| To assign students' work (about a particular topic)-I | | Such as an assignment on writing the equation of a line given different pieces of information about the line | |
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| A teacher would Choose or | Which means Assign | For example | Specifically Problems addressing conceptual understanding might |
| design problems for students to do (at the end of class and at home) | problems that provide for a balance between conceptual understanding and procedural fluency on the topic learned. | To exercise conceptual understanding one would assign problems or questions where students have to demonstrate that they know the purpose of particular calculations or the meaning of particular pieces of information, such as the slope. To exercise procedural fluency one would assign problems where students have to use the procedures learned; in this case, writing the equation of a line in slope intercept form. | include: a. Given the slope of a line, what else do you need to know to write an equation of that line? b. If two lines have the same slope what information will let you determine if they have the same x-intercept? c. Given the coordinates of three points, how could you check that they are on the same line without graphing? Problems addressing procedural fluency might include: • Write the equation of a line in slope intercept form:(a) given a slope and y intercept, such as a slope of -2/3 and a y intercept of 4; (b) given a slope and a pont on the line, such as a slope of -2/3 through the point (6, -4); (c) given two points on the line, such as (-3, 2) and (7, 8). • Given the coordinates of four points, find the equation of all the lines that they determine. |
| | Assign problems that you anticipate will present students with different degrees of difficulty in the | • To vary the difficulty you could assign problems that clearly identify what the given information is as well as problems that require the students to contribute some of the given information or choose the relevant information from what is given. | For example, the following problem gives students extra information. A line passing through the center of a circle divides a circle with a radius of 5 units into two semicircles. If the line intersects the circle at the points (2, 1) and (-6, 7) and the center of the circle is at (-2, 4), what is the equation of the line? |

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| application of the concepts or procedures to be learned. | • To vary the difficulty you could assign problems that clearly identify what students need to find as well as problems that require the student to determine what they need to find based on some broader question provided. | For example, the following problem requires students to determine that they are looking for an intercept. At a constant pressure, the relationship between temperature of a gas and its volume is linear. Write an equation for the relationship if the same amount of gas has a volume of 550 cubic cm at 60 degrees Celsius and 505 cubic cm at 30 degrees Celsius. Absolute zero is defined as the temperature at which gas will have no volume. What is absolute zero in degrees Celsius? |
| | • To vary the difficulty of problems you can assign problems that provide all the resources students need and also problems that count on the students to get some resources themselves. These resources can be artifacts, items of prior knowledge, or representations. Use variables to increase the conceptual and algebraic demands. Applied (real world) problems can have such difficulty-requiring students to find a mathematical model that describes them and then use the model to find the result. | The use of variables makes the following problems more difficult. a. A line goes through the points (2, -3) and (-6, a). If it has a positive slope, what values can a have? A slope of zero? b. A line has a slope of -2/3 and passes through the point (z, 7). If the x intercept of the line is 5, what is z? In addition to the temperature problem, the following two problems require students to create mathematical models, bu are less difficult. c. A cell phone company charges \$10 for 100 minutes of talking time and \$45 for 415 minutes of talking time. Write an equation to show the cost for x minutes. d. A gym charges an initial fee of \$30 dollars and then charges customers \$15 per month. Write an equation for the total cost of joining the gym for x months. |

For example: A teacher might ask her students to write the • These problems can also be made more equation of the lines that pass through points the pairs of or less difficult by changing the numbers points (a) (2,3) and (3,5) and (b) (17/25, 13/54) and (2/3, given and the result that will be 1/2). The second would be "more difficult" in the sense that obtained. Problems with fractional, students might take longer and make more mistakes. But negative, or decimal slopes may be more difficult than those with whole number their longer time and their potential mistakes would be calculating numbers, not thinking about slope or linear slopes, but this difficulty concerns functions. The problems are identical in regard to the students' skill with number and thinking about linear functions that they demand, they are operations, not necessarily with the both easy linear function problems. The second one should concepts and procedures of linear not be used for the sake of practicing linear functions, functions: The problem may be difficult to do, but success or failure on these because it masks students understanding of linear function under the veil of performance in arithmetic. Choose instead a problems does not say much about the different problem. sophistication of a students' understanding of anything else than number and operation. Problems can be easier or harder For example, the following problem requires students to analyze a novel situation using their knowledge of linear depending on their levels of cognitive demand. Bloom's taxonomy provides functions. Two lines both have slopes of -3. If the xresources for posing problems at several intercept of one of the lines is twice that of the other line, levels: Knowing, understanding, applying, what is the relationship between their y-intercepts? analyzing, synthesizing, and creating.

| To publicly review students' work (about a particular topic)-II | | Such as an assignment on writing the equation of a line given different pieces of information about the line |
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| A teacher would | Which means | For instance |
| Select a few key problems to review at the beginning of class (or at the time of homework review) | Select few problems to review that address key features of the concept or procedure at stake; they should be of varying degrees of difficulty. | The teacher could choose a procedural problem where students were expected to determine when they were done, an applied problem that required students to interpret real world information given, and a conceptual question. They might also choose problems that cover multiple concepts or procedures. |
| | Make your selection informed by students' difficulties. | The selection of problems to review should not be a matter of students' choice. But the teacher could poll the class about hard problems and use that information to confirm whether the problems chosen will take care of all the difficulties students' had. |
| | Identify the problems to be reviewed in public and request students' participation. | List the key problems that will be reviewed on the board and ask students to retrieve their work on these problems. |
| | Solicit students' participation in the homework review. | Problem by problem ask students what questions they have about those problems or what difficulties they had when they worked on them. Note those difficulties on the board (in general terms) so as to reinforce their applicability in other problems. |
| | Maintain the homework review focused on the material to be learned, not on the | Indicate to students that those problems on the board represent the material they were to learn and that other problems may be solved using the same ideas involved in the problems that were reviewed. |

| | problems themselves. | |
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| | Hold students accountable for attacking again the other problems in their own time. | Once the review of those problems is completed, tell students that if they had difficulty with other problems they should try them again after the review of the key problems and seek out help if they still have difficulty with them. |
| Strategically solicit students' input as you review each of the key problems. | For each problem, talk with students about the mathematical statement of each problem soliciting their input about what the problem says. | For example, the teacher could ask the class broad questions about what the problem is asking and what information is given. The teacher may also ask students to point out extraneous information. |
| | Use diverse solicitation strategies to incorporate all members of the class, not just those who are more vocal or have more difficulty. | After receiving a response, the teacher might ask if others in the class agree or continue to ask for another response or explanation. Use direct questioning of a student or of the class as a whole, pairing students up to think and then share, asking for volunteers, or randomly choosing a student, etc. |
| | Expect students to give reasons for their answers as much as possible and provide reasons when needed. | Ask students how they thought about the problem, what their solution method was, what things they got when they applied their method, what the answer was, and whether and why they thought that was the correct (or incorrect) solution. |
| | Invite different responses and | The teacher might also have students pair up to discuss how to solve and then call on pairs to explain their solution strategy. |

| | alternative processes. | Use wait time to encourage students to question each other's ideas and offer alternatives. |
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| Ensure that the solutions are clear, making sure that both the initial problem and the | Directly or through questions, focus students' attention on key steps or features of the problem that tie it to the topic being learned. | If a problem has been solved in multiple ways, the teacher should validate that a problem may be solved in many ways, but underscore the solution that uses the ideas learned (the ideas on whose behalf the problem was chosen). |
| mathematics involved are addressed | Avoid focusing too much attention on ancillary elements of the problem | For example if a linear functions problem involved doing a calculation with difficult numbers, this would not be the moment to review how one does that calculation. |
| | Use the problem as a context to stress the key ideas involved in the solution. | When students have correctly written an equation for the contextualized problems, the teacher might ask, "What does this answer mean?" to tie the mathematical aspects of the equations (slope, intercept) back into the problem context. This can make connections between the two problems more explicit. |
| | Focus students' attention on how the new concept or procedure makes for a better way of solving the problem posed. | This could be an occasion for comparing different solutions that might be just as correct. |
| Check for students' understanding of the homework | Using another homework problem, ask questions that relate the problem to the topic learned. | The teacher can check for understanding by having students identify another problem where the same strategy is usable, or by having students look again at one such problem and say how they would solve it. |
| review | Ask students how | For example students could be asked to raise hands if they feel they can do problems like the one |

| | they feel about their command of the topic learned after the homework review. | reviewed or if they feel they understand what they did on the board. |
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| | Give students a brief mathematical assessment on a related problem. | For example, the teacher could have students do a homework mini-quiz where they are given another homework problem that calls for work similar to what has been reviewed. |
| Use students' responses to the homework in future instruction | Use students' difficulties and successes with homework to inform instructional decisions about: • How, how soon, | The teacher should integrate areas of difficulty into future problems and assignments. • If students seem to lack mastery of the subject, the teacher may decide that more practice is |
| | and how fast to introduce a new topic • What problems | needed on the topic. What students thought was difficult in the day's assignment should inform what assignments |
| | to include in further homework assignments, review packages, and summative assessments. | are given for the following day. |
| | • What concepts or procedures to revisit in further instruction | If there are many difficulties, it might be better to tackle some, move on, and then come back to the remaining difficulties another day. |

Hold students accountable for the homework problems Hold students accountable for:

- Having the homework done or attempted
- Working for their learning: on the basis of how much they worked at the problems, the teacher could provide differential incentives and assistance.
- Using homework problems to develop understanding of underlying concepts.
- Completing the homework problems.
- Seeking help if they are having difficulty after going over the homework.

· The concepts in the homework problems will appear on assessments and be asked about in future lessons, where new material builds on prior conceptual understanding.

For example when students are sharing their ideas in response to the key problems, the teacher could ask students to put their work on their tables and could walk around jotting notes about who completed or attempted homework and who did not.

For example if students did not try the problems the teacher could provide as an incentive the opportunity to try them for the following day, using the present review as an added support. But if students tried and could not do some of the problems, the teacher could provide "effort" points and give worked out examples for students to study (or offer the opportunity to come after hours).

The teacher should refer back to and ask questions about homework problems when teaching the next topic.