Terminal-Year Investment in Finite-Horizon Planning Models

by

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Notes & Comments

TERMINAL-YEAR INVESTMENT IN FINITE-HORIZON PLANNING MODELS

by

RICHARD C. PORTER*

A common problem of finite-horizon planning models is that there is no logical determinant of investment in the final year (s). Where post-horizon production is not valued by a model, later-year investment, whose sole function is creation of capacity for post-horizon output, looks as incongruous as last rites for an atheist. A number of artificial devices have been developed to handle this difficulty[1], but one predominates: to assume that terminal-year investment is a function of terminal-year output. The purpose of this note is to show: 1) how varied and arbitrary are the assumed functions (Section I); 2) that the terminal-year variables and the apparent feasibility of the resulting Plan are highly sensitive to the choice of function (Section II); and 3) that the arbitrariness of functional form is inevitable in the sense that generally acceptable criteria do not much restrict the choice (Section III).

Throughout this note, we shall neglect four complexities that are not essential to the problem at hand. One, the marginal capital-output ratio (σ) is assumed fixed and known. Two, the most simple gestation-lag structure of investment is assumed, namely, that investment during period t permits an increase in the output of period (t+1) over the output of t. Three, net investment is considered and depreciation complexities are assumed away. And four, we neglect all intersectoral and foreign-trade complications. The addition of complexity on any of the above counts obscures, but does not alter, the basic problem of the choice of a terminal-year investment determinant.

I. TYPES OF FUNCTIONS ASSUMED

We begin with a review of the troops[2]:

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[1]Such as: 1) inclusion of the terminal-year capital stock in the objective function; 2) a constraint that precludes declines of investment in the final years; or 3) consideration for planning purposes of only the first few years of a model within an extended time horizon.

[2]While the various functional choices will be identified by reference to their users, no priority is implied; no attempt has been made to uncover either all users or all functions used. The models referred to have been selected because they are well constructed, readily accessible, and widely read.

1. *Bergsman and Manne* [2]. Terminal-year investment (I_T) is linked to the increase in output during the first post-Plan year (*i.e.*, $X_{T+1} - X_T$). If one asserts a post-horizon growth rate (g), the problem disappears:

$$I_T = \sigma\,(X_{T+1} - X_T) \qquad (1)$$

$$X_{T+1} - X_T = gX_T \qquad (2)$$

hence,

$$I_T = g\sigma X_T \qquad (3)$$

Though it appears simple and straightforward, this function (3) burdens its user with 1) the necessity of discovering (or assuming), as an input into the model, the post-Plan growth rate[3] of output, and 2) the usually implausible assumption that the post-Plan growth rate is independent of the intra-Plan growth rate. One way out of this problem (though an exit that remains unused) is to let the post-terminal growth rate (g) equal, or be a simple function of the intra-Plan growth rate[4]. The price is the loss of linearity between I_T and X_T.

2. *Tims* [9]. The Tims model was originally expressed entirely in terms of changes between the terminal-year values (T) and base-year values (0). Using the Bergsman and Manne argument, Tims then arrived at the following variation of function (3):

$$I_T - I_0 = g\sigma\,(X_T - X_0) \qquad (5)$$

or,

$$I_T = g\sigma X_T - g\sigma X_0 + I_0 \qquad (6)$$

For most relationships in flow models, it is quite reasonable to write equations in terms of changes over time, but in the stock-flow relationship, implicit in Equation (5), it is extremely difficult to justify. There is no obvious reason for requiring that $I_T = I$ when $X_T = X_0$; even if output failed to grow during the Plan, I_T could still be larger or smaller than I_0, depending upon the post-terminal growth rate being prepared for[5].

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[3]Or rates, where the model is multisector.

[4]For example, for g equal to the intra-Plan growth rate:

$$g = \left(\frac{X_T}{X_0}\right)^{1/T} - 1 \qquad (4)$$

where the subscript zero refers to the base year of the T-year Plan.

[5]Whether the I_T requirements are larger or smaller in the Tims formulation relative to that of Bergsman and Manne (for given g and σ), depends on whether:

$$I_0 \gtrless \sigma g X_0 \qquad (7)$$

In words, the Tims function leads to a lower terminal-year investment (I_T) if base-year investment (I_0) is too small to permit output to grow between years 0 and 1 at the assumed post-terminal rate (g). In later versions of the model, for example [8], Tims reverted to the Bergsman and Manne formulation.

3. *Sandee* [7]. Terminal-year investment can also be determined by demanding that the intra-Plan investment growth rates be continued into the terminal year. Sandee chooses the simplest assumption about intra-Plan investment, namely that it rises stepwise:

$$I_t = I_0 + ct \quad (t = 0, 1, \ldots, T) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

The virtue of Equation (8) lies in the fact that c can be readily calculated. Since total investment over years 0 to (T—1) equals the new capital required by the output growth over the Plan:

$$\sum_{t=0}^{T-1} I_t = \sigma (X_T - X_0) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

Then, with Equation (8), we have:

$$c = \frac{2}{T(T-1)} (\sigma X_T - \sigma X_0 - TI_0) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

and, substituting (10) into (8) for year T:

$$I_T = \frac{2\sigma}{T-1} (X_T - X_0) - \frac{T+1}{T-1} I_0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (11)$$

The Sandee function for I_T is quite different in appearance from the previous two, since its coefficients involve the length of the Plan (T). More important, however, is the fact that the X_T coefficient in Equation (11) is in general much larger than the X_T coefficients of the previous functions (*i.e.*, of Equations (3) and (6))[6]. This should give us pause since the stepwise intra-Plan growth of investment is really quite mild. If one assumed a constant growth *rate* for intra-Plan investment, terminal-year investment would be even more responsive to X_T.

4. *Khan* [4] and *Manne* [5 ; 6]. By assuming a fixed ratio (h) between terminal-year investment and total intra-Plan investment, they derive a still different I_T function. Let[7]

$$I_T = h \sum_{t=0}^{T-1} I_t \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (12)$$

[6]For plausible values of g and Plan horizons (T) of five to ten years.

[7]With the gestation assumption being used, the right-hand summation of Equation (12) is not exactly the total investment over the Plan—the difference lies in the exclusion of I_T and inclusion of I_0.

Since Equation (9) must hold, the requisite I_T relation is readily found:

$$I_T = h\sigma(X_T - X_0) \qquad \text{.......................................(13)}$$

So far, so easy; but their problem is to get a value for h that is not dependent on the Plan variables and solutions. The procedure is to assume a constant growth rate for I_t (over $t = 0, 1, \ldots, T$):

$$I_t = I_0 (1 + r)^t \qquad \text{.......................................(14)}$$

which, with Equations (9) and (12), yield:

$$h = \frac{r}{1 - (1 + r)^{-T}} \qquad \text{.......................................(15)}$$

Now a rough estimate of r (and knowledge of T) permits an estimate of the appropriate value of h. It is often claimed[8] this procedure has the advantage that h is not very sensitive to changes in r; for example, if r goes up or down 100 per cent from 0.10 at $T = 5$, the resulting value of h rises or falls by only 25 per cent from 0.26. All this, however, neglects a basic constraint on r (and h); summing I_t in Equation (14) over the years 0 to $(T - 1)$ and inserting this value in Equation (9) yields:

$$\frac{(1 + r)^T - 1}{r} = \sigma \frac{X_T - X_0}{I_0} \qquad \text{.......................................(16)}$$

or

$$\frac{X_T - X_0}{X_0} = \frac{I_0}{X_0} \left(\frac{(1 + r)^T - 1}{r\sigma} \right) \qquad \text{.......................................(17)}$$

This means that if one wishes to constrain the range of r for purposes of fixing h, consistency requires also constraining the range of the intra-Plan growth rate $\left(i.e., \frac{X_T - X_0}{X_0}\right)$ to be considered. For example, if $T = 5$, $\sigma = 2.4$, and $I_0/X_0 = 0.12$, then an r in the range 0.05 to 0.15 implies an intra-Plan growth rate in the range, 28 per cent to 34 per cent[9]. But even a narrowed range of Plan growth rates does not solve the logical problem that one assumes a value for h to help estimate r (or intra-Plan growth of output), while the resulting value of r in turn implies a value of h.

5. *Bruno* [3]. This approach differs from the others in that the capital stock is assumed to grow at a certain rate (k) in the terminal year of the Plan.

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[8] *E.g.*, [4, p. 151].

[9] Which implies per annum growth in the range 5.0 to 6.0 per cent, a range of 1.0 percentage points. Neither Khan nor Manne apparenty felt so constrained; they consider per annum growth ranges of 1.5 percentage points [6, p. 265], 2.2 percentage points [4, Pp. 175-176] and 2.5 percentage points [5, p. 383].

This, together with an assumption that the average capital-output ratio in the terminal year is equal to the marginal capital-output during the Plan, yields:

$$I_T = k\sigma X_T \qquad (18)$$

As it stands, this formulation is but trivially different from that of Bergsman and Manne (Equation (3)). A less simple (average vs. marginal) capital-output assumption in the Bruno model would, however, create a real difference.

Table I summarizes the various terminal-year investment functions discussed above. The variety is evident; the importance of the differences is examined in the next section.

TABLE I

THE TERMINAL-YEAR INVESTMENT FUNCTION

$(I_T = \alpha X_T + \beta X_0 + \gamma I_0)$

Model	Coefficient		
	α	β	γ
1. Bergsman and Manne [2, p. 255]	$g\sigma$	0	0
2. Tims [9]	$g\sigma$	$-g\sigma$	$+1$
3. Sandee [7, p. 22]	$\frac{2\sigma}{T-1}$	$\frac{2\sigma}{T-1}$	$\frac{T+1}{T-1}$
4. Khan [4, p. 151], Manne [5, p. 384; 6, p. 270]	$h\sigma$	$-h\sigma$	0
5. Bruno [3, p. 330]	$k\sigma$	0	0

Notes: g = assumed post-terminal output growth rate
h = ratio of terminal-year investment to total Plan investment
k = assumed terminal-year capital-stock growth rate
σ = capital-output ratio
T = length of Plan

II. SENSITIVITY TO THE FUNCTION ASSUMED

The sensitivity of the solution of the planning model to the kind of I_T function assumed can be readily seen through a simple arithmetical example. Let us assume that the known, fixed capital-output ratio (σ) is 2.4 and that the base-year ratio of investment to output (I_0/X_0) is 0.12; then the rate of growth of the economy at the *start* of the Plan[10] is 5 per cent. If the rate of growth throughout the Plan is to be 5 per cent then all of the functions discussed in Section I will yield the same value for I_T, namely, $I_T/X_T = I_0/X_0 = 0.12$, provided that the terminal or post-terminal parameter assumptions are consistent with the 5-per-cent growth figure[11]. It is interesting to see what happens to the implied terminal-year investment-output ratios (I_T/X_T) when intra-Plan growth rates above 5 per cent are considered. We will treat just two cases: 1) where output grows at 7 per cent during the Plan, but the post-terminal parameters continue to be based on 5-per-cent growth; and 2) where output grows at 7 per cent during the Plan, and the post-terminal parameters are also based on 7 per cent[12].

Assuming a five-year plan (*i.e.*, T = 5), we have in each of the two cases, $X_T = X_0 (1.07)^5$. For the post-terminal parameters, we have in the first case $g = k = 0.05$ and $h = 0.231$ and in the second case, $g = k = 0.07$ and $h = 0.244$. Table II summarizes the terminal-year investment-output ratios implied by the various functions under these two kinds of 7-per-cent growth.

TABLE II

IMPLIED I_T/X_T VALUES AT 7-PER-CENT PLAN GROWTH

Function	With post-terminal parameters keyed to	
	(1) 5% growth	(2) 7% growth
1. Bergsman and Manne	.120	.168
2. Tims	.120	.134
3. Sandee	.217	.217
4. Khan and Manne	.158	.170
5. Bruno	.120	.168

Note: $X_T = X_0 (1.07)^5$

Source: Col. (1): calculated at $g = k = .05$ and $h = .231$.
Col. (2): calculated at $g = k = .07$ and $h = .244$.
(For h calculations, *see* Equation (15)).

[10] *I.e.*, between years 0 and 1. Since $I_0 = \sigma (X_1 - X_0)$, we calculate, regardless of the Plan activities in year 1 (and after), that

$$\frac{X_1 - X_0}{X_0} = \frac{I_0 / X_0}{\sigma} = \frac{0.12}{2.4} = 0.05 \qquad (19)$$

[11] *I.e.*, consistency requires, in terms of earlier parameters, that g or k be equal to 0.05. The Khan, Manne h must be (for T = 5) equal to 0.231 to reflect 5-per-cent growth.

[12] It should be noted that continual growth at 7 per cent implies an investment-output ratio (eventually) of 0.168 (=0.07 × 2.4).

Not much needs to be said about Table II. The implied I_T/X_T ratios display an intolerable variation. To use models of this type, one must know more about the terminal-year investment-output ratio for any particular output growth rate than that it lies in the range 0.120 to 0.217. Note that this sensitivity is not a problem of inaccurate empirical information. Rather the model-builders have inserted implicitly, but inevitably, their biases into the results. Moreover, one of the first interests of "real-world planners" is the implied investment-output ratio (and hence, adjusted for "aid", the savings-income ratio) of different output growth rates. The model which says 7-per-cent growth can be achieved with a rise (between years 0 and T) in I_T/X_T of only 0.014 (*i.e.*, of 12 per cent) will have a very different impact than the one that says a rise in the investment-output ratio of 0.097 (*i.e.*, of 81 per cent) is necessary. The practical planner, if he listens, will conclude that the 7-per-cent target is pretty easy in the first case and quite infeasible in the second.

So much time is devoted to gathering data in the construction of these models, for we know that without good data they are useless. What many economists have not yet fully realized is how sensitive the results are to the assumed structure of these models. When an arbitrary choice of functional form can have such a serious impact on the results, as in the case of these terminal-year investment determinants, one worries.

III. TOWARD AN APPROPRIATE FUNCTION

Once one discovers how sensitive the results of the model are to the choice of the terminal-year investment function, the obvious question is: can we remove some of the arbitrariness in the choice. The answer is no.

In general, there can be no single "correct" function unless we are willing to specify exogenously the post-terminal behaviour of output[13]. If, for example, we include as one of the targets of the Plan exercise a requirement that the immediate post-terminal growth rate of output be g, then the simple Bergsman and Manne [2] formulation, Equation (3), is correct. If, on the other hand, one does not wish to fix the post-terminal data *a priori* (*e.g.*, if one feels that the intra-Plan path should be permitted to influence the post-Plan path), then no single function can be called correct.

To me, there are only two *generally acceptable* conditions that can be demanded of the terminal-year investment function. One, if past (*i.e.*, pre-Plan) growth rates are continued through the Plan years, the end-of-Plan investment-output ratio should emerge from the model unchanged from the start-of-Plan ratio. And two, if the intra-Plan growth rate is higher (lower) than the pre-Plan growth rate, the end-of-Plan investment-output ratio should be higher

[13]Or the terminal-year investment or capital stock.

(lower) than the start-of-Plan ratio. In symbols, this may be written:

$$\text{As } i \gtreqless i_0 \text{ , } I_T / X_T \gtreqless I_0/X_0 \qquad \text{(20)}$$

where $i_0 \left(= \frac{I_0 / X_0}{\sigma} \right)$ is the initial growth rate of output[14], and i is the Plan growth rate (*i.e.*, $X_T = X_0(1+i)^T$). Since the functions we are considering are linear, write [15]

$$I_T = aX_T + b \qquad \text{(21)}$$

where the only restriction on a and b is that neither may be a function of as-yet-unknown Plan variables. Equation (21) may be rewritten as:

$$\frac{I_T}{X_T} = a + \frac{b}{X_0 (1+i)^T} \qquad \text{(22)}$$

Then the equality part of condition (20) requires that:

$$(\sigma i_0 - a) X_0 (1 + i_0) = b \qquad \text{(23)}$$

And the inequality part of condition (20) requires that $d(I^T / X_T)/di$ be positive (where d represents a partial derivative). Together the criteria imply:

$$b < 0 \text{ and } a > \sigma i_0 \qquad \text{(24)}$$

It is clear from examination of Table I, where $a = \alpha$ and $b = \beta X_0 + \gamma I_0$, that those functions in which both β and γ are assumed zero fail these criteria. Thus, the Bergsman-Manne and Bruno formulations are unsatisfactory unless they are indeed based upon *ex ante*, exogenous knowledge of post-terminal growth rates (of output or capital stock). The other functions discussed fulfil the criteria[16].

Since it is difficult to feel conviction toward more, or more precise, criteria for the function, we are stuck with the acceptability of a wide variety of functional forms, and the biases implicit in them. In this age of improved access to better computers, there is much to be said for the introduction of nonlinear-

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[14]Essentially, the pre-Plan growth rate; since it depends only on year 0 variables and decisions, the Plan cannot affect it.

[15]The issue is not evaded, only complicated, by use of nonlinear terminal-year investment functions.

[16]Conditionally, for Tims, g must be greater than i_0; for Sandee, $2/(T-1)$ must be greater than i_0; and for Khan and Manne, h must be greater than i_0. All these conditions will generally be fulfilled in fact.

ities and explicit consideration of the intra-Plan data (*i.e.*, years 1, 2,. . . , T—1). But these extensions do not really meet the terminal-investment determination dilemma. The real solution, I feel, requires inclusion of the terminal-year capital stock as an explicit Plan objective and/or a model that functions through a longer time-horizon than that of the Plan to which the model is to be applied.

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