\theta_3] + (1 - \gamma)B}{2s\theta_1 - s[(2\gamma\theta_2 + (1 - \gamma)\theta_3] + (1 - \gamma)B}
\]

Clearly, what is needed is to establish the following fact:

\[
s[-2\gamma\theta_2 + (1 - \gamma)\theta_3] + (1 - \gamma)B \geq B
\]

Recall that \(s\) must satisfy the first order condition described before. From the first order condition, we have

\[
-s[\theta_2\gamma + (1 - \gamma)\theta_3] = \frac{-2B^2\theta_1s + B^2 + 4B^2\theta_1 - (2s\theta_1 + B)^3}{4\theta_1 B}
\]

\[
= \frac{-8B^2s + 4B^2 - 8s^2\theta_1 - 12s^2\theta_1B}{4B}
\]
Thus
\[ B - s[-2\gamma\theta_2 + (1 - \gamma)\theta_3 + (1 - \gamma)B] \]
\[ = B - s(1 - \gamma)B + s\theta_2 - \frac{8B^3s + 4B^2 - 8s^2\theta_1^2 - 12s^2\theta_1B}{4B} \]
\[ = \frac{4B^2s - 4B^2(1 - \gamma)s + [4B^2 + 4\gamma\theta_2B]s + 8s^2\theta_1^2 + 12s^2\theta_1B}{4B} > 0 \]
The last inequality was obtained because \( B + \theta_2 > 0 \) which implies that \( B + \gamma\theta_2 > 0 \)

Thus, we have proved that the first statement is true. As for the second one, it is enough to show that the first best choice does exclude some projects, since the public ownership system is not selective at all. Such a proof is obvious, since for any value of \( \gamma \), it is true that

\[-[2\gamma\theta_2 + (1 - \gamma)\theta_3] + (1 - \gamma)B > 0 \]

Q.E.D.

Proposition 3 can be explained in a non-technical way. Under private ownership, the creditor chooses an optimal profit sharing rule \( s \) so as to induce management to propose profitable projects. The cost of increasing \( s \) is that the creditor will have to give a bigger portion of the positive profit in state 1. On the other hand, an increase in \( s \) will attract more projects, i.e. a marginally larger number of projects will be proposed. To overcome the cost of increasing \( s \), these marginal projects have to bring positive expected profit for the creditor. However, by the first best principle, the expected profit of this marginal project should be 0. Thus, the first part of the proposition is explained intuitively. As for the second part of the proposition, intuition suggests that under public ownership management and labor are not subject to any financial and human capital loss and therefore will propose all projects including ones that have unduly low profit.

There are interesting implications of proposition 3. It implies that the demand for investment under private ownership is small compared to the socially desirable level; the demand for capital under public ownership is too big. In general, the lack of demand for investment is indeed one of several prominent characteristics of the modern capitalist economy. Since Keynes, this has been widely recognized. On the other hand, over-investment or the "investment hunger" (Kornai(1980)) have been widely recognized as a problem that plagued socialist economies for many decades. Many studies by economists such as Kornai (1980) have extensively documented this phenomena.

The demand for investment was the concern of the above discussion and proposition 3. The supply side of investment was ignored. That is part of the creditor's decision of whether to approve the investment proposal. The creditor does not know the exact parameter of each proposed project when she makes a decision to approve each project proposed to her. When the expectation of profit is too low, the creditor will not provide financing for all projects. The next proposition answers the question that under what conditions the creditor will approve the investment proposal.

**Proposition 4:** Under private ownership, after choosing the optimal profit sharing rule \( s \), the creditor always approves the proposed project. Under public ownership, the bureaucrat will approve the proposed project only if

\[ \theta_1 + \theta_2\gamma + \frac{1}{2}\theta_3(1 - \gamma) \geq \frac{1}{2}(1 - \gamma)B \]

**Proof:** For the first part, what needs to be shown is that the expected profit to the creditor is always positive when \( s \) is chosen properly. According to the termination
condition under private ownership, the expected profit to the creditor is

\[ Y_{C1} = E[2\theta_1(1 - s)q_1 + \theta_2 q_2 + \theta_3 q_3] \]

\[ = E[2\theta_1(1 - s)q_1 + \theta_2 \gamma(1 - q_1) + \theta_3 (1 - \gamma)(1 - q_1)] \]

where the expectation is taken with the condition that \( y_{C1} = 0 \) is \( q_1 \leq q \). Therefore,

\[ Y_{C1} = \theta_1(1 - s)(1 - q^2) + \frac{1}{2}[\theta_2 \gamma + \theta_3 (1 - \gamma)](1 - q) \]

This is because

\[ \int_q^1 q_1 dq_1 = \frac{1}{2}(1 - q^2) \]

and

\[ \int_q^1 (1 - q_1) dq_1 = \frac{1}{2}(1 - q)^2 \]

Further simplification on \( Y_{C1} \) gives

\[ Y_{C1} = \frac{1}{2}(1 - q) \{ 2\theta_1(1 - s)(1 + q) + [\theta_2 \gamma + \theta_3 (1 - \gamma)](1 - q) \} \]

In the above expression for \( Y_{C1} \), it is easy to see that when \( s \) is close to 0, \( Y_{C1} \) is positive. The reason is that \( s \) being close to 0 implies that \( q = \frac{\theta_2 \gamma}{\theta_2 \gamma + \theta_3 (1 - \gamma)} \) is close to 1. Thus, the negative term with \([\theta_2 \gamma + \theta_3 (1 - \gamma)]\) will be very close to 0 from the negative side and the term \(2\theta_1(1 - s)(1 + q)\) will be positive enough to make the whole expression positive.

Given that a small \( s \) makes \( Y_{C1} \) positive, the maximized \( Y_{C1} \) should be positive. Therefore, by approving the project, the creditor will definitely do better than other risk-free projects which bring in 0 profit.

The second statement of the proposition is very easy to show. The expected payoff to the bureaucrat is

\[ Y_{C2} = E[2\theta_1(1 - s)q_1 + 2\theta_2 q_2 + 2\theta_3 q_3 - B q_3] \]

This is because in both states 1 and 2 the project will last for 2 periods while in state 3, even though the project will be terminated at the end of period 1, the bureaucrat has to pay out \( B \) to management/labor. In other word,

\[ Y_{C2} = E[2\theta_1(1 - s)q_1 + 2\theta_2 \gamma(1 - q_1) + \theta_3 (1 - \gamma)(1 - q_1) - B(1 - \gamma)(1 - q_1)] \]

Under public ownership, the bureaucrat chooses \( s = 0 \) and all projects will be proposed. With these considerations, the expected payoff becomes:

\[ Y_{C2} = \theta_1(1 - s) + \theta_2 \gamma + \frac{1}{2}\theta_3 (1 - \gamma) - \frac{1}{2}B(1 - \gamma) \]

When \( Y_{C2} \geq 0 \), the bureaucrat will approve all investment proposals. This gives the condition in the proposition.

Q.E.D.

Proposition 4 has a social welfare implication. It can be interpreted as saying that when the overall unconditional expected profit \( \theta_1 + \theta_2 \gamma + \frac{1}{2}\theta_3 (1 - \gamma) \) is not high enough, financing risky projects is impossible under public ownership. This outcome is socially undesirable, since no matter how low the unconditional expected profit is, there are certainly numerous such investment projects which become highly profitable and are socially worthwhile. The private ownership system, meanwhile, is more robust. Some projects will be financed by the private creditor. These projects will bring both the creditor and the management better payoff than that of a riskless project. Therefore these projects will definitely improve social welfare.
To summarize this welfare implication, here is a corollary.

Corollary 1: When \( \theta_1 + \theta_2 \gamma + \frac{1}{2} \theta_2 (1 - \gamma) < \frac{1}{2} (1 - \gamma) B \) the private ownership system is more socially efficient than the public ownership system. In other words, under this condition, the private ownership system generates more social welfare than the public ownership system.

This conclusion on welfare comparison can be supported empirically. In the latter days of a socialist economy when clearly profitable investment projects are exhausted, there are fewer investment opportunities. There are plenty of data demonstrating that the investment efficiency deteriorated significantly relative to market economies.

In order to make more general social welfare comparisons, let us compute the achieved social welfare levels under both systems. The social welfare of an invested project consists of financial profit and returns to human capital. Under private ownership, if the state is \( 1 \), the project will last for 2 periods and yield a total social welfare of \( 2q_1 + 2B \). In state \( 2 \), the project will only last for 1 period and the social welfare is \( q_2 + B \). Similarly, in state \( 3 \), the welfare level is \( q_3 + B \). Thus, the total expected social welfare from a project \( (q_1, q_2, q_3) \) is:

\[
sw_1(q_1, q_2, q_3) = (2q_1 + 2B)q_1 + (q_2 + B)q_2 + (q_3 + B)q_3
\]

= \( (2q_1 + B)q_1 + \theta_2 q_2 + \theta_2 q_3 + B \)

Using the assumption on \( q_2 \) and \( q_3 \), we have

\[
sw_1(q_1, q_2, q_3) = (2q_1 + B)q_1 + \theta_2 \gamma (1 - q_1) + \theta_2 (1 - \gamma) (1 - q_1) + B
\]

(4.1)

Notice that in the above expression, \( q_1 \) must follow the condition given by \( PSC1 \).

In the case of public ownership, the expected social welfare associated with a project \( (q_1, q_2, q_3) \) has a different expression from equation (4.1), since the termination condition is different. The difference lies in state 2. Unlike the private ownership case, the project will last for 2 periods if the state is 2. The associated social welfare is \( 2q_2 + 2B \). Therefore under public ownership, project \( (q_1, q_2, q_3) \) is expected to yield a level of social welfare:

\[
sw_2(q_1, q_2, q_3) = (2q_1 + B)q_1 + (2q_2 + B) \gamma (1 - q_1) + \theta_2 (1 - \gamma) (1 - q_1) + B
\]

(4.2)

Comparing equations (4.1) and (4.2), it is not clear which ownership arrangement would yield higher social welfare. Given that the same project is invested, the public ownership case is better, since it allows the project to continue in state 2 when \( B + \theta_2 > 0 \). In other words, the second term on (4.2) is larger than the second term in (4.1). However, the project selection criteria under public ownership can be inferior, since many socially undesirable projects are invested. In other words, (4.2) is valid for many projects with very low \( q_1 \), while for such projects, (4.1) will not be valid. To compare the welfare level, one has to take expectations on both sides of (4.1) and (4.2) with respect to \( q_1 \). Here is the formal result.

Lemma 5: Suppose that

\[
\theta_1 + \theta_2 \gamma + \frac{1}{2} \theta_2 (1 - \gamma) \geq \frac{1}{2} (1 - \gamma) B
\]

then the difference between the expected social welfare from invested projects under private ownership and public ownership is

\[
DSW = E(sw_1) - E(sw_2)
\]

27
\[ -B\theta_1(1 - \gamma) - 2s\theta_1 B(1 - s) - 2s^2\theta_1^2 - (3 - \gamma)B^2 + 2s\gamma\theta_1 B + 2s\theta_1 B \]

Proof: (See the Appendix)

From the above discussion, one may expect that when the overall profitability is very high, the public ownership system would be superior to the private ownership system in terms of investment efficiency. As a matter of fact, it is not always true. However, under one special condition, the public ownership system is indeed more efficient.

**Proposition 5:** Assuming that \( \theta_1 + \theta_2 + \theta_3(1 - \gamma) \geq \frac{1}{2}(1 - \gamma)B \), that is, the bureaucrat will approve all projects under public ownership. If furthermore, \( \gamma \to 1 \) and \( \theta_2 + B \to 0^+ \), then a public ownership system is more socially efficient than a private ownership system in terms of investment decisions.

**Proof:** From the expression of \( DSW \) in the above lemma,

\[ DSW = [-B\theta_1(1 - \gamma)] - [2s\theta_1 B(1 - s) - 2s^2\theta_1^2 - (3 - \gamma)B^2] + 2s\gamma\theta_1 B \]

Notice that the first and the third term are both positive, while the second term in the square bracket is negative. When \( \gamma \to 1 \), the first term goes to 0. When \( \theta_2 + B \to 0 \) the last term also goes to 0. Thus, \( DSW \) is negative.

Q.E.D.

An intuitive interpretation for this proposition seems to be hard to find. However, a partial explanation is possible. When \( \gamma \to 1 \), state 3 is virtually non-existent. This is helpful to the public ownership case, since it means that the overall expected profitability is high. Therefore, the conjecture is partially true that the higher the overall expected profitability the more likely that the public ownership system is superior to the private ownership ones. In reality, it is a fact that in the early days of socialism, the socialist economies enjoyed seemingly higher investment efficiency and growth rate than market economies.

5. Conclusions

The starting point of this paper is that ownership of capital entitles the owner(s) to the decision rights regarding the disposition of their capital when such disposition of the capital under some circumstances is hard to specify beforehand. It is argued that bankruptcy is one of such circumstances.

The private ownership of capital gives rise to a "hard" budget constraint. That is, when the profit of the firm is found to be negative, the firm is liquidated. This is socially inefficient, since in many cases the firm should be saved in order to avoid the loss of human capital of management and labor. However, given that management and labor cannot absorb financial losses, such a "hard" budget constraint is necessary to discipline the management when it proposes an investment project at the beginning.

The public ownership of capital causes a soft budget constraint, since the bureaucrat cannot exclude management and labor from a decision to terminate the firm. Thus, the closing of the firm is much delayed until the profit is very low. This soft budget constraint is socially efficient ex post. However, ex ante it leaves management undisciplined when making investment proposals.

As consequences of this difference in "budget constraint", it is shown that the demand for investment under private ownership is in turn smaller than the socially desirable level which is smaller than the demand for investment under public ownership. Also, it is shown that when the expected profitability of all projects is not
big enough, the private ownership investment system is superior in efficiency to the public ownership investment system.

Appendix

A-I. Proof of The Existence of a Unique Root of the First Order Condition

Define \( t = 2s\theta_1 + B \), i.e. \( s = \frac{t-B}{2\theta_1} \). Substitue this expression into the first order condition:

\[
-t^3 + [2B^2(\theta_2 + (1-\gamma)\theta_3) - B^2] \frac{t-B}{2\theta_1} + B^3 + 4B^2\theta_1 = 0
\]

or

\[
t^3 - 2B(\theta_2 + (1-\gamma)\theta_3 - B) t + 2B^2(\theta_2 + (1-\gamma)\theta_3 - B) - B^3 - 4B^2\theta_1 = 0
\]

Define

\[
D = -2B(\theta_2 + (1-\gamma)\theta_3 - B)
\]

and

\[
E = 2B^2(\theta_2 + (1-\gamma)\theta_3 - B) - B^3 - 4B^2\theta_1 = 0
\]

Notice that \( D > 0 \) and \( E < 0 \). The FOC becomes

\[
t^3 + Dt + E = 0
\]

Let \( t_1, t_2, t_3 \) be the roots of the FOC, then

\[
(t - t_1)(t - t_2)(t - t_3) = t^3 + Dt + E
\]

Thus, comparing the coefficients, we have

\[
t_1 + t_2 + t_3 = 0
\]

\[
t_1t_2 + t_2t_3 + t_3t_1 = -E > 0
\]

From the second condition, we know that either there is one positive real root or
there are three positive real roots. (Recall that imaginary roots must appear in conjugate pairs.) From the first expression, we know that it is impossible to have three positive real roots. Therefore, there is a unique positive root. Let it be $t^*$.

A-II. Derivation of The Expression for DSW

From equation (5.1)

$$E(SW1) = E\{[(2\theta_1 + B)q_1 + \theta_2 \gamma(1 - q_1)\theta_3(1 - \gamma)(1 - q_1) + B]$$

$$= E\{[(2\theta_1 + B) - \theta_2 \gamma - \theta_3(1 - \gamma)]q_1 + \theta_2 \gamma + \theta_3(1 - \gamma) + B)\}$$

$$= [(2\theta_1 + B) - \theta_2 \gamma - \theta_3(1 - \gamma)]\frac{1}{2}(1 - \gamma^2) + [(\theta_2 + \theta_3(1 - \gamma) + B)(1 - q)]$$

The last step was derived using the project selection condition under private ownership. Namely, if $q_1 < q$ the project will bring 0 social welfare. Notice that

$$q = \frac{p}{2\theta_1 + B},$$

where $s$ is chosen by the creditor in maximizing her payoff:

$$s : \text{MAX}_s = [(2\theta_1 + B) - \theta_2 - 2\gamma(1 - \gamma)]\frac{1}{2}(1 - q_1) + [(\theta_2 + \theta_3(1 - \gamma) + B)(1 - q)] 5.3$$

In the case of public ownership, all projects will be invested. Therefore, $E(SW2)$ can be calculated as

$$E(SW2) = E\{([(2\theta_1 + B) - (2\theta_2 + B)\gamma - \theta_3(1 - \gamma)]q_1 + (2\theta_2 + B)\gamma - \theta_3(1 - \gamma) - B\}

= [(2\theta_1 + B) - (2\theta_2 + B)\gamma - \theta_3(1 - \gamma)]\frac{1}{2} + (2\theta_2 + B)\gamma - \theta_3(1 - \gamma) - B\]

Thus

$$DSW = E(SW1) - E(SW2)$$

$$= [(2\theta_1 + B) - \theta_2 \gamma - \theta_3(1 - \gamma)]\frac{1}{2}(1 - \gamma^2) + [(\theta_2 + \theta_3(1 - \gamma) + B)(1 - q)]$$

$$- [(2\theta_1 + B) - (2\theta_2 + B)\gamma - \theta_3(1 - \gamma)]\frac{1}{2} - (2\theta_2 + B)\gamma - \theta_3(1 - \gamma) - B\]

$$= -\frac{1}{2}g^2[\theta_1 + B - \theta_2 \gamma - \theta_3(1 - \gamma)] - \theta_2 \gamma + \theta_3(1 - \gamma) + B - \frac{1}{2}(-[\theta_2 + B] \gamma - \gamma B)$$

(5.4)

From the maximization problem (5.3), the first order condition can be written as:

$$\{[2\theta_1(1 - s) - \theta_2 \gamma - \theta_3(1 - \gamma)](\theta_3(1 - \gamma)) \theta_3 - \theta_3(1 - \gamma)\} \frac{1}{2} + \theta_3(1 - q) = 0 5.5$$

Utilizing the first order condition (5.5), equation (5.4) becomes

$$DSW = \frac{1}{2}[\theta_2 \gamma + \theta_3(1 - \gamma) + \theta_3(1 - \gamma) + \theta_3(1 - \gamma) - \theta_2 \gamma - \theta_3(1 - \gamma)] \frac{1}{2} - \frac{1}{2}(\theta_2 + B)$$

$$= \frac{1}{2} \frac{B}{2s\theta_1 + B} [\theta_2 \gamma + \theta_3(1 - \gamma) + \theta_3(1 - \gamma) + \theta_3(1 - \gamma) - \theta_2 \gamma - \theta_3(1 - \gamma)] - \frac{1}{2}(\theta_2 + B)$$

$$= \frac{1}{2} \frac{B}{2s\theta_1 + B} [\theta_2 \gamma + \theta_3(1 - \gamma) + \theta_3(1 - \gamma) + \theta_3(1 - \gamma) - \theta_2 \gamma - \theta_3(1 - \gamma)] - \frac{1}{2}(\theta_2 + B)$$

(5.6)

Clearly, $DSW$ can take both signs. For example, when $\theta_1 = 0$ and $\theta_3$ is negative enough such that $\theta_3(1 - \gamma) + (3 - \gamma)B < 0$, $DSW$ is positive. On the other hand, when $\theta_2$ and $\theta_3$ are close to 0, $DSW$ is negative.
References


