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Center for Research on Economic Development

University of Michigan
Ann Arbor, Michigan 48108

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Labor Migration and Urban Unemployment in Less Developed Countries: Comment

by

Richard C. Porter

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Although the growing seriousness of urban unemployment in the less developed countries has been long recognized, only recently has Todaro (1969) provided a concise economic explanation. Based upon the assumptions that the rate of migration to the cities depends upon the expected urban-rural wage differential and that the urban wage is pegged through noneconomic forces at a level above the rural wage, his model concludes that such urban unemployment is not transitory, but rather a long-run, equilibrium phenomenon.

In this note, the dynamics of the model are explicitly considered and it is shown that urban unemployment cannot exist in equilibrium if employment in the urban sector is growing at a more rapid rate than the population as a whole and other factors are unchanging. Unfortunately for practical purposes, this correction offers no ground for optimism -- the "transitory" urban unemployment rates are depressingly high and long-lived. Indeed, unemployment rates climb more than twice as high as the "equilibrium" rates estimated by Todaro (and his critic, Zarembka), for the same values of the parameters.

I. The Todaro Model

The Todaro model can be expressed in four equations. The rural labor force (R) grows at a rate, p, less the migration to the urban

* I wish to thank Elliot Berg, George Johnson and James Tobin for helpful comments.

areas (M):[1]

(1) $\dot{R} = pR - M.$

The urban labor force (U) also grows at a rate, p, plus the migration from the rural areas:[2]

(2) $\dot{U} = pU + M.$

The demand for urban labor (D) grows at a rate, g:

(3) $\dot{D} = gD.$

The core of the model is the migration function. The fraction of the rural labor force that migrates to the city (M/R) is a function of 1) the probability that an urban laborer can get a job, which in simplest form can be written as some monotonic function of the current urban employment rate (D/U), 2) the (assumed fixed) urban-rural real wage ratio (w, where w > 1), and 3) other factors that influence the migration decision (z):[3]

(4) $M/R = \Phi[D/U, w, z].$

As long as w and z are held constant, the function, Φ, can be written more simply as

(5) $\Phi[D/U, w, z] = f[D/U],$

where $f' \geq 0$ for all values of D/U between zero and one.[4]

Substitution of (4) and (5) into (2) yields the basic differential equation of the model:

(6) $\dot{U}/U = p + \frac{R}{U} f[D/U].$

I shall examine the time-path of this equation, in the next four sections, under various circumstances: 1) where the rate of growth of demand for urban labor exceeds the population growth rate (i.e. $g > p$); 2) where $g < p$, and out-migration from the city never occurs no matter how low D/U falls; 3) where $g < p$, and out-migration from the city is possible; and 4) where $g > p$, but w and/or z are changing in such a way as to stimulate an increased rate of migration to the city. Throughout, I assume -- for reasons of brevity and realism -- that the initial value of $\dot{U}/U$ is greater than both g and p.[5]

II. $\underline{g > p}$

When the rate of growth of the urban demand for labor (g) exceeds the natural rate of growth of the labor force (p), the qualitative aspects of the time-path of $\dot{U}/U$ and D/U are as illustrated in Figure 1. At the starting point, where $\dot{U}/U$ exceeds g (and hence p), the urban ~~un~~employment rate, D/U, will be falling. As a result, the migration rate, f[D/U], will be falling and since the rural-urban population ratio, R/U, is also falling, $\dot{U}/U$ must be falling.[6] This decline, of both $\dot{U}/U$ and D/U, continues until $\dot{U}/U$ reaches g; at that point, the decline in D/U is halted. But $\dot{U}/U$ continues to fall, since even though f[D/U] stops falling, the decline in R/U continues.

Once $\dot{U}/U$ falls below g, the urban employment rate, D/U, actually begins to rise. Since R/U continues to fall -- as long as $\dot{U}/U$ exceeds p -- whether $\dot{U}/U$ rises or falls depends upon the net effect of an upward force (i.e. the increased fraction of the rural population that migrates, or M/R) and a downward force (i.e. the relative decline of the rural source of migrants, or R/U). Nevertheless, the limits to the movements in $\dot{U}/U$ are clear: 1) it cannot fall below p since the rising urban

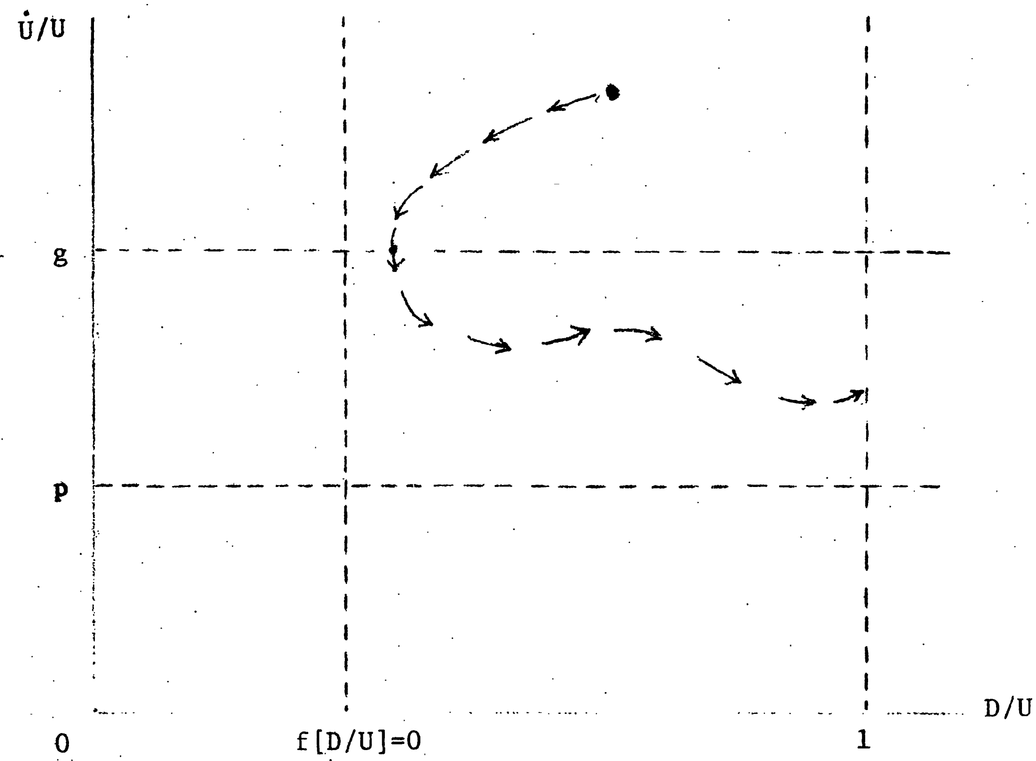


FIGURE 1

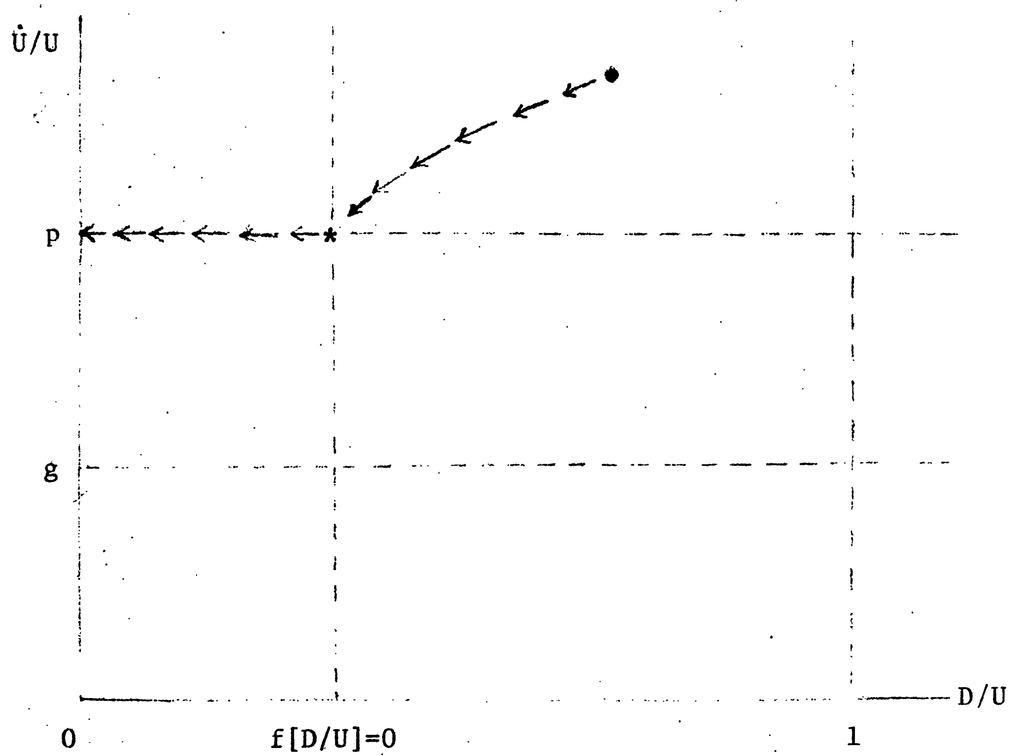


FIGURE 2

employment rate will always induce <u>some</u> migration to the cities; and 2) it cannot rise above g since, as it reaches g, the rise of D/U ceases and the downward force on migration (i.e. the decline of R/U) forces $\dot{U}/U$ down.

Once $\dot{U}/U$ has fallen below g, therefore, it must remain between g and p thereafter. But D/U rises, more or less rapidly, until eventually full employment of the urban labor force is attained.[7] Thus, in this case,[8] there is no equilibrium unemployment rate, but rather a phase of a rising urban unemployment rate followed by a phase of a falling urban unemployment rate, with the latter phase ending only when full employment is reached.

III. <u>$g < p$; Urban Out-migration Cannot Occur</u>

The results are quite different if the rate of growth of urban labor-force demand is less than the rate of growth of the labor force (i.e. $g < p$). Initially, as before, both $\dot{U}/U$ and D/U fall (see Figure 2). But $\dot{U}/U$ now falls to p, at which point migration to the cities has ceased (i.e. the point marked with an asterisk on Figure 2). But the natural growth of the urban labor force exceeds the growth of urban jobs, so the urban employment rate (D/U) continues to decline. If no out-migration from the cities occurs, $\dot{U}/U$ remains at p and D/U declines asymptotically toward zero.

Thus, there is no "equilibrium" urban unemployment rate (except, in a sense, at 0% or 100%) in the Todaro model. If $g > p$, urban unemployment eventually disappears; and if $g < p$, urban unemployment moves inexorably toward 100%. Only if $g = p$ is there an intermediate solution.

This can be seen in Figure 2; once the first phase of falling $\dot{U}/U$ and falling D/U is concluded (at the point marked with the asterisk), $\dot{U}/U$ equals not only p but also g. Thus, D/U falls no further, and an intermediate urban unemployment solution is reached, at the unemployment rate at which migration just ceases (i.e. f[D/U] = 0). Curiously, this is the "equilibrium" solution that Zarembka found and (implicitly) claimed as the general solution.[9]

IV. g < p; Out-migration Can Occur

The possibility of urban out-migration does not, of course, alter the time-path described in Section III (and shown in Figure 2) until $\dot{U}/U$ has fallen to p (and hence D/U has fallen to the level at which f[D/U] equals zero). Once D/U falls below that point, however, out-migration from the urban to the rural areas may begin to occur; when this happens, the time-path diverges from that of Figure 2, as $\dot{U}/U$ falls below p.

The time-path of $\dot{U}/U$ and D/U can no longer be deduced from the model of Section I, since the migration function there, equation (4), is clearly inappropriate for urban out-migration. While the rate of urban out-migration would be a function of the urban employment rate (D/U), the base population from which this migration occurs is certainly not the rural population (R). Most plausibly, the base is the urban unemployed (i.e. U - D), in which case, for a situation of out-migration, equation (4) would need to be rewritten as

(7) $(-M)/(U - D) = \Psi[D/U, w, z]$,

where M, as before, represents <u>in</u>-migration to the cities and hence $-M$ represents out-migration as a positive flow.

But the exact form of the out-migration function is not critical. That $(-M/U)$ can be written as some function, decreasing with D/U (for constant w and z), is sufficient, with equations (2) and (3), to solve for $\dot{U}/U$ in terms of D/U alone (i.e. R is irrelevant to the time-path). Then, the time-path of $\dot{U}/U$ must be (qualitatively) as pictured in Figure 3, where $\dot{U}/U$, after reaching p, continues to fall once out-migration commences.[10] D/U of course also falls steadily (since $\dot{U}/U > g$) until eventually the rate of out-migration $(-M/U)$ reaches $(p - g)$ and the rate of growth of the urban population just equals the rate of growth of urban employment.

Thus, the possibility of urban out-migration means that the urban employment rate (D/U) need not move asymptotically to zero (as in Section III) but might move to an equilibrium rate at which steady out-migration keeps the urban population growth rate down to the growth rate of the demand for urban labor.

V. <u>$g > p$; Other Factors Changing</u>

So far, I have maintained the assumption of constancy of w and z. Are the results altered if either of these parameters rises in once-and-for-all or secular fashion? Inspection of equations (5) and (6) indicates that, for any value of D/U, the value of $\dot{U}/U$ is higher if either w or z is rising,[11] and indeed, a rise of $\ddot{U}/U$ in the early stages is not impossible -- for a time. Eventually, however, rural-urban migration must slow down[12] in response to the declining D/U, and $\dot{U}/U$ must then begin to fall toward p. If $p > g$, the time-path eventually becomes that of Figure 2 or 3, though the first phase (i.e. the movement from the start to the point of the asterisk) may be less direct. If $g > p$, the process is less simple. Once $\dot{U}/U$ falls below g, D/U again begins to rise. The time-path does not, however, necessarily move steadily to full employment, because g no longer provides a ceiling to the range of $\dot{U}/U$. Sudden or secular shifts in w or z may lift $\dot{U}/U$ above g, which would renew the leftward movement (toward a falling rate of urban employment). Such a "looping" path is illustrated in Figure 4. Whether the economy can avoid initially, or escape eventually, such "loops" is a question that cannot be answered without specific information about the form of the migration function, Φ. Nevertheless, such things as rising urban wages or increased urban-oriented rural education clearly generate the possibility of a cyclically fluctuating but permanently large urban unemployment rate.[13]

If the agricultural sector is technologically stagnant, the natural course of the rural wage (assumed related to the marginal product of rural labor) will add to the possibility of such loops. As D/U declines, a time must be reached when M/R falls sufficiently that the absolute size of the rural labor force rises.[14] Then the marginal product (and wage) of rural labor falls and the urban-rural wage ratio rises. This places a continual upward pressure on migration and hence on $\dot{U}/U$ and makes loops more likely as long as D/U remains below the level at which $f[D/U] = p$.

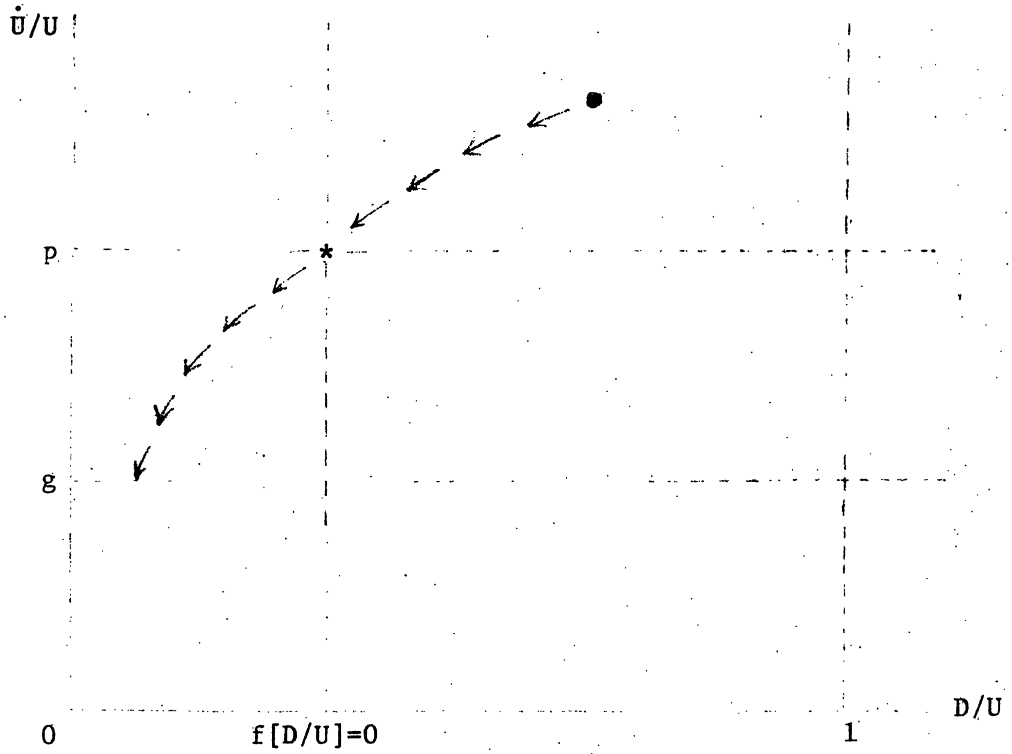


FIGURE 3

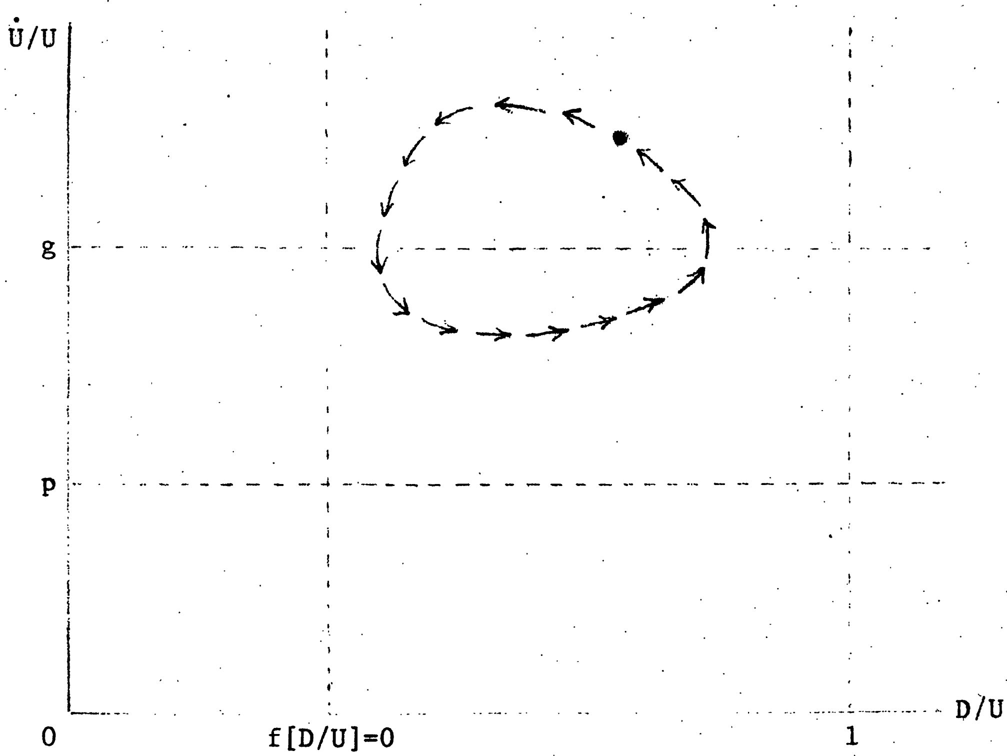


FIGURE 4

VI. How Long to Full Employment?

Thus, full employment is inevitable in the Todaro model, provided that employment in the modern sector grows more rapidly than the population (i.e. $g > p$) and that repeated "looping" is avoided (i.e. through upward shifts of w or z). But there is nothing in this qualitative inevitability to insure a rapid movement to urban full employment. If the urban unemployment rate rises or remains high for decades, it is of little solace to know that it will "eventually" decline and disappear.

In order to simulate a time-path of urban unemployment in the Todaro model, it is first necessary to specify the function, f, of equations (5) and (6). A plausible form, and one consonant with Todaro's formulation (as corrected by Zarembka), is

$$(7) \qquad \frac{M}{R} = k \frac{gD}{U - D} ,$$

where the constant, k, captures the influence of w and z and where $gD/(U - D)$ is the probability that the current urban unemployed will find jobs in the current period.[15]

Since the model, as represented in equations (1), (2), (3), and (7), does not yield an analytically solvable differential equation,[16] the only recourse is to convert the system into difference equations and simulate time-paths. Equations (1), (2), (3), and (7) can be written as

$$(9) \qquad R_t = (1+p)R_{t-1} - M_t ,$$

$$(10) \qquad U_t = (1+p)U_{t-1} + M_t ,$$

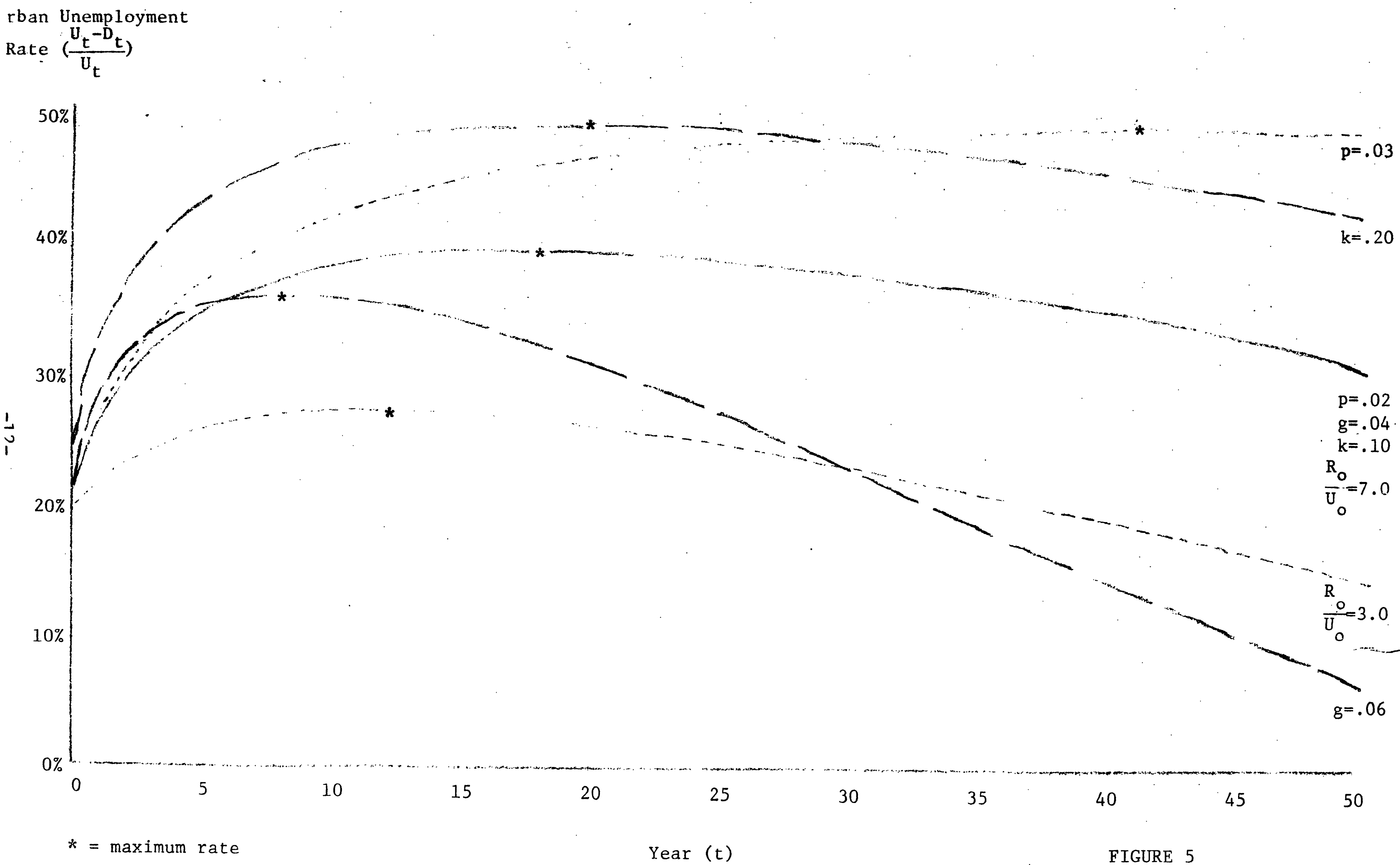
$$(11) \qquad D_t = (1+g)D_{t-1}, \text{ and}$$

$$(12) \qquad M_t = kg \frac{(A_{t-1})(D_{t-1})}{U_{t-1} - D_{t-1}} .$$

I adopt the values of the parameters which were used by Todaro and Zarembka -- i.e. $p = .02$, $g = .04$, and $k = .10$ -- and put the base-year ($t=0$) ratios between the variables arbitrarily at $D_o/U_o = 0.80$ and $R_o/U_o = 7.00$. The resulting half-century time-path of the urban unemployment rate is shown by the solid line in Figure 5.

The path is rather frightening. The urban unemployment rate rises, within three years, past 30%, and it eventually approaches 40%. But worse, the decline does not even begin for nineteen years and the rate has hardly fallen (i.e. only to 31%) by the end of the half century. The rate of annual rural-urban migration ranges between 0.6% and 1.6% of the rural population -- a modest flow -- and yet the resulting urban unemployment rate remains above 30% of the urban labor force for 50 years! Such "transitory" unemployment rates reach levels more than twice as high as the stationary rates which Todaro and Zarembka calculated.[17] Thus the logical error of the Todaro model pales before its essential truth -- if migration to the urban areas from a vast rural base continues in the face of high urban unemployment, the less developed countries will suffer distressing urban unemployment rates throughout the foreseeable future.

Some indication of the sensitivity of these results to the parameter values selected is also shown in Figure 5. A doubling of the rate of urban in-migration at each urban unemployment rate (i.e. raising k from .10 to .20), raises the urban unemployment rate by about ten percentage points throughout the half-century, but does not much alter the shape of the time-path. A 50% increase in the rate of population growth (i.e. a rise of p from .02 to .03) raises the peak urban unemployment rate by about ten percentage points and greatly postpones the beginning



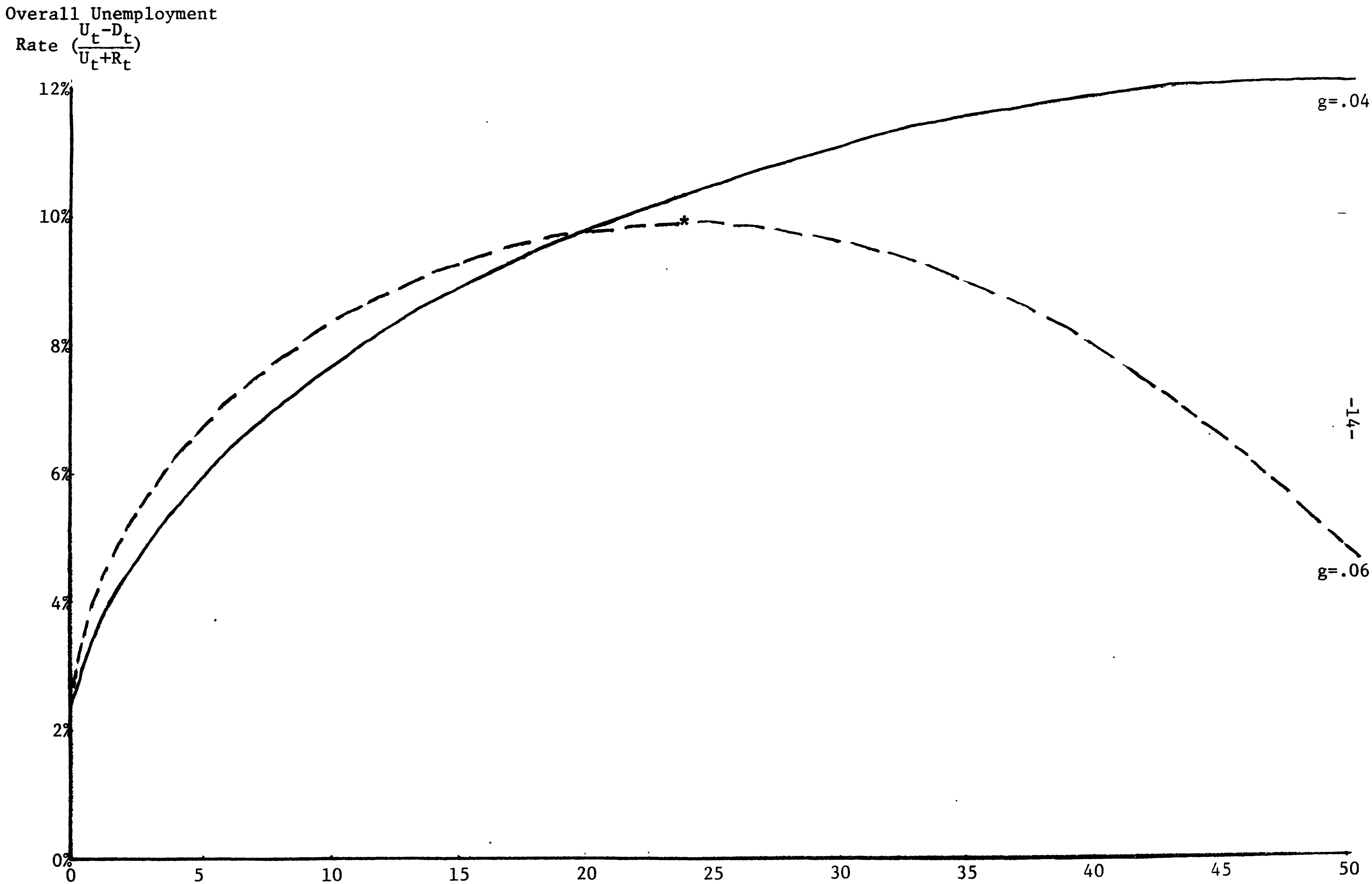
* = maximum rate

FIGURE 5

of its decline (i.e. from 19 to 42 years). A 50% increase in the growth rate of urban demand for labor (i.e. a rise of g from .04 to .06) dramatically reduces the urban unemployment rate in all but the first few years. A rise in the initial degree of urbanization of the population (i.e. a fall in R_o/U_o from 7.0 to 3.0) also dramatically reduces the urban unemployment rate.[18]

There is one final issue between Zarembka and Todaro on which these simulations offer evidence. Zarembka claimed -- and Todaro denied -- that in equilibrium "an improvement in employment opportunities in the urban sector, say through output expansion, will *increase* the unemployment rate..." (Zarembka, 1970, p. 186, his italics).[19] As inspection of the solid line and the dashed (g = .06) line of Figure 5 indicates, the higher g is likely to raise the urban unemployment rate only temporarily -- in Figure 5, it is higher for only five years; thereafter, the rate is increasingly lower.

Todaro seems to go too far, however, in suggesting that an increase in g may permanently increase the "*absolute* number of urban employed" (Todaro, 1970, p. 188, my italics). In Figure 6, the urban unemployment is shown as a percentage of the total population (i.e. of $R_t + U_t$) for g equal to .04 and .06.[20] The higher rate of growth of urban demand for labor does indeed raise the absolute numbers of unemployed, for a while (in this case, for 20 years), but eventually a higher value of g means fewer urban unemployed. Although such simulations cannot provide general proofs, the results accord with common sense; a higher rate of growth of urban employment may for a brief span raise the rate of urban unemployment, and may for a longer span raise the absolute number of urban unemployed, but eventually it must reduce both. Unfortunately, from a practi-



* = maximum rate

Year (t)

FIGURE 6

cal viewpoint, even a growth rate of urban employment several times the growth rate of population may be unable to reduce the urban unemployment rate to a tolerable level for an intolerably long time.

Footnotes

[1] A dot over a variable indicates its time derivative.

[2] A differential rural and urban population growth rate could be included, but would add little of qualitative interest to the model.

[3] Todaro originally wrote the left side of this equation, M/U. Zarembka (1970) persuasively argued that M/R is the more logical choice, and Todaro apparently accepted that revision -- at least his rebuttal of Zarembka's other criticisms (Todaro, 1970) contained no argument on this score.

[4] A prime indicates the first derivative.

[5] The paths are unaffected, in the limit, by this choice of initial conditions.

[6] As long as migration is positive, R grows at a rate less than p and U at a rate greater than p.

[7] At this point, the fixed urban wage rate will presumably begin to rise in response to excess demand for urban labor; and the Todaro model is no longer applicable.

[8] The only one considered by Todaro (1969), pp. 144-145.

[9] The accident occurred in the process of taking an unwarranted "close approximation" (Zarembka, 1970, p. 185), at which point the two $(g - p)$ terms were treated as zero. There is no easy explanation of Zarembka's error since elsewhere he recognized that the "unemployment rate approaches zero" (p. 186). Moreover, a later Todaro article (Harris and Todaro, 1970), while it added much to the model, unintentionally reinforced the myth of the equilibrium urban unemployment rate by its static nature (i.e. where $g = p = 0$).

[10] It need not start immediately at the point where f[D/U] reaches zero.

[11] "Other factors" (z) are defined in such a way that their rise stimulates in-migration.

[12] At the extreme, migration will surely cease as D/U approaches zero, no matter what rises in w and z may occur.

[13] But <u>not</u> an equilibrium rate unless the loop coincidentally becomes infinitesimal around some point at which $\dot{U}/U = g$.

[14] From equation (1), $\dot{R} > 0$ if $pR > M$, that is, if $p > f[D/U]$.

[15] The exact form of the migration function is of no consequence for the <u>qualitative</u> aspects of the Todaro model, provided only that the left side is written M/R rather than M/U. Zarembka's suggested function is quite inappropriate since it unrealistically generates urban <u>out</u>-migration at quite low rates of urban unemployment (i.e. less than 10% for the parameter values Zarembka considers). Of course, Todaro's function errs logically in the opposite direction, being incapable of generating urban out-migration at <u>any</u> urban unemployment rate, but it is not an unreasonable approximation in the range of urban unemployment rates to be considered.

[16] The equation is:

$$\dot{d} = \frac{Ad + Bd^2 + Ce^{-At}d^3}{1 - d}, \tag{8}$$

where $d = D/U$, t is time, and A, B, and C are parameters. Other plausible specifications of the migration function yield similarly awkward differential equations.

[17] They estimated equilibrium rates (for the same parameter values) of urban unemployment of 17% and 7%, respectively (Todaro, 1970, p. 187). Zarembka did note that the rate would be "slightly greater" than his "approximation" (Zarembka, 1970, p. 185n.).

[18] The effect of a change in the initial rate of urban unemployment (i.e. of D_o/U_o) is not shown in Figure 1, but is always small. For example, whether D_o/U_o is 0.80 or 0.90 never makes a difference over the subsequent fifty years, of as much as three percentage points.

[19] This claim has also been made by Knight (1971) p. 53, while Johnson (1971) has maintained that whether a higher g raise or lowers the urban unemployment rate depends on the speed of the reaction of migration to changes in the expected urban-rural income differential (p. 25). Knight and Johnson both assume that an equilibrium exists, Knight through not considering the dynamics and Johnson through a lapse similar to Zarembka's.

[20] Since the total population is the same for the two simulations, this (overall) unemployment rate also provides a comparison of the absolute numbers of unemployed.

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