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Differences of Opinion and the Volume of Trade

by

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Abstract. The standard models of financial markets assume that agents have identical probability beliefs but different utility functions. The volume of trade in such models is therefore due to these taste differences. In this paper I investigate the implications of reversing this setup: I assume that tastes are identical but probability beliefs differ across the agents. In this sort of model the volume of trade depends only on the differences of opinion, even in the presence of differential information. This is demonstrated in two models: a mean–variance model and an Arrow–Debreu model, which are closely related to earlier models by Grossman and Milgrom–Stokey. Having established the importance of differences of opinion I then consider their impact on asset markets in an Arrow–Debreu equilibrium. If risk tolerance grows less rapidly than it does in the case of logarithmic utility then increased dispersion of opinion will be associated with lower prices and higher volume of trade in equilibrium. If risk tolerance grows more rapidly than it does with logarithmic utility then increased dispersion of opinion will be associated with higher prices but the volume of trade may increase or decrease.

DIFFERENCES OF OPINION AND THE VOLUME OF TRADE

by
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"It's differences of opinion that make horse races" ... but you wouldn't know it from the literature on financial economics. Indeed the standards models of finance such as the Sharpe–Lintner mean variance model or the Rubinstein–Breeden–Litzenberger contingent consumption model both assume more–or–less homogeneous probability beliefs.[1]

There has been some work on extending the mean–variance model to allow for differences in beliefs across agents; see Cragg and Malkiel (1982), Jarrow (1980), Lintner (1969), Mayshar (1983), and Williams (1977). Differences in beliefs in contingent commodities models have received much less attention. The major references are Rubinstein (1975), (1976), Breeden–Litzenberger (1978), and Milgrom–Stokey (1982).

In this paper we consider two simple models of the effect of differences of opinion on asset prices and volume, with particular emphasis on the role of differential information. The clarification of the roles of differential *opinions* and differential *information* is one of the major themes of this paper. The first model is a simple mean variance model with differential information of the sort studied by Grossman (1976), (1978). The second is an Arrow–Debreu contingent consumption model of the sort studied by Milgrom and Stokey (1982). The message that emerges from each model is the same – it's differences of opinion that cause trade, not differences in public or private information.

Having established the importance of differences of opinion for trade, we then examine some of its consequences for asset market equilibrium. In the context of an Arrow–Debreu equilibrium, we show that if risk tolerance grows less rapidly than in the case of logarithmic utility, then more dispersed opinions will be associated with lower prices and more volume of trade. If this condition is not met, then more dispersion of opinion leads to higher prices and the difference in the volume of trade is ambiguous.

1. A Mean–Variance Model

We consider a simplified version of a model of the sort studied by Grossman (1976), (1978). There are two assets, one with an unknown payoff and one with a certain payoff. We let v denote the fixed but unknown value of the risky asset next period.

Each investor i has a subjective prior distribution for the value of the risky asset which we take to be Normally distributed with with mean v_i and precision α. (The precision of a random variable is the reciprocal of the variance.) In addition each investor observes some information which allows him to form an estimate of value of the asset.

In particular, we suppose that each agent i observes a signal V_i where:

$$V_i = v + \epsilon_i$$

and ϵ_i is IID Normal with mean zero and precision ω. Each agent has a different estimate of the value of the asset, but all agents have estimates of the same precision. Of course the information

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[1] More precisely, the mean variance model requires that agents agree on the mean returns and covariance matrix of the assets; the contingent consumption model requires that agents agree on the probability of occurrence of different levels of aggregate consumption.

set of agent i is not limited to just this piece of information; he or she will also use the market price in forming a final estimate of the value of the asset.

Given the n observations (V_i) an omniscient observer could form an estimate of the sample mean

$$\bar{V} = \sum_{i=1}^{n} \frac{V_i}{n}$$

which will have a Normal distribution with mean v and precision $\beta = n\omega$. In this model the equilibrium market price turns out to be fully revealing, so that each agent will be able to calculate $\bar{V}$ in equilibrium.

Being good Bayesians, each investor will then form a posterior distribution for value of the risky asset by combining his prior and his sample information in accord with Bayes' law. Given the distributional assumptions we have made, the posterior distribution of investor i will be Normal with precision of $\gamma = \tau + n\omega$ and mean of

$$\frac{\alpha}{\gamma} v_i + \frac{\beta}{\gamma} \bar{V}.$$

We assume that each investor has a constant absolute risk tolerance utility function with the *same* coefficient of risk tolerance a. In this case it can be shown that the demand function for the risky asset for each agent will have the linear form:

$$D_i = \frac{a(E(v|I_i) - P)}{\text{var}(v|I_i)}$$

where $E(v|I_i)$ is the expected value of the asset conditional on the information set of agent i, $\text{var}(v|I_i)$ is the variance of the asset conditional on i's information, and P is the market price.

Inserting the expressions for the expected value and the variance, the demand of agent i reduces to:

$$D_i = a(\alpha v_i + \beta \bar{V} - \gamma P).$$

Each agent has a supply S_i of the risky asset, and we suppose that the market price P adjusts so as to equate aggregate demand and supply which gives us:

$$P^* = \frac{\alpha \bar{v} + \beta \bar{V} - \bar{S}/a}{\gamma}$$

where bars over a variable denote the sample mean.

Let us consider the information structure in this market more closely. Suppose that all agents agree on the value of the *information*, (V_i), observed by the other agents but disagree about the value of the other agents' opinions, (v_i). Then knowing the average prior mean, $\bar{v}$, would not lead any agent to revise his own posterior mean, but knowing the average *information* $\bar{V}$ would induce an agent to change his posterior mean. That is, each agent would agree that, $\bar{V}$ would be a superior estimate of the true value of the asset compared to any agent's individual information V_i. The distinction being drawn here is the distinction between the other agents' *opinions* — which do not change a given agent's views — and other agents' *information* — which does change a given agent's views.

But, if each consumer knows the value of the coefficients in the aggregate demand function, he or she can estimate the value of $\bar{V}$ from the observed equilibrium price and the formula:

$$\bar{V} = \frac{\bar{S}/a + \gamma P^* - \alpha\bar{v}}{\beta}.$$

As in the original Grossman (1976) model, the equilibrium price aggregates all of the information in the economy and thus provides a superior estimate of the "true" expected value of the asset.

In Grossman's original paper all agents had the same prior beliefs but different tastes. In that paper he argues for the rational expectations equilibrium by appealing to a long run equilibrium where agents can tabulate the empirical distribution of the $(P^*, \bar{V})$ pairs and thus infer $\bar{V}$ from observations of P^*. The agents do not have to know the structure of the model, but only the reduced form relationship between P^* and $\bar{V}$.

This sort of argument does not work here. For if the events were repeated a large number of times the different prior beliefs would tend to converge to identical posterior beliefs. Instead we must assume that agents have some understanding of the structure of the model and are able to disentangle the information contained in the market price directly rather than to simply use a reduced form model. Thus this model with different prior opinions demands even more "rational expectations" than does Grossman's model.

Inserting the "rational expectations" estimate of $\bar{V}$ into the demand function for agent i and simplifying we have the equilibrium demand of agent i:

$$D_i = a\alpha(v_i - \bar{v}) + \bar{S}$$

Note the interesting feature of this equation: all of the information variables have dropped out — an agent's trade in equilibrium is determined solely by the deviation of his or her opinion from the average opinion.

The size of agent i's position in the risky asset depends positively on his risk tolerance and on his *prior* precision. This latter point is somewhat surprising. The sample precision of $\beta = n\omega$ may be much larger than the prior precision α, and one might have though that for large n it would have swamped the prior precision. Nevertheless the equilibrium demand for the risky asset is exactly the same as if the agent had no sample information at all! The explanation of this seeming paradox is that the market price adjusts to reveal all information in the economy and thus eliminates the value of the sample information to any one agent.

The sample information will affect agents' expected utility, even if it doesn't affect their behavior. The posterior expected utility will be higher if the sample information tends to confirm the prior information, and lower otherwise.

What are the implications for the volume of trade? Agent i's net demand for the risky asset is

$$T_i = a\alpha(v_i - \bar{v}) - (S_i - \bar{S})$$

so that in equilibrium volume depends on both the differences in opinions and the differences in endowments. Suppose for simplicity that each agent has an identical endowment $\bar{S}$ of the risky asset. Then the net trade of agent i is simply $T_i = a\alpha(v_i - \bar{v})$ and the overall volume of trade is given by:

$$\sum_{i=1}^{n} a\alpha|v_i - \bar{v}|/2$$

which clearly depends only on the 'differences of opinion.' An increase in the dispersion of opinion, as measured by the sum of the absolute deviations of the priors, will necessarily increase the volume of trade, regardless of the private information received by the market participants.

The same point can be made in a slightly different way that will tie in nicely with the Milgrom–Stokey results to be described in section 3. Suppose that agents initially have no information and trade only on the basis of their priors. Then the equilibrium net trade of agent i is simply $a\alpha(v_i - \bar{v})$ as given above. Now new "private" information arrives. According to the above model no new trade will take place because of the arrival of this new information.

Of course the equilibrium *price* will respond to arrival of information; if $\bar{V}$ is large in some particular realization then the market price will be large. Indeed, the price responds in such a way as to perfectly offset the desire to trade. But it has to be that way in a fully revealing equilibrium — if the price reveals all information, there is nothing left to be revealed by the volume of trade. Thus the volume of trade cannot depend in any independent way on the information signals received by the agents.[2]

In Grossman's original model all agents had identical priors but different risk tolerances; the equilibrium volume was therefore due solely to the differences in tastes. In the model above the agents all have the same tastes, but different priors, and the equilibrium volume depends only on the taste differences. What if agents have different tastes and different opinions? In this case it can be shown that the equilibrium trade of agent i is:

$$D_i = a_i\alpha(v_i - \sum_{j=1}^{n} w_j v_j) + w_i S$$

where

$$w_i = \frac{a_i}{\sum_{j=1}^{n} a_j}.$$

The volume of trade still depends on the deviation of agents opinion from the average opinion, but now it is a weighted average rather than the simple average we had before.

2. Differences in Interpretation

The no–trade result may appear to fly in the face of common sense; certainly the arrival of new information in real markets may contribute to volume. It seems that this view is held by those who have examined the empirical determinants of the volume of trade. For example:

"One reason to suggest a relationship between changes in price and transactions volume is that both are related to the type and flow of information in the market. New information can simultaneously spur trading and lead to new equilibrium prices." Karpoff (1985)

Casual empiricism (i.e. the nightly news) suggests that volume does react to new information. How can we reconcile this observation with the model described above?

One assumption of the above model was that all agents interpreted the information in the same way. That is, all agents agree that the observation of V_i contributed to their posterior estimate of v in the same way.

[2] Of course this model leads directly to the Grossman paradox as well: if the market price reveals all information, why does anyone bother to acquire information? Various resolutions of this paradox can be found in Grossman–Stiglitz (1980), Hellwig (1980), and Diamond–Verrecchia (1981).

For some kinds of information this may be plausible; but for other types it may be quite implausible. If OPEC were to break up tomorrow would everyone agree on the impact of this event on all asset prices? When Apple introduces a new computer line does everyone agree on the impact of this product on the market value of Apple stock?

We might model this by distinguishing between the *arrival* of the information and the *interpretation* of the information. Let Y_i be a random variable denoting the "magnitude" of a piece of information, and let $V_i = \delta_i Y_i$ be agent i's estimate of the impact of this information on the mean value of the risky asset. That is, agent i views $\delta_i Y_i$ as a signal about the value of the risky asset in the sense that he or she believes the model:

$$V_i = \delta_i Y_i = v + \epsilon_i.$$

All agents agree on the magnitude of the information Y_i but the "interpretation" of the information, δ_i, differs from agent to agent. Let us assume that δ_i and Y_i are distributed independently across the set of agents. Then $\bar{V} = \bar{\delta}\bar{Y}$ and the equation for the equilibrium price given above becomes:

$$P^* = \frac{\alpha\bar{v} + \beta\bar{\delta}\bar{Y} - \bar{S}/a}{\gamma}.$$

If we assume that the average value of δ is common knowledge, each agent can solve the above equation for $\bar{Y}$ to give:

$$\bar{Y} = \frac{\bar{S}/a + \gamma P^* - \alpha\bar{v}}{\beta\bar{\delta}}.$$

Thus after observing the equilibrium price of the asset, agent i should revise his own estimate of the expected value of the risky asset to be $\delta_i \bar{Y}$ which leads to an equilibrium demand of the form

$$D_i = a[\alpha v_i + \beta\delta_i\bar{Y} - \gamma P].$$

Substituting for $\bar{Y}$ we find:

$$D_i = a[\alpha v_i + (S/a + \gamma P^* - \alpha\bar{v})\delta_i/\bar{\delta} - \gamma P^*].$$

Substituting once more for the equilibrium price P^* we have:

$$D_i = a[\alpha(v_i - \bar{v}) + \beta(\delta_i - \bar{\delta})\bar{Y}] + \bar{S}.$$

The interpretation of this equation is rather nice: the final demand of the agent turns out to be a weighted average of his deviation from the mean opinion in both his prior beliefs and in his *interpretation* of the information. In this model, the agents can use the market price to estimate whether or not the events have occurred, but then put their own interpretation on the events themselves. In equilibrium the arrival of the information affects the volume of trade, but only through the differences of opinion about how the information should be interpreted.

3. An Arrow–Debreu Model

We now consider a contingent–consumption model which is a simplification of the model described in Milgrom–Stokey (1980). Let $s = 1, \ldots, S$ index states of nature and $i = 1, \ldots, n$ index the economic agents. Then $\pi_i(s)$ is agent i's prior probability belief about the occurrence of state s, and c_{is} is agent i's contingent consumption in state s. Finally, let $u_i(c)$ be agent i's vonNeuman–Morgenstern utility function and p_s the Arrow–Debreu price for consumption in state s.

We suppose that all agents trade to an equilibrium which will be characterized by the first order conditions for utility maximization:

$$\pi_i(s)u_i'(c_{is}) = \lambda_i p_s$$

where λ_i is agent i's marginal utility of wealth.

Now suppose that each agent observes a private signal y_i which is a realization of a random variable that affects the probability of occurrence of state s. We suppose that if agent i could observe the signals of all the agents, $y = (y_1, \ldots, y_n)$, he or she would calculate the probability of occurrence of state s via Bayes' law, and that all agents agree on the form of the likelihood function, which we denote by $\pi(y|s)$.

After each agent observes his private signal, markets are reopened and all agents can revise their contingent consumption plans. Given the new information we would expect that prices of the various states would change. As in the model presented above the agents can take these price changes into account and use them to infer something about what signals the other agents observed. What will the new post–information equilibrium look like?

Given this (simplified) framework, Milgrom–Stokey establish a remarkable result: there will always be a post–information equilibrium that reveals all the information observed by all the agents, and furthermore, the new equilibrium involves no trade by any agent.[3]

The proof is simply to simply consider what happens if the price of state s consumption changes to $\hat{p}_s = \pi(y|s)p_s$. In this case, each agent can extract the likelihood function $\pi(y|s)$ by dividing the post–information price by the pre–information price and use this likelihood to revise his or her prior probability via Bayes' law. The resulting equilibrium is characterized by the first order conditions:

$$\pi_i(s|y)u_i'(c_{is}) = \frac{\pi(y|s)\pi_i(s)}{\pi_i(y)}u_i'(c_{is}) = \frac{\lambda_i}{\pi_i(y)}\pi(y|s)p_s = \hat{\lambda}_i\hat{p}_s.$$

Unlike the Grossman model where the *level* of the price of the risky asset was a sufficient statistic for all the agents' information, here the *change* in the prices is a sufficient statistic to calculate the economy–wide likelihood function.

If agents recognize the information conveyed by the price changes they will presumably take it into account when they calculate their posterior probabilities. Thus we will write the likelihood function of an agent as depending on both their private information and the market price vector: $\pi_i(y_i, \hat{p}|s)$. (The initial price vector p can also be an argument but we suppress it so as not to clutter the notation.)

Just to fix ideas, let us consider a simple illustration of these ideas. Consider a futures market in wheat with Arrow–Debreu contracts that pay off based on the aggregate production. However, the aggregate production of wheat is influenced by the weather. In the pre–information equilibrium, each agent takes a position in the wheat market based on his prior probability assessments concerning the weather, and this determines some initial pattern of holdings of contingent consumption. Each agent then observes the weather in his own neighborhood. If each agent could observe the weather on every farm, each would agree on how the entire weather pattern would affect the wheat harvest.

[3] Related insights can be found in Marshall (1974), Rubinstein (1975), Hakansson, Kunkel and Ohlson (1982) and Verrecchia (1981).

Now markets are reopened and agents can revise their portfolios based on their own observations of their local weather and the new market price of wheat in the various states of nature. The Milgrom–Stokey result says that there is a new equilibrium where no trade occurs and where the price changes reveal the aggregate information of all the agents.

This is a very striking example of how the arrival of information can have large effects on prices, but have no effect at all on trade. But as before, it seems from casual empiricism that it might prove too much. If, in practice, the arrival of new information does lead to trade, something must be wrong with the model.

Again, one could question the assumption of a common likelihood function. If agents have different opinions about the effect of some given piece of information on the probability of occurrence of a state of nature, the Milgrom–Stokey result need no longer hold. Arrival of new information will in general cause agents to trade in this case, because agents *interpret* the information differently, not because it is different information.

In the context of the wheat example, if agents disagree about the effect of the weather on wheat, then information that they observe about the weather will cause them to trade. But the resulting trade is due to differences in opinion, not differences in information.

This observation can be sharpened; Milgrom–Stokey show that if everyone has the same likelihood function there will always exist an equilibrium which is fully revealing and which involves no trade. But there is a partial converse: if there is no trade after information is distributed, then all agents must have essentially the same likelihood function and equilibrium will be fully revealing. So the relationship between differences of opinions and the volume of trade is an if and only if relationship — common opinions about how information affects state probabilities means zero volume, and zero volume means essentially common opinions.

This proposition is a small extension of arguments given by Rubinstein (1975, Nonspeculation Condition), Milgrom–Stokey (1982, Theorem 3), and Hakansson, Kunkel and Ohlson (1982, Lemma 2). However, it seems worthwhile to give a brief exposition of the result.

Following Hakansson, Kunkel and Ohlson (1982), we will say that likelihood functions are *essentially homogeneous* if

$$\pi_i(y_i, \hat{p}|s) = k_1(y_i, \hat{p})\pi_1(y_i, \hat{p}|s)$$

for all $i = 1, \ldots, n$. If two likelihood functions are essentially homogeneous, then Bayes' law implies that they will generate the same posterior probabilities, since the $k_i(y_i, \hat{p})$ terms will cancel out from the numerator and the denominator. Thus agents that have essentially homogeneous likelihood functions will make the same inferences from the information they observe.

Theorem. *Let (c_{is}, p_s) be an equilibrium before the information y is revealed and let $(c_{is}, \hat{p}_s)$ be a no-trade equilibrium after y_i is revealed to each agent. Then the value of each agent's likelihood function is given by:*

$$\pi_i(y_i, \hat{p}|s) = k_i(y_i, \hat{p})\frac{\hat{p}_s}{p_s}$$

so that agents' likelihood functions must be essentially homogeneous.

Proof. Choose two states, s and t. Since the economy is in equilibrium before and after the information is revealed, we must have:

$$\frac{\pi_i(s)u_i'(c_{is})}{\pi_i(t)u_i'(c_{it})} = \frac{p_s}{p_t}$$

and

$$\frac{\pi_i(s|y_i,\hat{p})u_i'(c_{is})}{\pi_i(t|y_i,\hat{p})u_i'(c_{it})} = \frac{\hat{p}_s}{\hat{p}_t}.$$

By Bayes' law we can write the latter expression as:

$$\frac{\pi_i(y_i,\hat{p}|s)\pi_i(s)u_i'(c_{is})}{\pi_i(y_i,\hat{p}|t)\pi_i(t)u_i'(c_{it})} = \frac{\hat{p}_s}{\hat{p}_t}.$$

Dividing by the first equation gives:

$$\frac{\pi_i(y_i,\hat{p}|s)}{\pi_i(y_i,\hat{p}|t)} = \frac{\hat{p}_s/p_s}{\hat{p}_t/p_t}.$$

This implies that:

$$\pi_i(y_i,\hat{p}|s) = k_i(y_i,\hat{p})\frac{\hat{p}_s}{p_s}$$

for all s as required. []

As indicated above, this result is essentially a corollary to Theorem 3 in Milgrom–Stokey (1982). The focus however is different; Milgrom–Stokey show that if all agents have the same likelihood function, then in a post–information, no trade equilibrium, each agent's posterior probability must depend only on the post–information prices and be independent of his private signal. We show something a bit different: that *if* there is no trade in a post–information equilibrium, then all agents must have essentially the same likelihood value, and the equilibrium price change must reveal this common likelihood value. Hakansson, Kunkel and Ohlson (1982) have essentially the same result in a somewhat different setting.

The result is also closely related to Rubinstein's (1975) Nonspeculation Condition. Although Rubinstein's model is rather different than the Milgrom–Stokey model, the results are similar, and some of them carry over directly, at least with some reinterpretation. Rubinstein considers a three period model. In period 0, agents decide how many Arrow–Debreu securities to purchase that will pay off in the various states of nature that can occur in periods 1 and 2. In period 1, a state of nature is revealed and this may change the conditional probabilities that agents attach to outcomes in period 2. Agents can then use the proceeds from their period 1 investments to purchase different period 2 securities. Of course the price of the period 2 securities will change once the period 1 information is revealed, and Rubinstein is interested in determining when this sort of price change will eliminate incentive to trade. Under this interpretation, Rubinstein's model is like the Milgrom–Stokey model, except the signal is publically observable and agents are allowed to trade on it. In terms of the wheat example described above, we now have a market for the weather — i.e., we have contingent securities that pay off depending on the state of the weather.

Let us index the period 2 states by s and index the period 1 states by y. We are thinking of the period 1 states as "signals" in the Milgrom–Stokey sense, and use the same notation as before to emphasize this. The period 0 prices for Arrow–Debreu securities that pay off in period 1 will be denoted by p_y, and those those that pay off in period 2 will be denoted by p_s. After the information is revealed in period 1, the prices for securities that payoff in period 2 will change. Rubinstein denotes the post–information prices by $p_{s\cdot y}$, but they are the same as the prices $\hat{p}_s$ we defined earlier, since there is no time–preference in our model.

Rubinstein presents a condition which he calls the "Nonspeculation Condition" which he shows is a necessary and sufficient condition for no–trade. Using the notation established above, Rubinstein's condition can be stated as:

$$\frac{\pi_i(s)}{p_s} = \frac{\pi_i(y)\pi_i(s|y)}{p_y p_{s\cdot y}}.$$

We can rearrange this expression to get:

$$\frac{\pi_i(s|y)\pi_i(y)}{\pi_i(s)} = \frac{p_y p_{s\cdot y}}{p_s}.$$

But the laws of conditional probability show that the left hand side of this expression is just the likelihood function, $\pi_i(y|s)$. Hence:

$$\frac{\pi_i(y|s)}{p_y} = \frac{p_{s\cdot y}}{p_s} = \frac{\hat{p}_s}{p_s}.$$

Thus in period 1, after the information y is revealed and the period 2 prices equilibrate, each agent's likelihood function can be recovered by using only the publically observable variables, $\hat{p}_s$, p_s and p_y. As before, no trade implies no differences of opinion.

4. The Effect of Differences of Opinion on Volume and Price

The results of the last section suggest that the volume of trade in an Arrow–Debreu model is due primarily to the differences of opinion. In a one period model, these can be differences of opinion about the prior probabilities; in a two period model, trade requires differences of opinion about likelihood functions — that is, the interpretation of information.

How far can this insight be pushed? If two assets are otherwise identical but one has "more diverse" opinions, which asset will have the larger volume of trade? Which asset will have the higher price?

Varian (1985) has described some results concerning relative pricing in this framework; here we review these results and consider the implications for the volume of trade.

In order to do this, we must specialize the model by assuming that all agents have the same utility function. Letting $f(\cdot)$ be the inverse of $u'(\cdot)$ we can write the first order conditions as:

$$c_{is} = f(\lambda_i p_s/\pi_i(s))$$

where $\pi_i(s)$ is now interpreted as either a prior or a posterior probability. Let us define $q_{is} = \pi_i(s)/\lambda_i$ to be a "weighted" probability for agent i. In general we would expect wealthier agents to have lower marginal utilities of wealth in equilibrium so that their beliefs will get a higher weight in the q_{is} expression. (It is easy to show that if two agents have the same probability beliefs and the same utility function, the wealthier one will have a lower marginal utility of wealth.)

Then we can sum the first order conditions across the agents to derive an expression for aggregate consumption in state s:

$$C_s = \sum_{i=1}^{n} c_{is} = \sum_{i=1}^{n} f(p_s/q_{is}).$$

It is shown in Varian (1985) that $f(\cdot)$ is always an increasing function of q_{is}. It will be a concave or convex function of q_{is} as $r'(c)$ is greater or less than $-r(c)^2$ where $r(c) = -u''(c)c/u'(c)$ is the Arrow–Pratt measure of relative risk aversion. This condition is much more nicely expressed as requiring that the derivative of risk tolerance is less than 1. It is easily checked that the derivative of risk tolerance for a logarithmic utility is precisely one, so we can conclude that $f(\cdot)$ will be a concave function of q_{is} when risk tolerance increases less rapidly than it does in the case of logarithmic utility. This seems like a very natural assumption.

Following Varian (1985) we now consider two different states, s and t, such that $C_s = C_t$, but the probability beliefs for state t are a mean preserving spread of the probability beliefs for state s in the sense of Rothschild–Stiglitz (1970). It follows from the strict concavity of $f(\cdot)$ that:

$$\sum_{i=1}^{n} f(p_s/q_{it}) < \sum_{i=1}^{n} f(p_s/q_{is}) = C_s = C_t = \sum_{i=1}^{n} f(p_t/q_{it})$$

and therefore $p_t < p_s$.

Thus increased dispersion of beliefs should be associated with lower prices in equilibrium. What about the volume of trade?

In order to get a result here we must specialize the model some more. Since the final volume depends on the pattern of initial endowments across the agents we will further suppose that assets s and t have the same pattern of initial endowments: $\bar{c}_{is} = \bar{c}_{it}$ for all $i = 1, \ldots, n$. Suppose that we consider only a mean preserving spread that does not decrease q_{is} for any agent who is a net purchaser of the asset. That is, we assume that:

$$q_{it} \geq q_{is} \quad \text{for all } i \quad \text{such that } c_{is} > \bar{c}_{is}.$$

Then we have the situation illustrated in Figure 1. Here we have plotted the weighted probabilities q_{is} on the horizontal axis, and $c_{is} = f(p_s/q_{is})$ on the vertical axis. When we consider the movement from q_{is} to q_{it} we see that:

$$f(p_s/q_{is}) \leq f(p_s/q_{it}) < f(p_t/q_{it})$$

for all net purchasers of the asset where the first inequality follows from the monotonicity of $f(\cdot)$ and the second from the fact that $p_t < p_s$ which we established earlier. Since all of the net purchasers of the asset are now buying more, the net sellers of the asset must be selling more, and the volume of the trade is therefore larger. The volume of trade has increased for two reasons: each net purchaser wants to buy more at the same price, and the price decrease makes him or her want to buy even more again.

If it appears obvious that making buyers more optimistic will increase the volume of trade, it might be worth considering the case where $f(\cdot)$ is a *convex* function of q_{is}. For example take constant relative risk averse utility functions with $\rho < 1$. Then increasing the dispersion of opinion will *increase* asset prices, so the two effects described above work in opposite directions and the volume of trade can go either way. We give an example of this below.

5. Constant Relative Risk Aversion

The case of constant relative risk averse utility functions, $u(c_{is}) = c_{is}^{1-\rho}/(1-\rho)$, serves as a convenient example of the above discussion. The first order conditions take the form:

$$c_{is}^{-\rho} = \frac{p_s}{q_{is}}$$

so that

$$c_{is} = \left(\frac{q_{is}}{p_s}\right)^{\frac{1}{\rho}}.$$

This in turn implies

$$C_s = \sum_{j=1}^{n} \left(\frac{q_{js}}{p_s}\right)^{\frac{1}{\rho}}$$

and

$$p_s = C_s^{-\rho}\left(\sum_{j=1}^{n} q_{js}^{\frac{1}{\rho}}\right)^{\rho}.$$

Inserting this into the first equation, we see that the equilibrium demand of agent i takes the form:

$$c_{is} = w_{is} C_s$$

where

$$w_{is} = \frac{q_{is}^{\frac{1}{\rho}}}{\sum_{j=1}^{n} q_{js}^{\frac{1}{\rho}}}.$$

Thus the demand of agent i is completely determined by his weighted probability, relative to the weighted probabilities of the other agents.

We can use this expression to present the example promised above. Suppose that $\rho = .5$, $C_s = C_t = 1$ and (q_{is}) and (q_{it}) are as given in Table 1. It is straightforward to calculate the net trades (x_{is}) and (x_{it}) using the above expression. State s has a volume of .200 and state t has a volume of.190. Even though the dispersion of opinion has increased, and no net purchaser has become more pessimistic, the volume of trade has decreased.

Table 1. Example of decreased volume of trade

	q_{is}	q_{it}	$\bar{c}_{is}$	x_{is}	x_{it}
1	.250	.200	.300	−.050	−.143
2	.250	.300	.400	−.150	−.047
3	.250	.250	.100	.150	.145
4	.250	.250	.200	.050	.045

REFERENCES

Breeden, D. and M. Litzenberger (1978), "Prices of State–Contingent Claims Implicit in Option Prices," *Journal of Business*, **51**, 621–51.

Cragg, J. and B. Malkiel (1982), *Expectations and the Structure of Share Prices*, University of Chicago Press, Chicago.

Diamond, D. and R. Verrecchia (1981), "Informational Aggregation in a Noisy Rational Expectations Economy," *Journal of Financial Economics*, **9**, 221–235.

Grossman, S. (1976), "On the Efficiency of Competitive Stock Markets Where Traders Have Diverse Information," *The Journal of Finance*, **31**, 573–585.

Grossman, S. (1977), "The Existence of Futures Markets, Noisy Rational Expectations, and Informational Externalities," *Review of Economic Studies*, **44**, 431–449.

Grossman, S. (1978), "Further Results on the Informational Efficiency of Competitive Stock Markets," *Journal of Economic Theory*, **18**, 81–101.

Grossman, S. and J. Stiglitz (1980), "On the Impossibility of Informationally Efficient Markets," *American Economic Review*, **70**, 393–408.

Jarrow, R. (1980), "Heterogeneous Expectations, Restrictions on Short Sales, and Equilibrium Asset Prices," *Journal of Finance*, **35**, 1105–13.

Hakansson, N., J. Kunkel and J. Ohlson (1982), "Sufficient and Necessary Conditions for Information to have Social Value in Pure Exchange," *Journal of Finance*, **37**, 1169–1181.

Hellwig, M. (1980), "On the Aggregation of Information in Competitive Markets," *Journal of Economic Theory*, **22**, 477–498.

Karpoff, J. (1985), "The Relationship between Stock Prices and Volume," *Center for the Study of Banking and Financial Markets Digest*, University of Washington, Seattle.

Lintner, J. "The Aggregation of Investors' Diverse Judgements and Preferences in Purely Competitive Markets," *Journal of Financial and Quantitative Analysis*, **4**, 347–400.

Mayshar, J. (1983), "On Divergence of Opinion and Imperfections in Capital Markets", *American Economic Review*, **73**, 114–28.

Marshall, J. (1974), "Private Incentives and Public Information," *American Economic Review*, **64**, 373–390.

Milgrom, P. and N. Stokey (1982), "Information, Trade and Common Knowledge," *Journal of Economic Theory*, **26**, 17–27.

Rothschild, M. and J. Stiglitz (1970), "Increasing Risk I: a Definition," *Journal of Economic Theory*, **2**, 225–243.

Rubinstein, M. (1975), "Security Market Efficiency in an Arrow–Debreu Economy," *American Economic Review*, **65**, 812–824.

Rubinstein, M. (1976), "The Strong Case for the Generalized Logarithmic Utility Model as the Premier Model of Financial Markets," *Journal of Finance*, **31**, 551–571.

Varian, H. (1985), "Divergence of Opinion in Complete Markets," *Journal of Finance*, , – .

Verrecchia, R. (1981), "On the Relationship Between Volume Reaction and the Consensus of Investors: Implications for Interpreting Tests of Information Content," *Journal of Accounting Research*, **19**, 271–283.

Williams, J. (1977), "Capital Asset Prices with Heterogeneous Beliefs," *Journal of Financial Economics*, **5**, 219–39.

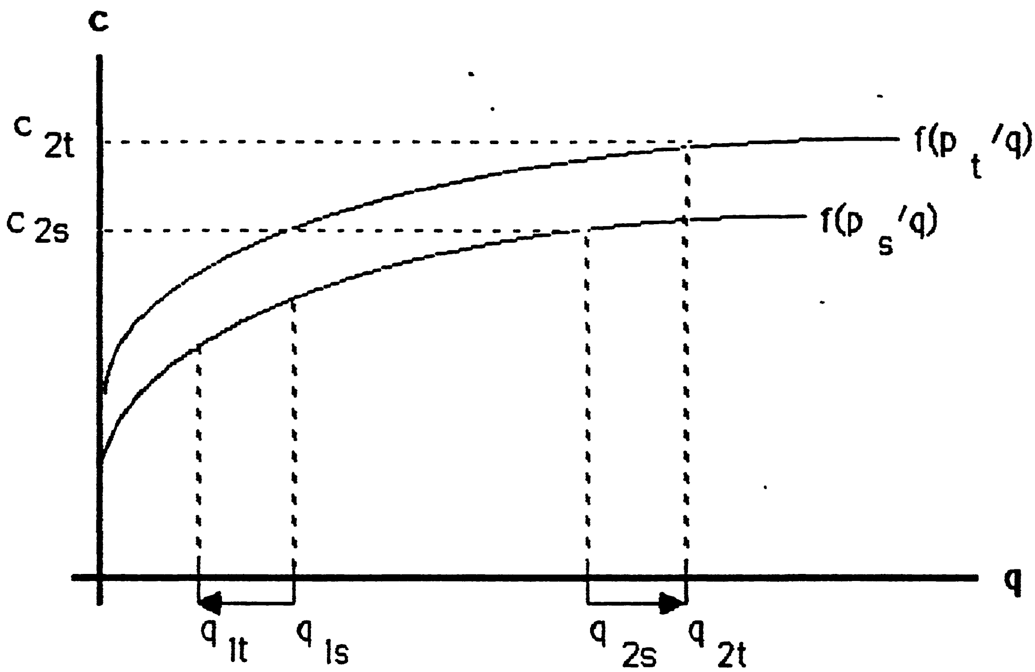

Figure 1. A mean preserving spread increases volume

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