C-25

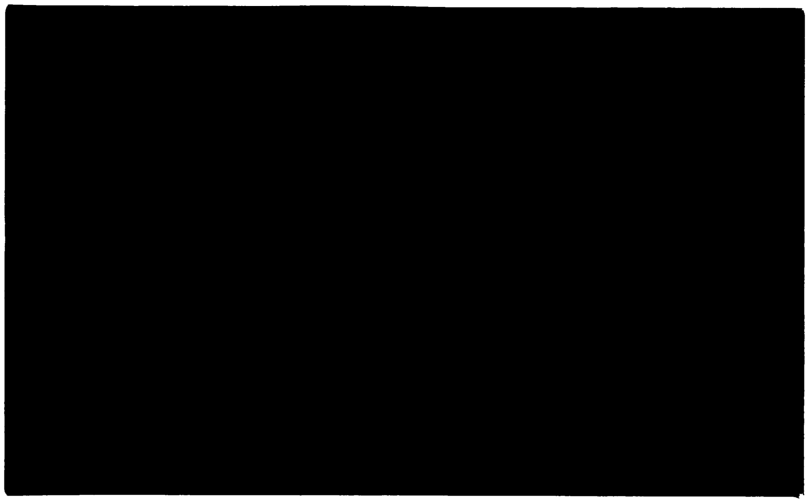
# Center for Research on Economic and Social Theory
# Research Seminar in Quantitative Economics

## Discussion Paper

DEPARTMENT OF ECONOMICS

**University of Michigan**

Ann Arbor, Michigan 48109



# Efficiency-Inducing Taxation for a Monopolistically Supplied Depletable Resource

Theodore C. Bergstrom
John G. Cross
Richard C. Porter
University of Michigan
48109

C-25

Where there is a fixed stock of a depletable resource, Pareto optimality requires that the difference between price and marginal cost of extraction rise at the interest rate. In competitive equilibrium, this condition is fulfilled. A monopolist, however, supplies the resource in such a way that the difference between marginal revenue and marginal extraction cost rises at the rate of interest.1/ As a result, a monopolist will not in general supply the resource efficiently. Depending on the nature of demand, he may supply either more or less rapidly than is required for Pareto optimality.

Here we address the question of how a government might specify in advance a time path of per-unit tax or subsidy rates on quantities supplied so as to induce the monopolist to supply the resource efficiently over time.2/ The principle employed is simple. If the tax or subsidy rate per unit of output changes over time in a prespecified way, then the monopoly can in general alter the present value of its total tax obligations or subsidy revenue by changing the time pattern in which it disposes of its stock of the resource. We show that if the competitive path is known and if the profit function is concave, then there is an easily described family of tax-subsidy policies that would induce a monopoly to follow the efficient production path of a competitive industry. Depending on the nature of demand functions it may or may not be possible to induce efficiency with a policy that yields a positive present value of net taxes collected by the government. We also show that the incidence of a tax is independent of whether it is nominally collected from buyers or from sellers, just as in the theory of static equilibrium. This fact has interesting implications for the case where buyers and sellers live in different countries and thus are not both subject to the same taxing authority.

The General Model

Let $p(t)$ and $q(t)$ denote respectively the price and quantity of the exhaustible resource supplied at time t. Let $f(q(t),t)$ be the inverse demand function, $c(q(t),t)$ be total extraction costs, and $c'(q(t),t)$ be marginal extraction costs at time t. Let Q be the stock of the resource available initially and let $\rho(t)$ be the instantaneous interest rate at time t. The standard intertemporal "arbitrage" condition informs us that competitive prices and quantities, $p^*(t)$ and $q^*(t)$, must satisfy the following equation so long as positive amounts of the resource are being extracted:

$$(1) \qquad \frac{d}{dt}[p^*(t) - c'(q^*(t),t)] = \rho(t)[p^*(t) - c'(q^*(t),t)].$$

Since $p^*(t) = f(q^*(t),t)$, equation (1) can be written as a differential equation in the single variable, $q^*(t)$:

$$(2) \qquad \frac{d}{dt}[f(q^*(t),t) - c'(q^*(t),t)] = \rho(t)[f(q^*(t),t) - c'(q^*(t),t)].$$

Assuming that in the competitive equilibrium the depletable resource is actually scarce, we have the additional constraint:

$$(3) \qquad \int_0^\infty q^*(t)dt = Q.$$

The differential equation (2) with the boundary condition (3) uniquely determines the competitive output path, $q^*(t)$.

For a monopoly, profit in the absence of taxes or subsidies is:

$$(4) \qquad R(q(t),t) = q(t)f(q(t),t) - c(q(t),t).$$

If the government pays a subsidy, $s(t)$, (possibly negative) per unit of output at time t, then the total profit of the monopolist at time t is

$$(5) \qquad \pi(q(t),t) = s(t)q(t) + R(q(t),t).$$

Throughout the remaining discussion we assume that $R(\cdot,\cdot)$ is continuous in both q and t and twice differentiable with respect to q. Let $R'(\cdot,\cdot)$, $R''(\cdot,\cdot)$, $\Pi'(\cdot,\cdot)$ and $\Pi''(\cdot,\cdot)$ denote derivatives with respect to q. Then we have

(6) $\Pi'(q(t),t) = s(t) + R'(q(t),t).$

We will assume that:

(7) $R''(q(t),t) < 0$ for all $q(t) > 0$ and $t > 0$.

From (6) and (7) it is immediate that:

(8) $\Pi''(q(t),t) < 0$ for all $q(t) > 0$ and $t > 0$.

Let $\Theta(t) = e^{-\int_0^t \rho(\tau)d\tau}$ be the discount rate applied to income in time t. The monopolist chooses a time path of output, q(t), so as to maximize the present value of profit

(9) $\int_0^\infty \Pi(q(t),t)\Theta(t)dt$

subject to the constraints

(10) $\int_0^\infty q(t)dt \leqq Q$ and $q(t) \geqq 0$ for all $t \geqq 0$.

Since the function $\Pi(q(t),t)$ is strictly concave in q for all $q > 0$ and all $t \geqq 0$, and since the constraint function (10) is linear in q(t), it is easy to see that there can be no more than one solution to this constrained maximization problem. Standard "Kuhn-Tucker theory" applies to this problem.[3/] Therefore necessary and sufficient conditions for $\tilde{q}(t)$ to maximize (9) subject to (10) are that for some $\lambda \geqq 0$

(11) $\Pi'(\tilde{q}(t),t)\Theta(t) \leqq \lambda$ for all $t \geqq 0$ with = if $\tilde{q}(t) > 0$.

(12) $\int_0^\infty \tilde{q}(t)dt \leqq Q$ with = if $\lambda > 0$.

Equations (11) and (12) are those derived by Lewis (1976) and Sweeney (1977) except for the incorporation of taxes in the profit function.

It follows from the discussion above that a time path of subsidy rates, $s^*(t)$, will induce a monopolist to behave competitively if and only if, when tax rates are set at $s^*(t)$, the competitive path $q^*(t)$ satisfies equation (11). (Notice that since $q^*(t)$ satisfies (3), the conditions for (12) are automatically satisfied.) From equations (6) and (11) we see that $q^*(t)$ will satisfy (11) if and only if $s^*(t)$ is of the form:

$$(13) \qquad s_K^*(t) = \frac{K}{\Theta(t)} - R'(q^*(t),t)$$

for some constant K. We also see that when $s_K^*(t)$ is chosen, the Lagrangean, $\lambda$, in (11) is equal to K. Conversely, if (12) and (13) are satisfied, then (11) holds and since (11) and (12) are sufficient (as well as necessary) conditions for profit maximization we conclude that for any $K \geqq 0$ the subsidy path $s_K^*(t)$ defined by (13) induces a monopolist to supply the competitive path $q^*(t)$. A government that knows the competitive path, $q^*(t)$, the profit functions, $R(\cdot,\cdot)$, and the discount rates, $\Theta(t)$, could therefore use (13) to calculate an efficiency-inducing tax rate $s_K^*(t)$ corresponding to any choice of $K \geqq 0$. In fact, we have shown that the family of tax rate schedules so defined exhausts the possibilities for inducing efficiency by linear tax schedules.[4/]

If the subsidy scheme $s_K^*(t)$ is applied, then the present value of the total flow of income to the monopolist is:

$$(14) \qquad \int_0^\infty [R(q^*(t),t) + q^*(t)s_K^*(t)]\Theta(t)dt =$$

$$KQ + \int_0^\infty [R(q^*(t),t) - q^*(t)R'(q^*(t),t)]\Theta(t)dt$$

Since $R(\cdot,\cdot)$ is assumed to be a concave function of q for all t and strictly concave when $q > 0$, it must be that $R(q^*(t),t) - q^*(t)R'(q^*(t),t) \geqq 0$ for all t, with strict inequality when $q(t) > 0$. Therefore the integral expression on the right hand side of equation (14) is positive. It is then easy to see that for all $\lambda \geqq 0$, the monopolist achieves a positive present value of profit by producing $q^*(t)$.

The Lagrangean multiplier, $\lambda$, in (11) represents the "shadow price" or marginal contribution of an additional unit of the depletable resource to the present value of net returns to the monopolist (including taxes or subsidies). Since $\lambda = K$ when the subsidy path is $\overset{*}{s}_K(t)$, the quantity KQ measures the value of the existing stock of the resource evaluated at its marginal value to the monopolist. Thus one might find it useful to think of the expression or the right side of (14) for the present value of the monopolist's income as consisting of a part KQ that represents "rents" to its ownership of the resource stock and a part

$$\int_0^\infty [R(q^*(t),t) - q^*R'(q^*(t),t)]\Theta(T)dt$$

attributable to its monopoly position. By setting $K = 0$, the government can drive the after-tax rent accruing to ownership of the resource to zero, but there will remain a monopoly profit equal in present value to the integral expression on the right side of (14).

The net present value of subsidies paid by the government is the amount:

$$(15) \qquad E_K = \int_0^\infty q^*(t)\overset{*}{s}_K(t)\Theta(t)$$

$$= KQ - \int_0^\infty q^*(t)R'(q^*(t),t))\Theta(t)dt$$

The path that minimizes the present value of government outlay from among the efficiency-inducing family, is then seen to be the path corresponding to $K = 0$. Then the present value of all government payments and receipts is:

$$(16) \qquad E_0 = -\int_0^\infty q^*(t)R'(q^*(t),t)\Theta(t)dt.$$

In general, along the competitive path, industry marginal net revenue $R'(q^*(t),t)$ can be either positive or negative and $E_0$ could also be of either sign. If $E_0$ is positive, then for all K the subsidy path, $s_K^*(t)$, requires a net expenditure (in present value terms) by the government. If $E_0$ is negative there would always exist a unique $\overline{K} > 0$ such that

(17) $$E_{\overline{K}} = \overline{K}Q + E_0 = 0$$

and thus the net present value of government transfers is zero.

The Case of Stationary Demand and Costless Extraction

Qualitative analysis of the optimal paths is much simplified if we assume that demand is stationary over time and extraction is costless. We can then write the inverse demand function as

(18) $$p(t) = F(q(t)).$$

We assume that:

(19) $$F'(q(t)) < 0 \text{ whenever } q(t) > 0$$

and

(20) $$c(q(t),t) = 0 \text{ for all } q(t) \geqq 0 \text{ and all } t \geqq 0.$$

Where extraction is costless, equation (1) can be solved to give the conventional result that the price, $p^*(t)$, rises at the discount rate, $\rho(t)$. Thus we have:

(21) $$p^*(t)\theta(t) = p^*(0) \text{ for all t with } q(t) > 0.$$

Since the price, $p^*(t)$, is increasing monotonically over time, and since the demand curve and the marginal revenue curve are both assumed to slope downward, it follows that marginal revenue must increase over time as prices and quantities follow the efficient paths $p^*(t)$ and $q^*(t)$. But since $s_0^*(t)$ is just the negative of marginal revenue at output $q^*(t)$, it must then be that $s_0^*(t)$ decreases monotonically over time. Thus if $s_0^*(0) < 0$ so that the program starts out as a tax on extraction, then extraction will continue to be taxed at ever higher rates at all times in the future. If $s_0^*(0) > 0$ so that the program starts out as a subsidy for extraction, then the rate of subsidy will decrease over time and the subsidy may ultimately turn into a tax.

The expression (16) for $E_0$ can be simplified in an interesting way when demand is stationary and extraction is costless. Recall the well-known relation between price and marginal revenue:

$$(22) \qquad R'(q^*(t)) = p^*(t)(1 + \frac{1}{\xi^*(t)})$$

where $\xi^*(t)$ is the price elasticity of demand at time t. Using equations (3), (16), (21), and (22) one obtains:

$$(23) \qquad E_0 = -p^*(0)Q[1 + \int_o^\infty \frac{q^*(t)}{Q} \frac{1}{\xi^*(t)}].$$

Thus $E_0$ is positive or negative depending on whether the quantity weighted average inverse elasticity of demand is larger or smaller than one in absolute value.

As Stiglitz (1976) observed, if the elasticity of demand for a depletable resource is constant (and greater than unity in absolute value) then a monopolist will offer the same quantities and charge the same prices as a competitive industry would. Marshall (1920) argued that, ordinarily, demand curves can be expected to have the property that the absolute value of the price elasticity of demand is an increasing function of price. While this is not true of all demand curves consistent with demand theory, it is true for an interesting class of cases (including the case of linear demand). We say that demand functions with this property have "Marshallian elasticity of demand". Lewis (1976) shows that if the elasticity of demand is Marshallian at all prices, then a monopolist will extract the resource more slowly than is Pareto efficient. Lewis, Matthews and Burness (1979) demonstrate that non-Marshallian elasticity of demand is consistent with concave profit functions. They also show that during time-periods when the elasticity of demand is non-Marshallian, a monopolist will extract more rapidly than is efficient.

The effects of a subsidy on the time path of the monopoly extraction rate can be usefully viewed as follows. By altering the time path of extraction, the monopolist can alter the present value of the total amount of subsidy received in the process of extracting the total amount Q. Thus if the present value of the subsidy rate, $s(t)\Theta(t)$, is constant over time he will receive the same present value of subsidy no matter when he extracts. If $s(t)\Theta(t)$ is an increasing (decreasing) function of t then he will receive more subsidy the sooner (later) he extracts his reserves.

For this reason we want to look at the time path of $s_K^*(t)\Theta(t)$. Using equations (13), (21) and (22), we find:

$$(24) \qquad s_K^*(t)\Theta(t) = K - p^*(0)\left(1 + \frac{1}{\xi^*(t)}\right).$$

Therefore:

$$(25) \qquad \frac{d}{dt}\, s_K^*(t)\Theta(t) = p^*(0)\frac{\dot{\xi}^*(t)}{(\xi^*(t))^2}\,.$$

Thus we see that the present value of subsidy paid per unit of extraction increases over time if $\dot{\xi}^*(t) > 0$, decreases over time if $\dot{\xi}^*(t) < 0$ and is constant if $\dot{\xi}^*(t) = 0$.

Where the elasticity of demand is a constant, there is no need to use government intervention to bring about an efficient output stream so long as $\xi < -1$. This is not surprising since as we remarked above, in this case the monopoly solution is the same as the competitive solution. Of course a tax schedule,

$s_K^*(t)$, which in this case is neutral, could be used to raise revenue for the government.

In the case of Marshallian elasticity of demand, the fact that $p^*(t)$ increases over time implies that $\dot{\xi}^*(t) < 0$. Thus the present value of the subsidy rate, $s_K^*(t)\theta(t)$ must be falling over time. This will encourage the monopolist to extract earlier than he would have if there were no subsidy. This direction of effect is to be expected since it is in the case where $\dot{\xi}^*(t) < 0$ that the untaxed monopolist extracts too slowly.

In the non-Marshallian case, studied by Lewis et al, we have $\dot{\xi}^*(t) > 0$ so that the present value of the subsidy rate increases over time. Thus the monopolist is encouraged to produce more slowly than he would have if there were no subsidies or taxes. This again seems reasonable since in this case Lewis et al, show that the monopolist, if left alone, would extract too quickly. As before, the undiscounted tax rate rises over time (since the marginal revenue even in this case falls over time); however, the rate of increase of the tax rate is lower than the rate of interest.

Taxing Consumers Rather than Producers

The discussion in the preceding sections is conducted in terms of a subsidy (tax) paid to (by) the supplier. Those familiar with the simple analytics of commodity taxation in static analysis will not be surprised to learn that in our case, the incidence of such a tax is the same whether it is collected from the supplier or from the consumer. To demonstrate this formally we observe the following. If the monopolist receives $\hat{p}(t)$ per unit sold at time t and the consumer receives a subsidy (possibly negative) of $s(t)$ per unit, then the net price to consumers is $\hat{p}(t) - s(t)$ per unit. The inverse demand function $f(q(t),t)$ registers the net price to consumers at which the quantity $q(t)$ can be sold in period t. Thus $\hat{p}(t) - s(t) = f(q(t),t)$. Therefore $\hat{p}(t) = f(q(t),t) + s(t)$ and the profit of the firm is

$$(26)\qquad \Pi(q(t),t) = \hat{p}(t)q(t) - c(q(t),t) = s(t)q(t) + R(q(t),t)$$

where $R(q(t),t)$ is defined as in (4). We see from (26) and (5) that monopoly profits are precisely the same function of the output path, $q(t)$, whether the tax is collected from the monopoly or from the consumers. Therefore the monopolist chooses the same time path, $q(t)$, of output and receives the same profit in each period in either case.

This fact has an interesting practical implication if consumers live in a different tax jurisdiction from the monopoly and its resource. Even if the consuming country has no legal authority to tax the monopolist directly, it can achieve the same results by taxing or subsidizing consumers. Typically, of course, consumers of a depletable resource will be located in many different countries. Therefore a unified subsidy-tax policy is probably not to be reasonably expected. In another paper, Bergstrom (1980) treats a model with non-cooperative behavior by consuming countries each of which has taxing authority over its own citizens.

## Appendix

### An Example - Linear Demand Functions

Suppose that we retain our assumptions that the demand function is stationary and extraction is costless, and add the restrictions that demand is linear and the interest rate is constant. With an appropriate choice of units of measurement for the depletable resource and for money, we can write the demand function as:

A.1. $$q(t) = \max\ [0, 1-p(t)] \text{ for } p(t) \geqq 0.$$

The competitive solution is as follows. With a constant discount rate, the arbitrage equation is:

A.2. $$p^*(t) = p^*_o e^{\rho t}$$

where $p^*_o$ is the v-lue of $p^*(0)$. It follows that:

A.3. $$q^*(t) = \max\ [0, 1-p^*_o e^{\rho t}]$$

Let T be the earliest date at which $q^*(t) = 0$. Then

A.4. $$p^*_o e^{\rho T} = 1 \quad \text{and}$$

A.5. $$T = -\frac{1}{\rho} \ln p^*_o$$

Since the total stock of resources must be just exhausted at time $T^*$, it follows that:

A.6. $$\int_0^T q^*(t)dt = Q$$

Substituting from A.3. - A.6. and integrating, we obtain

A.7. $$p^*_o - \ln p^*_o - 1 = \rho Q$$

The left side of A.7. is a monotone decreasing function of $p_0$ with domain (0,1] and range $(\infty,0]$. Therefore the solution, $p_0^*$, is uniquely determined by the product, $\rho Q \geqq 0$.

In the case of linear demand, any efficiency inducing tax subsidy scheme must take the form:

A.8. $$s_K^*(t) = Ke^{\rho t} - [2p^*(t) - 1] = (K - 2)p_0^* e^{\rho t} + 1$$

The present value of the total subsidy paid to the monopolist is:

A.9. $$E_K = KQ - \int_0^T [1 - p^*(t)][2p^*(t) - 1]e^{-\rho t}dt.$$

Performing the integration, and using A.4. and A.7.,

A.10. $$E_K = KQ + \frac{1}{\rho}[1 + p_0^{*2} + 3p_0^* \ln p_0^*].$$

Whether the tax/subsidy results in positive or negative net revenue to the government depends upon the value of the bracketed expression in A.13 and the value chosen for K. If the bracketed expression is positive, for example, then even with $K = 0$, the government is forced into a position of providing a net subsidy to the industry in present value terms. Some elementary calculus and computations with a pocket calculator show that the bracketed expression is positive if $p_0^* < .355$, (which occurs when $\rho Q > .391$) and negative if $p_0^* > .355$ and $\rho Q < .391$. Thus it is possible to accomplish efficiency by means of a tax-subsidy scheme at no net cost to the government only if Q is not "too large".

## Footnotes

1/ The behavior of a monopolistic supplier is thoroughly analyzed in Lewis (1976) and Sweeney (1977).

2/ This problem was first posed, as far as we know, by Simmons (1977). Although his paper contains a number of interesting remarks on the subject, he was not able to show the essential simplicity of the problem. In fact he was somehow led to the assertion that an optimal tax schedule would "involve initial taxation but with subsidization in later periods". We show that generally the reverse must be true.

3/ For a clear account of the applicability of Kuhn-Tucker theory in infinite dimensional spaces, see Hurwicz (1964).

4/ Of course there are other incentive schemes, including taxes which are not simply proportional to output, which also would induce competitive behavior.

## References


T. Bergstrom, On Capturing Oil Rents with a National Excise Tax. CREST discussion paper, C-20, Economics Department, University of Michigan, January 1980.

L. Hurwicz, Programming in Linear Spaces in Studies in Linear and Non-Linear Programming, K. Arrow, L. Hurwicz, H. Uzawa, eds., Stanford, 1964.

T. Lewis: "Monopoly Exploitation of an Exhaustible Resource", Journal of Environmental Economics and Management 3 (1976) pp. 198-201.

T. Lewis, S. Matthews and H. Burness, "Monopoly and the Rate of Extraction of Resources: Comment," AER, March 1979.

A. Marshall, Principles of Economics, eighth edition, MacMillan, London, 1920.

P. Simmons, "Optimal Taxation and Natural Resources", Rescherches Economiques de Louvain, June 1977.

J. Stiglitz, "Monopoly and the Rate of Extraction of Exhaustible Resources", AER, Sept. 1976.

J.L. Sweeney, "Economics of Depletable Resources: Market Forces and Intertemporal Bias", Rev. of Economic Studies, Feb. 1977, pp. 125-142.

## CREST Working Papers

C-1 The Ergodic Behavior of Stochastic Processes of Economic Equilibrium, by Lawrence E. Blume.

C-2 The Token Economy, Reinforcement and the Consumer Model, by John G. Cross.

C-3 Notes on a Theory of Learning to Search, by John G. Cross.

C-4 Redistributive Taxation as Social Insurance, by Hal R. Varian.

C-5 Catastrophe Theory and the Business Cycle, by Hal R. Varian.

C-6 Household Bequests, Perfect Expectations, and the National Distribution of Wealth, by John P. Laitner.

C-7 Inflationary Disequilibrium, by John G. Cross.

C-8 Ellet's Transportation Model of an Economy with Differentiated Commodities and Consumers, I: Generic Cumulative Demand Functions, by Carl P. Simon.

C-9 Cournot Equilibrium in Factor Markets, by Ted Bergstrom.

C-10 Is a Man's Life Worth More Than His Human Capital, by Ted Bergstrom.

C-11 A Model of Sales, by Hal R. Varian.

C-12 New Techniques for the Study of Stochastic Equilibrium Processes, by Lawrence E. Blume.

C-13 Consistent Expectations, by Lawrence E. Blume.

C-14 On Baubles and Bubbles, by John G. Cross.

C-15 When Does Majority Rule Supply Public Goods Efficiently?, by Ted Bergstrom.

C-16 "Rational" Duopoly Equilibria, by John P. Laitner.

C-17 The Natural Rate of Unemployment, by John Laitner.

C-18 Learning to be Rational, by Lawrence E. Blume and David Easley.

C-19 Notes on Cost-Benefit Analysis, by Hal R. Varian.

C-20 On Capturing Oil Rents With a National Excise Tax, by Ted Bergstrom.

C-21 Cournot, Novshek and the Many Firms, by Ted Bergstrom.

C-22 The Nonparametric Approach to Demand Analysis, Hal R. Varian.

C-23

C-24 The Taking of Land: When Should Compensation be Paid?, by Lawrence Blume, Daniel L. Rubinfeld and Perry Shapiro.

C-25 Efficiency-Inducing Taxation for a Monopolistically Supplied Depletable Resource, by Theodore C. Bergstrom, John G. Cross and Richard C. Porter.