CONSISTENT AGGREGATION OF FAMILY AND HIRED LABOR IN AGRICULTURAL PRODUCTION FUNCTIONS

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ABSTRACT

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An estimated production function for the Muda river valley in Malaysia is used to examine three issues: (1) whether or not labor marginal product is zero, (2) whether or not farm households allocate resources efficiently and (3) whether or not agricultural labor markets are characterized by dualism. In areas where an active labor market exists the first two of these issues may be closely related to the third if family and hired labor can be considered separate factors of production. This study shows that if the labor aggregate is defined as the sum of family and hired labor the resulting production function estimate will be subject to specification bias which will render empirical tests of the issues mentioned invalid. Using separate variables for family and hired labor it is shown that the marginal product of family labor is positive and significantly different from zero and that farms are approximately allocatively efficient. The study does find, however, some substantiation for a mild degree of dualism in the labor market.

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CONSISTENT AGGREGATION OF FAMILY AND HIRED LABOR IN AGRICULTURAL PRODUCTION FUNCTIONS [1/]

Introduction

Production function analysis has been widely used in the development economics literature to examine a variety of issues of major policy significance. Three issues in particular have been subject to extensive empirical research. These are,

(i) whether or not labor's marginal product in agriculture is zero. The answer to this question was originally thought to be the key in resolving the debate over the existence of surplus labor. Subsequent analysis has shown that zero marginal productivity is neither a sufficient nor a necessary condition for suplus labor.[2/] Nevertheless, the actual value of labor's marginal product is still of major importance in determining the social cost (or shadow wage rate) of withdrawing labor from agriculture;[3/]

(ii) whether or not rural households allocate resources efficiently. Employing a single-equation estimation procedure, researchers have compared the estimated marginal revenue product of labor with the observed wage-rate. Conclusions concerning allocative efficiency may then be

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1/ The authors acknowledge gratefully the research assistance of Leain-Hong Ding.

2/ See Amartya K. Sen, "Peasants and Dualism with or without Surplus Labor," The Journal of Political Economy, Vol. 74, No. 5 (October 1966): 425-50. In their model, however, Meghnad Desai and Dipak Mazumdar ("A Test of the Hypothesis of Disguised Unemployment," Economica, Vol. 37 (February 1970): 39-53) resurrect zero marginal productivity as a necessary condition for surplus labor.

3/ See Deepak Lal, "Disutility of Effort, Migration, and the Shadow Wage-rate," Oxford Economic Papers, Vol. 25, No. 1 (March 1973): 112-26.

drawn on the basis of the household's success in equating marginal revenue and cost;[4/]

(iii) whether or not agricultural labor markets are characterized by dualism. Among other things the concept of labor market dualism has been used by Mazumdar[5/] to explain the observed decrease in yield and labor intensity on Indian farms as farm size increases.

In areas where an active labor market exists the first two of these issues may be closely related to the third if family and hired labor can be considered separate factors of production. For instance, Desai and Mazumdar[6/] have estimated separate Cobb-Douglas production functions for rice growing farms in Bengal, first, for those using family and hired labor and, second, for those relying exclusively on family labor. They find that the elasticity of output with respect to labor for the farms using hired labor is positive while the elasticity (and, by deduction, the marginal product) for farms using family labor only is zero. Alternatively, for rice farms in Vietnam, Brown and Salkin[7/] have estimated a production

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4/ See, for instance, W. D. Hopper, "Allocational Efficiency in Traditional Indian Agriculture," Journal of Farm Economics, August 1965, pp. 611-624; Y. Huang, "Allocational Efficiency in a Developing Agricultural Economy in Malaya," American Journal of Agricultural Economics, 1971, pp. 514-516, or, more recently, A. A. Sidhu, "Economics of Technical Change in Wheat Production in the Indian Punjab," American Journal of Agricultural Economics, May 1974, pp. 217-226. This methodology has been critized by Vahid F. Nowshirvani ("Allocation Efficiency in Traditional Indian Agriculture: Comment," Journal of Farm Economics, Vol. 49 (February 1966): 218-21) and others on the grounds that the production function estimate is subject to simultaneous-equation bias. This criticism has been successfully answered by, among others, A. Zellner; J. Kmenta and J. Drèze ("Specification and Estimation of Cobb-Douglas Production Function Models," Econometrica, Vol. 34 (October 1966): 784-95) who demonstrate that, given the lag between input decisions and output which occurs in agriculture, ordinary least squares will give unbiased estimates of the production function.

5/ See Mazumdar, "Size of Farm and Productivity: A Problem of Indian Peasant Agriculture," Economica, Vol. 32 (May 1965): 161-73.

6/ Op cit.

7/ James A. Brown, Jr. and Jay S. Salkin, "Underemployment in Rural South Vietnam: A Comment and a Discussion of Family Labor," Economic Development and Cultural Change, Vol. 23, No. 1 (October 1974): 151-60.

function with separate family and hired labor inputs and also find that the elasticity (and therefore marginal product) for family labor is zero. The implication of these findings is that, since there are wage opportunities coexistent with a zero marginal product for family labor, there is a dual labor market and the farms in question are allocatively inefficient. A further implication is that it may be incorrect to sum family and fixed labor to form an aggregate labor variable for use in the estimation of a production function. Given Brown and Salkin's results family labor should be assigned a much smaller weight than hired labor in the construction of a labor aggregate or else the two types of labor should be introduced separately in the production function. Such results cast considerable doubt on the vast majority of empirical production function estimates, where, because of lack of data, researchers have had no alternative but to treat labor as a homogeneous factor of production.

In this paper we use estimates of a translog production function for rice production in Malaysia to test for the existence of a consistent aggregate of family and hired labor.[8/] Since our data are available in a disaggregated form we also derive separate marginal products for the two types of labor and check for a corroboration of labor market dualism. Our results indicate that a consistent aggregate does exist for family and hired labor, and that the marginal product of family labor is definitely positive, although our results are also consistent with a mild (but not statistically significant) degree of dualism in the labor market.

The Econometric Procedure

We estimate the translog production function, which, under various restrictions on the parameters, can be interpreted as an approximation to arbitrary production functions with characteristics of interest. Using the tests derived by Berndt and Christensen[9/] we examine parameters for conformity

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8/ E. R. Berndt and L. R. Christensen, "Testing for the Existence of a Consistent Aggregate Index of Labor Inputs," American Economic Review, Vol. LXIV, No. 3 (June, 1974), pp. 291-404 and same authors, "The Translog production Function and the Substitution of Equipment Structures, and Labor in U. S. Manufacturing 1929-68," Journal of Econometrics, Vol. 1, No. 1 (March 1973), pp. 81-113.

9/ Op cit.

with (1) constant returns to scale (CRTS), (2) separability of inputs,[10/] and (3) the Cobb-Douglas functional form. Finally, since we fit the trans-log production function directly, using ordinary least squares rather than deriving the parameters from simultaneous estimates of the profit maximizing conditions, we test the confirmed functions for allocational efficiency.[11/]

The production function to be estimated is of the form

$$\begin{aligned} \ln Q = {} & \ln\alpha_o + \alpha_A \ln A + \alpha_H \ln H + \alpha_F \ln F \\ & + 1/2\lambda_{AA}(\ln A)^2 + 1/2\lambda_{HH}(\ln H)^2 + 1/2\lambda_{FF}(\ln F)^2 \\ & + \lambda_{AH}\ln A.\ln H + \lambda_{AF}\ln A.\ln F + \lambda_{HF}\ln H.\ln F \end{aligned} \tag{1}$$

where Q = quantity of output, A = area cultivated, H = hired labor, F = family labor. For constant returns to scale to exist, it is necessary and sufficient that the following relations are satisfied:

$$\begin{aligned} & \alpha_A + \alpha_H + \alpha_F = 1 \\ & \lambda_{AA} + \lambda_{AH} + \lambda_{AF} = 0 \\ & \lambda_{AH} + \lambda_{HH} + \lambda_{HF} = 0 \\ & \lambda_{AF} + \lambda_{HF} + \lambda_{FF} = 0 \end{aligned} \tag{2}$$

For all of the inputs to be mutually separable (global separability) it is sufficient that:

$$\lambda_{AH} = \lambda_{AF} = \lambda_{HF} = 0 \tag{3}$$

If both CRTS (relation (2)) and global separability (relation (3)) exist, then the production function is of the Cobb-Douglas form.

Should global separability be rejected it is still possible that two of the three inputs are separable. For H and F to be separable from A it is sufficient that the following relation be satisfied:

10/ Two inputs are separable if the marginal rate of substitution between the two inputs remains unaffected by changes in the quantities of other factors used in production. An implication of separability is that a consistent aggregate exists, see H.A.J. Green, Aggregation in Economic Analysis, Princeton University Press, Princeton, 1964.

11/ See footnote 4 for references justifying this procedure.

$$\lambda_{AH} = \lambda_{AF} = 0 \quad (4a)$$

Similarly for A and H to be separable from F the critical relation is:

$$\lambda_{AF} = \lambda_{HF} = 0 \quad (4b)$$

and for A and F to be separable from H the critical relation is:

$$\lambda_{AH} = \lambda_{HF} = 0 \quad (4c)$$

The tests precede as follows: Given the estimated production function we first test for CRTS (relations (2) above). If constant returns to scale is not rejected we proceed to use restricted least squares to impose CRTS as a maintained hypothesis and test for global separability (relation (3)). If CRTS is rejected we test for global separability without first imposing the CRTS relations. Finally in either case if global separability is rejected we procede to test separability for pairs of inputs (relations 4a - 4c).[12/] In conducting the tests an overall significance level of .09 is used and .01 is allocated to each of the nine F - tests of joint hypothesis (four restrictions in relation (2), three in relation (3) and two in (4a - 4c)). Table 1 gives the critical F values for the tests.

12/ The tests we set forth above are for linear separability. Other tests are possible for nonlinear separability and for the conditions under which the production function is of the CES (constant elasticity of substitution) form. However, because of the nature of our results these linear tests were sufficient to examine the hypotheses we are considering. Since the linear separability relations are special cases of the nonlinear relations and since the Cobb-Douglas is a special case of the CES function it is sufficient to test for the linear and Cobb-Douglas cases first, then, if these cases are not rejected, no further tests are necessary. On the other hand, should these cases be rejected further tests regarding the non-linear characteristics of the production function are necessary. See Berndt and Christensen, *op. cit.*, and the refinements of the Berndt and Christensen tests made by Michael Denny and Melvyn Fuss, "The Use of Approximation Analysis to Test for Separability and the Existence of Consistent Aggregates," *American Economic Review*, Vol. 67, No. 3 (June 1977), pp. 404-418.

Table 1: SIGNIFICANCE LEVELS FOR RESTRICTIONS ON THE TRANSLOG PRODUCTION FUNCTION

Test	Significance Level	Restrictions	Degrees of Freedom	Critical Value of F
CRTS	.01	4	372	3.36
Complete Global Separability	.01	3	369	3.83
Three Linear Separability Tests	.01	2	369	4.66

DATA

The region from which the data are taken is in the extreme northwest of peninsular Malaysia and comprises the State of Perlis and four administrative districts in the State of Kedah. Padi cultivation is the main occupation of over 50,000 families in the area which contains about 30% of the padi land in Malaysia and accounts for almost 50% of total padi output. The recent switch from single to double-cropping as a result of new irrigation facilities (the Muda River Project) has been accompanied by a significant increase in output from 227,000 tons in 1965 to 678,000 tons in 1974.

The data are for individual farm households and were collected in 1973 as part of the FAO/IBRD cooperative program. For the purposes of our analysis, we have concentrated on those households which have been double-cropping for one or more years and which operate non-acid land. This concentration ensures a relatively homogeneous sample: it omits those operating on the qualitatively inferior acid soils (8% of the original sample) and those who were in the process of switching from single to double-cropping during the period of observation (20% of the original sample). The remaining sample was then further adjusted to exclude (i) all households which failed to report labor usage for land preparation, planting, harvesting or threshing; (ii) all households which failed to report padi ouput; and (iii) all households which did not participate in the labor markets. The final sample size is 386.

Because the survey commenced in the middle of the first crop production period, our study is confined to the second crop, that is, that crop which relies on irrigation water. The labor variable for our estimates is measured in hours and is disaggregated according to source (hired/family). The labor of household members less than 15 years of age was not included as information obtained from the survey indicated that the use of child labor is essentially nil. Area operated is measured in relong and the dependent variable is the quantity of harvested padi output, measured in gantang, which was produced in the second crop season.[13/]

Results

The estimated parameters for the unconstrained translog production function are given in column one of Table 2.[14/] The F statistic for the constant returns to scale and linear homogeneity restrictions (equations (2)) is 0.523; since the null hypothesis of CRTS cannot be rejected we conduct the remaining tests under a maintained hypothesis of CRTS. Column two presents the translog production function estimated under the restrictions necessary for CRTS.

The F statistic for the test of complete global separability (with a maintained hypothesis of CRTS) is 3.001 and the null hypothesis that the production function is of the Cobb-Douglas form cannot be rejected at the .01 significance level. We conclude that the inputs are separable and a consistent linear aggregate of hired and family labor exists.

A consistent aggregate of inputs in a Cobb-Douglas production function is obtained through the use of weights proportional to the elasticities of the respective inputs.[15/] If the elasticities are equal the weights would likewise be equal and a consistent aggregate can be obtained by simply multiplying the two inputs. For the Cobb-Douglas function (in

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13/ A relong equals .287 hectares and a gantang equals 2.41 kilograms.

14/ The regressions reported in Table 2 were examined for a heteroscedastic relationship between the residuals and area cultivated. Using the Spearman Rank Correlation test the hypothesis of homoscedasticity was not rejected at a 5% significant level.

15/ Zvi Griliches, "Specification Bias in Estimates of Production Functions," Journal of Farm Economics, Vol. 39 (February 1957), 8-20.

Table 2:[1/] PARAMETER ESTIMATES OF THE TRANSLOG PRODUCTION FUNCTION

Parameter	Unconstrained	CRTS Imposed	CRTS and Global Separability Imposed	Undifferentiated Labor Variable
α_A	-1.006 (0.662)	-.956 (0.448)	0.732 (0.060)	0.691 (0.067)
α_H	0.735 (0.313)	0.881 (0.202)	0.130 (0.031)	-
α_F	1.249 (0.530)	1.076 (0.277)	0.138 (0.041)	-
λ_{AA}	-0.834 0.241	-0.808 (0.221)	-	-
λ_{HH}	-0.080 (0.047)	-0.067 (0.046)	-	-
λ_{FF}	-0.246 (0.115)	-0.209 (0.093)	-	-
λ_{AH}	0.369 (0.105)	0.333 (0.100)	-	-
λ_{AF}	0.463 (0.148)	0.475 (0.138)	-	-
λ_{HF}	-0.253 (0.070)	-0.266 (0.064)	-	-
α_L	-	-	-	0.324 (0.069)
α_O	1.581 (1.437)	1.505 (0.453)	3.241 (0.124)	3.123 (0.138)
SSE	87.389	87.876	92.337	91.708
$\bar{R}^2$	.68	.68	.67	.65

1/ Standard errors are in parenthesis.

column three of Table 2), which we have confirmed with the analysis above, the elasticities of hired and family labor differ very little. In fact an F test of the equality of α_F and α_H gives a value for F of .035 which does not allow the rejection of the null hypothesis at significance levels far below .01. We conclude that either the Cobb-Douglas production function with disaggregated labor or the alternative function with total labor (defined as the multiple of hired and family labor) are acceptable. The corollary of this result is that, if the labor aggregate is defined as the sum of disparate labor types, the resulting production function estimate will be subject to specification bias.[16/]

Labor Market Duality and Allocational Efficiency

Although we have demonstrated that the elasticities of hired and family labor are the same it does not necessarily follow that the marginal products of each type of labor are equal. This is because the elasticities are determined by the state of technology while the marginal products are dependent not only on the technological relationships in the production function but also on the level of factor usage which is related to the economic and institutional environment. The basic premise of the dual labor market hypothesis is that the supply curve of family labor to off-farm, wage employment lies below that to on-farm self-employment. Reasons often put forward for this divergence relate to financial costs (for example, transport) associated with outside employment, the hypothesis that the probability of finding a job in any period may be less than unity, and the possibility that the return (in a utility sense) to outside work may be lower than that to on-farm work, because the former, but not the latter, may entail working in less pleasant circumstances or in supervised situations. This implies that use of family labor in on-farm work would be carried to a point where the difference between the marginal product of family labor and the market wage would be sufficient to offset the psychic and real cost of labor market participation.

With regard to hired labor it can be argued that the marginal product should exceed the market wage since there are, in the Muda River

16/ For purposes of comparison we have presented in column four of Table 2 an estimate of the production function for a labor agggregate defined as the sum of family and hired labor.

valley, other costs in the form of meals and supervisory time that are not reflected in the market wage. Thus the expectation is that there will be a divergence between the marginal products for family and hired labor, even though the production elasticities are nearly equal, and that the value of the marginal products will bracket the wage rate.

Table 3 gives the marginal products derived from the regression estimates in column three of Table 2 (that is for the regression with the Cobb-Douglas form imposed and the family and hired labor inputs separated). All marginal products are significantly different from zero at the .05 significance level. Although it can not be established using an F test that the marginal products are significantly different from each other (the value of F=1.32), this is not surprising given the lack of precision of the estimates and the fact that the theoretically predicted interval (using the arguments in the preceding paragraphs) is relatively small.[17/] We choose to note merely that the F statistic has increased considerably, compared to the test of equality of the elasticities (F=.035), and to interpret the marginal products as consistent with expectations.

With regard to allocative efficiency, the ratio of marginal revenue product to input price is given in the second column of Table 3. A value greater than one for this ratio indicates underutilization of the input and a marginal revenue product which exceeds the market wage; a value less than one indicates overutilization and a marginal revenue product less than the market wage; and a value equal to one indicates allocational efficiency with respect to the input in question. Both of the labor values have ratios which are not significatly different from one and thus we do not reject the hypothesis of allocational efficiency. In addition the difference between the ratios indicates that the marginal revenue product brackets the market wage as expected. We note then that although allocative efficiency is not rejected the results are consistent with a degree of labor market dualism attributed to the non-market costs of labor market participation mentioned earlier.

17/ A similar argument can be made for use of the estimated elasticities even though the difference is small. It is expected that family labor will work with more care than hired labor and that the slightly higher elasticity estimated for family labor reflects this difference. See Bardhan, "Size, Productivity and Returns to Scale, An Analysis of Farm-Level Data in Indian Agriculture," Journal of Political Economy, Vol. 81, December 1973, pp. 1370-1386, ref. to p. 1381.

Table 3: MARGINAL PRODUCTS AND ALLOCATIVE EFFICIENCY FOR HIRED AND FAMILY LABOR

Input	Marginal Products[1/]	Ratio of Marginal Revenue Product to Input Price[2/]
Hired Labor	0.94	1.3
	(0.23)	
Family Labor	0.67	0.95
	(0.30)	

1/ Marginal Products are computed at the geometric mean and are reported in gantang. Standard errors are in parenthesis beneath the estimates. The regression reported in column three of Table 2 is used for the analysis.

2/ The prices used are:

padi = M$0.54 per gantang,
labor = M$0.38 per hour.

Summary

Recent articles by Desai and Mazumdar and Brown and Salkin have served to put new life into the zero marginal product argument for surplus labor in the context of a dual labor market. By way of contrast, we find, for rice producing farms in Malaysia, that the marginal product of family labor is positive and significantly different from zero and that farms are approximately, allocatively efficient. We do find, however, some substantiation for a mild degree of dualism in the labor market with the result that, although neither marginal product is significantly different from the wage, the marginal product of family labor is about three fourths that of hired labor.