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HEIGHT VERSUS DIAMETER AS THE MOST SATISFACTORY BASIS FOR EXPRESSING STAND STOCKING IN FOREST MANAGEMENT

by

Forrest Dean Brunson

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INTRODUCTION

After all pertinent factors are considered and the purpose and product for which a stand of timber is to be placed under management has been decided upon, three fundamental questions in planning stand management must be answered. These questions are:

1. What constitutes full stocking for this stand under the plan proposed?

2. Does the stand need a thinning treatment at present and if it does, then to what degree?

3. When will the stand need treatment in the future and to what degree will it be?

The economically uncontrollable factors affecting a timber stand are the site factors of climate (such as temperature, light, and exposure), moisture, and soil (depth, profile, nutrients, and draingge). It must be noted here that this is a mere listing of some of the site factors and that actually many of these are closely interrelated such as climate with soil amd moisture with temperature, exposure, and drainage. Management plans must be constructed so as to fit these uncontrollable factors as favorably as possible economically and socially.

The controllable factors available in forest management are density, age, position in the stand, form, and to a limited extent species. The density factor is the most important controllable factor in management and is the chief means by which frequency and degree of stand cutting treatments are planned. It is in terms of density, or spacing and size of trees in a timber stand, that correct amounts of growing stock are computed. Therefore, much effort in years past has been devoted to the study and measurement of density or stocking in stands.

YIELD TABLES IN FOREST MANAGEMENT

The need for an easily applicable guide for quick determinations in the field of proper stocking in uniformly stocked, even-aged stands has steadily increased. For many years it was hoped by technically trained forest managers that yield tables would be the answer, but they have proved to be unsatisfactory for reasons discussed below. The stand density factor, even though so important in forest management, still requires much research work. Foresters cannot agree on concrete demarcations for overstocked, fully stocked, and understocked stands. The great obstacle to this in the United States is the lack of a workable definition of stocking range. This fact has particularly affected the reliability of yield tables in this country. In compiling the original data for yield tables many overstocked stands were included with resultant low average stand diameters where suppressed trees were erroneously included as growing stock. This inclusion of overstocked stands causes yield tables to indicate a higher degree of stocking than actually occurs in nature in stands just fully stocked. Also, some plots of species weak in expression of height dominance and stagnated in height growth by overcrowding were included in the data as a lower site class. The result has been unexplainable differences between yield tables of closely similar species that should normally be quite comparable in many characteristics.

An example of the inconsistencies in yield tables is graphically shown in Figure1. Tables 1 and 10 of U.S.D.A. Technical Bulletin Number 142 entitled <u>Yields of Second Growth Spruce and Fir in the Northeast</u> were used in constructing this graph. These two original tables in the bulletin were constructed by plotting age as the independent variable, the abscissa, with the other data plotted as dependent variables. In the

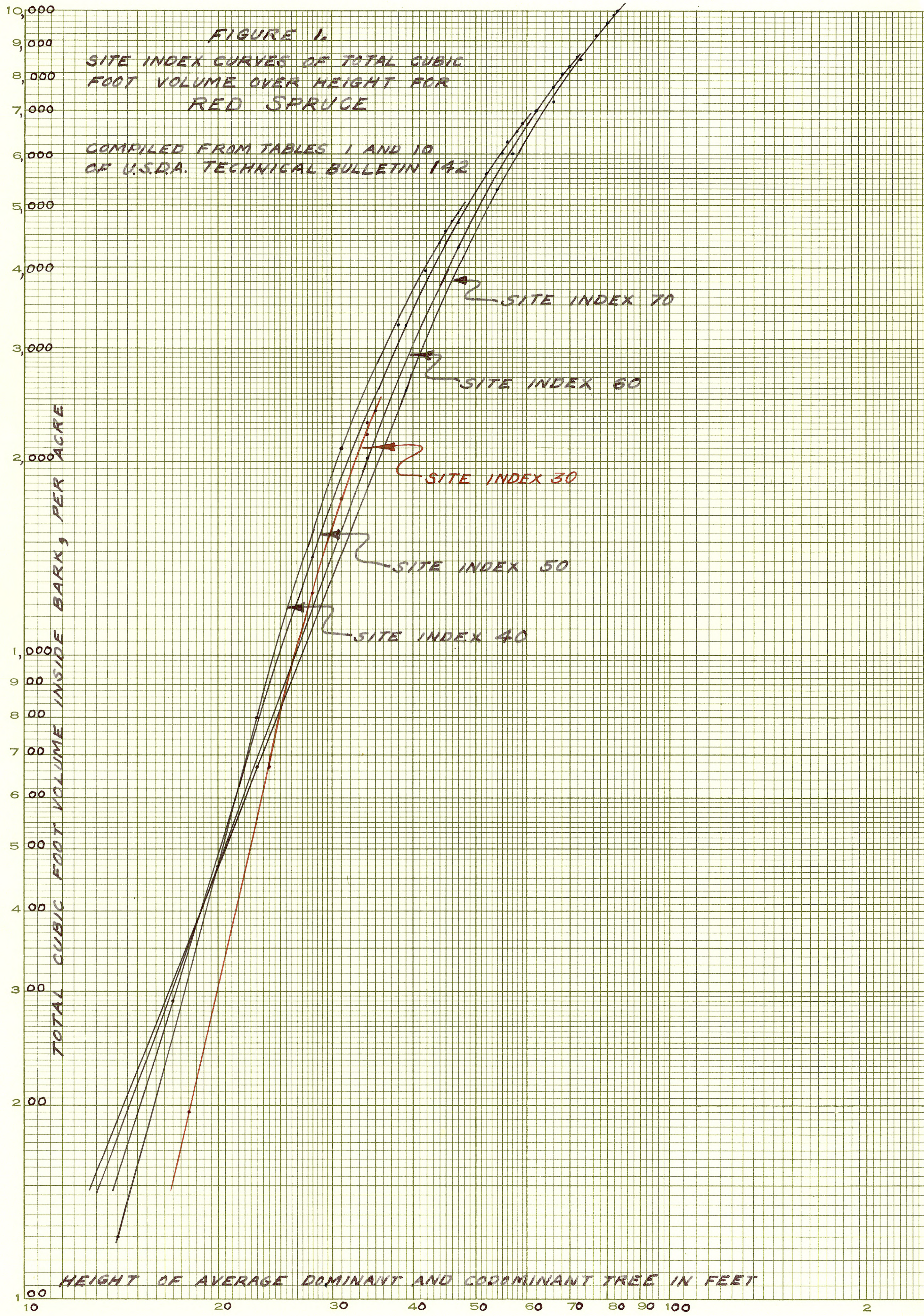
FIGURE 1.
SITE INDEX CURVES OF TOTAL CUBIC FOOT VOLUME OVER HEIGHT FOR
RED SPRUCE
COMPILED FROM TABLES 1 AND 10 OF U.S.D.A. TECHNICAL BULLETIN 142
SITE INDEX 70
SITE INDEX 60
SITE INDEX 30
SITE INDEX 50
SITE INDEX 40
TOTAL CUBIC FOOT VOLUME INSIDE BARK, PER ACRE
HEIGHT OF AVERAGE DOMINANT AND CODOMINANT TREE IN FEET
10,000
9,000
8,000
7,000
6,000
5,000
4,000
3,000
2,000
1,000
900
800
700
600
500
400
300
200
100
10
20
30
40
50
60
70
80
90
100
2

graph of Figure 1 height, from Table I of the bulletin, was plotted as the independent variable with volume, from Table 2 of the bulletin, as the dependent variable. Statistically this is not correct for reading specific data from the graph because field data was originally curved over age, but the principle is sound for showing the trend of harmonious relationship between the curves of the site indexes. Note in Figure 1 that all site indexes fit harmoniously into a trend except for site index 30 which definitely does not conform with the trend. The site index 30 line above twenty-seven feet in height seems to remain somewhere in the middle of the field of plotted curves. This is because data was not gathered for cubic foot volume of trees smaller than the four inch diameter breast high class. In site index 30 this meant a far higher percentage of volume not recorded than that whichwas not recorded in the other site indexes that are greater producers. Table 8 of the bulletin shows the number of trees per acre in site index 30 constantly increasing throughout the table and even at 110 years of age. The other site indexes decrease in number of trees per acre early with site index 40, the next poorest site, showing a marked decrease in number at the age of 70 years. These observations indicate that perhaps the original data for site index 30 included some overcrowded and stagnated plots and was not entirely compiled from normal, fully stocked stands. It is a difficult assignment to determine on a plot what numbers of the existing size trees that site can actually support without the stand becoming overcrowded. Poorer sites have lower ability to support populations of plants, and consequently overcrowding occurs easily and is more difficult to detect.

Yield tables have further disadvantages in the United States in that they are often unavailable or inapplicable for use in stand management. They are often awkward to use and offer even more difficulties when the site index of the stand to be put under management does not fall near the median of one of the site index bands in the yield table. Furthermore, present yield tables in the United States are based on natural stands accepted as the "normal". Stands will depart more and more from this assumed normal as they progressively undergo thinning treatments. Well managed stands will be slightly understocked through most of their life, and will have greater growth in the crop trees than the "normal" yield tables can predict. Another drawback in the use of yield tables is that they cannot be easily applied when making management decisions in the field and "on the spot" for individual groups of uniformly stocked, even-aged timber stands within a general forest area.

NUMERICAL EXPRESSIONS OF STAND STOCKING IN MANAGEMENT

The actual determination of what really constitutes understocked, fully stocked, and overstocked stands under a particular management plan must be made in measurable and easily applicable terms. This is necessary before it is possible to arrive at well substantiated decisions on frequency and degree of stand thinning treatments. Definite, quantitatively measured data must be had to support the loose terms such as "fully stocked" and "overstocked" that are used to indicate stand stocking. Otherwise, a concise and objective plan of good reliability cannot be formulated, and clear, unmistakable orders for the conduct of the thinning operations cannot be constructed without a great amount of study and experience in the limited area under consideration.

There are three quantitative units available for measuring stocking: 1) number of trees per unit ground area, 2) stand volume per unit ground area, and 3) stand basal area per unit ground area. Number of trees per unit area is obviously not a good method, per se, as there is no indication of either tree size or volume on the area. Volume per unit area does not show the real distribution of volume among age classes in the stand, and it can lead to particularly erroneous conclusions in using board foot volumes for young stands just before and just after reaching a size for tallying merchantable volumes. Basal area per unit ground area (acre) is quicker and easier to obtain than volume , is at the same time a good reflection of volume, and makes a good portrayal of stand stocking.

To fulfill the need for an easily applied and practical figure as a guide in the quick, on-the-spot determination of proper stocking in uniformly stocked, even-aged stands, the following methods have been devised: the spacing figure, spacing factor, and the height factor.

The use of basal area per acre which was described above as having good possibilities in the numerical portrayal of stand stocking is incorporated in the application of the first two of these methods. All three figures portray stand stocking in a concise numerical expression. In this thesis the spacing factor will be emphasized over the spacing figure since the former is actually a further development of the latter, embodying all the principles and possessing superior qualities over the latter. Therefore, this thesis will be primarily concerned with the height factor versus the spacing factor.

Relatively few contributions to these methods have been made because of the brief history of these numerical expressions of stand stocking. It is the purpose of the author in writing this thesis to set forth the first full discussion of these methods in one work, to discover and compare the advantages and disadvantages each has over the others, and to make further contributions in the application of these three methods.

THE SPACING FIGURE, SPACING FACTOR, AND HEIGHT FACTOR

AS TOOLS IN FOREST MANAGEMENT

THE SPACING FIGURE

The spacing figure is defined as the average distance between tree stems, D, in the uniformly stocked, even-aged stand divided by the diameter of the average tree stem, d, of the stand, both being expressed in terms of the same distance unit. (1)

$$\text{Spacing figure} = D/d$$

In utilizing the land to its fullest capacity, a complete tree-crown cover over ground area is normally required. This theory is employed in computing the formula for the spacing figure. The conception that in uniformly stocked, even-aged stands tree crowns can be considered as filling out squares whose sides are equal to the distance between tree stems is also fundamental in the development of the spacing figure formula. The derivation of the spacing figure equation can be followed on pages 27 and 28 of <u>Management of American Forests</u> by Donald M. Matthews. The spacing figure formula is as follows:

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1. Matthews, Donald M., <u>Management of American Forests</u>, McGraw-Hill Book Company Incorporated, 1935, p. 26.

$$D/d = \frac{185}{\sqrt{BA}}$$

The spacing figure was devised to answer the need for a more easily applied figure as a guide in quick determination of stand stocking. However, this method was still slightly awkward to use in the field for making quick decisions, an important factor particularly to consulting foresters hired to draw up management plans of large tracts of timber in only a few days. Therefore, the spacing factor has recently been developed as an improvement over the spacing figure for use in forest management work.

THE SPACING FACTOR

The spacing factor is defined as the average distance between tree stems in feet, D, in a uniformly stocked, even-aged stand divided by the average tree stem diameter breast high in inches. A simplification over the spacing figure is at once noted in the fact that the numerator and the denominator need not be converted into common distance units. Furthermore, its mathematical relationship to basal area per acre can be solved, and a relationship table can be easily constructed for making quick conversion to basal area per acre. Most important, it can be used quickly and directly for making "on the spot" determinations for present and future treatments of a uniformly stocked, even-aged stand if the correct full-stocking spacing factor for that stand has been decided upon, with the stand's purpose in consideration. Basal area in itself can then be ignored as it is represented in the spacing factor and stand stocking can be measured directly by this simple, numerical expression. A person with relatively little experience can be taught to estimate with sufficient accuracy for constructing management plans the average distance between tree stems in feet and the average diameter breast high of tree stems in inches in a uniformly stocked, even-aged timber stand immediately about him in the field. To have him attempt to estimate directly with any reasonable accuracy the basal area per acre represented in a particular stand to determine stand stocking would be quite another and far more difficult problem.

The relationship between crown and stem, fundamental in this method, cannot be relied upon unless the trees in the stand are thirty or more feet in height and of sufficient age so that the crown-stem diameter ratio will remain relatively constant thereafter. The crown-stem diameter ratio may lessen finally when the stand becomes very old, for the trees will continue to add stem diameter growth with no relative increase in crown spread.(2) The dominant and codominant trees in the stand must have definitely developed pole stems before the crown-stem relationship

2. Ibid., p.29.

is stoong enough for sufficiently accurate work.

The spacing factor expressed in formula form is as follows:

$$SF = \frac{D, \text{"diameter" of crown or distance between stems in feet}}{d, \text{diameter of stem breast high in inches}}$$

THE SPACING FACTOR -BASAL AREA FORMULA

The mathematical relationship between the spacing factor and basal area per acre must now be determined. This spacing factor formula was computed by Professor Donald M. Matthews of the School of Forestry and Conservation, University of Michigan, in developing the spacing factor as a tool in forest management. This original derivation of the formula will be described later. The author of this thesis developed another method for computing the spacing factor formula. The author solved for the corresponding basal areas of several spacing factor values with an assumed constant average stem diameter breast high of 12 inches. These results were plotted on logarithmic graph paper. The plotted values formed a simple straight line on the logarithmic paper, and the author computed the equation of the line. The resulting equation shows the relationship between the spacing factor of a uniformly stocked, even-aged stand and the stand's basal area per acre.

This spacing factor-basal area equation was computed in the following manner:

by definition $SF = D/d$

Let d be a constant of 12 inches.

Let $SF = 4.5$

Then $4.5 = D/12$

$D = 54$

The area of an acre is 43,560 square feet. As stated above, when the acre is considered as a whole, tree crowns can be thought of as filling out squares whose sides are equal to the average distance between stems. Thus, the number of trees per acre in this case can be computed as follows:

$$\frac{43,560}{D^2} = \text{number of trees per acre}$$

$$\frac{43,560}{54^2} = \text{number of 12" trees per acre when the spacing factor equals 4.5}$$

$$\frac{43,560}{2,916} = 14.938 \text{ trees per acre}$$

The basal area in square feet of each individual 12 inch (1 foot) tree is equal to $\frac{\pi d^2}{4}$ and is computed as follows:

$$\frac{\pi d^2}{4} = \frac{3.1416 \times 1^2}{4} = 0.785 \text{ sq.ft.}$$

The basal area per acre for the above given values thus becomes:

$$\text{basal area per tree} \times \text{no. trees per acre} = \text{basal area per acre}$$

$$0.785 \text{ square feet} \times 14.938 = 11.733 \text{ square feet basal area per acre}$$

This is the first value to be plotted on the logarithmic graph paper in Figure 2. The computations for other values to be plotted are as follows:

let $SF = 3$

then $SF = \frac{D}{d}$

$$3 = D/12$$

$$D = 36 \text{ feet}$$

$$\frac{43{,}560}{36^2} = \frac{43{,}560}{1{,}296} = 33.611 \text{ trees per acre}$$

$$0.785 \text{ square feet per tree} \times 33.611 \text{ trees per acre} = 26.398 \text{ sq.ft. B.A.}$$

Let $SF = 2$

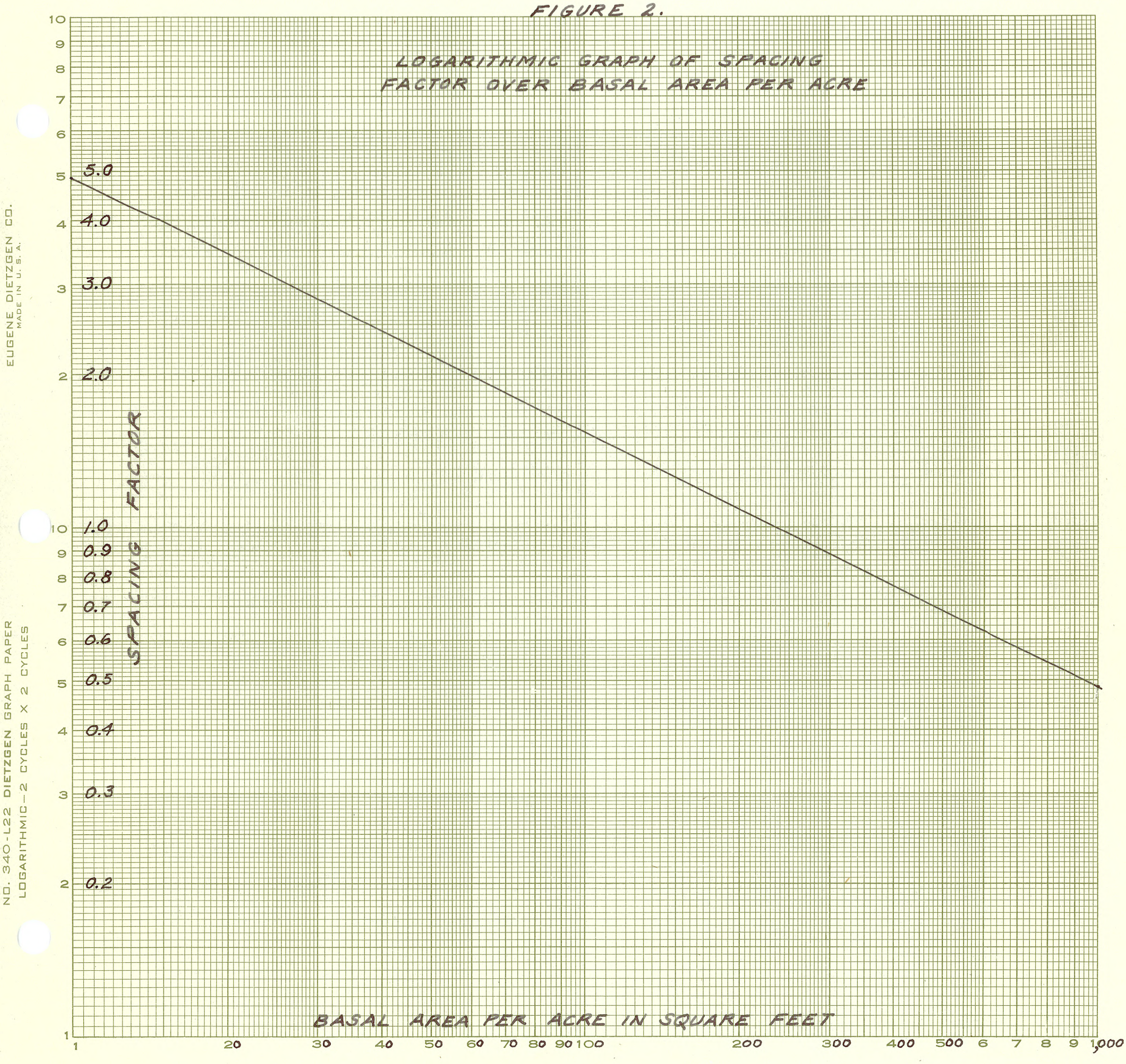
then $2 = \frac{D}{12}$

$$D = 24 \text{ feet}$$

$$\frac{43{,}560}{24^2} = \frac{43{,}560}{576} = 75.625 \text{ trees per acre}$$

$$0.785 \text{ square feet} \times 75.625 = 59.396 \text{ square feet basal area per acre}$$

FIGURE 2.


LOGARITHMIC GRAPH OF SPACING
FACTOR OVER BASAL AREA PER ACRE
SPACING FACTOR
BASAL AREA PER ACRE IN SQUARE FEET
5.0
4.0
3.0
2.0
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
1
20
30
40
50
60
70
80
90
100
200
300
400
500
1,000
EUGENE DIETZGEN CO.
MADE IN U. S. A.
NO. 340-L22 DIETZGEN GRAPH PAPER
LOGARITHMIC-2 CYCLES X 2 CYCLES

Let $SF = 1$

then $1 = \frac{D}{12}$

$D = 12$ feet

$\frac{43,560}{12^2} = \frac{43,560}{144} = 302.500$ trees per acre

0.785 square feet x 302.500 = 237.584 square feet basal area per acre

Let $SF = 0.5$

then $0.5 = \frac{D}{12}$

$D = 6$ feet

$\frac{43,560}{6^2} = \frac{43,560}{36} = 1,210$ trees per acre

0.785 square feet x 1,210 = 950.334 square feet basal area per acre

The degree of accuracy to which the above values are carried out is necessary for the careful computation of the spacing factor-basal area formula, and is not necessary in the normal use of applying these methods in practical stand management. Also, these assumed spacing factors for plotting the points in Figure 2 are theoretical in order to get the maximum range of values for the graph. The assumed spacing factor of 0.5 for one point of the graph could hardly be found in nature.

The equation of the straight line now plotted on the graph in Figure 2 can be computed by the use of logarithms. First, it can be noted that since the points fall in a straight line on the logarithmic graph, the equation is of the second degree. The equation of the line takes the form

$$\log SF = a \log BA + \log k,$$

letting "a" and "k" be constants and "BA" represent basal area per acre.

The solution for constants "a" and "k" is as follows:

$$\text{where } SF = 3, \quad BA = 26.398$$
$$SF = 1, \quad BA = 237.584$$

Using the logarithms of these, the observation equations become

$$0.47712 = 1.42160\ a + \log k$$
$$0.00000 = 2.37585\ a + \log k$$

Solving these simultaneous equations gives

$$a = -0.5000$$
$$\log k = 1.187925$$

Inserting these values into the logarithmic straight line equation gives

$$\log SF = -0.5000 \log BA + 1.187925$$

The antilog of 1.187925 is 15.414 and the final equation becomes

$$SF = \frac{15.414}{\sqrt{BA}} \quad \text{or} \quad \frac{15.4}{\sqrt{BA}}$$

Also

$$BA = \frac{15.41^2}{SF^2}$$

$$BA = \frac{237.47}{SF^2}$$

For any given basal area per acre the spacing factor can now be computed and vice versa.

The following is the original solution for the basal area relationship of the spacing factor as presented by Professor Donald M. Matthews in his classroom lectures of 1946 and 1947:

let d = average diameter breast high in inches

let c = constant to be added to diameter breast high to equal the average spacing of trees in feet

and $$SF = \frac{D}{d} = \frac{d+c}{d} = \frac{d}{d} + \frac{c}{d} = 1 + \frac{c}{d}$$

Then the formula becomes:

$$\text{number of trees per acre} = \frac{43{,}560 \text{ square feet per acre}}{(d+c)^2}$$

$$\text{basal area per tree in square feet} = \frac{d^2 \pi/4}{144} = 0.005454\ d^2$$

$$\text{basal area per acre} = \text{number of trees per acre} \times 0.005454 \; d^2$$

let BA = basal area per acre in square feet,

therefore, substituting:

$$BA = \frac{43560}{(d + c)^2} \times 0.005454 \; d^2$$

$$BA = \frac{237.5 \; d^2}{(d + c)^2}$$

$$BA = \left(\frac{15.4 \; d}{d + c} \right)^2$$

Taking the square root of both sides, the equation becomes:

$$\frac{15.4 \; d}{d + c} = \sqrt{BA}$$

and the procedure for isolating and solving for the constant, c, is as follows:

$$15.4 \; d = (\sqrt{BA} \times d) + (\sqrt{BA} \times c)$$

$$\sqrt{BA} \times c = 15.4 \; d - (\sqrt{BA} \times d)$$

$$c = \frac{(15.4 - \sqrt{BA}) \; d}{\sqrt{BA}}$$

and since

$$SF = 1 + \frac{c}{d}$$

$$SF = 1 + \frac{\dfrac{(15.4 - \sqrt{BA}) \; d}{\sqrt{BA}}}{d}$$

Cancelling the d's from the fraction gives

$$SF = 1 + \frac{15.4 - \sqrt{BA}}{\sqrt{BA}}$$

This equation is the formula for the use of the spacing factor as presented by Professor Matthews.(3)

The spacing factor formula developed by the author is $SF = \frac{15.4}{\sqrt{BA}}$ and appears superficially to be slightly different than the presently known and used formula $SF = 1 + \frac{15.4 - \sqrt{BA}}{\sqrt{BA}}$. However, these two equations must be equal if they are both correct.

The author discovered that the formula presented by Professor Matthews was actually not in its most simplified form. The author simplified this formula algebraicly as follows:

$$SF = 1 + \frac{15.4 - \sqrt{BA}}{\sqrt{BA}}$$

and putting all the right hand side of the equation over a common denominator gives

$$SF = \left(1 \times \frac{\sqrt{BA}}{\sqrt{BA}}\right) + \frac{15.4 - \sqrt{BA}}{\sqrt{BA}}$$

$$SF = \frac{\sqrt{BA} + 15.4 - \sqrt{BA}}{\sqrt{BA}}$$

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3. Matthews, Donald M., <u>The Spacing Factor as a Criterion of the Density of Stocking in Stands</u>, a mimeographed lecture paper, 1947, p.1.

Cancelling the $\sqrt{BA}$'s produces the same equation the author arrived at by the graphical method and is

$$SF = \frac{15.4}{\sqrt{BA}}$$

This formula can be arrived at in still another manner. By definition the spacing figure equals $\frac{D}{d}$ with both numerator and denominator in a common distance unit. The numerator "D" must be multiplied by 12 to convert crown diameter from feet to inches. In the spacing factor formula, $SF = \frac{D}{d}$, by definition no conversion is needed. Therefore, to make these two equations equal the spacing factor equation must be multiplied by 12 as follows:

$$12 \times \frac{D}{d} \text{ the spacing factor} = \frac{D}{d} \text{ the spacing figure}$$

The spacing figure-basal area formula was earlier stated to be

$$\text{Spacing figure} = \frac{185}{\sqrt{BA}}$$

Substituting the spacing factor gives $12 \times \frac{D}{d} = \frac{185}{\sqrt{BA}}$

$$\frac{D}{d} = \frac{185}{12\sqrt{BA}} = \frac{15.4}{\sqrt{BA}}$$

$$SF = \frac{15.4}{\sqrt{BA}}$$

For quick conversion from basal area per acre to the spacing factor and vice versa, the spacing factor–basal area formula is employed for basal area values ranging from thirty square feet per acre to two hundred square feet per acre, and the respective spacing factors are computed. These results are presented in Table 1 on the following page.(4) For graphical presentation of this relationship and for easy interpolation between values given in Table 1 the results are also shown in the curve in Figure 3 on page 26.

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4. Ibid., p.2.

TABLE I.----THE SPACING FACTOR VALUE CORRESPONDING TO EACH BASAL AREA VALUE PER ACRE

BA per acre in square feet	Spacing Factor
30	2.81
40	2.43
50	2.18
60	1.99
70	1.84
80	1.72
90	1.62
100	1.54
110	1.47
120	1.41
130	1.35
140	1.30
150	1.26
160	1.22
170	1.18
180	1.15
190	1.12
200	1.09

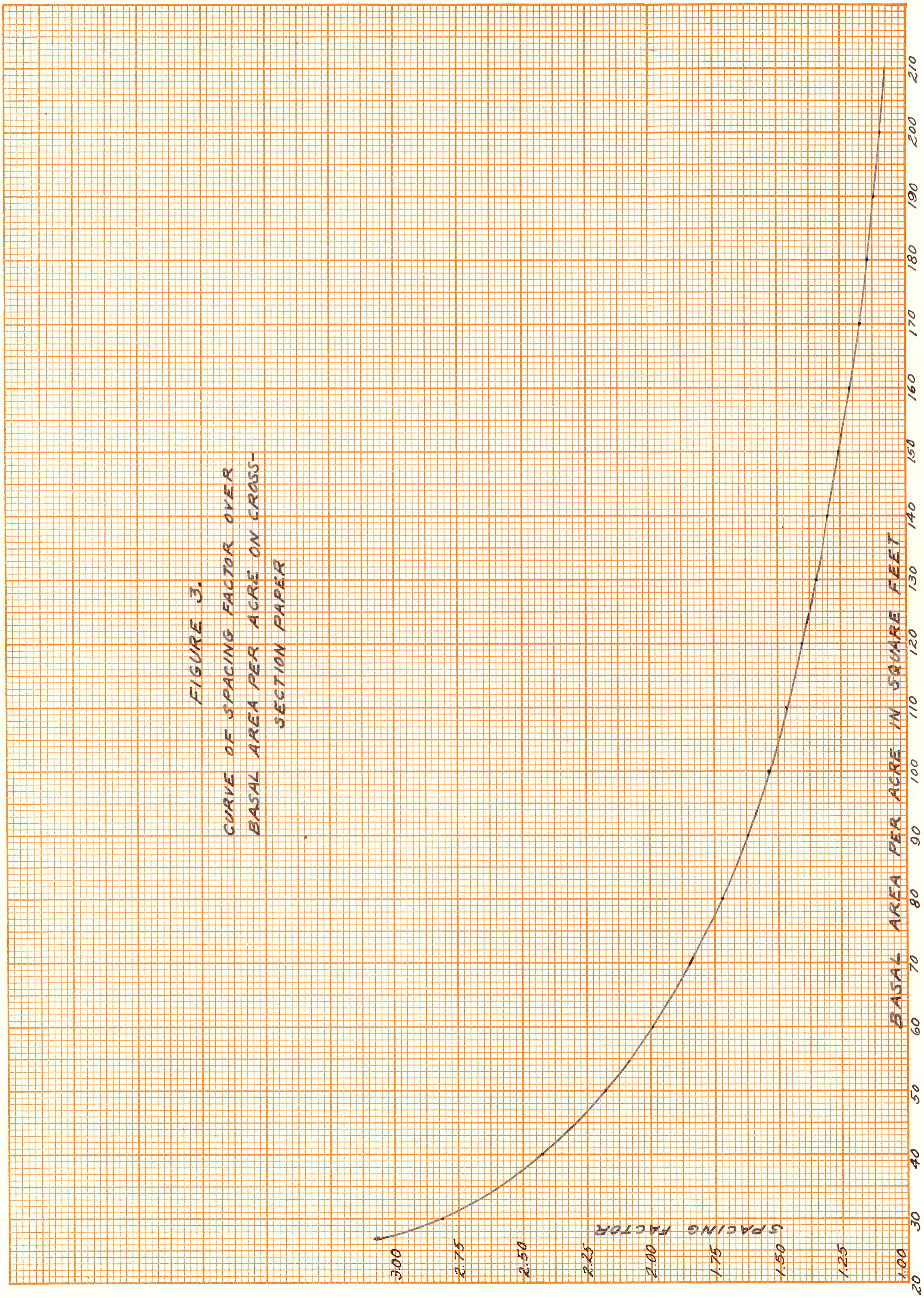
FIGURE 3.
CURVE OF SPACING FACTOR OVER
BASAL AREA PER ACRE ON CROSS-
SECTION PAPER
BASAL AREA PER ACRE IN SQUARE FEET
20
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170
180
190
200
210
SPACING FACTOR
3.00
2.75
2.50
2.25
2.00
1.75
1.50
1.25
1.00

THE SPACING FACTOR AS AN EXPRESSION OF STAND STOCKING

In order to make direct computations in spacing factor values alone for stand stocking and for determination of when and to what degree thinning treatments are to be made, it now becomes necessary to know what constitutes a fully-stocked stand for the particular stand of timber in question in terms of the spacing factor. In the United States there is still a great amount of research needed to determine the correct full stocking values for timber stands over the country. Work of this sort has been done in the Lake States, the Northeastern States, the Southeastern States, the Southern States, and in some other regions, but it is still far from complete. Correct stand stocking must be obtained from existing data, however unreliable that data is, until research produces new and more accurate and complete figures. Yield tables or other tables stating stand stocking in either basal area per acre or in spacing figure values for closely similar stands of timber must be the main souces of information at present. Data from these sources can be converted into spacing factor terms to obtain the proper values for full stocking.

Table IIshows the computed spacing factor values for stand species and ages for full stocking as converted from the table on page thirty of Management of American Forests by Professor D. M. Matthews. This conversion can be made in either of two ways as follows:

1. By conversion of basal area per acre to spacing factor by use of the formula or by use of either Table I or Figure 3

2. By converting the spacing figure to spacing factor values by use of the formula

$$SF = \frac{\text{spacing figure}}{12}$$

TABLE II.----SPACING FACTORS FOR VARYING AMOUNTS OF BASAL AREA PER ACRE FOR FULLY-STOCKED STANDS

BA per acre in square feet	Spacing Figure	Spacing Factor	Normal stocking for the following species and ages
50	26	2.18	Maple trees in
60	24	1.99	open, 6 to 8 inches
70	22	1.84	in diameter
80	20.6	1.72	
90	19.5	1.62	
100	18.5	1.54	
110	17.6	1.47	Range for forest-
120	16.9	1.41	grown intolerant coni-
130	16.2	1.35	fers and intolerant
140	15.6	1.30	hardwoods after
150	15.1	1.26	60 years
160-180	14.2	1.18	Forest-grown tolerant
180-200	13.4	1.12	conifers after 60 yrs.
200-240	12.5	1.04	Yellow pine and sugar
240-280	11.5	0.96	pine in Calif. and old
300	10.7	0.90	white pine in Lake States
400	9.2	0.77	Douglas fir in the
500	8.1	0.68	Northwest
1,000	5.8	0.48	Sequoia

APPLICATION OF THE SPACING FACTOR IN COMPUTING DENSITIES OF STOCKING AND DETERMINING STAND TREATMENTS

By the term "fully-stocked stand" is meant a stand of timber which completely utilizes to the best advantage the land area that it covers so that all facilities for the best diameter and height growth and natural pruning of the stems are put to the best possible use. "Complete crown closure" with no tree crowns interlocking with or badly overlapping other tree crowns, and with no spaces left and therefore wasted between adjacent tree crowns, is the condition that normally best exemplifies a fully-stocked stand. It is at once seen that if no thinning treatment is given a stand that is fully-stocked, the stand will soon become overstocked because of the procedure of normal growth, and a decrease in the growth rate will gradually come about. This is a condition which the forest manager attempts to prevent, and thinning treatments are thus timed to occur just as the stand reaches the desired full stocking, or are timed as close to this as is economically practicable.

It should be noted that the factor of proper stand density for the production of tall, straight, clear stems can properly be compensated for in the "fully-stocked" basal area or spacing factor value decided upon. Thus a timber stand can be maintained almost constantly a little under this chosen value of full stocking for optimum growth and still have sufficient closure of tree crowns to produce stems of desired straightness, clear length, and diameter. It should be remembered that normally the most profitable time of thinning is at the time of crown closure or shortly after.

After a timber stand has reached the age where good stem development is attained, the crown-stem relationship for this stand under fully-stocked conditions remains essentially the same throughout the remainder of the life span of the stand. Also, it can be said that the desirable fully-stocked basal area in square feet per acre will remain the same after the stand has reached the point of growth where definite stem development has been accomplished. Stands of different species but with similar silvical characteristics also show consistent crown-stem ratios that are equal for fully-stocked stands. As stated above, Table II shows these consistent values for fully-stocked stands of groups of tree species having certain similar silvical characteristics; thus it can be seen that loblolly pine, only of intermediate tolerance at best (5), probably would have a spacing factor of around 1.30 for a fully-stocked timber stand. In reality, it has been found that loblolly pine probably has a spacing factor of about 1.33 or a basal area of about 133 square feet for a fully-stocked stand on an average site.(6)

It should be noted from Table I that the spacing factor value varies inversely as the basal area per acre. Therefore, a spacing factor of 1.20 for a timber stand of loblolly pine indicates an overstocked stand, and a thinning treatment should be made as soon as is economically possible. If it is desired percentage stocking for various spacing factor values of different timber stands of any particular tree species can now be computed. The procedure is to look on the graph in Figure 1 for the basal area corresponding to the spacing factor of the fully-stocked stand. This basal area value is used as the denominator of the fraction.

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5. Toumey, James W. and Korstian, Clarence F., Foundations of Silviculture upon an Ecological Basis, John Wiley and Sons, Incorporated, 1937, p.341.

6. Matthews, Donald M., The Spacing Factor as a Criterion of the Density of Stocking in Stands, op.cit., p.3.

The corresponding basal area values for other spacing factor values are used as the numerators of the fraction consecutively. The resulting quotients are each multiplied by one hundred to give the percentage- of-stocking values. Thus, for loblolly pine, a fully-stocked stand has a spacing factor of 1.33 and the corresponding basal area of 133 square feet per acre. A stand of loblolly pine having a spacing factor of 1.54 has a basal area of 100 square feet per acre. The computation for the percentage stocking of the latter stand is as follows:

$$\frac{100 \text{ square feet BA (spacing factor 1.54)}}{133 \text{ square feet BA (spacing factor 1.33)}} \times 100 = \underset{\text{stocking}}{75\%}$$

In preparation for classifying stands for percentage density by spacing factor values in a timber cruise, these computations can be done in reverse and a table can be constructed. Thus, for a 90% stocking of loblolly pine, the computations are as follows:

$$.90 \times \begin{array}{l} \text{133 square feet BA} \\ \text{(spacing factor 1.33} \\ \text{fully-stocked)} \end{array} = \begin{array}{c} \text{120 square feet} \\ \text{BA} \end{array}$$

Corresponding spacing factor value for a BA of 120 square feet per acre is 1.41. Normally, all loblolly pine stands with spacing factor values less than 1.33 will be classified simply as "over-stocked". Density estimates can now be made by the crews right while they are out on the ground both on and off the

the plots being cruised, and such information noted down will produce better estimates of over-all stocking conditions.

By the method of computations explained a bove, Table III has been constructed and is presented here to show the corresponding spacing factor values (7) for the various percentages of stand stocking for crown diameter/stem ratios of fully-stocked stands of 16 to 1, 15 to 1, and 14 to 1.

TABLE III.----SPACING FACTOR VALUES FOR PERCENTAGES OF STAND STOCKING FOR DIFFERENT FULLY-STOCKED SPACING FACTOR VALUES AND CROWN-STEM RATIOS

Spacing Factor 1.33

16 to 1 Crown diameter-stem Ratio

Stocking	BA per acre square feet	Spacing Factor
Over-stocked	133 plus	1.33 minus
90% stocked	120	1.41
80% stocked	107	1.49
70% stocked	93	1.59
60% stocked	80	1.72
50% stocked	67	1.88
40% stocked	53	2.12
Under 30% stocked	40 minus	2.43 plus

7. Ibid., p.4.

TABLE III.---Continued:

Spacing Factor 1.25

15 to 1 Crown diameter-stem Ratio

Stocking	BA per acre square feet	Spacing Factor
Over-stocked	152 plus	1.25 minus
90% stocked	137	1.32
80% stocked	122	1.40
70% stocked	106	1.50
60% stocked	91	1.61
50% stocked	76	1.77
40% stocked	61	1.97
Under 30% stocked	46 minus	2.27 plus

Spacing Factor 1.17

14 to 1 Crown diameter-stem Ratio

Over-stocked	173 plus	1.17 minus
90% stocked	156	1.23
80% stocked	138	1.31
70% stocked	121	1.40
60% stocked	104	1.51
50% stocked	86	1.66
40% stocked	69	1.86
Under 30% stocked	52 minus	2.13 plus

In ascertaining the rate of growth by increment borings for a stand just being put under management, it must be remembered that under the stand treatment procedure indicated above of thinning the stand just as it reaches the fully stocked condition, the trees will be growing almost constantly in a stand slightly understocked. Therefore, increment borings should be taken from carefully chosen trees which are under the conditions right now that are planned to be produced in the future and maintained for the entire stand during its life, whether terminal or perpetual, when the management plan is put into effect. Trees of proper species, age, and crown closure conditions should be carefully selected for increment borings to make an accurate prediction of the growth rate that will be maintained in the stand after the stand comes into the conditions which the forest manager plans to produce and perpetuate in that stand.

Making quick and well-founded management plans for a particular stand of timber by means of the spacing factor now becomes a possibility. The following is a hypothetical problem presented here as an example to best explain the application of the spacing factor.

Cruise data for a timber stand of loblolly pine shows an average number of trees per acre of 450 with and average diameter breast high of eight inches. A basal area per acre of 133 square feet is the correct full stocking for loblolly pine timber stands, and 1.33 is the corresponding spacing factor value. Increment boring data indicates a future mean annual diameter growth of 0.3 inches when the stand is under proper management and not allowed to become overstocked.

It should be noted here that cruise data is not required when actually in the field and standing in a uniformly stocked, even-aged group of trees in a timber stand. The trained man can estimate "on-the-spot" with sufficient accuracy for making a management plan the average distance in feet between tree stems and the average diameter breast high in inches. From this he has the spacing factor directly, and can also compute the average number of trees per acre in that particular section of the timber stand by employing the formula

$$\text{Number of trees per acre} = \frac{43,560}{D^2}$$

He can also take a few, well chosen increment borings, and thus have all the stand data he needs as he looks at the situation while in the woods. Therefore, the spacing factor method enables the forest manager to draw up a reliable management plan while actually in the stand under consideration. This is one of the great advantages of the spacing factor method over other methods of expressing stand stocking and drawing up management plans.

The first decision in the above stated hypothetical problem is whether or not the stand needs treatment now, and if it is needed now, to what degree should the thinning treatment be. As noted above, tree crowns can be considered to conform to squares when taked for the acre as a whole, and therefore, the crown ~~basal~~ area per average tree equals 43,560 square feet per acre divided by 450 trees per acre, or 97 square feet per ~~acre~~ tree. The square root of this value is 9.84 feet, which is the average spacing between tree stems in feet. This is shown algebraicly as follows:

D = average spacing between tree stems in feet

therefore

D^2 = average crown ~~basal~~ area per tree in square feet

then

$$D^2 = \frac{43,560 \text{ square feet per acre}}{450 \text{ average number of trees per acre}} = 97 \text{ square feet}$$

$$D = \sqrt{97} = \pm 9.84 \text{ feet average spacing between tree stems}$$

The spacing factor can now be computed as follows:

$$SF = \frac{D \text{ in feet}}{d \text{ in inches}} = \frac{9.84}{8} = 1.23$$

Since this spacing factor value is below the full stocking value of 1.33, the stand is overstocked and needs immediate treatment.

It now must be decided when the next treatment in this stand can be made in the future. In this example it is assumed that for both economical and managerial reasons the best time for the next thinning will be ten years from now. The choice of the best thinning interval is another phase of forest management and cannot be covered here. Average diameter per tree ten years hence will equal the average diameter at present, plus the product of the number of years in the interval multiplied by the mean annual rate of diameter growth. The computations are shown as follows:

d = average diameter breast high in inches at present = 8 inches

d' = average diameter at time of next treatment

r = mean annual rate of diameter growth = 0.3 inches

y = number of years before the next treatment = 10 years

$d' = d + ry = 8'' + (0.3'' \times 10) = 11$ inches

As discussed above the stand should not be allowed to reach full stocking until close to the time scheduled for the next thinning treatment. Therefore, proper tree stem spacing must be computed for the timber stand of trees averaging eleven inches in diameter and with a fully-stocked spacing factor value of 1.33 as follows:

$$SF = \frac{D' \text{ in feet}}{d' \text{ in inches}}$$

$$D' = SF \times d' = 1.33 \times 11 = 14.63 \text{ feet}$$

The average spacing between tree stems after the present thinning treatment is completed should be approximately fifteen feet. Computations for the number of trees per acre that should be standing just before the thinning treatment of ten years from now are as follows:

$$\text{Number of trees per acre} = \frac{43{,}560 \text{ square feet per acre}}{\text{Average crown area per tree or } D^2}$$

$$D'^2 = 14.63^2 = 214 \text{ square feet}$$

$$\text{Number of trees per acre before treatment 10 years from now} = \frac{43{,}560 \text{ square ft.}}{214 \text{ square feet}} = 203 \text{ trees per acre}$$

Therefore, 203 trees per acre must be left in the stand after the present thinning treatment, and 450 trees minus 203, or 247, trees per acre must be removed on the average from the timber stand during the present thinning treatment. The average diameter of trees removed at present will probably be slightly less than eight inches, because normally the less vigorous trees will be selected for immediate cutting. The less vigorous trees in this sense are those that would die before the time of the next thinning or would be greatly suppressed by then. Only a very few of the larger diameter trees will be removed. These larger diameter trees to be cut in the stand thinning include those of poor form or other sufficiently bad characteristics, and those "wolf" trees that are giving, or soon will give serious competition to several trees of desirable form and development close around them.. Therefore, the average diameter breast high of the residual stand just after the present thinning treatment will be slightly greater than the eight inches to which growth predictions were added in the computations for the next stand treatment. This, in effect, introduces a slight element of conservatism as as a further protection from financially bad errors in judgment.

An order written as to what average crown opening to leave in the stand after the thinning treatment, would further clarify the thinning procedure and aid the man who subsequently selects the trees to be thinned from the stand. Since the average spacing in effect between tree stems is now 9.84, and the average spacing between tree stems after the present thinning treatment will be 14.63 feet, it would appear that the average crown opening left after the present thinning will be 4.79 feet. However, this is actually not the case since the crowns in the stand before the present thinning treatment are not just touching each other but are interlocking and overlapping due to the present over-stocked condition. The true crown opening, or average open spacing between crowns, after the present stand thinning can be computed by subtracting the product of the spacing factor of the fully-stocked stand multiplied by the present average diameter breast high in inches, from the average spacing in feet between tree stems created by the thinning treatment. Computations for this example problem are as follows:

O = average crown opening in feet immediately after thinning treatment

SF = spacing factor for stocking required as a fully-stocked stand

D' = average spacing in feet of tree stems after thinning treatment

d = average diameter breast high in inches at present

By the above reasoning the formula becomes

$$O = D' - (SF \times d)$$

$$O = 14.63 - (1.33 \times 8) = 14.63 - 10.64 -$$

$$O = 3.99 \text{ or } 4 \text{ feet}$$

The amount of crown opening to be left after the thinning treatment can be arrived at through another line of reasoning and procedure of calculation. (8) Since the average spacing between trees in feet, equals the average diameter breast high in inches multiplied by the spacing factor, the increase in desired spacing in feet between trees equals the product of the average increase in diameter breast high in inches during the intergal between thinning treatments multiplied by the spacing factor of the fully stocked stand. The computations for this approach in solving for the crown opening are as follows:

r = mean annual rate of diameter growth in inches

8. Ibid., p.5.

y = the number of years between thinning treatments

therefore

r x y = the average increase in diameter in inches between thinning treatments

and the formula becom es

$$O = SF \times r \times y \quad \text{or} \quad O = SF \times ry$$

The same crown opening is obtained, ofcousse, for this example problem as follows:

$$O = 1.33 \times 0.3 \times 10 = 3.99 \text{ or } 4 \text{ feet}$$

Actually through algebraic simplification of the first crown opening formula mentioned above the second equation can be arrived at as follows:

$$O = D' - (SF \times d)$$

D' = the average spacing between trees in feet "y" years from now

By definition

$$SF = \frac{D'}{d'}$$

D' = SF x the average diameter in inches of the trees in the stand "y" years from now or (ry + d)

$$D' = SF \times (ry + d)$$

Substituting in the above crown opening formula give

$$0 = SF \times (ry + d) - (SF \times d)$$

Algebraic factoring gives

$$0 = SF \left[(ry + d) - d \right] = SF\,(ry + d - d)$$

$$0 = SF \times ry$$

This is the same formula arrived at through the other channel of reasoning.

Orders for the management of this timber stand can now be compiled. From the average of 450 trees per acre an average of 247 trees per acre are to be removed in the present thinning. The trees cut at present should average slightly less than eight inches diameter breast high, and should be selected silviculturally so as to permit the most valuable part of the stand to produce the highest value increment possible during the next ten year period. An average of 203 trees per acre should be left in the residual stand after the present thinning treatment, and the average crown openings between these residual trees should be approximately four feet. The average distance between tree stems after treatment should be approximately 14.6 feet. The volume of the material removed in the thinning will depend on the heights and range of diameters of the trees cut, but quite an accurate estimate of this volume of cut can be made by the experienced man inspecting the stand when he knows the number of trees per acre to be removed and their approximate average diameter.

Ten years from now, just before the second treatment, the stand will have an average of 203 trees per acre with an average diameter breast high of eleven inches.

Plans for the degree of the second thinning can now be predicted if the time for the third thinning treatment, or harvest cut if there are to be no more thinning treatments, is decided upon. For this example it is assumed that the third treatment will come in twenty years from now. The computations in order to predict the degree of treatment for the second thinning are as follows:

d'' = average diameter breast high per tree in inches 20 years from now

y' = the number of years between the second and third treatments

Assuming that the rate of diameter growth predicted for the period between the first two thinning treatments can be maintained, the average diameter twenty years from now will be

$$d'' = d' + (r \times y') = 11'' + (0.3 \times 10)$$

$$d'' = 14 \text{ inches}$$

and, as for the first thinning above,

D'' = average spacing between tree stems in feet twenty years from now

$$SF = \frac{D''}{d''}$$

$$D'' = SF \times d'' = 1.33 \times 14 = 18.62 \text{ feet}$$

Number of trees per acre before treatment or harvest cut twenty years from now $= \dfrac{43{,}560 \text{ sq.ft. per acre}}{D^2}$

Number of trees per acre $= \dfrac{43{,}560}{18.62^2} = \dfrac{43{,}560}{347} = 126$ trees

Number of trees to cut in second treatment $= 203 - 126 = 77$ trees

$$O = SF \times r \times y' = 1.33 \times 0.3 \times 10$$

$$O = 3.99 \text{ or } 4 \text{ feet}$$

Orders for the second thinning treatment will contain the following instructions: of the average of 203 trees per acre, cut 77 trees per acre with an average diameter of slightly less than eleven inches, and leave an average of 126 trees per acre with an average opening between tree crowns of 3.99 or approximately four feet in the residual stand.

It can be seen that such predictions may thus be extended as far into the future as desired. However, greater caution must be used as each prediction is made extending farther into the future for the reliability in the prediction of each subsequent stand treatment is very greatly reduced. The practical limit for such specific plans for each succeeding thinning treatment probably lies somewhere around twenty years. By that time too many outside, uncontrollable factors can come to the fore to change somewhat the original plans so that such specific thinning treatment plans farther into the future would not be sufficiently reliable to warrant the effort. Large changes in market conditions is one influencing factor. Slightly more general plans can be extended as far into the future as desired.

The example problem above demonstrated the method of developing stand treatment plans for a stand in need of immediate treatment. If it is found that the timber stand in question is at present understocked, the procedure for computing the time at which the stand will become fully-stocked and need a thinning treatment is presented here in the form of another hypothetical problem as follows: cruise data shows that a particular timber stand of loblolly pine has an average of 250 trees per acre with an average diameter breast high of 8.5 inches. The predicted rate of growth is assumed to be the same as in the first hypothetical example, and the normal spacing factor value of 1.33 for a fully stocked stand of loblolly pine will be used also.

The first step is again to decide whether or not the stand needs immediate thinning treatment, and if not, then how soon will that need arise. The answer to the first question lies in solving for the present spacing factor as follows:

$$D^2 = \frac{43{,}560 \text{ square feet per acre}}{250 \text{ average number of trees per acre}}$$

$$D = \sqrt{\frac{43{,}560}{250}} = \sqrt{174.3} = 13.2 \text{ feet average spacing at present between tree stems}$$

$$SF = \frac{D}{d} = \frac{13.2}{8.5} = 1.55$$

Therefore, this stand is at present understocked since its present spacing factor value is greater than the spacing factor value for a fully-stocked stand. No treatment is needed at present. It now becomes necessary to compute the time that must elapse before the first stand treatment will be needed. The spacing factor at the time the thinning will be needed must be that of a fully-stocked stand or 1.33, and the average spacing in feet between tree stems will be the same just before thinning as it is now, 13.2 feet. As stated above, the formula for obtaining the future average diameter breast high in inches is $d' = d + ry$, or the present average diameter plus the product of the mean annual diameter growth multiplied by the number of years to the next thinning. Therefore, the equation is as follows:

$$SF = \frac{D}{d+ry}$$

in which the only unknown is "y", the number of years before the thinning treatment. The following is the algebraic solution for "y":

cross multiplying,

$$SF\ (d) + SF\ (ry) = D$$

$$SF\ (ry) = D - SF(d)$$

$$Y = \frac{D - SF(d)}{SF(r)}$$

The following is the solution for "y" in this hypothetical problem:

D = 13.2 feet

SF = 1.33, the spacing factor of a fully-stocked stand

d = 8.5 inches

r = 0.3 inches

$$y = \frac{13.2 - (1.33 \times 8.5)}{1.33 \times 0.3} = \frac{13.2 - 11.3}{0.399} = \frac{1.9}{0.399} = 4.77 \text{ years}$$

The first thinning treatment for this particular timber stand should be scheduled 4.77 or approximately four to five years from now. The degree of thinning will depend on the length of time interval between the first thinning and the second, and on the anticipated rate of growth during that period. It can be computed by the same procedure as in the first hypothetical problem described above. Computations for the second thinning treatment and other future thinning treatments can also be made in the same manner as those for the first example problem.

General tables can be constructed with the use of the crown opening formula, O = SF x ry, to further simplify the management plan computations in keeping with the goal of quickly and easily making concrete plans on-the-spot for timber stands.(9) A separate table must be constructed for each different full stocking spacing factor used. Table IV lists crown opening values for full stocking spacing factor values of 1.33 and 1.25. These values correspond respectively to crown-stem ratios of 16 to 1 and 15 to 1.

Table IV can be directly applied in computations for the construction of orders as to when and to what degree the thinning treatments should be made. The table indicates directly the amount of average crown opening there should be immediately after a specific stand thinning treatment is made. This crown opening value plus the product of the average diameter breast high in inches of the stand at the time of the thinning multiplied by the spacing factor of the fully-stocked stand will give average spacing in feet between tree stems that should exist after the thinning.

The procedure for the application of Table IV can better be demonstrated by an example problem as follows: The average spacing of trees in a loblolly pine stand is ten feet and the average diameter breast high is

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9. Ibid., p.6.

TABLE IV. ---AVERAGE CROWN OPENINGS IN FEET TO BE LEFT IN STAND THINNING TREATMENTS

Average annual d.b.h. increment in inches	Crown opening (or increase in spacing above the present full-stocking crown-stem ratio of 16 to 1) in feet for thinning intervals of: 5 years	10 years	15 years	20 years
Fully-stocked stand spacing factor 1.33 Crown-stem ratio 16 to 1				
0.10	0.67	1.33	2.00	2.67
0.15	1.00	2.00	3.00	4.00
0.20	1.33	2.66	4.00	5.33
0.25	1.67	3.33	5.00	6.67
0.30	2.00	4.00	6.00	8.00
0.35	2.33	4.67	7.00	9.33
0.40	2.66	5.33	8.00	10.66
Fully-stocked stand spacing factor 1.25 Crown-stem ratio 15 to 1				
0.10	0.62	1.25	1.88	2.50
0.15	0.94	1.88	2.81	3.75
0.20	1.25	2.50	3.75	5.00
0.25	1.56	3.12	4.69	6.25
0.30	1.88	3.75	5.62	7.50
0.35	2.19	4.38	6.56	8.75
0.40	2.50	5.00	7.50	10.00

nine inches. The predicted rate of diameter growth in inches under management is 0.25 and the spacing factor for the fully-stocked stand is 1.33. Therefore, the present spacing factor value is as follows:

$$SF = \frac{D}{d} = \frac{10 \text{ feet}}{9 \text{ inches}} = 1.11$$

The stand is overstocked and in need of immediate treatment since this spacing factor value is less than that of a fully-stocked stand spacing factor. It is assumed here that this stand cannot again be visited until 10 years hence. Immediately from the table we see that a crown opening averaging 3.33 feet must be left after the present thinning. The average spacing between tree stems after the present thinning will be this 3.33 feet plus the desired normal spacing for trees of the present average diameter, or, as stated in another way, the product of the present average diameter in inches multiplied by the spacing factor of the fully-stocked stand. The computations are as follows:

O = crown opening in feet

SF = spacing factor of fully-stocked stand

d = present diameter breast high in inches

Spacing between tree stems in feet after thinning = O + (SF x d)

Spacing between tree stems = 3.33 + (1.33 x 9) = 3.33 + 11.97 = 15.3 feet

Therefore, the stand should be thinned immediately and in such a manner as to leave an average spacing between tree stems of 15.3 feet and an average crown opening of 3.33 feet.

FURTHER APPLICATIONS OF THE SPACING FACTOR IN PORTRAYING STAND STOCKING

The spacing factor as it has been presented in the equation form is somewhat difficult to visualize in its effect on the timber stand, particularly since spacing factor values vary inversely with basal area per acre and number of trees per acre. The author of this thesis has, therefore, devised and constructed a system of tables and graphs to better portray the influence of spacing factor values on stand stocking and for better visualization of the spacing factor plan of management.

First, by definition we have in equation form

$$SF = \frac{D}{d}$$

and

$$\text{Number of trees per acre} = \frac{43,560}{D^2}$$

By cross multiplication in the first equation we have

$$D = SF \times d$$

and substituting in the second equation gives

$$\text{Number of trees per acre} = \frac{43,560}{(SF \times d)^2}$$

This resulting equation can now be used in constructing a table to show the number of trees per acre for each given spacing factor and average stand diameter. These figures are presented in Table V.

A demonstration of the procedure of computation for the construction of Table V is as follows: assume an average stand diameter breast high of 16 inches and a spacing factor of 1.4. Placing these values in the equation gives .

$$\text{Number of trees per acre} = \frac{43,560}{(SF \times d)^2} = \frac{43,560}{(1.4 \times 16)^2}$$

$$\frac{43,560}{(1.4 \times 16)^2} = \frac{43,560}{(22.4)^2} = \frac{43,560}{501.76}$$

$$\frac{43,560}{501.76} = 87 \text{ trees per acre}$$

It is possible to employ this table to simplify the drawing up of management plans for uniformly stocked, even-aged stands. This can be shown in the following hypothetical problem: a certain stand has an average diameter breast high of nine inches and an average spacing between tree stems of eleven feet. Because of management objectives, growth rate, and conditions in the stand it is planned to thin again after three more inches are added to the average stand diameter, and adopt and maintain a spacing factor of 1.3. The average number of trees per acre at present in the stand is computed as follows:

$$D = 11 \text{ feet}$$

$$\text{Number of trees per acre} = \frac{43{,}560}{D^2} = \frac{43{,}560}{11^2}$$

$$\frac{43{,}560}{121} = 360 \text{ trees per acre}$$

Referring to Table V for the number of trees per acre corresponding to a spacing factor of 1.3 and an average stand diameter of nine inches, gives directly 318 trees per acre. Since the stand now contains 360 trees per acre, we know the stand requires immediate treatment. To thin again after an addition of three inches in average diameter would mean to come to the stand when it has an average diameter of twelve inches. Referring to Table V again produces the information that for an

TABLE V.---NUMBER OF TREES PER ACRE FOR EACH SPACING FACTOR AND AVERAGE STAND DIAMETER

Average diameter of stand	Spacing Factor					
	0.60	0.70	0.80	0.90	1.00	1.10
Inches	Number of trees per acre					
4	7,562	5,556	4,254	3,361	2,722	2,250
5	4,840	3,556	2,722	2,151	1,742	1,440
6	3,361	2,469	1,891	1,494	1,210	1,000
7	2,469	1,814	1,389	1,098	889	735
8	1,891	1,389	1,063	840	681	562
9	1,494	1,098	840	664	538	444
10	1,210	889	681	538	436	360
11	1,000	735	562	444	360	298
12	840	617	473	373	302	250
13	716	526	403	318	258	213
14	617	454	347	274	222	184
15	538	395	302	239	194	160
16	473	347	266	210	170	141
17	419	308	235	186	151	125
18	373	274	210	166	134	111
19	335	246	189	149	121	100
20	302	222	170	134	109	90
21	274	202	154	122	99	82
22	250	184	141	111	90	74
23	229	168	129	102	82	68
24	210	154	118	93	76	62
25	194	142	109	86	70	58
26	179	132	101	80	64	53
27	166	122	93	74	60	49
28	154	113	87	69	56	46
29	144	106	81	64	52	43
30	134	99	76	60	48	40

TABLE V.---continued

Average diameter of stand	Spacing Factor 1.20	1.30	1.40	1.50	1.60
Inches	Number of trees per acre				
4	1,891	1,611	1,389	1,210	1,063
5	1,210	1,031	889	774	681
6	840	716	617	538	473
7	617	526	454	395	347
8	473	403	347	302	266
9	373	318	274	239	210
10	302	258	222	194	170
11	250	213	184	160	141
12	210	179	154	134	118
13	179	153	132	115	101
14	154	132	113	99	87
15	134	115	99	86	76
16	118	101	87	76	66
17	105	89	77	67	59
18	93	80	69	60	53
19	84	71	62	54	47
20	76	64	56	48	43
21	69	58	50	44	39
22	62	53	46	40	35
23	57	49	42	37	32
24	53	45	39	34	30
25	48	41	36	31	27
26	45	38	33	29	25
27	41	35	30	27	23
28	39	33	28	25	22
29	36	31	26	23	20
30	34	29	25	22	19

average diameter of twelve inches and a spacing factor of 1.3 the stand must have an average of 179 trees per acre. Therefore, it is quickly concluded in the management plan to thin immediately and remove an average of 360-179 or 181 trees per acre.

The author has also constructed Table VI for further simplification of the computations necessary in drawing up management plans by the spacing factor method. Table VI gives the average distance in feet between tree stems in a uniformly stocked, even-aged stand for each spacing factor value and average diameter of stand.

Table VIwas constructed in the following manner:

by definition

$$SF = \frac{D}{d}$$

Cross multiplication of the above equation gives

$$D = SF \times d$$

which is the formula used in compiling the table.

For the above hypothetical problem demonstrating the use of Table V, we can now obtain the average spacing in feet between tree stems in the stand that must be left after the present thinning by reading it directly from Table VI as follows: the spacing factor of the stand is to be maintained at 1.3 and the average

TABLE VI.---AVERAGE SPACING IN FEET BETWEEN TREE STEMS FOR EACH SPACING FACTOR AND AVERAGE STAND DIAMETER

Average diameter of stand	Spacing Factor 0.60	0.70	0.80	0.90	1.00	1.10
Inches	Average spacing between tree stems in feet					
4	2.4	2.8	3.2	3.6	4.0	4.4
5	3.0	3.5	4.0	4.5	5.0	5.5
6	3.6	4.2	4.8	5.4	6.0	6.6
7	4.2	4.9	5.6	6.3	7.0	7.7
8	4.8	5.6	6.4	7.2	8.0	8.8
9	5.4	6.3	7.2	8.1	9.0	9.9
10	6.0	7.0	8.0	9.0	10.0	11.0
11	6.6	7.7	8.8	9.9	11.0	12.1
12	7.2	8.4	9.6	10.8	12.0	13.2
13	7.8	9.1	10.4	11.7	13.0	14.3
14	8.4	9.8	11.2	12.6	14.0	15.4
15	9.0	10.5	12.0	13.5	15.0	16.5
16	9.6	11.2	12.8	14.4	16.0	17.6
17	10.2	11.9	13.6	15.3	17.0	18.7
18	10.8	12.6	14.4	16.2	18.0	19.8
19	11.4	13.3	15.2	17.1	19.0	2019
20	12.0	14.0	16.0	18.0	20.0	22.0
21	12.6	14.7	16.8	18.9	21.0	23.1
22	13.2	15.4	17.6	19.8	22.0	24.2
23	13.8	16.1	18.4	29.7	23.0	25.3
24	14.4	16.8	19.2	21.6	24.0	26.4
25	15.0	17.5	2010	22.5	25.0	27.5
26	15.6	18.2	20.8	23.4	26.0	28.6
27	16.2	18.9	21.6	24.3	27.0	29.7
28	16.8	19.6	22.4	25.2	28.0	30.8
29	17.4	20.3	23.2	26.1	29.0	31.9
30	18.0	21.0	24.0	27.0	30.0	33.0

TABLE VI.---continued

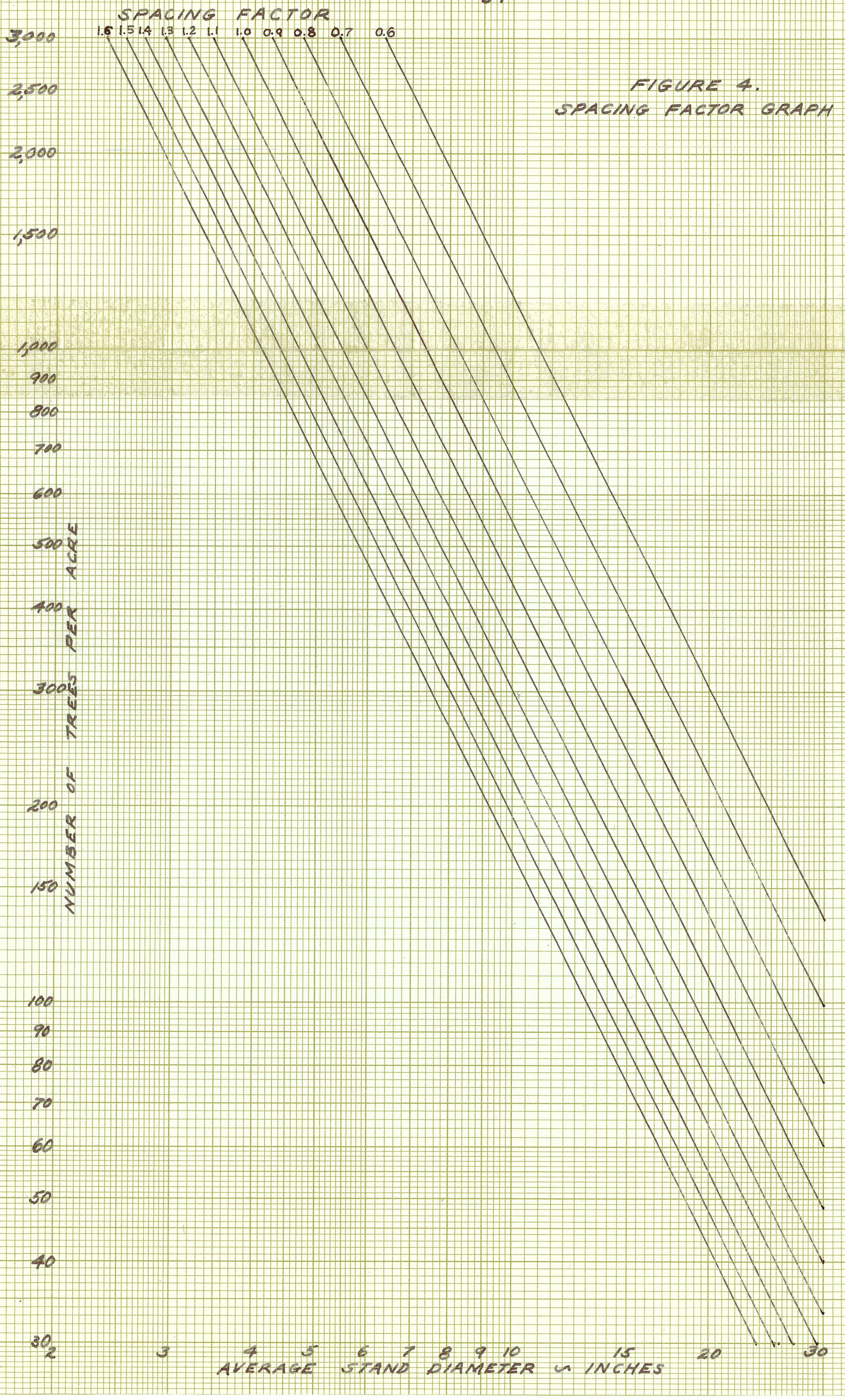
Average diameter of stand	Spacing Factor				
	1.20	1.30	1.40	1.50	1.60
Inches	Average spacing between tree stems in feet				
4	4.8	5.2	5.6	6.0	6.4
5	6.0	6.5	7.0	7.5	8.0
6	7.2	7.8	8.4	9.0	9.6
7	8.4	9.1	9.8	10.5	11.2
8	9.6	10.4	11.2	12.0	12.8
9	10.8	11.7	12.6	13.5	14.4
10	12.0	13.0	14.0	15.0	16.0
11	13.2	14.3	15.4	16.5	17.6
12	14.4	15.6	16.8	18.0	19.2
13	15.6	16.9	18.2	19.5	20.8
14	16.8	18.2	19.6	21.0	22.4
15	18.0	19.5	21.0	22.5	24.0
16	19.2	20.8	22.4	24.0	25.6
17	20.4	22.1	23.8	25.5	27.2
18	21.6	23.4	25.2	27.0	28.8
19	22.8	24.7	26.6	28.5	30.4
20	24.0	26.0	28.0	30.0	32.0
21	25.2	27.3	29.4	31.5	33.6
22	26.4	28.6	30.8	33.0	35.2
23	27.6	29.9	32.2	34.5	36.8
24	28.8	31.2	33.6	36.0	38.4
25	30.0	32.5	35.0	37.5	40.0
26	31.2	33.8	36.4	39.0	41.6
27	32.4	35.1	37.8	40.5	43.2
28	33.6	36.4	39.2	42.0	44.8
29	34.8	37.7	40.6	43.5	46.4
30	36.0	39.0	42.0	45.0	48.0

diameter breast high at the time of the next thinning will be 12 inches. Reference to Table VI at the point corresponding with these two values gives 15.6 feet average distance between tree stems after the present thinning treatment. This procedure saves no time for smaller average diameters, but is an aid when working with larger average stand diameters and larger spacing factors, and it provides less chance for error in computations.

For a graphical picture of the timber stand under management by the spacing factor method the author has devised the graph procedure as presented in Figure 4. The author constructed Table V with the employment of the formula

$$\text{Number of trees per acre} = \frac{43,560}{(SF \times d)^2}$$

Since this is a second degree equation, it should plot as a straight line on logarithmic paper if the spacing factor is taken as a constant. A spacing factor of 1.3 plots as a diagonal straight line. Since Table V was compiled by use of the same formula, its information can be used in locating all the desired spacing factor parallel diagonal lines, and this is


SPACING FACTOR
1.6 1.5 1.4 1.3 1.2 1.1 1.0 0.9 0.8 0.7 0.6
FIGURE 4.
SPACING FACTOR GRAPH
NUMBER OF TREES PER ACRE
3,000 2,500 2,000 1,500 1,000 900 800 700 600 500 400 300 200 150 100 90 80 70 60 50 40 30
2 3 4 5 6 7 8 9 10 15 20 30
AVERAGE STAND DIAMETER ~ INCHES

the manner in which Figure IV was constructed. The equation of the slope of these parallel, diagonal spacing factor lines is $d^2 = 1/n$ or, in logarithmic form for the graph, $2 \log d = -\log n$.

This method of plotting spacing factor lines on logarithmic graph paper corresponds closely to the plotting of stand density charts in which a constant basal area, or constant percentages of a chosen basal area, are plotted on logarithmic graph paper. (11) The spacing factor graph presents the same information as the stand density chart, but presents this information in terms of the spacing factor which, as discussed previously, is more easily employed in planning stand management.

Existing uniformly stocked, even-aged stands under management can be plotted on this graph by knowing the number of trees per acre and average diameter breast high before and after each thinning treatment. Existing managed stands can thus be studied in relation to how they conform to the parallel, diagonal spacing

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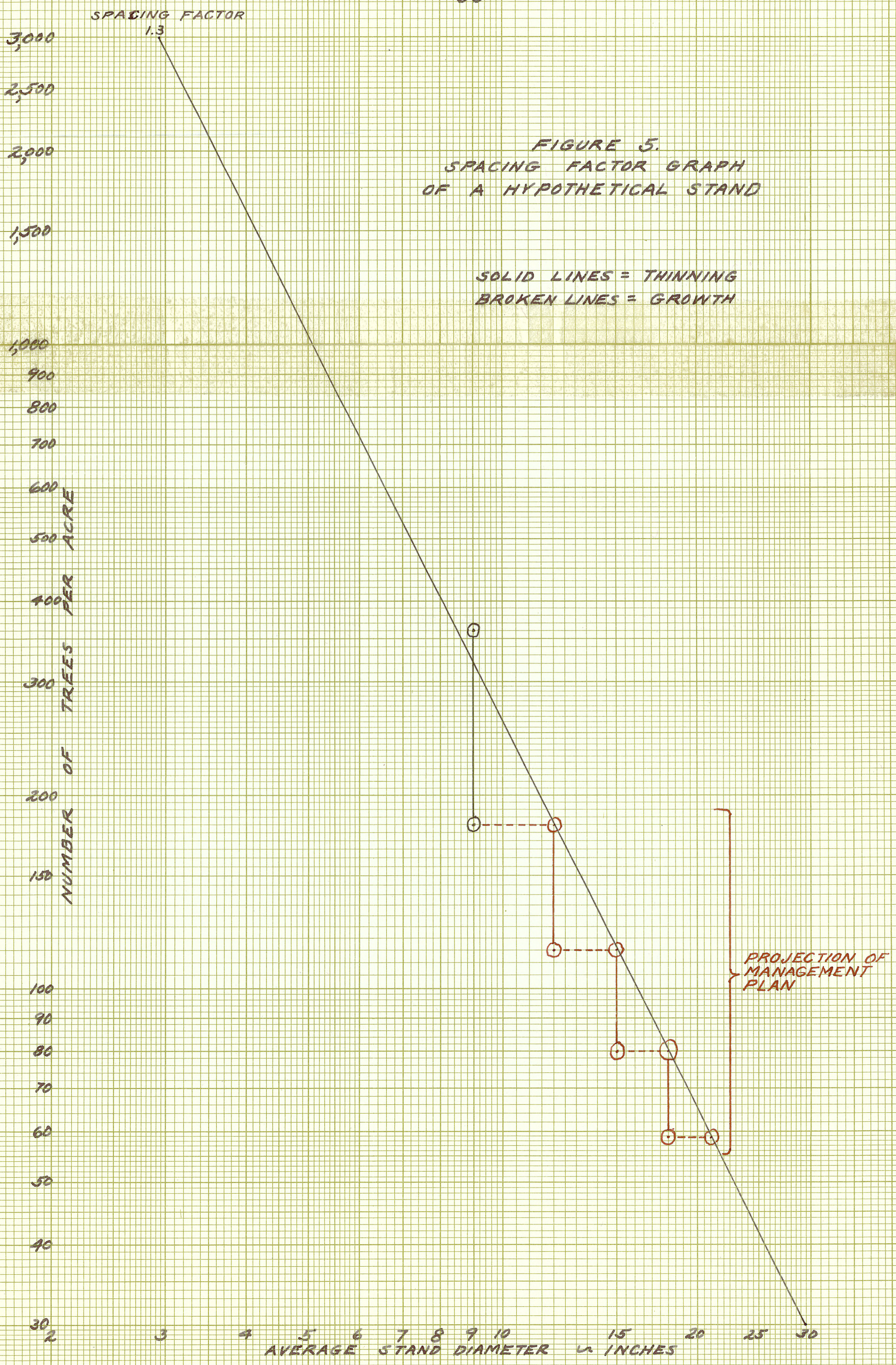
11. Schnur, G. Luther; Yield, Stand and Volume Tables for Even-aged Upland Oak Forests, United States Department of Agriculture, Technical Bulletin No. 560, April, 1937; p.38.

factor lines. Properly managed stands under the spacing factor method should plot in stair-step fashion parallel to the spacing factor lines. Desirable spacing factors for future stands to be placed under management can be decided upon by plotting a similar stand already being managed that has been deemed by technical foresters as having just about the right stocking. The spacing factor line, along which the existing managed stand plots on the graph, can be chosen as being a good spacing factor to be applied to the similar stand that is about to be put under management. Yield table data can also be plotted in this manner and may sometimes provide good information.

The stand data in the hypothetical problem employed to provide an example in the applications of Tables V and VI can now be plotted on logarithmic graph paper as shown in Figure V. The present plans for management of the stand are shown in black with solid lines indicating the effect of the thinning treatments and dashed lines denoting growth. Future management plans can be plotted on the graph to give a simple, complete picture. For this hypothetical stand, we will assume the plan is to thin after each additional three inches of average diameter growth and just as the stands reaches the spacing factor of 1.3. This projection of the management plan is shown in Figure 5 in red. We will also assume that either the harvest cut or the period of accretion cuttings will come when this stand reaches an average stand diameter of 21 inches. These also could be plotted on the graph.

Response of the timber stand to management by the spacing factor method can be recorded by plotting basal area values over average stand diameter breast high on regular graph paper. Equivalent spacing factor values can be plotted on the vertical axis in the same manner as basal areas, and showing both values on the same graph aids the forest manager in approximate conversions to either unit, and gives a more complete picture. In the hypothetical problem employed above to show the applications of Tables V and VI and Figures 4 and 5, the timber stand had not been managed before so no response to previous stand thinning treatments can be plotted. However, at the time the stand reaches 21 inches average diameter the stand's response to the past thinning treatments should theoretically plot somewhat as shown in Figure 6. Actually the stand's response will not conform exactly to the theoretical



SPACING FACTOR
1.3
FIGURE 5.
SPACING FACTOR GRAPH
OF A HYPOTHETICAL STAND
SOLID LINES = THINNING
BROKEN LINES = GROWTH
PROJECTION OF MANAGEMENT PLAN
NUMBER OF TREES PER ACRE
3,000
2,500
2,000
1,500
1,000
900
800
700
600
500
400
300
200
150
100
90
80
70
60
50
40
30
2
3
4
5
6
7
8
9
10
15
20
25
30
AVERAGE STAND DIAMETER ~ INCHES

picture shown in Figure 6, but well made and well carried out plans will force the stand's response to closely approach this ideal. The timber stand could actually show either greater or less response than is shown in Figure 6. The act of stand thinning in itself could affect the average stand diameter somewhat if average stand diameter for management of this stand is based on all trees in the stand instead of only final crop trees.

Note that in Figure 6 the practice of thinning under the spacing factor management method each time that a constant number of inches in average stand diameter is added to the timber stand causes the spacing factor values (and consequently basal area per acre also) that are produced just after each stand treatment to plot in a smooth curve for the entire period of thinnings. A similar, harmonious curve would occur under any management plan using the spacing factor method and a plan to thin after each addition of a constant diameter increment. These curves for spacing factors and constant average diameter increments could be plotted and used as an aid in governing the management of a stand. In fact, this method could be plotted on separated graphs and be used to project the

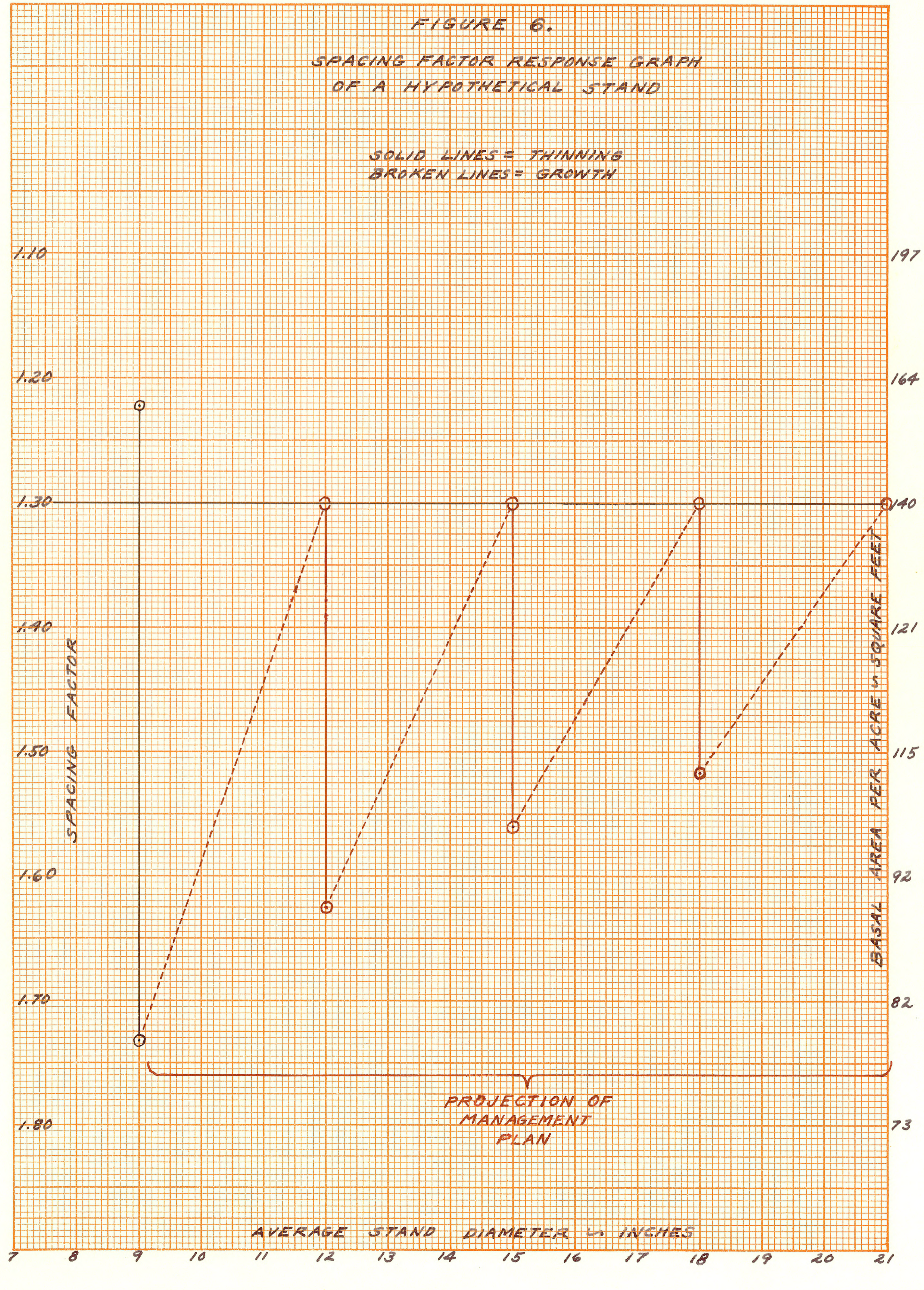


FIGURE 6.
SPACING FACTOR RESPONSE GRAPH
OF A HYPOTHETICAL STAND
SOLID LINES = THINNING
BROKEN LINES = GROWTH
SPACING FACTOR
BASAL AREA PER ACRE - SQUARE FEET
1.10
1.20
1.30
1.40
1.50
1.60
1.70
1.80
197
164
140
121
115
92
82
73
PROJECTION OF
MANAGEMENT
PLAN
AVERAGE STAND DIAMETER - INCHES
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21

management plans for the standas Figure 5 does. Actual stand response would then be pictured on a separate graph.

POSSIBLE APPLICATION OF THE SPACING FACTOR IN FOREST MENSURATION

It can be seen that the spacing factor also has a possible use in forest mensuration, but its most important application is in that phase of forest management for determining thinning treatments and expressing stand stocking. Its use in forest mensuration has been studied little as yet except for the one application in the estimation of timber volume which follows. (10)

The spacing factor of a timber stand can be converted to the corresponding basal area per acre by use of Table I or Figure 1. The cubic foot volume per acre can now be obtained by multiplying this basal area by the average tree height in the stand and then multiplying by the cubic foot form factor. This is shown in formula form as follows:

$$\text{Volume per acre in cubic feet} = FF \times BA \times H$$

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10. Matthews, Donald M., Illustration of Method of Using the Spacing Factor in Timber Estimation, mimeograph, 1947.

To convert cubic feet per acre to cords per acre, divide the number of cubic feet per acre by the number of cubic feet of solid rough wood per cord for that particular species, size of tree, and straightness of stem. Conversion of cubic feet per acre into board feet volume per acre is a little more difficult. A new spacing factor must first be computed. The average diameter breast high in inches is multiplied by the decimal percentage factor for that tree species to obtain the middle scaling diameter. The average spacing between tree stems in feet is now divided by this middle scaling diameter value to give the new, larger spacing factor value. The corresponding basal area per acre is now found by the use of Table I or Figure 1, and this basal area in square feet per acre is multiplied by the estimated merchantable cubic feet per acre. The number of cubic feet per acre is multiplied by the number of board feet per cubic foot obtained by one of the following formulae: (12)

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12. Matthews, Donald M., Basic Formulas for Forest Product Measurement, mimeograph, University of Michigan, p.3.

1) for Clark International 1/8" Rule

D = middle scaling diameter in inches

$$\text{Board feet per cubic foot} = 10 - \frac{32.5}{D}$$

2) for Clark International 1/4" Rule

$$\text{Board feet per cubic foot} = (10 - \frac{32.5}{D})0.905$$

3) for International 1/4" Rule

$$\text{Board feet per cubic foot} = (10 - \frac{32.5}{D})0.905$$

or

$$\text{Board feet per cubic foot} = 9.1 - \frac{29.5}{D}$$

Determining board foot volume by use of the spacing factor is demonstrated in the following example problem: the average spacing between tree stems for a certain timber stand is twenty feet and the average diameter breast high is fourteen inches. It is assumed that this is a stand of southern pine, and for such semi-tolerant pines the middle scaling diameter equals about 70% of the diameter breast high outside of the bark. The average saw log merchantable length is estimated to be three logs or 48 feet. The average middle scaling diameter inside bark equals 14 inches multiplied by 0.70 and is 9.8 inches. The spacing factor is the quotient of 20 divided by 9.8 and is 2.04. The corresponding basal area per acre for this spacing factor value of 2.04 is 57 square feet per acre. The number of cubic feet of merchantable saw timber equals 57 square feet multiplied by the 48 feet of merchantable height and is 2,735 cubic feet. Using the Clark International 1/8" Rule, the computation for the number of board feet per acre is as follows:

$$\text{Volume in board feet per acre} = 2{,}735\left(10-\frac{32.5}{9.8}\right)$$

$$\text{Volume in board feet per acre} = 2{,}735\,(10-3.32)$$

$$\text{Volume of the stand} = 18{,}270 \text{ board feet per acre}$$

Therefore, a timber stand of semi-tolerant southern pine averageing 14 inches diameter breast high, three logs (48 feet) high and 20 feet between tree stems will run 18,270 board feet measure per acre by Clark International 1/8" Rule.

THE HEIGHT FACTOR

The height factor is a recent development in the attempt to find a convenient numerical method of expressing stocking in uniformly stocked, even-aged stands. The height factor is defined as that constant fraction or percentage of the average dominant height of the uniformly stocked, even-aged stand at which the trees are spaced in the stand for the full stocking appropriate for the species.

The chief proponent in the United States of the height factor as an expression of stand stocking is F.G. Wilson, Superintendent of Cooperative Forestry, Wisconsin Conservation Department, Madison, Wisconsin. He states that height, within reasonable limits of stocking, is negligibly affected by spacing, and that height has the virtue of combining the components of age and site in one measurement. (13) He believes

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13. Wilson, F.G., "Numerical Expression of Stocking in Terms of Height", Journal of Forestry; Vol.44, No.10, October, 1946; pp.758-761.

that height is as suitable for the purpose of simple numerical expression of stocking as it is for the determination of site quality.

The height factor method of forest management is to thin uniformly stocked, even-aged stands to a number of trees per unit area based on the spacing of a chosen fraction of the height of the stand. For any species the height factor formula for normal stocking on the acre becomes

$$n = \frac{43,560}{(hf)^2}$$

n = number of trees per acre

h = average dominant height of uniformly stocked, even-aged stand

f = certain fraction of height appropriate for the species

Wilson has plotted this formula on logarithmic graph paper. This height factor formula is a second degree equation, and therefore, plots as a straight line on logarithmic graph paper if "f" is constant. Wilson observed that the lines for different values of "f" are parallel and their commom slope can be expressed by the logarithmic equation $2 \log h = -\log n$. The antilog of this equation is $h^2 = 1/n$. This equation indicates in mathematical form that the ratio of number of trees to the square of the average dominant height is the same for all ages and on all sites. Wilson states that this suggests a law of normal stocking for closed stands but that its validity can only be determined through experimentation. However, some support of the equation can be found through the plotting of good, existing yield tables.

Wilson employs the following method in plotting his height factor lines on logarithmic graph paper:

$$10 \text{ square chains} = 1\text{acre}$$

therefore

$$(66 \text{ feet})^2 \times 10 = 1 \text{ acre}$$

and

$$6^2 \times 11^2 \times 10 = 1 \text{ acre}$$

For number of trees per acre he now computes

$$6^2 = 11^2 \times 10$$ or 1,210 trees per acre at a spacing of 6 feet between tree stems

$$11^2 = 6^2 \times 10$$ or 360 trees per acre at a spacing of 11 feet between tree stems

$$\frac{11}{2} = 5\tfrac{1}{2} \text{ feet}$$

$(5\frac{1}{2})^2 = 360 \times 2^2$ or 1440 trees per acre at a spacing of $5\frac{1}{2}$ feet between tree stems

$11 \times 2 = 22$ feet

$22^2 = 360 \times (\frac{1}{2})^2$ or 90 trees per acre at a spacing of 22 feet between tree stems

Thus Wilson chooses 1440, 360, and 90 trees per acre as conveniently spaced values on the logarithmic graph paper for plotting the diagonal height factor spacing lines. These values in number of trees per acre can now be plotted over the height which is computed by multiplying the respective tree spacings of 5.5, 11 and 22 feet by the denominator of the height fraction or by dividing them by the percent of height. This can be expressed in formula form in the following manner:

$$n = \frac{43,560}{(hf)^2}$$

$$n = \frac{43,560}{(\text{average spacing between trees in feet})^2}$$

Therefore

Average spacing between trees in feet $= hf$

As an example, let the spacing between trees be 5.5 feet and the height factor be 1/5 or 20%. These substituted numerical values give

$$5.5 = \frac{1}{5} \times h$$

$$5.5 \times 5 = h$$

$$h = 27.5 \text{ feet}$$

and in percent form we have

$$5.5 = 0.20 \times h$$

$$\frac{5.5}{0.20} = h$$

$$h = 27.5 \text{ feet}$$

In this manner Table VII, the number of trees per acre over average dominant height at each height fraction, and Table VIII, the number of trees peracre over average dominant height at each height percentage value, were constructed.

Table VII is employed in plotting the parallel, diagonal lines of fractions of average dominant height on logarithmic graph paper as presented on Wilson's logarithmic sheet in Figure 7. Table VIII is employed

TABLE VII.---AVERAGE HEIGHT OF DOMINANT TREES CORRESPONDING TO THE NUMBER OF TREES PER ACRE AT EACH FRACTION OF HEIGHT

Fractions of Height	Number of Trees per Acre		
	1440	360	90
Feet	Average Height of Trees in Feet		
1/3	16.5	33	66
1/4	22	44	88
1/5	27.5	55	110
1/6	33	66	132
1/7	38.5	77	154
1/8	44	88	176
1/9	49.5	99	198

TABLE VIII.---AVERAGE HEIGHT OF DOMINANT TREES CORRESPONDING TO THE NUMBER OF TREES PER ACRE AT EACH PERCENTAGE OF HEIGHT

Percent of Height	Number of Trees per Acre		
Feet	1440	360	90
	Average Height of Trees in Feet		
10	55.0	110.0	220.0
11	50.0	100.0	200.0
12	45.8	91.7	183.3
13	42.3	84.6	169.2
14	39.3	78.6	157.1
15	36.7	73.3	146.7
16	34.4	68.8	137.5
17	32.3	64.7	129.4
18	30.6	61.1	122.2

TABLE VIII.---continued

Percent of Height	Number of Trees per Acre		
Feet	1440	360	90
	Average Height of Trees in Feet		
19	28.9	57.9	115.8
20	27.5	55.0	110.0
21	26.2	52.4	104.8
22	25.0	50.0	100.0
23	23.9	47.8	95.7
24	22.9	45.8	91.7
25	22.0	44.0	88.0
26	21.1	42.3	84.6
27	20.4	40.7	81.5
28	19.6	39.3	78.6
29	19.0	37.9	75.9
30	18.3	36.7	73.3
31	17.7	35.5	71.0
32	17.2	34.4	68.8
33	16.7	33.3	66.7
34.	16.2	32.4	64.7

similarly in plotting the lines of percents of height in Figure 8. It was stated above that the logarithmic equation of the slope of the parallel lines of the fractions or percents of height as shown in Figures 7 and 8 indicates that the ratio of the number of trees per acre to the square of the average dominant height

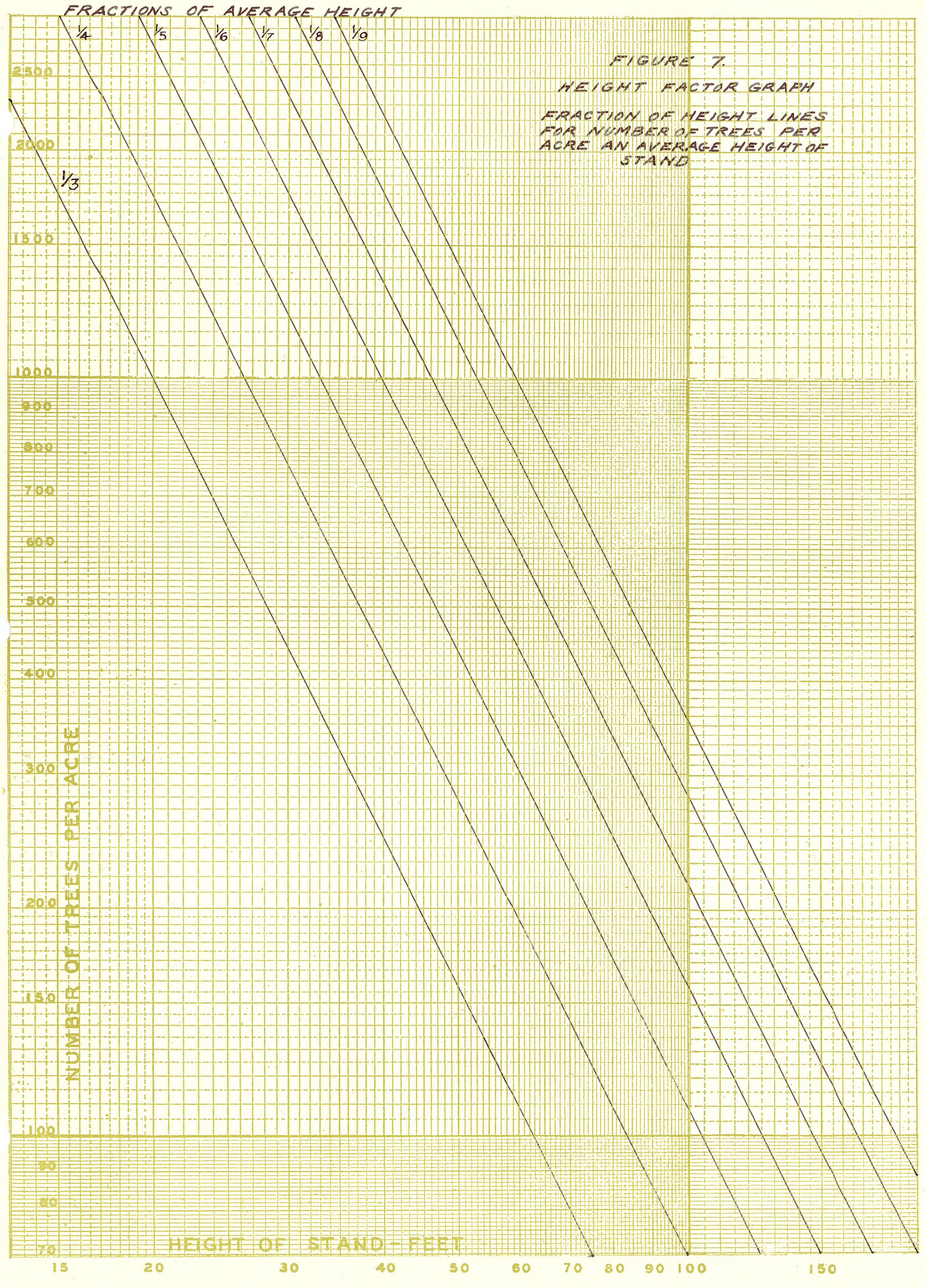
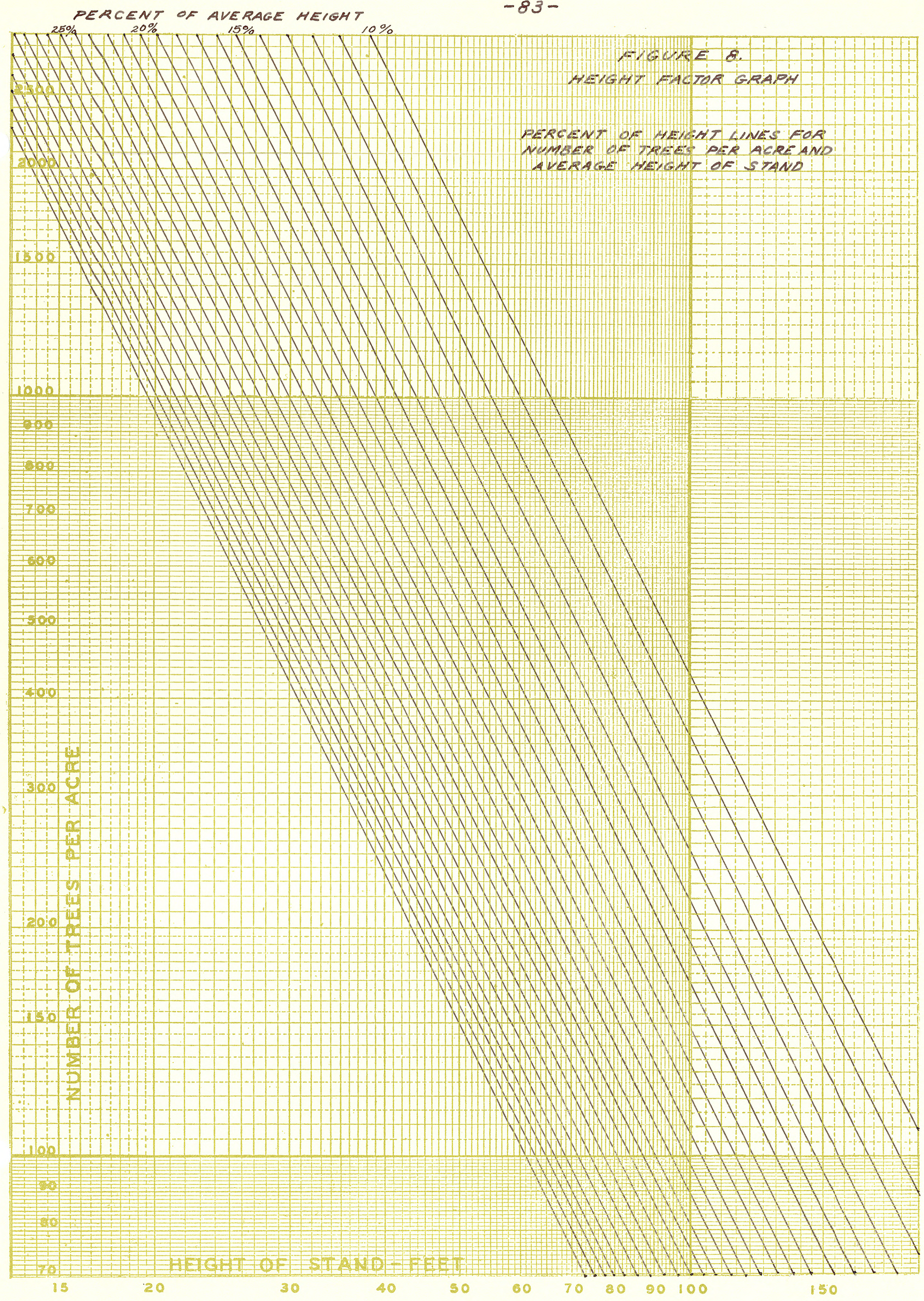

FRACTIONS OF AVERAGE HEIGHT
1/3
1/4
1/5
1/6
1/7
1/8
1/9
FIGURE 7.
HEIGHT FACTOR GRAPH
FRACTION OF HEIGHT LINES FOR NUMBER OF TREES PER ACRE AN AVERAGE HEIGHT OF STAND
NUMBER OF TREES PER ACRE
2500
2000
1500
1000
900
800
700
600
500
400
300
200
150
100
90
80
70
HEIGHT OF STAND - FEET
15
20
30
40
50
60
70
80
90
100
150

FIGURE 8.

HEIGHT FACTOR GRAPH

PERCENT OF HEIGHT LINES FOR NUMBER OF TREES PER ACRE AND AVERAGE HEIGHT OF STAND



is the same for all ages and sites. Also it was stated that the validity in this can only be determined through experimentation. Good, existing yield tables do give some support to this theory, however. An example of how yield tables plot on this logarithmic paper with sloping height factor lines is shown in Figure 9. The yield table employed was selected by R.C. Hawley as an example of good yield tables. (14) This yield table was plotted by the height factor method by F.G. Wilson to demonstrate the possible correlation between good yield tables and the height factor logarithmic slope theory.

F.G. Wilson sets forth two tables showing spacing of all trees in percent of average height of dominant and codominant trees for certain species in different site qualities and at various stand ages. (15) These tables are reproduced here in Tables IX and X.

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14. Hawley, Ralph C.; <u>The Practice of Silviculture</u>, John Wiley and Sons, Inc.; Fourth Edition, 1937; p. 192.

15. Wilson, F.G., op.cit., p. 759.

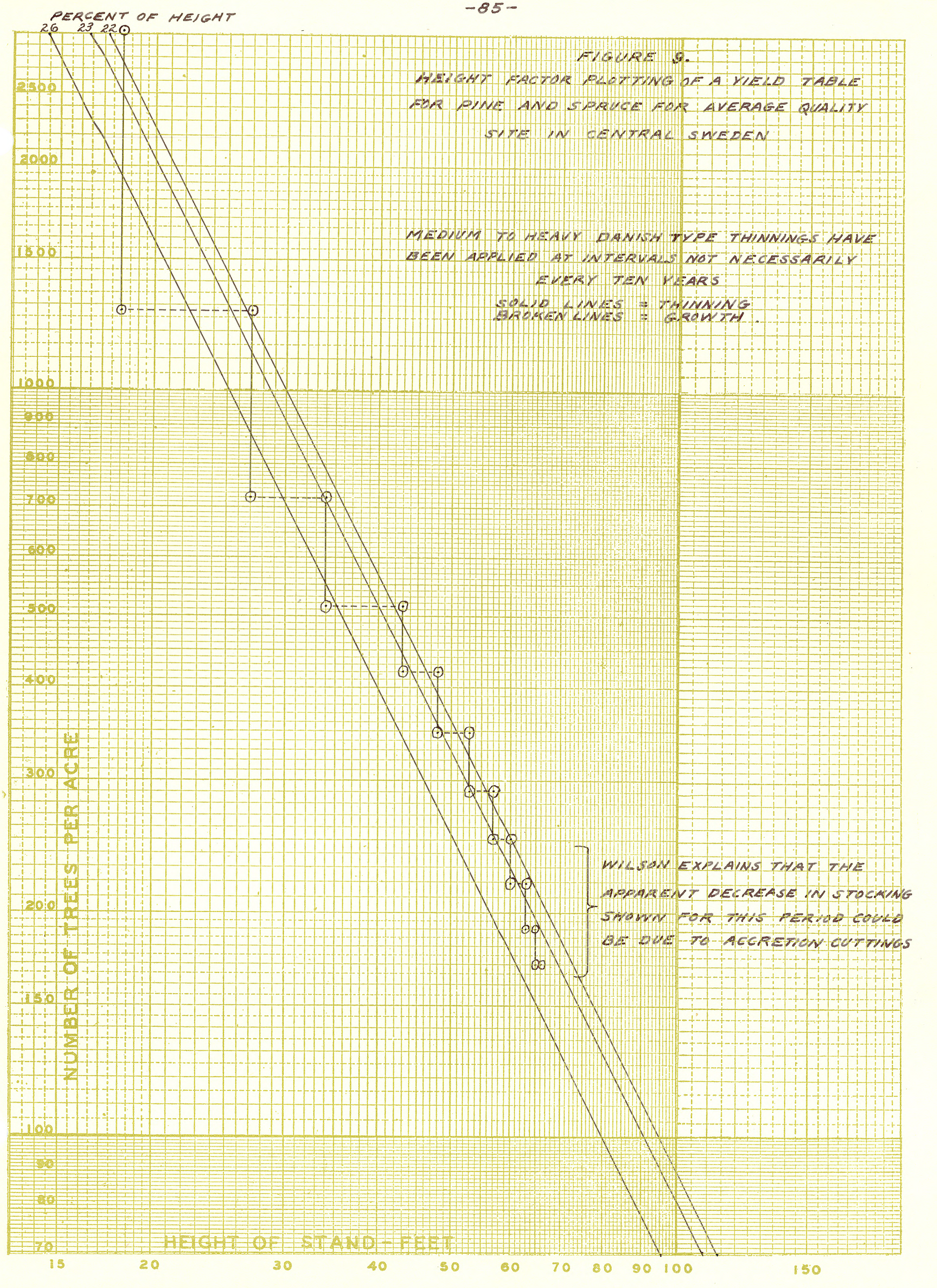
PERCENT OF HEIGHT
26 23 22
FIGURE 9.
HEIGHT FACTOR PLOTTING OF A YIELD TABLE
FOR PINE AND SPRUCE FOR AVERAGE QUALITY
SITE IN CENTRAL SWEDEN
MEDIUM TO HEAVY DANISH TYPE THINNINGS HAVE
BEEN APPLIED AT INTERVALS NOT NECESSARILY
EVERY TEN YEARS
SOLID LINES = THINNING
BROKEN LINES = GROWTH.
WILSON EXPLAINS THAT THE
APPARENT DECREASE IN STOCKING
SHOWN FOR THIS PERIOD COULD
BE DUE TO ACCRETION CUTTINGS
NUMBER OF TREES PER ACRE
2500
2000
1500
1000
900
800
700
600
500
400
300
200
150
100
90
80
70
HEIGHT OF STAND - FEET
15
20
30
40
50
60
70
80
90
100
150

TABLE IX.---SPACING OF ALL TREES IN PERCENT OF AVERAGE HEIGHT OF DOMINANT AND CODOMINANT TREES

Age of Stand	Site quality: Good	Medium	Poor
Years	f in percent		
	Jack pine		
30	18	20	22
40	18	19	21
50	18	19	21
60	17	19	21
70	18	19	21
80	18	20	22
	Aspen		
20	12	12	12
30	14	14	14
40	15	15	15
50	16	16	16
60	17	17	17
70	17	18	18
80	18	19	20
	Red pine		
40	20	21	20
60	18	18	19
80	17	17	19
100	17	17	19
160	18	18	20

TABLE X.---SPACING OF ALL TREES AND OF DOMINANT TREES IN PERCENT OF AVERAGE HEIGHT OF DOMINANT TREES

	Site quality					
Age	Good	Medium	Poor	Good	Medium	Poor
	All trees			Dominants		
Years	f in Percent					
			Black spruce			
40	13	12	13	18	18	20
60	12	12	12	17	17	18
80	12	11	12	16	16	17
100	12	11	12	15	16	17
160	11	11	11	15	15	16
			Red oak			
20	16	16	16	22	22	23
40	18	17	17	22	22	23
60	19	19	19	23	23	23
80	20	20	20	23	24	24
100	21	21	21	24	24	25
160	22	23	22	25	26	26

The conclusions which Wilson draws from the data presented in Tables IX and X are as follows:

"1. The factor 'f' decreases with tolerance. The discrepancy between red and jack pine is explained by the fact that jack pine, while definitely more intolerant, will endure extreme crowding from the side.

2. Site has no effect on 'f'.

3. The factor 'f' definitely increases where suppressed trees are excluded from the stand.

4. The factor 'f' varies with the natural ability of species to 'thin out with age'."

The apparent fact from Tables IX and X is that aspen and red oak are thinning out with age and their spacing increases, while black spruce and red pine show just the opposite trend in that they become more crowded with age. Wilson endeavors to justify this fact by suggesting that it may be a reflection of characteristic differences in the form of crowns. Wilson states that it appears from these two tables that perhaps species tending to become more crowded, especially when originating as dense stands, should benefit most from thinnings.

The height factor method of expressing stocking in uniformly stocked, even-aged stands is a method which has been greatly neglected in forest management research, and all too little is known about its actual advantages, disadvantages, and possibilities. Very little has been done to time thinning treatments by height growth.

Wilson, in his Star Lake Plantation, (16) set up

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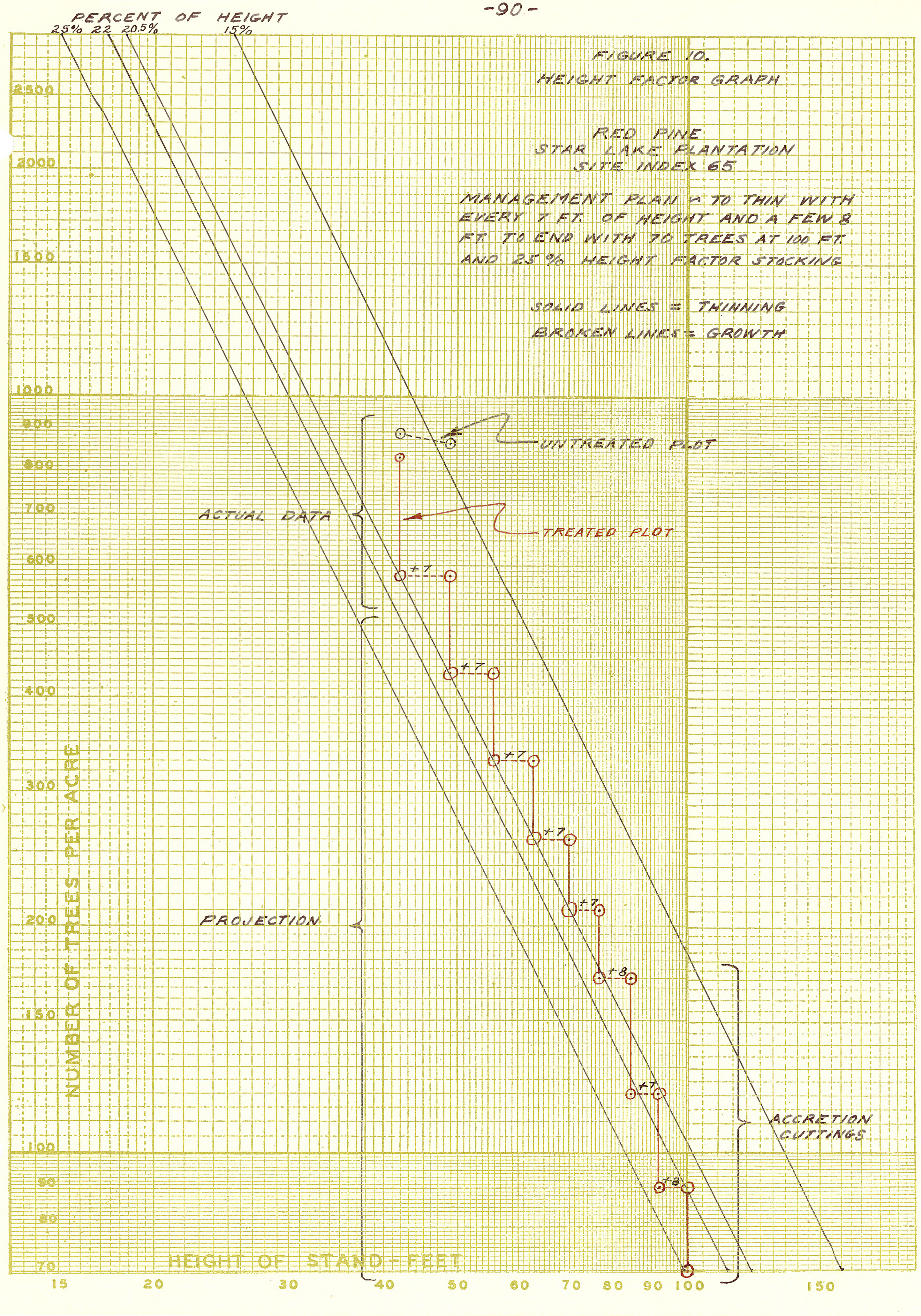
16. Wilson, F.G.; Thinning a Pine Plantation; State of Wisconsin Conservation Department, Madison, Publication 515, A-44.

two experimental plots with one to be an unthinned control plot and the other to be thinned to a height factor percentage of 20.5 every time the stand adds seven feet of height. The Star Lake Plantation was 32 years old when the plots were laid out and is 37 years old now. Only one thinning and growth period have occured to date, but the thinned plot added exactly the same amount of basal area per acre during the growth period as the unthinned plot. This indicates that the thinned plot has not been understocked by the treatment. The factor of 20.5% to express the stocking of the residual stand of the thinned plot was reached by the first thinning at a time when Wilson had not yet worked on the height factor concept, but the approval given by foresters for that stocking on his thinned plot encouraged him to maintain that residual stocking until the time of accretion cuttings. The present actual data and the projection of management plans for the plots at Star Lake are shown in Figure 10. Wilson depicts stand response to management with the height factor method of expressing stocking by plotting basal area peracre over average stand diameter. The response thus far at Star Lake is shown in Figure 11.

To determine correct height factor values for



PERCENT OF HEIGHT
25% 22 20.5% 15%
FIGURE 10.
HEIGHT FACTOR GRAPH
RED PINE
STAR LAKE PLANTATION
SITE INDEX 65
MANAGEMENT PLAN – TO THIN WITH EVERY 7 FT. OF HEIGHT AND A FEW 8 FT. TO END WITH 70 TREES AT 100 FT. AND 25% HEIGHT FACTOR STOCKING
SOLID LINES = THINNING
BROKEN LINES = GROWTH
UNTREATED PLOT
TREATED PLOT
ACTUAL DATA
PROJECTION
ACCRETION CUTTINGS
+7
+8
NUMBER OF TREES PER ACRE
HEIGHT OF STAND – FEET
2500
2000
1500
1000
900
800
700
600
500
400
300
200
150
100
90
80
70
15
20
30
40
50
60
70
80
90
100
150

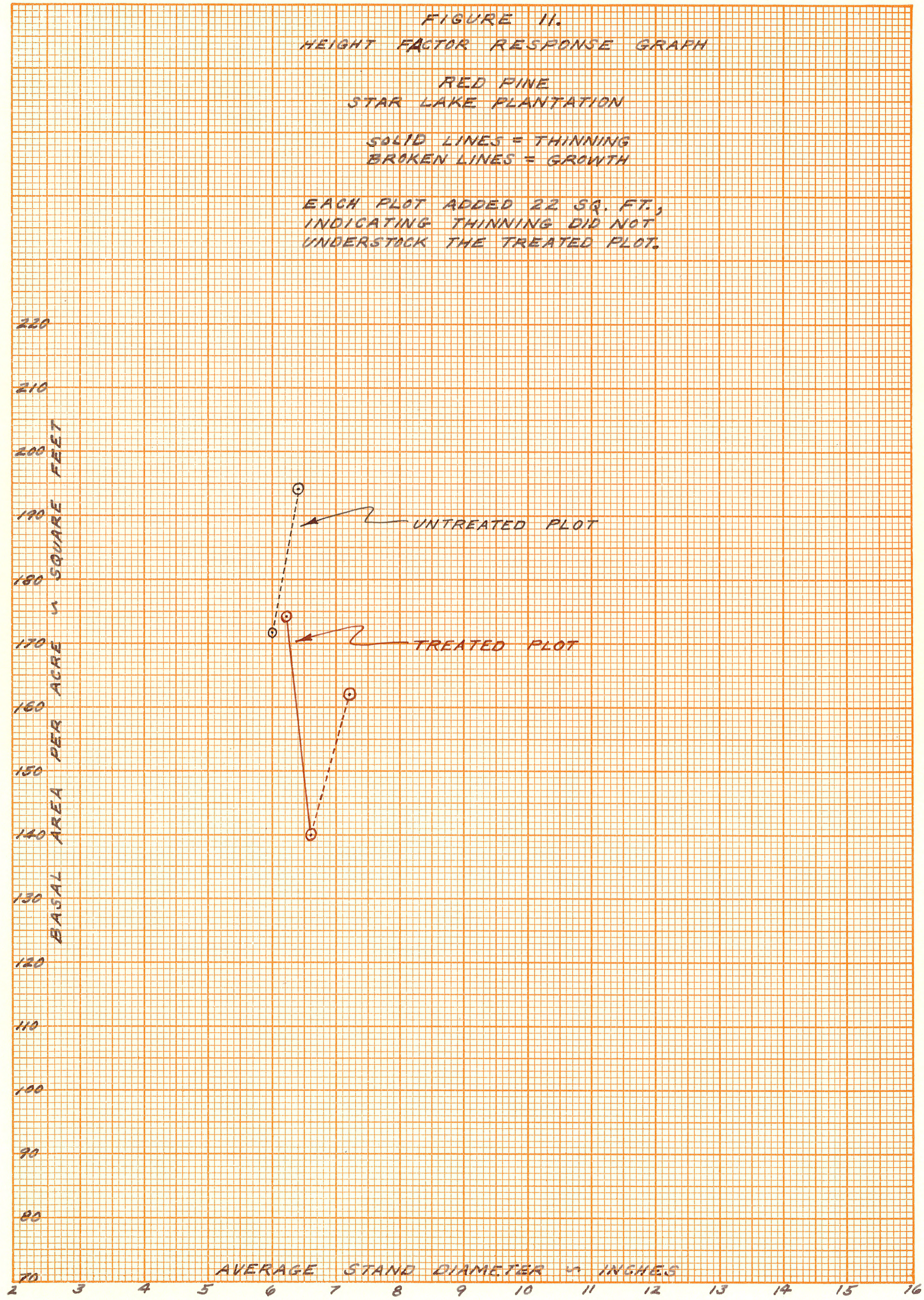


FIGURE 11.
HEIGHT FACTOR RESPONSE GRAPH
RED PINE
STAR LAKE PLANTATION
SOLID LINES = THINNING
BROKEN LINES = GROWTH
EACH PLOT ADDED 22 SQ. FT.,
INDICATING THINNING DID NOT
UNDERSTOCK THE TREATED PLOT.
UNTREATED PLOT
TREATED PLOT
BASAL AREA PER ACRE ~ SQUARE FEET
AVERAGE STAND DIAMETER ~ INCHES
220
210
200
190
180
170
160
150
140
130
120
110
100
90
80
70
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16

different tree species under management for various products, much research must be done on experimental plots set up for controlled thinning. Figures 12 and 13 are the height factor and response graphs of three experimental jack pine plots plotted originally by Wilson with data obtained from the Lake States Forest Experiment Station. (17) Plot No. 29 of these three plots was left unthinned while the other two plots, Numbers 26 and 27, were given thinning treatments. NO. 27 was thinned heavier than No. 26 by a considerable amount in order to bracket and determine the height fadtor level of stocking which produces the greatest increment value from the forest stand. The best stocking for jack pine in this case lies between spacings of one-fourth and one-fifth of height as indicated by a comparison of the responses of the three plots and returns from commercial thinnings. The heavy initial thinning of Plot 27 was noncommercial. This thinning removed many trees that could have been permitted to grow to pulpwood size for a greater commercial thinning later, yielding a larger cash income

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17. Wilson, F.G., Numerical Expression of Stocking in Terms of Height, op.cit., p.760.

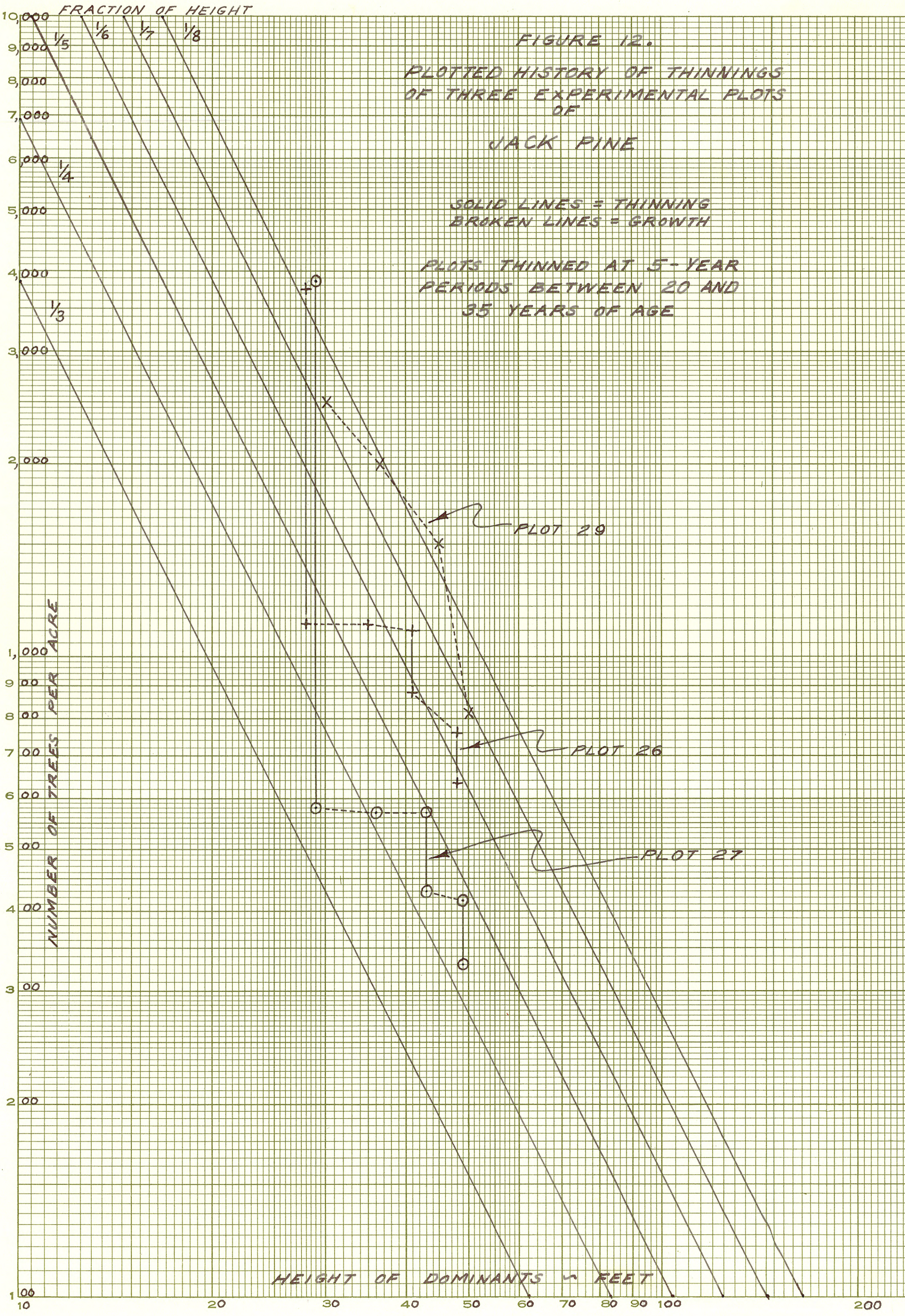
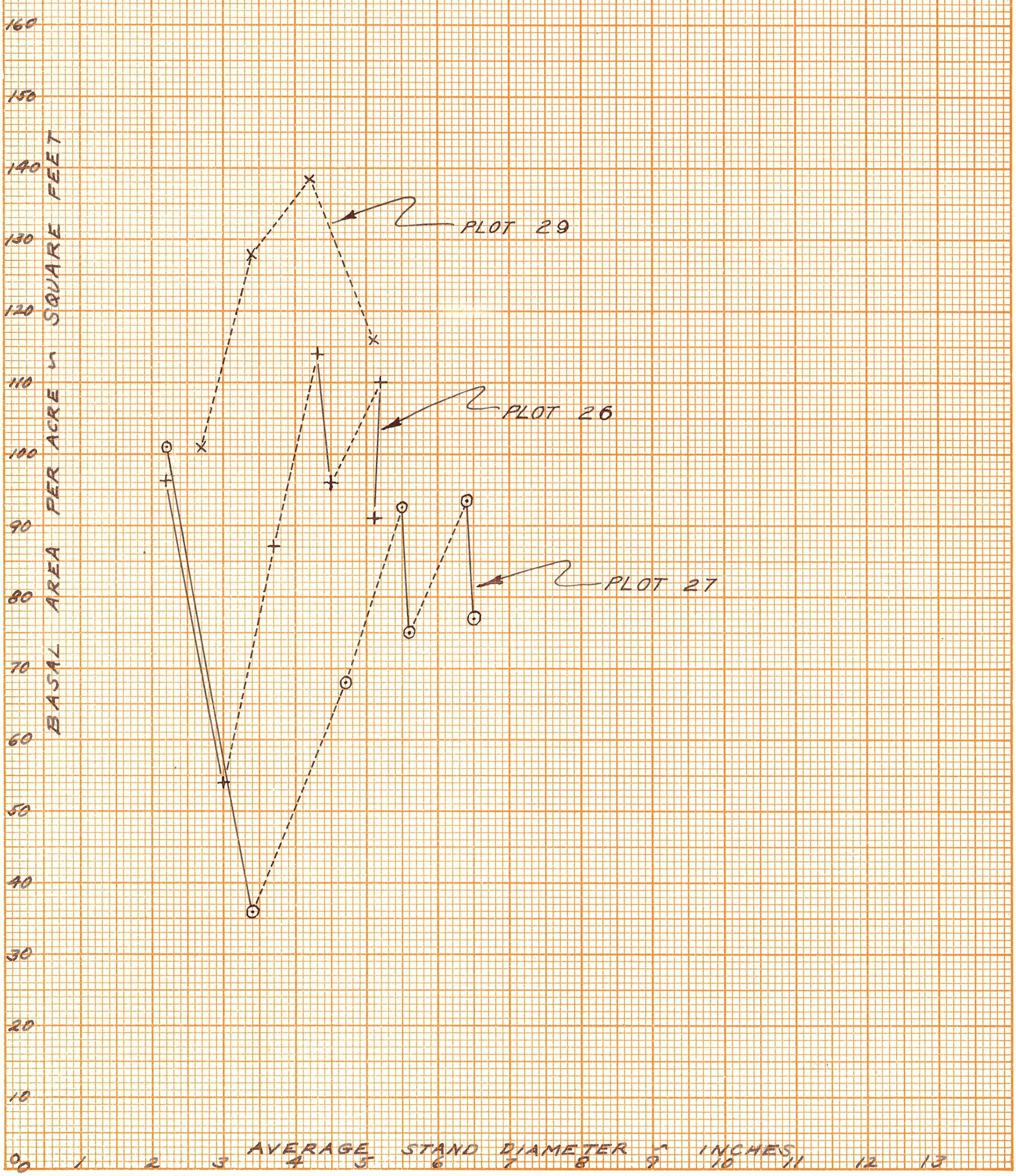


FIGURE 12.
PLOTTED HISTORY OF THINNINGS
OF THREE EXPERIMENTAL PLOTS
OF
JACK PINE
SOLID LINES = THINNING
BROKEN LINES = GROWTH
PLOTS THINNED AT 5-YEAR
PERIODS BETWEEN 20 AND
35 YEARS OF AGE
FRACTION OF HEIGHT
1/3
1/4
1/5
1/6
1/7
1/8
PLOT 29
PLOT 26
PLOT 27
NUMBER OF TREES PER ACRE
HEIGHT OF DOMINANTS ~ FEET
10,000
9,000
8,000
7,000
6,000
5,000
4,000
3,000
2,000
1,000
900
800
700
600
500
400
300
200
100
10
20
30
40
50
60
70
80
90
100
200

FIGURE 13.

STAND RESPONSE GRAPH OF
THREE EXPERIMENTAL PLOTS
OF
JACK PINE

SOLID LINES = THINNING
BROKEN LINES = GROWTH



from the stand. Setting up several series of experimental plots like this for particular species and with each series in a different site would determine the correct height factor for each species and would determine the effect of site. Thinning by height growth would mean thinning less often on poor sites than on good sites. Until enough data from experimental plots thinned and managed on the height factor basis has been compiled it will not be known how sound the height factor slope equation, $2 \log h = -\log n$, is.

As indicated above, jack pine probably should have stand stocking limits between one-fourth and one-fifth of height. Wilson believes a stocking between one-sixth and one-seventh of height would be good for spruce and true firs, and that eastern white pine probably should have a stocking between one-fifth and one-sixth of height.(18)

By the manner in which Wilson plots thinning and growth data on his height factor density sheets, it can be seen that the subject of proper density for the stand is approached from the concept of thinning down (on the graph) to a certain height factor line aftera certain

18. Ibid., p.761.

amount of height growth has taken place in the stand. Therefore, it is important in this procedure to reach a plan which always calls for a thinning befose enough height growth has taken place to cause overcrowding of the stand under the standards of a particular management plan.

To thin down to one particualr height factor line (as pictured on a height factor graph) whenever the stand arrives through growth at some smaller height factor (which would be pictured higher on the graph) is an unsatisfactory plan. This plan would mean, for example, that if it was planned to maintain a stand between a height factor of 20% and a height factor of 25%, the stand at the 1,440 trees peracre level would be thinned after 5.5 feet height growth, at the 360 trees per a cre level the stand would be thinned after 11 feet height growth, and at the 90 trees per acre level the stand would be thinned after 22 feet height growth. The more intensive the plan of management the smaller amount of height growth permitted between thinnings. The timing of thinnings should be by height growth itself.

Since by Wilson's method of planning future stand treatments, as shown in Figure 10, the stand to be placed under management in the manner shown on the height factor graph will be almost constantly of higher stocking than the height factor line chosen for proper stocking, it is important to choose a height factor line that will actually be an understocked value for that stand under a particular management plan. In this way the stand will not be allowed to become overcrowded, and growth can be maintained at the maximum desired rate. Frequency of thinnings will be determined by the intensity of management planned and the predicted rate of height growth during the life of the stand. This frequency of thinnings will be an important item in deciding on the proper height factor line to thin down to (as represented on a height factor graph) during each thinning treatment. The more frequent timing of thinnings allows the choice of a higher stocking height factor line that will be closer to the stocking value decided as the proper stocking at which that stand is still not overcrowded and growth is still at the desired rate.

This complicated approach to the height factor method of uniform, even-aged stand management in selecting a height factor line to thin down to at each stand treatment requires first a decision as to what stocking the stand must not exceed and then a decision as to what constant stocking the stand will be thinned to at each treatment. The height factor method of timing thinning treatments is to thin each time a constant increment of height is added to the average dominant and codominant height of the stand. By referring to Wilson's interpretation of this method in Figure 10, it can be seen that in timing thinnings by his variation a stand will reach the greatest stocking of its life history just before the first treatment, and its stocking just before each succeeding treatment will be progressively lower. In effect, this means the stand will not be thinned each time it reaches a constant, full-stocking, height factor value throughout its life history. Thus, from the stand stocking point of view a stand will have progressively shorter intervals between treatments.

THE DEVELOPMENT OF A MATHEMATICAL PROCEDURE FOR MANAGING STANDS BY THE HEIGHT FACTOR METHOD

To simplify the decision as to what height factor to use in managing a stand by the height factor method, and to put this method more on a comparable par with the spacing factor method, the author of this thesis has devised another approach in managing stands by the height factor system.

First the height factor at which a stand is fully-stocked must be decided upon. Then each time the stand arrives at this height factor value the stand is thinned, and therefore, is never allowed to become over-stocked with decreasing growth rate resulting. Timing and severity of thinning treatments are based on accessibility and value of the stand and its growth rate. The interval between thinnings should be measured in height growth increments. This positive approach of selecting just the one height factor line as represented on a height factor graph and thinning each time the stand reaches this stocking value offers a simplification of the height factor method. Otherwise the height factor method involves the task of determining a lower stocking height factor, which on a height factor graph lies below the chosen full-stocking value. This latter requires thinning down to the lower height factor on the graph, and at the same time scheduling future thinnings so that the stand during periods between thinnings will not exceed the full-stocking value and decrease the average growth rate that it was planned to maintain. The approach presented by the author of thinning each time a stand reaches its full-stocking is the same approach adopted by Professor Matthews in developing the spacing factor method of stand management.

This approach to the height factor method of stand management can be demonstrated by the use of the hypothetical problem which follows: it is decided to put a uniformly stocked, even-aged stand of jack pine under management and that a height factor of 1/5 or 0.20 will be the full-stocking value. It is predicted that under management the stand will maintain a height growth of ten feet in ten years. Because of the predicted rate of height growth to be maintained on that site while the stand is under management and because of the stand's accessibility and the value of its intended products, the management plan is drawn up to thin each time the stand adds ten feet in height. This will mean thinning approximately every ten years due to the predicted rate of height growth. Let "y" represent the number of years between thinnings and "r" represent the rate of height growth each year. Then "y" x "r" represents the height growth between thinnings. In this problem "yr" equals ten feet. Assume that the stand now averages 3,000 trees per acre and that the average height of dominant and codominant trees is 25 feet. It is planned to carry the stand through until its final crop trees are 75 feet tall.

The author of this thesis has devised a system to mathematically compute the management of a stand under the height factor method. This mathematical procedure closely parallels that followed under the spacing factor method.

Since the general height factor equation is

$n = \frac{43,560}{(hf)^2}$ and in this problem $n = 3,000$

and $h = 25$, the present height factor of the stand can be computed as follows:

$$n = \frac{43,560}{(hf)^2} = \frac{43,560}{h^2 f^2}$$

$$f^2 = \frac{43,560}{nh^2}$$

$$f = \sqrt{\frac{43,560}{nh^2}} = \frac{\sqrt{\frac{43,560}{n}}}{h}$$

$$\text{present height factor} = \frac{\sqrt{\frac{43,560}{3000}}}{25} = \frac{\sqrt{14.52}}{25} = \frac{3.8105}{25} = 0.15$$

The height factor varies inversely with the stocking. The present height factor of 0.15 is less than the fully-stocked height factor of 0.20, and therefore, the stand is overstocked and in immediate need of thinning.

The equation for the height factor of 0.20 is

$$n = \frac{43,560}{(0.2h)^2}$$ and can be reduced as follows:

$$n = \frac{43,560}{(0.20h)^2} = \frac{43,560}{0.04h^2} = \frac{1,089,000}{h^2}$$

Since it is planned to thin again when the stand adds "yr" or ten feet in height, the stand will be 35 feet tall at the time of the second thinning. Therefore, the number of trees that must exist in the stand just before the time of the second thinning can be computed in the following manner:

$$n = \frac{1,089,000}{(h+yr)^2} = \frac{1,089,000}{(25+10)^2} = 889 \text{ trees per acre}$$

During the present thinning, therefore, an average of 3,000 minus 889 or 2,111 trees per acre must be removed and an average of 889 trees per acre left standing.

The general equation for obtaining the proper average number of trees per acre at future thinnings is as follows:

$$n = \frac{43,560}{\left[(h+yr)f\right]^2}$$

In this problem we are dealing with a constant height factor of 20%, and therefore, simplify the mathematics by using the equation $n = \frac{1,089,000}{(h+yr)^2}$, which is for the specific height factor of 20%. Such a specific equation can be derived for any particular height factor.

Projecting this stand to the third thinning treatment is a simple matter. When the third thinning treatment comes the stand will have an average height of dominant and codominant trees of 45 feet. In order to just reach full stocking when the time of the third thinning arrives the stand must have the average number of trees per a cre computed by the equation,

$$n = \frac{1,089,000}{45^2},$$

and is 538 trees per acre at that time. Therefore, inthe second thinning an average of 889 minus 538 or 351 trees per acre will be removed from the stand. All subsequent thinning s can be computed in this manner.

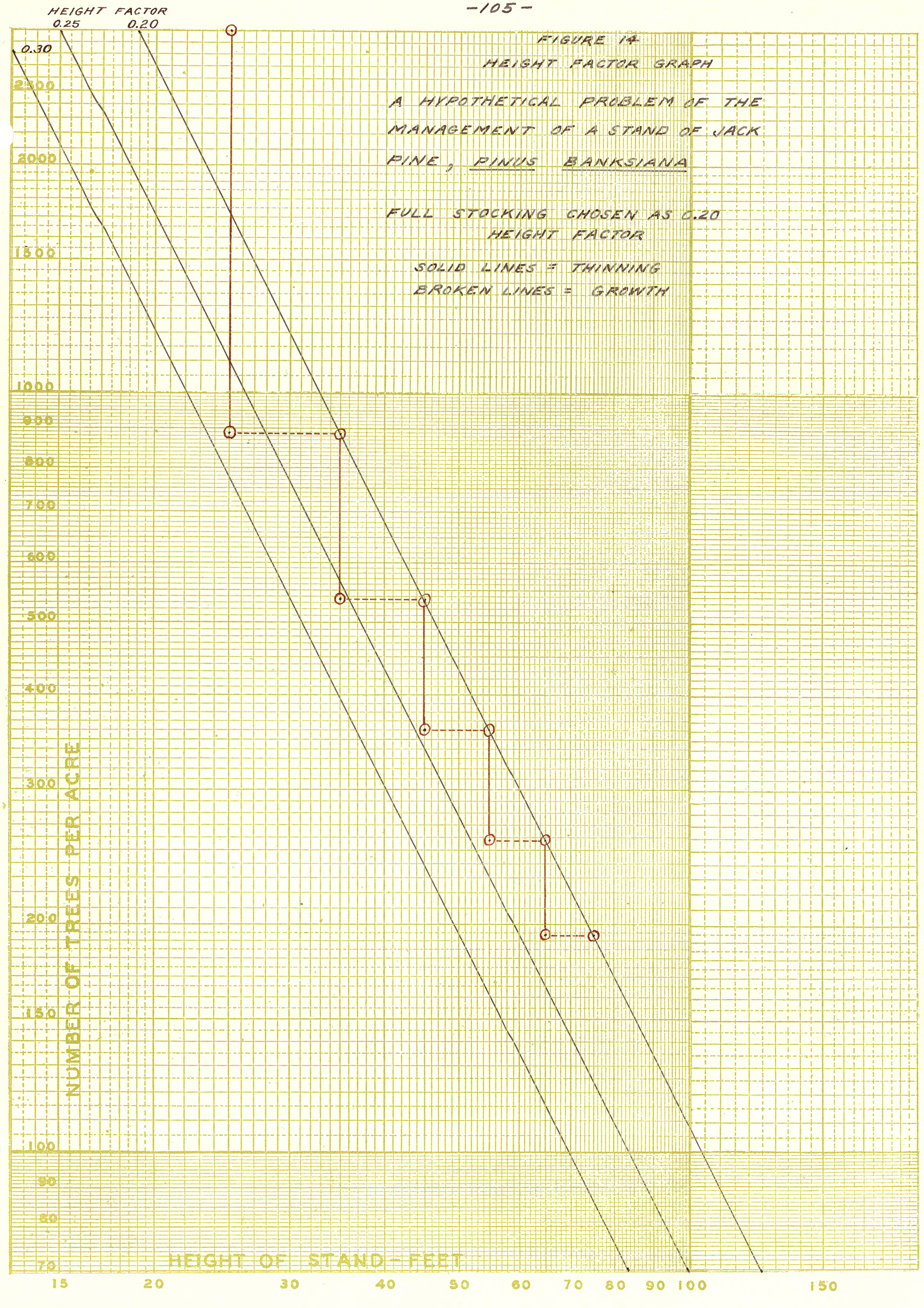
The projected management of this stand is plotted on the height factor graph of Figure 14. Note that the management history of this stand plots below the height factor line chosen as the full stocking value while in Figure 10 the plotted stand history of Wilson's Star Lake plot lies above the height factor line chosen as the constant stocking value by which that stand is managed. It can be seen in Figure 10 that if the height factor of 0.205 or 20.5% was intended to be the <u>full</u> stocking value for red pine the Star Lake stand would be almost constantly overstocked during its life history. In Figure 11 it is shown that the stand of jack pine will be almost constantly of lower stocking than the height factor line chosen to represent full stocking.

Assume that in this hypothetical problem the stand of jack pine at present has an average of 1,200 trees per acre and an average dominant and codominant tree height of 25 feet. All other data is taken as remaining the same as in the first case.

The first step with this problem is to again determine whether the stand is at present fully-stocked, overstocked or understocked. The present stocking is obtained as follows with the use of the equation employed in the first part of this hypothetical problem:

$$f = \frac{\sqrt{\frac{43,560}{n}}}{h} = \frac{\sqrt{\frac{43,560}{1200}}}{25} = \frac{\sqrt{36.300}}{25} = \frac{6.025}{25}$$

$$f = 0.241 \text{ or } 24.1\%$$


HEIGHT FACTOR
0.25
0.20
0.30
FIGURE 14
HEIGHT FACTOR GRAPH
A HYPOTHETICAL PROBLEM OF THE MANAGEMENT OF A STAND OF JACK PINE, PINUS BANKSIANA
FULL STOCKING CHOSEN AS 0.20 HEIGHT FACTOR
SOLID LINES = THINNING
BROKEN LINES = GROWTH
NUMBER OF TREES PER ACRE
2500
2000
1500
1000
900
800
700
600
500
400
300
200
150
100
90
80
70
HEIGHT OF STAND - FEET
15
20
30
40
50
60
70
80
90
100
150

The present height factor stocking of this jack pine stand is 24.1%. Therefore, this stand is now understocked since its present height factor value is greater than the chosen full stocking height factor value of 20.0%.

The determination as to when this stand will reach full stocking and be in need of a thinning treatment can be made in the following manner: the average number of trees per acre at present is 1,200 and the chosen full stocking height factor is 0.20 or 20%. Therefore, the average dominant and codominant height of the stand when full stocking is reached can be computed by use of the height factor equation. This computation is as follows:

$$f = \frac{\sqrt{\frac{43,560}{n}}}{h}$$

therefore,

$$h = \frac{\sqrt{\frac{43,560}{n}}}{f \text{ at full stocking}}$$

and, substituting the values in this problem,

$$h = \frac{\sqrt{\frac{43,560}{1200}}}{0.20} = \frac{6.025}{0.20} = 30.12 \text{ feet average height}$$

The stand will become fully stocked when the average height of its dominant and codominant trees reaches approximately 30 feet. Therefore, the first thinning treatment will come when the stand adds an average height of 30 minus 25 or 5 feet, and subsequent thinnings will come after each addition of ten feet in height. By means of the original data given in the first part of the problem this timing of thinnings by height growth can be converted to years. Thus, since it is predicted that the stand will maintain a height growth rate of ten feet every ten years, the first thinning will come about five years from now when the stand averages thirty feet tall. A general formula for directly obtaining the amount of height growth required before a stand becomes fully-stocked can bw derived as follows:

let yr = the amount of height growth to be added before a stand becomes fully-stocked

then $$n = \frac{43,560}{\left[(h+yr)f\right]^2}$$

$$(h + yr)^2 = \frac{43,560}{nf^2}$$

$$h + yr = \frac{\sqrt{\frac{43,560}{n}}}{f}$$

$$yr = \frac{\sqrt{\frac{43,560}{n}}}{f} - h$$

Subsequent thinnings in this hypothetical problem will come at approximately ten year intervals each time the stand adds ten feet in height. The average number of trees per acre to be removed in the thinning treatment five years from now can be computed. At the time of the second thinning the stand will average 30 plus "yr" or forty feet in height and will have just reached full stocking with a height factor of 20%. Therefore, the average number of trees per acre in the fully-stocked stand just before the second thinning can be computed as follows with the 20% height factor equation derived and employed in the first part of this problem:

$$n = \frac{1,089,000}{(30+yr)^2} = \frac{1,089,000}{40^2} = 681 \text{ trees per acre}$$

Thus, in the first thinning an average of 1,200 minus 681 or 519 trees per acre will be removed and 681 trees per acre will be left. The other subsequent thinnings can be computed in the same manner.

HEIGHT FACTOR CROWN OPENING FORMULA

The author has developed also a method of computing the average crown opening created by thinning a fully-stocked stand that is being managed under the height factor method.

When a stand is fully-stocked just before a thinning, it has, by definition, in the height factor equation, an average distance in feet between tree stems equal to "h f". The average spading between trees in this stand just before the next, subsequent thinning must be $(h + yr)f$. Therefore, this latter average spacing of $(h + yr)f$ must be created by this first thinning. Thus the full canopy of tree crowns, with no interlocking branches at proper full stocking, will be opened up an average distance between crowns equal to the difference between the average spacing of the stand just fully-stocked before the thinning and the average spacing of the stand after the thinning. This can be presented in formula form as follows:

let O = the average crown opening or open distance between crowns in feet

then $O = (h + yr)f - hf$

and simplifying the equation by factoring "f" gives

$$O = f(h + yr - h)$$

and $$O = fyr$$

This is the crown opening formula to be employed in the height factor method of forest management. This formula closely parallels the spacing factor crown opening formula.

The above hypothetical problem of putting a stand of jack pine under management can be employed to demonstrate the use of the crown opening equation. The height factor of the fully-stocked stand of jack pine was chosen to be 0.20 or 20%. The height increment between thinnings was chosen to be ten feet, which can be expressed as "yr". Therefore, the average crown opening created by properly thinning this stand when it reaches full stocking is computed as follows:

$$O = fyr$$

$$O = 0.20 \times 10 = 2.0$$ feet average open distance between crowns

This average crown opening of 2 feet should be created after each thinning in this problem.

For easy reference and determination of the amount of crown opening to be expected for various full stocking height factors and various size height increments between thinning treatments, a general table can be constructed. Table XI shows the average crown opening created by thinning treatments for each full stocking height factor fraction and average height growth increment between thinnings. This table can be constructed also with the height factors expressed as percentages. In addition to obtaining directly from Table XI the crown opening to be created by a thinning, Table XI also can be used in obtaining the average spacing between tree stems that will be created by the thinning. The average spacing between tree stems after a thinning will equal the average spacing before thinning, plus the average crown opening to be created by the thinning. This can be expressed in formula form as follows:

let hf = the average spacing between trees in feet as is defined by the height factor equation

let O = the average crown opening in feet as read from Table XI

let S = the average spacing between trees in feet after the thinning

then S = hf + O.

TABLE XI.---AVERAGE CROWN OPENINGS IN FEET TO BE LEFT IN STAND THINNING TREATMENTS UNDER HEIGHT FACTOR METHOD OF MANAGEMENT

Average stand height growth increment between thinnings in feet y r	Full stocking height growth factor 1/3	1/4	1/5	1/6	1/7	1/8	1/9
	Average crown opening (or increase in spacing above the full-stocking spacing existing before the thinning)in feet O = fyr						
3	1.00	0.75	0.60	0.50	0.43	0.38	0.33
4	1.33	1.00	0.80	0.67	0.57	0.50	0.44
5	1.67	1.25	1.00	0.83	0.71	0.62	0.56
6	2.00	1.50	1.20	1.00	0.86	0.75	0.67
7	2.33	1.75	1.40	1.17	1.00	0.88	0.78
8	2.67	2.00	1.60	1.33	1.14	1.00	0.89
9	3.00	2.25	1.80	1.50	1.29	1.12	1.00
10	3.33	2.50	2.00	1.67	1.43	1.25	1.11
11	3.67	2.75	2.20	1.83	1.57	1.38	1.22
12	4.00	3.00	2.40	2.00	1.71	1.50	1.33
13	4.33	3.25	2.60	2.17	1.86	1.62	1.44
14	4.67	3.50	2.80	2.33	2.00	1.75	1.56
15	5.00	3.75	3.00	2.50	2.14	1.88	1.67
16	5.33	4.00	3.20	2.67	2.29	2.00	1.78
17	5.67	4.25	3.40	2.83	2.43	2.12	1.89
18	6.00	4.50	3.60	3.00	2.57	2.25	2.00
19	6.33	4.75	3.80	3.17	2.71	2.38	2.11
20	6.67	5.00	4.00	3.33	2.86	2.50	2.22

A COMPARISON BETWEEN THE HEIGHT AND THE DIAMETER METHODS OF EXPRESSING STAND STOCKING, WITH CONCLUSIONS DRAWN

CORRELATION OF SPACING FACTOR LINES AND HEIGHT FACTOR LINES WITH ACTUAL STAND DATA AND YIELD TABLE DATA ON GRAPHS

While discussing above the fact that the height factor equation, $n = \frac{43,560}{(hf)^2}$, possibly reflects a law of normal stocking for uniform, even-aged, closed stands, it was mentioned that present support of the height factor equation must be found in the histories of well managed stands and in yield tables. Final proof of the merits of the equation must be obtained through experimentation. However, this suggestion of referring to existing managed stand histories and to existing yiedd tables for support of the height factor equation also offers a medium through which the height factor can be compared with the spacing factor. Thus, the height and the diameter methods of expressing stand stocking canbe compared in part on this basis. Histories of managed stands and data from yield tables can be plotted on both height factor and spacing factor graphs if all the required stand measurements are given. The correlations of these plottings with the height factor lines and spacing factor lines and the characteristic relationships shown by the graphic presentations can be studied to indicate the possible merits of each method and to compare the two methods.

The task of drawing conclusions from the manner in which the data from stand histories and yield tables plot on these graphs offers many difficulties. Thinning is done in many and various ways. It is an art, although it should be guided by proper management plans to utilize its full potential value. The different thinning methods will influence differently the general appearance presented by the plotted data of stands and the correlation of the plotted stand histories with the height factor lines and spacing factor lines.

If a stand is thinned from "underneath", taking out the suppressed and oppressed and less desirable trees, and the recorded height of the stand is the average height of all the trees in the stand, then the average height will be increased as the direct result of each thinning and this pattern will be reflected on a height factor graph. If the recorded height of the stand is of the average height of the dominant and codominant trees chosen to eventually become the final crop, then this dominant and codominant height will not jump as a direct result of each thinning treatment. This same relationship holds true in the case of the spacing factor and a verage stand diameter also. The diameter growth of the final crop trees can be maintained at quite a uniform rate throughout the life of a stand by proper management.

CORRELATION OF SPACING FACTOR LINES AND HEIGHT FACTOR LINES WITH DATA FROM INDIVIDUAL, UNIFORMLY STOCKED, EVEN-AGED STANDS

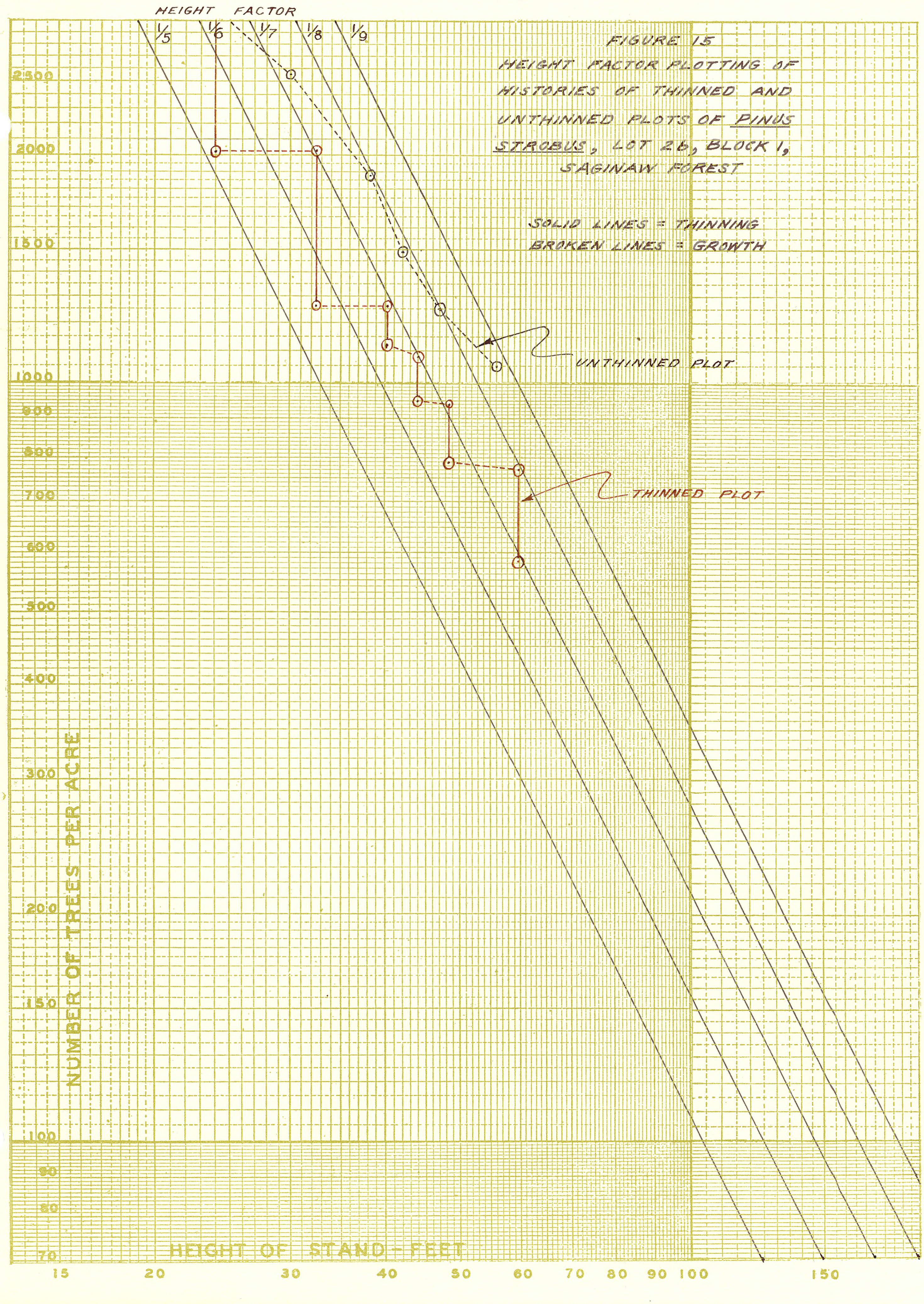
Eventually, many experimental plots should be established under varied sites and conditions and managed consistently throughout their life-spans under particualr height factor standards and spacing factor standards in order t o determine theactual worth of these methods, their advantages and limitations, and the possible scope of their applications. Until the time such experimental plots are established and be g in contributing data, some conclusions can be drawn by observing the correlation of data from existing, uniformly stocked, even-aged managed stands with the height factor and spacing factor lines even though these stands were not managed by the height factor or spacing factor method.

The author has plotted the stand data of several tree plantations of the University of Michigan on height factor and spacing factor graphs. The data for these managed stands, which are located on the Saginaw Forest, Ann Arbor, Michigan, is fairly complete and offers a means of studying the correlation of actual stand histories with the slopes of the height factor lines and spacing factor lines. Occasionally some specific data is lacking, which decreases even more the reliability of graphing this data by the height factor and spacing factor methods; But usually enough data is present to allow the author to make a close estimate of what the missing actual values should be. Some of these stands have had other trees planted within the stand at later dates. Other difficulties were occasionally confronted also. No specific conclusions should be drawn from the graphs of these stands presented below as to the actual stocking to which various species should be held nor should other specific conclusions be drawn. The purpose here in plotting this data is to compare the relative manner in which the same stand plots on each type of graph.

The data of the thinned and unthinned eastern white pine, Pinus strobus, plots of Lot 2b, Block 1, Saginaw Forest, is plotted on the height factor graph of Figure 15, the spacing factor graph of Figure 16, and the stand response graph of Figrue 17. The data from which these graphs were plotted is shown in Table 1 of the appendix. Photographs taken of this stand in December, 1948 are in Figure 1 of the appendix.

Note that the correlation of the stand data with the slope of the lines of stocking in both Figures 15 and 16 is quite good, but that the slope of the stand data is not quite as steep as that of the stocking lines. From the standpoint of both these graphs the stocking of this stand is steadily increasing. Figure 17 shows a constant increase in basal area to over two hundred square feet basal area per acre. The correlation of the stand data of the thinned plot with the slope of the stocking lines is very good on both the height factor graph and the spacing factor graph. However, the data from the unthinned plot correlates better with the spacing factor slope than with the height factor slope. The latter part of the stand history of the unthinned portion of the stand plots in what is very nearly a straight line on the spacing factor graph and is almost parallel with the stocking lines.

Note that on the height factor graph the average


HEIGHT FACTOR
1/5
1/6
1/7
1/8
1/9
FIGURE 15
HEIGHT FACTOR PLOTTING OF HISTORIES OF THINNED AND UNTHINNED PLOTS OF PINUS STROBUS, LOT 26, BLOCK 1, SAGINAW FOREST
SOLID LINES = THINNING
BROKEN LINES = GROWTH
UNTHINNED PLOT
THINNED PLOT
NUMBER OF TREES PER ACRE
2500
2000
1500
1000
900
800
700
600
500
400
300
200
150
100
90
80
70
HEIGHT OF STAND - FEET
15
20
30
40
50
60
70
80
90
100
150

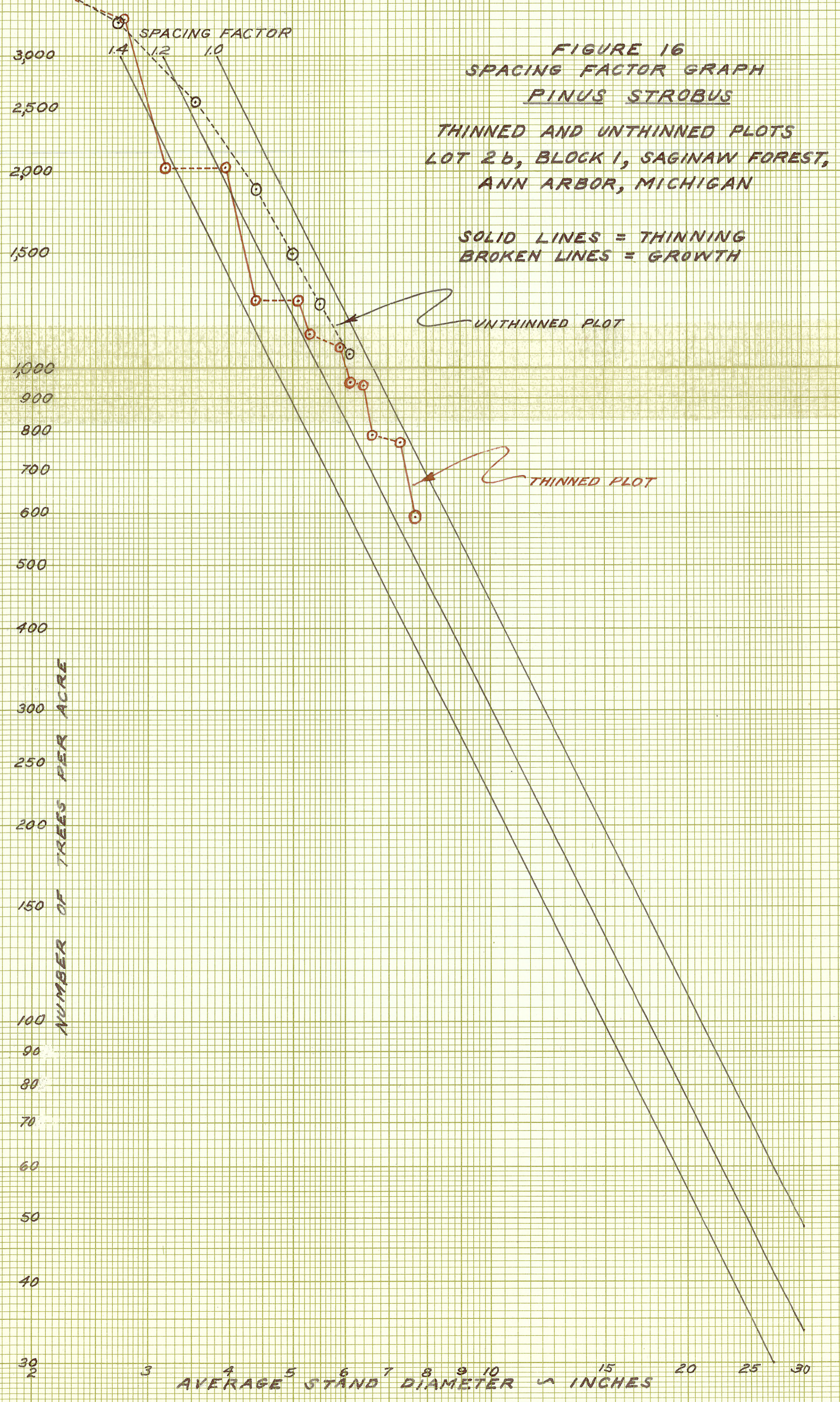


FIGURE 16
SPACING FACTOR GRAPH
PINUS STROBUS
THINNED AND UNTHINNED PLOTS
LOT 2b, BLOCK I, SAGINAW FOREST,
ANN ARBOR, MICHIGAN
SOLID LINES = THINNING
BROKEN LINES = GROWTH
SPACING FACTOR
1.4
1.2
1.0
UNTHINNED PLOT
THINNED PLOT
NUMBER OF TREES PER ACRE
3,000
2,500
2,000
1,500
1,000
900
800
700
600
500
400
300
250
200
150
100
90
80
70
60
50
40
30
2
3
4
5
6
7
8
9
10
15
20
25
30
AVERAGE STAND DIAMETER ~ INCHES

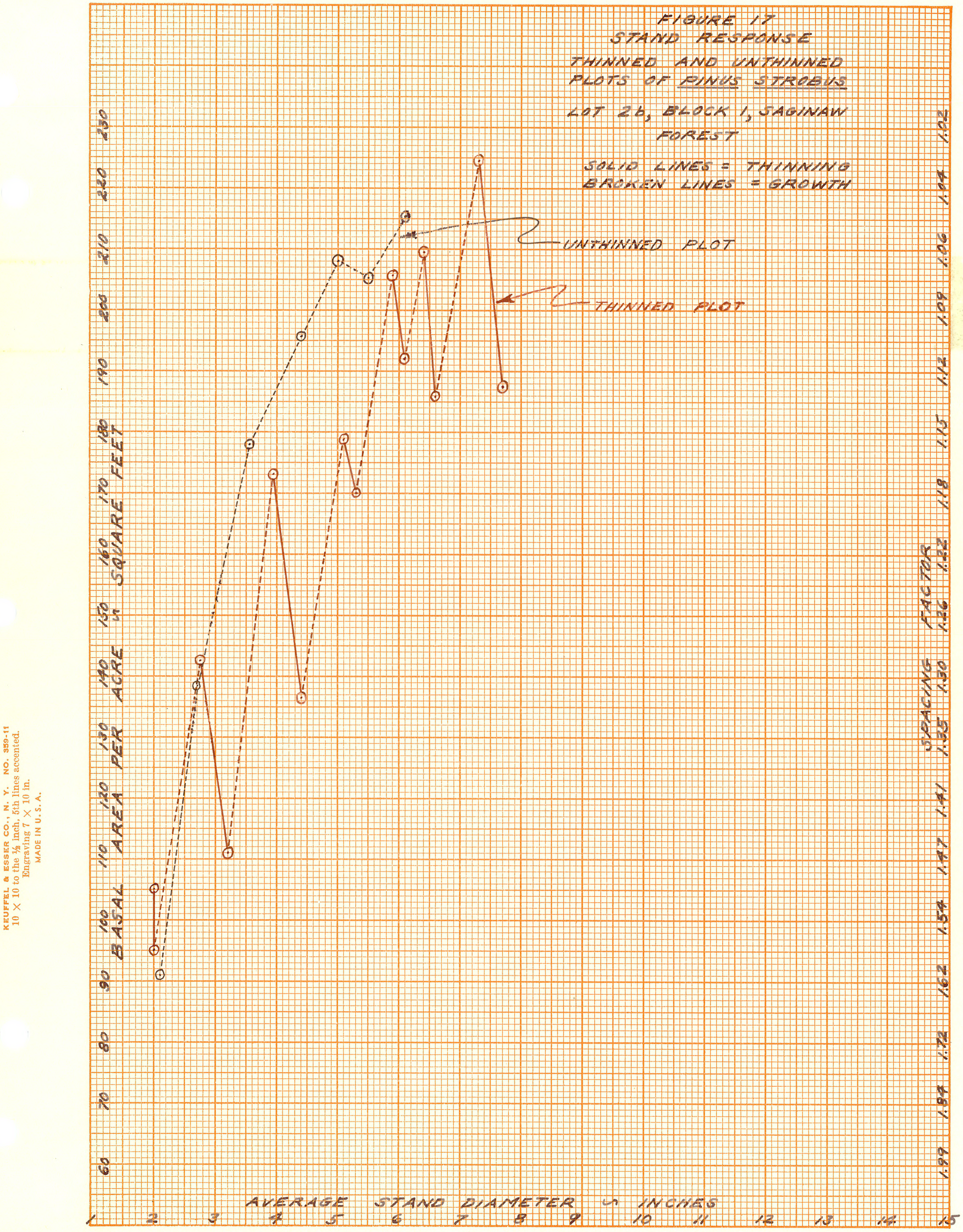


FIGURE 17
STAND RESPONSE
THINNED AND UNTHINNED PLOTS OF PINUS STROBUS
LOT 2b, BLOCK 1, SAGINAW FOREST
SOLID LINES = THINNING
BROKEN LINES = GROWTH
UNTHINNED PLOT
THINNED PLOT
BASAL AREA PER ACRE - SQUARE FEET
90 100 110 120 130 140 150 160 170 180 190 200 210 220 230
60 70 80
AVERAGE STAND DIAMETER - INCHES
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
SPACING FACTOR
1.99 1.84 1.72 1.62 1.54 1.47 1.41 1.35 1.30 1.26 1.22 1.18 1.15 1.12 1.09 1.06 1.04 1.02
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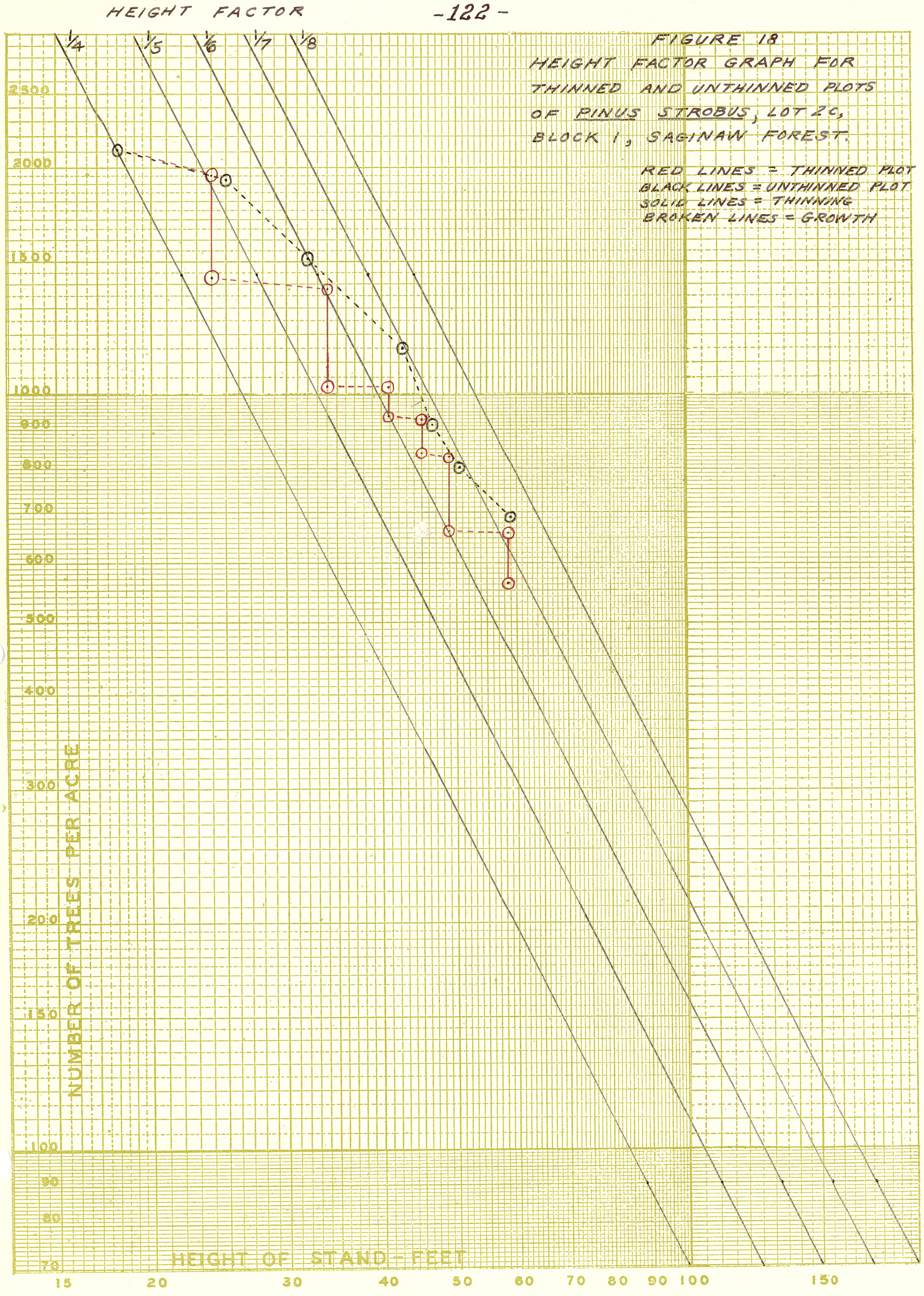
10 X 10 to the ½ inch, 5th lines accented.
Engraving 7 X 10 in.
MADE IN U. S. A.

height of the dominant and codominant trees does not increase directly as a result of each thinning, while the average stand diameter on the spacimg factor graph does increase as a result of each thinning. If the diameters recorded on the graph were the average of only the dominant and codominant trees to remain in the stand and not the average of all trees, then the diameters recorded would not imcrease directly as a result of each thinning but would present the same general picture on the spacing factor graph as the height recordings do on the height factor graph. This same condition exists in the data of all the other stands selected by the author from the Saginaw Forest and presented in graphic form below.

The stand data of the thinned and unthinned eastern white pine, Pinus strobus, plots of Lot 2c, Block 1, Saginaw Forest, is plotted on the height factor graph of Figure 18, the spacing factor graph of Figure 19, and the stand response graph of Figure 20. The data from which these graphs were plotted is compiled in Table 2 of the appendix. Photographs of this stand are in Figure 2 of the appendix.

The corrélation of the stand data for this white pine stand with the slope of the stocking lines


HEIGHT FACTOR
1/4
1/5
1/6
1/7
1/8
FIGURE 18
HEIGHT FACTOR GRAPH FOR THINNED AND UNTHINNED PLOTS OF PINUS STROBUS, LOT 2C, BLOCK 1, SAGINAW FOREST.
RED LINES = THINNED PLOT
BLACK LINES = UNTHINNED PLOT
SOLID LINES = THINNING
BROKEN LINES = GROWTH
2500
2000
1500
1000
900
800
700
600
500
400
300
200
150
100
90
80
70
NUMBER OF TREES PER ACRE
HEIGHT OF STAND - FEET
15
20
30
40
50
60
70
80
90
100
150

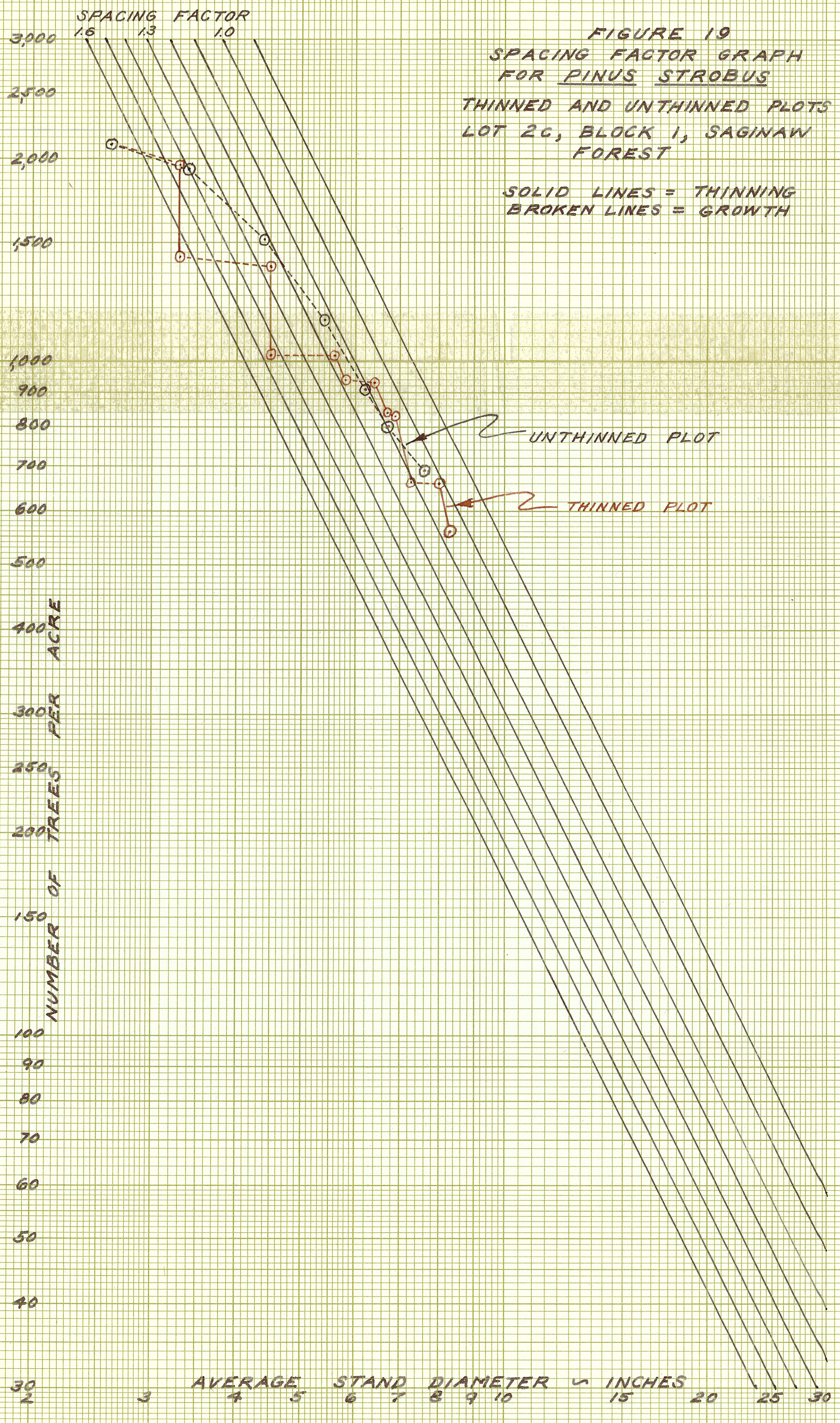
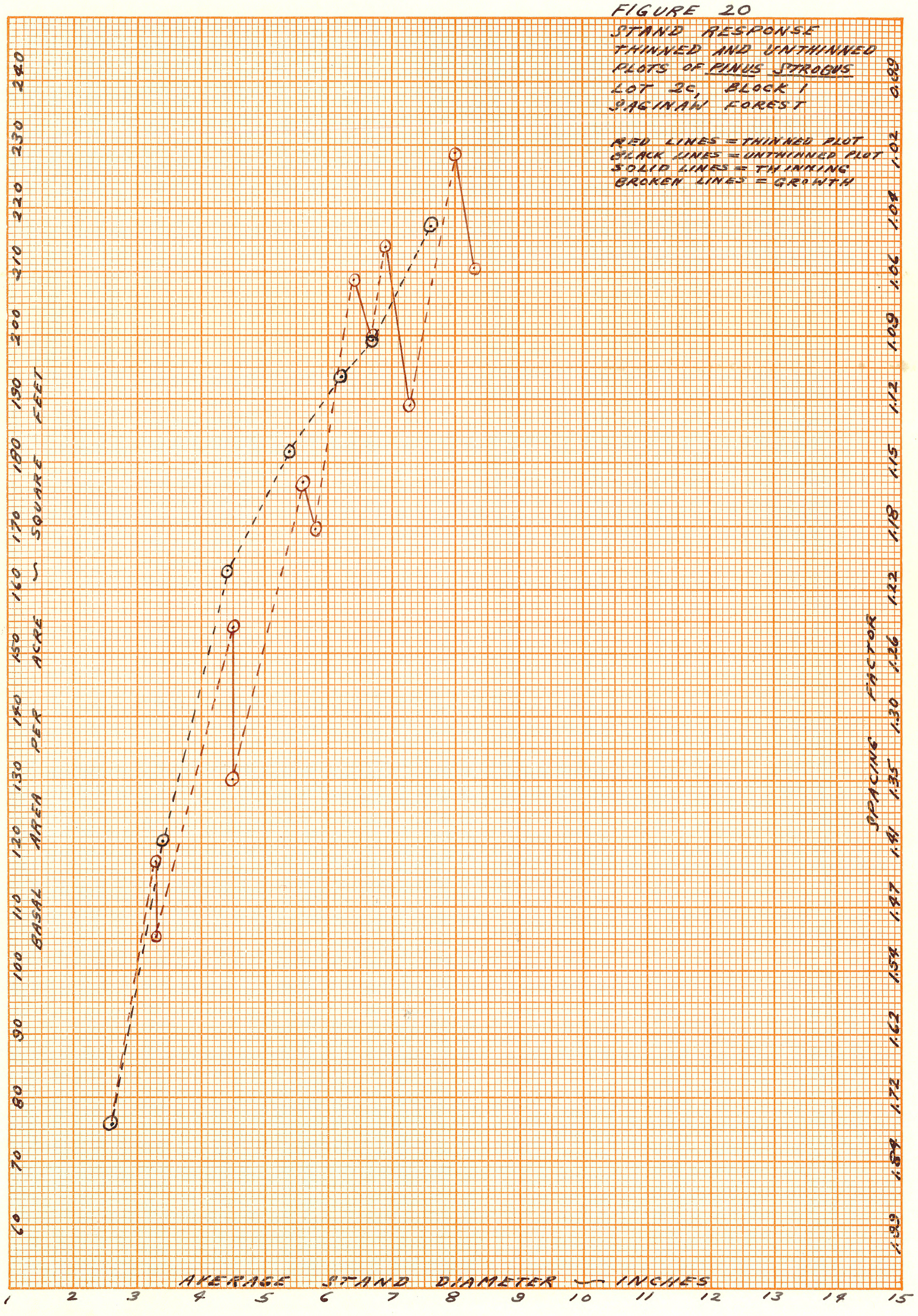


FIGURE 19
SPACING FACTOR GRAPH
FOR PINUS STROBUS
THINNED AND UNTHINNED PLOTS
LOT 2c, BLOCK 1, SAGINAW
FOREST
SOLID LINES = THINNING
BROKEN LINES = GROWTH
SPACING FACTOR
1.6
1.3
1.0
UNTHINNED PLOT
THINNED PLOT
NUMBER OF TREES PER ACRE
3,000
2,500
2,000
1,500
1,000
900
800
700
600
500
400
300
250
200
150
100
90
80
70
60
50
40
30
AVERAGE STAND DIAMETER ~ INCHES
2
3
4
5
6
7
8
9
10
15
20
25
30



KEUFFEL & ESSER CO., N. Y. NO. 359-11
10 × 10 to the ½ inch. 5th lines accented.
Engraving 7 × 10 in.
MADE IN U. S. A.

appears to be about the same on the spacing factor graph and the height factor graph. Again stand stocking increased steadily during the period for which there was data. The stand response graph of Figure 20 shows sharp increases in stand stocking after each treatment of the thinned plot.

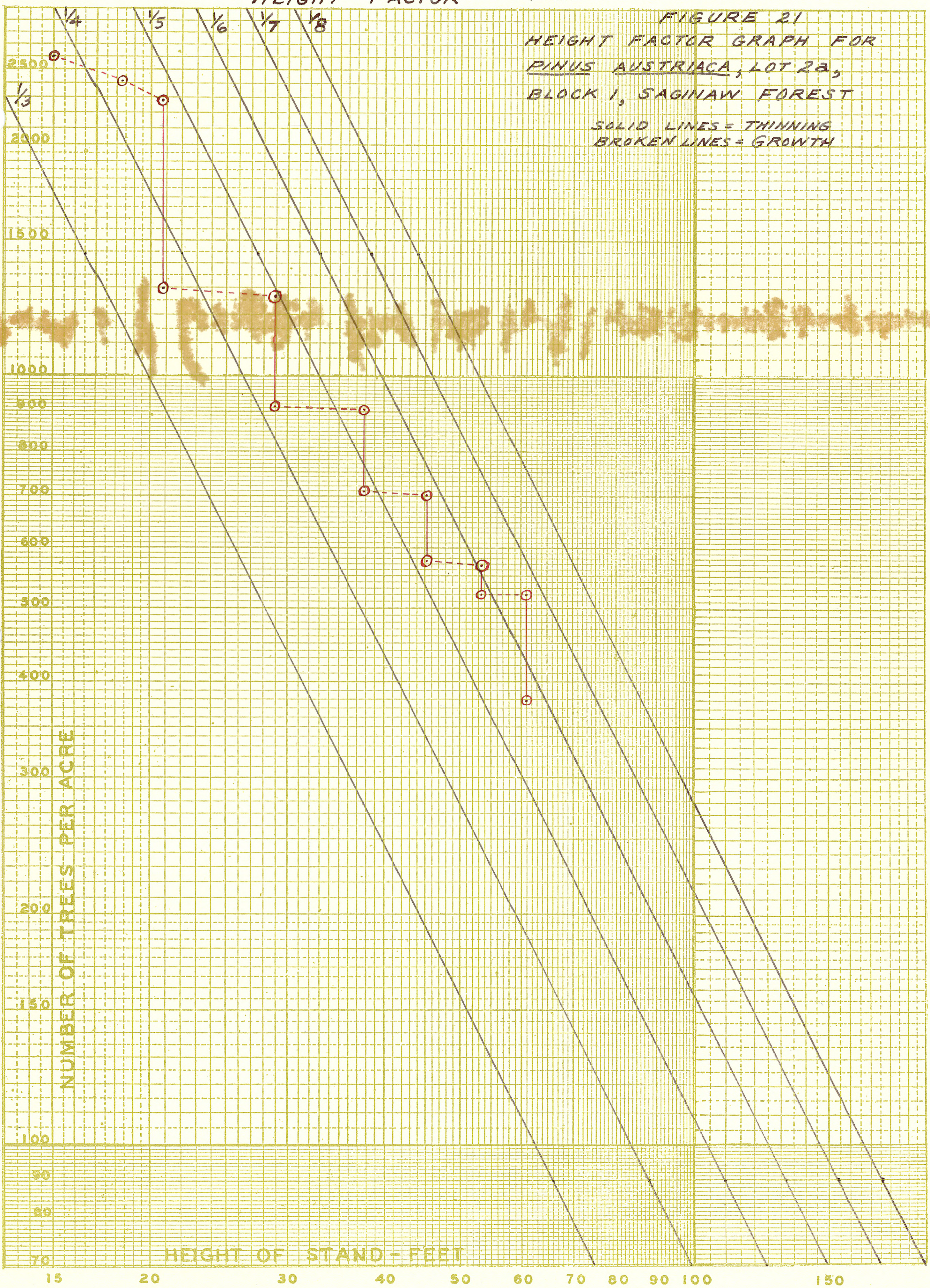
The stand data of the Austrian pine, Pinus austriaca, stand of Lot 2a, Block 1, Saginaw Forest, is plotted on the height factor graph of Figure 21, the spacing factor graph of Figure 22, and the stand response graph of Figure 23. The stand data used in these graphs is in Table 3 of the appendix and photographs of this stand are in Figure 3 of the appendix.

Correlation of the Austrian pine stand data with the slopes of both the height factor lines and the spacing factor lines is quite good. In both cases the data plots in a general line that does not have quite as steep a slope as the stocking lines. This same relationship was true in the case of both eastern white pine stands discussed above. The Austrian pine stand data plots in a general line that has very nearly a constant slope. The slope of this general line on the spacing factor graph is nearly equal to the slope of the spacing factor lines, while the slope of the

FIGURE 21

HEIGHT FACTOR GRAPH FOR PINUS AUSTRIACA, LOT 2a, BLOCK 1, SAGINAW FOREST

SOLID LINES = THINNING
BROKEN LINES = GROWTH



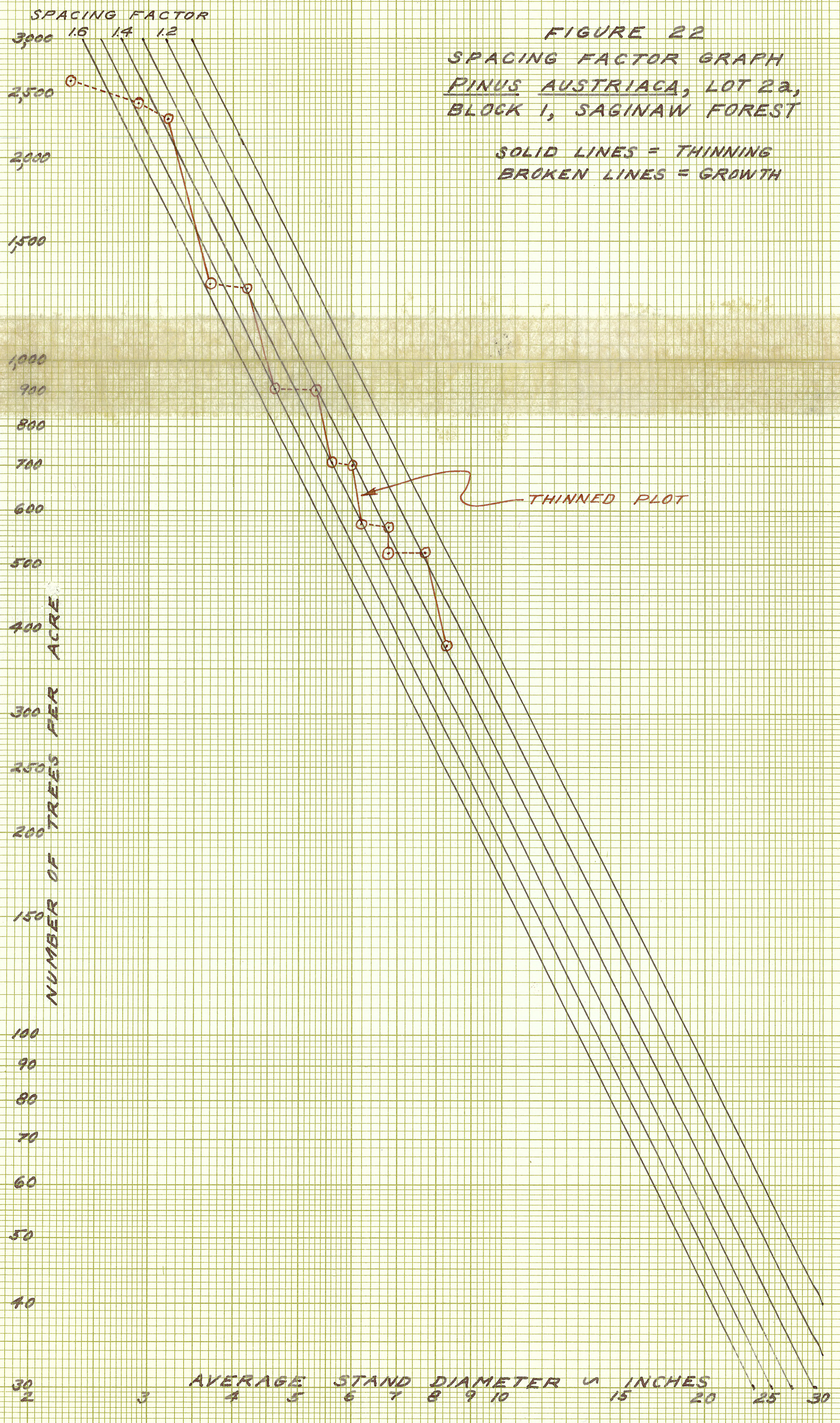


FIGURE 22
SPACING FACTOR GRAPH
PINUS AUSTRIACA, LOT 2a,
BLOCK 1, SAGINAW FOREST

FIGURE 23

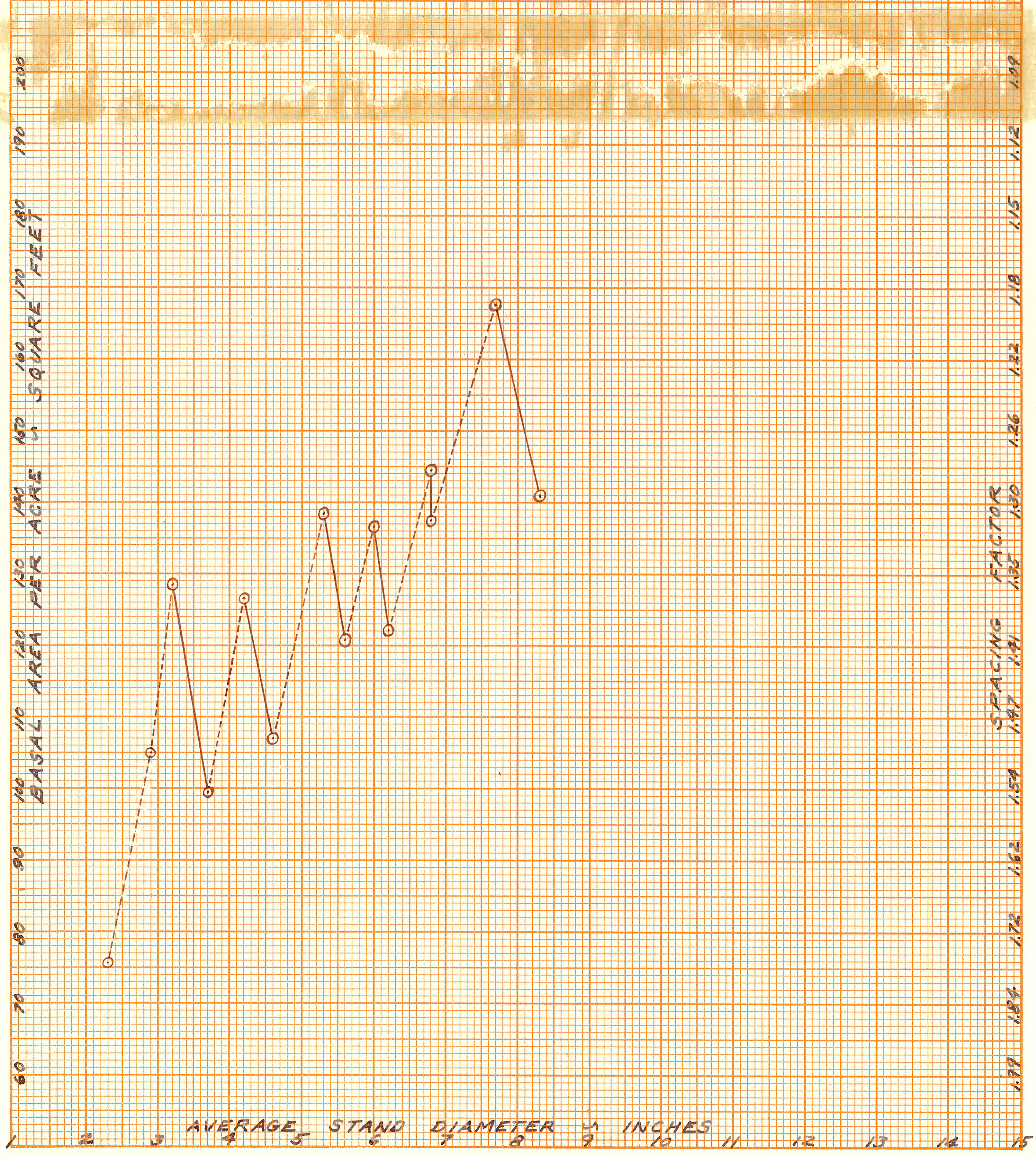
STAND RESPONSE

PINUS AUSTRIACA

LOT 2a, BLOCK 1

SAGINAW FOREST

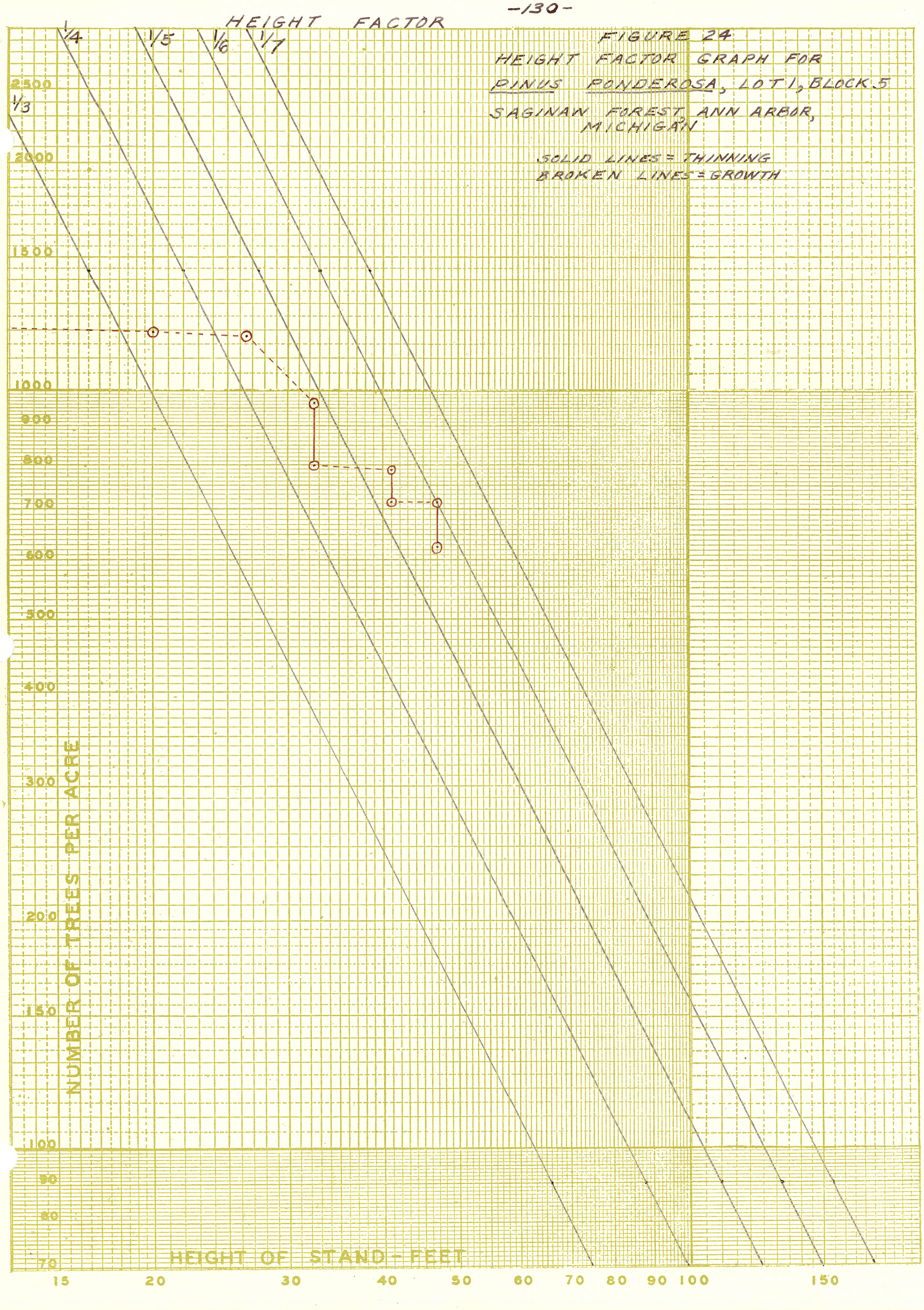
SOLID LINES = THINNING
BROKEN LINES = GROWTH



stand data on the height factor graph in relation to the height factor lines is not quite as close. The stand response graph of Figure 23 shows the basal area growth of the stand, and offers a good comparison with the stand response graphs, Figures 17 and 20, for the eastern white pine stands discussed above. The Austrian pine stand response graph shows not as great a rate of increase in basal area as is shown for the two white pine stands. Thus, the Austrian pine stand data plots more nearly parallel to the spacing factor lines on the spacing factor graph than do the stand data of the two white pine stands.

The stand data of the western yellow pine, Pinus ponderosa, plot of Lot 1, Block 5, Saginaw Forest, is plotted on the height factor graph of Figure 24, the spacing factor graph of Figure 25, and the stand response graph of Figure 26. The stand data used in these graphs is in Table 4 of the appendix and photographs of the stand are in Figure 4 of the appendix.

Note that this stand did not begin the trend of maintaining any constant slope on the graphs until it reached a stand height of about 27.5 feet on the height factor graph and a stand diameter of 4.6 inches on the spacing factor graph. The stand data of the plots


HEIGHT FACTOR
1/3
1/4
1/5
1/6
1/7
FIGURE 24
HEIGHT FACTOR GRAPH FOR
PINUS PONDEROSA, LOT 1, BLOCK 5
SAGINAW FOREST ANN ARBOR, MICHIGAN
SOLID LINES = THINNING
BROKEN LINES = GROWTH
2500
2000
1500
1000
900
800
700
600
500
400
300
200
150
100
90
80
70
NUMBER OF TREES PER ACRE
HEIGHT OF STAND - FEET
15
20
30
40
50
60
70
80
90
100
150

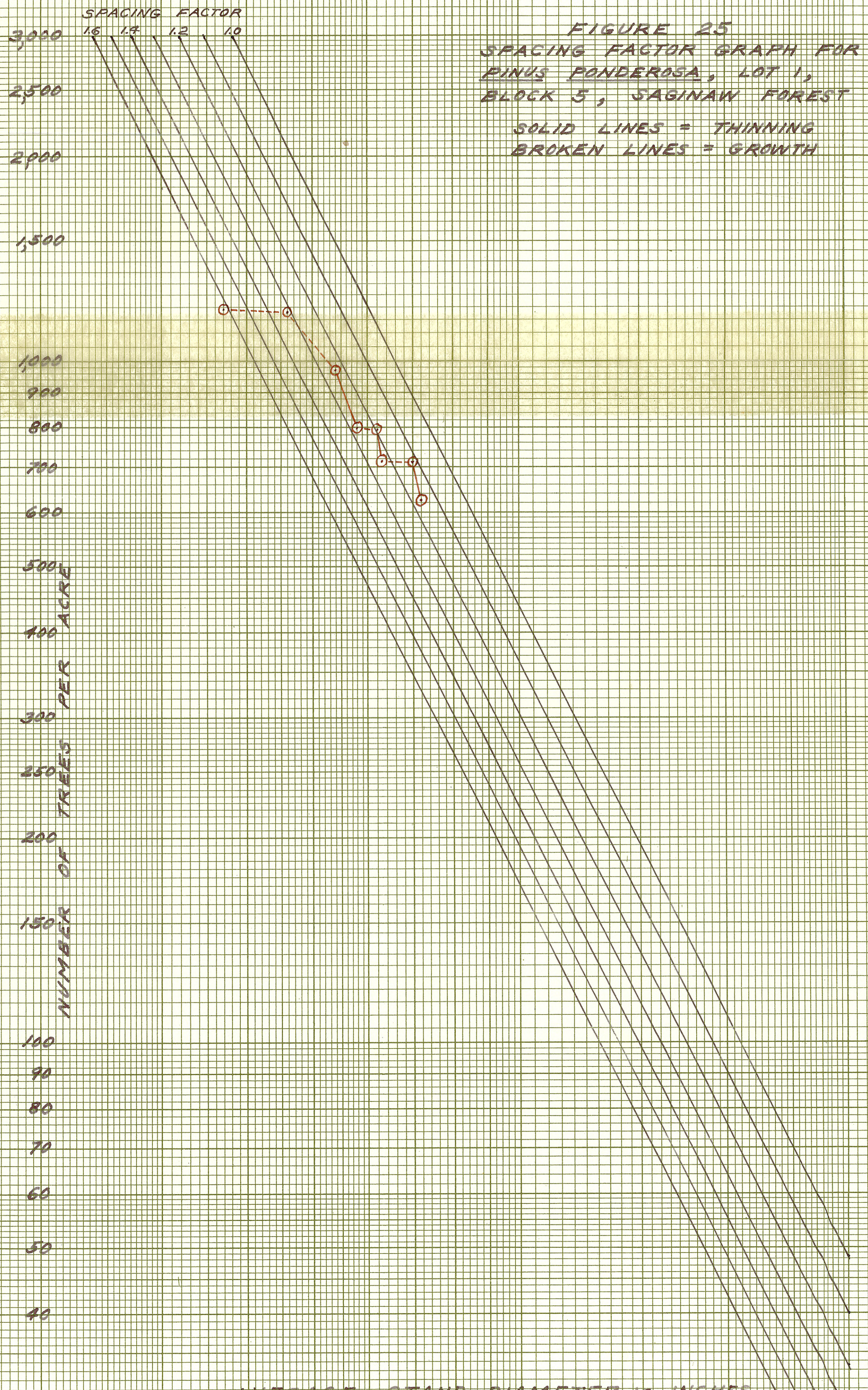


FIGURE 25
SPACING FACTOR GRAPH FOR *PINUS PONDEROSA*, LOT 1, BLOCK 5, SAGINAW FOREST

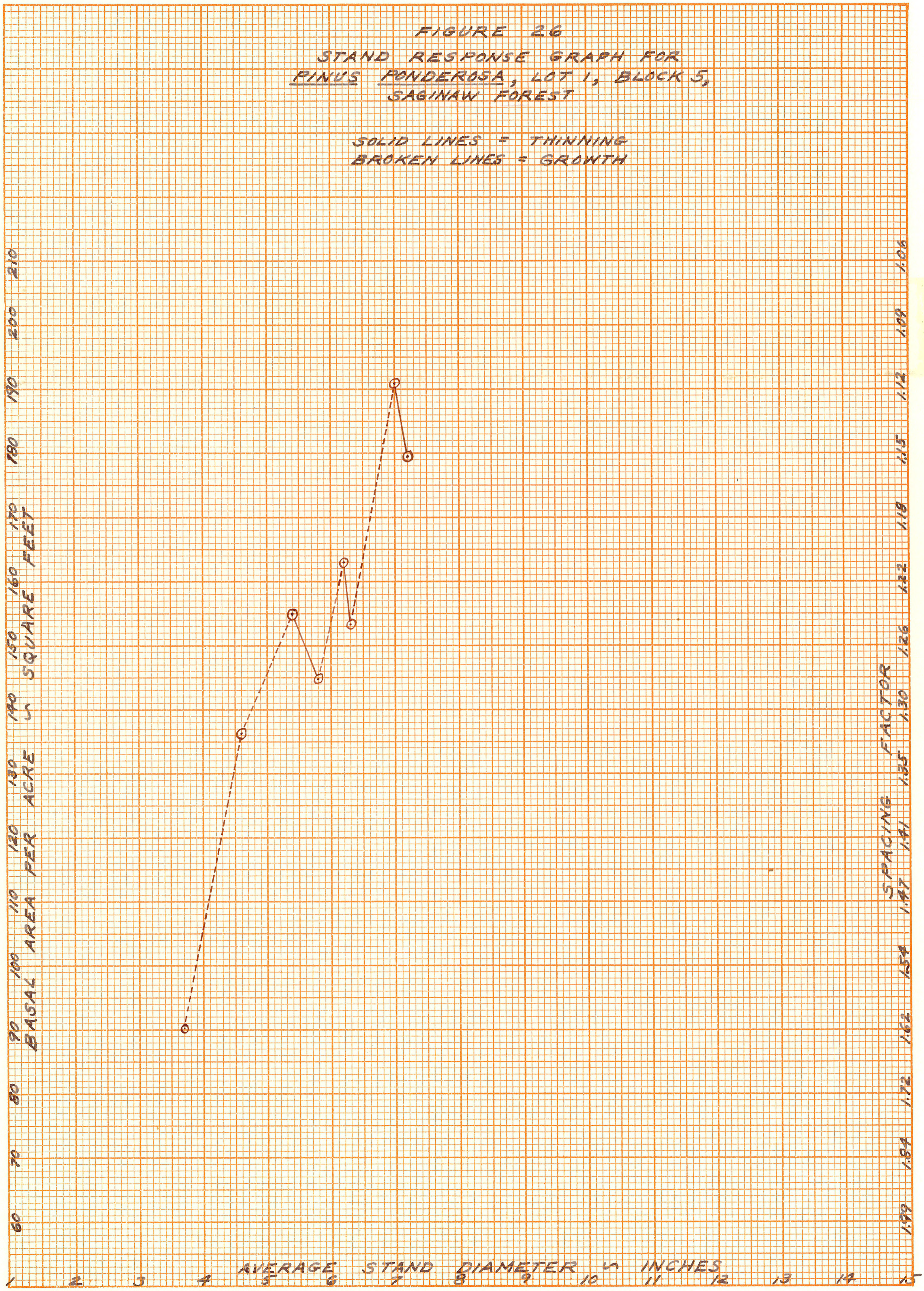


FIGURE 26

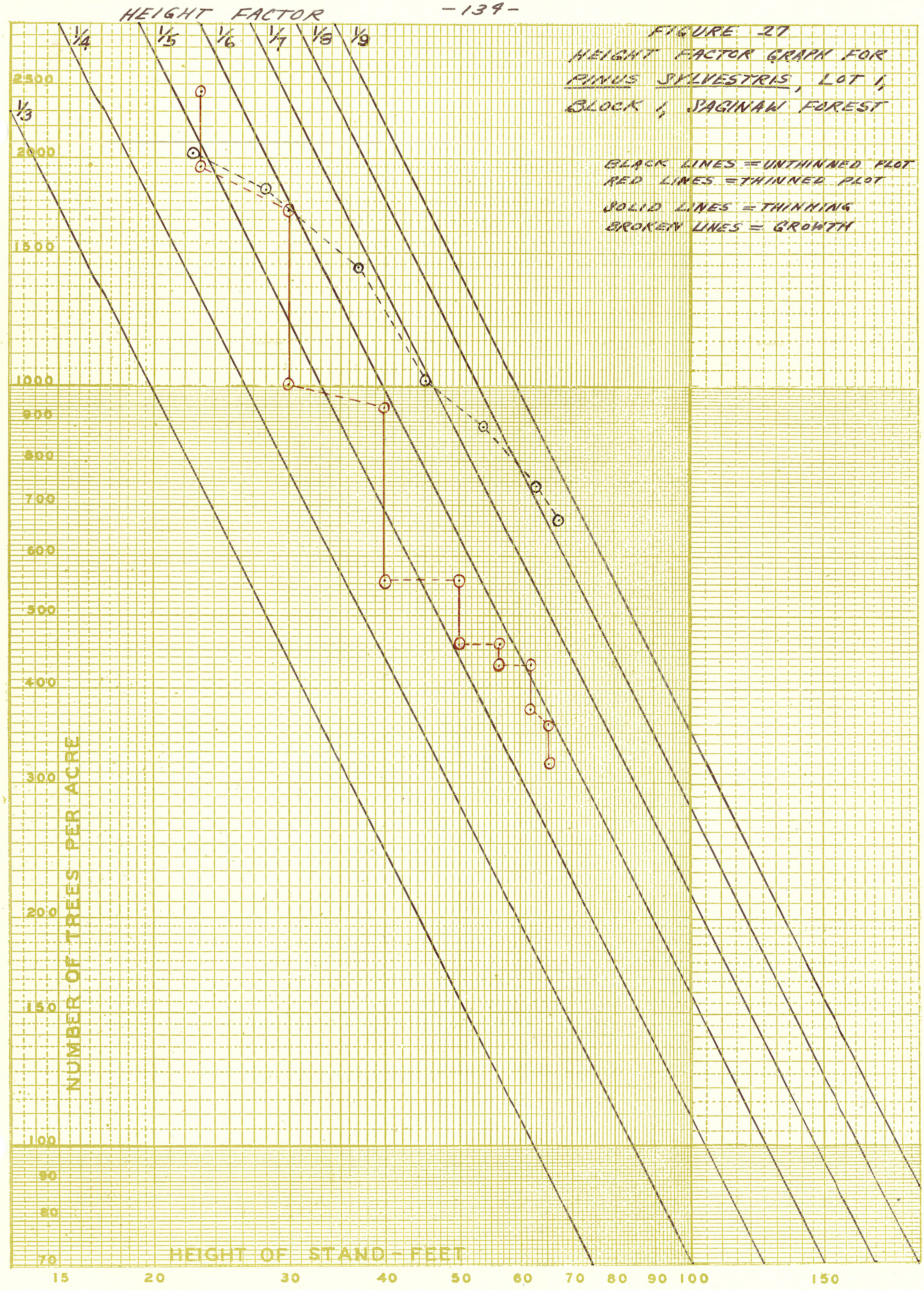
STAND RESPONSE GRAPH FOR
PINUS PONDEROSA, LOT 1, BLOCK 5,
SAGINAW FOREST

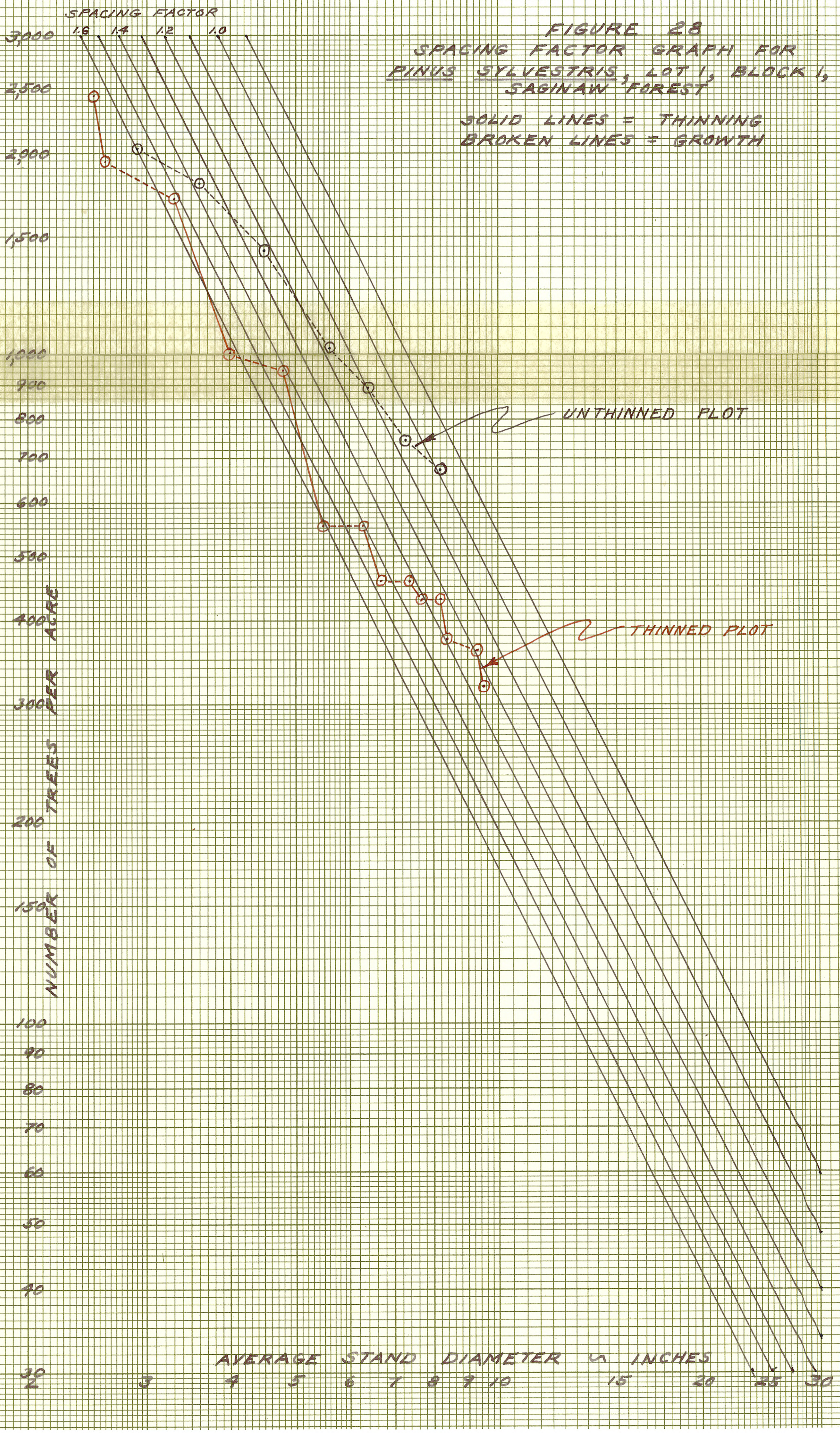
previously discussed appear to begin conforming to some constant slope earlier in the lives of the stands. The stand data of this ponderosa pine stand plots in a general line whose slope appears to correlate closer with that of the spacing factor lines in Figure 25 than that of the height factor lines in Figure 24. The stand response graph of Figure 26 shows the large increase in basal area per acre during the early life of the ponderosa pine stand. This reflects the same picture of the early life of the stand as portrayed in Figures 24 and 25.

The stand data of the thinned and unthinned plots of the Scotch pine, *Pinus sylvestris*, stand of Lot 1, Block 1, Saginaw Forest, is plotted on the height factor graph of Figure 27, the spacing factor graph of Figure 28, and the stand response graph of Figure 29. The stand data used in these graphs is in Table 5 of the appendix and a photograph of the stand is in Figure 5 of the appendix. Correlation of the Scotch pine stand data with the stocking lines is fair in both Figures 27 and 28 and appears to be somewhat better in the height factor graph than in the spacing factor graph for this particular stand. In both types of graphs the stand data again plots along a line with

FIGURE 27

HEIGHT FACTOR GRAPH FOR PINUS SYLVESTRIS, LOT 1, BLOCK 1, SAGINAW FOREST




SPACING FACTOR
1.6
1.4
1.2
1.0
FIGURE 28
SPACING FACTOR GRAPH FOR
PINUS SYLVESTRIS, LOT 1, BLOCK 1,
SAGINAW FOREST
SOLID LINES = THINNING
BROKEN LINES = GROWTH
UNTHINNED PLOT
THINNED PLOT
NUMBER OF TREES PER ACRE
3,000
2,500
2,000
1,500
1,000
900
800
700
600
500
400
300
200
150
100
90
80
70
60
50
40
30
AVERAGE STAND DIAMETER – INCHES
2
3
4
5
6
7
8
9
10
15
20
25
30

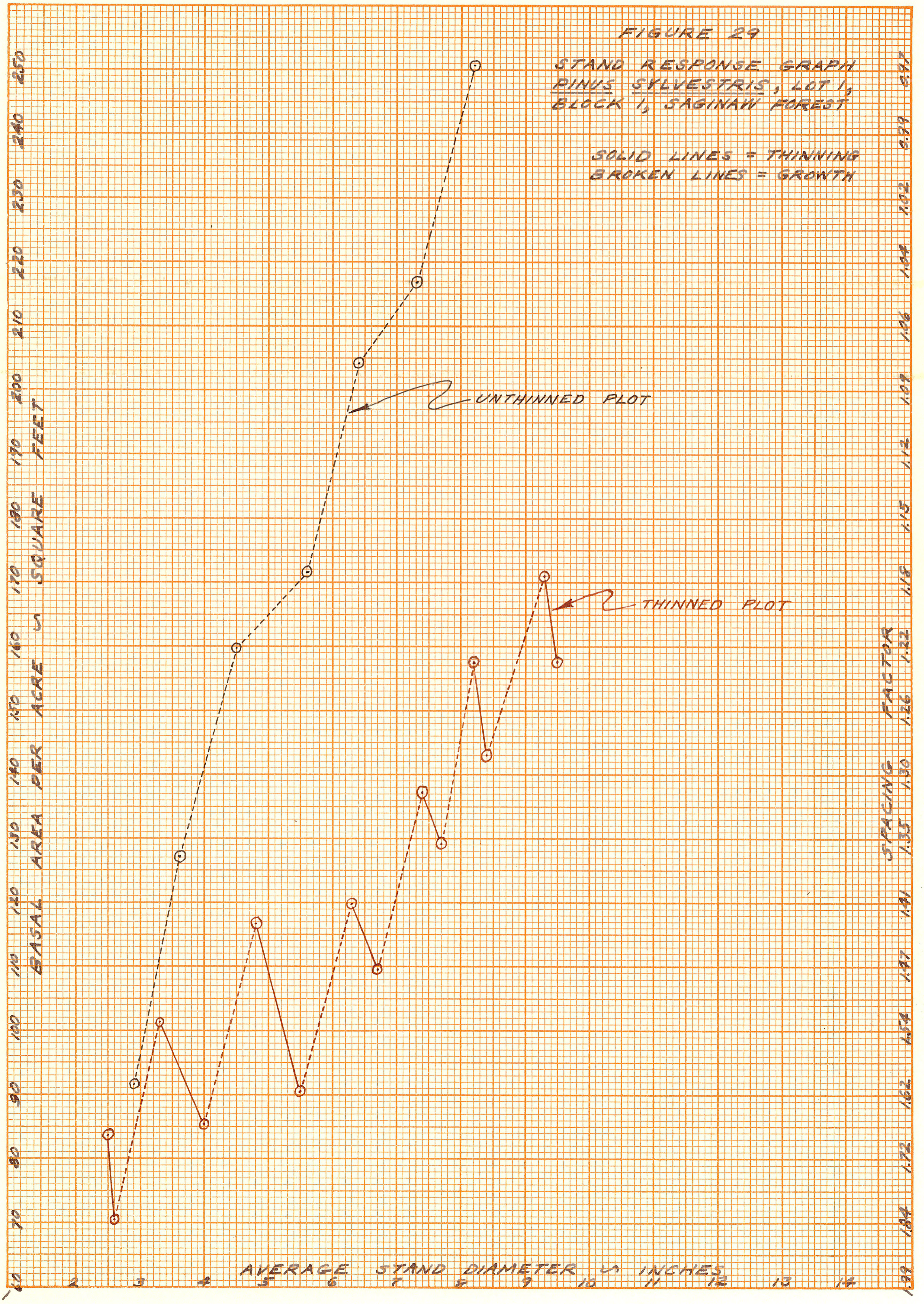


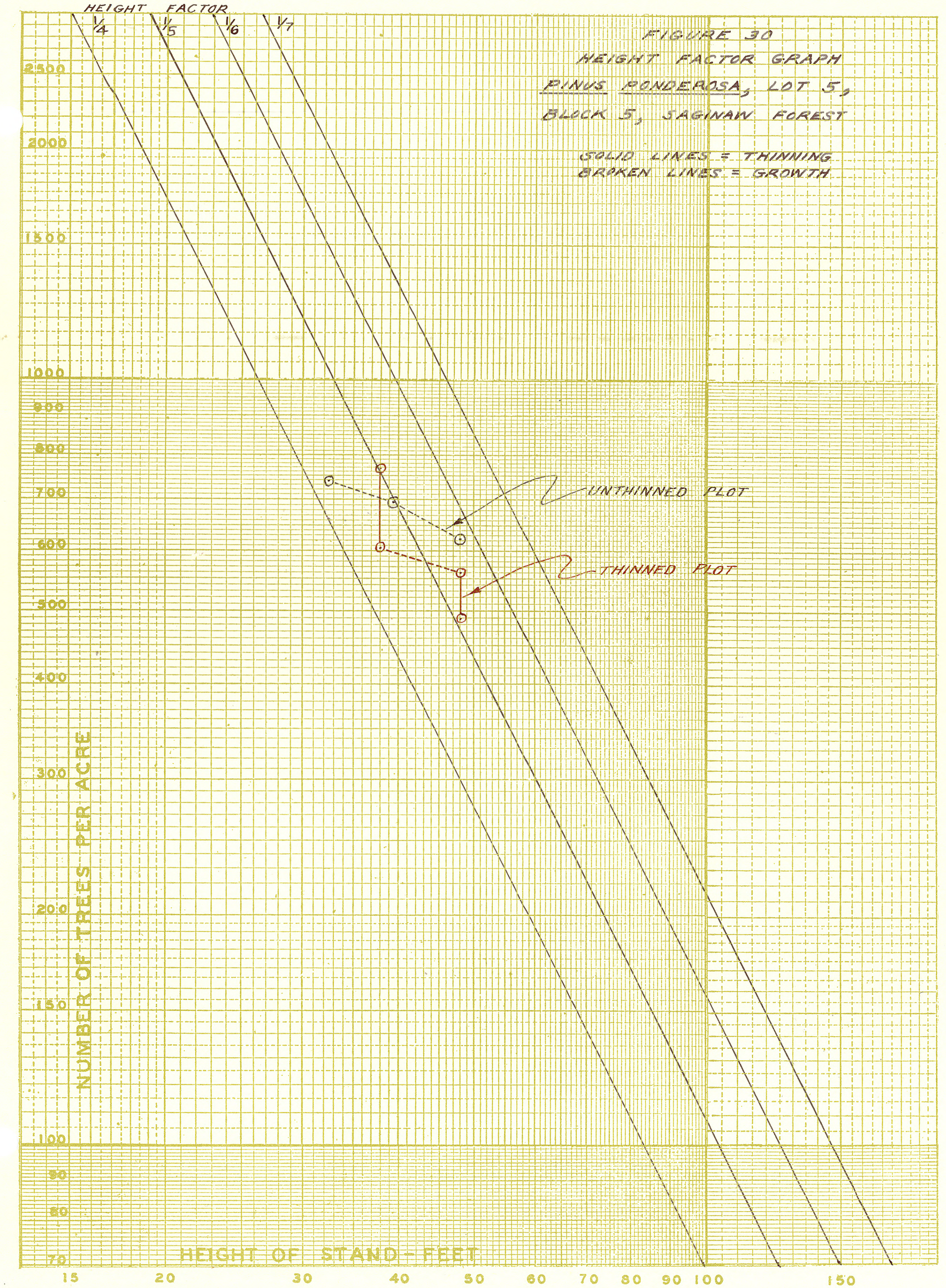
FIGURE 29
STAND RESPONSE GRAPH
PINUS SYLVESTRIS, LOT 1,
BLOCK 1, SAGINAW FOREST
SOLID LINES = THINNING
BROKEN LINES = GROWTH
UNTHINNED PLOT
THINNED PLOT
BASAL AREA PER ACRE IN SQUARE FEET
60
70
80
90
100
110
120
130
140
150
160
170
180
190
200
210
220
230
240
250
AVERAGE STAND DIAMETER IN INCHES
1
2
3
4
5
6
7
8
9
10
11
12
13
14
SPACING FACTOR
0.97
0.99
1.02
1.04
1.06
1.09
1.12
1.15
1.18
1.22
1.26
1.30
1.35
1.41
1.47
1.54
1.62
1.72
1.84
1.99

not quite as steep a slope as that of the graph's stocking lines. Note the considerably higher stocking of the unthinned Scotch pine plot in relation to the thinned plot. This difference is brought out best in the stand response graph of Figrue 29. The spacing factor graph and the stand response graph, however, show that the thinned plot has a higher average stand diameter than the unthinned plot, partly compensating for this large difference in stocking. The height factor graph of Figure 27 does not reflect this partial compensation since the average heights of the two plots are about equal, and in fact, the average height of the dominant and codominant trees in the stand is actually greater on the unthinned plot than on the thinned one. This can be construed as being a disadvantage for the height factor graph.

The brief history of the thinned and unthinned plots of the western yellow pine, <u>Pinus ponderosa</u>, stand of Lot 5, Block 5, Saginaw Forest, is plotted on the height factor graph of Figure 30, the spacing factor graph of Figure 31, and the stand response graph of Figure 32. The stand data used in these graphs are in Table 6 of the appendix, and photographs of this stand are in Figure 6 of the appendix.

The thinned plot of this stand is plotted only through two thinnings and one growth period, and the unthinned plot of this stand is plotted through only two growth periods. Note that it is apparent that even in this brief span of life the correlation of the stand data to the slope of the stocking lines is somewhat better on the spacing factor graph of Figure 31 than on the height factor graph of Figure 30. Again as in the cases of the other stands discussed above, the stocking of this stand is portrayed by the graphs as increasing. The stand response graph of Figure 32 brings this fact out in terms of basal area per acre.

From these graphs of the histories of individual stands it is noted that there is a fair correlation between the stand data and both the height factor and spacing factor lines. This is true even though these stands were not managed under these methods. Therefore, it can be assumed that stands managed under these methods will show even greater correlation and can probably be managed so as to follow very closely one particular stocking line. The height factor lines and spacing factor lines appear to give about the same degree of correlation with the slope of the plotted



HEIGHT FACTOR
1/4
1/5
1/6
1/7
FIGURE 30
HEIGHT FACTOR GRAPH
PINUS PONDEROSA, LOT 5,
BLOCK 5, SAGINAW FOREST
SOLID LINES = THINNING
BROKEN LINES = GROWTH
UNTHINNED PLOT
THINNED PLOT
NUMBER OF TREES PER ACRE
2500
2000
1500
1000
900
800
700
600
500
400
300
200
150
100
90
80
70
HEIGHT OF STAND - FEET
15
20
30
40
50
60
70
80
90
100
150

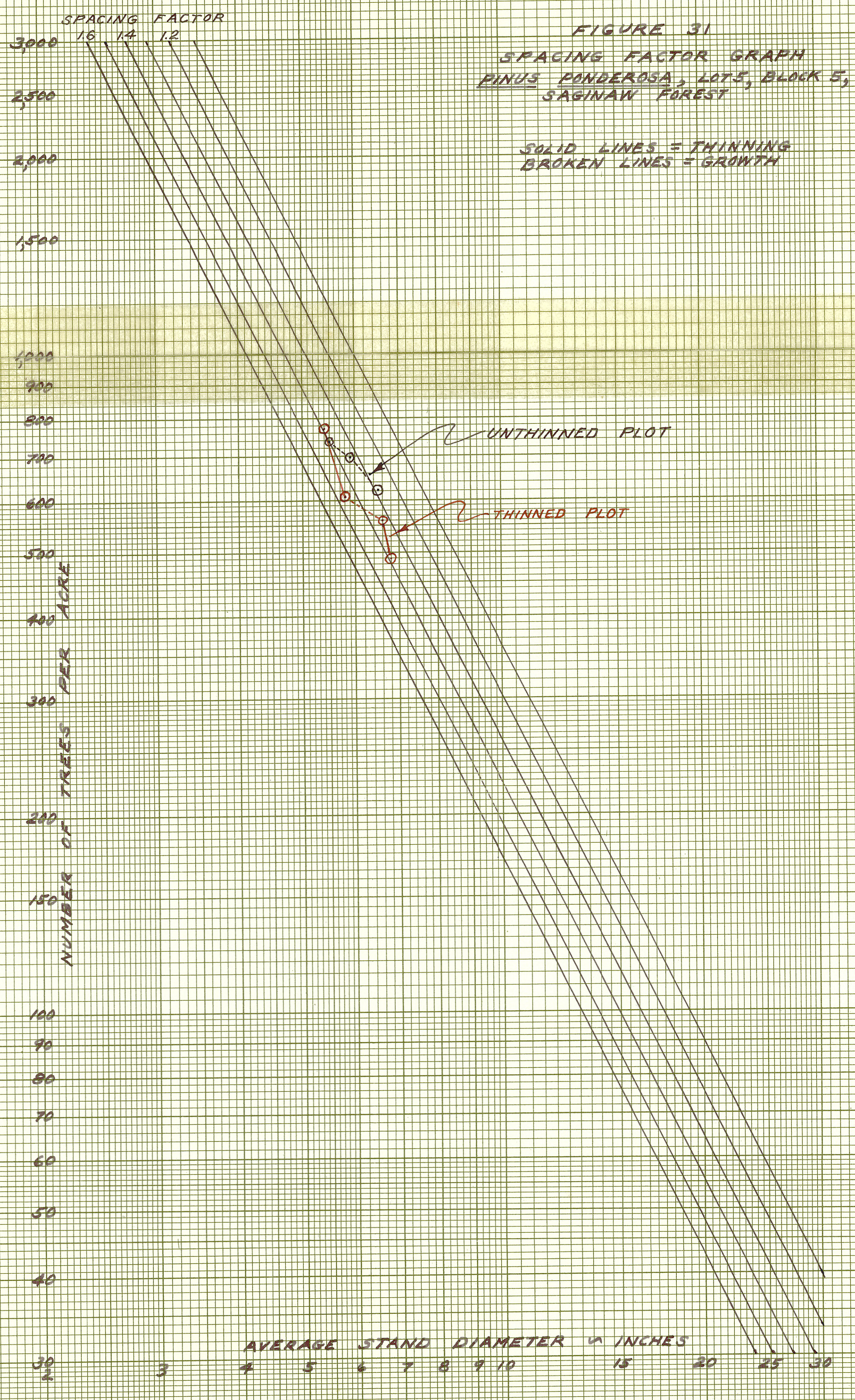


FIGURE 31
SPACING FACTOR GRAPH
PINUS PONDEROSA, LOT 5, BLOCK 5,
SAGINAW FOREST
SOLID LINES = THINNING
BROKEN LINES = GROWTH
SPACING FACTOR
1.6 1.4 1.2
UNTHINNED PLOT
THINNED PLOT
NUMBER OF TREES PER ACRE
3,000
2,500
2,000
1,500
1,000
900
800
700
600
500
400
300
200
150
100
90
80
70
60
50
40
30
AVERAGE STAND DIAMETER in INCHES
2
3
4
5
6
7
8
9
10
15
20
25
30

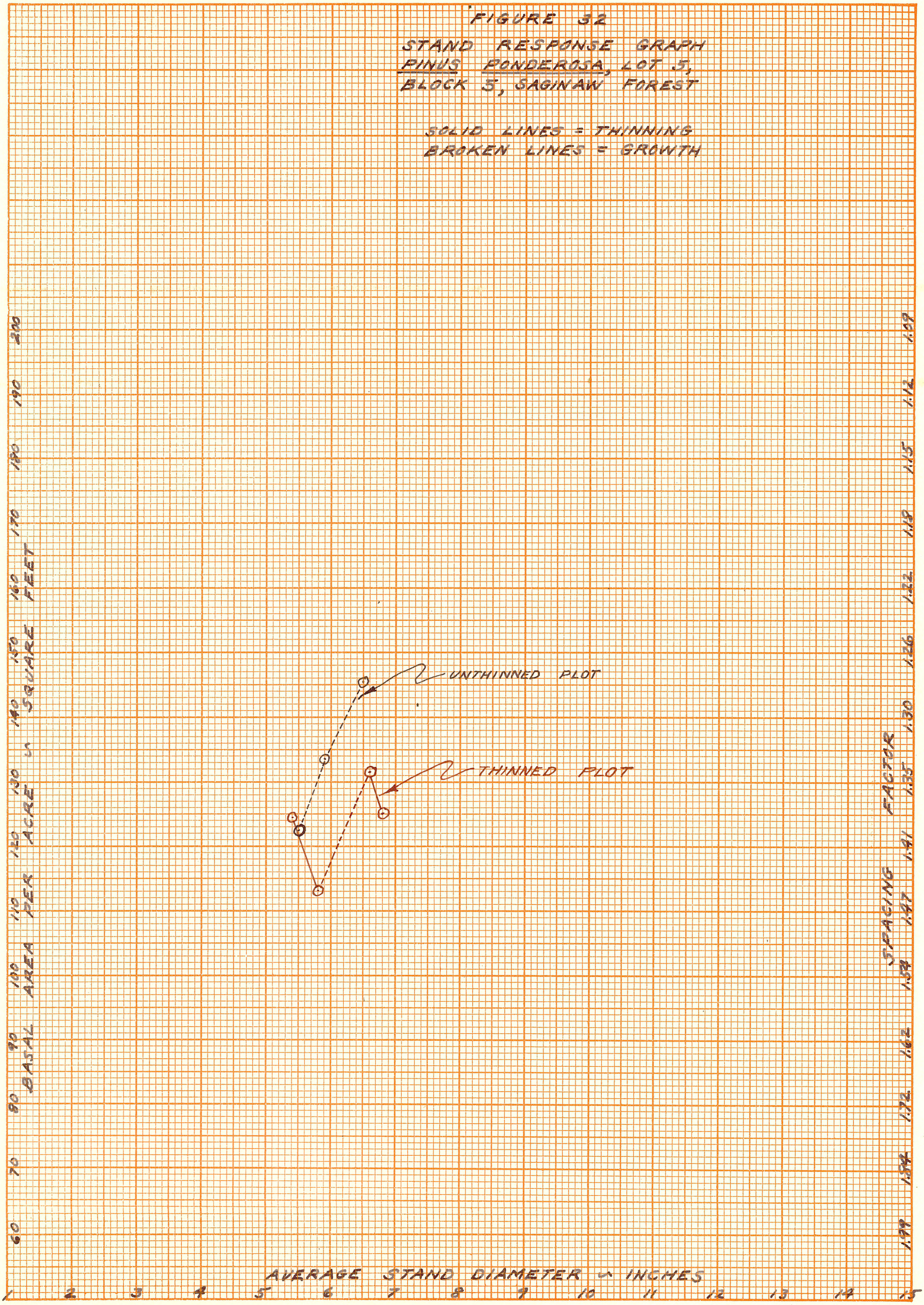


FIGURE 32
STAND RESPONSE GRAPH
PINUS PONDEROSA, LOT 5,
BLOCK 5, SAGINAW FOREST
SOLID LINES = THINNING
BROKEN LINES = GROWTH
UNTHINNED PLOT
THINNED PLOT
BASAL AREA PER ACRE - SQUARE FEET
60
70
80
90
100
110
120
130
140
150
160
170
180
190
200
SPACING FACTOR
1.98
1.84
1.72
1.62
1.54
1.47
1.41
1.35
1.30
1.26
1.22
1.18
1.15
1.12
1.09
AVERAGE STAND DIAMETER - INCHES
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15

stand data. In the case of the Scotch pine stand the correlation appears to be better between the slope of the height factor lines and that of the stand data. In the case of the Austrian pine stand and the two ponderosa pine stands the correlation appears to be somewhat better between the spacing factor line slope and the stand data. Therefore, it cannot be said from this limited data that either method is superior over the other in correlating with stand data. These above facts suggest the possibility that the height factor method might be best for managing certain tree species, and the spacing factor method might be best for managing other particular tree species. This is merely a suggestion, however, since this limited data with its inherent faults, described in several places above, is definitely not sufficient to draw any such conclusions. It may well be that one of the two methods is superior to the other in correlating with the stand data of all tree species, although this is not indicated here.

The plotting of the data of the Scotch pine stand brings out one disadvantage of the height factor graph, which the spacing factor graph does not possess in this case. The spacing factor graph showed that although the unthinned Scotch pine plot had a much higher stocking than the thinned plot, the thinned plot had a higher proportion of its stocking in trees of larger diameter since it possessed a larger average tree stem diameter breast high. The height factor graph did not show this relationship since the average height of the dominant and codominat trees was actually higher on the unthinned plot than on the thinned plot. However, the apparent advantage of the spacing factor graph in this case might possibly be lost if the average diameter of only the dominant and codominant trees were recorded instead of the average of all trees in the stand. If the average diameter of only the dominant and codominant trees were recorded on the spacing factor graph the solid thinning lines on the graph would be vertical as no increase in average diameter of dominant and codominant trees would result directly from the thinning treatment in the case of most types of thinning methods.

The average basal area per acre is a good reflection of volume. Since the spacing factor is merely an expression of basal area per acre in different terms, the spacing factor also is a good reflection of volume. Therefore, the stand response graph is unnecessary when managing a stand by the spacing factor method, because the information it portrays is already given in the spacing factor graph. In the case of the height factor method, the stand response graph does serve a valuable purpose, because the height factor graph does not give as good a reflection of volume as either basal area per acre or the spacing factor.

CORRELATION OF SPACING FACTOR LINES AND HEIGHT FACTOR LINES WITH DATA FROM YIELD TABLES FOR UNIFORMLY STOCKED, EVEN-AGED STANDS

It was stated above, while discussing the validity of the height factor equation, that good, existing yield tables can be used to give support to the height factor theory. A good yield table was plotted on the height factor graph in Figure 9 to demonstrate this fact. This same yield table plotted on a spacing factor graph provides a good comparison between the two methods of stand management by illustrating how well the slopes of the two types of stocking lines correlate with the slope of the line of the yield table data. For this type of comparison between the height factor method and the spacing factor method other yield tables for uniformly stocked, even-aged stands can be used also.

Since yield tables are the averages of many stands at different ages with different average heights and average diameters, they probably are not as good a means of comparing the correlation of stand data to the slope of the stocking lines of the height factor and spacing factor graphs as would be the historied of well managed, individual stands that have been kept within good stocking limits. The curving of the data from many stands in the construction of a yield table might possibly cast a bias into the yield table and result in a reflection of this bias when plotting the yield table data on height factor and spacing factor graphs. Nevertheless, with this possible fault kept in mine, it is still possible to make a comparison between the two methods of management by plotting the data from a yield table on both types of graphs and noting how well each method correlates with the yield table data.

Yield tables for managed stands will, ofcourse, plot with higher correlation on these graphs than will normal yield tables since normal yield tables are compiled from data taken from natural, unmanaged stands that were considered as fully-stocked under some chosen criteria. Managed stands, which the forest manager thins at intervals to maintain a high growth rate and concentrate as much growth as possible into chosen crop trees, will depart from the data given in normal yield tables more and more as the stands advance through each successive thinning.

Thus, the best correlation of stand data with the graphs of the height factor lines and spacing factor lines probably will be found in the data of individual, well managed stands that are uniformly stocked and even-aged stands. Correlation will probably be quite good in the data of yield tables of managed stands that are uniformly stocked and even-aged. Correlation will usually be poorer than the two cases above in the plotting of data from normal yield tables.

A yield table for managed pine and spruce stands in central Sweden is plotted on the height factor graph of Figure 9 (on page 85) and on the spacing factor graph of Figure 33. The data used in plotting these graphs is shown in Table 7 of the appendix. This yield table states that Danish type thinnings were applied to the stands from which this data was compiled. Information within the yield table indicates that the thinnings were not done just at ten year intervals but during the years spanning each interval as well. However, in the height factor graph of Figure 9 and the spacing factor graph of Figure 33, the data is plotted as though all trees removed during each ten year period were

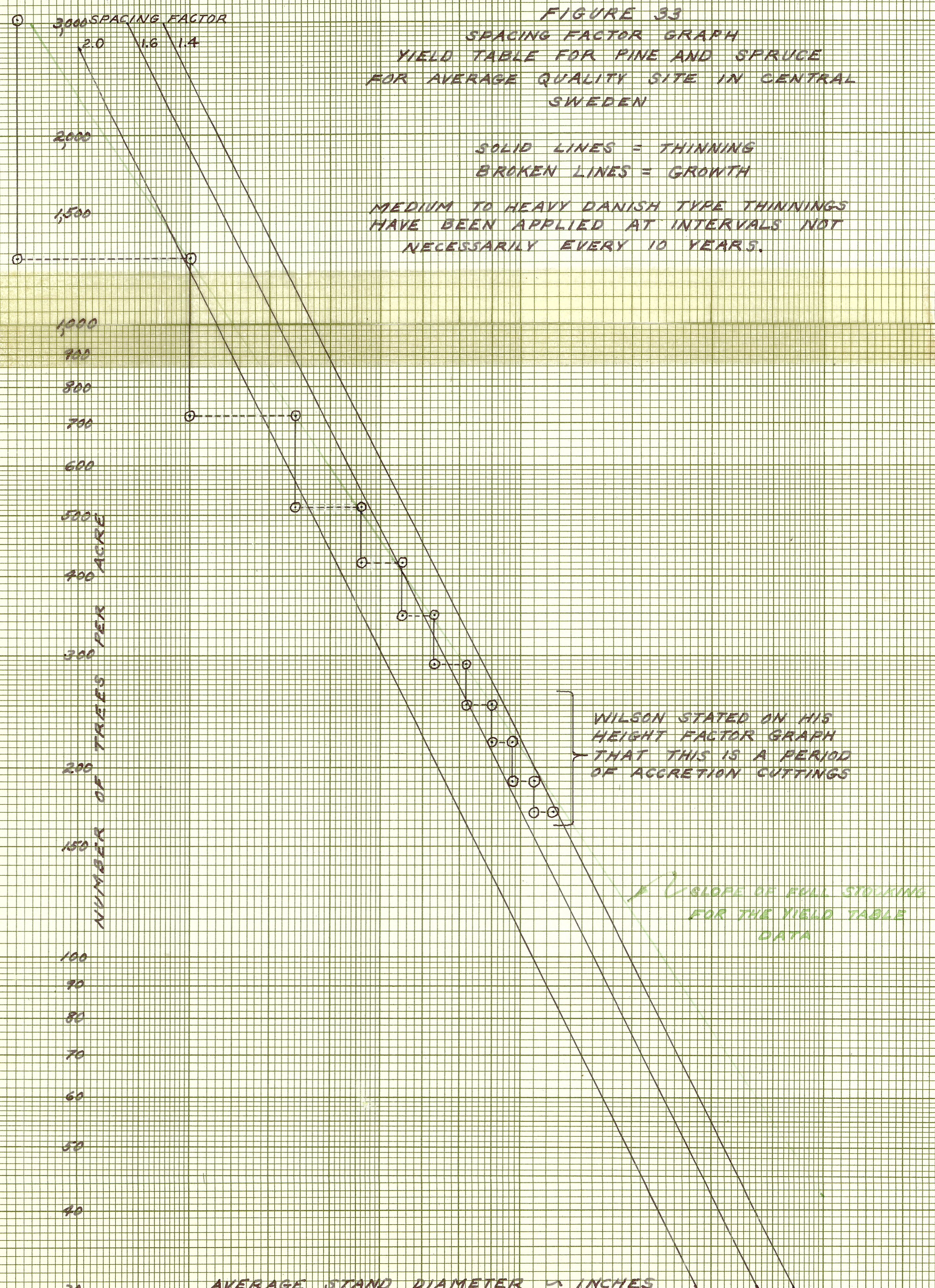


FIGURE 33
SPACING FACTOR GRAPH
YIELD TABLE FOR PINE AND SPRUCE
FOR AVERAGE QUALITY SITE IN CENTRAL SWEDEN

removed at one time at the beginning of each period since there is no detailed data in the yield table as to how many trees were removed each time the stands were visited for the purpose of a thinning treatment.

The comparison of the correlation between the slope of the stocking lines of each of these graphs with the slope of the yield table data offers an interesting contrast. The height factor lines on the graph on page 85 correlate very closely with the yield table data. A height factor of about 21.5% or 22.0% appears to be the correct upper limit of stocking. The data follows this stocking very closely until after a height of 63 feet is reached. F.G. Wilson attempts to explain this apparent drop in stocking when that height of stand is reached by calling it the period of accretion cuttings. The yield table data plots in just as constant a slope on the spacing factor graph of Figure 33 as it does on the height factor graph of Figure 9. However, this constant slope of the yieldtable data on the spacing factor graph does not closely approximate the slope of the spacing factor lines, while on the height factor graph the slope of the yield table data corresponds very closely with that of the height factor lines. On the spacing factor graph the slope of the yield table data is not as steep as the slope of the spacing factor lines. Thus, in terms of the spacing factor the yield table data indicates a steady increase in stocking, while in terms of the height factor the yield table data indicates nearly a constant stocking maintained through all the age classes. Since this is a yield table for managed stands in which the stands were probably not allowed to become overcrowded and stagnated in growth, the height factor method appears to be the better criterion on which to base stand stocking in this case. However, the spading factor graph does plot the yield table data along a constant slope. Note that the apparent period of accretion cuttings portrayed on the height factor graph is not reflected on the spacing factor graph. The average basal area per acre increases in each succeeding age class of the yield table throughout all the age classes. Therefore, there is no conclusive evidence that the stand treatments during the last three age periods in the yield table are accretion cuttings, but the height factor graph does portray them as such. In terms of the height factor they are accretion cuttings.

The stand data of two sites of a normal yield table for eastern white pine, <u>Pinus strobus</u>, in Massachusetts is plotted on the height factor graph of

Figure 34 and on the spacing factor graph of Figure 35. (19)

In the discussion of the height factor equation earlier in the thesis, it was stated that according to the equation, site should have no effect on "f". However, the height factor graph of this yield table in Figure 34 does not appear to bear out this hypothesis appreciably. There is a consistent difference in stocking between the two sites on the graph of between one and two percent in height factor value. In the case of the spacing factor graph of Figure 35, a similar hypothesis that site should have no effect on "S F" of the spacing factor equation would be substantiated by the graph as the stand data of the two sites plot very close together. It is interesting to note that on the height factor graph the stand data of Site II plots at a consistently lower stocking than that of Site I throughout the data; the stand data of Site II on the spacing factor graph plots close to but a little below the data of Site I in the early part

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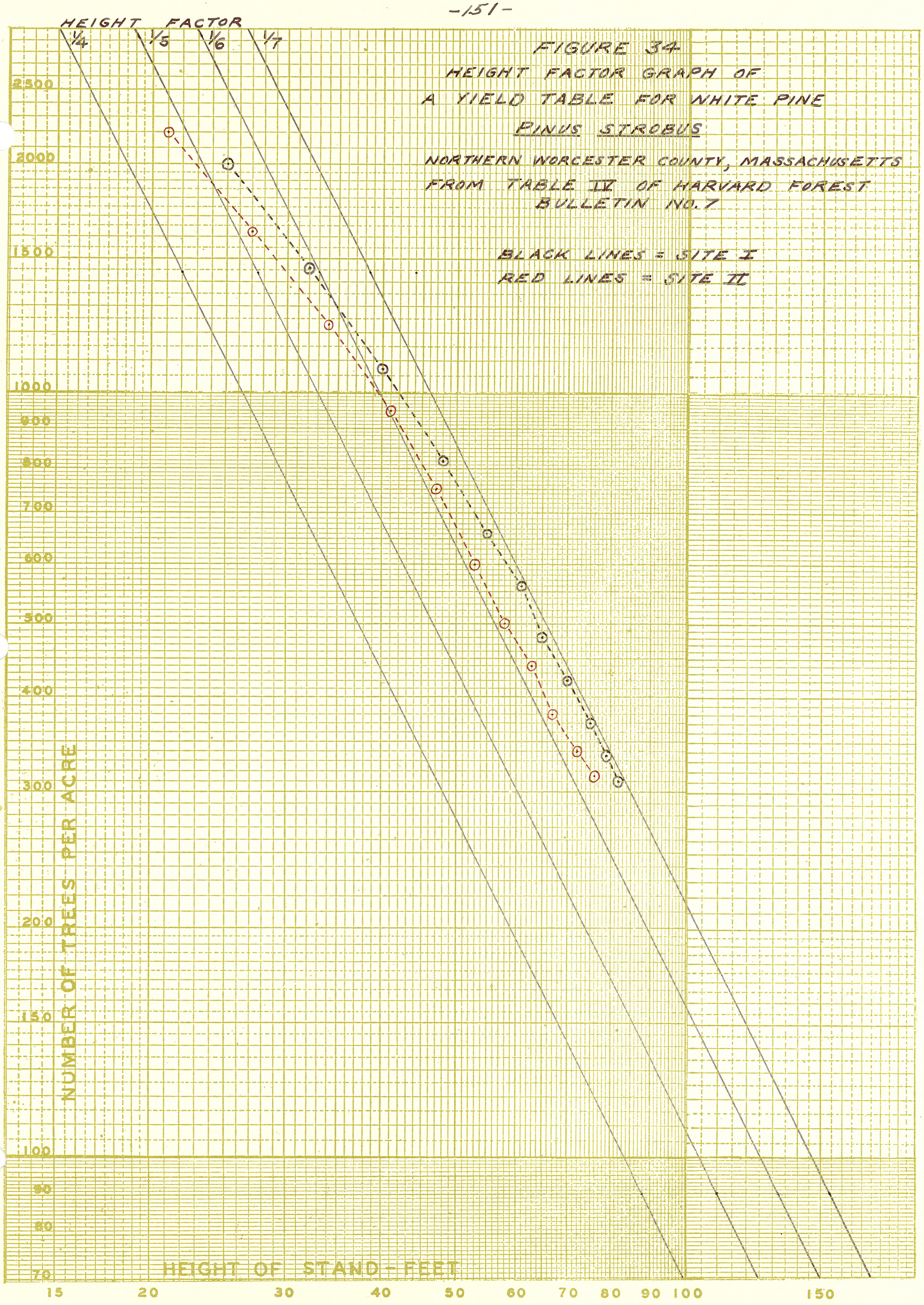
19. Reed, P.M. and Tarbox, E.E., "Table IV. Yield Table for White Pine", Harvard Forest Bulletin No.7, Harvard Forest, Petersham, Mass., 1924, p.17.


HEIGHT FACTOR
1/4
1/5
1/6
1/7
FIGURE 34
HEIGHT FACTOR GRAPH OF
A YIELD TABLE FOR WHITE PINE
PINUS STROBUS
NORTHERN WORCESTER COUNTY, MASSACHUSETTS
FROM TABLE IV OF HARVARD FOREST
BULLETIN NO.7
BLACK LINES = SITE I
RED LINES = SITE II
NUMBER OF TREES PER ACRE
2500
2000
1500
1000
900
800
700
600
500
400
300
200
150
100
90
80
70
HEIGHT OF STAND - FEET
15
20
30
40
50
60
70
80
90
100
150

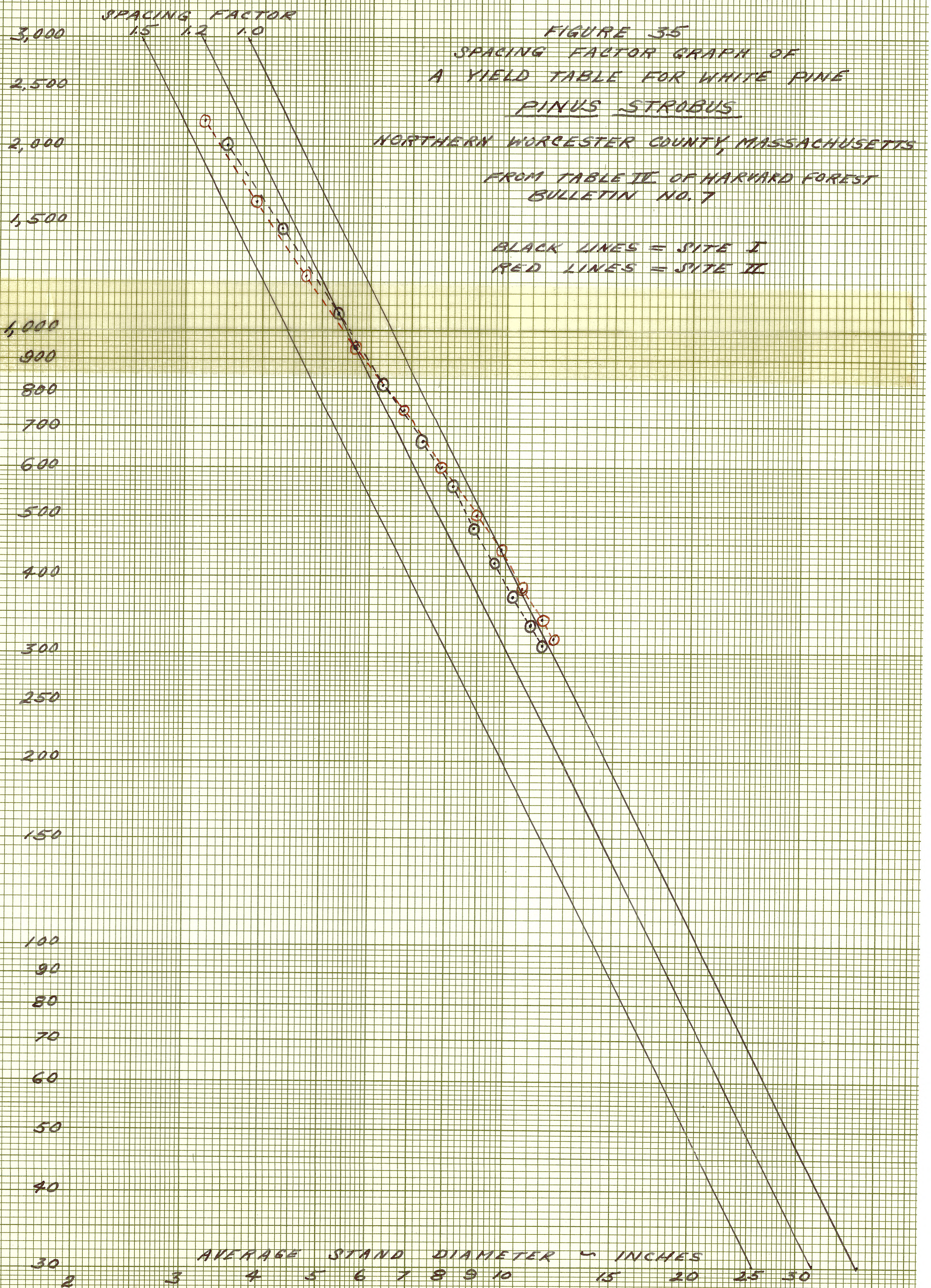


FIGURE 35
SPACING FACTOR GRAPH OF
A YIELD TABLE FOR WHITE PINE
PINUS STROBUS
NORTHERN WORCESTER COUNTY, MASSACHUSETTS
FROM TABLE IV OF HARVARD FOREST
BULLETIN NO. 7

of the stand history, and in the latter part of the stand history plots close to but is actually a little above that of Site I in stocking.

The data from this normal yield table for eastern white pine plots much closer to a straight line on the spacing factor graph than on the height factor graph. The data plots in a slight curve on the height factor graph and thus does not have as constant a slope as does the data plotted on the spacing factor graph. However, in the older age classes of the yield table, the data plots on a slope somewhat closer to the parallel height factor lines than does the slope of the data on the spacing factor graph in relation to the slope of the spacing factor lines. On the spacing factor graph the yield table table data is portrayed as steadily increasing instocking throughout all age classes.

The stand data of a poor site and a good site from normal yield tables of second-growth, eastern white pine, Pinus strobus, in Wisconsin is plotted on the height factor graph of Figrue 36 and on the spacing factor graph of Figure 37. (20)

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20. Brown, R.M. and Gevorkiantz, S.R., Volume, Yield, and Stand Tables for Tree Species in the Lake States, Technical Bulletin 39, University of Minnesota Agricultural Experiment Station, St. Paul, Minnesota, 1934, p.203 and 205.

Correlation between the slope of the plotted table data with the slope of the stocking lines on both the height factor graph and the spacing factor graph is very close and is better than the correlation of the yield table for the same tree species in Massachusettes in Figures 34 and 35. Figure 36, the height factor graph, again shows the yield table data with a slight curve in it as was shown in Figure 34 for Massachusetts. The spacing factor graph portrays the yield table data as being very nearly a straight line. The poor site is portrayed as of consistently less stocking than that of the good site. The poor site does not cross the plotted line of the good site, and does not show a higher stocking at any point in the spacing factor graph of Figure 37, while the two sites of the Massachusetts yield table did cross in the spacing factor graph of Figure 35. Figures 36 and 37 support the characteristic portrayed in Figure 34 that one quality site should plot consistently above or below a site of another quality. Note also that the good and poor sites of the Wisconsin (and applying to the Lake States) are portrayed by both the height factor graph and spacing factor graph as being of higher stocking than that shown in both the types of graphs in Figures 34 and 35 for

FIGURE 36

HEIGHT FACTOR GRAPH

NORMAL YIELD TABLES OF FULLY-STOCKED, PURE, EVEN-AGED STANDS OF

SECOND-GROWTH WHITE PINE

PINUS STROBUS

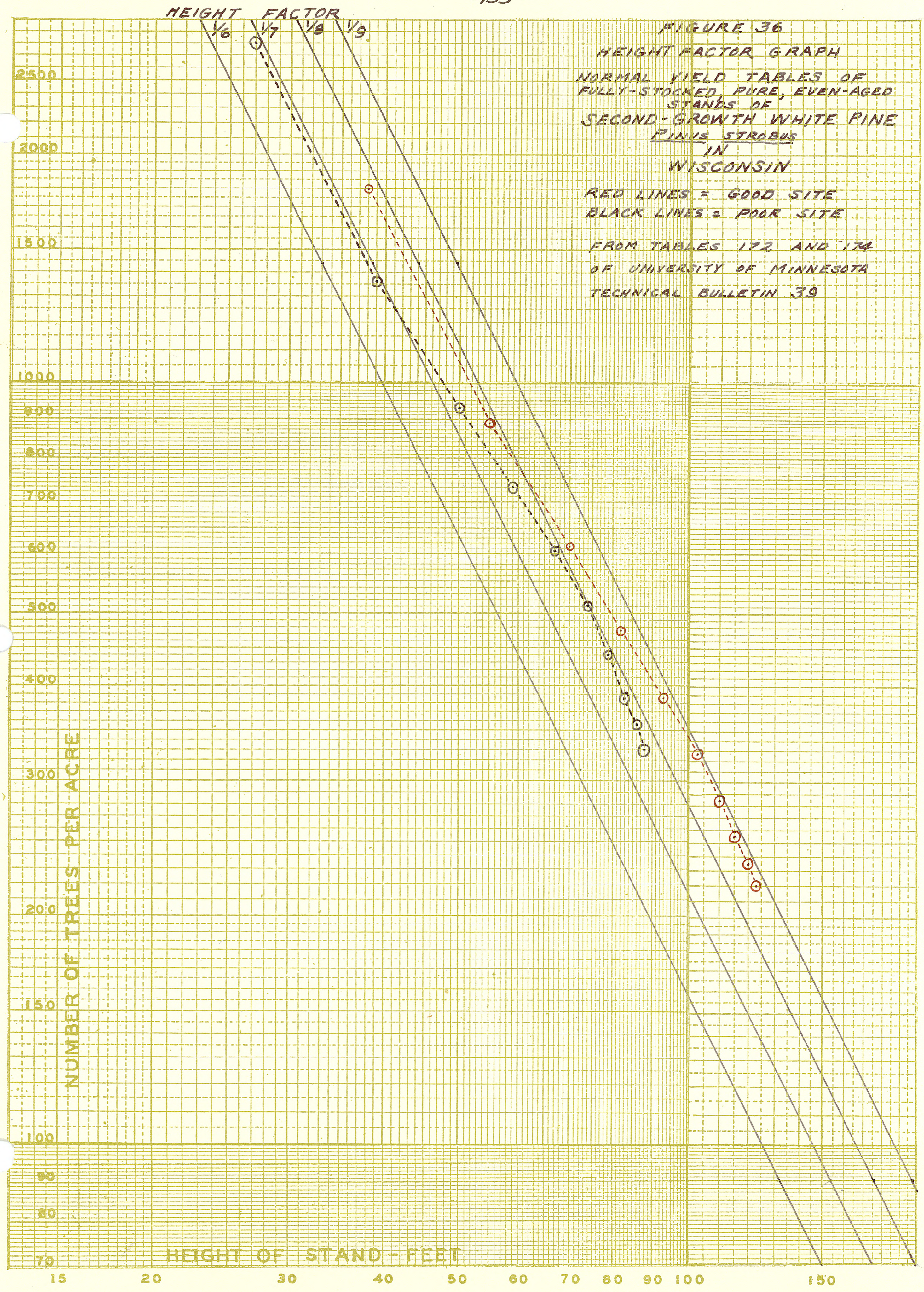
IN

WISCONSIN

RED LINES = GOOD SITE

BLACK LINES = POOR SITE

FROM TABLES 172 AND 174 OF UNIVERSITY OF MINNESOTA TECHNICAL BULLETIN 39



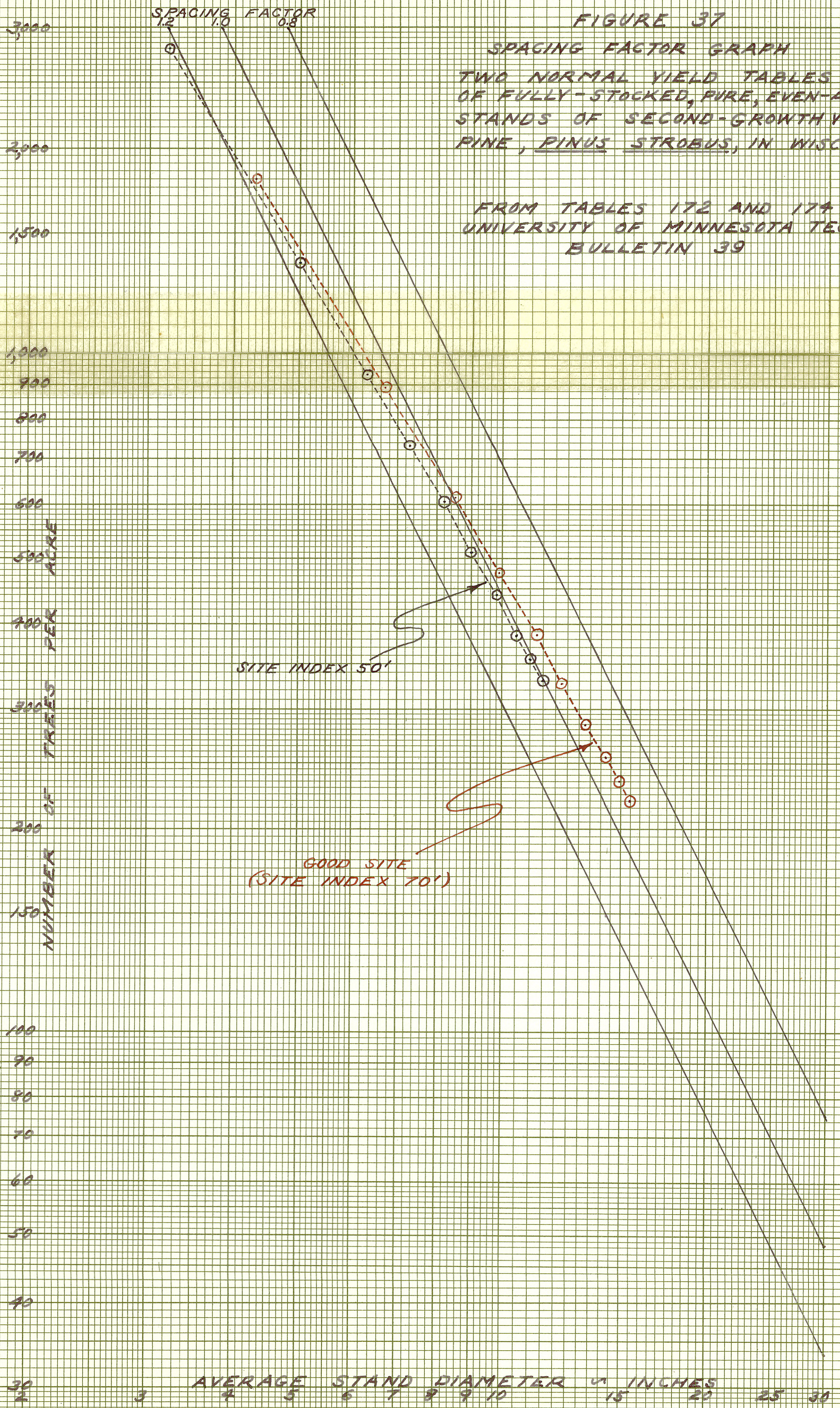


FIGURE 37

SPACING FACTOR GRAPH

TWO NORMAL YIELD TABLES OF FULLY-STOCKED, PURE, EVEN-AGED STANDS OF SECOND-GROWTH WHITE PINE, PINUS STROBUS, IN WISCONSIN

FROM TABLES 172 AND 174 OF UNIVERSITY OF MINNESOTA TECHNICAL BULLETIN 39 the Massachusetts yield table. As discussed earlier, Wilson stated that the apparent conclusions to be drawn from the height factor equation theory are that site has no effect on "f", but the consistent plotting of one site at a higher stocking level than another tends to dispute this theory.

Again, in Figures 35 and 36 it can be noted that the data plots along a slope less steep than that of the stocking lines. However, this difference in slope is very small on these two graphs.

As discussed earlier, Wilson states that white pine could be kept at a stocking between height factors one-fifth and one-sixth for proper stand stocking. Both height factor graphs of Figures 34 and 36 of white pine yield table data for Wisconsisn and Massachusetts show a higher stocking than this in all but the early age classes and for both good and poor sites.

The data for fully-stocked, pure, even-aged jack pine, Pinus banksiana, stands on good and poor sites from a normal yield table is plotted on the height factor graph of Figure 38 and the spacing factor graph of Figure 39. (21)

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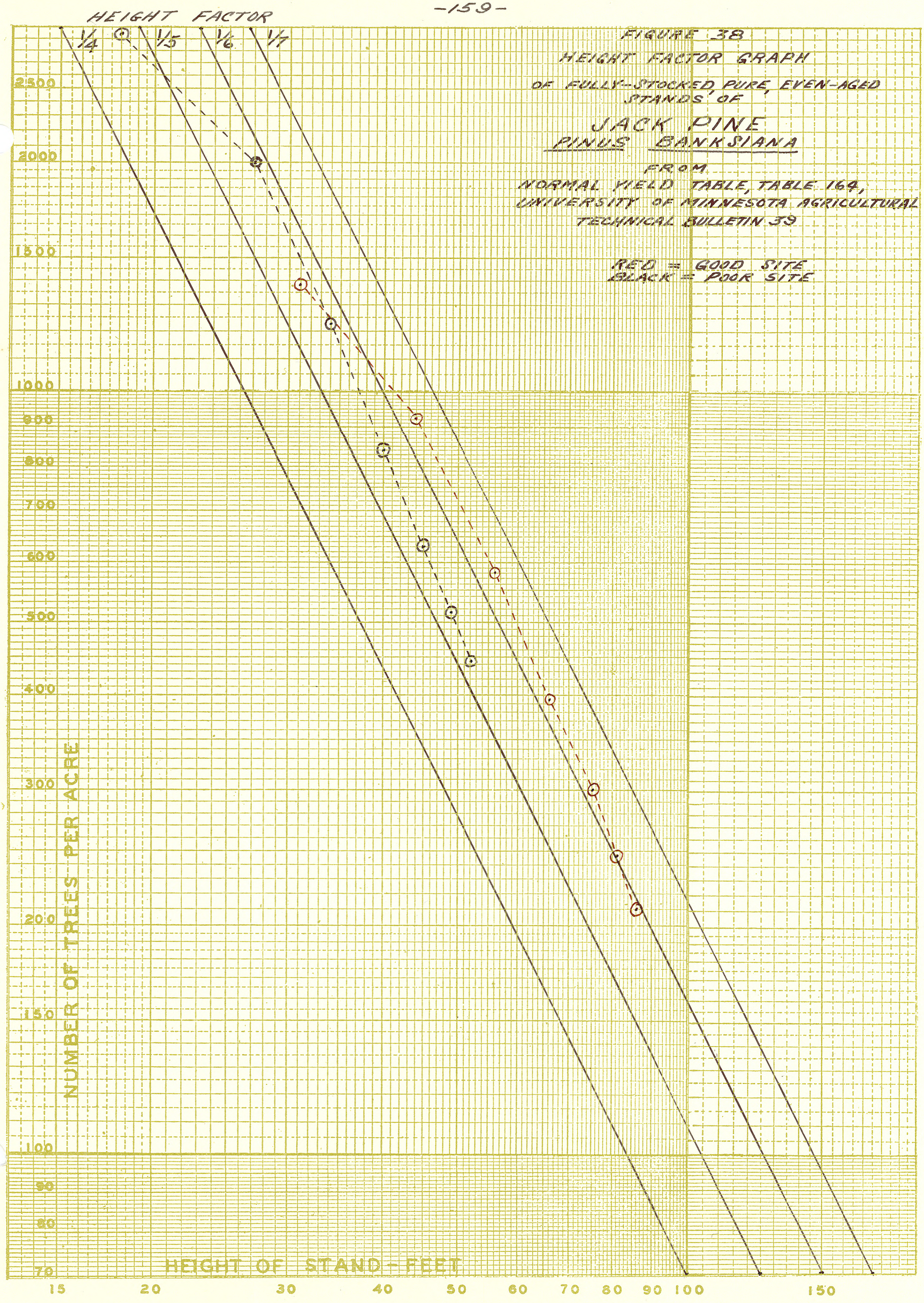
21. Ibid., p. 193.

After the first age class, the jack pine yield table data plots in a straight line for both good and poor sites on both the height factor and spacing factor graphs. Correlation of the slope of the plotted yield table data with the slope of the stocking lines is better on the spacing factor graph than on the height factor graph and is, in fact, almost perfect on the spacing factor graph. The spacing factor graph of Figure 39 portrays just a very small increase in stocking for the yield table data after the first age class. Note that on the height factor graph the slope of the yield table data is steeper than that of the stocking lines after the first age class. According to the height factor graph, the stocking of the yield table data gradually decreases after the first age class.

As mentioned earlier while describing the height factor method of management, Wilson stated in his article in the October, 1946, issue of the Journal of Forestry that jack pine could be kept at a height factor stocking of between one-fourth and one-fifth. The yield table data for jack pine plotted on the height factor graph of Figure 38 is shown as being of higher stocking. However, Figure 38 indicates the possibility that the stand data for the earlier age classes was at too high


HEIGHT FACTOR
1/4
1/5
1/6
1/7
FIGURE 38
HEIGHT FACTOR GRAPH
OF FULLY-STOCKED, PURE, EVEN-AGED STANDS OF
JACK PINE
PINUS BANKSIANA
FROM
NORMAL YIELD TABLE, TABLE 164,
UNIVERSITY OF MINNESOTA AGRICULTURAL
TECHNICAL BULLETIN 39
RED = GOOD SITE
BLACK = POOR SITE
NUMBER OF TREES PER ACRE
2500
2000
1500
1000
900
800
700
600
500
400
300
200
150
100
90
80
70
HEIGHT OF STAND - FEET
15
20
30
40
50
60
70
80
90
100
150

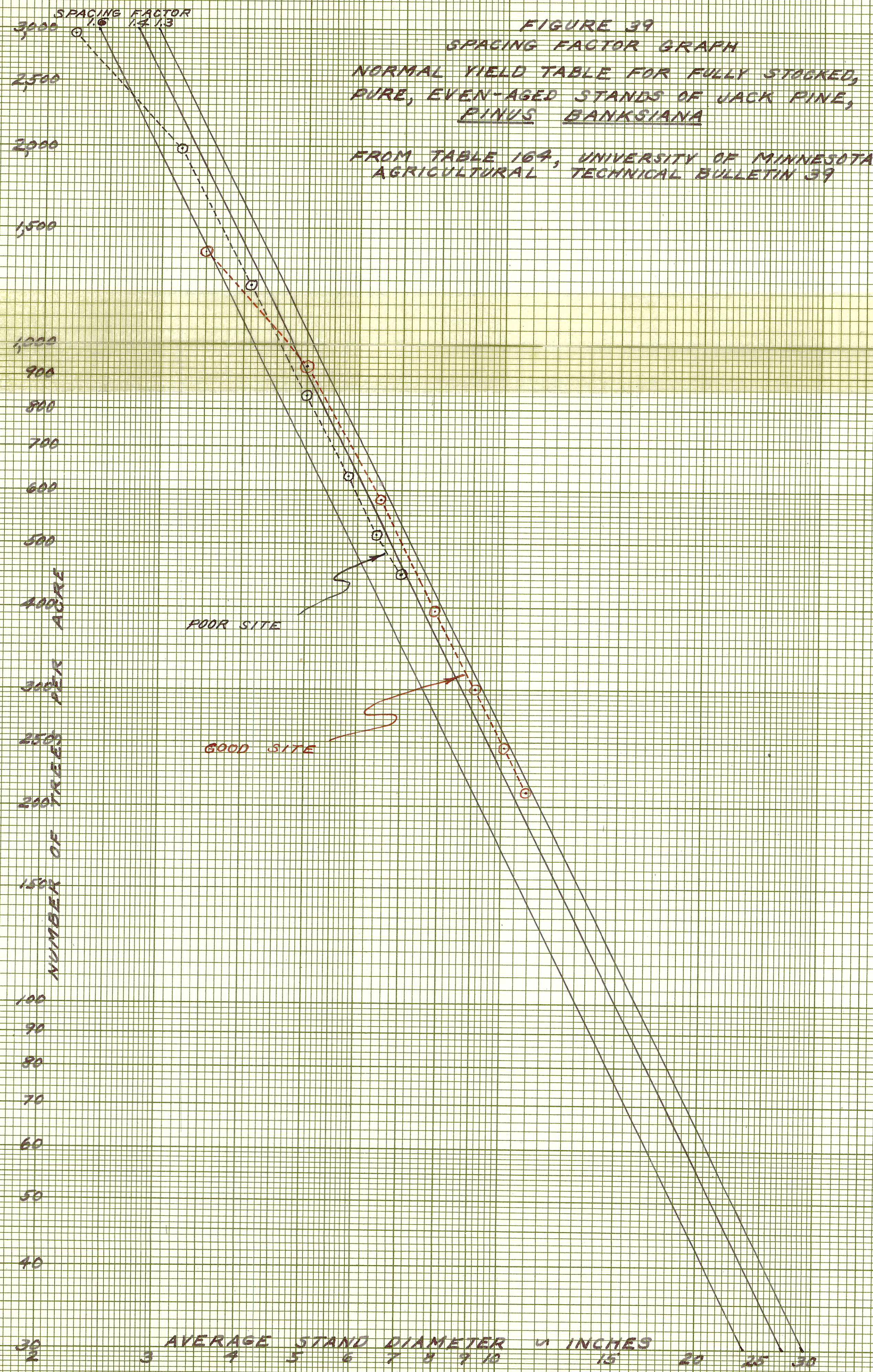


FIGURE 39
SPACING FACTOR GRAPH
NORMAL YIELD TABLE FOR FULLY STOCKED, PURE, EVEN-AGED STANDS OF JACK PINE, PINUS BANKSIANA
FROM TABLE 164, UNIVERSITY OF MINNESOTA AGRICULTURAL TECHNICAL BULLETIN 39
SPACING FACTOR
1.6
1.4
1.3
POOR SITE
GOOD SITE
NUMBER OF TREES PER ACRE
3,000
2,500
2,000
1,500
1,000
900
800
700
600
500
400
300
250
200
150
100
90
80
70
60
50
40
30
AVERAGE STAND DIAMETER - INCHES
2
3
4
5
6
7
8
9
10
15
20
25
30

a stocking because the stocking of subsequent age classes steadily decreases.

The data for fully-stocked, even-aged, normal stands of longleaf pine, Pinus palustris, for site index 50 and site index 100 is plotted on the height factor graph of Figure 40 and the spacing factor graph of Figure 41. (22) Table 8 of the appendix shows the stand data from which these two graphs were constructed.

Note that the correlation of the slope of the height factor lines with the slope of the plotted data in Figure 40 is very good for the middle age classes of both site indices. Between the first and second age classes on this graph the stocking increases, and then for the last two age classes the stocking decreases.

The slope of the spacing factor lines shows good correlation with that of the stand data in Figure 41. The data plots in more of a straight line on the spacing factor graph than on the height factor graph, and the two site indices are much closer in stocking on the spacing factor graph than on the height factor graph. As does the height factor graph, the spacing factor

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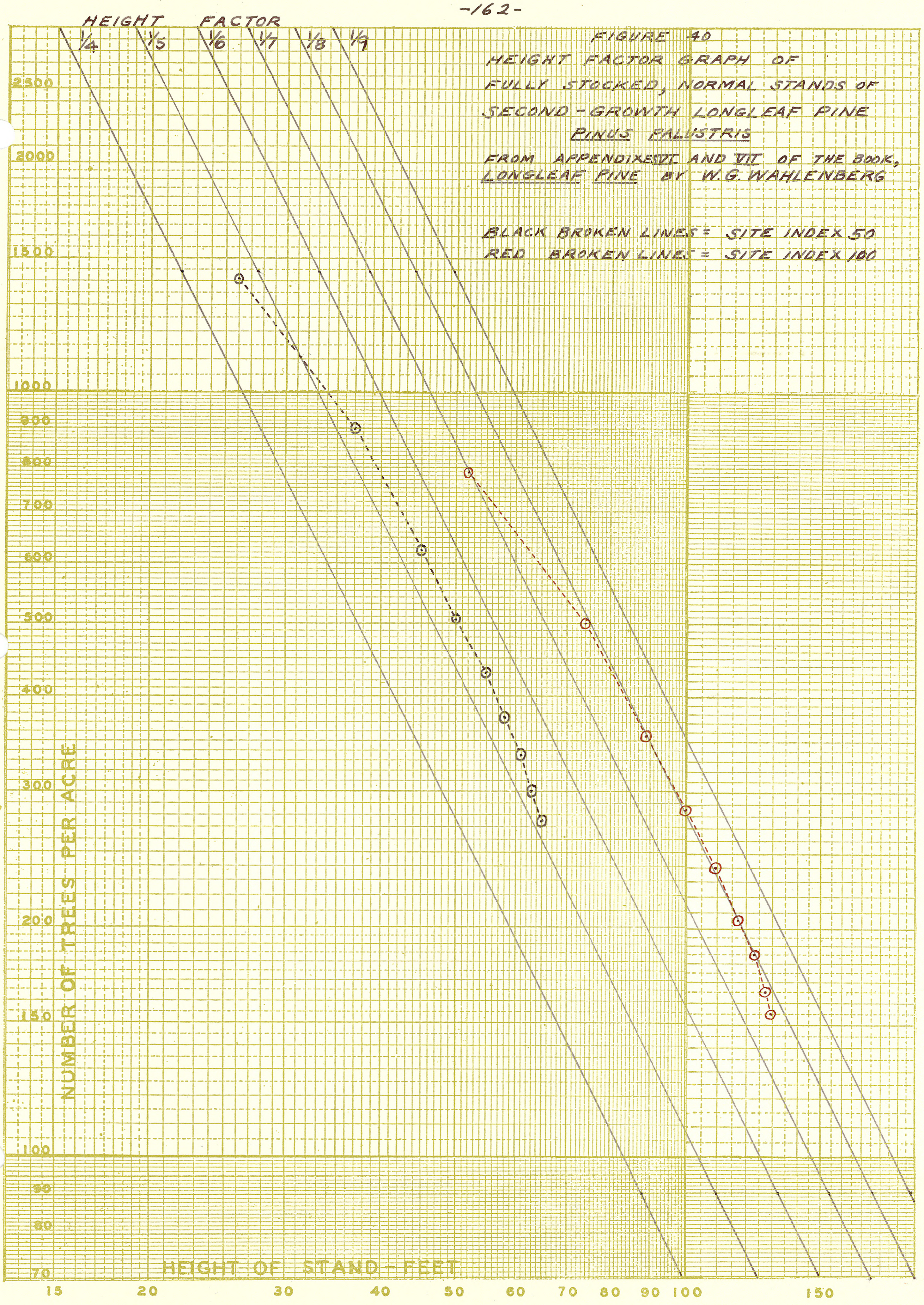
22. Wahlenberg, W.G., Longleaf Pine, Charles Lathrop Pack Forestry Foundation, publishers, Washington, D.C., First Edition, 1946, pp.290-295.


HEIGHT FACTOR
1/4
1/5
1/6
1/7
1/8
1/9
FIGURE 40
HEIGHT FACTOR GRAPH OF
FULLY STOCKED, NORMAL STANDS OF
SECOND-GROWTH LONGLEAF PINE
PINUS PALUSTRIS
FROM APPENDIXES VI AND VII OF THE BOOK,
LONGLEAF PINE BY W.G. WAHLENBERG
BLACK BROKEN LINES = SITE INDEX 50
RED BROKEN LINES = SITE INDEX 100
NUMBER OF TREES PER ACRE
2500
2000
1500
1000
900
800
700
600
500
400
300
200
150
100
90
80
70
HEIGHT OF STAND - FEET
15
20
30
40
50
60
70
80
90
100
150

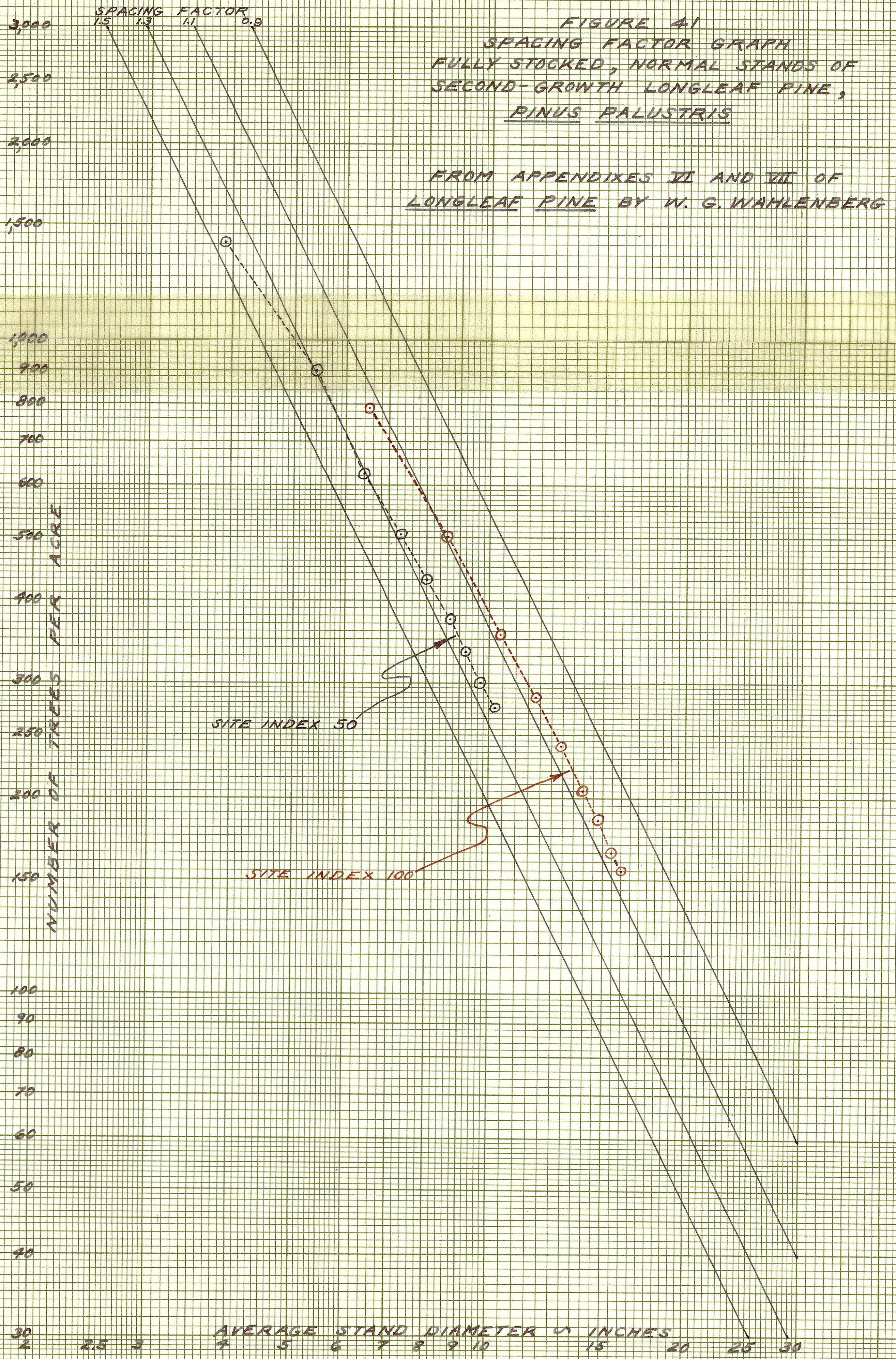


FIGURE 41
SPACING FACTOR GRAPH
FULLY STOCKED, NORMAL STANDS OF
SECOND-GROWTH LONGLEAF PINE,
PINUS PALUSTRIS
FROM APPENDIXES VI AND VII OF
LONGLEAF PINE BY W. G. WAHLENBERG
SPACING FACTOR
1.5
1.3
1.1
0.9
NUMBER OF TREES PER ACRE
3,000
2,500
2,000
1,500
1,000
900
800
700
600
500
400
300
250
200
150
100
90
80
70
60
50
40
30
SITE INDEX 50
SITE INDEX 100
AVERAGE STAND DIAMETER - INCHES
2
2.5
3
4
5
6
7
8
9
10
15
20
25
30

graph shows an increase in stocking between the first two age classes; however unlike the height factor graph, the spacing factor graph shows no decrease in stocking in the two oldest age classes. The slope of the plotted stand data on the spacing factor graph is not as steep as that of the stocking lines, and therefore, the graph shows a gradual increase in stocking for each older age class. On the height factor graph this relationship is not indicated.

The plotting of this stand data on the spacing factor graph of Figure 41 is unique from the spacing factor graphing of stand data from all the other yield tables and individual stands shown here in that the plotted average stand diameters are the average diameters of dominant and codominant trees alone and not that of all trees in the stand. The slope of this stand data was still about as close to that of the spacing factor lines as that of the data of the other stands.

It has been noted that the correlation between the slope of the stocking lines of the graph and the slope of the stand data of both yield tables for managed stands and for unmanaged, "normal" stand is good on both the height factor graphs and the spacing factor graphs. The stand data of these yield tables appear to plot consistently in more of a straight line of constant slope on the spacing factor graphs than on the height factor graphs. The yield table data is portrayed usually as gradually increasing in stocking on both the height factor graphs and the spacing factor graphs. The height factor graph shown here for jack pine yield table is an exception to this, and also the height factor graph of the longleaf pine data can be considered as an exception. The spacing factor graph nearly always portrays the data from these yield tables as gradually increasing in stocking as was similarly shown in the case of data from individual stands. It must be remembered that there are possible faults, as described above, in the plotting of yield table data, particularly those for unmanaged stands on these graphs.

The apparent increase in stocking shown in the early age classes of the stand data on the spacing factor graphs can be explained by the fact that during that early stage in the stand, while the stocking is increasing, the tree stems are not yet developed sufficiently, and the crown stem ratio of full stocking has not yet been reached. It may also be that the stands had not yet attained full crown cover and were understocked until the time that the stand data begins to plot on a constant slope. The importance of the development of the tree stems in the stand before the crown-stem ratio can be relied upon was stressed on page 13.

ANOTHER POSSIBLE THEORY FOR NUMERICALLY EXPRESSING THE STAND STOCKING OF UNIFORMLY STOCKED, EVEN-AGED STANDS

In the plotting of the stand data of both yield tables and individual stands on spacing factor graphs, it was noted above that the slope of the stand data was almost always not as steep as that of the spacing factor lines. Because of the weaknesses in plotting yield table data gathered and averaged from many stands of different ages, and in plotting the individual stand data of stands not managed by this method on spacing factor graphs, it cannot be concluded that the slope of the spacing factor lines is too steep. In fact, considering all these faults, the correlation of the data to the spacing factor lines is close. However, it is possible - and the above graphs do not reject it - that there should be some gradual increase allowed in the basal area per acre of the stand during its life. This would mean a gradual decrease in the spacing factor value.

This slope of the stand data on the spacing factor graph might possibly differ between species and might even be steeper than the spacing factor lines for some species after definite tree stem development has been established in the stand.

The author thought of the possibility that perhaps different stocking equations would better fit certain tree species in management while interpreting the characteristics of the plotted stand data on the graphs of the individual stands and the yield tables shown above. These graphs show that this theory has a possibility of significance; at least they do not disprove it.

It may well be that uniformly stocked, even-aged stands can be managed best by either the height factor method or the spacing factor method or just as well by both. However, if it is found through experimentation later that certain species should be managed so as to follow their own particular slopes on spacing factor graphs, then these slopes can be found for each species and the stocking equations can be computed by the following graphical method developed here. Stands of the same species but on different sites appear on the above graphs to plot along the same slope but at slightly different stocking levels; therefore it is a possibility that the same slope will apply to all uniformly stocked, even-aged stands of a given species, while some species have different slopes and, consequently, different stocking equations than others.

Suppose that the yield table for managed pine and spruce stands on average quality site in central Sweden, shown in Table 7 of the appendix, was accepted as the correct stocking amounts to be maintained during the life of a similar managed stand in like conditions. This yield table data can then be plotted on a graph similar to the spacing factor graph except that the spacing factor lines are omitted. This is done on the graph of Figure 42. It can be seen in Figure 33, on page 147, that the slope of this yield table data is not the same as that of the spacing factor lines.

The equation of stocking for this yield table and the equation of the slope of its data can be computed graphically from Figure 42 as follows:

let d = the average diameter breast high of the stand

let n = the average number of trees per acre

From the graph, where

$d = 2, \; n = 2{,}330$

$d = 10, \; n = 217$

FIGURE 42

GRAPH OF THE FULL STOCKING LINE OF A YIELD TABLE FOR PINE AND SPRUCE FOR AVERAGE QUALITY SITE IN CENTRAL SWEDEN

THINNINGS ARE MEDIUM TO HEAVY DANISH TYPE

The logarithmic equation of a straight line on this logarithmic graph is

$$\log n = a \log d + \log k$$

Using the logarithms of these values, the simultaneous observation equations become

$$3.36736 = 0.30103\,a + \log k$$

$$2.33646 = 1.00000\,a + \log k$$

Subtracting to eliminate log k gives

$$1.03090 = -0.69897\,a$$

$$a = -1.47490 \quad \text{or} \quad -1.475$$

Solving for log k gives

$$2.33646 = -1.47490 + \log k$$

$$\log k = 3.81136$$

Therefore, the equation of the line is

$$\log n = -1.475 \log d + 3.81136$$

and the antilog of this equation is

$$n = \frac{6,477}{d^{1.475}}$$

The equation of the slope of this line in logarithmic form is

$$1.475 \log d = -\log n$$

and the antilog is

$$d^{1.475} = \frac{1}{n}$$

Thus, the stocking equation can be computed for any species, and this equation can be employed through the use of logarithms in mathematically predicting the future management of the stand. The slope equation will be the same for all sites for a given species.

Ofcourse, this procedure is only correct if the theory itself is valid. The data gathered for this thesis indicates this possibility, but the data is from stand data of individual stands and of yield tables, and both of these groups contained no data from stands managed by either the height factor method or the spacing factor method. If this data had included some stands managed successfully by either the height factor or spacing factor method, and had the data of these stands shown that such a maintained degree of stocking was desirable, then this theory would have no foundation. This method of computing equations of stocking is presented in the possibility that this theory of different stocking equations for certain tree species might have some validity proven in the future, and consequently some method of determining stocking at different stages in the development of a stand would be needed.

It can be seen that in this particular case in Figure 33, the slope of this stocking line is not as steep as that of the spacing factor lines. Therefore, according to the spacing factor theory this data shows a constant increase in stocking and should be "overcrowded" during at least part of its life history. According to the spacing factor theory that the crown stem ratio should remain constant once the trees acquire sufficient stem development, this particular line of data will lead to unnecessary interlocking of crowns in the stand and a stagnation of growth. This is asssuming that the constant crown-stem relationship is correct.

The height factor graphs also portrayed many stocking lines of stand data at constant slopew that were different than the slope of the height factor lines. In the same manner as developed above, individual stocking equations in terms of average height and number of trees per acre can be computed for different stand data and applied to certain speciex, if it is shown that this theory is worth experimentation.

CONCLUSIONS

SPECIFIC CONCLUSIONS FROM THE COMPARISON OF THE HEIGHT FACTOR AND SPACING FACTOR METHODS

RELATIVE VALIDITY OF THE SPACING FACTOR AND HEIGHT FACTOR THEORIES

The spacing factor and height factor graphs of the data gathered for this thesis show that both methods correlate with existing stand data surprisingly well in many cases, indicating that they should correlate exdellently with the data of stands managed by these methods. With proper factors taken into account, both methods may enable the realization of close to the maximum possible economic return from a timber stand.

Both theories are of recent origin with the spacing factor theory undergoing the greatest amount of development and application until now.

Of the two theories, the spacing factor method is perhaps the most widely known and accepted for it embodies the theory of maintaining a constant basal area per acre within the stand, and foresters have been more and more interested in this method of management. Some experimental plots of uniformly stocked, even-aged stands have been thinned to a constant basal area per acre for quite a number of years to study the merits of maintaining a constant basal area per acre. The Yale University School of Forestry has been conducting experiments of this type on white pine plots. However, the height factor method of expressing stand stocking, even though more recent, is already receiving attention, and in some cases measurements are being taken in preparation for future employment of the method. (23) The method is being accepted as having good possibilities.

<u>THE CONSISTENCY OF RATE OF GROWTH IN TERMS OF BOTH HEIGHT AND DIAMETER FOR UNIFORM TIMING AND SEVERITY OF THINNINGS THROUGHOUT THE LIFE OF STANDS</u>

Bruce and Schumacher state that diameter growth is affected by the density of the stand. The more dense the stand the less the diameter growth. However, they also state that height growth behaves differently, and that height growth correlates very little if any with density within certain broad limits. (24) Height growth

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23. Gevorkiantz, S.R. and Scholz, H.F., <u>Timber Yields and Possible Returns from the Mixed-Oak Farmwoods of Southwestern Wisconsin</u>, Publication NO.521, U.S. Dept. of Agriculture, Forest Service, Feb. 1948, p.15.

24. Bruce, Donald and Schumacher, F.X., <u>Forest Mensuration</u>, McGraw Hill Book Co., Inc., N.Y. and London, Second Edition, 1942, p.365.

will not accelerate after a cutting, but diameter growth will. In fact, growth acceleration after cutting is often expressed as an increase in periodic diameter growth. (25)

It often occurs that the average height of dominant trees on experimental plots which have been thinned many times during their life span is no greater than that on the unthinned, control experimental plots. This fact is born out by the stand data of the thinned and unthinned experimental plots on the Saginaw Forest of the University of Michigan presented in this thesis. Other experimental forests show the same results. (26)

These facts represent an advantage of the height factor method over the spacing factor method. However, it is possible that in a stand managed well under the spacing factor method, the stand density will be confined to fairly narrow limits, and that diameter growth will not be allowed to stagnate from overcrowding and then to spurt after a thinning. The

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25. Ibid., p.400.

26. Hawley, Ralph C., Observations on Thinning and Management of Eastern White Pine in Southern New Hampshire, Bulletin No.42, Yale University School of Forestry, 1936, p.7.

object of management is to thin before overcrowding occurs and to maintain diameter growth at a fairly constant rate throughout the life of the stand after proper stem development has been reached. It is possible to maintain almost a constant rate of diameter growth over a broad spread of diameter classes from six inches to thirty inches. (27)

The spacing factor graphs of individual stands shown in this thesis all portray an increase in average stand diameter as a direct result of thinning. This will always occur with "low" thinnings, thinnings which primarily remove the smaller, poorer trees in the stand. However, if the spacing factor method employed the average diameter of only the dominant and codominant trees, as is the case with height for the height factor method, the increase in average diameter directly as a result of thinning probably would be eliminated.

MATHEMATICAL APPLICATIONS OF THE TWO METHODS OF EXPRESSING DENSITY IN FOREST MANAGEMENT

The height factor and spacing factor are both simple numerical expressions of stand density. Both

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27. Schnur, G. Luther, unpublished material on growth studies in Missouri.

have mathematical equations which can easily be applied in stand management. Prior to this, the spacing factor had been the most advanced in mathematical applications, but it was shown in this thesis that the height factor expression could be applied to stand management in a closely similar manner. As far as the particular field of forest management is concerned, the two methods are about equal.

COMPARATIVE EASE AND ACCURACY OF OBTAINING NECESSARY STAND MEASUREMENTS FOR APPLICATION OF THE TWO METHODS

The average spacing in feet between tree stems in the stand must be obtained when using either method. This is necessary in order to use this figure both with the average height of the dominant and codominant trees of the stand to obtain the height factor and with the average stand diameter breast high to obtain the spacing factor.

Obtaining the average height of the dominant and codominant trees in the stand for use in the height factor is a slower and more difficult process than obtaining an average diameter breast high for use in the spacing factor. The timber cruiser and the consulting forester can make direct measurements of stem diameters at breast height, for this is within easy reach of the man. Measuring the heights of trees is a more difficult problem. The top of the tree may be at quite a distance from the man. If the crown of a tree is very rounded it is difficult to estimate where the highest point of the crown is located.

In measuring diameters under favorable circumstances it is possible to measure accurately to the nearest tenth of an inch by means of a rule, caliper, or tape, but in many cases diameters will be rounded off to the nearest inch. (28) Height measurements, however, even when made with instruments accurate to the nearest foot, are handicapped by the fact that the base of the tree is a poorly defined target and that an imperceptible lean will cause an appreciable error of from three to five feet or even more. (29) Purely ocular estimates of total height cannot be attempted with an accuracy closer than by ten foot intervals or perhaps, with very short trees, by five foot intervals. (30) It is important to distribute height

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28. Bruce, Donald and Schumacher, F. X., op.cit.,p.11.
29. Ibid., p.24.
30. Ibid., p.24.

measurmments evenly over anarea, for there exists a human tendency to measure trees on level ground or lower slopes, resulting in a serious plus error. (31)

Recently there has been a rapid advancement in the use of aerial photographs in timber cruising. (32) It is possible to measure tree heights on a erial photographs and under favorable conditions to count the number of trees in a given area. This mounting importance of aerial photographs in measuring timber stands may increase the value of the height factor theory. Many factors affect the accuracy of tree height measurement on aerial photographs; but it is possible with a Harvard parallax wedge to measure tree heights on good photographs of a scale of 1:6,000 with an average error of three feet; about six feet at 1:12,000; and nine feet at 1:18,000. (33)

It is possible also to arrive at the diameter breast high of a tree by measurements on aerial photographs by use of the formula: (34)

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31. Chapman, Herman H. and Demeritt, Dwight B., Elements of Forest Mensuration, J.B. Lyon Company, Publishers, Albany, New York, Second Edition, 1936, p. 102.

32. ----,"Cruising and Mapping by Planeback",The Timberman, Portland,Oregon, Vol.XLIX,No.10,Aug.,1948,pp.55-57.

33. Spurr,Stephen H., Aerial Photographs in Forestry, The Ronald Press Company, New York, 1948, p.235.

34. Nash,A.J.,Some Volume Tables for Use in Air Survey, reprint from the Forestry Chronicle,Canada, Vol.XXIV.

$$\frac{k\,h}{d^2} = E$$

where d = diameter breast high

k = tree crown diameter

E = a coefficient which much be established for the tree species concerned

However, this is an indirect method for obtaining the stem diameter and the tree height must be measured in the process; therefore the height factor has the advantage in this type of application since it is easier to obtain and can be measured directly from the photographs. Also, except under the most favorable conditions, it is difficult to identify tree species on aerial photographs in order to apply the proper value for "E".

THE EASE OF FINDING REQUIRED DATA AND THE QUANTITY OF EXISTING REQUIRED DATA FOR THE APPLICATION OF EACH METHOD

In gathering the stand data from the records of the stands in the Saginaw Forest of the University of Michigan for use in this thesis, it was found that proper height data necessary for the application of the height factor method was often lacking. The tables of this Saginaw Forest data presented in the appendix indicate this frequent lack of height data. On the other hand, very adequate data on average stand diameters was almost always present.

Wilson, himself, admits that "forestry literature is replete with references to the close correlation between average stand diameter and total number of trees or basal area". (35) Besides extensive studies of this correlation, stand measurements in the past nearly always included the proper stand diameter measurements while often lacking proper height data. Therefore, it is more difficult now to apply the height factor theory to this existing stand data than it is to apply the spacing factor theory.

APPLICATION OF THE TWO METHODS IN "ON THE SPOT" MANAGEMENT DECISIONS IN THE FIELD

Both the height factor and spacing factor, as simple numerical expressions of stocking, can be applied quickly by the consulting forester who has only a limited time in the field to draw up tentative management plans for particular stands to be put under management. Their mathematical applications, as brought out earlier, are very similar and about equally developed. The spacing

35. Wilson, F.G., op. cit., p. 758.

factor does have possible application in mensuration as described above while discussing that theory. There has been no such application for the height factor as yet.

Though their mathematical applications are about the same, it is, however, easier to make diameter measurements than height measurements. It is also much easier to obtain the required data on diameter growth within a particular stand than it is to obtain height growth. The spacing factor method has the obvious advantage in this particular case.

THE PORTRAYAL OF VOLUME, STAND SIZE, AND TIMBER QUALITY IN EACH NUMERICAL METHOD OF EXPRESSING STOCKING

In comparing the correlation of the height factor and spacing factor theories with the stand data of individual stands earlier in this thesis, it was noted that a "stand response" graph was valuable when used in conjunction with the height factor method but was unnecessary when working with the spacing factor. The spacing factor portrays basal area in an exact mathematical relationship and also embodies the average stand diameter in its definition; therefore the "stand response" graph, involving just these two measurements, merely presents in another way what is already shown on the spacing factor graph.

Since basal area is a good reflection of timber volume, the spacing factor, which has an exact mathematical relationship with the basal area as explained above, is also a good refledtion of volume. When only the spacing factor is given for a stand, a quick reference to Table I or Figure 3 in this thesis will produce the corresponding basal area value. The height factor gives no such portrayal of volume.

Average stand diameter, required in the spacing factor method, gives the forester a picture of the saw timber size of the stand and is often employed as a measurement of stand merchantability. Lumber quality also is strongly and directly correlated with diameter. A forester has a clearer picture of a stand's merchantability, for example, if he knows the average stand diameter is 14 inches d.b.h. rather than that the stand's average height is 60 feet. Thus the spacing factor is more of an aid than the height factor in this particular application.

GENERAL CONCLUSION

Both the height factor and the spacing factor answer the need for a numerical expression of stocking for uniformly stocked, even-aged stands.

It was found that the two methods of expressing stocking are quite similar in their mathematical applications.

Because of the current lack of vital data for the application of the height factor method, because of the greater ease in gathering the necessary data for application of the spacing factor method, and because the spacing factor gives a much better portrayal of stand volume and merchantability, it appears that the spacing factor has the advantage over the height factor at present.

Both methods can serve as guides in the intelligent construction of forest management plans for timber stands. Much research on experimental plots with controlled thinnings is needed to determine the full value of these two theories, but it is already apparent that they will be of definite aid to the forest manager.

APPENDIX

TABLE 1.-HISTORY OF PINUS STROBUS STAND OF LOT 2b,

BLOCK 1, SAGINAW FOREST, ANN ARBOR, MICHIGAN.

Lot planted in 1904 with 2-year seedlings.
Lot 2b contains 0.54 acre.

Year	Average Height of Dominants and Codominants	Before or After Thinning	Number of Trees per Acre	Average D.B.H. of Stand	Basal Area per Acre
	Feet			Inches	Square Feet
		Thinned Plot			
1916	18.8	before	4,530	2.0	105.15
		after	3,936	2.0	95.066
1920	24.0*	before	3,407	2.75	142.628
		after	2,020	3.2	111.057
1925	32.4	before	2,020	3.95	173.199
		after	1,263	4.4	136.61
1930	40.2	before	1,260	5.1	178.836
		after	1,121	5.3	170.150
1935	44.0*	before	1,078	5.9	205.712
		after	947	6.1	192.203
1940	48.5*	Before	938	6.4	209.385
		after	788	6.6	185.933
1945	59.8	before	769	7.3	224.442
		after	583	7.7	187.301
		Unthinned, Control Plot			
1916	17.0*		3,904	2.1	90.888
1920	23.0*		3,376	2.7	138.3
1925	30.0*		2,552	3.55	178.005
1930	38.3		1,877	4.4	195.603
1935	42.0*		1,492	5.0	208.095
1940	47.0*		1,248	5.5	205.285
1945	56.0*		1,047	6.1	215.344

* Estimated values because average heights of dominant and codominant trees were not recorded for these periods. However, five of these periods do have the range of heights and average height of all trees in the stand recorded.

FIGURE 1

Eastern white pine, Pinus strobus, Lot 2b, Block 1, Saginaw Forest, Ann Arbor, Michigan. December, 1948

Plate 1.-Photograph of the stand
A foot-rule is attached to one stem for size comparison.

Plate 2.-Crown-closure photograph
Note interlacing branches between adjacent crowns.

TABLE 2.-HISTORY OF PINUS STROBUS STAND OF LOT 2c,

BLOCK 1, SAGINAW FOREST, ANN ARBOR, MICHIGAN.

Lot planted in 1904 with 2-year seedlings.
Lot 2c contains 0.57 acres.

Year	Average Height of Dominants and Codominants Feet	Before or After Thinning	Number of Trees per Acre	Average D.B.H. of Stand Inches	Basal Area per Acre Square Feet
		Thinned Plot			
1916	18.0*		2,108	2.6	76.024
1921	24.0*	Before	1,952	3.3	117.405
		After	1,430	3.3	104.84
1925	34.0*	Before	1,379	4.5	153.462
		After	1,020	4.5	130.693
1930	40.8	Before	1,020	5.6	177.122
		After	938	5.8	169.372
1935	45.0*	Before	929	6.4	208.750
		After	840	6.7	199.613
1940	49.0*	Before	830	6.9	214.019
		After	664	7.3	189.278
1945	58.6	Before	660	8.0	228.688
		After	561	8.3	210.325
		Unthinned, Control Plot			
1916	18.0*		2,108	2.6	76.024
1921	25.0*		1,924	3.4	120.302
1925	32.0*		1,517	4.4	162.314
1930	42.4		1,151	5.4	181.354
1935	46.5*		913	6.2	193.005
1940	50.5*		802	6.7	199.226
1945	59.0*		691	7.6	217.308

* Estimated values because average heights of dominant and codominant trees were not recorded for these periods. However, seven of these periods do have recorded the range of heights and average height of all trees in the stand.

FIGURE 2
Eastern white pine, Pinus strobus, Lot 2c,
Block 1, Saginaw Forest, Ann Arbor, Michigan. December, 1948

Plate 1.-Photograph of the stand
A foot-rule is attached to one stem
for size comparison.

Plate 2.- Cbown-closure photograph

TABLE 3.-HISTORY OF PINUS AUSTRIACA STAND OF LOT 2a,

BLOCK 1, SAGINAW FOREST, ANN ARBOR, MICHIGAN.

Lot planted in 1904 with 2-year seedlings.
Lot 2a contains 0.127 acre.

Year	Average Height of All Trees in the Stand Feet	Before or After Thinning	Number of Trees per Acre	Average D.B.H. of Stand Inches	Basal Area per Acre Square Feet
1915	15.0		2,600	2.3	75.4
1917	18.3		2,410	2.9	104.9
1919	20.8	Before	2,284	3.2	128.5
		After	1,299	3.7	99.45
1924	28.9	Before	1,267	4.2	126.73
		After	913	4.6	106.93
1929	37.8	Before	906	5.3	138.33
		After	709	5.6	120.88
1934	45.2	Before	701	6.0	136.409
		After	575	6.2	122.181
1939	53.6	Before	567	6.8	144.393
		After	520	6.8	137.4
1944	61.0	Before	520	7.7	167.6
		After	378	8.3	141.0

Average height of dominant and codominant trees was not recorded at any period for this stand; therefore average height of all trees had to be used. For 194 4 no height at all was recorded so 61 feet height was estimated.

This stand has a very small number of white pine mixed in. The white pine is not included in the above data. Actual total basal area per acre on the ground is greater than the above dada indicates.

FIGURE #3

Austrian pine, Pinus austriaca, Lot 2a, Block 1,
Saginaw Forest, Ann Arbor, Michigan. December, 1948

Plate 1. - Photograph of the stand
A foot-rule is attached to one stem
for size comparison.
Note presence of some undergrowth.

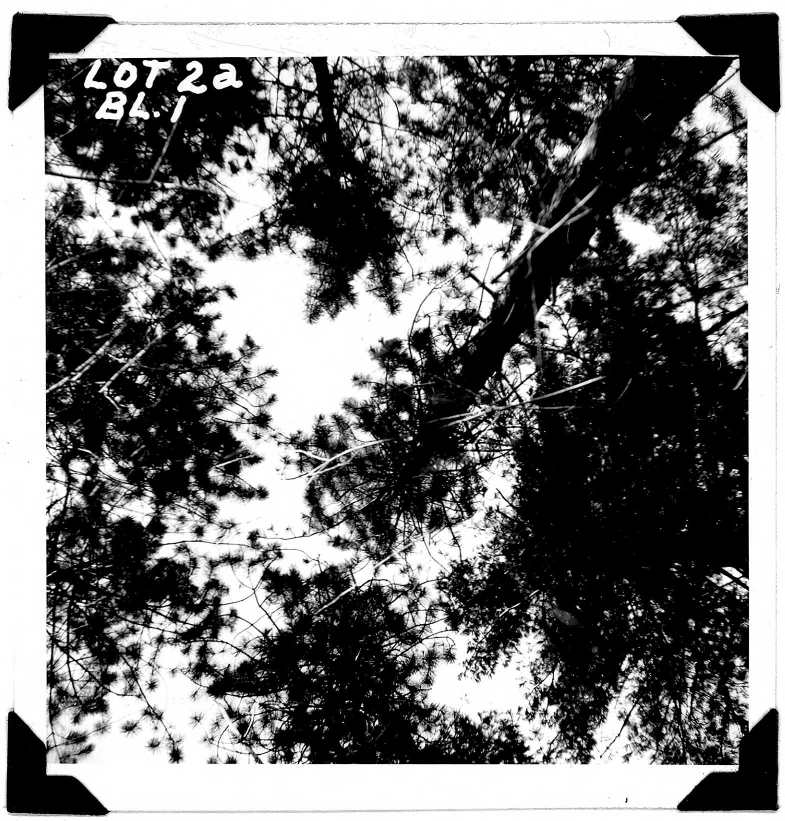

Plate 2. - Crown-closure photograph

TABLE 4.-HISTORY OF PINUS PONDEROSA OF LOT 1, BLOCK 5, SAGINAW FOREST, ANN ARBOR, MICHIGAN.

Lot planted in 1909 with 2-year seedlings.
Lot 1 contains 1.07 acres.

Year	Average Height of dominant and Codominant Trees (Feet)	Before or After Thinning	Number of Trees per Acre	Average D.B.H. of Stand (Inches)	Basal Area per Acre (Square Feet)
1920	12 *		1206		
1925	20 *		1192	3.7	90.15
1930	26.5 *		1179	4.6	136.215
1935	32.5 *	Before	965	5.4	154.953
		After	799	5.8	144.907
1940	41.0 *	Before	789	6.2	163.067
		After	713	6.3	153.154
1945	47.0	Before	713	7.0	190.95
		After	626	7.2	179.52

* Average heights of dominant and codominant trees were not recorded for these periods. However, average height of all trees in the stand and the ranges of heights among standing, live trees were recorded for all these periods,and dominant and codominant heights were estimated from this information.

FIGURE 4

Western yellow pine, Pinus ponderosa, Lot 1, Block 5, Saginaw Forest, Ann Arbor, Michigan. December, 1948

Plate 1.-Photograph of the stand
A foot-rule is attached to one stem for size comparison.
Note persistant dead branches.

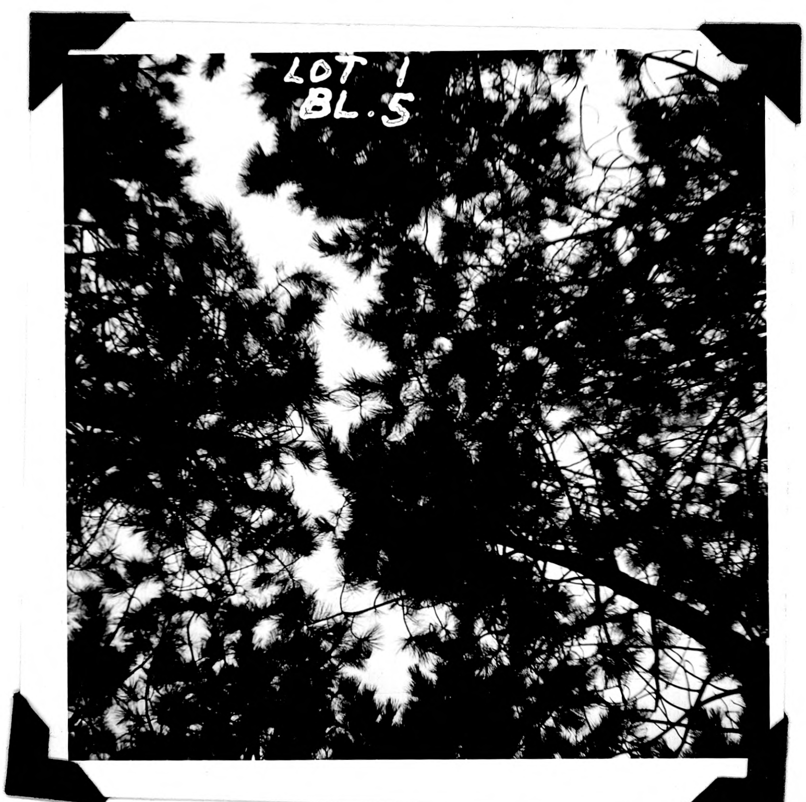

Plate 2.-Crown-closure photograph
Note complete use of overhead light, indicating high stocking.

TABLE 5.-HISTORY OF PINUS SYLVESTRIS STAND OF LOT 1, BLOCK 1, SAGINAW FOREST, ANN ARBOR, MICHIGAN.

Lot planted in 1904 with 2-year seedlings.
Lot 1 contains 0.24 acre.

Year	Average Height of Dominant and Codominant Trees	Before or After Thinning	Number of Trees per Acre	Average D.B.H. of Stand	Basal Area per Acre
	Feet			Inches	Square Feet
		Thinned Plot			
1916	23.0*	Before	2444	2.5	83.59
		After	1944	2.6	70.55
1920	30.0*	Before	1708	3.3	101.3
		After	1001	4.0	85.481
1925	40.0*	Before	944	4.8	116.68
		After	555	5.5	90.472
1930	50.0*	Before	555	6.3	119.848
		After	458	6.7	109.597
1935	56.5*	Before	458	7.4	137.194
		After	430	7.7	129.083
1940	62.1	Before	430	8.2	157.500
		After	374	8.4	142.986
1945	65.5	Before	361	9.3	170.972
		After	319	9.5	157.60
		Unthinned, Control Plot			
1916	22.5*		2032	2.9	91.691
1920	28.0*		1800	3.6	127.13
1925	37.0*		1435	4.5	159.818
1930	45.0*		1017	5.6	171.581
1935	54.0*		891	6.4	204.145
1940	63.0		745	7.3	216.927
1945	67.3		672	8.2	250.618

* Average heights of dominant and codominant trees were not recorded for these periods. However, average height of all trees in the stand and the range of heights of all trees in the stand were recorded for eight of these periods and average dominant and codominant heights were estimated from this data.

FIGURE 5

Scotch pine, Pinus sylvestris, Lot 1, Block 1,
Saginaw Forest, Ann Arbor, Michigan. December,1948

Plate 1.- Photograph of the stand
A foot-rule is attached to one stem for size comparison.
Note crooked form of stems. This is a defect apparently due to poor seed.

TABLE 6.-HISTORY OF PINUS PONDEROSA STAND OF LOT 5, BLOCK 5, SAGINAW FOREST, ANN ARBOR, MICHIGAN.

Lot planted in 1909 with 2-year seedlings.
Lot 5 contains 4.04 acres.

Year	Average Height of Dominant and Codominant Trees (Feet)	Before or After Thinning	Number of Trees per Acre	Average D.B.H. of Stand (Inches)	Basal Area per Acre (Square Feet)
		Thinned Plot			
1940	38.0*	Before	768	5.4	124.388
		After	607	5.8	113.064
1945	48.1	Before	560	6.6	131.616
		After	489	6.8	125.106
		Unthinned, Control Plot			
1935	32.5*		738	5.5	122.306
1940	39.5*		696	5.9	133.590
1945	48.0		622	6.5	145.475

* Average heights of dominant and codominant trees were not recorded for these periods. However, average height of all trees in the stand and the range of heights of all trees in the stand were recorded for all these periods and average dominant and codominant heights were estimated from this data.

FIGURE 6

Western yellow pine, Pinus ponderosa, Lot 5, Block 5, Saginaw Forest, Ann Arbor, Michigan. December, 1948

Plate 1.- Photograph of the stand
A foot-rule is attached to one stem for size comparison.

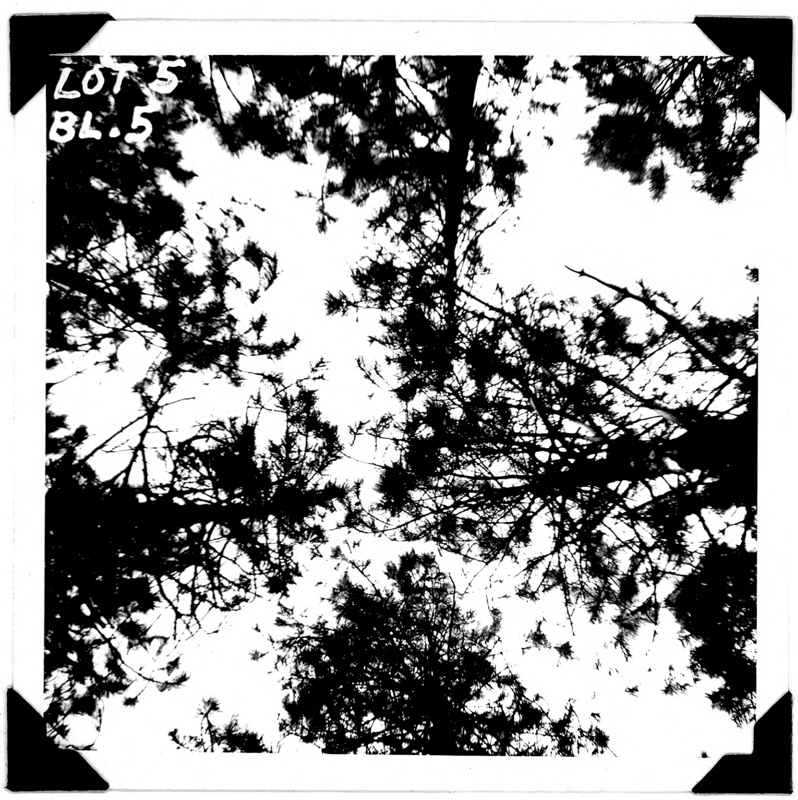

Plate 2.- Crown-closure photograph

TABLE 7.-YIELD TABLE FOR AVERAGE-QUALITY SITE,

PINE AND SPRUCE, CENTRAL SWEDEN *

Medium to heavy Danish type thinnings have been applied.

Reproduction method used: clearcutting with scattered seed trees and sowing in prepared spots.

Age in years	Average height in feet	Average diameter in inches	Basal area per acre in square feet	Total number of trees per acre	Number of trees per acre removed during fore-going 10 years
20	18	1.6	41	3,030	......
30	27	3.0	63	1,275	1,755
40	34	4.4	76	720	555
50	43	5.6	87	515	205
60	48	6.5	96	420	95
70	53	7.3	101	348	72
80	57	8.2	107	291	57
90	60	9.0	110	251	40
100	63	9.7	112	219	32
110	65	10.5	114	190	29
120	66	11.2	116	170	20

* Taken from <u>The Practice of Silviculture</u> by R.C. Hawley, p. 192.

TABLE 8.-STAND TABLES FOR SECOND-GROWTH LONGLEAF PINE IN FULLY-STOCKED, NORMAL STANDS *

Age in Years	Number of Trees per Acre	Basal Area per Acre in Square Feet	Average Diameter of Dominant Trees in Inches	Average Height of Dominant Trees in Feet
		SITE INDEX 50		
20	1410	64	3.9	26
30	900	78	5.4	37
40	625	88	6.4	45
50	505	95	7.3	50
60	430	100	8.0	55
70	375	104	8.7	58
80	335	106	9.2	61
90	300	108	9.7	63
100	275	109	10.2	65
		SITE INDEX 100		
20	790	114	6.5	52
30	500	140	8.6	74
40	355	158	10.4	89
50	285	170	11.8	100
60	240	179	12.9	109
70	205	185	13.9	117
80	185	189	14.7	123
90	165	192	15.4	127
100	155	194	16.0	129

* Wahlenberg, W.G., Longleaf Pine, Charles Lathrop Pack Forestry Foundation, publishers, Washington, D.C., First Edition, 1946, pp. 290-295.

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