On the Rationality of "Cascaded" Export Subsidies and Taxes

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In their efforts to encourage the export of manufactures, many less developed countries (LDCs) have introduced "cascaded" tax-subsidy schemes, whereby the export subsidy is higher (or export tax lower) the more highly processed is the exported product. While such cascading has been loosely defended on a variety of grounds—such as the encouragement of industrialization or of increased domestic content in exports—it's only rigorous rationale has derived from the perceived existence of monopoly power in the sale of primary products.

Recently, however, Scott [1970] has shown that cascading of specific taxes in order to exploit monopoly power is irrational. He proves that—"if there is good reason to tax the export of raw jute and cotton, then... one should tax the export of the raw jute and cotton contained in exports of manufactures." We maintain that Scott's call for equal specific taxes at all stages, while logically correct for the monopoly argument alone, neglects other, more important factors shaping the export tax and subsidy policies of LDCs. There are many more examples among LDCs of cascaded export subsidies than there are cases of monopoly power—real or perceived—in the export of primary products. We believe that the cascading of specific export taxes and subsidies often arises in an effort to overcome the more flagrant disadvantages of an overvalued exchange rate without drawing too heavily on scarce budgetary resources.

When seen in this light, cascading is not necessarily irrational. Rather, it emerges as second-best policy under plausible conditions on supply elasticities and planner preferences. The argument is developed below, first roughly (Section I), then for a single export product with many processing stages (Section II), and finally for the second-best optimum structure of both import and export taxes in an economy with many products (Section III). The conclusion (Section IV) summarizes the conditions for cascading.

I. The Basic Argument

Consider a series of competitive firms that sequentially process some primary product. At each stage of processing, each firm 1) uses as a raw material only the output of the previous stage, 2) creates value added subject to marginal costs which rise with output, and 3) either exports its output or sells it competitively to processors at the next stage. In the absence of taxes and subsidies, the private profit of each producer at the $i^{th}$ stage ($\pi_i$) is

$$\pi_i = p_i q_i - c_i(q_i) - p_{i-1} q_i,$$

where $p_i$ is the world price of the product of the $i^{th}$ stage converted to domestic currency at the official (overvalued) rate of exchange, $q_i$ is the volume of output defined such that the unit of output is arbitrary at the first stage but is defined for subsequent stages as requiring one unit of the output of the previous stage as its input, and $c_i(q_i)$ is the total value-
added cost function. \( p_0 \) is zero, i.e. the first stage involves only value added. The first and second derivatives of \( c_i(q_i)\), \( c'_i \) and \( c''_i \), are both assumed positive in the relevant regions.

Profit-maximization will lead producers at each stage to the output at which
\[
c'_i = p_i - p_{i-1} .
\]
(2)
Social profitability, however, requires the recognition that the output of the \( i^{th} \) stage is really worth \((1+\alpha)p_i\), where \( \alpha \) measures the extent of the overvaluation of the exchange rate. Social profit \( (\pi_i) \) is therefore
\[
\pi_i^* = (1+\alpha)p_i q_i - c_i(q_i) - (1+\alpha)p_{i-1} q_{i-1} ,
\]
(3)
and its maximization requires that
\[
(c'_i)^* = (1+\alpha) (p_i - p_{i-1}) .
\]
(4)
Comparison of equations (2) and (4) shows that as long as \( \alpha \) exceeds zero, \( (c'_i)^* \) will exceed \( c'_i \) by the proportion \( \alpha \). Since marginal costs are rising, this implies that the socially optimal output of any firm \( (q_i^*) \) will be larger than its actual output in the absence of taxes and subsidies \( (q_0^*) \). Moreover, summing horizontally the marginal cost of value added curves for all firms at any given stage of processing, allows the definition of the elasticity of the resulting industry marginal cost of value added curve—i.e. the industry supply curve of value added— as \( e_i = (\text{proportionate change in output})/(\text{proportionate change in the marginal cost of value added}) \). If this elasticity for the \( i^{th} \) stage is larger than the corresponding elasticity for the \( j^{th} \) stage, \( q_i^* \) will be relatively further above \( q_j^* \) than \( q_j^* \) is above \( q_j^* \) since both stages have the same proportionate divergence between \( (c'_i)^* \) and \( c'_i \). Thus if supply elasticities are believed to be higher at the stages of greater processing, the overvaluation will have reduced the output and export of "manufactures" proportionately more than of primary products—a particularly undesirable effect given the proclivities of LDCs and the teachings of UNCTAD.

The first-best solution to this sort of distortion is, of course, a uniform ad-valorem subsidy of \( \alpha \) applied to exports at all stages of production. Unfortunately, most LDCs resist such a subsidy for a variety of well-known reasons, chief among which is the heavy claim made upon the government budget. But improvements in the situation can still be made if, at the \( i^{th} \) stage, the privately perceived net marginal revenue \( ^5 \) can be raised toward the socially perceived net marginal revenue at that stage. Consider for instance the imposition of specific export taxes (or, if negative, subsidies) at the \( i^{th} \) and \((i-1)^{th}\) stages, \( t_i \) and \( t_{i-1} \). The privately perceived net marginal revenue at stage \( i \) becomes \( (p_i - t_i) - (p_{i-1} - t_{i-1}) \), and this net marginal revenue will be greater than that of the pre-tax situation to the extent that \( t_{i-1} > t_i \). In essence, this is the rationale for cascading: at any stage in the processing sequence, the export tax must be lower than the export tax at the preceding stage if the distortion (at that stage) due to the overvaluation is to be at least partially overcome. Similarly, if there is an export subsidy at the \((i-1)^{th}\) stage, there must be a larger subsidy at the \( i^{th} \) stage if the distortion is to be reduced.
The common sense of the above proposition is easily seen. The export
tax on the raw material used by the $i^{th}$ stage lowers the domestic price of
the raw material and hence increases the privately perceived net marginal
revenue at the $i^{th}$ stage. This induces a change in output and export toward
the higher level that is socially appropriate. If, however, the export tax
on the $i^{th}$ stage is not smaller than that on the raw material, the direction
of the inducement and the resulting movement of output and export is reversed
and the output distortion at the $i^{th}$ stage will be increased.

This "proof" is simple, but it is also quite inadequate. Only one stage
is analyzed at a time, with the tax at the preceding stage always assumed
given; no attention is directed at the question of how much of the distor-
tion is to be offset at each stage; and the budget constraint, which is the
reason for not offering the specific subsidy of $ap_i$ at each stage, which
would remove all distortion, is not explicitly considered at all. In the
next section, we will outline the more general determination of the optimal
structure of export taxes and subsidies.

II. The Model

Assume that planners wish to impose a set of export taxes and subsidies
so as to meet a budget constraint and to minimize their valuation of the
welfare losses caused by deviations from socially correct outputs. The
welfare loss at the $i^{th}$ stage is shown as the shaded area in Figure 1. If
the marginal cost of value added curve can be regarded as linear, the area
is readily measured in terms of the known 1) overvaluation of the exchange
rate ($\alpha$), 2) prices of the output and its raw material ($p_i$ and $p_{i-1}$), 3) pre-
tax output ($q_i$), and 4) elasticity of the marginal cost of value added ($e_i$).
The actual output at some set of export taxes or subsidies, $q_i$, and the
socially optimal output, $q_i^*$, can be determined from the definition of the
supply elasticity:

$$e_i = \left(\frac{q_i^0 - q_i'}{q_i^0}\right)\left(\frac{t_i - t_{i-1}}{p_i - p_{i-1}}\right), \text{ and}$$

$$e_i = \left(\frac{q_i^* - q_i^0}{q_i^0}\right) \alpha,$$

where $e_i$ is measured around the pre-tax price and quantity (i.e., at point
A in Figure 1). The welfare loss at the $i^{th}$ stage can then be measured as

$$\frac{e_i q_i^0}{2(p_i - p_{i-1})} \left[\alpha(p_i - p_{i-1}) + (t_i - t_{i-1})\right]^2.$$
Figure 1

![Diagram](image)

- Supply of value added
- Value added at $i^{th}$ stage

Mathematical expressions:
- $(1+\alpha)(p_i - p_{i-1})$
- $(p_i - p_{i-1})$
- $(p_i - t_i) - (p_{i-1} - t_{i-1})$

Variables:
- $q_i$
- $q_i'$
- $q_i^0$
- $q_i^*$
Planners need not weigh these losses equally at different stages, and for LDCs it is typically appropriate to recognize a weight \( w_i \) for each stage such that \( w_i \geq w_{i-1} \).

The budget constraint on the export taxes and subsidies may, for simplicity, be assumed to be

\[
\sum_{i=1}^{N} t_i (q_i' - q_{i+1}') = \sum_{i=1}^{N} (t_i - t_{i-1}) q_i' \geq B, \tag{8}
\]

where \( q_{N+1} \) and \( t_0 \) are understood to be zero.

The selection of the optimal structure of the export taxes and subsidies then reduces to the problem: minimize

\[
\sum_{i=1}^{N} \frac{w_i e_i q_i^o}{2(p_i - p_{i-1})} \left[ \alpha \left( p_i - p_{i-1} \right) + (t_i - t_{i-1}) \right]^2 \tag{9}
\]

with respect to the \( N \) taxes, \( t_i \), subject to the budget constraint (8) with its \( q_i' \) terms removed by use of equation (5), i.e.

\[
B - \sum_{i=1}^{N} \left( t_i - t_{i-1} \right) \left( 1 - \frac{t_i - t_{i-1}}{p_i - p_{i-1}} \right) q_i^o < 0. \tag{10}
\]

Partial differentiation of the Lagrangian equation formed by equations (9) and (10) yields \( N \) equations

\[
\alpha w_i e_i q_i^o + \frac{t_i - t_{i-1}}{p_i - p_{i-1}} - \lambda q_i^o + 2\lambda e_i q_i^o \frac{t_i - t_{i-1}}{p_i - p_{i-1}} = 0, \quad i=1, \ldots, N, \tag{11}
\]

where \( \lambda \) is the Lagrangian multiplier and the value of all parameters and variables is understood to be zero when subscripted zero or \((N+1)\). Equations (11) provide \( N \) linear equations in the \((N+1)\) variables, \( t_1, t_2, \ldots, t_N \), and \( \lambda \). The budget constraint, when binding, provides the necessary \((N+1)\)st equation.

A full solution to the system is not needed to solve equations (11) for the \( N \) values of \((t_i - t_{i-1})\) in terms of \( \lambda \) and parameters, \( \alpha, w_i, e_i, \) and \( q_i^o \); rather, equations (11) can be manipulated to yield:

\[
t_i - t_{i-1} = -\frac{\alpha w_i e_i - \lambda}{e_i(w_i + 2\lambda)} \left( p_i - p_{i-1} \right). \tag{12}
\]

Because \( t_0 \) is defined as zero, we can use equations (12) to solve in sequence for \( t_1 \), then \( t_2 \), and so on to \( t_N \). The possible values for the \( t \)'s must lie within two extremes:

1) When the budget constraint (8) is not binding and hence \( \lambda \) is zero, all stages receive equal ad valorem subsidies, i.e., \( t_i = -\alpha p_i \) for all \( i \); and
2) when the necessary budget revenue, B, is fixed at its maximum and hence \( \lambda \) is infinite, the tax differential at each stage depends on the elasticity of the supply of value added by the industry at that stage, i.e.

\[
\frac{p_i - p_{i-1}}{2e_i}
\]

But our primary interest is in the presence or absence of cascading. Direct inspection of equations (12) shows that \( t_i \leq t_{i-1} \) according to whether \( w_i e_i \leq \frac{\lambda}{a} \). (13)

Cascading, or \( t_i < t_{i-1} \), will therefore occur at those processing stages for which the values of \( e_i \) and \( w_i \) are sufficiently large. The first question is then whether \( e_i \) and \( w_i \) typically rise as production moves from lower to higher stages. Although there is no theoretical necessity that elasticities of the marginal cost of value added rise as higher stages of processing are reached, it seems empirically probable that they do. For instance the industrial labor and capital used at higher stages of processing are apt to be supplied more elastically in LDC's than the labor and land typical of lower stages. More importantly, and regardless of the empirical evidence, planners do seem to believe that these elasticities rise at higher stages. There is an even greater presumption that welfare weights will rise at higher stages of processing as the evidence of LDC policies and achievements strongly suggests that industry is preferred, ceteris paribus, to agriculture and mining, and that "higher" industry is preferred to "lower".

Of course, the fact that the product, \( w_i e_i \), rises at higher stages does not alone guarantee that cascading is optimal. But, as Figure 2 illustrates, for a "reasonable" value of \( \frac{\lambda}{a} \), generally rising values of \( w_i e_i \) -- or more precisely, values of \( w_i e_i \) that pass \( \frac{\lambda}{a} \) only once and then from below -- are sufficient to generate cascaded export taxes and subsidies for stages beyond that stage (i) at which \( w_i e_i \) passes \( \frac{\lambda}{a} \).

The absence of cascading at the initial stages may be explained by the observation that, for revenue purposes, the increase in product prices as one moves to higher stages of processing implies increasing specific taxes at higher stages if the elasticities do not also rise relatively quickly. Yet as \( e_i \) and/or \( w_i \) do rise, the welfare loss resulting from any given export tax will rise as well. Thus, when the values of \( w_i \) and \( e_i \) are rising, increasing taxes are imposed at early stages -- where \( w_i \) and \( e_i \) are still low -- in order to increase tax revenue, but decreasing taxes are imposed at later stages -- where \( w_i \) and \( e_i \) are high -- in order to reduce welfare losses.

Equations (12) yield several other interesting observations. They are stated here and derived in the Appendix:

1) The optimal value of any export tax, \( t_i \), depends upon the optimal value of all of the taxes at earlier stages of processing and thus upon, ultimately, the \( e_i \) and \( w_i \) of earlier stages but it depends upon none of the taxes at later stages of processing.
2) The optimal taxes (and subsidies) when stated in ad-valorem terms will cascade throughout, provided only that the values of $w_i$ and $e_i$ rise with higher stages of processing.

3) No export subsidy should ever exceed the first-best ad-valorem rate of $\alpha$.

III. The Extension to Many Products and to Imports

Finally, it is interesting to extend this analysis to a world of many exported and imported products, each with several stages of processing. Because the model rapidly becomes complex, we will assume that 1) domestic demand for each product at each stage is fixed, 2) there are no inter-industry flows, and 3) general-equilibrium problems can be safely ignored.

For the $i^{th}$ stage of processing of the $j^{th}$ product and with $d_{ij}$ representing the fixed level of domestic demand, exports occur when $(q_{ij} - q_{i+1,j} - d_{ij})$ is positive and imports occur when that expression is negative. As before, the planners wish to choose the values of $t_{ij}$ so as to minimize the welfare loss, subject to a budget constraint:

Minimize

\[
\sum_{i} \sum_{j} w_{ij} e_{ij} \left( q_{ij} - q_{i+1,j} - d_{ij} \right) + \left( t_{ij} - t_{i-1,j} \right)^2
\]

subject to

\[
B - \sum_{i} \sum_{j} t_{ij} (q_{ij} - q_{i+1,j} - d_{ij}) \leq 0 .
\]

A positive $t_{ij}$ is now seen as an export tax or import subsidy; a negative $t_{ij}$ is an export subsidy or import tax. Partial differentiation of the Lagrangian equation formed by equation (14), after first removing the $q_{ij}$ variables by means of expressions similar to equation (5), yields equations of the form,

\[
t_{ij} - t_{i-1,j} = - \frac{\alpha w_{ij} e_{ij} - \lambda \left( 1 - \sum_{x=1}^{N} d_{ij}^o / q_{ij}^o \right) \left( p_{ij} - p_{i-1,j} \right)}{e_{ij} (w_{ij} - 2\lambda)} .
\]

These equations are identical to equations (12) except for the terms occasioned by the introduction of domestic demand, i.e.

\[
1 - \sum_{x=1}^{N} \frac{d_{xj}^o / q_{ij}^o}{e_{ij} \left( w_{ij} - 2\lambda \right)} ,
\]

and this term is clearly positive if production at the $i^{th}$ stage of the $j^{th}$ product is more than sufficient to supply domestic consumption requirements at the $i^{th}$ and later stages, while it is negative if production is not sufficient.
Thus, the term is positive (but less than unity) for exports and negative for imports.

Cascading on both the import and export side is shown in equation (15) by negative values of \( t_{i+1} - t_{i-1,j} \). For imports, term (16) is negative and hence optimal import tariffs always cascade. The condition for cascading export subsidies is

\[
    w_{ij} e_{ij} > \frac{\lambda}{\alpha} \left[ 1 - \sum_{x=1}^{N} d^o_{ij} / q^o_{ij} \right],
\]

which is similar to, but less stringent than, condition (13). For exports, therefore, taxes and subsidies will cascade eventually if the values of \( w_{ij} \) and/or \( e_{ij} \) rise sufficiently at higher stages of processing. Finally, careful examination of equation (15) indicates that the optimal cascading of import tariffs must begin at an ad-valorem rate of \( \alpha \), while any cascading of export subsidies can never rise above that ad-valorem rate.

IV. Summary

There are then plausible conditions under which export taxes and import taxes could be cascaded in order to yield a second-best optimum. These conditions are generally characterized by budget stringency on the part of government, and by rising elasticities of the marginal cost of value added and/or rising planners' weights as the degree of processing increases.

Since there are forces tending to make the optimal set of export taxes cascaded, the frequency with which they are found in LDCs ought not to prove surprising nor should one be too quick to suggest that they are irrational. Nevertheless, by the criteria employed here, the second-best export subsidy never exceeds the degree of overvaluation of the exchange rate, suggesting that many LDC subsidy schemes have been overly cascaded.

Furthermore, we should note carefully what the calculation of the optimal set of taxes requires. As equations (12) and (15) show, for any product, the optimal tax at the \( i \)th stage will depend upon the prices at that and all preceding stages \( (p_1, \ldots, p_i) \), the supply elasticities at that and all preceding stages \( (e_1, \ldots, e_i) \), the planners' preferences with respect to that and all the preceding stages \( (w_1, \ldots, w_i) \), the degree of overvaluation of the exchange rate \( (\alpha) \), and value of the relaxation of the budget constraint \( (\lambda) \). While cascading may well be optimal in the second-best world of overvaluation and budgetary tightness, it is very unlikely that LDC planners have the knowledge, staff or patience required to implement the optimal structure of export taxes and subsidies. In the real, at least third-best world in which LDCs operate, the enforcement of a uniform, even if small, export subsidy may well be better policy.
Appendix

Several interesting observations can be made through the manipulation of equations (12).

1) Any single optimal value of $t_i$ will depend upon the optimal value of all the taxes at earlier stages of processing, or more precisely, upon the $e_j$ and $w_j$ of all previous stages. In particular, solving for successively earlier $t_i$ in equations (12) and noting that $t_0=0$,

$$t_i = \sum_{j=1}^{i} \frac{-\alpha w_j e_j + \lambda}{e_j (w_j + 2\lambda)} (p_j - p_{j-1})$$  (A 1)

2) Since most export subsidy schemes are of the ad-valorem type, it is useful to transform equations (A 1) into

$$T_i = \sum_{j=1}^{i} \frac{-\alpha w_j e_j + \lambda}{e_j (w_j + 2\lambda)} \frac{p_j - p_{j-1}}{p_i}$$  (A 2)

where $T_i$ is the ad-valorem equivalent of the specific $t_i$. The conditions for the cascading of the $T_i$ are less stringent than those for the $t_i$; cascading specific subsidies are a sufficient but unnecessary condition for cascading ad-valorem subsidies. Comparing equation (A 2) evaluated first for $T_i$ and then for $T_{i-1}$ yields as a sufficient condition for $T_i < T_{i-1}$,

$$\frac{\lambda - e_j \alpha}{e_j (w_j + 2\lambda)} > \frac{\lambda - e_{j-1} \alpha}{e_{j-1} (w_{j-1} + 2\lambda)}$$  (A 3)

This in turn will be satisfied if $e_j$ and $w_j$ rise continuously with increasing stages of processing. Ad-valorem subsidies may then begin cascading from the first stage, without the necessity of $we_i$ exceeding $\lambda/\alpha$.

3) Equations (A 2) may also be used to show that no export subsidy should exceed its first-best value, an ad-valorem rate of $\alpha$. This is most easily seen by noting that for any $T_i$, the partial derivative $\delta T_i/\delta \lambda$ is always positive, implying that $T_i$ is an increasing function of the degree of budget stringency. But when $\lambda = 0$, $T_i = -\alpha$; therefore, $\lambda > 0$ implies $T_i > -\alpha$ and no second-best export subsidy may exceed the degree of overvaluation. There is an upper limit on the optimal amount of cascading.
Footnotes

1 For examples of such cascading, see United Nations [1970] passim, but especially p. 41.


3 E.g. raw jute, jute thread, jute fabric, jute bags, etc.

4 For simplicity, we ignore the possibility of domestic final demand or imported raw materials. The assumption of competition is made to insure that terms-of-trade considerations do not intrude.

5 Hereafter, both net marginal revenue and marginal cost will be understood to refer to the marginal revenue of value added and the marginal cost of value added, respectively.

6 There is, of course, no reason why the post-tax private value added should be below either the optimal or the actual pre-tax value added. But the principle and the measurement of the welfare loss is in all cases the same.

7 There is no problem with the numeraire for welfare weights since a general scaling of the values of $w_i$ affects only, and proportionately, the value of $\lambda$.

References


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