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When Is a Man's Life Worth More Than His Human Capital?

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To most people life is exceedingly precious, perhaps even priceless. For many, no finite pecuniary payment would compensate for a death sentence to be carried out within a year. Few would place a finite price on the lives of their children. One is tempted to regard decisions about such matters as subject to a special "calculus of the heroic" which is somehow disjoint from the petty decisions usually discussed by economists. Still it is clear that everyone accepts small risks to his own life and the lives of his loved ones in return for small pleasures or small savings of money or effort. On a grander scale, public decision-makers must regularly make choices which involve exchanges of economic goods and human lives. Even the most affluent and humane government must reject some expenditures on items such as highway improvement or medical research, which, though almost certain to save lives, are "too expensive" for the amount of good they do. The necessity of such decisions suggests that even in "matters of life and death" there must be a logic of choice and thus a theory of "pricing the priceless".

If they intend to make roughly consistent choices, it is hard for policy-makers to avoid placing implicit or explicit "prices" on human lives saved or spent.[1] A procedure commonly used in benefit-cost analysis is to appraise a life at the value of its "human capital" -- the expected present value of its future earnings. Occasionally, economists have proposed an alternative "net output" approach. This approach suggests that the human capital valuation is too high because it neglects the fact that dead people do not consume. According to the net output approach, a life saved should be valued at the expected value of its human capital less the expected value of anticipated future consumption. Others have suggested that the appropriate valuation to put on a human life must in some way be related to the amount of life insurance that one purchases. All of these concepts have the advantage of being reasonably amenable to measurement. While it seems plausible that each might somehow be related to the appropriate valuation to place on saving a life, it is difficult without explicit analysis to know exactly how.

In this paper we use a simple one-period choice model to remove some of the sting from the paradox of pricing the priceless. We show that there is a simply described, though less easily measured concept of the "value of life" that is appropriate for measuring the benefits or costs of a broad class of public projects that save or expend human lives. This value can be decomposed in a simple way into the direct and pecuniary effects of a change in survival probability. The pecuniary effects, which are the consequence of the effects of a change in survival probability on the budget, can be related in a simple way to the human capital, net output, and insurance measures. We are able to find a reasonably easily interpreted condition on preferences that determines whether a man's human capital is too large or too small a value to place on a life saved. We argue that the presence of interpersonal benevolence would not in general imply that lives saved should be valued more highly than in a selfish world. Finally, we consider a model of private safety in which actuarially fair insurance is unavailable.

This paper is viewed as a contribution to the "subjectivist" theory of valuation of human life. The subjectivist approach has its roots in insightful articles by Shelling (1968) and Mishan (1971). Some of the notions of this paper are also drawn from Bergstrom (1974). Theoretical models which explicitly compare the value of saving a life to the value of human capital have been studied by Conley (1976) and Cook (1978). These papers suggest conditions sufficient (but not necessary) for the former to exceed the latter. However, Jones-Lee (1978) and Cook (1978) suggest that these conditions are not particularly plausible.[2] The results of our paper include a necessary and sufficient condition that helps to clarify on purely theoretical grounds the question

"When is a man's life worth more than his human capital?"

Rational Choice and the "Pricelessness of Life" -- Confessions of a jaywalker who wouldn't play Russian roulette for any price.

Throughout this section we deal with the simplest model that is rich enough to inform us about preferences toward risk of death. There is only one ordinary economic good -- call it bread. The protagonist is an individual who will either die immediately or will survive for a certain fixed amount of time. Being entirely selfish, he cares only about the amount of bread which he consumes if he survives and the probability that he is allowed to live out a full life span. His preferences, then, are defined over the set of pairs (π, c) where π specifies the probability that he will survive and c is his consumption of bread if he survives.

Here we show that entirely orthodox assumptions about preferences can explain the behavior of an individual who subjects his "priceless" life to small hazards. Suppose that someone has preferences represented by an expected utility function of the form

$$U(\pi, c) = \pi u(c)$$

where π is survival probability and c is bread consumption.[3] Assume that u is a non-negative,[4] continuous, strictly increasing real valued function of c and that there is a least upper bound b such that $u(c) < b$ for all $c \geqq 0$. Since u is non-negative and increasing in c, it is easily seen that $U(\pi, c) = \pi u(c)$ is a strictly increasing function of each of its arguments.

Let this individual be endowed with an initial survival probability $\bar{\pi}$ and with rights to consume $\bar{c}$ if he survives. He will voluntarily accept an exchange that puts him in the situation (π, c) if and only if $\pi u(c) \geq \bar{\pi} u(\bar{c})$. Since by assumption $u(c) \leq b$ for all $c \geq 0$, this can only occur where $\pi \geqq \frac{\bar{\pi}u(\bar{c})}{b}$. Thus no amount of consumption would be sufficient to compensate the individual for reducing his survival probability below $\frac{\bar{\pi}u(\bar{c})}{b}$. On the other hand, since u is assumed to be a continuous increasing function of c, there is always some small risk of death which he would accept in return for even a single loaf of bread.

It is instructive to draw an indifference curve representing the locus of combinations (π,c) such that $\pi u(c) = \bar{\pi}u(\bar{c})$. Points above the curved line are preferred to $(\bar{\pi},\bar{c})$ and points below are regarded as inferior. Notice that the indifference curve does not cross to the left of the vertical line on which $\pi = \frac{\bar{\pi}u(\bar{c})}{b}$.

To this individual, life is priceless. No amount of bread would induce him to accept certain death. In fact if $\frac{\bar{\pi}u(\bar{c})}{b} > \frac{1}{6}$, no finite amount of bread would compensate him for playing Russian Roulette. Still there are many points, such as $(\bar{\pi} + \Delta\pi, \bar{c} + \Delta c)$, on the indifference curve in Figure 1 where a reduction of $\Delta\pi$ in survival probability is compensated by a finite addition Δc to bread compensation. The rate $\frac{\Delta c}{\Delta \pi}$ at which compensation must be paid depends in general on the size of $\Delta\pi$. The slope of the indifference curve at $(\bar{\pi},\bar{c})$ is just the limit of such ratios as $\Delta\pi$ is small. This slope is the "marginal rate of substitution between survival probability and consumption". An individual is willing to make sufficiently small exchanges of consumption and survival probability so long as $\frac{\Delta c}{\Delta \pi}$ exceeds this marginal rate of substitution. Of course the individual will not be willing to sell all of his survival probability at his marginal valuation. Thus the "paradox" alluded to in the section heading is resolved in just the same way that neo-classical demand theory resolves the problem of (possibly infinite) consumer's surplus.[5]

A Model of Public Safety - Death and Taxes

Here we discuss a simple model in which public safety is a pure public good. Consumers have utility functions, $U_i(\pi_i, c_i) = \pi_i u_i(c_i)$ where $u_i'(c) > 0$. Each i has an initial endowment of k_i units of bread. If he survives, he will produce an additional h_i units of bread. The government collects t_i units of bread from i as taxes. This leaves him with an after-tax "non-human wealth of $w_i \equiv k_i - t_i$. In this simple model, the expected value, $\pi_i h_i$ of his earnings might reasonably be called his "human capital". The government spends its total tax revenue on "public safety". We choose units of measurement for public safety so that each unit of public safety costs one unit of bread. Thus the total amount of public safety is $s = \sum_i t_i$. Each consumer's survival probability is a function $\pi_i(s)$ of the level of public safety.

Since bread is useless to him if he does not survive, the individual will want to trade any positive after-tax wealth for an annuity. Here we assume that annuities can be purchased at actuarially fair prices.[6] Therefore i will use w_i to buy an annuity that pays him $\frac{w_i}{\pi_i(s)}$ units of bread if he survives and nothing if he dies. If he survives, he will be able to consume the yield on his annuity plus the amount of bread, h_i, that he produces. Therefore his consumption if he survives will be

$$(1) \quad c_i = \frac{k_i - t_i}{\pi_i(s)} + h_i = \frac{w_i}{\pi_i(s)} + h_i .$$

We can define the consumer's indirect utility function in terms of survival probability and wealth. This is

$$(2) \quad v_i(\pi_i, w_i) \equiv \pi_i u_i\left(\frac{w_i}{\pi_i} + h_i\right) .$$

Consider a small change Δs in public safety expenditures. The resulting change in i's survival probability is approximately $\pi_i'(s)\Delta s$. Therefore he is willing to pay an amount of non-human wealth equal to approximately

(3) $\ell_i(\pi_i(s),w_i)\pi_i'(s)\Delta s$

for the increment Δs, where we define

(4) $\ell_i(\pi_i(s),w_i) \equiv \dfrac{\partial v_i(\pi_i(s),w_i)}{\partial \pi_i} \div \dfrac{\partial v_i(\pi_i(s),w)}{\partial w_i}$

to be i's marginal rate of substitution between survival probability and wealth. A necessary condition for the allocation $(w_1,\ldots, w_n, s)$ of after-tax wealths and survival probabilities to be Pareto optimal is that the sum of individual marginal willingnesses to pay for a unit of public safety equals its marginal cost. Since public safety is measured so that its marginal cost is unity, this condition is:

(5) $\sum_i \ell_i(\pi_i(s),w_i)\pi_i'(s) = 1.$

Let n(s) be the expected number of survivors in the community if public safety is s. Then

(6) $n(s) \equiv \sum_i \pi_i(s)$ and $n'(s) = \sum_i \pi_i'(s).$

Expression (5) simplifies in a useful way if $\pi_i'(s)$ is uncorrelated with $\ell_i(\pi_i(s),w_i)$. If this is the case, then

(7) $\sum_i (\ell_i(\pi_i(s),w_i) - \bar{\ell}(s,w))\pi_i'(s) = 0$

where we define $\bar{\ell}(s,w)$ to be the mean of the terms $\ell_i(\pi_i(s),w_i)$. If (7) holds, then (5) is equivalent to

(8) $\bar{\ell}(s) = \dfrac{1}{n'(s)}$

But the right side of (8) is just the marginal cost per expected life saved. Therefore if $\pi_i'(s)$ and $\ell_i(\pi(s),w_i)$ are uncorrelated, then a necessary condition for Pareto optimality is that the marginal cost of an expected life saved equals the mean of the individual marginal rates of substitution between survival probability and wealth.

A Parable and an Answer to the Title's question

The argument of the previous section implies that for a large class of public projects, benefit-cost analysis should value an expected life saved at the mean $\bar{\ell}(s,w)$ of individual marginal rates of substitution between survival probability and wealth. For our single period model, we can calculate the individual marginal rates of substitution by taking derivatives of equation (2). We find that:

$$(9) \quad \ell_i(\pi_i,w_i) = \frac{u_i(c_i)}{u_i'(c_i)} + h_i - c_i$$

or equivalently:

$$(10) \quad \ell_i(\pi_i,w_i) = \frac{u_i(c_i) - c_i u_i'(c_i)}{u_i'(c_i)} + h_i$$

Expression (9) decomposes $\ell_i(\pi_i,w_i)$ into a direct or "compensated" effect and a pecuniary effect. The budget constraint (1) can be expressed as $\pi_i(c_i - h_i) = w_i$. Therefore $c_i - h_i$ is the change in the amount of non-human wealth needed to sustain the consumption c_i as π_i changes.[7] Stated differently, an increment in wealth of $(c_i - h_i)\,\Delta\pi_i$ will exactly compensate the consumer for the purely pecuniary effects of the change $\Delta\pi_i$. The direct effect, or "compensated marginal rate of substitution" is therefore equal to $\ell_i(\pi_i,w_i) + (c_i - h_i)$. From (9) we see that the compensated marginal rate of substitution is

$$(11) \quad \ell_i(\pi_i,w_i) + (c_i - h_i) = \ell_i(\pi_i,w_i) - (h_i - c_i) = \frac{u_i(c_i)}{u_i'(c_i)}$$

From (11) we see that the compensated marginal rate of substitution will be positive at consumption c_i if the consumer prefers the prospect of consuming c_i to the prospect of being dead and if he prefers more consumption to less. From (11) we also see that this straightforward and plausible condition guarantees that the marginal rate of substitution $\ell_i(\pi_i,w_i)$ exceeds the net output measure, $h_i - c_i$.

In order to discover when $\ell_i(\pi_i,w_i)$ exceeds human capital, h_i, we will need more subtle arguments. From equation (10) we see that the sign of $\ell_i(\pi_i,w_i) - h_i$ is the same as the sign of $u_i(c_i) - c_i u_i'(c_i)$. A simple mathematical argument shows that $u_i(c_i) - c_i u_i'(c_i) > 0$ for all $c_i > 0$ if $u_i(0) \geqq 0$ and if $u_i(\cdot)$ is an everywhere increasing, strictly concave function.[8] These conditions are sufficient, but not necessary for $\ell_i(\pi_i,w_i) > h_i$. Therefore unless we find them to be plausibly true, they are of no use. But these assumptions are not very plausible. To see this, imagine someone with consumption below the starvation level. Suppose that doubling his consumption would place him comfortably above starvation. It seems reasonable that such a person would willingly accept a bet in which with probability one-half he survives and his consumption is doubled and with probability one-half he dies. But if he is willing to accept such a bet, from the initial situation, (π,c) then $\frac{1}{2}\pi u(2c) > \pi u(c)$. But this is impossible if $u(\cdot)$ is concave and $u(0) \geqq 0$.

Conley (1976) and Cook (1978) propose the less restrictive assumption that u(c) is an increasing concave function while allowing the possibility that $u(c) < 0$ for small c. This assumption implies that there is some critical level c_i^* such that $u_i(c_i) - c_i u_i'(c_i) \gtreqless 0$ (and hence $\ell_i(\pi_i,w_i) \gtreqless h_i$ as $c_i \gtreqless c_i^*$). Conley asserts that the case $c_i > c_i^*$ is "the general case". But aside from Conley's proof by assertion, Conley and Cook are unable to find any way of verifying or falsifying this condition other than direct measurement of $\ell_i(\pi_i,w_i)$ and h_i.

Conditions involving global concavity of $u(\cdot)$ do not appear to be helpful in our search for independent evidence on the relative size of $\ell_i(\pi_i, w_i)$ and h_i. At any rate, there is something methodologically awkward about using such global conditions to derive local information about preferences. In this instance we are asked to learn about the choices of people who are pushed to the brink of starvation in order to draw inferences about the behaviour of prosperous people when faced with marginal adjustments in their environment.[9]

Fortunately, there is a sharper, more plausible condition that determines the relation of $\ell_i(\pi_i, w_i)$ to h_i. We will first motivate this condition by a parable. Imagine a tropical island populated by n identical people. There is a fixed supply of $n\bar{w}$ units of breadfruit which cannot be augmented by labor. It is known that the proportion π of the island population will survive and *ex ante* each islander believes his survival probability to be π. Survivors will each receive $\frac{\bar{w}}{\pi}$ units of breadfruit. The utility of each islander must then be $\pi u(\frac{\bar{w}}{\pi})$.

Now $\frac{\partial}{\partial \pi} \pi u(\frac{\bar{w}}{\pi}) = u(\frac{\bar{w}}{\pi}) - (\frac{\bar{w}}{\pi})\ u'(\frac{\bar{w}}{\pi}) = u(c) - cu'(c)$ where $c = \frac{\bar{w}}{\pi}$.

Therefore if $u(c) - cu'(c) < 0$, a small reduction in π would increase everyone's expected utility. If n is large this means that all expected utilities would be increased if there were a raffle where the unfortunate islander whose name is selected must forfeit his life while each of the survivors is comforted by a slightly larger portion of breadfruit. Such dismal circumstances must occasionally occur on lifeboats or in subsistence economies which have somehow become wretchedly over-populated. On more prosperous islands we would expect to find that an increase in survival probability would benefit everyone even though the breadfruit must then be divided among more survivors. On such islands, $u(c) - cu'(c) > 0$ and $\ell(\pi, w) > h$.

On the imaginary island, matters were simplified because there was no human capital. To find a corresponding condition for an individual with human capital, consider the following experiment. Let consumer i have non-human wealth w_i, survival probability π_i, and expected earnings $\pi_i h_i$. Imagine that he is given an additional endowment of non-human wealth equal to $\pi_i h_i$ and is forbidden to sell his labour. He therefore has non-human wealth $w_i^* = w_i + \pi_i h_i$ and no human capital. With this wealth he could buy an annuity that will give him $c_i = \frac{w_i^*}{\pi_i}$ units of consumption if he survives. Thus his utility will be $\pi_i u_i(\frac{w^*_i}{\pi_i})$. Now $\frac{\partial}{\partial \pi_i}(\pi_i u_i(\frac{w^*_i}{\pi_i})) = u_i(c_i) - c_i u_i'(c_i)$. Therefore $\ell_i(\pi_i, w_i) - h_i$ is of the same sign as $\frac{\partial}{\partial \pi_i}(\pi_i u(\frac{w^*_i}{\pi_i}))$. This gives us the following condition for deciding whether $\ell_i(\pi_i, w_i) > h_i$. Suppose that i exchanged his human capital for non-human capital of the same expected value. If he is willing to accept a reduction in his survival probability for no compensation other than a proportionate reduction in the cost of annuities, then $h_i \geqq \ell_i(\pi_i, w_i)$. If he is unwilling to make such an exchange then "his life is worth more than his human capital". This condition is not so obviously satisfied as the condition for $\ell_i(\pi_i, w_i)$ to exceed $h_i - c_i$. But it is difficult to imagine that many people in even modestly prosperous circumstances would accept an exchange of this kind. This argument, though far from conclusive, suggests that the answer to the title's question should be "usually".

Special Multiperiod Models

In a realistic model, there is no uncertainty about whether a person will die, but there is uncertainty about when he will die. In such a model, people would have preferences over probability distributions of the length of their lives and over their consumption levels at each period of life. Where there are many time periods, there are many interesting ways in which the time profile of survival probabilities can be perturbed. We begin this section with a quite general model and proceed to simplify its structure by a series of special assumptions.[10] In the process, we are able to glimpse the model at several intermediate levels of generality.

Let Π_t be the probability that an individual will survive for at least t periods. The probability that he survives for exactly t periods is then $\pi_t = \Pi_t - \Pi_{t+1}$. Let c_t be the vector of commodities that he will consume in period t if he survives for at least t years. An appealing case can be made for representing consumer preferences over alternative time patterns of survival probability and consumption by a state dependent expected utility function[11] of the form:

$$(12)\ \sum_{t=1}^{\infty} \pi_t u_t(c_1,\ldots,c_t).$$

Since $\pi_t = \Pi_t - \Pi_{t+1}$, (12) could be written equivalently as

$$(13)\ \sum_{t=1}^{\infty} \Pi_t\ \tilde{u}_t(c_1,\ldots,c_t)$$

where $\tilde{u}_t(c_1,\ldots,c_t) \equiv u_t(c_1,\ldots,c_t) - u_{t-1}(c_1,\ldots,c_{t-1})$

for $t > 1$ and $\tilde{u}_1(c_1) \equiv u_1(c_1)$.

Suppose that for any t, a person's preferences over alternative time paths of contingent consumption are additively separable between time periods. Then we can write:

$$(14)\ u_t(c_1,\ldots,c_t) = F_t(\sum_{i=1}^{t} u_{ti}(c_i))$$

where F_t is a monotone increasing function.

Suppose further that for any t the consumer's preferences over alternative consumption bundles for the first t years of his life would be the same if he knew that he would live exactly t periods as they would be if he knew that he would live for t + 1 periods. Then utility would take the simpler form:[12]

(15) $u_t(c_1,\ldots,c_t) = a_t + b_t \sum_{i=1}^{t} u_i(t_i)$

for all $t \geqq 2$.

It would be very convenient if the b_t terms were the same for all t. In addition to the previous assumptions, a necessary and sufficient condition for this to be the case is that the distribution of survival probability affects preferences about exchanging consumption in one period for consumption in another period only insofar as it affects the probability of being alive in each of these two periods.[12] If this is the case, then utility takes the form:

(16) $u_t(c_1,\ldots,c_t) = a_t + \sum_{i=1}^{t} u_i(c_i)$.

If we also assume that preferences are stationary over time in the sense of Koopmans (1960)

(17) $u_t(c_1,\ldots,c_t) = a_t + \sum_{i=1}^{t} \theta^i U(c_i)$

for some function $U(\cdot)$ and some real number $\theta > 0$.

From equations (13) and (17) it then follows that expected utility can be expressed as

(18) $E(U) = \sum_t \Pi_t(a_t + \theta^t u(c_t) - a_{t-1})$

where, by convention, $a_0 = 0$. Define

(19) $J(\Pi) \equiv \sum_t \Pi_t(a_t - a_{t-1})$.

Then (18) can be written as:

(20) $E(U) = J(\Pi) + \sum_t \Pi_t \theta^t u(c_t)$.

Let us assume that there is a single consumption good and a constant interest rate r. Define the expected present value of a person's earnings to be his human capital. This is denoted by

$$(21)\quad H(\Pi) \equiv \sum \Pi_t \left(\frac{1}{1+r}\right)^t h_t .$$

Let W be the present value of his after-tax non-human wealth. If he can buy actuarially fair annuities, then his budget simply requires that the expected present value of his contingent consumption plan equals the sum of his human and non-human wealth. That is:

$$(22)\quad \sum_t \Pi_t \left(\frac{1}{1+r}\right)^t c_t = W + H(\Pi).$$

We simplify the analysis further by assuming that the individual's personal rate of discount Θ is the same as the market rate of discount $\frac{1}{1+r}$. Then (22) can be written as:

$$(23)\quad \sum_t \Pi_t \Theta^t c_t = W + H(\Pi).$$

The consumer therefore chooses contingent consumption for each year so as to maximize (20) subject to (23). From the first order conditions equalizing marginal rates of substitution between the c_t's to their relative costs in (23), we see that if preferences are strictly convex, then (20) is maximized subject to (23) if and only if the c_t's are all equal to each other. Define

$$(24)\quad G(\Pi) \equiv \sum_t \Pi_t \Theta^t .$$

From (23) and (24) we see that the constant consumption level that satisfies the budget constraint is:

$$(25)\quad \bar{c} = \frac{W + H(\Pi)}{G(\Pi)}$$

From (20) and (25) it therefore follows that the highest utility the consumer can achieve if the vector of survival probabilities is Π and his non-human wealth is W is:

$$(26)\quad V(\Pi,W) = J(\Pi) + G(\Pi)U\left(\frac{W + H(\Pi)}{G(\Pi)}\right).$$

The expression (26) allows us to calculate the consumer's willingness to pay for any specified perturbation of survival probabilities. The vector Π enters (26) only through its effects on the three aggregates, $H(\Pi)$, $G(\Pi)$, and $J(\Pi)$. From its definition, we see that $G(\Pi)$ is the consumer's "discounted expected longevity". Stated another way, $G(\Pi)$ is the present value of a promise to deliver one unit of consumption good in each period so long as the individual is alive. The term $J(\Pi)$ allows for a variety of possible attitudes toward longevity. For example, if $a_t = t$ for each t, then $J(\Pi) = \sum_t \Pi_t$ which is just expected number of years of life. If $A_t = a \sum_{i=1}^{t} \Theta^i$, then $J(\Pi) = aG(\Pi)$. which would mean that the contribution of later years to $J(\Pi)$ would be discounted at the interest rate. "Risk averse" or "risk-preferring" attitudes toward gambles in which a small increase in current hazard is exchanged for an increase in later conditional probabilities of survival can be incorporated by making $a_t - a_{t-1}$ respectively decrease or increase as t increases.

Consider a postponement of a risk to ones life from period t to period t+1 with survival probabilities in other periods being unaltered. This amounts to an increase in Π_t with all other cumulative survival probabilities remaining constant. The consumer's willingness to pay for such a change is just the marginal rate of substitution.

$$(27)\quad \ell_t(\Pi,W) \equiv \frac{\partial V(\Pi,W)}{\partial \Pi_t} \div \frac{\partial V(\Pi,W)}{\partial W} .$$

Calculating the partial derivatives of $V(\Pi,W)$ and substituting into (27) we find:

$$(28)\quad \ell_t(\Pi,W) = \frac{a_t - a_{t-1} + \Theta^t U(\bar{c})}{U'(\bar{c})} + \Theta^t(h_t - \bar{c}).$$

From (26) we could also calculate the effects of other perturbations in survival probabilities. For example suppose we vary the amount of "hazard" to which an individual is subjected in the first period, while leaving conditional probabilities of survival in later periods unaltered. This amounts to a proportionate change in all cumulative survival probabilities. Thus if λ is the level of hazard, and Π is the original vector of cumulative survival probabilities, then indirect utility can be expressed as

$$(30)\quad V(\lambda\Pi,W) = G(\lambda\Pi)U\left(\frac{W + H(\lambda\Pi)}{G(\lambda\Pi)}\right) + J(\lambda\Pi).$$

From their definitions, it can be seen that the functions $G(\cdot)$, $H(\cdot)$, and $J(\cdot)$ are all homogeneous of degree one in Π. Therefore (30) can be expressed as:

$$(31)\quad V(\lambda\Pi,W) = \lambda G(\Pi)U\left(\frac{W + \lambda H(\Pi)}{\lambda G(\Pi)}\right) + \lambda J(\Pi).$$

The marginal rate of substitution between current hazard and wealth is then:

$$(32)\quad \frac{\partial V(\lambda\Pi,W)}{\partial\lambda} \div \frac{\partial V(\lambda\Pi,W)}{\partial W} = \frac{J(\Pi) + G(\Pi)U(\bar{c})}{U'(\bar{c})} + (H(\Pi)-G(\Pi)\bar{c})$$

Equations (29) and (32) are easily recognized as extensions of equation (9) which was derived for the single period model. As, before, these expressions can be partitioned into a direct effect and a pecuniary effect. Our earlier comparison between the value of risks to human life and human capital apply here as well.

Usher (1973) and Conley (1976) study stationary, additively separable utility functions like our (17) except that they neglect the term a_t and hence the expression $J(\Pi)$ in the expected utility function. Linnerooth (1979) criticized their formulation as inadequate to register the distinction between "living to consume" and the pure pleasure of living. Jones-Lee (1978) likewise expresses a suspicion that this form is unnecessarily limiting.

The point of Linnerooth's and Jones-Lee's argument can be seen if we notice that when $J(\Pi) = 0$

$$(33)\quad V(\Pi,W) = G(\Pi)U\left(\frac{W + H(\Pi)}{G(\Pi)}\right).$$

If utility took this special form, then any change in Π would affect utility only insofar as it affected the pecuniary variables human capital $H(\Pi)$, and the cost, $G(\Pi)$, of a unit annuity. As we have shown, the assumption that $J(\Pi) = 0$ is not warranted even by very strong assumptions of separability, independence, and stationarity. In fact, under these assumptions, $J(\Pi)$ can be any linear function, whatever, of the cumulative survival probabilities. Therefore the misgivings of Linnerooth and Jones-Lee are well-founded. We further illustrate this point with two examples.

Occasionally it has been suggested that knowledge of a consumer's preferences about purely financial gambles might, together with some reasonable assumptions about separability, allow one to deduce his willingness to exchange wealth for survival probability. This suggestion is evidently misguided. It is true that if the assumptions that lead to the functional form (17) are justified, then a fairly reasonable assumption implies that the function $U(\cdot)$ in (17) also serves as the von Neuman-Morgenstern utility function for bets not involving risk to life. Therefore, knowledge of preferences about such bets would imply knowledge of $U(\cdot)$. The snag is that such knowledge can tell us literally nothing about the function $J(\Pi)$ and therefore nothing about the marginal rates of substitution between survival probabilities and wealth.

Medical decisions frequently require choices between current and future survival probabilities. See, for example, Needleman (1976) and Fuchs (1981). An individual's marginal rate of substitution between survival probability in year s and in year t is just $\ell_s(\Pi,W) \div \ell_t(\Pi,W)$. From (29) it can be seen that $\ell_s(\Pi,W) \div \ell_t(\Pi,W) = \Theta^{s-t}$ if and only if $a_t - a_{t-1} = a\Theta^t$ for all t and some a. If this is the case, then future survival probability is discounted at the same rate as the interest rate. In general, however, there is no reason even with our strong independence and stationarity assumptions, for the a_t's to be related in this way. Therefore the intuitively appealing notion of discounting later years of life at the same rate as later dollars does not appear to be supported by fundamental assumptions.

A General Multiperiod Model with Inheritance and Life Insurance

Having explored the limits of results attainable by special assumptions, we now examine a general model. As it turns out, most of the results of the special cases apply in much greater generality. So far, our consumers have had no interest in their heirs. Now we will allow them an inheritance motive. We will make no assumptions about separability, independence or stationarity beyond the assumption that preferences are represented by a state dependent expected utility function.

We maintain the notation of the previous sections except that c_t is now interpreted as consumption in year t by the individual and his family if he is alive in year t and e_t is consumption by his family in year t if he is dead in year t. An appropriate expected utility representation will then take the form:

$$(34)\qquad \pi_0 u_0(e_1,\ldots,e_i\ldots) + \sum_{t=1}^{\infty} \pi_t u_t(c_1,\ldots,c_t,e_{t+1},\ldots).$$

If he can buy actuarially fair annuities and life insurance, the consumer's budget constraint is:

$$(35)\qquad \sum_{t=1}^{\infty} p_t[\Pi_t c_t + (1-\Pi_t)e_t] = W + H(\Pi)$$

where p_t is the present cost of goods to be delivered in period t.

If we wish to allow for the earnings capacities of the consumer's heirs we may interpret W as including the present value of the heirs' after-tax earnings as well as the consumer's after-tax non-human wealth.

Since $\pi_t = \Pi_t - \Pi_{t+1}$, we can express (34) in terms of the cumulative survival probabilities. This gives us an expected utility function:

$$(36) \qquad (1-\Pi_1)u_0(e_1,\ldots,e_{i1}\ldots) + \sum_{t=1}^{\infty}(\Pi_t - \Pi_{t+1})u_t(c_1,\ldots,c_t,e_{t+1},\ldots).$$

We define the indirect utility function $V(\Pi,W)$ to be the maximum of (36) subject to the budget constraint (35).

Let $\ell_t(\Pi,W)$ be defined as in (27). A simple envelope theory argument found in the appendix shows that:

$$(37) \qquad \ell_t(\Pi,W) = \frac{1}{\frac{\partial V}{\partial W}}[u_t(c_1,\ldots,c_t,e_{t+1},\ldots) - u_{t-1}(c_1,\ldots,c_{t-1},e_t,\ldots)] + p_t[h_t - (c_t - e_t)]$$

We also show in the appendix that the marginal rate of substitution of present hazard for wealth is:

$$(38) \qquad \frac{\partial V(\lambda\Pi,W)}{\partial\lambda} \div \frac{\partial V(\lambda\Pi,W)}{\partial W} = \frac{1}{\frac{\partial V}{\partial W}}\left[\sum_{t=1}^{\infty}(u_t(c_1,\ldots,c_t,e_{t+1},\ldots) - u_0(e_1,\ldots,e_t\ldots)\right] + H(\Pi) - \sum_{t=1}^{\infty}p_t(c_t - e_t).$$

Equations (37) and (38), like their counterparts for special cases decompose into a direct effect and a pecuniary effect. The pecuniary effect of a change in current hazard appears in (38) as $H(\Pi) - \sum_{t=1}^{\infty}p_t(c_t - e_t)$. The reason for this is clear. A proportionate increase in the probability of surviving to each future period increases the expected value of human capital at the rate $H(\Pi)$. It also increases proportionately the probability that the consumer's family will consume c_t rather than e_t in each period t. The expected present value of the family's consumption plan therefore rises at the rate $\sum_{t=1}^{\infty} p_t(c_t - e_t)$ as his survival probabilities increase proportionately. This is a cost which must be subtracted from the gain in human capital in the expression for pecuniary effects.

We can compare marginal willingness to pay for hazard reduction to net output and to human capital as we did in the single period model. Let us define a consumer's net output to be his human capital less the difference in the present value of the family consumption plan with and without his survival. This is exactly the pecuniary term in (38). Therefore willingness to pay will exceed net output if the first term of (38) is positive. Now the terms $u_t(c_1,\ldots,c_t,e_{t+1},\ldots) - u_0(e_1,\ldots,e_t,\ldots)$ will all be positive if, given the family's contingent consumption plan, he prefers surviving for t years to dying immediately. The term $\frac{\partial V}{\partial W}$ will be positive if more consumption in some period is preferred to less. Therefore under very weak conditions, marginal willingness to pay for hazard reduction exceeds net output.

There is also a simple extension of the single period comparison of marginal willingness to pay with $H(\Pi)$. From (38) we see that:

$$(39) \quad \frac{\partial V(\lambda\Pi,W)}{\partial\lambda} \div \frac{\partial V(\lambda\Pi,W)}{\partial W} - H(\Pi) = \frac{1}{\frac{\partial V}{\partial W}}\left[\sum_{t=1}^{\infty} u_t(c_1,\ldots,c_t,e_{t+1},\ldots) - u_0(e_1,\ldots,e_t,\ldots)\right] - \sum_{t=1}^{\infty} p_t(c_1 - c_t).$$

Suppose that this consumer is given non-human wealth equal to $W + H(\Pi)$ and is left with no human capital. His marginal rate of substitution between survival probability and wealth in this situation can be shown to equal the right side of equation (39). His marginal valuation of hazard reduction will therefore exceed his human capital if and only if he is unwilling, under these circumstances, to accept an increased present hazard in return for the net pecuniary benefits due to the reduction in cost of actuarially fair annuities. In this case, the pecuniary gain from cheaper annuities is partly counterbalanced by the pecuniary loss from more expensive life insurance.

The Effect of Benevolence on Valuing Lives

In the model of the previous section we considered the attitude of the head of the household toward consumption by himself and by his family and toward alternative patterns of survival probability for his own life. The model did not treat the possibility that the survival probabilities of his heirs might also be choice variables. To do this properly, we need a more explicit model of benevolence. The principal issues can be efficiently addressed in a single period model.

It seems plausible that if people are benevolent, more should be spent on saving lives than the amount that would be appropriate if benevolence is ignored. In fact, Mishan (1971), Needleman (1976) and Jones-Lee (1980) suggest that for the purposes of valuing lives saved, we should add to the average of individual marginal rates of substitution a term that expresses the valuation that people place on reducing risks to other people's lives. We argue here that such a procedure is inappropriate. What has been overlooked in these discussions, is that typically if one were benevolently disposed towards others, he would be interested not only in their survival probabilities, but also in their consumption. Suppose taxes are increased to pay for more public safety. If the benefits to Peter of the extra public safety must include Peter's valuation of increased safety for Paul, then the costs to Peter of the taxes that pay for increased safety must include Peter's regrets for Paul's reduced consumption. In general, the net effect of accounting for interpersonal sympathy could either increase or decrease the recommended level of public safety.

There is a nice case where these effects just balance; where despite the presence of benevolence, the appropriate marginal value of a life saved by public safety is just the average of private marginal rates of substitution between survival probability and wealth. Suppose that preferences of each i can be represented by a utility function of the form:

$$(40) \quad U_i(\pi_1 u_1(c_1),\ldots,\pi_n u_n(c_n))$$

where U_i is a non-decreasing function of each of its arguments and an increasing function of $\pi_i u_i(c_i)$. This is the case of pure benevolence. The function $\pi_i u_i(c_i)$ is seen to represent i's private preferences over his own survival probability and consumption where the circumstances of all other consumers are held constant. The assumption that each U_i is a non-decreasing function of $\pi_j u_j(c_j)$ for each j, means that the interpersonal regard of each individual respects the private preferences of the others.[14] Suppose that an allocation $(\bar{\pi}_1,\bar{c}_1,\ldots,\bar{\pi}_n,\bar{c}_n)$ of survival probabilities and wealth is Pareto optimal. Then this allocation would also be Pareto optimal if everyone were perfectly selfish and had the utility function $\pi_i u_i(c_i)$. To see this, suppose that there is another feasible allocation $(\bar{\bar{\pi}}_1,\bar{\bar{c}}_1,\ldots,\bar{\bar{\pi}}_n,\bar{\bar{c}}_n)$ such that $\bar{\bar{\pi}}_i u_i(\bar{\bar{c}}_i) \geqq \bar{\pi}_i u(\bar{c}_i)$ for all i with strict inequality for some i. Our assumption that U_i is monotonic in its arguments would then imply that:

$$(41) \quad U_i(\bar{\bar{\pi}}_1 u_1(\bar{\bar{c}}_1),\ldots,\bar{\bar{\pi}}_n u_n(\bar{\bar{c}}_n)) \geqq U_i(\bar{\pi}_1 u_1(\bar{c}_1),\ldots,\bar{\pi}_n u_n(\bar{c}_n))$$

for all i with strict inequality for some i. But this is impossible since the allocation $(\bar{\pi}_1,\bar{c}_1,\ldots,\bar{\pi}_n,\bar{c}_n)$ was assumed to be Pareto optimal. This proves our assertion. If every Pareto optimum is also a Pareto optimum for the selfish economy obtained by ignoring benevolence, then it follows that the necessary conditions for Pareto optimality in the presence of benevolence are the same as those for the selfish economy. But these conditions require us to value a marginal life saved at the average of private marginal rates of substitution between survival probabilities and wealth. Adding extra terms to this valuation would lead to inefficiency.

Provision of Private and Public Safety With and Without a Perfect Annuities Market

So far we have assumed the presence of perfect annuities markets. While this is a natural and useful simplification, it is also a strong assumption and one which deserves discussion. There are at least three reasons why this assumption is suspect. These are:

(i) There are real transactions and management costs in the selling of insurance;

(ii) the net earnings of an insurance company are of necessity random. For accepting such randomness in their incomes, stockholders of the insurance company must typically receive a risk premium;

(iii) different individuals may have different subjective probabilities of the relevant events and hence "actuarially fair" insurance is not even a well-defined concept.

We follow the usual practice of economic analysis in supposing that the costs mentioned in (i) are sufficiently small that our model remains a good approximation to reality. If the economy under consideration has many people and their survival probabilities are independent, then the objection raised in (ii) also becomes minor since the insurance company's total risk when divided among many stockholders becomes negligible for each. Objection (iii) is much more serious. Very commonly individuals' subjective probabilities of their own survival differ from the actuarial estimates that life insurance companies make. The cost to an insurance company of obtaining reliable information about individual health and safety practices precludes the possibility that these probabilities be identical. On the other hand, insurance rates need not be entirely invariant to such measures.

Here we explore one of many possible models of how prices of annuities might be imperfectly responsive to individual survival probabilities. Although the case treated is rather special, it illustrates the issues at stake. It is interesting that this model preserves the general qualitative results about the relative size of the human capital and the subjective values of incremental survival probability which were found in the "pure" case.

Let there be a large number, n, of individuals and a single tradeable commodity, bread. There is just one time period. Let c_i be the consumption of i's family if he dies. The probability that i survives is a function $\pi(q_i,s)$ of the amount q_i of bread that he spends on his personal safety and the amount s of bread spent by the government on public safety. There is a total stock of $n\bar{k}$ units of bread initially available and if he survives an individual can produce h units of bread. Preferences of i are represented by the utility function:

$$(42) \quad \pi(q_i,s)u(c_i) + (1-\pi(q_i,s))u^*(e_i).$$

The set of feasible allocations for the society is the set of vectors, $(c_1,\ldots c_n,e_1,\ldots e_n,q_1,\ldots,q_n,s)$ such that

$$(43) \quad \sum_i(\pi(q_i,s)c_i + (1-\pi(q_i,s))e_i + q_i) + s = n\bar{k} + \sum_i\pi(q_i,s)h.$$

Since preferences are identical, the Pareto optimum that treats everyone identically is of special interest. Thus we seek $\bar{c}$, $\bar{e}$, $\bar{q}$, and $\bar{s}$ to maximize:

$$(44) \quad \pi(q, s)u(c) + (1-\pi(q, s))\, u^*(c)$$

subject to:

$$(45) \quad q + \pi(q, s)(c-h) + (1-\pi(q,s))e = \bar{k} - \frac{s}{n}.$$

Define

$$(46) \quad \tilde{v}(\pi,w) \equiv \max_{c,e}.\ \pi u(c) + (1-\pi)u(e)$$

subject to $\pi c + (1-\pi)e = w$ and

$$(47) \quad \tilde{\ell}(\pi,w) \equiv \frac{\partial \tilde{v}}{\partial \pi} \div \frac{\partial \tilde{v}}{\partial w}$$

A bit of computation shows that the first order conditions for Pareto efficiency can be expressed as:

$$(48) \quad \tilde{\ell}(\pi(\bar{q}, \bar{s}), \bar{k} - \frac{\bar{s}}{n} - \bar{q})n \ \frac{\partial \pi}{\partial s} = 1$$

and

$$(49) \quad \tilde{\ell}(\pi(\bar{q}, \bar{s}), \bar{k} - \frac{\bar{s}}{n} - q) \ \frac{\partial \pi}{\partial q} = 1$$

The expected number of lives saved by an incremental expenditure Δs on public safety is $n\frac{\partial \pi}{\partial s}\cdot\Delta s$. Therefore condition (1) requires that public safety expenditures should be carried to the point where the cost per expected life saved is just $\tilde{\ell}[\pi(q,s),w-q]$. Similarly (2) requires that private expenditures be carried to the point where the expenditure per expected life saved is $\tilde{\ell}[\pi(q,s), w-q]$.

Suppose that each individual owns the same amount, $\bar{k}$, of bread and that the cost of public safety is shared equally. If for any level of expenditure on private safety, one can always buy insurance and annuities at actuarially fair rates, then each individual faces the budget constraint:

$$\frac{s}{n} + q_i + \pi(q_i, s)(c_i - h) + (1-\pi)(q_i, s))e_i = \bar{k} \tag{50}$$

Thus the constrained maximization problem faced by each individual is precisely the same as that which we posed above for finding an equalitarian Pareto optimum. Therefore, all individuals will agree on a level of public expenditures $\bar{s}$ and each will choose the same level of private expenditures $q_i = \bar{q}$ so that the resulting allocation is Pareto optimal and satisfies conditions (1) and (2) above.

Furthermore, the value $\ell[\pi(q,s), w-q]$ which should be placed on the marginal life saved due to public safety is just the same as the marginal rate of substitution displayed by individuals in decisions on private safety expenditures. This suggests that information about private decisions could be used in a benefit cost analysis to choose an efficient level of public safety.

If expenditures by individuals on private safety can not be cheaply and reliably observed, it is not reasonable to suppose that the cost of annuities will respond to changes in such expenditures so as to remain actuarially fair. Suppose, for instance, that the "insurance company" charges everyone the same price for annuities regardless of his private expenditure on safety and sets its price so that "on average" its annuities are actuarially fair. Then if the level of public safety is s and $\tilde{q} = (q_1, \ldots, q_n)$ is the vector of private expenditures on safety, then the average survival probability is $\bar{\pi}(q, s) = \frac{1}{n} \sum_{i=1}^{n} \pi(q_i, s)$. Annuities will be sold to all comers at a price of $\bar{\pi}(\tilde{q}, s)$ while the price of life insurance will be $1-\bar{\pi}(\tilde{q}, s)$.

Where the level of public expenditures is s, each individual chooses c_i, e_i, and q_i so as to maximize:

(51) $$\pi(q_i, s)u(c_i) + (1-\pi(q_i, s)u^*(e_i)$$

subject to:

(52) $$\frac{s}{n} + q_i + \bar{\pi}(\tilde{q}, s)(c_i-h) + (1-\pi(\tilde{q}, s) \leqq \bar{k}.$$

Computation shows that for each i, the first order conditions for this constrained maximization problem are

(53) $$\tilde{\ell}(\pi(q_i, s), \bar{k} - \frac{s}{n} - q_i)\ n\ \frac{\partial\pi(q_i, s)}{\partial s} = 1$$

(54) $$[\tilde{\ell}(\pi(q_i, s), \bar{k} - \frac{s}{n} - q_i) + \frac{n-1}{n}(c_i-h-e_i)]\ \frac{\partial\pi(q_i, s)}{\partial q_i} = 1.$$

When i spends more bread on safety, he increases the expected return from his annuities by increasing the likelihood that he will live to collect it. For the same reason he reduces theexpected return from his life insurance policies. Since his insurance rates are not adjusted the private return of a dollar spent on private safety differs from the social return by the amount $\frac{n-1}{n}(c_i-h-e_i)$. Since $c_i-h_i-e_i$ is the difference between the size of i's annuity and his life insurance policy, there will be under-expenditure or over-expenditure on private safety depending on whether one holds more life insurance than annuities or vice versa.

APPENDIX 1

NECESSARY AND SUFFICIENT CONDITIONS FOR OPTIMAL PUBLIC SAFETY

Here we establish an isomorphism between Samuelson's pure public goods problem and our formulation of the problem of optimal public safety.[15] This enables us to state a simple convexity condition that implies that our first-order conditions are sufficient as well as necessary for Pareto efficiency. It also gives us a rigorous demonstration of the first-order conditions derived more informally above. We establish these results for the model in the section *A General Multiperiod Model.*

The Samuelson public goods problem in its simplest form is as follows.

Problem S: Choose an allocation $(x_1,\ldots x_n,y)$ to optimize the vector of utilities $(u_1(x_1,y),\ \ldots,\ u_n(x_n,y))$ subject to the constraint that $y + \sum_i x_i = k$ for some $k > 0$.

First-order necessary conditions for a solution to Problem S are well-known from Samuelson (1964). That these conditions are also sufficient, given convex preferences, is proved by Bergstrom (1979). Thus we can state:

Lemma 1

If the functions $u_i(\cdot)$ are all differentiable and monotone increasing in x_i, then a necessary condition for the allocation $(x_1{}^*,\ldots,x_n{}^*,\ y^*)$ to be an interior solution of Problem S is $\sum_i \frac{\partial u_i(x_i{}^*,\ y^*)}{\partial y} \div \frac{\partial u_i(x_i{}^*,\ y^*)}{\partial x_i} = 1$. If $u_i(\cdot)$ is quasi-concave, then this condition is also sufficient.

The problem of finding efficient allocations for our model of public safety can be written:

Problem A:

Choose an allocation $(W_1,\ldots,W_n,s)$ to optimize the vector of indirect utilities $(V_1(\Pi_1(s),\ W_1),\ldots,V_n(\Pi_n(s),\ W_n))$ subject to the constraint $s + \sum_i W_i = k$. (Here k is the sum of initial holdings of non-human wealth).

Problem A is seen to be formally equivalent to Problem S where s and W_i correspond respectively to y and x_i and where the function $V_i(\Pi(s),W_i)$ corresponds to the function $u_i(x_i,y)$ when viewed as a function of the two variables s and W_i. Therefore the first-order conditions for solving Problem A are found by simple translation from Lemma 1. Furthermore, we will find conditions under which $V_i(\cdot)$ is concave in s and W_i. This will enable us to establish the following.

Proposition 1

Let $V_i(\Pi_i(s),\ W_i)$ be the maximum of $\sum_{t=0}^{\infty} \pi_{it}(s)u_{it}(c_{i1},\ldots,e_{it+1},\ldots)$ subject to the budget constraint $\sum_{t=0}^{\infty} p_t[\Pi_{it}(s)(c_{it}-h_{it}) + (1-\Pi_{it}(s))e_{it}] = W_i$. (Recall that $\pi_{it}(s) = \Pi_{it}(s) - \Pi_{it+1}(s)$ for $t \geq 1$ and $\pi_{i0}(s) = 1-\Pi_{i1}(s)$). Then a necessary condition for $(W_1^*,\ldots,W_n^*,\ s^*)$ to be an interior Pareto optimal allocation is $\sum_{i}\sum_{t} \ell_{it}(\Pi_i(s^*),W_i^*)\frac{\partial \Pi_{it}(s^*)}{\partial s} = 1$;

where $\ell_{it}(\Pi_i(s^*),\ W_i^*) \equiv \dfrac{\partial V_i(\Pi_{it}(s^*),W_i^*)}{\partial \Pi_{it}} \div \dfrac{\partial V_i(\Pi_{it}(s^*),W_i^*)}{\partial W_i}$.

If the functions $u_{it}(\cdot)$ are quasi-concave, and if the functions $\Pi_{it}(s)$ are concave, and the functions $V_i(\Pi,s)$ are increasing in Π , then this condition is also sufficient for Pareto optimality.

From Lemma 2 and the fact that an increasing concave function of a concave function is concave we have:

Lemma 3

If the functions $u_{it}(\cdot)$ are quasi-concave, the functions $\Pi_{it}(s)$ are concave and the functions $V_i(\Pi_i, W_i)$ are increasing in Π_i, then for each i, $V_i(\Pi_i(s), W_i)$ is a concave function of s and W_i.

From Lemmas 1 and 3, the sufficiency part of Proposition 1 then follows.

APPENDIX 2

A GENERALIZED ENVELOPE THEOREM AND ITS CONSEQUENCES

The expressions for marginal rate of substitution between survival probability and wealth in the section *A General Multiperiod Model* are all simple consequences of the following generalized envelope theorem.

Let $f(\Pi,c,e)$ and $g(\Pi,c,e)$ be functions of the vector of "parameters" Π and the vectors of "decision variables", (c,e). Define $V(\Pi,W) = \max.\ \{f(\Pi,c,e) \mid (c,e) \geqq 0 \text{ and } g(\Pi,c,e) \leqq W\}$. If $f(\cdot)$ and $g(\cdot)$ are differentiable functions and if the feasible set $\{(c,e) \geqq 0 \mid g(\Pi,c,e) \leqq W\}$ is non-empty and compact, then (subject to a constraint qualification)

$$\frac{\partial V}{\partial \Pi_t} \quad \frac{\partial f(\Pi,\bar{c},\bar{e})}{\partial \Pi_t} - \frac{\partial g(\Pi,\bar{c},\bar{e})}{\partial \Pi_t} \quad \frac{\partial V(\Pi,\bar{c},\bar{e})}{\partial W}$$

where $\bar{c},\bar{e}$ solves the constrained maximization problem at Π, W.

The proof of this theorem is an entirely straightforward extension of the well-known proof of the standard envelope theorem which would apply if $\frac{\partial f(\Pi,\bar{c},\bar{e})}{\partial \Pi_t} = 0$. Equations (37) and (38) are immediate results of this lemma where $f(\cdot)$ and $g(\cdot)$ are defined by equations (36) and (35) respectively.

FOOTNOTES

1 This point is cleverly and persuasively argued by Thomas (13).

2 Conley (1978) in reply to Jones-Lee suggests that it is inappropriate to judge his theory on the plausibility of its assumptions. Rather, he argues that his model is validated by the accuracy of its predictions. The "predictions" referred to are evidently that the subjective value of human life exceeds human capital. This argument seems to be a thoroughly misconceived application of Chicago positivism. The difficulty is that there are as yet no really persuasive estimates of peoples' valuations of risk to human life. The direct evidence is fragmentary at best. (See Blomquist (1981). It is for precisely this reason that it is of interest to use theory to see whether other independently verifiable propositions imply a relation between the value of a life and the value of human capital. Such conditions are of interest only if their plausibility can somehow be evaluated.

3 With slightly more apparent generality we could write $U(\pi, c) = \pi u(c) + (1 - \pi)u^*$ where u^* is the "utility of being dead". But then where $\tilde{u}(c) \equiv u(c) - u^*$, the above is equivalent to $U(\pi, c) = u^* + \pi\tilde{u}(c)$. Since the utility representation of preferences is unique only up to positive monotone transformations, these same preferences could also be represented by $\pi\tilde{u}(c)$. Thus there is no real gain in generality over assuming that $U(\pi, c) = \pi u(c)$ in the first place.

4 More generally we could allow $u(c)$ to take negative values for sufficiently small c. At such low levels of consumption the individual would prefer dying to surviving.

5 In fact, the analogy to Adam Smith's celebrated "diamond-water paradox" is close. When a consumer's total water consumption is valued at his marginal valuation, the "value" of his water consumption is relatively small. On the other hand, the cost of inducing him to give up water altogether might well be infinite.

6 Since this is a model of public safety, it is reasonable to assume that s and the survival rates $\Pi(s)$ are public knowledge. Therefore, to assume the availability of actuarially fair annuities seems more reasonable than to assume that annuity prices do not respond to s. In a later section, we consider a model of private safety in which individual precautions are not public knowledge and where actuarially fair insurance is not available.

7 From (1) we see that $h_i - c_i$ may be positive or negative depending on whether tax obligations t_i are greater or less than non-human capital k_i.

8 This result is also stated by Cook (1978).

9 In fact if we were to try to investigate whether $u(\cdot)$ is concave at very low levels of consumption or to determine the smallest amount of consumption that is better than being dead, we would need to build a multiperiod "theory of misery" in which the technology of gradual starvation was explicitly recognized.

10 Consideration of inheritance motives is postponed to the next section. Other multiperiod models have been studied by Conley (1976), Jones-Lee (1981) and Arthur (1981). Arthur's paper is particularly interesting since he embeds the individual decisions in a steady-state growth model.

11 Satisfactory foundational axioms for state-dependent expected utility theory can be found in Luce and Krantz (1971) or Balch and Fishburn (1974). For our application, the main idea is that events are chosen so as to partition all possible outcomes. Preferences are defined over lotteries that include as "prizes" both a specific length of life and a time pattern of consumption. With the prizes thus defined, the theory becomes isomorphic to a model of state independent expected utility theory. The standard assumptions of expected utility theory seem as reasonable when applied to the model thus recast as they do in its usual application. The functional form of the "expected utility representation" thus implied is then (12).

12 This assumption requires that if $u_t(c_1,\ldots,c_t) \geqq u_t(c_1',\ldots,c_t')$, then $u_{t+1}(c_1,\ldots,c_t,\bar{c}_{t+1}) \geqq u_t(c_1',\ldots,c_t',\bar{c}_{t+1})$ for any $\bar{c}_{t+1}$. From equation (14) it follows that $F_{t+1}(\sum_{i=1}^{t} u_{t+1\,i}(c_i) + u_{t+1\;t+1}(\bar{c}_{t+1})) = H(\sum_{i=1}^{t} u_i(c_i), \bar{c}_{t+1})$ where the function H is monotone increasing in its first argument. A standard result of consumer theory is that two additively separable representations of the same preferences must be affine transformations of each other. Applied in this instance, this result implies that preferences can be represented as in (15).

13 Formally we assume the following. Let c and c' be two time paths of consumption that differ only in periods s and t. Suppose that when the vector of cumulative survival probabilities is $(\Pi_1,\ldots,\Pi_t,\ldots)$, the consumption path c is preferred to c'. Let $(\Pi_1',\ldots,\Pi_t',\ldots)$ be another vector of survival probabilities such that $\Pi_t = \Pi_t'$ and $\Pi_s = \Pi_s'$. Then the consumption path c will also be preferred to c' when the cumulative survival probabilities are $(\Pi_1',\ldots,\Pi_t',\ldots)$.

To see that this condition is sufficient for the representation (16), observe that if utility is of the form (15), then expected utility can be written: $\sum_t \pi_t(a_t + b_t \sum_{i=1}^{t} u_i(c_i)) = \sum_t \pi_t a_t + \sum_t \gamma_t u_t(c_t)$ where we define $\gamma_t = \sum_{i=1}^{t} \pi_i b_i$. If the b_i's are not all the same, then it is always possible to change the survival probabilities in such a way as to change the ratio $\gamma_t \div \gamma_s$ without changing either Π_t or Π_s. Therefore it would be possible to change preferences between c and c' without changing Π_t or Π_s where c and c' differ only in periods s and t. It follows that the assumed condition implies that the b_t's in (15) all be the same. Without loss of generality we can set them all equal to unity. The converse result that the functional form (16) implies our formal condition is a matter of straightforward verification.

14 This assumption is analyzed in some detail in Bergstrom (1970) and (1971).

15 Dreze and Dehez (1981) also exploit the observation that public safety can be treated as a pure public good.

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