APPENDIX

A. COORDINATION SCENARIO

A.1 Bidding Strategies

We identify the bidding equilibria where no agent plays weakly dominated strategies (Assumption 1) for each possible combination of agents' choices of signals to observe.

When neither agent observes a signal, the other agent's bid conveys no information about value. In consequence, bidding the *a priori* expected value

$$\text{bid}_{\varnothing\varnothing} = \mathbb{E}[v] = \frac{1}{4}(1+2g)$$

is a weakly dominant strategy, and uniquely so.

When one agent observes a signal, the unique weakly dominant strategy for the observing agent is to bid the expectation of value given its signal:

$$\begin{aligned}\text{bid}_{j\varnothing}(L) &= \mathbb{E}[v \mid s_i^j = L] \\ &= v(G,G)p(G)p(G \mid L) + v(G,B)p(G)p(B \mid L) \\ &\quad + v(B,G)p(B)p(G \mid L) + v(B,B)p(B)p(B \mid L) \\ &= \frac{1}{2}ag + \frac{1}{2}(1-a)(1+g), \\ \text{bid}_{j\varnothing}(H) &= \mathbb{E}[v \mid s_i^j = H] = \frac{1}{2}a(1+g) + \frac{1}{2}(1-a)g.\end{aligned}$$

The weakly undominated bids for the unobserving coincide with the range of possible expected values,

$$\text{bid}_{\varnothing j} \in \left[\mathbb{E}[v \mid s_{-i}^j = L], \mathbb{E}[v \mid s_{-i}^j = H]\right].$$

Every bid in this range is an equilibrium bid in response to the observing agent's weakly dominant bid, because the unobserving makes zero profit regardless.

When both agents observe the same attribute (j), Assumption 1 restricts possible bids to the ranges

$$\text{bid}_{jj}(L) \in \left[\mathbb{E}[v \mid s_i^j = L, s_{-i}^j = L], \mathbb{E}[v \mid s_i^j = L, s_{-i}^j = H]\right],$$

$$\text{bid}_{jj}(H) \in \left[\mathbb{E}[v \mid s_i^j = H, s_{-i}^j = L], \mathbb{E}[v \mid s_i^j = H, s_{-i}^j = H]\right].$$

In these ranges, only one equilibrium exists. Each agent bids its expected value conditioned on its opponent signal being equal to its own (e.g. conditioned on its opponent seeing *High* if it saw *High*),

$$\begin{aligned}\text{bid}_{jj}(L) &= \mathbb{E}[v \mid s_i^j = L, s_{-i}^j = L] \\ &= v(G,G)p(G)p(G \mid LL) \\ &\quad + v(G,B)p(G)p(B \mid LL) \\ &\quad + v(B,G)p(B)p(G \mid LL) \\ &\quad + v(B,B)p(B)p(B \mid LL) \\ &= \frac{1}{2}\frac{(1-a)^2}{a^2+(1-a)^2} + \frac{1}{2}g, \\ \text{bid}_{jj}(H) &= \mathbb{E}[v \mid s_i^j = H, s_{-i}^j = H] \\ &= \frac{1}{2}\frac{a^2}{a^2+(1-a)^2} + \frac{1}{2}g.\end{aligned}$$

This corresponds to the famous result of Milgrom and Weber [6] that a symmetric equilibrium for SPSB with interdependent values is for each agent to bid its expected value, conditional on the most favorable opponent signal being equal to its own.[9] To see that it is the only equilibrium in weakly undominated strategies, suppose that one agent deviates to a point in the interior of one or both ranges. The ranges above and symmetry of signals ensure that its bid given a *High* observation is at least that of its bid given *Low*. The agent's opponent now can gain advantage by slightly underbidding it in the *Low* case, and/or slightly overbidding it in the *High* case. The agent's bid is clearly worse than matching the opponent, and so not in equilibrium. Indeed, the only stable point lies at the extremes of these ranges, as concluded above.

When the agents observe different attributes (j and $-j$), then the possible bids are restricted to the same ranges, and due to equal signal accuracy the situation is analogous to that above. Thus we conclude that the unique solution bids are the expected valuation conditioned on the opponent observing the same signal value:

$$\text{bid}_{j-j}(L) = \mathbb{E}[v \mid s_i^j = L, s_{-i}^{-j} = L] = (1-a)^2 + 2a(1-a)g,$$

$$\text{bid}_{j-j}(H) = \mathbb{E}[v \mid s_i^j = H, s_{-i}^{-j} = H] = a^2 + 2a(1-a)g.$$

A.2 Expected Utilities

See Appendix A.1 for the bid expressions. When neither agent observes a signal, they bid identically and make no utility:

$$u_{\varnothing\varnothing} = 0.$$

When one agent observes a signal, the unobserving agent wins only when its bid is equal to its expected value conditioned on winning, in which case it makes no utility:

$$u_{\varnothing 1}, u_{\varnothing 2} = 0.$$

The observing agent's profit depends on which equilibrium bidding strategy the unobserving agent employs. If the unobserving agent bids less than $\mathbb{E}[v \mid s_{-i}^j = H]$ then the observing agent wins and makes a profit every time it observes a *High* signal. If the unobserving agent bids $\mathbb{E}[v \mid s_{-i}^j = H]$ then the observing agent never makes a profit.

$$\begin{aligned}u_{1\varnothing}, u_{2\varnothing} &\le p(H)\big(\mathbb{E}[v \mid s_i^j = H] - \text{bid}_{\varnothing j}\big) \\ &\le \frac{1}{4}(2a-1).\end{aligned}$$

When agents observe the same signal, an agent profits only when it observes a *High* signal and its opponent observes a *Low* signal, since both agents bid identically when they observe equal signals,

$$\begin{aligned}u_{11}, u_{22} &= p(H,L)\big(\mathbb{E}[v \mid s_i^j = H, s_{-i}^j = L] - \text{bid}_{jj}(L)\big) \\ &= \frac{1}{4}(2a-1)\frac{a(1-a)}{a^2+(1-a)^2}.\end{aligned}$$

When both agents observe different signals, and agent similarly profits only when it observes a *High* signal and its opponent observes a *Low* signal,

$$\begin{aligned}u_{12}, u_{21} &= p(H,L)\big(\mathbb{E}[v \mid s_i^j = H, s_{-i}^{-j} = L] - \text{bid}_{j-j}(L)\big) \\ &= \frac{1}{4}(2a-1)[(1-a)(1-g) + ag].\end{aligned}$$

B. PRIVATE VERSUS COMMON SCENARIO

[9]See also discussions of this setting by Menezes and Monteiro [5, Theorem 5], Krishna [3, Section 6.2], and Wellman [10, Section 3.3.3.1].

B.1 Bidding Strategies

We identify the bidding equilibria where no agent plays weakly dominated strategies (Assumption 1) for each possible combination of agent' choices of signals to observe. When neither agent observes one of their private signals, all of the bidding strategies from the coordination scenario apply here (See Appendix A.1). This is clear when one of the common attributes from the coordination scenario is marginalized out, and both private attributes from the private versus common scenario are marginalized out.

When neither agent observes a signal, the weakly dominant bid is to bid their expected value:

$$\text{bid}_{\varnothing\varnothing} = \mathbb{E}[v] = \frac{1}{4}(1+p+c).$$

When one agent observes the common signal, and the other observes nothing, the observing agent bids its expected value given its signal. The unobserving agent bids in the range between its expected value given the observing agent saw a *Low* signal amd its expected value given the observing agent saw a *High* signal:

$$\begin{aligned}
\text{bid}_{C\varnothing}(L) &= \mathbb{E}[v \mid s_C^C = L] = \frac{1}{2}ap + \frac{1}{2}(1-a)(1+c),\\
\text{bid}_{C\varnothing}(H) &= \mathbb{E}[v \mid s_C^C = H] = \frac{1}{2}a(1+c) + \frac{1}{2}(1-a)p,\\
\text{bid}_{\varnothing C} &= \mathbb{E}[v \mid s_C^C = L] \geq \text{bid}_{C\varnothing}(L)\\
\text{bid}_{\varnothing C} &= \mathbb{E}[v \mid s_C^C = H] \leq \text{bid}_{C\varnothing}(H).
\end{aligned}$$

When both agents observe the common signal, an agent's equilibrium bid is to bid its expected value conditioned on its opponent seeing the same signal (Appendix A.1):

$$\begin{aligned}
\text{bid}_{CC}(L) &= \mathbb{E}[v \mid s_i^C = L, s_{-i}^C = L]\\
&= \frac{1}{2}\frac{(1-a)^2}{a^2+(1-a)^2}(1+c) + \frac{1}{2}\frac{a^2}{a^2+(1-a)^2}p,\\
\text{bid}_{CC}(H) &= \mathbb{E}[v \mid s_i^C = H, s_{-i}^C = H]\\
&= \frac{1}{2}\frac{a^2}{a^2+(1-a)^2}(1+c) + \frac{1}{2}\frac{(1-a)^2}{a^2+(1-a)^2}p.
\end{aligned}$$

When one agent observes its private signal, and the other agent observes nothing, each agent's unique weakly dominant bidding strategy is to bid its expected value conditioned on its signal or lack of signal. This result follows from the independence between an agent's value and its opponent's signal in this subgame,

$$\begin{aligned}
\text{bid}_{\varnothing P} &= \mathbb{E}[v] = \frac{1}{4}(1+p+c),\\
\text{bid}_{P\varnothing}(L) &= \mathbb{E}[v \mid s_P^P = L] = \frac{1}{2}ac + \frac{1}{2}(1-a)(1+p),\\
\text{bid}_{P\varnothing}(H) &= \mathbb{E}[v \mid s_P^P = H] = \frac{1}{2}a(1+p) + \frac{1}{2}(1-a)c.
\end{aligned}$$

When both agents observe their private signal, an agent's unique weakly dominant strategy is still to bid it expectation of value given its signal, due to value being independent of the opponent's signal,

$$\begin{aligned}
\text{bid}_{PP}(L) = \text{bid}_{P\varnothing}(L) &= \frac{1}{2}ac + \frac{1}{2}(1-a)(1+p),\\
\text{bid}_{PP}(H) = \text{bid}_{P\varnothing}(H) &= \frac{1}{2}a(1+p) + \frac{1}{2}(1-a)c.
\end{aligned}$$

When one agent observes its private signal and the other agent observes the common signal, the common-signal agent has a unique weakly dominant strategy, to bid its expected value conditioned on the signal it observed, due to value being independent of the opponent's signal,

$$\begin{aligned}
\text{bid}_{CP}(L) &= \mathbb{E}[v \mid s_C^C = L] = \frac{1}{2}ap + \frac{1}{2}(1-a)(1+c),\\
\text{bid}_{CP}(H) &= \mathbb{E}[v \mid s_C^C = H] = \frac{1}{2}a(1+c) + \frac{1}{2}(1-a)p.
\end{aligned}$$

The private-signal agent's bids are more complicated. Assumption 1 restricts possible bids to the ranges

$$\begin{aligned}
&\text{bid}_{PC}(L) \in \left[\mathbb{E}[v \mid s_P^P = L, s_C^C = L], \mathbb{E}[v \mid s_P^P = L, s_C^C = H]\right],\\
&\text{bid}_{PC}(H) \in \left[\mathbb{E}[v \mid s_P^P = H, s_C^C = L], \mathbb{E}[v \mid s_P^P = H, s_C^C = H]\right].
\end{aligned}$$

Given the unique weakly dominant bids of the common-signal agent, any strategy where the private-signal agent overbids the common-signal agent when the common-signal agent observes a *High* signal, and underbids when it gets a *Low* signal is an equilibrium:

$$\begin{aligned}
\text{bid}_{PC}(L) &< E[v \mid s_C^C = L] = \text{bid}_{CP}(L),\\
\text{bid}_{PC}(H) &> E[v \mid s_C^C = L] = \text{bid}_{CP}(H).
\end{aligned}$$

Therefore the upper bound on potential *Low* bids is the minimum of $\mathbb{E}[v \mid s_i^P = L, s_{-i}^C = H]$ and $E[v \mid s_c^C = L]$, and similarly for the lower bound on potential *High* bids. For simplicity we keep both bounds instead of taking the intersection,

$$\begin{aligned}
\text{bid}_{PC}(L) &\leq \mathbb{E}[v \mid s_P^P = L, s_C^C = H]\\
&\leq a^2c + (1-a)^2p + a(1-a),\\
\text{bid}_{PC}(L) &< \text{bid}_{CP}(L),\\
\text{bid}_{PC}(L) &\geq \mathbb{E}[v \mid s_P^P = L, s_C^C = L]\\
&\geq (1-a)^2 + a(1-a)(c+p),\\
\text{bid}_{PC}(H) &\leq \mathbb{E}[v \mid s_P^P = H, s_C^C = H]\\
&\leq a^2 + a(1-a)(c+p),\\
\text{bid}_{PC}(H) &\geq \mathbb{E}[v \mid s_P^P = H, s_C^C = L]\\
&\geq a^2p + (1-a)^2c + a(1-a),\\
\text{bid}_{PC}(H) &> \text{bid}_{CP}(H).
\end{aligned}$$

B.2 Expected Utilities

See Appendix B.1 for the bid expressions in this scenario. When neither agent observes a signal, they bid identically and make no utility:

$$u_{\varnothing\varnothing} = 0.$$

When one agent observes the common signal, and the other observes nothing, the unobserving agent wins only when its bid is equal to its expected value conditioned on winning, and it makes no utility:

$$u_{\varnothing C} = 0.$$

The observing agent may make a profit every time it observes a *High* signal, depending on how high the unobserving agent bids:

$$\begin{aligned}
u_{C\varnothing} &\leq p(H)\big(\mathbb{E}[v \mid s_C^C = H] - \text{bid}_{\varnothing C}\big),\\
&\leq \frac{1}{4}(2a-1)(c-p+1).
\end{aligned}$$

When both agents observe the common signal, they bid identically if they both observe the same signal value. Therefore, an agent only profits when it observes a *High* signal and its opponent observes a *Low* signal,

$$\begin{aligned} u_{CC} &= p(H,L)\big(\mathbb{E}[v \mid s_i^C = H, s_{-i}^C = L] - \mathrm{bid}_{CC}(L)\big) \\ &= \frac{1}{4}(2a-1)\frac{a(1-a)}{a^2+(1-a)^2}(c-p+1). \end{aligned}$$

When one agent observes its private signal, and the other agent observes nothing, then the private-signal agent wins the auction every time it observes a *High* signal,

$$\begin{aligned} u_{P\varnothing} &= p(H)\big(\mathbb{E}[v \mid s_P^P = H] - \mathrm{bid}_{\varnothing P}\big) \\ &= \frac{1}{2}\left(\frac{1}{2}a(1+p) + \frac{1}{2}(1-a)c - \frac{1}{4}(1+p+c)\right) \\ &= \frac{1}{8}(2a-1)(p-c+1). \end{aligned}$$

The unobserving agent wins the auction every time the private-signal agent observes a *Low* signal,

$$u_{\varnothing P} = p(L)\big(\mathbb{E}[v] - \mathrm{bid}_{P\varnothing}(L)\big) = \frac{1}{8}(2a-1)(p-c+1).$$

When both agents observe their private signal, an agent profits only when it observes a *High* signal and its opponent observes a *Low* signal,

$$\begin{aligned} u_{PP} &= p(H,L)\big(\mathbb{E}[v \mid s_i^P = H] - \mathrm{bid}_{PP}(L)\big) \\ &= \frac{1}{8}(2a-1)(p-c+1). \end{aligned}$$

When one agent observes the common signal and the other its private signal, then due to the equilibrium condition requiring the private-signal agent to underbid with a *Low* signal and over bid with a *High* signal, the private-signal agent makes profit only when it observes a *High* signal,

$$\begin{aligned} u_{PC} &= p(s_P^P = H, s_C^C = L) \\ &\quad \cdot \big(\mathbb{E}[v \mid s_P^P = H, s_C^C = L] - \mathrm{bid}_{CP}(L)\big) \\ &\quad + p(s_P^P = H, s_C^C = H) \\ &\quad \cdot \big(\mathbb{E}[v \mid s_P^P = H, s_C^C = H] - \mathrm{bid}_{CP}(H)]\big) \\ &= \frac{1}{8}(2a-1)(p-c+1). \end{aligned}$$

Due to the range of possible bids the private-signal agent can make, the profit of the common-signal agent can vary depending on the equilibrium bids of the private-signal agent. The following inequalities are required by Assumption 1 and equilibrium bidding:

$$\begin{aligned} u_{CP} &\le p(s_C^C = H, s_P^P = L) \\ &\quad \cdot \big(\mathbb{E}[v \mid s_C^C = H] - \mathbb{E}[v \mid s_C^C = L, s_P^P = L]\big) \\ &\quad + p(s_C^C = L, s_P^P = L) \\ &\quad \cdot \big(\mathbb{E}[v \mid s_C^C = L] - \mathbb{E}[v \mid s_C^C = L, s_P^P = L]\big) \\ &\le \frac{1}{8}(2a-1)^2(c+p-1) + \frac{1}{4}(2a-1), \\ u_{CP} &\ge p(s_C^C = H, s_P^P = L) \\ &\quad \cdot \big(\mathbb{E}[v \mid s_C^C = H] - \mathbb{E}[v \mid s_C^C = H, s_P^P = L]\big) \\ &\quad + p(s_C^C = L, s_P^P = L) \\ &\quad \cdot \big(\mathbb{E}[v \mid s_C^C = L] - \mathbb{E}[v \mid s_C^C = H, s_P^P = L]\big) \\ &\ge \frac{1}{8}(1+p+c) - \frac{1}{2}(a^2c + (1-a)^2p + a(1-a)), \\ u_{CP} &> p(s_C^C = H, s_P^P = L) \\ &\quad \cdot \big(\mathbb{E}[v \mid s_C^C = H] - \mathrm{bid}_{CP}(L)\big) \\ &\quad + p(s_C^C = L, s_P^P = L) \\ &\quad \cdot \big(\mathbb{E}[v \mid s_C^C = L] - \mathrm{bid}_{CP}(L)\big) \\ &> \frac{1}{8}(2a-1)(c-p+1). \end{aligned}$$

B.3 Efficiency

The efficiency of a profile is the ratio of the social welfare of that profile to the Maximum Social Welfare (MSW). The maximum social welfare is equal to the expected value of the winner in the subgame where one agent observes its private signal, and the other agent observes nothing (See Section 4.3 for details),

$$\begin{aligned} \mathrm{MSW} &= \mathbb{E}[\text{winner's } v_{P\varnothing}] \\ &= p(H)\,\mathbb{E}[v \mid s_P^P = H] + p(L)\,\mathbb{E}[v] \\ &= \frac{1}{4}(a(1+p) + (1-a)c) + \frac{1}{8}(1+c+p) \\ &= \frac{1}{8}((3-2a)c + (1+2a)(p+1)). \end{aligned}$$

The CC social welfare can be calculated in a similar manner,

$$\begin{aligned} \text{CC Social Welfare} &= \mathbb{E}[\text{winner's } v_{CC}] \\ &= \mathbb{E}[v] \\ &= \frac{1}{4}(1+c+p). \end{aligned}$$

The efficiency of the CC equilibrium is therefore

$$\begin{aligned} \text{CC Efficiency} &= \frac{\text{CC Social Welfare}}{\text{Maximum Social Welfare}} \\ &= \frac{2c + 2(p+1)}{(3-2a)c + (1+2a)(p+1)}. \end{aligned}$$

B.4 Symmetric Accuracy Relaxation

To relax the constraint that the private and common signals have the same accuracy, we split the accuracy into private and common accuracy:

$$\begin{aligned} \Pr[s_i^P = H \mid \omega = G] = \Pr[s_i^P = L \mid \omega = B] = a_P, \\ \Pr[s_i^C = H \mid \omega = G] = \Pr[s_i^C = L \mid \omega = B] = a_C. \end{aligned}$$

In every subgame except PC, agents observe only one type of signal (either *common* or *private*). Therefore the only accuracy that factors into the expected utility is the accuracy of the type of observed signal ($a \to a_C$ *or* a_P). For example, C from Table 2 goes from

$$C = \frac{1}{4}(2a-1)\frac{a(1-a)}{a^2+(1-a)^2}(c-p+1)$$

to

$$C = \frac{1}{4}(2a_C-1)\frac{a_C(1-a_C)}{a_C^2+(1-a_C)^2}(c-p+1).$$

The only case that qualitatively changes is when one agent observes the common signal while the other agent observes its private signal. In the PC bidding subgame, the bidding strategies change slightly, but retain the same structure as the bidding strategies when the accuracy was symmetric,

$$\begin{aligned}
\text{bid}_{CP}(L) &= \mathbb{E}[v \mid s_C^C = L] = \frac{1}{2}a_C p + \frac{1}{2}(1-a_C)(1+c),\\
\text{bid}_{CP}(H) &= \mathbb{E}[v \mid s_C^C = H] = \frac{1}{2}a_C(1+c) + \frac{1}{2}(1-a_C)p,
\end{aligned}$$

$$\begin{aligned}
\text{bid}_{PC}(L) &< \text{bid}_{CP}(L),\\
\text{bid}_{PC}(L) &\le \mathbb{E}[v \mid s_P^P = L, s_C^C = H]\\
&\le (1-a_P)a_C + (1-a_P)(1-a_C)p + a_P a_C c,\\
\text{bid}_{PC}(L) &\ge \mathbb{E}[v \mid s_P^P = L, s_C^C = L]\\
&\ge (1-a_P)(1-a_C) + (1-a_P)a_C p\\
&\quad + a_P(1-a_C)c,\\
\text{bid}_{PC}(H) &> \text{bid}_{CP}(H),\\
\text{bid}_{PC}(H) &\ge \mathbb{E}[v \mid s_P^P = H, s_C^C = L]\\
&\ge a_P(1-a_C) + a_P a_C p + (1-a_P)(1-a_C)c,\\
\text{bid}_{PC}(H) &\le \mathbb{E}[v \mid s_P^P = H, s_C^C = H]\\
&\le a_P a_C + a_P(1-a_C)p + (1-a_P)a_C c.
\end{aligned}$$

As in the symmetric accuracy case, the private-signal agent makes profit only when it observes a *High* signal,

$$\begin{aligned}
P \equiv u_{PC} &= p(s_P^P = H, s_C^C = L)\\
&\quad \cdot \big(\mathbb{E}[v \mid s_P^P = H, s_C^C = L] - \mathbb{E}[v \mid s_C^C = L]\big)\\
&\quad + p(s_P^P = H, s_C^C = H)\\
&\quad \cdot \big(\mathbb{E}[v \mid s_P^P = H, s_C^C = H] - \mathbb{E}[v \mid s_C^C = H]\big)\\
&= \frac{1}{8}(2a_P - 1)(p-c+1).
\end{aligned}$$

The common-signal agent only makes profit when the private-signal agent observes a *Low* signal,

$$\begin{aligned}
C \equiv u_{CP} &\le p(s_C^C = H, s_P^P = L)\\
&\quad \cdot \big(\mathbb{E}[v \mid s_C^C = H] - \mathbb{E}[v \mid s_C^C = L, s_P^P = L]\big)\\
&\quad + p(s_C^C = L, s_P^P = L)\\
&\quad \cdot \big(\mathbb{E}[v \mid s_C^C = L] - \mathbb{E}[v \mid s_C^C = L, s_P^P = L]\big)\\
&\le \frac{1}{8}(p+c+1) - \frac{1}{2}[(1-a_P)(1-a_C)\\
&\quad + p(1-a_P)a_C + c a_P(1-a_C)],
\end{aligned}$$

$$\begin{aligned}
u_{CP} &\ge p(s_C^C = H, s_P^P = L)\\
&\quad \cdot \big(\mathbb{E}[v \mid s_C^C = H] - \mathbb{E}[v \mid s_C^C = H, s_P^P = L]\big)\\
&\quad + p(s_C^C = L, s_P^P = L)\\
&\quad \cdot \big(\mathbb{E}[v \mid s_C^C = L] - \mathbb{E}[v \mid s_C^C = H, s_P^P = L]\big)\\
&\ge \frac{1}{8}(p+c+1) - \frac{1}{2}[a_C(1-a_P)\\
&\quad + c a_P a_C + p(1-a_P)(1-a_C)],\\
u_{CP} &> p(s_C^C = H, s_P^P = L)\\
&\quad \cdot \big(\mathbb{E}[v \mid s_C^C = H] - \mathbb{E}[v \mid s_C^C = L]\big)\\
&\quad + p(s_C^C = L, s_P^P = L)\\
&\quad \cdot \big(\mathbb{E}[v \mid s_C^C = L] - \mathbb{E}[v \mid s_C^C = L]\big)\\
&> \frac{1}{8}(2a_C - 1)(c-p+1).
\end{aligned}$$