Center for Research on Economic and Social Theory
CREST Working Paper

Differences of Opinion in Financial Markets

Hal R. Varian

March, 1985
Number 87--23

DEPARTMENT OF ECONOMICS
University of Michigan
Ann Arbor, Michigan 48109

DIFFERENCES OF OPINION IN FINANCIAL MARKETS

by

Hal R. Varian
University of Michigan

March 1985
Current version: January 28, 1987

Abstract. Agents trade because of differences in endowments, tastes and beliefs. In the standard models of finance beliefs are assumed to be identical across agents so that trade is due only to differences in endowments and tastes. In this paper I investigate trade due to different beliefs. Differences in equilibrium beliefs may be due to different *opinions* (i.e., prior probabilities) or different *information* (i.e., different values of the likelihood function). Using modified versions of a mean-variance model due to Grossman and an Arrow-Debreu model due to Milgrom-Stokey I argue that differences in information will not in general cause trade. Rather it is only differences in *opinion* that generate stock market volume. I then go on to examine the effect of different opinions on asset prices. I show that if tastes are identical, and if risk tolerance does not grow too rapidly, then assets that have more dispersed opinions will, other things being equal, have lower prices and a greater volume of trade. In general the effect of differences of opinion on asset prices will depend on the *curvature* of asset demand functions with respect to the opinions of the agents.

Address. Prof. Hal R. Varian, Department of Economics, University of Michigan, Ann Arbor, MI 48109, U.S.A.

DIFFERENCES OF OPINION IN FINANCIAL MARKETS

Hal R. Varian

The standard models of financial markets such as the Sharpe-Lintner mean variance model or the Rubinstein-Breeden-Litzenberger contingent consumption model both assume more-or-less homogeneous probability beliefs.[1] There has been some work on extending the mean-variance model to allow for differences in beliefs across agents; see Jarrow (1980), Lintner (1969), Mayshar (1983), and Williams (1977). Differences in beliefs in contingent commodities models have received much less attention. The major references are Rubinstein (1975), (1976), Breeden-Litzenberger (1978), Hakansson-Kunkel-Ohlson (1982) and Milgrom-Stokey (1982).

Cragg and Malkiel (1982) have done some empirical work concerning the effect of the diversity of beliefs on asset prices. According to them:

> "We found that the best single risk measure available for each company was the extent to which different forecasters were not in agreement about that company's future growth ... [These results] suggest that the variance of analysts' forecasts may represent the most effective risk proxy available ..." Cragg and Malkiel (1982, p.4).

The strong empirical relationship between dispersion of forecasts and share performance found by Cragg and Malkiel suggests that it is appropriate to examine more deeply the theoretical relationship between dispersion of beliefs and asset prices.

One issue that must be faced at the outset is how there can be any differences of belief in equilibrium. Several authors have argued that differences in beliefs that are due solely to differences in information should tend to be eliminated in equilibrium. I briefly discuss this literature in Section 1 and argue that to explain observed trading volumes, one must allow for differences in *opinions*—that is, differences in beliefs that are not shared by other agents, even when they are known to other agents. Equilibrium models that allow for both differences in opinions and differences in information are explored in Sections 2–5.

In Sections 2–4 I study a simple mean-variance model with differential information of the sort examined by Grossman (1976), (1978). I generalize Grossman's model to allow for different prior probabilities and for two periods of trade and find that even when the equilibrium prices reflect all available information, the volume of trade is determined entirely by the differences of opinion. In Section 5 I examine an Arrow-Debreu contingent consumption model of the sort studied by Milgrom and Stokey (1982). A similar result emerges there: prices are determined by information, but the pattern of trade is determined by the differences in opinion.

Having established the importance of differences of opinion for trade, I then examine some of its consequences for asset market equilibrium in Sections 6–10. In the context of an Arrow-Debreu equilibrium I show that if risk aversion does not decrease too rapidly, then assets with more dispersed opinion will have lower prices, other things being equal. If risk aversion does decrease too rapidly, then more dispersion of opinion will be associated

This work was supported by the National Science Foundation. I wish to thank Sudipto Bhattacharya, Fisher Black, Robert Holbrook, Jeff Mackie-Mason, Dale Morse, Paul Milgrom, Tom Russell, Duane Seppi, Sarab Seth, Joseph Swierzbinski, and Jean Tirole for helpful discussions and references.

[1] More precisely, the mean variance model requires that agents agree on the mean returns and covariance matrix of the assets; the contingent consumption model requires that agents agree on the probability of occurrence of different levels of aggregate consumption.

with higher prices.[2] Under general conditions assets with more dispersion of opinion will have more equilibrium trade, other things being equal.

It is important to understand that the results in Sections 6–10 involve comparisons of asset prices and trading volumes in a single Arrow-Debreu equilibrium; they are emphatically *not* comparative statics results about how an equilibrium price changes as the overall dispersion of opinion changes. Rather they are *comparative asset pricing* results that compare the prices of two different assets in the *same* equilibrium. These results are analogous to those in standard asset pricing models that compare the equilibrium relationship between the prices of assets and various characteristics of the assets such as their betas, the covariance with aggregate consumption, and so on. We have simply added a new characteristic of assets, namely, the dispersion of opinion.

Since the results describe the relationship between the *equilibrium* price and volume of trade in assets and the *equilibrium* probability beliefs about those assets, they are independent of the exact model of how equilibrium beliefs are formed. As long as one admits that probability beliefs may differ in equilibrium, the results in Sections 6–10 of this paper describe how those differences in belief will be related to asset prices, regardless of precisely *why* these beliefs differ.

1. No Trade Theorems

The main results of this paper described in Sections 6–10 are independent of the exact model of why equilibrium beliefs may differ. However, it is worthwhile investigating conditions in which equilibrium probability beliefs differ and agents actually trade on the basis of these different beliefs, since a number of authors have shown that in a speculative market composed of fully rational agents with identical priors there will be no trade in equilibrium. The basic structure of the argument has been summarized by Tirole (1982):

> "Consider a purely speculative market (i.e., a market where the aggregate monetary gain is zero and insurance plays no role). Assume that it is common knowledge that traders are risk averse, rational, have the same prior and that the market clears. Then it is also common knowledge that a trader's expected monetary gain given his information must be positive in order for him to be willing to trade. The market clearing condition then requires that no trader expect a monetary gain from his trade." Tirole (1982, p.1164).

Put in more familiar terms, if one agent has information that induces him to want to trade at the current asset price, then other rational agents would be unwilling to trade with him, because they realize that he must have superior information. Papers that explore this kind of No Trade Theorem in a variety of contexts include Rubinstein (1975), Bhattacharya (1976), Hakannsson, Kunkel and Ohlson (1982), and Milgrom and Stokey (1982).

Hence, if we to are examine models with different equilibrium beliefs and non-zero trading volume, we must consider models that lack one of the necessary hypotheses for the No Trade Theorems described above. Tirole (1982, p. 1167) describes the possibilities: (1) there may be some risk loving or irrational traders; (2) insurance and diversification considerations may play a significant role; or (3) agents may have different prior beliefs.

The first option is compelling on grounds of casual empiricism; clearly *some* participants in speculative markets behave in apparently irrational ways. Black (1986) explores some of the implications of this observation. It has sometimes been argued that introducing a "fringe" of irrational traders into traditional models will not significantly alter the

[2] The meaning of "too rapidly" is made precise below.

analysis of these models, since the irrational traders will only introduce a non-systematic "error term" on top of the traditional results. However, if the rational traders know that irrational traders are present in a market they would rationally attempt to exploit them. *The introduction of irrational traders will, in general, alter the behavior of the rational players and thereby change the nature of the equilibrium.* The problem with pursuing this approach lies in deciding what kinds of irrational behavior are plausible. Some interesting leads are being examined in this area, but, as yet, little progress has been made.

The second way to get around the No Trade Theorems is the route that most of the literature in finance has taken. All trade in speculative markets is taken to be due to differences in endowments and risk aversion, and observed portfolio holdings are taken to be the outcome of pure "hedging" rather than "speculation" per se.

However, attributing all trade to hedging seems implausible empirically and doesn't really get around the No Trade Theorems. Suppose that agents do have different risk aversion and different endowments, so that there is some gain from trade on these characteristics. After a single round of trading based on hedging and insurance considerations, there is no further reason to trade when new information arrives for exactly the same reasons described by Tirole: in a market of rational individuals there would be no one to trade with. Trading on the arrival of new information can only arise if agents *interpret* the information differently—i.e., if the information affects their posterior beliefs differently. This point will be explored in greater detail below.

This leaves us with the third option: different prior beliefs. This is the motive force for trade that we will explore in this paper. If differences in prior beliefs can generate trade, then these differences in belief can not be due to *information* as such, but rather can only be pure differences in *opinion.* Let us consider the difference between information and opinion more closely.

The distinction between what counts as differences in information and differences of opinion must lie in the eyes of the beholder. According to Bayesian decision theory, a rational agent will combine his prior probability beliefs and his likelihood function via Bayes' law to determine his posterior probability beliefs. If I convey a probability belief to another agent and he updates his posterior just as though this probability belief were objective evidence, then he has interpreted my probability beliefs as information: he has accepted my beliefs as being *credible.* If he doesn't update his posterior at all, then he has interpreted my beliefs as opinion—he has interpreted my beliefs as being noncredible. The extent that one agent's beliefs are capable of influencing another agent's beliefs determines to what extent one agent conveys information or opinion to the others. We will refer to an agent's belief about what fraction of other agents' beliefs are opinion and what fraction is information as the first agent's beliefs about the *credibility* of the other agents' views.

One hardly needs to consider speculative markets to encounter this distinction. In any kind of human communication it is necessary for one party to determine how credible the other's "information" is. When is someone really conveying information and when is he just conveying his opinion? How much weight should I attach to another individual's pronouncements?

The same sort of question arises in financial markets. If a firm introduces a new product which I find attractive, but the stock market value of the firm falls, what am I to conclude? How much of the price fall is due to superior information of others, and how much is due to differences in opinion? What credibility should we attach to market price movements? As we will see below, the nature of the market equilibrium will depend crucially on the degree of credibility that agents' attach to the "information" conveyed by the market price.

Allowing for differences of opinion in this sense can be viewed as allowing for a certain kind of irrational agents. If each agent truly believed that all other agents were rational, doesn't that mean that each agent would accept the other agents' beliefs as being due solely to different information? If I "weight" another agent's beliefs differently than hard evidence, doesn't this mean that I view the other agent as having some degree of irrational behavior? Harsanyi (1983), for example, has argued that under certain conditions fully rational agents must have the same prior beliefs. According to this view, having different priors—differences of opinion—is tantamount to irrationality in some cases. However, Harsanyi's discussion certainly allows that there also exist cases where rational people can agree to disagree at least with respect to prior probability beliefs.[3]

I prefer to remain agnostic on this issue. It seems to me that agents do have different beliefs in practice, and that they do have different degrees of credibility. This is enough to generate equilibrium trade, and nothing in particular hinges on whether we want to call this "rational" or "irrational", at least for the purposes of this paper.

2. A Mean-Variance Model

We first consider a simplified version of a model of the sort studied by Grossman (1976), (1978). There are two assets, one with an unknown payoff and one with a certain payoff. We let v denote the unknown value of the risky asset next period. For simplicity we assume that the certain asset has a zero rate of return.

Each investor i has a subjective prior distribution for the value of the risky asset which we take to be Normally distributed with mean v_i and precision α.[4]

We assume that investor i has a constant absolute risk tolerance utility function with coefficient of risk tolerance τ.[5] For simplicity we assume for now that all agents have the same risk tolerance; this assumption will be relaxed in Section 3. It can be shown that the demand function for the risky asset for each agent will have the linear form

$$D_i = \frac{\tau[E_i(v) - P]}{\text{var}_i(v)} \tag{1}$$

where $E_i(v)$ is the expected value of v in the opinion of agent i, $\text{var}_i(v) = 1/\alpha$ is the variance v in the opinion of agent i, and P is the market price. Thus the demand function for agent i is

$$D_i = \tau\alpha[v_i - P]. \tag{2}$$

Each agent has a supply S_i of the risky asset, and we suppose that the price P adjusts so as to equate aggregate demand and supply which yields an equilibrium price of

$$P^* = \overline{v} - \overline{S}/\tau\alpha \tag{3}$$

where bars over a variable denote the arithmetic mean of that variable across the agents.

Inserting this into the demand function of agent i we find the equilibrium demand for the risky asset to be

$$D_i^* = \tau\alpha[v_i - \overline{v}] + \overline{S}. \tag{4}$$

[3] Aumann's (1976) famous paper on this topic establishes only that rational individuals with *identical* priors and likelihoods who communicate their posteriors cannot disagree.

[4] The precision of a random variable is the reciprocal of the variance.

[5] Risk tolerance is given by $-u'(c)/u''(c)$; it is the reciprocal of the Arrow-Pratt measure of absolute risk aversion. For a discussion of the Arrow-Pratt measure see Varian (1984).

Note that each agent's equilibrium demand depends on the deviation of his opinion from the average opinion.

Up until now, the trade has been due entirely to differences in opinion. Now suppose that some information arrives that allows each agent to improve his estimate of the value of the asset. In particular, we suppose that each agent i observes a signal V_i where

$$V_i = v + \epsilon_i \tag{5}$$

and ϵ_i is IID Normal with mean zero and precision ω. Each agent will have a different piece of information about the value of the asset, but all agents will have information of the same precision. Of course the information set of agent i is not limited to just this piece of information; he or she will also use the market price in forming a final estimate of the value of the asset.

Given the n observations (V_i) an omniscient observer could calculate the sample mean

$$\overline{V} = \sum_{i=1}^{n} \frac{V_i}{n} \tag{6}$$

which will have a Normal distribution with mean v and precision $\beta = n\omega$. In this model the equilibrium price turns out to be fully revealing, so that each agent will be able to calculate $\overline{V}$ in equilibrium.

Being good Bayesians, each investor will then form a posterior distribution for the value of the risky asset by combining his prior and his sample information in accord with Bayes' law. Given the distributional assumptions we have made, the posterior distribution of investor i will be Normal with precision of $\gamma = \alpha + n\omega$ and mean of

$$\frac{\alpha}{\gamma} v_i + \frac{\beta}{\gamma}\overline{V}. \tag{7}$$

Using the assumption of constant absolute risk tolerance, the demand for the risky asset will have the linear form

$$D_i = \frac{\tau[E(v|I_i) - P]}{\text{var}(v|I_i)} \tag{8}$$

where $E(v|I_i)$ is the expected value of the asset conditional on the information set of agent i, $\text{var}(v|I_i)$ is the variance of the asset conditional on i's information, and P is the market price.

Inserting the expressions for the expected value and the variance given in equation (7) in equation (8), the demand of agent i reduces to

$$D_i = \tau[\alpha v_i + \beta\overline{V} - \gamma P]. \tag{9}$$

We suppose that the market price P adjusts so as to equate aggregate demand and supply which gives us a post-information equilibrium price of

$$\hat{P} = \frac{\alpha\overline{v} + \beta\overline{V} - \overline{S}/\tau}{\gamma} \tag{10}$$

where, as before, bars over a variable denote the mean across the population.

Let us consider the information structure in this market more closely. Suppose that all agents agree on the value of the *information*, (V_i), observed by the other agents but disagree about the value of the other agents' opinions, (v_i). Then knowing the average prior mean, $\overline{v}$, would not lead any agent to revise his own posterior mean, but knowing the average *information* $\overline{V}$ would induce an agent to change his posterior mean. That is, each agent would agree that $\overline{V}$ would be a superior estimate of the true value of the asset compared to any agent's individual information V_i. The distinction being drawn here is the distinction between the other agents' *opinions*—which do not change a given agent's beliefs—and other agents' pieces of *information*—which do change a given agent's beliefs.

But, if each consumer knows the value of the coefficients in the aggregate demand function, he or she can estimate $\overline{V}$ from the observed equilibrium price and the formula:

$$\overline{V} = \frac{\overline{S}/\tau + \gamma\hat{P} - \alpha\overline{v}}{\beta}. \tag{11}$$

As in the original Grossman (1976) model, the equilibrium price aggregates all of the information in the economy and thus provides a superior estimate of the "true" expected value of the asset.

In Grossman's original paper all agents had the same prior beliefs but different tastes. In that paper he argues for the rational expectations equilibrium by appealing to a long run equilibrium where agents can tabulate the empirical distribution of the $(\hat{P}, \overline{V})$ pairs and thus infer $\overline{V}$ from observations of $\hat{P}$. The agents do not have to know the structure of the model, but only the reduced form relationship between $\hat{P}$ and $\overline{V}$.

This sort of argument does not work here. For if the events were repeated a large number of times the different prior beliefs would tend to converge to identical posterior beliefs. Instead it appears that we must assume that agents have some understanding of the structure of the model and are able to disentangle the information contained in the market price directly rather than to simply use a reduced form model. It appears that a model with different prior opinions demands even more "rational expectations" than does Grossman's model.

Luckily this is not the case. For we have an extra feature not present in the original Grossman model. We have assumed that the agents can observe an equilibrium price prior to the arrival of the information. And certainly the *change* in the equilibrium price when new information arrives should reveal more about the new information than simply the *level* of the equilibrium price. In particular, if we rearrange expressions (3) and (10) which characterize the pre-information and post-information equilibrium prices, we have

$$\alpha P^* = \alpha\overline{v} - \overline{S}/\tau \tag{12}$$

$$\gamma\hat{P} = \alpha\overline{v} + \beta\overline{V} - \overline{S}/\tau. \tag{13}$$

These equations can be solved for $\overline{V}$ to yield:

$$\overline{V} = \frac{\gamma\hat{P} - \alpha P^*}{\beta}. \tag{14}$$

Using the definitions of the precisions, $\beta = n\omega$, and $\gamma = \alpha + n\omega$, this expression can be rewritten as

$$\overline{V} = \hat{P} + \frac{\alpha}{n\omega}(\hat{P} - P^*). \tag{15}$$

Thus if the agents are able to observe the *change* in the equilibrium prices when information arrives, they only need to know the precisions α and ω in order to estimate the aggregate information. The number ω/α is the ratio of the precision of the likelihood to the precision of the prior opinion, and can be thought of as a measure of credibility. A credible agent gives a larger weight to the evidence than to his opinions; therefore the ratio ω/α should be large. If all agents believe that all agents are very credible—or that there is a very large number of not-so-credible agents—then the market price will essentially be equal to $\overline{V}$, the average information in the population.

If agents perceive the other agents' beliefs as not being particularly credible in the sense that ω/α is small then they will attempt to use the change in the equilibrium price, appropriately weighted, to try to sort out the "information" from the "opinion".[6]

Inserting the "rational expectations" estimate of $\overline{V}$ into the demand function for agent i and simplifying we have the equilibrium demand of agent i:

$$D_i = \tau\alpha(v_i - \overline{v}) + \overline{S} \tag{16}$$

Note the interesting feature of this expression: all of the information variables have dropped out—an agent's trade in equilibrium is determined solely by the deviation of his or her opinion from the average opinion.

The size of agent i's position in the risky asset depends positively on his risk tolerance, τ, and on his *prior* precision, α. This latter point is somewhat surprising. The sample precision of $\beta = n\omega$ may be much larger than the prior precision α, and one might have thought that for large n it would swamp the prior precision. Nevertheless the equilibrium demand for the risky asset is exactly the same as if the agent had no sample information at all! An agent's posterior *beliefs* may be dominated by the sample information but his *trades* will depend only on the prior information. The explanation of this seeming paradox is that the market price adjusts to reveal all information in the economy and thus eliminates the value of the sample information to any one agent. Trade can only occur when people are different; and in this model the only difference that people have are in their prior opinions.

Note that the equilibrium demand for each agent in the post-information equilibrium is exactly the same as in the pre-information equilibrium. The arrival of the information will have zero impact on the volume of trade. This is because each agent is able to extract the same information from the market price. The only differences among the agents are the differences in their prior opinions, and trade on the differences in opinions has already taken place. Thus there is nothing left to trade on when the information arrives.

Of course the equilibrium *price* will respond to arrival of information; if $\overline{V}$ is large in some particular realization then the market price will be large. Indeed, the price responds in such a way as to perfectly offset the desire to trade. But it has to be that way in a fully revealing equilibrium—if the price reveals all information, there is nothing left to be revealed by the volume of trade. Thus the volume of trade cannot depend in any independent way on the information signals received by the agents.[7]

These observations become even more striking if we consider what happens as the information becomes more and more precise, relative to the prior. According to equation (10)

---

[6] Milgrom and Stokey (1982) also make use of the change in equilibrium prices to convey information. We will examine their model in shortly. I would like to thank Sarab Seth for suggesting that a similar approach would be useful in the mean-variance context considered here.

[7] Of course this model leads directly to the Grossman paradox as well: if the market price reveals all information, why does anyone bother to acquire information? Various resolutions of this paradox can be found in Grossman and Stiglitz (1980), Hellwig (1980), and Diamond and Verrecchia (1981).

as n goes to infinity, for example, the precision of the information goes to infinity, and the equilibrium post-information price becomes $\overline{V}$. Thus as the sample information becomes more and more informative, the price of the asset approaches its average perceived value: the post-information equilibrium price reflects all available information. Nevertheless, the equilibrium trade of each agent, as given in equation (16), is entirely determined by the initial dispersion of opinion and remains unaffected by a change in the accuracy of the information. When information is very informative the equilibrium price is determined almost entirely by the information, while the equilibrium trade is determined entirely by the differences in opinion.

What are the implications of opinion differences for the volume of trade? Agent i's net trade in the risky asset is

$$T_i = \tau\alpha(v_i - \overline{v}) - (S_i - \overline{S}) \tag{17}$$

so that the equilibrium volume depends on both the differences in opinions and the differences in endowments. Suppose for simplicity that each agent has an identical endowment $\overline{S}$ of the risky asset. Then the net trade of agent i is simply $T_i = \tau\alpha(v_i - \overline{v})$ and the overall volume of trade is given by:

$$\sum_{i=1}^{n} \tau\alpha|v_i - \overline{v}|/2 \tag{18}$$

which clearly depends only on the dispersion of opinion. An increase in the dispersion of opinion, as measured by the sum of the absolute deviations of the priors, will necessarily increase the volume of trade, regardless of the private information received by the market participants.

3. Different Tastes

In Grossman's original model all agents had identical priors but different risk tolerances; the equilibrium volume was therefore due solely to the differences in tastes. In the model above the agents all have the same tastes, but different prior opinions, and the equilibrium volume depends only on the differences of opinion. What if agents have different tastes and different opinions?

Letting τ_i denote the risk tolerance of agent i, the pre-information price is determined by solving:

$$\sum_{i=1}^{n} \tau_i\alpha[v_i - P] = \sum_{i=1}^{n} S_i \tag{19}$$

and the post equilibrium price is determined by solving:

$$\sum_{i=1}^{n} \tau_i[\alpha v_i + \beta\overline{V} - \gamma P] = \sum_{i=1}^{n} S_i. \tag{20}$$

Subtracting equation (19) from (20) and solving for $\overline{V}$, we find that the rational expectations estimate of $\overline{V}$ is exactly the same that given in equation (15):

$$\overline{V} = \frac{\gamma\hat{P} - \alpha P^*}{\beta} = \hat{P} + \frac{\alpha}{n\omega}(\hat{P} - P^*). \tag{21}$$

The agents do not need to know anything about the taste differences in the population to estimate $\overline{V}$, as long as they can observe the pre-information and post-information prices. Everything hinges on the magnitude of "credibility" of the market price, as determined by α relative to $n\omega$.

Using this estimate of $\overline{V}$ it can be shown that the equilibrium trade of agent i is:

$$D_i = \tau_i \alpha (v_i - \sum_{j=1}^{n} w_j v_j) + w_i S \tag{22}$$

where

$$w_i = \frac{\tau_i}{\sum_{j=1}^{n} \tau_j}.$$

The volume of trade still depends on the deviation of agents opinion from the average opinion, but now it is a weighted average rather than the simple average we had before.

4. Differences in Interpretation

The no-trade result may appear to fly in the face of common sense; certainly the arrival of new information in real markets may contribute to volume. It seems that this view is held by those who have examined the empirical determinants of the volume of trade. For example:

> "One reason to suggest a relationship between changes in price and transactions volume is that both are related to the type and flow of information in the market. New information can simultaneously spur trading and lead to new equilibrium prices." Karpoff (1985)

Casual empiricism (i.e., the nightly news) suggests that volume does react to new information. Empirical work by Epps and Epps (1976), Tauchen and Pitts (1983), Harris (1983) and others suggest that volume is highly correlated with the absolute value of the price change for daily stock returns. How can we reconcile this observation with the model described above?

One assumption of the above model was that all agents interpreted the information in the same way. That is, all agents agree that the observation of V_i contributed to their posterior estimate of v in the same way. For some kinds of information this may be plausible; but for other types it may be quite implausible. If OPEC were to break up tomorrow would everyone agree on the impact of this event on all asset prices? When Apple introduces a new computer line does everyone agree on the impact of this product on the market value of Apple stock?

We might model this by distinguishing between the *arrival* of the information and the *interpretation* of the information. Let Y_i be a random variable denoting the "magnitude" of a piece of information, and let $V_i = \delta_i Y_i$ be agent i's estimate of the impact of this information on the mean value of the risky asset. That is, agent i views $\delta_i Y_i$ as a signal about the value of the risky asset in the sense that he believes the model

$$V_i = \delta_i Y_i = v + \epsilon_i. \tag{23}$$

All agents agree on the magnitude of the information Y_i but the "interpretation" of the information, δ_i, differs from agent to agent. That is, each agent i has a potentially different belief, δ_i, about how the information Y_i will affect the equilibrium value of the asset.

Let us assume that the interpretations, (δ_i), and the observations, (Y_i), are distributed independently across the set of agents so that $\overline{V} = \overline{\delta}\,\overline{Y}$. Substituting this into equation (15) and solving for $\overline{Y}$ yields

$$\overline{Y} = \frac{\hat{P}}{\overline{\delta}} + \frac{\alpha}{n\omega}\frac{(\hat{P} - P^*)}{\overline{\delta}}. \tag{24}$$

Thus as long as the average value of δ is common knowledge, each agent will be able to estimate $\overline{Y}$ from the observed change in equilibrium price. After observing the equilibrium price of the asset, agent i should then revise his own estimate of the expected value of the risky asset to be $\delta_i\overline{Y}$. This leads to an equilibrium demand of the form

$$D_i = \tau[\alpha v_i + \beta\delta_i\overline{Y} - \gamma P]. \tag{25}$$

Solving for the post-information equilibrium price we have

$$\hat{P} = \frac{\alpha\overline{v} + \beta\overline{\delta}\overline{Y} - \overline{S}/\tau}{\gamma}. \tag{26}$$

Substituting this back into the demand function of agent i and subtracting off his endowment to get his equilibrium net trade, T_i, we have

$$T_i = \tau\left[\alpha(v_i - \overline{v}) + \beta(\delta_i - \overline{\delta})\overline{Y}\right] + (S_i - \overline{S}). \tag{27}$$

The interpretation of this equation is rather nice. The equilibrium net trade of an agent depends on the difference in his opinion, his interpretation, and his endowment from the averages of these variables. In this model, the agents can use the market price to estimate whether or not the events have occurred, but then put their own interpretation on the events themselves. In equilibrium the arrival of the information affects the volume of trade, but only through the differences of opinion about how the information should be interpreted, not through the difference in information itself.

5. An Arrow-Debreu Model

The model described in the first part of this paper is a mean-variance model. However, the ideas described there are robust; we will demonstrate that by considering the same issues in the context of a contingent-consumption model which is a simplification of the model described in Milgrom-Stokey (1980). Let $s = 1, \ldots, S$ index states of nature and $i = 1, \ldots, n$ index the economic agents. Use $\pi_i(s)$ to denote agent i's prior probability belief about the occurrence of state s, and c_{is} to denote agent i's contingent consumption in state s. Finally, let $u_i(c)$ be agent i's von Neumann-Morgenstern utility function and p_s the Arrow-Debreu price for consumption in state s.

We suppose that all agents trade to an equilibrium which will be characterized by the first-order conditions for utility maximization:

$$\pi_i(s)u_i'(c_{is}) = \lambda_i p_s \tag{28}$$

where λ_i is agent i's marginal utility of wealth.

Now suppose that each agent observes a private signal y_i which is a realization of a random variable that affects the probability of occurrence of state s. We suppose that if agent i could observe the signals of all the agents, $y = (y_1, \ldots, y_n)$, he would calculate the probability of occurrence of state s via Bayes' law, and that all agents agree on the form of the likelihood function, which we denote by $\pi(y|s)$.

After each agent observes his private signal, markets are reopened and all agents can revise their contingent consumption plans. Given the new information we would expect that prices of the various states would change. As in the model presented above the agents can take these price changes into account and use them to infer something about what signals the other agents observed. What will the new post-information equilibrium look like?

Given this framework, Milgrom-Stokey establish a remarkable result: there will always be a post-information equilibrium that reveals all the information observed by all the agents, and furthermore, the new equilibrium involves no trade by any agent.[8]

The proof is simply to consider what happens if the price of state s consumption changes to $\hat{p}_s = \pi(y|s)p_s$. In this case, each agent can extract the likelihood function $\pi(y|s)$ by dividing the post-information price by the pre-information price and use this likelihood to revise his or her prior probability via Bayes' law. The resulting equilibrium is characterized by the first-order conditions:

$$\pi_i(s|y)u_i'(c_{is}) = \frac{\pi(y|s)\pi_i(s)}{\pi_i(y)}u_i'(c_{is}) = \frac{\lambda_i}{\pi_i(y)}\pi(y|s)p_s = \hat{\lambda}_i\hat{p}_s \tag{29}$$

where $\hat{\lambda}_i = \lambda_i/\pi_i(y)$. Since the new prices, $(\hat{p}_s)$, the new beliefs $(\pi_i(s|y))$, and the original consumption levels, (c_{is}), satisfy the appropriate first-order conditions, we do indeed have an equilibrium in the post-information market that involves no trade due to the arrival of the information.

As before, one could question the assumption of a common likelihood function. If agents have different opinions about the effect of a given piece of information on the probability of occurrence of a state of nature, the Milgrom-Stokey result need no longer hold. Arrival of new information will in general cause agents to trade in this case, but only because agents *interpret* the information differently, not because it is different information.

This observation can be sharpened; Milgrom-Stokey show that if everyone has the same likelihood function there will always exist an equilibrium which is fully revealing and which involves no trade. But there is a partial converse: if there is no trade after information is revealed, then all agents must have essentially the same likelihood function and equilibrium will be fully revealing. So the relationship between differences of opinions and the volume of trade is an if-and-only-if relationship—common opinions about how information affects state probabilities means zero volume, and zero volume means essentially common opinions.

This proposition is a small extension of arguments given by Rubinstein (1975, Nonspeculation Condition), Milgrom-Stokey (1982, Theorem 3), and Hakansson, Kunkel and Ohlson (1982, Lemma 2). However, it seems worthwhile to give a brief exposition of the result.

Following Hakansson, Kunkel and Ohlson (1982), we will say that likelihood functions are ***essentially homogeneous*** if

$$\pi_i(y_i, \hat{p}|s) = k_i(y_i, \hat{p})\pi_1(y_i, \hat{p}|s)$$

[8] **Related insights can be found in Marshall (1974), Rubinstein (1975), Bhattacharya (1976), Hakansson, Kunkel and Ohlson (1982) and Verrecchia (1981).**

for all $i = 1, \ldots, n$. If two likelihood functions are essentially homogeneous, then Bayes' law implies that they will generate the same posterior probabilities, since the $k_i(y_i, \hat{p})$ terms will cancel out from the numerator and the denominator. Thus agents that have essentially homogeneous likelihood functions will make the same inferences from the information they observe.

THEOREM 1. *Let (c_{is}, p_s) be an equilibrium before the information y is revealed and let $(c_{is}, \hat{p}_s)$ be a no-trade equilibrium after y_i is revealed to each agent. Then the value of each agent's likelihood function is given by:*

$$\pi_i(y_i, \hat{p}|s) = k_i(y_i, \hat{p}) \frac{\hat{p}_s}{p_s}$$

so that agents' likelihood functions must be essentially homogeneous.

Proof. Choose two states, s and t. Since the economy is in equilibrium before and after the information is revealed, we must have:

$$\frac{\pi_i(s) u_i'(c_{is})}{\pi_i(t) u_i'(c_{it})} = \frac{p_s}{p_t}$$

and

$$\frac{\pi_i(s|y_i, \hat{p}) u_i'(c_{is})}{\pi_i(t|y_i, \hat{p}) u_i'(c_{it})} = \frac{\hat{p}_s}{\hat{p}_t}.$$

By Bayes' law we can write the latter expression as:

$$\frac{\pi_i(y_i, \hat{p}|s) \pi_i(s) u_i'(c_{is})}{\pi_i(y_i, \hat{p}|t) \pi_i(t) u_i'(c_{it})} = \frac{\hat{p}_s}{\hat{p}_t}.$$

Dividing by the first equation gives:

$$\frac{\pi_i(y_i, \hat{p}|s)}{\pi_i(y_i, \hat{p}|t)} = \frac{\hat{p}_s / p_s}{\hat{p}_t / p_t}.$$

This implies that:

$$\pi_i(y_i, \hat{p}|s) = k_i(y_i, \hat{p}) \frac{\hat{p}_s}{p_s}$$

for all s as required. ∎

As indicated above, this result is essentially a corollary to Theorem 3 in Milgrom-Stokey (1982). However, the focus is different; Milgrom-Stokey show that if all agents have the same likelihood function, then in a post-information, no trade equilibrium, each agent's posterior probability must depend only on the post-information prices and be independent of his private signal. We show something a bit different: that *if* there is no trade in a post-information equilibrium, then all agents must have essentially the same likelihood value, and the equilibrium price change must reveal this common likelihood value. Hakansson, Kunkel and Ohlson (1982) have essentially the same result in a somewhat different setting while Bhattacharya (1976) has shown that this result is closely related to Rubinstein's (1975) Nonspeculation Condition.

6. The Effect of Differences of Opinion on Asset Prices

The results of the last section suggest that the volume of trade in an Arrow-Debreu model is due primarily to the differences of opinion. In a one period model, these can be differences of opinion about the prior probabilities; in a two period model, trade requires differences of opinion about likelihood functions—that is, the interpretation of information.

How far can this insight be pushed? If two assets are otherwise identical but one has "more diverse" beliefs, which asset will have the larger volume of trade? Which asset will have the higher price? It is important to note that these questions can be addressed independently of the particular model of information transfer. For the questions are phrased in terms of *equilibrium* differences in beliefs; they are questions about comparative asset pricing, not about comparative statics.

Varian (1985a) has described some results concerning relative pricing in this framework; here we review these results and consider the implications for the volume of trade. In this section we will consider only the case of identical utility functions; the case of different utility functions will be considered in Sections 9 and 10.

Letting $f(\cdot)$ be the inverse of $u'(\cdot)$ we can write the Arrow-Debreu first-order conditions in equation (28) as:

$$c_{is} = f(\lambda_i p_s / \pi_i(s))$$

where $\pi_i(s)$ is now interpreted as either a prior or a posterior probability. Let us define $q_{is} = \pi_i(s)/\lambda_i$ to be a "weighted" probability for agent i. In general we would expect wealthier agents to have lower marginal utilities of wealth in equilibrium so that their beliefs will get a higher weight in the q_{is} expression. (It is easy to show that if two agents have the same probability beliefs and the same utility function, the wealthier one will have a lower marginal utility of wealth.)

Then we can sum the first-order conditions across the agents to derive an expression for aggregate consumption in state s:

$$C_s = \sum_{i=1}^{n} c_{is} = \sum_{i=1}^{n} f(p_s/q_{is}).$$

THEOREM 2. *The function $f(p_s/q_{is})$ is always an increasing function of q_{is}. It will be a concave (convex) function of q_{is} as the derivative of risk tolerance is less (greater) than 1.*

Proof. The first part of the theorem is proved in Varian (1985a). The same paper shows that $f(p_s/q_{is})$ will be concave in q_{is} iff $r'(c) > -r(c)^2$ where $r(c) = -u''(c)/u'(c)$ is the Arrow-Pratt measure of absolute risk aversion. This condition can be rearranged to give:

$$-\frac{r'(c)}{r(c)^2} = \frac{d}{dc}\left(\frac{1}{r(c)}\right) < 1$$

which establishes the second statement. ∎

It is easy to check that the derivative of risk tolerance for a logarithmic utility is precisely one, so we can conclude that $f(p_s/q_{is})$ will be a concave function of q_{is} when risk tolerance increases less rapidly than it does in the case of logarithmic utility. This seems like a very natural assumption.

Following Varian (1985a) we now consider two different states, s and t, such that $C_s = C_t$, but the probability beliefs for state t are a mean preserving spread of the probability beliefs for state s in the sense of Rothschild-Stiglitz (1970). It follows from the strict concavity of $f(\cdot)$ that:

$$\sum_{i=1}^{n} f(p_s/q_{it}) < \sum_{i=1}^{n} f(p_s/q_{is}) = C_s = C_t = \sum_{i=1}^{n} f(p_t/q_{it})$$

and therefore $p_t < p_s$.

This argument proves:

THEOREM 3. *Consider two Arrow-Debreu assets that pay off in states with the same level of aggregate consumption. Then if all agents have identical tastes and risk tolerance does not increase too rapidly, the asset with the more dispersion of weighted probability beliefs will have the lower equilibrium price.*

Thus increased dispersion of beliefs should be associated with lower prices in equilibrium. It is important to understand that this is ***not*** a comparative statics statement. We are not examining two different equilibria. Rather we are examining two different assets in a ***given*** equilibrium. Theorem 3 followed solely from the first-order conditions and the assumption about risk tolerance; it is therefore compatible with any model of how equilibrium beliefs are formed. Equilibrium beliefs could be due to differences in likelihoods, non-rational behavior, or whatever, as long as the first-order conditions for an Arrow-Debreu equilibrium hold.

7. The Effect of Differences of Opinion on Volume

In order to get a result concerning the relative volume of trade in different assets we must refine the meaning of "other things being equal" and use a more restrictive notion of "increased dispersion of opinion." Since the final volume of trade depends on the pattern of initial endowments across the agents we will further assume that assets s and t have the same pattern of initial endowments: $\bar{c}_{is} = \bar{c}_{it}$ for all $i = 1, \ldots, n$.

In a given equilibrium and state s, let D_s be the set of net demanders—those agents for whom $f(p_s/q_{is}) > \bar{c}_{is}$—and let S_s be the set of net suppliers, defined in a similar manner. Since the amount bought equals the amount sold in equilibrium, we can express the volume of trade in state s by

$$V_s = \sum_{i \in D_s} [f(p_s/q_{is}) - \bar{c}_{is}] = \sum_{i \in S_s} [\bar{c}_{is} - f(p_s/q_{is})].$$

Now consider two states, s and t, and suppose that all of the agents who purchase asset s feel that state t is at least as likely as state s, and that all agents who sell asset s feel that t is less likely that s. That is:

$$q_{it} \geq q_{is} \quad \text{for all } i \in D_s$$
$$q_{it} \leq q_{is} \quad \text{for all } i \in S_s.$$

Informally, we are assuming that all of the net demanders for state s securities are even more optimistic about state t occurring than they are about state s occurring and all of the net suppliers of state s securities are even more pessimistic about state t occurring that they are about state s occurring. Thus there is certainly more dispersion of equilibrium beliefs about state t than about state s.

THEOREM 4. *If all of the net demanders in state s are more optimistic about state t occurring than state s occurring and all of the net suppliers in state s are more pessimistic about state t occurring than state s occurring, then the volume of trade in state t must be higher than the volume of trade in state s.*

Proof. Suppose first that $p_t > p_s$, and let agent i be a net supplier of asset s in equilibrium. By the hypothesis of the theorem, $q_{it} < q_{is}$. The function $f(p_s/q_{is})$ is increasing in q_{is} and decreasing in p_s by Theorem 2, so we have that

$$f(p_s/q_{is}) > f(p_t/q_{it}).$$

Since $\overline{c}_{is} = \overline{c}_{it}$ by assumption we have

$$\overline{c}_{is} - f(p_s/q_{is}) < \overline{c}_{it} - f(p_t/q_{it}).$$

Summing over the net suppliers shows that $V_s < V_t$.

Now suppose that $p_t < p_s$. Then let agent i be a net *demander* of asset s in equilibrium. By exactly the same argument, we have

$$f(p_s/q_{is}) - \overline{c}_{is} > f(p_t/q_{it}) - \overline{c}_{it}.$$

Summing shows that $V_t > V_s$, as required. ∎

It is worth observing that this argument does not depend on agents having identical tastes or on the earlier assumption about risk tolerance: it is true in complete generality. Furthermore, the argument holds for the unweighted beliefs, $\pi_i(s)$, as well as the weighted beliefs, q_{is}. In fact, the argument has a very simple graphical proof which is depicted in Figure 1a. The assumption that all net demanders become more optimistic implies that the aggregate demand curve shifts to the right and the assumption that all net suppliers become more pessimistic implies that the aggregate net supply curve shifts to the right. Hence the volume of trade must increase, as shown in Figure 1a.[9]

However, this particular kind of increase in the dispersion of opinion is rather special. We can get a different result using a weaker definition of an increase in the dispersion of belief. In particular, let us consider a mean preserving spread that does not decrease q_{is} for any agent who is a net purchaser of the asset. That is, we assume that the mean preserving spread in beliefs is such that:

$$q_{it} \geq q_{is} \quad \text{for all } i \text{ such that } c_{is} > \overline{c}_{is}.$$

Let us also assume that the derivative of risk tolerance is less than 1, so that $f(p_s/q_{is})$ is concave. Then we have the situation illustrated in Figure 2. Here we have plotted the weighted probabilities q_{is} on the horizontal axis, and $c_{is} = f(p_s/q_{is})$ on the vertical axis. When we consider the movement from q_{is} to q_{it} we see that:

$$f(p_s/q_{is}) \leq f(p_s/q_{it}) < f(p_t/q_{it})$$

for all net purchasers of the asset where the first inequality follows from the monotonicity of $f(\cdot)$ and the second from the fact that $p_t < p_s$ which we established in Theorem 3. Since

[9] The fact that Theorem 4 has an elementary graphical argument was pointed out to me by Duanne Seppi.

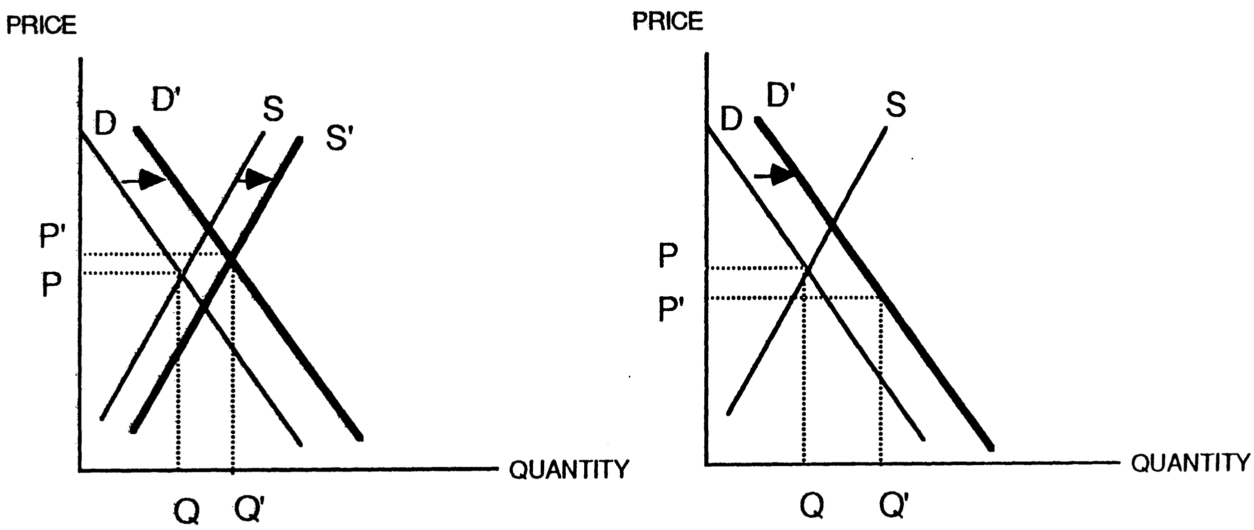

Figure 1. (a) An increase in the dispersion of beliefs of both demanders and suppliers must increase volume. (b) If the buyers become more optimistic and the equilibrium price decreases, then the volume must increase, regardless of what happens to the beliefs of suppliers.

all of the net purchasers of the asset are now buying more, the net sellers of the asset must be selling more, and the volume of the trade is therefore larger. The volume of trade has increased for two reasons: each net purchaser wants to buy more at the same price, and the price decrease makes him or her want to buy even more again.

This can also be demonstrated using the supply and demand graph in Figure 1. If the demand increases and the equilibrium price decreases, then it follows that the equilibrium volume must be larger, as shown in Figure 1b. This result holds regardless of the change in the beliefs of the sellers of the asset, although it must be the case that on the average their beliefs have become more pessimistic.

If it appears obvious that making buyers more optimistic will increase the volume of trade, it might be worth considering the case where $f(\cdot)$ is a *convex* function of q_{is}. For example, take constant relative risk averse utility functions with $\rho < 1$. Then increasing the dispersion of opinion will *increase* asset prices, so the two effects described above work in opposite directions and the volume of trade can go either way. We give an algebraic example of this below.

An example can also be constructed using the supply and demand framework in Figure 2. Simply imagine the case where the demand curve shifts to the right as illustrated in Figure 2b, but the supply curve shifts far enough to the left so that the equilibrium price rises and the equilibrium transactions decrease.

8. Constant Relative Risk Aversion

The case of constant relative risk averse utility functions, $u(c_{is}) = c_{is}^{1-\rho}/(1-\rho)$, serves as

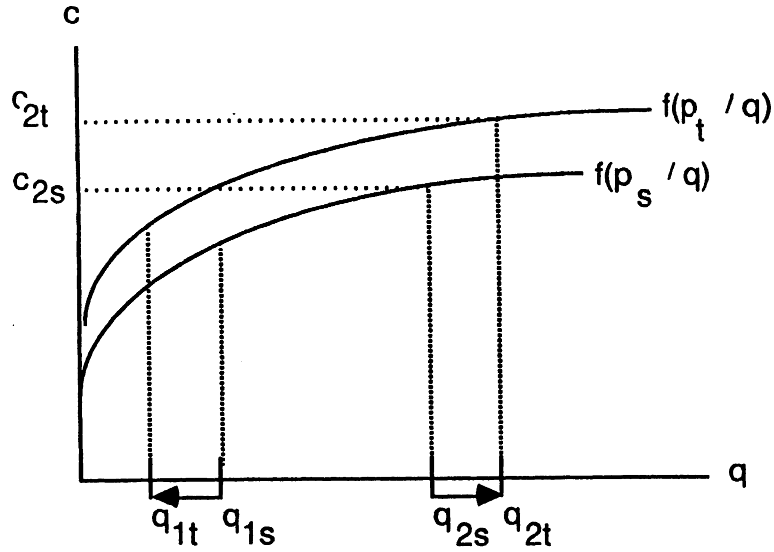

Figure 2. A mean preserving spread in weighted beliefs increases volume.

a convenient example of the above discussion. The first-order conditions take the form:

$$c_{is}^{\rho} = \frac{\lambda_i p_s}{\pi_{is}}$$

Straightforward but tedious calculations show that the equilibrium value of λ_i is given by:

$$\lambda_i = \left(\frac{\sum_{s=1}^{S} p_s^{\frac{\rho-1}{\rho}} \pi_{is}^{\frac{1}{\rho}}}{m_i} \right)^{\rho}$$

where m_i is agent i's equilibrium wealth. As indicated earlier, λ_i is a decreasing function of equilibrium wealth. If $\rho = 1$, λ_i is simply $1/m_i$.

Letting $q_i = \pi_{is}/\lambda_i$ the first-order conditions may be rewritten as:

$$c_{is}^{-\rho} = \frac{p_s}{q_{is}}$$

so that

$$c_{is} = (\frac{q_{is}}{p_s})^{\frac{1}{\rho}}.$$

This in turn implies

$$C_s = \sum_{j=1}^{n} (\frac{q_{js}}{p_s})^{\frac{1}{\rho}}$$

and

$$p_s = C_s^{-\rho}(\sum_{j=1}^{n} q_{js}^{\frac{1}{\rho}})^{\rho}.$$

This function gives us the explicit equilibrium relationship between aggregate consumption, probability beliefs, and equilibrium prices. Inserting this into the first equation, we see that the equilibrium demand of agent i takes the form

$$c_{is} = w_{is}C_s$$

where

$$w_{is} = \frac{q_{is}^{\frac{1}{\rho}}}{\sum_{j=1}^{n} q_{js}^{\frac{1}{\rho}}}.$$

Thus the equilibrium demand of agent i is completely determined by his weighted probability, relative to the weighted probabilities of the other agents.

We can use this expression to present the example promised above. Suppose that $\rho = .5$, $C_s = C_t = 1$ and (q_{is}) and (q_{it}) are as given in Table 1. Note that in state s all agents have the same value of q_{is} so that any mean-preserving change in the equilibrium beliefs about state s will be a mean preserving spread. In the case illustrated in the table, both of the net demanders have the same probability beliefs about state t as state s while one of the net suppliers is more optimistic and one is more pessimistic about state s as compared to state t.

Using the formula given above it is straightforward to calculate the net trades (x_{is}) and (x_{it}). We find that state s has a volume of .200 and state t has a volume of .190. Even though the dispersion of opinion has increased, and no net purchaser has become more pessimistic, the volume of trade has decreased. This is not of course a violation of Theorem 4, since one of the net demanders of the Arrow-Debreu security is more optimistic about state t than about state s. Nevertheless, the example is somewhat surprising since it shows that without the curvature assumption on demands somewhat perverse results can arise.

Table 1. Example of decreased volume of trade

	q_{is}	q_{it}	$\bar{c}_{is}$	x_{is}	x_{it}
1	.250	.200	.300	–.050	–.143
2	.250	.300	.400	–.150	–.047
3	.250	.250	.100	.150	.145
4	.250	.250	.200	.050	.045

9. Different Tastes and Different Opinions

Here we discuss the general case where tastes differ across the agents. Letting $f_i(\cdot)$ be the inverse of agent i's marginal utility function, and following the derivation in Section 5 we have:

$$C_s = \sum_{i=1}^{n} f_i(p_s/q_{is}).$$

For fixed values of $(q_{1s}, \ldots, q_{ns})$, $f_i(\cdot)$ is a decreasing function of p_s. Thus we can invert it to get $p_s = F(C_s, q_s)$ where $q_s = (q_{1s}, \ldots, q_{ns})$. It is easy to show, following Varian (1985a), that $F(C_s, q_s)$ is a decreasing function of C_s and an increasing function of q_{is} in a given Arrow-Debreu equilibrium. What about the curvature of $F(C_s, q_s)$ The results of Varian (1985b) show that increasing risk tolerance (i.e., decreasing absolute risk aversion) implies that $F(C_s, q_s)$ is a convex function of C_s. With some more effort we can show:

THEOREM 5. *If risk tolerance increases less rapidly than in the case of logarithmic utility, $F(C_s, q_s)$ will be a quasiconcave function of q_s.*

Proof. Fix C at $\overline{C}$, say, and consider three different sets of prices and weighted probabilities such that:

$$\sum_{i=1}^{n} f_i(p''/q_i'') = \sum_{i=1}^{n} f_i(p'/q_i') = \sum_{i=1}^{n} f_i(p/q_i) = \overline{C}$$

where $q_i'' = tq_i + (1-t)q_i'$.

In order to establish quasiconcavity, we need to show that:

$$F(C, q'') \geq \min\{F(C, q), F(C, q')\}$$

or, equivalently, that

$$p'' \geq \min\{p, p'\}.$$

Without loss of generality let $p = \min\{p, p'\}$.

The assumption that risk tolerance does not increase too rapidly implies that $f_i(p/q)$ is concave in q. This implies that

$$f_i(p/q_i'') \geq tf_i(p/q_i) + (1-t)f_i(p/q_i').$$

By the negative monotonicity of $f_i(p/q_i)$ in p and the assumption that $p' \geq p$ we have:

$$tf_i(p/q_i) + (1-t)f_i(p/q_i') \geq tf_i(p/q_i) + (1-t)f_i(p'/q_i').$$

Putting these two inequalities together and summing we have:

$$\begin{aligned}\sum_{i=1}^{n} f_i(p/q_i'') &\geq t\sum_{i=1}^{n} f_i(p/q_i) + (1-t)\sum_{i=1}^{n} f_i(p'/q_i') \\ &= t\overline{C} + (1-t)\overline{C} = \overline{C} \\ &= \sum_{i=1}^{n} f_i(p''/q_i'').\end{aligned}$$

It follows that $p \leq p''$, as was to be shown. ∎

This result suggests that even if preferences are different, there is a tendency for divergence of opinion to lead to lower asset prices. Consider Figure 3 where we have illustrated a level set of $F(C, q_1, q_2)$. In this example we have two states of nature with the same value of aggregate consumption and state prices but different weighted subjective probabilities, (q_1, q_2) and (q_1', q_2'). As established above, the upper contour set is convex and monotonic.

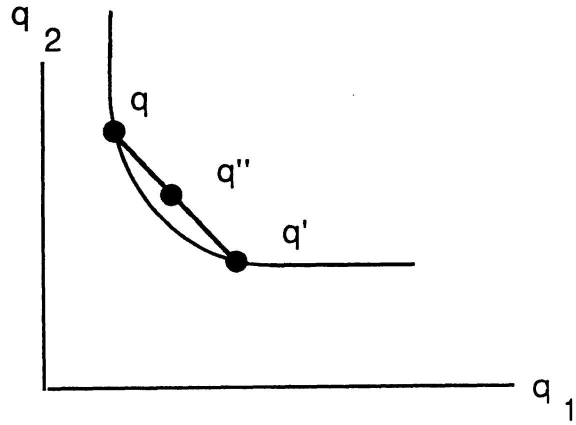

Figure 3. Averaging beliefs will tend to lead to lower asset prices.

Therefore, if we take a weighted average of (q_1, q_2) and (q_1', q_2') to get (q_1'', q_2''), the equilibrium value of p'' must increase. In this sense, taking an average of "extreme" distributions of beliefs will tend to increase state prices.

It is instructive to consider this geometry when the tastes are identical. In Figure 4*a* we have illustrated a typical level set of $F(C, q_1, q_2)$. We can also describe this level set implicitly as the set of all (q_1, q_2) such that:

$$C = f(p/q_1) + f(p/q_2)$$

where p and C are fixed.

Consider the slope of this level set along the diagonal, where $q_1 = q_2$. Since $f(p/q_i)$ is independent of i along the diagonal, the slope of the level set must be -1. Thus if we take a "spread" in (q_1, q_2) away from the diagonal that preserves the mean of (q_1, q_2) we are moving along a 45 degree line and must therefore move to a lower level set. At any other point on the curve a mean preserving spread will necessarily move us to a lower level set, as illustrated.

Now consider the general case with different utilities as depicted in Figure 4*b*. Here the level curve does not necessarily have slope of -1 at the diagonal so that a spread in beliefs may easily move us to a higher level set.

10. Moving from Homogeneous to Diverse Beliefs

The geometric analysis of the last section can be used to describe another sense in which increases in the dispersion of opinion will tend to lower asset prices. Consider Figure 4 once again. It is true that the level curve may not have a slope of -1 at the diagonal, but we can compute its slope easily enough. The level curve is defined by the identity

$$f_1(p/q_1) + f_2(p/q_2) = C.$$

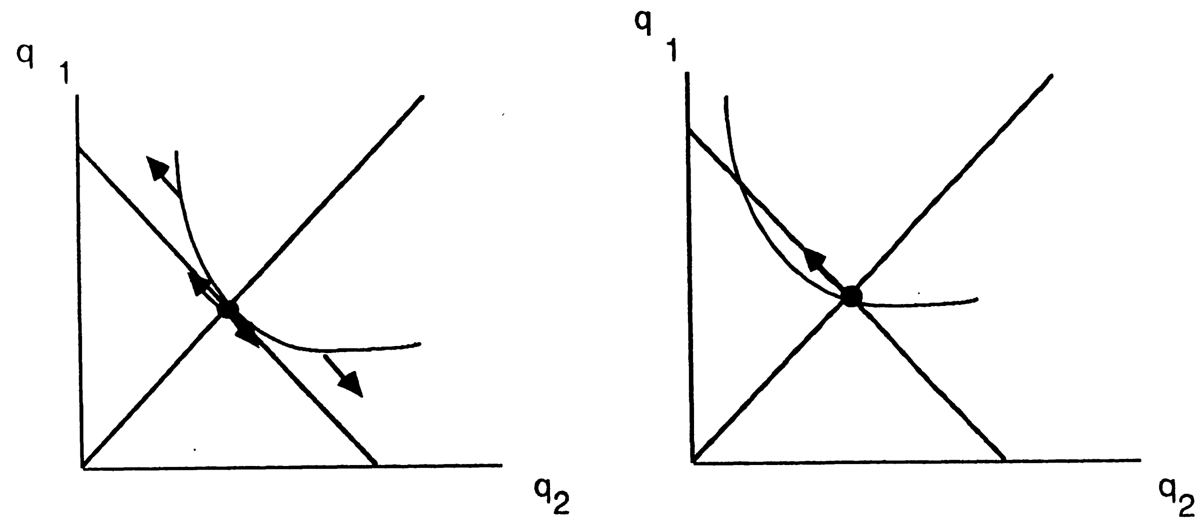

Figure 4. A mean preserving spread with identical tastes (a) and different tastes (b)

Totally differentiating this expression gives us

$$-f_1'(p/q_1)\frac{p}{q_1^2}dq_1 - f_2'(p/q_2)\frac{p}{q_2^2}dq_2 = 0.$$

Now use the facts that $f_i' = 1/u_i''$ and $u_i' = p/q_i$ to write

$$\left(-\frac{u_1'(c_1)}{u_1''(c_1)}\right)\frac{dq_1}{q_1} + \left(-\frac{u_2'(c_2)}{u_2''(c_2)}\right)\frac{dq_2}{q_2} = 0. \tag{30}$$

Using τ_i to represent individual i's risk tolerance, and evaluating this expression where $q_1 = q_2$ we have

$$\tau_1 dq_1 + \tau_2 dq_2 = 0 \tag{31}$$

or

$$\frac{dq_2}{dq_1} = -\frac{\tau_1}{\tau_2}. \tag{32}$$

This gives us the slope of the level set when the weighted beliefs are the same. A change in (q_1, q_2) that keeps the *weighted* sum in equation (31) constant represents an increase in the dispersion of beliefs which preserves the *weighted* mean. By the convexity of the level set, any such change must reduce the state price, starting from equal weighted beliefs.

In this expression an agent's beliefs are weighted by both his marginal utility of wealth and his risk tolerance. Generally speaking, the beliefs of wealthy and risk tolerant agents will have a larger weight in the above expression. Thus an increase in the dispersion of those agents' beliefs will have a larger impact than an increase of the dispersion of beliefs of poorer and more risk averse agents, as one might expect.

This result can also be stated in terms of the absolute beliefs rather than the weighted beliefs. By definition of q_i, we have that $dq_i/q_i = d\pi_i/\pi_i$ since the λ_i terms will cancel from the numerator and denominator. Thus if we evaluate (30) at a state where $\pi_1 = \pi_2$, we have:

$$\tau_1 d\pi_1 + \tau_2 d\pi_2 = 0. \tag{33}$$

In this expression the risk tolerances τ_1 and τ_2 are evaluated at a different level of consumption than before—the consumption in the state associated with homogeneous probabilities rather than homogeneous weighted probabilities.

Since the upper contour set is convex in (π_1, π_2) space as well as in (q_1, q_2) space, any movement along tangent line defined in (33) will necessarily decrease the state price. Thus moving from a state with homogeneous beliefs to a state with heterogeneous beliefs but the same level of aggregate consumption will necessarily lower the state price, as long as the *weighted* average of probability beliefs remains constant—where the weights are given by the risk tolerances.

Summarizing the above discussion we have:

THEOREM 6. *Assume that risk tolerance decreases less rapidly than in the case of logarithmic utility. Suppose the agents have homogeneous beliefs about the probability of occurrence of state s. Then if state t has identical aggregate consumption, and identical "weighted mean opinion," $\overline{\pi}$, but more dispersed probability beliefs, it must have a lower state price.*

11. Other Equilibrium Models

The basic analytic tool used in the last few sections is the relationship between a mean preserving spread in opinions and the concavity of the "demand functions". This observation can be applied to a variety of other equilibrium models.

The simplest case is that of a single risky asset. Let v_i be agent i's estimate of the expected value of this asset, and let p be the market price. Suppose that all agents have identical demand functions $D(p, v_i)$ and that the equilibrium price is determined by demand and supply:

$$\sum_{i=1}^{n} D(p, v_i) = S.$$

Suppose that the demand function is a concave function of v_i. Then a mean preserving spread in v_i will decrease the sum of the demands. In order to restore equilibrium, the price must rise. The reverse result holds if the demand function is convex in v_i. These statements, unlike those of the last few sections, *are* comparative statics results since they refer to how the equilibrium values change when opinions become more dispersed, rather than how different asset values compare in a given equilibrium.

So the problem of how an increase in the dispersion of some variable affects equilibrium prices can be reduced to the question of whether demand functions are concave or convex in that variable. How can that be determined The answer comes from examining the structure of the maximization model that lies behind the demands.

The standard comparative static technique to determine the slope of a demand function with respect to some parameter is to differentiate the first-order conditions. In order to determine the *curvature* of a demand function, it is necessary to take the *second derivative* of the first-order conditions which will typically involve the third derivatives of the utility function. In the case investigated in this paper we were able to give a simple interpretation of the conditions for concavity of the demand function in terms of how risk tolerance changes as wealth changes. In general these third-order terms are rather messy, but it seems that there is hope of interpreting them in special cases.

Of course in some models one can derive explicit forms for demand functions. For example, a constant relative risk aversion model with a single, Normally distributed asset gives rise to a demand function of the form

$$D_i = \frac{\tau_i[v_i - P]}{\sigma_i^2},$$

where τ_i is agent i's risk tolerance, v_i is his expected value, and σ_i^2 is his variance. In this case, demand is linear in the "weighted beliefs" $\tau_i v_i / \sigma_i^2$, so that a mean-preserving spread in these weighted beliefs will leave asset prices unchanged. However, demand is a *convex* function of the variances, (σ_i^2), so a mean preserving spread in the agents' beliefs about the variance of the risky asset will tend to *increase* the equilibrium price.[10]

Similar conclusions emerge in a CAPM or continuous time model.[11] Since asset demand functions are linear functions of expected values for each agent, the equilibrium prices of assets will simply depend on the average expected values; assets with different degrees of dispersions of opinion will have the same equilibrium prices, other things being equal.

12. Summary

In the first part of this paper I examined the distinction between information and opinions and showed that in equilibrium the pattern and volume of trade would be determined by differences of opinion, not differences in information.

In the second part of the paper, I examined a model with pure opinion differences and derived the implications for comparative asset pricing. Maintaining the assumption that risk tolerance decreases, but not too rapidly, I showed that if tastes are identical, asset prices would be decreasing functions of the dispersion of opinion. If tastes are different, asset prices would be quasiconcave function of the vector of weighted probability beliefs, so that averaging beliefs would tend to increase asset prices. The curvature of the demand function is the crucial feature in determining how asset prices relate to different degrees of dispersion of beliefs.

---

[10] For an interesting analysis of an equilibrium in a futures market in which traders have different variances, see Stein (1986).

[11] See the Appendix for a discussion of dispersion of beliefs in a continuous time framework.

Appendix: A Continuous Time Model

In this appendix, we examine a continuous time model in which there are heterogeneous beliefs about asset returns. Models of this sort have been extensively investigated by Williams (1977) and Grossman and Shiller (1982), with a somewhat different emphasis. By combining the insights of these authors we can adapt their results to the problem at hand.

Let r_s be the (instantaneous) random return on security s, and let r_0 be the (instantaneous) risk free rate. Suppose that all asset returns and the optimal consumption of each agent follow an Ito process. Then in our notation Grossman and Shiller's equation (11a) takes the form:

$$r_{is} = r_0 + A_i \mathrm{cov}_i(r_s, dc_i/c_i)$$

where r_{is} is agent i's expected return on security s, $\mathrm{cov}_i(r_s, dc_i/c_i)$ is agent i's belief about the covariance of the rate of growth of his consumption and the rate of return on security s, and

$$A_i = -\frac{u_i''(c_i)}{u_i'(c_i)} c_i$$

is agent i's coefficient of *relative* risk aversion. This is simply the first order condition for optimal choice in a continuous time model, combined with Ito's lemma. Using the linearity of the covariance operator, and that fact that c_i is nonstochastic at the time the optimal choice is made we have:

$$r_{is} = r_0 + \frac{1}{\tau_i} \mathrm{cov}_i(r_s, dc_i). \tag{34}$$

where $\tau_i = -u'(c_i)/u_i''(c_i)$ is agent i's absolute risk tolerance.

Williams (1977) and Mertón (1980) have argued that if security returns follow an Ito process, it is possible to estimate the covariance matrix of security returns arbitrarily accurately in an arbitrarily short time interval. The expected returns of the assets, on the other hand, will always be estimated imprecisely. Thus, following the argument of Williams (1977), we will assume that agents agree about the covariance matrix of the returns, and disagree only in their beliefs about the expected returns. This allows us to drop the subscript i on the covariance term in (34) and simply write it as $\mathrm{cov}(c_i, r_s)$.

Cross multiply (34), sum over the agents $i = 1, \ldots, n$, and use the linearity of the covariance operator to get:

$$\sum_{i=1}^{n} \tau_i r_{is} = r_0 \sum_{i=1}^{n} \tau_i + \mathrm{cov}(r_s, dC) \tag{35}$$

where $dC = \sum_{i=1}^{n} dc_i$ is the change in aggregate consumption.

Now define

$$\gamma_i = \frac{\tau_i}{\sum_{j=1}^{n} \tau_j}$$

and

$$\kappa = \frac{C}{\sum_{j=1}^{n} \tau_j}$$

and rewrite this expression as:

$$\sum_{i=1}^{n} \gamma_i r_{is} = r_0 + \kappa \text{ cov}(r_s, dC/C). \tag{36}$$

In order to interpret this equation, we note that the left hand side is the weighted average of expected returns, where the weight given to agent i's beliefs is proportional to his risk tolerance. The right hand side of the equation is the risk free return plus a constant depending on the average risk tolerance in the economy, times the covariance of the asset return with the rate of growth of aggregate consumption.

This calculation is similar to the derivation of Grossman and Shiller (1982) but with a different interpretation. They take expectations with respect to the set of common information available to the agents at the time their consumption decisions are made. The left hand side of equation (36) in their framework is the expected return on security s based on the publicly available information at a given time. In our framework, the left hand side of (36) is a weighted average of subjective beliefs about the expected returns.

Following Williams (1977) we have assumed that agents have homogeneous beliefs about the covariance structure of the asset returns, but differ in their expected values for asset returns. The expectations in equation (36) are the actual expectations of the agents, based on their own private information and beliefs, not the expectations based on some common information as in Grossman and Shiller.

Equation (36) is the analog of Williams (1977) continuous time CAPM with heterogeneous beliefs, but because we use the covariance with consumption rather than the covariance with respect to wealth, the "hedging portfolios" in his equation are not needed here. (See Breeden (1979) for further discussion of why "hedging portfolios" are not needed in general in consumption based pricing models in continuous time.)

For our purposes, the interesting thing about equation (36) is what it says about comparative asset pricing. If two securities have the same "consumption beta" then they have to have the same "average expected return." This in turn implies that the pricing of securities is independent of the dispersion of opinion about their expected values. To see this, let v_s be the market value of security s at a specific time and let dv_s be the random change in its value in the next instant. Then since the instantaneous return is given by $r_s = dv_s/v_s$, we can solve (36) to get:

$$v_s = \frac{\sum_{i=1}^{n} \gamma_i dv_{is} + \kappa \text{ cov}(dC/C, dv_s)}{r_0}.$$

Here dv_{is} is agent i's belief about the expected change in value of asset s. Thus two securities that have the same "consumption beta" and the same *average* expected return must have the same price.

It is interesting to compare this result with equation (33). In that equation π_1 and π_2 are the expected returns on the Arrow–Debreu asset under consideration, since it pays zero in all states but one. Theorem 6 established that an increase in dispersion of opinion that keeps the weighted average of opinions constant will decrease the state price. In the continuous time framework examined above, such an increase in dispersion of opinion will leave the state price unchanged.

The insensitivity of asset prices to differences of opinion arise in this framework because of the linearity of the asset demand functions in the expected returns. Since only the first two derivatives of utility enter into the portfolio choice problem, the higher order curvature properties of utility considered earlier will not affect asset prices in equilibrium.

A similar result occurs in a discrete time model, if we are willing to postulate that individuals believe that their optimal consumption in future periods and asset returns will be Normally distributed. Since the algebra is almost the same as that given above, we will only sketch the details.

The first order conditions for utility maximization by individual i will take the form:

$$E_i u_i'(c_i)(r_s - r_0)$$

where E_i stands for expectation with respect to agent i's subjective beliefs about the asset returns. (The expectations are at time 0 for some given time in the future, but we omit the time subscript so as not to clutter the notation.)

Using the standard covariance identity, and rearranging, we can write this as:

$$r_{is} = r_0 - \frac{\text{cov}_i(u_i'(c_i), r_s)}{E_i u'(c_i)}$$

where $r_{is} = E_i r_s$, as before. Assume now that c_i and r_s are perceived as bivariate Normal by investor i. Applying a theorem due to Rubinstein (1976b) we have:

$$r_{is} = r_0 + \left(-\frac{E_i u_i''(c_i)}{E_i u_i'(c_i)}\right) \text{cov}_i(c_i, r_s).$$

Letting $\tau_i = -E_i u_i'(c_i)/E_i u''(c_i)$ we can write this as:

$$r_{is} = r_0 + \frac{1}{\tau_i}\text{cov}_i(c_i, r_s). \tag{37}$$

This equation is virtually the same as (34). If we assume that agents have the same beliefs about the covariance structure of the asset returns, the we can apply the same algebraic manipulations as before gives us:

$$\sum_{i=1}^{n} \gamma_i r_{is} = r_0 + \kappa \ \text{cov}(C, r_s). \tag{38}$$

The only difference between (36) and (38) is that γ_i has a slightly different interpretation in the two equations, and that (38) is expressed in levels rather than instantaneous rates of change.

Equations (36) and (38) are on the verge of being estimable, at least given survey data on expectations and the time series estimates of the covariance between asset returns and aggregate consumption that are beginning to appear. Perhaps the addition of heterogeneous beliefs about asset returns will improve the performance of the empirical estimates of consumption based asset pricing models.

Finally, we consider the volume of trade in this kind of model. If we increase dispersion of opinion in the way described earlier — where the net demanders become more optimistic and the endowments of the two assets across the consumers are the same — and we keep the weighted sum of expected returns constant then it is easy to see that the volume of trade must increase. For an increase in the expected return on an asset must increase net demand for it by an individual, and we have established above that the asset price remains unchanged. Thus the transactions volume in the asset with the more dispersed beliefs must unambiguously be larger.

REFERENCES

Aumann, R. (1976), "Agreeing to Disagree," *Annals of Statistics*, **4**, pp. 1236–39.

Bhattacharya, S. (1976), "Information Efficiency and Inter-temporal Rates of Return," Sloan School mimeo.

Black, F. (1986), "Noise," *Journal of Finance*, , .

Breeden, D. and M. Litzenberger (1978), "Prices of State-Contingent Claims Implicit in Option Prices," *Journal of Business*, **51**, 621–51.

Breeden, D. (1979), "An Intertemporal Asset Pricing Model with Stochastic Consumption and Investment Opportunities," *Journal of Financial Economics*, 7, 265–296.

Cragg, J. and B. Malkiel (1982), *Expectations and the Structure of Share Prices*, University of Chicago Press, Chicago.

Diamond, D. and R. Verrecchia (1981), "Informational Aggregation in a Noisy Rational Expectations Economy," *Journal of Financial Economics*, **9**, 221–235.

Epps, T. and M. Epps (1976), "The Stochastic Dependence of Security Price Changes and Transaction Volumes: Implications for the Mixture-of-Distribution Hypothesis," *Econometrica*, **44**, 305–321.

Grossman, S. (1976), "On the Efficiency of Competitive Stock Markets Where Traders Have Diverse Information," *The Journal of Finance*, **31**, 573–585.

Grossman, S. (1977), "The Existence of Futures Markets, Noisy Rational Expectations, and Informational Externalities," *Review of Economic Studies*, 44, 431–449.

Grossman, S. (1978), "Further Results on the Informational Efficiency of Competitive Stock Markets," *Journal of Economic Theory*, **18**, 81–101.

Grossman, S. and R. Shiller (1982), "Consumption Correlatedness and Risk Measurement in Economies with Non–Traded Assets and Heterogeneous Information," *Journal of Financial Economics*, 10, 195–210.

Grossman, S. and J. Stiglitz (1980), "On the Impossibility of Informationally Efficient Markets," *American Economic Review*, **70**, 393–408.

Hakansson, N., J. Kunkel and J. Ohlson (1982), "Sufficient and Necessary Conditions for Information to have Social Value in Pure Exchange," *Journal of Finance*, **37**, 1169–1181.

Harris, L. (1983), "The Joint Distribution of Speculative Prices and of Daily Trading Volume," University of Southern California Working Paper.

Harsanyi, J. (1983), "Bayesian Decision Theory, Subjective and Objective Probabilities, and Acceptance of Empirical Hypotheses," *Synthese*, **57**, pp. 341–365.

Hellwig, M. (1980), "On the Aggregation of Information in Competitive Markets," *Journal of Economic Theory*, **22**, 477–498.

Jarrow, R. (1980), "Heterogeneous Expectations, Restrictions on Short Sales, and Equilibrium Asset Prices," *Journal of Finance*, **35**, 1105–13.

Karpoff, J. (1985), "The Relationship between Stock Prices and Volume," *Center for the Study of Banking and Financial Markets Digest*, University of Washington, Seattle.

Lintner, J. "The Aggregation of Investors' Diverse Judgements and Preferences in Purely Competitive Markets," *Journal of Financial and Quantitative Analysis*, **4**, 347–400.

Marshall, J. (1974), "Private Incentives and Public Information," *American Economic Review*, **64**, 373–390.

Mayshar, J. (1983), "On Divergence of Opinion and Imperfections in Capital Markets", *American Economic Review*, **73**, 114–28.

Merton, R. (1980), "On Estimating the Expected Return on the Market," *Journal of Financial Economics*, 8, 323–361.

Milgrom, P. and N. Stokey (1982), "Information, Trade and Common Knowledge," *Journal of Economic Theory*, **26**, 17–27.

Rothschild, M. and J. Stiglitz (1970), "Increasing Risk I: a Definition," *Journal of Economic Theory*, **2**, 225–243.

Rubinstein, M. (1975), "Security Market Efficiency in an Arrow-Debreu Economy," *American Economic Review*, **65**, 812–824.

Rubinstein, M. (1976), "The Strong Case for the Generalized Logarithmic Utility Model as the Premier Model of Financial Markets," *Journal of Finance*, **31**, 551–571.

Stein, J. (1986), "Real Effects of Futures Speculation: Asymptotically Rational Expectations," *Economica*, **53**, pp. 159–180.

Tauchen, G. and M. Pitts (1983), "The Price Variability–Volume Relationship on Speculative Markets," *Econometrica*, **51**, 485–506.

Varian, H. (1984), *Microeconomic Analysis*, W. W. Norton & Co., New York.

Varian, H. (1985a), "Divergence of Opinion in Complete Markets," *Journal of Finance*, **40**, 1, 309—317.

Varian, H. (1985b), "Observable Implications of Changes in Risk Aversion," University of Michigan Working Paper. To appear in *Econometrica*.

Verrecchia, R. (1981), "On the Relationship Between Volume Reaction and the Consensus of Investors: Implications for Interpreting Tests of Information Content," *Journal of Accounting Research*, **19**, 271–283.

Williams, J. (1977), "Capital Asset Prices with Heterogeneous Beliefs," *Journal of Financial Economics*, **5**, 219–39.

Recent CREST Working Papers

87-1: Jeffrey K. MacKie-Mason, "Nonlinear Taxation of Risky Assets and Investment, With Application to Mining" September, 1984.

87-2: Jeffrey K. MacKie-Mason, "Sequential Decision Problems and Asymmetric Information" September, 1985.

87-3: Michelle J. White, "Contract Breach and Contract Discharge due to Impossibility: A Unified Theory" July 29, 1987.

87-4: Ted Bergstrom, "Systems of Benevolent Utility Interdependence" May 19, 1987.

87-5: Ted Bergstrom, "A Fresh Look at the Rotten Kid Theorem—and Other Household Mysteries" November, 1986.

87-6: Michelle J. White, "The Corporate Bankruptcy Decision" July, 1987.

87-7: Michelle J. White, "Location Choice and Commuting Behavior in Cities with Decentralized Employment" July, 1987.

87-8: Lawrence E. Blume and David Easley, "Implementation of Walrasian Expectations Equilibria" December, 1985.

87-9: Lawrence E. Blume, "Lexiocographic Refinements of Nash Equilibrium" April, 1986.

87-10: David Lam, "Lorenz Curves, Inequality, and Social Welfare Under Changing Population Composition" June 16, 1987.

87-11: Mark Bagnoli and Naveen Khanna, "Equilibrium with Debt and Equity Financing of New Projects: Why More Equity Financing Occurs When Stock Prices are High" June, 1987.

87-12: Mark Bagnoli and Barton L. Lipman, "Provision of Public Goods: Fully Implementing the Core through Private Contributions" March, 1987.

87-13: Mark Bagnoli and Barton L. Lipman, "Successful Takeovers without Exclusion" August, 1987.

87-14: Mark Bagnoli and Michael McKee, "Controlling the Game: Political Sponsors and Bureaus" May, 1987.

87-15: Mark Bagnoli and Michael McKee, "Can the Private Provision of Public Goods be Efficient?—Some Experimental Evidence" March, 1987.

87-16: Mark Bagnoli, "Non-Market Clearing Prices in a Dynamic Oligopoly with Incomplete Information" January, 1986.

87-17: John Laitner, "Bequests, Gifts, and Social Security" February 28, 1986.

87-18: John Laitner, "Dynamic Determinacy and the Existence of Sunspot Equilibria" May 12, 1986.

87-19: David Lam, "Does a Uniform Age Distribution Minimize Lifetime Wages?" August 12, 1987.

87-20: David Lam, "Assortative Mating with Household Public Goods" April, 1987.

87-21: Jeffrey A. Miron and Stephen P. Zeldes, "Production, Sales, and the Change in Inventories: An Identity that Doesn't Add Up" June 1987.

87-22: Jeffrey A. Miron and Stephen P. Zeldes, "Seasonality, Cost Shocks, and the Production Smoothing Model of Inventories" December, 1986.

87-23: Hal R. Varian, "Differences of Opinion in Financial Markets" March, 1985.

87-24: Roger H. Gordon and Hal R. Varian, "Taxation of Asset Income in the Presence of a World Securities Market" August, 1986.

87–25: Hal R. Varian, "Measuring the Deadweight Costs of DUP and Rent Seeking Activities" November, 1982.

87–26: Hal R. Varian, "Price Discrimination" January, 1987.

87–27: Roger H. Gordon and Hal R. Varian, "Intergenerational Risk Sharing" October, 1985.

87–28: Hal R. Varian, "Three Papers on Revealed Preference" August, 1987.

87–29: Hal R. Varian, "Optimal Tariffs and Financial Assets" April, 1987.

87–30: Jonathan Cave and Stephen W. Salant, "Cartels That Vote: Agricultural Marketing Boards and Induced Voting Behavior" August, 1987.

87–31: Stephen W. Salant and Donald H. Negri, "Pastures of Plenty: When is the Standard Analysis of Common Property Extraction Under Free Access Incorrect?" July 10, 1987.

87–32: Stephen W. Salant, "When is Inducing Self-Selection Sub- optimal for a Monopolist?" February, 1987.

87–33: Stephen W. Salant, "Treble Damage Awards in Private Lawsuits for Price-Fixing" August, 1987.

87–34: Stephen W. Salant and Roy Danchick, "Air Force Academy Attrition: A New Perspective on the College Dropout Problem" August, 1987.

87–35: Stephen W. Salant and Eban Goodstein, "Committee Voting Under Alternative Procedures and Preferences: An Experimental Analysis" April 20, 1987.

87–36: Robert B. Barsky and Jeffrey A. Miron, "The Seasonal Cycle and the Business Cycle" June, 1987.

87–37: Robert B. Barsky, N. Gregory Mankiw, Jeffrey A. Miron and David N. Weil, "The Worldwide Change in the Behavior of Interest Rates and Prices in 1914" July, 1987.

87–38: Jeffrey K. MacKie-Mason, "Taxes, Information and Corporate Financing Choices" April 1986.