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TIME CONSISTENT TAXATION BY A GOVERMENT WITH REDISTRIBUTIVE GOALS*

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ABSTRACT

In a dynamic economy whose government is interested in both equity and efficiency, time consistency problems arise even if the government has access to nondistortionary tax instruments. Moral hazard in production leads to a nondegenerate distribution of income, which the government would like to "flatten" ex post. Self-enforcing social agreements can mitigate the tendency toward excessive redistribution. We investigate the nature of the distortions caused by the time consistency problem, and show that in the constrained-optimal equilibrium, usually a *linear* tax schedule is imposed. This remains true if renegotiation of the social agreement is possible.

1. INTRODUCTION

This paper studies the nature and consequences of time consistent taxation in a society whose government is concerned with both efficiency and equity. In the early work of Fisher (1980) on dynamic tax policy, problems of time consistency arise only because instruments such as poll taxes are for some reason not available to the government. While it is unattractive to rule out poll taxes and other nondistortionary taxes on an ad hoc basis, one might conjecture that even a government with unrestrictive tax instruments would face costly time consistency problems if the government had *redistributive goals* (and hence found poll taxes unattractive). We show that this conjecture is correct, and examine the structure of constrained efficient equilibria in which the government's behavior is disciplined by its desire not to lose the public's confidence.[1] An unexpected result is that *linear* tax schedules often play a prominent role in constrained efficient equilibria. Toward the end of the paper the effects of the possibility that the government and the public will "renegotiate" the terms of the implicit agreement they share are considered.

The economy described in Section 2 is perhaps the simplest model of taxation in which there is a conflict between efficiency and equity, and in which there is scope for certain self-enforcing promises by the government to arise in equilibrium. All individuals are identical and risk-averse. In each of a countable infinity of periods, competitive risk-neutral firms bid for the services of each worker, who enters into a principal-agent relationship (with moral hazard) with the firm. There is no borrowing, lending, or saving. At the end of any period the government can confiscate or redistribute income as it chooses; there could be an exogenous revenue requirement, but this is immaterial for our results. The government's rate of time preference is expressed by a discount factor strictly between zero and one. The government seeks to maximize the expected present discounted value of social welfare, which in turn is the utilitarian sum of individual utilities.[2]

The payment schedules (specifying salaries as functions of the observed outcome of a worker's effort) that each principal and agent would agree to in the absence of government intervention represent the optimal (second-best) tradeoff between the need to give the agent

[1] Following the seminal work of Kydland and Prescott (1977) that pointed out the time consistency problem for government policy, many papers have considered reputation as a device for disciplining the behavior of the government. See especially Backus and Driffill (1985), Barro (1986), Barro and Gordon (1983), Chari and Kehoe (1990), Rogoff (1987), and Stokey (1989).

[2] Our model is reminiscent of the economy in Phelan and Townsend (1991), from which it differs in two essential ways: Phelan and Townsend allow for multiperiod contracting, but they do not consider the time consistency of government policy.

incentives to work hard, and the desirability of having the agent bear as little risk as possible. Thus, in any period, the redistribution of income that arises in the economy if no intervention is expected is the best solution to the moral hazard problem associated with production. Ideally, then, the government should not redistribute income at all. But at the end of the period, the government is tempted to equalize individuals' final compensations, since productive decisions for that period are over. If the government is sufficiently patient, the prospect of losing the public's confidence (and hence moving to an inferior equilibrium) suffices to deter it from interfering with the second-best distribution, period by period. For lower values of the discount factor, the time consistency problem "bites": the government wishes it could tie its hands and commit to nonintervention. Since no commitment mechanism is available, the public correctly anticipates that income will be redistributed, and adjust their production contracts accordingly.

Section 3 investigates the distortionary effects of government intervention. Let a^* and c^* denote the agent's second-best action and the accompanying consumption schedule. In principle, the distortion might involve a^* being supported by a less efficient schedule c', or a different action a' being supported. We show that the former case cannot occur: if there is an inefficiency loss due to the government's inability to commit to nonintervention, it *always* results in the implementation of an action different from a^*. Under substantial but reasonable regularity conditions, this action, while not the second-best, is supported in a constrained efficient way. In other words, the imposition of a tax schedule distorts the choice of productive action but not the way the firm induces the worker to take that action. Moreover, the tax schedule imposed in each period of the best equilibrium is *linear* (but not proportional).

Section 4 briefly states some "strategic dynamic programming" results for this economy that are straightforward analogs of propositions in Abreu, Pearce and Stacchetti (1990) concerning "self-generating sets" and supergame equilibria. This enables us in Section 5 to apply to the strategic economy the definition of renegotiation-proof equilibrium suggested by Pearce (1987). Regardless of the value of the discount factor, it turns out that typically the second-best action a^* cannot be sustained in equilibrium. The results of Section 3 on the efficacy of linear tax schedules continue to hold for renegotiation-proof equilibria.

Section 6 presents an example whose best equilibria are computed for various parameter values, with and without renegotiation. Section 7 contains concluding remarks.

2. THE DYNAMIC ECONOMY

The economy is comprised of a continuum of firms and a continuum of identical individuals, who own the firms. In each period $t = 0, 1, \ldots$, a firm hires the services of individuals for that period. The firm and each worker enter into a standard principal-agent relationship with moral hazard: the firm does not observe the agent's effort (or action) a, but promises a salary $s(\omega)$ that depends on a signal ω that is publicly observable and stochastically related to effort. Let $A \subset \mathbf{R}$ be the action space and $\Omega \subset \mathbf{R}$ be the signal space, a finite set, and assume that the signal ω has density $f(\omega \mid a)$ and c.d.f. $F(\omega \mid a)$. There are no interaction effects among workers: the signals ω for different workers in a firm are stochastically independent (for any actions chosen) and a worker's contribution $p(\omega)$ to the firm's gross profits depends only on the realization of his own signal ω. For each $a \in A$, let

$$P(a) := \sum_{\omega \in \Omega} p(\omega) f(\omega \mid a)$$

be the expected contribution to the firm's gross profits of a worker taking action a.

A worker's utility function is time-separable. His utility $U(c, a)$ in period t depends on his consumption c and the action a he takes at t. Each worker maximizes expected utility. Firms maximize the present discounted value of expected net profits according to a discount factor $\delta \in (0, 1)$. This rate of time preference is also shared by the government and the workers. No one in this economy (workers, firms, or government) can borrow, lend or save. The government is benevolent and utilitarian: it seeks to maximize the discounted sum of the average utilities of individuals. The government's primary policy instrument is a (potentially redistributive) tax imposed on individual incomes at the end of each period, before consumption occurs. Although consumption cannot be negative, a worker's salary $s(\omega)$ may be negative for some signals (that is, the worker is required to make a payment to the firm). This is possible only to the extent that the government is willing to make a transfer to the worker of at least the size of the required payment.[3]

Accordingly, at the beginning of each period, the government announces a "minimum wage" $\underline{s} \leq 0$ to be respected by all firms. Each firm j simultaneously offers a salary

[3] Another source of the worker's payment to the firm could be his income from profits, but as we shall see, in equilibrium each firm's realized profits will be zero. For this reason it has not been necessary to specify the precise pattern of ownership of firms.

contract $s_j : \Omega \to [\underline{s}, \infty)$. Each worker then chooses which contract to accept and selects an action from the set A. Once the signals are realized, the government implements an income tax which effectively redistributes incomes. Formally, it chooses a redistribution function $R : [\underline{s}, \infty) \to \mathbf{R}_+$. Thus a worker with signal ω and contract s will enjoy consumption $R(s(\omega))$.

Let individuals be indexed by the interval $[0, 1]$. The index set is endowed with the Lebesgue measure. In equilibrium we will require that the set of workers hired by any active firm is uncountable, although of measure 0; the informal interpretation is that there are many more workers than firms. Thus, we can assume (see Judd (1985)) that if all individuals in a firm take action a, then with probability 1, the fraction of them who receive a signal less than or equal to ω is $F(\omega \mid a)$. This implies that there is no uncertainty in the firm's profits (or, for that matter, in the net revenues of the government). Moreover, the lack of aggregate uncertainty means that in a symmetric equilibrium, the payoff of the government in any period coincides with the expected utility of an individual.

We make the following assumptions:

(A1) $A = [0, 1]$.

(A2) $U : \mathbf{R}_+ \times A \to \mathbf{R}$ is continuous, and $U(c, a)$ is strictly concave and increasing in c, and decreasing in a.

(A3) $P : A \to \mathbf{R}$ is continuous, and $P(0) > 0$.

(A4) Ω is a *finite* subset of $\mathbf{R}$, $f(\omega \mid a)$ is continuous in a, and $f(\omega \mid a) > 0$ for all $\omega \in \Omega$ and $a \in A$.

(A5) $U(P(a), a) > U(P(0), 0)$ for all $a \neq 0$.

Assumption (A1) is made for convenience; we could also easily work with a discrete action space. Assumption (A2) introduces risk aversion and regularity conditions that, along with (A3) and (A4), guarantee the existence of constrained optimal contracts. Requiring $P(0) > 0$ ensures that it is feasible to give workers positive consumption levels. The continuity of $f(\omega \mid a)$ implies that $f(\omega \mid a)$ is uniformly bounded away from 0; this avoids existence problems of the kind illustrated by the famous example of Mirrlees (1974). Assumption (A5) says that if income were redistributed evenly, 0 would be the least socially desirable action. It is used later in this Section in the characterization of worst credible equilibria.

All actors in the economy except the government are "strategically anonymous". By this we mean that firms act competitively, and that the government does not react to changes in contractual arrangements involving only a set of workers of measure 0. Consequently, it is optimal for each worker to optimize "myopically" period by period, because an individual's choice of contract is effectively unobserved by the government.

We wish to analyse solutions to the dynamic economy that are analogous to public equilibria (Fudenberg, Levine and Maskin (1990)) of extensive games. In the pure strategy symmetric equilibria we are interested in, the *same* contract governs the relationship between all firms and workers (even in periods where deviations from equilibrium have occurred in earlier periods). Consequently, for the purposes of checking incentives for conformity to equilibrium behavior, it suffices to consider only "contract distributions" in which at most two different contracts have been accepted by (different) workers in the economy. Formally, a *contract distribution* is a finite collection $\{(s^k, q_k) \mid k = 1,...,K\}$, where for each k, s^k is a contract, $q_k \geq 0$, and $\Sigma q_k = 1$. The interpretation is that a fraction q_k of the population has accepted contract s^k. A *public history* for the periods 0 through t is a sequence $h^t = (h_0, ..., h_t)$, where for each τ, h_τ specifies the government's choices $\underline{s}_\tau$ and R_τ and the contract distribution that occurred in period τ.

A *symmetric strategy profile* σ for the economy specifies for every period t and public history h^{t-1},

(1) a minimum salary $\underline{s}_t$.

(2) a contract s_t (which depends on the minimum salary actually announced by the government).

(3) a contract s_t' from the contracts available, and an action a_t. (Both s_t' and a_t depend on the contracts available and $\underline{s}_t$.)

(4) a redistribution function R_t that depends on $\underline{s}_t$ and the contract distribution that arose in period t.

Any symmetric profile σ generates a unique path $\{(\underline{s}_t, s_t, a_t, R_t)\}_{t \geq 0}$. The value of this path for the government is

$$\psi(\sigma) := (1-\delta)\sum_{t=0}^{\infty} \delta^t \Phi(s_t, a_t, R_t), \quad \text{where}$$

$$\Phi(s_t, a_t, R_t) := \sum_{\omega \in \Omega} U(R_t(s_t(\omega)), a_t) f(\omega \mid a_t)$$

is the "indirect utility function".

Denote by $\sigma|_{h^t}$ the symmetric profile induced by σ following public history h^t. We now define an equilibrium of the economy. A verbal discussion follows the formal definition. A symmetric strategy profile σ is an ***equilibrium*** of the dynamic economy if, after any history h^{t-1}, if $H_t = (\underline{s}_t, s_t, a_t, R_t)$ denotes the corresponding symmetric behavior prescribed by σ following h^{t-1}, then

(1) (s_t, a_t) solves the following principal-agent problem:

$$\begin{aligned} &\max_{(s,a)} \Phi(s, a, R_t) \\ &\text{s.t. } \Phi(s, a, R_t) \geq \Phi(s, \alpha, R_t) \quad \text{for all } \alpha \in A \\ &\qquad P(a) - C(s, a) = 0 \\ &\qquad s(\omega) \geq \underline{s}_t \quad \text{for all } \omega \in \Omega, \end{aligned}$$

where $C(s, a) := \sum_{\omega \in \Omega} s(\omega) f(\omega \mid a)$ is a firm's expected salary payment to a worker taking action a under contract s.

(2) For any $\hat{\underline{s}}_t \neq \underline{s}_t$, let $\hat{H} = (\hat{\underline{s}}_t, \hat{s}_t, \hat{a}_t, \hat{R}_t)$ be the symmetric behavior prescribed by σ after h^{t-1} and deviation $\hat{\underline{s}}_t$ by the government, $\hat{h}_t := (\hat{\underline{s}}_t, \hat{s}_t, \hat{R}_t)$, and $\hat{h}^t := (h^{t-1}, \hat{h}_t)$. Then

$$\psi(\sigma|_{h^{t-1}}) \geq (1-\delta)\Phi(\hat{s}_t, \hat{a}_t, \hat{R}_t) + \delta\psi(\sigma|_{\hat{h}^t}).$$

(3) Let $h_t := (\underline{s}_t, s_t, R_t)$, $h^t := (h^{t-1}, h_t)$, and for each $R \neq R_t$, let $\bar{h}_t := (\underline{s}_t, s_t, R)$ and $\bar{h}^t := (h^{t-1}, \bar{h}_t)$. Then

$$(1-\delta)\Phi(s_t, a_t, R_t) + \delta\psi(\sigma|_{h^t}) \geq (1-\delta)\Phi(s_t, a_t, R) + \delta\psi(\sigma|_{\bar{h}^t}) \text{ for all } R \neq R_t.$$

Condition (1) says that, given the redistribution function R_t they expect, firms and workers devise an optimal contractual arrangement. The first constraint in the maximization problem in (1) is the incentive constraint corresponding to the moral hazard problem between firm and worker. The second constraint is a zero expected profit condition that reflects the perfect competition among the firms. The third constraint simply enforces the government's minimum wage law. We implicitly assume that each firm has a capacity constraint that precludes it from hiring a positive measure of workers. Along with the assumption that the government is insensitive to any change in the contractual arrangements of a set of workers of measure zero, this implies that the firm takes R_t as fixed when it decides what contract to offer. Throughout the definition of equilibrium, we simplify by ignoring histories that are irrelevant for checking the intertemporal credibility of the path generated by σ from each period onward. For example, condition (1) does not address the choice problem faced by a worker who is offered an arbitrary array of out-of-equilibrium contracts in period t.

Conditions (2) and (3) require that the government cannot gain by imposing a different minimum salary or redistribution function, respectively. With slightly more cumbersome notation, one could combine (2) and (3), exploiting the fact that nothing happens between the government's choice of R_t and its choice of $\underline{s}_{t+1}$. For a related reason, it turns out to be unneccesary to check for (2) explicitly. Why is this the case? As in Abreu (1988), it suffices to consider only "severest credible punishments" for deviations from equilibrium behavior. We show next that there is a severest punishment (for an unexpected choice of redistribution function) in which firms and workers always expect the government will henceforth flatten salaries completely, and consequently workers choose $a = 0$ (exert minimum effort). In this regime, which we call the "no trust equilibrium", it makes no difference for average utility what $\underline{s}$ is chosen by the government: firms all offer the same constant contract specifying $s(\omega) = \max\{P(0), \underline{s}\}$ for each ω. Think now about the government's problem on the original equilibrium path (not the punishment path). If it is tempted to deviate from $\underline{s}_t$, say, it might as well flatten the income distribution at the end of period $t-1$ as well (the no trust equilibrium will ensue one way or the other). But the second part of this two-part deviation will then be pointless, as just explained, so the deviation is worthwhile only if its first part is worthwhile on its own. The latter is guarded against by (3) alone.

It is easy to see that the so-called "no trust equilibrium" is indeed an equilibrium. If firms and workers anticipate a completely egalitarian redistribution of incomes, implementing $a = 0$ is the rational response. The government, in turn, has no reason to resist flattening the income distribution, because it has already lost its reputation. Notice

that this equilibrium yields the government a payoff of $U(P(0)), 0)$. No other equilibrium is worse for the governemnt, because under any profile and in any contingency, the government can flatten the distribution, thereby attaining a payoff of $U(P(a)), a)$ for that period, where a is the action that was taken. By (A5), $U(P(a)), a) \geq U(P(0)), 0)$ for all a. Henceforth, the no trust equilibrium is the only punishment regime we consider.

Second-Best Equilibria

A useful benchmark is the solution of the one-period principal-agent problem in an environment without government intervention , that is, when $\underline{s} = 0$ and $R = I$, the identity function. For any fixed $a \in A$, consider the agency problem of maximizing the agent's expected utility subject to implementing action a (the incentive constraint (IC)) and giving the firm expected net profit of zero (the budget constraint (BC)). The problem's optimal value is

$$\varphi(a) := \max_{s} \Phi(s, a, I)$$
$$\text{s.t. } \Phi(s, a, I) \geq \Phi(s, \alpha, I) \quad \text{for all } \alpha \in A \qquad \text{(IC)}$$
$$C(s, a) = P(a), \qquad \text{(BC)}$$
$$s(\omega) \geq 0 \quad \text{for all } \omega \in \Omega.$$

Denote by $s^*(\omega, a)$, $\omega \in \Omega$, the optimal contract that implements action a (if a is not implementable, we will make $\varphi(a) = -\infty$, and the optimal contract does not exist). Let $a^* := \operatorname{argmax} \varphi(a)$.[4] We are interested in the nondegenerate case in which $a^* > 0$:

(A6) $\quad \varphi(a^*) > U(P(0), 0) = \varphi(0).$

DEFINITION: *The salary schedule* s budget implements *the action* a *if* (s, a) *satisfies constraints (IC) and (BC).*

As noted in the Introduction, the salary schedule and corresponding action that would be implemented in the absence of government intervention comprise the optimal (second-best) tradeoff between the need to motivate the agent to work hard, and the desirability of having the agent bear as little risk as possible. Thus, in any period the distribution of income that arises in the economy if no intervention is expected is the best solution to the moral hazard problem associated with production. Whether or not the government can credibly promise nonintervention (that is, to make R the identity function) depends on

[4] For simplicity, assume $s^*(\omega, a)$ and a^* are unique.

how patient it is. Consider the strategy profile in which (s^*, a^*) is implemented in every period as long as the government refrains from redistributing income, and in which any redistribution triggers the no trust equilibrium for the remainder of the game. The "trust constraint" for the government is:

$$\varphi(a^*) \geq (1-\delta)U(P(a^*), a^*) + \delta\varphi(0).$$

On the right side, $U(P(a^*), a^*)$ is the current period payoff to the government's best deviation, that is, flattening the income distribution, and $\varphi(0)$ is the supergame payoff in the resulting no trust equilibrium. This is satisfied if and only if

$$\delta \geq \underline{\delta} := \frac{U(P(a^*), a^*) - \varphi(a^*)}{U(P(a^*), a^*) - \varphi(0)}.$$

When $\delta < \underline{\delta}$, the second-best arrangement is not sustainable, but the constrained-optimal symmetric equilibrium is still stationary: along the equilibrium path, the same (s, a) pair is implemented in every period. We omit the argument, which is identical to that given by Abreu (1986) for the stationarity of best strongly symmetric subgame perfect equilibria.

When $\delta < \underline{\delta}$, it is not only the second best way of supporting a^* that violates the trust constraint: no other way of implementing a^* is compatible with the government's incentives either. To see this, notice that the government's payoff from flattening the income distribution depends only on the *aggregate* income to be allocated, which is simply $P(a^*)$ regardless of how a^* was implemented. The payoff to "good behavior" by the government, on the other hand, is lower when a^* is implemented in a way that is inferior to the second-best.

3. The Distortionary Effects of Government Intervention.

We now study the effects of redistributive intervention on the actions chosen in equilibrium, and on the way those actions are implemented by final consumption schedules.

Let $\sigma(\underline{s}, s, a, R)$ be the stationary symmetric profile in which $(\underline{s}, s, a, R)$ occur on the equilibrium path and deviations trigger the no trust equilibrium. A necessary condition for any profile to be an equilibrium is that it satisfy the trust constraint for the government. Thus, in looking for the best equilibrium when $\delta < \underline{\delta}$, an optimistic starting point is to consider a profile of the form $\sigma(\underline{s}, s, \bar{a}, R)$, where $\bar{a}$ solves the optimization problem:

$$\max \varphi(a)$$
$$\text{s.t. } \varphi(a) \geq (1-\delta)U(P(a), a) + \delta\varphi(0).$$

Even if the government can choose $\underline{s}$ and R so that firms and workers respond with s and $\bar{a}$, the final consumption schedule $R(s(\cdot))$ might not equal $s^*(\cdot, \bar{a})$. Then the value of $\sigma(\underline{s}, s, \bar{a}, R)$ would be $\Phi(s, \bar{a}, R) < \varphi(\bar{a})$, and the government trust constraint would be violated. The problem is that in specifying R, the government is trying to distort the choice of action without distorting the way in which the new action is implemented. It is for this reason that it is optimistic to focus on $\bar{a}$.

It turns out that for a wide range of specifications, the action $\bar{a}$ *can* be supported in every period along the equilibrium path with a stationary strategy $\sigma(\underline{\bar{s}}, \bar{s}, \bar{a}, \bar{R})$, where the optimal redistribution function $\bar{R}$ is *linear*, and the contract $\bar{s}$ offered by the firms is such that $\bar{R}(\bar{s}(\omega)) = s^*(\omega, \bar{a})$ for all $\omega \in \Omega$.

For a fixed action a consider the following family of linear redistribution functions parameterized by the slope $m \in (0, \infty)$:

$$L(s, a, m) := ms + (1-m)P(a) \quad \text{if } s \geq \underline{s}_m := -\left(\frac{1-m}{m}\right)P(a).$$

Note that when $m = 1$ the redistribution function becomes the identity I (that is, the government does not intervene). For any contract s with expected cost $P(a)$ when the worker chooses a, the government expects to break even when the workers choose action a. That is, if $c(\omega) = L(s(\omega), a, m)$, $\omega \in \Omega$, then

$$\sum_{\omega\in\Omega} c(\omega)f(\omega \mid a) = P(a) = C(s, a) = \sum_{\omega\in\Omega} s(\omega)f(\omega \mid a).$$

For short, we will write $L_m(s)$ instead of $L(s, a, m)$, when the action a has been fixed.

Fix a budget implementable action a; for each action α and slope m, let $K(\alpha, a, m)$ be the minimal cost of implementing action α and delivering an expected utility of at least $\varphi(a)$, when the government imposes the redistribution function L_m (if α cannot be implemented with a contract that satisfies the "participation constraint", $K(\alpha, a, m) := \infty$). Then

$$K(\alpha, a, m) := \min_{s} \sum_{\omega \in \Omega} s(\omega) f(\omega \mid \alpha)$$

$$\begin{aligned} \text{s.t.} \quad & \Phi(s, \alpha, L_m) \geq \varphi(a) \\ & \Phi(s, \alpha, L_m) \geq \Phi(s, \beta, L_m) \quad \text{for all } \beta \in A \\ & s(\omega) \geq \underline{s}_m \quad \text{for all } \omega \in \Omega. \end{aligned}$$

LEMMA 1:

(i) *If s_1 is the optimal solution of the above problem when $m = 1$, then the optimal solution s_m for any other slope $m > 0$ is defined by*

$$s_m(\omega) = L_m^{-1}(s_1(\omega)) = \frac{1}{m} s_1(\omega) - \left(\frac{1-m}{m}\right) P(a), \text{ and}$$

$$K(\alpha, a, m) = \frac{1}{m} K(\alpha, a, 1) - \left(\frac{1-m}{m}\right) P(a).$$

(ii) *When $\alpha = a$, $s_1(\omega) = s^*(\omega, a)$ for all $\omega \in \Omega$, and $K(a, a, 1) = P(a)$.*

(iii) *$K(a, a, m) = P(a)$ for all $m > 0$.*

PROOF: Observe that there is a one-to-one linear relationship between the feasible contracts in the problem defining $K(\alpha, a, 1)$ and those in the problem defining $K(\alpha, a, m)$: s is feasible for the former if and only if $\tilde{s}(\omega) = L_m^{-1}(s(\omega))$, $\omega \in \Omega$, is feasible for the latter. Therefore, the costs of feasible contracts for the problem defining $K(\alpha, a, m)$ are just linear transformations of the costs of those feasible contracts for the problem defining $K(\alpha, a, 1)$, and thus the optimal contract for $K(\alpha, a, m)$ is exactly that obtained by the linear transformation from the optimal contract for $K(\alpha, a, 1)$. This concludes the proof of (i). When $\alpha = a$ and $m = 1$ (i.e., when $R = I$), $s^*(\cdot, a)$ is feasible, and from the definition of $\varphi(a)$, it must also be optimal. Part (iii) is obtained by substituting (ii) in the second formula of (i). *Q.E.D.*

Lemma 1 shows that for a firm determined to implement a particular action a, a linear redistribution scheme has no effect on *how* a is implemented: the worker's consumption schedule will be unchanged. But the tax schedule *will* affect a firm's decision regarding which action to implement. Can the government choose a linear scheme that induces the firm to implement $\bar{a}$ (defined at the beginning of this Section)? A reasonable guess is that if the model is well-behaved, the government should be able to vary a continuously by varying the slope of the redistribution function, and that consequently it can, by judicious

choice of m, induce firms to implement $\bar{a}$. (A7) and (A8) provide the convexity needed to generate the desired continuous response from the private sector.

(A7) P is concave

(A8) For each budget implementable $a \in A$, $K(\alpha, a, 1)$ is convex and increasing in α.

Although the convexity of a cost function is a plausible assumption, (A8) is an "endogenous" assumption, and it is awkward to phrase it in terms of primitives. It is likely to hold, for example, when the utility function is very convex in the effort level a.

Let $a \neq a^*$ be a budget implementable action, and define $\bar{K}(\alpha) := K(\alpha, a, 1)$, $\alpha \in A$. Since $s^*(\omega, a^*)$ implements action a^* at zero net profit for the firm (so at an expected cost of $P(a^*)$), and $\varphi(a^*) > \varphi(a)$, the firm could modify $s^*(\omega, a^*)$ to implement a^* at a lesser cost, by reducing the worker's expected payoff (but still keeping it above $\varphi(a)$). Therefore, $\bar{K}(a^*) < P(a^*)$. Also, by definition, $\bar{K}(a) = P(a)$. There are two possible cases, corresponding respectively to $a < a^*$ and $a^* < a$. Figure 1 illustrates the case $a < a^*$.

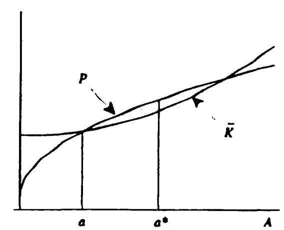

FIGURE 1

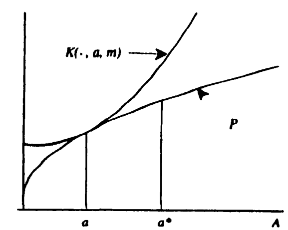

FIGURE 2

Let $m := \bar{K}'(a)/P'(a)$. If $a < a^*$, $\bar{K}'(a) < P'(a)$ and $m < 1$, while if $a^* < a$, $m > 1$. From (i) in Lemma 1, we obtain

$$\frac{\partial K}{\partial \alpha}(a, a, m) = \frac{\bar{K}'(a)}{m} = P'(a).$$

That is, $K(\cdot, a, m)$ is tangent to P at a. Figure 2 shows that when the government imposes the redistribution function $L(\cdot, a, m)$, and the firm assumes that the workers can get $\varphi(a)$ elsewhere, the cost of implementing any action α other than a is higher than the corresponding gross profit to the firm, and the best the firm can do to attract workers is to implement a at a zero net profit. These observations lead to the following main result. The slope m that makes $K(\cdot, a, m)$ tangent to P at a appears often in the analysis below. Accordingly we denote it by

$$\mu(a) := \frac{1}{P'(a)} \frac{\partial K}{\partial \alpha}(a, a, 1).$$

THEOREM 1: *Let a be a budget implementable action, and $m := \mu(a)$. If the government imposes the linear redistribution function $R := L(\cdot, a, m)$ and $\underline{s} := -(1-m)P(a)/m$, the optimal choice for the firm is to implement action a with the contract $s(\omega) = L_m^{-1}(s^*(\omega, a))$, $\omega \in \Omega$, and the resulting consumption schedule for the workers is $c(\omega) = R(s(\omega)) = s^*(\omega, a)$. The corresponding expected utility for the worker is $\varphi(a)$, and $\sigma(\underline{s}, s, a, R)$ is an equilibrium with discounted average payoff $\varphi(a)$ for the government, in which the action a is implemented optimally in every period along the equilibrium path.*

COROLLARY: *Let $m := \mu(\bar{a})$, $\bar{\underline{s}} := -(1-m)P(\bar{a})/m$, $\bar{s}(\omega) := L_m^{-1}(s^*(\omega, \bar{a}))$, $\omega \in \Omega$, and $\bar{R} := L(\cdot, \bar{a}, m)$. Then $\sigma(\bar{\underline{s}}, \bar{s}, \bar{a}, \bar{R})$ is an optimal equilibrium with value $\varphi(\bar{a})$.*

For problems that satisfy the additional assumptions (A7) and (A8), we have been able then to construct a very simple *optimal* equilibrium. As we observed earlier, the outcome of this equilibrium yields a payoff $\varphi(\bar{a})$ smaller than what could be attained, $\varphi(a^*)$, if the government could not intervene, or had mechanisms to commit its actions at the beginning of every period. For problems that do not satisfy (A7) and (A8), in addition to this efficiency loss, there might be another loss: any redistribution function inducing the firm to implement $\bar{a}$ is nonlinear and the corresponding contract offered by the firm (maximizing the worker's utility at zero net profit) does not provide after redistribution the second-best consumption schedule $s^*(\cdot, \bar{a})$. Hence, the government trust constraint cannot be satisfied at $\bar{a}$.

4. STRATEGIC DYNAMIC PROGRAMMING

In Section 5 we shift attention to equilibria that are renegotiation-proof in a sense that will be made precise. The analysis there is most easily carried out using the language of "self-generating sets" and some associated results on "strategic dynamic programming" developed in Abreu, Pearce and Stacchetti (1986, 1990). This Section collects the definitions and propositions that we wish to use, adapted to the present context. The treatment here is not intended to be self-contained; some readers may find it useful to refer to one of the original papers for more detail.

In conventional dynamic programming, one looks for a fixed point of a function that maps vectors of values (one for each state) into vectors of values. Here we look for fixed points of a function that maps *sets* of supergame values into value sets. The latter function, unlike its nonstrategic counterpart, takes incentive constraints into account. The "largest" fixed point of the set-valued function turns out to be the set of equilibrium values of the dynamic economy.

Let V denote the set of payoffs to the government (and, ex ante, to any consumer) in all possible equilibria. For any bounded set $W \subset \mathbf{R}$, let $\underline{W} := \inf W$, and $\bar{W} := \sup W$. We have argued above that $\underline{V} = \varphi(0)$, and $\bar{V} = \varphi(\bar{a})$.

DEFINITION: *Let $W \subset \mathbf{R}$ be bounded. A tuple $(\underline{s}, s, a, R, w)$ is* admissible *with respect to W if the following constraints are satisfied:*

(i) *Given $(\underline{s}, R)$, (s, a) solves the principal-agent problem*

$$\begin{aligned} \max_{(s', a')} \ & \Phi(s', a', R) \\ \text{s.t.} \ & \Phi(s', a', R) \geq \Phi(s', \alpha, R) \text{ for all } \alpha \in A \\ & P(a') - C(s', a') = 0 \\ & s'(\omega) \geq \underline{s} \text{ for all } \omega \in \Omega. \end{aligned}$$

(ii) $\sum_{\omega \in \Omega} R(s(\omega)) f(\omega \mid a) = \sum_{\omega \in \Omega} s(\omega) f(\omega \mid a)$.

(iii) $w \in W$ *and* $(1-\delta)\Phi(s, a, R) + \delta w \geq (1-\delta)U(P(a), a) + \delta \underline{W}$.

Admissibility is used to help compute or characterize the equilibrium value set V. The definition has the following interpretation. The set W is a stand-in for V (or a subset thereof), which is initially unknown. The worst equilibrium value available in this set is

$\underline{W}$. An admissible tuple $(\underline{s}, s, a, R, w)$ summarizes critical information implied by an equilibrium profile: $(\underline{s}, s, a, R)$ is the behavior prescribed for the first period, and w is the continuation value when the outcome of the first period agrees with this recommendation. Deviations are punished with the continuation value $\underline{W}$. Condition (i) states that given $\underline{s}$ and R, firms and workers want to implement (s, a). Condition (ii) guarantees that the government does not tax or subsidize the workers (in expected terms). Finally, condition (iii) represents the trust constraint for the government.

Obviously, if $(\underline{s}, s, a, R, w)$ is admissible with respect to W, then the following tuples are also admissible with respect to W: (i) $(\underline{s}_m, s', a, R', w)$, where $m := \mu(a)$, $R' := L(\cdot, a, m)$ and $R'(s'(\omega)) = s^*(\omega, a)$, $\omega \in \Omega$; and (ii) $(\underline{s}, s, a, R, w')$ for each $w' \in W$ with $w' > w$.

DEFINITION: *The value set* generated *by W is*

$$B(W) := \{\ (1-\delta)\Phi(s, a, R) + \delta w \mid (\underline{s}, s, a, R, w) \text{ is admissible w.r.t. } W\}.$$

DEFINITION: *A bounded set* $W \subset \mathbf{R}$ *is* self-generating *if* $W \subset B(W)$.

THEOREM 2 (Self-generation): *If* W *is self-generating*, $B(W) \subset V$.

THEOREM 3 (Factorization): $V = B(V)$.

THEOREM 4 (Compactness): *If* W *is self-generating, so is* $cl(W)$.

These theorems combined imply that V is the largest self-generating set, and that V is compact. Thus, best and worst equilibria with respective values $\underline{V}$ and $\bar{V}$ do exist.

For a symmetric strategy profile γ and a symmetric behavior $(\underline{s}, s, a, R)$, let $\sigma(\underline{s}, s, a, R, \gamma)$ be the stationary symmetric profile in which $(\underline{s}, s, a, R)$ occur on the equilibrium path and deviations trigger the equilibrium γ.

LEMMA 2: *Let* W *be self-generating. Then*

(i) *There exists an equilibrium* γ *with value* $\underline{W}$, *all of whose continuation values are in* W.

(ii) *There exists* $(\underline{s}, s, a, R, \gamma)$, *with* $R := L(\cdot, a, \mu(a))$ *and* $s(\omega) := R^{-1}(s^*(\omega, a))$ *for all* $\omega \in \Omega$, *such that* $\sigma(\underline{s}, s, a, R, \gamma)$ *is an equilibrium with value* $\sup B(W) = \varphi(a)$.

5. RENEGOTIATION

In the equilibria studied in Sections 2 and 3 in which the government refrains from flattening the distribution of income, it does so because if it were to break its promise, a "punishment regime" would follow in which the government would never again be trusted. Is this "no trust equilibrium" unnecessarily severe, and if so, would it actually be carried out following a deviation by the government?

More generally, suppose that the government has deviated from some social agreement, and the equilibrium σ stipulates a continuation equilibrium with value x, say (for the government and, in expected terms, for each consumer). In the spirit of the definition of renegotiation-proof equilibrium proposed by Pearce (1987), we say that the original equilibrium σ is vulnerable to renegotiation if there exists another equilibrium γ all of whose continuation values (including the value of γ itself) are at least y, for some $y > x$. The idea is that the punishment with value x is needlessly harsh, because there is another social arrangement (that the government could propose) that *never* requires a punishment that is so severe.[5]

In light of the close connection between equilibria and self-generating sets, it is easy to show (see Pearce (1987)) that instead of working with renegotiation-proof equilibria, one can equivalently look for a self-generating set W such that $\underline{W}$ is at least as great as $\underline{X}$ for every bounded self-generating set X.

DEFINITION: $\underline{r} := \sup\ \{\ \underline{X} \mid X$ *is a self-generating set*$\}$.

DEFINITION: W *is a* renegotiation-proof set *if it is self-generating and* $\underline{W} = \underline{r}$.

It is easy to see that if $\{W_\alpha\}$ is a collection of renegotiation-proof sets, their union is also a renegotiation-proof set. And, if W is renegotiation-proof, so is its closure $\mathrm{cl}\ W$. Let

$$\bar{r} := \max\ \{\bar{W} \mid W \text{ is renegotiation-proof}\}.$$

The previous observations imply that this maximum is indeed attained. Any renegotiation-proof set that attains the maximum will be called a *maximal* renegotiation-proof set. From

[5] The application of Pearce's notion of renegotiation-proofness to government policy games was first suggested by Matsuyama (1988). Readers may consult Bernheim and Ray (1989) and Farrell and Maskin (1989) for a different approach to renegotiation-proofness from the one applied here.

Lemma 2, there exists an action a_R such that $\varphi(a_R) = \bar{r}$ and the tuple $(\underline{s}_R, s_R, a_R, R, \bar{r})$, where $m := \mu(a_R)$, $\underline{s}_R := -(1-m)P(a_R)/m$, $R := L(\cdot, a_R, m)$, and $R(s_R(\omega)) = s^*(\omega, a_R)$ for all $\omega \in \Omega$, is admissible with respect to any maximal renegotiation-proof set.

The *temptation function* $T : A \to \mathbf{R}$ is defined by $T(a) := U(P(a), a) - \varphi(a)$, $a \in A$. This function evaluates for each action a the maximal one period gain for the government from optimally redistributing income when the firms and the workers implement action a optimally. We invoke a further regularity assumption that allows us to establish the convexity of the maximal renegotiation-proof value set. This in turn simplifies the analysis to follow.

(A9) $\quad \varphi : [0, a_R] \to \mathbf{R}$ is continuous and $T : [0, a_R] \to \mathbf{R}$ is increasing.

THEOREM 5 (Convexity): *Suppose the tuple $(\underline{s}, s, a, R, r)$ satisfies conditions (i) and (ii) of admissibility, and the following trust constraint:*

(iii') $\Phi(s, a, R) \geq (1-\delta)U(P(a), a) + \delta r$, and $r \geq \varphi(0)$.

Then the set $W := [r, \varphi(a)]$ is self-generating.

PROOF: Since $\varphi(a) \geq \Phi(s, a, R)$, the tuple $(\underline{s}_m, s', a, R', r)$, where $m := \mu(a)$, $R' := L(\cdot, a, m)$ and $R'(s'(\omega)) = s^*(\omega, a)$, $\omega \in \Omega$, also satisfies the conditions of the Theorem, because $\Phi(s', a, R') = \varphi(a)$. Let $v_0 := (1-\delta)\varphi(0) + \delta\varphi(a)$. For each $v \in [v_0, \varphi(a)]$, there exists $\alpha \in [0, a]$ such that $(1-\delta)\varphi(\alpha) + \delta\varphi(a) = v$. Let $m := \mu(\alpha)$, $R'' := L(\cdot, \alpha, m)$, and the contract s'' be such that $R''(s''(\omega)) = s^*(\omega, \alpha)$, $\omega \in \Omega$. We will show that the tuple $(\underline{s}_m, s'', \alpha, R'', \varphi(a))$ is admissible w.r.t. W, and since by construction v is the value of this tuple, $v \in B(W)$. Clearly this tuple satisfies (i) and (ii) of admissibility. By definition we have that $T(a) \leq \frac{\delta}{1-\delta}(\varphi(a) - r)$, and by (A9), $T(\alpha) \leq T(a)$; this implies (iii) of admissibility. We have then shown that $[v_0, \varphi(a)] \subset B(W)$. It remains to show that $[r, v_0] \subset B(W)$. If $v \in [r, v_0]$, there exists $w \in [r, \varphi(a)]$ such that $(1-\delta)\varphi(0) + \delta w = v$. The tuple $(0, s^*(\cdot, 0), a, I, w)$ is clearly admissible w.r.t. W and its value is v; so $v \in B(W)$. Q.E.D.

Applying Lemma 2 to $\bar{r}$ and any maximal renegotiation-proof set, we see that there exists a tuple $(\underline{s}, s, a, R, \bar{r})$ satisfying the conditions of Theorem 5. Consequently, the set $[\underline{r}, \bar{r}]$ is self-generating. This implies the following corollary.

COROLLARY: $V_R := [\underline{r}, \bar{r}]$ *is the largest renegotiation-proof set.*

Readers familiar with Abreu, Pearce and Stacchetti (1993) will recognize the strong similarity between Theorem 6 below and the characterization of "consistent bargaining equilibria" provided in that paper. The result gives a simple way to compute $\underline{r}$ and a_R (and hence $\bar{r}$, which is just $\varphi(a_R)$). The required calculations are no more sophisticated than those associated with the solution of a standard static moral hazard problem.

THEOREM 6: $\underline{r} = \max_{a \in A} \frac{1}{\delta}[\varphi(a) - (1-\delta)U(P(a), a)]$, *and a_R is an optimal solution of this problem.*

PROOF: Temporarily, denote $\hat{r} := \max_{a \in A} \frac{1}{\delta}[\varphi(a) - (1-\delta)U(P(a), a)]$, and let $\hat{a}$ be an optimal solution of this problem. If W is any maximal renegotiation-proof set, by Lemma 2, there exists $a \in A$ such that $\varphi(a) = \bar{W}$ and $\varphi(a) \geq (1-\delta)U(P(a), a) + \delta\underline{r}$. Therefore $\underline{r} \leq \hat{r}$. Conversely, consider the set $W := [\hat{r}, \varphi(\hat{a})]$. By Theorem 5, W is self-generating. Hence, $\hat{r} \leq \underline{r}$, and therefore $\hat{r} = \underline{r}$.

Let $r := \frac{1}{\delta}[\varphi(a_R) - (1-\delta)U(P(a_R), a_R)]$. By the definiton of a_R we have $r \geq \underline{r}$. Since $[r, \varphi(a_R)]$ is self-generating, we must have $r = \underline{r}$. Q.E.D.

Our next result establishes that unless a^* coincides with the first-best action, a^* cannot be sustained in a renegotiation-proof equilibrium.

THEOREM 7: Suppose $\frac{dU}{da}(P(a^*), a^*) \neq 0$. Then $a_R \neq a^*$.

PROOF: For each $a \in A$, let $r(a) := \frac{1}{\delta}[\varphi(a) - (1-\delta)U(P(a), a)]$; $r(a)$ represents the least severe punishment value that would make the government incentive compatible when workers and firms implement action a optimally in every period in equilibrium. We will show that this function cannot have a global maximum at a^*. By contradiction, if a^* maximizes r, then $r'(a^*) = 0$. But, since a^* also maximizes φ, we have that $\varphi'(a^*) = 0$. This implies that $\frac{dU}{da}(P(a^*), a^*) = U_c(P(a^*), a^*)\frac{dP}{da}(a^*) + U_a(P(a^*), a^*) = 0$, which contradicts our assumption. Q.E.D.

The intuition for Theorem 7 is very simple: consider the function r near a^*. Since a^* maximizes φ, moving away from a^* has only a second order effect on $\varphi(a)$. However, if a^* is not a critical point of $U(P(a), a)$ (in particular, if it is not a first-best action), moving away from a^* has a first order effect in $U(P(a), a)$. Thus one can change the action from a^* to a nearby action a (possibly smaller) and reduce $r(a^*)$.

6. Example

In this section we present an example which has been solved, for the most part, in closed form. Our principal purpose is to show how the theory plays out in one concrete example, both with and without renegotiation of the social agreement. As noted below, solutions for some of the nonlinear equations characterizing a^*, $\bar{a}$, and a_R have been computed numerically with Mathematica.

The worker's action space is $A = [0, 2]$,[6] and his utility function is

$$U(s, a) := \sqrt{s} - \gamma a^n, \text{ where } \gamma > 0 \text{ and } n > 1.$$

Technology is given by the probability density

$$f(\omega \mid a) := \lambda(a)\, e^{-\lambda(a)\omega}, \ \omega \geq 0,$$

where $\lambda(a) := \frac{1}{1 + ka}$, with $k > 0$, and the gross profit function $p(\omega) := \omega$, $\omega \geq 0$. Therefore, the expected revenue function is $P(a) = \frac{1}{\lambda(a)} = 1 + ka$. All but one condition in assumptions (A1) - (A5) are satisfied: $\Omega = \mathbf{R}_+$ is not finite. But, as we noted when it was introduced, this condition is required only to guarantee the existence of optimal contracts; here we prove their existence by direct construction.

The density $f(\omega \mid a)$ has the monotone likelihood ratio property, but its corresponding distribution function does not satisfy the convexity condition[7] sufficient to guarantee the validity of the first-order approach. Nevertheless, we use the first-order approach to compute the optimal contracts supporting each action, and verify directly that the worker's incentive constraints are indeed satisfied. Consequently, for each action $a \in A$, we solve the modified problem:

[6] Although we have assumed throughout that $A = [0, 1]$, this change is immaterial. To build examples we have found it easier to allow the action space to be an arbitrary compact interval.
[7] See, for example, Grossman and Hart (1983).

$$\begin{aligned}
\varphi(a) := \max_s \ & \int_\Omega \sqrt{s(\omega)}\, f(\omega \mid a) d\omega \\
\text{s.t.} \ & \int_\Omega \sqrt{s(\omega)}\, f_a(\omega \mid a) d\omega - \gamma n a^{n-1} = 0 && \text{(IC')} \\
& \int_\Omega s(\omega) f(\omega \mid a) d\omega \leq 1 + ka && \text{(BC)} \\
& s(\omega) \geq 0 \ \text{ for all } \omega \geq 0,
\end{aligned}$$

where $f_a(\omega \mid a)$ denotes the partial derivative of f with respect to a. The optimal solution of this problem is

$$s^*(\omega, a) = (e(a) + b(a)\omega)^2, \text{ where}$$

$$b(a) := \frac{\gamma n a^{n-1}}{k}, \text{ and } e(a) := \frac{-b(a)}{\lambda(a)} + \sqrt{P(a) - \left[\frac{b(a)}{\lambda(a)}\right]^2}.$$

When the contract offered by the firm is $s^*(\omega, a)$, $\omega \in \Omega$, the expected utility for the worker when he takes action α is

$$\int_\Omega (e(a) + b(a)\omega) f(\omega \mid \alpha) d\omega - \gamma\alpha^n = e(a) + b(a)(1 + k\alpha) - \gamma\alpha^n,$$

which is a *concave* function of α; hence condition (IC') guarantees that condition (IC) is satisfied. Substituting the optimal solution $s^*(\omega, a)$ into the objective function, we obtain

$$\varphi(a) = \sqrt{P(a)}\sqrt{1 - b(a)^2 P(a)} - \gamma a^n.$$

We make the following choice of parameters: $k = 4$, $\gamma = \frac{1}{2}$, $n = \frac{3}{2}$. The function $\varphi : A \to \mathbf{R}$ is concave, and has a unique maximum $\varphi(a^*) = 1.56727$ at $a^* = 0.774169$ (which was computed numerically).

When the discount factor is $\delta := \frac{1}{2}$, the government's trust constraint is satisfied at a^*:

$$\varphi(a^*) > (1 - \delta) U(P(a^*), a^*) + \delta\varphi(0).$$

Therefore, $\bar{a} = a^*$, and in the optimal equilibrium, the government does not intervene at all in any period – the second-best is attained. As we argued in Section 5, with renegotiation, this is never the case.

We turn now to the computation of the optimal renegotiation-proof solution to the dynamic economy. Since $P(a)$ is affine, it obviously satisfies (A7). The temptation function $T(a) = \sqrt{P(a)} - \gamma a^n - \varphi(a) = \sqrt{P(a)}\ \{1 - \sqrt{1 - b(a)^2 P(a)}\ \}$ is convex and increasing in A, and $T(0) = 0$. Clearly φ is continuous in A; hence, (A9) is also satisfied. We now use the first-order approach to simplify the problem defining the function K:

$$K(\alpha, a, m) := \min_{s} \int_{\Omega} s(\omega) f(\omega \mid a) d\omega$$

$$\text{s.t.} \int_{\Omega} \sqrt{ms(\omega) + (1-m)P(a)}\, f_a(\omega \mid \alpha) d\omega - \gamma n \alpha^{n-1} = 0$$

$$\int_{\Omega} \sqrt{ms(\omega) + (1-m)P(a)}\, f(\omega \mid \alpha) d\omega - \gamma \alpha^n \geq \varphi(a)$$

$$s(\omega) \geq -(1-m)P(a)/m \quad \text{for all } \omega \geq 0.$$

The optimal solution of this problem is given by

$$s(\omega) = \frac{1}{m}[c(\omega) - (1-m)P(a)], \text{ where}$$

$$c(\omega) := (e + b\omega)^2,\ b := \frac{\gamma n a^{n-1}}{k},\ \text{and}\ \ e := \varphi(a) + \gamma a^n - b(1 + k\alpha).$$

Similarly to the previous problem, one can verify directly the validity of the first-order approach. Substituting the optimal solution one obtains

$$K(\alpha, a, m) = \frac{1}{m}\left\{[\varphi(a) + \gamma\alpha^n]^2 + \left[\frac{\gamma n \alpha^{n-1}(1 + k\alpha)}{k}\right]^2 - (1-m)P(a)\right\}.$$

Given our choice of parameters (including $n = \frac{3}{2}$), this function is *convex* in α, as required by assumption (A8).

To compute the optimal renegotiation-proof action a_R, we solve the problem

$$\max_{a \in A} \frac{1}{\delta}[\varphi(a) - (1-\delta)U(P(a), a)]$$

numerically. We obtain $a_R = 0.616216$, and thus $\underline{r} = \varphi(a_R) - (1-\delta)U(P(a_R), a_R) = 1.4771$ and $\bar{r} = \varphi(a_R) = 1.54833$. The optimal redistribution function is

$$R(s) := ms + (1-m)P(a_R) = 0.782623s + 0.753182,$$

where $m := \mu(a_R) = 0.782623$ and $P(a_R) = 3.46486$, and the minimum wage is $\underline{s}_m := -0.753182$.

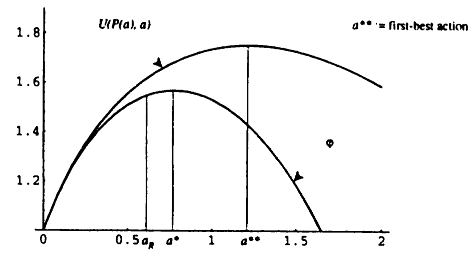

FIGURE 3

7. CONCLUSION

We have exploited recent developments in the theory of repeated games to characterize constrained-optimal social agreements in a simple dynamic economy with moral hazard and a government plagued with time consistency problems. The possibility that the government may attempt to renegotiate the "social contract" was shown to simplify the analysis, although it is welfare-decreasing (ex ante). With or without renegotiation, government intervention has social costs and no social benefits; indeed, the government in all instances wishes it could tie its hands and commit to nonintervention. This is an artifact of the oversimplified nature of the model: in an economy with sources of inequality other than the moral hazard problem considered here, redistributive government intervention may, of course, increase expected welfare.

Similarly, the startling result that in a highly nonlinear model the optimal choice of income tax schedule in equilibrium is usually linear, would not survive the introduction of adverse selection or a variety of other complications. But apart from serving as a simple benchmark for comparison with outcomes of more realistic models, the linear solution may be of interest in other areas. For example, in certain models in which a regulator wishes to affect the level of some productive activity that is hard to monitor directly, but prefers to do so without distorting compensation schemes in the regulated firm, linear schemes may be useful.

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