Three Essays on Managing Competitive Bid Procurement

by

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A dissertation submitted in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy
(Business Administration)
in The University of Michigan
2013

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ACKNOWLEDGEMENTS

I am fortunate to have Prof. Damian R. Beil and Prof. Hyun-Soo Ahn as my advisors. When I entered Ross Business program, I was a stubborn one who was always thinking and communicating in a mathematician way. Damian and Hyun-Soo convinced me to switch focus from detailed math techniques to fundamental managerial insights. Throughout the years, they have been helping me enhance my abilities as an OM researcher. Without their patience, encouragement, support and guidance, this dissertation would not have been possible.

I would like to thank my committee members. Prof. Xiuli Chao taught me stochastic techniques that I have been constantly using, and I appreciate his constructive advice for my research. Prof. Xun Wu has raised many valuable comments from the perspective of a strategy specialist, which clearly improves the quality of this dissertation work. I would also like to thank other OM academicians, inside and outside Ross. Their questions and suggestions urge me to think through more practical concerns which have made this work a better one.

I have also benefited from other faculty members in the department. From introducing various research topics to helping me with presentations, their help has been in many aspects. And many thanks to my fellow PhD students. I am grateful for their good company in this long journey.

Last but not least, I thank my family for their love, especially my husband, Lei Qiao, who is always on my side through the ups and downs.

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Introduction

Procurement has emerged as a critical function, yet a challenging topic, since in many firms procurement operations are complex, and laced with misaligned incentives and information asymmetry between buyers and suppliers. Although economic (auction) theory provides a basic foundation, the richness of the operational context residing in supply chains offers numerous opportunities for research.

In my dissertation research, I focus on creating novel models to capture important phenomena in sourcing that are under-studied and analytically challenging. In particular, my thesis spans three different contexts that arise from a buyer's lack of information: (1) The role of cost modeling, (2) Consequence and remedies of having an error in initial request for quotes (RFQ), and (3) The strategic use of an incumbent supplier as an outside option when negotiating with entrant suppliers.

Essay 1: The role of cost modeling in competitive bid procurement

For firms who rely extensively on external suppliers for production, it is natural to experience atrophying capabilities in manufacturing, as process engineers, production technology experts, material experts, etc. are shed, which eventually leaves the firms with little insight into current production trends and technologies. A firm who lacks such information is at a disadvantage when buying from external suppliers, since suppliers can earn outsized profits by exploiting the buyer's lack of knowledge about the true production cost. To mitigate this, many buyers engage in rigorous activities (known as cost modeling) seeking to estimate the minimum price a supplier is willing to accept for a contract. These activities include sending experts to supplier sites, gathering data, and engineering and process analysis, all of which are expensive and time-consuming. However, in settings where the buyer plans to source through competitive bidding, the benefit of cost modeling is not clear since the bidding competition among suppliers can itself reveal cost information. This essay examines if, how, and when cost modeling should be deployed. The value of cost modeling depends on depth and breadth of the modeling, as well as on the amount of parity (or disparity) that exists between the minimum prices suppliers are willing to accept for the contract, information which the buyer does not have a priori. I show that although bid competition sometimes duplicates the information gleaned by cost modeling, the latter can still be beneficial when it helps the buyer set an effective reserve price. I use stochastic ordering to characterize when and how the buyer gains the most through cost modeling. Specifically, I characterize which supplier(s) to learn about, which portion(s) of the costs to learn, and how deeply and broadly the buyer should learn for general supply base topologies. I show that, when it comes to cost modeling, approaches that rely on naive intuition and conventional wisdom about the benefit of learning do not necessarily work. For instance, learning about the supplier whose cost is the most uncertain is not necessarily optimal, nor is learning about the cost portion that contributes most to the total cost. I also show that conventional intuition that the benefit of additional information has a diminishing rate of return does not apply. To the best of my knowledge, this essay is the first to model and analyze how two very common procurement tools — cost modeling and competitive bidding — interact. It has the potential to help procurement managers make better decisions about how to apply cost modeling in practice.

Essay 2: Procurement auctions with error-prone RFQs

While most procurement research assumes that the buyer's RFQ is perfect as given, in reality the RFQ can be flawed due to an error of omission and misspecification. Moreover, experienced suppliers can be more knowledgeable than the buyer and have the ability to, at the time of bidding, recognize such an error and anticipate windfall profit consequent to it. This makes the buyer vulnerable to massively expensive change orders during the life of the contract. For example, Northrop Grumman is currently suing USPS for $180 million for imposing numerous change orders to the contract for a mail sorting machine (*Federal Times*, 2012). I model the case in which the buyer is prone to making an error in initial RFQ, which in turn may or may not be detected by each supplier. I then ask how the buyer's total cost is affected by RFQ error and how this cost should be managed. I find that RFQ error encourages suppliers to submit lower bids with anticipation of future windfall profit, and suppliers' disparity in their abilities to detect RFQ error generally hurts the buyer. Furthermore, I propose a creative approach of "pre-pay" to stem windfall profit; in this new mechanism, the buyer can use a nominal bounty to induce knowledgeable suppliers to divulge the existence of RFQ error. To the best of my knowledge, this essay is the first in operations to consider the practical case that the buyer's RFQ can be misspecified, as well as the fact that experienced suppliers can be more knowledgeable than the buyer and the buyer can leverage suppliers' knowledge. Analyzing the problem is challenging, and I use a problem transformation method leveraging optimal mechanism design to prove results about common auction formats like open-bid descending and first-price sealed-bid auctions.

Essay 3: Should the buyer strategically exclude the incumbent supplier from the procurement auction?

In summer 2011, I interacted with a Tier-1 auto manufacturer who suspected that they were paying too much to their incumbent supplier for a particular part, and consequently they planned to hold an on-line auction with several entrant suppliers. A primary concern of the procurement manager was whether the incumbent supplier should be invited to the auction or not. On one hand, when withholding the incumbent and using him as an outside option, the buyer can set an aggressive reserve price for the entrant suppliers, but the auction itself is less fierce with one fewer bidder. On the other hand, when including the incumbent in the auction, the buyer drums up competition with one more bidder but is exposed to the risk of paying more than current incumbent price. I model this situation and find that the buyer's optimal decision depends on the incumbent's and entrants' cost distributions: When the incumbent's cost is expected to be substantially different from the entrants' (either much lower or much higher) and has low uncertainty, withholding the incumbent supplier is the better option; Conversely, when the incumbent's cost is comparable to the entrants' costs and has similar amount of uncertainty, the buyer prefers inviting the incumbent to the auction.

CHAPTER 2

The Role of Cost Modeling in Competitive Bid Procurement

2.1 Introduction

The average US manufacturer spends roughly 57% of its revenue on purchases from external suppliers (*U.S. Department of Commerce*, 2011). Procurement has emerged as a critical function, particularly for manufacturers in highly competitive industries. Seeking to manage their input costs, many such firms select external suppliers via competitive bidding (reverse auctions). Ideally, competitive bidding between potential suppliers drives the contract price to nearly the minimum the winning supplier would be willing to accept, including of course a necessary operating profit for the supplier. When this does not happen the buyer is leaving money on the table.

Leaving money on the table is not likely a major concern if the buyer's competitive bidding event is for a highly standardized, non-critical good/service that is easy to produce and for which it is relatively simple to qualify suppliers (e.g., paperclips). But in practice buyers often auction off contracts for non-standardized items for which supplier costs can be heterogeneous (e.g., driven by the supplier's production technology). Moreover, in addition to price, supplier qualification is a paramount concern to buyers. The process of locating and qualifying a supplier who is capable of producing the needed good is typically very time-consuming. For example, at automotive and electronics Fortune 500 manufacturers we interacted with, the supplier qualification processes can take months, even for "commodity-type" goods like cables, connectors, and screw machine parts. At these firms, functional areas such as engineering invest considerable time and resources to verify supplier capabilities and approve the supplier(s).

It is standard practice for buyers to invite only qualified suppliers to a competitive bidding event and announce that the lowest bidding supplier will win the contract. But due to the costly nature of supplier qualification (in terms of time, monetary costs of testing, and political costs of imposing on the engineering department's time) it is common to have a relatively small supply pool. For example, at one manufacturer we worked with many parts had just two suppliers who would be asked to bid. Unfortunately, a small supply pool can result in high prices for the buyer. As a buyer at Pfizer put it (quoted by *Atkinson* (2008)) "In our online auctions, suppliers can see what each other bids. If they were willing to go much lower, but none of the other participants forced them to do so, [the buyer] can lose out."

How are buyers attempting to avoid leaving money on the table while still ensuring that they only deal with qualified suppliers? A very common approach in practice is the use of a reserve price. A reserve price is the price at which the buyer opens the bidding — it is the maximum amount the buyer is willing to pay for the contract. Setting the reserve price is important because the closer it is to the eventual winner's minimum price, the less money the buyer will leave on the table. As the practitioner survey *Beall et al.* (2003) points out (page 44), a critical issue when procuring an item is "understanding the unique characteristics of the item's supply market structure, degree of competitiveness, key cost drivers, and current open capacity (i.e., is it a buyer's, suppliers', or neutral market?). This information is essential for setting an appropriately aggressive reserve price."

To set a more informed reserve price, it is a growing trend for buyers to actively engage in gathering market intelligence about suppliers. At an electronics firm we worked with, a procurement department employee with a background in corporate finance spent an enormous amount of time combing through information about the supply pool. Using any data he could find (like financial statements) the employee formed an understanding the supplier's financial status, what their operating profits were (if they were publicly listed), etc. Procurement managers at the firm would also perform reconnaissance on the ground, by visiting suppliers and gauging the supplier's capacity utilization (for example, seeing if they could spot a recently installed new production line that was under-utilized). Similarly, at an automotive firm we worked with, two employees were allocated for more than one month to painstakingly collect cost information about an auto part being procured. They tracked down a production engineer formerly employed at the firm's now shuttered internal plant to learn how the part had been made. They also interviewed industry experts to learn about current manufacturing practices, scoured reports on prevailing labor and utility costs, gathered, cleaned and analyzed data, etc. In industry these activities are often referred to as *cost modeling*, however what is actually being estimated is the minimum price at which the supplier is wiling to accept the contract.

Although a small qualified supply pool is one good reason to set an informed reserve price, even with a large supply pool money can still be left on the table due to supplier collusion. In fact, at the automotive firm we dealt with, this was an additional concern. Several potential suppliers submitting bids were co-located within a few hours' drive of each other in Asia. The buyer knew that the suppliers were in close contact with one another. The buyer was concerned about collusion, which would dampen bidding competition and make it less effective. In this case, the buyer decided to invest time and energy to create cost models to help her obtain price concessions.

Cost modeling is expensive as it involves skilled employees' time and travel. It should only be used when the firm is confident that it will get its money's worth out of the exercise. In this essay we explore the value of cost modeling in competitive bid procurement, to understand if, how and when cost modeling should be deployed. The answer is not obvious, since the bidding competition itself acts as a tool to discover suppliers' cost information — suppliers may have to bid close to their minimum acceptable prices in order to win the competitive bid. The value of cost modeling depends on the amount of parity (or disparity) that exists between the minimum prices suppliers are willing to accept for the contract, information which the buyer does not have a priori.

We examine a buyer who, after deciding to source (e.g., after winning a new production contract from a downstream customer), needs to select an upstream supplier in a timely fashion to allow adequate ramp-up time to avoid production delays. Since cost models take many weeks or possibly months to develop, we take the perspective of a buyer who, at the beginning of a bidding cycle, must decide which supplier(s) to learn about, which portion(s) of their costs to learn, and how deeply to learn. More specifically, we investigate the following research questions:

Research Question 1: How does one characterize the benefit of cost modeling? Is cost modeling beneficial to the buyer even when suppliers compete?

Research Question 2: If the buyer chooses to develop a cost model, which supplier(s) should the buyer learn about?

Research Question 3: If the buyer can choose to learn about various portions of the cost, e.g., labor cost, input cost, packaging cost, etc., which portion(s) should the buyer learn about?

Research Question 4: If the buyer chooses to develop a cost model, how deeply into supplier(s)' costs should she learn? Would the buyer ever want to learn about multiple competing suppliers?

Research Question 5: How do answers to the above questions depend on underlying business characteristics, such as correlations across suppliers' costs, and supplier collusion?

Answers to these research questions are summarized in §2.9. We review related literature in §2.2, and introduce the model in §2.3. Sections 2.4 and 2.5 study the two-supplier and N-supplier cases, respectively. We extend our analysis to include the cost of learning and provide an optimal learning strategy incorporating it in §2.6. Section 2.7 studies correlations across suppliers' costs and collusion among suppliers. Section 2.8 compares the benefit of learning to the benefit of adding suppliers, and examines how the buyer's power in the supply chain affects the benefit of learning. Proofs of results are furnished in §2.10.

2.2 Literature Review

There is a vast economics literature on competitive bidding (auctions). *Krishna* (2009) provides an excellent introduction to the early literature. There is a growing literature on procurement auctions, examining various operational aspects such as the need to consider quality as well as price (*Che*, 1993; *Beil and Wein*, 2003; *Kostamis et al.*, 2009), the need to perform costly supplier qualification screening (*Wan and Beil*, 2009; *Wan et al.*, 2012), auctions with quantity-flexible pricing (*Chen*, 2007; *Duenyas et al.*, 2013; *Tunca and Wu*, 2009), post-award audits and profit-sharing (*Chen et al.*, 2008), supplier capacity decisions (*Li*, 2013), unreliable suppliers (*Chaturvedi et al.*, 2011), double auctions (*Chu*, 2009), and large-scale combinatorial auctions (*Olivares et al.*, 2012). However, none of these papers study cost modeling prior to competitive bidding, which is the focus of our paper.

A number of papers apply econometric analyses on historical bid data to estimate bidders' private information. For example, *Paarsch* (1997) uses historical auction field data to derive an estimate of the optimal reserve price in English auctions for government timber sales (see the recent text by *Paarsch and Hong* (2006) for more examples). In contrast to these papers, we take the buyer's prior beliefs about suppliers' costs as our point of departure. We examine how the buyer can deploy a totally different approach to refine her prior information, namely creating a cost model to derive suppliers' costs using a bottom-up approach. The main aim of our paper is to examine if the buyer should engage in the time-consuming cost modeling prior to competitive bidding, and how and to what extent the buyer should deploy it. To our knowledge, ours is the first paper in the literature to examine cost modeling in this context.

A practitioner literature exists on how to construct cost models. For example, chapter 3 in *Laseter* (1998) describes various ways that buyers can decompose a supplier's cost into a number of cost drivers, each of which can then be estimated through various means of data collection, as well as the challenges of doing so. One way of gathering data about suppliers firsthand is Rapid Plant Assessment; *Goodson* (2002) describes how this careful approach to factory tours allows a trained team of procurement specialists to estimate a plant's cost of sales based on key observations made while touring the plant. There is also a literature on using cost modeling to inform new product development decisions where the designer must balance functionality against cost (see *Locascio* (2000) and references therein).

2.3 Model Description and Preliminaries

We consider a buyer seeking to allocate a single, indivisible production contract to a supplier. We assume the buyer can set a reserve price, which she uses to help avoid leaving money on the table. This setup reflects industry practice; see the quote from the practitioner survey *Beall et al.* (2003) in the Introduction, which discusses the fact Fortune 500 companies in practice use reserve prices informed by what they know about the supply market.

Our model also captures the fact that, when setting a reserve price, the buyer must be careful. As *Beall et al.* (2003) points out (page 9), "misreading the market and setting a reserve price that is too far below the market price, resulting in no bidder responses" is considered a "dysfunctional outcome" of an auction. To understand why practitioners are loathe to set a reserve price that no bidder can meet, consider the following: If the reserve price is not met, the buyer has to exclude the original bidders from further consideration. (If suppliers believe that the buyer will increase the reserve price and re-invite them to bid if the original reserve price is not met, then no supplier would ever bid in an auction with a reserve price.) The buyer is then faced with the prospect of: (i) forgoing the business altogether and abandoning the purchase; (ii) turning to its own internal production facilities; or (iii) finding new qualified suppliers to offer the business to. In reality, forgoing the business is often not palatable — for example, an electronics firm is unlikely to discontinue its production of laptops simply because it could not strike a deal in an auction for the needed USB connectors. Turning to internal production facilities is also often not available — for example, the electronics and automotive parts manufacturers we dealt with did not have internal production facilities for many of the parts they needed.

This leaves the last option, finding new suppliers. With this option, the procurement manager faces many challenges. First, because the procurement manager's job is striking a deal with a supplier (not vetting suppliers' technical qualifications), she has to turn to her internal customers (e.g., manufacturing and engineering) and ask the internal customers to help qualify additional suppliers, due to the fact that the suppliers that the internal customers had originally qualified were eliminated from further consideration because she set a reserve price so low that it was below all the suppliers' true costs. Understandably, internal customers would not necessarily react well to this imposition on their time and work which the procurement manager's actions had caused. Moreover, due to the time required to identify and qualify a new supplier, finding qualified new suppliers in time may not even be feasible within a procurement cycle (e.g,. some parts may require very time-consuming testing).

To reflect the above aspects of reality, in our model we capture the realistic situation that non-transaction is extremely costly for the buyer. Namely, we model a buyer who wishes to avoid non-transaction, but still use a reserve price that avoids (to the extent possible) leaving money on the table. To present our model and build intuition, we first examine a setting where the buyer has a single supplier.

2.3.1 One supplier

Let c denote the minimum price the supplier would accept for the contract. Following auction theoretic convention we simply refer to c as the supplier's "cost", which of course is the supplier's private information. The value c is a realization of random variable C drawn from a continuous distribution H. We assume that C is bounded above by $\bar{c}$. The buyer can employ a reserve price, r, as a take-it-or-leave-it offer to the supplier, and a supplier will accept this offer if this price can cover his production cost, i.e., $r \geq c$.

Since the buyer wishes to avoid non-transaction, but still use a reserve price that avoids (to the extent possible) leaving money on the table, the best reserve price the buyer can offer is $r^o = \bar{c}$. This reserve price leaves the supplier a sizeable windfall if the upper bound $\bar{c}$ is well above the supplier's true cost c. As mentioned in §2.1, to get a better estimate of the supplier's cost the buyer can build a cost model and collect data on the supplier before deciding her reserve price. For instance, for a manufactured good, a buyer might decompose the supplier's production cost into the sum of several attributes — such as raw material cost, direct labor cost, electricity cost, technology cost, overhead cost, and minimum acceptable profit margin — and learn some of these attribute costs through data collection. For example, a buyer can contact raw material providers to learn the cost of material that a supplier uses, estimate labor cost using local labor cost reports, or send specialists to visit a supplier's plant to learn the type of machines used and hence the technology cost. Of course, not all attributes can be learned; for example, overhead cost is usually difficult to learn.

To model this, we assume that the supplier's cost C can be decomposed into two portions: A, the portion of cost attributes that can be learned via cost modeling, and X, the portion that cannot be learned. We assume that A follows a continuous distribution F and is bounded above by $\bar{a}$, X follows a continuous distribution G and is bounded above by $\bar{x}$. As such, the supplier's cost C's distribution H is the convolution of F and G ($H = F \otimes G$) and the upper bound of C satisfies $\bar{c} = \bar{a} + \bar{x}$. Let a and x denote the realizations of A and X, respectively.

After learning the supplier's portion, $A = a$, the buyer can update her knowledge about the supplier's cost: $[C|(A = a)] = a + X$. This random variable $C|(A = a)$ has a new upper bound $\left[\bar{c}^l|(A = a)\right] = a + \bar{x}$, so the buyer can set a new reserve price $\left[r^l|(A = a)\right] = \left[\bar{c}^l|(A = a)\right] = a + \bar{x}$, which is the new contract payment. The benefit of learning is the difference in the contract payment without learning and with learning, denoted by ψ:

$$\psi = r^o - r^l = \bar{c} - (A + \bar{x}) = \bar{a} - A.$$

Obviously, learning brings the buyer a positive benefit, as the following shows:

Lemma 2.1. *If the buyer's supply base has only one supplier,* $P(\psi > 0) = 1$ *and* $E[\psi] = \bar{a} - E[A]$.

2.3.2 Two suppliers

Now we consider the case when the buyer has two suppliers. It is less clear whether learning about the supplier(s)' cost information can bring the buyer a positive benefit, because competition between suppliers itself acts as a cost-discovery tool.

To explore this interplay, we assume the two suppliers' costs, C_1 and C_2, are independent (we extend our analysis to correlated costs in §2.7.1) and follow distributions H_1 and H_2 with upper bounds $\bar{c}_1$ and $\bar{c}_2$, respectively. The buyer first sets a reserve price r as a starting point and then asks the suppliers to give price offers at or below price r. The buyer can play the suppliers' offers against one another by treating the current lowest offer as the winner unless this offer is unseated by a lower offer from the other supplier. As we have observed in industries ranging from electronics to automotive parts, such competitive bid processes are often administered via an online platform where the bidding takes place over a short period of time (e.g., 30 minutes). The ensuing process drives down the contract price until it becomes so low that only one supplier (the contract winner) remains. This competitive process can be modeled as a reverse open-descending auction for the contract, for which the dominant strategy for a bidder is to continue bidding until reaching their true cost or winning the auction. The predicted equilibrium outcome is as follows: The lower-cost supplier wins the contract at a price equalling the cost of the other supplier. The reserve price set by the buyer, r, acts as a price ceiling; hence, the buyer's contract payment with two suppliers is $\pi = \min(r, \max(C_1, C_2))$.

The buyer must transact with one of the suppliers qualified to produce the good, but seeks to set a reserve price that minimizes her expected contracting payment:

$$\min_r E[\pi] \qquad s.t.\ P(\min(C_1, C_2) \leq r) = 1.$$

The solution of the above problem, and the buyer's contracting payment are, respectively:

$$r^o = \min(\bar{c}_1, \bar{c}_2), \text{ and } \pi^o = \min(r^o, \max(C_1, C_2)).$$

Notice that r^o is the lowest reserve price that can avoid the risk of non-transaction since the supplier with the lower upper bound can generate non-negative surplus and will surely be interested in the contract.

Similar to the one-supplier case, the buyer can create cost models to better estimate the suppliers' costs. For supplier $i = 1, 2$, we let random variable A_i denote the learnable portion, with distribution F_i whose domain is bounded above by $\bar{a}_i$. Likewise, let $X_i \sim G_i$ denote the unlearnable portion, which is bounded above by $\bar{x}_i$. After learning supplier i's portion $A_i = a_i$, the buyer can update her knowledge of supplier i's cost $[C_i|(A_i = a_i)] = a_i + X_i$ which has a new upper bound $\left[\bar{c}_i^l|(A_i = a_i)\right] = a_i + \bar{x}_i$. Hence, if the buyer learns about a subset of suppliers S (S can be $\{1\}, \{2\}$, or $\{1, 2\}$), the buyer can set a new reserve price:

$$r^l = \min_{i \in S, j \notin S} (\bar{c}_i^l, \bar{c}_j) = \min_{i \in S, j \notin S} (A_i + \bar{x}_i, \bar{c}_j).$$

Accordingly, the buyer's contract payment after learning is:

$$\pi^l = \min(r^l, \max(C_1, C_2)).$$

The benefit of learning is:

$$\psi = \pi^o - \pi^l. \tag{2.1}$$

Unlike the one-supplier case, when there exists competition between suppliers, learning does not always bring a positive benefit for the buyer.

Lemma 2.2. *If there are two competing suppliers and one supplier's cost is not dominated by the other supplier's cost (i.e.,* $0 < P(C_1 > C_2) < 1$*), then with strictly positive probability learning brings no benefit at all (i.e.,* $P(\psi = 0) > 0$*).*

To understand the reason why competition between the two suppliers can make learning redundant, suppose the buyer learns about just one of the two suppliers. Competition will automatically identify the lower-cost supplier as the contract winner and can push the price down to the cost of the other supplier, so a good reserve price needs to be set close to the winner's true cost (or the minimum price the winner would accept) so that the buyer can squeeze the winner's surplus. Simply setting a lower reserve price does not guarantee it will be effective. Indeed, the buyer may be able to set a new, lowered reserve price even when the supplier she learns about eventually loses. However, in this case, the new reserve price is never low enough to affect the final payment. Even if the buyer learns about the contract winner, learning can again be useless since if the new reserve price is not low enough it will not bind the payment. It should be noted that learning results in a positive benefit to the buyer only when the buyer learns about the contract winner *and* can set a new reserve price low enough to be the new contract payment.

However, this makes the buyer's problem difficult since suppliers' true costs are their private information and the buyer can not *ex ante* predict with certainty who will be the contract winner or how close the suppliers' costs will be. In the next section, we will discuss when to learn and how to learn so as to get the maximum expected benefit. We define the expected benefit of learning as $\Psi = E[\psi]$. In this paper, the sample path value, ψ, and its expectation, Ψ, will be subscripted to denote which supplier(s) are learned about.

Thus far we have introduced the expected benefit of learning, which we analyze in Sections 2.4 and 2.5. The models in these sections do not consider the cost of learning. Of course, cost modeling is time-consuming and expensive as the buyer may need to visit the supplier's site, interview industry experts, analyze industry reports and forecasts, etc. Practitioners need to consider the tradeoff between the expense and the benefit of cost modeling. Building on our base model analyses, we incorporate the cost of learning in §2.6. Sections 2.7 and 2.8 offer further generalizations and extensions.

2.4 Analysis – Two Suppliers

We now answer our research questions when the buyer has two competing suppliers. We allow the possibility that the two suppliers are heterogeneous. For example, the cost structure of a supplier using labor-intensive production processes will differ from that of a supplier whose production is highly automated. Suppliers located in different regions may have different energy and shipping cost structures. In these cases, cost models are supplier-specific and a single cost model is developed to estimate the cost information of one supplier. Without loss of generality, we consider learning about supplier 1's learnable portion A_1, and let Ψ_1 denote the expected benefit of learning.

We also allow the other possibility, namely the two suppliers are homogeneous. In such cases, the suppliers can have different cost realizations but both share the same cost distributions, i.e., C_1, C_2 follow H with upper bound $\bar{c}$, A_1, A_2 follow F with upper bound $\bar{a}$ and X_1, X_2 follow G with upper bound $\bar{x}$. For example, the suppliers could have similar, labor-intensive production processes, so one cost model can be used for both suppliers. Of course, even if they share the same cost drivers (e.g., labor) the suppliers' costs can be different (due to differences in exact minutes of direct labor per unit, suppliers' labor wage rates, or fringe benefits for employees). Let $\Psi_{1\&2}$ denote the expected benefit of learning about portion A of both suppliers.

The following result characterizes the benefit of cost modeling prior to competitive bidding. This result and Lemma 2.2 characterize the benefit of learning and therefore answer research question 1.

Proposition 2.1. [Relation between learning and cost distributions]

(i) *When the two suppliers are heterogeneous, the expected benefit of learning A_1 is*

$$\Psi_1 = E[\min(\bar{a}_1 - A_1, (C_2 - (A_1 + \bar{x}_1))^+)]. \tag{2.2}$$

As long as the domains of F_1 and G_1 do not change,

(*a*) Ψ_1 *decreases as F_1 becomes stochastically larger;*

(*b*) Ψ_1 *is independent of G_1;*

(*c*) Ψ_1 *increases as H_2 becomes stochastically larger.*

(*ii*) *When the two suppliers are homogeneous, the expected benefit of learning A_1 and A_2 is*

$$\Psi_{1\&2} = E[(\max(C_1, C_2) - (\min(A_1, A_2) + \bar{x}))^+].$$

As long as the domains of F and G do not change,

(*a*) $\Psi_{1\&2}$ *is not necessarily monotonic as F becomes stochastically larger;*

(*b*) $\Psi_{1\&2}$ *increases as G becomes stochastically larger.*

Proposition 2.1 shows that the benefit of learning depends on the suppliers' cost distributions in interesting ways. Consider the case in which the buyer builds a cost model for a single supplier in a heterogeneous supply base. As the distribution of the learnable portion, F_1, becomes stochastically larger, the benefit of learning decreases, as shown in part (i.a). This is intuitive because the new reserve price (which will stochastically increase in F_1) is less likely to bind the contract payment. Likewise, the benefit grows as the cost distribution of supplier 1's opponent whom we do not learn about increases, explaining part (i.c). However, part (i.b) implies that the expected benefit of learning is independent of the distribution that governs supplier 1's unlearnable cost, G_1. This is because after learning, the new reserve price, $a_1 + \bar{x}_1$, depends on G_1 only through its upper bound.

Next, consider the case where the buyer creates a cost model that applies to both suppliers in the supply base. Interestingly, the results for this case differ from what we saw above. As the learnable cost decreases, the reserve price, $\min(A_1, A_2) + \bar{x}$, decreases as well. But at the same time, the cost the buyer would have paid without learning, $\max(C_1, C_2)$, also decreases. As a result, the benefit of learning — the difference between these two — does not necessarily grow or shrink, as stated in part (ii.a). Moreover, part (ii.b) shows that the benefit of learning increases as the unlearnable portion becomes stochastically larger. This is because the cost the buyer would have paid without learning increases, but the reserve price remains unchanged.

2.4.1 Which supplier(s) should the buyer learn about?

For the heterogeneous suppliers case in the previous section, we assumed that the buyer learned about supplier 1. However, would the buyer have been better off learning instead about supplier 2? To help answer this, we need to compare the expected benefit of learning either supplier. We now examine this issue, addressing research question 2.

Intuitively, one may expect that the buyer wishes to resolve the maximal amount of cost uncertainty and so she will optimally choose whichever supplier has the most uncertain learnable cost. This logic is true when the buyer has only one supplier in her supply base. Suppose supplier 1's learnable cost $A_1 \sim U[5, 10]$ and unlearnable cost $X_1 \sim U[0, 5]$, while supplier 2's cost $A_2 \sim U[0, 10]$ and $X_2 \sim U[10, 11]$. Were the buyer facing a single-supplier situation with supplier 1, learning would bring an expected benefit of $\bar{a}_1 - E[A_1] = 2.5$ (Lemma 2.1). This is smaller than $\bar{a}_2 - E[A_2] = 5$, which would be her expected benefit from learning if supplier 2 was her only supplier. The intuition in the single-supplier case is that the buyer would gain more benefit from learning when the learnable portion has more uncertainty.

However, this result is no longer true when the buyer has two competing suppliers. The benefit of learning does not solely depend on the A_i's. In fact, in the example above, learning about supplier 1 brings a much larger benefit (per equation (2.1), $\Psi_1 = 1.79 > 0.04 = \Psi_2$). Competition is what differentiates the one-supplier and two-supplier cases. Competition itself works as a cost-discovery tool, so learning brings a positive benefit only when it collects information that *cannot* be duplicated by competition. More precisely, the buyer needs information about the contract *winner's* cost that helps the buyer to set an effective new reserve price. In this example, supplier 1's cost C_1 is stochastically smaller than supplier 2's cost C_2; this means that supplier 1 has a better chance of winning the contract. In addition, the new reserve price set by learning about supplier 1 (i.e., $A_1 + \bar{x}_1$) is more likely to be smaller than that for supplier 2 (i.e., $A_2 + \bar{x}_2$). As a result, learning about supplier 1 is more beneficial. Our result below formalizes.

Proposition 2.2. [Preferred supplier to learn about] *For any distributions F_i, G_i and H_i, $i = 1, 2$, the buyer prefers to learn about supplier 1 rather than supplier 2 if*

$$A_1 + \bar{x}_1 \leq_{st} A_2 + \bar{x}_2 \text{ and } C_1 \leq_{st} C_2.$$

It should be noted that both conditions in Proposition 2.2 are necessary to draw the conclusion. Even if learning is likely to dramatically reduce the reserve price, it will be useless if it does not bind the contract payment to the contract winner.

Instead of learning about just one supplier, the buyer could instead choose to undertake learning about both suppliers. If suppliers are homogeneous, the buyer can create a single cost model that applies to both suppliers. If suppliers are heterogeneous, the buyer needs to develop two different cost models concurrently. Since cost models take several months to develop, in most cases, it is not feasible to conduct multiple rounds of learning in a single bidding cycle. Thus, decisions on whom to learn about should be made judiciously at the beginning.

Utilizing the same machinery, our next result characterizes the benefit of learning about multiple suppliers. One may intuitively reason that the marginal benefit of learning will diminish in the number of suppliers that the buyer learns about — after all, the buyer ultimately only sets one new reserve price which applies to all suppliers, thus more learning will likely just generate duplicative information. In other words, the benefit of learning about two suppliers should be smaller than the sum of the benefits of learning about either supplier individually. Surprisingly, the next result shows that this is not the case: The benefit is actually additive.

Proposition 2.3. [Benefit of learning about multiple suppliers]

(i) For two heterogeneous suppliers, $\psi_{1\&2} = \psi_1 + \psi_2$ *almost surely; hence* $\Psi_{1\&2} = \Psi_1 + \Psi_2$.

(ii) For two homogeneous suppliers, $\Psi_{1\&2} = 2 \cdot \Psi_1$.

To understand the intuition behind Proposition 2.3, recall that the buyer realizes a benefit from learning only if she learns about the contract winner. For a given sample path, at most one supplier can yield a positive benefit, i.e., $\psi_{1\&2} = \psi_1, \psi_2 = 0$ or $\psi_{1\&2} = \psi_2, \psi_1 = 0$, which explains part (i). Part (ii) follows from part (i) and the fact that the two suppliers have the same cost distributions.

2.4.2 Which portion(s) of cost should the buyer learn?

Another interesting challenge for the buyer is to determine which portion(s) of the suppliers' costs she should learn. To address this question, in this section we assume that portions A_i and X_i are both learnable. Note that this setup is without loss of generality: our results still hold when supplier i's cost is $C_i = A_i + X_i + \epsilon_i$, where ϵ_i is unlearnable. To address research question 3, we first ask which portion, if learned, would bring more benefit to the buyer.

Suppose two cost portions, labor and utility, comprise roughly 50% and 20% of the total cost, respectively. Intuitively, one might expect that the buyer would prefer to learn the labor cost, since it represents a larger share of the total cost. However, this intuition does not tell the whole story. What is more important for the buyer to consider is the amount of cost uncertainty which will be resolved by learning, not the absolute magnitude of the cost. This is the intuition behind the following proposition.

Proposition 2.4. [Preferred portion to learn]

(i) When the two suppliers are heterogeneous, for any distributions F_i, G_i and H_i, $i = 1, 2$, the buyer prefers to learn portion A_1 rather than portion X_1 if

$$\bar{a}_1 - A_1 \geq_{st} \bar{x}_1 - X_1.$$

(ii) When the two suppliers are homogeneous and the buyer learns about one portion of both suppliers, then for any distributions F, G and H, the buyer prefers to learn portion A rather than portion X if

$$\bar{a} - A \geq_{st} \bar{x} - X.$$

Thus, it is not the case that the buyer only wishes to learn whichever portion contributes most to the supplier's overall total cost. Instead, Proposition 2.4 reveals that the buyer prefers to learn whichever portion will reduce the reserve price the most.

Having discussed which portion (A or X) the buyer should learn, we now address research question 4 by considering how deep the buyer should learn, that is, the buyer's preference between learning one portion (A or X) or both portions (A and X). To examine the buyer's preference, suppose again that the two cost portions are labor and utilities. The buyer's expected benefit is \$10,000 from learning only the labor cost, and her expected benefit is \$17,000 from learning only the utility cost. Intuitively, one might expect that "doubling down" on the supplier by learning the supplier's labor *and* utility costs would yield an expected benefit less than the sum of the individual benefits (i.e., the expected benefit should be smaller than \$27,000)—after all, there is a chance that supplier 1 will not win the contract and then learning will be useless. However, the following result shows that, when it comes to the depth of learning, the benefit of learning the whole is greater than the sum of the benefits of learning its parts (*superadditivity*).

Proposition 2.5. [Benefit of learning multiple portions] *Consider three cost-learning models: Model A in which the buyer learns about portion A, Model X in which the buyer learns about portion X, and Model AX in which the buyer learns about both portions (A and X) simultaneously.*
(i) When the two suppliers are heterogeneous and cost modeling is applied only to supplier 1*, we have* $\psi_1^{AX} \geq \psi_1^A + \psi_1^X$ *almost surely; hence* $\Psi_1^{AX} \geq \Psi_1^A + \Psi_1^X$.
(ii) When the two suppliers are homogeneous and the same cost model can be applied to both suppliers, we have $\psi^{AX} \geq \psi^A + \psi^X$ *almost surely; hence* $\Psi^{AX} \geq \Psi^A + \Psi^X$.

Both competition and cost modeling are tools that the buyer can use to reduce the winning supplier's surplus. Our earlier results establish that cost modeling prior to competition is only beneficial when the new reserve price squeezes the winning supplier's surplus more than competition alone would have. In other words, to yield a positive benefit, the new reserve price must be lower than the second-lowest supplier's cost. The new reserve price is much more likely to accomplish this if two cost portions are learned instead of just one, because the reserve price reduction is cumulative in cost portions learned. To use a golf analogy, when the buyer only learns one portion, she has to "hit a hole-in-one" but when she learns two portions she only needs to hit a hole in two strokes; the proposition means that hitting a hole-in-one in two attempts is much less likely to occur than hitting a hole in two strokes. Combining Propositions 2.3 and 2.5, the takeaway for procurement managers is that, all else equal, depth in cost modeling is likely to be more beneficial than breadth.

2.5 More than Two Suppliers

We now generalize our results to the case of $N > 2$ suppliers. Some of the N suppliers may share the same cost drivers (i.e., are homogeneous), in which case the buyer can learn about those suppliers simultaneously by developing a cost model that applies to all of them. We also allow supplier heterogeneity, meaning that not all N suppliers have the same cost structure. Formally, we divide the N suppliers into S groups, where group s ($s = 1, 2, ..., S$) contains N_s homogeneous suppliers whose cost is represented by random variable $C_s = A_s + X_s$. As in the two-supplier case, random variables C_s, A_s, and X_s respectively follow distributions H_s, F_s, and G_s, whose domains are bounded above by $\bar{c}_s$, $\bar{a}_s$ and $\bar{x}_s$. When referring to the cost realizations for a specific supplier i within group s we will use notation c_s^i, a_s^i and x_s^i. Suppliers in the same group are homogeneous and suppliers in different groups are heterogeneous. For example, suppose the buyer has a total of eight suppliers: five are located in northeastern China and use very similar labor-intensive production techniques; three are located in the midwestern US and all these suppliers utilize very similar automated production lines. We can divide the eight suppliers into two separate groups: group 1 has five suppliers in China and group 2 has three suppliers in the US.

We find that all of §2.4's propositions for the two-supplier case can be generalized: Suppliers across different groups are heterogeneous (analogous to part (i) of the two-supplier propositions) and suppliers within a group are homogeneous (analogous to part (ii) of the two-supplier propositions). Building on the two-supplier case results, we find that the buyer follows similar strategies when deciding which group(s) of suppliers to learn about and which portion(s) of cost to learn. To denote the N-supplier generalizations of earlier propositions, we append their label with "G" for general.

Characterizing the benefit of cost modeling. Addressing research question 1, Lemma 2.2 directly applies to the N-supplier case, and shows that learning might not be beneficial due to the fact that learning and competition can act as substitutes. Moreover, we extend Proposition 2.1:

Proposition. 2.1-G. [Relation between learning and cost distributions] *Let Ψ_1 denote the expected benefit of learning all suppliers in group 1. As long as the domains of F_1 and G_1 do not change,*

(i) *when group 1 has only one supplier, i.e., $N_1 = 1$,*

 (a) *Ψ_1 decreases as F_1 becomes stochastically larger;*

 (b) *Ψ_1 is independent of G_1;*

 (c) *Ψ_1 increases as H_s $(s \neq 1)$ becomes stochastically larger;*

(ii) *when group 1 has more than one supplier, i.e., $N_1 \geq 2$,*

 (a) *Ψ_1 is not necessarily monotonic as F_1 becomes stochastically larger;*

 (b) *Ψ_1 increases as G_1 becomes stochastically larger;*

 (c) *Ψ_1 increases as H_s $(s \neq 1)$ becomes stochastically larger.*

The intuition is similar to that for Proposition 2.1. Learning about group 1 brings a positive benefit only when group 1 contains the winner. Furthermore, when learning does bring a positive benefit, the benefit's magnitude is governed by the gap between the reserve price (which is determined by F_1 and the domain of X_1) and the second-lowest supplier's cost (determined by all the non-winning suppliers' costs). For example, the benefit of learning is not affected by G_1 when there is one supplier in group 1 (i.b), but is affected when there are two suppliers in group 1 (ii.b). This is because when group 1 contains the winner, the second-lowest supplier's cost does not change in the former case but depends on G_1 in the latter case.

Choosing which group(s) of suppliers to learn about. Addressing research question 2, we have:

Proposition. 2.2-G. [Preferred group to learn about] *For any distributions F_s, G_s and H_s, $s = 1, 2, ..., S$, the buyer prefers to learn about group 1 rather than group 2 if*

$$A_1 + \bar{x}_1 \leq_{st} A_2 + \bar{x}_2, \; C_1 \leq_{st} C_2 \text{ and } N_1 \geq N_2.$$

Compared to Proposition 2.2, in Proposition 2.2-G we have an additional condition, $N_1 \geq N_2$. The intuition is similar to that for Proposition 2.2. As before, the buyer's preference about which group to learn about depends on how low the new reserve price can be and how likely it is that a supplier in the group will win the contract. Both factors improve as the group's *size* increases: Larger groups offer a bigger chance to set a small reserve price and are more likely to contain the contract winner. The next result characterizes how the breadth of learning across groups affects the buyer's benefit.

Proposition. 2.3-G. [Benefit of learning about multiple groups] *Let $\Psi_{1\&2}$, Ψ_s, and Ψ_s^i denote, respectively, the expected benefits of learning about all suppliers in groups 1 and 2, all suppliers in group s, and supplier i in group s. We have:*

(i) $\psi_{1\&2} = \psi_1 + \psi_2$ *almost surely; hence* $\Psi_{1\&2} = \Psi_1 + \Psi_2$.

(ii) $\Psi_s = N_s \cdot \Psi_s^i$.

One may reason that, particularly with N suppliers, the expected marginal benefit of learning will diminish in the number of groups that the buyer learns about — after all, the buyer ultimately only sets one new reserve price which applies to all N suppliers. However, the expected benefit of learning multiple groups is in fact additive across groups and linear in the number of learned suppliers within each group. The intuition behind this is the same as for Proposition 2.3: Along any sample path only one supplier wins the contract, and the buyer only benefits when she learns about the winner.

Choosing which portion(s) of cost to learn and how deeply to learn. Addressing research questions 3 and 4, we have:

Proposition. 2.4-G. [Preferred portion to learn] *Suppose the buyer learns about all suppliers in group 1. For any distributions F_s, G_s and H_s, $s = 1, 2, ..., S$, the buyer prefers to learn portion A_1 rather than portion X_1 if*

$$\bar{a}_1 - A_1 \geq_{st} \bar{x}_1 - X_1.$$

The proposition reveals that what matters to the buyer is the amount of cost uncertainty that learning will resolve: She prefers to learn about whichever portion will (stochastically) reduce the reserve price the most. The takeaway is similar to what we found with two suppliers, namely it is not the case that the buyer only wishes to learn about whichever cost portion contributes most to the suppliers' overall total costs. Next we examine how the benefit changes in depth of learning.

Proposition. 2.5-G. [Benefit of learning multiple portions] *Let Ψ_1^A, Ψ_1^X, and Ψ_1^{AX} denote, respectively, the expected benefits when cost modeling is applied to portion A, portion X, and both portions A and X for all suppliers in group 1. We have*

$$\psi_1^{AX} \geq \psi_1^A + \psi_1^X \quad \textit{almost surely; hence} \quad \Psi_1^{AX} \geq \Psi_1^A + \Psi_1^X.$$

The intuition is akin to that for Proposition 2.5: To be effective the reserve price must bind the winner, and this is particularly encouraged by deeper learning because reserve price reductions are cumulative in cost portions learned. Combining this result with Proposition 2.3-G, which shows that the benefit across groups is linear, we can see that, all else equal, the buyer gets greater benefit from learning a single group in more depth than she does by learning two groups more superficially.

2.6 Optimal Learning Strategy

The previous sections characterized the buyer's benefit of learning without explicitly considering the cost of learning. However, as established earlier, cost modeling is time-consuming and expensive. If we consider the cost of learning, should the buyer learn at all? If so, which suppliers should the buyer learn about, and how deep into suppliers' costs should the buyer learn when deeper learning incurs higher cost? The buyer's cost of learning can depend on which suppliers she learns about, how many suppliers she learns about, and how deeply she learns about them; we will show how the previous sections' analytical results can directly be applied in all these cases to provide insight on the structure of the buyer's optimal learning.

Learning cost is fixed for all suppliers in the same group. To begin, suppose there is simply a fixed cost K_s to create a cost model for each supplier group s. Recall that Ψ_s is the expected benefit of learning about group s without considering the cost of learning. Thus, the net benefit of learning about group s is $\Psi_s - K_s$. Label the groups such that $\Psi_1 - K_1 \geq \Psi_2 - K_2 \geq \cdots \geq \Psi_S - K_S$. If the buyer can learn about just one group, it is optimal to learn about group 1 (provided that $\Psi_1 - K_1 > 0$). If the buyer can learn about multiple groups, it is optimal for her to learn about the first s groups, where s is largest value such that $\Psi_s - K_s > 0$. In fact, although it may seem intractable to consider the buyer's cost versus benefit of learning while taking into account the possibility that she learns from multiple cost models at once, the problem in fact decomposes and can be solved by considering each supplier group in isolation. This is a direct consequence of Proposition 2.3-G(i), which proves that the benefit of learning is additive across groups.

Learning cost increases in the number of suppliers learned. Next suppose that there is a cost associated with the number of suppliers learned in any group s, $K_s(n)$. Let $\Psi_s(n)$ be the expected benefit of learning about n suppliers in group s.

Putting these two together, the net benefit of learning is $\Psi_s(n) - K_s(n)$.

If the cost function $K_s(n)$ is concave in n, Proposition 2.3-G(ii) directly applies to prove that the optimal number of suppliers to learn about must be $n_s^* = 0$ or N_s. This comes from the fact that the benefit of learning is linear in the number of suppliers learned, which makes the net benefit function convex in n. On the other hand, if $K_s(n)$ is convex, Proposition 2.3-G(ii) directly applies to prove that the net benefit function, $\Psi_s(n) - K_s(n)$, is concave in n. Hence, one can easily determine the optimal number of suppliers to learn about. For general increasing functions, the optimal number of suppliers to learn about depends on the particular shape of the cost function $K_s(n)$.

Once we determine the optimal number of suppliers to learn about for each group, label the groups such that $\Psi_1(n_1^*) - K_1(n_1^*) \geq \Psi_2(n_2^*) - K_2(n_2^*) \geq \cdots \geq \Psi_S(n_S^*) - K_S(n_S^*)$. If the buyer can learn about just one group, it is optimal to learn about group 1 if $\Psi_1(n_1^*) - K_1(n_1^*) > 0$. If the buyer can learn about multiple groups, it is optimal to learn about the first s groups, where s is largest value such that $\Psi_s(n_s^*) - K_s(n_s^*) > 0$. Once again, what initially appears to be an intractable problem — considering the buyer's cost versus benefit of learning while taking into account the possibility that she learns from multiple cost models at once and the cost of learning depends on how many suppliers she chooses to learn — the problem in fact decomposes and again we can solve it by examining each supplier group separately.

Learning cost increases in the depth of learning. Suppose the buyer can learn portions A and X, but the cost of learning increases with the portions learned. (For simplicity, we focus on just two cost portions, but the results easily extend to multiple portions.) Our previous sections' analytical results also apply in this case to determine the optimal depth of learning.

If $K_s(A) + K_s(X) \geq K_s(AX)$ (i.e., the cost of learning is subadditive in depth), it immediately follows from Proposition 2.5-G that the optimal depth to learn in group

s is $d_s^* = 0$ or AX. This is because, as we proved, the expected benefit of learning is superadditive in the portions learned, so subadditive learning cost makes the net benefit superadditive in the portions learned. On the other hand, if $K_s(A)+K_s(X) < K_s(AX)$ (superadditive), then it is possible that an intermediate level of learning becomes optimal, i.e., $d_s^* \in \{A, X\}$. In this case, the optimal depth to learn depends on the particular shape of the learning cost function.

Regardless of the learning cost function's shape, once we determine the optimal depth of learning for each group, label the groups such that $\Psi_1(d_1^*) - K_1(d_1^*) \geq \Psi_2(d_2^*) - K_2(d_2^*) \geq \cdots \geq \Psi_S(d_S^*) - K_S(d_S^*)$. If the buyer can learn about just one group, she prefers to learn about group 1 if $\Psi_1(d_1^*) - K_1(d_1^*) > 0$. If the buyer can learn about multiple groups, she prefers to learn about the first s groups, where s is largest value such that $\Psi_s(d_s^*) - K_s(d_s^*) > 0$.

Propositions 2.3-G and 2.5-G can be used to render similar results when the cost of learning is a function of both the number of suppliers and portions learned. For example, for any given group s, Proposition 2.3-G implies that the optimal number of suppliers to learn about will be 0 or N_s if the cost is concave in the number of suppliers for any given depth d_s. Similarly, from Proposition 2.5-G, the optimal depth to learn about each group will be 0 or AX if the cost of learning is subadditive in depth for any number of suppliers learned. Finally, after ordering the groups by their optimal benefits of learning, if the buyer can only learn about one group she prefers to learn about the highest benefit group, and if she can learn about multiple groups she learns about all the groups with positive individual benefit.

Example: In this example, all costs are in thousands of dollars. Suppose there are two groups of suppliers: group 1 has two suppliers, where $A_1 \sim U[200, 575], X_1 \sim U[300, 360]$, only portion A_1 is learnable and the cost of learning is a function of the number of suppliers learned, $K_1(n) = 10 + 2\sqrt{n}$ $(n = 1, 2)$; group 2 has only one supplier, where $A_2 \sim U[360, 530], X_2 \sim U[270, 600]$, both portions A_2 and X_2 are learnable and the cost of learning is $K_2(A) = 12$, $K_2(X) = 13$, $K_2(AX) = 15$.

We first determine the optimal learning strategy for group 1. Since $K_1(n)$ is concave in n, the optimal number of suppliers to learn about is either 0 or 2. The benefit of learning 2 suppliers in group 1 is $\Psi_1(2) = 83.80$ which is greater than the cost of learning $K_1(2) = 12.828$, so $n_1^* = 2$ and $\Psi_1(n_1^*) - K_1(n_1^*) = 70.972$. As for group 2, since $K_2(A) + K_2(X) \geq K_2(AX)$, the optimal depth to learn is either 0 or AX. The benefit of learning AX is $\Psi_2(AX) = 3.31$ which can not cover the cost of learning $K_2(AX) = 15$, so the optimal depth to learn is $d_2^* = 0$. Thus, the optimal learning strategy that the buyer should follow is to learn about the two suppliers in group 1 and not to learn about group 2. In this example, the expected payment without learning is 764.73. Through learning, the buyer reduces the expected payment by 83.80/764.73 = 10.96% and the net saving is 70.972/764.73 = 9.28%.

2.7 Relaxing Independence

2.7.1 Correlated costs

Thus far our models have assumed that suppliers' costs are statistically independent from each other. However, it is easy to imagine that in practice some portion of a supplier's cost may be correlated to the corresponding portion of another supplier's cost. For example, all suppliers might rely on a certain key material or component from the same upstream source, or perhaps suppliers who are located in the same industrial zone (e.g., southern China) incur similar labor rates. Unlike the independent cost case, when costs are correlated learning about one supplier may reveal cost information about his competitors, which is relevant to the buyer's decision about cost modeling (e.g., whether to learn or not, how much to learn, etc.). In this section we show how our results extend to the correlated cost setting.

To extend our model, we assume that the cost of a supplier in group s is represented by a random variable $C_s = A_s + X_s$, where $A_s = A^0 + A^s + A'_s$, and $X_s = X^0 + X^s + X'_s$. Random variables A^0 and X^0 represent portions of A and X that are common for all suppliers. Random variables A^s and X^s represent portions that are common for all the suppliers in group s, but independent from all other suppliers in other groups (if a group s consists of just one supplier, we take $A^s \equiv X^s \equiv 0$). Finally, A'_s and X'_s represent portions that are independent across all suppliers. Let F^0, F^s and F'_s be the distributions of A^0, A^s, and A'_s, respectively. Likewise, $X^0 \sim G^0$, $X^s \sim G^s$ and $X'_s \sim G'_s$. As before, $H_s = F_s \otimes G_s$, where $A_s \sim F_s$, $X_s \sim G_s$. Additionally, we define $H'_s = F'_s \otimes G'_s$.

With this setup, supplier cost realizations are arrived at as follows: one draw from F^0 and one draw from G^0 are made and applied to all suppliers; for each group s, one draw from F^s and one draw from G^s are made and applied to just suppliers in group s; and for each supplier in group s, one draw from F'_s and and one draw from G'_s are made and applied to just that individual supplier. As a check, note that the case with $A^0 \equiv X^0 \equiv A^s \equiv X^s \equiv 0$ corresponds to our model of independent supplier costs. Recall that N_s is the number of suppliers in group s. We let random variable C_{-s} denote the minimum cost among $N_s - 1$ draws from H_s and N_t draws from H_t for each $t \neq s$ (these $N-1$ draws may be correlated). Let r^o denote the original reserve price set without learning, $r^o = \min(\bar{c}_1, \bar{c}_2, ..., \bar{c}_s)$. We have the following result.

Proposition 2.6. *All previous results hold with the following changes to accommodate the correlation model:*

(i) Adapting Proposition 2.1-G [Relation between learning and cost distributions]: *F_1, G_1 and H_s are replaced with F'_1, G'_1 and H'_s, respectively. Moreover, the benefit of learning Ψ_1 decreases in F^0 and F^1, but increases in G^0 and G^1.*

(ii) Adapting Proposition 2.2-G [Preferred group to learn about]: *Condition* $A_1 +$

$\bar{x}_1 \leq_{st} A_2 + \bar{x}_2$ *is replaced by* $A_1' + \bar{x}_1 \leq_{st} A_2' + \bar{x}_2$, *and condition* $C_1 \leq_{st} C_2$ *is replaced by* $\min(r^o, C_{-1}) - A^0 - A^1 \geq_{st} \min(r^o, C_{-2}) - A^0 - A^2$*;*

(iii) Adapting Proposition 2.4-G [Preferred portion to learn]: *Condition* $\bar{a}_1 - A_1 \geq_{st} \bar{x}_1 - X_1$ *is replaced by* $\bar{a}_1 - A_1' \geq_{st} \bar{x}_1 - X_1', \min(r^o, C_{-1}) - A^0 - A^1 \geq_{st} \min(r^o, C_{-1}) - X^0 - X^1$*;*

(iv) Propositions 2.3-G [Benefit of learning about multiple groups] *and* 2.5-G [Benefit of learning multiple portions] *hold without any changes.*

Under the cost correlation model, some supplier cost portions are independent across suppliers while others are correlated. It turns out that the effect these costs' distributions have on the buyer's benefit of learning depends on whether the cost distribution governs an independent or correlated portion. For the independent portions, the effects are exactly as we saw before for the case with completely independent supplier costs (Proposition 2.1-G). For the correlated portions, the benefit of learning decreases with F^0 and F^1, the distributions governing the portions of learned cost that are common across multiple suppliers. The intuition is that the new reserve price stochastically increases in F^0 and F^1 and therefore recovers less profit for the buyer when it binds. Moreover, increasing F^0 and F^1 may increase the payment without learning, but unlike the independent cost case, this change "washes out" due to correlation because the new reserve price would increase by exactly the same amount. Analogously, the benefit of learning increases with G^0 and G^1, the distributions governing the portions of the unlearned cost that are common across multiple suppliers, since the new reserve price is unchanged by this but the contract price in the absence of learning becomes higher. To summarize, we find that learning is less valuable when the learned cost portions exhibits more substantial correlation (F^0 and F^1 become larger), but learning becomes more valuable when the unlearned cost portions exhibit more substantial correlation (G^0 and G^1 become larger). The managerial takeaway is that, although one may intuitively expect that more correlation will always reduce the benefit of learning since supplier costs will be more closely matched and competition will be fiercer, increasing the magnitude of correlated cost drivers can actually increase the benefit of learning.

In choosing which group to learn about, we need to account for correlation when determining how effective the revised reserve price will be in lowering the buyer's payment. In adapting Proposition 2.2-G for correlation, the first condition focuses on the supplier-specific uncertainty (A is replaced by A'). This condition is just like the condition we had in the independent case, capturing the fact that — all else equal — the buyer prefers learning that will lead to a smaller new reserve price. In the second condition we subtract out the correlated portions ($A^0 + A^s$) from the original price, $\min(r^o, C_{-s})$. The reason is that in reaching the original price (without learning), the correlated portions will be revealed "for free" through competition, so intuitively the buyer prefers to learn about groups that have less correlation in the costs that will be learned; as a check, notice that if group 2 has a large within-group correlation governed by A^2, the second condition is more likely to hold and the buyer would prefer to learn about group 1 instead of group 2. We make a similar modification when determining which cost portion to learn (adapting Proposition 2.4-G). The managerial insight here is that, all else equal, the buyer prefers learning about groups and/or cost portions that exhibit weaker cost correlations in the learned portion because strongly correlated costs are more apt to be revealed via competition regardless of learning.

For the fully independent costs case we saw that the benefit of learning about multiple suppliers was additive, and we next ask whether this insight changes in the presence of supplier cost correlation. One might expect it would, since we just saw how sensitivity of the benefit of learning and the decision of which group or portion to learn should account for whether or not the cost portions are correlated across suppliers. However, Proposition 2.6 shows that the value of cost modeling remains additive across suppliers and groups, even with correlation. The reason is that, regardless of cost correlation, the buyer must still learn about the contract winner in order for the learning to be beneficial. Similar reasoning gives our final result, namely that the value of cost modeling is superadditive in cost portions that are learned about, even under correlation.

2.7.2 Supplier collusion

Cost correlation is not the only reason that supplier bids are not independent. At firms we have interacted with — in industries ranging from electronics to automotive parts — buyers have related suspicions of supplier collusion. Collusion dampens bidding competition, making it less effective. We now extend our analysis to study the effect of cost modeling in cases with supplier collusion.

In this subsection we allow the possibility that a group $t \in \{1, \ldots, S\}$ can be a bidding ring. Literature on bidding rings dates back to the seminal paper *Graham and Marshall* (1987). *Krishna* (2009) offers a nice distillation of the canonical theory as follows: (1) the bidders in a ring identify which bidder among them has the lowest cost; (2) only this bidder submits meaningful bids for the contract (other bidders submit bids that do not affect the final winning price, e.g., drop out at the reserve price); (3) after the bidding, the ring shares any surplus gained from collusion. This 3-step distillation describes so-called *efficient* collusion, meaning only the "best" bidder from a colluding ring submits a serious bid for the contract. *Graham and Marshall* (1987) identified a simple way that the ring can coordinate its members to achieve efficient collusion.

Suppose that T of the S supplier groups are bidding rings, and label the indices such that the bidding rings are groups $\{1, \ldots, T\}$. Since a ring can maximize its gain by colluding efficiently, we assume that each ring colludes efficiently and so effectively collapses into a single "best bidder" who represents that entire ring in the bidding process. Not surprisingly, it is a dominant strategy of each ring's representative bidder to lower his bid until winning or reaching his true production cost. It is also a dominant strategy of bidders in groups $\{T+1, \ldots, S\}$ to lower their bid until winning the contract or reaching their true production cost. All other bidders (members of rings who are not ring representatives) submit non-serious bids (meaning they intentionally drop out early, e.g., at the reserve price). For each ring t, define cost distributions $F_t^{\text{ring}} = 1-(1-F_t)^{N_t}$, $G_t^{\text{ring}} = 1-(1-G_t)^{N_t}$, and $H_t^{\text{ring}} = 1-(1-H_t)^{N_t}$. We have the following result.

Proposition 2.7. *Propositions 2.1-G, 2.2-G, 2.3-G and 2.4-G hold if each bidding ring t is replaced with a synthetic group, t^{ring}, containing a single supplier whose learnable, unlearnable, and total costs are governed by distributions F_t^{ring}, G_t^{ring}, and H_t^{ring}, respectively.*

In words, synthetic group t^{ring} consists of a single supplier whose costs equal the first order statistic of costs from all suppliers in the bidding ring t. The implication behind Proposition 2.7 is that buyers can understand the role of cost modeling in the presence of supplier collusion simply by considering the colluding groups to be collapsed to representative suppliers.

However, there are some interesting differences that arise. First, note that we no longer get super-additivity of benefits of learning multiple portions for a colluding group (namely, Proposition 2.5-G does not hold under collusion, see proof of Proposition 2.7(ii)). This is because in Proposition 2.7 we have collapsed the ring to a single representative supplier. In fact, if we look at individual suppliers within a ring, we recover the super-additivity result (see proof of Proposition 2.7(iii)). Thus, unlike the non-collusion case where super-additivity across portions applied at the individual supplier level *and* at the group level, under collusion it only applies at the individual level. The opposite situation arises for the additivity of benefits of learning multiple groups (Proposition 2.3-G): Under collusion additivity of benefits holds at the group level (per Proposition 2.7), but it turns out that it does not hold at an individual supplier level (see proof of Proposition 2.7(i); recall that Proposition 2.7 had collapsed the ring into a single representative bidder). Thus, under collusion there is an interesting difference between group and individual-level learning benefits.

The reason behind this is that collusion changes the way benefits accrue across suppliers. Recall that when suppliers do not collude, losing suppliers' cost information would always be revealed through the bidding competition, so the buyer only benefitted when she learned about whoever eventually won the contract. By contrast, if a losing supplier is in a winning bidding ring, their cost information will *not* be revealed through the competition. In fact, under collusion the buyer can gain a positive benefit from learning even if the supplier she learns about is not the eventual contract winner. The upshot is that under collusion the successive benefits of learning about additional suppliers in the same bidding ring need not be additive. However, more importantly, the benefit of any amount of learning in the presence of collusion is always at least as large as it would have been were the collusion absent.

Proposition 2.8. *Compared to the non-colluding case, the presence of collusion can only increase the buyer's benefit of learning about supplier costs.*

2.8 Other Extensions

2.8.1 Learning versus more bidders

Thus far we have shown that learning about suppliers can decrease the buyer's procurement cost. Of course, the buyer could also lower her cost by locating new suppliers who will bid for the contract. In practice, this option is very expensive and time-consuming; at Fortune 500 companies we interacted with it typically takes months to locate and qualify new suppliers to bid in an auction. This raises an important question: When designing a supply base, should the buyer focus on fostering her knowledge about her existing suppliers, or instead focus on locating and qualifying new suppliers? We address this question by comparing the benefits of adding new suppliers to the benefit of learning about existing suppliers.

We consider a buyer who has a supply base consisting of a group of N non-colluding suppliers whose costs are *ex ante* symmetric but possibly correlated. We use the correlation model from §2.7.1, but drop the group index s since there effectively is only one group; thus, for example, $H = F \otimes G$ represents the suppliers' cost distribution. (With multiple groups (ex ante asymmetric suppliers) and/or collusion, the key insights do not change but the expressions are more complicated.)

As in §2.6, suppose that there is a cost to learn about each supplier. We compare the following two options: (a) The buyer learns about m out of N suppliers (and learns the realization of portion A for each of these m suppliers); (b) The buyer locates y more suppliers, for a total of $N + y$ suppliers. The benefit of option (a), learning m suppliers, is given by $m\Psi$, where Ψ is the benefit of learning about a single supplier (see Proposition 2.6 part (iv)). Let $\Phi(y) \triangleq E[C_{2:N} - C_{2:N+y}]$ denote the benefit of adding y suppliers, where $C_{2:n}$ denotes the second-lowest order statistic from n draws from distribution H.

We compare the benefit of learning, $m\Psi$, and the benefit of locating, $\Phi(y)$, in the following proposition. We let $\lceil s \rceil$ denote the smallest integer that is greater than or equal to s.

Proposition 2.9. *The buyer prefers learning about existing suppliers to locating new suppliers if and only if* $m \geq m^* = \left\lceil \frac{\Phi(y)}{\Psi} \right\rceil$.

As one can see, if the benefit of learning each individual supplier, Ψ, is very small then $m^* > N$ and learning will not be better than locating even if the buyer learns about all N existing suppliers. Likewise, the same is true if the number of additional suppliers, y, is very large. To make the comparison between learning and locating more meaningful, we consider a case where learning is effective and y is not extremely large. Namely, suppose that the buyer can learn a supplier's exact cost through learning, and compare this to the benefit of adding one new supplier.

Proposition 2.10. *Let* $\gamma_N = \frac{\int H'(c)^2 \bar{H}'(c)^{N-1} \mathrm{d}c}{\int H'(c) \bar{H}'(c)^{N-1} \mathrm{d}c}$, *and* $m^* = \lceil \gamma_N \cdot N \rceil$. *Learning about* m *suppliers completely is better than locating one more supplier if and only if* $m \geq m^*$. *Moreover, for any cost distribution* H', $\gamma_N \in (0, 1)$, *and thus* $m^* \leq N$.

Costs that are common to all suppliers "wash out" in competition and therefore do not affect the benefit of learning. This is why the threshold m^* in Proposition 2.10 only depends on the cost portion that is independent across suppliers, which is governed by H' (whose tail is denoted by $\bar{H}'$). The proposition implies that, when cost modeling is precise enough, learning can provide larger cost saving than locating a new supplier.

It is interesting to note that the minimum number of existing suppliers that the buyer needs to learn about in order to prefer it to locating one new supplier can be quite small. See Table 2.1, which gives the minimum (rightmost column) for several well-known distributions. For all the cases in the table, the benefit from learning the costs of just two (out of N) suppliers exceeds the benefit from locating a new supplier. In fact, when supplier costs follow a uniform or power function distribution, our result shows that for a buyer with small existing supply base ($N = 2$ for uniform and $N \leq \nu + 1$ for power distribution), learning the cost of just one supplier is better than locating a new supplier. Our result contrasts with the famous finding of *Bulow and Klemperer* (1996), who showed it is optimal for a buyer to locate one more supplier (and intensify competition) rather than set an optimal reserve price with the existing supply base. This is because a buyer in our paper has ability to learn about suppliers' cost (which is certainly true in procurement settings), and this capability enables the buyer to set a more effective reserve price than a buyer in *Bulow and Klemperer* (1996).

Distribution	CDF	γ_N	m^*
Uniform	$H'(c) = c \cdot I_{0 \le c \le 1}$	$\frac{2}{N+2}$	$1(N=2); 2(N \ge 3)$
Exponential	$H'(c) = (1 - e^{-\lambda c}) \cdot I_{c \ge 0}$	$\frac{2}{N+1}$	2
Power-function ($\nu > 1$)	$H'(c) = c^{\nu} \cdot I_{0 \le c \le 1}$	$\frac{1+1/\nu}{N+1+1/\nu}$	$1(N \le \nu+1); 2(N > \nu+1)$

Table 2.1: Examples of Proposition 2.10

2.8.2 The effects of buyer power on the benefits of Learning

In this paper, we focused our analysis on the reverse open-descending auction where the buyer sets a reserve price as a starting point and then the suppliers compete by lowering their bids until only one bidder remains. We study this mechanism because it is analytically tractable and extremely prevalent in practice. However, one may wonder what the buyer's learning preferences will be under other mechanisms.

The buyer's power relative to her suppliers determines which mechanism she can choose. Following the reasoning of *Bulow and Klemperer* (1996), we can classify a buyer's power into three levels.

- Zero power: A buyer with zero power can not set a reserve price and the contract payment is completely determined by suppliers' bid competition in a reverse open-descending auction *without* a reserve price. In reality, this could correspond to a small mom and pop firm who cannot scare suppliers by setting a reserve price because she has little clout over her large manufacturer suppliers. In this case, one can show that learning provides no benefit since the buyer has no power to apply any cost information.

- Moderate power: A buyer with moderate power can set a reserve price. This was the case documented in *Beall et al.* (2003), which surveyed large (Fortune 500) firms, many of whom spend billions of dollars on procurement each year. Through learning, the buyer can apply the cost information to set a more aggressive reserve price and reduce the contract payment. This is the case that we analyzed in this paper.

- Absolute power: A buyer with absolute power relative to her suppliers can design and commit to any mechanism. This allows her to design, for example, an optimal mechanism. This category of buyer is of theoretical interest, but in reality few buyers have such power (firms like Apple may be an exception). One can show that a buyer with absolute power does not have to exert effort to learn about suppliers' costs before the auction. Instead, she can induce the suppliers to voluntarily divulge any learnable cost information practically for free before the auction, simply by employing random post-auction audits on learnable portions and levying large penalties on suppliers found to have lied about their costs. In reality, these penalties could take the form of disallowing future bids from the supplier, which might be significant if the buyer comprises a substantial portion of market demand.

Thus, although one might expect that buyers with more power will have more incentive to utilize cost modeling since a powerful buyer can utilize the information better than a weak buyer, we conclude from the above discussion that a buyer's inclination to undertake cost modeling on her suppliers is not monotonic in her power. Since buyers with zero power are too weak to apply any information and buyers with absolute power essentially obtain learnable information for free without cost modeling, learning via cost modeling is only interesting for buyers with moderate power. This is why our paper focused on this latter case.

2.9 Conclusions

In this paper, we ask whether a buyer should deploy cost modeling to learn about suppliers who will compete for a contract. Intuitively, because competitive bidding and cost modeling are *both* ways that a buyer can discover information about suppliers' costs, and moreover since cost modeling itself is an expensive undertaking, careful analysis is needed to understand if and when cost modeling should be used in conjunction with supplier competition. To our knowledge, this paper is the first study of cost modeling and its interactions with competitive bidding.

Addressing research question 1 (How does one characterize the benefit of cost modeling?), we show that learning prior to competitive bidding enables the buyer to set a tighter reserve price. In doing so the buyer seeks to truncate suppliers' surplus. However, when there are competing suppliers, the buyer does not always receive a positive benefit from learning. In fact, the benefit of learning is positive only when the new reserve price actively truncates the surplus of the contract winner, and learning about suppliers who wind up losing the contract provides no useful information to the buyer, since these suppliers' costs will be revealed during the competitive bid process anyway. Of course, this makes the buyer's problem difficult because she cannot *ex ante* predict which supplier will win — after all, the suppliers' costs (which the buyer does not know) determine who wins the contract. Summarizing the above, learning is more valuable to the extent that the reserve price is likely to truncate the surplus of the contract winner.

Turning to research question 2 (If the buyer chooses to develop a cost model, which supplier(s) should she learn about?), we find that the key criterion is not to learn about the supplier whose learnable cost is more uncertain, but to learn about the supplier who is more likely to win and whose learnable cost, once learned, will enable the buyer to lower the reserve price the most. Moreover, one might intuitively expect it to be redundant to learn about several competing suppliers. However, we find that the benefit of learning about multiple suppliers is actually additive across suppliers. Hence, managers should strongly consider learning about multiple suppliers at once, when the cost of learning is linear or concave in the number of suppliers learned.

Addressing research question 3 (Which cost portion(s) should the buyer learn?), we find that the buyer prefers to learn about the portion which can resolve more uncertainty. Surprisingly, we also find that the benefit of learning multiple portions at once is even greater than the sum of the benefits of learning each portion in isolation. Coupled with our insights regarding research question 2, this suggests that — all else equal — learning fewer suppliers in depth is preferable to learning more suppliers superficially. Thus, staffing a procurement department with specialists having deep but narrow domain expertise (e.g., of a particular type of production method used by some suppliers) may be preferable to having generalists with broader but more limited knowledge about the industry's general cost drivers.

Addressing research question 4 (What is the optimal learning strategy?), we find that the buyer can first divide all suppliers into several groups based on their cost structures, next figure out the optimal level of learning for each group, and then the optimal learning strategy is to learn about the groups with a positive net benefit of learning. Depending on the cost to learn various groups to varying depths, we show that the buyer can adopt a mix-and-match strategy of learning, whereby she learns some groups deeply, others superficially, and some not at all.

Finally, research question 5 considers the effect of the underlying business context. Our insights are robust to correlations across supplier costs, and surprisingly — even though correlation tends to make competition fiercer by increasing the parity between suppliers' costs — larger correlated costs across suppliers can actually increase the benefit of learning. We show that supplier collusion short circuits the price discovery of the competitive bidding process and makes cost modeling aimed at learning about supplier costs even more valuable for the buyer. We also show that learning just a few suppliers deeply can be preferable to adding another bidder. Lastly, we examined how the buyer's power relative to her suppliers affects her benefit of learning. Interestingly, we find that buyers with moderate power benefit more from learning than buyers with zero or absolute power.

For simplicity our discussion focused on two cost portions, A and X. However, our results extend to multiple cost portions. Suppose that supplier i's cost is given by $C_i = Z_1^i + Z_2^i + \cdots + Z_m^i + \epsilon_i$, where the Z_j^i's are learnable cost portions and ϵ_i is unlearnable. For any subsets $R, T \subseteq \{1, 2, \ldots, m\}$, $R \cap T = \emptyset$, we can apply all the paper's propositions by defining $A_i = \sum_{j \in R} Z_j^i$, $X_i = \sum_{j \in T} Z_j^i$, and $\epsilon_i = C_i - A_i - X_i$.

Our paper is an important first step in understanding how two very common procurement tools — cost modeling and competitive bidding — interact. It has the potential to help procurement managers make better decisions about how to apply cost modeling in practice. We hope that it spurs further research into the role of cost modeling in supply chain and sourcing strategies.

2.10 Proofs

In our proofs we let A_i, X_i, and C_i denote supplier i's cost random variables, and let a_i, x_i, and c_i denote the cost realizations. This notation suppresses the group index s, but it is understood that if suppliers i and j are in the same group (i.e., are homogeneous) then A_i and A_j have the same distribution, X_i and X_j have the same distribution, and C_i and C_j have the same distribution. Also, we let ψ^1, ψ^2, and $\psi^{1\&2}$, respectively, denote the benefits of learning about supplier 1, supplier 2, and both supplier 1 and supplier 2, where $\Psi^1 = E[\psi^1]$, $\Psi^2 = E[\psi^2]$, and $\Psi^{1\&2} = E[\psi^{1\&2}]$ denote the expected benefits of learning.

Proof of Proposition 2.3-G

The proof of Proposition 2.1-G and 2.2-G utilizes Proposition 2.3-G; therefore we prove Proposition 2.3-G first.

It suffices to prove that when there are N suppliers, the benefit of learning any two of them is additive. Consider any suppliers 1 and 2, who may or may not be in the same group. We want to show $\psi^{1\&2} = \psi^1 + \psi^2$. We consider the sample paths such that $C_i \neq C_j (\forall i \neq j)$. Since $C_i (i = 1, 2, ..., N)$ are continuous random variables, the probability that $C_i \neq C_j (\forall i \neq j)$ is 1. Let $C_{[k]}$ denote the k^{th} order statistic of $C_1, C_2, \ldots, C_N$, and define $C_{-i} \triangleq \min_{j \neq i}\{C_j\}$. We have

$$\begin{aligned}
\psi^{1\&2} =& [\min(r^o, C_{[2]}) - \min(A_1 + \bar{x}_1, A_2 + \bar{x}_2)]^+, \\
=& I_{\{C_1 = C_{[1]}\}}[\min(r^o, C_{[2]}) - \min(A_1 + \bar{x}_1, A_2 + \bar{x}_2)]^+ \\
&+ I_{\{C_2 = C_{[1]}\}}[\min(r^o, C_{[2]}) - \min(A_1 + \bar{x}_1, A_2 + \bar{x}_2)]^+ \\
&+ \sum_{i=3}^{N} I_{\{C_i = C_{[1]}\}}[\min(r^o, C_{[2]}) - \min(A_1 + \bar{x}_1, A_2 + \bar{x}_2)]^+, \\
=& I_{\{C_1 = C_{[1]}\}}[\min(r^o, C_{[2]}) - \min(A_1 + \bar{x}_1, A_2 + \bar{x}_2)]^+ \\
&+ I_{\{C_2 = C_{[1]}\}}[\min(r^o, C_{[2]}) - \min(A_1 + \bar{x}_1, A_2 + \bar{x}_2)]^+, \\
=& I_{\{C_1 = C_{[1]}\}}[\min(r^o, C_{-1}) - \min(A_1 + \bar{x}_1, A_2 + \bar{x}_2)]^+ \\
&+ I_{\{C_2 = C_{[1]}\}}[\min(r^o, C_{-2}) - \min(A_1 + \bar{x}_1, A_2 + \bar{x}_2)]^+, \\
=& I_{\{C_1 = C_{[1]}\}}[\min(r^o, C_{-1}) - (A_1 + \bar{x}_1)]^+ + I_{\{C_2 = C_{[1]}\}}[\min(r^o, C_{-2}) - (A_2 + \bar{x}_2)]^+.
\end{aligned}$$

The third equality follows since $C_i = C_{[1]}, i \geq 3$ implies $C_1 \geq C_{[2]}, C_2 \geq C_{[2]}$, so $\min(A_1 + \bar{x}_1, A_2 + \bar{x}_2) \geq \min(C_1, C_2) \geq C_{[2]} \geq \min(r^o, C_{[2]})$. The final equality follows since $C_1 = C_{[1]} \Rightarrow C_{-1} \leq C_2 \leq A_2 + \bar{x}_2 \Rightarrow \min(r^o, C_{-1}) - (A_2 + \bar{x}_2) \leq 0$.

By similar arguments,

$$\begin{aligned}
\psi^1 &= [\min(r^o, C_{[2]}) - (A_1 + \bar{x}_1)]^+ = I_{\{C_1 = C_{[1]}\}}[\min(r^o, C_{-1}) - (A_1 + \bar{x}_1)]^+, \\
\psi^2 &= [\min(r^o, C_{[2]}) - (A_2 + \bar{x}_2)]^+ = I_{\{C_2 = C_{[1]}\}}[\min(r^o, C_{-2}) - (A_2 + \bar{x}_2)]^+.
\end{aligned}$$

Hence $\psi^{1\&2} = \psi^1 + \psi^2$ almost surely.

Proof of Proposition 2.1-G

Suppose supplier 1 is in group 1. We have the benefit of learning about supplier 1

$$\psi^1 = [\min(r^o, C_{[2]}) - (A_1 + \bar{x}_1)]^+ = [\min(r^o, C_{-1}) - (A_1 + \bar{x}_1)]^+.$$

This is because: when $C_1 = C_{[1]}$, $C_{[2]} = C_{-1}$; when $C_1 \geq C_{[2]}$, $A_1 + \bar{x}_1 \geq C_1 \geq C_{[2]} \geq C_{-1} \Rightarrow \min(r^o, C_{-1}) \leq \min(r^o, C_{[2]}) \leq A_1 + \bar{x}_1$.

Since ψ^1 increases in C_{-1} and decreases in A_1, the expected benefit of learning group 1, $\Psi_1 = N_1 \cdot E[\psi^1]$, increases as the distribution of C_{-1} becomes larger and decreases as the distribution of A_1 becomes larger. This explains part (i.a), (i.b), (i.c), (ii.b) and (ii.c). Part (ii.a) is because when group 1 has multiple suppliers, F_1 affects not only A_1 but also C_{-1}.

Proof of Proposition 2.2-G

Suppose supplier 1 is in group 1 and supplier 2 is in group 2. We have

$$\psi^1 = [\min(r^o, C_{[2]}) - (A_1 + \bar{x}_1)]^+ = [\min(r^o, C_{-1}) - (A_1 + \bar{x}_1)]^+, \quad \text{and}$$
$$\psi^2 = [\min(r^o, C_{[2]}) - (A_2 + \bar{x}_2)]^+ = [\min(r^o, C_{-2}) - (A_2 + \bar{x}_2)]^+.$$

Since r^o is constant and $C_1 \leq_{st} C_2$, we have $\min(r^o, C_{-1}) \geq_{st} \min(r^o, C_{-2})$ (see page 7 in *Müller and Stoyan* (2002)). Also, since $A_1 + \bar{x}_1 \leq_{st} A_2 + \bar{x}_2$, we have $-(A_1 + \bar{x}_1) \geq_{st} -(A_2 + \bar{x}_2)$. Because (C_{-1}, A_1) are independent random variables and (C_{-2}, A_2) are independent random variables, from Theorem 1.2.17 in *Müller and Stoyan* (2002), we can conclude that $\min(r^o, C_{-1}) - (A_1 + \bar{x}_1) \geq_{st} \min(r^o, C_{-2}) - (A_2 + \bar{x}_2)$ and $\psi^1 \geq_{st} \psi^2$. It then follows that $\Psi^1 \geq \Psi^2$. Finally, by applying Proposition 2.3-G, the expected benefit of learning group 1 is $\Psi_1 = N_1 \cdot \Psi^1$ and the expected benefit of learning group 2 is $\Psi_2 = N_2 \cdot \Psi^2$. Since $N_1 \geq N_2$ and $\Psi^1 \geq \Psi^2$, we have $\Psi_1 \geq \Psi_2$.

Proof of Proposition 2.4-G

Suppose supplier 1 is in group 1. The benefit of learning supplier 1's portion A_1, or X_1, are

$$\psi^{1,A} = [\min(r^o, C_{[2]}) - (A_1 + \bar{x}_1)]^+ = [\min(r^o, C_{-1}) - (A_1 + \bar{x}_1)]^+,$$
$$\psi^{1,X} = [\min(r^o, C_{[2]}) - (X_1 + \bar{a}_1)]^+ = [\min(r^o, C_{-1}) - (X_1 + \bar{a}_1)]^+.$$

Since C_{-1}, A_1, X_1 are independent,

$$\bar{a}_1 - A_1 \geq_{st} \bar{x}_1 - X_1 \Rightarrow A_1 + \bar{x}_1 \leq_{st} X_1 + \bar{a}_1 \Rightarrow \psi^{1,A} \geq_{st} \psi^{1,X},$$

which implies $\Psi^{1,A} = E[\psi^{1,A}] \geq \Psi^{1,X} = E[\psi^{1,X}]$.

The result extends to the case of any number of suppliers in the same group. By Proposition 2.3-G, the expected benefit of learning portion A of group 1 is $\Psi_1^A = N_1 \cdot \Psi^{1,A}$ and the expected benefit of learning portion X of group 1 is $\Psi_1^X = N_1 \cdot \Psi^{1,X}$. Thus we have $\Psi_1^A \geq \Psi_1^X$.

Proof of Proposition 2.5-G

The proof uses the following technical lemma, which is stated and proved below.

Lemma 2.3. *If Y, Z, T are positive, then $(Y + Z - T)^+ \geq (Y - T)^+ + (Z - T)^+$.*

Proof of Lemma 2.3.

If $Y \geq T, Z \geq T$, then $(Y + Z - T)^+ = Y + Z - T \geq (Y - T) + (Z - T) = (Y - T)^+ + (Z - T)^+$;

if $Y \geq T, Z < T$, then $(Y + Z - T)^+ = Y + Z - T \geq Y - T = (Y - T)^+ + (Z - T)^+$;

if $Y < T, Z \geq T$, then $(Y + Z - T)^+ = Y + Z - T \geq Z - T = (Y - T)^+ + (Z - T)^+$;

if $Y < T, Z < T$, then $(Y + Z - T)^+ \geq 0 = (Y - T)^+ + (Z - T)^+$.

Hence, $(Y + Z - T)^+ \geq (Y - T)^+ + (Z - T)^+$.

Suppose supplier 1 is in group 1. The benefit of learning supplier 1's portion A_1 is

$$\psi^{1,A} = [\min(r^o, C_{-1}) - (A_1 + \bar{x}_1)]^+ = [(\bar{a}_1 - A_1) - (\bar{c}_1 - \min(r^o, C_{-1}))]^+.$$

Similarly, the benefit of learning supplier 1's portion X_1 is

$$\psi^{1,X} = [(\bar{x}_1 - X_1) - (\bar{c}_1 - \min(r^o, C_{-1}))]^+,$$

and the benefit of learning supplier 1's both portions A_1 and X_1 is

$$\psi^{1,AX} = [(\bar{c}_1 - C_1) - (\bar{c}_1 - \min(r^o, C_{-1}))]^+ = [(\bar{a}_1 - A_1) + (\bar{x}_1 - X_1) - (\bar{c}_1 - \min(r^o, C_{-1}))]^+.$$

Since the reserve price without learning is $r^o = \min_{i=1}^N(\bar{c}_i) \leq \bar{c}_1$, we have $\bar{c}_1 - \min(r^o, C_{-1}) \geq 0$. Also, because $\bar{a}_1 - A_1 \geq 0$ and $\bar{x}_1 - X_1 \geq 0$, Lemma 2.3 implies $\psi^{1,AX} \geq \psi^{1,A} + \psi^{1,X}$. By Proposition 2.3-G, we have $\psi_1^{AX} \geq \psi_1^A + \psi_1^X$.

Proof of Propositions 2.1-2.5

Propositions 2.1, 2.2, 2.3, 2.4 and 2.5 are direct consequences of the corresponding generalized propositions.

Proof of Proposition 2.6

(i) Adapting Proposition 2.1-G: Suppose supplier 1 is in group 1. We have the benefit of learning about supplier 1

$$\begin{aligned}\psi^1 &= [\min(r^o, C_{[2]}) - (A_1 + \bar{x}_1)]^+ = [\min(r^o, C_{-1}) - (A_1 + \bar{x}_1)]^+ \\ &= [\min(r^o - A^0 - A^1, C_{-1} - A^0 - A^1) - (A_1' + \bar{x}_1)]^+.\end{aligned}$$

Since $\min(r^o - A^0 - A^1, C_{-1} - A^0 - A^1)$ decreases in A^0, A^1 and increases in X^0, X^1, the expected benefit of learning group 1, $\Psi_1 = N_1 \cdot E[\psi^1]$, decreases in F^0, F^1 and increases in G^0, G^1.

(ii) Adapting Proposition 2.2-G: Suppose supplier 1 is in group 1 and supplier 2 is in group 2. We have

$$\psi^1 = [\min(r^o, C_{[2]}) - (A_1 + \bar{x}_1)]^+ = [\min(r^o, C_{-1}) - A^0 - A^1 - (A'_1 + \bar{x}_1)]^+, \quad \text{and}$$
$$\psi^2 = [\min(r^o, C_{[2]}) - (A_2 + \bar{x}_2)]^+ = [\min(r^o, C_{-2}) - A^0 - A^2 - (A'_2 + \bar{x}_2)]^+.$$

Because $\min(r^o, C_{-i}) - A^0 - A^i$ and $A'_i + \bar{x}_i (i = 1, 2)$ are *independent* random variables, from Theorem 1.2.17 in *Müller and Stoyan* (2002), we can conclude that $\min(r^o, C_{-1}) - A^0 - A^1 - (A'_1 + \bar{x}_1) \geq_{st} \min(r^o, C_{-2}) - A^0 - A^2 - (A'_2 + \bar{x}_2)$ and $\psi^1 \geq_{st} \psi^2$.

(iii) Adapting Proposition 2.4-G: Suppose supplier 1 is in group 1. The benefit of learning supplier 1's portion A_1, or X_1, are

$$\psi^{1,A} = [\min(r^o, C_{[2]}) - (A_1 + \bar{x}_1)]^+ = [\min(r^o, C_{-1}) - A^0 - A^1 - (A'_1 + \bar{x}_1)]^+,$$
$$\psi^{1,X} = [\min(r^o, C_{[2]}) - (X_1 + \bar{a}_1)]^+ = [\min(r^o, C_{-1}) - X^0 - X^1 - (X'_1 + \bar{a}_1)]^+.$$

Since $\bar{a}_1 - A'_1 \geq_{st} \bar{x}_1 - X'_1$, $A'_1 + \bar{x}_1 \leq_{st} X'_1 + \bar{a}_1$. Because $\min(r^o, C_{-1}) - A^0 - A^1$ and $A'_1 + \bar{x}_1$ are independent, and $\min(r^o, C_{-1}) - X^0 - X^1$ and $X'_1 + \bar{a}_1$ are independent, from Theorem 1.2.17 in *Müller and Stoyan* (2002), we can conclude that $\min(r^o, C_{-1}) - A^0 - A^1 - (A'_1 + \bar{x}_1) \geq_{st} \min(r^o, C_{-1}) - X^0 - X^1 - (X'_1 + \bar{a}_1)$ and $\psi^{1,A} \geq_{st} \psi^{1,X}$.

(iv) Proofs of Proposition 2.3-G and 2.5-G can be directly applied to the correlation case.

Proof of Proposition 2.7

For a bidding ring t, the representative bid is the lowest of all suppliers' costs, which is governed by distribution H_t^{ring}. If the buyer learns about portion A of all suppliers in bidding ring t, then the new reserve price is $\min(r^0, A_{t[1:Nt]}+\bar{x}_t)$, in which $A_{t[1:Nt]}$ is governed by distribution F_t^{ring}. Similarly for portion X, the new reserve price set by learning about portion X is $\min(r^0, X_{t[1:Nt]}+\bar{a}_t)$, in which $X_{t[1:Nt]}$ is governed by distribution G_t^{ring}. This explains why Propositions 2.1-G, 2.2-G, 2.3-G and 2.4-G hold if each bidding ring is replaced with a representative supplier.

Next, we will show that Proposition 2.5-G only applies at individual supplier level but not at the group level, and Proposition 2.3-G does not hold at individual level.

(i) Proposition 2.3-G does not hold at individual level. We provide a counter-example. Suppose there are two colluding suppliers who have the same cost structure with upper bounds $\bar{a}=1, \bar{x}=1, \bar{c}=2$. The cost realizations are $a_1=0.2, x_1=0.5, c_1=0.7; a_2=0.5, x_2=0.3, c_2=0.8$. Since the two suppliers are colluding, supplier 2 will not submit meaningful bid and the contract price without learning is $\bar{c}=2$, so the benefits of learning are $\psi^1=[\bar{c}-(a_1+\bar{x})]^+=0.8$, $\psi^2=[\bar{c}-(a_2+\bar{x})]^+=0.5$ and $\psi^{1\&2}=[\bar{c}-(\min(a_1,a_2)+\bar{x})]^+=0.8$. The additivity does not hold since $\psi^{1\&2}<\psi^1+\psi^2$.

(ii) Proposition 2.5-G does not hold at group level. Consider the same example as in (i), the benefits of learning both suppliers are $\psi^A=[\bar{c}-(\min(a_1,a_2)+\bar{x})]^+=0.8$, $\psi^X=[\bar{c}-(\min(x_1,x_2)+\bar{a})]^+=0.7$ and $\psi^{AX}=[\bar{c}-\min(c_1,c_2)]^+=1.3$. The super-additivity does not hold since $\psi^{AX}<\psi^A+\psi^X$.

(iii) Proposition 2.5-G applies at individual level. We can use the proof of Proposition 2.5-G, while C_{-1} in that proof stands for the minimum cost of all other suppliers not in bidding ring 1.

Proof of Proposition 2.8

When suppliers collude, fewer bids are submitted and thus the contract price without learning is higher. Since the new reserve price is the same, the benefit of learning (which equals to the gap between the contract price without learning and the new reserve price) is higher in the presence of collusion.

Proof of Proposition 2.9

The benefit of learning m suppliers is $m\Psi$, and the benefit of locating y suppliers is $\Phi(y)$. Hence, the buyer prefers learning to locating if and only if $m\Psi \geq \Phi(y) \Longleftrightarrow m \geq \frac{\Phi(y)}{\Psi} \Longleftrightarrow m \geq m^* = \lceil \frac{\Phi(y)}{\Psi} \rceil$.

Proof of Proposition 2.10

We only need to prove that $\gamma_N = \frac{\Phi(1)}{N\Psi} = \frac{\int H'(c)^2 \bar{H}'(c)^{N-1}\mathrm{d}c}{\int H'(c)\bar{H}'(c)^{N-1}\mathrm{d}c} \in (0,1)$.

Let $\mu_{k:M}$ denote $E[C'_{k:M}]$. Since $\Phi(1) = E[C_{2:N} - C_{2:N+1}] = E[C'_{2:N} - C'_{2:N+1}] = \mu_{2:N} - \mu_{2:N+1}$ and $\Psi = E[C_{2:N} - C_1]^+ = P(C_1 = C_{1:N}) \cdot E[C_{2:N} - C_{1:N}] = \frac{1}{N}E[C'_{2:N} - C'_{1:N}] = \frac{1}{N}(\mu_{2:N} - \mu_{1:N})$, we have $\gamma_N = \frac{\Phi(1)}{N\Psi} = \frac{\mu_{2:N} - \mu_{2:N+1}}{\mu_{2:N} - \mu_{1:N}}$.

Suppose C' is distributed over interval $[\underline{c}', \bar{c}']$ with density function h' and CDF H', then the density function of $C'_{1:M}$ is $p(c) = Mh'(c)\bar{H}'(c)^{M-1}$ and the expected value of the first-order statistic is

$$\mu_{1:M} = \int_{\underline{c}}^{\bar{c}} cMh'(c)\bar{H}'(c)^{M-1}\mathrm{d}c = -\int_{\underline{c}}^{\bar{c}} c\ \mathrm{d}(\bar{H}'(c)^M)$$
$$= -c\bar{H}'(c)^M|_{\underline{c}}^{\bar{c}} + \int_{\underline{c}}^{\bar{c}} \bar{H}'(c)^M\mathrm{d}c = \underline{c} + \int_{\underline{c}}^{\bar{c}} \bar{H}'(c)^M\mathrm{d}c.$$

We utilize the following recursive relationship from *David and Joshi* (1968):

$$\mu_{2:N} = N\mu_{1:N-1} - (N-1)\mu_{1:N}, \text{ and } \mu_{2:N+1} = (N+1)\mu_{1:N} - N\mu_{1:N+1}.$$

Then, we have

$$\gamma_N = \frac{\mu_{2:N} - \mu_{2:N+1}}{\mu_{2:N} - \mu_{1:N}} = \frac{\mu_{1:N-1} - 2\mu_{1:N} + \mu_{1:N+1}}{\mu_{1:N-1} - \mu_{1:N}}$$
$$= \frac{\int \bar{H}'(c)^{N-1}\mathrm{d}c - 2\int \bar{H}'(c)^N\mathrm{d}c + \int \bar{H}'(c)^{N+1}\mathrm{d}c}{\int \bar{H}'(c)^{N-1}\mathrm{d}c - \int \bar{H}'(c)^N\mathrm{d}c} = \frac{\int H'(c)^2\bar{H}'(c)^{N-1}\mathrm{d}c}{\int H'(c)\bar{H}'(c)^{N-1}\mathrm{d}c} < 1.$$

CHAPTER 3

Procurement Auctions with Error-Prone RFQs

3.1 Introduction

As suppliers become increasingly sophisticated and move up the value chain, firms can enjoy cost savings by outsourcing "non-core" aspects of their business. For example, the average US manufacture spends roughly 57% of its revenue purchasing from external suppliers (*U.S. Department of Commerce*, 2011). However, while relying on the external suppliers, these firms gradually lose expertise in internal design and production of the outsourced parts.

Due to this loss of expertise, a buyer might find it difficult to always design a perfect request for quotes (RFQ) that takes into account every design specification detail, and thus is susceptible to making an "error" such as incorrect design, omission, and misspecification. An RFQ error may only be discovered after the contract has been signed with one supplier and even after the production process has begun. For example, a buyer trying to outsource a newly designed item provides a list of specifications in the RFQ and selects a supplier via competitive bidding; then after a few months into production, the buyer might start to experience field failures, and realize that the item needs to be redesigned for better fitness of use. This scenario also happens for capital equipment. When the U.S. Postal Service (USPS) signed a $874 million fixed-price contract for high-tech mail sorting machine systems with one of its largest contractors, Northrop Grumman Corporation, USPS failed to provide accurate design specifications and later imposed numerous after-the-fact changes to the design requirement (*Federal Times*, 2012).

In the event of a flawed RFQ and corresponding change orders, the buyer incurs direct cost, such as the cost of delay due to redesign. In fact, in the USPS and Northrop example, the machine systems were supposed to be deployed in 47 cities but were delayed for almost 10 months, which caused the USPS extra labor costs of $394 million. Costs also are incurred by the supplier, such as the cost of adapting to the new design and the opportunity cost of extended project time. For example, Northrop claimed $179 million for adapting to all the change orders. Unfortunately, the USPS and Northrop failed to agree on the responsibility and compensation for the redesign, which led to a lawsuit of hundreds of millions of dollars (*Federal Times*, 2012).

While the lawsuit between the USPS and Northrop is still unresolved, in other cases when redesign is completely and clearly due to the buyer's fault, the buyer has to pay the supplier a redesign fee which typically should cover not only the redesign cost incurred by the supplier but also some extra profit. This is because the redesign is due to no fault of the supplier (it is the buyer who designed a flawed RFQ) and it's unfair to force the supplier to undertake additional work and switch to new production procedure for which he earns no profit. In fact, in government contracting, this type of compensation is referred to as "equitable adjustment". In a typical equitable adjustment, the supplier documents all his cost associated with the redesign and submits it to the buyer, who should then pay the redesign cost plus a profit margin to compensate the supplier for the fact that the buyer changes the original contract specifications.

When RFQ error results in high redesign expenses (in the form of direct cost incurred by the buyer and the redesign fee paid to the supplier as well), the buyer may have incentive to discover the RFQ error earlier. However, it is usually difficult for the buyer to discover the RFQ error by herself since she does not retain the in-house production expertise which can be critical in designing a perfect RFQ. For example, many automotive OEMs have reduced their engineering staff. Because production technology changes rapidly, it is difficult for those automotive OEMs to control every detailed specification and thus are prone to making an error. The buyer can choose to rehire engineering staff or hire an outside expert to vet RFQ specifications. Those approaches are expensive: We want to investigate whether it is worthwhile to pursue them and whether other ways exist to substitute buyer's lost expertise — for example, leveraging suppliers' expertise.

Suppliers can be more knowledgeable than the buyer since they are experts in production. This type of asymmetric information is present in all kinds of industries: Northrop probably knows better than USPS about how to build a machine; A construction contractor knows better than the house owner about the eventual problems before tearing down the walls. After receiving a flawed RFQ, some suppliers can recognize the error right away and thus foresee an opportunity to earn windfall profit out of the inevitable redesign. Returning to the construction example, once the house walls are opened up, there is usually little a house owner can do and the contractor gets opportunities for lucrative change orders. Therefore, even if suppliers detect the error beforehand, they have incentive to keep silent. Suppliers will bid more aggressively in order to win the business, hoping that they will recoup the profit margin later. As a senior certified public accountant *Baker* (2000) puts it "many of them [suppliers] will submit low-price bids in order to secure the job and then sit in their offices and pray for change orders to arise — that is where they make their money." One may wonder whether it is illegal for a supplier who knows the existence of error but choose not to tell. The bottom line is that it is often really difficult to prove what people knew or should have known. For example, the contractor could feign ignorance about "surprises" that are uncovered during the construction process by claiming that every house is different (which is true) and one never knows what will arise (which is maybe less true).

Not every supplier is so capable of detecting an error due to limited experience or production capability. Suppliers can have different levels of error-detecting expertise and thus follow different bidding strategies when they compete for a contract. We analyze how these dynamics affect the buyer's procurement decisions.

To summarize, we examine the problem where a buyer holds liability and pays dearly for possible RFQ error, and it is possible that some suppliers are able to detect the error ahead of time. More specifically, we investigate the following three research questions:

Research Question 1: How is the buyer's payment (including contract price and possible redesign expenses) affected by the possibility of RFQ error and suppliers' expertise in detecting the error?

Research Question 2: Can the buyer design some alternative mechanism to stem supplier windfall profit out of redesign?

Research Question 3: Can the buyer leverage suppliers' error-detecting expertise to preempt RFQ error?

Answers to these research questions are summarized in §3.7. We review related literature in §3.2, and introduce the model in §3.3. Section 3.4 analyzes a motivating example with two suppliers and supplier production cost following two-type distribution. In Section 3.5, we generalize the results to N-supplier case with general cost distribution, and propose the idea of "pre-pay" to stem supplier windfall profit out of redesign. We incorporate the redesign cost incurred by both the buyer and the supplier in Section 3.6, and propose the idea of "error-bounty" to preempt RFQ error. Proofs of results are furnished in §3.8.

3.2 Literature Review

Many papers examine various operational aspects of how procurement processes might go "wrong". For example, some suppliers are not capable of producing the required item, or even worse, they provide defective items that later require recall resulting in huge loss. Consequently, buyers need to take into account supplier quality as well as cost (*Che*, 1993; *Beil and Wein*, 2003; *Kostamis et al.*, 2009) and consider exerting efforts to qualify suppliers (*Wan and Beil*, 2009; *Wan et al.*, 2012). Suppliers also may be unreliable in their delivery schedule due to natural disasters like hurricanes or earthquakes. In this case, buyer can consider multisourcing to mitigate supply risk (*Parlar and Perry*, 1996; *Tomlin*, 2006). Unreliable suppliers themselves can also provide price and quality guarantees to better compete against more reliable suppliers (*Gümüş et al.*, 2012). Most of the existing literature investigates the potential non-performance from the supplier side assuming buyer's RFQ is perfect as given; however, our paper considers the story from the buyer side — a buyer lacking production expertise is prone to designing a flawed RFQ.

One type of imperfect RFQ is studied in the literature of incomplete contract, where the contract does not cover all contingencies and future duties are intentionally left unspecified. There exist theoretical papers (*Hart and Moore*, 1988) as well as empirical papers (*Bajari et al.*, 2006; *Crocker and Reynolds*, 1993). Incomplete contract is imperfect in the sense that some specifications are not explicitly defined since there is too much uncertainty and neither the buyer nor the suppliers can exactly foresee the future changes. However, our paper models a buyer making an error (i.e., omission, misspecification) in an RFQ, and experienced suppliers recognize the existence of the error. Suppliers' error-detecting expertise is an important factor captured in our paper. We analyze how supplier expertise affects the buyer's sourcing strategy and whether the buyer can leverage supplier expertise to make improvements in her RFQs.

Bidders' obtaining further information about the auction item is also observed in practice. For example, in U.S. timber auctions (*Athey and Levin*, 1999), after the Forest Service publicly announces the tract characteristics, each bidder can further inspect the tract and get his/her own estimate. Hence, experienced bidders can be more informed than their competitors and the auctioneer. A similar phenomenon is also observed in other natural resource auctions. *Hendricks and Porter* (1988) examine off-shore oil drilling and discover that neighbor firms are better informed than non-neighbor firms. In these forward auctions, bidders' advanced information cannot change the auction item (e.g., the characteristics of tract is determined by nature). However, in procurement auctions, suppliers' expertise can take a role in improving the RFQ. We propose a creative approach to induce suppliers to reveal their knowledge to the buyer and consequently make the entire supply chain more efficient. To our best knowledge, our paper is the first to analyze how a buyer can leverage supplier error-detecting expertise to preempt an RFQ error.

We model suppliers have different production costs as well as different levels of error-detecting expertise, which also relates our paper to the literature of multidimensional type bidders, such as *Maskin* (1992); *Pesendorfer and Swinkels* (2000); and *Jehiel and Moldovanu* (2001). As a consequence of multidimensional type, the existence of bidding equilibrium cannot be proved and none of these papers explicitly characterize the bidding functions for the general case. We tackle this problem by first considering a special case where a supplier's production cost can either be low or high, deriving the bidding functions (although complicated enough even for this two-type distribution, see full version of Lemma 3.1 in Section 3.8) and gaining some insights. Then we extend the insights to a more general case using an indirect approach that does not rely on closed-form bidding functions.

3.3 Model

We consider a buyer who wants to allocate a single, indivisible production (service) contract to one supplier. At the beginning of a procurement cycle, a buyer sends out an RFQ (which we call the *initial* RFQ) to N *ex ante* identical suppliers who will compete for the contract in an auction. Specifically, let $X_i, i = 1, \ldots, n$, be a random variable that represents supplier i's production cost (or the minimum acceptable price which includes a profit margin) to fulfill the contract as specified by the initial RFQ. We assume that X_i are independently drawn from a distribution $F_X(\cdot)$.

A distinctive feature of our model is that the buyer's initial RFQ may contain an error, which needs to be rectified through redesign and rework. This RFQ error originates from the buyer's lack of production expertise and results in omission or misspecification of critical requirements and design flaws. For example, the buyer incorrectly indicates the required material strength for a part to function, or the buyer incorrectly specifies the physical dimensions (e.g., weight, size) of a component. Let T be a random variable representing the severity of the buyer's RFQ error and $F_T(\cdot)$ be its distribution. We assume that the larger T is, the more severe (and costly) the buyer's error. Without loss of generality, $T = 0$ represents the case where the initial RFQ does not contain an error.

When the initial RFQ is error-free, the winning supplier fulfills the contract at the price determined by the auction. However, if the buyer discovers an error in the initial RFQ after a supplier partially or completely fulfills the original contract (e.g., a machine produced according to the initial RFQ fails in a field test), a considerable amount of work (which includes redesign, refurbishing, and building a new machine or part) must follow to rectify the error. As the USPS/Northrup example illustrates, redesign and rework can be costly to both buyer and supplier. The redesign is costly to the buyer because she may incur direct cost of designing a new RFQ, lose profit opportunity or incur cost associated with delay in the contract fulfillment. The supplier also incurs cost associated with a redesign, which includes direct cost of labor, material, and opportunity cost for using production resources. Let C_b and C_s be redesign costs incurred by the buyer and the supplier, respectively. We assume that both C_b and C_s are dependent on the severity of the error, T (we omit the dependence of C_b and C_s on T when it is obvious). Let $F_{b|T}$ and $F_{s|T}$ be the corresponding distributions. In addition to the cost that the supplier incurs, the buyer may have to guarantee compensation for the supplier to perform extra work to correct the error, which we call *windfall profit*. We use a random variable, R, to represent the windfall profit, and a function, $F_{R|T}(\cdot)$, to represent its distribution. We assume that both redesign costs and windfall profit stochastically increase in the error severity, T.

Suppliers are heterogeneous in their abilities to detect the RFQ error before submitting bids. In other words, some suppliers have better technical expertise than others to recognize the buyer's RFQ error. To model this, we assume that there are two types of suppliers in the marketplace — *informed* and *uninformed* suppliers: An *informed* supplier is capable of detecting the error and its severity, $T = t$; An *uninformed* supplier knows that the initial RFQ may have an error, but is unable to detect it at the time of bidding. An supplier's type is his private information and is independent from others. Let p ($\in [0, 1]$) be the probability that a supplier is the informed type. Two special cases are important in our analysis: $p = 0$ represents a situation where all suppliers are uninformed, and $p = 1$ represents a situation where all suppliers are informed.

In our base model, the buyer holds an open-bid reverse auction to award the contract. The open-bid reverse auction is a widely used practice where each supplier can observe his competitors' bids and continue to lower his own bid until either winning the auction or dropping out. Although our model uses an open-bid reverse auction, none of our results depend on the auction format. In fact, all of our results hold in other auctions such as a sealed-bid first price auction, a dutch auction, and

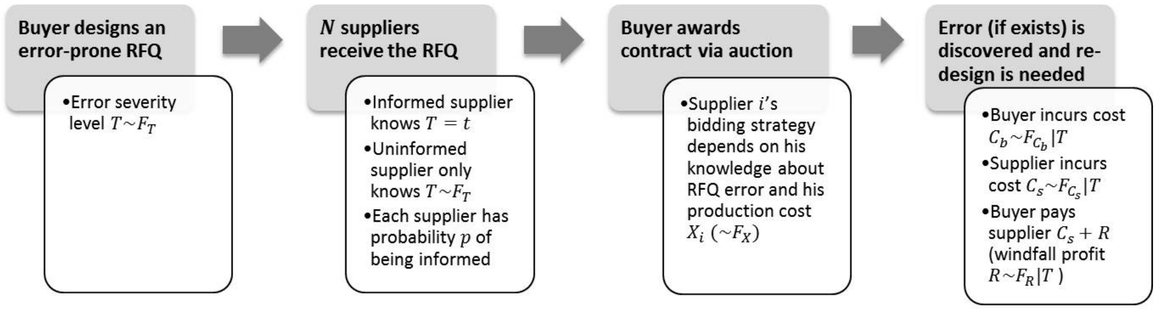


Figure 3.1: Model Schematic

so on: See Section 3.7.

Given this set-up, a supplier's bidding strategy (the drop-out price) depends not only on his production cost to fulfill the initial RFQ but also on his knowledge about the RFQ error. An informed supplier knows whether the RFQ contains an error, and if so, how severe it is. Consequently, he has better information about costs and windfall profit associated with the redesign than an uninformed buyer who cannot predict the presence and severity of the RFQ error. Due to this difference, informed suppliers and uninformed suppliers will use different bidding strategies in an auction.

The time line of our model is summarized in Figure 3.1.

To build up intuition, we first analyze how supplier's redesign profit (R) affects buyer's payment by assuming redesign is costless ($C_s = C_b = 0$). We start from a two-supplier example assuming fairly simple distributions in Section 3.4, and then consider N-supplier case with more general distributions in Section 3.5. Later in Section 3.6, we incorporate redesign cost ($C_s > 0, C_b > 0$) to investigate whether previous insights change if redesign is indeed costly.

3.4 Effect of Windfall Profit on Bidding Strategies: Case with Two Suppliers.

We first analyze how the presence of windfall profit affects the suppliers' bidding strategies and the buyer's payment. To build up intuition, we start with a simple version of the problem: Two suppliers are in the market place where each supplier's production cost to complete the initial RFQ, $X_i, i = 1, 2$, can be either low (x_L with probability s) or high (x_H with probability $1 - s$). To focus on the impact of windfall profit, we start with the model where redesign is costless for both buyer and supplier: $C_s = C_b = 0$. We also assume that error severity, T, follows a Bernoulli distribution with $P(T = 1) = q$, and the associated windfall is $R = T \times \bar{R}$ where $\bar{R}$ is a positive constant. This means that with probability q, the RFQ has an error and the contracted supplier will receive a windfall profit ($T = 1$ and $R = \bar{R}$); with probability $1-q$, the RFQ contains no error and the supplier earns no windfall profit.

In an open-bid reverse auction, the price decreases as suppliers continue to undercut each other. Each supplier's bidding strategy can be captured by his drop-out price (denoted as β), which is the lowest price the supplier is willing to accept. Consequently, the auction closes when only one supplier stays and the auction price equals to the second lowest suppliers' drop-out price.

$$\text{Auction price} = \beta_{2:N} \quad (= \beta_{2:2} \text{ when } N = 2) \tag{3.1}$$

where $\beta_{i:N}$ is the ith smallest bid from N suppliers' bids.

If the initial RFQ is flawless ($T = 0$ with probability 1), it can be shown with a standard argument (see Proposition 2.1 in *Krishna* (2009)) that a supplier i's dominant bidding strategy is to set his drop-out price to match his true production cost, X_i. However, when the RFQ may contain an error ($q > 0$), this is no longer the case. In fact, suppliers are strategic and they factor into the potential windfall associated with RFQ error when determining their bids. For instance, consider an informed supplier: When the RFQ is indeed flawed, an informed supplier sees that there will be additional profit ($\bar{R}$) after the RFQ error is discovered, and thus is willing to lower his drop-out price (as much as $\bar{R}$) to increase his chance of winning the contract; On the other hand, if there is no error, the informed supplier does not gain from further lowering his drop-out price. The situation is different for an uninformed buyer: Since an uninformed buyer cannot predict the error, he cannot adjust his bid the same way an informed supplier would. The following lemma characterizes the suppliers' bidding strategies, which can be complicated under different scenarios. The full version is relegated to Section 3.8.

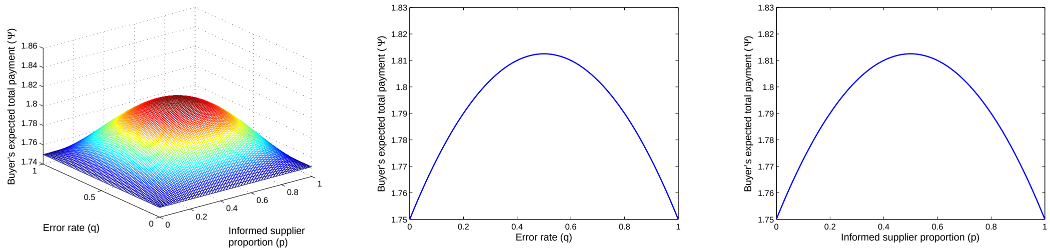
Lemma 3.1. *An informed supplier's dominant strategy is to bid down to his production cost x if the RFQ has no error, and to bid down to $x - \bar{R}$ when the RFQ has an error. For uninformed suppliers, an equilibrium exists in which an uninformed supplier with cost x will bid down to $x - E(R)$ when $\bar{R} \leq x_H - x_L$, and may deploy a mixed strategy when $\bar{R} > x_H - x_L$.*

Lemma 3.1 highlights the difference between informed and uninformed suppliers. If the RFQ indeed has an error, an informed supplier will anticipate the extra profit, $\bar{R}$, and will lower his drop-out price by the amount of windfall. If the RFQ does not have an error, he will bid as in the standard auction (where RFQ is perfect as given). On the other hand, an uninformed supplier cannot predict this additional gain with certainty, he cannot lower his bid as much as the informed supplier would. When the windfall profit is small, it is optimal for an uninformed supplier to lower the drop-out price by the expected windfall $E(R) = q\bar{R}$. However, this strategy is no longer optimal when the windfall is large. Consider the case where there is one informed supplier with high cost and one uninformed supplier with low cost in the market place. When the error exists, the informed supplier will bid down to $x_H - \bar{R}$. If the uninformed supplier only bids down to $x_L - q\bar{R}$, then he could lose the contract although he has lower cost, since when windfall size is large, informed supplier's drop-out price could be lower than that of the uninformed supplier ($x_H - \bar{R} < x_L - q\bar{R}$). Taking this possibility into account, the uninformed supplier with low cost would bid more aggressively than just lower the drop-out price by the expected amount of windfall (See full version of Lemma 3.1 in §3.8 for more details). Similarly, consider the case where there is one informed supplier with low cost and one uninformed supplier with high cost. When there is no error, the informed supplier will bid down to x_L. If the uninformed supplier bids down to $x_H - q\bar{R}$, then he could win the contract because the informed supplier's drop-out price could be higher than that of the uninformed supplier ($x_L > x_H - q\bar{R}$). However in this case, the uninformed supplier loses money since the contract price (x_L) cannot even cover his cost (x_H) and there is no future compensation. To avoid this situation, the uninformed supplier with high cost should bid more conservatively.

With suppliers' bidding strategies provided in Lemma 3.1, we can compute the buyer's expected total payment (denoted as Ψ) which includes the contract price determined by the auction and (possible) redesign windfall payment.

$$\Psi = E(\text{Auction price} + \text{Redesign windfall payment}) = E(\beta_{2:N}) + E(R). \tag{3.2}$$

To address research question 1, we first analyze how the buyer's payment Ψ changes in error severity level T. Because error is associated with windfall payment, one might expect that the buyer always pays more when the RFQ error is more severe (which is equivalent to a higher error rate q since $T \sim Bernoulli(q)$). To check whether this intuition is correct, we consider the following example: $x_L = 1, x_H = 2, s = 0.5$ and $\bar{R} = 1$. Figure 3.2(b) plots buyer's payment as a function of the error rate q when fixing informed supplier probability $p = 0.5$. Surprisingly, buyer's payment does not monotonically increase in error rate. When error rate is



(a) (b) (c)

Figure 3.2: Example with parameters $x_L = 1, x_H = 2, s = 0.5$ and $\bar{R} = 1$. Panel (a) shows how buyer's total payment change in error rate q and informed supplier probability p; Panel (b) shows the slice about how buyer's total payment changes in error rate q when fixing informed supplier probability $p = 0.5$; Panel (c) shows the slice about how buyer's total payment changes in informed supplier probability p when fixing error rate $q = 0.5$.

already high ($q > 0.5$ in this example), lowering the error rate may even result in a higher payment, because the buyer's payment includes not only redesign windfall but also contract price determined via auction. While the expected redesign windfall payment indeed increases in error rate q, auction price *decreases* since suppliers bid more aggressively if they anticipate a higher chance of gaining windfall. It is worth mentioning that the buyer's payment decreasing in error rate does not mean the buyer would intentionally design a flawed RFQ, because here we assume redesign is costless and high redesign costs (which will be introduced in Section 3.6) make a flawed RFQ unappealing.

Next we analyze how the buyer's payment Ψ changes in informed supplier probability p. Intuitively, informed suppliers can take advantage of their expertise to squeeze the buyer, so one might expect that the buyer has to pay more with higher informed supplier probability. However, this intuition is also incorrect. We refer to the previous example when $x_L = 1, x_H = 2, s = 0.5$ and $\bar{R} = 1$. Figure 3.2(c) plots buyer's payment as a function of the informed supplier probability p when fixing the error rate $q = 0.5$. We observe that the buyer's payment also does not monotonically increase in informed supplier probability. The underlying reason is suppliers' competition: Informed suppliers' advantage is not against the buyer but rather against other competing suppliers and therefore windfall profit is competed away (at least partially).

Furthermore, we observe in Figure 3.2(a) that in the extreme cases when the RFQ is perfect ($q = 0$), the RFQ is always flawed ($q = 1$), all suppliers are uninformed ($p = 0$) or all suppliers are informed ($p = 1$), the buyer has the same lowest expected total payment.

- When the RFQ is perfect, supplier with production X will bid down to $\beta(X) = X$. The buyer's expected payment is $\Psi|_{q=0} = E(X_{2:N})$.
- When the RFQ is always flawed, both types of suppliers (whether informed or uninformed) know for sure there will be additional payment $\bar{R}$, so all will lower their bids by amount $\bar{R}$, i.e., $\beta(X) = X - \bar{R}$. The contract price is thus lowered by $\bar{R}$. The buyer later pays the winning supplier additional $\bar{R}$ for redesign, so the buyer's total payment does not change. Recall Formula (3.2), the buyer's expected payment is $\Psi|_{q=1} = E[(X - \bar{R})_{2:N}] + \bar{R} = E(X_{2:N})$.
- When all suppliers are uninformed, they value the expected windfall at $E(R)$ and lower their bids by amount $E(R)$, i.e., $\beta(X) = X - E(R)$. The contract payment is lowered by $E(R)$ which cancels out the expected redesign windfall. The buyer's expected payment is $\Psi|_{p=0} = E[(X - E(R))_{2:N}] + E(R) = E(X_{2:N})$.
- When all suppliers are informed, they can correctly anticipate the error and adjust their bids accordingly, i.e., $\beta(X) = X - R$. Again the windfall profit is completely washed away. The buyer's expected payment is $\Psi|_{p=1} = E[(X - R)_{2:N}] + E(R) = E(X_{2:N})$.

We are also interested in interior cases when there is a mixture of informed and uninformed suppliers ($0 < p < 1$) and uninformed suppliers are not sure whether there is error or not ($0 < q < 1$). Given the fact that an uninformed supplier will lower his bid even when there is actually no error and the buyer does not pay windfall, one might wonder whether it is better for the buyer to be in an interior case ($0 < p < 1$ and $0 < q < 1$) rather than the extreme cases ($p = 0, p = 1, q = 0$ or $q = 1$). It is true that the buyer may end up paying less in some sample paths when there is no error and the auction price is set to some uninformed supplier's lowered bid. However, in terms of expected value, the buyer will never be better off in interior cases, since a rational uninformed supplier will not submit an overly aggressive bid due to winner's curse and thus the buyer cannot benefit by uninformed suppliers' incorrectly foreseeing the windfall profit. We summarize our findings in the following proposition.

Proposition 3.1. *When $N = 2$, $T \sim Bernoulli(q)$, and supplier production cost X follows a two-type distribution, the buyer's expected total payment Ψ is not monotonic in error rate q or informed supplier probability p. It has the minimal value $\Psi_o = E(X_{2:2})$ when $p = 0, p = 1, q = 0$ or $q = 1$.*

3.5 Analysis: N Suppliers with General Cost Distribution

We addressed research question 1 in the last section for the two-supplier case with simple distributions. In this section, we consider a more general case with N suppliers. Each supplier's production cost follows distribution F_X which can be any regular distribution[1]. Regular distribution is a standard assumption in the auction literature; examples of it include uniform, normal, logistic and exponential distributions. Error severity level T follows distribution F_T, which no longer only has two states (no error, error) but can take multiple values with higher values indicating more severe errors (e.g., no error, small error, medium error, large error). In the previous two-supplier

[1] In the context of procurement auction, a regular distribution satisfies the condition that $x + \frac{F_X(x)}{f_X(x)}$ increases in x.

example, we assumed that the windfall profit R equals to a constant $\bar{R}$ when RFQ contains error. We also generalize this assumption by allowing uncertainty even for a fixed error severity $T = t$, namely, windfall profit R follows a conditional distribution $F_{R|T=t}$. A more severe error is usually associated with higher profit, so we assume that $F_{R|T=t}$ stochastically increases in t. We continue to focus the analysis on how supplier's redesign windfall profit affects the buyer's payment by assuming redesign is costless ($C_s = C_b = 0$); this assumption will be relaxed in Section 3.6.

3.5.1 How RFQ error and supplier expertise affect buyer's payment

We address research question 1 in the general setting. The key arguments for the two-supplier example still work. Informed suppliers can foresee future windfall profit due to RFQ error, and the uninformed suppliers are aware of the possibility of a windfall profit. Suppliers are strategic; they will price in future profit attached to the contract. Furthermore, multiple suppliers competing for one contract means that suppliers might have to give up some profit to win the contract. For example, imagine one wants to remodel a house. If there is only one contractor to choose, the house owner may have to pay a high price and pay for change orders down the line. But if there are multiple competing contractors, each contractor may be willing to offer a low price quote that takes into account of getting more money in future.

In the context of competing suppliers, a lowered contract price (determined by suppliers' competitive bidding) offsets (at least partially) future windfall payment due to RFQ error. Therefore, naive intuitions about RFQ error does not hold — an RFQ with higher error severity level does not always result in a higher buyer's payment; a supply base with more informed suppliers does not always squeeze more profit from the buyer. We extend the result in Proposition 3.1 to the general setting.

Theorem 3.1. *The buyer's expected total payment Ψ is not strictly monotonic in error severity level T or informed supplier probability p. It has the minimal value*

$\Psi_o = E(X_{2:N})$ *when* $p = 0, p = 1$ *or when* T *is a degenerate random variable.*

The error severity level T is a random variable, and we are comparing two error severity levels by stochastic order. For example, one RFQ has half the chance of containing an error, $T_1 \sim Bernoulli(0.5)$, and another RFQ always contains an error, $T_2 \equiv 1$. The first RFQ is less severe than the second RFQ in terms of possibility of error ($T_1 < T_2$). When all else is equal, the buyer pays more for the first RFQ, because with the second RFQ ($T_2 \equiv 1$), all suppliers know the exact error severity level and lower their bids by the same amount, canceling out the future windfall payment. Note that we do not mean a buyer should purposely design a bad RFQ because we have not consider redesign cost, which will be incorporated in Section 3.6.

One way to prove Theorem 3.1 is to explicitly derive the suppliers' bidding functions, then compute the buyer's expected total payment using Formula (3.2), as how we handled the two-supplier example in Section 3.4. However, this direct approach is not applicable for the general case here. The suppliers in our model not only have their own different production costs, but also have different error-detecting expertise as well. As a consequence of this multidimensional bidder type, closed-form bidding equilibrium is analytically intractable. Even for the two-supplier example (see full version of Lemma 3.1 in §3.8), it is already complicated enough when $\bar{R} > x_H - x_L$, which involves mixed strategy and discrete monetary value space. There are papers considering multidimensional bidder type, such as *Maskin* (1992), *Pesendorfer and Swinkels* (2000), *Jehiel and Moldovanu* (2001). As explained in these papers, the analytical difficulty lies in the fact that unlike one-dimensional space where bidders can easily be ranked according to their valuations, there is no natural way to rank bidders in multidimensional space. None of these papers explicitly characterize the bidding equilibrium for the general case.

To overcome this difficulty, we develop a new, *indirect* approach to prove the results without deriving the suppliers' bidding functions.

Sketch of Proof

Step 1: We prove that in any extreme cases ($p = 0, p = 1$ or T is a degenerate random variable), the buyer's expected total payment equals $\Psi_o = E(X_{2:N})$.

We can prove that any informed supplier's bidding strategy is to bid down to his production cost minus the windfall profit he expects on the back end, conditioning on knowing $T = t$, i.e., $\beta(X) = X - E(R|T = t)$. When all suppliers are informed ($p = 1$), all adjust their bids by the same amount $E(R|T = t)$, so the auction price will be lowered by $E(R|T = t)$ which cancels out the expected future windfall payment $E(R|T = t)$. Recall Formula 3.2, the buyer's expected total payment is $\Psi|_{p=1} = E[(X - E(R|T))_{2:N}] + E(R) = E(X_{2:N})$.

Similarly, when all suppliers are uninformed ($p = 0$) or when RFQ only has one severity level ($T \equiv t$), all suppliers have the same knowledge about the windfall profit. Every supplier is willing to bid down to his production cost minus the expected future windfall profit, i.e., $\beta(X) = X - E(R)$. Thus, the buyer's payment is $\Psi|_{p=0} = \Psi|_{T\equiv t} = E[(X - E(R))_{2:N}] + E(R) = E(X_{2:N})$.

Step 2: We prove that any interior case ($0 < p < 1$ and T is a non-degenerate random variable) cannot compete with the extreme cases discussed in Step 1. That is, $\Psi \geq \Psi_o$.

We consider a new model: The RFQ is perfect as given; supplier type is one-dimensional with only production cost $X \sim F_X$; buyer can design any auction mechanism. In this new model, according to *Myerson* (1981), the optimal mechanism[2] results in the buyer's payment equal $E(X_{2:N})$, which is the same value as in the extreme cases of our model. Since the optimal mechanism results in lower payment than any feasible mechanism, we need only to design a feasible mechanism for this new model that has the same payment as in our model. We construct an equivalent mechanism in the new model:

[2]The optimal mechanism depends on the assumption that supplier production cost X follows a regular distribution.

(i) The buyer generates a random variable T, which follows distribution F_T;

(ii) For each supplier, the buyer flips a coin which has probability p for the head. If it is the head, let this supplier know the value of T, otherwise, only let him know the distribution F_T;

(iii) The buyer announces to all suppliers that whoever wins the auction will get an additional amount R, which follows a conditional distribution $F_{R|T}$;

(iv) Suppliers compete in an open-bid reverse auction;

(v) The buyer generates a random variable R which follows distribution $F_{R|T}$, and pays the winning supplier an additional amount R above the auction price.

The key point for creating this equivalent mechanism is: The realized values for error severity, supplier type, and redesign windfall profit, which were previously decided by nature are controlled by the buyer in the new model; however, the suppliers in the new model are given the same information as in our model and thus should follow the same bidding strategies. Therefore, the auction clears at the same price and the buyer has the same expected total payment in the equivalent mechanism as in our model.

3.5.2 Pre-paying supplier windfall

Although supplier windfall profit is fully competed away in the extreme cases ($p = 0, p = 1$ and T is degenerate), it is not always so in interior cases ($0 < p < 1$ and T is non-degenerate). We consider research question 2 and analyze if there is a way the buyer can completely remove supplier windfall profit.

To answer this question, we first analyze the extreme cases to see why windfall profit can be fully competed away. We find the extreme cases have something in common: All suppliers have the same level of anticipation regarding to the future windfall profit — either the suppliers have the same capability of detecting the error severity ($p = 0$ or $p = 1$), or the error severity has no uncertainty (T is degenerate). Based on this observation, we are wondering whether there exists a way to "mimic" the extreme cases — *equalize* the levels of anticipation for windfall profit between uninformed and informed suppliers.

Our base model is actually a contingent contract since supplier's windfall profit is paid after the error is discovered and depends on the error severity level. Because informed suppliers and uninformed suppliers have different knowledge about the error severity level, and thus have different anticipations for windfall profit, the windfall profit is not always competed away.

Next we propose an alternative contract called "pre-pay" and show how a pre-pay contract levels suppliers' anticipation for windfall profit. We define $\bar{R}$ as the maximal possible value of windfall R that a buyer can ever get for any error severity level. Before holding the auction, the buyer announces to all suppliers that the contract winner will be paid an additional payment $\bar{R}$ as compensation for potential design flaws, regardless of the RFQ error severity. We call this new contract "pre-pay" because the additional amount is paid to the auction winner immediately after the auction is closed and before any RFQ error is discovered. That is, even if it turns out that redesign is unnecessary or the error is minor and easy to fix, the supplier keeps the pre-paid amount. When redesign does occur, the supplier cannot ask for more compensation since the maximal possible profit $\bar{R}$ is already paid to cover this uncertainty. With a pre-pay contract, the winning supplier gets a guaranteed payment $\bar{R}$ that is not contingent on the presence or the size of RFQ error. Therefore, both informed and uninformed suppliers have the exact same anticipation for windfall profit — the winning supplier is guaranteed a fixed amount, $\bar{R}$.

We choose $\bar{R}$ as the pre-paid amount for two reasons. First, since $\bar{R}$ is the maximal possible amount a supplier can get from redesign, after the buyer pre-pays $\bar{R}$, the supplier has no excuse to ask for more profit later. Second, although we model the cases where the buyer decides the contract type, in practice suppliers may also participate in determining the contract type, especially if the buyer wants to switch from a traditional contingent contract to a new pre-pay contract. Since a guaranteed amount of $\bar{R}$ is obviously better than the possibility of getting profit $R(\leq \bar{R})$, even if a supplier can choose his own contract type, he will choose a pre-pay contract over a contingent contract.

The pre-pay contract may seem harmful for the buyer at first glance, because he must pay the maximal possible windfall amount regardless of the existence or severity of the RFQ error. However, this intuition is incorrect; the pre-paid amount eventually will be competed away. We support this argument by considering the supplier's bidding strategy. Since the contract comes with a guaranteed additional value $\bar{R}$, suppliers will price in this value and lower their bids by $\bar{R}$. Hence the auction price is lowered by amount $\bar{R}$ which completely cancels out the pre-paid amount. What is interesting and seemingly paradoxical about this pre-pay idea is that by committing to pay windfall profit, the buyer eventually removes the suppliers' capability of gaining any windfall. We formalize the result in the following theorem.

Theorem 3.2. *Let Ψ^{pp} be the buyer's expected total payment with the pre-pay contract. It is always lower than that with the contingent contract. For any $0 \leq p \leq 1$, any distribution F_T and $F_{R|T}$, and any regular distribution F_X, $\Psi^{pp} = E(X_{2:N}) \leq \Psi$.*

It is worth mentioning that pre-pay is effective only in the context of competing suppliers, and re-arranging the payment alone does not solve the problem. If there is only one supplier, pre-pay actually penalizes the buyer. Only when competition exists, suppliers have to give away the profit associated with redesign and this is when the buyer benefits from pre-pay.

We also want to point out that pre-pay is not meant to squeeze the supplier too hard, but rather to prevent the supplier from extracting too much windfall profit out of redesign. The supplier still receives profit from finishing the requirement on the RFQ (with production cost $X_{1:N}$, which is smaller than auction price $X_{2:N}$). As for the profit associated with redesign, the supplier is willing to compete it away in order to win the contract. Recall that even with a traditional contingent contract, when all suppliers are equally informed or uninformed, the profit associated with redesign is also competed away.

Example

Suppose a homeowner wants to remodel her kitchen and lists the requirements on the RFQ. There are two contractors whose costs of completing the work are $x_1 = \$10,000$ and $x_2 = \$12,000$. If the RFQ is perfect and there is no change order, then the contract will be given to contractor 1 with price $\max(x_1, x_2) = \$12,000$.

Suppose the buyer knows that the RFQ is not perfect and asks the contractors how much change orders might run. The contractors says the change order fee R (on top of costs, since for now we assume redesign is costless) can range from \$500 to \$4,000 depending on the severity. Of course, the contractors tend to give a wide range just to cover themselves.

The buyer can sign a pre-pay contract with the contract winner and pay him an extra \$4,000 up-front to cover all possible change orders. Of course, the contractor needs to promise to complete the necessary change orders without asking for more profit later. The contractors would accept this pre-pay contract since a guaranteed \$4,000 dominates a possible amount between \$500 and \$4,000.

The two contractors compete for the contract. Since the contract comes with an extra value of \$4,000, each contractor is willing to bid down to his true production cost minus this extra value. That is, contractor 1 bids down to $\beta_1 = x_1 - \bar{R} = \$6,000$ and contractor 2 bids down to $\beta_2 = x_2 - \bar{R} = \$8,000$, so the contract will be given to contractor 1 with price $\max(\beta_1, \beta_2) = \$8,000$. Then the buyer pays contractor 1 an additional amount \$4,000, and thus the buyer's total payment is \$12,000 which is the same payment as if there is no error in RFQ.

3.6 Incorporating Redesign Cost

So far we have discussed how supplier redesign profit affects the buyer's payment and how a pre-pay contract eliminates the supplier's redesign profit. But we have not considered the additional costs associated with redesign yet, and this is what we are going to discuss in this section.

Redesign can be costly for the supplier, because he needs to adapt the production procedure and may incur additional overhead cost, material cost, labor cost and opportunity cost by taking this extra work. Redesign can also be very costly for the buyer. For example, if the RFQ is for some part of a new product, redesign may delay the product launch date. Things get even worse if the buyer discovers the error after the new product has been launched to the market, for the buyer needs to recall the product and incurs a huge loss of goodwill. In the USPS/Northrop example, Northrop incurred the cost of \$179 million to compensate for all change orders, and USPS incurred the cost of \$394 million for losses caused by retesting and late delivery (*Federal Times*, 2012). In the house remodel example, the supplier redesign cost is the contractor's cost of paying sub-contractors, hiring a plumber expert, and extra material costs, and the buyer's redesign cost is the disruption and displacement costs of having the house wall torn up for more time. To capture these practical concerns, we incorporate the supplier's redesign cost C_s and the buyer's redesign cost C_b. The redesign costs C_s and C_b depend on the severity T, and follow conditional distributions $F_{C_s|T}$ and $F_{C_b|T}$, respectively. Since a more severe error usually is more expensive to fix, we assume $F_{C_s|T}$ and $F_{C_b|T}$ increase in T. In the event of redesign (which is consequent to the buyer making an error), the buyer reimburses the supplier's redesign cost C_s besides windfall payment R.

3.6.1 How RFQ error and supplier expertise affect buyer's payment

We consider the contingent contract where the buyer pays the supplier redesign cost and redesign windfall profit contingent on the existence and size of RFQ error. Since we incorporate redesign costs, the buyer's expected total payment (denoted as Ψ_G) should be

$$\Psi_G = E(\text{Auction price+Redesign profit+Redesign cost}) = E(\beta_{2:N})+E(R)+E(C_s+C_b).$$

From a supplier's perspective, redesign cost C_s is a contingent cost and will be reimbursed by the buyer anyway, so the amount C_s should not affect his bidding strategy. In fact, suppliers will submit the same bid as if there is no redesign cost, so the auction price is the same as in the case $C_s = 0$.

From the buyer's perspective, in the event of redesign, she has to reimburse the supplier's redesign cost C_s and also incurs redesign cost C_b to herself, so the buyer's expected total payment is increased by the amount of expected redesign cost, $E(C_s + C_b)$, compared to the case where redesign is costless (discussed in Section 3.5). That is, the buyer's total payment including redesign costs is

$$\Psi_G = \Psi + E(C_s + C_b), \tag{3.3}$$

in which Ψ is the buyer's payment assuming redesign is costless.

The property of Ψ has been analyzed in Theorem 3.1, and $E(C_s+C_b)$ increases in T since a more severe error is usually more difficult to fix. Based on this relationship, Theorem 3.1 can be extended.

Theorem. 3.1-G. *The buyer's expected total payment Ψ_G is not monotonic in p. For $p = 0$, $p = 1$ or degenerate T, $\Psi_G = E(C_s + C_b) + \Psi_o$; For any $0 < p < 1$ and non-degenerate T, $\Psi_G \geq E(C_s + C_b) + \Psi_o$, where $\Psi_o = E(X_{2:N})$.*

Based on Formula (3.3) and redesign costs being independent in informed supplier probability p, the buyer's payment is still not monotonic in p. The naive intuition that the buyer should avoid having many informed suppliers who can take advantage of their expertise is also incorrect. Actually, a supply base with all informed suppliers results in the same payment as a supply base with all uninformed suppliers, and a mixed supply base (consisting of both informed and uninformed suppliers) results in a higher payment for the buyer.

However, whether the buyer's payment is monotonic in error severity level T depends on the magnitude of redesign cost $C_s + C_b$. When $C_s + C_b$ is high, the effect of high redesign cost dominates the benefit of suppliers bidding aggressively, so the buyer's payment does increase in error severity level T. In this case, the buyer should consider improving design expertise, such as rehiring engineering staff or hiring an outside expert to vet RFQ specifications.

Again, in the extreme cases when suppliers have the same anticipate for future windfall — either all suppliers have the same error-detecting capability ($p = 0$ or $p = 1$) or the error severity has no uncertainty (T is degenerate), the redesign windfall is completely washed away through suppliers' competition.

3.6.2 Pre-paying supplier windfall

Next we analyze how to remove supplier windfall profit when redesign is costly. We assume that supplier's redesign cost is perfectly verifiable in Section 3.6.2 and 3.6.3. In practice, buyers require suppliers report cost information related to redesign. Suppliers cannot ask for an arbitrarily high amount of money, but have to justify their request. For example, this reflects what happens in government contracting. Government requires suppliers to submit equitable adjustment documentation stating all costs associated with redesign; any misrepresentation in the actual costs is illegal.

When redesign cost is verifiable, we propose a slightly altered pre-pay which can remove supplier windfall completely: The buyer pre-pays the maximal amount of windfall $\bar{R}$ but keeps supplier redesign cost C_s as a contingent payment. That is, the contracted supplier is guaranteed to receive the maximal amount windfall $\bar{R}$, but the cost is reimbursed later to the actual amount associated with redesign. The contingent payment C_s does not affect suppliers' bidding strategy since it is reimbursed by the exact amount anyway. As for the pre-paid amount $\bar{R}$, similar to the earlier model, all suppliers will lower bid by the amount $\bar{R}$, lowering the auction price and canceling out the pre-paid amount.

This altered pre-pay is as effective as the pre-pay strategy introduced in Section 3.5.2 in removing supplier redesign windfall profit, but the buyer still has to pay redesign cost. If we use Ψ_G^{pp} to denote buyer's expected total payment incorporating redesign cost (recall that Ψ^{pp} is buyer's payment with pre-pay when redesign is costless), we have the following relationship:

$$\Psi_G^{pp} = \Psi^{pp} + E(C_s + C_b).$$

Therefore, Theorem 3.2 can be extended.

Theorem. 3.2-G. *When the buyer pre-pays $\bar{R}$ and keeps C_s as a contingent payment, the buyer's expected total payment is always lower than that with the contingent contract. Actually, $\Psi_G^{pp} = E(C_s + C_b) + \Psi_o \leq \Psi_G$.*

3.6.3 Extract knowledge from informed suppliers

While the buyer can remove windfall profit completely utilizing a pre-pay contract, she still has to pay redesign costs. Therefore, the buyer has an incentive to discover the error earlier and avoid redesign, especially when redesign is expensive (e.g. recalling a flawed product hurts the brand image). Because the buyer no longer retains production expertise, it is difficult for her to design a perfect RFQ or detect

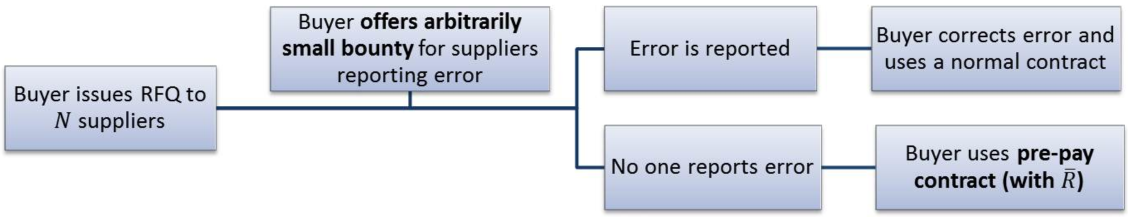


Figure 3.3: Error-bounty approach when the supplier's redesign cost is verifiable

the error in a flawed RFQ. She may rehire engineering staff or hire outside expert to vet RFQ specifications, but those options are expensive. However, the informed suppliers are experts in detecting RFQ error. Now we address research question 3: Can the buyer leverage suppliers' error-detecting expertise to preempt RFQ error?

Recall that with a pre-pay contract, suppliers compete away the redesign windfall profit. Since any informed supplier cannot make a profit from RFQ error, he has no incentive to hide the information. Therefore, the buyer can offer a nominal "error-bounty" to induce an informed supplier to report the RFQ error essentially for free. Figure 3.3 shows the steps to implement the error-bounty approach with a pre-pay contract.

If a supplier reports the existence of RFQ error, the buyer can correct it and reissue a new perfect RFQ. In this case, the buyer uses a normal contract and the contract price is determined by the suppliers' competition. When no one reports the error and the RFQ is still possibly flawed, the buyer uses a pre-pay contract. One may wonder why a supplier would report the error for a small bounty that eliminates the guaranteed pre-paid amount $\bar{R}$. The reason is with a pre-pay contract, the supplier does not actually receive any profit due to competition. The bounty, however small, is actual money that a supplier can receive. Therefore, an informed supplier foregoes the pre-pay amount (it does not translate into money in his pocket) but chooses to report the error to get the bounty (which is money in his pocket).

However, when all suppliers are uninformed (with probability $(1-p)^N$), no supplier can detect the error and the buyer has to pay the inevitable redesign costs $(C_s + C_b)$ when RFQ contains an error. We formalize our finding in the following theorem.

Theorem 3.3. *In the context of a pre-pay contract, the buyer can pick an arbitrarily small bounty, offer it to the first supplier who shows that the RFQ is flawed, and correct the RFQ before holding an auction. Then the buyer's expected total payment will be* $\hat{\Psi}_G^{pp} = (1-p)^N E(C_s + C_b) + \Psi_o \leq \Psi_G^{pp}$.

Theoretically any arbitrarily small bounty would be enough to induce an informed supplier to report RFQ error. In reality, the bounty should not be just one penny but need to be large enough to act as an incentive. Nevertheless, the bounty is a negligible amount compared to the size of the entire contract.

By offering an error-bounty with a pre-pay contract, the buyer can leverage suppliers' superior knowledge and at the same time avoid paying windfall profit or reimbursing redesign costs. Actually, this creates a win-win solution for both parties: The supplier can receive a bounty and the buyer can avoid the pain of redesign. Consequently, the entire supply chain becomes more efficient.

3.6.4 Supplier's redesign cost is not perfectly verifiable

Our analysis in Section 3.6.2 and 3.6.3 is based on the assumption that supplier redesign cost is perfectly verifiable. However, sometimes the cost is fuzzy and suppliers might potentially lie about the redesign cost. For example, suppose redesign requires extra labor time from the supplier, he may report 200 hours while it actually costs him 190 hours; the labor cost of 10 hours is the profit that a supplier gets from lying about the actual redesign cost. To capture this, we still let C_s be the actual redesign cost, but write $R = R_l + R_a$ where R_l is the profit from lying and R_a is the mutually agreed profit when redesign happens.

This new structure of redesign profit does not change our answer to research question 1 about how the RFQ error and informed supplier probability affect the

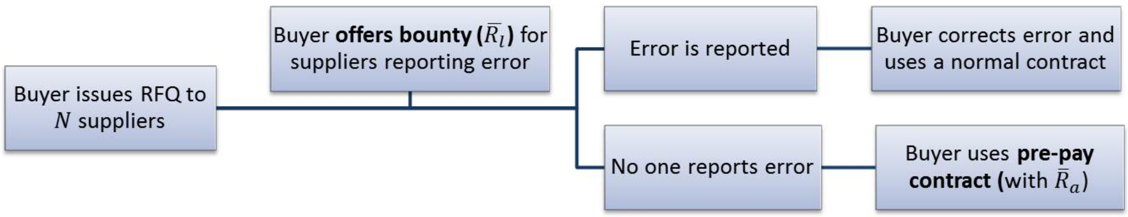


Figure 3.4: Error-bounty approach when the supplier's redesign cost is not verifiable

buyer's payment, i.e., Theorem 3.1-G still holds, because the suppliers still price in windfall profit R and bid aggressively to win the contract.

As for research question 2 (how to remove supplier windfall profit from RFQ error), the buyer can remove the agreed windfall payment (R_a) by pre-paying the maximal amount of agreed windfall $\bar{R}_a$. All suppliers lower their bids by $\bar{R}_a$ and the auction price is lowered which cancels out the pre-paid amount completely. However, supplier's profit from lying (R_l) cannot be avoided through pre-pay, because even if the buyer pre-pays the maximal profit a supplier can get from lying, the supplier can still "lie" about his true redesign cost.

To address this issue and answer research question 3 (how to leverage supplier expertise to preempt RFQ error), we use an altered error-bounty approach: The buyer uses the maximum amount of R_l (denoted as $\bar{R}_l$) as a bounty for suppliers to report RFQ error. With this incentive, any informed supplier prefers reporting the error (receiving bounty $\bar{R}_l$) to hiding the error (getting a profit less than R_l since part of R_l will be competed away). The size of the bounty ($\bar{R}_l$) depends on how much a supplier can lie about his redesign cost, which is usually small relative to the actual redesign cost. Hence, by using pre-pay and error-bounty, the buyer avoids paying redesign costs ($C_s + C_b$), avoids paying windfall profit ($R_a + R_l$), and only has to pay a relatively small amount that cannot be ex post verified ($\bar{R}_l$). Figure 3.4 shows the steps to implement the error-bounty approach with pre-pay contract when redesign cost is not verifiable.

3.7 Conclusions

While most procurement research assumes that a buyer's RFQ is perfect as given, in reality the RFQ can be flawed due to errors of omission and misspecification. This makes the buyer vulnerable to expensive future change orders during the life of the contract. However, experienced suppliers may be more knowledgeable than the buyer and possess the knowledge to recognize errors and anticipate windfall profit at the time of bidding. We model the case where the buyer is prone to making an error in the initial RFQ (accompanied by future redesign and supplier windfall), which in turn may or may not be detected by each supplier depending on each supplier's expertise.

To address research question 1, we first analyze how a buyer's payment changes with the severity of the RFQ error. Although intuitively one might expect that a more severe error would always increase the buyer's payment because the buyer has to pay a higher redesign payment, we find this intuition is not always correct. The reason behind this is that while the redesign payment indeed increases, the auction price decreases because suppliers compete more aggressively to secure the contract if they anticipate higher future windfall profit.

We then analyze how the buyer's payment changes with the probability of informed suppliers who can detect the RFQ error. One might expect that a higher informed supplier probability hurts the buyer since it means there are more informed suppliers who can take advantage of their expertise to squeeze the buyer. Again, this intuition is also incorrect. A supply base with all informed suppliers will result in the same payment as that of a supply base with all uninformed suppliers, because, while informed suppliers are more knowledgeable, they still need to compete against each other and their information advantage is against other suppliers rather than against the buyer. The managerial takeaway is that supplier disparity does not benefit the buyer. When the buyer designs a supply base, she should choose either all informed suppliers or all uninformed suppliers. In a mixed supply base consisting of both informed and uninformed suppliers, windfall is not washed away completely.

Addressing research question 2 (how to remove supplier redesign windfall profit), we propose a creative approach called "pre-pay". By pre-paying the maximum possible windfall, the buyer can stem any windfall payment. It benefits the buyer by equalizing suppliers' anticipation for redesign windfall, since all suppliers will lower their bids by the exact pre-paid amount and the lowered auction price cancels out the pre-paid amount.

Furthermore, we address research question 3 (how to leverage supplier expertise to preempt RFQ error). Since pre-pay removes the possibility for suppliers to benefit from redesign, suppliers have no incentive to hide information about the error in the initial RFQ. By offering a bounty, the buyer can induce the informed suppliers to reveal their information. This creates a win-win situation for both the buyer and the suppliers, since both will be better off by avoiding redesign. Through the approach of pre-pay and error-bounty, the buyer creates a reciprocal environment where the buyer and suppliers have incentive to communicate and share expertise. Consequently, the entire supply chain becomes more efficient.

Our model allows the redesign cost and windfall profit to be correlated since they both depend on error severity level. In addition, our results also go through if the redesign cost and supplier production cost are positively correlated. For example, a supplier with low production cost tends to have low redesign cost. Suppose supplier i's redesign cost is $C_s(X_i, \theta_i)$ where θ_i is a commonly distributed random term, and suppose $x_i \leq x_j$ implies that $E[C_s(x_i, \theta_i)] \leq E[C_s(x_j, \theta_j)]$. This assumption assures that, if the buyer awards the contract to the supplier with lowest production cost, this supplier is also the one with lowest expected redesign cost. This allows us to conclude that regardless of the existence of the RFQ error, the buyer's decision for contract allocation will not change (but her total payment would be inflated by the redesign cost). We extend Theorem 3.1-G, 3.2-G and 3.3 by replacing C_s with $C_s(X_{1:N}, \theta)$.

We consider open-bid reverse auction in our model because it is commonly used in practice. However, all our results can directly apply to other auction mechanisms such as first-price sealed-bid auction, Vickrey auction, Dutch auction, or optimal mechanism. This is because when suppliers have the same cost distribution, all those auctions are equivalent resulting in the same expected buyer payment (see Proposition 3.1 in *Krishna* (2009)). Therefore, the indirect approach we use to prove Theorem 3.1 goes through even with other auction formats and other theorems also hold accordingly.

To our best knowledge, this paper is the first in operations to consider the practical case that the buyer's RFQ can be misspecified, as well as the fact that experienced suppliers can be more knowledgeable than the buyer, and the buyer can leverage suppliers' knowledge. Analyzing the problem is challenging, and we use a problem transformation method applying optimal mechanism design to prove results about common auction formats. This paper has potential to help procurement managers make decisions about improving expertise in RFQ and designing the supply base. This paper also provides an alternative strategy to make the entire supply chain more efficient. We hope it can spur more research into this important yet under-studied area about error-prone RFQs.

3.8 Proofs

Proof of Lemma 3.1

Lemma. 3.1. *An informed supplier with production cost $X = x$ has a dominant bidding strategy: biding down to $x - \bar{R}$ when RFQ is flawed and biding down to x otherwise.*

When $p = 0, q = 0$ or $q = 1$, an uninformed supplier with production cost $X = x$ will bid down to $x - q\bar{R}$; when $0 < p < 1, 0 < q < 1$, an uninformed supplier has a *symmetric equilibrium: Set* $\tilde{R} = \frac{\bar{R}}{x_H - x_L}$*;*

1. *If* $X = x_L$*, then*

   (a) *when* $\tilde{R} < \frac{1}{1-q}$*, bid down to* $\beta_L = x_L - q\bar{R}$*;*

   (b) *when* $\frac{1}{1-q} \leq \tilde{R} < \frac{(1-s)pq}{s(1-p)(1-q)} + \frac{1}{1-q}$*, bid down to* $\beta_L = x_H - \bar{R} - \epsilon$[3]*;*

   (c) *when* $\tilde{R} \geq \frac{(1-s)pq}{s(1-p)(1-q)} + \frac{1}{1-q}$*, follow a mixed strategy: Set* $\alpha_L = \frac{(1-s)pq}{s(1-p)((1-q)\tilde{R}-1)}$*, bid down to* $\beta_L = x_H - \bar{R}$ *with probability* α_L *and bid down to* $\beta_L = x_L - q\bar{R}$ *with probability* $1 - \alpha_L$*.*

2. *If* $X = x_H$*, then*

   (a) *when* $\tilde{R} < \frac{1}{q}$*, bid down to* $\beta_H = x_H - q\bar{R}$*;*

   (b) *when* $\frac{1}{q} \leq \tilde{R} < \frac{sp(1-q)}{(1-s)(1-p)q} + \frac{1}{q}$*, bid down to* $\beta_H = x_L + \epsilon'$*;*

   (c) *when* $\tilde{R} \geq \frac{sp(1-q)}{(1-s)(1-p)q} + \frac{1}{q}$*, follow a mixed strategy: Set* $\alpha_H = \frac{sp(1-q)}{(1-s)(1-p)(q\tilde{R}-1)}$*, bid down to* $\beta_H = x_L$ *with probability* α_H *and bid down to* $\beta_H = x_H - q\bar{R}$ *with probability* $1 - \alpha_H$*.*

Proof: Suppose supplier 2 follows the strategy described in Lemma 3.1. We will argue that it is optimal for supplier 1 to also follow the strategy described in the lemma.

We can easily verify that $\beta_L \leq x_L - q\bar{R} < x_H - q\bar{R} \leq \beta_H$.

1. When supplier 1 is uninformed and with low cost $X_1 = x_L$, the expected

[3]Here we consider possible bids in discrete space. Assume in 1(b), $x_H - R - \epsilon$ is the closest bid lower than $x_H - R$; in 2(b), $x_L + \epsilon'$ is the closest bid higher than x_L. Otherwise, there is no equilibrium in continuous space either for the case 1(b) or 2(b).

payment for supplier 1 to bid down to b is

$$
\begin{aligned}
\Pi_L(b) =& pE_{X_2}[q(X_2 - \bar{R} - x_L + \bar{R})(I_{b<X_2-\bar{R}} + 0.5I_{b=X_2-\bar{R}}) \\
& + (1-q)(X_2 - x_L)(I_{b<X_2} + 0.5I_{b=X_2})] \\
& + (1-p)sE_{\beta_L}[(\beta_L - x_L + q\bar{R})(I_{b<\beta_L} + 0.5I_{b=\beta_L})] \\
& + (1-p)(1-s)E_{\beta_H}[(\beta_H - x_L + q\bar{R})(I_{b<\beta_H} + 0.5I_{b=\beta_H})] \\
=& p(1-s)[q(x_H - x_L)(I_{b<x_H-R} + 0.5I_{b=x_H-\bar{R}}) \\
& + (1-q)(x_H - x_L)(I_{b<x_H} + 0.5I_{b=x_H})] \\
& + (1-p)sE_{\beta_L}[(\beta_L - x_L + q\bar{R})(I_{b<\beta_L} + 0.5I_{b=\beta_L})] \\
& + (1-p)(1-s)E_{\beta_H}[(\beta_H - x_L + q\bar{R})(I_{b<\beta_H} + 0.5I_{b=\beta_H})] \\
=& (1-s)p(1-q)(x_H - x_L)(I_{b<x_H} + 0.5I_{b=x_H}) \\
& + (1-s)(1-p)E_{\beta_H}[(\beta_H - x_L + q\bar{R})(I_{b<\beta_H} + 0.5I_{b=\beta_H})] \\
& + (1-s)pq(x_H - x_L)(I_{b<x_H-R} + 0.5I_{b=x_H-\bar{R}}) \\
& + s(1-p)E_{\beta_L}[(\beta_L - x_L + q\bar{R})(I_{b<\beta_L} + 0.5I_{b=\beta_L})] \\
=& \Upsilon_1(b) + \Upsilon_2(b) + \Upsilon_3(b)
\end{aligned}
$$

Here, we define $\Upsilon_1(b) = (1-s)p(1-q)(x_H - x_L)(I_{b<x_H} + 0.5I_{b=x_H}) + (1-s)(1-p)E_{\beta_H}[(\beta_H - x_L + q\bar{R})(I_{b<\beta_H} + 0.5I_{b=\beta_H})]$. Since $\beta_H > x_L - qR$, any $b < \min(x_H, \beta_H)$ maximize $\Upsilon_1(b)$. And $\Upsilon_2(b) = (1-s)pq(x_H - x_L)(I_{b<x_H-\bar{R}} + 0.5I_{b=x_H-\bar{R}})$ is maximized by any $b < x_H - \bar{R}$, $\Upsilon_3(b) = s(1-p)E_{\beta_L}[(\beta_L - x_L + q\bar{R})(I_{b<\beta_L} + 0.5I_{b=\beta_L})]$ is maximized by $b = x_L - q\bar{R}$.

(a) When $\tilde{R} < \frac{1}{1-q}$, i.e, $x_L - q\bar{R} < x_H - \bar{R}$, $b = x_L - q\bar{R}$ maximizes $\Upsilon_1(b), \Upsilon_2(b), \Upsilon_3(b)$ and thus maximizes $\Pi_L(b) = \Upsilon_1(b) + \Upsilon_2(b) + \Upsilon_3(b)$.

(b) Set $A = (1-s)pq(x_H - x_L), B(t) = s(1-p)(x_L - q\bar{R} - t)$. $B(t)$ is strictly decreasing in t. When $\frac{1}{1-q} \leq \tilde{R} < \frac{(1-s)pq}{s(1-p)(1-q)} + \frac{1}{1-q}$, we have $x_H - \bar{R} \leq x_L - q\bar{R}, B(x_H - \bar{R}) \geq 0, A > B(x_H - \bar{R})$. Since $\beta_L = x_H - \bar{R} - \epsilon$, when ϵ is small enough, $A > B(\beta_L) > 0$.

$$\Upsilon_2(b) + \Upsilon_3(b) = \begin{cases} A - B(\beta_L) & \text{if } b < \beta_L \\ A - 0.5B(\beta_L) & \text{if } b = \beta_L \\ 0.5A & \text{if } b = x_H - \bar{R} \\ 0 & \text{if } b > x_H - \bar{R} \end{cases}$$

Since β_L is the highest possible bid lower than $x_H - \bar{R}$, the above equation has considered all possible bids. We can see that $b = \beta_L$ maximize $\Upsilon_2(b) + \Upsilon_3(b)$, and it also maximize $\Upsilon_1(b)$ since $\beta_L < x_H - \bar{R} \leq x_H - q\bar{R} \leq \min(x_H, \beta_H)$. Thus $b = \beta_L$ maximizes $\Pi_L(b)$.

(c) When $\tilde{R} \geq \frac{(1-s)pq}{s(1-p)(1-q)} + \frac{1}{1-q}$, we have $x_H - \bar{R} \leq x_L - q\bar{R}, B(x_H - \bar{R}) \geq 0, A \leq B(x_H - \bar{R})$. As described in the lemma, $\alpha_L = \frac{A}{B(x_H - \bar{R})}$.

$$\Upsilon_2(b) + \Upsilon_3(b) = \begin{cases} A - \alpha_L B(x_H - \bar{R}) = 0 & \text{if } b < x_H - \bar{R} \\ 0.5A - 0.5\alpha_L B(x_H - \bar{R}) = 0 & \text{if } b = x_H - \bar{R} \\ 0 & \text{if } b > x_H - \bar{R} \end{cases}$$

For any b, $\Upsilon_2(b) + \Upsilon_3(b) = 0$. Since $x_H - \bar{R} \leq x_L - q\bar{R} < \min(x_H, \beta_H)$, the mixed strategy described in 1(c) also maximize $\Upsilon_1(b)$. Thus β_L is the optimal bidding strategy.

2. Similarly, when supplier 1 is uninformed and with high cost $X_1 = x_H$, the

expected payment for supplier 1 to bid down to b is

$$\begin{aligned}\Pi_H(b) =& pE_{X_2}[q(X_2 - \bar{R} - x_H + \bar{R})(I_{b<X_2-\bar{R}} + 0.5I_{b=X_2-\bar{R}})\\ &+ (1-q)(X_2 - x_H)(I_{b<X_2} + 0.5I_{b=X_2})]\\ &+ (1-p)sE_{\beta_L}[(\beta_L - x_H + q\bar{R})(I_{b<\beta_L} + 0.5I_{b=\beta_L})]\\ &+ (1-p)(1-s)E_{\beta_H}[(\beta_H - x_H + q\bar{R})(I_{b<\beta_H} + 0.5I_{b=\beta_H})]\\ =& pr[-q(x_H - x_L)(I_{b<x_L-\bar{R}} + 0.5I_{b=x_L-\bar{R}})\\ &- (1-q)(x_H - x_L)(I_{b<x_L} + 0.5I_{b=x_L})]\\ &+ (1-p)sE_{\beta_L}[(\beta_L - x_H + q\bar{R})(I_{b<\beta_L} + 0.5I_{b=\beta_L})]\\ &+ (1-p)(1-s)E_{\beta_H}[(\beta_H - x_H + q\bar{R})(I_{b<\beta_H} + 0.5I_{b=\beta_H})]\\ =& -spq(x_H - x_L)(I_{b<x_L-\bar{R}} + 0.5I_{b=x_L-\bar{R}})\\ &+ s(1-p)E_{\beta_L}[(\beta_L - x_H + q\bar{R})(I_{b<\beta_L} + 0.5I_{b=\beta_L})\\ &- sp(1-q)(x_H - x_L)(I_{b<x_L} + 0.5I_{b=x_L})\\ &+ (1-s)(1-p)E_{\beta_H}[(\beta_H - x_H + q\bar{R})(I_{b<\beta_H} + 0.5I_{b=\beta_H})]\\ =& \Gamma_1(b) + \Gamma_2(b) + \Gamma_3(b)\end{aligned}$$

Here, we define $\Gamma_1(b) = -spq(x_H - x_L)(I_{b<x_L-\bar{R}} + 0.5I_{b=x_L-\bar{R}}) + s(1-p)E_{\beta_L}[(\beta_L - x_H + q\bar{R})(I_{b<\beta_L} + 0.5I_{b=\beta_L})$. Since $\beta_L < x_H - q\bar{R}$, any $b > \max(x_L - \bar{R}, \beta_L)$ maximize $\Gamma_1(b)$. And $\Gamma_2(b) = -sp(1-q)(x_H - x_L)(I_{b<x_L} + 0.5I_{b=x_L})$ is maximized by any $b > x_L$, $\Gamma_3(b) = (1-s)(1-p)E_{\beta_H}[(\beta_H - x_H + q\bar{R})(I_{b<\beta_H} + 0.5I_{b=\beta_H})]$ is maximized by $b = x_H - q\bar{R}$.

(a) When $\tilde{R} < \frac{1}{q}$, i.e, $x_H - q\bar{R} > x_L$, $b = x_H - q\bar{R}$ maximizes $\Gamma_1(b), \Gamma_2(b), \Gamma_3(b)$ and thus maximizes $\Pi_H(b) = \Gamma_1(b) + \Gamma_2(b) + \Gamma_3(b)$.

(b) Set $C = sp(1-q)(x_H - x_L), D(t) = (1-s)(1-p)(t - (x_H - q\bar{R}))$. $D(t)$ is strictly increasing in t. When $\frac{1}{q} \le \tilde{R} < \frac{sp(1-q)}{(1-s)(1-p)q} + \frac{1}{q}$, we have $x_H - q\bar{R} \le x_L, D(x_L) \ge 0, C > D(x_L)$. Since $\beta_H = x_L + \epsilon'$, when ϵ' is small enough, $C > D(\beta_H) > 0$.

$$\Gamma_2(b)+\Gamma_3(b)=\begin{cases} D(\beta_H)-C & \text{if } b<x_L \\ D(\beta_H)-0.5C & \text{if } b=x_L \\ 0.5D(\beta_H) & \text{if } b=\beta_H \\ 0 & \text{if } b>\beta_H \end{cases}$$

Since β_H is the lowest possible bid higher than x_L, the above equation has considered all possible bids. We can see that $b=\beta_H$ maximize $\Upsilon_2(b)+\Upsilon_3(b)$, and it also maximize $\Gamma_1(b)$ since $\beta_H > x_L \geq \max(x_L-\bar{R},\beta_L)$. Thus $b=\beta_H$ maximizes $\Pi_H(b)$.

(c) When $\tilde{R} \geq \frac{sp(1-q)}{(1-s)(1-p)q}+\frac{1}{q}$, we have $x_H-q\bar{R}\leq x_L, D(x_L)\geq 0, C\leq D(x_L)$. As described in the lemma, $\alpha_H=\frac{C}{D(x_L)}$.

$$\Gamma_2(b)+\Gamma_3(b)=\begin{cases} -C+\alpha_H D(x_L)=0 & \text{if } b<x_L \\ -0.5C+0.5\alpha_H D(x_L)=0 & \text{if } b=x_L \\ 0 & \text{if } b>x_L \end{cases}$$

For any b, $\Gamma_2(b)+\Gamma_3(b)=0$. Since $x_L\geq x_H-q\bar{R}>\max(x_L-\bar{R},\beta_L)$, the mixed strategy described in 1(c) also maximize $\Gamma_1(b)$. Thus β_H is the optimal bidding strategy.

Proof of Proposition 3.1

It is easy to verify that when $q=0$ or $q=1$ or $p=0$ or $p=1$, the buyer's expected total payment equals to $\Psi_0=E[\max(X_1,X_2)]$. Next, we will prove that for any fixed p $(0<p<1)$, the buyer's expected total payment $\Psi(q)$ satisfies $\Psi(q)\geq\Psi_o$. For simplicity, we let $x_L=0, x_H=1$ and hence $\bar{R}=\tilde{R}$. This will not affect our analysis, because if we view the expected payment as a function $\Phi(x_L,x_H)$, then $\Phi(x_L,x_H)=x_L+(x_H-x_L)\Phi(0,1)$.

Suppose an uninformed supplier with production cost X will bid $\beta(X)$. We can write the buyer's expected total payment as

$$\begin{aligned}\Psi(q) =& p^2 E[\max(X_1, X_2)] + (1-p)^2 (E[\max(\beta(X_1), \beta(X_2))] + q\bar{R}) \\ & + 2p(1-p)(q(E[\max(X_1 - \bar{R}, \beta(X_2))] + \bar{R}) + (1-q)E[\max(X_1, \beta(X_2))]) \\ =& p^2 \Psi_0 + (1-p^2)q\bar{R} + (1-p)^2 E[\max(\beta(X_1), \beta(X_2))] \\ & + 2p(1-p)(qE[\max(X_1 - \bar{R}, \beta(X_2))] + (1-q)E[\max(X_1, \beta(X_2))])\end{aligned}$$

For any $0 < p < 1, 0 < q < 1$, according to the bidding strategy described in Lemma 3.1 [4], we have:

- When $X_1 < X_2$, $\max(\beta(X_1), \beta(X_2)) = \beta(X_2)$, so

  $$\begin{aligned}E[\max(\beta(X_1), \beta(X_2))] =& 2s(1-s)E(\beta_H) + s^2 E[\max(\beta_L, \beta_L')] \\ & + (1-s)^2 E[\max(\beta_H, \beta_H')];\end{aligned}$$

- When $X_1 \leq X_2$, $\max(X_1 - \bar{R}, \beta(X_2)) = \beta(X_2)$, so

  $$E[\max(X_1 - \bar{R}, \beta(X_2))] = (1-s)E(\beta_H) + s^2 E(\beta_L) + s(1-s)E[\max(1 - \bar{R}, \beta_L)];$$

- When $X_1 \geq X_2$, $\max(X_1, \beta(X_2)) = X_1$, so

  $$E[\max(X_1, \beta(X_2))] = (1-s) + s(1-s)E[\max(0, \beta_H)].$$

[4] In cases 1(b) and 2(b), we use $\beta_L = x_H - \bar{R}$ and $\beta_H = x_L$ instead. When ϵ and ϵ' are sufficiently small, $\Psi(p, q)$ calculated below will be accurate enough to show the buyer's expected payment.

Hence, we can rewrite the buyer's expected total payment

$$
\begin{aligned}
\Psi(q) =& p^2\Psi_0 + (1-p^2)q\bar{R} \\
&+ (1-p)^2(2s(1-s)E(\beta_H) + s^2E[\max(\beta_L, \beta'_L)] + (1-s)^2E[\max(\beta_H, \beta'_H)]) \\
&+ 2p(1-p)q((1-s)E(\beta_H) + s^2E(\beta_L) + s(1-s)E[\max(1-\bar{R}, \beta_L)]) \\
&+ 2p(1-p)(1-q)((1-s) + s(1-s)E[\max(0, \beta_H)]) \\
=& p^2\Psi_0 + (1-p^2)q\bar{R} + 2(1-s)p(1-p)(1-q) \\
&+ 2s^2p(1-p)qE(\beta_L) + 2s(1-s)p(1-p)qE[\max(1-\bar{R}, \beta_L)]) \\
&+ s^2(1-p)^2E[\max(\beta_L, \beta'_L)] + 2(1-s)(1-p)(s(1-p) + pq)E(\beta_H) \\
&+ 2s(1-s)p(1-p)(1-q)E[\max(0, \beta_H)] + (1-s)^2(1-p)^2E[\max(\beta_H, \beta'_H)] \\
=& C_o(p,q) + C_L(p,q) + C_H(p,q)
\end{aligned}
$$

Here,

$$
\begin{aligned}
C_o(p,q) =& p^2\Psi_0 + (1-p^2)q\bar{R} + 2(1-s)p(1-p)(1-q) \\
C_L(p,q) =& 2s^2p(1-p)qE(\beta_L) + 2s(1-s)p(1-p)qE[\max(1-\bar{R}, \beta_L)] \\
&+ s^2(1-p)^2E[\max(\beta_L, \beta'_L)] \\
C_H(p,q) =& 2(1-s)(1-p)(s(1-p) + pq)E(\beta_H) \\
&+ 2s(1-s)p(1-p)(1-q)E[\max(0, \beta_H)] \\
&+ (1-s)^2(1-p)^2E[\max(\beta_H, \beta'_H)].
\end{aligned}
$$

We now analyze C_L. Since C_L is continuous in q and differentiable in q almost everywhere (actually, C_L has at most two non-differentiable points), we consider the first order and second order derivatives in the differentiable points.

$$\begin{aligned}\frac{\partial C_L(p,q)}{\partial q} =& 2s^2p(1-p)E(\beta_L) + 2s(1-s)p(1-p)E[\max(1-\bar{R},\beta_L)] \\ &+ s^2(1-p)^2\frac{\partial E[\max(\beta_L,\beta_L')]}{\partial q} \\ &+ 2s^2p(1-p)q\frac{\partial E(\beta_L)}{\partial q} + 2s(1-s)p(1-p)q\frac{\partial E[\max(1-\bar{R},\beta_L)]}{\partial q}\end{aligned}$$

$$\begin{aligned}\frac{\partial C_L^2(p,q)}{\partial q^2} =& 4s^2p(1-p)\frac{\partial E(\beta_L)}{\partial q} + 4s(1-s)p(1-p)\frac{\partial E[\max(1-\bar{R},\beta_L)]}{\partial q} \\ &+ s^2(1-p)^2\frac{\partial^2 E[\max(\beta_L,\beta_L')]}{\partial q^2} \\ &+ 2s^2p(1-p)q\frac{\partial^2 E(\beta_L)}{\partial q^2} + 2s(1-s)p(1-p)q\frac{\partial^2 E[\max(1-\bar{R},\beta_L)]}{\partial q^2}\end{aligned}$$

We define $\Omega_L^a = \{q : \bar{R} < \frac{1}{1-q}\}, \Omega_L^b = \{q : \frac{1}{1-q} \leq \bar{R} < \frac{(1-s)pq}{s(1-p)(1-q)} + \frac{1}{1-q}\}, \Omega_L^c = \{q : \bar{R} \geq \frac{(1-s)pq}{s(1-p)(1-q)} + \frac{1}{1-q}\}$. Then we have

$$E(\beta_L) = \begin{cases} -q\bar{R} & \Omega_L^a \\ 1-\bar{R} & \Omega_L^b \\ \alpha_L(1-\bar{R}) + (1-\alpha_L)(-q\bar{R}) & \Omega_L^c \end{cases}$$

$$E[\max(1-\bar{R},\beta_L)] = \begin{cases} 1-\bar{R} & \Omega_L^a \\ 1-\bar{R} & \Omega_L^b \\ \alpha_L(1-\bar{R}) + (1-\alpha_L)(-q\bar{R}) & \Omega_L^c \end{cases}$$

$$E[\max(\beta_L,\beta_L')] = \begin{cases} -q\bar{R} & \Omega_L^a \\ 1-\bar{R} & \Omega_L^b \\ \alpha_L^2(1-\bar{R}) + (1-\alpha_L^2)(-q\bar{R}) & \Omega_L^c \end{cases}$$

Take first order derivative:

$$\frac{\partial E(\beta_L)}{\partial q} = \begin{cases} -\bar{R} & \Omega_L^a \\ 0 & \Omega_L^b \\ -(\bar{R} + \frac{(1-s)p}{s(1-p)}) & \Omega_L^c \end{cases} \leq 0$$

$$\frac{\partial E[\max(1-\bar{R}, \beta_L)]}{\partial q} = \begin{cases} 0 & \Omega_L^a \\ 0 & \Omega_L^b \\ -(\bar{R} + \frac{(1-s)p}{s(1-p)}) & \Omega_L^c \end{cases} \leq 0$$

$$\frac{\partial E[\max(\beta_L, \beta_L')]}{\partial q} = \begin{cases} -\bar{R} & \Omega_L^a \\ 0 & \Omega_L^b \\ -((1+\alpha_L^2)\bar{R} + 2\frac{(1-s)p}{s(1-p)}\alpha_L) & \Omega_L^c \end{cases} \leq 0$$

Take second order derivative:

$$\frac{\partial^2 E(\beta_L)}{\partial q^2} = 0$$

$$\frac{\partial^2 E[\max(1-\bar{R}, \beta_L)]}{\partial q^2} = 0$$

$$\frac{\partial E^2[\max(\beta_L, \beta_L')]}{\partial q^2} = \begin{cases} 0 & \Omega_1^a \cup \Omega_L^b \\ -2(\alpha_L \bar{R} + \frac{(1-s)p}{s(1-p)})\frac{\partial \alpha_L}{\partial q} < 0 & \Omega_L^c \end{cases}$$

Thus, we have

$$\begin{aligned} \frac{\partial C_L^2(p,q)}{\partial q^2} =& 4s^2p(1-p)\frac{\partial E(\beta_L)}{\partial q} + 4s(1-s)p(1-p)\frac{\partial E[\max(1-\bar{R}, \beta_L)]}{\partial q} \\ &+ s^2(1-p)^2\frac{\partial^2 E[\max(\beta_L, \beta_L')]}{\partial q^2} \\ &+ 2s^2p(1-p)q\frac{\partial^2 E(\beta_L)}{\partial q^2} + 2s(1-s)p(1-p)q\frac{\partial^2 E[\max(1-\bar{R}, \beta_L)]}{\partial q^2} \\ \leq& 0 \end{aligned}$$

Hence C_L is concave in each set Ω_L^a, Ω_L^b and Ω_L^c. When $\bar{R} \leq 1$, $\Omega_L^a = \{q : q \in (0,1)\}, \Omega_L^b = \emptyset$ and $\Omega_L^c = \emptyset$, so C_L is concave in q for any $0 < q < 1$. When $\bar{R} > 1$, $\Omega_L^c = \{q : 0 < q \leq \frac{s(1-p)(\bar{R}-1)}{s(1-p)\bar{R}+(1-s)p}\}$, $\Omega_L^b = \{q : \frac{s(1-p)(\bar{R}-1)}{s(1-p)\bar{R}+(1-s)p} < q \leq \frac{\bar{R}-1}{\bar{R}}\}$ and $\Omega_L^a = \{q : q > \frac{\bar{R}-1}{\bar{R}}\}$, so C_L has two non-differentiable points at $q_1 = \frac{s(1-p)(\bar{R}-1)}{s(1-p)\bar{R}+(1-s)p}$ and $q_2 = \frac{\bar{R}-1}{\bar{R}}$.

Similarly for C_H. When $\bar{R} \leq 1$, C_H is concave in q for any $0 < q < 1$. When $\bar{R} > 1$, we define $\Omega_H^a = \{q : q < \frac{1}{\bar{R}}\}$, $\Omega_H^b = \{q : \frac{1}{\bar{R}} \leq q < \frac{sp+(1-s)(1-p)}{sp+(1-s)(1-p)\bar{R}}\}$ and $\Omega_H^c = \{q : q \geq \frac{sp+(1-s)(1-p)}{sp+(1-s)(1-p)\bar{R}}\}$, then we can prove that C_H is concave in each set Ω_H^a, Ω_H^b and Ω_H^c. Also, C_H has two non-differentiable points at $q_3 = \frac{1}{\bar{R}}$ and $q_4 = \frac{sp+(1-s)(1-p)}{sp+(1-s)(1-p)\bar{R}}$.

Since C_o is linear in q, $\Psi = C_o + C_L + C_H$ is concave in $q \in \Omega_L^w \cap \Omega_H^v$ for any $w, v \in \{a, b, c\}$. When $\bar{R} \leq 1$, $\Omega_L^a = \{q : q \in (0,1)\}, \Omega_H^a = \{q : q \in (0,1)\}$, so Ψ is concave in q for any $0 < q < 1$. Therefore, Ψ get the minimal value either at $q = 0$ or $q = 1$. Since $\Psi(0) = \Psi(1) = \Psi_o$, $\Psi(q)$ has the minimal value Ψ_o. When $\bar{R} > 1$, Ψ has 4 possible non-differentiable points. We can verify that for each non-differentiable points $\Psi(q_i) > \Psi_o,\ i = 1, 2, 3, 4$. Also because $\Psi(q)$ is concave in $q \in \Omega_L^w \cap \Omega_H^v$ for any $w, v \in \{a, b, c\}$, $\Psi(q) \geq \Psi_o$ for any q.

Proof of Theorem 3.1

The 2-step proof is given in Section 3.5. Here I will just prove the supplier's bidding strategies in extreme cases as shown in Step 1.

When all suppliers are uninformed ($p = 0$), suppose we want to solve for supplier i's bidding strategy. Since no supplier can detect the error severity T, suppliers' bids do not depend on T. We let y_{-i} denote the highest bid of the other $N-1$ suppliers. Suppose supplier i has production cost $X_i = x_i$ and bids b_i. Then supplier i's expected payoff is $E[I_{\{b_i<y_{-i}\}}(y_{-i} - x_i + R)] = I_{\{b_i<y_{-i}\}}[y_{-i} - (x_i - E(R))]$. To maximize this expected payoff, we should have $b_i < y_{-i}$ if $y_{-i} - (x_i - E(R)) > 0$ and $b_i > y_{-i}$ is $y_{-i} - (x_i - E(R)) < 0$. Therefore, one dominant strategy is to bid $b_i = x_i - E(R)$.

Hence, the buyer's expected total payment is $E[(X - E(R))_{2:N} + R] = E(X_{2:N}) = \Psi_o$.

Similar arguments hold when $p = 1$ or T is degenerate.

Proof of Theorem 3.1-G

If a supplier wins the contract, his total cost is $X + C_s$, and he receives auction payment, additional windfall R and reimbursed payment C_s as well. From the supplier's perspective, this is equivalent with the case having cost X and receiving only auction payment and windfall R. Since the redesign cost C_s is reimbursed anyway, it should not affect the supplier's bidding strategy. Therefore, the contract is assigned to the same supplier with the same auction price as in the case when $C_s = 0$. From the buyer's perspective, in the event of redesign, she has to reimburse the supplier's redesign cost C_s and also incurs redesign cost C_b to herself, so the buyer's expected total payment is $\Psi_G = E(C_s + C_b) + \Psi$, in which Ψ is buyer's payment analyzed in Theorem 3.1. We can directly apply the result in Theorem 3.1.

Proof of Theorem 3.2-G

This proof covers Theorem 3.2 by assuming $C_s = C_b = 0$.

The winning supplier spends additional redesign cost C_s and receives additional payment C_s which cancel out each other. The profit gaining from redesign is moved up-front, so knowing how severe the error is does not benefit the supplier. Hence, informed or uninformed suppliers will apply the same bidding strategy. Whoever wins the auction will get additional payment $\bar{R}$, so every supplier is willing to lower the bid by $\bar{R}$. Therefore, the buyer' expected total payment is $\Psi_G^{pp} = E(C_s + C_b) + \bar{R} + E[(X - \bar{R})_{2:N}] = E(C_s + C_b) + E(X_{2:N}) = E(C_s + C_b) + \Psi_o$.

Proof of Theorem 3.3

From the proof of Theorem 3.2-G, we know that knowing how severe the RFQ is does not benefit an informed supplier. Hence, an informed supplier has no incentive to hide this information. When the buyer offers an arbitrarily small bounty for reporting the error, an informed supplier is willing to do so. If the buyer knows the error beforehand, then she can correct the RFQ and avoid redesign, so the expected payment is Ψ_o. There is still a probability $(1-p)^N$ that no supplier can detect the error. In this case, the buyer has to pay the redesign cost $E(C_s + C_b)$.

Proof of Results when C_s and X are positively correlated

We need to modify the proof of Theorem 3.1-G slightly. When $p = 0, p = 1$ or T is degenerate, the contract is given to the supplier with lowest production cost, hence in the event of redesign, the buyer need to reimburse supplier redesign cost $E[C_s(X_{1:N}, \theta)]$ and the buyer's expected total payment $\Psi_G = E[C_s(X_{1:N}, \theta) + C_b] + \Psi_o$. For $0 < p < 1$ and non-degenerate T, the contract is not necessarily given to the supplier with lowest production cost, but the expected redesign cost must be greater than or equal to $E[C_s(X_{1:N}, \theta)]$ since $E[C_s(x, \theta)]$ increases in x. Hence $\Psi_G \geq E[C_s(X_{1:N}, \theta) + C_b] + \Psi \geq E[C_s(X_{1:N}, \theta) + C_b] + \Psi_o$.

For Theorem 3.2-G and Theorem 3.3, because of pre-pay, the contract is always given to the supplier with lowest production cost. The expected supplier redesign cost is $E[C_s(X_{1:N}, \theta)]$.

CHAPTER 4

Should the Buyer Strategically Exclude the Incumbent Supplier from the Procurement Auction?

4.1 Introduction

In summer 2011, the authors interacted with a Tier-1 auto manufacturer. This manufacturer had signed a contract with one incumbent supplier stating that the incumbent supplier would reduce the price annually. However, in that year, the incumbent asked the manufacturer for a higher price than agreed per the contract clause. The incumbent claimed that they were not able to meet the annual price reduction target due to increased material cost, increased shipping cost, etc. The buyer had been working with this incumbent for a few years and had lost track of the true production cost, so she could not figure out if the supplier was telling the truth. To resolve this issue, the buyer decided to market-test the price by holding an online auction with several entrant suppliers. A primary concern of the procurement manager was whether the incumbent supplier should be invited to the auction or not.

On one hand, the buyer can withhold the incumbent and treat him as an outside option. The buyer can set an aggressive reserve price to the competing entrant suppliers, and get price reduction either through the low reserve price or the entrant suppliers' competition. If no one meets the reserve price, the buyer can go back to the incumbent supplier. In this way, the buyer can make sure that she gets a better price than (or at least no worse than) the current incumbent price.

On the other hand, the buyer can invite the incumbent to the auction and drum up competition with one more bidder. However, by doing so, the buyer is exposed to the risk of paying more than current incumbent price. When the incumbent is in the auction, he knows the other bidders' cost information revealed through the competition. If all entrant suppliers' costs turn out to be higher than the current incumbent price, the incumbent obtains proof that the market price is actually higher than his current price and will persist in raising the price. This puts the buyer in a weak position to negotiate with the incumbent supplier.

We investigate the tradeoff between the two alternatives: (a) withholding the incumbent and use him as an outside option; (b) including the incumbent in the auction. We introduce the model in §4.2. Section 4.3 analyzes an example with one incumbent and one entrant. In §4.4, we generalize the results to the N-entrant case with general cost distributions. We discuss the extensions in §4.5 and conclude the paper in §4.6. Proofs of results are furnished in §4.7.

4.2 Model Description and Preliminaries

We consider a buyer seeking to allocate a production contract to a single supplier. We assume the buyer has one incumbent supplier who claims that he cannot do better than price $\bar{x}_0$. Although the buyer does not know the incumbent's true minimum acceptable price, she suspects that it could be lower than the incumbent's claimed price $\bar{x}_0$. We use random variable X_0 to denote the incumbent's true minimum acceptable price, which follows distribution F_0 with support $[\underline{x}_0, \bar{x}_0]$. The buyer does not know how low the incumbent's true minimal acceptable price compared to the claimed price. If it is indeed much lower (i.e., $X_0 = x_0 << \bar{x}_0$), the buyer will lose money by simply trusting the incumbent and signing a contract with price $\bar{x}_0$.

In such situations, bringing in the market can be a powerful tool for a buyer. We model this as the buyer running an open-bid reverse auction, which is a common mechanism used in practice. The buyer hopes that by reopening the contract for bid she will get a price below current incumbent price from the auction winner. Suppose the buyer locates N qualified entrant suppliers. Supplier i's true minimal acceptable price X_i follows distribution F_i with support $[\underline{x}_i, \bar{x}_i]$. For simplicity, from now on, we refer to one supplier's minimum acceptable price as the supplier's "cost" although it, of course, includes a necessary profit margin. Following the logic of Proposition 2.1 in *Krishna* (2009) and other classic auction literature, if supplier i participates in the open-bid reverse auction, his dominant bidding strategy is to bid down to his true cost X_i.

The buyer holds the auction including all entrant suppliers. Whether the incumbent supplier also should be invited to the auction is the research question we are going to analyze. We assume the buyer is risk-neutral and her goal is to minimize the expected contract payment.

4.2.1 Alternative (a): withhold the incumbent as an outside option

The buyer holds the auction only among the entrants. The incumbent does not know the existence of the auction, so the buyer can always go back to the incumbent if the auction fails. In other words, the buyer can view the incumbent as an outside option and accordingly set a reserve price to truncate the entrants' prices. If some entrants are interested, the buyer gives the contract to the winning entrant. If no entrant meets the reserve price, the buyer transacts with the incumbent supplier at current incumbent price $\bar{x}_0$.

We use $X_{k:N}$ to denote the kth lowest value of random variables $X_1, X_2, ..., X_N$ (whose distributions are not necessarily identical). We use $\mu_{k:N}, \underline{x}_{k:N}, \bar{x}_{k:N}$ and $F_{k:N}$

to denote its mean, lower bound, upper bound, and distribution, respectively. The buyer's expected contract payment with reserve price r is

$$EP_a(r) = \bar{x}_0 \cdot P(X_{1:N} > r) + r \cdot P(X_{1:N} < r < X_{2:N}) + E[X_{2:N}|X_{2:N} < r]P(X_{2:N} < r) \tag{4.1}$$

The first term on the right hand side is the case when no entrant meets the reserve price and the buyer has to turn back to the incumbent at price $\bar{x}_0$. The second term is when only one supplier meets the reserve price and the reserve price sets the auction price. The third term is when more than one supplier meets the reserve price and suppliers' competition decides the auction price.

From Equation (4.1), we observe that the buyer's payment with alternative (a) depends on only the entrants' cost distributions (in fact, only $F_{1:N}$ and $F_{2:N}$ matter) and the incumbent's cost upper bound ($\bar{x}_0$) but does not depend on the incumbent's cost distribution F_0. By excluding the incumbent from the auction, the buyer gives up the opportunity to explore more about the incumbent's true cost X_0. Instead, the buyer hopes to strike a good deal with one of the entrant suppliers by using the incumbent's current price $\bar{x}_0$ to set an aggressively low reserve price, which is a credible threat to the entrants since the buyer can always turn back to the incumbent.

Intuitively, the reserve price should not be higher than the outside option cost, i.e., $r \leq \bar{x}_0$. Moreover, the reserve price is effective only when it is between $X_{1:N}$ and $X_{2:N}$: When $r \leq \underline{x}_{1:N}$, no entrant can meet the reserve price, $EP_a(r) = \bar{x}_0$; When $r \geq \bar{x}_{2:N}$, the reserve price never binds, $EP_a(r) = E[X_{2:N}]$. Therefore, we need to focus only on $r \in [\underline{x}_{1:N}, \min(\bar{x}_0, \bar{x}_{2:N})]$. The optimal reserve price is

$$r^* = \arg \min_{r \in [\underline{x}_{1:N}, \min(\bar{x}_0, \bar{x}_{2:N})]} EP_a(r).$$

Later when we discuss the buyer's payment with alternative (a), we use the buyer's expected payment with the optimal reserve price r^*:

$$EP_a = EP_a(r^*) = \min_{r\in[\underline{x}_{1:N},\min(\bar{x}_0,\bar{x}_{2:N})]} EP_a(r).$$

When the entrants' cost distributions (in fact, only $F_{1:N}$ and $F_{2:N}$ matter) are C^1-continuous, the function $EP_a(r)$ is C^1-continuous on interval $[\underline{x}_{1:N}, \min(\bar{x}_0, \bar{x}_{2:N})]$. Therefore, the minimal value should be reached either at one of the endpoints or where the first order condition is satisfied.

Lemma 4.1. *When $F_{1:N}$ and $F_{2:N}$ are C^1-continuous, the optimal reserve price r^* either equals to $\underline{x}_{1:N}$ or $\min(\bar{x}_0, \bar{x}_{2:N})$, or satisfies the first order condition*

$$\frac{\mathrm{d}EP_a(r)}{\mathrm{d}r}|_{r=r^*} = F_{1:N}(r^*) - F_{2:N}(r^*) - (\bar{x}_0 - r^*)f_{1:N}(r^*) = 0. \tag{4.2}$$

We can use this lemma to solve the optimal reserve price for special cost distributions. Furthermore, when entrants' costs are *i.i.d.* with distribution F, we can simplify Equation (4.2) and have the following lemma.

Lemma 4.2. *When N entrants are symmetric and the cost distribution F is C^1-continuous with support $[\underline{x}_1, \bar{x}_1]$, the optimal reserve price r^* either equals to $\underline{x}_1$ or $\min(\bar{x}_0, \bar{x}_1)$, or satisfies*

$$r^* + \frac{F(r^*)}{F'(r^*)} = \bar{x}_0. \tag{4.3}$$

4.2.2 Alternative (b): include the incumbent in the auction

Alternatively, the buyer can invite the incumbent to compete with the N entrants, i.e., holding an open-bid reverse auction among all $N+1$ suppliers with no reserve price. In this way, the buyer gives the incumbent a chance to retain the business. If the entrants are very competitive and push the auction price low enough, it is possible that the incumbent will have to bid below the current incumbent price $\bar{x}_0$ in order to win the business. However, the buyer also carries a risk: The auction may reveal that the market price is actually above the current incumbent price, so the incumbent has more reasons to ask for an even higher price.

One might wonder why the buyer cannot set a reserve price at the current incumbent price. Suppose the buyer sets this reserve price and only the incumbent meets it, then the incumbent is going to realize that the buyer has no other options at this price, and will persist in his demands for a price increase — essentially, the auction has just proved the incumbent's point that the current price $\bar{x}_0$ is too low. While one may imagine many ways a price between the incumbent and buyer could be reached, our approximation is to model the price as the second-best supplier cost in the auction. This corresponds to the buyer's next-best alternative to the incumbent, and we believe this would be a natural price that the incumbent and the buyer would settle on. By agreeing to market-test the item (i.e., reopen the contract and hold an auction), the buyer is implicitly putting credibility on the line that the incumbent is pricing above market, and once this is disproved the buyer is in a much weaker position to demand a price concession. Therefore, the reserve price $\bar{x}_0$ is not credible and the buyer gets the same result as if there is no reserve price.

With alternative (b), the contract price is determined by all suppliers' competition and thus the buyer's expected payment depends on all suppliers' cost distributions. We use $X_{k:N+1}$ to denote the kth lowest value of random variables $X_0, X_1, X_2, ..., X_N$ (whose distributions are not necessarily identical). The buyer's expected payment is

$$EP_b = E[X_{2:N+1}] \tag{4.4}$$

4.3 One Incumbent and One Entrant

Whether to exclude or include the incumbent lies in the tradeoff between setting a reserve price versus bringing more competition. To build a more concrete intuition, we analyze the case with one incumbent and one entrant. The entrant is labeled Supplier 1, whose cost follows uniform distribution with mean μ_1 and support length 1, i.e., $X_1 \sim U[\mu_1 - 0.5, \mu_1 + 0.5]$, $\underline{x}_1 = \mu_1 - 0.5$ and $\bar{x}_1 = \mu_1 + 0.5$. The incumbent is labeled Supplier 0, whose cost follows uniform distribution with mean μ_0 and support length l, i.e., $X_0 \sim U[\mu_0 - 0.5l, \mu_0 + 0.5l]$, $\underline{x}_0 = \mu_0 - 0.5l$ and $\bar{x}_0 = \mu_0 + 0.5l$. We define the mean difference $\Delta\mu = \mu_0 - \mu_1$. We introduce a parameter $z = \bar{x}_0 - \underline{x}_1 = \Delta\mu + 0.5(l + 1)$, then $X_1 \sim \underline{x}_1 + U[0, 1]$ and $X_0 \sim \underline{x}_1 + U[z - l, z]$.

Alternative (a): withhold the incumbent as an outside option

The buyer employs a reserve price, r, as a take-it-or-leave-it offer to the entrant. The entrant will accept this offer if it can cover his cost, i.e., $X_1 \leq r$. Otherwise, the entrant turns down the offer and the buyer transacts with the incumbent with price $\bar{x}_0$. The expected payment with reserve price r is

$$EP_a(r) = r \cdot P(X_1 \leq r) + \bar{x}_0 \cdot P(X_1 > r)$$

We can prove that the optimal reserve price is

$$r^* = \underline{x}_1 + \begin{cases} 0 & \text{if } z \leq 0 \\ 0.5z & \text{if } 0 < z < 2 \\ 1 & \text{if } z \geq 2 \end{cases} \tag{4.5}$$

Consequently, the expected payment with the optimal reserve price is

$$EP_a = \underline{x}_1 + \begin{cases} z & \text{if } z \leq 0 \\ z - 0.25z^2 & \text{if } 0 < z < 2 \\ 1 & \text{if } z \geq 2 \end{cases} \tag{4.6}$$

Alternative (b): include the incumbent in the auction

The incumbent and the entrant compete in an open-bid reverse auction. The auction price is the maximum of the two suppliers' costs. We can prove that the buyer's expected payment is

$$\begin{aligned} EP_b &= E[X_{2:2}] \\ &= \underline{x}_1 + \begin{cases} 0.5 & \text{if } z \leq 0 \\ 0.5 + \frac{z^3}{6l} & \text{if } 0 < z < \min(l,1) \\ 0.5 + \frac{1}{2l}(z^2 - z + \frac{1}{3}) & \text{if } 1 \leq z \leq l \\ 0.5 + \frac{1}{6l}[z^3 - (z-l)^3] & \text{if } l \leq z \leq 1 \\ 0.5 + 0.5z(z-l) + \frac{1}{6l}[(1-z)^3 + l^3] & \text{if } \max(l,1) < z < l+1 \\ z - 0.5l & \text{if } z \geq l+1 \end{cases} \end{aligned} \tag{4.7}$$

4.3.1 How the size of incumbent cost affects the decision

From Equations (4.6) and (4.7), we can see that whether the buyer should withhold the incumbent as an outside option (alternative: a) or include the incumbent in the auction (alternative: b) depends on the two parameters, l and z. The interval length l captures the incumbent's cost uncertainty since $Var(X_0) = \frac{l^2}{12}$, and parameter z captures the size of the incumbent's cost since $E[X_0] = E[X_1] + z - 0.5l$.

We analyze how the size of incumbent cost affects the buyer's decision by fixing $l = 1$. In this case, the incumbent and the entrant have the same amount of cost uncertainty but with different cost means. Parameter $z = \Delta\mu + 0.5(l+1) = \Delta\mu + 1$.

The mean difference $\Delta\mu = \mu_0 - \mu_1$ shows the size of incumbent cost compared to the entrant cost: $\Delta\mu < 0$ means that the incumbent cost tends to be smaller than the entrant cost ($X_0 \leq_{st} X_1$[1]); $\Delta\mu > 0$ means the incumbent cost tends to be larger

[1] Random variables $A \leq_{st} B$ means A is stochastically smaller than B.

than the entrant cost ($X_0 \geq_{st} X_1$); $|\Delta\mu|$ captures how different the incumbent cost is compared to the entrant cost.

We compare the buyer's expected payment with alternative (a) (given in Equation 4.6) and that with alternative (b) (given in Equation 4.7).

$$EP_b - EP_a = \begin{cases} -\Delta\mu - 0.5 & \text{if } \Delta\mu \leq -1 \\ \frac{\Delta\mu^3}{6} + 0.75\Delta\mu^2 - \frac{1}{12} & \text{if } -1 < \Delta\mu < 0 \\ -\frac{\Delta\mu^3}{6} + 0.75\Delta\mu^2 - \frac{1}{12} & \text{if } 0 < \Delta\mu < 1 \\ \Delta\mu - 0.5 & \text{if } \Delta\mu \geq 1 \end{cases}$$

Proposition 4.1. *Suppose the incumbent's cost interval has the same length as that of the entrant cost, $l = 1$. When $|\Delta\mu| > 0.347$, the buyer prefers to withhold the incumbent as an outside option. When $|\Delta\mu| < 0.347$, the buyer prefers to invite the incumbent to compete with the entrant.*

This proposition reveals that the buyer wishes to invite the incumbent to compete with the entrant only when their costs have similar sizes. When the incumbent cost is either too high or too low compared to the entrant cost, the buyer would rather exclude the incumbent and treat him as an outside option. This is because when the incumbent cost is too low, the entrant cost tends to set the auction price which can even be higher than the current incumbent cost; when the incumbent cost is too high, including the incumbent to the auction will not intensify the competition much. In either case, the buyer would rather withhold the incumbent as an outside option. Only when the incumbent cost and the entrant cost are of similar sizes, is the buyer more likely to reach the price concession through suppliers' competition.

4.3.2 How the uncertainty of incumbent cost affects the decision

We now analyze how the uncertainty of incumbent cost affects the buyer's decision by fixing $\Delta\mu = 0$. In this case, we view the incumbent cost and the entrant cost as having the same size since $\mu_1 = \mu_0$. We change the interval length l to see how the buyer's decision changes in the incumbent's cost uncertainty. Larger l indicates higher uncertainty since $Var(X_0) = \frac{l^2}{12}$.

We compare the buyer's expected payment with alternative (a) (given in Equation 4.6) and that with alternative (b) (given in Equation 4.7).

$$EP_b - EP_a = \begin{cases} \frac{5l^2}{48} - \frac{3l}{8} + \frac{3}{16} & \text{if } l \leq 1 \\ \frac{l^2}{16} - \frac{l}{4} + \frac{1}{16} + \frac{1}{24l} & \text{if } 1 < l < 3 \\ \frac{l}{8} + \frac{1}{24l} - 0.5 & \text{if } l \geq 3 \end{cases}$$

Proposition 4.2. *Suppose the incumbent and the entrant have the same mean (i.e., $\mu_0 = \mu_1$). When the length of the incumbent cost interval, $l < 0.6$ or $l > 2 + \sqrt{\frac{11}{3}}$ ($\approx$ 3.915), the buyer prefers to withhold the incumbent as an outside option. When $0.6 < l < 3.915$, the buyer prefers to invite the incumbent to compete with the entrant.*

Auction is a mechanism to reveal cost information. When the incumbent's cost uncertainty is very low, there is little room for the buyer to explore the incumbent cost, and thus, the buyer benefits little by including the incumbent in the competition.

With one incumbent and only one entrant, when the incumbent's cost range is very high ($l > 3.915$), the buyer is worse off by inviting the incumbent to compete with the entrant, because with only two bidders, the auction price is set by the maximum of two bidders' costs. With very high l, the incumbent cost has a bigger chance of taking a high value, which sets a high auction price. The buyer would rather withhold the incumbent as outside option and transact with the entrant instead.

4.3.3 When incumbent's cost upper bound $\bar{x}_0$ is fixed

In the last section, since the mean of the incumbent cost is fixed, as the cost uncertainty changes, the cost upper bound also changes, which also contributes to the buyer's decision between the two alternatives. To remove this factor and focus on the cost uncertainty, we now fix the incumbent cost upper bound $\bar{x}_0 = \bar{x}_1$, and change the interval length l. As l increases, the incumbent cost is stochastically smaller and has higher uncertainty.

In this case, $z = 1$. We compare the buyer's expected payment with alternative (a) (given in Equation 4.6) and that with alternative (b) (given in Equation 4.7).

$$EP_b - EP_a = \begin{cases} \frac{l^2}{6} - \frac{l}{2} + 0.25 & \text{if } l \leq 1 \\ \frac{1}{6l} - 0.25 & \text{if } l > 1 \end{cases}$$

Proposition 4.3. *Suppose the incumbent and the entrant have the same cost upper bound (i.e., $\bar{x}_0 = \bar{x}_1$). When $l < \frac{3-\sqrt{3}}{2}(\approx 0.634)$, the buyer prefers to withhold the incumbent as an outside option. When $l > 0.634$, the buyer prefers to invite the incumbent to compete with the entrant.*

As we discuss in §4.2, the buyer's payment with alternative (a) depends on the incumbent's cost upper bound but not the incumbent cost distribution. Therefore, by fixing the incumbent cost upper bound, the buyer's payment does not change even when the interval length changes. On the other hand, with alternative (b), the auction price is determined by maximal value of the incumbent cost and entrant cost. When length interval l increases, the incumbent cost is stochastically smaller and thus the auction price tends to be smaller. Therefore, as l increases, the buyer prefers more to choose alternative (b): invite the incumbent to compete with the entrant.

4.4 Analysis: N Entrants with General Cost Distributions

Now we consider a more general case with N entrants. Suppliers' cost distributions do not have to be uniform and can be any continuous distributions over bounded intervals. We also allow suppliers' costs to be asymmetric, i.e., different suppliers can have different cost distributions. We compare the two alternatives: (a) [withhold the incumbent as an outside option] The buyer holds an open-bid reverse auction among N entrants with an optimally selected reserve price r^*; (b) [include the incumbent in the auction] The buyer holds an open-bid reverse auction among all $N+1$ suppliers with no reserve price. We want to investigate if the insights we derived in last section hold in general.

4.4.1 How the size of incumbent cost affects the decision

In the one incumbent and one entrant example, Proposition 4.1 reveals that when incumbent cost is either relatively low or high (compared to the entrant cost), the buyer is better off excluding the incumbent rather than inviting him in the auction. This insight still holds in general case, as shown in the next Theorem.

Theorem 4.1. *For any cost distributions $F_0, F_1, ..., F_N$, if $\bar{x}_0 \leq \mu_{1:N}$ or $\underline{x}_0 \geq \bar{x}_{2:N}$, the buyer prefers to withhold the incumbent as an outside option.*

Recall that $\mu_{1:N}$ is the mean of the lowest value of $X_1, X_2, ..., X_N$, and $\bar{x}_{2:N}$ is the second-lowest upper bound. This theorem gives two threshold points $\mu_{1:N}$ and $\bar{x}_{2:N}$ to show how low and how high the incumbent cost should be so that the buyer will choose to exclude the incumbent from the auction. For specific distributions, we can get tighter threshold points.

When the incumbent cost is low, the buyer should not reveal this information to the incumbent because the incumbent will have more reasons to ask for a higher price. Therefore, the buyer should not invite the incumbent to the auction, which is a mechanism to reveal cost information to bidders. The buyer is better off to sign a contract with the low-cost incumbent, which is equivalent to alternative (a) by setting a very low reserve price for the entrants.

When the incumbent cost is high, including the incumbent in the auction will not intensify the competition. Even if the incumbent sees the threat from the entrants, he cannot lower the price because he needs a price to cover his high cost. However, withholding this incumbent will provide the entrants a credible threat and thus get a better contract price from the entrants (by setting an interior reserve price which is possible to cut the bid price).

In the other cases when the incumbent cost is in between — not too high, not too low compared to the entrant cost — the buyer may choose alternative (b) over alternative (a). The decision depends on the distribution setting.

4.4.2 How the uncertainty of incumbent cost affects the decision

We first analyze the extreme case where the incumbent is telling the truth that there is indeed no room for him to lower his price, that is, $P(X_0 = \bar{x}_0) = 1$.

We can prove that when the buyer knows the incumbent's exact cost (no matter how low or how high the incumbent cost is), she always prefers to withhold the incumbent as an outside option. We formalize this in the following lemma.

Lemma 4.3. *For any entrants' cost distributions $F_1, F_2, ..., F_N$, if the buyer knows the incumbent's exact cost (i.e., $Var(X_0) = 0$), the buyer prefers to withhold the incumbent as an outside option.*

An auction reveals the bidders' cost information, so if the buyer already knows the incumbent's true cost, there is no need to include the incumbent in the auction. With alternative (a), if the buyer sets the reserve price just equal to the incumbent cost, her payment is lower than (or at least no greater than) that with alternative (b). The buyer can do even better by choosing the optimal reserve price.

Furthermore, we find that when the incumbent cost uncertainty is sufficiently low, the buyer prefers to withhold the incumbent as an outside option. To remove the effects of the size of the incumbent cost and focus on the cost uncertainty, we create a mean-preserve contraction. Suppose for any distribution G, random variable $Y \sim G$ with mean μ_Y. Define $Y_\epsilon = \epsilon(Y - \mu_Y) + \mu_Y$ and its distribution G_ϵ is a mean-preserve contraction of distribution G.

Theorem 4.2. *For any distribution G, there exists $\epsilon_G > 0$ such that for any $\epsilon \in [0, \epsilon_G]$, when the incumbent's cost X_0 follows distribution G_ϵ, the buyer prefers to withhold the incumbent as an outside option.*

Theorem 4.2 captures Lemma 4.3 by setting $\epsilon = 0$. The reason is similar. Auction is a mechanism that reveals cost information. When the incumbent cost uncertainty is low, there is little room for the buyer to explore the incumbent's true cost through auction.

Theorems 1 and 2 reveal that the buyer chooses to withhold the incumbent when the incumbent cost is not comparable to entrant cost and when the incumbent's cost is less uncertain. On the other hand, it is possible that the buyer chooses to include the incumbent in the auction when the incumbent cost is similar in size to the entrant cost and has a certain amount of uncertainty. We analyze the case when the incumbent is as competitive as the entrant and has the same amount of uncertainty, i.e., all suppliers follow the same cost distribution.

Theorem 4.3. **[Symmetric Suppliers]** *When all suppliers' costs are i.i.d., the buyer always prefers to hold an auction among all suppliers.*

4.4.3 When incumbent's cost upper bound $\bar{x}_0$ is fixed

When withholding the incumbent, the buyer's payment depends on the incumbent's cost upper bound (which is the outside option cost) but does not depend on the incumbent's cost distribution. Therefore, when the incumbent's cost upper bound is fixed and only the distribution changes, the buyer's payment with alternative (a) does not change but the payment with alternative (b) does change. We have the following theorem.

Theorem 4.4. *When the incumbent's cost upper bound($\bar{x}_0$) is fixed, as the incumbent's cost distribution F_0 becomes stochastically smaller, $EP_b - EP_a$ is smaller and thus the buyer prefers more to invite the incumbent to the auction.*

As the incumbent cost becomes stochastically smaller, the auction price with alternative (b) tends to be smaller, so the buyer prefers more to invite the low-cost incumbent supplier to the auction.

This finding seems to contradict the insights in Section 4.4.1, which indicates that the buyer prefers to withhold the incumbent if the incumbent cost is very low. However, in Section 4.4.1, the incumbent cost upper bound is also very low and the buyer benefits from alternative (a) with a low outside option cost. When the incumbent's cost upper bound is fixed, even if the incumbent cost distribution is stochastically small, the buyer's payment with alternative (a) is not necessarily low since it depends on the incumbent's cost upper bound but not the cost distribution.

4.5 Extension

4.5.1 Alternative (c): withhold the entrants as an outside option

We propose a third alternative: The buyer makes a take-it-or-leave-it offer to the incumbent first; If the incumbent is not interested in this offer, the buyer then holds an open-bid reverse auction (with no reserve price) among the N entrants. The auction is held only among the entrants. However, in contrast to alternative (a), the outside option is not transacting with the incumbent but holding the auction among the entrants.

The buyer makes the offer r to the incumbent, then the buyer's expected payment is

$$EP_c(r) = E[X_{2:N}]P(X_0 > r) + r \cdot P(X_0 < r)$$

The first term on the right hand side is the case when the incumbent rejects the offer and the buyer has to run an auction among the entrants to award the contract. The second term is when the incumbent accepts the offer, the buyer gives the contract to the incumbent and does not have to run an auction.

Following similar logic as in §4.2.1, we need to focus only on $r \in [\underline{x}_0, \min(\mu_{2:N}, \bar{x}_0)]$. The optimal offer r^* either equals to the two endpoints, or satisfies

$$r^* + \frac{F_0(r^*)}{F_0'(r^*)} = \mu_{2:N}.$$

The advantage of using alternative (c) is that the buyer may save the trouble of running an auction with the entrants, while with alternative (a) or (b), running an auction (either among only the entrants or among all suppliers) is inevitable. However, alternative (c) also brings some practical concern: Since the offer given to the incumbent is below the expected market price (i.e., $r^* < \mu_{2:N}$), the incumbent may view this offer as unfair. As the buyer and the incumbent are currently in a contract, according to some contract clauses, the buyer may not be allowed to walk away without showing evidence that the incumbent is indeed pricing above the market.

For now let us suppose the buyer is very powerful and can use this alternative. We use some numerical examples to illustrate the tradeoff among the three alternatives.

Example: Suppose there are two entrants whose costs both follow uniform distribution, $X_i \sim U[5, 6]$ for $i = 1, 2$. The incumbent's cost follows uniform distribution with mean μ_0 and support length l, i.e., $X_0 \sim U[\mu_0 - 0.5l, \mu_0 + 0.5l]$. The mean difference $\Delta\mu = \mu_0 - \mu_1 = \mu_0 - 5.5$.

• How the size of incumbent cost affects the decision

We fix the interval length l and only change the mean difference $\Delta\mu$. Figure 4.1 shows how the buyer's payment changes in the mean difference. We observe: When the incumbent cost is much lower than the entrant costs, running an auction among all

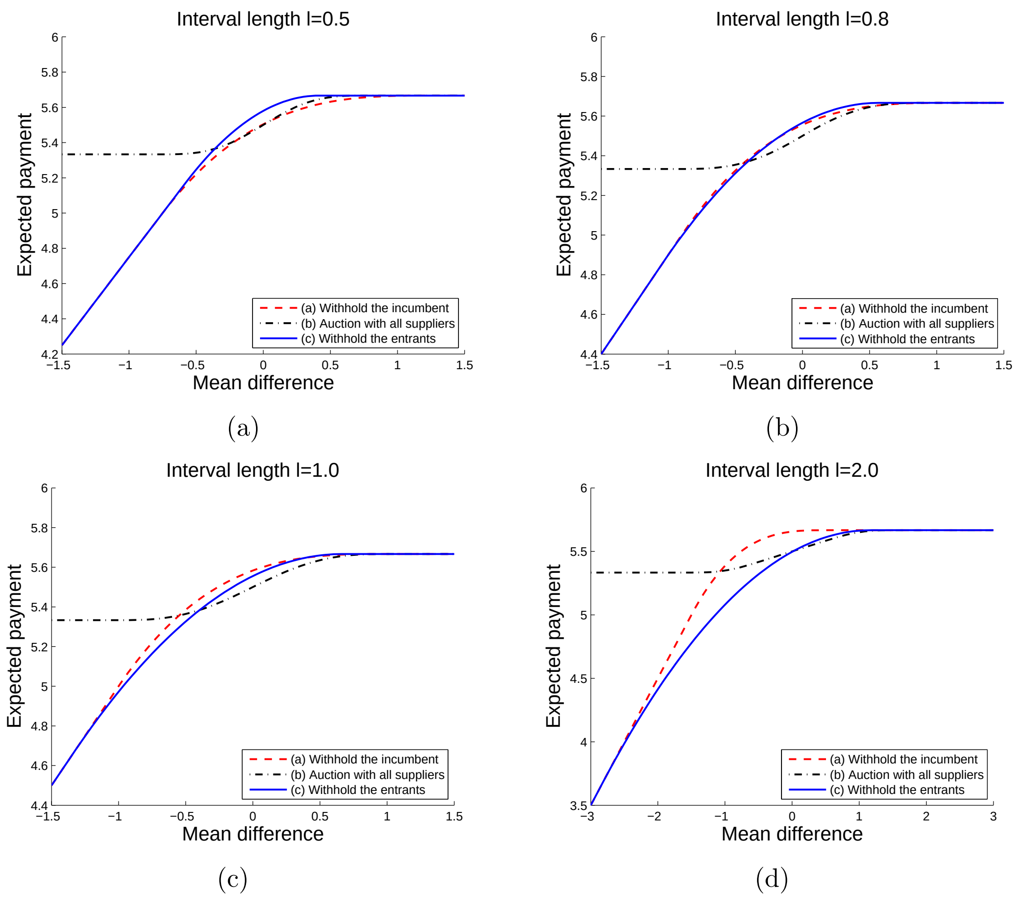


Figure 4.1: $X_i \sim U[5,6]$ for $i = 1,2$, $X_0 \sim U[\mu_0 - 0.5l, \mu_0 + 0.5l]$, $\Delta\mu = \mu_0 - 5.5$. Fix the length of the incumbent cost interval. Panel (a): $l = 0.5$; Panel (b): $l = 0.8$; Panel (c): $l = 1$; Panel (d): $l = 2$.

suppliers is not preferred, because the buyer would rather transact with the incumbent directly than have the incumbent know (through auction) that he is pricing below the market price; When the incumbent cost is much higher than the entrant costs, all three alternatives result in the same expected payment, because a very high-cost incumbent can neither affect the fair market price nor serve as a credible threat to push the entrants to further lower the price.

- **How the uncertainty of incumbent cost affects the decision**

We fix the mean of the incumbent cost and change only the interval length l. Figure 4.2 shows how the buyer's payment changes in the cost uncertainty while the mean is fixed. In Panel (a), the incumbent cost is very low compared to the entrant ($\Delta\mu = -0.7$), so the buyer prefers to directly negotiate with the incumbent and withhold the entrants as an outside option. In the other panels, we observe: When the incumbent cost uncertainty is very low, the buyer prefers to withhold the incumbent as an outside option; When the incumbent has moderate cost uncertainty, the buyer prefers to include the incumbent in the auction; When the incumbent cost uncertainty is very high, the buyer prefers to withhold the entrants as an outside option. This is because when the incumbent and the entrants have similar amounts of cost uncertainty, the competition reveals the most information and the buyer should include all suppliers to the auction. If the buyer chooses to withhold the incumbent, the payment depends only on the incumbent cost upper bound, regardless of the possibility that the incumbent cost can be low, so alternative (a) is more effective when the incumbent cost uncertainty is low. If the buyer chooses to withhold the entrants and sets a reserve price for the incumbent, it is more effective when the incumbent cost uncertainty is high so that there is more room for the reserve price to truncate the incumbent price.

- **When incumbent's cost upper bound $\bar{x}_0$ is fixed**

We fix the incumbent cost upper bound and change only the interval length l. Figure 4.2 shows how the buyer's payment changes in the interval length while the interval upper bound is fixed. We can see that the buyer's payment when withholding the incumbent is constant, because it depends only on the cost upper bound not the incumbent cost distribution. When the interval length is high, the incumbent cost has a high chance of taking a small value. In this case, the buyer prefers to directly negotiate with the incumbent, using entrants as an outside option rather than including the incumbent in the auction so that the incumbent knows that he is already pricing below the fair market price.

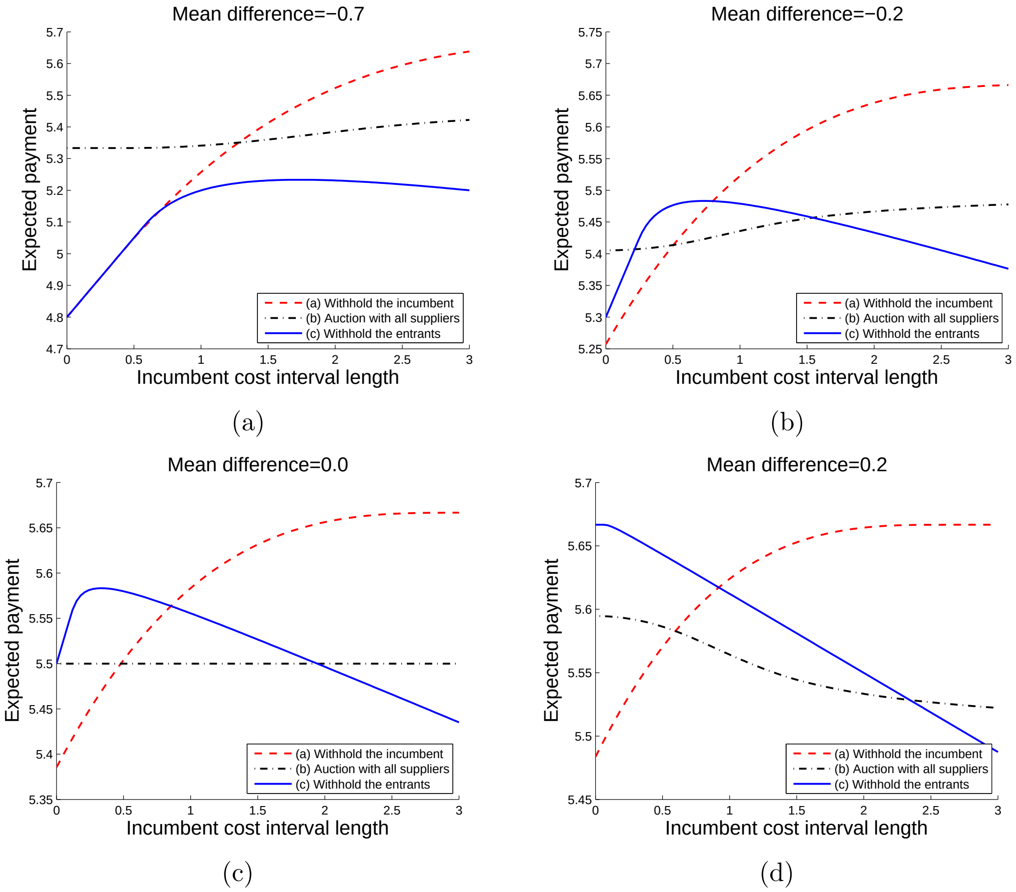


Figure 4.2: $X_i \sim U[5,6]$ for $i = 1,2$, $X_0 \sim U[\mu_0 - 0.5l, \mu_0 + 0.5l]$, $\Delta\mu = \mu_0 - 5.5$. Fix the mean of the incumbent cost. Panel (a): $\Delta\mu = -0.7$; Panel (b): $\Delta\mu = -0.2$; Panel (c): $\Delta\mu = 0$; Panel (d): $\Delta\mu = 0.2$.

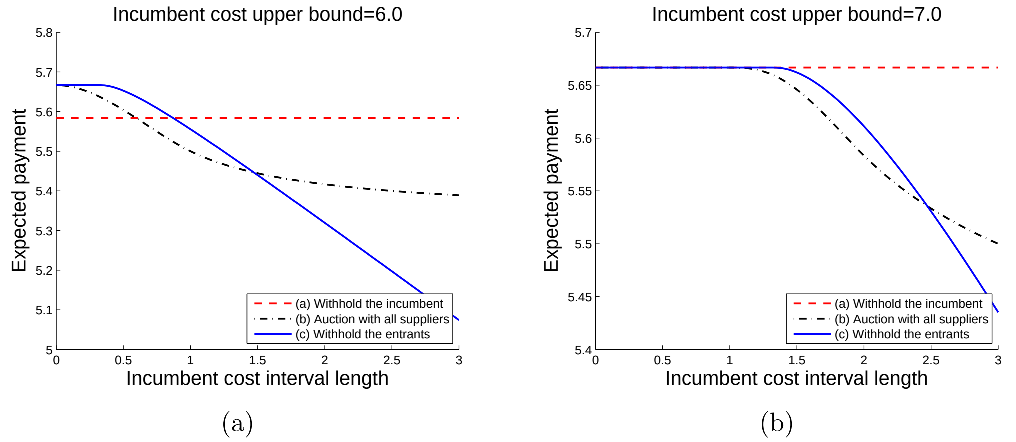


Figure 4.3: $X_i \sim U[5,6]$ for $i = 1,2$, $X_0 \sim U[\bar{x}_0 - l, \bar{x}_0]$. Fix the incumbent cost upper bound. Panel (a): $\bar{x}_0 = 6$; Panel (b): $\bar{x}_0 = 7$.

4.5.2 Others approaches: withhold a subset of suppliers

In fact, withholding bidder(s) would be possible whenever there is more than one supplier. We can think about a more general problem with total M suppliers — the buyer can withhold some suppliers and run an auction with the rest. There is no incumbent or entrant in this general model setting, and suppliers are labeled as supplier $1, 2, ..., M$.

We can build such a recursive version: Denote $S_0 = \{1, 2, ..., M\}$, set $k = 1$; The buyer withholds suppliers in $S_k \subset S_{k-1}$, and runs an auction with suppliers in $S_{k-1} \setminus S_k$ with reserve price r_k; If auction fails, then $k = k + 1$ and repeat the last step.

Algorithm for the general case

We use $V(S)$ to denote the minimum expected payment that a buyer can make with suppliers in set S. We define $X_{k:T}$ as the kth lowest cost for suppliers in set T. When T only contains one supplier i, define $X_{2:T} = \bar{x}_i$.

For set S which contains only one supplier i, $V(S) = \bar{x}_i$. When $|S| \geq 2$,

$$
\begin{aligned}
V(S) = \min_{\emptyset \neq S' \subset S, r} \{ & E[X_{2:S}], V(S')P(r < X_{1:S \setminus S'}) \\
& + E[\min(r, X_{2:S \setminus S'}) | r > X_{1:S \setminus S'}] P(r > X_{1:S \setminus S'}) \}
\end{aligned}
$$

We can solve this by backward induction. First solve $V(S)$ for any $|S| = 2$, then solve for any $|S| = 3$, $|S| = 4$,..., and finally solve $V(S_0)$ where $S_0 = \{1, 2, ..., M\}$. In this process, we can also record how to reach the minimum value, i.e., what the optimal reserve prices are and which supplier(s) to withhold.

Algorithm when suppliers' costs are ordered

Proposition 4.4. *If $X_i \geq_{st} X_j$ and $\bar{x}_i = \bar{x}_j$, then the buyer prefers to withhold supplier i rather than supplier j.*

This result allows us to reduce the algorithm complexity if all suppliers have the same cost upper bound ($\bar{x}_i = \bar{x}$ for any i) and can be ordered $X_1 \geq_{st} X_2 \geq_{st} ... \geq_{st} X_M$. We use $V(m)$ to denote the minimum expected payment that a buyer can make with the first m suppliers. Define $V(1) = \bar{x}_1$.

$$
\begin{aligned}
V(m) = \min_{0<i<m,r} \{ & E[X_{2:m}], V(i)P(r < X_{1:\{i+1,...,m\}}) \\
& + E[\min(r, X_{2:\{i+1,...,m\}})|r > X_{1:\{i+1,...,m\}}]P(r > X_{1:\{i+1,...,m\}})\}
\end{aligned}
$$

We can solve this by backward induction. First solve $V(2)$, then $V(3), V(4)$,..., and finally $V(M)$. In this process, we can record how to reach the minimum value, i.e., what the optimal reserve prices are and which supplier(s) to withhold.

4.6 Conclusions

In this essay, we ask whether a buyer should exclude the incumbent supplier from the auction and use him as an outside option or invite the incumbent supplier to compete with other suppliers. On one hand, when withholding the incumbent as an outside option, the buyer can set an aggressive reserve price for the entrant suppliers but the auction itself is less fierce with one fewer bidder. On the other hand, when including the incumbent in the auction, the buyer drums up competition with one more bidder, but is exposed to the risk of paying more than current incumbent price.

We find that the buyer's decision depends on the size of the incumbent cost. When the incumbent cost is expected to be substantially different from the entrants' (much lower or much higher), withholding the incumbent supplier is the better choice. This is because if the incumbent has very low cost, including him in the auction proves the incumbent is pricing below the market price and the incumbent has more reasons to persist in raising the price. If the incumbent has very high cost, including him in the auction will not intensify the competition, so the buyer would rather withhold him as an outside option to set a reserve price and push the entrants to lower the price.

The buyer's decision also depends on the incumbent's cost uncertainty. An auction serves as a way to reveal cost information. Therefore, when the incumbent's cost uncertainty is very low, there is little room for the auction to take effect in exploring the incumbent cost information, so the buyer will not benefit from inviting the incumbent to the auction. When the incumbent and the entrants have similar cost uncertainty (for example, as shown in Theorem 4.3, when all suppliers are symmetric), an auction among all suppliers is more effective. We also consider another alternative in Section 4.5.1: withholding the entrants as an outside option. This alternative is more effective when the incumbent cost uncertainty is high, since there is more room for the reserve price to truncate the incumbent price.

In this essay, we study approaches that are very natural and would be familiar to suppliers in practice. Of course, one could imagine other approaches which may also work in certain circumstances. For example, the buyer could hold an auction with no reserve price among all entrants; then based on the auction price, the buyer makes a take-it-or-leave-it offer to the winning entrant; if the winning entrant rejects this offer, the buyer turns back to the incumbent. This approach works like alternative (a) but with a more informed reserve price, which makes withholding the incumbent even more attractive. However, it is questionable whether the buyer can apply this approach in practice, since the buyer needs to persuade the entrant suppliers to participate in an auction where the winner may not get the contract. The buyer may use this approach when the entrant suppliers are very hungry for new business.

Another possible approach is that the buyer could hold an auction among all entrants with a reserve price equal to the current incumbent price, and then take the winner of this auction (if there is one) to compete with the incumbent in the second round auction. This approach would have the same result as alternative (b) with a reserve price equal to the current incumbent price. The limitation of this approach is similar to the former one: It requires the buyer to persuade the entrants to compete in an initial auction where nothing is awarded before being allowed to compete in a second auction with a new bidder.We leave the analysis about these alternative approaches to future work.

4.7 Proofs

Proof of Lemma 4.1

We use $f_{k:N}$ to denote the density function of $X_{k:N}$. Recall Equation (4.1),

$$\begin{aligned}EP_a(r) &= \bar{x}_0 \cdot P(X_{1:N} > r) + r \cdot P(X_{1:N} < r < X_{2:N}) + E[X_{2:N}|X_{2:N} < r]P(X_{2:N} < r) \\ &= \bar{x}_0 \cdot [1 - F_{1:N}(r)] + r[F_{1:N}(r) - F_{2:N}(r)] + \int_{\underline{x}_{2:N}}^{r} t \cdot f_{2:N}(t)\mathrm{d}t.\end{aligned}$$

Therefore,

$$\begin{aligned}\frac{\mathrm{d}EP_a(r)}{\mathrm{d}r} &= -\bar{x}_0 \cdot f_{1:N}(r) + r[f_{1:N}(r) - f_{2:N}(r)] + [F_{1:N}(r) - F_{2:N}(r)] + r \cdot f_{2:N}(r) \\ &= F_{1:N}(r) - F_{2:N}(r) - (\bar{x}_0 - r)f_{1:N}(r)\end{aligned}$$

Proof of Lemma 4.2

When $X_1, X_2, ..., X_N$ are *i.i.d.* with distribution F and density function f, we have

$$f_{1:N}(r) = Nf(r)(1 - F(r))^{N-1},$$

$$F_{1:N}(r) - F_{2:N}(r) = P(X_{1:N} < r < X_{2:N} = NF(r)(1 - F(r))^{N-1}.$$

Therefore, Equation (4.2)

$$\frac{\mathrm{d}EP_a(r)}{\mathrm{d}r}|_{r=r^*} = F_{1:N}(r^*) - F_{2:N}(r^*) - (\bar{x}_0 - r^*)f_{1:N}(r^*) = 0$$
$$= N(1 - F(r^*))^{N-1}(F(r^*) - (\bar{x}_0 - r^*)f(r^*))$$

Since $1 - F(r) \neq 0$ and $f(r) \neq 0$ when $r < \bar{x}_1$, we have Equation (4.3).

Proof of Equation (4.5), (4.6) and (4.7)

Since $X_1 \sim \underline{x}_1 + U[0,1]$ and $X_0 \sim \underline{x}_1 + U[z-l, z]$, the optimal reserve price (r^*) and buyer's payments (EP_a and EP_b) with $\underline{x}_1 = C$ equal to those with $\underline{x}_1 = 0$ plus C, respectively. Therefore, for simplicity, we let $\underline{x}_1 = 0$, then $X_1 \sim U[0,1]$ and $X_0 \sim U[z-l, z]$.

Proof of Equation (4.5): We only need to focus on $r \in [0,1]$. The payment with reserve price $r \in [0,1]$ is

$$EP_a(r) = r \cdot P(X_1 \leq r) + z \cdot P(X_1 > r) = r^2 + z(1-r). \tag{4.8}$$

The first order condition is

$$0 = \frac{\mathrm{d}EP_a(r)}{\mathrm{d}r} = 2r - z \Longleftrightarrow r = 0.5z.$$

Therefore, the optimal reserve price is

$$r^* = \begin{cases} 0 & \text{if } z \leq 0 \\ 0.5z & \text{if } 0 < z < 2 \\ 1 & \text{if } z \geq 2 \end{cases} \tag{4.9}$$

Proof of Equation (4.6): We replace r in Equation (4.8) with r^* given in Equation (4.9).

Proof of Equation (4.7): For $0 < x < 1$, define

$$h(x) = E[X_{2:2}|X_0 = x] = xP(X_1 < x) + E[X_1|X_1 > x]P(X_1 > x) = x^2 + (\frac{x+1}{2})(1-x)$$

$$H(x) = \int_0^x h(t)\mathrm{d}t = \frac{x}{2} + \frac{x^3}{6}$$

The buyer's payment is

$$\begin{aligned}
EP_b =& E[X_{2:2}] \\
=& E[X_1]P(X_0 < 0) + E[X_{2:2}|0 < X_0 < 1]P(0 < X_0 < 1) \\
&+ E[X_0|X_0 > 1]P(X_0 > 1) \\
=& 0.5\frac{[\min(0, z) - \min(0, z - l)]}{l} \\
&+ \frac{1}{l}\{H[\max(\min(1, z), 0)] - H[\min(\max(0, z - l), 1)]\} \\
&+ 0.5[\max(1, z) + \max(1, z - l)]\frac{[\max(1, z) - \max(1, z - l)]}{l} \\
=& \begin{cases}
0.5 & \text{if } z \leq 0 \\
0.5(\frac{l-z}{l}) + \frac{1}{l}(H(z) - H(0)) & \text{if } 0 < z < \min(l, 1) \\
0.5(\frac{l-z}{l}) + \frac{1}{l}(H(1) - H(0)) + 0.5(1+z)\frac{z-1}{l} & \text{if } 1 \leq z \leq l \\
\frac{1}{l}[H(z) - H(z-l)] & \text{if } l \leq z \leq 1 \\
\frac{1}{l}[H(1) - H(z-l)] + 0.5(1+z)\frac{z-1}{l} & \text{if } \max(l, 1) < z < l+1 \\
0.5(2z - l) & \text{if } z \geq l+1
\end{cases}
\end{aligned}$$

Proof of Proposition 4.1

We simplify Equation (4.7) minus Equation (4.6) when $l = 1$ and $z = \Delta\mu + 1$.

$$EP_b - EP_a = \begin{cases} -\Delta\mu - 0.5 & \text{if } \Delta\mu \leq -1 \\ \frac{\Delta\mu^3}{6} + 0.75\Delta\mu^2 - \frac{1}{12} & \text{if } -1 < \Delta\mu < 0 \\ -\frac{\Delta\mu^3}{6} + 0.75\Delta\mu^2 - \frac{1}{12} & \text{if } 0 < \Delta\mu < 1 \\ \Delta\mu - 0.5 & \text{if } \Delta\mu \geq 1 \end{cases}$$
$$= \begin{cases} -\frac{|\Delta\mu|^3}{6} + 0.75|\Delta\mu|^2 - \frac{1}{12} & \text{if } |\Delta\mu| < 1 \\ |\Delta\mu| - 0.5 > 0 & \text{if } |\Delta\mu| \geq 1 \end{cases}$$

We define $g(x) = -\frac{x^3}{6} + 0.75x^2 - \frac{1}{12}$. Function $g(x)$ has three roots $x = -0.322$, 0.347, 4.475. Hence, $g(x) > 0$ when $x \in (0.347, 1)$, and $g(x) < 0$ when $x \in (0, 0.347)$. Therefore, $EP_b - EP_a > 0$ when $|\Delta\mu| > 0.347$ and $EP_b - EP_a < 0$ when $|\Delta\mu| < 0.347$.

Proof of Proposition 4.2

We simplify Equation (4.7) minus Equation (4.6) when $z = 0.5(l + 1)$.

$$EP_b - EP_a = \begin{cases} \frac{5l^2}{48} - \frac{3l}{8} + \frac{3}{16} & \text{if } l \leq 1 \\ \frac{l^2}{16} - \frac{l}{4} + \frac{1}{16} + \frac{1}{24l} & \text{if } 1 < l < 3 \\ \frac{l}{8} + \frac{1}{24l} - 0.5 & \text{if } l \geq 3 \end{cases}$$

We define $g_1(l) = \frac{5l^2}{48} - \frac{3l}{8} + \frac{3}{16}$, $g_2(l) = \frac{l^2}{16} - \frac{l}{4} + \frac{1}{16} + \frac{1}{24l}$ and $g_3(l) = \frac{l}{8} + \frac{1}{24l} - 0.5$. Function $g_1(l)$ has two roots $l = 0.6, 3$, so $g_1(l) > 0$ when $l \in (0, 0.6)$ and $g_1(l) < 0$ when $l \in (0.6, 1)$. Function $g_2(l) = (\frac{l}{4} - 0.5)^2 - \frac{3}{16} + \frac{1}{24l} < -\frac{1}{12} < 0$ when $l \in (1, 3)$. Also, $g_3'(l) = \frac{1}{8} - \frac{1}{24l^2} > 0$ when $l \geq 3$ and $g_3(l)$ has root $l = 3.915$. Hence, $g_3(l) < 0$ when $3 \leq l < 3.915$ and $g_3(l) > 0$ when $l > 3.915$. To summarize, $EP_b - EP_a > 0$ when $l < 0.6$ or $l > 3.915$, and $EP_b - EP_a < 0$ when $0.6 < l < 3.915$.

Proof of Proposition 4.3

We simplify Equation (4.7) minus Equation (4.6) when $z = 1$.

$$EP_b - EP_a = \begin{cases} \frac{l^2}{6} - \frac{l}{2} + 0.25 & \text{if } l \leq 1 \\ \frac{1}{6l} - 0.25 & \text{if } l > 1 \end{cases}$$

Since $\frac{l^2}{6} - \frac{l}{2} + 0.25$ has two roots $\frac{3\pm\sqrt{3}}{2}$, it is greater than zero when $l < \frac{3-\sqrt{3}}{2}$ and lower than zero when $\frac{3-\sqrt{3}}{2} < l < 1$. Also, $\frac{1}{6l} - 0.25 < 0$ when $l > 1$. Therefore, $EP_b > EP_a$ when $l < \frac{3-\sqrt{3}}{2}$ and $EP_b < EP_a$ when $l > \frac{3-\sqrt{3}}{2}$.

Proof of Theorem 4.1

The second lowest of all $N + 1$ suppliers' cost is no less than the lowest of N entrants' costs, so $X_{2:N+1} \geq_{st} X_{1:N}$ and thus $EP_b = E[X_{2:N+1}] \geq \mu_{1:N}$. Since $\bar{x}_0$ is the outside option cost, $EP_a \leq \bar{x}_0$.

When $\bar{x}_0 \leq \mu_{1:N}$, $EP_a \leq \bar{x}_0 \leq \mu_{1:N} \leq EP_b$.

When $\underline{x}_0 \geq \bar{x}_{2:N}$, the incumbent's cost is no less than the second lowest entrant cost, so $X_{2:N+1} = X_{2:N}$ and thus $EP_b = E[X_{2:N+1}] = \mu_{2:N}$. Also, $EP_a \leq EP_a(\underline{x}_0) = \mu_{2:N}$. Hence, $EP_a \leq EP_b$.

Proof of Lemma 4.3

For alternative (a), if we set reserve price equaling to the incumbent's cost $X_0 = x_0$, then recall Formula (4.1),

$$\begin{aligned} EP_a(x_0) =& x_0 \cdot P(X_{1:N} > x_0) + x_0 \cdot P(X_{1:N} < x_0 < X_{2:N}) \\ &+ E[X_{2:N}|X_{2:N} < x_0]P(X_{2:N} < x_0) \\ =& x_0 \cdot P(X_{2:N} > x_0) + E[X_{2:N}|X_{2:N} < x_0]P(X_{2:N} < x_0) \end{aligned}$$

With alternative (b), the buyer's payment

$$\begin{aligned}
EP_b =& E[X_{2:N+1}] \\
=& E[X_{1:N}|X_{1:N} > x_0]P(X_{1:N} > x_0) + x_0 \cdot P(X_{1:N} < x_0 < X_{2:N}) \\
&+ E[X_{2:N}|X_{2:N} < x_0]P(X_{2:N} < x_0) \\
\geq & x_0 \cdot P(X_{2:N} > x_0) + E[X_{2:N}|X_{2:N} < x_0]P(X_{2:N} < x_0) \\
=& EP_a(x_0)
\end{aligned}$$

Therefore, $EP_b \geq EP_a(x_0) \geq EP_a$.

Proof of Theorem 4.2

If $\mu_Y > \bar{x}_{2:N}$, we can set $\epsilon_G = \mu_Y - \bar{x}_{2:N}$, then for any $\epsilon \in [0, \epsilon_G]$ and $X_0 \sim G_\epsilon$, $\underline{x}_0 = \mu_Y - \epsilon \geq \bar{x}_{2:N}$. According to Theorem 4.1, $EP_a \leq EP_b$.

Define $H(\epsilon) = EP_b - EP_a$. When all random variables are continuous, $H(\epsilon)$ is continuous. If we can prove $H(0) > 0$ when $\mu_Y \leq \bar{x}_{2:N}$, then according to the definition of continuous function, such ϵ_G exists and the proof is complete.

We consider $X_0 \sim G_0$ (i.e., $P(X_0 = \mu_Y) = 1$). When $\mu_Y \leq \bar{x}_{2:N}$, $\frac{\mathrm{d}EP_a(r)}{\mathrm{d}r}|_{r=\mu_{Y-}} < 0$. Hence $EP_a < EP_a(\mu_Y) \leq EP_b$.

Proof of Theorem 4.3

The optimal mechanism with symmetric suppliers is to hold an open-bid reverse auction among all suppliers. The optimal mechanism dominates other mechanisms including alternative (a).

Proof of Theorem 4.4

Buyer's payment with alternative (a) EP_a does not change. As X_0 becomes stochastically smaller, $X_{2:N+1}$ becomes stochastically smaller, so the buyer's payment with alternative (b) $EP_b = E[X_{2:N+1}]$ is smaller. The proof is complete.

Proof of Proposition 4.4

When $\bar{x}_i = \bar{x}_j$, withholding supplier i and withholding supplier j have the same outside option cost. However, since $X_i \leq_{st} X_j$, the auction price when withholding supplier j (supplier i participates in the auction) is stochastically smaller than that with withholding supplier i (supplier j participates in the auction). The proof is complete.

CHAPTER 5

Conclusion

This dissertation research consists of three essays examining different aspects in competitive bid procurement.

The first essay explores the value of cost modeling in competitive bid procurement, to understand if, how and when cost modeling should be deployed. The answer is not obvious, since the bidding competition itself acts as a tool to discover suppliers' cost information — suppliers may have to bid close to their minimum acceptable price in order to win. I show that although bid competition sometimes duplicates the information gleaned by cost modeling, the latter can still be beneficial when it helps the buyer set an effective reserve price. Then I analyze how the buyer can gain the most benefit through cost modeling. Specifically, I characterize which supplier(s) to learn about, which portion(s) of the costs to learn, and how deeply and broadly the buyer should learn for general supply base topologies.

Many requests for quotes (RFQs) issued by firms can be flawed due to errors of omission and misspecification, which triggers redesign and associated supplier windfall profit. The second essay studies the problem of error-prone RFQ. Surprisingly, I find that RFQ error encourages suppliers to submit lower bids with anticipation of future windfall profit due to redesign. I also find that supplier disparity in error-detecting expertise generally hurts the buyer. Furthermore, I propose a creative approach, "pre-pay", to stem supplier windfall profit; under this new mechanism, the buyer can offer a small bounty to induce knowledgeable suppliers to divulge the existence of RFQ error. This pre-pay and error-bounty approach helps avoiding the cost of redesign, creates a win-win solution for both parties, and makes the entire supply chain more efficient.

In the third essay, I analyze a buyer firm's decision about whether to invite an incumbent supplier to compete with entrant suppliers. On one hand, when excluding the incumbent from auction and use him as an outside option, the buyer can set an aggressive reserve price to truncate the entrants' prices but the auction is less fierce with one fewer bidder; On the other hand, when including the incumbent in the auction, the buyer drums up competition with one more bidder but is exposed to the risk of paying more than current incumbent price. I find that the buyer's decision depends on the incumbent's and entrants' cost distributions: When the incumbent's cost is expected to be substantially different from the entrants' (either much lower or much higher) and has low uncertainty, withholding the incumbent supplier is the better option; Conversely, when the incumbent's cost is comparable to the entrants' costs and has similar amount of uncertainty, the buyer prefers inviting the incumbent to the auction.

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