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For Sale By Owner: When to Use a Realtor and How to Price the House

Stephen W. Salant

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University of Michigan
Ann Arbor, Michigan 48109

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Stephen W. Salant
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1. Introduction

A home owner wishing to sell his house can either utilize the services of a realtor or, alternatively, can attempt to sell the house himself. The disadvantage of realtors is well-known: they charge a commission which is typically 6% of the sales price. There are, however, some offsetting advantages. If the owner has moved to another city, the cost of holding open-houses or showing the house to prospective buyers may be prohibitive. But even if it were not inconvenient for the owner to show the house himself, there remains the problem of calling the availability of the house to the attention of prospective buyers. Without a realtor, owners cannot get access to the multiple-listing service. Instead, they must rely on "for sale" signs, classified ads and the neighborhood rumor mill. Since realtors can utilize computerized listings as well as these other channels of communication, they are more likely to contact prospective buyers. Hence, by retaining a realtor the owner can sample prospective buyers more rapidly.

Owners attempting to sell their own houses are often bombarded with fliers, phone calls, and visits from realtors eager to get the listing should the owners change their minds. The abundance of these solicitations—as well as direct observation of discouraged sellers—suggest that, after an unsuccessful interval of "by owner" sales, owners frequently do retain realtors.

If a realtor is eventually engaged, the question naturally arises as to how the house should be *re-priced*. On the one hand, the realtor may argue that it would be irrational to raise the asking price since no buyer had been found at the lower price when the owner was selling it himself. This argument is less than compelling, however, since it assumes that every prospective buyer was already sampled during the by-owner phase. Even if it is optimal in some circumstances to raise the asking price, there does nonetheless seem to be a limit on the magnitude of the increase. Intuition suggests that if the seller is at all times rational both about the price he charges and about the time when he engages the realtor, then the asking price should never be raised to the point where the seller would earn more from a sale with the realtor than in the by-owner phase. For if that were ever optimal, bringing in the realtor earlier would have permitted the seller not only to sample more frequently but also to get a better price. Finally, there remains the question of whether it is ever optimal to lower the asking price when the realtor is retained.

The purpose of this paper is to construct a tractable model which can be used to address these somewhat subtle pricing issues. We consider the following "search model." The seller has T periods in which to sell his house after which he receives a default bequest—say, the value of renting. This assumed finite horizon induces a nonstationarity in the seller's decision problem which causes the expected value of continued search to decline over time.[1] As we will discover, under some conditions

[1] In reality, there exists a widespread preference among families for not moving during the school year and this induces a nonstationarity into sellers' decision problems. A second source of nonstationarity, from which we abstract here, is that the seller is initially uncertain about relevant market parameters and learns about them as he samples.

the seller will prefer to sell the house himself when the expected value of continued search is high but will prefer to engage a broker when the expected value of continued search is low. In each period, the seller must decide whether or not to list his house with a realtor and what the asking price should be. If the house is ever listed with a realtor, we assume it cannot be "unlisted" since standard realtor contracts require that the commission be paid for an interval of time (assumed to exceed T) even if the seller terminates the realtor's services. It is assumed that the house is sold to the first sampled buyer willing to pay at least the asking price and that the transaction takes place at the seller's asking price.

We characterize the optimal time to retain a realtor and the optimal pricing strategy with and without a realtor. Although the asking price monotonically declines over time before and after the introduction of the realtor, the asking price should be *raised* (above what it would have been in the absence of a realtor) when the realtor is first enlisted. However, this increase covers only part of the commission.

Although a "search model," our model differs in one important respect from the standard model of "job search." In the familiar job-search model, the seller (the worker) samples buyers (firms) as does the seller of a house. Once a buyer is encountered, however, the two models differ. In the standard model of the labor market, buyers typically set the price and the seller accepts or rejects it. In the housing market, on the other hand, the seller typically sets the price and it is the buyer who must decide whether to accept it. In the familiar job search model, it is never optimal for a worker to reject a job worth a given amount if the expected discounted value of continuing to search optimally is strictly smaller. In our model, the seller sets the asking price with incomplete information about what the current buyer will accept or reject. It is then always optimal to set the asking price higher than the expected discounted value of continued search.

In the past, antitrust authorities have expressed concern that the seemingly invariant 6% commission rate reflects collusion at the national or at least the local level [Federal Trade Commission 1984]. These hypotheses could in principle be tested given a derived demand curve for realtor services. Properly refined and estimated, our model can provide such derived demands. Hence, it may eventually be helpful in addressing this important question in industrial organization.

2. The Value of Optimal Behavior

We consider the following model. In each period, the seller can engage a realtor if he has not done so already (if a realtor has previously been engaged, we assume he must be retained). Given the presence or absence of a realtor, the seller then sets an asking price for the period and awaits a buyer. We assume that the length of each period is sufficiently short that at most one buyer will arrive. Moreover, the probability that a buyer arrives is assumed to be strictly higher if a realtor has been engaged. Given that a buyer arrives, however, the likelihood of his having a reservation price in any interval is assumed to be the same with or without a realtor. That is, a realtor is viewed as someone who speeds up the sampling process—not as someone who brings in better prospects. If a buyer arrives and if he is willing to pay more than the seller asks, then a sale takes place at the seller's asking price. If, however, no buyer arrives or—alternatively—if one does arrive but is unwilling to pay the asking price, the house remains unsold and the next period is entered. At that time, the seller who has so far tried to sell his house "by owner" can once again exercise his option to engage a realtor and so on ... If the house has not been sold after T periods, the seller accepts an exogenous terminal bequest—for example, the discounted value of renting. It is assumed that the seller is risk neutral, discounts future payoffs at rate β and pays a fixed percentage of the price as a commission if a sale occurs with a realtor engaged.[2]

We adopt the following notation:

V_t is the maximized expected value to the seller if he enters period t and uses a realtor;

P_t is the asking price in period t with a realtor engaged;

α is the fraction ($\alpha \in (0,1)$) of the price retained by the seller if the house is sold by the realtor (α is typically 94%);

γ_S is the probability per period that one buyer is sampled when no realtor is engaged;

γ_R is the probability per period that one buyer is sampled when a realtor is engaged;

$g(y)$ is the probability density of reservation prices with or without a realtor;

$h(p)$ is the hazard function associated with this density of reservation prices $(g(p)/\int_p^u g(y)dy)$;

[2] To explain why some owners begin selling "by owner" but later switch to a realtor, we have simplified by assuming that the cost of sampling buyers is zero. In reality, of course, such costs sometimes govern whether an owner uses a realtor. For example, if the seller has moved to a city far from where his house is located he will retain a realtor because trying to sell the house himself would be very costly. On the other hand, an owner who has moved around the corner from the house he wishes to sell may forego a realtor since, *ceteris paribus*, his cost of sampling buyers will be relatively low. We have omitted the cost of sampling from the functional equations since its effect seems straightforward. This cost should be introduced, however, before estimating the model.

$[l, u]$ is the support of the density;

W_t is the expected value to the seller if he enters period t with no realtor engaged previously (through $t-1$) and proceeds optimally;

$\hat{P}_t$ is the asking price in period t if in period t the owner does not utilize a realtor;

$\bar{V}$ is the terminal bequest if there is no sale up through period T.

We begin by analyzing the optimal policy when a realtor is engaged. In that case,

$$V_t = \max_{u \geq P_t \geq l} \left\{ \alpha P_t \gamma_R \int_{P_t}^{u} g(y)dy + \beta V_{t+1} \left[1 - \gamma_R \int_{P_t}^{u} g(y)dy \right] \right\} \qquad t = 1, \ldots, T$$

and

$$V_{T+1} = \bar{V} \ .$$

That is, the expected value at t when a realtor is engaged equals the probability-weighted sum of the payoff (αP_t) from selling the house today and the payoff (βV_{t+1}) from continuing. To sell the house at t, the seller must (a) sample a buyer and (b) draw someone willing to pay at least the asking price. Defining $\gamma_R g(y) = f(y)$, we can simplify:

$$V_t = \max_{u \geq P_t \geq l} \left\{ \alpha P_t \int_{P_t}^{u} f(y)dy + \beta V_{t+1} \left[1 - \int_{P_t}^{u} f(y)dy \right] \right\} \qquad t = 1, \ldots, T \qquad (1)$$

and

$$V_{T+1} = \bar{V} \ .$$

Define the function $R(V)$ as follows:

$$R(V) = \max_{u \geq P \geq l} \left\{ \alpha P \int_{P}^{u} f(y)dy + \beta V \left[1 - \int_{P}^{u} f(y)dy \right] \right\} \ . \qquad (2)$$

$R(\cdot)$ exists and is single-valued since P lies in a compact set and the objective function (in braces) is continuous. Moreover, by the "theorem of the maximum, " $R(V)$ is continuous. Denote by P^* a maximizer of (2). To emphasize that P^* depends implicitly on V we shall sometimes write $P^*(V)$. We can rewrite (1) compactly as follows:

$$\begin{aligned} V_t &= R(V_{t+1}) \qquad t = 1, \ldots, T \\ V_{T+1} &= \bar{V} \ . \end{aligned} \qquad (3)$$

The sequence $\{V_t\}$ is, therefore, generated by a first-order, nonlinear difference equation. To analyze this equation, we note some properties of $R(V)$. Clearly, $R(0) > 0$ since even a nonoptimal asking price, $P \in (l, u)$, would result in a strictly positive expected payoff. Moreover, $R(\alpha u) < \alpha u$ since $R(V)$ would be the weighted average of two terms each of which is strictly smaller than αu. Finally, when the

derivative exists, $R'(V) \in [0, \beta]$ since in principle the maximizer $P^*(V)$ must be either interior or at the upper or lower limit of the control set. When the derivative exists

$$R'(V) = \begin{cases} \beta[1 - \int_{P^*(V)}^{u} f(y)dy] & \text{if } P^*(V) \in (l, u) \\ 0 & \text{if } P^*(V) = l \\ \beta & \text{if } P^*(V) = u \ . \end{cases} \qquad (4)$$

As we will show later, only the first case is relevant since $P^*(V) \in (l, u)$.

We illustrate the properties of $R(\cdot)$ in Figure 1:

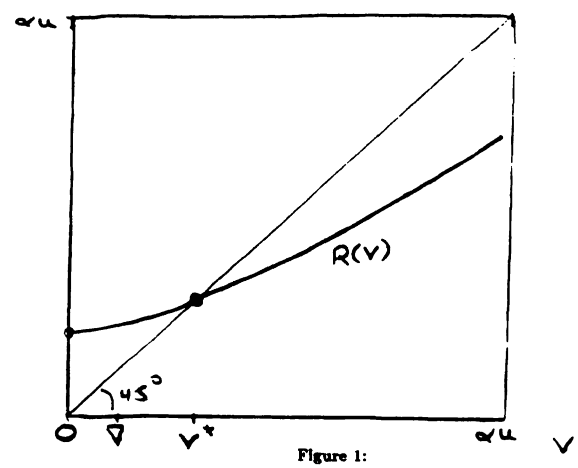


Figure 1:

These properties imply that $R(V)$ has a unique fixed point. $V^* = R(V^*) \in (0, \alpha u)$. Assuming that $0 < \bar{V} < V^*$, the induced sequence of V's $(\bar{V}, R(\bar{V}), R^2(\bar{V}) \ldots)$ strictly increases as we work backwards from $\bar{V}$.[3] Hence, the value of continuing optimally would decline as time runs forward:

$$V_{t+1} < V_t \ . \qquad (5)$$

We now investigate the associated sequence of asking prices. $\{P_t\}$ where $P_t = P^*(V_{t+1})$. Since P^* is optimal, it must solve at least one of the following conditions:

$$\begin{aligned} &\text{a.} \quad P^* \in (l, u) \text{ and } \alpha \textstyle\int_P^u f(y)dy - \alpha P f(P) + \beta V f(P) = 0 \\ \text{or} \quad &\text{b.} \quad P^* = l \text{ and } \alpha + \beta V f(l) \leq 0 \\ \text{or} \quad &\text{c.} \quad P^* = u \text{ and } \beta V - \alpha u \geq 0 \ . \end{aligned} \qquad (6)$$

Since in our application $V \in (0, \alpha u)$, neither (b) nor (c) can hold. Hence, the optimal asking price is always an interior solution and must satisfy (a). Recall that

[3] By $R^k(\bar{V})$ is meant the application of the function R to the real number $\bar{V}$ to produce the real number $R(V)$, the application of the function R to this result, and so forth for a total of k applications of the function. Thus, $R^2(\bar{V}) = R(R(V))$.

$h(p) = g(p)/\int_p^u g(y)dy$. That is, $h(p)$ is the hazard function of the reservation price density. Since $\gamma_R g(y) = f(y)$, $h(p) = f(p)/\int_p^u f(y)dy$. We can therefore rewrite (a) as follows:

$$f(P)\left\{\frac{\alpha}{h(P)} - \alpha P + \beta V\right\} = 0\,. \tag{7}$$

If $h'(P) > -h^2(P)$, the maximand is strictly concave and this first–order condition has a unique solution, $P^*(V)$.

Hence, if the density has a hazard function which increases (or does not decrease too rapidly), then the second-order condition is satisfied for all P. Many common densities—among them the uniform, exponential and normal—satisfy this second–order condition for all P. We will assume henceforth that it is satisfied globally.

Given the sequence $\{V_t\}$, we can now investigate optimal decision–making when *no* realtor has so far been enlisted. In that case,

$$W_t = \max\left(V_t, \max_{u\geq \hat{P}_t \geq l} \hat{P}_t \gamma_S \int_{\hat{P}_t}^u g(y)dy + \beta W_{t+1}[1-\gamma_S\int_{\hat{P}_t}^u g(y)dy]\right) \qquad t=1,\ldots,T$$

$$\text{and } W_{T+1} = \bar{V}.$$

If the seller enlists a realtor at t, he receives V_t. If he does not enlist a realtor at t, he receives the probability–weighted average of the payoff if he sells his house this period (without a realtor) and the payoff expected if $t+1$ is entered without a realtor. To sell his house this period, he must (a) sample a buyer and (b) draw someone willing to pay at least the asking price. Define $\gamma = \gamma_S/\gamma_R$. By assumption, $\gamma \in (0,1)$. We can rewrite the foregoing equation as:

$$W_t = \max\left(V_t, \max_{u\geq \hat{P}_t \geq l} \hat{P}_t \gamma \int_{\hat{P}_t}^u f(y)dy + \beta W_{t+1}[1-\gamma\int_{\hat{P}_t}^u f(y)dy]\right) \qquad t=1,\ldots,T \tag{8}$$

$$\text{and } W_{T+1} = \bar{V}.$$

Using (8) and the sequence $\{V_t\}$, we can work backwards from $\bar{V}$ to construct the sequence $\{W_t\}$. Three possibilities may arise: (1) $W_1 = V_1$, (2) $W_t > V_t$ for all t, *or* (3) $W_1 > V_1$ but $W_t = V_t$ for *some t*. In the first case, it is optimal to use a realtor from the outset. In the second case, it is optimal to sell "by owner" throughout the season. In the third case, it is optimal to sell by owner at the outset but to *switch* to a realtor at a certain date if the house remains unsold at that point. Suppose, given the constructed sequences, that case (3) arose. Let t^* denote the *earliest* date at which $W_t = V_t$. Then the realtor should be retained at t^*. By definition of t^*,

$$\begin{aligned} W_t &= V_t \quad \text{if } t = t^* \\ W_t &> V_t \quad \text{if } t = 1,\ldots,t^*-1 \end{aligned} \tag{9}$$

Define the function $S(W)$ as follows:

$$S(W) = \max_{u\geq\hat{P}\geq l}\left\{\gamma\hat{P}\int_{\hat{P}}^u f(y)dy + \beta W[1-\gamma\int_{\hat{P}}^u f(y)dy]\right\}\,. \tag{10}$$

Denote by $\hat{P}^*(W)$ the maximizer. We can rewrite (8) compactly as:

$$W_t = \max(V_t, S(W_{t+1})) = \max(R(V_{t+1}), S(W_{t+1})) \text{ and } W_{T+1} = \bar{V}\,. \tag{11}$$

A sufficient condition for the owner to avoid engaging a realtor at any time is that $\alpha \leq \gamma$. For, comparing (10) and (2), it is obvious that $S(W) > R(W)$. Hence, the seller would not retain a realtor on the last period if he had not engaged one earlier. Moreover, on the penultimate period, the owner would not engage a realtor. For, even if he were forced to engage one on the last period, selling by owner on the penultimate period would be more profitable than engaging a realtor on the penultimate period. Since, in fact, the seller is free to sell in the final period without a realtor and would choose to do so, the advantage to a by-owner sale in the penultimate period is even greater. Repeating this argument at every stage, it follows that the seller would never want to engage a realtor if $\alpha \leq \gamma$.

Suppose, on the other hand, that $S(\bar{V}) < R(\bar{V})$ (a necessary condition for which is that $\alpha > \gamma$). Then even if no realtor had been engaged prior to the final period, it would be optimal to retain one on the last period. Hence, the condition is sufficient for a realtor to be engaged *sometime*. If, in addition, $S(V^*) > R(V^*) = V^*$ and the time horizon is sufficiently long, by-owner sales will occur at the outset and the realtor will be engaged only subsequently. For, if not, the value of entering the initial period with a realtor engaged would be approximately V^* (the fixed point of $R(\cdot)$) while the value of entering without one would be strictly higher.

Recall that once a realtor is enlisted, the value of continuing strictly declines (equation 5) as time runs forward. A similar phenomena occurs during the by–owner phase. To see this, recall that $W_{t^*} = V_{t^*}$. Since $W_{t^*-1} > V_{t^*-1}$, equation (11) implies that $S(V_{t^*}) > R(V_{t^*})$. But $R(V_{t^*}) > V_{t^*}$. Therefore,

$$S(V_{t^*}) > V_{t^*}\,. \tag{12}$$

Now $S(\cdot)$ is monotone: if $c > b$, $S(c) > S(b)$.[4] Hence, by applying $S(\cdot)$ iteratively to the left and right sides of (11) we obtain:

$$S^{k+1}(V_{t^*}) > S^k(V_{t^*})\,.$$

But from (9) and (11)

$$S^k(V_{t^*}) = W_{t^*-k} \qquad \text{for } k = 1,\ldots,t^*-1\,.$$

Hence we have established that $W_{t^*-k-1} > W_{t^*-k}$ for $k = 1,\ldots,t^*-1$. As in the second phase, the value of continuing strictly decreases prior to t^*.

The asking price in the by–owner phase, $\hat{P}^*$, must solve at least one of the following conditions:

$$\begin{aligned} &\text{a.}\quad \hat{P}^* \in (l,u) &&\text{and}\quad \textstyle\int_{\hat{P}}^u f(y)dy - \hat{P}f(\hat{P}) + \beta W f(\hat{P}) = 0 \\ \text{or}\ &\text{b.}\quad \hat{P}^* = l &&\text{and}\quad 1 + \beta W f(l) \leq 0 \\ &\text{c.}\quad \hat{P}^* = u &&\text{and}\quad \beta W - u \geq 0\,. \end{aligned} \tag{13}$$

[4]For $S(c)$ must be at least as large as the right-hand side of (10) evaluated at the suboptimal $\hat{P}^*(b)$ and that strictly exceeds $S(b)$.

It is straightforward to show that $W_t \in (0, u)$. For, equation (12) implies that $S(0) > 0$ and $u > S(u)$. Moreover, $0 < V_{t^*} < u$. Since $S(\cdot)$ is monotone, it follows that $S^k(V_{t^*}) \in (0, u)$. Since $W_t \in (0, u)$, neither (b) nor (c) can hold. The asking price ($\hat{P}^*$) is always an interior solution and must satisfy (a). We can rewrite (a) as follows:

$$f(\hat{P})\left\{\frac{1}{h(\hat{P})} - \hat{P} + \beta W\right\} = 0\,. \tag{14}$$

As before, if $h'(P) > -h^2(P)$ the maximand is strictly concave and (a) has a unique solution ($\hat{P}^*(W)$ is a function rather than a correspondence).

In the next section we investigate properties of the optimal sequence of asking prices in the by–owner phase, the realtor phase, and at the transition. We then illustrate our findings by means of an example.

3. Properties of Optimal Asking Prices

In both the by–owner phase and the realtor phase, the optimal asking price always exceeds the discounted expected value from continuing. In the by–owner phase, $\hat{P}_t^* > \beta W_{t+1}$; and in the realtor phase, $\alpha P_t^* > \beta V_{t+1}$. These conclusions follow directly from the pair of first-order conditions (equations (7) and (14)). The intuition behind this result may not be immediately obvious; it implies, for example, that if the owner randomly sampled a buyer with reservation price $y^* \in (\beta W_{t+1}, \hat{P}_t^*)$, no sale would occur *despite* the fact that the seller expects a lower payoff from continuing optimally.

To understand this result, suppose the seller in the by–owner phase *did ask* only βW_{t+1} in period t. Then his payoff would be the same whether or not he sold the house at t and variations in the complementary probability weights would be of no consequence. Now suppose he *raised* his asking price marginally. Although the induced change in the probability weights would not alter the expected payoff, the expected payoff would nonetheless increase because any sales which transpired would occur at a higher price. Hence, it is optimal to precommit to an asking price at t strictly higher than the discounted value expected at $t+1$. Like any price-setter constrained to set a uniform price (e.g. a textbook monopolist), our house seller willingly abandons some potential sales which would be profitable so as to enhance the profitability of other potential sales.

This tradeoff is forced on him because he lacks the information necessary to price discriminate. In particular, he does not observe each buyer's willingness to pay. If he did, the seller would in each period demand the larger of two amounts: the expected discounted value of continuing and the current buyer's reservation price. Hence, the seller would never let a buyer escape whose reservation price exceeded his own expected discounted value of continuing and would fully extract the surplus of the purchaser. Our uninformed seller is unable to capture all of the surplus because he lacks the information necessary to price discriminate.

It is interesting to note that no corresponding results appear in the standard model of job search with which most readers are familiar because of the timing assumed in that model. In the standard job-search model, the seller (the worker) moves after the buyer (the firm) and either accepts or rejects the buyer's observed bid. In these circumstances, it is optimal for the seller to accept any bid which exceeds the expected discounted value of continuing. Hence, the unemployed seller of labor services never rejects a bid if his expected discounted value of continuing is lower—much like the price discriminator. But since the bid of the buyer is smaller than his reservation price, the seller does not extract all of the buyer's surplus.

In the last section, we verified that the value of continuing optimally strictly decreases during each phase as time runs forward. Differentiating the conditions defining the optimal asking price in each phase (respectively, equation (6a) and (13a)), we obtain:

$$P^{*\prime}(V) = \frac{\beta/\alpha}{\Delta(P^*)} > 0$$

$$\hat{P}^{*\prime}(W) = \frac{\rho}{\Delta(\hat{P}^*)} > 0$$

where $\Delta(x) = 1 + h'(x)/h^2(x) > 0$. Since during the by-owner phase $P_t^* = P^*(V_{t+1})$ while in the realtor phase $\hat{P}_t^* = \hat{P}^*(W_{t+1})$, the asking price within each phase must strictly decrease as time runs forward.

Of particular interest is the behavior of the optimal asking price at the *transition* between the two phases. We consider the following question: Is the final price in the by-owner phase $(\hat{P}_{t^*-1}^*)$ higher or lower than the price one period later $(P_{t^*}^*)$ when the realtor is retained?

It is convenient to think about this question in two steps. In the first step, we compare the final price in the by-owner phase to the price which would then (at $t^* - 1$) have been optimal if the realtor had been retained. That is, we consider how the asking price would have changed if the broker had been engaged without time elapsing. We then compare this hypothetical price, the optimal asking price at time $t^* - 1$ with the realtor engaged, to the optimal price one period later.

As to the first step, it is straightforward to show that:

$$\alpha P_{t^*-1}^* < \hat{P}_{t^*-1}^* < P_{t^*-1}^*.$$

That is, if a realtor were enlisted at $t^* - 1$ (one period before it is optimal), the seller should *raise* the asking price above what he would otherwise charge; but this markup in the asking price would be insufficient to cover the realtor's commission.

To verify this claim, recall that $W_{t^*} = V_{t^*}$. At $t^* - 1$, therefore, the seller will ask $\hat{P}_{t^*-1}$ solving equation (14):

$$\frac{1}{h(\hat{P}_{t^*-1})} - \hat{P}_{t^*-1} + \beta V_{t^*} = 0\ . \qquad (15)$$

If, instead, he were to utilize a realtor at $t^* - 1$, it would be optimal to ask P_{t^*-1} solving equation (7):

$$\frac{1}{h(P_{t^*-1})} - P_{t^*-1} + \frac{\beta V_{t^*}}{\alpha} = 0\ . \qquad (16)$$

A comparison of (15) and (16) indicates that $P_{t^*-1} > \hat{P}_{t^*-1}$. Therefore, if the realtor were enlisted at $t^* - 1$, the asking price would be higher than is optimal with no realtor.

Indeed, (15) and (16) imply that:

$$\hat{P}_{t^*-1} - \alpha P_{t^*-1} = \frac{1}{h(\hat{P}_{t^*-1})} - \frac{\alpha}{h(P_{t^*-1})}\ .$$

Hence, if $h(\cdot)$ is increasing, $\hat{P}_{t^*-1} > \alpha P_{t^*-1}$.

We now proceed to the second step. The optimal asking price at $t^* - 1$ strictly exceeds the asking price one period later since—once the realtor is engaged—the asking price monotonically decreases. The price therefore jumps up in the first step and declines in the second step. This suggests what we now verify—that the asking

price can either rise or fall between the final period of the by-owner phase and the first period of the realtor phase.

Since $V_{t^*} = R(V_{t^*+1})$, we can rewrite equation (15) as:

$$\frac{1}{h(\hat{P}_{t^*-1})} - \hat{P}_{t^*-1} + \beta R(V_{t^*+1}) = 0\ . \qquad (17)$$

Moreover, the price in the first period of the realtor phase must satisfy:

$$\frac{1}{h(P_{t^*})} - P_{t^*} + \frac{\beta V_{t^*+1}}{\alpha} = 0\ . \qquad (18)$$

Hence, assuming that a realtor is retained at t^*, the asking price then will strictly exceed the price one period earlier if and only if $V_{t^*+1}/\alpha > R(V_{t^*+1})$; it will decline if the inequality is reversed.

Either case can be constructed for any $f(y), \beta$, and α. To produce an upward jump, one merely sets $\bar{V}$ within α percent of V^* and adjusts γ so that it is optimal to switch to a realtor after a by-owner phase.[5]

To produce a decline in price for any $f(y), \beta$, and α, one must set $\bar{V}$ such that $R(\bar{V})$ lies below α percent of V^*. One can then set γ so that the transition occurs to a realtor while V remains below this boundary.

To summarize, when the realtor is first retained the asking price can be either higher or lower than it was on the previous period. The change in the asking price at the transition point is the net effect of two opposing tendencies—each of which we have studied in isolation. If the realtor were enlisted prematurely at $t^* - 1$, the asking price would necessarily jump up then. But the asking price in the realtor phase decreases between period $t^* - 1$ and period t^*.

Parameter values chosen for plausibility produce an upward jump in the asking price when the realtor is first retained. Consider the benchmark case reported in Table 1. As indicated at the top of the table, the realtor's commission is assumed to be 6%, the discount factor (per week) is 1, buyer reservation prices are assumed to be uniformly distributed between 150 and 220 (thousand dollars), the probability of drawing a buyer in a given week with no realtor is assumed to be 25% [6] and the terminal bequest is 183.6 (thousand dollars). The seller is assumed to have at most 17 weeks to sell his house ($T = 17$). If the house still remains unsold at that point, the seller collects the terminal bequest.

The optimal strategy is to begin selling by-owner at an asking price of $207,970 (which exceeds the discounted value of continuing, $196,150). The seller should then drop the price each week if no one purchases it. After 5 weeks, the owner should charge $207,260. On the following week (week 6), he should retain a realtor

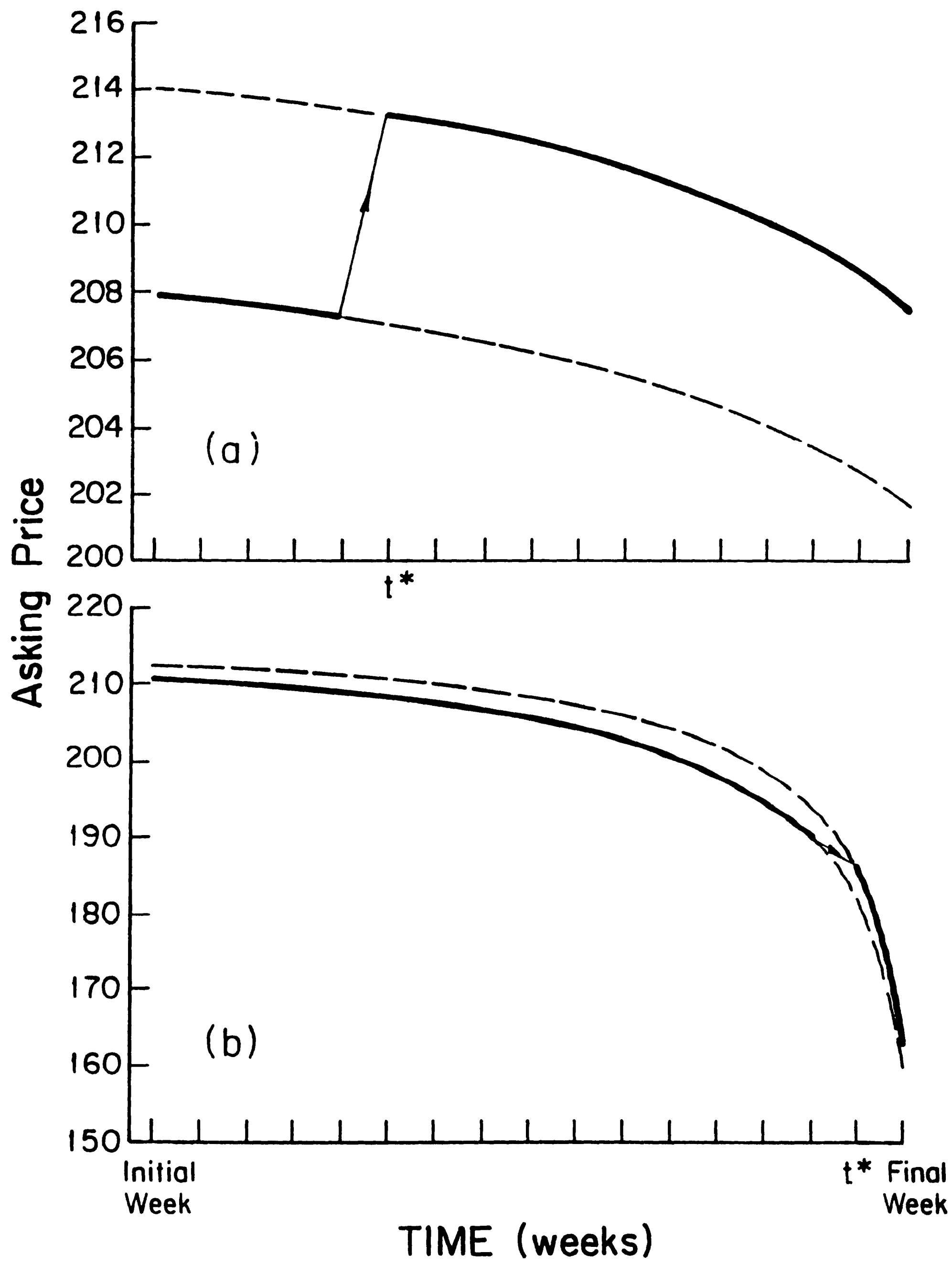
[5] In terms of Figure 1, an upward jump would be inevitable if $\bar{V}$ lies to the right of the intersection of the convex curve $(R(V))$ and a ray through the origin with slope $1/\alpha$. Varying γ to insure the optimality of a two-phase program will shift neither the ray nor $R(V)$.

[6] For simplicity, we have assumed in the simulations that one buyer always arrives during the period if the realtor is engaged ($\gamma_R = 1$). Hence, $\gamma = \gamma_S$ and $f(y) = g(y)$.

and initially ask \$213,240—an upward jump of 2.9%. The price jump is insufficient to cover the commission. In this example, the final asking price in the realtor phase is \$207,560—not significantly lower than the owner charged at the outset. Table 2 illustrates the case where the price *declines* when the realtor is introduced. As before, the parameters are listed at the top of the table. Note the low value for $\bar{V}$. In this example, the realtor is retained only for the final two weeks of the season. The price drops from \$189,990 on the final week of the by-owner phase to \$186,050 in the initial week of the realtor phase—a decline of 2.1%. We can regard this \$3,940 decline as the result of two changes. If the realtor had been introduced a week earlier, the price would have then been \$4,210 higher (\$194,200). However, in the following period of the realtor phase, the price would have declined by \$8,150 (to \$186,050) so that on balance the price declines by \$3,940. We illustrate each case in the two panels of Figure 2:

Tables 3 and 4 illustrate the sensitivity of the results to changes in the realtor commission. In each of these tables, all other parameters are the same as in the benchmark case (Table 1). As Table 3 indicates, if the commision were instead 5%, the realtor would be enlisted at the outset; on the other hand, as Table 4 indicates, at a 8% commission rate, the realtor would be utilized only in the final three weeks.

A similar sensitivity arises with respect to changes in γ. A reduction in γ has no effect on the elements of $\{V_t\}$ but reduces the elements of $\{W_t\}$. The earliest date when $W_t = V_t$ therefore occurs further from the terminal date T and t^* weakly decreases. Thus, if the frequency of drawing buyers in the absence of a realtor were smaller, it would be optimal to terminate the by-owner phase sooner. The case where the realtor is utilized from the outset arises for sufficiently low γ ($\gamma \leq .21$ with the other assumptions as in the benchmark case). Similarly, the case where the realtor is never utilized arises for sufficiently high γ ($\gamma \geq .44$).



Realtor retained at t^*

4. Future Research

Since only realtors have access to computerized listings, their comparative advantage over by-owner sales should vary widely by city size. In terms of our model, γ should be high in small towns and low in major cities. On the other hand, casual observation of the real estate market suggests that realtor commissions vary little across different regions of the country. The model therefore predicts that realtor sales should never occur in small towns (e.g. Duck, North Carolina) while by-owner sales should never occur in major cities (e.g. Manhattan). In towns of intermediate size like Ann Arbor, one would expect to see both forms of selling.

The absence of much variation of realtor commissions across regions of the country is itself a longstanding and unsolved puzzle [Owen 1977 and Wachter 1987] It is not at all uncommon to see a reduction in the standard 6% commission to 5% of the value of the house and this may in fact not be "small" since it reduces realtor earnings by 17%. The antitrust authorities, however, have been concerned that the commission rate may reflect collusion at the national or at least the local level [Federal Trade Commission 1984]. To investigate this hypothesis, a better understanding is needed of the derived demand for realtor services. This is precisely what our model can provide. As we have seen, an increase in the commission causes some sellers to forego a realtor altogether and others to use one more sparingly; but it also affects asking prices and hence the probability that the house will be sold before the realtor would be retained. It would be useful to quantify these effects.

Techniques now exist to *estimate* dynamic programming models such as ours from micro data on advertised asking prices and on dates of sale.[7] Once estimated, the model could be used study the demand and profit consequences of variations in the commission rate.

[7] See, for example, Gotz-McCall (1984), Pakes (1986), and Rust (1987).

Parameters	
alpha	0.94
beta	1.00
u	220.00
l	150.00
gamma	0.25
vbar	183.60

Table 1

Periods	W	R	S	Realtor P	Self $\hat{P}$	Phase*
1	196.47	196.29	196.47	214.17	207.97	1.00
2	196.15	196.03	196.15	214.01	207.81	1.00
3	195.81	195.74	195.81	213.85	207.63	1.00
4	195.46	195.43	195.46	213.67	207.45	1.00
5	195.09	195.09	195.09	213.46	207.26	1.00
6	194.71	194.71	194.69	213.24	207.05	0.00
7	194.29	194.29	194.25	212.99	206.81	0.00
8	193.82	193.82	193.76	212.72	206.55	0.00
9	193.30	193.30	193.20	212.41	206.26	0.00
10	192.72	192.72	192.58	212.06	205.94	0.00
11	192.07	192.07	191.88	211.67	205.57	0.00
12	191.33	191.33	191.08	211.22	205.15	0.00
13	190.48	190.48	190.16	210.70	204.66	0.00
14	189.51	189.51	189.10	210.10	204.10	0.00
15	188.38	188.38	187.86	209.40	203.44	0.00
16	187.06	187.06	186.38	208.57	202.65	0.00
17	185.49	185.49	184.61	207.56	201.71	0.00
	183.60	183.60	183.60			

* Periods when it is optimal to enlist a realtor are denoted by "0.00"; periods when it is optimal to continue with no broker are denoted by "1.00".

Parameters	
alpha	0.95
beta	1.00
u	220.00
l	150.00
gamma	0.75
vbar	100.00

Table 2

Period	W	R	S	Realtor P	Self $\hat{P}$	Phase*
1	202.27	195.73	202.27	212.63	210.67	1.00
2	201.53	195.19	201.53	212.31	210.26	1.00
3	200.72	194.58	200.72	211.95	209.80	1.00
4	199.80	193.89	199.80	211.54	209.29	1.00
5	198.77	193.11	198.77	211.07	208.70	1.00
6	197.60	192.22	197.60	210.53	208.03	1.00
7	196.26	191.19	196.26	209.90	207.26	1.00
8	194.72	190.00	194.72	209.16	206.36	1.00
9	192.92	188.59	192.92	208.28	205.30	1.00
10	190.80	186.92	190.80	207.21	204.03	1.00
11	188.26	184.88	188.26	205.88	202.48	1.00
12	185.15	182.36	185.15	204.19	200.55	1.00
13	181.28	179.15	181.28	201.96	198.06	1.00
14	176.30	174.91	176.30	198.87	194.73	1.00
15	169.63	169.01	169.63	194.20	189.99	1.00
16	160.14	160.14	159.77	186.05	182.25	0.00
17	144.65	144.65	138.54	162.58	159.95	0.00
	100.00	100.00	100.00			

* Periods when it is optimal to enlist a realtor are denoted by "0.00"; periods when it is optimal to continue with no broker are denoted by "1.00".

Parameters	
alpha	0.95
beta	1.00
u	220.00
l	150.00
gamma	0.25
vbar	183.60

Table 3

Periods	W	R	S	Realtor P	Self $\hat{P}$	Phase*
1	198.16	198.16	198.12	214.04	208.84	0.00
2	197.87	197.87	197.82	213.87	208.68	0.00
3	197.56	197.56	197.49	213.69	208.51	0.00
4	197.22	197.22	197.13	213.50	208.32	0.00
5	196.84	196.84	196.73	213.28	208.11	0.00
6	196.42	196.42	196.29	213.03	207.88	0.00
7	195.96	195.96	195.80	212.76	207.62	0.00
8	195.45	195.45	195.25	212.46	207.34	0.00
9	194.87	194.87	194.63	212.12	207.01	0.00
10	194.22	194.22	193.93	211.73	206.65	0.00
11	193.49	193.49	193.13	211.29	206.23	0.00
12	192.65	192.65	192.22	210.79	205.75	0.00
13	191.69	191.69	191.17	210.20	205.19	0.00
14	190.58	190.58	189.94	209.52	204.54	0.00
15	189.28	189.28	188.49	208.71	203.77	0.00
16	187.73	187.73	186.74	207.73	202.85	0.00
17	185.88	185.88	184.61	206.53	201.71	0.00
	183.60	183.60	183.60			

* Periods when it is optimal to enlist a realtor are denoted by "0.00"; periods when it is optimal to continue with no broker are denoted by "1.00".

Parameters	
alpha	0.92
beta	1.00
u	220.00
l	150.00
gamma	0.25
vbar	183.60

Table 4

Periods	W	R	S	Realtor P	Self $\hat{P}$	Phase*
1	194.80	192.64	194.80	214.48	207.11	1.00
2	194.40	192.43	194.40	214.35	206.90	1.00
3	193.99	192.20	193.99	214.22	206.68	1.00
4	193.55	191.96	193.55	214.07	206.44	1.00
5	193.08	191.69	193.08	213.91	206.20	1.00
6	192.59	191.39	192.59	213.74	205.94	1.00
7	192.08	191.07	192.08	213.54	205.67	1.00
8	191.54	190.71	191.54	213.33	205.39	1.00
9	190.97	190.31	190.97	213.09	205.09	1.00
10	190.36	189.88	190.36	212.83	204.77	1.00
11	189.73	189.39	189.73	212.53	204.43	1.00
12	189.05	188.85	189.05	212.20	204.07	1.00
13	188.33	188.23	188.33	211.82	203.69	1.00
14	187.57	187.54	187.57	211.40	203.29	1.00
15	186.76	186.76	186.72	210.91	202.84	0.00
16	185.86	185.86	185.75	210.34	202.32	0.00
17	184.82	184.82	184.61	209.67	201.71	0.00
	183.60	183.60	183.60			

* Periods when it is optimal to enlist a realtor are denoted by "0.00"; periods when it is optimal to continue with no broker are denoted by "1.00".

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