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Center for Research on Economic Development

University of Michigan
Ann Arbor, Michigan 48104

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A General Equilibrium Model
Of Tariffs in a Non-Competitive Economy

by

Charles P. Staelin

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Errata sheet for Discussion Paper 26

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on page 17 and on page 22 equation 34 should read

$$\hat{\eta}_i^D = \frac{1}{\eta_i^D} \left(\frac{\eta_i^C C_i}{D_i} [\hat{C}_i - \hat{D}_i] + \frac{\eta_i^I I_i}{D_i} \hat{\eta}_i^I + \eta_i^I \frac{I_i}{D_i} [\hat{I}_i - \hat{D}_i]\right). \quad (34)$$

on page 18 and on page 22 equation 35 should read

$$\hat{\eta}_i^I = \frac{1}{\eta_i^I} \sum_{j \epsilon N} \sigma_{ii}^j \Theta_{ij} \gamma_{ij} (2\hat{a}_{ij} + \hat{X}_j - \hat{I}_i - \hat{p}_j + \hat{p}_i - \hat{\lambda}_j^D), \; i \epsilon N \quad (35)$$

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A General Equilibrium Model
Of Tariffs in a Non-Competitive Economy

The recognition of the many theoretical defects in the partial-equilibrium measures of resource pulls and resource costs under different tariff and exchange rate policies[1] has led to a renewed interest in general-equilibrium models of these phenomenon. Recent works in this area utilizing general-equilibrium systems (in particular Taylor and Black [1972] and Evans [1972]) have assumed perfectly competitive economies as their starting points. Yet, there are two major difficulties with the use of such competitive models.

The first difficulty arises from the use of the competitive model in a non-competitive world. The degree of monopoly, oligopoly and other non-competitive behavior observed in the real world need not be detailed here. And non-competitive behavior is especially prevalent in most less developed countries (LDC's) where the small size of many industries allows only a few firms to operate, and where those firms are shielded from foreign competition by restrictive commercial policies and high transport costs. Yet it is also in LDC's that the need for general-equilibrium models of the impact of the tariffs is most felt.

The second difficulty arises when competitive models use constant-returns-to-scale (CRS) production functions.[2] For under these circumstances the model

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[1] E.g. Tan [1970], Bruno [1972], Travis [1972], Leith [1968], Mayer [1971], Ruffin [1969], Ramaswami and Srinivasan [1971].

[2] The use of CRS production functions is normally justified on three grounds: 1) CRS production functions are generally observed in most empirical studies of industry and agriculture, 2) CRS production functions contain at least one less parameter to determine empirically than do either increasing- or decreasing-returns functions and 3) increasing returns is normally thought to be incompatible with competitive assumptions because of the tendency toward monopoly, while decreasing returns may be inconsistent with the "small units" assumption of perfect competition.

of a small economy with infinite import-supply elasticities (finite export-demand elasticities may be the rule for even small countries) will not be determinate in output and trade volumes unless there are at least as many identifiable factors as there are import industries. And in most economies, the empirical identification of so many separable factors is not feasible.

The non-competitive model can offer a solution to both these difficulties. The specification of some given non-competitive behavior--out of the many theoretical possibilities--has at least the potential for more accurately describing the real world than does the specificaiton of competitive behavior. In addition, most non-competitive specifications allow the relative number of identifiable factors and imported goods to be ignored.

Section II briefly describes a simple competitive model in order to demonstrate the factor versus import-industry balance problem and to indicate how it might be solved. Section III describes one particular non-competitive model, one employing monopolistic pricing behavior at the industry level, while Section IV briefly offers an alternate model employing satisficing behavior. Finally, Section V looks at the empirical applicability of the two models and indicates a more simplistic alternative.

II

The competitive, general-equilibrium model of an open economy has been well analyzed by Samuelson [1953] (and more recently by Travis [1972] and Melvin [1968]) and this section will serve only to extend his analysis to cover non-traded goods and finite export demand elasticities and to illustrate some of his conclusions. The model presented here is very simple, but the characteristics it exhibits carry through to more complex competitive models.

The economy produces exports, import substitutes and non-traded goods.

Exports face finite export demand elasticities but, because of the small-country assumption, imports are supplied at fixed prices in foreign currency. The domestic prices of imports and exports, p_i, are given by world prices, ω_i, times an exogenous exchange rate, r, plus import tariffs, τ_i^M, or export subsidies, τ_i^E, respectively.[1]

$$p_i = \omega_i r(1 + \tau_i^M) \quad , \qquad i\varepsilon M \quad , \tag{1}$$

$$p_i = \omega_i r(1 + \tau_i^E) \quad , \qquad i\varepsilon E \quad , \tag{2}$$

where M is the set of import substitution industries and E is the set of export industries. The exchange rate will be made endogenous below, but this change has little significance for the results. Perfect competition in goods and factor markets coupled with CRS production functions insure that the prices of all goods equal costs, i.e.:

$$p_i = \sum_{j\varepsilon F} a_{ji} v_j + \sum_{j\varepsilon N} a_{ji} p_j \qquad i\varepsilon N \tag{3}$$

where F and N are the sets of all factors and all goods respectively, v_j is the wage of factor j and a_{ji} is the physical input of j into a physical unit of i. These input coefficients may be variables, but under CRS production functions they can only be functions of relative prices and not of output levels. Therefore,

$$a_{ij} = A^{ij}\,(p_1, \ldots, p_n, v_1, \ldots, v_f) \qquad i\varepsilon F,\ N \qquad j\varepsilon N \tag{4}$$

The economy is constrained by (fixed) factor supplies and the normal supply and demand relationships:

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1 The definitions of all variables and parameters are also given in Table 2.

$$\sum_{j\varepsilon N} a_{ij} X_j = H_i \qquad i\varepsilon F \tag{5}$$

$$X_i - \sum_{j\varepsilon N} a_{ij} X_j - C_i = \begin{cases} 0 & , i\varepsilon M^c \cap E^c \\ -M_i & , i\varepsilon M \\ E_i & , i\varepsilon E \end{cases} \tag{6}$$

$$C_i = C^i (p_1, \ldots, p_n, v_1, \ldots, v_f, X_1, \ldots, X_n), \quad i\varepsilon N \tag{7}$$

$$E_i = E^i (\omega_i) \quad , \qquad i\varepsilon N \tag{8}$$

Output, consumption, import and export volumes are given by X_i, C_i, M_i and E_i respectively; factor supplies are fixed at H_i; consumption is a function of prices and income, i.e., prices, wages and output; and exports are a function of the world price. The balance-of-payments constraint is ignored for the moment as it serves little purpose with the exchange rate exogenous.

Overall, there are as many equations as variables. After substituting the expression for the a_{ij}'s (equation 4) into equations 3, 5, and 6, one is left with 3n + m + 2e + f equations in 3n + m + 2e + f variables--the ω_i, i ε M are fixed by the small country assumption. Although the counting of equations and unknowns is not foolproof, there is usually a presumption that the system is determinate if there is an equality between the two. Unfortunately, this is <u>not</u> one of the cases where a simple enumeration is applicable, for this system is normally block angular.

Looking at equations 1, 2, 3, 7 and 8 (again after the substitution of equation 4 in equation 3) one finds 2n + m + 2e equations in the n + f variable domestic prices and wages, the 2e variable world prices and volumes of exports, and the n consumption levels. If, as is usually the case in most empirical specifications, there are fewer factors than there are import industries, the number of equations will exceed the number of variables and all prices and wages, and all export and consumption volumes will be completely (indeed, perhaps overly) determined by the subsystem of equations 1, 2, 3, 7 and 8.[1] And given these prices and export volumes, equations 5 and 6 form n + f equations in the n + m output and import volumes, yielding (again for $f < m$) a system which is in general under determined in these variables.[2]

The inclusion of an endogenous exchange rate and a balance-of-payments constraint does not significantly help in reducing the indeterminacy of the volumes of output and imports in the system. The endogenous r results in 2n + f + 2e + 1 variables in the 2n + m + 2e equations 1, 2, 3, 7 and 8; the problem of more import substituting industries than factors is "reduced" only by 1. Moreover, the addition of the balance-of-payments constraint

$$\sum_{j \varepsilon M} M_j - \sum_{j \varepsilon E} E_j = 0 \tag{9}$$

(assuming net capital flows to be zero for simplicity) to the system of equations 5 and 6 reduces the underdeterminacy of output and import volumes by only one. Only if m - f = 1 will the inclusion of an endogenous exchange

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1 Notice that strictly speaking it is only prices and wages which are overdetermined for there are "too many" equations 1, 2 and 3 in the p_i, v_i and ω_i ($j \varepsilon E$). The addition of equations 7 and 8 add as many unknowns as variables and thus does not affect the overdeterminancy. The reasons for the occurrence of this overdeterminacy are noted below.

2 Samuelson also discusses the case where the number of factors exceeds the number of import industries, but this does not seem to be an empirically relevant case.

rate and the balance-of-payments constraint make the total system determinate.

The reason for this problem of indeterminate outputs is quite clear.[1] CRS production functions coupled with the no-profit assumptions of perfect competition, allow no link between prices and output. Thus prices need merely be consistent with each other, satisfying the price-equal-to-cost criterion of equation 3, without being constrained by output relationships. In general with $m > f$, such consistency is not possible; there are too many constraints. Furthermore, since domestic output is not tied to prices, and since it is only the final consumption of import substitutes which is determined by prices, there is no mechanism in the model for fixing the relative volumes of imports and domestic production in the import substituting industries; many combinations of imports and domestic production can satisfy both the factor and the final consumption constraints as long as the number of import industries exceeds (by more than one if the exchange rate is endogenous) the number of factors. Moreover, because the domestic production of import substitutes cannot be determined, neither can the production of exported and non-traded goods used as inputs, directly or indirectly, in the import substituting industries.

There are basically two means by which competitively-based models can overcome the problems of indeterminacy.

The first is to allow the economy to specialize in as many import substitutes as there are factors (or one more if the exchange rate is endogenous)--a solution toward which Samuelson maintains the economy would gravitate.[2] Yet this seems to have been an unacceptable solution in the

[1] Travis [1972] gives a much more rigorous explanation of the problem.

[2] See Samuelson [1953]. If a sufficient number of redundant equations of type 3 can be found to reduce the number of independent equations without reducing the number of industries (i.e. without specialization), prices may be "just" determined rather than overdetermined; yet outputs will still not be determinate. Redundant equations are cited by Samuelson as a distinct possibility in the real world where similar production techniques are shared by many countries.

two recent empirical studies cited earlier, perhaps because of the unrealistic policy assumptions this solution implies.

The second means is to "create" more factors so that the number of factors will be as large as or larger than the number of import industries--an overabundance of factors leads to no particular problems in determinacy.[1] Yet the creation of these factors presents several problems. Since it is generally not possible to identify enough "real" factors, empirical studies normally postulate (in addition to the "shared" factors such as labor) the existence of n factors, each specific to one and only one of the n industries. Unfortunately, it is difficult to identify such industry-specific factors in real life, except perhaps in the relatively few instances of mineral deposits or other such truly specific, fixed factors. Almost all factors are transferable among industries to some extent, in the sense that their wage when used in one industry is influenced by their wage when used in others.

Moreover, the Black and Taylor [1972], and Evans [1972] studies, for example, include only labor as a shared factor, making the capital employed in each industry <u>specific</u> to that industry. Thus, no transfer of capital among industries is allowed in response to price changes and capital earns rents rather than wages. The varying return to capital in different industries then has no impact on resource allocation in the economy, i.e., an equilibrium is possible with widely varying returns (rents) to capital in different industries and with no tendency toward equalization. Not only can one quarrel with the realism of such a model, but a model of this type seems

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1 Again, see Samuelson [1953].

to obviate one of the major purposes behind general equilibrium models of tariffs: the reflection of the impact of changing factor prices (due to changing tariffs) on resource allocation.[1]

A non-competitive model will, of course, allow for profits and their impact on resource allocation. But more importantly, by tying domestic output to profits and thus to prices, the non-competitive model eliminates the block angular nature of the general equilibrium model <u>even</u> when the number of import industries exceeds the number of factors. For when output is tied to prices, the relative volume of imports and domestic production will be determined by prices, thus eliminating the source of the indeterminacy of outputs. In addition, by separating profits and the returns to capital and by obviating the need for industry-specific capital, capital returns may be equalized (and capital redistributed) among all industries without requiring that profits also be so equalized.

The next two sections present non-competitive models which are neither troubled by the number of factors versus import industries, nor ignorant of the role of profits in resource allocation. Interestingly enough, they are not particularly harder to use empirically than are competitive models, as is indicated in Section V.

III

The model presented below is designed to overcome many of the drawbacks associated with the typical competitive-economy-based model described above.

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[1] The Black and Taylor [1972] model also ties the labor wage rate to the price level, holding real wages constant. This essentially removes labor as a factor in their model allowing no role for changing factor prices, and defeating, it would seem, the purpose of their "general-equilibrium" model.

More importantly however, it is felt that the model can, at least potentially, come closer to approximating reality as it exists in the "small" country and particularly the "small" LDC.

Since this is a model of a non-competitive economy, it relies heavily on assumptions regarding behavior. However, the model will first be presented in a "pure" form in order to demonstrate exactly how the specific behavioral assumptions operate within the context of the model.

Throughout, production functions are assumed to demonstrate the properties of constant return to scale (CRS) and constant elasticities of substitution (CES). As seen below, the former property is the more crucial to the model as presented here. All variables are summarized in Table 2 .

The economy is presumed to consist of n industries, elements of the set of industries N. A given subset of m industries, M, competes with imports. Firms in this industry are assumed to accept the C.I.F. price plus tariff[1] as the domestic market price for the import substitute; i.e., there is assumed to be no incentive for these firms to sell at a price below the C.I.F. price plus tariff.[2] Finally, another subset of e industries, E, is presumed to export some of its production. Over the relevant time period these industries are assumed to face finite (although possibly high)

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[1] Throughout this paper, "tariffs" will refer to effective tariffs, that is, the difference between domestic and world prices whether due to nominal tariffs, quotas, or other restrictions.

[2] This is an important assumption. It guarantees that the effective tariff as the measured difference between the domestic and C.I.F. price is attributable only to trade policies (e.g. tariffs and quotas) and not to any "redundancy" of restrictions. This makes the τ_i^M direct policy variables. More importantly it distinguishes between non-traded goods and import substitutes. Throughout, it is assumed that non-traded goods do <u>not</u> become traded as commercial policies change. However, with more data and a more complex model this contingency could be handled.

export demand elasticities and are able to price discriminate between domestic and export markets. Subsets E and M are presumed mutally exclusive and the subset $M^c \cap E^c$ is the subset containing non-traded goods industries.

There are f factors in the economy, members of the set F. Factor markets are assumed to be competitive in the sense that factors are paid their marginal revenue products as defined below.

This model is constructed in terms of the proportionate changes of all relevant variables rather than in terms of their levels as was done in Section II. Models utilizing proportionate changes are not uncommon, e.g. Johanson [1960], Black and Taylor [1972] and Mayer [1971]. The reason for the use of this form is the reduction in the amount of data required for empirical specification. However, it should be noted that the specification is correct only for "small" changes. Proportionate changes are signified by a circumflex ($\hat{}$), e.g., $\hat{X}_i \equiv \frac{dX_i}{X_i}$. The equations in proportionate changes are in most cases derived from equations in the levels of the same variables, however, the derivations will be shown only when necessary for clarity.

World and domestic prices are related by equations 10 and 11.

$$\hat{p}_i = \hat{\tau}_i^M + \hat{r} \qquad i \varepsilon M \tag{10}$$

$$\hat{q}_i = \hat{\tau}_i^E + \hat{r} + \hat{\omega}_i \qquad i \varepsilon E \tag{11}$$

The domestic prices of imports and exports are equal to their world prices times the exchange rate, plus the effective import tariffs or export subsidies. These tariffs and subsidies are the "driving" exogenous variables of the model. The C.I.F. prices of imports in foreign currency are assumed constant and exogenous. The F.O.B. prices of exports are endogenous.

The non-competitive nature of the economy is typified by the pricing equations 12 and 13:

$$p_i = \lambda_i^D \left(\sum_{j\varepsilon F} a_{ji} v_j + \sum_{j\varepsilon N} a_{ji} P_j \right) \qquad i\varepsilon N \tag{12}$$

$$q_i = \lambda_i^E \left(\sum_{j\varepsilon F} a_{ji} v_j + \sum_{j\varepsilon N} a_{ji} p_j \right) \qquad i\varepsilon E \quad . \tag{13}$$

All industries are assumed to mark-up price, i.e., to sell their output at a given proportion (greater than one) of marginal (equals average) cost. These markups need not be constant and are endogenous variables in the model; thus, the equations 12 and 13 are quite general in spite of their appearance. The markups, λ_i^D and λ_i^E are behavioral variables and they can admit almost any form of non-competitive pricing behavior. The set of equations 12 and 13, together with the CRS production functions 14,

$$X_i = X^i \, (I_{ji}) \qquad i\varepsilon N,\ j\varepsilon F,\ N \quad . \tag{14}$$

is compatible with the assumptions that all industries minimize cost and, furthermore, pay factors their marginal physical products times the price of output <u>less</u> profits (which for profit-maximizing firms is equivalent to marginal revenue products), eg.,

$$v_j = \frac{\delta X^i}{\delta I_{ji}} \; \frac{P_i}{\lambda_i^D} \quad . \tag{15}$$

Total differentiation of equation 12 yields

$$\hat{p}_i = \hat{\lambda}_i^D + \left(\sum_{j\varepsilon F} \Theta_{ji} \hat{v}_j + \sum_{j\varepsilon N} \Theta_{ji} \hat{p}_j \right) \tag{16}$$

$$+ \left(\sum_{j\varepsilon F} \Theta_{ji} \hat{a}_{ji} + \sum_{j\varepsilon N} \Theta_{ji} \hat{a}_{ji} \right) \quad .$$

But for CRS production functions such as equations 14,

$$1 = X^{i}\ (a_{ji}) \qquad i\varepsilon N,\ j\varepsilon F,\ N,$$

and taking derivatives:

$$0 = \sum_{j\varepsilon F,N} \frac{\delta X^{i}}{\delta I_{ji}}\ d(a_{ji})\ .$$

Using the factor and input pricing equations such as equation 15,

$$0 = \sum_{j\varepsilon F} \frac{v_j \lambda_i^D}{p_i} d\ a_{ji} + \sum_{j\varepsilon N} \frac{p_j \lambda_i^D}{p_i}\ d\ a_{ji}\ ,$$

$$0 = \sum_{j\varepsilon F,N} \Theta_{ji}\ \hat{a}_{ji}$$

and, for $\lambda_i^D \neq 0$

$$0 = \sum_{j\varepsilon F,N} \Theta_{ji}\ \hat{a}_{ji} \qquad i\varepsilon N,\ j\varepsilon F,\ N \quad . \tag{17}$$

Equation 16 may then be reduced to

$$\hat{p}_i = \hat{\lambda}_i^D + \quad (\sum_{j\varepsilon F} \Theta_{ji}\ \hat{v}_j + \sum_{j\varepsilon N} \Theta_{ji}\ \hat{p}_j) \qquad i\varepsilon N \quad . \tag{18}$$

Likewise, equation   may be transformed into equation 19,

$$\hat{q}_i = \hat{\lambda}_i^E + \quad (\sum_{j\varepsilon F} \Theta_{ji}\ \hat{v}_j + \sum_{j\varepsilon N} \Theta_{ji}\ \hat{p}_j) \qquad i\varepsilon E \quad . \tag{19}$$

Equations 18 and 19 relate the change in the price of output to the changes in the prices of inputs and in profits, after substitution effects have been accounted for. The system of equations 10, 11, 18 and 19 is hereinafter referred to as System I and in the normal general-equilibrium system is sufficient to determine all domestic prices and factor wages when the exchange rate is held exogenous. The system contains m + 2e + n equations in 3e + f + 2n variables. However, if the $\hat{\lambda}_i^D$ and the $\hat{\lambda}_i^E$ can be reduced to constants or to functions of simply the goods and factor prices, the number of variables falls to 2e + f + n. As long as m $\geq$ f, system I can determine all domestic and export prices and all factor wages. The condition $m > f$ is of course the same as saying that the number of imported goods exceeds the number of factors, a typical case in empirical work. As mentioned in Section II, Samuelson [1953] has shown that in this case as it applies to the competitive economy, there is a tendency for either traded goods to be squeezed out of production, reducing m to f, or for equations to be redundant, reducing the number of <u>independent</u> equations to f. But in this non-competitive system which allows positive profits there is no such former tendency and it is quite possible that the excess of equations over variables will persist. The economy described here overcomes this difficulty by adding more variables to system I; not through the use of additional factors as is done in most empirical studies using competitive models, but by making markups, and thus prices, functions of output.

There are many constraints which operate on the economy, the most important being the factor constraints:

$$\sum_{i\varepsilon N} \gamma_{ji} \hat{X}_i + \sum_{i\varepsilon N} \gamma_{ji} \hat{a}_{ji} = \hat{v}_j \eta_j^F \qquad j \varepsilon F \qquad (20)$$

That is, the change in the use of each factor must equal the change in its supply. The $\hat{a}_{ji}$ in the second term on the LHS of equation 20 are not so easily disposed of as were the $\hat{a}_{ji}$ in equation 16. However, as shown in a footnote,[1]

$$\hat{a}_{ki} = \sum_{j\varepsilon F} \Theta_{ji} \sigma_{kj}^{i} \hat{v}_j + \sum_{j\varepsilon N} \Theta_{ji} \sigma_{kj}^{i} \hat{p}_j , \quad i \varepsilon N,\ k \varepsilon F, N . \qquad (21)$$

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[1] From equation and for some given element k of F, N

$$\sum_{j\neq k} \Theta_{ji} \hat{a}_{ji} = -\Theta_{ki} \hat{a}_{ki} , \quad i\varepsilon N,\ j\varepsilon F, N \quad .$$

(continued)

All the σ_{ij} are constants under the assumptions of CES production functions and the $\hat{a}_{ji}$ are therefore linear functions of the price changes alone.

Every domestically produced good is also subject to constraints:

$$\hat{X}_i = ({}^{C_i}/X_i)\, \hat{C}_i + ({}^{I_i}/X_i)\, \hat{I}_i \;, \quad i\varepsilon M^c \cap E^c \;, \tag{22}$$

$$\hat{X}_i = ({}^{C_i}/X_i)\, \hat{C}_i + ({}^{I_i}/X_i)\, \hat{I}_i - ({}^{M_i}/X_i)\, \hat{M}_i \;, \quad i\varepsilon M, \tag{23}$$

$$\hat{X}_i = ({}^{C_i}/X_i)\, \hat{C}_i + ({}^{I_i}/X_i)\, \hat{I}_i + ({}^{E_i}/X_i)\, \hat{E}_i \;, \quad i\varepsilon E. \tag{24}$$

These constraints assure that the supply and demand for each good are in balance. $\hat{C}_i$, $\hat{I}_i$ and $\hat{E}_i$ are further defined as:

$$\hat{C}_i = \hat{p}_i\, \eta_i^C + \sum_{j\neq i} \hat{p}_j\, \eta_{ji}^C + \hat{Y}\, \eta_{Yi}^C \;, \quad i\varepsilon N, \; j\varepsilon N \tag{25}$$

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[1] (continued from previous page)

The elasticity of substitution is defined along an isoquant as

$$\sigma_{kj}^i = (\hat{a}_{ki} - \hat{a}_{ji})/(\hat{p}_j - \hat{p}_k);$$

therefore (using p_j in place of the v_j for $j\varepsilon F$),

$$\sum_{j\neq k} \Theta_{ji}\, (\hat{a}_{ki} - \sigma_{kj}^i\, (\hat{p}_j - \hat{p}_k)) = -\Theta_{ki}\, \hat{a}_{ki},$$

$$\hat{a}_{ki}\, (\sum_{j\neq k} \Theta_{ji} + \Theta_{ki}) = \sum_{j\neq k} \Theta_{ji}\, \sigma_{kj}^i\, \hat{p}_j - \hat{p}_k \sum_{j\neq k} \Theta_{ji}\, \sigma_{kj}^i \;.$$

But $\sum_j \Theta_{ji} \equiv 1$ and from Allen [1938, p. 504],

$$\sum_{j\neq k} \Theta_{ji}\, \sigma_{kj}^i = -\Theta_{ki}\, \sigma_{kk}^i \;.$$

Therefore,

$$\hat{a}_{ki} = \sum_{j\varepsilon F} \Theta_{ji}\, \sigma_{kj}^i\, \hat{v}_j + \sum_{j\varepsilon N} \Theta_{ji}\, \sigma_{kj}^i\, \hat{p}_j \;.$$

$$\hat{I}_i = \sum_{j\varepsilon N} \gamma_{ij} \hat{I}_{ij} = \sum_{j\varepsilon N} \gamma_{ij} \hat{X}_j + \sum_{j\varepsilon N} \gamma_{ij} \hat{a}_{ij} , \ i\varepsilon N; \tag{26}$$

$$\hat{E}_i = \eta_i^E \hat{\omega}_i , \quad i\varepsilon E . \tag{27}$$

The change in disposable income destined for consumption, Y, is equal to the weighted sum of the changes in the total income of each factor, the weights being the base income of each factor, multiplied by unity minus its marginal savings propensity and divided by the base Y.

Thus,

$$\hat{Y} = \sum_{i\varepsilon F} (1 - \phi_i) \Psi_i (1 + \eta_i^F) \hat{v}_i , \ i\varepsilon F . \tag{28}$$

Taxes are ignored in this model or, alternatively, included in the savings propensity and not spent by the government.

Equations 20 through 28 comprise equation system II. After substituting equations 21 in equations 20 and26 , system II contains $3n + f + e + 1$ equations and, <u>if</u> the price and wage changes are given from system I, $3n + m + e + 1$ variables. There will be m-f "too many" variables (assuming the number of imports exceeds the number of factors) and the solution will not be determinate. It might seem that the "excess" equations from system I would be useful here, but this is not so for reasons discussed in Section II.

If the exchange rate, r, is to be endogenous one further equation corresponding to the balance of payments may be added.

$$\sum_{i\varepsilon E} (\omega_i E_i)(\hat{\omega}_i + \hat{E}_i) - \sum_{i\varepsilon M} (\omega_i M_i) \hat{M}_i + \Pi \hat{\Pi} = 0 \tag{29}$$

This adds one equation and one variable to the system as a whole.

To this point the analysis has been quite general, relying on no specific behavioral assumptions. However, for the full system to be solved, behavior for the $\hat{\lambda}_i^D$ and $\hat{\lambda}_i^E$ must now be specified. For expositional purposes profit maximization at the industry level shall be the assumed behavior for those industries with control over the price of their output, i.e. for all non-import substituting industries. Import substituting industries are assumed to have no choice in the price of their output and thus move into and out of production as their profit rates rise and fall.

For the first set of industries,

$$\hat{\lambda}_i^D = - \frac{1}{(\eta_i^D - 1)} \hat{\eta}_i^D \qquad i \varepsilon M^C \tag{30}$$

typifies profit maximization in the domestic market.[1] η_i^D is the elasticity of domestic demand, D_i, defined positively, where

$$D_i = I_i + C_i \tag{31}$$

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[1] The derivation is quite simple:

marginal cost, $MC_i = \sum_{j \varepsilon F} a_{ji} V_j + \sum_{j \varepsilon N} a_{ji} p_j$

marginal revenue, $MR_i = p_i \, (1 - 1/\eta_i^D)$;

but $p_i = \lambda_i^D MC_i$; thus when $MC_i = MR_i$

$$p_i / \lambda_i^D = p_i \, (1 - 1/\eta_i^D) \; ,$$

$$\lambda_i^D = \frac{\eta_i^D}{\eta_i^D - 1} \; .$$

Taking derivatives of the above equations yields equations 30.

The domestic and export markets are assumed separable, therefore

$$\hat{\lambda}_i^E = - \frac{1}{(\eta_i^E - 1)} \hat{\eta}_i^E \qquad i\varepsilon E \tag{32}$$

In addition, the elasticity of export demand, η_i^E, is assumed constant so

$$\hat{\lambda}_i^E = 0 , \qquad i\varepsilon E \quad . \tag{33}$$

However, η_i^D may be expressed as

$$\eta_i^D = ({}^{C_i}/D_i) \ \eta_i^C + ({}^{I_i}/D_i) \ \eta_i^I , \quad i\varepsilon N$$

and although η_i^C is constant, η_i^I is not, thus

$$\hat{\eta}_i^D = \frac{1}{\eta_i^D} \left(\frac{\eta_i^C C_i}{D_i} [\hat{C}_i - \hat{D}_i] + \sum_{j\varepsilon N} \frac{\eta_i^I I_i}{D_i} \hat{\eta}_i^I + \eta_i^I \frac{I_i}{D_i} [\hat{I}_i - \hat{D}_i] \right) . \tag{34}$$

In solving for $\hat{\eta}_i^D$, $\hat{\eta}_i^I$ is defined as

$$\eta_i^I = \frac{\delta I_i}{\delta p_i} \frac{P_i}{I_i} = \frac{\delta \sum_{j\varepsilon N} a_{ij} X_j}{\delta p_i} \frac{p_i}{I_i}$$

$$= {}^{P_i}/I_i \left[\sum_{j\varepsilon N} X_j \frac{\delta a_{ij}}{\delta p_i} + a_{ij} \frac{\delta X_j}{\delta p_i} \right] .$$

Since $\delta X_j/\delta p_i$ depends upon $\delta p_j/\delta p_i$, and since the latter is a function of industry-specific behavior, we assume that industry j takes $\delta X_j/\delta p_i$ as zero in evaluating the relevant η_i^I . Therefore

$$\eta_i^I = \sum_{j\varepsilon N} \mu_{ij} \gamma_{ij}$$

where

$$\mu_{ij} = \frac{\delta a_{ij}}{\delta p_i} \frac{p_i}{a_{ij}} \quad .$$

But μ_{ij} is equivalent to

$$\mu_{ij} = \hat{a}_{ij}/\hat{p}_{ij}$$

when all other prices and outputs are held constant, and from equation 21 one knows that under such circumstances

$$\hat{a}_{ij} = \hat{p}_i \, \Theta_{ij} \, \sigma_{ii}^j \quad .$$

Therefore

$$\mu_{ij} = \Theta_{ij} \, \sigma_{ii}^j \quad .$$

and

$$\eta_i^I = \sum_{j \epsilon N} \gamma_{ij} \, \Theta_{ij} \, \sigma_{ii}^j$$

Taking derivatives yields

$$\hat{\eta}_i^I = \frac{1}{\eta_i^I} \sum_{j \epsilon N} \sigma_{ii}^i \, \Theta_{ij} \, \gamma_{ij} \, (2\hat{a}_{ij} + \hat{X}_j - \hat{I}_i + \hat{p}_j - \hat{p}_i) \ , \quad i \epsilon N \tag{35}$$

Combining equations 21 and 26 in 35, 35 in 34 and 34 in 30 gives an expression for λ_i^D, $i \epsilon M^c$, which is a function of the endogenous $\hat{X}_j$, $\hat{p}_j$ and $\hat{v}_j$. The inclusion of the $\hat{X}_j$ is important since by introducing the $\hat{X}_j$ in system I, the solutions of systems I and II are explicitly tied together.

The $\hat{\lambda}_i^D$ for the import industries are given by

$$\hat{\lambda}_i^D - (\sum_{j \epsilon N} \lambda_j^D \, \hat{\lambda}_j^D \, / \sum_{j \epsilon N} \lambda_j^D) = \beta_i (\hat{X}_i - \hat{D}_i) \tag{36}$$

implying that change in the proportion of total demand supplied from domestic production varies proportionately with the change in the profit rate in industry i relative to the profit rates in all other domestic industries.

The total system is now complete. Substituting equations 12 and 26 in 35, 35 in 34, 34 in 30, 30 and 36 in 18, 33 in 19 and using 31 where necessary yields n + e pricing equations in the form of 18 and 19 with variables in the endogenous $\hat{X}_j$, $\hat{p}_j$, $\hat{q}_j$ and $\hat{v}_j$ ($\hat{C}_i$ and $\hat{I}_i$ being functions of the same variables). Equations 10, 11, 18 and 19 are then a system of m + 2e + n equations in 2e + 2n + f + 1 variables (the $\hat{X}_j$, $\hat{p}_j$, $\hat{q}_j$, $\hat{\omega}_j$ and r).

The substitution of equations 21 into 20 and 26, 25, 26 and 28 into 22 and 23, and 25, 26, 27 and 28 into 24, yields n + f equations of type 20, 22, 23 and 24 in the same variables as above plus the $\hat{M}_i$, $i \varepsilon M$. These form the system's constraints. The combination of the pricing equations of type 18 and 19 and the constraints of type 20, 22, 23 and 24 plus equations 10 and 11 yield m + 2e + 2n + f equations in the m + 2e + 2n + f + 1 variables. The system needs only equation 29 to close the system and make all changes in prices, wages, outputs, exports and imports determinate.[1]

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[1] For those who wish to do more counting before the substitutions, equations 10, 11, 18 through 31, and 33 through 36 form (f+n)n + 8n - m + 4e + f + 2 equations in the same number of endogenous variables listed in Table 2.

Table 1

List of Equations

Price equations

$$\hat{p}_i = \hat{\tau}_i^M + \hat{r} \qquad i \varepsilon M \tag{10}$$

$$\hat{q}_i = \hat{\tau}_i^E + \hat{r} + \hat{\omega}_i \qquad i \varepsilon E \tag{11}$$

$$\hat{p}_i' = \hat{\lambda}_i^D + \left(\sum_{j \varepsilon F} \Theta_{ji} \hat{v}_j + \sum_{j \varepsilon N} \Theta_{ji} \hat{p}_j \right) \qquad i \varepsilon N \tag{18}$$

$$\hat{q}_i = \hat{\lambda}_i^E + \left(\sum_{j \varepsilon F} \Theta_{ji} \hat{v}_j + \sum_{j \varepsilon N} \Theta_{ji} \hat{p}_j \right) \qquad i \varepsilon E \tag{19}$$

Factor constraints:

$$\sum_{i \varepsilon N} \gamma_{ji} \hat{X}_i + \sum_{i \varepsilon N} \gamma_{ji} \hat{a}_{ji} = \hat{v}_j \eta_j^F \qquad j \varepsilon F \tag{20}$$

Input and factor substitution relations

$$\hat{a}_{ki} = \sum_{j\varepsilon F} \Theta_{ji}\, \sigma^{i}_{kj}\, \hat{v}_j + \sum_{j\varepsilon N} \Theta_{ji}\, \sigma^{i}_{kj}\, \hat{p}_j \,, \quad i\varepsilon N,\ k\varepsilon F, N \,. \tag{21}$$

Supply and demand balances:

$$\hat{X}_i = (C_i/X_i)\, \hat{C}_i + (I_i/X_i)\, \hat{I}_i \,, \quad i\varepsilon M^c \cup E^c \,, \tag{22}$$

$$\hat{X}_i = (C_i/X_i)\, \hat{C}_i + (I_i/X_i)\, \hat{I}_i - (M_i/X_i)\, \hat{M}_i \,, \quad i\varepsilon M \,, \tag{23}$$

$$\hat{X}_i = (C_i/X_i)\, \hat{C}_i + (I_i/X_i)\, \hat{I}_i + (E_i/X_i)\, \hat{E}_i \,, \quad i\varepsilon E \,. \tag{24}$$

$$\hat{C}_i = \hat{p}_i\, \eta^C_i + \sum_{j\neq i} \hat{p}_j\, \eta^C_{ji} + \hat{Y}\, \eta^C_{Yi}, \quad i\varepsilon N, \quad j\varepsilon N \tag{25}$$

$$\hat{I}_i = \sum_{j\varepsilon N} \gamma_{ij}\, \hat{I}_{ij} = \sum_{j\varepsilon N} \gamma_{ij}\, \hat{X}_j + \sum_{j\varepsilon N} \gamma_{ij}\, \hat{a}_{ij} \,, \quad i\varepsilon N \tag{26}$$

$$\hat{E}_i = \eta^E_i\, \hat{\omega}_i, \quad i\varepsilon E \tag{27}$$

$$\hat{Y} = \sum_{i\varepsilon F} (1 - \phi_i)\, \Psi_i\, (1 + \eta^F_i)\, \hat{v}_i \,, \quad i\varepsilon F \,. \tag{28}$$

Balance-of-payments constraint:

$$\sum_{i\varepsilon E} (\omega_i\, E_i)(\hat{\omega}_i + \hat{E}_i) - \sum_{i\varepsilon M} (\omega_i\, M_i)\, \hat{M}_i + \Pi\, \hat{\Pi} = 0 \tag{29}$$

Behavioral relations for profit maximization:

$$\hat{\lambda}_i^D = -\frac{1}{(\eta_i^D - 1)}\ \hat{\eta}_i^D \qquad i\varepsilon M^c \tag{30}$$

$$D_i = I_i + C_i \tag{31}$$

$$\hat{\lambda}_i^E = -\frac{1}{(\eta_i^E - 1)}\ \hat{\eta}_i^E \qquad i\varepsilon E \tag{32}$$

$$\hat{\lambda}_i^E = 0 , \qquad i\varepsilon E . \tag{33}$$

$$\hat{\eta}_i^D = \frac{1}{\eta_i^D}\ \left(\frac{\eta_i^C\ C_i}{D_i}\ [\hat{C}_i - \hat{D}_i] + \sum_{j\varepsilon N} \frac{\eta_i^I\ I_i}{D_i}\ \hat{\eta}_i^I + \eta_i^I\ \frac{I_i}{D_i}\ [\hat{I}_i - \hat{D}_i] \right) \tag{34}$$

$$\hat{\eta}_i^I = \frac{1}{\eta_i^I}\ \sum_{j\varepsilon N} \underset{\uparrow\ \Theta_{ij}}{\sigma_{ii}^{i}}\ \gamma_{ij}\ (2\hat{a}_{ij} + \hat{X}_j - \hat{I}_i + \hat{p}_j - \hat{p}_i) , \qquad i\varepsilon N \tag{35}$$

Behavioral relations for import industries:

$$\hat{\lambda}_i^D - \left(\sum_{j\varepsilon N} \lambda_j^D\ \hat{\lambda}_j^D \Big/ \sum_{j\varepsilon N} \lambda_j^D \right) = \beta_i (\hat{X}_i - \hat{D}_i) \tag{36}$$

Table 2

List of Variables

Endogenous Variables

$\hat{a}_{ki}$ -- the change in the physical input coefficient of k into i

$\hat{C}_i$ -- the change in the domestic final consumption of i

$\hat{D}_i$ -- the change in the total domestic demand for i

$\hat{E}_i$ -- the change in the exports of i

$\hat{I}_i$ -- the change in the domestic use of i as an intermediate input

M_i -- the change in the import of i

$\hat{p}_i$ -- the change in the price of good i

$\hat{q}_i$ -- the change in the ex factory price of export i

$\hat{r}$ -- the change in the exchange rate

$\hat{v}_i$ -- the change in the wage of factor i

$\hat{\omega}_i$ -- the change in the world price of good i in foreign currency

$\hat{X}_i$ -- the change in the output of industry i

$\hat{Y}$ -- the change in disposable money income less savings, i.e., total consumption

$\hat{\lambda}_i^D$ -- the change in the markup on domestic sales of i

$\hat{\lambda}_i^E$ -- the change in the markup on export sales of i

$\hat{\eta}_i^D$ -- the change in the elasticity of domestic demand, D_i

$\hat{\eta}_i^I$ -- the change in the elasticity of intermediate demand, I_i

Exogenous Variables

$\hat{\pi}$ -- the change in net foreign capital inflows

$\hat{\tau}_i^M$ -- the change in the effective tariff on import i

$\hat{\tau}_i^E$ -- the change in the effective tariff on export i

Parameters

Θ_{ji} -- the proportion of the cost of input or factor j in the total cost of i

γ_{ji} -- the proportion of the total intermediate use of input or factor j used in good i

σ_{kj}^i -- the elasticity of substitution between inputs and/or factors k and j in producing good i

η_i^C -- the price elasticity of consumption, C_i

η_i^E -- the price elasticity of export demand, E_i

η_{jk}^C -- the cross elasticity of demand of j and k in consumption

η_{Yi}^C -- the income elasticity of the consumption of good i, C_i

η_i^F -- the elasticity of supply of factor i

ϕ_i -- the marginal savings propensity of factor i

Ψ_i -- the ratio of the total income of each factor to total consumption y.

β_i -- the parameter of response of the production of import substitutes to profit rates

H_i -- the base period supply of factor i

$\Delta_i^{\pm}$ -- the proportion of increased (+) or decreased (-) cost which are passed on to the price of outputs (used only in the satisficing behavior of Section IV).

also, the base period values of π and the ratios of C_i, D_i, E_i, I_i, and M_i to X_i

IV

The preceding section used profit maximization at the industry level as its behavioral assumption. Yet, the model is not in any way tied to this assumption as may be seen by employing a different one, that of the satisficing industry.

Industries are assumed to satisfice by passing on a proportion λ_i $(1-\Delta_i^{\pm})$ of any increase or decrease in costs, Δ^+ if costs increase and Δ^- if costs decrease. The markup equations for industries with control over their price then become[1] :

$$\hat{\lambda}_i^D = -\Delta_i^{\pm} \left(\sum_{j\varepsilon F} \Theta_{ji} \hat{v}_j + \sum_{j\varepsilon N} \Theta_{ji} \hat{p}_j \right) \qquad i\varepsilon N \tag{37}$$

$$\hat{\lambda}_i^E = -\Delta_i^{\pm} \left(\sum_{j\varepsilon F} \Theta_{ji} \hat{v}_j + \sum_{j\varepsilon N} \Theta_{ji} \hat{p}_j \right) \qquad i\varepsilon E \tag{38}$$

Industries which satisfice completely would have $\Delta_i^+ \equiv 0$ and $\Delta_i^- \equiv 0$ although $\Delta_i^+ = 0$ and $1 > \Delta_i^- > 0$ is not implausible. For import competing industries caught between exogenous output prices and uncontrollable changes in input costs, equation 36 would continue to hold.

Comparing the satisficing and the profit maximizing systems, it is evident that the former system is less complex--substituting two equations, 37 and 38, for five equations, 30 and 32 through 35--yet the solution is still determinate in all variables. This may be seen by noting that the domestic price of imports still influences the domestic production of imports through equation 36, determining thereby the relative volumes of

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[1]The assumption of equal $\Delta_i^{\pm}$ for the domestic and export activities is for convenience only.

domestic production and imports. Looked at a bit differently, equations 10, 11, 18, 19, 37, 38 and 36 yield (after the use of equation 21 where necessary and assuming $\hat{r}$ to be exogenous) a system of 2n + 3e + m equations in the 2n + 3e + f prices and profit rates and the 2m import volumes (M_i) and domestic usages (D_i) of the import industries. Prices are, therefore, not overdetermined but underdetermined, and the system is not block angular. The requirement for determinacy of the full model can quickly be seen to be that at least as many domestic outputs be tied directly to profits-- as in equation 36 or as in equations 30, 34 and 35--as there are import industries less factors.[1]

The various $\Delta_i^{\pm}$ must be estimated or assumed; however, even assuming plausible values may be no worse than assuming perfect competition, or for that matter, monopolistic profit maximization. It remains to be seen in empirical studies whether the degree of satisficing can be easily estimated, and how sensitive the model is to errors in estimation.

V

The non-competitive model is in general more complex than a competitive model of comparable detail--compare, for example, the system of equations in Table 1 with the competitive model of Taylor and Black--yet it does not have significantly greater data requirements.

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[1] One less is sufficient when the exchange rate is endogenous. Also, all this assumes no block angularity in the a_{ij} matrix structure itself.

Indeed, the only parameters which appear in the profit-maximization model which would not appear in a comparable competitive model are the β_i, the parameters measuring the production response of import-substitution industries to changing relative profit rates. And, if one were to assume that the X_i/D_i were directly proportional to relative profit rates in industry i, the β_i would all be identically equal to unity.

Of course, the satisficing variant of the non-competitive model introduces still more parameters, the $\Delta_i^{\pm}$, relative to the competitive model. Profit-maximizing behavior is perhaps the simplest non-competitive behavior to describe and any other behavior--even one as simple as satisficing--is likely to require relatively more data. Yet, it seems that this kind of behavioral data is precisely the kind which LDC's (at least) should be gathering. For as these countries become more industrialized, this information will become more and more necessary for effective planning.

Needless to say, the data requirements could be much simplified through a simplification of the model. Many such simplifications are possible where the cost of accuracy-through-completeness-and-complexity is too high. The cross-price consumption elasticities and factor-supply elasticities can be assumed as zero, the factor savings propensities can be assumed equal and the elasticities of substitution can be set individually at zero or unity, without seriously violating the usual empirical practices or even, perhaps, reality itself. And the remaining information is normally available in some form from input-output flow tables and miscellaneous demand studies.

The use of unsophisticated data in sophisticated models can be counterproductive. But it remains for empirical studies to determine just what kind of inputs are most vital to models of this type. It also remains to be seen precisely what kind of behavioral assumptions are the most appropriate in any given case. Further study and empirical testing of non-competitive models will hopefully not only prove their worth, but indicate their most appropriate forms.

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