RESEARCH SEMINAR IN INTERNATIONAL ECONOMICS

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SEMINAR DISCUSSION PAPER NO. 264

TOLERANCE OF ARREARAGES: HOW IMF LOAN POLICY CAN EFFECT DEBT-REDUCTION

by

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July 1990

Abstract: A model of bargaining with asymmetric information is applied to the case of sovereign debt and IMF intervention. Debtors and creditors are assumed to bargain over the "price" to be paid by the debtors for their outstanding debt, and the ability of the debtors to delay an agreement is private information. It is shown that an IMF policy of "no lending in arrears" increases the price of the debt, increases any delay to agreement, and increases the likelihood that the creditor will demand the highest price. An IMF policy of "lending in arrears", in contrast, decreases the price of the debt, decreases the delay time to agreement, and my increase the likelihood that the creditor accepts the lowest price as it shifts the locus of bargaining power. Predictions of the model are consistent with results from recent debt agreements, and implications for current debate on debt-reduction strategies are discussed.

I thank Alessandra Casella, Dick Eckaus, Rob Kahn, Paul Krugman, Richard Lindsey, and Ken Rogoff for providing insight, Dimitrios Vayanos for excellent research assisstance, and I accept responsibility for all mistakes within.

1. INTRODUCTION

Few would argue that the actions of official agencies, although not direct participants in sovereign debt negotiations, have not influenced the outcomes of those negotiations. The nature of that influence is a matter of debate. Some claim that official actions will facilitate negotiations, enabling participants to come to faster and more mutually beneficial agreements. Others insist that indirect official participation has actually hindered the negotiations, making the participants more intransigent as they seek to increase the transfers they receive from official agencies.

Which side of this debate is correct has important implications for current proposals for a "Debt Facility" which would, it is claimed by its proponents, quickly effect the needed debt reduction which has so far been unattainable. The same sides have been drawn in this debate, with the oppposition claiming that a debt facility would largely benefit creditors without helping debtors, and stultify the negotiating process.[1]

As the academic debate goes on, events proceed apace. In some countries, the IMF has instituted a policy change called for in the Brady Plan of "lending in arrears", disbursement of funds whether an agreement has been reached with creditors or not. Those countries are Mexico and the Philippines, Jordan, Argentina, Cote d'Ivoire, Ecuador, Poland and Venezuela. This change has been greeted with dismay by

[1]See Kenen (1990), Sachs (1990), Dornbusch and Makin (1989) for pro-debt facility arguments, and Bulow and Rogoff (1990) and Eaton (1990) for contra-debt facility arguments.

bankers[2]. A menu of favorable debt reduction mechanisms has been agreed upon by creditors for Mexico in July of 1989, for Venezuela in February 1990, and others are coming down the pipe. Mexico's agreement was recently analyzed by Wijnbergen and Claessens (1989). They conclude, quite surprisingly, that Mexico captured between 76% and 97% of the enhancement value of the new debt instruments, which means that practically all of the enhancement monies went towards real debt reduction. This result is surprising because it implies that Mexico struck a much more advantageous deal than any other debtor has been able to strike so far. Adding to this puzzle is the fact that the IMF has publicly committed itself to lending to Mexico in the presence of arrears, even though Mexico has not run arrears for several years and it is not anticipated to run arrears in the future. Why announce a policy for an event which is not expected to occur?

This paper modifies the Admati and Perry model of model of bargaining with asymmetric information and applies it to this debate on the effect of third-party intervention in the bargaining process. The model is used to analyze a scenario in which debtors have differing ability to withstand lack of access to international financial markets, and this ability is private knowledge. It is shown that the policy of "no lending in arrears", that is loans are not disbursed until an agreement is reached with creditors, has a very different effect on the outcome of the bargaining game than a policy of "lending in arrears".

[2]Viz., the statement by John Haseltine, a senior banking executive, in Dornbush, Makin and Zlowe, "The banks are also concerned with the lessening of the linkages betweeen IMF disbursements and actions by banks. We have heard the expression "tolerance of arrearages," whch puts fright into a banker's heart and is the kiss of death for general waivers."

It is shown that an <u>announced</u> policy of lending in arrears, as well as the actual policy of lending in arrears, can effect debt reduction before the bargaining process even begins. These results support the spirit of the Bulow and Rogoff critique on debt-reduction schemes and fully explain the response of bankers to the change in policy as well as the conundrums posed by the Mexico agreement. Unlike previous models in either the sovereign debt literature or the bargining literature, this model specifically analyzes the effects of disbursements made by third-parties on the equilibrium outcomes.

The following describes the structure of the paper. Section 2 presents a benchmark model of bargaining under asymmetric information when there are two types of debtors. Section 3 analyzes the effect of a loan to the debtor, made contingent upon agreement being reached between the debtor and creditor, on the bargaining game between debtor and creditor. Section 4 analyzes the effect of non-contingent lending upon the bargaining game. Section 5 discusses the implications of the model for debate on debt-reduction schemes, and the recent Mexico agreement. Section 6 concludes/ both the effect of loan disbursements and the announcement of loan dibursements made before the bargaining game begins, and discusses important implications for debt-reduction schemes. Section 5 concludes.

2. BARGAINING UNDER ASYMMETRIC INFORMATION WITH TWO TYPES OF DEBTORS

Assume that a single bank, denoted B, is bargaining with a single debtor, denoted D, for repayment. One can think of B as offering to "sell" its lien against D's assets and a bargaining game determines the

price at which this lien is sold. The value of the lien to B, if it were executed upon immediately, is S, the amount which B could recover from D in event of default, or the salvage value of the lien. S is common knowledge. The value of the termination of the lien to D, if it were terminated immediately, is K. K is also common knowledge. S and K are assumed to be constant during the negotiating process, an assumption which, though somewhat implausible, makes the problem much more tractable. One could instead think of S and K as constant shares of the debtor's GNP (e.g., a "fixed penalty technology") and the rest of the analysis will follow through unperturbed. Trade is assumed to be efficient, so S < K. Given this, S is normalized to equal 0. Both B and D have a positive rate of time preference. B's discount rate, δ_B, is equal to the market discount rate, $(1 + r_M)^{-1}$, where r_M = LIBOR. Hence the net present value to B of K if it were delivered to him in the next period is $\delta_B K$. $(1 - \delta_B)K$, B's per period loss on uncollected funds represents his opportunity cost of those uncollected funds, and is a function only of r_M as it is assumed that B has unlimited access to world capital markets. D's discount rate can take either of two values, δ_L or δ_H, where $0 < \delta_H < \delta_L < 1$, $\delta_H = (1 + r_H)^{-1}$, $\delta_L = (1 + r_L)^{-1}$. Whether D is the low type (low cost to waiting/high patience) or the high type (high cost to waiting/low patience) is private information. Let π denote B's prior probability that $\delta = \delta_H$, and π is common knowledge. For D, the share of the surplus lost per period until an agreement is reached are due to lack of access to world financial markets. Hence it is assumed that $\delta_H < \delta_L < \delta_B$. One may think of D losing the return on all projects with a return less than D's real interest rate which is higher than the market rate. With a higher discount rate, the bank is effectively more patient than either type of debtor.

Bargaining commences at $t = 0$, in which B makes the first offer[3]. B and D then alternate making offers until an agreement is reached. Time periods are endogenous and are defined as commencing immediately after an offer is made. A party can delay in making a counteroffer, however, so a single round of bargaining can last an indefinite amount of time. Let P^i denote the price on the table at round i, and α^i denote the time delay in round i. An agreement after N rounds is reached at $t = N + \sum_{i=1}^{N}\alpha^i$. For now it will be assumed that D is not wealth-constrained in the sense that is is able to pay any price agreed upon. This is a relatively innocuous assumption because if one assumes instead that due to wealth constraints only a portion of the debt is negotiated at a time, the structure of the model and its results still hold. Denoting an outcome by the pair (P,t), the payoff to B associated with (P,t) is $\delta_B^{t-1}P$, and the payoff to D is $\delta_i^{t-1}(K - P)$, $i \in \{L,H\}$.

From the standard Rubinstein game, if D's type were common knowledge and one restricted equilibria to the set of perfect equilibria[4], agreement would immediately be reached at the following prices:

$$Q_L = \frac{1 - \delta_L}{1 - \delta_B\delta_L}K = s_L K \quad \text{and} \quad Q_H = \frac{1 - \delta_H}{1 - \delta_B\delta_H}K = s_H K \qquad (1)$$

[3]A disconcerting feature of bargaining models is that a larger share of the surplus goes to the party which makes the first offer relative to the share received the by the same party if she moved second. Binmore (1980) randomizes over the choice of the first mover, and Sutton (1986) lets time delay between rounds converge to 0 to eliminate the first-mover advantage.

[4]A perfect equilibrium is one in which the strategies for the entire game and for every subgame (i.e., round) are an equilibrium.

Q_L and Q_H are thus the unique perfect equilibrium prices in the full information game. Note $Q_H > Q_L$, a result of H's greater impatience. Also note that D's surplus from the agreement, $(1 - s_L)K$ for type L and $(1 - s_H)K$ for type H, is less than B's surplus from the agreement, s_LK and s_HK respectively, again owing to B's relatively greater patience.

With private information and restricting the set of equilibrium outcomes to lie in the set of sequential equilibria[5], several equilibrium outcomes are still possible. Lemma 1 in Appendix I characterizes the set of all possible equilibrium outcomes. A tie-breaking assumption and a Cho-type refinement are used to further restrict the set of possible equilibria and are described in Appendix II.

In general, there are three types of equilibrium outcomes: pooling, separating, and semi-separating. In a pooling equilibrium, no private information is revealed and both types pay Q_L. In a separating equilibrium, all private information is revealed: D_H pays either Q_H or δ_BQ_H[6] and D_L pays δ_BQ_L. In the semi-separating equilibrium outcome, private information is partially revealed: D_H pays $P \in [\bar{P},Q_H]$[7] and D_L pays δ_BQ_L. Which type of equilibrium occurs is determined by the initial offer made by B, which is determined by π_0. For $\pi_0 > \pi^*$ (π^* is defined in Section 2.2.), the separating equilibrium is the unique equilibrium outcome. For $\pi_0 < \pi^{**}$ (π^{**} is defined in Section 2.2.), the pooling

[5]In a sequential equilibrium, for every history the set of strategies and corresponding beliefs (π) form an equilibrium. See Kreps and Wilson (1982) or Rubinstein (1985).

[6]D_H pays Q_H when the separating equilibrium is the unique equilibrium, and may pay δ_BQ_H or Q_H when B plays the separating strategy but the separating equilibrium is not a unique equilibrium. See Sections 2.2. and 2.3.

[7]$\bar{P}$ is defined in Lemma 1(v).

equilibrium is the unique equilibrium outcome. For $\pi^{**} < \pi_0 < \pi^*$, there is no unique equilibrium outcome and a pooling, separating or semi-separating equilibrium can occur.

2.1. THE DEBTOR'S EQUILIBRIUM STRATEGIES

The equilibrium strategies chosen by both types of debtors will depend upon B's offer. In response to an offer, a debtor can accept or counteroffer immediately, or counteroffer with a delay. The following definition is essential to deriving equilibrium strategies:

<u>Definition</u>: For a given offer P, let $\Gamma^*(P)$ satisfy the following preference ordering:

$$(P,0) \sim_H (\delta_B Q_L, \Gamma^*(P)) \tag{2}$$

When $\alpha = \Gamma^*(P)$, D_H is indifferent between accepting P immediately and pretending to be D_L by counteroffering the minimum acceptable (to D) offer, $\delta_B Q_L$.[8] Hence by delaying at least as long as $\Gamma^*(P)$, D_L can 'separate' himself from D_H. Observe that $\Gamma^*(P)$ is increasing in P.

Theorem 1 in Appendix III characterizes the equilibrium strategies of both types of debtor, which are summarized as follows. If B offers Q_L

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[8] $\Gamma^*(P)$ serves the function of establishing an incentive compatability constraint. That is, for an offer P, the following must hold:

$$K - P = \delta_H^{\Gamma^*(P)}(K - \delta_B Q_L)$$

Thus D_H will not wish to imitate D_L by signalling $\Gamma^*(P)$. An individual rationality constraint is automatically satisfied by the fact that $K > 0$ and $0 < s_L, s_H < 1$.

($P \geq Q_L$ from Lemma 1 in Appendix I), both D_H and D_L accept immediately. This is the pooling equilibrium. If $P > Q_L$, D_H may accept P or counteroffer $\delta_B Q_H$ at time t + 1, which B accepts. If D_H accepts P, D_L counteroffers $\delta_B Q_L$ at time max {t + 1, t + $\Gamma^*(P)$}, which B accepts. If D_H counteroffers $\delta_B Q_H$, D_L counteroffers $\delta_B Q_L$ at time t + 1 + $\Gamma^*(\delta_B Q_H)$. Thus if $P > Q_L$, the result is either a separating or semi-separating equilibrium.

2.2. THE UNIQUE SEPARATING EQUILIBRIUM AND THE UNIQUE POOLING EQUILIBRIUM

Note from Lemma 1(v) in Appendix.II. the definition of $\bar{P}$:

$$(\bar{P},0) \sim_H (\delta_B Q_L, 1)$$

$\bar{P}$ is the benchmark price at which D_H is indifferent between accepting immediately and counteroffering $\delta_B Q_L$ immediately. Suppose that at t = 0, B's prior assessment is π, and B offers $P \in [\bar{P}, Q_H]$ which D_H accepts at t = 1. D_L then counteroffers $\delta_B Q_L$ at t = 1 + $\Gamma^*(P)$, which B accepts. The expected payoff to B is then

$$W(P,\pi) = \pi P + (1 - \pi)\delta_B^{\Gamma^*(P)}\delta_B Q_L \tag{3}$$

If B instead offered $P \in (Q_L, \bar{P})$, D_H accepts at t = 1, and D_L counteroffers $\delta_B Q_L$ at t = 1, which B accepts. The expected payoff to B is then

$$\hat{W}(P,\pi) = \pi P + (1 - \pi)\delta_B^2 Q_L \tag{4}$$

In choosing the level of an initial offer, B must trade-off the return from that offer if D = D_H versus the loss due to delay if D = D_L. Differentiation of (3) with respect to P and using the fact that

$$\delta_H^{\Gamma^*(P)} = \frac{K - P}{K - \delta_B Q_L} \tag{5}$$

obtains the result that $W(P,\pi)$ is increasing in P if and only if $\pi > \hat{\pi}$ where

$$\pi > \hat{\pi} = -\frac{\dfrac{\partial \delta_B^{\Gamma^*(P)}}{\partial P}\,\delta_B Q_L}{1 - \dfrac{\partial \delta_B^{\Gamma^*(P)}}{\partial P}\,\delta_B Q_L} \tag{6}$$

and
$$\frac{\partial \delta_B^{\Gamma^*(P)}}{\partial P} = -\left(\frac{1}{K - \delta_B Q_L}\right)\left(\frac{\delta_B}{\delta_H}\right)^{\Gamma^*(P)} \frac{\ln \delta_B}{\ln \delta_H} < 0 \tag{7}$$

Simple algebra shows that $W(Q_H,\pi) > Q_L$ if and only if $\pi > \pi^{**}$, where

$$\pi^{**} = \frac{Q_L\left(1 - \delta_B^{\Gamma^*(Q_H)}\delta_B\right)}{Q_H - \delta_B^{\Gamma^*(Q_H)}\delta_B Q_L} \tag{8}$$

Using the fact that $\bar{P} = K - \delta_H(K - \delta_B Q_L)$, more simple algebra will show that $W(\bar{P},\pi) > Q_L$ if and only if $\pi > \pi^*$, where

$$\pi^* = \frac{Q_L\left(1 - \delta_B^2\right)}{K - \delta_H\left(K - \delta_B Q_L\right) - \delta_B^2 Q_L} \tag{9}$$

The following theorem states the conditions under which the separating equilibrium and the pooling equilibrium are the unique equilibrium outcomes:

<u>Theorem 2</u>: If (i) $\pi_0 > \pi^*$, and (ii) $\left(\dfrac{\delta_H}{\delta_B}\right)^{\Gamma^*(Q_H)} > \left|\dfrac{\ln \delta_B}{\ln \delta_H}\right|$, then the unique equilibrium is the separating equilibrium in which B offers Q_H at $t = 0$, D_H accepts, and D_L counteroffers $\delta_B Q_L$ at $t = 1 + \Gamma^*(P)$, which B accepts.[9] If $\pi_0 < \pi^{**}$, then the unique equilibrium is the pooling equilibrium in which B offers Q_L at $t = 0$, which D_H and D_L accept at $t = 1$.

<u>Proof</u>: See Appendix IV.

To summarize the results of this section, if B's prior assessment that she is facing a high-cost debtor is sufficiently high and potential delays are not too large, B will offer the highest possible price, Q_H. If D is the high-cost type, he will accept immediately. If, however, D is the low-cost type, he will delay by $\Gamma^*(P)$ to signal his type, and then counteroffer the minimum acceptable price, $\delta_B Q_L$. B will accept this price as he is convinced that D is truely the low-cost type and knows that a counteroffer of a higher price will always be rejected. The fully separating equilibrium is the unique equilibrium in this case. If, however, B's prior assessement that he is facing high cost debtor is sufficiently small (less than π^{**}), B will offer the minimum acceptable price, Q_L, and both D_H and D_L will accept immediately. The pooling

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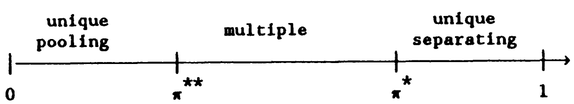
[9] An intuitive interpretation of condition (ii) is that the value taken by $\Gamma^*(Q_H)$ must not be too large, which implies that $\delta_L - \delta_H$ must not be too large. Otherwise, it is too risky for B to play the separating strategy (offer Q_H) because the cost associated with the potential delay is large relative to the return from this strategy. (ii) is a monotonicity condition which assures that $W(\bar{P},\pi)$ is monotonically increasing in P.

equilibrium is the unique equilibrium in this case.

2.3. MULTIPLE EQUILIBRIA

If $\pi^{**} \le \pi_0 \le \pi^{*}$, the equilibrium outcome may be a pooling, separating, or semi-separating (that is B offers $P \in [\bar{P}, Q_H])$ equilibrium. The reason that D_H may be able to pay $P \in [\bar{P}, Q_H]$ is that for this intermediate range of π_0, D_H can credibly threaten to play the pooling strategy (counteroffer $\delta_B Q_L$) if B's offer is too high. Thus if π_0 lies in this intermediate range, D_H's payoff may be better than his separating equilibrium payoff, and certainly will not be any less than his separating equilibrium payoff.

The following diagram summarizes the relationship between π_0 and the equilibrium outcomes:



(D.1)

We will now focus the rest of this paper upon how different IMF loan policies effect these results.

3. APPLICATION OF THE MODEL TO THE POLICY OF "NO LENDING IN ARREARS"

Up until now, the discussion on IMF loan policy towards troubled debtors has centered around the Fund's policy of 'conditionality': that is, loans are not disbursed until a debtor has undertaken a program of austerity measures and corrective policies. However, another ingredient of the Fund's loan policy toward debtor's has been its making disbursements of loans contingent upon an agreement being reached between

the debtor and its creditors. That is, funds are not disbursed to the debtor until it is current with its creditors. This is the policy of "no lending in arrears." The effect of this policy upon the bargaining process between debtors and creditors has not been analyzed until this now.

Bulow and Rogoff (1988) model a full information three-way bargaining game between a debtor, creditors, and creditor-country taxpayers. Creditor-country taxpayers are effectively "gamed" into making side-payments to the debtor and creditors owing to the taxpayers interest in ensuring that an agreement is reached because they gain from trade with the debtor. I choose not to employ a three-way bargaining game in modelling the effect of IMF policy because the IMF has not operated as an independent bargaining party in debtor-creditor negotiations. Rather, it has behaved like a "residual party" in the sense that its policies may influence the bargaining game between debtor and creditor but it has not entered into the bargaining game as an independent agent. Rather, I assume that the IMF makes loans of a predetermined amount (that is, not subject to negotiation) at the market interest rate, and has the choice of making those loans contingent upon agreement between debtor and creditor (in which case the funds are disbursed once the agreement is reached) or making a loan independent of the agreement (in which case funds are disbursed prior to the bargaining process). The former case is analyzed in this section using the the asymmetric information bargaining game with two types. The latter case is analyzed in Section 5. The equilibria resulting from two types of policies are shown to be quite different.

Since the the IMF loan is made to the debtor at LIBOR, it has a zero net present value for the IMF but a positive net present value for both types of debtors since $r_H > r_L > r_M$. It is shown that the positive net present value of the contingent loan becomes part of the surplus of the agreement to be divided between the debtor and creditor, hence some of the benefit of the loan accrues to the creditor. Furthermore, the contingent loan does not alter the equilibrium shares which result from the bargaining game and neither does it change the form of the game or the manner in which the creditor's prior beliefs determine the equilibrium outcome.

Assume that at $t = 0$, the IMF announces a policy that it will disburse a loan in the amount τ at LIBOR to D once agreement has been reached with B. The value of this loan to D if it were received immediately is $(r_H - r_M)\tau = A_H$ for D_H and $(r_L - r_M)\tau = A_L$ for D_L, and is discounted at δ_H and δ_L respectively per period. Note that $A_H > A_H$ for any τ. The existence of this policy does not change K as it was originally defined because the policy does not change the value of B's lien against D's assets. B realizes, however, that an agreement now has a greater value to D owing to the positive value of the contingent loan to D. Hence B will increase the price it offers to D to capture part of the extra surplus accruing to D from an agreement. If D_i does not offer a share of A_i, $i \in \{H,L\}$, initially to B but only offer a share of K, bargaining will continue until a round is reached in which D_i prefers to accept immediately a price which captures part of A_i rather than counteroffer only a share of K. But as this fact is known at the beginning of bargaining, the full information prices will necessarily contain a share of A_i:

$$Q_L^c = \frac{1 - \delta_L}{1 - \delta_B \delta_L}(K + A_L) = s_L(K + A_L)$$

$$Q_H^c = \frac{1 - \delta_H}{1 - \delta_B \delta_H}(K + A_H) = s_H(K + A_H) \tag{10}$$

τ simply becomes part of the surplus resulting from an agreement. The full information equilibrium prices are now (8).

The question remains, however, of whether τ changes the equilibrium strategies and outcomes of the signalling game between D and B. B will now receive a higher price for the debt from both types of debtors. τ has also changed the relative benefit to being a D_L versus being a D_H, hence the delay necessary to signal a D_L type should change. Changes in the full information equilibrium prices and delay times will cause B to alter his equilibrium strategies.

For a given τ, a new preference ordering for D_H is given by the following:

Definition: For a given offer P, let $\Gamma^{*c}(P)$ solve the following equation:

$$(P,0) \sim_H (\delta_B Q_L^c, \Gamma_C^*(P)) \tag{11}$$

and

$$(\bar{P}^c, 0) \sim_H (\delta_B Q_L, 1) \tag{12}$$

What is the effect of the contingent loan upon $\Gamma^{*c}(P)$, the delay necessary to signal? It must be true that

$$K + A_H - P = \delta_H^{\Gamma_C^*(P)}(K + A_H - \delta_B Q_L^c) \tag{13}$$

Comparing (5) and (10), it is true that for a given offer $P \neq Q_H^C$,

$$\frac{K + A_H - P}{K + A_H - \delta_B Q_L^C} > \frac{K - P}{K - \delta_B Q_L} \tag{14}$$

hence $\Gamma_c^*(P) < \Gamma^*(P)$. Note, however, that this does not hold when $P = Q_H^C$; in fact $\Gamma_c^*(Q_H^C) < \Gamma^*(Q_H)$ as algebra shows that

$$\delta_H^{\Gamma_c(Q_H^C)} = \frac{1 - s_H}{1 - \delta_B s_L \dfrac{K + A_L}{K + A_H}} < \frac{1 - s_H}{1 - \delta_B s_L} = \delta_H^{\Gamma^*(Q_H)}. \tag{15}$$

The intuition behind $\Gamma_c^*(Q_H^C) < \Gamma^*(Q_H)$ is that, despite the fact that $A_H > A_L$, τ has made D_L *relatively better off* compared to D_H when B demands the high price. This is because D_L pays a smaller share of τ to B than D_H pays. Thus D_L must delay longer in order to separate himself when B demands Q_H^C. Thus one gets the rather perverse result that a side payment contingent upon an agreement being reached can actually *increase* the delay until the agreement is reached. It is easily verifiable that:

$$\text{sign}\left(\frac{\partial \Gamma_c^*(Q_H^C)}{\partial \tau}\right) > 0 \tag{16}$$

Thus the delay by D_L when B demands the high price is *increasing* in the size of the contingent loan. For a given P, however, $\Gamma_C^*(P) < \Gamma^*(P)$ as D_H is relatively better off with τ. But comparing Γ_c^* and Γ^* for a given P does not take into account any effect upon B's offer by τ.[10]

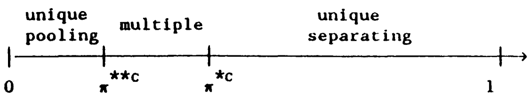
Now we ask what is the effect of the contingent loan upon B's equilibrium strategies and the ensuing outcomes. Let π^{*c} satisfy $\pi > \pi^{*c} \Leftrightarrow W(\bar{P}^c,\pi) > Q_L^C$, and π^{**c} satisfy $\pi > \pi^{**c} \Leftrightarrow W(Q_H^C,\pi) > Q_L^C$. The following

[10] In fact, if $P > Q_H^C - \left(\frac{\delta_B}{1 - \delta_B s_L}\right)\left(A_H - A_L\right)$, then $\Gamma_C^*(P) > \Gamma^*(P)$.

theorem, proved in Appendix V, summarizes the strategies and outcomes for the unique pooling and unique separating equilibria.

Theorem 3: If (i) $\pi_0 > \pi^{*C}$, where $\pi^{*C} < \pi^*$, and (ii) $\left(\frac{\delta_H}{\delta_B}\right)^{\Gamma_c^*(Q_H^C)} > \left|\frac{\ln \delta_B}{\ln \delta_H}\right|$, then the separating equilibrium is the unique equilibrium in which B offers Q_H^C at $t = 0$, D_H accepts, and D_L counteroffers $\delta_B Q_L^C$ at $t = 1 + \Gamma_c^*(Q_H^C)$, which B accepts. If $\pi_0 < \pi^{**C}$, where $\pi^{**c} < \pi^{**}$, then the unique equilibrium is the pooling equilibrium in which B offers Q_L^C at $t = 0$, which D_H and D_L accept at $t = 1$.

There are several important implications to be drawn from the contingent loan case. First, as the contingent loan does not change the discount rate during the bargaining process, s_H and s_L, the shares of the surplus going to B, are unchanged by the contingent loan. Secondly, the surplus is increased by the positive value of the loan to D; hence, D_H and D_L both pay a higher price for their debt. B is able to capture a share of the future loan to D by its ability to hold-up its disbursement. Thirdly, the range of π in which B offers the high price, Q_H^C, increases, and the range of π in which B offers the low price, Q_H^C, decreases. Thus, the contingent loan makes the likelihood of the high price being offered (the separating strategy) greater, and the likelihood of the low price being offered (the pooling strategy) smaller. Fourthly, the delay when B offers the high price is increased by the contingent loan (and for other prices as well; see footnote 10). Thus a contingent loan can increase the inefficiency (the delay) caused by asymmetric information.



(D.2)

These results are in the spirit of the argument put forward in Bulow and Rogoff (1990) in response to calls for a debt facility. They state, "The mere creation of some paper claims will not change the fundamental bargaining factors which govern relations between governments." It has been shown here that enlarging the surplus from an agreement does not change the fundamental bargaining strength of the two parties, as evinced by the fact that their equilibrium shares remain unchanged. The use of a debt facility, in the absence of current new lending, necessarily makes debt relief contingent upon an agreement being reached with creditors because the debt facility can only operate once an agreement is reached. Thus creditors will capture a share of the net benefit to the debtor of the debt facility and of any new monies disbursed upon agreement. Furthermore, contingent debt relief will make creditors more likely to demand the highest possible price for the debt, and less likely to offer the lowest acceptable price for the debt. Lastly, contingent debt relief can increase inefficiency by increasing the delay a low-valuation debtor must use to obtain a low price for its debt.

4. APPLICATION OF THE MODEL TO THE POLICY OF "LENDING IN ARREARS"

As previously mentioned, the IMF has recently changed its policy of "no lending in arrears" to one of "lending in arrears" for several debtor countries. Lending in arrears is simply a policy of disbursing new loans regardless of whether an agreement is reached between a debtor and its

creditors. The reason for this change is formally obscure, but it is hinted that "the IMF wishes to put some pressure on the banks." But if this is truely the reason, the mechanism by which lending in arrears pressures the banks has not been yet made clear.

In this section, we analyze the effect of the change in policy to one of lending in arrears on the bargaining game between debtor and creditor. In contrast to a policy of contingent lending, we show that a policy of lending in arrears will in fact achieve debt reduction by lowering the price paid by the debtor for his debt. Furthermore, the creditor will be unable to capture any of the surplus from the loan. We also show that a policy of lending in arrears will lessen the delay time until agreement for any price which incurs a delay. We derive a relationship between the equilibrium strategies of the creditor and the convexity of the opportunity cost schedule of funds to the credit-constrained debtor. We find that the more convex cost schedule, the less likely the creditor will demand the highest possible price and the more likely he will accept the lowest possible price. Lastly, owing to the effect of lending in arrears upon the creditor's equilibrium strategies, we discuss the fact that an <u>announced</u> policy of lending in arrears, without the actual disbursement of the loan, may also achieve debt reduction before bargaining even begins.

Unlike a contingent-disbursement loan, none of the positive value to D of a loan made regardless of the state of the bargaining game between D and B can be captured by B. This is because B cannot use the threat of diminishing the value of the loan to D by holding up an agreement. Hence the surplus to be divided is only K, as in the original problem.

If the IMF followed a policy of lending to D as much as D wanted at r_H and this was common knowledge, the bargaining problem is now trivial. The discount factor for both D_H and D_L is now one as there is no cost to D of waiting because she effectively has full access to international capital markets. Hence the entire surplus K will go to D. If, however, the IMF followed a policy of limited lending to D at r_H, one returns to the asymmetric information framework. A limited loan would decrease D's real interest rate but would generally leave it above r_H. Hence D's discount factor would be less than δ_H, and whether he is a low-valuation debtor or a high-valuation debtor would still be private information.

Suppose, for simplicity, that the IMF makes a limited size loan at r_H to D and that this loan lowers r_H and r_L by Δr_H and Δr_L respectively. D's new discount rates are then:

$$\delta_H^{NC} = (1 + r_H - \Delta r_H)^{-1} > \delta_H$$

$$\delta_L^{NC} = (1 + r_L - \Delta r_L)^{-1} > \delta_L \tag{17}$$

δ_H^{NC} and δ_L^{NC} are assumed to be common knowledge. We assume that D faces an increasing, convex opportunity cost of funds, consistent with the assumption that D is credit-constrained. Hence $\frac{\Delta r_H}{\Delta r_L} = \lambda > 1$.

Notice that the full information prices for K are now lower for both types:

$$Q_L^{NC} = \left(\frac{1 - \delta_L^{NC}}{1 - \delta_B \delta_L^{NC}}\right) K = s_L^{NC} K \quad \text{and} \quad Q_H^{NC} = \left(\frac{1 - \delta_H^{NC}}{1 - \delta_B \delta_H^{NC}}\right) K = s_H^{NC} K \tag{18}$$

as $s_L^{NC} < s_L$ and $s_H^{NC} < s_H$. D_L and D_H are effectively made more patient by the disbursement of the loan, hence they pay a smaller share of K.

Furthermore, B is unable to capture any of the surplus to D from the loan. For both D_H and D_L, the following inequalities hold:

$$Q_{\text{LENDING IN ARREARS}} < Q_{\text{NO LOAN}} < Q_{\text{CONTINGENT LENDING}} \tag{19}$$

As before, for any offer $P > Q_L^{NC}$, D_L's equilibrium strategy is determined by D_H's cost to waiting. Define $\Gamma^*_{NC}(P)$, where $(P,0) \sim_H (\delta_B^{NC} Q_L^{NC}, \Gamma^*_{NC}(P))$. For a given offer, P, it is clear that

$$\Gamma^*_{NC}(P) < \Gamma^*(P) \quad \text{as} \quad \frac{K - P}{K - \delta_B Q_L^{NC}} > \frac{K - P}{K - \delta_B Q_L} \tag{19}$$

Furthermore, comparing D_L's delay time when B demands the highest price, we find that $\Gamma^*_{NC}(Q_H^{NC}) < \Gamma^*(Q_H)$ (proof in Appendix VI). Thus lending in arrears lowers the delay time to agreement between D_L and B for all prices (except Q_L, of course). The intuitive explanation of this result is simple: lending in arrears makes D_H relatively better off than D_L compared to the case of no lending. Hence D_L does not have to delay as long to prove his type.

$$\Gamma^*(Q_H)_{\text{LENDING IN ARREARS}} < \Gamma^*(Q_H)_{\text{NO LOAN}} < \Gamma^*(Q_H)_{\text{CONTINGENT LENDING}} \tag{20}$$

B's equilibrium strategies are determined by his priors in precisely the same manner as in Theorems 2 and 3 (hence the reader is referred to the proof of theorem 2 for the proof of the following theorem). Let π^{*NC} satisfy $\pi > \pi^{*NC} \Leftrightarrow W(\bar{P}^{NC},\pi) > Q_L^{NC}$, and π^{**NC} satisfy $\pi > \pi^{**NC} \Leftrightarrow W(Q_H^{NC},\pi) > Q_L^{NC}$.

Theorem 4: If (i) $\pi_0 > \pi^{*NC}$, and (ii) $\left(\frac{\delta_H^{NC}}{\delta_B^{NC}}\right)^{\Gamma^*_{NC}(Q_H^{NC})} > \left|\frac{\ln \delta_B^{NC}}{\ln \delta_H^{NC}}\right|$, then the separating equilibrium is the unique equilibrium in which B offers Q_H^{NC} at $t = 0$, D_H accepts, and D_L counteroffers $\delta_B^{NC} Q_L^{NC}$ at $t = 1 + \Gamma^*_{NC}(Q_H^{NC})$, which B accepts. If $\pi_0 < \pi^{**NC}$, then the unique equilibrium is the pooling equilibrium in which B offers Q_L^{NC} at $t = 0$, which D_H and D_L accept at $t = 1$.

The likelihoods of B demanding the highest possible price and of B offering the lowest possible price in the case of lending in arrears, versus the cases of no lending and contingent lending, depend sensitively upon the *relative* change between δ_H and δ_L. The following lemma summarizes the effect upon lending in arrears upon the ranges of the separating and pooling equilibria:

Lemma 3:

(i) If $\frac{d\delta_H}{d\delta_L} > \frac{1 - \delta_H}{1 - \delta_L}$, then π^{*NC} is increasing in τ.

(ii) If

$$\frac{d\delta_H}{d\delta_L} > \frac{\left(1-\delta_B\delta_H\right)^2}{\left(1-\delta_B\delta_L\right)^2} \frac{\pi^{**} + (1-\pi^{**})\left(\frac{d\delta_H}{ds_H} B - \frac{1}{1-\delta_B s_L}\right)\delta_B s_L}{(1 - \pi^{**})\left(\delta_B^{\Gamma^*(Q_H^{NC})+1} + \frac{\delta_B^2 s_L \Gamma^*(Q_H^{NC})}{1 - \delta_B s_L}\right) - 1},$$

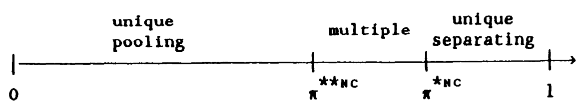
then π^{**NC} is increasing in τ.

The proof of the lemma is contained in Appendix VII. Thus we get the result that if $\frac{d\delta_H}{d\delta_L}$ is sufficiently large, the range of the separating equilibrium shrinks and the range of the pooling equilibrium

enlarges. This is equivalent to saying that B is less likely to demand the highest possible price and more likely to accept the lowest possible price. Whether $\frac{d\delta_H}{d\delta_L}$ is large enough for these two conditions to hold depends upon the convexity of the opportunity cost of funds schedule at δ_H and δ_L: the more credit-constrained D is, the more convex the schedule and the larger is $\frac{d\delta_H}{d\delta_L}$.

The intuition behind Lemma 3(i) is relatively simple. As $\frac{d\delta_H}{d\delta_L}$ increases, the expected payoff to demanding Q_H^{NC} relative to other strategies diminishes, and thus the range of the unique separating equilibrium diminishes. Lemma 3(ii) is more complicated as we must consider that the delay time has now decreased for every $P > Q_L^{NC}$, which makes demanding such a P relatively more attractive. The decrease in the expected surplus from any $P > Q_L^{NC}$ as $\frac{d\delta_H}{d\delta_L}$ increases must be balanced against the lower expected loss due to delay as $\Gamma^*_{NC}(P)$ decreases. As $\frac{d\delta_H}{d\delta_L}$ gets sufficiently large, the first effect will dominate and the range of the unique pooling equilibrium increases.

B's EQUILIBRIUM STRATEGIES WHEN $\frac{d\delta_H}{d\delta_L}$ SUFFICIENTLY LARGE (Lemma 3)



(D.3)

An important implication of the preceding analysis is that debt reduction can occur when τ is disbursed, before any bargaining begins. First, it occurs because B knows that he is now facing an adversary with increased patience, and so the equilibrium price drops for both types of debtors. Secondly, if the debtor is sufficiently credit-constrained, $\left(\frac{d\delta_H}{d\delta_L}\right.$ is sufficiently large$\left.\right)$, the creditor is more likely to open negotiations with the lowest possible price and less likely to open with the highest possible price, thereby providing debt reduction for D_H. Furthermore, the original result that once bargaining has begun D_L will achieve debt reduction through delay still holds. Notice that the first two types of debt reduction will occur if the creditor *simply believes* he is facing a stronger D. Thus a credible announcement of lending in arrears will also achieve debt reduction.

5. IMPLICATIONS FOR CURRENT DEBATE ON DEBT-REDUCTION SCHEMES

Current policy debate has focused upon the desirability of a debt facility which would "manage and finance the debt-reducing process" (Kenen, 1990). Market-based debt reduction schemes, such as exit bonds and debt-for-equity swaps, have had very limited success. Their lack of success has been partly due to lack of resources to finance such deals on the part of the debtors, and the incentive among creditors to hang back from such deals in anticipation that debt-forgiveness by other creditors will increase the value of the claims of those who hold back. This free-rider problem has largely frustrated market-based approaches in light of the Bolivian experience, in which the total debt outstanding remained constant after a market buyback due to an increase in the price of the remaining debt (Bulow and Rogoff 1988a). Proponents of a debt facility claim that the provision of "sweeteners", such as loan-guarantees, by creditor-country governments to creditors in exchange for debt-reduction, would overcome the free-rider problem. In the

absence of a legal mechanism to enforce participation, however, it is unclear that such a scheme would eliminate the free-rider problem. Furthermore, the cost of these sweeteners could exhaust the willingness to pay of creditor-country governments' well before a significant degree of debt-reduction is achieved (Krugman, 1989). Bulow and Rogoff (1990) have claimed the contrary, that "Far from speeding compromise, the presence of official creditors has tended to ossify the negotiating position of the banks and countries." It is asserted that a debt facility will simply result in a substantial portion of its funds being transferred to creditor banks without helping debtors.

The preceding analysis has important implications for the choice of debt-reduction schemes. It is clear that the use of a debt-facility for debt-reduction is analogous to the case of no lending in arrears or contingent lending: all surplus to the debtor from debt-reduction is made contingent upon the agreement of creditors. Thus creditors will increase the price of the debt to capture a share of the surplus, creditors are more likely to demand the highest possible price, and any delay to agreement will increase. This validates the Bulow-Rogoff critique that a debt-facility will transfer much of the gains from debt-reduction to creditors and harden the bargaining positions of debtor and creditors.

In constrast, a policy of lending in arrears will lower the price of debt, reduce the delay time to agreement, and will increase the likelihood that creditors will accept the lowest feasible price if the debtor is sufficiently credit-constrained. Debt-reduction can then be accomplished through either market mechanisms or concerted programs such as the 1989 Mexico agreement. It is interesting that prior to the agreement, the IMF announced it would lend to Mexico in the future if it should run into arrearages, although Mexico had not run arrears for some time and it was not anticipated to in the future. Such an announcement can be viewed as a promise by the IMF "to step into the breach" in case Mexico's future bargaining strength diminished, a promise which would strengthen Mexico's current bargainng strength. This would explain the surprising findings of Van Wijnbergen and Claessens that Mexico was able to capture up to 97% of the sweeteners to the agreement. This could only happen if the underlying real price of Mexico's debt had declined, a prediction of our model.

These results, however, immediately pose another question: if the IMF actually began lending in arrears, wouldn't it be making loans which are extremely difficult to collect as debtors realize that the IMF is less likely to move against them than creditor banks in the event of difficulties? There are two answers to this question. The first answer is yes: to the extent that the IMF is less willing to punish than banks, it will have a harder time collecting its loans. The other answer is that this question may be irrelevant. That is, to the extent that such imperfectly-collectable loans are made, they are "buying debt-reduction" for the debtors. The relevant question then is whether these loans effect debt-reduction in the least costly way. We believe they do in the sense that the benefit from these loans accrues entirely to debtors rather then some of it being diverted to creditors. The total cost involved is unclear as it depends upon the debtor's discount factor, and as emphasized previously, the creditors' perceptions of the debtor's discount factor.

6. CONCLUSION

This paper has developed a modified version of the Admati and Perry model of asymmetric information in bargaining to sovereign debt negotiations, and analyzed the effect of two IMF policies upon the bargaining process: no lending in arrears or contingent lending, versus lending in arrears or non-contingent lending. It is shown that a high-patience (less credit-constrained) debtor will use delay to decrease the price he pays for his debt, and that the creditor's offer price depends upon his perception of the debtor's patience. We show that a policy of contingent lending increases the price both high-patience and low-patience debtors debtor pay for their debt as the creditor captures a share of the surplus of the contingent IMF loan to the debtor, increases the delay time for the high-patience debtor, and increases the likelihood that the debtor will demand the highest possible price. A policy of non-contingent lending, in contrast, decreases the price paid by both types of debtors, does not transfer a share of the surplus from the loan to the creditors, decreases the delay time for the high-patience debtor, and may increase the likelihood that the creditor accepts the lowest feasible price for the debt.

This paper points to a critical distinction in the manner of third-party intervention in asymmetric information bargaining games and its effect. If such intervention makes transfers to one party *contingent* upon an agreement being reached between the two main parties, it will simply aid the other party, make bargaining less efficient by increasing delay time, and harden the other party's bargaining position. If transfers are *not contingent* upon agreement, however, all of the benefit of the transfer will accrue to the receiving party, bargaining will be made more efficient as delay time decreases, and the bargaining position of the other party may be softened.

APPENDIX

I. SET OF POSSIBLE EQUILIBRIA

The following lemma characterizes the set of possible equilibria:

<u>Lemma 1</u> (Admati and Perry):

In any sequential equilibrium:

(i) B never accepts an offer P if $P < \delta_B Q_L$.

(ii) B always accepts an offer P if $P \geq \delta_B Q_H$.

(iii) D never accepts an offer P if $P > Q_H$.

(iv) D always accepts an offer P if $P \leq Q_L$.

(v) Define $\bar{P}$ such that $(\bar{P},0) \sim_H (\delta_B Q_L,1)$. D_H always accepts an offer $P \leq \bar{P}$.

(vi) Let (P_1,t_1) be the equilibrium outcome for D_H, and (P_2,t_2) be the equilibrium outcome for D_L. Then $P_1 \geq P_2$, and $t_1 \leq t_2$.

(vii) An acceptance of an offer occurs with no delay.

<u>Proof</u>: Parts (i)-(v) are straightforward implications of the assumption of perfection. (vi) is a result of D_H's greater impatience. (vii) is straightforward.

II. EQUILIBRIUM REFINEMENTS

1) Tie-Breaking Assumption

If an agent can obtain the same payoff by making fewer offers, then he makes few offers.

2) Cho-Type Refinement

Let H^N denote the history of the first N rounds of the game:

$$H^N = ((P^1,P^2,\ldots,P^N),\ (\alpha^1,\alpha^2,\ldots,\alpha^N))$$

Let $\sigma = (\sigma_H,\sigma_L,\sigma_B)$ be the set of sequential equilibrium strategies for the players. Suppose an action (P^N,α^N) is bad for type D_H given history H^N and σ, but it is not bad for type D_L. Then B's assessment of

D_H after (H^N, P^N, α^N), $\pi^{N+1} = 0$.

The Cho-type refinement will ensure the following lemma holds:

<u>Lemma</u> 2 (Admati and Perry):

For any history H^N which ends in an offer P by D, in any equilibrium,

(i) If $\pi(H^N) = 0$, B accepts P if and only if $P \ge \delta_B Q_L$.

(ii) If $\pi(H^N) = 1$, B accepts P if and only if $P \ge \delta_B Q_H$.

<u>Proof</u>: See Admati and Perry.

III. THE DEBTOR'S EQUILIBRIUM STRATEGIES

With the above refinements, the following theorem characterizes the complete set of equilibria of this game:

<u>Theorem</u> 1 (Admati and Perry): Suppose along an equilibrium path that B makes the first offer, P, at time t ($P \ge Q_L$ by Lemma 1). Then the sequential equilibria in the subgame that follows are:

(i) If $P = Q_L$, then $t = 0$ and both D_H and D_L accept Q_L at $t + 1$.

(ii) If $P > Q_L$, then

(iia) If D_H accepts P, then D_L offers $\delta_B Q_L$ at $t + \Gamma^*(P)$, which B accepts.

(iib) If D_H rejects P, then D_H offers $\delta_B Q_H$ at $t + 1$, which B accepts. D_L offers $\delta_B Q_L$ at time $t + 1 + \Gamma^*(\delta_B Q_H)$, which B accepts.

<u>Proof</u>:

(i) This is obvious given Lemma I.

(iia) Suppose that D_H accepts P at $t + 1$, and that along the equilibrium

path D_L offers P_2 at t_2. Then $(P, t+1) \gtrsim_H (P_2, t_2)$. Suppose $P_2 > \delta_B Q_L$. Then there exists $t_3 > t_2$ such that $(P_2, t_2) \gtrsim_H (\delta_B Q_L, t_3)$ and $(P_2, t_2) \lesssim_L (\delta_B Q_L, t_3)$. D_L can offer $\delta_B Q_L$ at t_3, which B will accept by the Cho refinement and Lemma 2. Therefore, $P_2 = \delta_B Q_L$. Suppose $t_2 > t + \Gamma^*(P)$. By a similar argument it can be shown that Q_L will be better off by offering $\delta_B Q_L$ earlier, at $t + \Gamma^*(P)$. This equilibrium is supported by the beliefs $\pi(H^{t+1}) = 1$ for any deviations from the strategy $(\delta_B Q_L, t + \Gamma^*(P))$.

(iib) If $P > Q_L$ and D_L and D_H paid the same price, then this price must be $\delta_B Q_L$ at some $t \ge 2$ by (iia) above. But B would have preferred instead to offer Q_L at time 0. Hence D_L and D_H cannot pay the same price if $P > Q_L$. Hence D_H will reveal his type in his next offer, and thus cannot do any better than offering $\delta_B Q_H$ at t+1. D_L's equilibrium strategy is derived by an argument similar to that in (iia). ▮

IV. PROOF OF THEOREM 2

Claim (i): $W(P,\pi)$ is strictly increasing in P if and only if

$$\pi \ge \hat{\pi} = \frac{\left(\frac{\delta_B}{\delta_H}\right)^{\Gamma^*(Q_H)} \frac{\delta_B Q_L}{K-\delta_B Q_L} \frac{\log \delta_B}{\log \delta_H}}{1 + \left(\frac{\delta_B}{\delta_H}\right)^{\Gamma^*(Q_H)} \frac{\delta_B Q_L}{K-\delta_B Q_L} \frac{\log \delta_B}{\log \delta_H}} \qquad (1')$$

Proof: Substituting (5) into (3), it can be shown that

$$W(P,\pi) = \pi P + (1 - \pi)\left(\frac{K - P}{K - \delta_B Q_L}\right)^{\frac{\log \delta_B}{\log \delta_H}} \delta_B Q_L \qquad (2')$$

The derivative of $W(P,\pi)$ with respect to P is

$$\pi - (1-\pi)\left(\frac{K - P}{K - \delta_B Q_L}\right)^{\frac{\log \delta_B}{\log \delta_H} - 1} \delta_B Q_L \frac{\log \delta_B}{\log \delta_H} \frac{1}{K - \delta_B Q_L} \qquad (3')$$

In Eq.(3′), π is a decreasing function of π. Thus if Eq. (3′) is to be greater or equal to 0, it must be greater or equal to 0 at Q_H, where it is minimized. Thus the condition on π is

$$\pi - (1-\pi)\left(\frac{K - Q_H}{K - \delta_B Q_L}\right)^{\frac{\log \delta_B}{\log \delta_H} - 1} \delta_B Q_L \frac{\log \delta_B}{\log \delta_H} \frac{1}{K - \delta_B Q_L} \geq 0 \qquad (4')$$

Rearranging Eq. (4′) and using the fact that

$$\left(\frac{K - Q_H}{K - \delta_B Q_L}\right)^{\frac{\log \delta_B}{\log \delta_H} - 1} = \left(\frac{\delta_B}{\delta_H}\right)^{\Gamma^*(Q_H)}$$

produces the condition on π found in Eq. (1′).

Claim (ii): $W(\tilde{P},\pi) > Q_L$ if and only if

$$\pi > \pi^* = \frac{Q_L\left(1 - \delta_B^2\right)}{K - \delta_B\left(K - \delta_B Q_L\right) - \delta_B^2 Q_L} \qquad (5')$$

Proof: Substitute $\tilde{P} = K - \delta(K - \delta_B Q_L)$ into (4) and rearrange.

Claim (iii): $W(Q_H,\pi) > Q_L$ if and only

$$\pi > \pi^{**} = \frac{Q_L\left(1 - \delta_B^{\Gamma^*(Q_H)}\delta_B\right)}{Q_H - \delta_B^{\Gamma^*(Q_H)}\delta_B Q_L} \qquad (6')$$

Proof: Rearrange (3).

Claim (iv): $\pi^* > \pi^{**}$.

Proof: The proof will proceed by showing first that numerator(π^*) > numerator(π^{**}), and then that denominator(π^*) < denominator(π^{**}).

a) Clearly $\delta_B^{\Gamma^*(P)} > \delta_B$ as $Q_H > \tilde{P}$. Hence Num(π^*) > Num(π^{**}).

b) Denom(π^*) = $\tilde{P} - \delta_B^2 Q_L$. If Denom($\pi^{**}$) > Denom($\pi^*$), then it must be that $Q_H - \tilde{P} > \left(\delta_B^{\Gamma^*(P)} - \delta_B\right)\delta_B Q_L$. Clearly it is true that $Q_H - \tilde{P} > 0 > \left(\delta_B^{\Gamma^*(P)} - \delta_B\right)\delta_B Q_L$.

Claim (v): $\pi^{**} > \hat{\pi}$ if $\left(\frac{\delta_H}{\delta_B}\right)^{\Gamma^*(P)} > \left|\frac{\ln \delta_B}{\ln \delta_H}\right|$.

Algebra shows that the condition $\pi^{**} > \hat{\pi}$ is equivalent to requiring that

$$\left(\delta_L - \delta_H\right)\left[\left(\frac{\delta_B}{\delta_H}\right)^{\Gamma^*(Q_H)} \frac{\log \delta_B}{\log \delta_H} - 1\right] \frac{\delta_B}{\left(1-\delta_B\delta_H\right)\left(1-\delta_B\delta_L\right)} < 1 \qquad (7')$$

This is necessarily satisfied if $\left(\frac{\delta_H}{\delta_B}\right)^{\Gamma^*(P)} > \left|\frac{\ln \delta_B}{\ln \delta_H}\right|$, which we will assume from now on and will be referred to as the "monotonicity condition". Thus the relationship between $\hat{\pi}$, π^* and π^{**} is

$$0 \;\text{——}\; \hat{\pi} \;\text{——}\; \pi^{**} \;\text{——}\; \pi^{*} \;\text{——}\; 1 \qquad (D.1')$$

Claim (vi): (a) if $\pi_0 < \pi^{**}$, the pooling equilibrium (B offers Q_L at t = 0) is the unique equilibrium, and (b) if $\pi_0 > \pi^*$, the separating equilibrium (B offers Q_H at t = 0, D_H accepts at t = 1, and D_L counteroffers $\delta_B Q_L$ at $\Gamma^*(\delta_B Q_L)$.

Proof: (a) Obvious by definition of π^{**}. (b) By Lemma 3, the only equilibrium after B offers Q_H is the one in which D_H accepts Q_H and D_L counteroffers $\delta_B Q_L$ at $\Gamma^*(\delta_B Q_L)$. Therefore B prefers to offer Q_H.∎

V. PROOF OF THEOREM 3

The proof of the claim that, given conditions (i) and (ii), the separating equilibrium is the unique equilibrium is analogous to the proof of Theorem 2, so we will not repeat it. Similarly, we will not repeat the proof that, given $\pi_0 < \pi^{**c}$, the pooling equilibrium is the unique equilibrium.

Claim (vii): $\pi^{*C} < \pi^{*}$.

As in Claim (ii), $W(\bar{P}^C,\pi) > Q_L^C$ if and only if

$$\pi > \pi^{*C} = \frac{Q_L^C\left(1 - \delta_B^2\right)}{\bar{P}^C - \delta_B^2 Q_L^C} \tag{8'}$$

Substituting in the definitions of Q_L^C, Q_L, $\bar{P}$, and $\bar{P}^C$ into π^* and π^{*c}, and simple algebra shows $\pi^{*c} < \pi^*$.

Claim (viii): $\pi^{**c} < \pi^{**}$.

Let $x = \dfrac{K + A_H}{K + A_L}$. π^{**c} is defined by

$$s_H \pi^{**C} x + (1 - \pi^{**C})\delta_B s_L \delta_B^{\Gamma_c^*(s_H(K+A_H))} = s_L \tag{9'}$$

Differentiating (9') with respect to x and evaluating at x = 1, we get

$$\left(s_H - s_L\delta_B^{\Gamma_c^*+1}\right)d\pi^{**C} + \left(s_H\pi^{**C} + (1-\pi^{**C})\delta_B s_L A \frac{1 - s_H}{(1-\delta_B s_L)^2}(-\delta_B s_L)\right)dx = 0 \tag{10'}$$

where $A = \dfrac{\delta_B^{\Gamma_c^*}}{\delta_B^{\Gamma_c^*}} \dfrac{\log \delta_B}{\log \delta_H}$. The sign of the coefficient of dx is greater than 0 if and only if

$$s_H(1 - \delta_B^{\Gamma_c^*}) > (s_H - s_L)\delta_B A \frac{1 - s_H}{(1-\delta_B s_L)^2}\delta_B s_L \tag{11'}$$

This is true by the monotonicity condition ((7') in the contingent loan case).█

VI. PROOF OF $\Gamma_{NC}^*(Q_H^{NC}) < \Gamma^*(Q_H)$

Simple algebra shows that

$$\Gamma_{NC}^*(Q_H^{NC}) < \Gamma^*(Q_H) \Leftrightarrow \frac{\Delta s_H}{\Delta s_L} > \frac{\delta_B(1 - s_H)}{(1 - \delta_B s_L)} - \delta_B\delta_H^{\Gamma^*(Q_H)} \tag{10'}$$

The RHS of the second inequality in (10') is necessarily between 0 and 1. It is easy to show that

$$\frac{\Delta s_H}{\Delta s_L} = \frac{\Delta r_H}{\Delta r_L}\left(\frac{\delta_H}{\delta_L}\right)^2\left(\frac{1 - \delta_B\delta_L}{1 - \delta_B\delta_H}\right)^2 > \frac{\Delta r_H}{\Delta r_L} \tag{11'}$$

As it has been assumed that $\dfrac{\Delta r_H}{\Delta r_L} > 1$, $\Gamma_{NC}^*(Q_H^{NC}) < \Gamma^*(Q_H)$ is proved. █

VII. PROOF OF LEMMA 3

(i) We know by (5') that

$$\pi^* = \frac{Q_L(1 - \delta_B^2)}{K - \delta_H(K - \delta_B^2 Q_L) - \delta_B Q_L} = (1 + \delta_B)\frac{1 - \delta_L}{1 + \delta_B - \delta_B\delta_L - \delta_H} \tag{12'}$$

If we take the total derivative of π^* with respect to δ_H amd δ_L, we obtain

$$d\pi^* = \frac{1 + \delta_B}{\left(1 + \delta_B - \delta_B\delta_L - \delta_H\right)^2}\left[\left(1 - \delta_H\right)\left(-d\delta_L\right) + \left(1 - \delta_L\right)\left(d\delta_H\right)\right] \tag{13'}$$

(13') has the same sign as $\left(1 - \delta_H\right)\left(-d\delta_L\right) + \left(1 - \delta_L\right)\left(d\delta_H\right)$. Rearranging, one obtains the result that

$$d\pi^* > 0 \Leftrightarrow \frac{d\delta_H}{d\delta_L} > \frac{1 - \delta_H}{1 - \delta_L} \tag{14'}$$

(ii) π^{**} is defined by the following relation:

$$\pi^{**} s_H + (1 - \pi^{**})\delta_B^{\Gamma^*(Q_H)}\delta_B s_L - s_L = 0 \qquad (15')$$

Taking the total derivative of (15'), we obtain

$$\left[s_H - \delta_B^{\Gamma^*(Q_H)}\delta_B s_L\right] d\pi^{**} + \left[\pi^{**} - (1-\pi^{**}) s_L \delta_B \left(\frac{A}{1-\delta_B s_L} - E \frac{d\delta_H}{ds_H}\right)\right] ds_H$$

$$\left[(1 - \pi^{**})\delta_B \left(\delta_B^{\Gamma^*(Q_H)} + \delta_B s_L A \frac{\delta_B (1 - s_H)}{\left(1 - \delta_B s_L\right)^2}\right) - 1\right] ds_L \qquad (16')$$

where $A = \dfrac{\delta_B^{\Gamma^*(Q_H)} \log\delta_B}{\delta_H^{\Gamma^*(Q_H)} \log\delta_H}$, $E = \delta_B^{\Gamma^*(Q_H)-1}A$. Consider first the coefficient of ds_H. Using the definition of π^{**} and the facts that $A < 1$ by the monotonicity condition, and that $\frac{d\delta_H}{ds_H} < 0$, it can be shown that $\frac{\partial\pi^{**}}{\partial s_H}$. Similarly it can be shown that $\frac{\partial\pi^{**}}{\partial s_L} > 0$. Finally, using the fact that $\frac{ds_H}{ds_L} = \frac{d\delta_H}{d\delta_L}\frac{(1 - \delta_B\delta_L)^2}{(1 - \delta_B\delta_H)^2}$ and rearranging, one obtains the expression in Lemma 3(ii).∎

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