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Markets in Twentieth Century Sweden

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Large, short-run fluctuations in the birth rate have been an important demographic feature of industrialized, low-fertility populations in the twentieth century. Since females normally marry men who are two or three years older than themselves, these fluctuations result in large imbalances between the size of male and female cohorts who would normally marry each other. These imbalances must somehow be resolved, either by a change in traditional patterns of age at marriage or by changes in the proportions of the population of one sex or the other who ever marry.

Following a suggestion of Becker (1973,1974,1981), we have developed a developed an implementable general equilibrium model of marriage assignments, which can be used to predict the way in which marriage patterns adjust to change in the numbers of males and females in each cohort. This model poses equilibrium in the marriage market as an application of the *linear programming assignment problem*, which was introduced to economics by Koopmans and Beckman (1957). For the purposes of this paper, we suppose that persons of the same sex differ only by the year in which they were born. Each individual has a preferred age of marriage. Any two people who marry each other must, of course, marry at the same time. Therefore, the total payoff to a marriage between any male and female is a function of the age difference between them. The more their age difference diverges from the difference between their preferred ages at marriage, the greater must be the loss of utility to one or both from marrying at an age that is not ideal. If we posit a particular payoff structure to marriages as a function of the age of marriage of each partner, then given the size of each cohort, we can compute the optimal assignment of marriage partners by cohort. The fit of the predicted assignments from our model can then be compared with actual marriage patterns.

Swedish marriage patterns during the twentieth century are ideal for testing our assignment model. Excellent data exist on marriages and cohort size for single years of age and single calendar years going back to the 1890's. Dramatic fluctuations in fertility in Sweden have created large differences between the size of male and female cohorts who would normally marry each other. Nevertheless, the median age difference between husbands and wives has remained in the range of two or three years. We hope to use our assignment model to shed some light on the way that the marriage market absorbed these cohort size changes. As we will show, the simple form of the assignment model that we present in this paper is partially, but not entirely successful in this endeavor.

The first section of the paper presents descriptive statistics of marriage patterns in twentieth century Sweden. We follow this with an outline of our equilibrium theory of marriage assignment. Finally, we compare actual marriage patterns with those predicted by a simple implementation of our model which assumes that the difference between preferred age of marriage for the two sexes did not change over the entire period.

Cohort Size and Age-at-Marriage in Twentieth Century Sweden

Sweden experienced sharp fluctuations in fertility during the twentieth century. This is demonstrated in Figure 1, which shows Swedish cohort size fluctuations for the years 1883 to 1942. The figure plots the size of female birth cohorts in Sweden, evaluated when the females are age 15. As the figure demonstrates, Sweden experienced a sharp peak in fertility in 1920 and a major trough in the early 1930's. This decline was followed by a rapid increase into the 1940's.

The large changes in cohort size in Sweden over this period cause dramatic imbalances in the sex ratios typically used to analyze marriage squeezes. Figure 2 shows the sex ratio that we will use throughout the paper, the number of potential husbands available to women born in a given year if every woman married a man three years older. Specifically, the sex ratio shown is the ratio of the number of males born in year $t - 3$ (measured at age 15) to the number of females born in year t (also measured at age 15).¹ The figure shows, for example, that if every woman married a man three years older, the 1923 cohort of women would have had more than 1.25 potential husbands for every woman, while the 1942 cohort would have had less than .9 potential husbands for every woman.

Figure 3 and Figure 4 demonstrate secular trends in Swedish marriage patterns, trends which are similar to those of many other European populations the twentieth century. Figure 3 shows the mean age at marriage for male and female birth cohorts, conditional on marriage by age 40.² The mean age at marriage fell sharply for both men and women, declining by around three years for both sexes between the 1900 cohort and the 1940 cohort. In spite of the large changes in the mean age at marriage for both sexes, the difference between these ages remained in the range of two to three years through the entire period. This persistent difference in mean age at marriage must imply that husbands are on average older than their wives by two to three years of age throughout this period. The consistency of the gap between the mean age at marriage for males and the mean age at marriage for women is especially striking when confronted with the sharp fluctuations in sex ratios shown in Figure 2.

Figure 4 shows the cumulative proportions of men and women in each cohort who married before age 40. Over time, these proportions increase sharply for both sexes, with a larger increase

¹ For example, the point plotted for year of birth 1920 is the size of the 1917 male birth cohort at age 15 (i.e. the number of 15 year old males in 1932) divided by the size of the 1920 female birth cohort at age 15 (i.e. the number of 15 year old females in 1935). If all females married men three years older, the graph shows the ratio of eligible females to eligible males for each female birth cohort.

² We truncate the cohort marriage histories at age 40 for calculation of both the mean age at marriage and the proportions marrying in the results presented here. We use age 40 in order to maximize the number of cohorts with comparable histories while still including the greatest part of each cohort's marriage experience. We have compared these results with results using age 45 and 50 as the truncation for those cohorts which permit such comparisons. None of the general patterns we indicate here are sensitive to the use of age 40 as the truncation point.

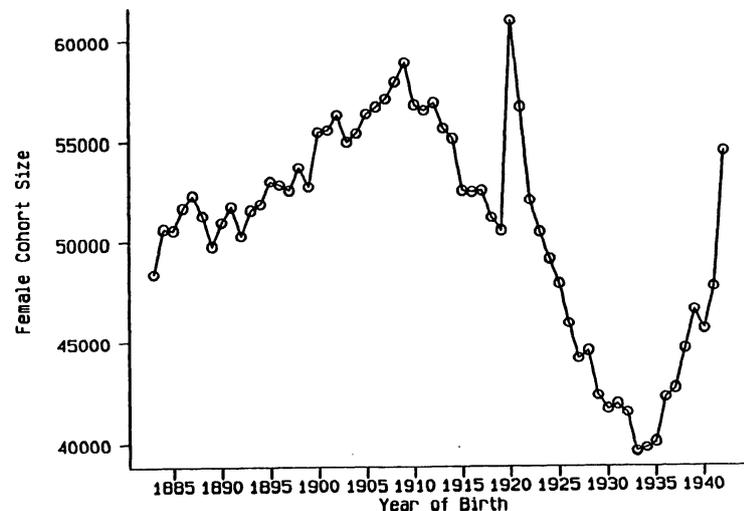


Figure 1. Cohort size fluctuations in Sweden, 1883-1942. Size of female cohort born in year t , evaluated at age 15.

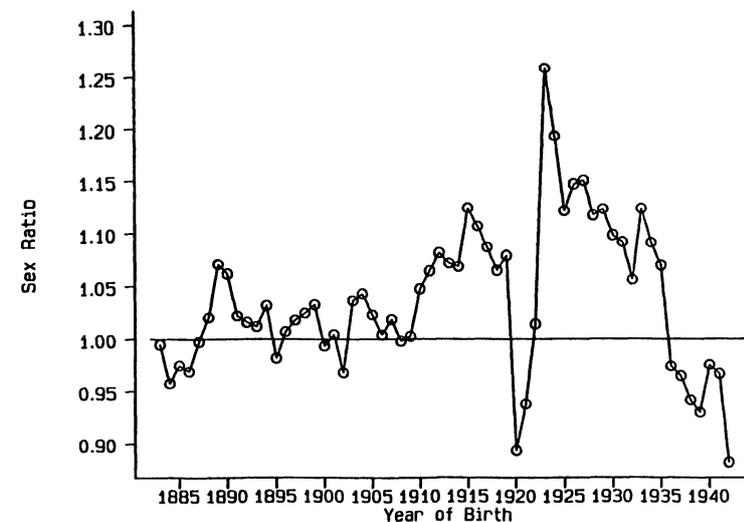


Figure 2. Male/female sex ratios in Sweden, 1883-1942. Number of males born in year $t - 3$ divided by number of females born in year t ; both sexes evaluated at age 15.

for women than for men. The proportion of women married by age 40 (a number only slightly smaller than the proportion of women who eventually marry) rose from slightly over 60% before 1900 to over 95% in the late 1920's. After 1930, there is a decline in the proportion of females who marry, while the proportions of males who marry stays roughly constant.

The fluctuations of cohort size and of sex ratios documented in Figure 1 and Figure 2 could be resolved in the marriage market in two possible ways. One response is that the fluctuations lead to changes in the proportions of males or females marrying for each cohort. For example, one simple adjustment would be that the number of females who marry is determined entirely by the supply of potential husbands, with the proportion of females marrying absorbing all fluctuations in sex ratios. A second way of resolving these imbalances is through changes in the age difference between husbands and wives.

The following analogy may help one to see how the adjustment works. Suppose that men and women are entering a ballroom through separate doors, and we have the task of assigning partners to those that enter. If the males and females file into the room at the same speed then we can simply pair up each man with the woman that enters at the same time. But if the men begin to flow into the room faster than the women, this won't work. We have two options. One is to let the number of matches be determined by the slower flow of entering women. "Surplus" males might be turned away (or put in storage near the refrigerator). Alternatively, we might try to reach back into the line of women to find partners for every man that enters. Although this could not work as a permanent solution if the men continued to enter faster than the women, it could be a successful strategy if the speeds of the two lines will eventually adjust so that the women enter faster than the men. Thus we might succeed in assigning a partner to each man and woman in spite of the short-run problem caused by the different speeds at which the two groups enter the room.

When husbands tend to be older than their wives, fluctuating cohort sizes create exactly this kind of problem. If cohort sizes are relatively constant then men and women can form marriages with a two or three year age difference between spouses without difficulty. A temporary decrease in fertility, however, will be analogous to women beginning to enter the ballroom at a slower speed, since younger cohorts will be smaller than their predecessors. We would predict then, that the marriage market would have to respond with some combination of a decrease in the proportion of males marrying and an increase in the age difference between spouses. The increase in the age difference corresponds to reaching further down the line of women to find matches for all of the men in the ballroom analogy above. An increase in fertility would have the opposite effect, leading to increases in the relative proportion of males from previous cohorts marrying and a decrease in the age difference between spouses.

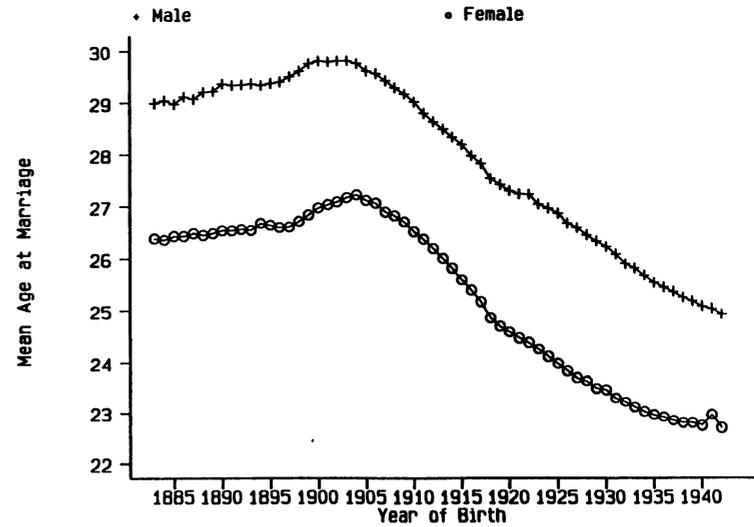


Figure 3. Mean age at marriage for males and females, Sweden, 1883-1942, by year of birth, conditional on marriage by age 40.

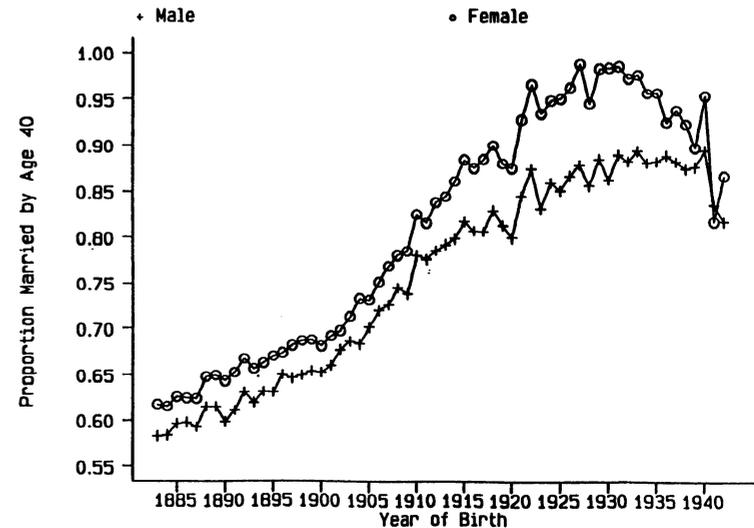


Figure 4. Proportions marrying by age 40, males and females, Sweden, 1883-1942, by year of birth; conditional on marriage by age 40.

Swedish marriage data make it possible to analyze adjustments in both the age difference between spouses and in proportions marrying in response to cohort size fluctuations. Figure 5 presents evidence on the potential contribution of changes in the age difference between spouses in absorbing the large fluctuations in cohort size in Sweden over the period 1883–1942. Published data do not exist giving cross-tabulations of age of husband by age of wife in single years of age. An approximate time series for the age difference between husbands and wives can be constructed by comparing the mean age of marriage for women with the mean age of marriage for men born three years earlier, roughly corresponding to the average cohort these women marry into. Figure 5 presents a time series of the difference between the mean age at marriage of female cohorts and the mean age at marriage of males from the cohort three years older. Figure 5 also shows the sex ratio (the ratio of males born in year $t - 3$ to females born in year t) for comparison.

Figure 6 presents a series that compares the proportion of females married by age 40 in each cohort with the proportion of males married by age 40 in the cohort born three years earlier. Specifically, the graph shows the ratio of the proportion of women married by age 40 in the to the proportion of men married by age 40 in the cohort born three years earlier. The figure also shows the sex ratio (the ratio of males born in year $t - 3$ to females born in year t) for comparison.

Figure 5 and Figure 6 indicate that both husband-wife age differences and relative proportions marrying move in directions consistent with the adjustments implied by long term and short term changes in sex ratios. The male-female sex ratio begins to increase above unity beginning around 1900. At the same time, the age gap between spouses begins to rise and the relative proportion of males marrying begins to decrease. The effects of short term fluctuations can be seen in the experience of the unusually large 1920 cohort. The reduced male-female sex ratios caused by the fertility boom around 1920 are associated with a decline in the age gap between spouses and a decrease in the relative proportion of females marrying, in comparison to adjacent cohorts. The decrease in the male/female sex ratio beginning in the mid 1920's, a result of the slowdown and eventual reversal of the major fertility decline of previous decades, also leads to a decline in the age difference between spouses and a decrease in the relative proportion of females marrying.

In order to look more closely at the relationship between the sex ratio and the age difference, we present estimates of a regression using the time series of the age difference between spouses and the male-female sex ratio. Table 1 shows the results of the regression for the set of cohorts born from 1883 to 1942, the cohorts for which our data provide complete marriage histories up to age 40. The results show a statistically significant effect in the predicted direction. Higher sex ratios (i.e. ratios of males born in year $t - 3$ to females born in year t) lead to *increases* in the age

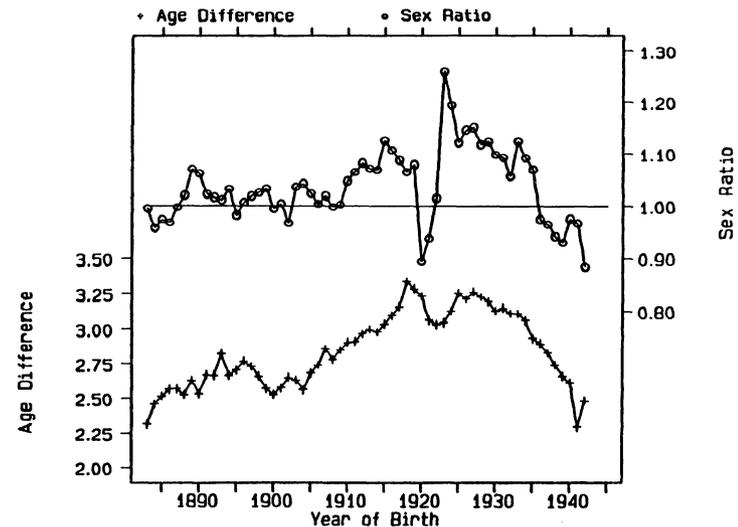


Figure 5. Sex ratio and husband-wife age differences, Sweden, 1883–1942. Sex Ratio: Number of males born in year $t - 3$ divided by number of females born in year t . Age Difference: Mean age at marriage of men born in year $t - 3$ minus mean age at marriage of women born in year t .

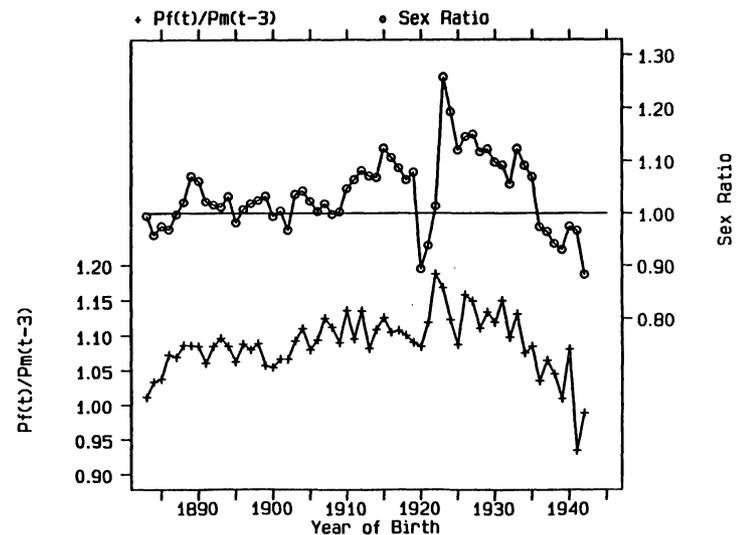


Figure 6. Sex ratio and relative proportions of males and females marrying, Sweden, 1883–1942. Sex Ratio: Number of males born in year $t - 3$ divided by number of females born in year t . Relative Proportions Marrying: Proportion married by age 40 of women born in year t divided by proportion married by age 40 of men born in year $t - 3$.

difference between husbands and wives for that pair of cohorts.³ Specifically, the results imply that an increase in the sex ratio by .1 will raise the age difference between spouses by .23 years. The predicted mean age difference between husbands and wives when the sex ratio is 1 is about 2.75 years. The R^2 for the regression indicates that fluctuations in sex ratios alone can explain 35% of the variance in the age difference between spouses over this 60 year period.

The relationship between the sex ratio and the age difference between spouses can be seen more directly in Figure 7, which shows a scatterplot of the age difference plotted against the sex ratio. The figure also shows the ordinary least squares regression line for the regression of the age difference on the sex ratio shown in Table 1. The plot shows a clear relationship in the predicted direction, with increases in the relative supply of potential husbands leading to increases in the age gap between husbands and wives, a result consistent with the demographic constraint on marriage market equilibrium.

A similar regression demonstrates that proportions marrying also respond to changes in the sex ratio in Sweden during this period. Table 2 presents estimates of an OLS regression in which the dependent variable is the ratio of the proportion of males married in the cohort born in year $t - 3$ to the proportion of females married in the cohort born in year t . This is the inverse of the ratio shown in Figure 6. Once again, the results show a statistically significant effect in the predicted direction. In this case a higher sex ratio (males born in year $t - 3$ to females born in year t) leads to a decrease in the relative proportion of males marrying for that pair of cohorts. The R^2 for the regression indicates that fluctuations in sex ratios alone can explain 43% of the variance in the relative proportions of males and females marrying over the period 1883–1942.

Figure 8 plots the ratio of proportions marrying and the predicted proportions implied by the regression in Table 2. The results imply that at a sex ratio of unity only 93.2% as many men marry as women. This is partly a result of the fact that only marriage histories up to age 40 are used, although the discrepancy between proportions of males and females marrying persists when marriage histories up until age 50 are used. The regression implies that an increase in the sex ratio from 1 to 1.1 would decrease the relative proportion of men marrying from .932 to .889. If all of the 10% “surplus” of men were to go unmarried, then the relative proportion of men marrying would decline to $.9 \times .932 = .838$.⁴ The change in the proportion marrying, then, accounts for something

³ More precisely, it leads to increases in the difference between the mean age at marriage of men born in year $t - 3$ and the mean age at marriage of women born in year t , our proxy for the mean age difference between husbands and wives for women born in year t .

⁴ The ratio shown is the ratio $p = \frac{M_m/N_m}{M_f/N_f}$, where M_i is the number married from sex i and N_i is the total number eligible for sex i . If we increase N_m by 10% and leave all other values constant, p will decrease to $p/1.1 \approx .9p$.

Table 1
Ordinary Least Squares Regression
Age Difference Between Husband and Wife
By Male-Female Sex Ratio
Sweden, 1883–1942

Variable	Coefficient	Standard Error	T Statistic	Mean	Standard Deviation
Age Difference (<i>Dependent Variable</i>)				2.842	0.268
Sex Ratio	2.2608	0.4036	5.601	1.037	0.070
Constant	0.4979	0.4194	1.187		
R^2	.3510				
N	60				

Notes: Dependent variable is the difference between the mean age at marriage of males born in year $t - 3$ and the mean age marriage of females born in year t . The sex ratio is the ratio of the number of males born in year $t - 3$ (evaluated at age 15) to the number of women born in year t (evaluated at age 15).

Table 2
Ordinary Least Squares Regression
Ratio of Proportion of Males and Females Marrying
By Male-Female Sex Ratio
Sweden, 1883–1942

Variable	Coefficient	Standard Error	T Statistic	Mean	Standard Deviation
$P_m(t - 3)/P_f(t)$ (<i>Dependent Variable</i>)				.920	0.038
Sex Ratio	-.3458	0.0536	-6.447	1.037	0.070
Constant	1.2786	0.0557	22.944		
R^2	.4262				
N	60				

Notes: Dependent variable is the ratio of the proportion married by age 40 for males born in year $t - 3$ to the proportion married by age 40 for females born in year t . The sex ratio is the ratio of the number of males born in year $t - 3$ (evaluated at age 15) to the number of women born in year t (evaluated at age 15).

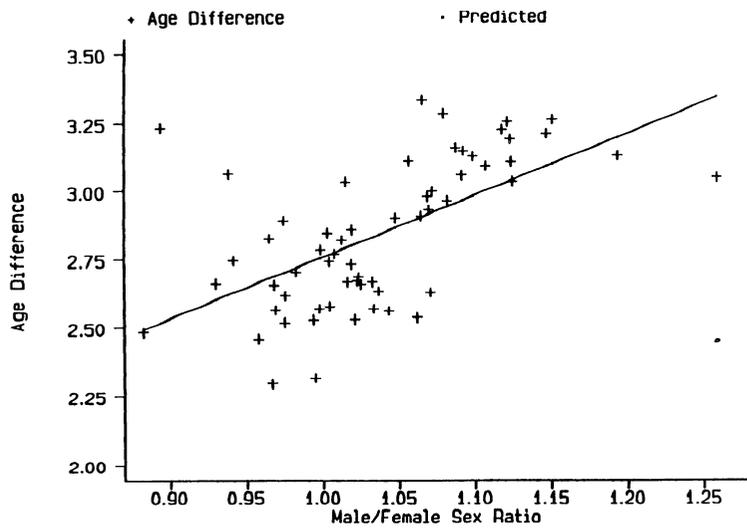


Figure 7. Scatter plot, sex ratio and husband-wife age differences, Sweden, 1883-1942. Mean age at marriage of men born in year $t - 3$ minus mean age at marriage of women born in year t , and least squares regression line.

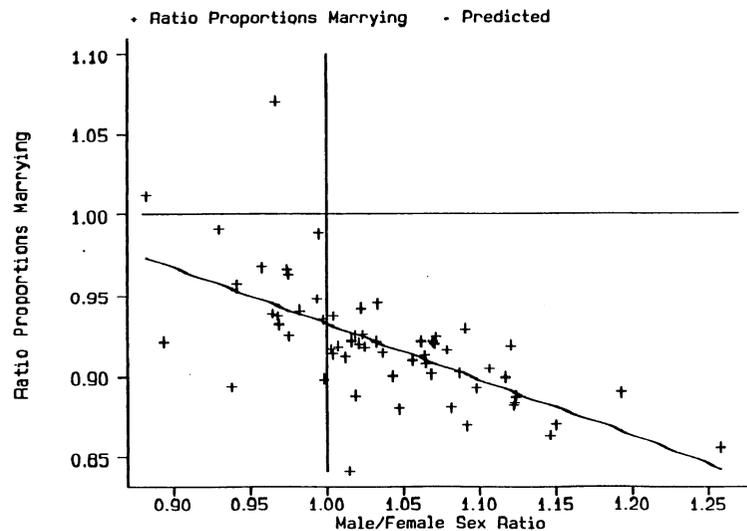


Figure 8. Scatter plot, sex ratio and relative proportions of males and females marrying, Sweden, 1883-1942. Proportion married by age 40 of men born in year $t - 3$ divided by proportion married by age 40 of women born in year t , and least squares regression line.

less than half of the response to the fluctuating sex ratio, with adjustments in the age difference between spouses working to absorb the remainder.

Marriage Markets and The Linear Programming Assignment Algorithm

We believe that the descriptive data presented in the previous section can be better understood as the equilibrium of a marriage market in which marriage behavior must respond to the supply and demand for each sex and each cohort. In such a model, we would expect that persons of the same sex but different ages would be substitutes, *albeit* imperfect substitutes, with persons being closer substitutes the closer their ages. In contrast to our equilibrium model, earlier, "one-sex" models of marriage would find historical averages of age-specific marriage rates for one sex, (usually female) and apply these rates to later age distributions in order to project the number of marriages that will take place in subsequent periods. As should be evident from the descriptive statistics that we have just presented, a severe weakness of this approach is that a male one-sex model would predict very different numbers of marriages in any period than a female one-sex model.⁵

Pollak (1986, 1987) has reformulated the "two-sex problem" by replacing the constant age-specific fertility schedule of the classical theory with more fundamental relationships. These are a "birth matrix" and a "mating rule." The birth matrix postulates an expected number of births per period from a marriage of an age i male to an age j female. The mating rule is a function that determines the number of marriages of type i males to type j females for all i and j as a function of the vector listing the numbers of males and females of each age and sex in the population. Pollak shows that if these relationships remain constant over time and if the mating rule follows certain natural conditions, the dynamical system so defined will converge to a constant equilibrium growth rate, yielding a constant equilibrium age structure. Pollak imposes only certain very general conditions on the mating function such as nonnegativity, homogeneity, continuity, and that the number of persons of a given sex who marry must not exceed the number of persons of that age and sex in the population.

Pollak's mating rule can be thought of as a "reduced form" description of the dependence on the outcome of a marriage market on supplies and demands of the two sexes from various cohorts. Our model looks behind this reduced form by posing an explicit structure of payoffs to the possible patterns of mating. We can then analyze marriage market equilibrium under this structure. In particular, we take advantage of an idea proposed by Becker (1981), who suggested that the problem of finding an efficient and stable assignment of marriage partners can be usefully viewed as an application of the linear programming assignment problem.

⁵ One-sex models, and their weaknesses are discussed in detail by Keyfitz (1971), McFarland (1972), Mahsam (1974), Das Gupta (1974), Schoen (1981), Sanderson (1983), Pollak (1986, 1987) and Caswell and Weeks (1986.)

The assignment problem was originally devised as a model for the efficient assignment of workers to jobs. For each worker, i , in job, j , there is a money value of output a_{ij} which could be produced if worker i is assigned job j . The assignment problem finds the assignment of workers to jobs that maximizes the total value of output subject to the constraint that each worker has only one job and each job is done by only one worker. The solution to the assignment problem not only reveals an optimal assignment, but it also imputes “shadow prices” to each worker and job in such a way that if each worker were paid his shadow price as a wage and each job received its shadow price as a rent, the optimal assignment would be a competitive equilibrium.

It would be reasonable to apply the assignment problem to the case of marital sorting if the “value” of a marriage could be measured by a single number like money income. On the face of it, this seems an outrageous simplification of what marriage is about. In a marriage there are many joint decisions to be made about many matters that are far removed from money. There also may be substantial differences in tastes, in skills, and in initial wealth between potential marriage partners. As it turns out, we have been able to show that the complexity of interaction in a marriage can be quite well modeled by the presence of a large number of shared public goods in the marriage. For a broad and interesting class of preferences over public and private goods, it happens that there is “transferable utility”. This means that although many complex joint decisions must be reached about household public goods, the efficient amount of public goods in a marriage is determined independently of the distribution of private goods within the marriage. When this is true, the assignment problem model of marriage as proposed by Becker can be applied directly.⁶

We present here a simple model of the marriage market that is empirically implementable and yet rich enough to capture much of the character of the marriage squeeze. Suppose that an individual’s utility depends only on his or her age at marriage and on consumption of private goods. Of course, any two people who choose to marry each other must marry on the same day. This effect can be nicely modeled by treating the date of the wedding as a local public good, entering into both of the potential partners’ utility functions. Each person’s wedding date enters their utility function because this date determines their age of marriage.

Let \bar{A}_i be person i ’s preferred age at marriage. To enable us to empirically fit our model, suppose that individual i has a quadratic loss function for marrying at a less than ideal age. In particular, let C_i be person i ’s consumption of private goods, let W_i be the year of i ’s marriage and B_i the year of person i ’s birth. Then $A_i = W_i - B_i$ is i ’s age at marriage. Let \bar{A}_i be i ’s most

preferred age of marriage. Then let person i ’s utility be given by

$$U_i(W_i, B_i, C_i) = C_i - [(W_i - B_i) - \bar{A}_i]^2.$$

Suppose now that male i and female j are contemplating marriage. Their utility functions belong to the class of utility functions for which the optimal choice of public goods is independent of how the private goods are distributed in the marriage. As demonstrated in Bergstrom and Lam (1989), the implication of these special functions is that the optimal time for i and j to marry if they do marry is exactly half way between the favorite date of i and the favorite date of j .

Consider, for example, a male i born in 1924 and a female j born in 1926. Suppose that i ’s preferred age at marriage is 25 and j ’s preferred age at marriage was 21. Then i would prefer to marry in 1949 and j would prefer to marry in 1947. The efficient time for this couple to marry if they do marry would be in 1948. In this case, each person would be missing his or her favorite age of marriage by one year. Because of the special utility function that we have assumed, the model displays transferable utility. That is, for any two people i and j of opposite sexes, there is a number A_{ij} which is the total utility generated by an optimally timed marriage between i and j . Persons i and j can divide this utility in any way that adds up to A_{ij} by redistributing private goods between them.

Suppose that all males preferred to marry at age \bar{A}_m and that all females preferred to marry at age \bar{A}_f . Then, given the assumption of a quadratic loss function, we can find the payoff matrix which reports the value A_{ij} of a marriage between a cohort i male and a cohort j female. This is the structure we will impose in order to apply our model to Sweden. Since we know from our historical series the number of persons of each cohort and sex, once we impose a payoff matrix we can solve a linear program to assign the cohorts to each other in an optimal way.

Comparing Predicted to Actual Swedish Marriage Patterns

The equilibrium model that we will compare with the Swedish historical data is one of extreme simplicity. We assume that individuals do not care *who* they marry, but only care about *when* they marry. While we will allow the possibility that the preferred age of marriage for each sex has changed over time, we will assume that the difference between the preferred marriage age of males and the preferred marriage age of females has been constant throughout the entire period and is equal to 3 years. Specifically, we assume that matrix of payoffs to possible pairings of male and female cohorts is that shown in Figure 9. The zeros along the third super-diagonal indicate that marriages of males from a given cohort to females from the cohort born three years later are the “best” marriages. The negative numbers indicate a utility loss compared to the ideal marriage

⁶ A detailed description of how this works out can be found in Bergstrom and Cornes (1981,1983). For an application to the issue of assortative mating see Lam (1988).

for all other marriages, with the penalty increasing quadratically away from the diagonal of ideal marriages.

Given this payoff matrix, we will use a linear programming algorithm to solve for the assignment of partners that maximizes the total payoff from marriages in the population of men and women born in Sweden between 1895 and 1945. More precisely, these cohorts were measured by the number of persons of each cohort and sex who reached age 15 in Sweden between 1910 and 1960. In this way we avoid statistical problems caused by sex differences in infant mortality. This procedure also allows us to consider only persons who were in Sweden after 1910, at which time emigration from Sweden had essentially ceased. Given this payoff matrix, we calculate the optimal assignment of partners, assuming that all males and females who reach the age of 15 are assigned marriage partners. One difficulty that has to be resolved is what to do about people born between 1895 and 1945 who married partners who were born outside of this interval. We chose to follow the simple procedure of assuming that all women born in 1895, 1896, and 1897 married men who were born three years earlier than they and to assume that all men born in 1943, 1944, and 1945 married women who were three years older than themselves. Therefore the population of people actually matched to each other by our assignment algorithm are the set of men born in the years from 1895 to 1942 and the set of women born in the years from 1898 to 1945.

The solution to our assignment problem is a 50 by 50 matrix whose ij th element is the number of males born in year i who marry females born in year j . In fact this assignment matrix is very sparse, with positive numbers of marriages concentrated close to the third super-diagonal, the diagonal corresponding to "ideal" marriages. Actual demographic data on the distribution of age differences between marriage partners will be much more dispersed than our predicted optimal assignments. The lack of dispersion predicted by our model comes from the fact that we have assumed that persons of the same age are all identical. If we were to allow a distribution of maturity levels among people of the same chronological level, we would obtain more realistic predictions on this account. But even our very simple model will make interesting predictions about the time path of the mean age difference between husbands and wives. For a given cohort of men, for example, we calculate the mean year of birth of the wives assigned to that cohort by our linear program. Using this to calculate the mean age difference between spouses for each cohort, we can compare our results to actual Swedish experience.

Figure 10 shows the mean age difference for husbands and wives for every female birth cohort resulting from our linear program. The line marked with circles corresponds to our assignments. The line marked with triangles reproduces the actual data for Sweden for the same years, based on the series shown above in Figure 5. The graph has both good news and bad news regarding the ability of our simulated assignments to capture the actual history of age differences between spouses

		Female Birth Cohort j								
		...	1920	1921	1922	1923	1924	1925	1926	...
Male Birth Cohort i	1920	...	-9	-4	-1	0	-1	-4	-9	...
	1921	...	-16	-9	-4	-1	0	-1	-4	...
	1922	...	-25	-16	-9	-4	-1	0	-1	...
	1923	...	-36	-25	-16	-9	-4	-1	0	...
	1924	...	-49	-36	-25	-16	-9	-4	-1	...
	1925	...	-64	-49	-36	-25	-16	-9	-4	...
	1926	...	-81	-64	-49	-36	-25	-16	-9	...

Figure 9. Payoff matrix for Swedish marriage assignments. Total utility from a marriage between male born in year i and female born in year j given quadratic loss function and three year difference in ideal age at marriage for males and females.

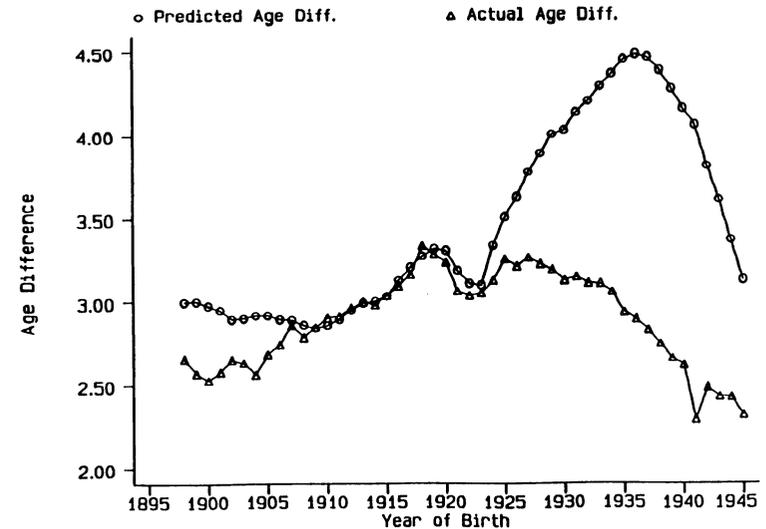


Figure 10. Husband-wife age differences, Sweden, 1895-1945. Actual differences and differences predicted by linear program assignment algorithm.

in Sweden. For the period 1905 to 1923 our assignments track actual experience remarkably well. The most dramatic result is the similarity in the way our assignments deal with the extreme sex ratio fluctuations between 1915 and 1925. Since these are the largest short-run fluctuations in the age difference we actually observe in Sweden, our ability to replicate them is encouraging.

As can be seen in Figure 10, our assignments begin to diverge from actual Swedish experience, starting with the cohorts born in the late 1920's. These are the people whose marriages took place in the 1950's and later. Our assignments predict that the age difference between husbands and wives would increase during this period, when in fact the actual age difference declined quite steadily until the end of our period of observation.

Evidently our model is not capturing something that happened during the last part of the period. We can think of at least two important effects that we have left out. One omission is that we fitted our optimal assignments based on the assumption that all members of both sexes choose to marry. This would make no difference to the accuracy of our predictions if the proportions of each sex who marry were constant throughout the period. As Figure 8 indicates, the proportion of members of a given sex and cohort who marry is inversely related to the abundance of its members relative to size of the sex-cohort group from which it would normally select marriage partners. These effects would tend to moderate the fluctuations in the age gap between marriage partners that are induced by changes in the relative cohort sizes.

The most suspect of our assumptions is probably the assumption that the optimal age differential for couples remained constant over the entire period. As Figure 3 shows, there was a steady secular decline in the mean age of marriage for the cohorts born after 1905, with a slightly faster decline for males than for females. If the preferred age of marriage for females fell at exactly the same rate as the preferred age of marriage for males, then there would be nothing wrong with our assumption that the optimal age differential remained constant. But, quite possibly, these preferred ages have not fallen at the same rate and the optimal age difference has therefore diminished. This issue, too will be addressed in later work.

Conclusions

Swedish cohort size and marriage experience during the twentieth century present intriguing questions. The age difference between husbands and wives remained in the range of 2.5 to 3.5 years for all birth cohorts born between 1885 and 1940. Yet cohort size fluctuations over this period were so large that if every woman married a man three years older than her, the number of potential husbands per woman would fluctuate dramatically, ranging from .9 to 1.25 in periods as short as five years.

The first part of this paper describes the changes in patterns of age at marriage for each sex. We show that changes in the age difference between spouses worked in the direction that we predict to respond to changes in sex ratios. Relative proportions of males and females marrying also move in the direction we predict in response to changing sex ratios.

We discuss a model of marriage market assignments that can be directly tested against the Swedish experience. In this model, the efficient assignment of marriage partners across birth cohorts can be interpreted as a linear programming assignment problem. We apply a very simple version of our model to the Swedish historical data. This model assumes a payoff matrix in marriages have husbands three years older than their wives, with deviations from this ideal penalized according to a quadratic loss function. The model attempts to assign partners to all men and women born during the period 1895–1945, using only adjustments in the age difference between spouses to respond to fluctuating cohort sizes. Using the linear program to find the optimal set of marriage assignments between cohorts, we are able to almost perfectly replicate the rapid fluctuations in the age difference between spouses actually observed in Sweden for the cohorts born between 1915 and 1925. This is the period of the most dramatic fluctuations in cohort size in recent Swedish history. The success of our much simplified model of marriage assignments in replicating actual Swedish marriage patterns during this period indicates that we have captured fundamental features of the way the marriage market responds to changes in the relative supplies of males and females.

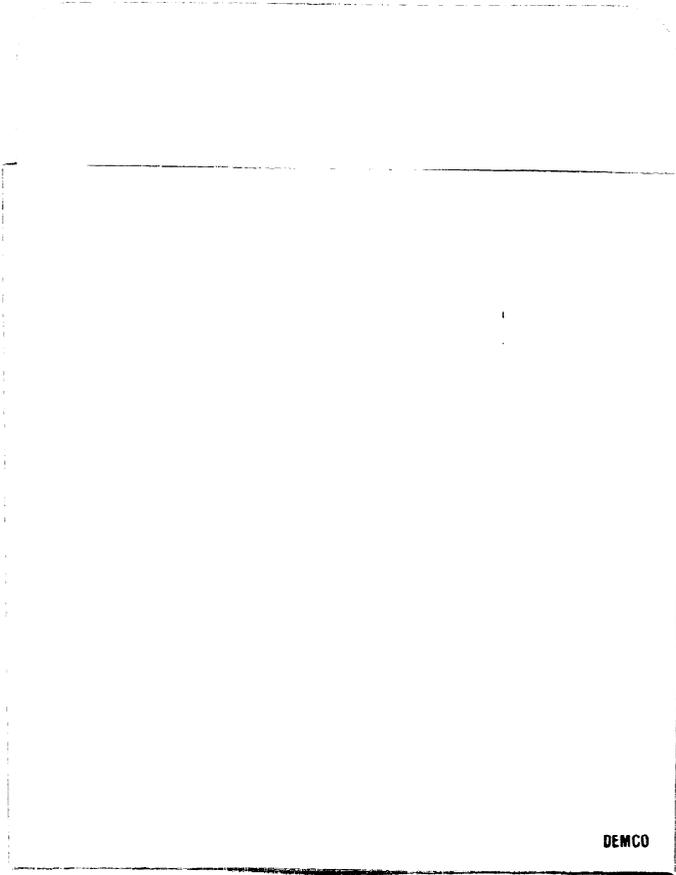
Our model is less successful at tracking the steady decline in the age difference between spouses beginning with the late 1920's cohorts. We believe that there are two major sources of divergence between the predictions of our model and historical outcomes. First, our implementation of the assignment problem did not allow the proportions marrying to adjust to excess supplies or demands. Second, we assumed that the optimal age difference between husband and wives has remained constant over the entire period. Encouraged by our ability to capture some of the major features of Swedish marriage experience using a model in which adjustments occur only in the age difference between spouses, we will attempt to enrich our model to incorporate nonstationarity and adjustments in proportions marrying.

References

- Becker, G. (1973) A theory of marriage, part I. *Journal of Political Economy*, July/August, 813-847.
- Becker, G. (1974) A theory of marriage, part II. *Journal of Political Economy*, S11-S26.
- Becker, G. (1981) *A Treatise on the Family*, Cambridge: Harvard University Press.
- Bergstrom, T. and Cornes, R. (1981) Gorman and Musgrave are dual—an antipodean theorem on public goods. *Economic Letters*, 371-378.
- Bergstrom, T. and Cornes, R. (1983) Independence of allocative efficiency from distribution in the theory of public goods. *Econometrica*, 1753-1765.
- Bergstrom, T. and Lam, D. (1989) The two-sex problem and the marriage squeeze in an equilibrium model of marriage markets, presented at Annual Meetings of Population Association of America, Baltimore, April 1989.
- Caswell, H., and Weeks, D.E. (1986) Two-Sex models: chaos, extinction, and other dynamic consequences of Sex. *The American Naturalist*, 128(5): 707-735.
- Das Gupta, P. (1974) On two-sex models leading to stable populations. *Demography*.
- Keyfitz, N. (1971) The mathematics of sex and marriage. *Proceedings of the Sixth Berkeley Symposium on Mathematical Statistics and Probability*, Vol. IV, Berkeley: University of California Press.
- Koopmans, T. and Beckman, M. (1957) Assignment problems and the location of economic activities. *Econometrica*, January, 53-76.
- Lam, D. (1988) Marriage markets and assortative mating with household public goods. *Journal of Human Resources*, 23(4): 462-487.
- Mahsam, H. (1974) The marriage squeeze. *Demography*, 291-299.
- McFarland, D. (1972) Comparison of alternative marriage models. in T.E. Greville, ed. *Population Dynamics*, New York: Academic Press, 89-106.
- Pollak, R.A. (1986) A reformulation of the two-sex problem. *Demography* 23(2): 247-259.
- Pollak, R.A. (1987) The two-sex problem with persistent unions: a generalization of the birth matrix-mating rule model, *Theoretical Population Biology* 32(2): 176-187.
- Sanderson, W. (1983) A two-sex general equilibrium marriage model. in Allen C. Kelley, Warren C. Sanderson, and Jeffrey G. Williamson, eds., *Modeling Growing Economics in Equilibrium and Disequilibrium*, Duke Press Policy Studies: Durham, N.C.
- Schoen, R. (1981) The harmonic mean as the basis of a realistic two-sex marriage model. *Demography* 18: 201-216.

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