Center for Research on Economic and Social Theory
CREST Working Paper

A Solution to the Problem of Externalities and Public Goods when Agents are Well-Informed

Hal R. Varian

January 17, 1990
Number 90-1

DEPARTMENT OF ECONOMICS
University of Michigan
Ann Arbor, Michigan 48109

Preliminary Draft

A Solution to the Problem of Externalities and Public Goods when Agents are Well-Informed.

by

Hal R. Varian
University of Michigan

November, 1989
Current version: January 17, 1990

Abstract. I consider economic environments involving externalities and public goods where agents have full information but the regulator does not. For these environments I present a class of simple two-stage games whose subgame perfect equilibria are efficient allocations. In the case of two-party externalities, the equilibria involve compensation for the party upon whom the externality is inflicted. In the case of public goods, the equilibria are Lindahl allocations.

Keywords. externalities, public goods, sequential games, Lindahl allocations, subgame perfect equilibria

Address. Hal R. Varian, Department of Economics, University of Michigan, Ann Arbor, MI 48109

A Solution to the Problem of Externalities and Public Goods when Agents are Well-Informed.

Hal R. Varian

Consider an economic environment in which some agents may take actions which impose benefits or costs on other agents. All agents are aware of the relevant technology and the tastes of all other agents. However the "regulator," who has the responsibility for determining the final allocation, does not have this information. How can the regulator design a mechanism so that the agents will have the proper incentives to reveal their information and achieve an efficient allocation?

In addition to implementing an efficient allocation, one might also want the mechanism to achieve some distributional goals. For example, one might want agents who are injured by an externality to be compensated for that injury. Or, in the case of public goods, one might want the public goods to be paid for by a system of Lindahl taxes.

In this paper I describe a simple two-stage game that implements efficient allocations in this sort of environment. The mechanism also achieves the distributional goals just described. In the subgame perfect equilibria of this game, parties injured by the externality are compensated and in the case of public goods, the mechanism implements Lindahl allocations. The mechanism appears to work in a broad variety of economic environments and does not involve substantial restrictions on tastes or technology. In addition, the mechanism is very simple to describe and analyze.

The fact that sequential games and subgame perfect equilibria may be very useful in implementation problems was first suggested by Moore and Repullo (1988). They show that in economic environments, almost any choice rule can be implemented by subgame perfect equilibria. However, as Moore and Repullo point out, "...the mechanisms we construct ... are far from simple; agents move simultaneously at each stage and their strategy

---

This work was supported by National Science Foundation grant SES-8800114. I wish to thank Mark Bagnoli, Ted Bergstrom, Ken Binmore, Arthur Robson, and Michael Whinston for their comments and suggestions.

sets are unconvincingly rich. We present such mechanisms to show what is possible, not what is realistic." (p. 1198)

However, Moore and Repullo also show that in certain "economic environments" it is possible to use somewhat simpler mechanisms. I pursue this idea and construct mechanisms that appear to be quite simple and practical for the sorts of the externality problems of interest to economists.

It should be emphasized that these mechanisms only work in the case where the agents are perfectly informed about the technology and tastes of the other agents. This is, of course, more restrictive than one would like. However, there is a broad set of cases for which such mechanisms may be useful. For example, consider a set of agents who must design a mechanism to make group decisions in situations that will arise in the future. At the time the mechanism is chosen, the agents do not know the relevant tastes and technologies, but they will know these things when the mechanism is actually used.

This latter example is sometimes known as a "constitutional choice" problem: we must choose a decision-making procedure without knowing much about the tastes and technologies available to the agents who will use the procedure. However, we do know that the agents who will actually use the procedure will be reasonably well-informed about these matters when the decision must be made. I suggest that the mechanism described below may be useful for such problems.

In section 1 I describe a very simple example of the mechanism in a two-firm externalities problem and discuss in an intuitive way why the method works. The following sections show how the method can be extended to work in more general environments. Next, I describe a generalization of the mechanism which implements Lindahl allocations in a standard public goods problem. Finally, I show how the method may be used to solve very general problems in resource allocation.

1. An example of the mechanism

Consider the following externality problem involving two firms. Firm 1 produces output x in order to maximize its profit

$$\pi_1 = rx - c(x).$$

Here r is the competitive price of output and the cost function $c(x)$ is a differentiable, positive, increasing, and convex function.

We assume that firm 1's choice of output imposes an externality on firm 2. For any choice of x, firm 2's profits are

$$\pi_2 = -e(x),$$

where $e(x)$ is a differentiable, positive, increasing, and convex function of x. All of this information is common knowledge among the agents, but is not known to the regulator.

One class of solutions to this externality problem involves re-assigning the property rights. For example, one firm could buy out the other, internalize the externality, and eliminate the inefficiency. Another class of solutions involves negotiation between the agents with respect to the externality. A third class of solutions involves intervention by a regulator who imposes a Pigouvian tax.

We will assume that the property rights are fixed, so that one firm cannot buy out the other or move away from the externality. Hence "property rights" solutions to the externality are not available and some type of negotiation between the firms or intervention by the regulator is necessary to encourage efficient outcomes.

If the regulator had full information the problem would be easy. One solution would be for the regualtor to impose the costs of the externality on firm 1 by charging it a "tax" of $e(x)$ if it produces x units of output. Firm 1 would then solve the problem

$$\max_x \; rx - c(x) - e(x).$$

Let x^* be solution to this problem; then x^* satisfies the first-order condition

$$r - c'(x^*) - e'(x^*) = 0.$$

Given our curvature assumptions on $e(x)$, we could just as well set a "Pigouvian tax," $p^* = e'(x^*)$ and let firm 1 solve the problem

$$\max_x \; rx - c(x) - p^* x.$$

However, we have assumed that the regulator doesn't know the size of the externality and therefore cannot determine p^*. The regulator's problem is to design a mechanism that will induce the agents to reveal their information about the magnitude of the externality and achieve an effient level of production.

Here is a mechanism that solves the regulator's problem.

Announcement stage. Firm 1 and firm 2 simultaneously announce the magnitude of the appropriate Pigouvian tax; denote the announcement of firm 1 by p_1 and the announcement of firm 2 by p_2.

Production stage. Firm 1 chooses x to maximize the following payoff

$$\Pi_1 = px - c(x) - p_2 x - \alpha_1 (p_1 - p_2)^2.$$

Firm 2 receives a payoff of

$$\Pi_2 = p_1 x - e(x) - \alpha_2 (p_2 - p_1)^2.$$

The parameters α_1 and α_2 are arbitrary positive numbers.

In this mechanism firm 1 is forced to pay a penalty based on the marginal social cost of the externality as reported by firm 2, and firm 2 receives compensation based on the marginal social cost as reported by firm 1. Each firm must also pay a penalty based on the square of the difference between the two reports. This penalty can be any increasing function of difference between the two reports, but we have chosen a quadratic penalty for simplicity. Since the α's can be arbitrary positive constant, the penalties can be arbitrarily small.

Analysis of the mechanism

There are many Nash equilibria of this game; essentially any triple (p_1, p_2, x) such that $p_1 = p_2$ and x maximizes firm 1's objective function is a Nash equilibrium. However, if we use the stronger concept of subgame perfect equilibrium we get a much smaller set of equilibria. In fact, the *unique* subgame perfect equilibrium of this game has each agent reporting $p_1 = p_2 = p^*$ and firm 1 producing the efficient amount of output.

In order to verify this, we must work backwards through the game. We begin with the production stage. Firm 1 maximizes its profits, given the reports generated in stage 1, which implies that firm 1 will choose x to satisfy the first-order condition

$$r = c'(x) + p_2.$$

This determines the optimal choice, x, as a function of p_2, which we denote by $x(p_2)$. Note that $x'(p_2) < 0$—the higher the cost that firm 2 announces, the less firm 1 will want to produce.

We now examine the price-setting stage of the game. Consider first firm 1. If firm 1 believes that firm 2 will announce p_2, then firm 1 will want to announce $p_1 = p_2$. This is clear since p_1 has no effect on firm 1's payoff except through the quadratic penalty.

Consider now firm 2's decision. Although firm 2's announcement has no *direct* effect on firm 2's profits, it does have an *indirect* effect through the influence of p_2 on firm 1's output choice in stage 2. Differentiating the profit function of firm 2 with respect to p_2, we have

$$\Pi_2'(p_2) = [p_1 - e'(x)]x'(p_2) - 2\alpha_2(p_2 - p_1). \tag{1}$$

We have seen that $p_1 = p_2$ in equilibrium. Hence if firm 2 is at an *interior* equilibrium we must have $p_1 = e'(x)$.

The second derivative of firm 2's profit is

$$\Pi_2''(p_2) = [p_1 - e'(x)]x''(p_2) - x'(p_2)^2 e''(x) - 2\alpha_2 < 0.$$

Here we have used the fact that $p_1 = e'(x)$ and $x'(p_2) < 0$. Hence the second-order condition must be satisfied at any interior solution.

If $p_1 = e'(x)$, then the first order condition for production is

$$r = c'(x) + e'(x),$$

which is simply the condition for social optimality. Hence, the unique subgame perfect equilibrium to this game involves firm 1 producing the socially optimal amount of the externality. Note that this is a *strict* equilibrium in the sense that each firm's maximization problem achives a strict maximum at this equilibrium.

The only thing remaining to check is that there are no boundary solutions. Suppose that p can range between zero and $\bar{p}$. Since $p_1 = p_2$ in any equilibrium, the derivative of firm 2's profit at $p_2 = 0$ is

$$\Pi_2'(0) = -e'(x)x'(0) > 0.$$

Similarly, at $p_2 = \bar{p}$, we have

$$\Pi_2'(\bar{p}) = [\bar{p} - e'(x)]x'(\bar{p}).$$

If $\bar{p}$ is larger than the largest possible marginal externality cost, this derivative will certainly be negative.

Note that in this mechanism, firm 2 is compensated for the externality. Since the externality cost function is convex, firm 2 in fact is *overcompensated* for the externality, in the sense that the amount of money it collects exceeds the cost of the externality.

2. Discussion of the example

The intuition behind the mechanism is not particularly difficult. In the first stage each firm announces the (marginal) cost of the externality: firm 1 announces a cost that will be used to compensate firm 2, and firm 2 announces a cost that will be used to tax firm 1.

It is more or less obvious that firm 1 will never want to say the externality is not as costly as it really is, since then firm 2 will then want to set a large tax for firm 1 so as to make the level of production as small as possible. This will certainly harm firm 1.

Will firm 1 will ever want to overstate the cost of the externality by announcing that the externality is larger than it really is? If it does, the firm 2 will be overcompensated on the margin and will want set $p_2 = 0$ so that firm 1 will produce as much output as possible. But then firm 1 will have to pay a penalty based on the difference between their two reports. Hence firm 1 can only lose by exaggerating the size of the externality.

Collusion

We have seen that each firm independently has an incentive to reveal the truth in equilibrium. Furthermore, in equilibrium the government's budget balances in the sense that the compensation paid to firm 2 is just equal to the tax paid by firm 1. But out of equilibrium the budget will not necessarily balance. Is it possible that the two firms can collude in some way so as to exploit the regulator?

The sum of the profits of the two firms using the mechanism is:

$$(r + p_1 - p_2)x - c(x) - e(x) - (\alpha_1 + \alpha_2)(p_1 - p_2)^2. \tag{2}$$

Ignore the quadratic penalty term in (2) for the moment. Then without this term, the firms would like to set $p_1 = \bar{p}$ and $p_2 = 0$. That is, firm 1 would want to exaggerate the magnitude of the externality in order to encourage the regulator to pay a large compensation to firm 2. But if firm 2 gets overcompensated for the externality, it wants to report $p_2 = 0$ so as to encourage firm 1 to produce as much as possible.

However, this strategy involves making highly divergent reports. Can we use the penalty term to discourage such collusion?

The quadratic penalty is not very good for this purpose since it has a derivative of zero when $p_1 = p_2$. However an absolute value penalty works reasonably well. To see this, let the penalty term be given by $\alpha_i|p_1 - p_2|$. If we choose $\alpha_2 = 0$ the proof given earlier goes through: $p_1 = p_2$ is still the unique subgame perfect equilibrium of the game.

Suppose that we consider increasing p_1 and decreasing p_2. Then the profits of the firm increase by x, and the penalty increases by α_1. Certainly for large enough values of α_1 this will not be a profitable move for the coalition.

Strict equilibria and information costs

We have assumed that each firm has full information. It is natural to suppose that firm 2 knows the cost of the externality, but firm 1 may not know the magnitude of the costs it imposes, at least with certainty. However, let us suppose that firm 1 may learn the size of the externality by incurring some costs. The question is, will firm 1 have proper incentives to actually make this investment?

Without modeling the information acquisition in detail we cannot give a precise answer to this question. However, roughly speaking, it appears that the answer is yes. Since the efficient output is a strict equilibrium, firm 1 incurs a penalty if it announces $p_1 \neq p_2$. The size of this penalty depends on the magnitude of the α_1. The larger the value of α_1 the more incentive firm 1 has to match firm 2's announcement. Hence, firm 2 does have an incentive to invest in information acquisition.

Income effects

The above example used payoff functions that are linear in money, so that the objective functions of the agents are quasilinear utility functions. This is reasonable in the case of firm behavior, but somewhat restrictive in the case of externalities involving consumers.

However, the mechanism can easily be generalized to include income effects. Since this is a special case of the public goods problem and general resource allocation problem presented later, we defer the argument until we treat those topics.

3. Relation to the literature

There is a broad literature that is concerned with the design of games to implement desired allocations. See Laffont and Maskin (1982) and Maskin (1985) for overviews of this literature. There are three crucial aspects in the mechanism described above: first, that agents report on the type of other agents, second, that there is a penalty based on the difference in the reports, and third, that we use subgame perfection as an equilibrium concept.

Each of these features has appeared in other mechanisms available in the literature, but, to my knowledge, they have never been used together in the same way that I use them. For example, Matsushima (1988) and Palfrey and Srivastava (1989) describe mechanism where agents report on each others type. Each of these mechanisms involves penalties when the types are different.

Moore and Repullo (1989) use all three devices in their basic mechanism, but the mechanism is very complex. It is however, designed for more general problems than simply correcting externalities. I discuss the relationship with Moore and Repullo in more detail below.

4. Public goods

The externality described above is a very simple one: there are only two agents involved. It might be supposed that if property rights can be freely transferred, or other sort of negotiations can be undertaken, then it would be possible to internalize simple two-party externalities of this sort. The vast literature on the Coase theorem discusses this point in detail.

But problems involving several agents are not so easily dismissed. If the externality is a public good, then transferring property rights may be difficult or impossible. And totally unstructured negotiations may be very complex or costly to implement. However, a generalization of our mechanism may still be used effectively. We loosely describe the mechanism here, and give a more detailed description in the next section.

In the case of public goods, the relevant parameter for describing the tastes of an agent is simply his Lindahl price—his marginal rate of substitution at a particular Pareto efficient allocation. In stage 1 of our game, each agent i will simultaneously announce a "price of the public good," p_{ij}, for each other agent j. The "price" that agent j will actually face is the average of the prices chosen for him by the other agents.

In the second stage of the game, each agent j determines how much he wants to contribute to the public good based on his personalized price determined in the first stage of the game. The amount that each agent has to pay for his contribution will depend on his own contribution and on the total contributions by the other agents.

Agents recognize in the first stage that the prices that they set will influence the Nash equilibrium of the contribution game in the second stage. We will show that in the unique subgame perfect equilibrium, each agent will announce a set of Lindahl prices, and the contributions in the second stage will comprise the associated Lindahl allocation!

5. A simple mechanism for the public goods problem

There are n agents and two goods, a private good, x_i, and a public good, G. Agent i has a differentiable quasiconcave utility function $u_i(G, x_i)$ and initial wealth w_i. We assume that the demand function for the public good is a differentiable function of its price; see the appendix for discussion of this point.

A unit of the private good can be transformed into a unit of the public good on a one-for-one basis. Hence if each agent j sacrifices g_j of the private good, the utility of agent i will be $u_i(\sum_{j=1}^n g_j, w_i - g_i)$. This is not a serious restriction on the technology that generates the public good. If G is produced from the private good by a concave production function $h(x)$, then the utility function of agent i is $u_i(h(\sum_{j=1}^n g_j), w_i - g_i)$, which simply involves reinterpreting the original utility function.

A Lindahl allocation for this problem is a set of personalized prices (p_i^e) for the public good, such that the amount of the public good that each consumer i demands at his personalized price is equal to a Pareto efficient amount of the public good. Since a Pareto efficient amount of the public good satisfies the Samuelson condition that the sum of the marginal rates of substitution equals the marginal cost, we must have $\sum_{i=1}^n p_i^e = 1$.

Our mechanism for solving this public goods problem consists of two stages.

The price setting stage. Each agent i announces $n-1$ numbers p_{ij} for all $j \neq i$. The personalized price facing agent j is the average of the numbers named by the other agents:

$$p_j = \frac{1}{n-1} \sum_{i \neq j} p_{ij}.$$

Let p denote the vector of prices $(p_1, \ldots, p_n)$.

The contribution stage. Each agent i chooses a nonnegative amount g_i to contribute to the public good. If the total amount of contributions equals G, then agent i must pay an amount $p_i G + Q(p)$, where $Q(p) = \alpha(\sum_{i=1}^n p_i - 1)^2$ and α is an arbitrary (small) positive constant.

I claim that the *unique* subgame perfect equilibrium of this game will be a Lindahl allocation.

It is easy to show that the Lindahl allocation is *an* equilibrium of this game. To do this we only need to show that if every other agent announces the Lindahl prices, agent i can not increase his utility by announcing some other set of prices.

To prove this we first observe that since the Lindahl prices generate an efficient amount of the public good, G^e, since $\sum_{i=1}^n p_i^e = 1$. Hence, $Q(p) = \alpha(\sum_{i=1}^n p_i^e - 1)^2 = 0$. Suppose that agent i, say, announces some numbers that change the price vector p^e to p' and that this change results in some possibly different amount of the public good, G'. Note that the price facing agent i does not change since agent i can only affect the prices facing the other agents.

We have

$$u_i(G^e, w_i - p_i^e G^e) \geq u_i(G', w_i - p_i^e G') \geq u_i(G', w_i - p_i^e G' - Q(p')).$$

The first inequality arises from the fact that we start with a Lindahl allocation so that agent i's demand for the public good at his personalized price is the actual amount of the public good. The second inequality arises from the fact that $Q(p') \geq 0$. This shows that agent i is at least as well off announcing numbers that lead to the Lindahl prices (p_j^e) as any other numbers.

The proof that there are no other subgame perfect equilibria to the game is somewhat more complicated and is given in the next section. However, it is worthwhile giving an intuitive argument here to get a feel for the mechanism. In order to do this we make the strong assumption that each agent contributes a positive amount in the second stage of the game. This will not generally be true, and dealing with this difficulty is what generates the complications in the formal proof given below.

Let p_i be an arbitrary set of prices that do not sum to 1. In the first stage of the game agent i gets to influence agent j's price, p_j. We will show that agent i can change p_j in a way that will increase agent i's utility.

The derivative of agent i's utility with respect to p_j is

$$\frac{\partial u_i}{\partial p_j} = \frac{\partial u_i}{\partial G}\frac{\partial G}{\partial p_j} - \frac{\partial u_i}{\partial x_i}\left[p_i \frac{\partial G}{\partial p_j} + 2\alpha\left(\sum_{j=1}^{n} p_j - 1\right)\right].$$

Since by assumption agent i is contributing a positive amount in the contribution stage of the game,

$$\frac{\partial u_i}{\partial G} - \frac{\partial u_i}{\partial x_i} p_i = 0,$$

and we are left with

$$\frac{\partial u_i}{\partial p_j} = -2\alpha\left(\frac{\partial u_i}{\partial x_i}\sum_{i=1}^{n} p_j - 1\right) \neq 0.$$

Hence agent i can always change p_j in a way that will increase his utility. This completes the informal argument.

This argument shows in an intuitive way why the mechanism works. However, it is not a rigorous argument. The main problem is that in general we will not have an interior equilibrium in the contribution stage of the game. For arbitrary prices, it will typically be in the interest of some of the agents to free ride and contribute zero. However, this problem in the above argument can be patched up; there will always be *some* agent who makes a positive contribution to the public good and we only need show that he has an incentive to change his behavior if we are not at a Lindahl allocation.

In Varian (1989) I describe a related, but much simpler, mechanism that implements a Lindahl allocation in the case of quasilinear utility and two agents. In stage 1 each agent announces the rate at which they will subsidize the contributions of the other agent. In stage 2 each agent makes a contribution, paying for his own contribution at the subsidized rate, and paying the subsidy promised to the other agent. I show that the unique subgame perfect equilibrium to this game is the Lindahl allocation. Note that no penalty term is required in the case of two agents.

6. The proof that the mechanism works

We make the following assumptions about the agent's utility functions.

Assumption. The utility function $u_i(G, x_i)$ is a differentiable, quasiconcave function that is increasing in both arguments. The marginal-rate-of-substitution function, defined by

$$m_i(G, x_i) = \frac{\partial u_i / \partial G}{\partial u_i / \partial x_i},$$

is infinite when $G = 0$.

This assumption ensures that some agent will always want to contribute in the contribution stage, regardless of the prices set by the other agents. It can be relaxed significantly, but at the cost of complicating the argument.

The contribution stage

We begin by analyzing the second stage of the game. Given arbitrary prices $p_i > 0$, and arbitrary contributions by the other agents, $G_{-i} = \sum_{j \neq i} g_j$, agent i wants to solve the problem

$$\max_{x_i, g_i} u_i(G_{-i} + g_i, x_i)$$
$$\text{such that } x_i + p_i g_i = w_i - p_i G_{-i} - Q(p)$$
$$g_i \geq 0.$$

In interpreting the budget constraint, remember that each agent must pay an amount that depends on the sum of the contributions by the other agents, $p_i G_{-i}$, as well as a quadratic penalty $Q(p)$ depending on the announced prices.

If we add G_{-i} to both sides of these constraints and use the definition $G = G_{-i} + g_i$ we can rewrite this problem as

$$\max_{x_i, G} u_i(G, x_i)$$
$$\text{such that } x_i + p_i G = w_i - Q(p)$$
$$G \geq G_{-i}.$$

This is just like a standard two-good consumer maximization problem except for the inequality constraint. Although each agent i only gets to choose the level of his contribution g_i, he is effectively choosing level of the public good, since agent i can choose to contribute zero and get G_{-i} or to contribute a positive amount and get his preferred amount.

Let G_m be the maximum amount of the public good demanded by any of the agents at the prices (p_i). Then I claim that G_m must be the Nash equilibrium amount of the public good. To prove this, simply note that G_m must solve each individual's utility maximization problem.

Note that the pattern of contributions among the agents who prefer to contribute G_m is arbitrary. To see this, suppose that agents i and j want the same amount of the public good, G_m, and that they are the only two agents making contributions to the public good. Let g_i and g_j denote these contributions; of course, $g_i + g_j = G_m$. However, i's total *payment* is $p_i g_i + p_i G_{-i} = p_i(g_i + g_j) = p_i G_m$. Similarly agent j's total payment is $p_j G_m$.

It follows that any reallocation of the contributions between i and j does not affect their total payments, which only depend on G_m and their personalized prices.

This has an important consequence which we will use below. If each individual's demand for the public good is a differentiable function of the price he faces, then the *equilibrium* amount of the public good will be left-differentiable function of the price faced by any contributor.

To see this, observe that there is no difficulty with differentiability when only one agent is contributing to the public good. The only difficulties arise when two or more agents each contribute. But we have just seen that when two agents contribute, they are indifferent about reallocations of the contributions between them—since the amount that they have to pay is independent of such reallocations. Suppose that we have an equilibrium in which agents i and j are both contributing. If we increase p_i a little bit, then agent i will want to stop contributing and free-ride on agent j. However, agent j will want to increase his contribution by exactly the amount that i was contributing. Hence the same amount of the public good will be provided, and the same payments will be made by each agent.

Suppose that we decrease p_i a little bit. Then agent i will want to increase his contribution to the public good by a small amount, and agent j will no longer contribute. Since we have assumed that agent i's demand for the public good is a differentiable function of the price he faces, the equilibrium amount of the public good will only change by a small amount.

The price-setting stage

Consider now the price-setting stage of the game. We have already seen that each agent naming the Lindahl prices (p_i^e) is a Nash equilibrium.[1]

So we only need to show that at any price vector other than the Lindahl prices, some agent can increase his utility. Let (p_i^*) the the set of prices that result from the announcements (p_{ij}^*) and let G^* be the Nash equilibrium amount of the public good in the second stage of the mechanism. We have to investigate three possibles cases.

Case 1. $\sum_{k=1}^{n} p_k^* > 1$.

[1] More precisely, if each agent i names prices p_{ij} such that $\frac{1}{n-1}\sum_{i \neq j} p_{ij} = p_j^e$, then this is an equilibrium choice. All that matters is that the numbers p_{ij} named by the agents average out to the Lindahl prices. A small penalty based, say, on the variance of the p_{ij}'s would eliminate this degeneracy.

We have seen that at least one agent i must be contributing to the public good. Suppose that he changes his p_{ij}^* for some other agent j in such a way that the price facing agent j *decreases* by a small amount dp_j. This results in some change in the amount of the public good in the second stage dG, which may be zero.

The impact of this change on agent i's utility is

$$du_i = \frac{\partial u_i}{\partial G} dG - \frac{\partial u_i}{\partial x_i} \left[p_i^* dG + 2\alpha \left(\sum_{k=1}^{n} p_k^* - 1 \right) \right] dp_j.$$

We can write this expression as

$$du_i = \left[\frac{\partial u_i}{\partial G} - \frac{\partial u_i}{\partial x_i} p_i^* \right] dG - 2\alpha \left(\frac{\partial u_i}{\partial x_i} \sum_{k=1}^{n} p_k^* - 1 \right) dp_j.$$

Since agent i is contributing a positive amount in the contribution stage of the game the bracketed expression vanishes, leaving us with

$$du_i = -2\alpha \frac{\partial u_i}{\partial x_i} \left(\sum_{k=1}^{n} p_k^* - 1 \right) dp_j > 0.$$

The inequality is due to the fact that dp_j is negative and all other terms are positive. Hence a small decrease in p_j^* must increase agent i's utility.

Case 2. $\sum_{k=1}^{n} p_k^* < 1$.

Again, let i be an agent who is making a positive contribution, and now let him *increase* agent j's price by dp_j. Let dG be the associated change in the equilibrium amount of the public good. Repeating the argument given for case 1, we are left with

$$du_i = -2\alpha \frac{\partial u_i}{\partial x_i} \left(\sum_{k=1}^{n} p_k^* - 1 \right) dp_j > 0.$$

Case 3. $\sum_{k=1}^{n} p_k^* = 1$.

Suppose first that everyone is contributing to the public good in the contribution stage of the game. Then the equilibrium amount of the public good, G^*, must satisfy the n first-order conditions

$$m_i(G^*, x_i^*) = p_i^*.$$

Summing these conditions gives us

$$\sum_{i=1}^{n} m_i(G^*, x_i^*) = 1.$$

But these are the conditions that characterize the Lindahl allocation, in which case were are done.

We are left with the case where some agent i is not contributing to the public good. If agent i is just on the verge of contributing, so that $m_i(G^*, x_i^*) = p_i^*$ the argument given above applies, so we may assume that $m_i(G^*, x_i^*) < p_i^*$.

Let agent i choose announcements (p_{ij}) such that the price for each agent j who is contributing to the public good increases by a small amount and thus the equilibrium level of the public good decreases by a small amount dG. The resulting change in utility for agent i is

$$du_i = \left[\frac{\partial u_i}{\partial G} - \frac{\partial u_i}{\partial x_i} p_i^*\right] dG > 0.$$

The inequality is due to the fact that the bracketed expression is negative since $m_i(G^*, x_i^*) < p_i^*$ and dG is negative by construction. Hence agent i can increase his utility by such a move.

This completes the argument: at any prices other than the Lindahl prices, there is some agent i that can increase his utility by naming different prices.

7. Related literature

There is a large literature on mechanisms that "solve the public goods problem." A solution generally means that one can exhibit a mechanism which has Nash equilibria that are efficient allocations. This literature up until 1979 is nicely surveyed by Groves (1979).

Clarke (1971) and Groves (1976) examine the public goods problem when all agents have quasilinear utility functions and exhibit a mechanism for which the dominant strategy of each agent is to report his true valuation of the public good. However, the resulting allocation is not in general Pareto efficient.

Groves and Ledyard (1977) present a game whose Nash equilibria are Pareto efficient but not Lindahl allocations. Hurwicz (1979) and Walker (1981) have described games whose Nash equilibria are Lindahl allocations. The Hurwicz (1979) mechanism is in much the same spirit as our mechanism in that agents announce "prices" that in equilibrium turn out to be Lindahl prices. However, in the Hurwicz mechanism, each agent announces his *own* Lindahl price, not that of the other agents.

Moore and Repullo (1988) describe a mechanism whose subgame perfect equilibria implement an arbitrary choice correspondence—including the Lindahl correspondence. Moore and Repullo describe their construction in the case of quasilinear preferences, but indicate that it can be used for more general preferences.

In the Moore and Repullo mechanism, one agent announces his type and an other agent can challenge. If the second agent challenges, then we move to a subgame where agent 1 must choose between two allocations. The allocations in the subgame are chosen so that both agents will have an incentive to be truthful in the announcement and challenge stages.

This scheme has the advantage that it will work in very general environments. However, it has the disadvantage that one needs to specify the payoffs in advance for all the possible subgames. Although payoffs that satisfy the appropriate conditions will exist under general conditions, it may be quite complicated to actually specify them. In our mechanism, by contrast, the payoffs are very simple to specify.

8. A general treatment

We now consider a general treatment of the method. Suppose that there are n agents. Let x denote an allocation of k goods to the n agents. Each agent has a utility function defined over *allocations* denoted by $u_i(x)$. Since the utility functions are defined over allocations of the goods, each individual's utility can depend in an arbitrary way on other individual's consumption.

Let p_i be an nk-vector of personalized prices for agent i. Let x^* be an efficient allocation. We assume that the allocations x^* can be decentralized by a set of personalized prices, p_i^* in the sense that x^* is the solution to each agent's maximization problem

$$\begin{gathered}\max_x u_i(x) \\ \text{such that } p_i^* x \le p_i^* \omega_i.\end{gathered} \tag{3}$$

Here ω_i is agent i's initial endowment of goods. This assumption essentially requires that preferences be convex.

Given *any* set of personalized prices (p_i), denote the solution to agent i's maximization problem (3) by x_i. Note that x_i is an *allocation*, not a consumption bundle. For simplicity, assume that x_i is unique.

For an arbitrary set of personalized prices, the set of proposed allocations will not necessarily be feasible. Even if everyone proposes the same feasible alloction, it will not necessarily be efficient. Let $F(p)$ denote a measure of how far the allocations fail to be feasible, and let $E(p)$ denote a measure of by how much the allocation fails to be efficient. These functions should be differentiable, nonnegative functions that equal zero if and only if the allocations are feasible or efficient, respectively, and are positive otherwise. The quadratic forms used earlier are a natural choice.

We can now state the appropriate mechanism to solve our resource allocation problem.

Announcement stage. Each agent i announces a personalized price vector p_{ij} for each other agent j. The price announcements are averaged to construct the personalized prices facing agent j.

Consumption stage. Each agent chooses x_i to maximize his utility, given his personalized prices p_i. However, each agent must also pay a penalty of $\alpha[E(p) + F(p)]$ where p is the vector of all of the personalized prices and α is a strictly positive number.

Hence agent i's maximization problem is:

$$\begin{gathered}\max_x \ u_i(x) \\ \text{such that } p_i^* x \leq p_i^* \omega_i - \alpha[E(p) + F(p)].\end{gathered} \tag{4}$$

I claim that the unique subgame perfect equilibrium of this game is an efficient allocation.

The proof is a simple variation on the earlier arguments. Let x^* be an efficient allocation. Then, by hypothesis, this allocation is supported by a set of personalized prices (p_i^*). At the equilibrium set of (p_i^*) the penalty term $E(p^*) + F(p^*)$ equals zero. Changing his price announcements for the other agents doesn't affect agent i's price, and can only increase the value of the penalty term. Hence (p_i^*, x^*) is a subgame perfect equilibrium for the game.

Consider any other set of prices that do not minimize $E(p) + F(p)$. Then every agent has the incentive to adjust his price announcement in a way that decreases the value of this penalty term. Hence, no other set of prices can be an equilibrium of the game. This completes the argument.

Discussion of the general result

The logic behind the general mechanism is simple: we express our desired allocation as the zero of some penalty function, and then give each agent an incentive to minimize that penalty function. Stated in this way, the result is trivial. However, as we have seen earlier, this general principle can be tailored to specific resource allocation problems in a way that makes the mechanism much more useful.

For example, in the case of a simple externality discussed earlier, one firm controls the level of the externality directly. In this case there is no problem with feasibility, and we only need to ensure that the efficiency condition is met; i.e., that the marginal cost of the externality be equated across the firms.

In the case of the public goods problem, we can use the idea of a Lindahl allocation to ensure that the feasibility condition is automatically satisfied. I expect that this can be done in a large number of cases when the resource allocation problem has sufficient structure.

9. Summary

We have exhibited a class of mechanisms that implement efficient outcomes in classical environments involving externalities and public goods. Several questions remain for future research. Of course, it would be nice to have a mechanism that works when each agent only knows his own type. However, it seems unlikely that such a mechanism exists.

First, it would be nice to have a simple mechanism for the public goods case that does not require simultaneous moves. Second, it would be nice to find a mechanism that works when agents have incomplete information about the types of the other agents. Third, it would be useful to examine applications of this method to other problems of resource allocation. Finally, it would be very interesting to see how well this mechanism performs in real life. I suspect that some progress will be possible on each of these fronts.

References

Bergstrom, T., L. Blume, and H. Varian (1986), "On the Private Provision of Public Goods," *Journal of Public Economics,* 29, 25–49.

Clarke, E. (1971), "Multipart Pricing of Public Goods," *Public Choice,* 11, 17–33.

Groves, T. (1976), "Information, Incentives, and the Internalization of Production Externalities," in Lin, S. (ed.) *Theory and Measurement of Economic Externalities,* Academic Press, New York.

Groves, T. and J. Ledyard (1977), "Optimal Allocations of Public Goods: A Solution to the 'Free Rider Problem'," *Econometrica,* 45, 783–809.

Groves, T. (1979), "Efficient Collective Choice when Compensation is Possible," *Review of Economic Studies,* 227–241.

Laffont, J.-J. and E. Maskin (1982), "The Theory of Incentives: An Overview," in *Advances in Economic Theory: Invited Papers for the Fourth World Congress of the Econometric Society, 1980,* ed. by W. Hildenbrand, Cambridge University Press, Cambridge, 31–94.

Maskin, E. (1985), "The Theory of Implementation in Nash Equilibrium," in *Social Goals and Social Organization,* ed. by L. Hurwicz, D. Schmeidler, and H. Sonnenschein, Cambridge Univeristy Press, Cambridge, 173–204.

Matsushima, Hitoshi (1988), "A New Approach to the Implementation Problem," *Journal of Economic Theory,* 45, 128–144.

Palfrey, Thomas and Sanjay Srivastava (1989), "Implementation with Incomplete Information in Exchange Economies," *Econometrica*, 57, 115–134.

Hurwicz, L. (1979), "Outcome Functions Yielding Walrasian and Lindahl Allocations at Nash Equilibrium Points," *Review of Economic Studies,* 217–225.

Moore, J. and R. Repullo (1988), "Subgame Perfect Implementation," *Econometrica,* 56, 1191–1220.

Varian, H. (1989), "Sequential Provision of Public Goods," University of Michigan CREST Working Paper.

Walker, M. (1981), "A Simple Incentive Compatible Scheme for Attaining Lindahl Allocations," *Econometrica*, 48, 56–73.

Recent CREST Working Papers

89-01: Mark Bagnoli, Severin Borenstein, "Carrot and Yardstick Regulation: Enhancing Market Performance with Output Prizes," October, 1988.

89-02: Ted Bergstrom, Jeffrey K. MacKie-Mason, "Some Simple Analytics of Peak-Load Pricing," October, 1988.

89-03: Ken Binmore, "Social Contract I: Harsanyi and Rawls," June, 1988.

89-04: Ken Binmore, "Social Contract II: Gauthier and Nash," June, 1988.

89-05: Ken Binmore, "Social Contract III: Evolution and Utilitarianism," June, 1988.

89-06: Ken Binmore, Adam Brandenburger, "Common Knowledge and Game Theory," July, 1989.

89-07: Jeffrey A. Miron, "A Cross Country Comparison of Seasonal Cycles and Business Cycles," November, 1988.

89-08: Jeffrey A. Miron, "The Founding of the Fed and the Destabilization of the Post-1914 Economy," August, 1988.

89-09: Gérard Gaudet, Stephen W. Salant, "The Profitability of Exogenous Output Contractions: A Comparative-Static Analysis with Application to Strikes, Mergers and Export Subsidies," July, 1988.

89-10: Gérard Gaudet, Stephen W. Salant, "Uniqueness of Cournot Equilibrium: New Results from Old Methods," August, 1988.

89-11: Hal R. Varian, "Goodness-of-fit in Demand Analysis," September, 1988.

89-12: Michelle J. White, "Legal Complexity," October, 1988.

89-13: Michelle J. White, "An Empirical Test of the Efficiency of Liability Rules in Accident Law," November, 1988.

89-14: Carl P. Simon, "Some Fine-Tuning for Dominant Diagonal Matrices," July, 1988.

89-15: Ken Binmore, Peter Morgan, "Do People Exploit Their Bargaining Power? An Experimental Study," January, 1989.

89-16: James A. Levinsohn, Jeffrey K. MacKie-Mason, "A Simple, Consistent Estimator for Disturbance Components in Financial Models," April 25, 1989.

89-17: Hal R. Varian, "Sequential Provision of Public Goods," July, 1989.

89-18: Hal R. Varian, "Monitoring Agents with Other Agents," June, 1989.

89-19: Robert C. Feenstra, James A. Levinsohn, "Distance, Demand, and Oligopoly Pricing," July 17, 1989.

89-20: Mark Bagnoli, Shaul Ben-David, Michael McKee, "Voluntary Provision of Public Goods," August, 1989.

89-21: N. Gregory Mankiw, David Romer, Matthew D. Shapiro, "Stock Market Forecastability and Volatility: A Statistical Appraisal," August, 1989.

89-22: Arthur J. Robson, "Efficiency in Evolutionary Games: Darwin, Nash and the Secret Handshake," 1989.

89-23: Mark Bagnoli, Ted Bergstrom, "Log-Concave Probability and Its Applications," September 7, 1989.

89-24: Gérard Gaudet, Stephen W. Salant, "Towards a Theory of Horizontal Mergers," July, 1989.

89-25 (evolved from 87-35): Stephen W. Salant, Eban Goodstein, "Predicting Committee Behavior in Majority-Rule Voting Experiments," July, 1989.

89-26: Ken Binmore, Martin J. Osborne, Ariel Rubinstein, "Noncooperative Models of Bargaining," 1989.

89-27: Avery Katz, "Your Terms or Mine? The Duty to Read the Fine Print in Contracts," February 19, 1989.

Recent CREST Working Papers

90–01: Hal R. Varian, "A Solution to the Problem of Externalities and Public Goods when Agents are Well–Informed," January 17, 1990.

90–02: Hal R. Varian, "Sequential Provision of Public Goods," January 17, 1990.

90–03: Hal R. Varian, "Goodness-of-Fit Optimizing Models," December 11, 1989.

90–04: Paul N. Courant, Alan Deardorff, "International Trade with Lumpy Countries," August 22, 1989.

90–05: Dilip K. Bhattacharyya, Jan Kmenta, "Testing Hypotheses about Regression Coefficients in Misspecified Models," December 15, 1989.

90–06: Greg Shaffer, "Slotting Allowances and Resale Price Maintenance: A Comparison of Facilitating Practices," November 1989.

90–07: Michelle J. White, "Why are Taxes So Complex and Who Benefits?" January 19, 1990.

90–08: Daniel P. O'Brien, "Endogenous Price Leadership: A Bargaining Model of International Telecommunications Settlements," May 1989.

90–09: Mark Bagnoli, Barton L. Lipman, "Stock Price Manipulation Through Takeover Bids," November 1989.

90–10: Ken Binmore, "Bargaining Theory Without Tears," December 1, 1989.

90–11: Hyun Song Shin, "Non-Partitional Information On Dynamic State Spaces and the Possibility of Speculation," December 1989.