Air Force Academy Attrition:
A New Perspective on the
College Dropout Problem

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AIR FORCE ACADEMY ATTRITION: A NEW PERSPECTIVE ON THE COLLEGE DROPOUT PROBLEM

by Stephen W. Salant and Roy Danchick

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The importance of the academy environment to attrition takes on particular significance because most recent attrition studies performed by the academies...have concentrated on the relationship between attrition and the characteristics of the students who enter. Comparatively little research was directed to critically examining the effect the academy environment on attrition. Because of the importance of the academy environment to attrition, we see a need to redirect some of the academies' research so that it is more balanced in scope. There should be more emphasis on evaluating the impact of the academy environment on attrition, especially how the environment interacts with the students' characteristics to cause attrition.

[GAO Report, 1976, p.41]
I. INTRODUCTION

The purpose of this report is to summarize the conceptual underpinnings and computerized implementation of a model of Air Force Academy attrition. The policy question which prompted development of this model concerns the effects of moving the commitment point—the date after which dropouts are obligated to serve on active duty in an enlisted status. This date is currently set at the beginning of the junior year. Prior to that date, a cadet may leave the Academy—having paid nothing for tuition, room, or board—without incurring any service obligation or financial penalty.

We began our research by reviewing several surveys of the enormous literature on student attrition at the college level. There are actually two distinct but complementary strands to this literature that have developed somewhat independently: one for civilian schools and the other for military academies. While the best of the literature provides very useful information about attrition behavior in an existing environment, none of it is directly applicable to the policy question we were asked to examine. For, the literature deals almost exclusively with the selection question: what will be the effect on attrition of selecting from a given applicant pool candidates with alternative sets of entry characteristics. None of it concerns the effects of a change in the college environment on 1) the behavior of a given set of entrants or on 2) the composition of the applicant pool from which entrants are selected.

To study the effects on cadet attrition of any change in the college environment requires a fundamentally different approach than has been taken in other studies of student attrition. We hypothesize that cadet attrition behavior is not random or invariant, but rather is responsive to the circumstances in which the cadet finds himself. In particular, we regard the student as behaving in a self-interested manner—albeit in an uncertain environment and one about which he continues to learn.
The hypothesis that students behave "as if" in a purposeful manner may strike some readers as sensible but others as bizarre. Hopefully, its merits will soon be demonstrated by a statistical test of our model's predictions. We were unable to procure the requisite Academy data to estimate our model or to perform such a test ourselves; however, later in this report we discuss the data needed to estimate the model and outline the principles of the estimation. Although at this point we lack the data to estimate all the parameters in our model, whenever possible we have incorporated data for the class of 1985 at the Air Force Academy.

The hypothesis of goal-seeking behavior underlies virtually all microeconomic studies of adults and has been found repeatedly to be consistent with observed behavior. However, it is reasonable to question whether such a hypothesis is equally applicable to youths of college age. Experimental evidence strongly supports this presumption. Over the last twenty years, a variety of controlled experiments have been conducted [Plott, 1982; Smith, 1982] to test propositions in microeconomic theory. These experiments invariably use college students [drawn from California Institute of Technology, University of Arizona, Pasadena Community College, etc.] as subjects. Very rarely do subjects in such experiments deviate systematically from self-interested behavior. It is, of course, possible in principle that these same students are less purposeful in the conduct of their own lives. But the experiments also suggest that when the relative payoffs for self-interested behavior increase, subjects make fewer--not a greater--number of errors.

The policy simulation model we have built is based on the hypothesis of self-interested student behavior. If changes in the student's environment (such as the commitment obligation) make the previous pattern of behavior less in a cadet's interest than some alternative pattern of behavior, we expect his behavior will adjust to whatever best serves his interests in the new environment. The function of the computer model is to calculate--for each of a heterogeneous group of students--the behavior most suitable for the environment specified and to summarize the predicted effects on attrition, cost, and quality of graduate.
The report is divided into two parts. In part one, we provide an overview of attrition at the Air Force Academy, then summarize several widely accepted findings of the attrition literature which have influenced our modeling, then describe our model's structure, and then turn to illustrative simulations of the effects of policy changes. Two types of policy changes are illustrated: 1) selection policies which alter the composition of the entering class but not student incentives and 2) environmental policies which alter cadet (and applicant) incentives. Part one concludes with a discussion of how the model can be estimated. Part two, which is self-contained, describes the main program and the six subroutines which comprise the computerized implementation of the model. The report contains two appendices. The appendix to part one is a historical summary of the service obligations faced by dropouts from the Class of 1959 onwards. The appendix to part two is a listing of the computer program.
II. OVERVIEW OF ATTRITION AT THE AIR FORCE ACADEMY

In this report, the attrition rate is defined as the percentage of entering cadets who fail to graduate with their class four years later. By this definition, the attrition rate at the Academy has ranged from a low of 27.8% for the class of 1970 to a high of 46.2% for the class of 1975. The classes of 1966-1985 had an average (entrant-weighted) attrition rate of 38.8%. The attrition experience of each class is reported in Chart 1.

Shocking as these numbers may be to the uninitiated, they compare favorably with the attrition experience of most civilian colleges. According to Iffert [1957], for the nation as a whole, 60% of an entering class fails to graduate from the average college four years later. Some of these students do graduate from another school or graduate later from their college of entry but--for purposes of comparison with Academy data--it seems appropriate to count these students as dropouts. Such a procedure may overstate the percentage of college students who would have attrited if their colleges had severely discouraged taking a year or more off. But this group constituted only 10% of entrants in Iffert's study. Hence, even if every one of these students graduated in four years from his college of entry, attrition rate would still have been 50%. Iffert's findings were subsequently confirmed by Summerskill [1962] in his review of 35 attrition studies over a forty year period from 1913-1953. According to Summerskill, the percentage of students lost to a college over a four-year period had not changed significantly in four decades. The median attrition rate of the graduating classes in his study was 63% although he noted substantial variation across schools.

Perhaps a better standard of comparison, however, is the attrition performance of the other federal service academies. Chart 2 compares the attrition rates at each of the five service academies for the classes of 1964-1975. Clearly, the attrition experience of the Air Force Academy is not an outlier.
ATTRITION RATE BY GRADUATING CLASS YEAR

PERCENT ATTRITION

GRADUATING CLASS YEAR

Chart 1
FEDERAL SERVICE ACADEMIES
ATTRITION RATE BY GRADUATING CLASS YEAR

PERCENT ATTRITION

Source: GAO Report

Chart 2
These comparisons are intended to put the Academy's attrition problem in perspective, not to deny its existence. To determine the severity of the problem, it is useful to ask (1) if dismissals account for most attrition; (2) if most attrition occurs before substantial resources are wasted; and (3) if a disproportionate number of poorly qualified students are among the voluntary dropouts. To the extent that the Academy attrition had these characteristics, its effects on resource costs and quality of graduate would generate little basis for concern.

Chart 3 indicates, however, that most attrition is voluntary and occurs subsequent to pretraining. In recent years (for the classes of 1982-5), involuntary attrition (mostly academic dismissals) has ranged from 7.9% to 9.8%. The remaining attrition is voluntary. Typically only 5-6% of an entering class leaves during Basic Cadet Training (BCT); 18-24% of entrants leave subsequently.

**CHART 3**

<table>
<thead>
<tr>
<th>Class</th>
<th>Aggregate Attrition</th>
<th>Voluntary Attrition</th>
<th>Involuntary Attrition</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>BCT POST BCT</td>
<td>ACAD NONACAD</td>
</tr>
<tr>
<td>1982</td>
<td>41.91</td>
<td>4.83 27.28</td>
<td>8.57 1.23</td>
</tr>
<tr>
<td>1983</td>
<td>36.52</td>
<td>7.70 20.72</td>
<td>7.17 .93</td>
</tr>
<tr>
<td>1984*</td>
<td>33.7</td>
<td>5.42 20.38</td>
<td>6.48 1.42</td>
</tr>
<tr>
<td>1985*</td>
<td>33.7</td>
<td>5.9 18.4</td>
<td>8.64 .76</td>
</tr>
</tbody>
</table>

*As of September, 1983.

Moreover, as will be shown below, a disproportionate share of these dropouts are highly qualified. These characteristics suggest the importance of understanding the magnitude, timing, and incidence of attrition at the Academy.
III. LITERATURE FINDINGS INCORPORATED IN THE MODEL

What variables do past studies indicate are likely to influence Air Force Academy attrition? According to Astin [1975, p.45], who surveyed 40,000 students at 358 nonmilitary institutions in 1968 and then tracked them longitudinally four years later, "By far the greatest predictive factor [in dropout proneness] is the student's past academic record and academic ability." High school grades and aptitude scores (SAT or ACT) have been shown in countless studies to be associated with low attrition rates.¹

Their influence is clearly evident in Air Force Academy data. The Academy aggregates various measures of high school performance into a composite statistic called ACACOMP. Chart 4 indicates the clear differences in attrition performance for the different ACACOMP groups in the entering class of 1985. The average attrition rate of cadets with scores in the highest performance interval is less than half the rate of those in the lowest ACACOMP interval.² Given this pronounced relationship, we distinguish entrants by ACACOMP group in our model.

Attrition is also strongly correlated with performance in college as measured by cumulative grade point average (GPA). Since students who perform well in high school tend to have higher GPA's in college, this correlation is to be expected. However, Astin [1976, p.99-100] investigated whether cumulative GPA exerted an independent influence on college attrition using linear multiple regression techniques. Chart 5 reports his findings. The higher a student's cumulative GPA, the lower his attrition rate. Moreover, Astin found that even after high school performance and other variables observable at entry are controlled for, the actual attrition for students with high GPAs is lower than predicted and the actual attrition for students with low GPAs is higher than

¹For a wide variety of references, see Astin [1975, p.30 or Pantages and Creedon [1978, p.62].
²Since a higher percentage of the attrition of cadets with high ACACOMP scores is voluntary, reductions in voluntary attrition are likely to raise the mean quality of the graduates.
### CLASS OF '85 ATTRITION RATE

<table>
<thead>
<tr>
<th>ACACOMP</th>
<th># OF ENTRANTS</th>
<th>ACTUAL ATTRITION RATE (THRU JUNE OF SOPHOMORE YR.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3240+</td>
<td>515</td>
<td>25.4</td>
</tr>
<tr>
<td>2940-3240</td>
<td>492</td>
<td>30.0</td>
</tr>
<tr>
<td>2640-2940</td>
<td>398</td>
<td>46.5</td>
</tr>
<tr>
<td>2640-</td>
<td>43</td>
<td>55.8</td>
</tr>
<tr>
<td>AGGREGATE</td>
<td>1448</td>
<td>33.7</td>
</tr>
</tbody>
</table>

Chart 4
predicted. From this, he concluded:

"The effects of academic performance on attrition cannot be attributed entirely to differences among students when they enter college. College grades appear to influence persistence directly, independent of initial variations in ability and family background, financial aid and work during college, freshman residence, and type of institution. (These variables were included in computing expected investigation by service dropout rates)."

While we were unable to find any systematic investigation by service academies of the independent influence of GPA on attrition, the GAO on attrition, the GAO Report [1976, enclosure B, p.74] cites several studies suggestive of Astin's conclusions. Given these findings, we allowed in our model for the possibility that the cumulative GPA of cadets in each ACACOMP group will have an independent influence on attrition as they pass through the academy.  

A final potential influence on attrition is the accuracy of the student's expectations about the Academy and the Air Force. These expectations are presumably formed prior to entry or in the first few months at the Academy and are likely to be influenced by information received during recruitment as well as by impressions formed during BCT. Support for the plausible hypothesis that some cadets are simply uninformed (or misinformed) about what they are getting into is informal but persuasive. There is not only the self-selected testimonial of one dropout [Penthouse Magazine, Oct. 1979, p.119] that, "What the recruiters say and what actually goes on at the [Air Force] academy are two entirely different stories...;" there have also been informal interviews by a trained psychologist of all BCT dropouts of a given class. If taken at face value, these interviews indicate a surprising ignorance on the part of some entering cadets about the nature and goals of the Academy and the military: "Cultural shock," "didn't understand the military," "military not for him, wants to be a doctor," "doesn't want to be an officer, "religious conflict over

3Although we expect GPA to exert an influence on attrition independent of characteristics at entry, the relationship might be complex since students who do better academically are likely also to have better outside opportunities if they attrite (and conversely).
### EFFECT OF CUMULATIVE GPA IN COLLEGE ON ATTRITION

**ACTUAL AND EXPECTED DROP OUT RATES OF STUDENTS AT SIX GRADE LEVELS (PERCENTAGES)**

<table>
<thead>
<tr>
<th>GRADEPOINT AVERAGE</th>
<th>ACTUAL</th>
<th>EXPECTED</th>
<th>ACTUAL - MINUS EXPECTED+</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>WHITE MEN</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.25-4.00</td>
<td>24</td>
<td>15</td>
<td>-9</td>
</tr>
<tr>
<td>2.75-3.24</td>
<td>32</td>
<td>22</td>
<td>-10</td>
</tr>
<tr>
<td>2.25-2.74</td>
<td>39</td>
<td>31</td>
<td>-7</td>
</tr>
<tr>
<td>1.75-2.24</td>
<td>47</td>
<td>69</td>
<td>22</td>
</tr>
<tr>
<td>1.25-1.75</td>
<td>53</td>
<td>97</td>
<td>44</td>
</tr>
<tr>
<td>LESS THAN 1.25</td>
<td>54</td>
<td>100</td>
<td>45</td>
</tr>
<tr>
<td><strong>WHITE WOMEN</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.25-4.00</td>
<td>24</td>
<td>19</td>
<td>-5</td>
</tr>
<tr>
<td>2.75-3.24</td>
<td>28</td>
<td>23</td>
<td>-6</td>
</tr>
<tr>
<td>2.25-2.74</td>
<td>35</td>
<td>34</td>
<td>-1</td>
</tr>
<tr>
<td>1.75-2.24</td>
<td>45</td>
<td>72</td>
<td>27</td>
</tr>
<tr>
<td>1.25-1.75</td>
<td>48</td>
<td>92</td>
<td>44</td>
</tr>
<tr>
<td>LESS THAN 1.25</td>
<td>42</td>
<td>100</td>
<td>58</td>
</tr>
<tr>
<td><strong>BLACKS IN BLACK COLLEGE</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.25-4.00</td>
<td>27</td>
<td>19</td>
<td>-8</td>
</tr>
<tr>
<td>2.75-3.24</td>
<td>28</td>
<td>17</td>
<td>-11</td>
</tr>
<tr>
<td>2.25-2.74</td>
<td>38</td>
<td>31</td>
<td>-7</td>
</tr>
<tr>
<td>1.75-2.24</td>
<td>44</td>
<td>67</td>
<td>23</td>
</tr>
<tr>
<td>1.25-1.75</td>
<td>60</td>
<td>88</td>
<td>28</td>
</tr>
<tr>
<td>LESS THAN 1.25</td>
<td>--*</td>
<td>--*</td>
<td>--*</td>
</tr>
<tr>
<td><strong>BLACKS IN WHITE COLLEGES</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.25-4.00</td>
<td>28</td>
<td>21</td>
<td>-7</td>
</tr>
<tr>
<td>2.75-3.24</td>
<td>42</td>
<td>28</td>
<td>-15</td>
</tr>
<tr>
<td>2.25-2.74</td>
<td>45</td>
<td>39</td>
<td>-6</td>
</tr>
<tr>
<td>1.75-2.24</td>
<td>61</td>
<td>72</td>
<td>12</td>
</tr>
<tr>
<td>1.25-1.75</td>
<td>66</td>
<td>96</td>
<td>30</td>
</tr>
<tr>
<td>LESS THAN 1.25</td>
<td>--*</td>
<td>--*</td>
<td>--*</td>
</tr>
</tbody>
</table>

**SOURCE:** A. ASTIN, PREVENTING STUDENTS FROM DROPPING OUT.

*NUMBER OF STUDENTS TOO SMALL TO COMPUTE RELIABLE ESTIMATES.

+"EXPECTED" AFTER CONTROLLING FOR VARIATIONS IN STUDENT ABILITY, BACKGROUND, RESIDENCE, FINANCIAL STATUS AS WELL AS CHARACTERISTICS OF THE INSTITUTION. SEE ASTIN, PP. 98-100.
Were continuing students concerned about impending commitment obligation?

Source: GAO Report

Chart 6
Were dropouts concerned about impending commitment obligation?

Source: GAO Report

Chart 7
war/killing"...[Resignee's Reasons, collected by John Swiney for the class of 1987]. In recognition that attrition may sometimes occur because students enter the Academy with incorrect expectations, we allowed in our model for the possibility that prior to and shortly after entry, cadets continue to process information in an attempt to learn both about the Academy and the military career beyond.

The literature also suggests several policy instruments which the military can use to influence voluntary attrition. According to the GAO Report, the enlisted service obligation for attrition after the commitment point exerts a distinct influence on sophomore attrition. The GAO administered a questionnaire to all students (about 13,000) who were at the five service academies as of May 1974, all those who left the academies before graduating between July 1970 and May 1974 (about 7,300), and all graduates of the class of 1973 (about 3,000). Charts 6 and 7 from the GAO Report [1976, p.78] contrast the responses of continuing sophomores to those who attrited just prior to the commitment point. Strong concern about the service obligation appears to distinguish the sophomore dropouts from the continuing students.

These GAO findings for service academy cadets are closely related to what Astin discovered about college students on ROTC scholarships. Using his linear multiple regression approach, Astin computed the expected dropout rate for those students in his sample receiving ROTC benefits. Since ROTC students are a select group, it should not be surprising that their expected dropout rate was substantially smaller (one third less) than the expected attrition rate for non-recipients. Astin then went on, however, to examine the actual dropout rate of the ROTC scholarship recipients. He found their attrition rate to be dramatically less than he had predicted on the basis of entry characteristics and other variables. His results are reported in Chart 8.
In attempting to explain why the actual ROTC attrition rate was less than half what he predicted, Astin [1975, p.66] observed:

Participation in ROTC may represent a commitment that greatly decreases the chances that the student will leave college. Among other things, ROTC is contractual: students who receive benefits normally make a commitment to continue in the program and to serve on active duty once they finish college.

He might have added that a ROTC scholarship recipient who does drop out incurs a significant service or--at his option--financial obligation as specified in the ROTC contract. For example, the AFROTC contract specifies:
That as a condition of receiving advanced education as defined in 10 United States code 2005, [I agree]:

To complete the educational requirements specified in this contract and to serve on active duty for the period specified in this contract; and

That if I fail to complete the specified educational requirements, I will serve on active duty for the period specified in this contract; and

That if I voluntarily or because of misconduct, fail to complete the period of active duty specified in this contract, I will reimburse the United States in an amount that bears the same ratio to the total cost of my advanced education as the unserved portion of active duty bears to the total period of active duty that I agreed to serve...

The Applicant understands and agrees that discharge in bankruptcy under Title 11, United States Code, if less than 5 years after the last day of the specified period of active duty, will not release Applicant from the obligation to reimburse the United States as provided in this contract.

Unlike a service academy cadet, a ROTC scholarship recipient incurs a service obligation--or its financial substitute--from the moment he receives any Government support.

In reviewing the hard-out policy at the Air Force Academy prior to August 1970, the GAO Report [1976, p.15-17] provided a final piece of evidence that voluntary attrition behavior is affected by attrition penalties. The Superintendent during the period from 1965 to 1970 reportedly made it extremely difficult to resign prior to October of the freshman year. According to the Report, "Even after October of the first year, we were told, 'resignation was a time-consuming process involving considerable counseling by psychiatrists, officers, and senior cadets, and some potential dropouts were thus discouraged.' The consequences of the hardout policy on first-summer attrition are indicated in Chart 9. Whether this policy reduced voluntary attrition or simply delayed it, the policy clearly altered cadet attrition behavior.
AIR FORCE ACADEMY
FIRST SUMMER ATTRITION

PERCENT ATTRITION

"HARD-OUT" POLICY

"EASY-OUT" POLICY

ACADEMIC YEAR


Source: GAO Report

Chart 9
Given the apparent responsiveness of cadet attrition behavior to incentives, we allow in our model for the possibility that—when attrition occurs either before the end of the hardout period or after the beginning of the commitment period—cadets' payoffs are reduced.

Having reviewed those empirical findings which influenced our modeling choices, we turn to a description of our model.
IV. MODEL DESCRIPTION

The cadet attrition model computes the attrition behavior which self-interested students would adopt in response to any specified Academy environment and in addition reports statistics summarizing this behavior. The model consists of two parts: a model of student attrition behavior and a characterization of Academy selection policy.

A. THE BEHAVIORAL MODEL

The behavioral model calculates the optimal decisions of high school seniors in each ACACOMP group. Each student must decide: 1) whether to set up an interview with the Academy, 2) whether to apply given the interview, 3) whether—if accepted and enrolled—to continue beyond BCT given the additional information acquired during pretraining and 4) whether to attrite or continue at the end of each semester's marking period given the current cumulative GPA and outside alternatives. The first two of these decisions determine the composition of the applicant pool while the last two determine the magnitude and timing of voluntary attrition.

Each cadet is regarded as facing a sequential decision problem under uncertainty. Information acquired at the interview and in the first months at the Academy is regarded as providing a way to reduce the uncertainty. As a simplification, it is assumed that this information-processing activity (modeled here as Bayesian learning) ends before receipt of the first report card. At that point, uncertainty about the Academy is resolved but some (possibly reduced) uncertainty about the match with the Air Force may remain. In what follows, we first describe the cadet’s decision problem and then turn to its solution.¹

¹A comprehensive but elementary exposition of such "Bayesian decision problems" may be found in Stokey and Zeckhauser [1978, p.201-54]; a more advanced treatment is located in Raiffa [1970].
The Decision Problem

Prior to entry, each high school senior must decide whether to go to an interview with an Air Force Academy Liaison Officer (LO) and, if so, whether to apply to the Academy. Subsequent to entry, each cadet must decide---after BCT and after receipt of each semester's grades---whether to drop out or whether to continue. The point in time when a decision must be made is referred to as a "decision point" or "stage." There are, therefore, eleven stages where binary decisions must be made: two prior to entry and nine subsequent to entry. Chart 10 depicts this sequence of decision points.\(^2\)

At the time a student must make a decision, certain information will be available to him. The cadet may want to take this information into account when comparing the value of attriting at the current stage to the expected value of continuing to the next stage. Although there are only eleven decision points in the model, a wide variety of situations (referred to as "states" or "decision nodes") may occur at each stage. Hence, there is not merely one potential binary decision to be made at each of the eleven stages but many---one for each of the situations which might at that point potentially confront the cadet.

For example, the cadet making an attrition decision after receipt of his third semester's grades might base his choice on his interview, his BCT experience, each of his prior report cards, and the best outside opportunity available at that point. He could certainly observe all of this information; and some of it would be likely to affect his comparison of the expected value of attriting at that stage to the expected value of continuing to the next stage. Suppose when the third marking period is reached, one of a thousand situations must then prevail. To fully specify a cadet's behavior at that stage would require a description of what he would do in each of these thousand potential situations.

\(^2\)When the model is estimated a third stage prior to entry should be added. At this stage, students who have been accepted decide whether or not to go to the Academy. See p. ___.
PRE-ENTRY STUDENT DECISIONS IN MODEL

GO TO INTERVIEW? → INTERVIEW → DIGEST INFO. APPLY? → ADMISSIONS/ENTRY

Chart 10a
POST-ENTRY STUDENT DECISIONS IN MODEL

BASIC CADET TRAINING → DIGEST INFO, CONTINUE? → INVOLUNTARY ATTRITION OR CONTINUE AFTER 1ST GRADES? → INVOLUNTARY ATTRITION OR CONTINUE AFTER 2ND GRADES? → INVOLUNTARY ATTRITION OR CONTINUE AFTER 3RD GRADES? → INVOLUNTARY ATTRITION OR CONTINUATION

Chart 10b
At least in principle, a student's attrition behavior can be characterized by a description of whether he would attrite or continue in each situation which might confront him at each of the eleven stages.

Given any such specification of student behavior, the ultimate outcome (how, when, and in what state the process terminates) would then depend entirely on the realized path of situations which arises. The student is assumed to be able to assess the probability attached to each path and to assign to each a number or payoff indicating how he values that path. Any specification of student behavior therefore induces a probability distribution across the various payoffs. The hypothesis that each student behaves in a self-interested manner then translates into the hypothesis that he adopts whatever behavior maximizes his expected payoff.

The larger the number of potential situations which the cadet might confront at each stage, the more difficult it is to compute self-interested behavior. As a practical matter, therefore, a priori modeling choices are customarily made about the relevance of available information to the decision at each stage. Distinguishable situations which nonetheless would likely always result in a common decision are aggregated together in an attempt to cope with the "curse of dimensionality." We have assumed, for example, that the cadet's attrition decision at a given point is based on his cumulative GPA at that point rather than on a more refined description of his grade history. Moreover, we have grouped the continuum of possible GPA averages at any point into seven grade intervals. Finally, we have also assumed that the student's interview and BCT experiences each resulted in one of two reactions (positive or negative).

The cadet's decision problem has the following structure. At each of the eleven stages, the cadet finds himself in one of a finite number of situations. If he has just been dismissed, he has no choice to make. If, however, he has not just been dismissed he can choose whether or not to continue. If he does not continue he receives an expected payoff and the process terminates. If he does continue, he transits stochastically to the next stage, where he again finds himself in one of a finite number of situations... The process terminates in one of three ways:
1) voluntary termination, 2) dismissal, or 3) graduation. Payoffs depend on which of these three types of termination occurs, its time of occurrence, and the state at the time of termination. The cadet's decision problem prior to and subsequent to the start of academics is depicted in Charts 11 and 12.

As Chart 11 indicates, there are three decision points in the model prior to the start of academics. At these points the prospective cadet must decide: 0) whether to go to an interview, 1) whether to apply, and 2) whether— if admitted— to continue beyond BCT. These decisions are denoted DZERO, DONE, and DTWO. At the time the application decision must be made, the cadet has been to the interview and can base his choice on his impressions. Similarly at the time the cadet must decide whether to continue on and receive his first report card, he has information both from his interview and from his BCT experience. There are, therefore, two potential situations confronting him at the application stage and four immediately following BCT. Hence, there are seven potential situations in which he will be called to make a decision: DZERO, DONE(1), DONE(2), DTWO(1), DTWO(2), DTWO(3), and DTWO(4). Each of these seven decision nodes is represented in the decision tree by a square. Following the cadet's decision, the process transits stochastically to some state at the next stage. That is, following his decision, some event occurs which the cadet regards as beyond his control. Such transitions occur at "chance nodes," which are represented in the decision tree by circles.

The cadet's decision problem after the start of academics is too complex to depict in its entirety. Instead, Chart 12 depicts the choice which would confront the cadet at a "representative" decision node.

Since for each of seven marking periods there are seven GPA states in which a decision is required (the other two GPA categories represent involuntary dismissals) and since each is characterized by one of four end-states to the learning process and one of three realizations of a "transient disturbance" there are $7 \times 7 \times 4 \times 3 = 588$ decision nodes of the

---

3 Since the current GPA at the time of a dismissal is seldom computed, the payoff following a dismissal is assumed to depend on GPA in the prior marking period.

4 This notation is used in most treatments of decision problems.
Decisions About Joining the Applicant Pool and Continuing Beyond BCT
(All 7 Decisions Prior to Receipt of Grades are Represented)

Chart 11
Voluntary Attrition Decision After Receipt of Grades
(One of the 588 Decision Nodes of This Type)

State of the System:
- Semester = $t$
- Cumulative GPA = $G$
- Current Disturbance = $\epsilon$
- Signal Pair = $i$ ($i = 1, \ldots, 4$)

$V_i(G, \epsilon)$

$d_i(G, \epsilon) = 1$
Continue to semester $t + 1$

$d_i(G, \epsilon) = 0$
Voluntarily attrite

$\tilde{U} + \theta(G - \bar{G})$ - Hardout or Commitment-Point Penalty if Either is Applicable

*Note: In output the single column "value of attritlng" is reported assuming $\epsilon = 0$.
To compute the corresponding columns if $\epsilon = +a$ ($\epsilon = -a$) add (subtract) $a$ from each entry.
type represented in Chart 12.

The idea of a "transient disturbance" term merits discussion. It is intended to capture those transient events which alter the value of outside opportunities but which only the cadet observes: changes in family responsibilities, illnesses, accidents, exceptional job offers, unprecedented transfer opportunities, and so forth. Although it is assumed that such shocks are unobserved by the analyst it is still possible to estimate the common distribution from which they are drawn by using detailed longitudinal attrition records of individual cadets."

The disturbance distribution is included in simplified form here in the simulation model merely to illustrate where it enters the cadet's calculations. Its real use is in estimation. When estimating the model, it is recommended that this "dummy" distribution be replaced by one which allows for more realizations. As discussed in Section VI, this requires only a trivial modification which will not increase computational difficulties.

• Solution to the Decision Problem

To determine optimal behavior given the decision problem sketched in Charts 11 and 12, information is required about various

-- Probability Distributions;
-- Commitment and Hardout Policies;
-- Terminal Payoffs; and,
-- Transition Costs.

Table 1 summarizes the requisite information and introduces the notation we will use henceforth. The student confronting the sequential decision problem under discussion is assumed to know all of the information listed. How the analyst can estimate parameters he does not know is discussed in Section VI.

For a brilliant application of this methodology to a closely related model, see Gotz and McCall's [1980, 1984] estimation of their policy model of Air Force officer retention.
Given the information, the student can solve the decision problem as follows. Since he knows that the signals he will observe are correlated with the satisfaction (in monetary equivalents) he will ultimately experience if he graduates from the Academy and becomes an Air Force officer, he will use his observation of the signal pair to revise his expectation about the psychic reward of such a career. Denote this conditional expectation as \( \xi_i(i=1, 4) \). Since the expected monetary compensation of a career as an officer is \( \bar{M} \), for each possible end-state of the learning process (signal pair \( i,i = 1, 4 \)), he expects to receive \( \bar{M} + \xi_i \) in total by graduating.\(^6\) Therefore,

\[
V^i_8(\Delta, \varepsilon) = \bar{M} + \xi_i.
\]

In addition, it is assumed that the last voluntary attrition occurs in January of the senior year.\(^7\)

Given this "terminal condition," the payoff has been specified for the endpoints of every branch of the tree reachable from the seventh marking period. To solve the decision problem, two rules are used repeatedly, starting in June of the senior year and working backwards. The rules permit the valuation of chance nodes (represented in the diagrams by circles) and decision nodes (represented in the diagrams by squares):

\(^6\)This simplification that the payoff to graduation is independent of his overall GPA incorporates what we were told on visits to the Academy about a "clean slate" policy for graduates.

\(^7\)While it might have been preferable aesthetically to permit voluntary attrition in June of the senior year, choosing to leave at that stage would be suboptimal in all of the policy environments we considered. Hence as a practical matter, our convention of treating January of the senior year as the last decision point is innocuous.
Table 1

I. PROBABILITIES

<table>
<thead>
<tr>
<th>Probability</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>PA(j), j = 1, 3</td>
<td>Disturbance probability distribution</td>
</tr>
<tr>
<td>PDF^k(G), k=1,10; G=1,9</td>
<td>Initial grade distribution for k\textsuperscript{th} ACACOMP group</td>
</tr>
<tr>
<td>P_t(G,G'), t=1,7; G=1,7; G'=1,9</td>
<td>Markov probabilities for the t\textsuperscript{th} transition</td>
</tr>
<tr>
<td>P_1</td>
<td>Probability of a positive first signal (at interview)</td>
</tr>
<tr>
<td>P_2</td>
<td>Probability of a positive second signal (at BCT) given a positive first signal (at interview)</td>
</tr>
<tr>
<td>P_3</td>
<td>Probability of a positive second signal (at BCT) given a negative first signal (at interview)</td>
</tr>
</tbody>
</table>

II. COMMITMENT AND HARDOUT POLICIES

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>j_{COM} (= 0, 1, ..., 9)</td>
<td>The earliest semester when the commitment obligation applies; j_{COM} = 9 means no commitment obligation is in effect</td>
</tr>
<tr>
<td>j_{HO} (= -1, 0, ..., j_{COM})</td>
<td>The last semester when the hardout penalty applies; j_{HO} = -1 means no hardout policy is in effect</td>
</tr>
<tr>
<td>γ_{COM}</td>
<td>The commitment penalty</td>
</tr>
<tr>
<td>γ_{HO}</td>
<td>The hardout penalty</td>
</tr>
</tbody>
</table>

\[
PLTY(t) = \begin{cases} 
γ_{COM} & \text{if } t \geq j_{COM} \\
γ_{HO} & \text{if } t \leq j_{HO} \\
0 & \text{otherwise} 
\end{cases}
\]
III. UNDISCOUNTED TERMINAL PAYOFFS

\[ \bar{U}_k \]
Payoff to high school senior in the \( k^{th} \) ACACOMP group who chooses the best alternative to the Air Force Academy.

\[ \bar{U}_k - \text{PLTY}(0) \]
Payoff to freshman in the \( k^{th} \) ACACOMP group who attrites following BCT.

\[ \bar{U}_k - \text{PLTY}(1) - X_j \ (j = 1, 2) \]
Payoff to freshman in the \( k^{th} \) ACACOMP group who is dismissed for academic \((j = 1)\) or nonacademic \((j = 2)\) reasons in January of the freshman year.

\[ \bar{U}_k + \theta(G_t - \bar{G}_k) - \text{PLTY}(t) + \varepsilon \]
Payoff to cadet in the \( k^{th} \) ACACOMP group who attrites in the \( t^{th} \) semester with GPA \( G_t \) and disturbance \( \varepsilon \); \( \bar{G}_k \) is the mean initial grade of this ACACOMP group and \( \theta \) is a parameter.

\[ \bar{U}_k + \theta(G_t - \bar{G}_k) - \text{PLTY}(t + 1) - X_j \ (j = 1, 2) \]
Payoff to cadet in the \( k^{th} \) ACACOMP group who is dismissed for academic \((j = 1)\) or nonacademic \((j = 2)\) reasons in the \( t + 1^{st} \) semester.

\[ \bar{M} + \xi_i \]
Payoff to graduation: monetary reward \( (M) \) plus the monetary equivalent \( (\xi_i) \) of the satisfaction of an Air Force career expected conditional on signal pair \( i \) \((i = 1, 4)\).

\[ \beta \]
The factor used to discount payoffs.

IV. TRANSITION COSTS

\[ B^i(t, G) \]
Psychic reward or cost to cadet of one more semester at Academy given GPA \( G \) in marking period \( t \) after signal pair \( i \).

\[ \text{CST} \]
The cost of enduring BCT.
V. OPTIMAL DECISIONS* (1 MEANS CONTINUE; 0 MEANS STOP)

DZERO  
Go to interview

DONE (1)  
Apply given positive signal at interview

DONE (2)  
Apply given negative signal at interview

DTWO (1)  
Continue past BCT given two positive signals

DTWO (2)  
Continue past BCT given mixed signals--the negative one at BCT

DTWO (3)  
Continue past BCT given mixed signals--the positive one at BCT

DTWO (4)  
Continue past BCT given two negative signals

\[ d_t^i (G, \varepsilon), t = 1, 7; G = 1, 7; \varepsilon = -a, o, +a \]

VI. VALUES*  

\[ V \]
Value of the optimal strategy discounted to beginning of the program

\[ V^i \]
Value of reaching first marking period (discounted to that point) and proceeding optimally given signal path i

\[ V_{t}^{i}(G, \varepsilon) \]
Value of reaching \( t^{th} \) marking period (discounted to that point) and proceeding optimally given signal path i, grade G, and disturbance \( \varepsilon \)

*The superscript \( k \) (to denote the \( k^{th} \) ACACOMP group) should be attached to each decision and value in the table but has been omitted to simplify the notation.
At each chance node nearest the terminus, compute the probability-weighted average of the payoffs of its branches. Enter the computed expected value in the circle on the decision-tree.

At each decision node nearest the terminus, pick the branch with the larger payoff (net of any transition costs) and enter its associated net payoff (denoted below as $V_t^i$) in the square on the decision-tree.

Having evaluated these nodes, we are then in a position to repeat the process at the two types of nodes at the previous stage and so on... Eventually, the first semester of the freshman year is reached and a value ($V_t^i(G, \xi)$, for $G = 1, 7$ and $\xi = -a, 0, a$) is assigned to each of the twenty-one decision nodes. The entire process can then be repeated for the three other signal pairs ($i = 1, 4$).

Since the computer model follows precisely this procedure, we can use its output to illustrate the process. Consider in Job 298 how a cadet of quality type 1 on signal path 2 would behave if he reached the fifth semester with $G = 4$ and $\xi = 0$. The data needed to determine $V_5^2(4, 0)$ and $d_5^2(4, 0)$ are summarized in Table 2. If the cadet continues, he pays a transition cost of -5.5 and encounters a chance node. Using the foregoing rule for valuing chance nodes, it is valued at 12.146. If the cadet attrites, he receives a payoff of -986.05 (this negative value reflects the fact that the commitment point has been passed). Since $12.146 - 5.5 > -986.05$, the net expected payoff from continuing exceeds the value of attriting; hence, the cadet would decide to continue: $d_5^2(4, 0) = 1$. Using the foregoing rules, the decision nodes can then be evaluated $V_5^2(4, 0) = 12.146 - 5.5 = 6.646$. These calculations are summarized in Chart 13.

There are twenty other decision nodes for path two, semester five. Each is evaluated using the foregoing procedure to obtain $d_5^2(G, \xi)$ and $V_5^2(G, \xi)$ for $G = 1, 7$ and $\xi = -a, 0, a$. 
Voluntary Attrition Decision After Receipt of Grades: A Numerical Illustration

Source: Job 298, Quality Type 1

State of System:
- Semester = 5
- Cumulative GPA = 4
- Current disturbance = 0
- Signal Pair = 2

Exogenous Data:
- \( \bar{B} \) = 5.5
- \( P_{5}(4, 3) = 0.2 \)
- \( P_{5}(4, 5) = 0.2 \)
- \( P_{5}(4, 6) = 0 \)
- \( P_{5}(4, 7) = 0 \)
- \( P_{5}(4, 8) = 0 \)
- \( P_{5}(4, 9) = 0 \)
- \( \beta = 1 \)
- \( X_1 = 25 \)
- \( X_2 = 50 \)

Data Computed Previously for Subsequent Semester 6

\[
[V(G', e)] = \begin{pmatrix}
1001 & 1001 & 1001 \\
527.1 & 527.1 & 527.1 \\
13.27 & 13.27 & 13.27 \\
18.50 & 18.50 & 18.50 \\
18.50 & 18.50 & 18.50 \\
18.50 & 18.50 & 18.50 \\
\end{pmatrix}
\]

Expected Pseudo Rewards from 3 Possible Career (\( \xi_2 \))

\( \xi_1 = 500 \)
\( \xi_2 = 7.5 \)
\( \xi_3 = 500 \)
Table 3 is a xerox from Job 298 and shows precisely how the results of such calculations appear in the print-out. The location of each number used in the calculation of \( d_5^2(4, 0) \) and \( V_5^2(4, 0) \) is highlighted.

Using the matrix \( V^2(G, \varepsilon) \), the procedure can then be repeated to obtain \( d_4^2(G, \varepsilon) \) and \( V_4^2(G, \varepsilon) \) and so on... Ultimately the \( 7 \times 3 \) matrix, \( V_1^2(G, \varepsilon) \) is calculated and the process can be repeated for the other three signal pairs \( (i = 1, 4) \).

The entire sequence of value functions \( \{V_t(G, \varepsilon)\} \) must (by construction) solve the following functional equation:

\[
V_t^i(G, \varepsilon) = \max \left\{ \left[ \sum_{k=1}^{7} \left( \theta(G_t - \overline{G}_k) - \text{PLTY}(t) - B^i(t, G_t) \right) + \beta \sum_{E} \sum_{G_t^i} P_A(\varepsilon) \sum_{G_t+1} P(G_t, G_{t+1}) V_{t+1}(G_{t+1}, \varepsilon) \right] 
+ \beta \sum_{j=1}^{2} P_t(G_j, j + 7) \left[ \sum_{k=1}^{7} \left( \theta(G_t - \overline{G}_k) - \text{PLTY}(t + 1) - X_j \right) \right] \right\}
\]

for \( i = 1, 4 \)

\( G = 1, 7 \)

\( \varepsilon = -a, 0, +a \)

\( t = 1, 7 \)

and \( k = 1, 10 \)

where \( V_8^i(G, \varepsilon) = \overline{N} + \xi_1 \).

We now use \( V_1^i(G, \varepsilon) \), which we have obtained for signal path \( i \), to compute the expected value--on that path--of drawing a grade and disturbance in January of the freshman year. The value of this chance node is simply

\[
V^i = \left[ \sum_{E} P_A(\varepsilon) \sum_{G=1}^{7} \text{PDF}_k^2(G) V_1^i(G, \varepsilon) \right] + \left[ \sum_{j=1}^{2} \text{PDF}_k^2(j + 7) \left( \sum_{k=1}^{7} \left( \theta(G_t - \overline{G}_k) - \text{PLTY}(t + 1) - X_j \right) \right) \right].
\]

---

*Whether or not PLTY(\( t + i \)) is deducted from the payoffs of students who are dismissed (as in the functional equation in the text) is controlled by an input parameter IPUN. If IPUN = 1, the penalty is imposed on dismissed students; if IPUN = 0, it is not imposed on them.*
### Table 3

<table>
<thead>
<tr>
<th>VALUE ARRAY</th>
<th>VALUE OF ATTRITION</th>
<th>TRANSITION COST</th>
<th>DISCOUNTED VALUE OF CONTINUING</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.1001E+04</td>
<td>-0.1001E+04</td>
<td>0.5500E+01</td>
<td>-0.1026E+04</td>
</tr>
<tr>
<td>-0.5271E+03</td>
<td>-0.9960E+03</td>
<td>0.5500E+01</td>
<td>-0.5216E+03</td>
</tr>
<tr>
<td>-0.1327E+02</td>
<td>-0.9910E+03</td>
<td>0.5500E+01</td>
<td>-0.7767E+01</td>
</tr>
<tr>
<td>0.1850E+02</td>
<td>-0.9860E+03</td>
<td>0.5500E+01</td>
<td>0.2400E+02</td>
</tr>
<tr>
<td>0.1850E+02</td>
<td>0.1850E+02</td>
<td>0.5500E+01</td>
<td>0.2400E+02</td>
</tr>
<tr>
<td>0.1850E+02</td>
<td>0.1850E+02</td>
<td>0.5500E+01</td>
<td>0.2400E+02</td>
</tr>
<tr>
<td>0.1850E+02</td>
<td>0.1850E+02</td>
<td>0.5500E+01</td>
<td>0.2400E+02</td>
</tr>
</tbody>
</table>

DECISION ARRAY FOR PSI(2) = 7.500 PERIOD = 5 FOR QUALITY TYPE = 1

<table>
<thead>
<tr>
<th>0</th>
<th>0</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
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<td>1</td>
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<tr>
<td>1</td>
<td>1</td>
<td>1</td>
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</tbody>
</table>

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<th>TRANSITION COST</th>
<th>DISCOUNTED VALUE OF CONTINUING</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.1001E+04</td>
<td>-0.1001E+04</td>
<td>0.5500E+01</td>
<td>-0.1026E+04</td>
</tr>
<tr>
<td>-0.6279E+03</td>
<td>-0.9960E+03</td>
<td>0.5500E+01</td>
<td>-0.6224E+03</td>
</tr>
<tr>
<td>-0.6379E+02</td>
<td>-0.9910E+03</td>
<td>0.5500E+01</td>
<td>-0.5829E+02</td>
</tr>
<tr>
<td>0.6646E+01</td>
<td>-0.9860E+03</td>
<td>0.5500E+01</td>
<td>0.1215E+02</td>
</tr>
<tr>
<td>0.1300E+02</td>
<td>-0.9810E+03</td>
<td>0.5500E+01</td>
<td>0.1850E+02</td>
</tr>
<tr>
<td>0.1300E+02</td>
<td>-0.9760E+03</td>
<td>0.5500E+01</td>
<td>0.1850E+02</td>
</tr>
<tr>
<td>0.1300E+02</td>
<td>-0.9710E+03</td>
<td>0.5500E+01</td>
<td>0.1850E+02</td>
</tr>
</tbody>
</table>
Transition Cost: -5.5

\[ v^2_{4,0} = 6.646 \]

\[ d^2_{4,0} = 1 \]

Discounted Value of Continuing:

\[ (0.2)(-13.27) + (0.6)(18.50) + (0.2)(18.50) = 12.146 \]

Value of Attriting:

\[ 25 + 5(4 - 6.21) - 1000 + 0 = -986.05 \]

Implied Optimal Decision and Its Value

\[ d^2_{4,0} = 1 \]

\[ V^2_{4,0} = 6.646 \]
Exogenous Data

\[ PA(j) = \frac{1}{3}, \ j = 1, \ldots, 3 \]
\[ U_j = 25 \]
\[ X_1 = 25 \]
\[ X_2 = 50 \]

Commitment Point = 4
Commitment Penalty = -1,000
Hardout Point = -1
Hardout Penalty = -10,000

\[ PDF^1(1) = .0047 \]
\[ PDF^1(2) = .0094 \]
\[ PDF^1(3) = .00141 \]
\[ PDF^1(4) = .0329 \]
\[ PDF^1(5) = .1174 \]
\[ PDF^1(6) = .3474 \]
\[ PDF^1(7) = .4507 \]
\[ PDF^1(8) = .00470 \]
\[ PDF^1(9) = .0188 \]

Data Calculated Previously for Quality Type 1, Signal Pair 2

\[
\begin{pmatrix}
0.049 & 0.049 & 0.049 \\
3.951 & 3.951 & 3.951 \\
8.951 & 8.951 & 8.951 \\
13.95 & 13.95 & 13.95 \\
18.95 & 18.95 & 18.95 \\
23.95 & 23.95 & 23.95 \\
28.95 & 28.95 & 28.95 \\
0 & 0 & 0 \\
-25 & -25 & -25 \\
\end{pmatrix}
\]

Implied Expected Value of Receiving Initial Grades

\[ V^2 = \sum_{j=1}^{3} \sum_{o=1}^{3} PA(j) PDF^1(G) V^i(G, o) = 23.62651 \]
This value should be entered in the circle on the left of Chart 14. The computation can then be repeated for the other signal paths to obtain $V^i$ for $i = 1, 4$.

Returning to the illustration of Job 298, the information relevant to computing $V^2$ for quality type one is summarized in Table 4. The computed value of this chance node is: $V^2 = 23.62651$. These calculations are reported in the computer output following the period one decision array for quality type one on path two. Table 5 is a xerox from Job 298 and shows precisely how such calculations appear in the print-out.

The computer calculated $V^2$ as $0.2363E+02$, which agrees with our computations. Similar calculations—for the other signal paths—are reported after period one is reached for the particular path.

This information is then summarized after the fourth path has been evaluated for the quality type under the heading "$V$-Values for Continuing Beyond BCT." In Job 298, these values are

$$V^1 = .417E+03 \quad V^2 = .236E+02 \quad V^3 = .417E+03 \quad V^4 = .236E+02$$

Note that in this particular run, $V^1 = V^3$ and $V^2 = V^4$. This is a consequence of two input assumptions for Job 298:

- The signal at the LO interview (unlike the BCT signal) was assumed uninformative—that is, it provided the student with no information about the psychic reward of an Air Force career. Hence, $\xi_1 = \xi_3$ and $\xi_2 = \xi_4$.
- Neither signal was regarded as influencing the cost of transiting from one semester to the next ($B^i_t(G)$ was assumed independent of $i$; in fact, it was constant independent of the stage or GPA).

To complete the analysis, the computed $V^i$ values are entered at the tips of the decision tree in Chart 15. The reader should now be able to calculate for this decision tree the value of each of the seven decision nodes, the value of each of the three chance nodes, and
Expected Value of Receiving Initial Grades (and Continuing Optimally) Given Signal Pair i \((i = 1, \ldots, 4)\)

Chart 14
Table 5

| DECISION MATRIX FOR PS1(2) = 1.500 PERIOD = 1 FOR QUALITY TYPE = 1 |
|---------------------------------|---------------------------------|-------------------------------|-------------------------------|
| VALUE ARRAY                     | VALUE OF ATTRITING              | TRANSITION COST               | DISCOUNTED VALUE OF CONTINUING |
|                                  |                                |                               |                               |
| -0.1049E+01                     | 0.3951E+01                     | 0.9500E+01                    | 0.3494E+01                    |
| 0.3951E+01                      | 0.3951E+01                     | 0.3951E+01                    | 0.3951E+01                    |
| 0.8951E+01                      | 0.8951E+01                     | 0.8951E+01                    | 0.8951E+01                    |
| 0.1395E+02                      | 0.1395E+02                     | 0.1395E+02                    | 0.1395E+02                    |
| 0.1895E+02                      | 0.1895E+02                     | 0.1895E+02                    | 0.1895E+02                    |
| 0.2395E+02                      | 0.2395E+02                     | 0.2395E+02                    | 0.2395E+02                    |
| 0.2895E+02                      | 0.2895E+02                     | 0.2895E+02                    | 0.2895E+02                    |

EXPECTED INITIAL VALUES TAKEN OVER DISTURBANCES

| INITIAL GRADE AND ATTRITION PROBABILITIES |
|---------------------------------|---------------------------------|-------------------------------|-------------------------------|
| 0.4700E-02                      | 0.9400E-02                     | 0.1400E-02                    | 0.3200E-02                    |

EXPECTED VALUE OF ENTERING ACADEMICS FOR QUALITY TYPE 1 + PS1(2)

0.2363E+02
Decisions About Joining the Applicant Pool and Continuing Beyond BCT: A Numerical Illustration

Source: Job 298
Exogenous Data
\[ G = 0.25 \]
\[ P_1 = 0.5 \]
\[ P_2 = 0.8 \]
\[ P_3 = 0.8 \]
\[ CST = +1 \]
\[ \beta = 1 \]
Commitment Point = 4 (attrition after end of sophomore year incurs an obligation)
Commitment Penalty = -1,000
Hardout Point = -1 (there is no hardout policy in effect)
Hardout Penalty = -10,000

Data Calculated Previously for Quality Type 1
\[ V_1' = 417 \]
\[ V_2 = 23.6 \]
\[ V_3 = 417 \]
\[ V_4 = 23.6 \]

Consider AFA? 
Interview 
Apply? 
BCT 
Continue to 1st Grades? 

Implied Optimal Decisions
\[ DZERO = 1 \]
\[ DONE(1) = 1 \]
\[ DONE(2) = 1 \]
\[ DTWO(1) = 1 \]
\[ DTWO(2) = 0 \]
\[ DTWO(3) = 1 \]
\[ DTWO(4) = 0 \]
the optimal decision at each of the seven decision nodes. The answers are printed in smaller type on Chart 15. These same calculations are reported in the computer output after the fourth signal path for the particular quality type. Table 6 is a xerox of the relevant part of the printout and indicates how these calculations are reported.

The last decision calculated is the first decision the high school senior will face: whether even to consider the Air Force Academy. If the expected value of going to the LO interview is lower than the "Value of the Best Non-AF Career Alternative," the cadet will not pursue the Academy. Otherwise, he will go to the interview.

This completes the solution to the decision problem of the cadet of a particular quality type.

B. Selection by the Academy

Our computerized model contains a rudimentary characterization of the Academy selection process. The user must designate whether the Academy's goal is--on average--(1) to admit a specified number of students or, alternatively, (2) to graduate a specified number of students. The model then solves the foregoing optimization problem for cadets in the highest ACACOMP group and computes the expected number of entrants and the expected number of graduates for that group. If the expected number falls short of the target, the computer repeats the procedure for the next highest ACACOMP group... As soon as the target is exceeded, the model selects from the last ACACOMP group considered enough students that the expected number of entrants (or, alternatively, graduates) equals the specified target.\footnote{For further details, see p. __.}

In fact, the Academy does not select its candidates in this way. Instead, a complex process is used involving Congressional and Presidential nominations. Such nominees are often carefully selected and of high quality; nonetheless, the Academy is occasionally forced to admit candidates with lower ACACOMP scores when students with higher ACACOMP scores are available. To this extent, the actual admissions process differs from the one we have modeled.
INITIAL GRADE AND ATTITUDES PROBABILITIES

<table>
<thead>
<tr>
<th>Probabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4700E-02</td>
</tr>
<tr>
<td>0.9400E-02</td>
</tr>
<tr>
<td>0.1410E-02</td>
</tr>
<tr>
<td>0.3290E-01</td>
</tr>
<tr>
<td>0.1174E+00</td>
</tr>
<tr>
<td>0.3474E+00</td>
</tr>
<tr>
<td>0.4507E+00</td>
</tr>
<tr>
<td>0.4700E-02</td>
</tr>
<tr>
<td>0.1880E-01</td>
</tr>
</tbody>
</table>

EXPECTED VALUE OF ENTERING ACADEMICS FOR QUALITY TYPE 1 \( \psi(4) \)

\[ \psi \] = 0.2363E+02

STAY/LEAVE AFTER ACT DECISION FOR THE FOUR SIGNAL PAIRS

<table>
<thead>
<tr>
<th>V-VALUES FOR CONTINUING PAST BCT</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.417E+03 0.236E+02 0.417E+03 0.236E+02</td>
</tr>
</tbody>
</table>

VALUE OF DROPPING AFTER BCT

0.2500E+02

OPTIMAL DECISIONS/VALUES FOR APPLYING GIVEN (+), (-) SIGNALS

<table>
<thead>
<tr>
<th>1 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.338E+03 0.338E+03</td>
</tr>
</tbody>
</table>

VALUE OF NOT APPLYING GIVEN TO INTERVIEW

0.2500E+02

LO INTERVIEW DECISION/VALUE

<table>
<thead>
<tr>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.338E+03</td>
</tr>
</tbody>
</table>

VALUE OF BEST NON AF CAREER ALTERNATIVE

0.2500E+02
The existing process does, however, given the Academy some freedom of action and it is our understanding that this freedom is used to select applicants with the highest ACACOMP scores. Hence, "at the margin," the actual selection process does resemble the one we have modeled.

As a consequence, the two processes respond in identical ways to marginal changes in policy. For example, an expansion of the academy will--under either process--require the Academy to "dig deeper down in the barrel." As a consequence, the ACACOMP score of the marginal cadet will fall.

Moreover, it may be possible to "trick" our submodel of the selection process into mimicking a given admissions policy.

If, for example, the Academy selects no one from designated ACACOMP groups, the user can input to the model that there exist no seniors in such groups. Alternatively, if the Academy is forced to select a limited number of students (S₁) from designated ACACOMP groups the required number of students in each group can be inputted as belonging to the three highest ACACOMP groups (but assigned their actual $\bar{U}_k$ and PDF $^k(G)$). Our admissions submodel will then give them priority over candidates who in fact have higher ACACOMP scores.

In short, while the selection process we have modeled differs from the actual process used by the Academy, it does respond in the same way to policy changes and, in addition, can often be tricked into mimicking reality more closely. While developing a more complex submodel of the selection process might ultimately prove worthwhile, we felt our time was better spent on other aspects of the problem.
V. POLICY SIMULATIONS

For a model to be useful, it should capture the essential aspects of the process under consideration without being so complex that its behavior in response to policy changes is no more comprehensible than the process it is intended to illuminate. We believe our model meets this standard. Below, we review several policy simulations which illustrate its usefulness.

* Data Inputs

Although we lack the data to estimate many parameters in the model, we have incorporated available Academy data wherever possible.

Students are divided into the following ten ACACOMP intervals:

<table>
<thead>
<tr>
<th>Quality Type</th>
<th>ACACOMP Interval</th>
<th>Percentage of High School Seniors</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3400+</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>3240+ - 3400</td>
<td>1.5</td>
</tr>
<tr>
<td>3</td>
<td>3140+ - 3240</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>3040+ - 3140</td>
<td>1.5</td>
</tr>
<tr>
<td>5</td>
<td>2940+ - 3040</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>2840+ - 2940</td>
<td>4</td>
</tr>
<tr>
<td>7</td>
<td>2740+ - 2840</td>
<td>4</td>
</tr>
<tr>
<td>8</td>
<td>2640+ - 2740</td>
<td>9</td>
</tr>
<tr>
<td>9</td>
<td>2540+ - 2640</td>
<td>7</td>
</tr>
<tr>
<td>10</td>
<td>0 - 2540</td>
<td>65</td>
</tr>
</tbody>
</table>

The percentage of high-school seniors in each ACACOMP category (the right-hand column) could not be determined directly since some students did not take all of the tests aggregated into the ACACOMP score. Instead, the missing information was "constructed" using prior academic records (PAR)—a variable highly correlated with ACACOMP.

The GPA intervals were defined as follows:

<table>
<thead>
<tr>
<th>Grade</th>
<th>GPA Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0 - 1.75</td>
</tr>
<tr>
<td>2</td>
<td>1.75+ - 2</td>
</tr>
<tr>
<td>3</td>
<td>2+ - 2.25</td>
</tr>
<tr>
<td>4</td>
<td>2.25+ - 2.45</td>
</tr>
</tbody>
</table>
5 2.45+ - 2.75
6 2.75+ - 3.40
7 3.40+
8 academic dismissal
9 nonacademic dismissal

Using these definitions and data for the first two years' performance of the Class of 1985, an initial grade distribution \( PDF^k(G) \) for \( k = 1, 10 \) and \( G = 1, 9 \) was estimated for each ACACOMP group. After a careful review, it was assumed that the stochastic process governing subsequent transitions of the cumulative GPA did not differ significantly across ACACOMP categories. Accordingly, a common sequence of Markov transition matrices \( P_t(G, G') \) for \( t = 1, 8; G = 1,7; \) and \( G' = 1, 9 \) was estimated from data on the partial grade histories of the Class of 1985. While these estimates are rough and could undoubtedly be refined if more complete data were made available, they do at least insure that both the grade evolution and the involuntary attrition of each ACACOMP group in the Class of 1985 are appropriately represented in the model. These estimated probabilities are hardwired into the code. Since they are echoed in the output, they will not be reported here. Most of the other parameters were assigned judgemental (i.e. arbitrary) values pending estimation of the model (see Section VI). In general, we tried to simplify the remaining inputs so that the simulation results would be easier to interpret. Thus, for example, the disturbance term is set equal to zero, the outside opportunities of different ACACOMP groups are assumed identical, the LO interview is assumed not to affect expectations about the value of a career of an Air Force officer, and so forth. A complete description of the data inputs is reported in Appendix __; the output of each simulation is available upon request.

* Illustrative Simulations

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1 These data were incomplete since they stopped in June of their sophomore year.
Policy changes at the Academy may affect (1) who applies; (2) who is selected; and (3) how those selected behave after entry. The least complicated policies to analyze have only a single effect; the most complicated have all three effects. When a policy has multiple effects, it is useful to distinguish (1) the effects if only the applicant pool had been altered; (2) the effects if only the selection of entering classes had been altered; and (3) the effects if only the voluntary attrition behavior of the cadets had changed. The aggregate effect is then approximated by the sum of the three individual effects. For simplicity, we begin with policy changes which have a single effect and then consider more complex cases.

- Academy Expansion

In the late 1960's, the Air Force Academy began to increase the number of freshmen admitted and soon doubled the size of the student body. The principal effect of this policy change was to alter the composition of the entering class. To simulate the effects of such a change, we specify various target numbers of entrants and simulate the consequences if no other inputs changed. Table 6 reports the results:

Table 7

<table>
<thead>
<tr>
<th>Target Number of Entrants</th>
<th>Simulated Attrition</th>
</tr>
</thead>
<tbody>
<tr>
<td>800</td>
<td>34.82</td>
</tr>
<tr>
<td>1000</td>
<td>36.24</td>
</tr>
<tr>
<td>1200</td>
<td>37.64</td>
</tr>
<tr>
<td>1400</td>
<td>38.77</td>
</tr>
<tr>
<td>1600</td>
<td>40.55</td>
</tr>
</tbody>
</table>

Doubling the size of the entering class raises the simulated attrition rate by sixteen percent. Expansion affects attrition behavior (and also cost and quality of graduates) because it alters the ACACOMP composition of the entering class. In expanding, the Academy admits at the margin students with lower ACACOMP scores. As illustrated previously in Chart 4, such students have higher attrition rates.

\(^2\)To a first approximation, the expansion is assumed to reduce
The hypothesis that attrition at the Air Force Academy was adversely affected by the substantial expansion which began in the late nineteen sixties seems to us both plausible and worthy of further investigation. All that is required is data on the attrition rate of each ACACOMP group (such as that reported in Chart 4) and a detailed understanding of who would have been selected had the Academy sought to limit freshman classes to their former sizes. To illustrate, suppose that if the Academy had not expanded, only the 800 students in the Class of 1985 with the highest ACACOMP scores would have been admitted. Since the attrition rate of these students was only 27%, the rate for that Class would have been 20% lower than was observed. Since some of the poorer students would presumably have been forced on the Academy by the actual nomination process, this calculation overstates the adverse consequences of the expansion. But the actual effects can be more accurately assessed by using a more refined description of who would have been selected if class sizes had not increased. An analogous procedure could be used to determine the effects of the expansion on cost, quality, and other variables of policy relevance. This information would permit a clearer distinction between the effects of the substantial expansion and the effects of other policy changes which have occurred since the late nineteen sixties.

- Changes in the Commitment Point

Since admissions policies affect only the composition of an entering class rather than the incentives of students, they can be analyzed using a variety of models besides our own. Our model is indispensible, however, in predicting the consequences of policies which alter the Academy environment (the GAO term quoted on the coversheet) and through it student behavior. Such policies can, in principle,

neither the rewards of being a cadet \( (B(t,G)) \) nor the rewards of being a graduate of the Academy \( (\bar{H} + \xi) \).

Surprisingly, although it was the sharp increase in attrition at the Air Force Academy during the early 1970s which prompted the GAO Report on the five service academies, this otherwise invaluable analysis makes no mention of the potential effects of the substantial expansion there.
induce changes in 1) the applicant pool, 2) Academy selection, and 3) cadet behavior. To illustrate, we discuss in this subsection the simulated effects of moving the commitment point. For simplicity, the input parameters were set so that no changes in the applicant pool were induced. Hence only two effects of the policy change are considered in this subsection: the "behavioral effect" and the "selection effect." Discussion of the applicant-pool effect will be deferred until the next subsection.

The selection effect arises because the model was instructed in each simulation to admit enough applicants to achieve an unchanged target number of graduates (1000 graduates on average). The selection effect magnifies the behavioral effect on attrition of any policy change. If, for example, a policy change would raise the attrition rate of a given set of cadets, then more cadets would have to be admitted to maintain the same expected number of graduates. Since the additional students would have lower ACACOMP scores, admitting them would cause a secondary rise in the aggregate attrition rate. Decreases in the attrition rate would be magnified in an analogous manner.

Table 7 reports the results of simulations in which the commitment point was moved in one-semester increments from the entry point (t=0) to graduation (t=8) and then removed entirely (t=9). As the table reflects, such policy changes are expected to influence the class size at entry, the attrition rate, the mean length of spells ending before graduation, the mean ACACOMP scores of entrants and of graduates, and the cost per thousand graduates.

The effect of commitment point changes on the attrition rate is plotted in Chart 16. The attrition rate is lowest (12.44%) when the commitment obligation is incurred at entry. As the commitment point is moved further toward graduation, the attrition rate rises sharply and then falls slightly (to 30.96%). The simulated rate associated with the current policy of commitment at the 4th marking period is 32.15%. Ironically, because of the U-shaped curve, the attrition rate could be reduced by moving the commitment point in either direction although--in the absence of applicant-pool effects--substantially larger improvements in attrition, cost, and quality could be achieved by requiring commitment at entry.
Table 8
SIMULATED EFFECTS OF MOVING THE COMMITMENT POINT

<table>
<thead>
<tr>
<th>Commitment Point</th>
<th>Class Size</th>
<th>Attrition Rate</th>
<th>Mean Spell Length (in semesters)</th>
<th>Mean ACACOMP of Entrants</th>
<th>Mean ACACOMP of Graduates</th>
<th>Cost in Man-Semesters</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1142</td>
<td>12.44</td>
<td>3.388</td>
<td>3276</td>
<td>3281</td>
<td>8481</td>
</tr>
<tr>
<td>1</td>
<td>1448</td>
<td>30.93</td>
<td>1.460</td>
<td>3234</td>
<td>3240</td>
<td>8654</td>
</tr>
<tr>
<td>2</td>
<td>1617</td>
<td>38.14</td>
<td>1.035</td>
<td>3208</td>
<td>3226</td>
<td>8638</td>
</tr>
<tr>
<td>3</td>
<td>1503</td>
<td>33.47</td>
<td>1.257</td>
<td>3225</td>
<td>3234</td>
<td>8633</td>
</tr>
<tr>
<td>4</td>
<td>1473</td>
<td>32.15</td>
<td>1.384</td>
<td>3229</td>
<td>3237</td>
<td>8655</td>
</tr>
<tr>
<td>5</td>
<td>1467</td>
<td>31.76</td>
<td>1.412</td>
<td>3230</td>
<td>3237</td>
<td>8658</td>
</tr>
<tr>
<td>6</td>
<td>1464</td>
<td>31.64</td>
<td>1.444</td>
<td>3231</td>
<td>3238</td>
<td>8669</td>
</tr>
<tr>
<td>7</td>
<td>1448</td>
<td>31.00</td>
<td>1.419</td>
<td>3234</td>
<td>3240</td>
<td>8637</td>
</tr>
<tr>
<td>8</td>
<td>1448</td>
<td>30.96</td>
<td>1.442</td>
<td>3234</td>
<td>3240</td>
<td>8646</td>
</tr>
<tr>
<td>9</td>
<td>1448</td>
<td>30.96</td>
<td>1.442</td>
<td>3234</td>
<td>3240</td>
<td>8646</td>
</tr>
</tbody>
</table>
SIMULATED EFFECT ON ATTRITION OF CHANGING COMMITMENT POINT

Attrition Rate

32.15

12.44

0 1 2 3 4 5 6 7 8 9
date of commitment

Chart 16
The attrition paths for individual ACACOMP groups are also \( \Omega \)-shaped. Two opposing forces account for the shapes of these paths. To understand these forces, consider the consequences of postponing the commitment point. The decision nodes in the model can then be partitioned into three sets: 1) those nodes following the later commitment point; 2) those nodes preceding the earlier commitment point; and 3) those nodes in between. Chart 17 is a representation of these three sets of nodes. For concreteness, assume the commitment point is moved from the fourth to the fifth semester. No decision rules would change from the fifth semester onward since the cadet would at those points face the same payoffs under either policy. At every decision node prior to the fourth semester, however, the increased flexibility of a later service obligation would (weakly) increase the value of continuing at the Academy while leaving unchanged the payoff to dropping out. Consequently, the cadet might decide to continue in some situations where he formerly would have attrited. If this were the only force at work, it would reduce the attrition rate. However, the postponement of the commitment point would also mean that a cadet would no longer incur a service obligation by attriting in the fourth semester. Consequently, he might decide to attrite at some of those decision nodes in the fourth semester where he formerly would have continued—a force tending to increase the attrition rate. One implication of this analysis is that a marginal postponement of the commitment point from an initial position at entry must increase the attrition rate. This follows because there are then no attrition decisions preceding the earlier commitment point and hence only the latter force discussed above is present.

It has not been established for all parameter values that the attrition path of an ACACOMP group must be single-peaked or that it must achieve its minimum when commitment occurs at entry; nonetheless, our simulations all had these characteristics. In any case, the simulations do clearly demonstrate that the relationship between the commitment point and the attrition rate need not be monotonic. Hence, even though the Coast Guard Academy reportedly experienced some reduction in attrition when it moved its commitment point from the junior year to
EFFECT OF POSTPONING COMMITMENT ON INDIVIDUAL'S DECISION TO LEAVE

Chart 17
graduation as did West Point when it moved its commitment point to the senior year, nonetheless larger reductions might have been achieved by moving these commitment points in the opposite direction.  

Besides the attrition rate, movements in the commitment point also influence other variables of policy relevance. Charts 18 and 19 depict the simulated effects on the quality of graduates (measured by their mean ACACOMP score) and the cost of producing them (measured in terms of man-semesters per thousand graduates). Policy changes which increase the aggregate attrition rate induce the Academy to admit additional students. As a result, the mean ACACOMP score of the graduates falls. The quality path therefore decreases whenever the aggregate attrition path increases (and vice versa) as Charts 16 and 18 illustrate. Thus, policies which reduce the aggregate attrition rate simultaneously improve the quality of graduates. In the absence of applicant-pool effects, commitment at entry not only secures the lowest attrition rate but also the highest quality graduates.

The cost consequences of changes in the commitment point are illustrated in Chart 19. Costs are reported in terms of the man-semesters required per thousand graduates and must therefore equal or exceed 8000 man-semesters. If costs exceed this benchmark, the interaction of two factors must be responsible: the magnitude of attrition and the mean length of spells ending in attrition. Any policy change which reduces the number of attritees by some percentage but at the same time increases the mean length of their spells by a larger percentage will increase costs. Consequently, as a comparison of Charts

---

4Interview (July 29, 1984) with John D. Pinto, a former instructor at the Coast Guard Academy and letter (January 19, 1984) from Carlton E. Bacon, Director of Institutional Research at the USMA. According to the latter, "During the 1970's we experienced many resignations during the summer between the sophomore and junior years from cadets who were not ready to face a commitment to complete USMA vis-a-vis active duty enlisted service. Now cadets are more willing to stick with USMA through the junior year and, based on our data, are graduating instead of resigning."

5This improvement is magnified if the policy change happens in addition to selectively deter voluntary attrition since involuntary attrition tends to be more specific than voluntary attrition in weeding out cadets with lower ACACOMP scores.
SIMULATED EFFECT ON QUALITY OF CHANGING COMMITMENT POINT

Quality of Graduate

Chart 18
16 and 19 illustrates, changes in the commitment point which reduce attrition do not always reduce costs. Nonetheless in our simulations commitment at entry results in the lowest costs as well as the lowest attrition and the highest quality graduates.

- Penalizing Early Attrition

The foregoing simulations suggest that—in the absence of applicant-pool effects—penalizing early attrition may be beneficial from a number of standpoints. One way to impose such penalties is by moving the commitment point to entry. Alternative policies exist, however, which are superior. Suppose, for example, that the commitment point remained at the fourth marking period but that a penalty of size \( x \) was imposed for earlier attrition. The proposed penalty could be a service obligation the length of which is increasing in \( x \) or a monetary penalty of \( x \) dollars. It is treated as a monetary penalty in the discussion below.

If \( x \) is set equal to the monetary equivalent of the current obligation to serve on active duty in enlisted status, the new policy will have exactly the same effects as setting the commitment point at entry. Hence the new policy must be at least as effective. If, on the other hand, \( x \) is set equal to zero, the new policy is equivalent to the current practice of commitment after the fourth semester. Intermediate values of \( x \) will result in a policy which is less harsh than commitment at entry but shares many of its attributes.

Chart 20 reports the simulated effect on the dropout rate of increasing the penalty \( (x) \) for early attrition.\(^6\) The attrition rate

---

\(^6\)The simulations were run by retaining the commitment obligation from the fourth semester onward but adding a hardout penalty of variable size \( (x) \) for attrition up to and including the third semester. Simulating the consequences of leaving a "loophole" by ending the hardout penalty in the second semester instead is also instructive. The availability of the loophole stimulates attrition in the third semester as is evident from the drop in the third-semester continuation rates of each ACACOMP group. The current policy, which contains a two-year grace period before any attrition penalty is imposed, can be regarded as a loophole of quadruple the size.
SIMULATED EFFECT ON COST OF CHANGING COMMITMENT POINT

Cost in Man-Semesters

8655

8481

0 1 2 3 4 5 6 7 8 9 date of commitment

Chart 19
falls monotonically from 32.15% (the rate previously associated with a fourth semester commitment point) to 12.44% (the rate previously associated with commitment at entry).

A warning sign is also implicit in Chart 20. In those simulations where the magnitude of the penalty for early attrition was set too high, there were not enough applicants to produce the desired number of graduates. More generally, policies which change the environment of the Academy may alter the applicant pool. Such "applicant-pool effects" did not arise in the previous simulations because of the input parameters chosen.

In general, as the penalty (x) for early attrition is increased, the maximized value (V) of attending the Academy will fall and those ACACOMP groups who were previously on the verge of choosing their next best alternative instead of applying will withdraw from the applicant pool. These are likely to be precisely the cadets who previously dropped out as soon as new information, a better job, or a favorable transfer opportunity came along. The removal of these weakly-attached cadets is likely to lower the attrition rate since their choice of the non-Academy alternative prior to entry removes them from the attrition statistics.

That some cadets enter the Academy with plans to leave soon after seems evident from survey information in the GAO Report. Chart 21 indicates the responses of first summer dropouts and "continuing" students to a question about their plans at that early point to transfer. Assuming an attrition rate at BCT of 6%, these responses imply that more than 53% of those students indicating that there is "a very good chance" they will transfer do in fact leave before the end of the first summer; some of the others may leave subsequently. It would be extremely difficult for the admissions office at the Academy to identify these weakly-attached students from the information they provide on their applications. Nonetheless, even a very modest penalty for early attrition will remove from the applicant pool those most

---

7That is, they will either not go to the interview or will not apply afterwards when they previously would have. Formally, one or both of the products DZERO DONE(1) and DZERO DONE(2) change from one to zero.
VARIATIONS IN PENALTY FOR EARLY ATTRITION

ATTRITION RATE

32.15
EQUIV. TO COMMITMENT AFTER 4TH SEMESTER

12.44
EQUIV. TO COMMITMENT AFTER ENTRY

INSUFFICIENT APPLICANTS

MAGNITUDE OF PENALTY

Chart 20
MANY EARLY DROPOUTS PLAN IN ADVANCE TO LEAVE*

"VERY GOOD CHANCE WILL TRANSFER"

35% OF 1ST SUMMER DROPOUTS

2% OF CONTINUING STUDENTS

PLANS REPORTED UPON ENTRY

FIRST-SUMMER ATTRITION RATE FOR ENTRANTS EXPRESSING PLANS TO TRANSFER: 53%

SOURCE: GAO REPORT

Chart 21
likely to transfer.

For a sufficiently large penalty, however, there is a risk that high quality students will also cease to apply and the Academy will be forced to admit lower quality applicants to achieve its target number of graduates. Chart 22 illustrates how the path in Chart 20 would likely change in the presence of applicant-pool effects. As the penalty is increased, the weakly-attached students withdraw from the applicant pool, reducing the attrition rate. Hence, the applicant-pool effect would reinforce the behavioral and selection effects considered previously. Eventually, however, students with high ACACOMP scores withdraw from the applicant pool and the applicant-pool effect works in the opposite direction. Since the attrition rate no longer falls monotonically as the penalty for early attrition is increased, care must be exercised in setting the size of the penalty. While a modest penalty (e.g. $5000) for early attrition is unlikely to deter high quality applicants who wish not only to enter but also to complete the program at Academy, consideration of larger penalties should probably be deferred until the model is estimated.

Once estimated, the model can also be used to evaluate the quantitative effects of other penalty structures. The GAO Report [1976, p. 63], for example, proposed an interesting alternative to the two-step penalty function we have been discussing:

One alternative which could reduce attrition involves establishing a more gradual buildup of commitment rather than forcing a decision at the beginning of the second-class year. This buildup could be accomplished by making academy attendance a financial obligation which must be repaid if the student does not complete the program. This system would be similar to the proposed conversion of Reserve Officer Training Corps scholarships to loans for dropouts of that program...

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*Chart 22 is not based on actual simulations.

*The increase in the attrition rate would be discontinuous because of the assumption that ACACOMP scores vary discretely rather than continuously. Since any continuous distribution can be approximated in this way, our simplification is innocuous.
INDUCED CHANGES IN APPLICANT POOL AND THEIR PROBABLE CONSEQUENCES

Attrition Rate

Deterrence and Selection Effects

Favorable Applicant-Pool Effect

Unfavorable Applicant-Pool Effect

Size of Penalty

Chart 22
This GAO proposal may be generalized as follows: cadets who attrite before the commitment point would pay a financial penalty which starts at \( a_o \) and rises linearly at rate \( s \) per semester; once the commitment point is reached at semester \( t \), the active duty obligation would replace this financial penalty. By setting the three parameters \((a_o, s, t)\) appropriately, any of the policies we have so far considered can be reproduced. Hence, this policy must be at least as good as the best of them.

Historically, the academies have set \( a_o \) and \( s \) at zero and have varied only \( t \). It seems intuitively clear, however, that setting a strictly positive \( a_o \) is preferable since such a penalty would induce weakly-attached applicants to self-select themselves out of the applicant pool. Similarly, setting a strictly positive \( s \) would induce students whose eventual departure becomes a certainty to attrite sooner --thus lowering costs. These changes need not deter desirable candidates from applying since the commitment point could be postponed (\( t \) increased) in an offsetting way. Once the model is estimated, it can be used to determine quantitatively the best of this flexible class of attrition policies.\(^{11}\)

We have focused attention in this section on movements in the commitment point and alternative policies which penalize attrition. We have done so because the Academy expressed interest in this issue. Had the Academy asked instead about the consequences of some other policy--such as providing high school seniors more accurate information about the Academy--we would have constructed essentially the same model but in this section would have reported on entirely different simulations. We conclude the section with a reminder: the model is very versatile and--when estimated--can be used to evaluate quantitatively the consequences of a wide variety of proposed changes in Academy policy.\(^{12}\)

\(^{10}\)We have so far assumed that the Air Force prefers lower to higher attrition, lower to higher costs, and higher to lower quality graduates. Unless guidance is provided about "tradeoffs" among these three goals, however, no policy can be considered "best." Nonetheless, some policies are clearly superior (technically, "Pareto superior") to others since they would induce an outcome which is simultaneously an improvement in terms of attrition, cost, and quality.

\(^{11}\)A trivial change in the code (i.e. in PLTY(t)) would be required before the model could be used to evaluate this generalization of the GAO proposal.

\(^{12}\)In principle, the model could also be used to forecast the
policies which each of the various service academies would adopt in the long run assuming each competes for the same pool of high school seniors and pursues its own goals. Rules to restrain such interservice competition could then be evaluated. This analysis would use the methodology of "industrial organization" to determine the benefits of cooperation in industries with a small number of firms.
VI. READYING MODEL FOR THE ESTIMATION

Estimation of this model will not be an easy task, but with sufficient ingenuity (and appropriate data) it can be accomplished. An essential preliminary step is to reduce the number of parameters involved. To accomplish this, parameter estimates should be incorporated wherever possible from other sources and "inessential" parameters in the model should be eliminated. Below, we specify how the number of parameters can be reduced and then outline how the remainder of them can be estimated.

The initial grade distribution for each ACACOMP group (PDF\(_k\)(G)) and the sequence of transition matrices (\(P_t(G, G')\)) can be estimated independently using data on grade histories.

The joint (and hence the derived conditional) probabilities (\(P_1, P_2, P_3\)) that the signal pair at the interview and BCT falls in one of the four mutually exclusive and exhaustive categories can be obtained from LO interview reports, BCT evaluations, and psychological tests administered to incoming cadets.

In our simulation model, the expected reward from graduating (\(M + \xi_i\) for \(i = 1, 4\))--conditional on the four signal pairs--is computed from eleven more fundamental parameters. These eleven parameters indicate the value of three possible outcomes subsequent to graduation (H, L, M) and the eight joint probabilities (and hence the derived conditional probabilities \(P_4\) through \(P_{11}\)) that either H or L occurs in conjunction with one of the four signal pairs.\(^1\)

To estimate the model, we suggest eliminating these eleven parameters, and specifying \(\xi_i\) (\(i = 1, 4\)) directly. This constitutes a net reduction of seven in the number of parameters to be determined.

\(^1\)For details, see p. ___
Furthermore, we suggest that $\bar{M} + \xi_i$ ($i = 1, 4$) be estimated separately using the Gotz-McCall [1984] model on Air Force officer retention. Their model computes the expected value—both monetary and subjective—of a career as an Air Force officer and can be applied to distinct groups of Academy graduates. A group would be distinguished by classifying of its interview and its BCT experience. This approach would eliminate five more parameters ($\bar{M}$ and four $\xi_i$) from further consideration.

Finally, we suggest that the four-parameter probability distribution of the disturbance term $(a, PA(j)$ for $j = 1, 3)$ be replaced by a one-parameter distribution. This will permit the elimination of three other parameters (and will also permit greater flexibility in estimation).

To accomplish this, let $Pr(\epsilon)$ be the new distribution with $\sigma$ its one parameter. Assume $E(\epsilon) = 0$ as before. In the final decision period ($t = 7$), determine for each $G$ the largest disturbance consistent with retention by the Academy and denote this borderline number as $\epsilon^i_7(G)$. That is,

$$d^i_7 (G, \epsilon) = \begin{cases} 
0 & \text{for } \epsilon > \epsilon^i_7(G) \\
1 & \text{for } \epsilon \leq \epsilon^i_7(G) 
\end{cases}$$

Since $\epsilon$ may take on a large (possibly infinite) set of values, it is no longer practical to print out or store the matrix $d^i_t(G, \epsilon)$. Nor is it necessary. Instead, the seven-component column vector $(\epsilon^i_t(G))$ completely characterizes the optimal decision in each state. Hence, for $t = 7$ the computer would compute and store $\epsilon^i_7(G)$.

Analytically, $V^i_7(G, \epsilon)$ is simply:

$$V^i_7(G, \epsilon) = \begin{cases} 
\bar{U}_k + \theta(G_t - \bar{G}_k) - \text{PLTY}(7) + \epsilon & \text{if } \epsilon^i > \epsilon^i_7(G) \\
\beta \sum_{j=i}^{2} P^i_7(G_7, j+7)[U_k + \theta(G_j - \bar{G}_k) - \text{PLTY}(8) - X_j] & \text{if } \epsilon \leq \epsilon^i_7(G) 
\end{cases}$$

Since $V^i_t(G, \epsilon)$ is no longer small enough to store in its entirety, we
instead extract from it in advance the information we will subsequently need: the expected value of entering stage 8:

\[ E_\varepsilon[V_7(G, \varepsilon)] = \Pr(\varepsilon > \varepsilon_7(G)) \cdot \left[ \bar{U}_k + \theta(G-G_k) - \text{PLTY}(7) + E(\varepsilon | \varepsilon > \varepsilon_7(G)) \right] + \sum_{\varepsilon \leq \varepsilon_7(G)} \Pr(\varepsilon) V_7^1(G, \varepsilon). \]

Since \( V_7(G, \varepsilon) \) is independent of \( \varepsilon \) for \( \varepsilon \leq \varepsilon_7(G) \) (denote it \( V_7(G) \)), the last term in the foregoing sum is simply \( V_7(G) \cdot \Pr(\varepsilon \leq \varepsilon_7(G)) \).

Hence, in general, the proposed modification requires the calculation of \( \varepsilon_t(G) \), \( \Pr(\varepsilon > \varepsilon_t(G)) \) and \( E(\varepsilon | \varepsilon > \varepsilon_t(G)) \) (for \( t = 1, 7 \); \( i = 1, 4 \); and \( G = 1, 7 \)). The \( 7 \times 3 \) decision matrix is replaced by a \( 7 \times 1 \) column vector (\( \varepsilon_t(G) \)) indicating the magnitude of the borderline disturbance and the \( 7 \times 3 \) value matrix is replaced by a \( 7 \times 1 \) column vector (\( E_\varepsilon[V_t(G, \varepsilon)] \)) for \( G = 1, 7 \).²

A disturbance term of this form could also usefully be included at the two-pre-entry decisions of whether to go to the interview and whether to apply. In any case, it must be included at the third pre-entry decision (omitted in the current version of the model) of whether to enroll if admitted. Otherwise, every student who applied would be predicted to enroll if accepted.³ With these minor changes, the recursion of the model would proceed exactly as in the existing code.

The following parameters remain to be estimated: \( \sigma, \beta, \theta, U_k, X_1, B^i(t, G) \) and CST. Of these, the last three seem the least important and could provisionally be set equal to zero if further reductions in the number of parameters are required. Finally, by choosing a sample period where no hardout policy is in effect, we can set \( \gamma_{HO} \) to zero. There remain, therefore, \( k + 4 \) parameters to be estimated (\( \sigma, \beta, \theta, \gamma_{COM} \) and \( U_k \)).

²A minor modification must also be made in the calculation of continuation rates. See footnote 1.
³In fact, we understand that roughly 80% of admitted students enroll. The remainder—in this formulation—draw disturbances making another alternative more attractive.
These parameters can be estimated by maximum likelihood using our model. The "sample" would consist of longitudinal records on individual students. Each record would classify his interview and BCT experience and would include his cumulative GPA path and the time and manner of termination (with graduation, academic dismissal, nonacademic dismissal, or voluntary attrition). Our model could then be used to calculate the joint probability—conditional on the vector of unknown parameters—that a sample would behave in the observed way. The parameters would then be varied to maximize the likelihood function.

The estimation procedure outlined here was developed and implemented by Gotz and McCall [1984] for their optimal-stopping model of Air Force officer retention. The success of their project and the similarity of their model to ours leads us to believe that our model can be estimated once the relevant data are secured. After this is accomplished, the predictions of our model can be assessed and the model used to predict behavioral responses to policy-induced changes in the Academy environment.
Commitment Obligations by Class

The service obligations of dropouts from various classes (1959 - present) were as follows:

--Early classes 1959-1962.

No commitment or obligations. Had to sign statement promising to complete program.

--Class 1963

Obligation to serve in active or reserve military status for 6 years from date of entry to Academy.

Upon resignation transferred to ready reserve in an enlisted grade.

--Class 1964 to 1971

If discharged from Academy will be transferred to Air Force reserve in an enlisted grade to complete total of 6 year obligation.

--Classes 1972 to 1979

Discharged in 4th or 3rd class year, no active duty obligation incurred.

Discharged in 2nd class year, active duty obligation for not more than 2 years.

Discharged in 1st class year, active duty obligation for not more than 4 years (classes 72 to 77) or not more than 3 years (classes of 78 and 79).

Refuse to accept commission upon graduation, obligation of 4 years of active duty.

--Classes of 80 to 82

Discharged in 4th, 3rd, or 2nd class years, no active duty obligation incurred.

Discharged in 1st class year or refuse commission upon graduation, obligation of 3 or 4 years on active duty, respectively.

--Classes of 83 and following
Same rules as used for classes of 78 and 79.

A distinction must be drawn between the self-interested behavior of (1) students who correctly anticipate from the outset that the Academy will follow a given policy and (2) students who are surprised subsequent to entry by an unanticipated change in policy. For example, when the Class of 1983 entered the Academy, it was assured the commitment point would be the beginning of the senior year—as it had been for the three previous classes. However, during the summer before the start of their junior year, members of that class received the following memo:

AF/MP

Change in the Air Force Active Duty Service Commitment Policy
(AF/MP Ltr, 15 Dec 80)

USAFA/SUPT

1. The Assistant Secretary of the Air Force (Manpower, Reserve Affairs and Installations) has approved establishing an active duty service commitment for cadets at the beginning of their second class year, effective with the Class of 1983. Second class cadets who disenroll after the start of academics, either voluntarily or involuntarily, will be transferred to the reserve in the appropriate enlisted grade and will normally be called to active duty in an enlisted status for a period of two years. Exceptions will be made for humanitarian reasons and those few cases in which it is not in the best interest of the Air Force to call a cadet to active duty who is physically disqualified, unfit, or unsuited for military service in an enlisted status.

2. This letter supersedes AF/MP letter, dated 15 Dec 80, subject as above, with the exception of paragraphs 2 and 3 therein, which remain in effect.

A similar modification of the estimation procedure (see Section VI) is required if data from the Class of 1983 is used to estimate the parameters of the model.
REFERENCES


Porazzo, E. M., "Give Us the Man and We'll Give You the Boy," Penthouse Magazine, October 1979, pp. 118-119.


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