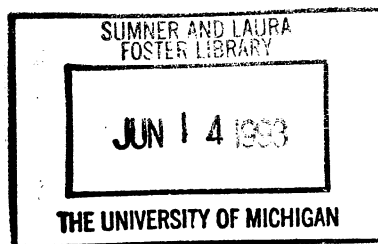Center for Research on Economic and Social Theory

and

Department of Economics

Working Paper Series

Rationalizable Predatory Pricing

David Roth

January, 1993
Number 93-12

DEPARTMENT OF ECONOMICS
University of Michigan
Ann Arbor, Michigan 48109-1220

RATIONALIZABLE PREDATORY PRICING

by David Roth*

Department of Economics

University of Michigan

Ann Arbor, MI 48109

January 1993

This paper shows that predatory pricing is *rationalizable*, and is in this sense rational even when there is no uncertainty regarding structural parameters such as cost or demand.

The paper models predatory pricing as a war of attrition: The entrant gives up by leaving the market while the incumbent acquiesces through pricing high to share the market. In this framework, all strategies are rationalizable. The paper provides strong characterizations of the relationship between the firms' conjectures over the strategies their rivals may be playing, and best responses to those conjectures. This characterization revolves around the mixing probabilities associated with the mixed strategy equilibria of the game. Since rationalizable behavior is a best response only to conjectures and not generally to actual rivals' strategies being employed, entry may not be ultimately profitable, nor must predatory pricing lead to eventual monopolization of the market. The ultimate success of any strategy depends upon how accurately a firm anticipates the play of its rival.

The paper then considers a variety of common knowledge restrictions beyond common knowledge of rationality, and examines their implications. The particular quit/continue structure of the war of attrition leads seemingly innocuous common knowledge restrictions to have drastic implications for the play of the game. Finally, the paper proposes a notion of reputation-building—that a firm's seeing tough behavior must lead it to lowered expectation of its rival's acquiescence—and shows that common knowledge of reputation-building by either or both firms does not restrict play of the game.

1. INTRODUCTION

Predatory pricing—aggressive pricing behavior by dominant firms in markets in order to induce the exit of weaker rivals and deter potential rivals from competing—has long been a concern of both policymakers and industrial economists. Despite the long history of apparent prevalence of such pricing (see, for example, Burns (1986)), and its potential implications for antitrust as a monopolizing practice, the paradoxical question long remained: "Why would a potential entrant to a market, perfectly anticipating the predatory response to its entry, and knowing that the market would therefore be found unprofitable, enter the market, incur losses, and then leave?"

Developing models based upon *imperfect information*, theorists have recently been quite successful in rationalizing predatory pricing. Post-entry learning about market conditions may convince an entrant whose initial prospects were attractive to exit after incurring losses, exactly because losses are expected to continue. Unlike competitive models of entry and exit, for example, the evolutionary models of Jovanovic (1982) and Hopenhayn (1992), the hysteresis-inspired model of Dixit (1989), as well as models with neither hidden information nor hidden actions (Ericson and Pakes (1989)), these models have firms which are able to affect the learning process of the entrant through their pricing decisions.

Each of the models has focussed upon a different source of imperfect information in order to study a particular type of predatory pricing. Milgrom and Roberts (1982) first suggested that small amounts of incomplete information concerning costs or demand could release us from the "Chain-Store" paradox (Selten (1978)) and allow incumbent firms of different types to build a reputation for preying when they faced potential entry and competition in a long but finite sequence of markets. Salop and Shapiro (1980), Scharfstein (1984), and Roberts (1986) explicitly model the pricing decision and suggest that an incumbent in a single small market may distinguish or "separate" itself from a high-demand incumbent by pricing low, signalling that the market is too small to support both firms profitably. In such a separating equilibrium, the entrant leaves the market

2

exactly when it would under full information, the low price being used only as a signalling device.

Saloner (1987) extends this model to include the possibility of merger. He finds that predatory pricing may be rational not only in order to induce exit from the market, but also to improve the terms of a buyout of an unsuccessful rival. Saloner cites Burns' (1986) work on the cigarette industry, where predatory pricing by the dominant American Tobacco Company led to lower purchase prices of upstart rivals. Saloner interprets his model and Burns' evidence in light of McGee's (1958, 1980) skepticism about the efficacy of predatory pricing for *exit* when merger is less destructive for both firms: Predatory pricing may soften up rivals for purchase as well.

Fudenberg and Tirole (1986) have also noted that imperfect information need not be asymmetrically distributed for predatory pricing to be rational. If first-period profits provide the entrant with only a noisy estimate of its profitability, then, although the entrant is not fooled by the 'signal-jamming,' the dominant firm has a first-order incentive to lower its price from its optimal one-period level in order to affect the entrant's inferences. See also the related papers by Benoit (1984) on finance constraints and incomplete information, by Easley, Masson, and Reynolds (1985) on incomplete information and the relationship of profitabilities across markets, and by Bolton and Scharfstein (1990), where low-demand incumbents prey in order to exacerbate agency problems inherent in the financing of rivals.

The models of single-market predatory pricing all share one rather unattractive element: Although, as we expect, low pricing is strategically set by the incumbent in order to achieve information transmission, the information concerns the *inherent profitability* of the industry for the entrant. Due to the finite horizons, the full-information profits are generally unique; small firms exit when it becomes clear enough that the market cannot support two firms. However, when we concern ourselves with predatory pricing as a monopolization issue in public policy analysis, we are seldom concerned with markets which cannot support two firms in any event. We are rather interested in markets which are large enough that two firms may coexist profitably, yet in which a dominant firm is simply

unwilling to allow any rival a share of the substantial pie.

Consider an infinite-period model where two firms compete in such a market. From folk theorems for repeated games (e.g., Fudenberg and Maskin (1986)), we might see (essentially) any market shares and any prices from competitive to monopolistic in subgame perfect equilibrium. Milgrom and Roberts (1982, p. 282) point out that it is "trivial" that in a model with an infinite sequence of entrants, and by analogy one with infinite-period interaction between two rivals in one market, the threat of low pricing as a response to entry is credible. However, in such a model, this predatory response to entry only happens off the equilibrium path, the entrant does not actually enter, and we are led back to the question posed earlier concerning why the entrant would enter the market, lose money, and then exit.

This paper suggests a resolution to this paradox in the context of an infinite-period model of duopolistic competition where the market is large enough to support both firms profitably. I model the predatory battle as a war of attrition (Maynard-Smith (1974), Riley (1980), Chatterjee and Samuelson (1987)). I use the perfect-information version of the model: There is no uncertainty in the extensive form of the game; both firms know the nature of demand and the cost structures. The only uncertainty is *strategic*: Firms are uncertain about the strategies that will be played by their rivals. The extreme multiplicity of possible market-sharing arrangements suggested by the folk theorems makes it seem rather exceptional that the rivals might perfectly coordinate their expectations upon any of these equilibria; but such coordination is exactly the basis of Nash equilibrium and its refinements.

Coordination is particularly unlikely in the types of markets in which predatory pricing may be a concern. The firms are greatly asymmetric, and there is therefore no focal point (Schelling (1960)) for the rivals to use for division of the market. The firms have no experience competing in the same market, and thus have little historical guide to likely future rival behavior. Interpreting predatory pricing as a "battle for market share" (e.g., Roberts (1987), Ordover and Saloner (1988)) focuses our attention on the fact that

4

3

determination of market share is of fundamental importance, and that there is no reason to expect the market ever to settle down to a stable, profitable sharing arrangement for the two firms.

In order to study strategic uncertainty I utilize the solution concept of *extensive-form rationalizability* (Pearce (1984); see also Bernheim (1984), Börgers (1992)). From the construction of the set of rationalizable strategies given by Pearce, it is clear that each rationalizable strategy is a perfect best response to *conjectures*—probability distributions—the players have over which of their rivals' own rationalizable strategies may be played. Although Pearce and Bernheim were concerned with the coordination problems inherent in general games with a multiplicity of reasonable behaviors, other authors have discussed the problem with particular reference to oligopolistic games (see, e.g., Milgrom and Roberts (1987, p. 188)).

Consider predatory pricing as a war of attrition: The first firm to acquiesce ends the game, with the entrant acquiescing by leaving the market, the incumbent by raising prices. When both entry and non-entry can be supported in perfect equilibria of the war of attrition, there exist mixing probabilities for each firm which make each firm indifferent between between fighting and acquiescing. There are many mixed-strategy equilibria which focus on these probabilities. Proposition 1 shows that, due to the existence of the mixed strategy equilibria, all strategies are rationalizable.

The mixing probabilities in the mixed strategy equilibria form a critical point of optimism for the players: Theorems 1–3 of Section 4 draw the relationships out in their starkest form. If a firm will never again be optimistic enough about its rival's acquiescence as the critical level, then the firm must itself have given up by that point (Theorem 1). When it is more likely that one's rival will acquiesce in the following period than this critical level, one should continue to fight in hopes of such acquiescence, regardless of what one might expect thereafter (Theorem 2). Finally, if a firm is pessimistic about the prospects of its rival's acquiescence in the very next period, then the firm prefers quitting now to fighting for another period if it were only to give up at that point (Theorem 3).

The results are based completely upon the recursive nature of the problem: Because the only difference from one decisionmaking point in time to the next for a firm lies in its *expectations* about its rival's play on the remainder supergame, we may apply standard dynamic programming techniques to the problem, using conjectures rather than physical quantities as the state variables.

That all strategies are rationalizable is the same as saying that common knowledge of the structure of the game and rationality alone provide no restrictions on the play of the game. In characterizing the relationship between conjectures and best responses, therefore, Theorems 1–3 provide us with all the structure possible in analyzing the market with such limited common knowledge restrictions.

It is the purpose of section 5 to suggest stronger common knowledge restrictions and examine their implications. In particular, it is concerned with the beliefs that, for whatever rational reason, a firm were not going to play particular strategies, or had certain conjectures. I utilize the solution concept of rationalizability in the extensive form presented by Pearce (1984), and I contrast the results obtained with those based upon other definitions of rationalizability in the extensive form (e.g., Börgers (1992)).

Theorem 4 examines the implications of common knowledge that one firm, say the entrant, believes that its rival will not acquiesce in the first T stages of the game. The entrant will not *first* acquiesce in $T + 1$, then, as, if I were certainly not going to acquiesce in period T, then E would not wish to continue through T in order to give up in $T + 1$, as suggested by Theorem 3. The process continues now *forward* rather than back, with each firm not first acquiescing in later and later periods. After some point in the logic, E would have to then wait too long through this phase of nonacquiescence to make it worthwhile to continue at the beginning. It therefore will acquiesce before waiting through this phase. But since I does not acquesce in the first T stages, the beginning of this phase is stage 0, and E never enters.

Suppose that it were common knowledge that E believed that I would never acquiesce with at least some probability $x > 0$. In playing best responses to all such conjectures

E might entertain, the *longest* E might wish to remain in the market may be readily characterized in terms of x and E's critical optimism level π_E^*, the mixing probability for I in the mixed strategy equilibria. This is the content of Lemma 5.1. Call this length of time $T_E(\pi_E^*, x)$. This leads directly to Theorem 5: Under common knowledge of such beliefs, the entrant will never enter. When I's decision in stage $T_E(\pi_E^*, x)$ is reached, I rationalizes this knowing that E will at most remain for $T_E(\pi_E^*, x)$ periods. Therefore I fights at $T_E(\pi_E^*, x)$. Knowing this, having only strategies still undeleted in which it acquiesces somewhere in the first $T_E(\pi_E^*, x) + 1$ periods, E will acquiesce at $T_E(\pi_E^*, x)$, by Theorem 3. Working in this fashion backwards, we see that E never enters.

Section 6 suggests a weak notion of reputation-building suggested by strategic uncertainty rather than incomplete information. Notice that (Bayesian) updating of conjectures through play of the game does not necessarily lead players to find it more likely that a long stream of tough rival behavior will be followed with toughness, than the firms had originally conjectured for the rival's first moves. Suppose we were to make such a restriction, however, and that this restriction were common knowledge. From Theorems 1–2 and the fact that all structures on updated conjectures are possible, becoming more pessimistic through time about the likelihood of one's rival's acquiescence says *nothing* about one's own best response: It may still be after any number of periods T (or never) that one's conjectures fall below the critical level defined by the mixing probabilities.

Section 7 relates the present analysis to reputation formation across markets, finance constraints, and predation for merger. Each of these cases has an intuitive relation to strategic uncertainty; I discuss this in more detail in Section 7. At the same time, the general agnosticism displayed within the present model about predictions for the play of the game carries over to these situations as well.

The remainder of the paper is structured as follows. Section 2 introduces the model and the assumptions under which all strategies are rationalizable. Section 3 introduces notation for strategies and conjectures. Section 4 develops the relationship between conjectures and best responses to the conjectures. Section 5 considers the implications of further common knowledge restrictions. Section 6 considers reputation-building. Section 7 discusses the relevance of the present approach for the study of predatory pricing in different contexts. Section 8 concludes.

2. THE MODEL, EQUILIBRIA, AND RATIONALIZABILITY

Consider the following infinite-period market with two price-setting competitors: An incumbent is at present in a monopoly position in the market, yet faces the threat of entry. A potential entrant may enter costlessly at the beginning of the game. The model proceeds in an infinite sequence of stages, beginning at stage 1. In each stage, the entrant E decides whether or not to participate. The incumbent I then decides whether or not to prey. Should either choose to acquiesce at any point, the game terminates. That is, any occurrence of exit or accomodating pricing ends the game. In stage t the discounted flow payoffs to the firms (E, I) are as follows: If I is in a monopoly position, payoffs are $\delta^t(0, M)$, if I fights, $\delta^t(\underline{v}, \underline{w})$, and if I acquiesces, $\delta^t(\overline{v}, \overline{w})$. Make the following assumptions on payoffs:

(A1) $\underline{v} < 0 < \overline{v}$,

(A2) $\underline{w} < \overline{w} < M$

(A3) $\delta M + (1 - \delta)\underline{w} > \overline{w}$

Since $\underline{v} < \overline{v}, \underline{w} < \overline{w}$, it is destructive of both firms' profits for I to prey. (A1) implies that sharing the market is profitable for the entrant while being preyed upon imposes losses. Assumption (A2) merely reflects the fact that I prefers to be a monopolist in any event. That I should be willing to incur one period of low profits by preying in order to then monopolize the market with certainty is captured by assumption (A3).

Under (A1)–(A3), there are two pure-strategy subgame perfect equilibria of the game.

In the first, E enters after every history and I acquiesces. The entrant never enters in the other pure-strategy equilibrium, faced with the credible threat of a predatory response. There also exist many mixed strategy equilibria. Since fighting and acquiescing for each firm leads its respective rival to different best responses, there exist mixing probabilities which make each firm indifferent at each point between fighting and acquiescence.

Let π_E^*, π_I^* be the mixing probabilities *expected* by E and I, respectively, which lead to indifference. Then

$$\pi_E^* \bar{v} + (1 - \pi_E^*)(1 - \delta)\underline{v} = 0.$$

With probability π_E^*, I acquiesces, leading to a flow payoff for E of $\bar{v}$, and with probability $(1 - \pi_E^*)$, I fights, providing a loss of $(1 - \delta)\underline{v}$ for one period and a continuation of 0, as E is again indifferent in the following period. For I to be indifferent,

$$(1 - \delta)\underline{w} + \delta(\pi_I^* M + (1 - \pi_I^*)\overline{w}) = \overline{w}.$$

If I preys, it receives low profits of $(1-\delta)\underline{w}$ today, but has the chance of receiving monopoly profits M starting tomorrow with probability π_I^* if E were to exit, and the chance of again becoming indifferent and receiving an expected $\overline{w}$. Solving for π_E^*, π_I^*, we get

$$\pi_E^* = \frac{-\underline{v}}{\bar{v}/(1 - \delta) - \underline{v}},$$

$$\pi_I^* = \frac{(1 - \delta)(\overline{w} - \underline{w})}{\delta(M - \overline{w})}.$$

There is therefore a mixed strategy equilbrium in which each firm mixes at each point in time with the probability necessary to support mixing and indifference on the part of its rival. Each strategy is played in this mixed strategy equilibrium, and therefore all information sets are reached with positive probability. Therefore, all strategies must be rationalizable:

PROPOSITION 1: *Assume* (A1)–(A3). *Then all strategies are rationalizable.*

This provides the basis for the rest of the analysis.

3. STRATEGIES AND CONJECTURES

Let us restrict attention to pure strategies. Since all strategies are rationalizable, we will assume that the firms have conjectures over all rival pure strategies. Full strategies specify for each firm not only whether and when it will first acquiesce, but rather whether or not it acquiesces after each *history*. In each stage, $t = 1, 2, 3, \ldots$, each firm has a contingency plan of whether to acquiesce. A pure strategy, then, is a subset of the natural numbers N, with the set of all strategies for each firm being 2^N. For example, a firm playing N acquiesces after each history, one following $\{t_1, t_2, \ldots\}$ gives up only in stages $t_1, t_2, \ldots$, and a firm fighting after every history plays $\emptyset$.

A *conjecture* for a firm is a sequence of subjective probability distributions the firm has over which strategies its rival may be playing, one for each of the firm's own information sets. I provide a definition which only differs from Pearce's (1984) with respect to the accounting for updating: Pearce does not ask that the conjectures at each point in time be probability distributions and sum to 1. I alter this convention so that the relationships between conjectures, updating, and the critical mixing probabilties π_E^* and π_I^* come out as clearly as possible. For $t \in Z^+$, let $N + t := \{t + 1, t + 2, \ldots\}$.

DEFINITION: $C := (C^1, C^2, \ldots)$ is a *conjecture* if, $\forall t, C^t$ is a probability distribution on 2^{N+t-1}, and if, $\forall t_1 < t_2 \in N$, if C^{t_1} reaches t_2, then, $\forall \sigma$ s. t., $\forall t < t_2, t \notin \sigma$,

$$C^{t_2}(\sigma) = \frac{C^{t_1}(\sigma)}{(1 - \sum_{t=t_1}^{t_2-1} \sum_{\sigma'|t\in\sigma'} C^{t_1}(\sigma'))}$$

Conjectures C^t held at any point in time t must be probability distributions over all strategies that reach t, and they must be consistent with conjectures at previous periods if the previous conjectures reach the later period, that is, with positive proabability.

In terms of expected profits today and therefore today's fight/acquiesce decision, each firm is only concerned about the very next time that its rival might acquiesce. It may then group together all strategies in which its rival *first* acquiesces at various dates. This motivates the following definitions of sets of strategies and conjectures over these sets:

DEFINITION: For $t \in N, \Sigma^t := \{\sigma \in 2^N | t = \min(\sigma)\}, \Sigma^\infty := \{\emptyset\}$, and $\Sigma := \{\{\Sigma^t\}_{i=1}^\infty, \Sigma^\infty\}$.

DEFINITION: For $t \in N, \pi_s^t := \sum_{\sigma \in \Sigma^s} C^t(\sigma), \pi_\infty^t := C^t(\emptyset)$ and $\Pi^t := \{\pi_s^t\}_{s=t}^\infty$.

DEFINITION: The *hazard rate at t* is $h^t := \pi_t^t$.

The set Σ is a partition of all strategies: It groups the strategies into those which first acquiesce at each time t, Σ^t, and that which never acquiesces Σ^∞. Given this partition, the probability π_s^t is the probability that, having reached t, the rival will first acquiesce in stage $s \geq t$; π_∞^t denotes the probability that one's rival will *never* acquiesce given that t periods of tough rival behavior have been played. Because it represents the probability that one's rival will acquiesce in period t given that t has been reached, we call π_t^t h^t and denote it the *hazard rate at t*.

Notice that if C is a conjecture, then the probability distributions Π^t on Σ must be Bayesian updates of previous probabilities if C reaches t:

PROPOSITION 2: *If $C = \{C^t\}_{i=1}^\infty$ is a conjecture and if for $t_1 < t_2$, C^{t_1} reaches t_2, then, $\forall s \geq t_2$:*

$$\pi_s^{t_2} = \frac{\pi_s^{t_1}}{1 - \sum_{r=t_1}^{t_2-1} \pi_r^{t_2}}.$$

Given this pattern of updating, there exist conjectures which give rise to *any* hazard rate sequence $H := \{h^t\}_{i=1}^\infty$ in $\times_{i=1}^\infty [0,1]$:

PROPOSITION 3: *Let $H = \{h^t\}_{i=1}^\infty \in \times_{i=1}^\infty [0,1]$. Then $\exists$ a conjecture C with hazard rate sequence H.*

Of particular interest is that there is no tendency to expect, merely as a result of seeing tough behavior, that the future is more likely to provide a fight than one had expected in the past. This is because firms are learning about entire supergame strategies being played by similarly purposeful firms, rather than about parameters of concern as in bandit models or sequences of one-period moves in games where partners are drawn from a large population. For example, a firm may well expect with very high probability that its rival will fight for exactly T periods, conjecturing that that will drive the firm from the market, but then be very likely to give up thereafter if unsuccessful.

4. BEST RESPONSES TO CONJECTURES

This section provides a characterization of best responses to conjectures firms have over what their rivals may be playing. Consider the entrant's decisionmaking problem. At each stage t, it must decide whether to continue participation in the market. As the market is one of discounted, infinitely-repeated interaction with the same payoff structure over time, the remainder supergame is structurally identical at all points at which the entrant must choose. However, its decisionmaking problem differs substantially from period to period, as its expectations regarding the remainder supergame strategies generally evolve

with the play of the game. The fact that the only difference to the entrant which bears upon its decision from any point to another lies in its conjectures suggests that we may analyze the problem using standard dynamic programming techniques with conjectures as the state variables.

Let us abuse terminology and for the purposes of this section call each of the probability sequences Π^t *conjectures*. The following functional equation represents the entrant's choice in period t, where Π^t_E and Π^{t+1}_E are the entrant's conjectures for periods t and $t+1$, respectively:

$$V(\Pi^t_E) = \max\{0, h^t_E\overline{v} + (1 - h^t_E)[(1 - \delta)\underline{v} + \delta V(\Pi^{t+1}_E)]\}.$$

The entrant may leave the market and receive nothing, or it may choose to continue. Participating in the market provides an expected $h^t_E\overline{v}$ due to the possibility of immediate acquiescence by the incumbent, and $(1 - h^t_E)[(1 - \delta)\underline{v} + \delta V(\Pi^{t+1}_E)]$ based upon the incumbent's predation. The first term represents the loss today due to predation. The second term is the discounted value of the maximized value function based on the updated stage-$t+1$ conjectures.

A similar functional equation applies to the problem the incumbent faces:

$$W(\Pi^t_I) = \max\{\overline{w}, (1 - \delta)\underline{w} + \delta[h^t_I M + (1 - h^t_I)W(\Pi^{t+1}_I)]\}.$$

The incumbent can choose to receive its duopolistic profits $\overline{w}$ forever by sharing the market, or it can fight and receive a period of low profits $\underline{w}$ before taking its chances on the entrant's next move. The tradeoffs are clear and intuitive.

From this point on, I will focus only upon the *entrant's* problem and leave the completely analogous incumbent's problem to the reader. For the entrant, make the following

definitions. Let Ω be the set of all conjectures. Notice that conjectures are now probability distributions on $N \cup \{\infty\}$ rather than on 2^N. Let $C := \{V : \Omega \to \Re\}$ be the set of value functions on the space of conjectures. Let E's dynamic programming transformation $B : C \to C$ be defined as follows:

DEFINITION: $B(V(\Pi)) := \max\{0, \pi_1\overline{v} + (1 - \pi_1)[(1 - \delta)\underline{v} + \delta V(\Pi'(\Pi))]\}$,

where $\Pi = \{\pi_t\}^\infty_{t=1}$, and $\Pi'(\Pi)$ is the update of Π. This update only exists if $\pi_1 < 1$; but if $\pi_1 = 1$, then E is certain that I will acquiesce and thus has expected profits of $\overline{v}$.

An analysis of the entrant's decisions through the functional equation is exactly an analysis of the shapes of the conjectures. Recall the probability π^*_E which led to E's indifference in the mixed-strategy equilibria of the game. This probability structures our analysis when E is only making conjectures as to how I *might* behave as well. Suppose first that after a particular point in time, E is chronically *pessimistic*: E never again will expect I to acquiesce in the very next period, given that the period has been reached, with a probability as great as π^*_E. Then E must acquiesce by that point:

THEOREM 1: *Fix $T \geq 1$, and suppose that, $\forall t \geq T, h^t_E < \pi^*_E$. Then $\forall t \geq T, t \in \sigma^*$.*

PROOF: Set $V_0 : \Pi \to \Re$ equal to zero everywhere, let $V_{t+1} := B(V_t), \forall t \geq 0$, and let $V := \lim_{t\to\infty} V_t$. Then, $\forall t \geq T, V_1(\Pi^t_E) = B(V_0(\Pi^t_E)) = \max\{0, h^t_E\overline{v} + (1 - h^t_E)[\underline{v} + \delta V_0(\Pi^{t+1}_E)]\} = \max\{0, h^t_E\overline{v} + (1 - h^t_E)\underline{v}\} = 0$, first by assumption on V_0, then since $h^t_E < \pi^*_E$. By induction, then, for all iterations s, for $t \geq T, V_s(\Pi^t_E) = 0$. Therefore, for $t \geq T, V(\Pi^t_E) = 0$. And since $h^t_E < \pi^*_E$, E *strictly* prefers $t \in \sigma^*_E$. Q. E. D.

The proof focuses on the following decomposition of the decisionmaking problem for E: If the entrant expects to exit in period $t+1$ and receive 0 from that point on, it cannot wish to stay through t, as it then loses money in expectation if $h_t < \pi^*_E$.

The following are immediate implications of Theorem 1; I state them without proof:

COROLLARY 1.1: *Fix $T \geq 1$, and suppose that, $\forall t \geq T, h_E^t < \pi_E^*$. Then $\exists s \leq T$ s. t. $\sigma_E^* \in \Sigma^s$, and E exits by stage T.*

COROLLARY 1.2: *Suppose that, $\forall t \geq 1, h_E^t < \pi_E^*$. Then $\sigma_E^* = N$, and E never enters.*

If E is forever pessimistic, it will never participate in a single period of pessimism in hopes of future gains since it will not participate later either.

On the other hand, E must only be *optimistic* for one period in order to wish to stay in the market. Whatever E may choose to do in future periods, if $h_E^t > \pi_E^*$ it makes positive expected profits today by staying in the market:

THEOREM 2: *Fix $t \geq 1$, and suppose that $h_E^t > \pi_E^*$. Then $t \notin \sigma_E^*$.*

PROOF: Since $V(\Pi_E^t) = \max\{0, h_E^t \bar{v} + (1 - h_E^t)[(1-\delta)\underline{v} + \delta V(\Pi_E^{t+1})]\}$, the continuation value $V(\Pi_E^{t+1}) \geq 0$, and since $h_E^t > \pi_E^*, h_E^t \bar{v} + (1 - h_E^t)(1 - \delta)\underline{v} > 0$. Thus $h_E^t \bar{v} + (1 - h_E^t)[(1-\delta)\underline{v} + \delta V(\Pi_E^{t+1})] > 0$, so $t \notin \sigma_E^*$. Q. E. D.

Since E would never leave upon reaching stage t, it certainly will not *first* exit in period t:

COROLLARY 2.1: *Fix $t \geq 1$, and suppose that $h_E^t > \pi_E^*$. Then $\sigma_E^* \notin \Sigma^t$.*

Finally, suppose that E will be pessimistic in a particular stage t. Suppose it plans upon exiting if stage $t + 1$ were reached. Then it must also be planning to exit at t, as it will not lose money today knowing that it will then simply exit tomorrow. Put differently, if E were planning upon leaving tomorrow, staying today provides it with no option value of continuing, and thus the negative expected profits associated with today's pessimism are sufficient to drive it from the market:

THEOREM 3: *Fix $t \geq 1$, and suppose that $h_E^t < \pi_E^*$. Then, if $t+1 \in \sigma_E^*$, then $t \in \sigma_E^*$.*

15

PROOF: The value of staying in at t is $h_E^t \bar{v} + (1-h_E^t)[\underline{v} + \delta V(\Pi_E^{t+1})] = h_E^t \bar{v} + (1-h_E^t)\underline{v} < 0$, first, since $t + 1 \in \sigma_E^*$, and then because $h_E^t < \pi_E^*$. So E strictly prefers to exit at t, so $t \in \sigma_E^*$. Q. E. D.

Again, this provides strong implications not only for the structure of the entrant's perfect best response to its conjectures, but also for the play of the game. If E is pessimistic today, it will not first exit tomorrow:

COROLLARY 3.1: *Fix $t \geq 1$, and suppose that $h_t < \pi_E^*$. Then $\sigma_E^* \notin \Sigma^{t+1}$.*

Figure 1 summarizes the relationships between conjectures and best responses developed in Theorems 1–3 and their Corollaries. It displays a variety of shapes of hazard rate sequences and the relationship between the hazard rates and the critical level of optimism π_E^*. Along the horizontal axis lies the stage of the game, along the vertical, the hazard rate. Figure 1 (a) plots two hazard rate sequences. H_1 lies above π_E^* at all stages and H_2 below. From Theorem 2, an entrant with hazard rate H_1 will never exit, while Corollary 1.2 to Theorem 1 suggests that an entrant whose conjectures give it the pessimistic hazard rate H_2 will never enter the market.

In Figure 1 (b), E becomes more pessimistic over time about the prospects of market-sharing, and exits at t^*, exactly when its hazard rate falls below π_E^*, by Theorems 1, 2.

Figure 1 (c) displays the full strength of the propositions put forth in this section. The hazard rates move above and below the critical level π_E^* in some pattern. Theorem 2 states that the entrant will not exit in a stage of optimism, that is, where the hazard rate lies above π_E^*. By Corollary 3.1, E will not exit in any but the first stage of a phase of pessimism. Therefore, based upon the scant information contained in Figure 1 (c), we know that the entrant might exit only in periods $t_1^*, t_2^*, \ldots$, the starting points of the pessimistic phases. Exactly which of the periods is the first at which E would exit must be found through a full solution to its infinite-period decisionmaking problem.

16

Note that since all strategies are rationalizable, forward induction has no bite: Having seen a stream of tough behavior by one's rival in the past puts no restrictions on what the rival might do in the future. All that can be done then is to analyze the relationships between beliefs and best responses. Also implied is that all information sets may be reached in behavior consistent with Bayesian rationality, and that we therefore need not concern ourselves with the differences in extensive-form solution concepts based upon what players should believe when information sets unreached according to the theory actually are reached.

5. BELIEF RESTRICTIONS

Common knowledge of rationality alone provides no predictive restrictions on the play of the game; all strategies are rationalizable. This section addresses the issue of whether, combined with common knowledge of rationality, further restrictions on beliefs might provide stronger implications.

I utilize the solution concept of *rationalizability in the extensive form* as suggested by Pearce (1984). This solution concept combines notions of backward induction—players must behave rationally based upon their expectations for future play *at each information set*—with notions of forward induction (e.g., Kohlberg and Mertens (1986))—players must be able to explain how a particular information set was reached, based upon rationality of the other players, when possible. The solution concept is one of iterative deletion of dominated strategies, and in that sense mimics the more widely accepted method of solving for rationalizable strategies in *normal form* games.

The deletion proceeds as follows. Consider a particular round of deletion. At all information sets in the game *which can still be reached* by strategies not yet deleted, players must have conjectures about what strategies might have brought them to the particular information sets, the conjectures must have support only on remaining strategies,

conjectures must be consistent with Bayes' rule across information sets, and the players must play best responses to the conjectures at each information set. One restriction made is that no further deletion occurs at information sets not reached by remaining strategies. Similarly, no *reintroduction* of strategies may occur: There are generally information sets which are not reached, but the solution concept makes no provision for the reintroduction of previously deleted strategies on the basis that the theory would then be refuted and new beliefs would have to be generated, leading to the further reintroduction of strategies.

Suppose that it were common knowledge that one firm, say E, believed that its rival I were, for whatever rational reason, not going to acquiesce by a particular stage T. Consider E's decision at T between acquiescence at T and at $T + 1$. Since I will not acquiesce at T, $h_E^T = 0$. By Theorem 3, E would exit at T if it were planning to do so at $T + 1$; it cannot *first* quit at $T + 1$. Now reconsider I's decision at T in light of the fact that E has both not yet exited by T, and that E only exits at $T + 1$ if it would have at T as well. I's hazard rate at T must be zero, and since I preys at T, it must be because it would do so at $T + 1$ again, by Theorem 3. So $\sigma_I^* \notin \Sigma^{T+1}$.

As this process continues, E reevaluates its incentive to acquiesce before this period of toughness: Is it worthwhile to wait for such a long period without any hope of acquiescence? Define the number of stages that E would be willing to wait if I were certain not to acquiesce as follows:

DEFINITION: $\overline{T}_E := \max\{T \in Z^+ | -(1 - \delta^T)\underline{v} < \delta^T \overline{v}\}$.

Once E has become certain through deletion of dominated strategies that I will not acquiesce for $\overline{T}_E$ periods, it must itself give up before this point. But E expects I to fight from the very beginning; therefore, E never enters:

THEOREM 4: *Fix $T \geq 1$. Suppose it were common knowledge that E believed that, $\forall t \leq T, \sigma_I^* \notin \Sigma^t$. Then $1 \in \sigma_E^*$, that is, the entrant never enters.*

PROOF: The result is immediate if $T > \overline{T}_E$. Otherwise, at stage T, E expects I to fight with certainty. Therefore $h_E^T = 0$, and by Corollary 3.1, $\sigma_E^* \notin \Sigma^{T+1}$. This is the only deletion made in round 1. At stage T, in order for I to both explain having arrived at T and believe that $\sigma_E^* \notin \Sigma^{T+1}$, it must be that $h_I^T = 0$. So, again by Corollary 3.1, $\sigma_I^* \notin \Sigma^{T+1}$. Let $S \in N$. After $2S$ rounds of deletion, $\forall t, T < t \leq T+S, \sigma_E^*, \sigma_I^* \notin \Sigma^t$; after $2S + 1$ rounds, we also have $\sigma_E^* \notin \Sigma^{T+S+1}$. Consider the $(2(\overline{T}_E - T) + 1)^{st}$ round of deletion. For all $t, 0 \leq t \leq \overline{T}_E, \sigma_I^* \notin \Sigma^t$. So, by the definition of $\overline{T}_E$, E's expected profits by entering in stage 1 are less than 0. Therefore, all remaining strategies σ_E such that $1 \notin \sigma_E$ are deleted. So $1 \in \sigma_E^*$. Q. E. D.

REMARK: This result is based upon the fact that forward induction provides such *precise* information to the players about what strategies might have been, and therefore will be used. In particular, if I must choose in T between quitting at T and at $T+1$, it must be that E has not chosen to acquiesce in the past—an eventuality consistent with many strategies that have not yet been deleted—and that therefore I, knowing that E does not give up at $T+1$, will not itself give up only at $T+1$, as from T the same strategy that dominated leaving at T will certainly dominate leaving one period later.

Consider instead a form of extensive-form rationalizability based upon the notion of *trembles* (e.g., Selten (1975), Börgers (1992)). Rather than having players consider what they might do at unreached information sets through their having conjectures at each point in time, trembles force all information sets to be reached with positive probability under all strategy profiles. Consider the following notion of trembles for the problem at hand: Suppose that whenever a firm attempted to end the game through acquiescence, Nature chose instead with some small probability $\epsilon > 0$ to continue the game.

Consider the common knowledge that E believes that I will not acquiesce in the first T periods. It still holds that E will not first acquiesce in $T+1$, since $h_E^t = 0$. Yet consider I's decision at T. Having arrived at its $T + 1^{st}$ stage no longer assures I that E did not

yet choose to acquiesce: It may be that E did so yet Nature chose to continue the game. Thus, although E does not first acquiesce in $T+1$, it may indeed acquiesce in $T+1$ when reached—from the point of view of I in T—for E's tough behavior through T and therefore through $T+1$ is no longer the only explanation of I having arrived at T.

The differences in restrictions placed upon behavior in the two forms of the game are quite plausible when the solutions are interpreted in terms of the thought processes the players use in analyzing the game(s). When using Pearce's solution concept, players need not think again past particular points in time if they are ruled out through deletion. If period T is reached in the game under consideration, it is *first* noted that no firm will acquiesce in succeeding periods, *then* it is noted that E would never enter due to its impatience. However, when no such strong beliefs could be held regarding past behavior, giving forward induction no bite, E must simply consider whether it would be worthwhile to lose $(1 + \delta + \cdots + \delta^{T-1})(1 - \delta)\underline{v}$ for certain in periods 1 through T in the prospect of receiving as much as $\delta^T \overline{v}$ thereafter. E may then enter only if $T \leq \overline{T}_E$.

Next, suppose that it were common knowledge that the entrant, let's say, believed that the incumbent would never acquiesce with at least a small positive probability $x > 0$. That is, it is common knowledge that $\pi_{\infty,E}^1 \geq x$. Theorem 5 states that E will then never enter the market. The reasoning is as follows. As play proceeds, more and more of the remaining probability weight *remaining* in the conjectures, approaching all of the weight, falls on the prospect of I's never acquiescing. Therefore, past some point, E will never again be sufficiently optimistic about I's acquiescence to warrant continuing. From there, I will know that if the period is reached, E will acquiesce in the following period, and so I will not acquiesce. Working backward from this point, E will never enter.

The longest that the entrant might be willing to fight before exit can be readily computed in terms of its critical optimism level π_E^* and $\pi_{\infty,E}^1$. Given $\pi_{\infty,E}^1$, the entrant will stay longest if at each stage in which it stays, it is indifferent between staying and leaving, that is, if $h_E^t = \pi_E^*$ as long as E does not exit. This allows it the greatest possible

optimism in future periods, given that it has been willing to stay until a given stage. These conjectures Π_E^{*1} are $(\pi_E^*, (1-\pi_E^*)\pi_E^*, \ldots, (1-\pi_E^*)^t\pi_E^*, \ldots, (1-\pi_E^*)^T\pi_E^*, 0, \ldots)$. For example, suppose that $\pi_E^* = 1/4$ and $\pi_{\infty,E}^1 = 1/2$. Then the conjectures $(1/4, 3/16, 0, \ldots)$ allow the entrant to be optimistic enough for two periods, as the hazard rate is $1/4$ for periods 1, 2. In general:

LEMMA 5.1: *For $0 < x < 1$, let the entrant's conjectures be such that $\pi_\infty^1 \geq x$. Then if σ_E^* is a best response to Π_E^1, $\exists t \leq \text{int}(\ln x / \ln(1 - \pi_E^*)) + 1$ such that $t \in \sigma_E^*$.*

PROOF: See the Appendix.

All entrant strategies in which E does not acquiesce by $\text{int}(\ln x / \ln(1 - \pi_E^*)) + 1$ are eliminated immediately due to the restriction on E's beliefs. At stage $\text{int}(\ln x / \ln(1 - \pi_E^*))$, then, I expects E to acquiesce with certainty in the very next stage, and therefore must not itself acquiesce. Working backward from this point:

THEOREM 5: *Let there exist $x \in (0,1)$ such that the firms have common knowledge that the entrant's conjectures are such that $\pi_{\infty,E}^1 \geq x$. Then $1 \in \sigma_E^*$, that is, E never enters.*

PROOF: From Lemma 5.1, $\exists t \leq \text{int}(\ln x / \ln(1 - \pi_E^*)) + 1$ such that $t \in \sigma_E^*$. Define the point by which E must have acquiesced as $T_E(\pi_E^*, x) := \text{int}(\ln x / \ln(1 - \pi_E^*)) + 1$. Consider then I's choice at $T_E(\pi_E^*, x)$. Having reached $T_E(\pi_E^*, x)$ tells I that E will acquiesce with certainty from that point; that is, $h_I^{T_E(\pi_E^*, x)} = 1$. Therefore, $T_E(\pi_E^*, x) \notin \sigma_I^*$, by Theorem 2. Therefore, $h_E^{T_E(\pi_E^*, x)} = 0$. By Theorem 3 then, if $T_E(\pi_E^*, x) + 1 \in \sigma_E^*$, then $T_E(\pi_E^*, x) \in \sigma_E^*$. Having reached $T_E(\pi_E^*, x) - 1$, then, I is certain that E will acquiesce; that is, $h_I^{T_E(\pi_E^*, x)-1} = 1$, and $T_E(\pi_E^*, x) - 1 \notin \sigma_I^*$, again by Theorem 2. Working backwards in this fashion, we see that $1 \in \sigma_E^*$. Q. E. D.

No matter how small the probability E must place on I's playing $\emptyset$ and thereby never acquiescing, this puts a *known* limit on how long E might wish to stay in. I then will not acquiesce one stage earlier, expecting E's acquiescence should it fight itself, and E in turn knows this, and so on. Notice that in the infinite-period version of the chain-store paradox model with a small probability of a "crazy" incumbent, or at least one whose demands or costs are such that it prefers each period to price low, there is only one sequential equilibrium outcome, as there will be some point at which the sane type would wish to mimic the crazy in any candidate for an equilibrium which had the sane incumbent ever acquiesce.

6. REPUTATION FORMATION

Strategic uncertainty seems a natural place to study reputation formation. If one's rival consistently reacts to certain of one's own actions with a particular response, perhaps one could learn that the rival was acting in such a way, and respond accordingly. Reputation formation has previously been studied in economics, on the other hand, as the updating of distributions within the context of imperfect information models. See the seminal papers by Kreps and Wilson (1982), Milgrom and Roberts (1982), and Kreps, Milgrom, Roberts, and Wilson (1982). See also the application of the KMRW methodology to general finitely-repeated games of incomplete information in the context of a folk theorem by Fudenberg and Maskin (1986).

One of the primary motivations for the KMRW analysis was the *chain-store paradox*. Selten (1978) suggested that, despite the fact that the unique backward-induction solution to a finite chain-store game was entry and acquiescence at each location, we should expect it to be possible for the incumbent to generate a *reputation* for tough behavior by fighting in early markets, losing money there but hoping to suggest to later potential entrants that such aggression will continue. Selten thought it unlikely that an incumbent facing a long

yet finite stream of entrants should behave noticeably differently from one facing an infinite sequence. A related finite game in which the unique subgame-perfect equilibrium seems to many as particularly implausible is the centipede game, where players alternately choose whether to take a pile of money or leave it for their opponents to choose. (The pile of money grows at each stage.) (See Rosenthal (1982), Binmore (1987), Reny (1988)).

It seems as though in studying reputation formation in these perfect-information models, the authors have been thinking of the updating of strategic uncertainty: There is no imperfect information, yet the incumbent's logic for preying in an early market is that this might make later entrants believe that it is more likely that the incumbent will follow with predation in their markets than they would have expected had the incumbent not preyed. Unfortunately, it is difficult to reconcile this intuition for reputation-building within a finite-period, perfect information model, where backward-induction pins down the strategies.

An infinite-period model in which many strategies are rational seems to be a natural place to study reputation formation and strategic uncertainty. Recall that updating in the present model does not place any restrictions upon future expectations of rival acquiescence: All hazard rate sequences are consistent with some conjectures. Suppose, however, we were to make the following restriction, and that this restriction was believed to hold for each player by each rival: Hazard rate sequences decline. That is, a stream of tough behavior by a firm will always lead to its rival's increasing expectation of continued tough behavior:

DEFINITION: A firm *builds a reputation for tough behavior* if its rival R has a hazard rate sequence $H_R = \{h_R^t\}_{t=1}^{\infty}$ such that $\forall t_1 < t_2, h_R^{t_1} > h_R^{t_2}$.

Does this form of reputation formation have any implications for the play of the game? We might expect it to be the case that if one firm were to be able to build a reputation for tough behavior, then there should be some point beyond which its rival would never

fight. Then the firm itself, knowing this, would not acquiesce immediately prior, and so on, leading to immediate acquiescence of the rival.

This logic is flawed. Knowledge that the firm becomes more pessimistic over time does not pin down the point at which it becomes pessimistic enough to acquiesce. That is, there exist decreasing hazard rate sequences for each firm which lead the firm to acquiescence at any point of time t, or to permanent nonacquiescence. That is:

THEOREM 6: *Common knowledge that either or both firms build reputations for tough behavior provides no implications for the play of the game.*

Therefore, reputation formation as suggested here is extremely weak. Its weakness stems precisely from the fact that the firms, despite knowing that they are building reputations over time, know neither how strong their reputations were at the beginning—what their rivals' Rs' h_R^1s were—nor how rapidly their rivals are becoming convinced thay they will continue their toughness.

7. RELATED ISSUES IN PREDATORY PRICING

Might the analysis herein presented shed light upon related issues in predatory pricing, such as credit constraints, multimarket reputation-building, and predation for merger, which have not been formally modelled? The question is not only of academic concern. Well-known historical instances of predatory pricing often concern large, well-established firms and sequences of entrants attempting to make inroads into various parts of the incumbents' wider markets. McGee (1958), Elzinga (1970), Burns (1986), and Weiman and Levin (1992), for example, document the monopolization of petroleum refining, gunpowder, tobacco products, and telephone markets in the turn-of-the-century age of the trusts, where foreclosure, multiple markets, and merger were common concerns.

Finance constraints arise quite naturally in the present model. Suppose a bank or

other financier were to provide capital to the entrant. The firm's employees then become agents of the financier, who may be closely supervising the running of the firm. Barring agency problems in financial contracting (see, on the other hand, Bolton and Scharfstein (1990)), it becomes the conjectures held by the *financier* concerning the toughness of the incumbent that determine the observed behavior of the entrant. The active decisionmaker for E is its bank, rather than its manager. Although the firm's employees are perfectly happy to continue in their present positions, the firm will not continue operations past the point where the financier has become pessimistic about the incumbent's acquiescence.

Reputation-building across markets may be incorporated into the present approach as well. Consider an incumbent facing entry and the potential of infinite-period rivalry in a *finite* sequence of markets. Recall the chain-store paradox notion of reputation-building across markets. It may very well be, although it need not be, that later potential entrants believe that it is more likely that the incumbent will fight longer in the later markets if it fights longer in early markets. However, as we expect from Theorem 6, such common knowledge of reputation-building will not restrict behavior.

When merger is an option, the incentives to engage in predatory pricing must be based upon the greater bargaining power the incumbent would have after a bout of predatory pricing than before. McGee (1958) suggests that this is unlikely, while Yamey (1972) provides a historical disputation, and Saloner (1987) one theoretical. Extend the present model to one in which some bargaining may occur between each pair of stages of competition. The game again will be one in which the structure of the game is similar from each similar decisionmaking point for each firm, and therefore, the difference from one stage to the next lies completely in the firms' conjectures. Here, however, conjectures will also extend to the bargaining process, a topic which has only recently received attention in the literature (see Roth and Bayer (1992)). It is clear still that predation to soften up the rival has an intuitive representation here: Rather than exiting as a response to a bout of predatory pricing, the entrant may be willing to acquiesce by accepting a low bid for its assets.

8. CONCLUSION

The present paper has shown that predatory pricing is *rationalizable* and is therefore rational even when there is no uncertainty in the structure of the game. Previous models of predatory pricing have been based upon such structural rather that strategic uncertainty.

The present analysis of predatory pricing suggests a number of directions for future research. Utilizing strategic rather than structural uncertainty may prove to be a more intuitive road toward the understanding of many economic phenomena. For example, price wars may not be optimal forms of enforcing collusion in oligopolistic supergames with imperfect monitoring (e.g., Porter (1983), Green and Porter (1984), Abreu, Pearce and Stacchetti (1986)), but rather coordination failures in the setting of market shares. If an oligopoly is pricing monopolistically, each firm will be uncertain of exactly how much of the market its rivals may be willing to concede. A price war may be brought about, then, as a reaction to a firm's slight output expansion, or as a firm's attempting to convince its rivals that it should have a larger share of the market directly through its own protracted output expansion.

In work closely related to the present, Roth and Bayer (1992) have shown that delay in bargaining can arise as a coordination failure. Again, such a result runs counter to the thinking prevalent in the literature that strikes and delays in bargaining are information-constrained optimal (second best) (see, e.g., Sobel and Takahashi (1983), Fudenberg and Tirole (1983), Cramton (1984), Rubinstein (1985), Chatterjee and Samuelson (1987), Admati and Perry (1987)). (Other models displaying delay in bargaining are based upon supergame reward and punishment schemes in variant's of Rubinstein's (1982) that display multiple prefect equilibria (e.g., van Damme, Selten, and Winter (1990), Fernandez and Glazer (1991), Haller and Holden (1990)), and externalities (Jehiel and Moldovanu (1992)).

Finally, the fact that outcomes occur in rationalizable behavior which are not *individually rational* suggests a striking difference between Nash and rationalizable behavior

in repeated games. In the present model, the entrant's reservation profit level is zero, but this level is not achieved when entry, predation, and exit is the outcome. Although it is a common interpretation of folk theorems for repeated games that "anything can happen," there is nevertheless the strong restriction to individual rationality (see Aumann and Shapley (1976), Rubinstein (1979), Fudenberg and Maskin (1986)). The work here on predatory pricing suggests the basis for a folk theorem for rationalizable behavior: Each player must *expect* at each point in time to receive at least her reservation level, yet, she may not, as her expectations may not be realized. ▄▄▄▄▄▄▄▄▄▄▄▄▄▄▄▄▄▄▄▄▄▄▄ ▄▄▄▄▄▄▄▄▄▄▄.

APPENDIX

Proof of Lemma 5.1

The proof proceeds by showing that conjectures must have at least a particular sum, leaving over a particular amount for $\pi^1_{\infty,E}$, that this sum approaches 1 as t rises, and therefore exceeds $1 - x$ after some point. Therefore, there exists a maximum number of periods of staying in as a response to conjectures with $\pi^1_{\infty,E} \geq x$.

We need only show the following: The conjectures with the lowest sum $\sum_{t=1}^{\infty} \pi_t$ and allow E to stay in for T periods are $\Pi^{*1} = \{\pi^{*1}_t\}_{t=1}^{\infty} := (\pi^*_E, (1 - \pi^*_E)\pi^*_E, \ldots, (1 - \pi^*_E)^{T-1}\pi^*_E, 0, \ldots)$. These conjectures make E indifferent at all stages up until T, and put no weight on acquiescence thereafter.

Suppose not. Let $\Pi^{\prime 1} = \{\pi^{\prime 1}_t\}_{t=1}^{\infty}$ be conjectures under which E may choose to stay for T stages yet for which $\sum_{t=1}^{\infty}(\pi^{*1}_t - \pi^{\prime 1}_t) > 0$. We will derive a contradiction.

We will need the following result, which states that it is better for E, in terms of its expected supergame profits, to have probability weighted at the beginning than later in its conjectures:

LEMMA 5.1.1: *Let* $\Pi^1 = \{\pi^1_t\}_{t=1}^{\infty}$ *be conjectures with* $\pi^1_2 > 0$, *let* $\epsilon \in (0, \pi^1_2]$, *and let* $\hat{\Pi}^1 := (\pi_1 + \epsilon, \pi_2 - \epsilon, \pi_3, \ldots)$. *Then:*

(a) $V_f(\hat{\Pi}^1) > V_f(\Pi^1)$, *and*

(b) $V(\hat{\Pi}^1) \geq V(\Pi^1)$,

where V_f *is the value function for fighting or staying in.*

PROOF: Notice that $\hat{\Pi}^3 = \Pi^3$, and that therefore $V(\hat{\Pi}^3) = V(\Pi^3)$. If $V(\Pi^2) = 0$, the result is immediate. For $V(\Pi^2) > 0$, we have:

$$V_f(\hat{\Pi}^1) = (\pi^1_1 + \epsilon)\bar{v} + (1 - \pi^1_1 - \epsilon)[\underline{v}(1 - \delta) + \delta V(\hat{\Pi}^2)]$$
$$= (\pi^1_1 + \epsilon)\bar{v} + (1 - \pi^1_1 - \epsilon)[\underline{v}(1 - \delta) +$$
$$\delta[\max\{0, (\pi^1_2 - \epsilon)\bar{v}/(1 - \pi^1_1 - \epsilon) + ((1 - \pi^1_2 + \epsilon)/(1 - \pi^1_1 - \epsilon))[\underline{v}(1 - \delta) + \delta V(\hat{\Pi}^3)]\}]]$$
$$\geq (\pi^1_1 + \epsilon)\bar{v} + (1 - \pi^1_1 - \epsilon)[\underline{v}(1 - \delta) +$$
$$\delta[(\pi^1_2 - \epsilon)\bar{v}/(1 - \pi^1_1 - \epsilon) + ((1 - \pi^1_2 + \epsilon)/(1 - \pi^1_1 - \epsilon))[\underline{v}(1 - \delta) + \delta V(\hat{\Pi}^3)]]]$$
$$= (\pi^1_1 + \epsilon)\bar{v} + (1 - \pi^1_1 - \epsilon)\underline{v}(1 - \delta) + \delta[(\pi^1_2 - \epsilon)\bar{v} + (1 - \pi^1_2 + \epsilon)[\underline{v}(1 - \delta) + \delta V(\Pi^3)]].$$

Similarly, and since $V(\Pi^2) > 0$,

$$V_f(\Pi^1) = \pi^1_1\bar{v} + (1 - \pi^1_1)\underline{v}(1 - \delta) + \delta[\pi^1_2\bar{v} + (1 - \pi^1_2)[\underline{v}(1 - \delta) + \delta V(\Pi^3)]].$$

Therefore,

$$V_f(\hat{\Pi}^1) - V_f(\Pi^1) \geq \epsilon\bar{v} - \epsilon\underline{v}(1 - \delta) + \delta[-\epsilon\bar{v} + \epsilon[\underline{v}(1 - \delta) + \delta V(\Pi^3)]]$$
$$= \epsilon[\bar{v}(1 - \delta) - \underline{v}(1 - \delta)^2 + \delta^2 V(\Pi^3)]$$
$$\geq \epsilon[\bar{v}(1 - \delta) - \underline{v}(1 - \delta)^2]$$
$$> 0,$$

completing part (a). Part (b) follows immediately. *Q. E. D.*

Next, create conjectures which place all of the weight that falls on stages $T + 1$ and

later in Π^1 on the Tth stage; by discounting and the successive application of Lemma 5.1.1, this will have at least the same total value of staying in at stage T and therefore at all previous points. Therefore, the newly constructed conjectures also have E (potentially) wish to stay in until T, without raising $\sum_{t=1}^{\infty} \pi_t$.

DEFINITION: For $T \geq t$, $\Pi^t(T) = \{\pi^t_s(T)\}_{s=t}^{\infty} := (\pi^t_t, \pi^t_{t+1}, \ldots, \pi^t_{T-1}, \sum_{s=T}^{\infty} \pi^t_s, 0, \ldots)$.

LEMMA 5.1.2: $\forall T \geq 1$,

(a) $V(\Pi^{\prime t}(T)) \geq V(\Pi^{\prime t}), \forall t \leq T$,

(b) $\exists \sigma \in \Sigma^{T+1}$ such that σ is a perfect best response to $\Pi^{\prime 1}(T)$, and

(c) $\sum_{s=t}^{\infty} \pi^{\prime t}_s(T) = \sum_{s=t}^{\infty} \pi^{\prime t}_s$.

Parts (a) and (b) follow immediately from Lemma 5.1.1 and discounting; part (c) is a direct consequence of the construction of $\Pi^{\prime t}(T)$. Finally, we will construct conjectures with the same sum as $\Pi^{\prime 1}(T)$, but which have a smaller sum than does Π^{*1}, a contradiction. Since the firm with conjectures $\Pi^{\prime 1}(T)$ may stay in until T by Lemma 5.1.2, and must leave at $T+1$ by construction and Theorem 1, it must be that $\pi^T_T(T) \geq \pi^*_E$. But it must be stronger: It must be that $\pi^T_T(T) > \pi^*_E$, since otherwise either the entrant with conjectures $\Pi^{\prime 1}$ would not stay in at T or $\Pi^{\prime 1}$ would coincide with $\Pi^{\prime 1}(T)$ everywhere after T, which leads to a contradiction of the sum of conjectures in $\Pi^{\prime 1}$ being less than in Π^{*1}.

Define then ϵ_t as follows: Consider the conjectures created by taking $\Pi^{\prime 1}(T)$ and moving the excess weight making the hazard rate at any point greater than π^*_E forward as follows:

$$\Pi^{\prime 1}(T, \epsilon) := (\pi^{\prime 1}_1 + \epsilon_2, \pi^{\prime 1}_2 - \epsilon_2 + \epsilon_3, \ldots, \pi^{\prime 1}_{T-1} - \epsilon_{T-1} + \epsilon_T, (\sum_{t=T}^{\infty} \pi^{\prime 1}_t) - \epsilon_T, 0, \ldots)$$

where for all $t \leq T$, $\pi^{\prime t}_t(T, \epsilon) = \pi^{\prime}_E$. This is possible since E stays in at all $t \leq T$ under $\Pi^{\prime 1}$. But since $\pi^{\prime 1}_{\infty}(T, \epsilon) = \pi^{\prime 1}_{\infty} > \pi^{*1}_{\infty}$, it must be that, $\forall t \leq T, \pi^{\prime 1}_t(T, \epsilon) > \pi^{*1}_t$ in order for

the hazard rates to be at least π^*_E. But then both $\pi^{\prime 1}_{\infty}(T, \epsilon) > \pi^{*1}_{\infty}$ and $\forall t, \pi^{\prime 1}_t(T, \epsilon) \geq \pi^{*1}_t$, a contradiction.

We must finally characterize the maximum number of stages E might wish to fight given $\pi^*_{\infty} \geq x$. Conjectures of the form $(\pi^*_E, \pi^*_E(1 - \pi^*_E), \ldots, \pi^*_E(1 - \pi^*_E)^T, 0, \ldots)$ maximize this number. We must therefore find the greatest T such that $\sum_{t=1}^{T} \pi^*_E(1 - \pi^*_E)^{t-1} \leq 1 - \pi^*_{\infty} \leq 1 - x$. But

$$\sum_{t=1}^{T} \pi^*_E(1 - \pi^*_E)^{t-1} = \sum_{t=1}^{\infty} \pi^*_E(1 - \pi^*_E)^{t-1} - \sum_{t=T+1}^{\infty} \pi^*_E(1 - \pi^*_E)^{t-1}$$

$$= 1 - (1 - \pi^*_E)^T \leq 1 - x.$$

Simplifying, we get $T \leq \ln x / \ln(1 - \pi^*_E)$. $Q.E.D.$

FIGURE 1
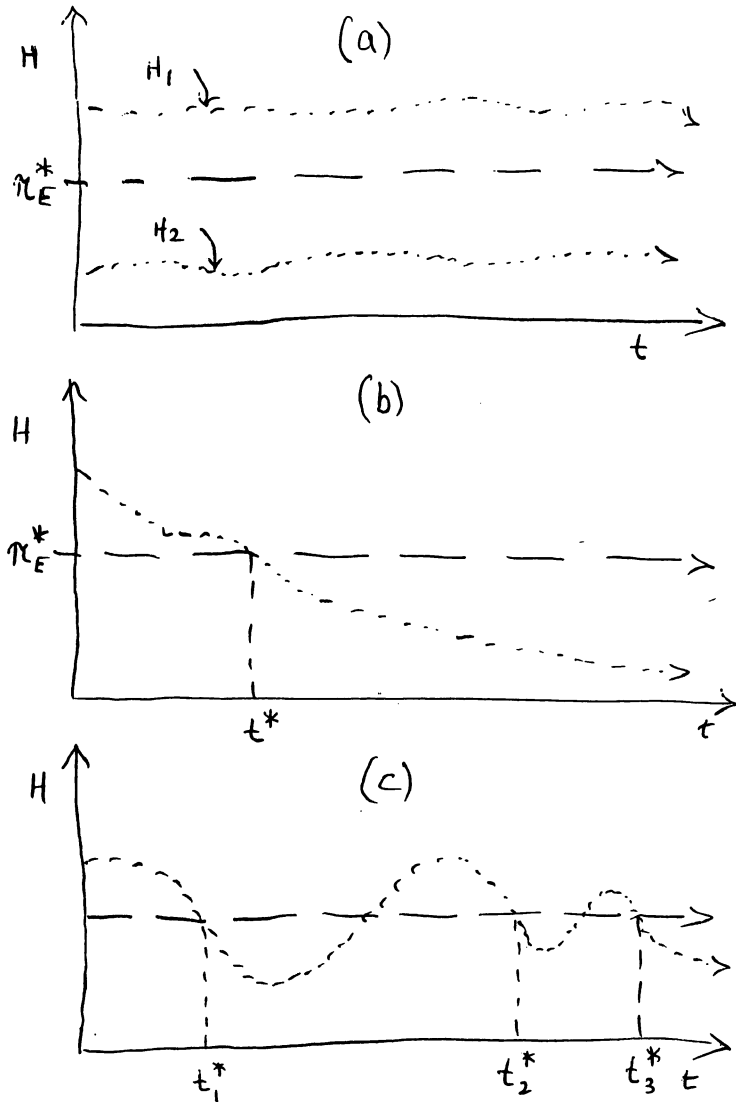
REFERENCES

Abreu, D., D. Pearce, and E. Stacchetti (1986): "Optimal Cartel Equilibria with Imperfect Monitoring," *Journal of Economic Theory* 39: 251-269.

Admati, A., and M. Perry (1987): "Strategic Delay in Bargaining, *Review of Economic Studies* 54: 345-364.

Aumann, R., and L. Shapley (1976): "Long Term Competition: A Game Theoretic Analysis," mimeo, Hebrew University.

Benoit, J.P. (1984): "Financially Constrained Entry in a Game with Incomplete Information," *Rand Journal of Economics* 15: 490-499.

Bernheim, B. D. (1984): "Rationalizable Strategic Behavior," *Econometrica* 52: 1007-1028.

Bolton, P, and D. Scharfstein (1990): "A Theory of Predation Based on Agency Problems in Financial Contracting," *American Economic Review* 80: 93-106.

Börgers, T. (1992): "On the Definition of Rationalizability in Extensive Games," mimeo.

Burns, M. (1986): "Predatory Pricing and the Acquisition Costs of Competitors," *Journal of Political Economy* 94: 266-296.

Chatterjee, K., and L. Samuelson (1987): "Bargaining with Two-sided Incomplete Information: An Infinite Horizon Model with Alternating Offers," *Review of Economic Studies* 54: 175-192.

Cramton, P. (1984): "Bargaining with Incomplete Information: An Infinite-Horizon Model with Two-Sided Uncertainty," *Review of Economic Studies* 51: 579-593.

Dixit, A. (1989): "Entry and Exit Decisions under Uncertainty," *Journal of Political Economy* 97: 620-638.

Easley, D., R. Masson, and R. Reynolds (1985): "Preying for Time," *Journal of Industrial Economics* 33: 445-460.

Ericson, R., and A. Pakes (1989): "An Alternate Theory of Firm and Industry Dynamics," Mimeo, Columbia University.

Fudenberg, D., and E. Maskin (1986): "The Folk Theorem in Repeated Games with Discounting and with Incomplete Information," *Econometrica* 54: 533-554.

Fudenberg, D., and J. Tirole (1983): "Sequential Bargaining with Incomplete Information," *Review of Economic Studies* 50: 221-247.

Fudenberg, D., and J. Tirole (1986): "A 'Signal-Jamming' Theory of Predation," *Rand Journal of*

Economics 17: 366-376.

Grossman, S., and M. Perry (1985): "Sequential Bargaining under Asymmetric Information," *Journal of Economic Theory* 39: 120-154.

Green, E., and R. Porter (1984): "Non-cooperative Collusion under Imperfect Price Information," *Econometrica* 52: 87-100.

Jehiel, P., and B. Moldovanu (1992): "Cyclical Delay in Bargaining with Externalities," mimeo.

Jovanovic, B. (1982): "Selection and the Evolution of Industry," *Econometrica* 50: 649-670.

Kohlberg, E., and J. F. Mertens (1986): "On the Strategic Stability of Equilibria," *Econometrica* 54: 1003-1037.

Kreps, D., and R. Wilson (1982): "Reputation and Imperfect Information," *Journal of Economic Theory* 27: 253-279.

Kreps, D., P. Milgrom, J. Roberts, and R. Wilson (1982): "Rational Cooperation in the Finitely-Repeated Prisoners' Dilemma," *Journal of Economic Theory* 27: 245-252, 486-502.

Maynard Smith, J. (1974): "The Theory of Games and the Evolution of Animal Conflicts," *Journal of Theoretical Biology* 47: 209-221.

McGee, J. (1958): "Predatory Price Cutting: The Standard Oil (N. J.) Case," *Journal of Law and Economics* 1: 137-169.

McGee. J. (1980): "Predatory Pricing Revisited," *Journal of Law and Economics* 23: 289-330.

Milgrom, P., and J. Roberts (1982): "Predation, Reputation, and Entry Deterrence," *Journal of Economic Theory* 27: 280-312.

Milgrom, P., and J. Roberts (1987): "Informational Asymmetries, Strategic Behavior, and Industrial Organization," *American Economic Review Papers and Proceedings* 77: 184-193.

Ordover, J., and G. Saloner (1989): "Predation, Monopolization, and Antitrust." In: R. Schmalensee and R. Willig, eds., *Handbook of Industrial Organization*. Amsterdam: North-Holland.

Pearce, D. (1984): "Rationalizable Strategic Behavior and the Problem of Perfection," *Econometrica* 52: 1029-1050.

Porter, R. (1983): "Optimal Cartel Trigger Price Strategies," *Journal of Economic Theory* 29: 313-338.

Riley, J. (1980): "Strong Evolutionary Equilibrium and the War of Attrition," *Journal of Theoretical Biology* 82: 383-400.

Roberts, J. (1986): "A Signaling Model of Predatory Pricing," *Oxford Economic Papers* (supplement) 38: 75-93.

Roberts, J. (1987): "Battles for Market Share: Incomplete Information, Aggressive Strategic Pricing, and Competitive Dynamics." In: T. Bewley, ed., *Advances in Economic Theory: Invited Papers for the Fifth World Congress of the Econometric Society*. Cambridge University Press.

Rosenthal, R. (1981): "Games of Perfect Information, Predatory Pricing, and the Chain-store Paradox," *Journal of Economic Theory* 25: 92-100.

Roth, D., and A. Bayer (1992): "Delay in Bargaining as a Coordination Failure," mimeo.

Rubinstein, A. (1985): "A Bargaining Model with Incomplete Information about Preferences," *Econometrica* 53: 1151-1172.

Saloner, G. (1987): "Predation, Merger, and Incomplete Information, *Rand Journal of Economics* 18: 165-186.

Salop, S., and C. Shapiro (1980): "A Guide to Test-Market Predation," mimeo.

Scharfstein, D. (1984): "A Policy to Prevent Rational Test-Market Predation," *Rand Journal of Economics* 15: 229-243.

Schelling, T. (1960): *The Strategy of Conflict*. Cambridge, MA: Harvard University Press.

Selten, R. (1975): "Reexamination of the Perfectness Concept for Equilibrium Points in Extensive Games," *International Journal of Game Theory* 4: 25-55.

Selten, R. (1978): "The Chain-store Paradox," *Theory and Decision* 9: 127-159.

Sobel, J., and I. Takahashi (1983): "A Multi-Stage Model of Bargaining," *Review of Economic Studies* 50: 411-426.

Yamey, B. (1972): "Predatory Price Cutting: Notes and Comments," *Journal of Law and Economics* 15: 129-142.