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**Application of system identification in analysis of automobile
crash**

Gandhi, Umesh Nandlal, Ph.D.

The University of Michigan, 1993

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300 N. Zeeb Rd.
Ann Arbor, MI 48106

Application of System Identification in Analysis of Automobile Crash

by

Umesh N. Gandhi

A dissertation submitted in partial fulfillment
of the requirements for the degree of
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(Mechanical Engineering)
in The University of Michigan, Ann Arbor
1993

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TABLE OF CONTENTS

ACKNOWLEDGEMENTS	ii
LIST OF FIGURES	vii
LIST OF TABLES	xii
LIST OF ACRONYMS	xiv
CHAPTER	
1 INTRODUCTION	1
1.1 Problem definition and motivation	1
1.2 Thesis Organization	2
2 CRASHWORTHINESS ENGINEERING	8
2.1 Overview of crashworthiness requirements	8
2.1.1 Brief history of safety improvements in automobiles	8
2.1.2 Development of crash testing of automobiles	12
2.1.3 Development of injury criteria	14
2.1.4 Development of occupant simulators	17
2.1.5 NHTSA and the full car crash tests	18
2.1.6 Accident reconstruction	21
2.2 Analytical tools to evaluate crashworthiness	22
2.2.1 Mechanisms of crashes	22
2.2.1.1 Frontal impact mechanisms	23
2.2.1.2 Rear Impact mechanisms	26
2.2.1.3 Side Impact mechanisms	26
2.2.2 Analytical tools in crash analysis	28
2.2.2.1 Why analytical tools are needed	28
2.2.3 Tools for vehicle structure modeling and analysis	28
2.2.3.1 Fundamental principles of mechanics	29
2.2.3.2 Lumped parameter modeling (LPM)	30
2.2.3.3 Finite element modeling (FEM)	32
2.2.4 Occupant modeling	35

2.2.5	Combined models	36
3.	SYSTEM IDENTIFICATION METHODS IN CRASH ANALYSIS	38
3.1	Introduction	38
3.2	Importance of data based models in crash analysis	40
3.3	Overview of system identification methods	42
3.4	Design of experiments and data collection	42
3.5	Model selection	42
3.5.1	Black box model structure	44
3.5.2	Model structure with physical parameters	45
3.6	Parameter estimation methods	47
3.6.1	Prediction-error identification methods (PEM).....	47
3.6.2	Prediction error correlation	50
3.6.3	Recursive parameter estimation methods	50
3.7	Model validation	52
3.8	Review of applications of system identification methods in crash analysis	54
4.	DATA BASED APPROACH IN MODELING AUTOMOBILE CRASH	57
4.1	Introduction	57
4.2	Development of model structure	58
4.3	Correlating the model parameters with the structural characteristics	59
4.4	Estimation of parameters in data based models.....	62
4.4.1	Estimation of lumped masses	62
4.4.2	Estimation of stiffness and damping parameters	63
4.4.3	Estimation method	63
4.5	Applications	67
4.5.1	Verification of the approach	68
4.5.2	Frontal impact	69
4.5.3	Effect of sampling	76
4.5.4	Side impact	81
4.6	Use of data based model in designing for crashworthiness	84
4.7	Chapter summary	86

5.	ADDRESSING PROBLEMS IN ESTIMATING MODEL PARAMETERS FROM THE CRASH DATA	88
5.1	Introduction	88
5.2	Complexities in estimation of parameters from crash data	88
5.3	Simplify the model structure	89
5.4	Make use of physical insights	92
5.5	Modify the estimator	92
5.5.1	Modified recursive least squares (MRLS)	93
5.5.2	Intuitive explanation of the proposed algorithm	94
5.6	Applications	97
5.6.1	Verification of the MRLS	97
5.6.2	Application to test data	104
5.6.3	Comments on the results	113
5.7	Chapter summary	122
 6.	 DEVELOPMENT OF ANALYTICAL MODELS FOR SIDE IMPACT FROM TEST MEASUREMENTS	 124
6.1	Introduction	124
6.2	Side impact mechanisms	125
6.3	Review of lumped parameter models in the side impact	129
6.4	Model structure and estimation	130
6.4.1	Model structure	130
6.4.2	Estimation of parameters	133
6.4.3	Verification of KF in estimation of interface parameters	137
6.4.4	Application to the test data	142
6.5	Use of estimated model in understanding the crash event and design improvements	149
6.6	Application in design	153
6.7	Chapter summary	163
 7.	 CONCLUSIONS AND RECOMMENDATIONS	 166
7.1	Summary	166
7.2	Conclusions	168
7.3	Future work recommended	169

APPENDIX.	171
BIBLIOGRAPHY	172

LIST OF FIGURES

Figures in Chapter 2

2.1	Leading causes of occupant injury.	10
2.2	Schematic of a sled test.	15
2.3	Methods of determining human tolerance to impact.	16
2.4	Frontal impact test procedure (FMVSS 208).	20
2.5	Rear impact test procedure (FMVSS 301).	20
2.6	Side impact test procedure (FMVSS 214).	21
2.7	Schematic of unitized body structure of an automobile.	24
2.8	Frontal impact velocity distribution.	25
2.9	Lateral packaging space in side impact.	27
2.10	Lumped parameter model used in frontal impact.	31

Figures in Chapter 3

3.1	The system identification procedure.	43
3.2	Generalized transfer function of a dynamic system.	44

Figures in Chapter 4

4.1	Concept of lumped mass in side impact model.	61
4.2	Front impact Model.	68
4.3	Assumed parameter used to generate simulated data.	69
4.4	Comparison of assumed and estimated parameter #1.	70
4.5	Comparison of assumed and estimated acceleration for parameter #1.	70
4.6	Comparison of assumed and estimated parameter #2.	71
4.7	Comparison of assumed and estimated acceleration for parameter #2.	71
4.8	Measured data in frontal impact for three different tests.	72

4.9	Measured data in frontal impact – velocity.	72
4.10	Measured data in frontal impact – displacement.	72
4.11	Measured barrier force in frontal impact.	74
4.12	Estimated mass in frontal impact.	74
4.13	Estimation of discontinuities in parameters using AFMM approach for test#2.	74
4.14	Estimated parameter $K/M \cdot \text{crush}$, force per unit mass for three different tests.	75
4.15	Comparison of crush dependent force $K \cdot \text{crush}$, with the measured barrier force.	77
4.16	Comparing simulated acceleration with the measured and estimated acceleration, $p,q=1,2$. (test #1)	78
4.17	Comparing simulated acceleration with the measured and estimated acceleration, $p,q=1,2$. (test #2)	78
4.18	Comparing simulated acceleration with the measured and estimated acceleration, $p,q=3,4$. (test #3)	78
4.19	Comparison of simulated velocity and measured velocity at rocker.	79
4.20	Autocovariance of the residue and cross-covariance of the residue with the input for frontal impact. (e is the residue or error)	77
4.21	Measured accelerations sampled at two different sampling rates.	80
4.22	Estimated force ($K/M \times \text{crush}$) for the data with sampling interval $dt/t_0 = 1$ and $dt/t_0 = 0.1$, for test #4 and test #5.	80
4.23	Side impact measures data – accelerations.	82
4.24	Side impact measured data – velocity.	82
4.25	Side impact measured data – displacement.	82
4.26	Estimated lumped mass in side impact.	83
4.27	Estimated crush dependent force $K_1 (X_1 - X_2)$ between MDB and door, for $p,q = 0,0$, and $p,q=0,3$.	83
4.28	Estimated crush dependent force $K_2 (X_2 - X_3)$ between MDB and door, for $p,q = 0,0$, and $p,q=0,3$.	83

4.29	Comparison of measured and simulated accelerations.	85
4.30	Comparison of measured and simulated velocity.	85

Figures in Chapter 5

5.1	Flow chart of MRLS algorithm.	95
5.2	The model structure used for frontal impact.	98
5.3(a)	Simulated data in frontal impact at rocker and engine.	99
5.3(b)	Simulated data in frontal impact – velocity.	99
5.3(c)	Simulated data in frontal impact – displacement.	99
5.3(d)	Simulated barrier force in frontal impact.	101
5.4	Comparing assumed and estimated parameters K_1 and K_2 using RLS and MRLS.	101
5.5(a)	Barrier force, Comparison of RLS and MRLS estimates.	102
5.5(b)	Car acceleration, comparison of RLS and MRLS estimates.	102
5.5(c)	Engine acceleration, comparison of RLS and MRLS estimates.	102
5.6	The model structure used for side impact.	104
5.7(a)	Simulation : Acceleration at MDB and test car rocker.	105
5.7(b)	Simulation : Measured velocity in side impact.	105
5.7(c)	Simulation : Measured displacement in side impact.	105
5.7(d)	Simulation : Acceleration at door.	106
5.8	Comparing assumed and estimated parameters using RLS and MRLS.	106
5.9(a)	MDB acceleration, comparison of RLS and MRLS estimates.	107
5.9(b)	Door acceleration, comparison of RLS and MRLS estimates.	107
5.9(c)	Car acceleration, comparison of RLS and MRLS estimate.	107
5.10(a)	Measured data in frontal impact at rocker and engine.	111
5.10(b)	Measured data in frontal impact - velocity.	111

5.10(c)	Measured data in frontal impact - displacement.	111
5.10(d)	Measured barrier force in frontal impact.	112
5.11	Estimated parameters K_1 and K_2 .	112
5.12(a)	Barrier force, comparison of RLS and MRLS estimates.	114
5.12(b)	Car acceleration, comparison of RLS and MRLS estimates.	114
5.12(c)	Engine acceleration, comparison of RLS and MRLS.	115
5.13	Autocovariance of the residue and cross-covariance of the residue with the input for barrier.	115
5.14	Autocovariance of the residue and cross-covariance of the residue with the input for car.	116
5.15	Autocovariance of the residue and cross-covariance of the residue with the input for engine.	116
5.16(a)	Measured data in side impact on MDB and on car at the non struck side rocker.	117
5.16(b)	Measured data in side impact - velocity.	117
5.16(c)	Measured data in side impact - displacement.	117
5.16(d)	Measured acceleration at door in side impact.	118
5.17	Comparing assumed and estimated parameters using RLS and MRLS from test data.	118
5.18(a)	MDB acceleration, comparison of RLS and MRLS estimates for test data.	119
5.18(b)	Door acceleration, comparison of RLS and MRLS estimates for test data.	119
5.18(c)	Car acceleration, comparison of RLS and MRLS estimates for test data.	119
5.19	Autocovariance of the residue and cross-covariance of the residue with the input for MDB, side impact.	120
5.20	Autocovariance of the residue and cross-covariance of the residue with the input for door, side impact.	121

5.21	Autocovariance of the residue and cross-covariance of the residue with the input for car, side impact.	121
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Figures in Chapter 6

6.1	Side impact test procedure.	126
6.2	Schematic of dummy and the car structure in side impact.	127
6.3	Model structure for side impact.	131
6.4(a)	Model for structure only.	131
6.4(b)	Occupant to interior interface model.	131
6.5	Loading and unloading characteristics (schematic)	136
6.6	Flow chart for modified KF algorithm.	138
6.7	Forced displacement time history.	139
6.8	Verification : simulated response with and without added noise.	140
6.9	Verification : comparing estimated force–deflection data with the assumed data.	141
6.10	Measured dummy data for test #1, #2, #3 and #4.	143
6.11	Derived dummy input from the measured data.	144
6.12	Estimated force–deflection data for test #1, #2, #3 and #4.	147
6.13	Comparison of simulated response with measured data – padded interior.	150
6.14	Comparison of simulated response with measured data un–padded interior.	151
6.15	Comparison of forces on the dummy from upper door, lower door and car.	152
6.16	Comparison of forces on the spine from the rib and pelvis.	152
6.17(a)	Detail of structural changes in modifies car.	154
6.17(b)	Detail of trim changes in modifies car.	155
6.17(c)	Comparison of estimated structural stiffness of side structure for base car and modified car.	156

6.17(d)	Comparison of estimated interior stiffness of door for base car and modified car.	156
6.18	Measured dummy accelerations for test case A and B.	157
6.19	Input used in estimation, derived from measured data for test cases A and B.	158
6.20	Estimated parameters for test cases A and B.	160
6.21	Comparison of simulated acceleration response with measured test data.	161
6.22	Prediction in side impact for test cases A,B,C and D.	162
6.23	Comparing prediction of car with modified interior with the test case E, i.e. a car with similar (not same) content.	164

LIST OF TABLES

Tables in Chapter 2

2.1	Sequence of events and timing for zero degree frontal impact at 30 mph.	24
-----	---	----

Tables in Chapter 3

3.1	Comparison of system identification methods.	49
-----	--	----

Tables in Chapter 5

5.1.	Frontal impact model : mean and variance.	103
5.2.	Side impact model : mean and variance.	108
5.3.	The detail of the known parameters and the control parameters used in the MRLS estimation.	109
5.4.	Cross correlation for simulated data used in verification.	110

Tables in Chapter 6

6.1.	Verification : covariance of prediction error.	139
6.2.	MVMA test data, covariance of prediction error.	146
6.3.	Covariance of prediction error.	159

LIST OF ACRONYMS

AFMM	:	Adoptive forgetting through multiple modeling.
ARMA	:	Autoregressive moving average.
ARMAX	:	Autoregressive moving average exogenous.
AIS	:	Abbreviated injury scale.
BIOSID	:	BIOfeedlick side impact dummy.
EUROSID	:	European side impact dummy.
FE(M)	:	Finite element (Model).
FMVSS	:	Federal Motor Vehicle Safety Standards.
GM	:	General Motors.
KF	:	Kalman Filter
kg	:	Kilograms
LP(M)	:	Lumped parameter (Model).
MDB	:	Moving Deformable Barrier.
mph	:	Miles per hour.
MRLS	:	Modified recursive least squares.
MVMA	:	Motor vehicle manufacturers association.
ms	:	millisecond.
N	:	Newtons.
NHTSA	:	National highway traffic safety administration.
PEM	:	Prediction–error identification methods.
RLS	:	Recursive least squares.
RML	:	Recursive maximum likelihood.
SAE	:	Society of automotive engineers.
SID	:	Side Impact Dummy.
TTI	:	Thoracic trauma index.

CHAPTER 1

INTRODUCTION

1.1 Problem definition and motivation

In 1991, 54,724 motor vehicles were involved in 36,895 fatal crashes resulting in 41,462 deaths, IIHS[1992]. The major cause of occupant injury during an automobile crash is the high accelerations of the occupants or a severe intrusion type loading of the occupant. It is desirable to design the automobile structure and interior such that injury to the occupant during the crash is minimized. The crash performance of the automobile largely depends on the ability of its structure to smoothly absorb the kinetic energy and maintain the integrity of the occupant compartment. To ensure the desired crash performance of the automobiles, extensive testing as well as analysis are performed during the early stages of design. In the last few years, emphasis on the use of analytical tools in design for crashworthiness has increased as a result of the rising cost of building prototypes and the shortening of the product development cycle.

Presently, lumped parameter modeling (LPM) and finite element modeling (FEM) are the most commonly used analysis techniques in the industry. Both of these techniques are 'ground up methods', in which the knowledge available at the lowest level of the system is integrated to represent the whole system using mathematical relationships. Both of these techniques have been used in automotive analysis for more than twenty years.

A full car crash is a very complicated event. Therefore, most of the analytical models have limited success in duplicating the mechanisms involved in a full car crash. As a result, testing in the early stages of design is usually considered necessary. The

purposes of crash tests are to verify the design objectives, to validate the analytical tools and to increase the understanding of the crash performance of the design. Although the current analytical tools are not sophisticated enough to replace the actual testing, they are helpful in providing 'design directions' and therefore reducing the number of tests.

The purpose of this thesis is to develop analytical models for crashworthiness analysis of automobiles from the readily available crash test data, and then demonstrate the use of the analytical models in predicting crash performance and improving the understanding of the crash event.

Crashworthiness can be defined as the property of a vehicle that contributes to the vehicle performance under crash conditions. The analytical models are the mathematical representation of the crashworthiness properties of an automobile that can be used to simulate the crash event. Crashworthiness engineering refers to designing cars for improved crash performance using analytical methods, testing and understanding of the crash event.

The objectives of the thesis are accomplished by developing data based analytical models (also called data based models) with physically meaningful structure and then estimating the model parameters from the available test data using system identification techniques.

1.2 Thesis organization

The thesis is organized into seven chapters. This chapter, the introduction, explains the purpose of the thesis and its organization. Chapter 2 provides an insight into crashworthiness engineering. It includes the objectives of crash design and an overview of existing practices in crash testing and analysis. Chapter 3 presents a literature review of the system identification methods. Data based models for crash simulation are developed in Chapter 4. In Chapter 5, the problems in developing data

based models for crash are recognized and approaches to address those problems are presented. A data based model for side impact analysis is developed and its use in design is demonstrated in Chapter 6. More details for each chapter are presented below.

Chapter 2 consists of two sections. Section 2.1 presents a brief history of the development of crashworthiness concerns, testing procedures and government regulatory activities. A brief overview of the development of the injury criteria, occupant simulators, full car test procedure required by the NHTSA (National Highway Traffic Safety Administration) and post accident analysis techniques is also presented. Since a single comprehensive reference dealing with the crashworthiness engineering is unavailable outside this thesis, Section 2.1 of Chapter 2 becomes a necessary and valuable part of the thesis.

An overview of the analytical techniques currently used in crash analysis is presented in Section 2.2 of Chapter 2. First, the injury causing mechanisms involved in the frontal, rear and side crashes are discussed and then the related occupant protection criteria are presented. Currently used analysis techniques are presented along with a detailed discussion of their advantages, disadvantages and limitations. The 'physical insights' on the crash performance of automobiles provided in Chapter 2 are useful in developing analytical models for crash simulation in the later chapters.

In Chapter 3, the need for the data based models in crashworthiness is first established in view of the existing testing and analysis procedures in the industry. The system identification methods are ideally suitable mathematical tools for developing data based models. An overview of the system identification theory, modeling, parameter estimation methods and their use in crashworthiness are also presented in Chapter 3.

In Chapter 4, the proposed approach is demonstrated for the development of a simple data based model for frontal and side impacts using existing parameter identification tools. The selected model structure consists of a lumped parameter model structure appended with a transfer function type autoregressive moving average (ARMA) model. The lumped parameter part of the model represents the source of rigid body response in the measurements i.e. major structural behavior while the ARMA part of the model represents the source of vibration response in the measurements. A generalized approach to correlating the lumped parameters like mass, stiffness and damping with the physical characteristics is established and then the lumped parameters and ARMA parameters are identified simultaneously by minimizing the prediction error using a Gauss-Newton iterative algorithm. The non-linearity of the parameters is addressed using recursive estimation with the forgetting factor. The following objectives are achieved in this chapter:

- 1) Development of a physically meaningful model structure.
- 2) Development of a generalized approach to establish the correlation between the model structure and physical parts of the system.
- 3) Demonstration of the proposed approach for simple examples.

Chapter 5 addresses the complexities involved in developing data based models from crash test data. A review of these complexities is first presented. Based on the review it is recognized that the crash is a highly nonlinear event with rapidly changing parameters and high noise content in the measurements. The part of the crash of interest is transient and passes rapidly. Therefore it is desirable to track the parameters as soon as they change. The interaction among various components during a crash is extensive and therefore a detailed model is necessary.

It can be seen that conflicting requirements need to be addressed while developing the data based models. For example, a detailed model with a large number

of unknown parameters is desired to include the complexities of the event. On the other hand, the increase in the number of unknown parameters creates difficulties in identifying them. Similarly, the rapidly changing parameters demand an 'alert' estimation procedure, whereas high noise content in the output demands a 'sluggish' parameter tracking system. Otherwise the noise will 'misguide' the 'alert' estimator. Therefore Chapter 5 essentially deals with the development of 'compromise' procedures for modeling and parameter identification. These are addressed by:

- 1) Simplifying the model structure,
- 2) Making use of physical insights, and
- 3) Modifying the estimator to meet the specific needs.

It was realized, in Chapter 4, that the presence or absence of an ARMA part of the model did not make a significant change in the estimation of the lumped parameters. Therefore, if only structural parameters are of interest, the ARMA part of the model can be dropped. This results in a simpler - linear model structure (though the parameters are time varying). A computationally efficient recursive least squares (RLS) technique was applied for the estimation of time varying parameters on the linearized model structure. The physical insight or the known information about the event was used in selecting initial values for the unknown parameters and in selecting noise characteristics of the event.

RLS worked well with the examples considered in Chapter 4, but with a more complex model the prediction error continued to remain high. This was because of the difficulties in choosing an adequate forgetting factor. The presence of a high noise/signal ratio required a low forgetting factor while the changing parameters required a higher forgetting factor. Modifications to the well known RLS algorithm are implemented to address these difficulties and was called Modified Recursive Least Squares (MRLS). In the MRLS algorithm, along with minimization of the sum of the

squares of prediction error, the moving average of prediction error is maintained within a pre-specified limit. This is accomplished as follows: the moving average or the prediction error is monitored and when it exceeds the pre-specified limits the forgetting factor is altered and the estimation is repeated on the last few observations. This corrective action is repeated until the moving average of the prediction error is within the pre-specified limits. This modification is based on the assumption that the frequency of change in the parameters is much lower compared with the frequency of change in the noise (noise has zero mean). The improvement in the estimation with MRLS over RLS is demonstrated with both simulated and test data.

Chapter 6 presents an application of system identification techniques in the side impact analysis in an actual design environment. A lumped parameter model for the structure-to-dummy interface is developed. The unknown model parameters are estimated using a Kalman filter (KF) based estimator. The KF based estimator is similar to the RLS estimator except for the parameter correction mechanism. The formulation is such that incorporating the known information and physical insights is easier. Knowledge of structural properties during loading and unloading is also included.

The model structure considered in Chapter 6 is intended for use in the actual design. The model is verified using simulated data and then used on a number of test data. An example is also presented to demonstrate the use of this model in developing 'design directions' for an actual design.

Another use of the data based models demonstrated in Chapter 6 is increased insight into the test data. The model enables the analyst to estimate contributions of individual components to the global performance of the car. This is a helpful insight in developing design directions.

Summary and conclusions of the thesis are presented in Chapter 7. The limitations of the approach are recognized and a list of recommendations for the future research are also presented.

CHAPTER 2

CRASHWORTHINESS ENGINEERING

The design efforts to improve occupant protection in an automobile crash based on the understanding, testing and analysis of crash events are called crashworthiness engineering. In this chapter an overview of the current status of crashworthiness engineering in the automotive industry is presented.

2.1 Overview of crashworthiness requirements

In this section a brief overview of the developments of crashworthiness concerns, developments in the crash testing procedures, an understanding of the injury criteria and a review of governmental involvement in automobile crash testing is provided.

2.1.1 Brief history of safety improvements in automobiles

The automobile revolution began in the US in the early 20th century. By 1930, largely due to the developments in mass production techniques, the number of automobiles on the road increased to 25 million, Kamal et al. [1982]. The increased number of vehicles resulted in increased traffic and hence an increased number of accidents.

In the early days (1900-1937) engineering efforts were aimed at improving the comfort, handling, reliability and durability of a vehicle. This resulted in a number of developments such as self starter, safety glass, improved tires, four wheel brakes, all steel bodies, etc. Stonex [1970]. Though these developments were not directly aimed at improving occupant safety, they indirectly helped crashworthiness by improving crash

avoidance and structural performance. As a result of these developments the fatality rate dropped from 30 per 100 million miles driven in 1909 to 15.9 per 100 million miles driven in 1937, Cheng et al. [1987].

Along with improvements in the vehicle, the amount of miles driven also increased. This resulted in an increase in the total number of fatalities. Conferences on street and highway safety were held in 1924 and 1926. These conferences looked at the traffic problems and proposed guidelines for pedestrian and vehicle behavior on the road. As a result, limited regulatory efforts began in these early days, mainly implemented by state governments.

In 1953, under a grant from the US army, the Cornell University Accident Research Center started investigating automotive injuries. By 1955, based on studies of over 70,000 accident reports, major causes of injury in automobile accidents (see Figure 2.1) were identified, Nader [1972]. This study indicated a need for a restraint system to minimize the relative movement of the occupant within the automobile. It also expressed the need for an improved hinge and greater lock strength between the door and the vehicle body.

Major car manufacturers readily accepted the findings of this study and made modifications in their automobiles. By 1956, seat belts (lap belts only) were offered by most car manufacturers, Kamal et al. [1982]. Improvements like safety padded interiors, recessed panel knobs on the door, etc. were also implemented. To maintain the integrity between the door and the car body the door latch and hinge were designed with increased strength.

'Unsafe at any speed', written by Ralph Nader, first appeared in 1965 and was updated in 1972. Nader alleged that the major car manufacturers were not making sufficient efforts in improving the safety of the occupants in the automobiles. Many of

LEADING CAUSES OF INJURY **RANKED BY TWO METHODS**

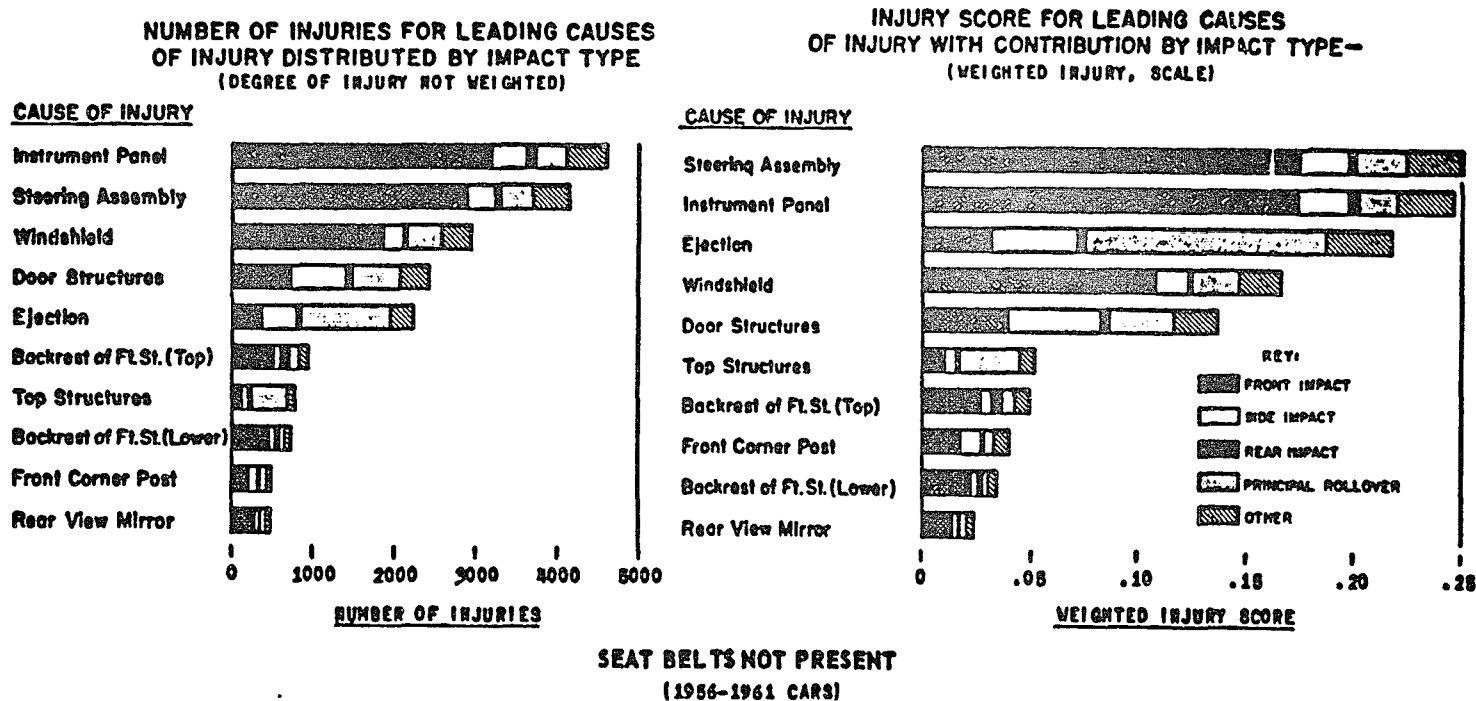


Fig 2.1 Leading causes of occupant injury, Nader[1972]

his allegations may have been incorrect. Nevertheless, they caused enough of a public uproar to make the US Congress pass the 'National Traffic and Motor Vehicle Safety Act' in 1966. Soon an administrative body called the NHTSA (National Highway Traffic Safety Administration) was established. Since then, NHTSA has played a very active role in developing regulations to improve the safety of occupants in automobiles, Kahane [1984]. Some of the safety related improvement are listed below.

- 1) Collapsible steering columns: By the late 1950's it was realized that the steering column was one of the major causes of injury in a frontal impact. After extensive testing and development, the energy absorbing steering columns became available at GM from 1967, Horsch et al. [1991].
- 2) Shatter proof laminated windshield: Improved windshield with a laminate structure that will not allow the glass to shatter came into production in 1967, Yanik [1983].
- 3) Side guard beam: As an improvement in the side structure crashworthiness the side guard beam was introduced by GM in 1969, Yanik [1983].
- 4) Air bag: The concept of the air bag evolved in the early 1950's. However, after some experiments both GM and Ford gave up on the idea due to practical difficulties like sensing a real crash and triggering the air-bag with the correct pressure within a few milliseconds, Lundstrom [1970]. It took some time for the engineers to overcome these problems. In 1974 GM offered an optional airbag. According to recent regulations by NHTSA, air-bags will be mandatory for all passenger cars beginning in the model year 1995.
- 5) Seat belts: A regulation requiring a three point restraint system (FMVSS 208, where FMVSS stands for Federal Motor Vehicle Safety Standard) for the front seat and lap restraints for all the other seating positions became

effective in 1970. The field study indicated that there was a reduction in severe or fatal injuries with the use of a restraint system. The field study also indicated that the use of passive restraint systems was limited. As a result, after a long deliberations, NHTSA passed a regulation requiring automatic restraint systems or a supplemental inflatable restraint system (SIR) to be phased in from the 1987-90 model years.

Many other improvements like four wheel drive, anti-lock brakes, improved steering, head-up displays etc. have been offered on many models in the 1980's. Lately, emphasis is being shifted on accident prevention by designing intelligent transportation systems, i.e., cars that can sense the possibility of a collision and take corrective actions.

As a result of all the above mentioned and many other improvements, the fatality rate dropped to 4 per/100 million miles driven in 1987.

2.1.2 Development of crash testing of automobiles

Since the early 1960's, it was realized that by optimizing the design of the structure, the crashworthiness of automobile can be greatly improved. The idea behind structural improvement is to manage the dissipation of the vehicle kinetic energy in such a way that the structural intrusion into the occupant area and the deceleration of the structure and occupant are minimized. This offers a great challenge to today's structural engineers. When this work began in the 1950's and 1960's, analysis techniques were quite rudimentary and engineers largely depended on testing to develop an understanding and make improvements to the crashworthiness of automobiles.

Testing for crashworthiness can be divided into three categories :

- 1) **Full car testing:** These tests are performed to evaluate the overall occupant and structural performance and interaction of various components with each other. See Wilson [1970], Sinke and Prevost [1970] for details.
- 2) **Sled testing:** These tests are mainly performed to study occupant to automotive interior interactions. A dummy and the limited part of the interior are accelerated at very high rate on a rigid sled (see Figure 2.2). See Kipp [1970] and Veenstra [1968] for details.
- 3) **Component testing:** Normally performed to evaluate the force - deflection characteristic of a particular component. Static crushes and drop-silo techniques are commonly used. See Wilson [1970] for details.

The crash test procedure basically involves three major tasks :

- 1) **Test setup:** This involves making fixtures for the static components and guiding the moving barrier or vehicle in the desired direction and speed.
- 2) **Taking the measurement:** This involves recording various parameters like displacement, deceleration, forces, time, and using high speed photography (500 to 2000 frames/sec).
- 3) **Interpretation of test observations and post crash analysis.**

A description of a typical test facility is available from Sinke and Prevost [1970]. The development of instrumentation is discussed by Wilson [1970] and photography techniques have been presented by Staffeld [1970]. A short history of the development of automobile testing is presented by Wilson [1970], Lundstrom [1970] and Barr [1970].

The first barrier impact tests began at GM in 1934. A vehicle was directed at a low speed into a rigid wall to evaluate its structural integrity. By 1940 high speed motion

pictures were being taken to record events in the barrier impact tests. During this early period very little data was measured during the test. The deformed shape after the crash was studied to develop an understanding for design improvements.

In the 1950's, as a result of statistical studies of the accident data, the emphasis was put on improving the interior. To evaluate the interaction between the occupant and the interior, sled rigs were developed as an alternative to the full car barrier test. The cost of a sled test is much less than a full car crash test. Sled rigs are designed to produce an acceleration (instead of a deceleration) on a cart carrying a dummy in the car interior, Wilson [1970] (see Figure 2.2).

The first completely enclosed barrier impact test facility was operational at GM by 1969. The facility is described in detail by Sinke and Prevost [1970]. Since 1970 the basic test setup has not changed significantly. Improvements in instrumentation and recording devices have of course been made along with new development in electronics and computer technology.

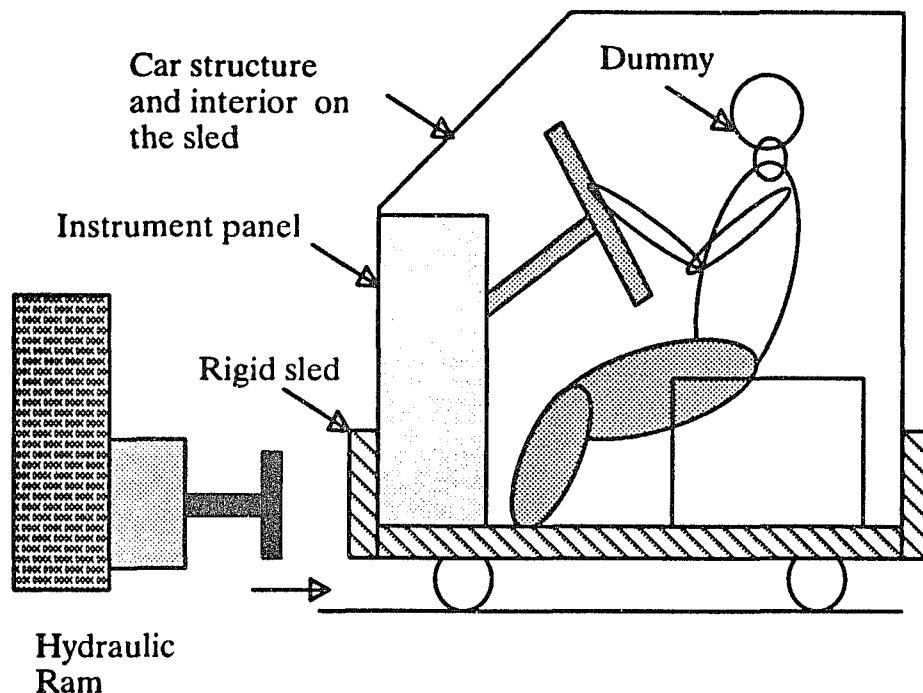
2.1.3 Development of injury criteria

While considering human safety in automotive crashes the fundamental question is, "How do we correlate physically measurable quantities like acceleration, deformation, force, etc. with injury to human organs?".

In the early 1950's, Stapp [1956,1957] and Dehaven [1969] did pioneering work in improving the understanding of injury causing mechanisms in automobile accidents. Snyder [1970] and Petrucelli [1981] provide a good survey of the developments in finding the human tolerance to impact. The basic methodology is presented in Figure 2.3. Voluntary human subjects were used for light non-harming impacts. Tests with increased severity were performed on anesthetized animals and human cadavers. Measurements like acceleration and deformation recorded during the tests were

correlated with the severity of injury index determined through the post impact surgery. The Abbreviated Injury Scale (AIS) is one of the popularly used injury indices. Basically, the AIS is a subjective numerical expression (on a 1 to 6 scale) of the severity of injury (established through post impact surgery).

In the last decade D. Viano [1978, 1987, 1989] at GM has conducted several experiments to correlate human injury with the measurable parameters such as rib deformation and rib velocity in the side impact. It should be noted that because most human organs are highly viscous, the rate of loading highly influences the injury, therefore many of the injury criteria developed are time dependent.



The hydraulic ram is designed to create a predetermined velocity profile on the rigid sled. The sled is designed to have a representative car interior relative to the dummy. The idea is to simulate the dummy-to-interior interaction as in a frontal collision.

Fig. 2.2 Schematic of a sled test

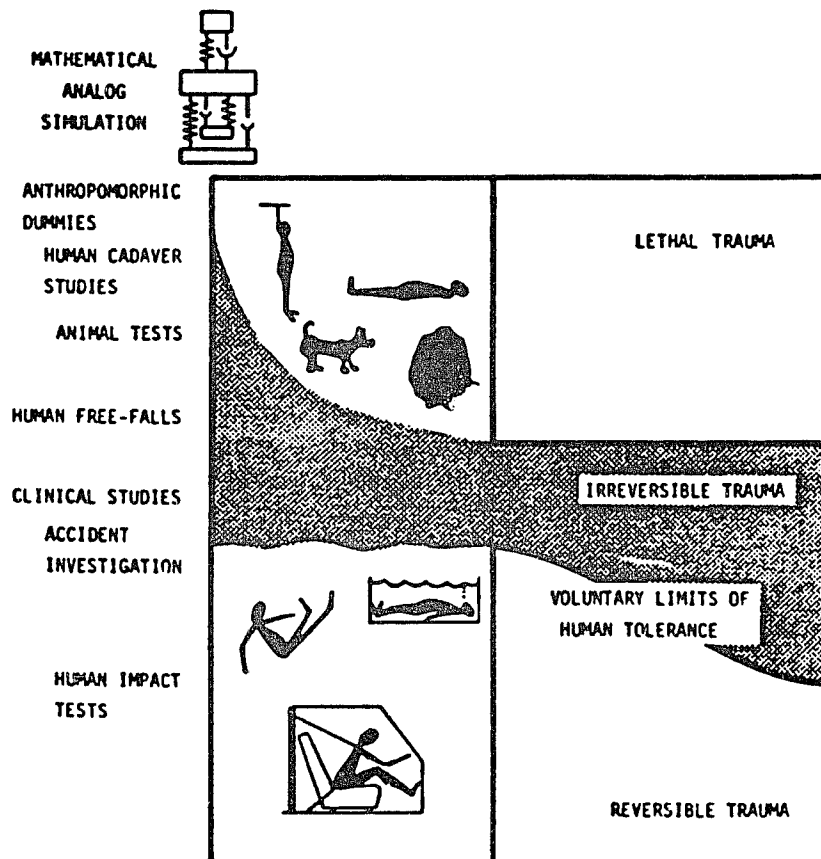


Fig. 2.3 Methods of determining human tolerance to impact.
Snyder[1970]

2.1.4 Development of occupant simulators

The presence of an occupant in the test cars in the early days was often simulated using gravel bags. Their purpose was to get a proper mass distribution. With improvements in the technology more sophisticated occupant simulators, popularly known as dummies, were developed. The dummies were designed to have a human like response in terms of stiffness and weight distribution. Also, the dummies were instrumented to measure various physical quantities like acceleration, deflection, force etc.

The development of dummies is discussed in detail by Alderson [1967], Cheng et al. [1987] and Wilson [1970]. Alderson Laboratories, Inc. was one of the first to develop an anthropomorphic dummy, which they called Mark I. Mark I was available for sale by 1954. Sierra Engineering Co. developed a highly articulated dummy called "Sophisticated Sam" in 1967 for GM, Bloom et al. [1968]. An SAE task group collected the available information and provided an SAE recommended practice to standardize dummy development, SAE [1972]. GM completed the development of Hybrid II in 1972 and it was accepted as a standard dummy for the evaluation of restraint systems in the frontal impact certification tests, FMVSS208, GM [1973]. Hybrid III, the improved version of Hybrid II was developed at GM in 1976. The 5th percentile female dummy was developed at GM in 1982.

The dummies discussed so far were designed to produce a human-like response in the frontal impact. The dummies in current use, Hybrid II and Hybrid III, are designed to measure the head and chest accelerations and femur loads. In addition, Hybrid III allows a number of other measurements, (e.g. sternum deformation, neck and ankle load and tibia load).

In a side impact the injury criteria are different from that in a frontal impact. Therefore, different dummies have been developed. After extensive testing and study of available crash data, the NHTSA developed SID (Side Impact Dummy), Viano [1987a]. The European car manufacturers developed a side impact dummy and called it EUROSID. The Biomedical Engineering Laboratory at GM developed its own side impact dummy and called it BIOSID, Beebe [1990]. These three dummies are different in construction and have different injury criteria. The injury criteria used with SID is pelvic accelerations and TTI (thoracic trauma index). TTI is the average of the peak acceleration at the thorax rib (upper or lower) and spine. The recommended injury criteria for BIOSID and EUROSID are deflection and rate of deflection of the thoracic ribs. A comparison of these three dummies is presented by Kaninthara et al. [1991].

2.1.5 NHTSA and the full car crash tests

NHTSA developed a number of safety standards which were mandatory for the automobile manufacturers. The purposes of these standards were to improving the crash performance while maintaining practicality in usage of the vehicles within the limitations of the existing manufacturing technology.

The initial standards set by NHTSA formalized the existing safety related design practises used by the auto companies. Gradually, based on the accident studies and in cooperation with the industry, more regulations were implemented. A study of the influence of the NHTSA standards on improving safety is available in Kahane [1984]. The NHTSA safety standards, also called FMVSS, (Federal Motor Vehicle Safety Standards) includes regulations for accident prevention (FMVSS 100 series), injury protection (FMVSS 200 series) and post accident protection (FMVSS 300 series). There are also many other standards aimed at improving the safety of the occupants.

Full car dynamic tests as well as component level static and dynamic tests are necessary to verify the compliance with the above safety standards. Only the full car tests are discussed below :

- 1) **Frontal Impact:** The concerns of these tests are occupant protection, fuel leakage and structural integrity (FMVSS 208, 301, 212 and 219). In full car frontal test, a test vehicle with two instrumented dummies (Hybrid II or Hybrid III) in the front seat strike a stationary rigid barrier at 30 mph (see Figure 2.4). The injury parameters of concern are measured as HIC (head injury criteria), chest acceleration and femur load.
- 2) **Rear Impact:** The concern here is fuel leakage (FMVSS 301). A flat moving barrier weighing 8800 lb hits a stationary car from the rear at 30 mph. No dummies are used since the test is for fuel system integrity only (see Figure 2.5).
- 3) **Side Impact:** The concern here is occupant protection. This is a recently added requirement to the existing FMVSS 214 standard, scheduled to be phased in during the 1994-97 model years. In the full car test a deformable moving barrier representing a typical car strikes the side of the test vehicle at 33.5 mph. Two side impact dummies (SID) are located on the struck side in front and rear seats of the vehicle (see Figure 2.6). The injury criteria are the thoracic trauma index (TTI) and pelvic acceleration. An additional test required by NHTSA is the FMVSS 301 side impact test. For this test a flat moving wall, weighing 8800lb, strikes the stationary test car on the side at 20 mph. The purpose of this test is to check the fuel system integrity.

The full car test procedures are based on extensive field studies by various government agencies, research organizations and the auto industry. The NHTSA required test procedures are supposed to represent the most probable mode of

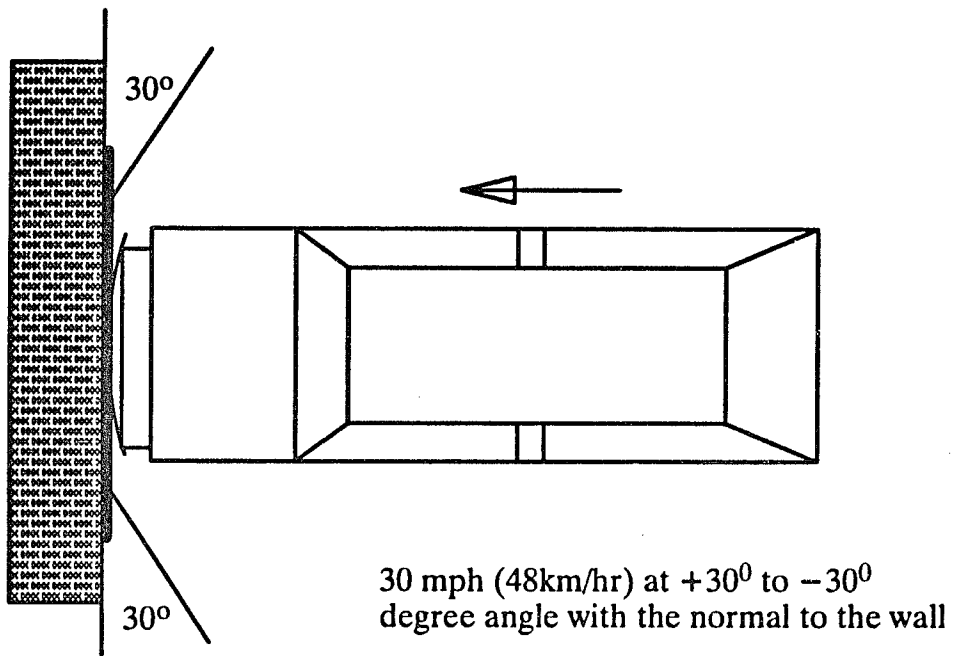


Fig. 2.4 Frontal impact test procedure (FMVSS 208)

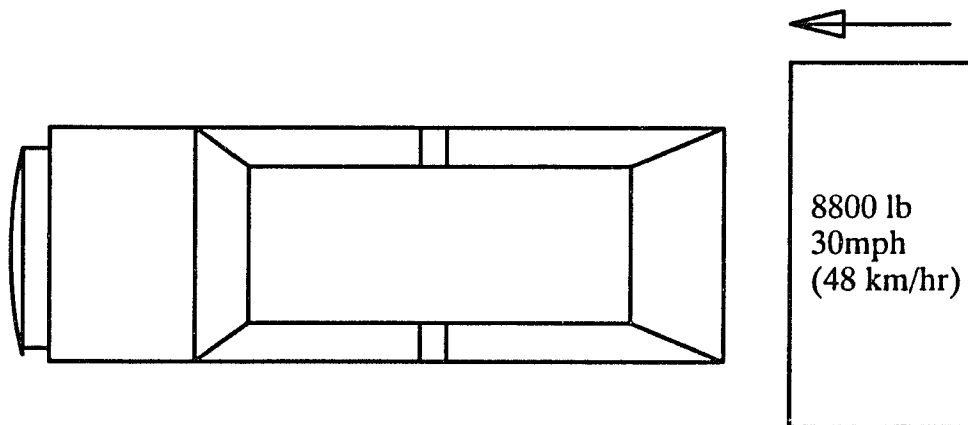


Fig. 2.5 Rear impact test procedure (FMVSS 301)

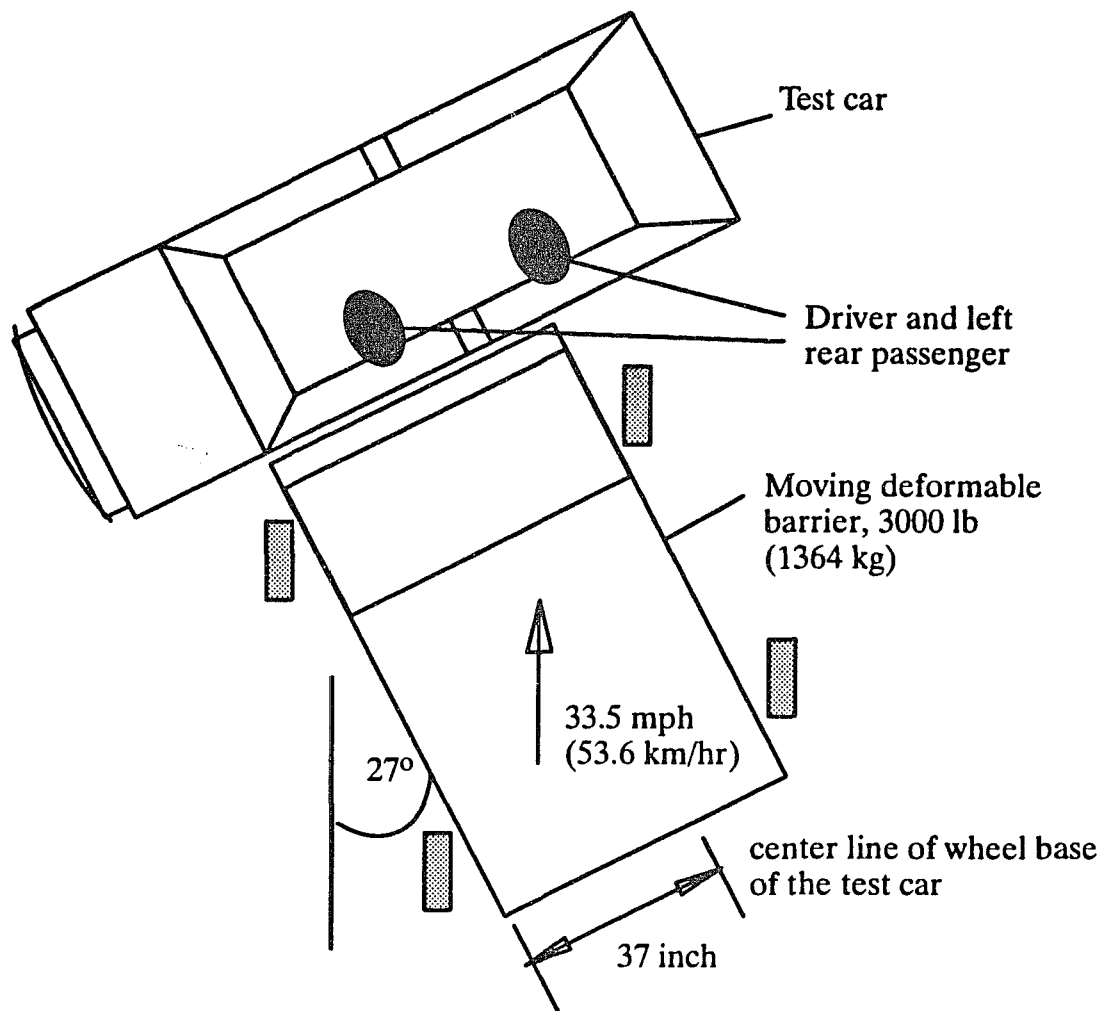


Fig. 2.6 Side impact test procedure (FMVSS 214)

collision. Further details about the FMVSS test procedures and requirements can be obtained from Title 49 of the United States Code of Federal Regulation parts 510 through part 582.

2.1.6 Accident reconstruction

Accident reconstruction has been one of the major sources of learning for safety engineers, Keskin et al. [1992], Martinez [1985]. Studies of traffic behavior and accidents have been conducted for a long time and extensive data with statistical analyses are available from the NHTSA.

CRASH and SMAC, developed through funding from NHTSA, are the most popular vehicle accident analysis programs in use today. Both these programs have been developed in the last 20 years. They are discussed in detail by McHenry [1973,1975], Cheng et al. [1987], Day and Hargens [1985].

2.2 Analytical tools to evaluate crashworthiness

With increased customer consciousness regarding safety, the auto manufacturers are not only working hard to meet the mandatory FMVSS requirements, but also are making additional efforts to improve the crashworthiness of automobiles under worst case accident conditions. At the same time, stringent corporate average fuel economy (CAFE) standards from the Environmental Protection Agency (EPA) require weight reduction, resulting in the down-sizing of automobile structures. These two conflicting design requirements have demanded additional performance from the structure to meet the crashworthiness goals.

A full car test is a very complicated event. It lasts for only about 100-150 ms (millisecond) and involves extensive interaction between various components of the structure and the occupant. Due to the complexities of the test and the number of variables involved, the task of understanding and improving the crash performance of an automobile is quite monumental.

2.2.1 The mechanisms of crashes

The purpose of frontal, rear and side impact tests, based on FMVSS test procedures, is to evaluate structural performance and occupant injury (for side and frontal) in real-world accidents. The mechanisms involved in frontal, rear or side impact have very little in common. The crush modes are different. Performance goals are also different and hence different design and analysis approaches are needed.

2.2.1.1 Frontal impact mechanisms

According to FMVSS 208, the test procedure for the frontal impact, the test vehicle moves at 30 mph (48 km/hr) and impacts a rigid stationary wall. Assuming the vehicle mass to be approximately 3300 lb (1500 kg, which is typical for a mid-size car with two passengers), the kinetic energy will be approximately 133 kN.m Assuming the barrier wall to be rigid, all the kinetic energy needs to be dissipated through the deformation of the front end of the automobile structure. This is comparable to a head collision with a similar vehicle traveling at 30 mph in opposite direction.

The sequence and timing for a frontal crash of a typical mid-size car is presented in Table 2.1. The vehicle reaches 0 velocity from 30 mph in approximately 80 to 100 milliseconds(ms). The bumper is the foremost component and hence it gets loaded first. Within a few milliseconds the upper rail, mid rail and the suspension rails also get loaded. A schematic of a unitized body structure is presented in Figure 2.7. The volume occupied by the non-structural components like the engine, tires, brake booster etc. plays an important role in the load distribution and influences the crush modes of various structural components. Therefore, packaging of these (non-crush) members is one of the most important crashworthiness design considerations.

When the vehicle strikes the barrier and starts to decelerate, the occupant, (a test dummy, weighing 186 lb) if not restrained to the vehicle, keeps moving forward relative to the vehicle and hits the instrument panel or steering wheel in about 80 to 100ms (i.e. when the vehicle is rebounding). This sudden encounter of the occupant with the vehicle interior, also called a 'secondary impact', is considered one of the major causes of injury in a frontal impact. The simplest approach to reduce the occupant injury is to restrain the dummy to the car, so that the occupant also follows the deceleration of the vehicle and hard contact with the interior can be avoided. The other

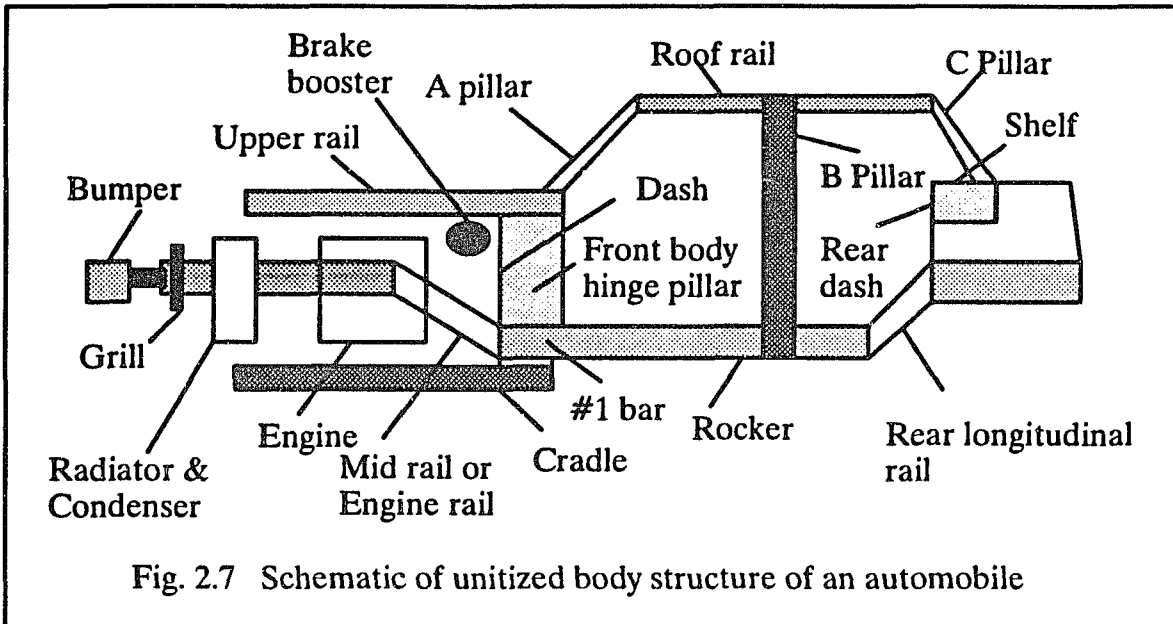


Table 2.1 Sequence of events and timing in zero degree frontal impact test at 30 mph.

Approximate contact timing in 30 mph 0 degree frontal hit to a rigid barrier	Normalized-Time t/t_0	Approximate crush at vehicle CG in % of total dynamic crush
Bumper to barrier	0.0	0
Grill to barrier	8	17
Hood to barrier	14	30
Upper rail to barrier	21	44
Mid rail to barrier	22	46
Engine to radiator	25	50
Cradle to barrier	31	57
Engine to dash	42	63
Tire to bumper	50	87
Car velocity becomes '0'	80	100
Occupant maximum HIC	100	92

possibility (in addition to the previous approach) is to add force limiting devices like an air-bag, a collapsible steering column, a padded instrument panel, etc. to the vehicle.

It should be noted that as the crush progresses, more and more components hitting the barrier become stationary. As a result the location of the CG of the vehicle as well as the total moving mass of the vehicle keeps changing. Also the velocity is in a state of transition and varies along the longitudinal direction, as shown in Figure 2.8. To

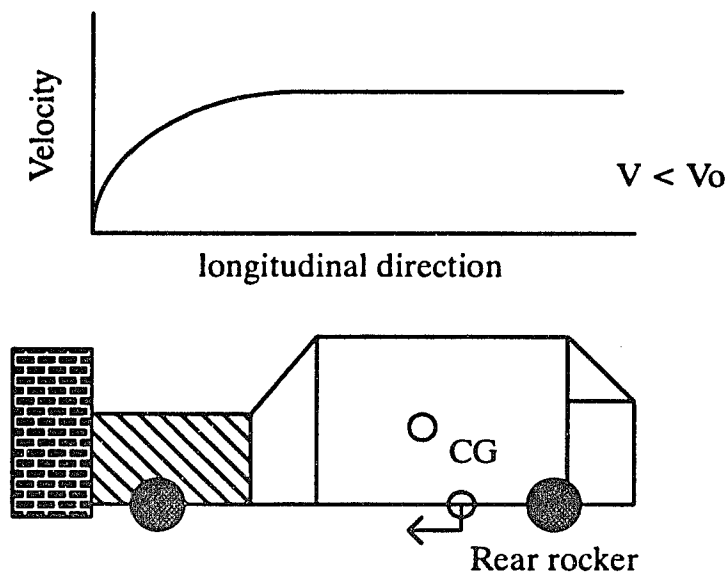


Fig. 2.8 Frontal impact velocity distribution
(Note that the velocity is not constant at points along the longitudinal axis of the car during the transition period)

simplify these complications it is customary to consider the acceleration measured at the rear rocker as the deceleration of the center of gravity of the vehicle. The deceleration response at the rocker is one of the most frequently used piece of test data to describe the structural performance of an automobile in a frontal crash.

2.2.1.2 Rear impact mechanisms

In a rear impact the test car is stationary and is struck by a moving barrier. The initial kinetic energy of the moving barrier is dissipated in two ways:

- 1) Through deformation of the struck vehicle and the striking barrier.
- 2) Through transfer of momentum to the struck vehicle.

In a rear impact the concern is for fuel leakage, since the fuel tank is generally located in the rear. The rear longitudinal rail provides the major load carrying path. Components like the spare wheel in the trunk, rear wheels and suspension system also play an important role in distributing the load. Since no occupants are involved, a rear impact is relatively less complex. The discussion in this thesis is limited to frontal and side impacts only.

2.2.1.3 Side impact mechanisms

In the side impact test, a moving deformable barrier (MDB) weighing 3000 lb (1360 kg) and moving at a velocity of 33.5 mph (54 kmh) strikes a stationary test car on the side as shown in Figure 2.6. After the impact, both the test vehicle and the moving barrier move at a velocity of about 20-25 kmh. Applying the laws of conservation of momentum and conservation of energy, it is estimated that only about 60-70% of the energy is dissipated in the deformation of the vehicle and the barrier.

The lateral packaging for a typical mid size car including SID is presented in Figure 2.9. It can be seen that there is 100-150 mm of clearance between the door inner and the occupant. Typically, the door inner deforms 300-400 mm inboard, hence contact between the door inner and the occupant is unavoidable. The severity with which the door inner contacts the occupant is considered one of the major cause of injury.

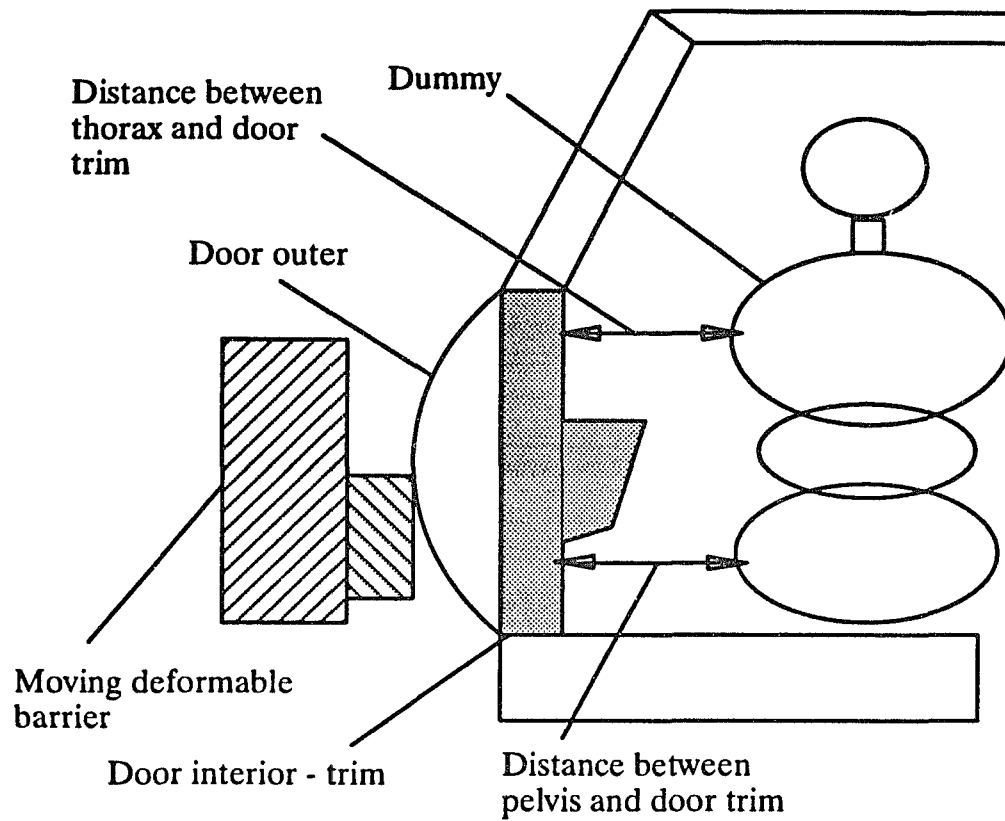


Fig. 2.9 Lateral packaging space in side impact

The injury to the occupant during side impact can be reduced by :

- 1) Increasing the lateral force carrying characteristics of the door and side structure. This will reduce the inboard intrusion of the door inner and also improve the transfer of momentum to the struck car.
- 2) Improving the load transfer mechanism between the door interior and the occupant. This can be achieved by designing a sufficient width of a force limiting material like foam or hexcell so that the magnitude of the load transferred to the occupant can be limited.

2.2.2 Analytical tools in crash analysis

The crashworthiness analysis of an automobile can be divided into three major tasks :

- 1) Analysis of vehicle structure,
- 2) Analysis of occupant kinematics, and
- 3) Analysis of the interaction between the occupant and the vehicle.

Currently, the most commonly used analytical techniques in the auto industry are lumped parameter (LP) analysis, Kamal [1970 and 1982], and finite element (FE) analysis, Haung et al. [1983].

2.2.2.1 Why analytical tools are needed

A structural designer is faced with the problem of understanding the mechanisms involved in the crash and then designing the car such that the desired crashworthiness performance can be achieved. In the early years engineers depended heavily on tests, design handbooks and hand calculations to evaluate the crash performance of a vehicle. However, with the globalization of the automotive market and dynamic market conditions the product development cycle has become shorter. Also the cost of building multiple prototype vehicles for the tests has increased in the recent years. Both these resulted in an increased demand for analytical tools in designing, evaluating and proving the automotive designs.

2.2.3 Tools for vehicle structure modeling and analysis

Most automotive structures are made of sheet metal (mild steel). The use of composites and plastics have been limited to the outer skin or non-structural members. Only in some high performance cars like the Chevrolet Corvette, Honda NSX or Lamborghini Contessa, which are manufactured in small quantities, materials like

aluminum, magnesium or glass fibre composites etc. are used in the structure. Therefore, in this work only the analysis concerned with steel structure is reviewed.

The analysis techniques used in the structural analysis can be categorized as follows :

- 1) Fundamental principals of mechanics.
- 2) Lumped parameter modeling.
- 3) Finite element modeling.

2.2.3.1 Fundamental principles of mechanics

Consider the case of a typical mid-size car weighing 1500 kg discussed earlier. Assume that the crushable space designed in front of the dash is about 500 mm (typically, 500-700 mm of crush is observed). If this car hits a rigid barrier at 30 mph at 0 degrees, from Newton's laws of motion it can be estimated that the average deceleration will be approximately 20G's. This means that the average axial crush force will be approximately 300 kN.

The challenge to engineers is to design the individual structural members like the engine rail, upper rail, roof rail, cradle etc. to work as a part of the front end system and offer an average crush force of 300 kN. In the 1950's and early 1960's there were no computers available to most engineers. They mainly depended on actual testing and hand calculations. Analytical techniques to evaluate the axial load carrying characteristics of thin walled structures were developed to help in the early phases of design, see Dedow [1961], Lin [1984]. Mahmood et al. [1984] reported that, "In thin walled members the stresses rarely reach the yield level and the force deflection relationship is usually controlled by the local buckling and subsequent collapse of the structure". A number of researchers have worked on developing analytical and

empirical expressions to estimate the failure load for various cross sections and geometries. Among them Mahmood et al. [1988], Grimm et al. [1988], Murray [1983], and Mahmood and Tang [1989], are of special interest. Beam elements based on local buckling and subsequent failure due to plastic hinging have also been implemented in the finite element formulations. Thompson [1972] developed a finite element implementation of beam elements, which helped in the analysis of designs with complex geometry and boundary conditions.

The axial load carrying capacity of the front structure members evaluated as described above is helpful during the initial stages of design for determining the initial packaging requirements of various components.

2.2.3.2 Lumped parameter modeling (LPM)

The use of lumped parameter modeling in crashworthiness began in the aircraft industry and was gradually extended to the auto industry. The first successful lumped parameter model for a frontal crash was developed by Kamal [1970]. This technique has since been extensively used throughout the auto industry.

A typical lumped parameter model used for a frontal crash is shown in Figure 2.10. The vehicle can be represented as a combination of mass and springs. The dynamic relationship among the lumped parameters is established using Newton's laws of motion and then the set of differential equations is solved using numerical techniques. The biggest advantages of this technique are the simplicity of modeling and the low demand on computer resources. Because of these characteristics, the lumped parameter models are helpful in parametric studies.

Various computer codes like LSD (language for structure dynamics) from Boeing Co. and GENPACT (developed at GM and used in GM only) are extensively used for lumped parameter analysis.

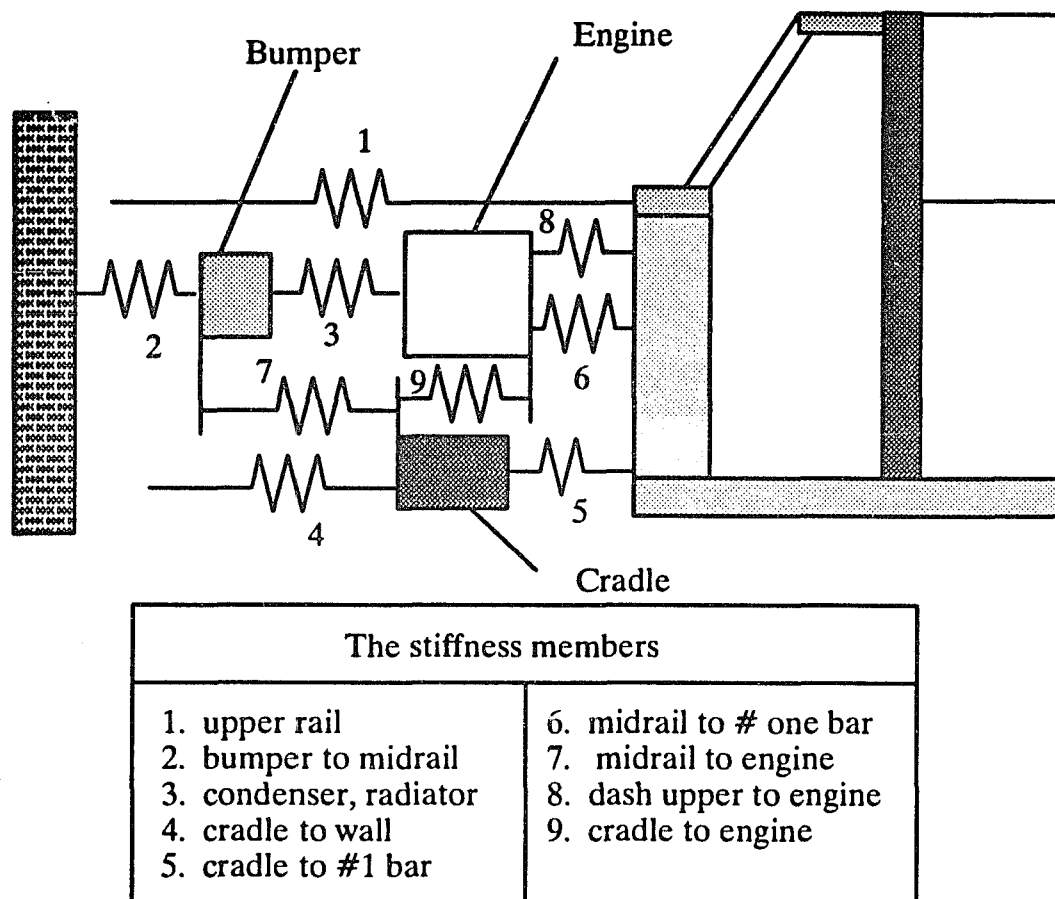


Fig. 2.10 Lumped parameter model used in frontal impact

The difficulty with the lumped parameter method is in estimating the lumped parameters like stiffness, damping and mass. The current approach is to crush the structural components using a static crush test or drop silo technique (dynamic impact test) to generate its force-deflection characteristics. Lumping the mass is based on the experience and judgment of the analyst.

It has been observed in many cases that the crush mode for a particular component during the static component crush is quite different from that seen in the full car test. This is attributed to the different end conditions during the component

tests. To get proper end conditions, special fixtures need to be designed. This adds complexity and cost to the crush test.

The lumped parameter technique has been in use for a long time. Along with the frontal barrier impact it has been successfully applied to car-to-car frontal impact, side impact, and rear impact, see Struble and Piganell [1992] .

2.2.3.3 Finite element modeling (FEM)

The finite element technique has been developed in last forty years. In the early 1960's, the theoretical background for the linear theory of elasticity was readily available, see Zienkiewicz [1979], Bathe and Wilson [1987]. Rapid development also occurred in computers technology, resulting in increased computing capacity at most of the research and development centers, Ginsberg [1989]. A NASA funded general purpose computer program NASTRAN (Nasa STRuctural ANalysis) was released for public use by 1969. All these ingredients, together with the increased need for engineers to depend on the analyses, resulted in the explosive growth in the use of the finite element technique for linear stress-deflection estimation and vibration analysis.

However, the use of finite element techniques in crashworthiness did not see a similar growth. This was due to the lack of development of a general purpose theory for nonlinear plasticity, the lack of a general purpose computer program for crash simulation and the lack of needed computer resources.

Thompson [1972], working on his Ph.D dissertation at the University of Detroit, developed a nonlinear beam element model for the automobile structure. He also developed a simple lumped parameter occupant model and proposed a technique to integrate these two models. Young [1972] worked on a similar approach and developed a general purpose finite element program for crash analysis. Both of these FE programs were based on an implicit integration scheme. As pointed out by Belytschko

[1988], due to the use of the implicit technique these models lacked robustness and needed excessive computer resources. Also, to keep the degrees of freedom to a reasonably low number (as computer resources were limited) the representation of the structural complexity of the model was quite limited. As a result, correlation of the analytical simulations with the tests was limited.

In 1973, the NHTSA started looking into developing analytical techniques of crash simulation to improve the occupant safety. As a result of the funding from NHTSA, Belytschko and co-workers developed a general purpose crash simulation program called WRECKER, Belytschko [1988]. This code based on explicit integration techniques, required very small time steps but the explicit integration scheme increased the robustness of the program. The simulation cost of this program was quite high and hence it did not gain much usage.

Meanwhile, at Grumman Aerospace Corporation, Winter et al. [1984] developed a general purpose crash simulation program called DYCAST. DYCAST was successfully used in the frontal impact analysis of 1984 Chevrolet Corvett, and the 1984 Dodge Caravan, Pifko and Winter [1981]. In this analysis, the acceleration response correlated quite well with the test data, however the correlation of the crush modes was not satisfactory.

In the short period from 1981 to 1985, a number of developments occurred simultaneously. Belytschko and his co-workers developed a practical four-node quadrilateral thin shell element, see Belytschko et al. [1984]. This new element increased computation speed. Hughes and Liu [1981] developed a robust shell element for highly non-linear analysis. John Hallquist [1984] at Lawrence Livermore National Laboratory (LLNL) developed a general purpose explicit integration code called DYNA3D. DYNA3D was very efficiently written to save computation time and had robust contact algorithms. Both Belytschko et al. [1984] and Hughes-Liu shell

elements were implemented in DYNA3D. By this time the super fast CRAY computers were available to the engineers at major car manufacturers.

The above developments, fueled by need, resulted in the tremendous growth of the use of finite element technique in crash simulation, see Khalil and Vander Lugt [1989], Hallquist and Benson [1989], Shkolnikov et al. [1989] and Walker and Dallard [1991]. During this period (1985-1990) many developments based on the original work of Hallquist, came onto the market (e.g. PAMCRASH, RADIOS etc.) In 1990, John Hallquist released an improved version of DYNA3D called LS-DYNA. LS-DYNA has a number of additional features to increase the robustness, computing efficiency and the ease of modeling.

Finite element codes based on implicit integration, (e.g., ABAQUS from HKS and ADINA from Bathe at MIT) are also popular in crash analysis. However, as pointed out by Belytschko [1988], due to inherent characteristics of implicit integration schemes and their lack of robust contact algorithms, their use in the full car crash analysis is limited.

Most car manufacturers have started using FE models for crash simulation in their new car development programs. A full car FE model for the crash simulation, consists of all major structural components and many other non structural components like the engine, radiator etc. Most components are modeled using shell elements, and have approximately 60,000 to 100,000 degrees of freedom (10000 to 15000 nodes). In the next few years, more complex models with 200,000 to 300,000 degrees of freedom, representing most of the complex mechanism involved in the crash, are expected.

The biggest advantage of the FE model is that they are first principle models, i.e., the models are built upon the basic principles of physics and mathematics. Therefore, the geometry of the car and material properties of the structure are directly

represented. No other analysis technique can replace this feature of the FEM's. FEM's have proven to be more useful and accurate than the lumped parameter models. They have shown a remarkable capability of capturing major mechanisms involved in a crash and good correlation with the test data. The major drawback of FEM's are cost and time. To get a good correlation of the FEM simulation with the crash test data inclusion of extensive details in the model is necessary adding cost and time used in modeling and analysis. For example, a typical front end crash model takes about 3 to 6 months in developing the model alone. Each simulation can take 10 to 30 hours of CPU time on supercomputers.

2.2.4 Occupant modeling

Since the injury to the occupant is the major safety concern, understanding occupant behavior is very important. The occupant models are lumped parameter models with the ability to simulate the rigid body movements. Many of the current occupant modeling programs have a number of added capabilities like airbags, seat belts etc. to simplify the modeling task.

When developing an occupant (dummy) model, much time is spent in estimating the dummy properties and validating the model. Once a dummy model is validated, the dummy response for different cars can be obtained by defining the surrounding structure. In the following paragraphs the development history of occupant modeling and the features of popular occupant simulation programs are presented.

One of the earliest occupant models was developed at Cornell Aeronautical Laboratory by McHenry [1966]. This was an eleven degree of freedom nonlinear model. Improvements on McHenry's model resulted in the CAL-2D model (CAL stands for Cornell Aeronautical Laboratory). Segal [1971] published the improved

version of CAL-2D and called it the ROS (Revised Occupant Simulator) model. GM modified this model further and called it MODROS.

In 1974, Robbins et al. [1974] revised the CAL-2D model under support from the Motor Vehicle Manufacturer's Association (MVMA) and therefore called it MVMA-2D. The model had a good representation of non-linear properties and gave good correlation with experimental results.

In 1973, Bartz developed a three dimensional, 40 degree of freedom model with algorithms to simulate contact interfaces between the occupant and the vehicle interior, calling it CAL-3D. CAL-3D has been debugged and improved over the years. Various versions of this program are available and marketed by different companies. As reported by Prasad [1984], the latest version available to the public is called ATB40A, version 37. Various features like airbag, seat belts and various type of joints are incorporated in this improved version of CAL-3D.

In 1979 Maltha and Wismans, (Prasad [1984]) developed a general purpose computer program for occupant simulation - named MADYMO. MADYMO is more flexible and user friendly than any of the occupant simulation programs discussed so far. Since the initial offering, MADYMO has been periodically updated to include the latest developments.

2.2.5 Combined models

It has been realized that the interaction of an occupant with the interior of the vehicle in a side impact and with the air-bag in a frontal impact play an important role in determining occupant injury. Therefore, mathematical models representing both the occupant and the structure are desirable. Occupant simulation codes like CAL-3D have the capability to represent the interior geometry and stiffness. However, this capability is quite limited as geometry can only be represented by simple shapes like

planes and ellipses. The stiffness of a component is represented by a predetermined force-deflection curve used as an input to the model. FE models provide detailed representations of the interaction: The capabilities to define various joints and rigid bodies are also available in LS-DYNA. In recent years, FE models representing both structure and occupant have been reported, see Walker and Dallard [1991]. The direct interaction of the occupant and structure is expected to give a closer correlation and a better insight into injury mechanisms.

CHAPTER 3

SYSTEM IDENTIFICATION METHODS IN CRASH ANALYSIS

3.1 Introduction

One of the challenges faced by engineers and scientists is to develop mathematical expressions to represent a system from the available observations and the existing knowledge of the system. A set of such mathematical expressions is the analytical model of that system. In general there are two possible approaches to building analytical models.

One is a 'ground up' approach, such as finite element (FE) models. In this approach the system is divided into simpler subsystems for which the properties are known based on the past experiences, which essentially are laws of nature or well accepted empirical relationships (e.g., Hook's law). The analytical model for the whole system is built by integrating these subsystems using mathematical relationships. As a result, while building a complex model, the analyst is burdened with the laborious task of modeling most of the major mechanisms involved in the event. This is a very time consuming task. The FE models used in the crash simulation discussed in Section 2.2.3.3 of Chapter 2 are a good example of these problems. In the last few years, numerous mechanisms involved in the crash have been added to the FE models. This has resulted in an improved correlation of the model prediction with the test. However, cost and time needed for modeling and analysis has also increased.

The other approach is to develop data based models. These models are built based on the input and output data of the systems. The structure of the model can be

any transfer function type model, such as Autoregressive Moving Average (ARMA) models. The advantage of this technique is in relief from modeling all of the mechanisms involved in the event because the data being used to estimate the parameters includes that information.

When comparing data based model with the ground-up models some caution is necessary. The data based models are only a partial representation of a system, i.e., only the measured part is represented in the model and therefore the model has limitations in duplicating the response of the system under drastically different conditions. For example quantities like temperature, Dow Jones economic indicator and acceleration response are simple realizations of complex systems of weather, the economy and the crush of a vehicle. As the measured parameters are only a partial perception of the underlying event, the data based models developed are limited representation of the system.

It is desirable to develop a modeling technique that makes use of both, the observed data from the test and the physical insights of the system. The physical insights are the understanding of the physical structure and the behavior of the system. Section 2.2.1 in Chapter 2, presented the physical insights in the crash behavior of the automobiles. The benefit of the combined approach in developing analytical models is described by Ljung [1987] as, “It is often advantageous to try to combine the approaches of modeling and identification in order to maximize the information obtained from identification experiments and to make the data analysis as sensible as possible ”.

In the next sections the importance of data based models in crash simulation is discussed and the use of system identification methods in developing data based models is presented.

3.2 Importance of data based models in crash analysis

Consider the design of the automobiles for crashworthiness. To ensure a desired performance in the crash tests several cars are tested and extensive analysis is carried out during the early phases of design. The purposes of crash tests are to verify the design objectives, increase understanding of the crash performance and verify compliance to Federal Motor Vehicle Safety Standards (FMVSS). The purpose of using analytical models is to identify the relative importance of design variables and hence reduce the number of full car crash tests. As the analytical models - often - have limited correlation with the physical tests they are not used for FMVSS certification.

In a typical full car test the acceleration, displacements and forces are measured at a number of locations. A careful study of these measurements has been done in the past and a general understanding of the mechanisms involved in the crash is available. See Section 2.2.1 of Chapter 2 for details. The measurements obtained during the crash tests can be considered a representation of the structural properties during the crash event. Therefore the test measurements can be treated as very useful instruments in developing analytical models for crashworthiness analysis. The analytical models that are in current usage, i.e., finite element (FE) models and lumped parameter (LP) models, are discussed in Section 2.2.3 of Chapter 2. These type of models are essentially 'ground up' models and their development is not directly dependent on the measurements from physical crash tests of the automobiles.

The purpose of this section is to emphasize the use of data based models developed directly from the crash data for use in crash simulations. When compared with the FE models the data based models for crash are expected to be very time and cost efficient to model and simulate. However the data based models will not have direct representation of material and geometry details as in the FE models. Therefore the data based models are not intended to replace the FE models but are intended to

compliment the FE models in certain applications. In addition to simulating the crash event the data based models are expected to provide insights into the crash test i.e., contributions of certain structural members in an actual test.

System identification methods are the mathematical tools for developing a data based model from experimental or operational data. In Sections 3.3 through 3.7, a detailed review of system identification methods is presented. A review of the existing literature on the applications of system identification methods in identifying structural parameters in crash analysis is presented in Section 3.8.

3.3 Overview of system identification methods

The idea behind system identification is to develop mathematical models for a system based on the input and output observations. In general, 'system' refers to any dynamic system in which input and output are correlated. Among the variables involved, the variables that can be directly manipulated are called inputs while the observable but non-manipulatable variables are called outputs. The unobservable variables are called disturbances or noise. A system with unobservable inputs is called a stochastic system and the observed outputs are called time series. Thus a system is a highly generalized concept and has wide applications. System identification refers to the techniques to develop a mathematical representation of a system based on the input and output observations. Over the years system identification techniques have proven to be very useful in such a diverse areas as engineering, economics, weather forecasting, medicine etc.

A lot of published literature on system identification is available. Goodwin and Payne [1977] is one of the most widely used texts in this area. Box and Jenkins[1976] deals with time series and its application in diverse areas. Pandit and Wu [1983] present a practical approach to applying time series to engineering problems. Sinha and Kuszta

[1983] and Eykhoff [1974] present a comprehensive coverage of the subject from an engineering perspective. More recently, Ljung [1987] provides a comprehensive overview of theoretical and practical aspects of system identification techniques. As the system identification techniques are applied in diverse areas, the definitions, notations and approaches differ among the published literature. Ljung [1987] presents a generalized theory which provides a common base, from which most of the popularly known system identification techniques can be derived. In this work the generalized approach is used in system identification.

The system identification procedure according to Isermann [1979] and Ljung[1987] is presented in Figure 3.1 The basic tasks involved are:

1. Design of experiment and data collection.
2. Model selection.
3. Identification of parameters.
4. Model validation.

3.4 Design of experiments and data collection

This is an important aspect of system identification. The main considerations are input signals (of sufficient amplitude and spectra to excite all the modes of interest in the system), sampling interval, measuring time period, open and closed loop identification, off-line or on-line identification, equipment for signal generation, data storage and computation, signal filtering etc. A detailed discussion on all the above concerns is available in Isermann [1979].

3.5 Model selection

Parameterized models used in system identification can be broadly divided into two major groups:

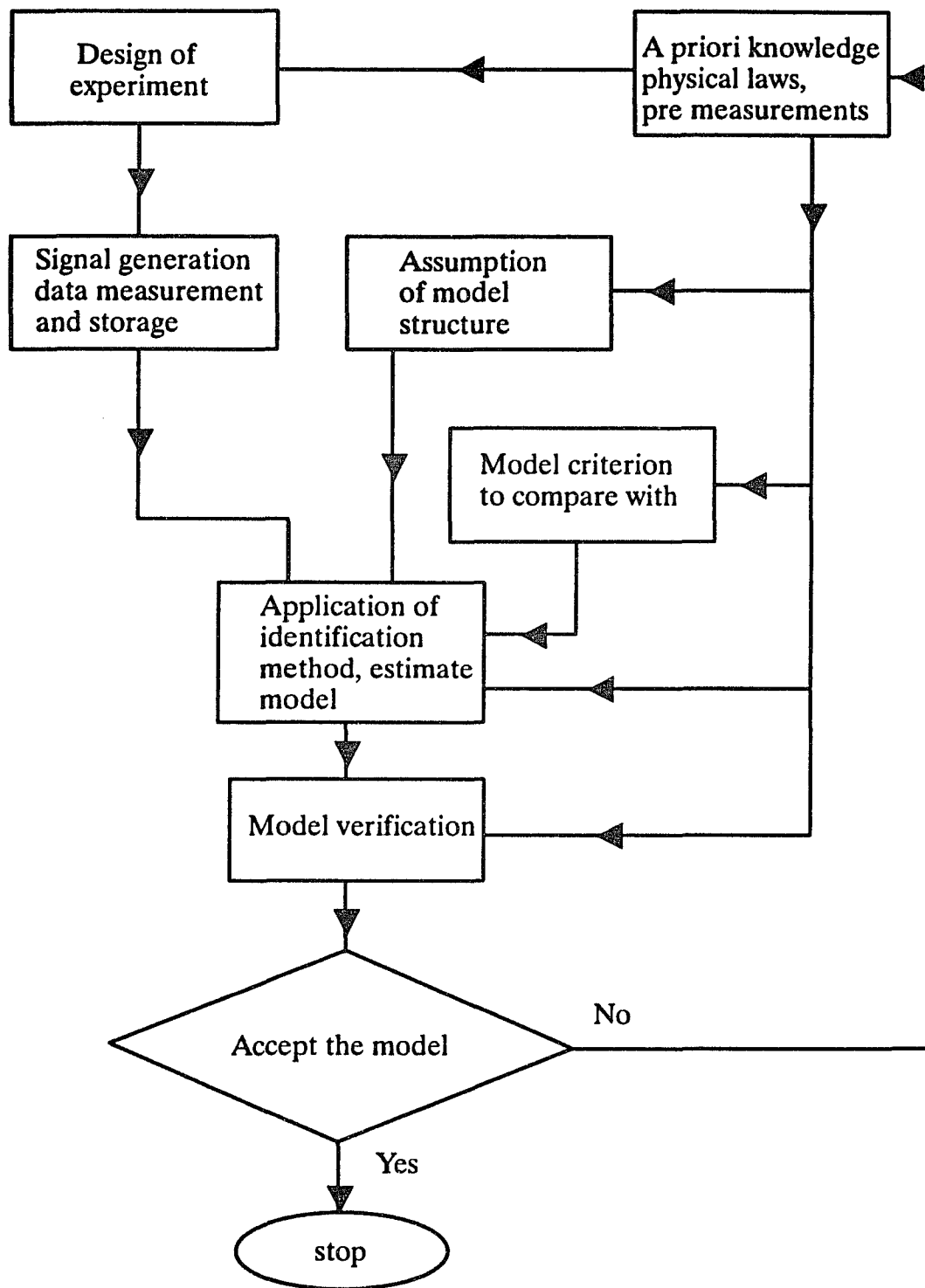


Fig. 3.1 The system identification procedure .
based on Ljung [1987]

- 1) **Black box model structure:** The idea is to obtain flexible model sets that can accommodate a variety of systems, without looking into their physical structures.
- 2) **Model structure with physical parameters:** The idea is to develop a model which is based on a physical understanding of the system. The model parameters are not abstract numbers but physical quantities like mass, stiffness etc.

The models can be also classified as:

Time invariant or time varying parameters.

Linear or non linear model structure.

3.5.1 Black box model structure

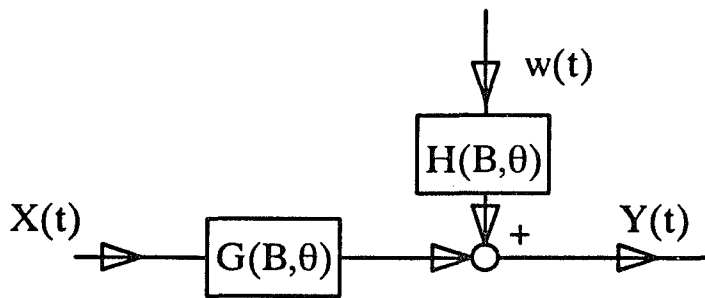


Fig. 3.2 Generalized transfer function of a dynamic system

The most popular black box model structure is the transfer function family of models. A generalized transfer function for a time invariant linear system, presented in Figure 3.2, can be presented as

$$Y(t) = G(B,\theta) X(t) + H(B,\theta) w(t)$$

Where $G(B,\theta)$ is the transfer function of the system and $H(B,\theta)$ is the transfer function of the added noise, $X(t)$ is the input, $Y(t)$ is the output and $w(t)$ is white noise

with a known probability density function. θ is the parameter vector and B is back-shift operator.

The transfer functions can be expressed in general form as:

$$G(B, \theta) = E(B)/A(B)$$

$$H(B, \theta) = D(B)/C(B) \quad (\text{note: } \theta \text{ dropped for simplicity in writing})$$

This is also called the Box-Jenkins model. A , E , C and D are polynomial functions of B given by

$$A = A_0 + A_1 B + A_2 B^2 + \dots$$

$$E = E_0 + E_1 B + E_2 B^2 + \dots$$

$$C = C_0 + C_1 B + C_2 B^2 + \dots$$

$$D = D_0 + D_1 B + D_2 B^2 + \dots$$

Different black box models are obtained by choosing various forms of the transfer function A , E , C and D , e.g.

If $A=D=C=1$ then it is called a finite impulse response model

If $D = 1$ and $C = A$ then it is called an equation error model.

If $C = A$ then it is called an ARMAX model.

If $E=0$ then it is called an ARMA model

Detailed discussions of the properties of these models are available in Ljung [1987], Box & Jenkins [1976], Isermann [1979] and Strejc [1979].

3.5.2 Model structure with physical parameters

The purpose here is to develop a mathematical representation of the system based on the physical understanding of the system. Therefore characteristics of the system are expressed based on the laws of physics, i.e., equilibrium of forces, conservation of energy etc. As the laws of physics are expressed in continuous time the

model consists of a set of differential equations with lumped or distributed parameters in the continuous time rather than in discrete time.

State space representation in continuous time is one of the most popular examples of this type of model.

$$\frac{dZ(t)}{dt} = A_c Z(t) + B_c X(t)$$

$$Y(t) = C_c Z(t)$$

Where, A_c , B_c and C_c are matrices of the parameters that can be related back to the physical quantities in the underlying system. $Z(t)$ is a state variable vector and $\frac{dZ(t)}{dt}$ is the first derivative of $Z(t)$, $X(t)$ is input, and $Y(t)$ is output.

However, in practice sampled data are available. State space representation of a discrete system can then be presented as,

$$Z_{t+T} = A_d Z_t + B_d X_t + V_{1t}$$

$$Y_t = C_d Z_t + V_{2t}$$

Where A_d , B_d and C_d are parameter matrices of the discrete data. V_{1t} is the process noise and V_{2t} is the measurement noise. T is the sampling interval. As the computers used are digital, algorithms to estimate parameters from discrete state space representations are readily available. Therefore it is desirable to estimate the parameters from the discrete data.

The transformation of a discrete time model parameters to a continuous time model parameters under certain restriction is described by Sinha and Kuszta [1983,pp98-101] and Ljung [1987,pp82-83].

There are two approaches; direct and indirect. With the direct approach both sides are directly integrated using the trapezoidal rule,

$$Z_{k+1} = Z_k + A_c \int_{kT}^{kT+T} Z dt + B_c \int_{kT}^{kT+T} X dt.$$

which becomes

$$Z_{k+1} = Z_k + T/2 A_c [(Z_{k+1} + Z_k)] + T/2 B_c [(X_{k+1} + X_k)]$$

and rearranging

$$Z_{k+1} - Z_k = (T/2) [A_c(Z_{k+1} + Z_k) + B_c(X_{k+1} + X_k)]$$

With this formulation, A_c and B_c , the parameter matrices for the continuous time case, are directly evaluated from the sampled data. The sampling interval, T , is assumed to be very small.

For the indirect approach the transformation can be presented as follows,

$$\begin{aligned} A_d &= e^{A_c T} \\ &= I + A_c T + (1/2!) (A_c T)^2 + (1/3!) (A_c T)^3 + \dots \\ B_d &= \int_0^T e^{A_c t} B_c dt \\ &= (IT + (1/2!) A_c T^2 + (1/3!) A_c^2 T^3 + \dots) B_c \end{aligned}$$

Knowing A_d and B_d , A_c and B_c can be evaluated.

3.6 Parameter estimation methods

Once a model structure is chosen, the next task is to estimate the unknown parameters of the model. The criteria for a good model and methods to estimate parameters are discussed next.

3.6.1 Prediction-error identification methods (PEM)

One of the criteria is to choose the model parameters such that the one step ahead prediction error is minimized, Ljung [1987]. Consider a linear time-invariant model of a dynamic system.

$$Y_{(t)} = G(B, \theta) X_{(t)} + H(B, \theta) w_{(t)}$$

If the one step ahead prediction of output vector is expressed as $Y_{p(t/t-1)}$, then the prediction error vector will be,

$$e_{(t/t-1)} = Y_{(t)} - Y_{p(t/t-1)}$$

We are looking for a parameter vector which can minimize

$$\theta = \arg \min [V_n(e)]$$

Where V_n is some function of prediction error vector.

If $V_n(e)$ is expressed as $V_n(e) = E\{^{1/2} e_{(t/t-1)}^T \Lambda e_{(t/t-1)}\}$, where $E\{ \}$ is an averaging operator and Λ is the weighing matrix, then mathematically, the problem becomes a minimization of the average of the squares of the prediction error with respect to the parameter vector θ . Based on the selected model structure, weighing matrix and available priori information, there are a number of techniques available to solve this problem:

Least Squares methods (LS),

Weighted Least Squares methods (WLS), and

Generalized Least Squares methods (GLS),.

These methods are discussed in detail in Ljung [1987], Isermann [1979] and Sinha and Kuszta [1983]. The advantages and disadvantages of the various estimation techniques are compared in Table 3.1.

If $V_n(e)$ is expressed as $V_n(e) = E\{^{1/2} e_{(t/t-1)} e_{(t/t-1)}^T\}$, then mathematically the problem becomes a minimization of the covariance matrix of the prediction error. The estimator predicting the parameters based on this criteria is called a minimum variance estimator. Kalman filter(KF) is an example of such an estimator. Details of the KF are available in Chui and Chen [1991] and Ljung [1987].

Table 3.1 Comparison of system identification methods.
Condensed from Isermann[1979]

Parameter Estimation Method	Process Model	Noise Model	Properties and comments
Least Squares (LS)	--	1/C	<ul style="list-style-type: none"> # Biased estimate for real noise # Applicable for short identification time if noise acts on the process # Relative small computational expense # Good starting method for IVA or ML # Knowledge of output mean is necessary # RLS has reliable convergence
Generalized Least Squares (GLS)	E/A	1/A	<ul style="list-style-type: none"> # Biased estimate possible # Relative large computational expense # Noise model identified
Instrumental Variable (IVA)	E/A	D/C	<ul style="list-style-type: none"> # Good performance for wide range of noise model # Medium/small computational expense # Output mean not significant if input mean =0 # RIV convergence is not reliable so start with RLS
Maximum Likelihood (ML)	E/A	D/C	<ul style="list-style-type: none"> # Good performance for special noise model D/C # Large computational expense # Noise model identified # Problems if local minima of loss function # RML has slow convergence but more reliable than RLS
Correlation based LS COR-LS	E/A	D/C	<ul style="list-style-type: none"> # Good performance for wide class of noise models # Relative small computational expense # Access to intermediate results # Search of model order is not very expensive # Easy verification # Output mean not significant if input mean =0 # RCOR-LS has reliable convergence
<pre> graph LR noise --> DC[D/C] input --> EA[E/A] DC --> sum(()) EA --> sum sum --> output </pre>			

3.6.2 Prediction error correlation

Another criteria for a good model is to estimate model parameters such that the prediction error becomes independent of the past measured data. This means that the cross-correlation of the prediction error with the measured input and output data becomes negligible, Ljung [1987].

Consider a finite-dimensional vector sequence, S_t , derived from the measured observations of the system. Then choose the parameters such that the cross correlation of the prediction error vector ' e_t ' with the vector S_t is minimized, i.e.,

$$\theta = \arg \min \left\{ \frac{1}{n} \sum_{t=1}^n S_t \cdot e_t \right\}$$

The parameter estimation techniques based on this criteria are:

- 1) Pseudo Linear Regression.
- 2) Instrumental Variable.

Both of these methods are discussed in detail by Ljung [1987] and Isermann [1979]. Their comparison with the other methods is also presented in Table 3.1.

3.6.3 Recursive parameter estimation methods

The recursive parameter estimation techniques are useful when the parameter identification has to be done on-line or when the parameters are time varying. The criteria of a good model are the same as for the off line methods - the difference is in the method of estimating parameters. The parameters being estimated are sequentially updated for every new observation. A comprehensive discussion of the recursive identification methods is presented in Ljung and Soderstrom [1983]. Various recursive estimation techniques are compared by Matko and Schumann [1982].

Ljung and Soderstrom [1983] observed that there are numerous techniques of recursive estimation with very similar structure which can be given by

$$\theta_t = \theta_{t-1} + \lambda_t P_t n_t [Y_t - Y_{pt}] \quad (3.1)$$

where θ_t is the parameter vector being estimated at time t and λ_t is a weighing function based on a forgetting factor. P_t and n_t are matrices based on the past observation and are updated for every new observation. n_t in some manner represents the gradient of Y_{pt} with respect to θ . Y_t is the measured output at time t and Y_{pt} is the predicted output based on the measurements up to time $t-1$.

Variations in this generic recursive algorithm results in different recursive algorithms, e.g. recursive least squares, recursive instrument variable, recursive generalized least square etc. These different recursive algorithms are based on the type of model structure for which they are developed, the error criteria being minimized and the variations in the parameter updating mechanisms. Detail of various recursive algorithms are available in Ljung and Soderstrom [1983].

The weighing function based on a forgetting factor (which is a user's choice) differentiates between the contribution of the most recent observations and the past observations. This feature makes it possible to track time varying parameters. The high forgetting factor means a low weighing function hence larger influence of past observations. This is useful in tracking slowly varying parameters in presence of noise. Whereas a low forgetting factor means high weighing function and hence larger influence of recent observations. This is useful in tracking quickly changing parameters.

In real life conditions the dynamic systems have two types of parameter changes – continuous and abrupt. Continuously changing parameters vary slowly while the abrupt parameters make step changes at infrequent intervals. There is also presence of noise in the measurements. This creates problems in the selection of a forgetting

factor. The basic recursive algorithm is altered to address these problems. Discussions of estimation techniques for continuously changing parameters were presented in Hagglund [1985], Ljung [1983], Anderson and Johnstone [1983]. Discussion on abrupt parameters is available in Andersson [1985] and Ljung [1988]. The proposed methods essentially consists of multiple recursive estimation running in parallel with different weighing factors. A survey of recursive algorithms to estimate rapidly changing parameters is presented by Evans et al. [1988]. The algorithms discussed include methods with variable forgetting factors, directional forgetting and covariance resetting.

3.7 Model validation

Model validation is discussed by Isermann [1979], Ljung [1987] and Sinha and Kuszta [1983]. The purpose of validation is to check the adequacy of the model in predicting real process behavior. It is understood that the model is only a partial representation of the system and valid over a limited specific range. Therefore, while validating the model two concerns need to be addressed:

- a) Does the model agree well with the observed data and is it good enough for the purpose it was developed?
- b) Are the a priori assumptions used in deriving the identification method true?

The adequacy of the model for the intended purpose can be verified by considering a number of factors, Ljung [1987].

- 1) Estimating parameters from the simulated data where the underlying parameters are known.
- 2) Comparing model simulation with the system output - preferably using inputs, different from the ones used for estimation of parameters.

- 3) Check if the estimated parameters make physical sense.
- 4) Apply model reduction technique to check if the model is over parameterized.

The validity of the priori assumption can be verified as follows,

Time invariance and linearity: By comparing the model obtained for different input signals.

Covariance matrix of parameters estimated: Are the covariances constant with change in time?

Residues analysis: The check on residues is an important one. The auto-correlation and cross correlation of the residues with the input can be evaluated and compared with the confidence limits. The confidence limits are evaluated as follows, Pandit and Wu [1983].

A sequence of residue r_k ($k=1 \dots N$) with auto-correlation given by ' ζ_{rr} ' is said to be white with 'p' percentage confidence limits if

$$|\zeta_{rr}| \leq p_{\text{limt}} / \sqrt{N} \quad \text{for all } r \geq 1$$

where sequence r_k is assumed to be normally distributed. Hence

$$\text{for } p=90\% \quad p_{\text{limt}} = 1.65$$

$$\text{for } p=95\% \quad p_{\text{limt}} = 1.96$$

$$\text{for } p=99\% \quad p_{\text{limt}} = 2.58$$

Similarly the limits for cross correlation, $\zeta_{ru}(i)$ between the residue 'r' and the input 'u', with lag 'i', can be given by, Ljung [1987],

$$|\zeta_{ru}(i)| \leq p_{\text{limt}} \sqrt{P/N}$$

where,

$$\mathcal{P} = \sum_{i=-\infty}^{\infty} \zeta_{rr}(i) \zeta_{uu}(i)$$

N is the number of observations.

3.8 Review of applications of system identification methods in crash analysis

Estimation of structural parameters for dynamic systems has been discussed by a number of researchers. Hart and Yao [1977] presents a survey of system identification approaches used in estimating structural parameters like mass, stiffness and damping matrices. Leuridan et al. [1985] describes a time domain parameter identification method in which the system is modeled directly using constitutive differential equations with multiple input multiple output data. Karnik and Sinha [1985], Michelberger et al. [1985] Ibrahim and Mikulcik [1977], Carvani et al. [1977], and Beliveau [1976] are some of the interesting references in this area. Most of this literature is on the estimation of parameters under steady state conditions. A crash is a highly nonlinear event, where the parameters change rapidly and the noise content in the measurement is quite high. Very limited work has been done in estimating structural parameters under such severe conditions.

Huang et al. [1977] parameterized the deceleration response in the frontal crash. He applied a general function of time to fit the deceleration response, e.g.

$$a = \sum_{i=0}^n a_i t^i \quad \text{and} \quad a = \sum_{i=0}^n a_i \sin(w_i t)$$

The purpose of his work was to provide a simple platform for comparison of deceleration characteristics of different automobiles in a frontal crash. The use of this approach is limited because the parameters a_i have no physical meaning.

Trella and Kanintra [1985] worked on the so called 'inverse problem' in dynamics to estimate the stiffness parameters of a lumped parameter model used in side impact. A lumped parameter model was chosen such that the inverse deterministic solution was possible (i.e. the number of stiffness parameters are the same as the number of lumped masses). This approach restricted the choice of model structure.

Hollowell et al. [1988] started with a lumped parameter model and developed a method for determining the stiffness and damping characteristics of a structure under a crash loading environment using an adaptive time domain, constrained minimization technique. The structural parameters were considered to be piecewise linear and the adaptivity was incorporated in the formulation to identify the discontinuity. The model validation was performed on noise-free simulated data. With this approach the parameters were not allowed to change continuously and knowledge of variations in the variance was necessary.

The data measured during a crash test are result of contributions from various sources, such as the rigid body motion of the component, vibratory response at the accelerometer location and instrumentation noise. However, the lumped parameter model considered, based on physical insight, may not be comprehensive enough to represent all of the dynamics included in the measurements. As a result the equation error is a combination of correlated and uncorrelated noises. The correlated part of the equation error is the dynamic response not represented by the assumed model structure. Therefore, in this thesis an ARMA (autoregressive moving average) model is attached to the lumped parameter model to capture the 'missed dynamics' of the system. The parameters are assumed to be varying continuously and are estimated

using a recursive estimation approach. The detail of the model structure and the estimation method is presented in the next chapter.

CHAPTER 4

DATA BASED APPROACH IN MODELING AUTOMOBILE CRASH

4.1 Introduction

In Chapter 2, mechanisms involved in the crash event were discussed and then the test procedures and analytical tools used to analyze the car structure were presented. In the last chapter, first the importance of data based models for crash simulation from the test data was established and then system identification methods helpful in developing such models were presented. The purpose of this chapter is to investigate the development of data based models for crash simulation directly from the crash test measurements.

Typically, accelerations measured on the structure during a full car crash test consists of two components. One, rigid body response, is assumed to represent crush of the structure and the other, zero mean high frequency autocorrelated response, is assumed to represent the structural vibrations and instrumentation errors. The data based models developed in this chapter are intended to represent both these components in the simulation. Therefore two components are used in the data based model. One is a lumped parameter model which represents the crush of the structure and hence contributes for rigid body response in the simulation. The other is a transfer function type model (i.e., autoregressive moving average model—ARMA) which represents structural oscillations and hence contributes for the vibratory response in the simulation.

Details of the data based model structure are presented in the Section 4.2. The model parameters are correlated with the structural characteristics in Section 4.3. This

is necessary to use the model in design. In Section 4.4, methods to estimate parameters for the selected model structure are presented. Finally, in Section 4.5 the proposed approach is verified using simulated data and then its use in physical crash test data is demonstrated. Use of these approach is design is discussed in Section 4.6 and a chapter summary is presented in Section 4.7.

4.2 Development of model structure

Consider a lumped parameter model with l masses and n stiffness and damping members. The mass represents the inertia-dependent force and the stiffness and damping members represents the crush dependent and rate of crush dependent forces.

The equilibrium of the k^{th} mass, M_k , connected with 'n' sets of stiffness and damping members can be expressed as follows,

$$M_k a_k = f_{xk} (\text{crush dependent force}) \\ + f_{vk} (\text{rate of crush dependent force}) + e_k$$

If the crush and rate of crush dependent forces are defined as :

$$f_{xk} = \sum_{i=1}^n \sum_{j=1}^{\text{ord}} K_{ij} X_i^j \delta_{ki} \\ f_{vk} = \sum_{i=1}^n \sum_{j=1}^{\text{ord}} C_{ij} V_i^j \delta_{ki}$$

then the equilibrium expression for k^{th} mass, M_k can be written as :

$$M_k a_k = \sum_{i=1}^n \sum_{j=1}^{\text{ord}} K_{ij} X_i^j \delta_{ki} + \sum_{i=1}^n \sum_{j=1}^{\text{ord}} C_{ij} V_i^j \delta_{ki} + e_k \quad \text{for } k=1,2 \dots l \quad (4.1)$$

where,

- a_k is the acceleration of the k^{th} mass M_k ,
- X_i^j is the j^{th} order of the crush of i^{th} spring,
- K_{ij} is the corresponding stiffness parameter, which is a nonlinear function of crush X_i^j ,

- V_i^j is the j^{th} order of the rate of crush of the i^{th} damper and C_{ij} is the corresponding damping parameter,
- C_{ij} is the corresponding damping parameter, which is a nonlinear function of rate of crush V_i^j ,
- δ_{ki} is the connectivity parameter defining the model structure and has values of $-1, 0$ or 1 ,
- n is the number of stiffness and damping parameters,
- ord is the highest order of X_i^j or V_i^j , i.e. $J_{\text{maximum}} = \text{ord}$,
- e_k is the equation error, which can be modeled as an ARMA(p,q) process of white noise e_k' ,

$$e_k = \frac{A_k(B)}{D_k(B)} e_k'$$

where,

$$A_k(B) = A_{k0} + A_{k1} B + A_{k2} B^2 + \dots + A_{kq} B^q$$

$$D_k(B) = D_{k0} + D_{k1} B + D_{k2} B^2 + \dots + D_{kp} B^p$$

B is the back-shift operator.

e_k represents the acceleration response that is not represented by the lumped parameter model. e_k is assumed to be made up of the vibration response of the structure at the accelerometer location, instrumentation error and the noise added from unknown sources. The parameters K_{ij} and C_{ij} are considered to be nonlinear functions of crush and rate of crush. For simplicity 'j', the order of crush and rate of crush is assumed to be 1. In the next sections, j in the K and C parameter is dropped from the subscript notations.

4.3 Correlating the model parameters with the structural characteristics

The question is : How do we associate the needed lumped parameters mass, spring and damper (M, K and C) in the model with the structural characteristics? The structure is three dimensional with infinite degrees of freedom in all the six directions, whereas the lumped parameter model considered is unidirectional with only a few degrees of freedom. This leads to questions such as "which points on the structure

should be represented in the model?” and “How should the distributed mass in the actual structure be lumped in the model?”.

In general the lumped mass (M) can be considered as a multiplier to the measured acceleration (a) such that the inertia force on that component is $M \cdot a$. The lumped mass is assumed to be located at the point of acceleration measurement. The stiffness (K) and damping (C) used in the model can be considered as nonlinear functions of crush (X) and rate of crush (V) respectively, such that the expression ' $K \cdot X + C \cdot V$ ' defines the structural resistance to crush. This structural resistance is assumed to be acting at the location of the accelerometer and is considered as a crush characteristic of the structure between the two corresponding accelerometer locations.

To understand the characteristics of the lumped mass consider the side impact test shown in Figure 4.1. Assume that accelerations are measured at three locations i.e. on the moving deformable barrier (MDB), the door and the non struck side rocker of the test car. A lumped parameter model with three mass components (see Figure 4.1), can be used to represent the event. As the structure of the test car, the MDB and door are deforming, the acceleration on any of these components (i.e. MDB, door or car) at any instant are different at different physical locations. To find the resultant force or the unbalanced external force, F_k , on any component k , assume that the volume of the component k is divided into 'nvol' very small sections of volume δv_{ki} with mass δm_{ki} , ($i=1 \dots nvol$). The volume δv_{ki} is chosen such that the acceleration a_{ki} in the volume δv_{ki} can be considered to be uniform at all the points at any instant. Then, the resultant force on the component k , at any instant can be expressed as,

$$F_k = \sum_{i=1}^{nvol} \delta m_{ki} a_{ki} \quad (4.2a)$$

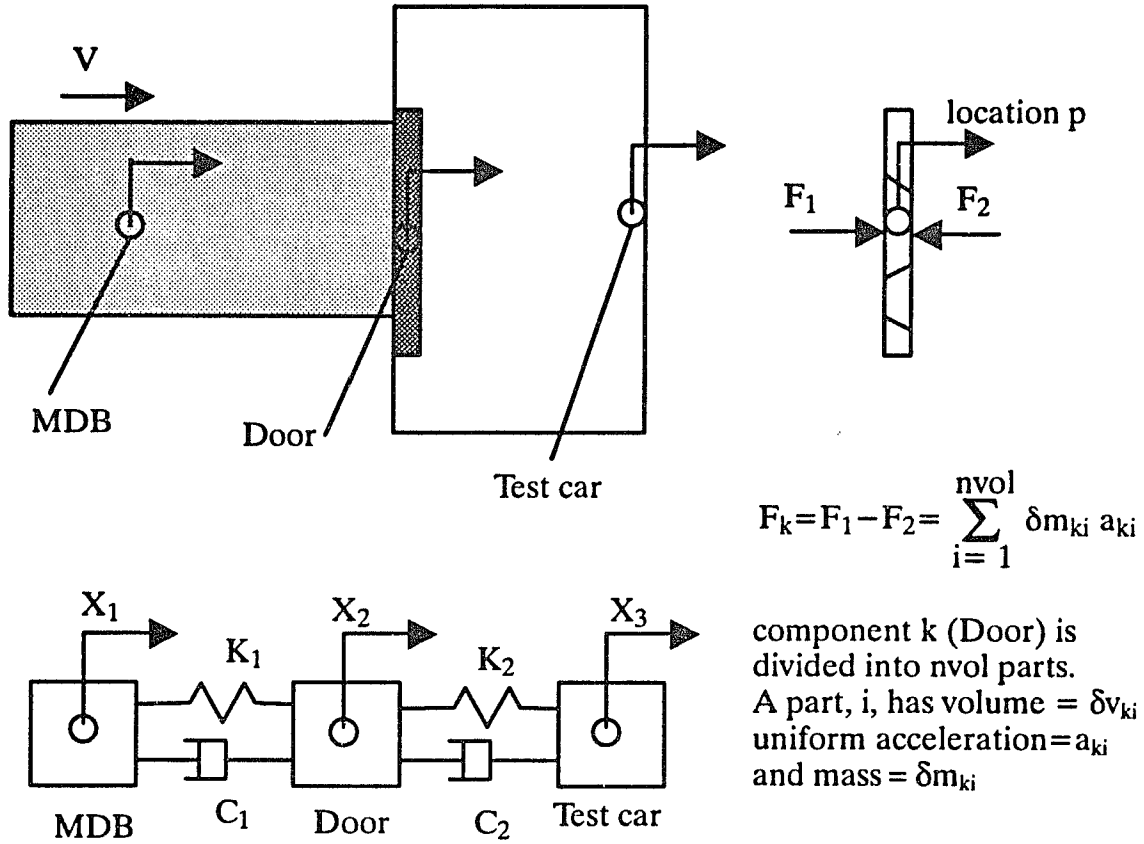


Fig. 4.1 Concept of lumped mass in side impact model

Note that accelerations $a_{k1}, a_{k2} \dots$ are different and keep changing with time. As acceleration is not measured at 'nvol' locations but only one location on a component, say a_{kp} , the equivalent mass, M_{kp} , attached at that accelerometer location, p, on body, k, can be estimated by,

$$M_{kp} = \sum_{i=1}^{nvol} \delta m_{ki} \frac{a_{ki}}{a_{kp}} \quad (4.2b)$$

This expression indicates two important characteristics of the lumped mass. First, the magnitude of M_{kp} depends on the location of the accelerometer. This is because for a component undergoing deformation the acceleration (a_{kp}) will be

different at different locations at any instant, and as a result the corresponding multiplier M_{kp} will be different. Second, for a given accelerometer the lumped mass M_{kp} will be time varying. This is because as the component is deforming the mass corresponding to the acceleration a_{kp} for that instance will change.

4.4 Estimation of parameters in the data based models

In the next sections estimation methods for the parameters in data based models are developed. The development of estimation methods is based on the system identification methods presented in Chapter 3.

4.4.1 Estimation of lumped masses

The conventional approach to lumping mass in a model is to divide a system into a number of components based on intuition and lump the mass equivalent to the actual mass of that component. This approach is quite arbitrary and is likely to cause unbalanced forces in the system. Also, as discussed in the previous section, the magnitude of the lumped mass for a component depends on the location of the accelerometer and is a time varying quantity. Therefore in this chapter, the following approach is used in estimating the lumped mass.

Consider the equilibrium of the lumped parameter model with 'n' masses, at any instant of time:

$$M_1 a_1 + M_2 a_2 + \dots + M_n a_n = \text{external force} + \text{error} \quad (4.3a)$$

where a_i ($i=1 \dots n$) are accelerations measured by accelerometers and M_i ($i=1 \dots n$) are the lumped masses attached at the accelerometer locations. For the known values of a_i ($i=1 \dots n$) and external force, M_i can be estimated by minimizing the error in the equation (4.3a). When there is no external force, (e.g. side impact model) an additional constraint can be used, i.e. the total mass is conserved. Mathematically,

$$M_1 + M_2 + \dots + M_n = M \text{ total (known and constant)} \quad (4.3b)$$

As the estimated lumped mass is different from the conventional concept of mass it is called 'effective mass'.

4.4.2 Estimation of stiffness and damping parameters

To demonstrate the approach used in estimating stiffness and damping parameters consider the side impact model in Figure 4.1. The equation of motion, using expression (4.1), can be written as follows :

$$\begin{aligned} M_1 a_1 &= -K_1 (X_1 - X_2) - C_1 (V_1 - V_2) + e_1 \\ M_2 a_2 &= K_1 (X_1 - X_2) - K_2 (X_2 - X_3) + C_1 (V_1 - V_2) - C_2 (V_2 - V_3) + e_2 \\ M_3 a_3 &= K_2 (X_2 - X_3) + C_2 (V_2 - V_3) + e_3 \end{aligned} \quad (4.4)$$

where e_i can be expressed as,

$$e_i = \frac{A_i(B)}{D_i(B)} e_i'$$

where e_i' is white noise, D_i and A_i are p,q polynomials of the back-shift operator B.

The estimate of mass x acceleration, $M_i a_i$ on left hand side, is available from the previous section, i.e. equations (4.3a) and (4.3b). The velocity and displacement can be estimated from the acceleration measurements. The unknown lumped parameters K_1 , K_2 , C_1 , C_2 and transfer function parameters $A_{i0}, A_{i1} \dots A_{iq}$, $D_{i0}, D_{i1} \dots D_{ip}$ for $i=1, 2$ and 3 can be estimated by minimizing the errors, e_1, e_2 and e_3 . The methods to estimate the unknown parameters are discussed in the next section.

4.4.3 Estimation method

While estimating the unknown parameters for a model, expressed using equation (4.1), two types of nonlinearities are encountered. One is due to the

nonlinearity of the parameters in the model structure, i.e. moving average part of the transfer function model. This type of nonlinearity is addressed using nonlinear estimation algorithms. The other type of nonlinearity is due to the nonlinear relationship of the structural parameters (K and C) with the crush and rate of crush. The crush and rate of crush are readily available as a function of time from the test measurements. Also the mathematical expression for the model is in the time domain, see equations (4.1) and (4.4). As a result, the structural parameters are estimated first as time varying parameters using recursive estimation methods and then they are related to the crush and rate of crush.

In the following paragraphs a recursive estimator based on the minimization of one step ahead prediction error is discussed. The estimation algorithm presented is based on Ljung and Soderstrom [1983].

For a selected model structure, $M(\theta)$, the one step ahead predictor, $Y_p(t/\theta)$, at time, t , can be expressed as,

$$M(\theta) : Y_p(t/\theta) = G(\theta; t, \psi^{t-1}) \quad (4.5)$$

Here G is a nonlinear function of input-output measurements ψ^{t-1} , up to time $t-1$ and parameter vector θ .

A good model is one that makes a good prediction. Mathematically this can be described as :

$$E(t, \theta) = Y(t) - Y_p(t/\theta) \quad t = 1, 2 \dots n \quad (4.6)$$

Here θ is chosen such that the prediction error, $E(t, \theta) \quad t=1, 2 \dots n$, is minimized.

One popular approach is to minimize the quadratic norm of the prediction error

$$V(\theta) = \frac{1}{n} \sum_{t=1}^n E^T(t, \theta) \Lambda E(t, \theta) \quad \text{where, } \Lambda \text{ is a weighing matrix.} \quad (4.7)$$

To address the time varying nature of the parameters the estimation is carried out using a recursive estimation technique. A survey of existing literature on the recursive estimation methods was presented in Section 3.6 of Chapter 3 (see Hagglund [1985], Matko and Schumann [1982], Strejc [1979], Evans et al. [1988] are of special interest). As proposed by Ljung and Soderstrom [1983] most of the different recursive estimation techniques can be represented as special cases of a generalized framework.

A recursive estimator that minimizes the quadratic criterion of the prediction error, $V(\theta)$, can be presented as,

$$\theta_t = \theta_{t-1} + \gamma_t R^{-1}(t) \Psi(t, \theta_{t-1}) \wedge E(t, \theta_{t-1}) \quad (4.8)$$

where

$R(t)$ is an approximation of second derivative of the quadratic criterion $V(\theta)$,
 $\Psi(t, \theta_{t-1})$ is the negative gradient of $E(t, \theta_{t-1})$ with respect to θ_t (column wise),
 γ_t is the weighing function based on forgetting factor.

There are different methods to estimate $R(t)$. For the Gauss-Newton method, $R(t)$ can be approximated as

$$R(t) = \frac{1}{t} \sum_{k=1}^t \Psi(k) \wedge \Psi^T(k).$$

Now we must estimate $\Psi(t, \theta)$. A model for a dynamic system can be considered as a link between the observed past and the unknown future. Therefore a linear predictor model can be obtained by filtering the input-output data through a finite-dimensional linear filter:

$$\phi(t+1, \theta) = F(\theta) \phi(t, \theta) + S(\theta) Z(t) \quad (4.9)$$

$$Y_p(t/\theta) = H(\theta) \phi(t, \theta)$$

where

$Z(t)$ is the input-output vector, and

$F(\theta)$, $S(\theta)$, $H(\theta)$ are matrices dependent on the parameter θ and model structure.

Also note that the gradient of the prediction output will be the same as the gradient of the prediction error, i.e.

$$\Psi(t, \theta) = d/d\theta[Y_p(t/\theta)] \quad (\text{column wise}),$$

Then, a straight forward derivative of equation (4.9) with respect to θ and a rearrangement of the matrices yields

$$\zeta(t+1, \theta) = A_r(\theta) \zeta(t, \theta) + B_r(\theta) Z(t)$$

$$\begin{Bmatrix} Y_p(t/\theta) \\ \text{Col. } \Psi(t, \theta) \end{Bmatrix} = C_r(\theta) \zeta(t, \theta)$$

The above structure can be used to estimate the gradient of prediction $\Psi(t, \theta)$. Matrices $A_r(\theta)$, $B_r(\theta)$ and $C_r(\theta)$ are based on parameter θ and the model structure.

The recursive algorithm for the estimation of parameters can be presented as,

$$\begin{aligned} E(t) &= Y(t) - Y_p(t) \\ \Lambda(t) &= \Lambda(t-1) + \gamma(t) [E(t) E^T(t) - \Lambda(t-1)] \\ R(t) &= R(t-1) + \gamma(t) [\Psi(t) \Lambda^{-1}(t) \Psi^T(t) - R(t-1)] \\ \theta(t) &= \theta(t-1) + \gamma(t) R^{-1}(t) \Psi(t) \Lambda^{-1}(t) E(t) \\ \zeta(t+1) &= A_r(\theta) \zeta(t) + B_r(\theta) Z(t) \\ \begin{Bmatrix} Y_p(t+1) \\ \text{Col. } \Psi(t+1) \end{Bmatrix} &= C_r(\theta) \zeta(t+1) \end{aligned} \tag{4.10}$$

where

- $E(t)$ is the prediction error,
- $Y(t)$ is the output measurement,
- $\Psi(t)$ is the gradient of predicted output,
- $Z(t)$ input-output measurement,
- $\theta(t)$ is the estimated parameter vector,

$\Lambda(t)$ is the error covariance matrix,

$R(t)$ is the input-output covariance matrix, and

$\gamma(t)$ is the weighing function given by ;

$1/\gamma(t) = (\lambda / \gamma(t-1)) + 1$, where λ is forgetting factor.

For a linear model structure, given as, $Y_p(t/\theta) = Z^T(t)\theta(t-1)$, the prediction error will be $E(t, \theta) = Y(t) - Y_p(t/\theta)$. For such a structure the last part of the algorithm becomes trivial as the gradient, $\Psi(t) = Z^T(t)$. This is the well known recursive least squares estimator.

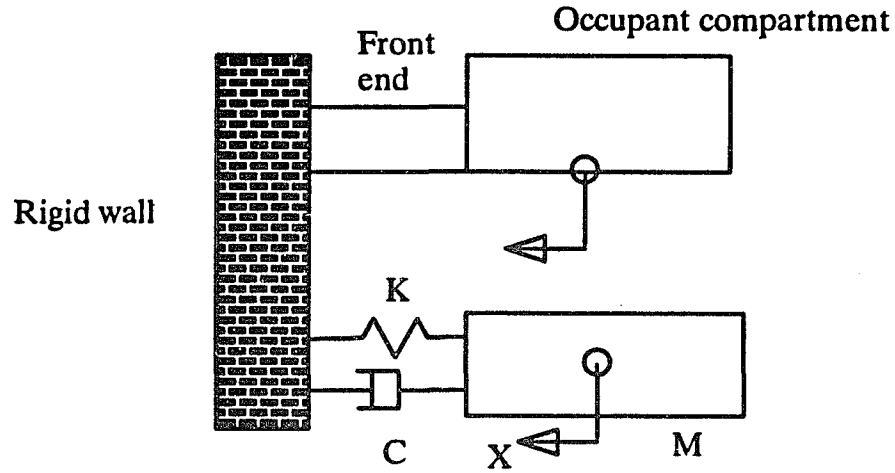
The detail of the implementation of the above algorithm is available in Ljung and Soderstrom [1983] and Ljung [1991].

The crash event being modeled is highly non linear in time and there are possibilities of abrupt changes in the parameters. Recursive algorithms to estimate such a parameter are discussed by Andersson [1985] and Ljung [1988]. Andersson [1985] called his algorithm “adaptive forgetting through multiple models (AFMM)”. AFMM is ideally suited for jumping or rapidly changing parameters. The proposed method essentially consists of multiple recursive least squares estimations running in parallel with different weighing factors. AFMM is useful in estimating discontinuities in the parameters.

In this chapter, first sudden changes in the parameters using AFMM are identified and then slowly changing parameters within these intervals are estimated using a recursive version of the Gauss-Newton algorithm. All the parameters are considered nonlinear and are estimated simultaneously.

4.5 Applications

In the following section the proposed approach is verified on simulated data and then applied to actual test data.



Mathematical expression for the model from equation (4.1)

$$M a + C V + K X = e$$

Fig. 4.2 Front impact model

4.5.1 Verification of the approach

To verify the capability of the proposed identification method in estimating parameters, a frontal impact model shown in Figure 4.2 is considered. The model consists of a mass, spring and damper. The mass represents the total mass of the car. The spring and damper represent the structural properties of the car in the longitudinal direction. Two different sets of parameter (K/M and C/M) as shown in Figure 4.3, are selected. The simulated response is obtained by integrating the differential equations in Figure 4.2 with the assumed parameters, initial velocity = 13.3 m/sec and $e=0$.

Next, e is added in the simulated acceleration response, i.e. autocorrelated noise ($10 \sin 0.3t$) and random noise (mean=0, variance=10, normal distribution).

Parameters estimated from the corrupted data, and fits with the estimated acceleration, are presented in Figures 4.4, 4.5, 4.6 and 4.7. Also the autocorrelation and cross correlation of residue with input is evaluated. It is observed that for model structures with $p,q=0,0$ and $p,q=1,2$ the residue does not cross correlate with the the

input, therefore the parameters K and C estimated using both the models are quite close even though the residue for model with $p, q = 0, 0$ is autocorrelated.

Based on the above verification it was concluded that the estimation of parameters using a recursive Gauss-Newton algorithm is acceptable in the presence of combined autocorrelated noise and random noise.

4.5.2 Frontal impact

A simple model, presented in Figure 4.2, used in the previous section for frontal impact is considered. The purpose is to estimate structural parameters like mass, stiffness and damping from the actual test data.

Test data: Frontal impact

The frontal impact test data were measured at the rear rocker location. The data from three different tests of a typical large car, with different initial velocities of impact are obtained and presented in Figure 4.8, 4.9 and 4.10. All the actual test data sets used in this Chapter are normalized, as explained in the Appendix. It is assumed

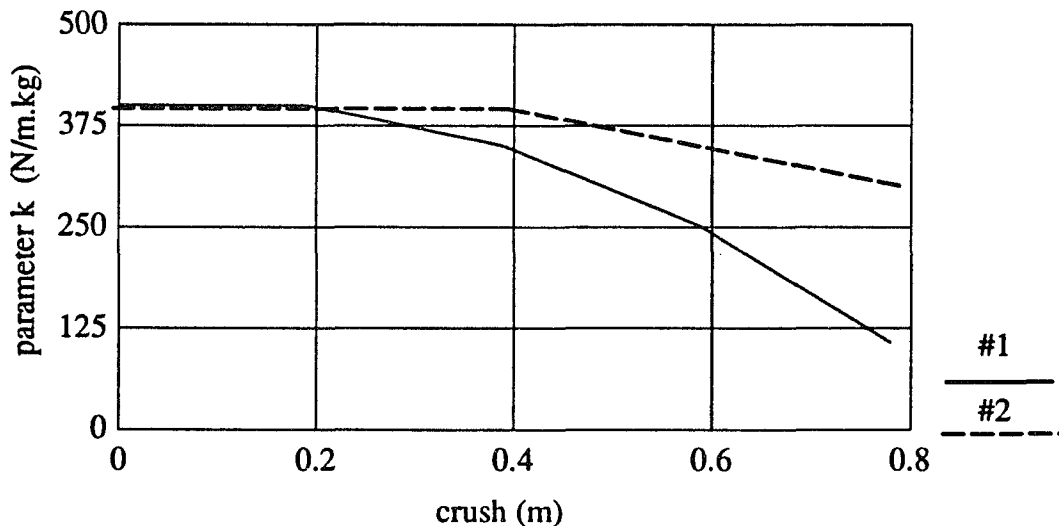


Fig. 4.3 Assumed parameter used to generate simulated data
(K/M are #1 and #2 as shown, $C/M = 1.0$)

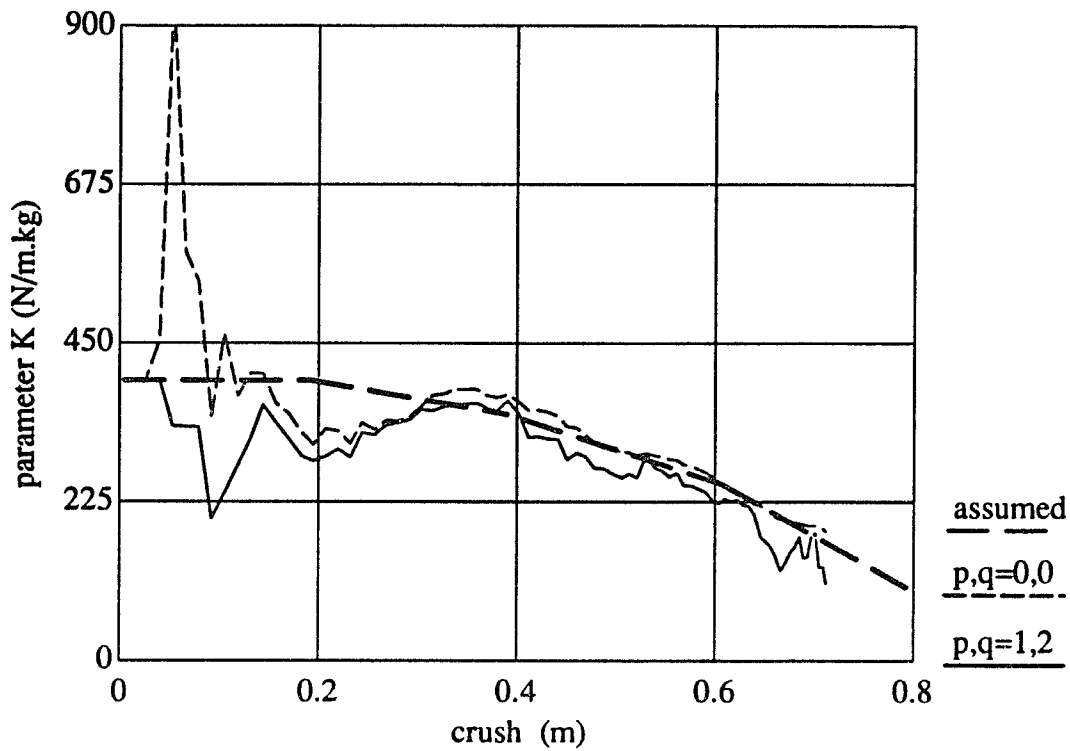


Fig. 4.4 Comparison of assumed and estimated parameter #1

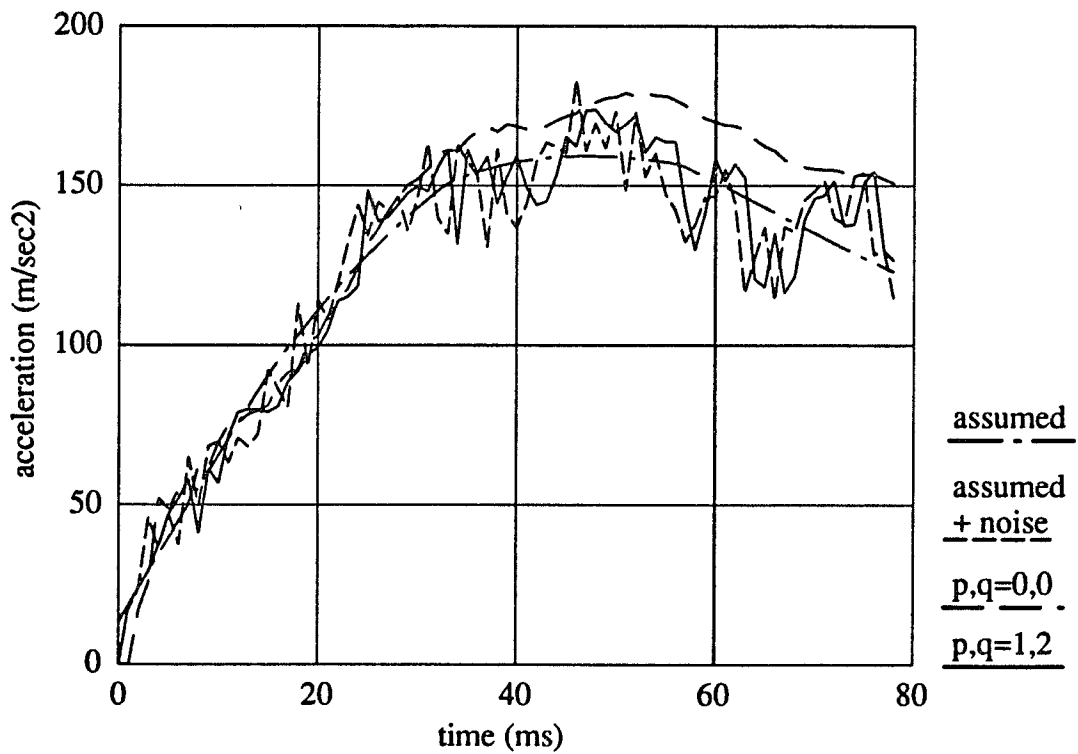


Fig. 4.5 Comparison of assumed and estimated acceleration for parameter #1.

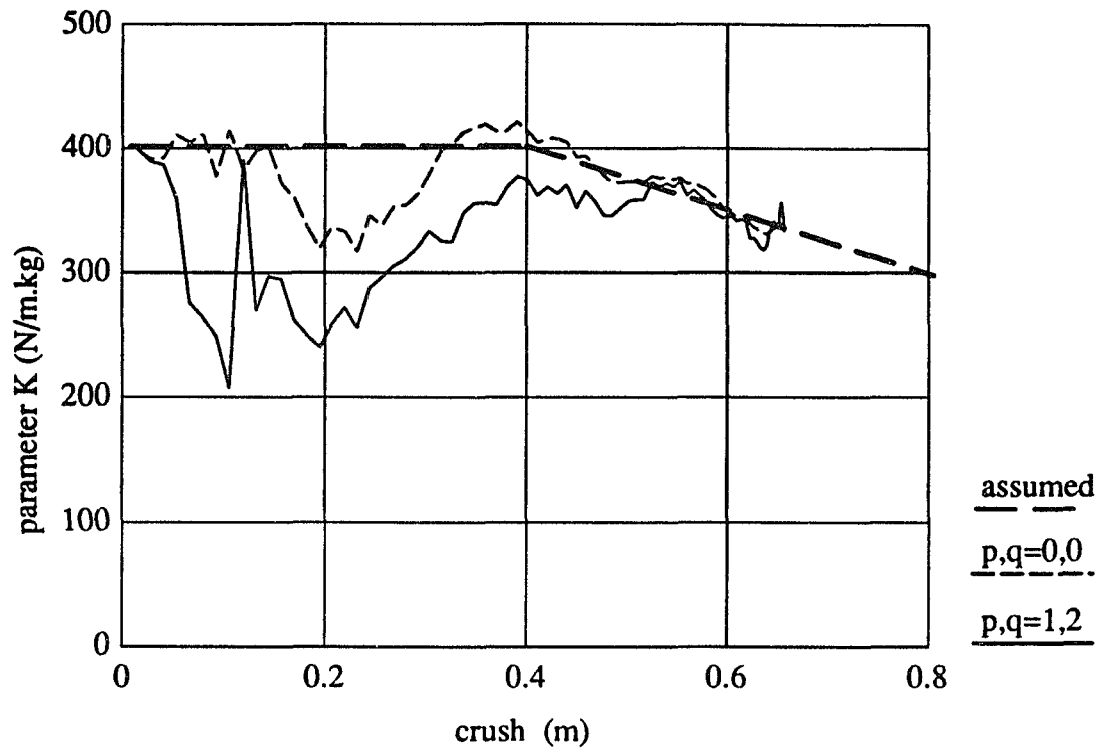


Fig. 4.6 Comparison of assumed and estimated parameter #2

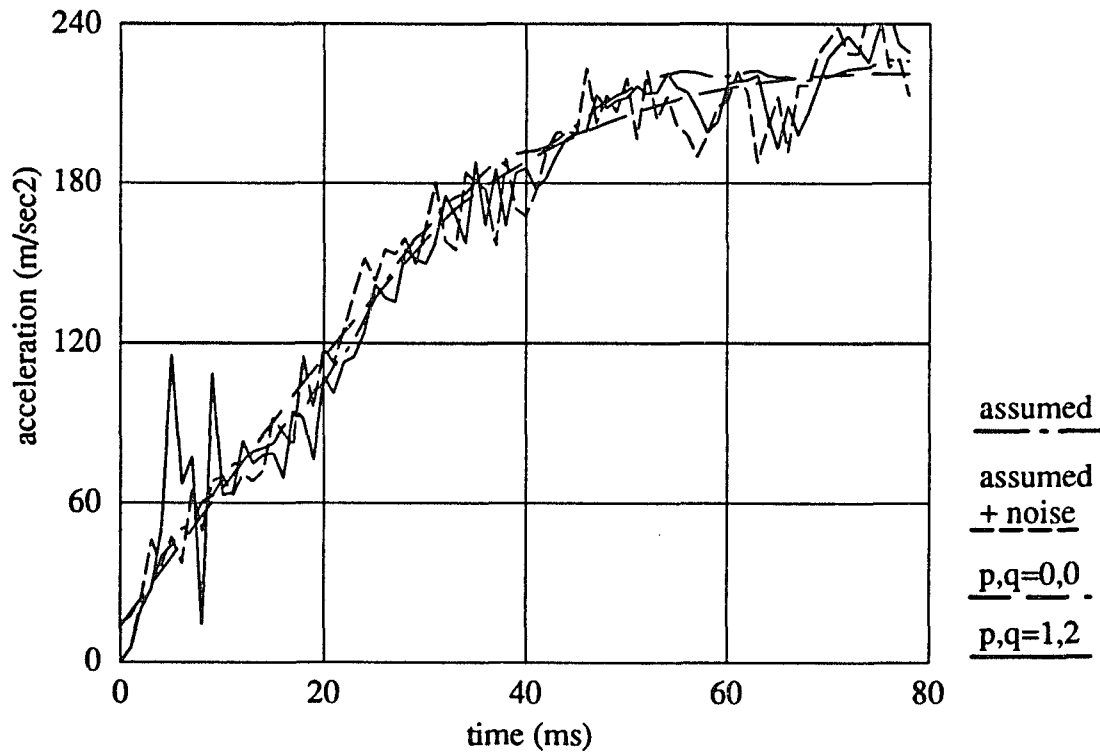


Fig. 4.7 Comparison of assumed and estimated acceleration for parameter #2.

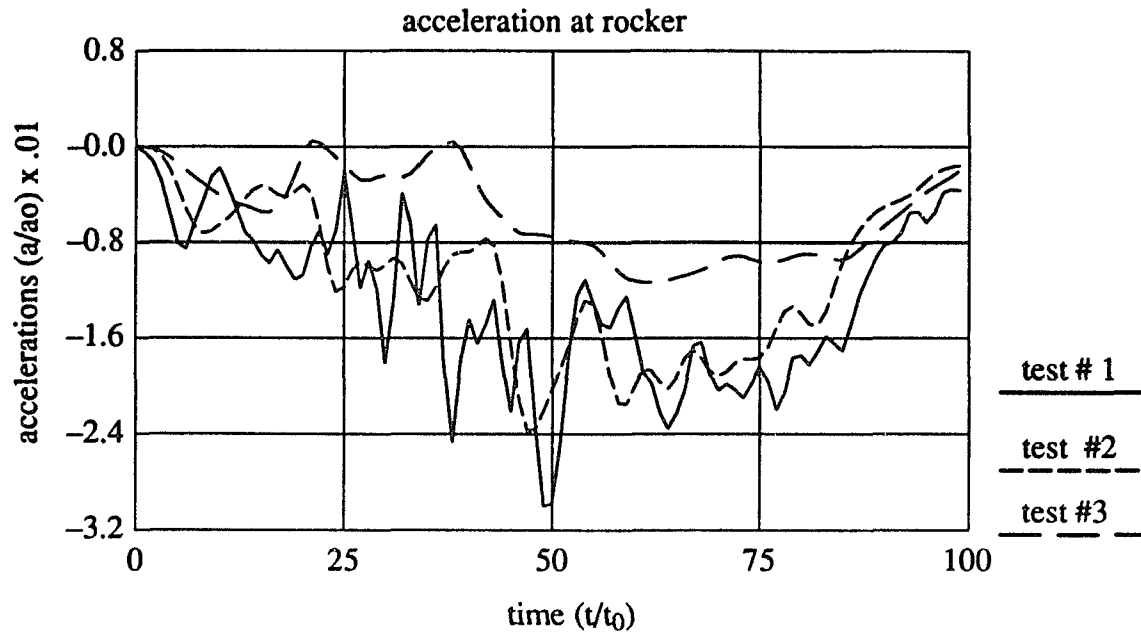


Fig. 4.8 Measured data in frontal impact for three different test

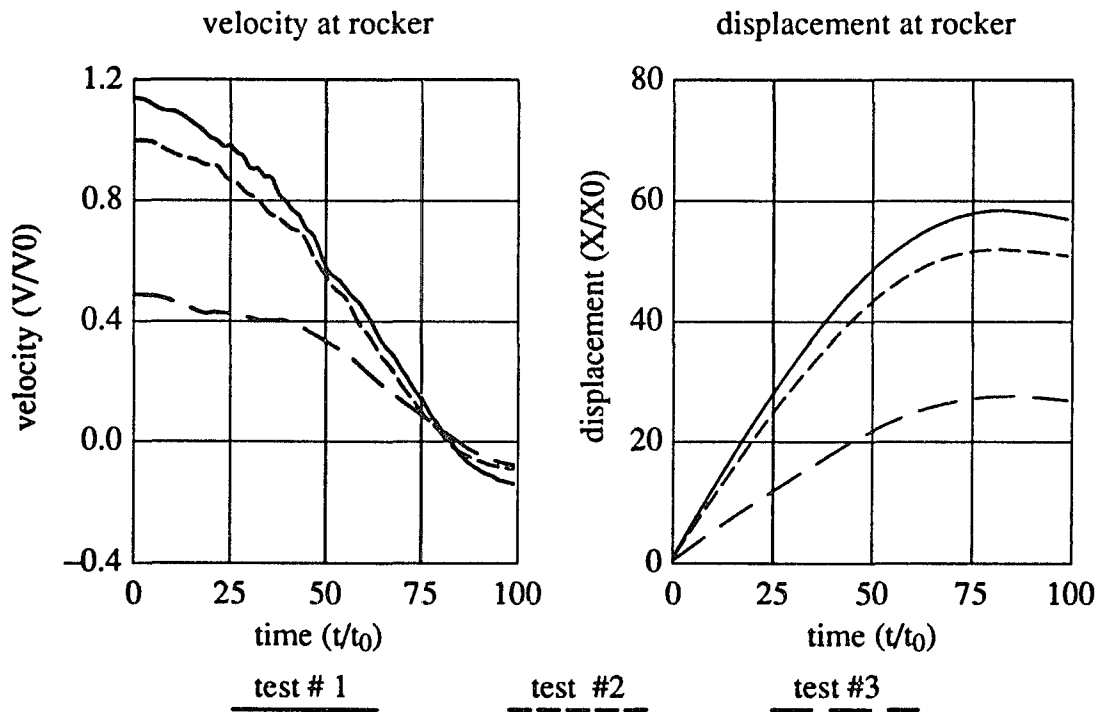


Fig. 4.9 Measured data in frontal impact

Fig. 4.10 Measured data in frontal impact

that the occupant compartment is a rigid body and its acceleration is measured at the rear rocker of the car.

Estimation of mass

For a frontal impact the external force will be the force measured on the barrier wall. Therefore the effective mass can be estimated using expression (4.3a). The barrier forces for three different tests (#1, #2, #3) are presented in Figure 4.11. The estimated mass is presented in Figure 4.12. As discussed earlier the estimated mass or effective mass is not expected to be constant.

Estimation of stiffness and damping parameters

The equation of motion, using expression (4.1) can be written as:

$$M a + C V + K X = e \quad (4.11)$$

The acceleration is available from the test. The velocity and displacement are evaluated from the acceleration. The unbalanced force e can be represented as ARMA(p,q) process of white noise e' , i.e.

$$e = \frac{A(B)}{D(B)} e' \quad (4.12)$$

The “Adaptive forgetting through multiple model (AFMM)” approach as proposed by Andersson [1985] is applied to estimate the major discontinuities in the parameters. As shown in Figure 4.13, for test #2 the change in the parameter is observed at approximately $t/t_0 = 80$. This makes physical sense because at this time the velocity becomes zero and the car starts to rebound. Similar behavior is observed for the tests #1 and #3.

The crush dependent force ($F/M = K/M \cdot \text{crush}$), estimated using recursive Gauss–Newton algorithm is presented in Figure 4.14. The mass compensated crush

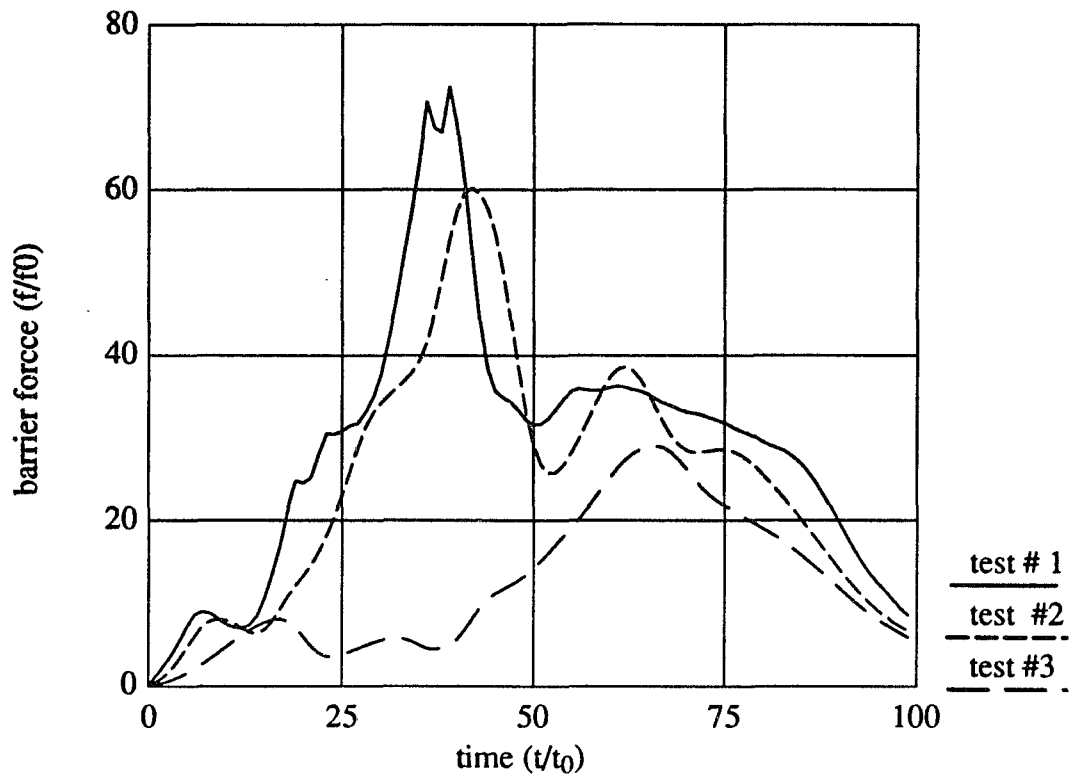


Fig. 4.11 Measured barrier force in frontal impact

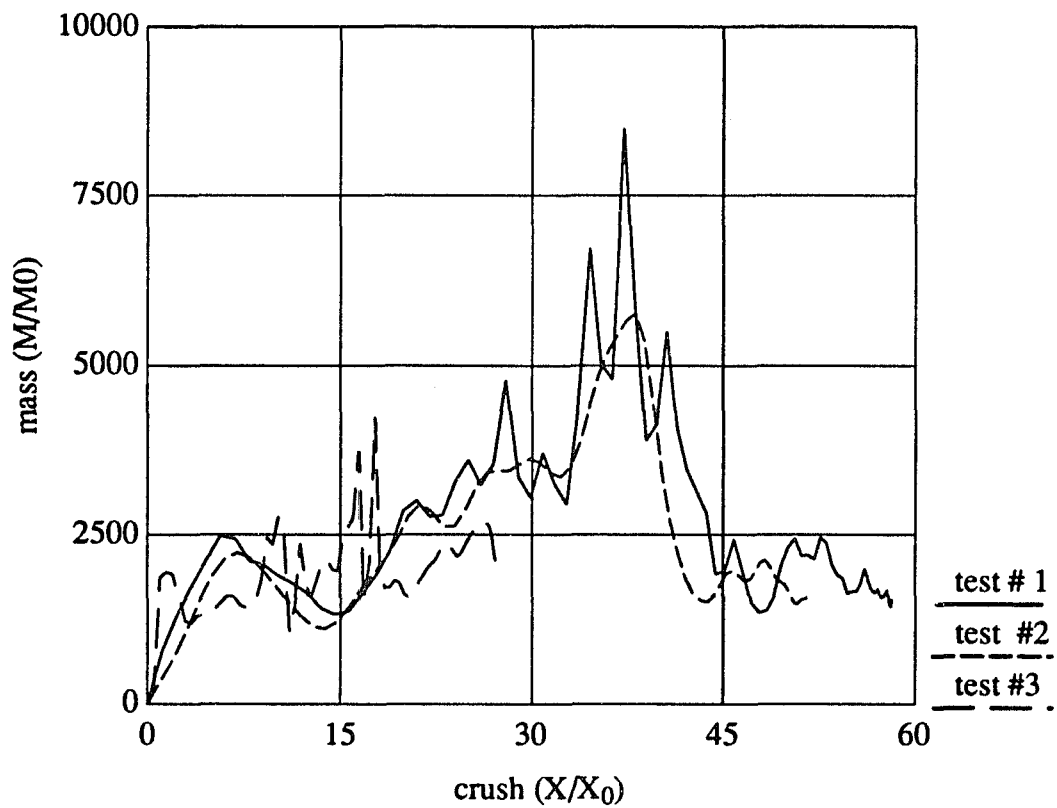


Fig. 4.12 Estimated mass in frontal impact

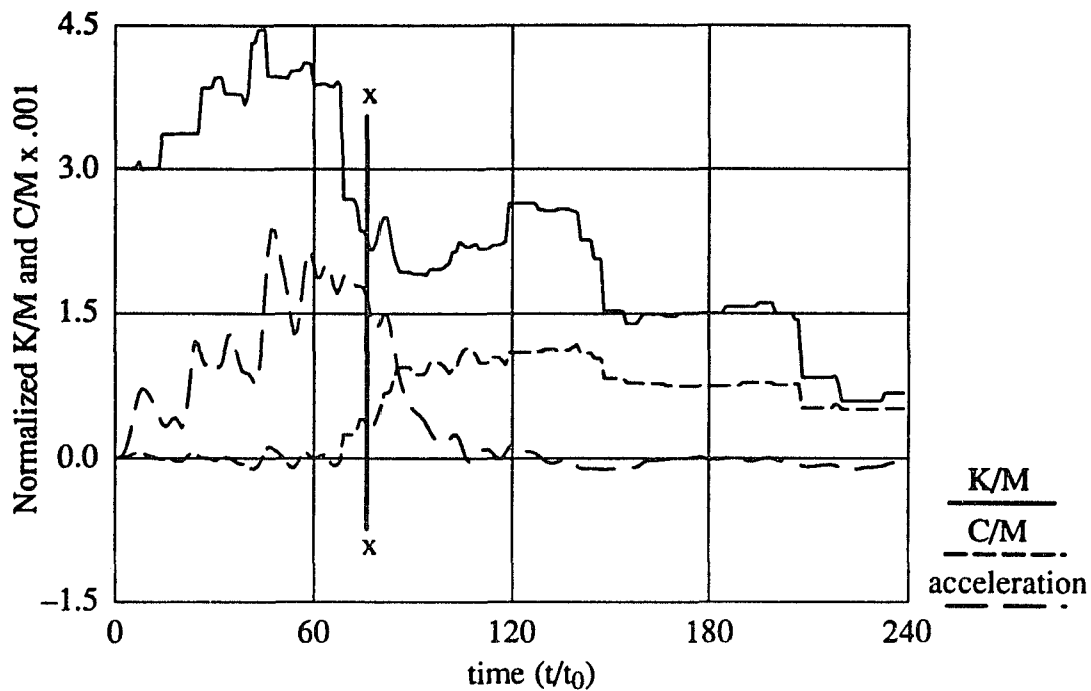


Fig. 4.13 Estimation of discontinuities in the parameters using AFMM approach for test #2. Acceleration is for reference only-not scaled. Major discontinuity (x-x) in the parameters at approximately $t/t_0 = 80$.

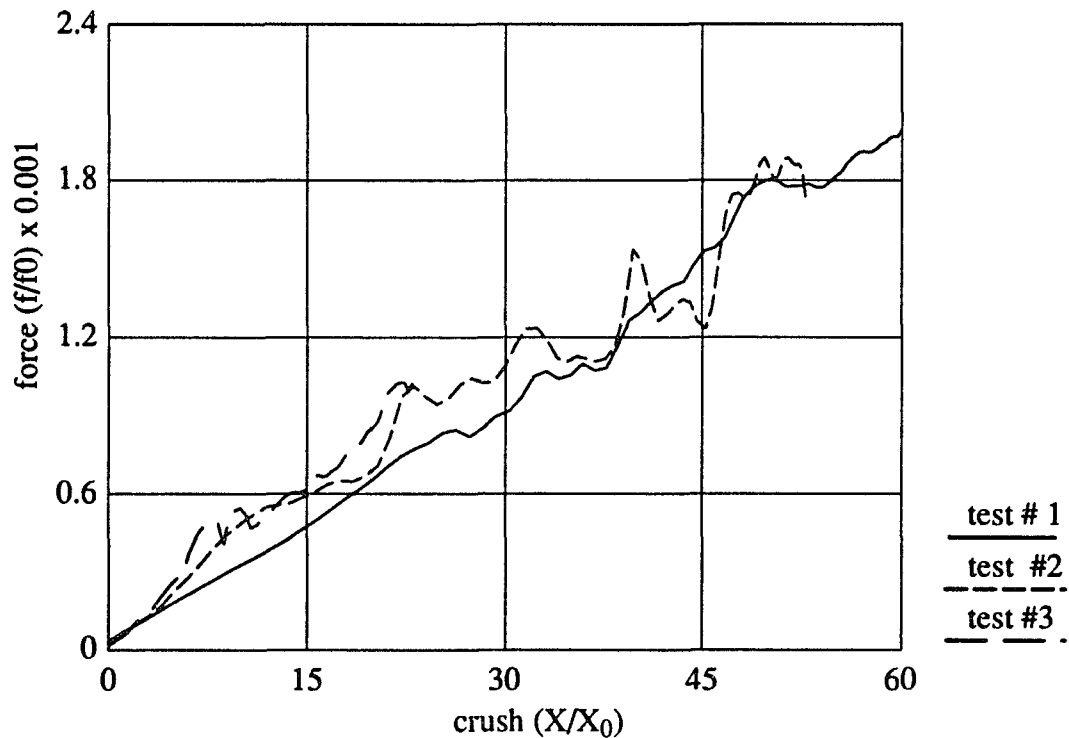


Fig. 4.14 Estimated parameter K/M x crush, force per unit mass for three different test.

The crush dependent force ($F/M=K/M \cdot \text{crush}$), estimated using recursive Gauss–Newton algorithm is presented in Figure 4.14. The mass compensated crush dependent force [$F=M (K/M \cdot \text{crush})$] is presented in Figure 4.15. It can be seen that the crush dependent force is about the same for all three tests. This is physically meaningful as the underlying structure in the three tests is about the same. The mass compensated force is close to the measured barrier force.

The estimated acceleration is compared with the test data in Figures 4.16, 4.17, 4.18 and the auto-correlation of the residue and cross correlation of residue with input is presented in Figure 4.20. It can be seen that the residue is quite small and is uncorrelated.

For further verification, the differential equations of motion are simulated with the estimated parameter. The simulated acceleration and velocity are compared with the test data in Figures 4.16, 4.17, 4.18, 4.19. It can be seen that the simulation is quite close to the test measurements.

4.5.3 Effect of sampling

How would the estimation be affected with higher sampling rate? This question is important as the crash is a relatively short event and the structure is greatly altered in the short duration. In the previous examples the parameters were estimated from only 80 set of data points. Therefore it is believed that with more sampling more information about the event will be available. On the other hand with a higher sampling rate more noise is introduced, and this is likely to make estimation difficult.

In order to study the effect of a higher sampling rate, data from two similar tests (test #4 and test #5) are obtained. For each test there are two sets of measurements available, one sampled at 1/ms and the other at 10/ms. The accelerations for both tests

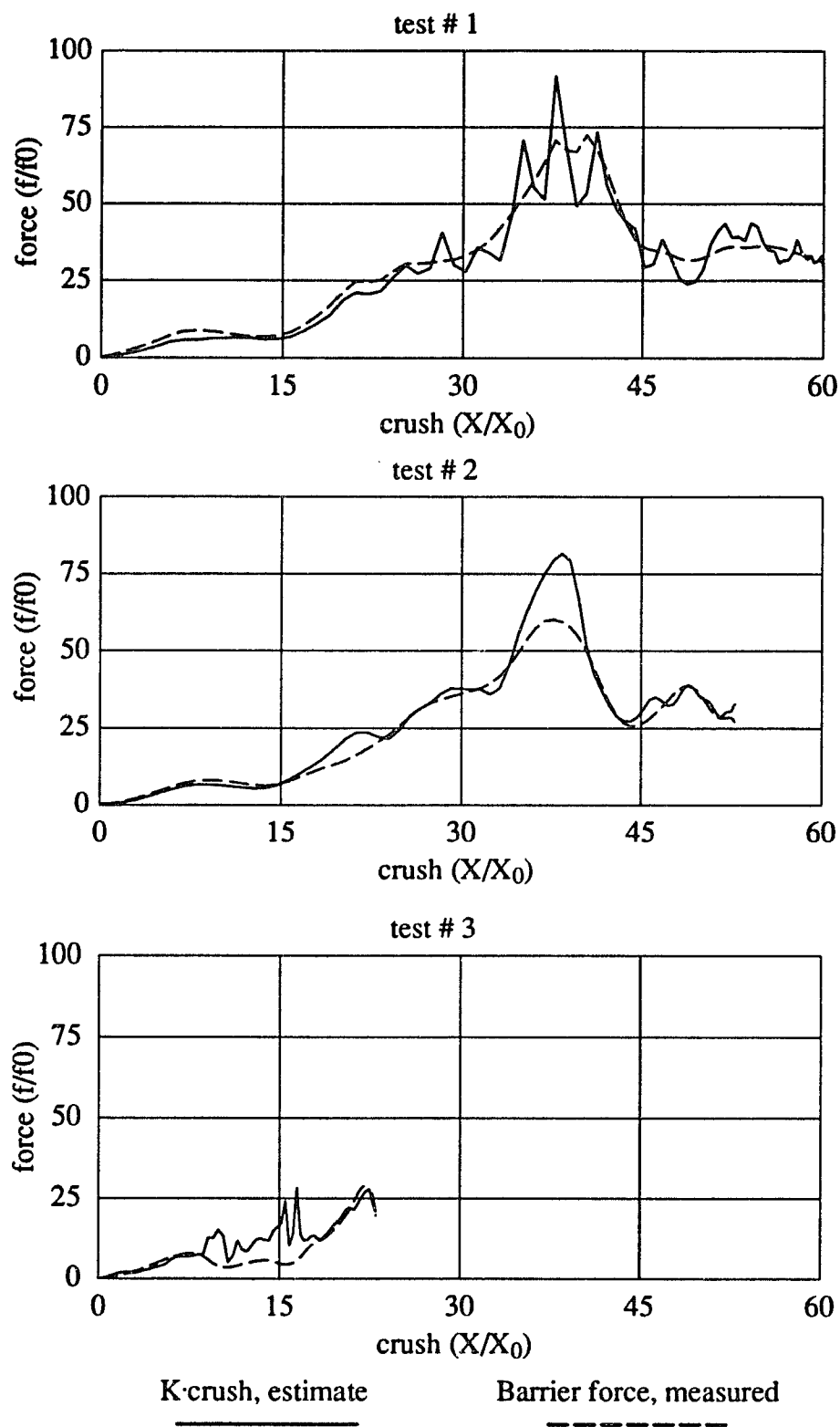


Fig. 4.15 Comparison of crush dependent force $K \cdot \text{crush}$ with the measured barrier force.

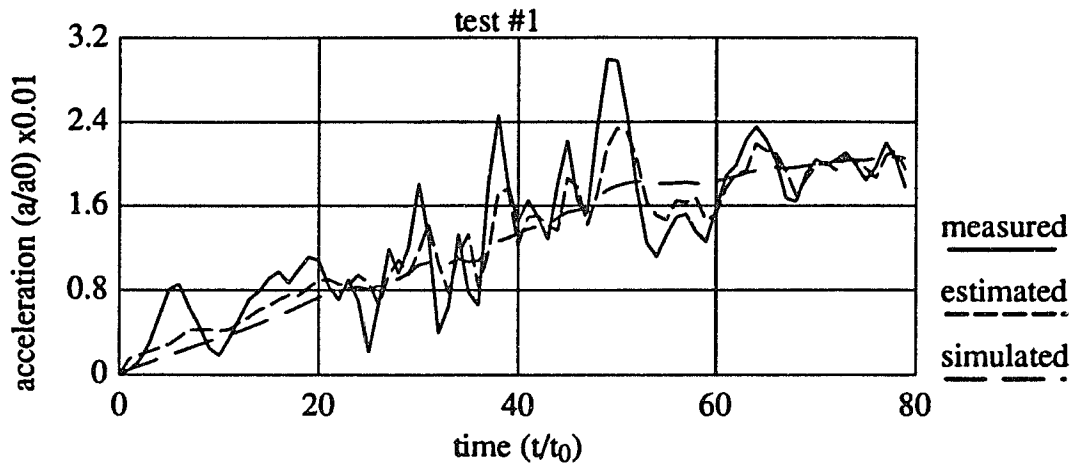


Fig. 4.16 comparing simulated acceleration with the measured and estimated acceleration, $p,q=1,2$

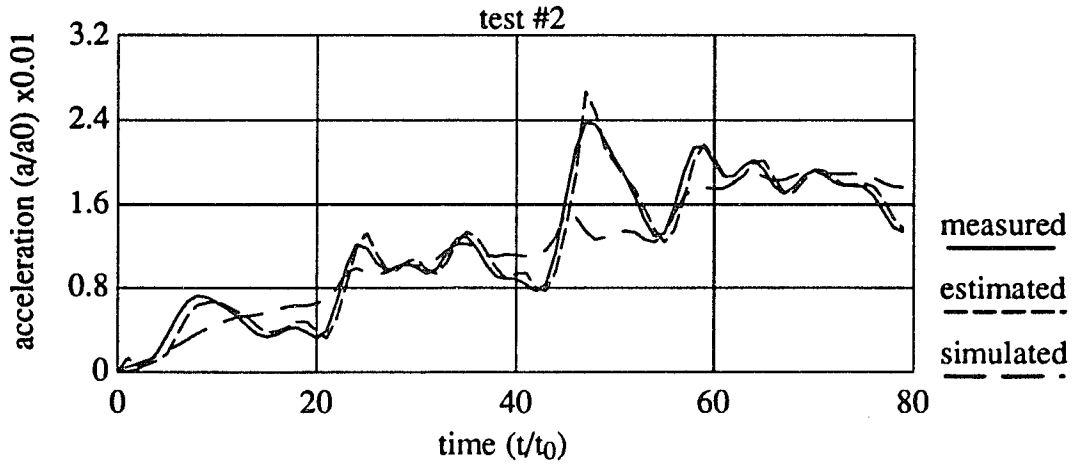


Fig. 4.17 comparing simulated acceleration with the measured and estimated acceleration, $p,q=1,2$

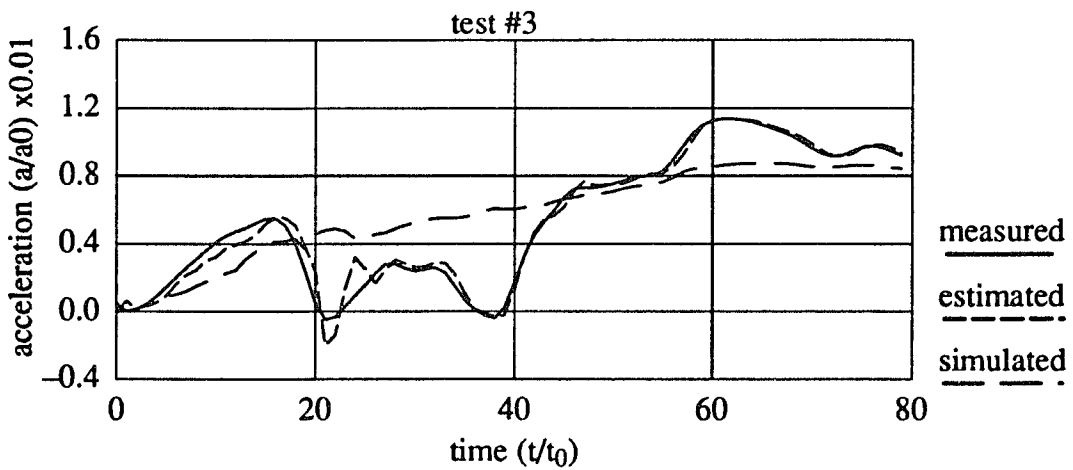


Fig. 4.18 comparing simulated acceleration with the measured and estimated acceleration, $p,q=3,4$

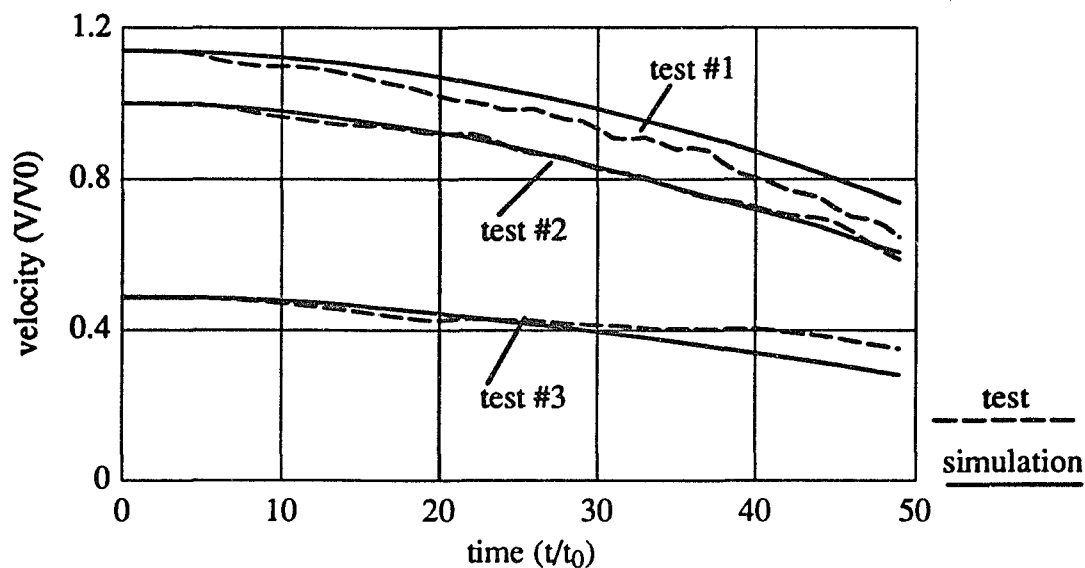


Fig. 4.19 Comparison of simulated velocity and measured velocity at rocker

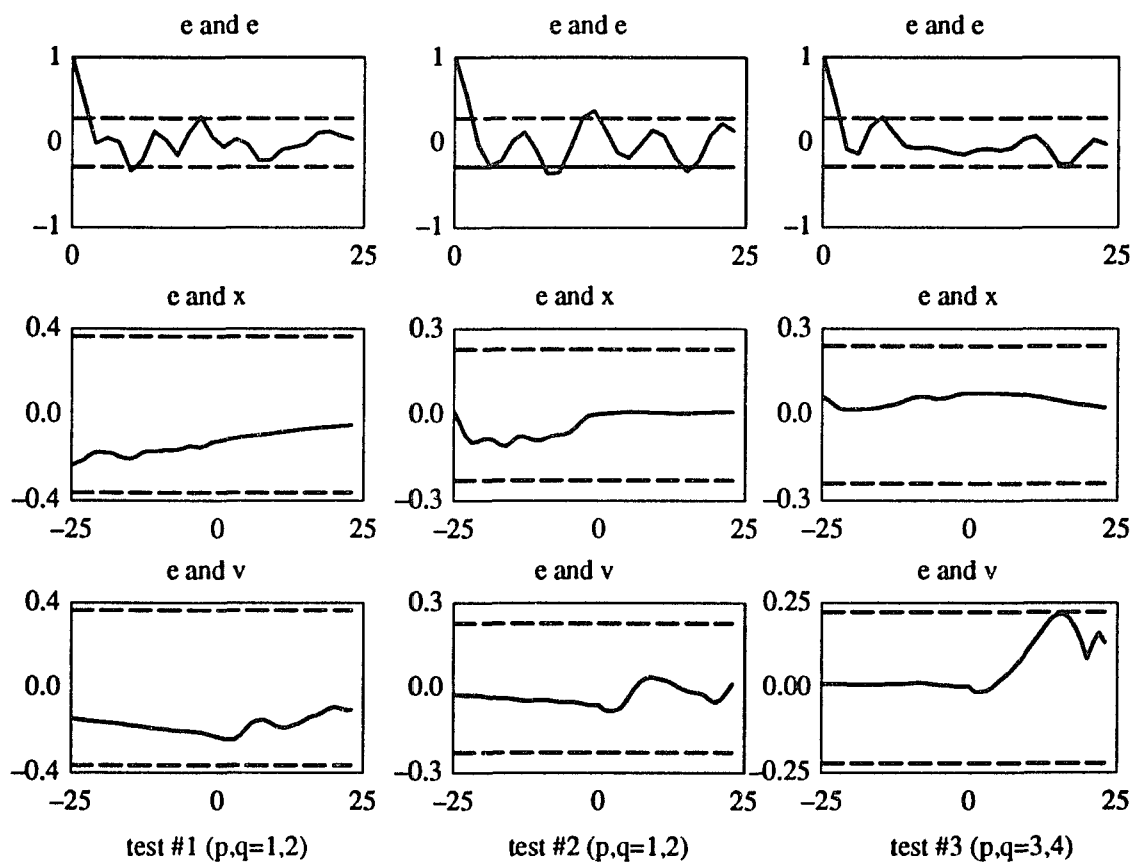


Fig. 4.20 Autocovariance of the residue and cross-covariance of the residue with the input for frontal impact. (e is the residue)

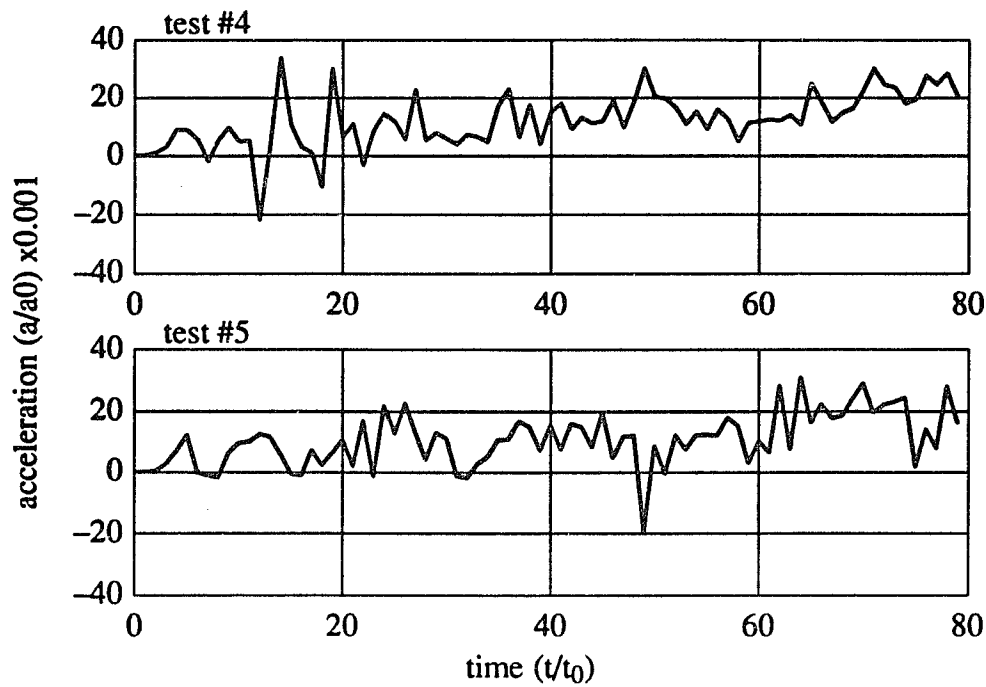


Fig. 4.21 Measured acceleration sampled at two different sampling rates

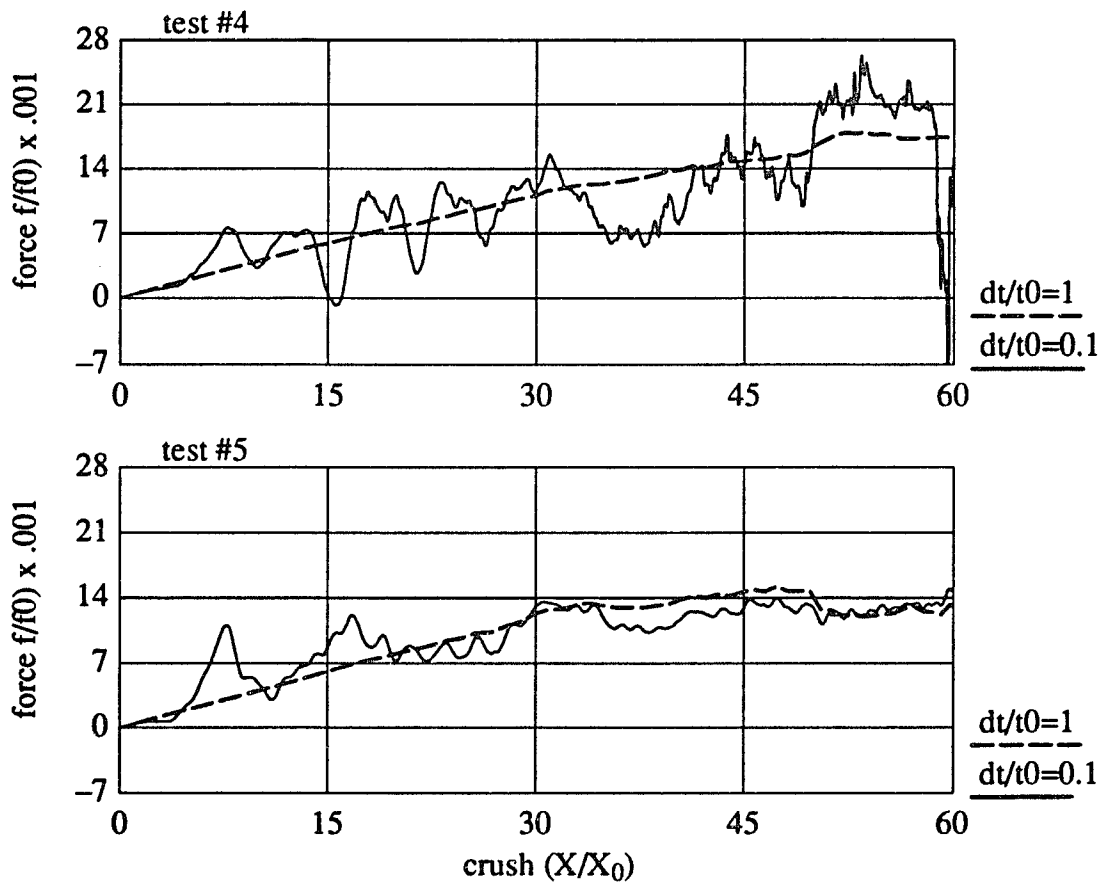


Fig. 4.22 Estimated force ($K/M \times$ crush) for the data with sampling interval $dt/t_0 = 1$ and $dt/t_0 = 0.1$, for test #4 and test #5.

The estimations of crush dependent forces are presented in Figure 4.22. It can be seen that the crush dependent force for both tests are quite close except that the estimated values for stiffness with higher sampling rates are a little lower than those with lower sampling rates. It is also observed that due to the presence of high frequencies at higher sampling rate the ARMA part of the model has higher values of p and q .

Based on the above it is concluded that for the particular application (slowly changing parameters) the sampling rate of 1/ms is acceptable.

4.5.4 Side impact

A simple model for side impact was presented in Figure 4.1. The model consists of a test car MDB and door connected by two set of stiffness and damping parameters

Test data: side impact

The test measurements for a typical mid size car are presented in Figure 4.23. MDB acceleration was measured at the CG. The car acceleration was measured at the right rocker and the acceleration on the door was measured at a location that was close to the occupant's chest. Relative velocity and displacement between the MDB and the door, and the door and the car, are presented in Figures 4.24, 4.25.

Estimation of mass

Using the formulation developed in previous section i.e. equation (4.3a) and (4.3b) the time varying mass distribution is estimated using recursive least square method. The estimated mass distribution is presented in Figure 4.26. It should be noted that the mass here is a multiplier to the acceleration such that the error in equation (4.3a) is minimized while satisfying equation (4.3b).

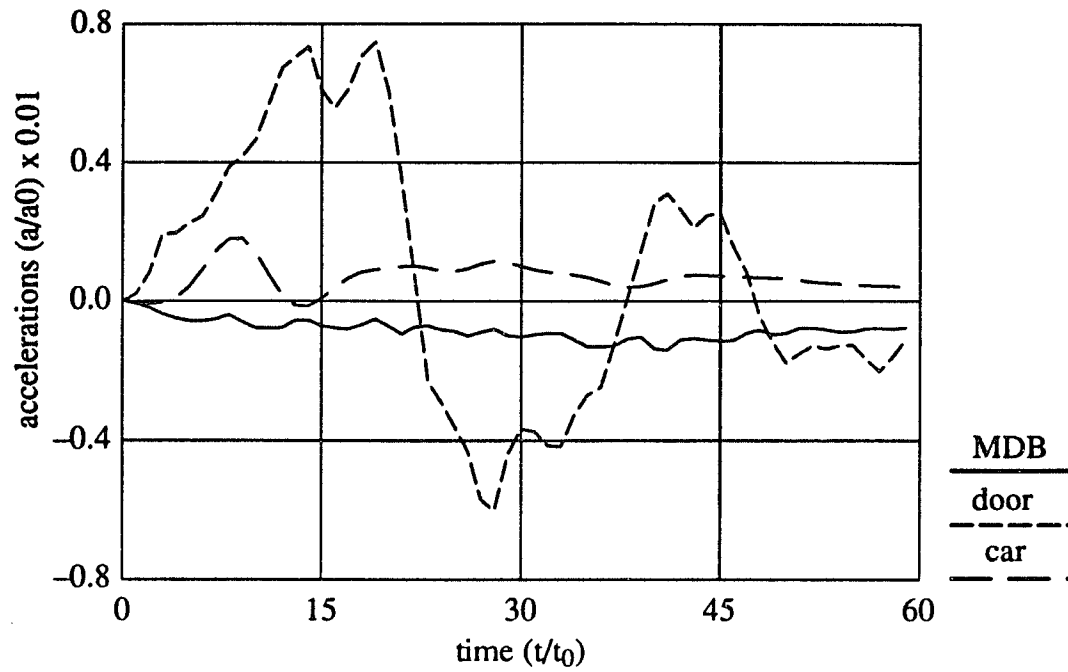


Fig. 4.23 Side impact measured data - accelerations

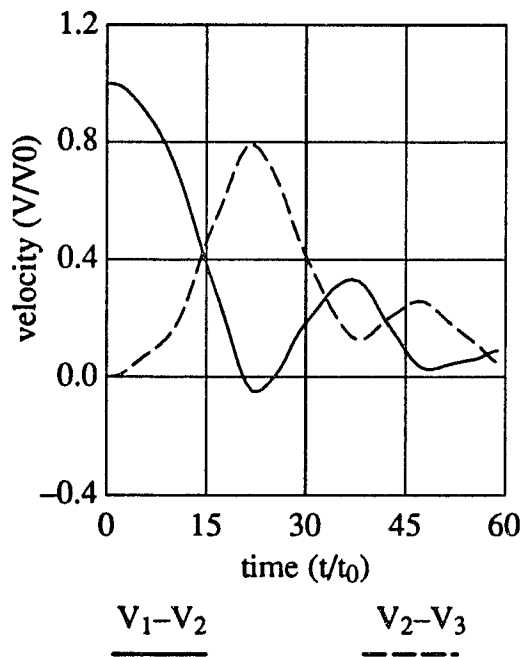


Fig. 4.24 Side impact measured data-velocity

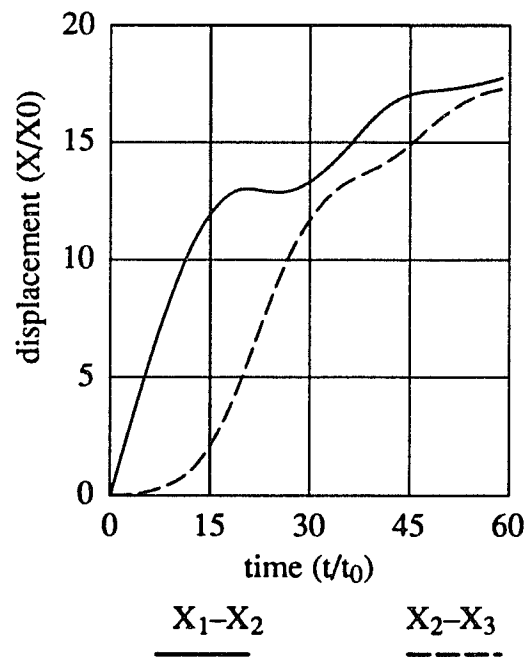


Fig. 4.25 Side impact measured data-displacement

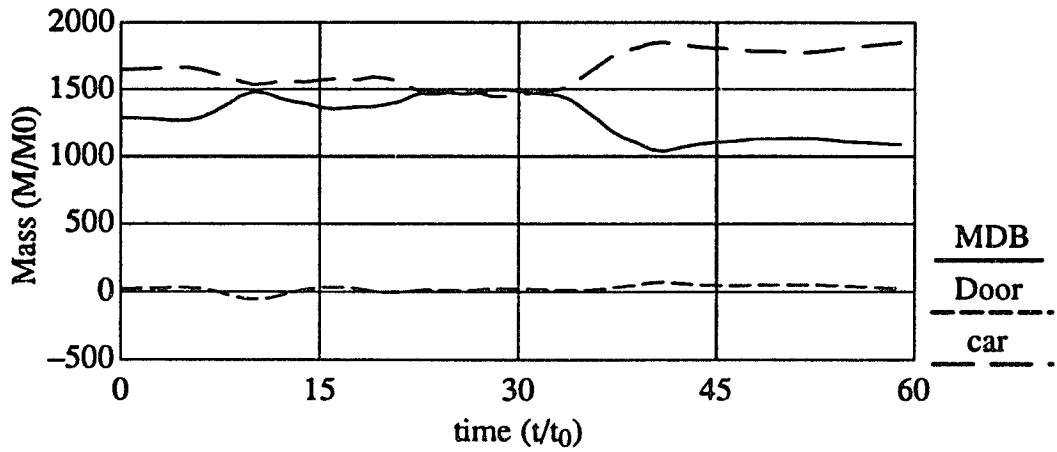


Fig. 4.26 Estimated lumped mass in side impact

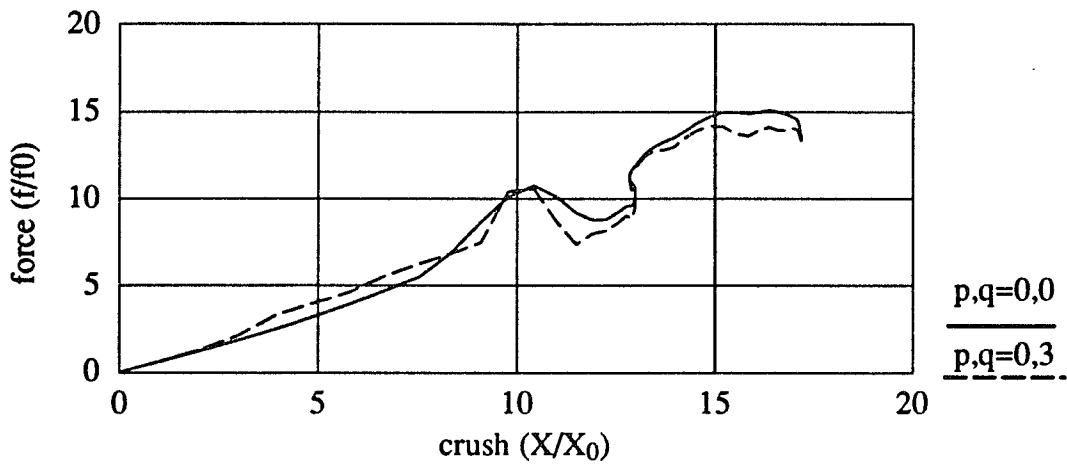


Fig. 4.27 Estimated crush dependent force $K_1 (X_1 - X_2)$ between MDB and door, for $p,q=0,0$ and $p,q=0,3$.

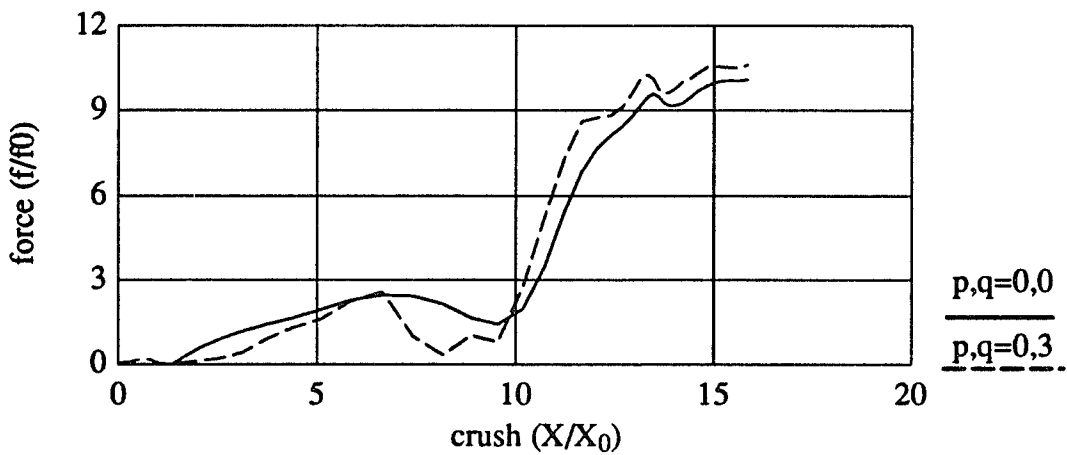


Fig. 4.28 Estimated crush dependent force $K_2 (X_2 - X_3)$ between door and car, for $p,q=0,0$ and $p,q=0,3$.

Estimation of stiffness and damping parameters from test data

The equation of motion for the system shown in Figure 4.1 can be represented using the equation set (4.4). The estimated mass \cdot acceleration, ($M_i \cdot a_i$, on the left hand side) is available from the estimation of mass. The velocity and displacement are evaluated from the acceleration measurements. The unknown parameters are evaluated using the recursive Gauss-Newton algorithm, for various values of p, q in the ARMA part of the model.

The estimated crush dependent force ($K_i \cdot \text{crush}$) is presented in Figures 4.27, 4.28. The rate of crush dependent force is also estimated and is found to be quite small compared to the crush dependent forces. The autocorrelation of the residue for the model structures showed that the residue for the linear structure ($p, q=0,0$) has a high autocorrelation compared to an estimate with nonlinear model structure ($p, q=0,3$). However, as the inputs are uncorrelated with the residue, the structural parameters estimated from both the models are quite close. The simulation with the estimated parameters is presented in Figures 4.29 and 4.30.

4.6 Use of data-based model in designing for crashworthiness

The system identification based approach can be very useful in developing an analytical model for crash simulation as well as in understanding the test event. In this chapter the approach was demonstrated with simple models, but more complex models, such as those shown in Figure 2.10, are possible. The stiffness and damping parameters were estimated under dynamic loading and realistic boundary conditions. This eliminates the problem of incorrect crush modes often encountered with conventional methods like static crusher or drop-silo test.

Once a data-based model is developed from the available test data, the model will be very useful in performing parametric study and evaluation of design trade-offs.

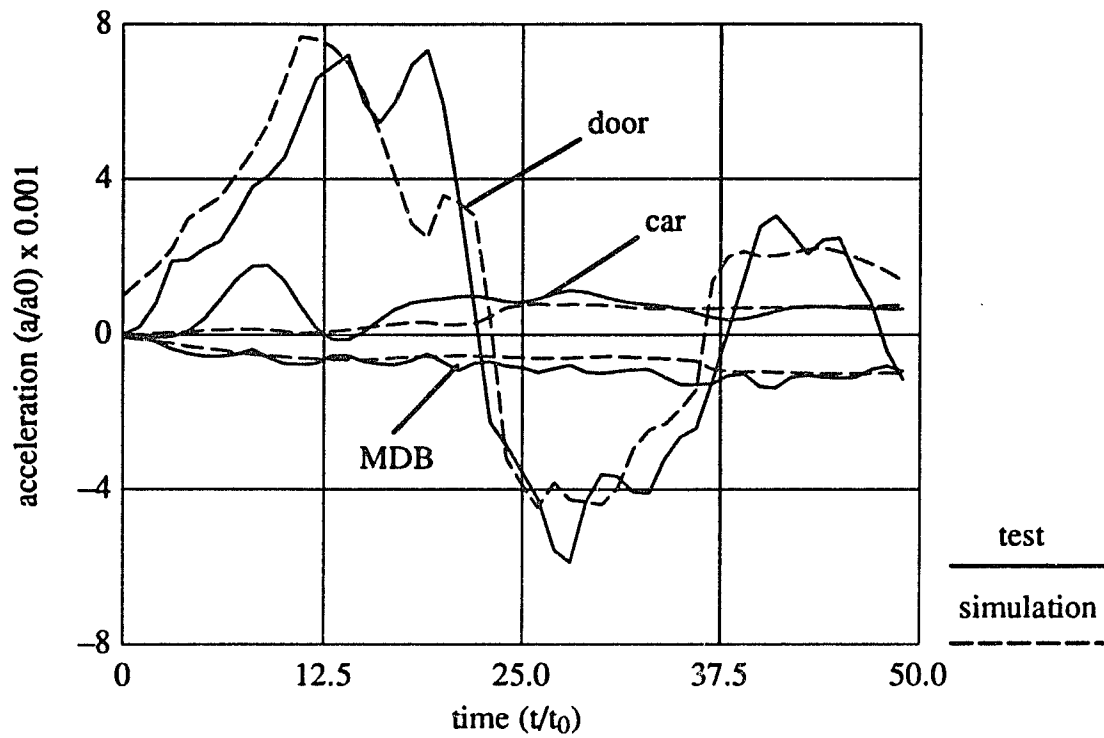


Fig. 4.29 Comparison of measured and simulated acceleration

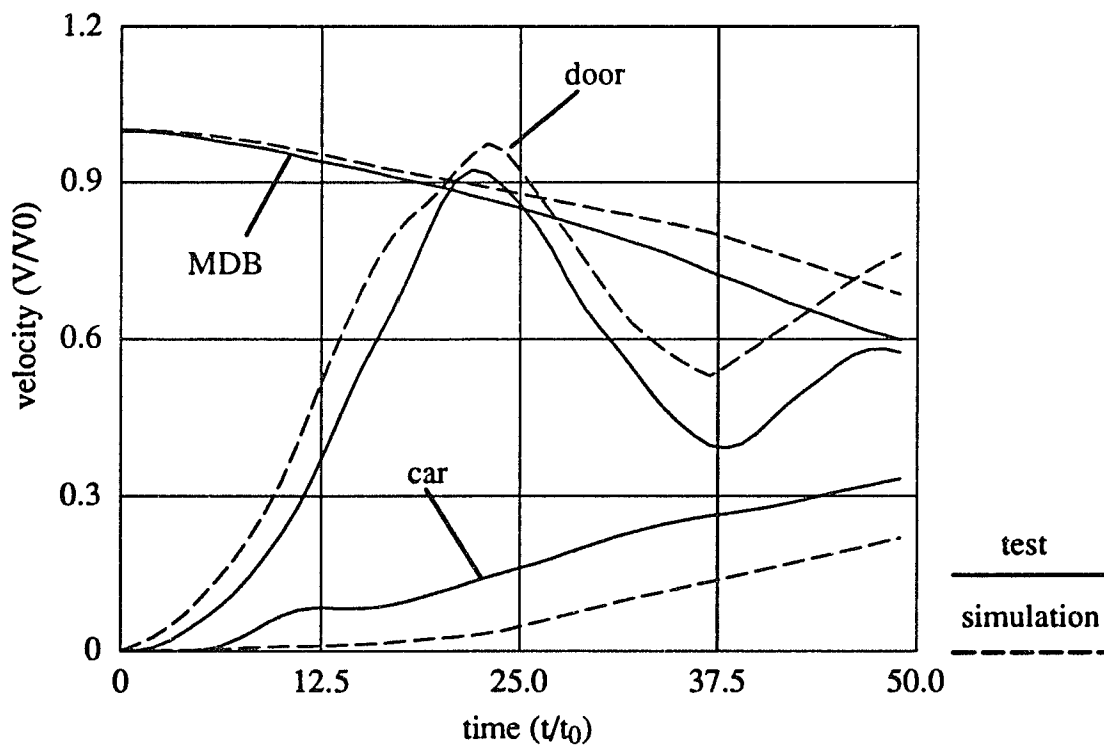


Fig. 4.30 Comparison of measured and simulated velocity

The data based model will also be very useful in the development of air-bag trigger mechanisms, since the acceleration response of a vehicle for various initial velocities and loading (mass) can be estimated from a single test.

The other important use of the data based model will be in analyzing test data. The method allows us to lump mass at any location where an accelerometer can be located. If accelerometers are placed at two points on a structural member then the in-position structural characteristic of that member along the line joining the two accelerometers can be estimated. This is very useful in analyzing the test data, as with a judicious use of accelerometers, the contribution of a particular structural member in the crash test can be identified.

4.7 Chapter summary

- 1) Analytical models for crash simulation of an automobile were developed from the crash test measurements. The rigid body response of the crash test measurements were represented using physically meaningful lumped parameter model. The part of the measured crash test data not represented by the lumped parameter model response was represented by an autoregressive moving average (ARMA) model appended to the lumped parameter model.
- 2) The model structure was correlated with the physics of the crash event. The mass can be considered as an inertia dependent characteristic and assumed to be a nonlinear quantity lumped at the location of the acceleration measurements. Stiffness and damping in the model can be treated as the crush and rate of crush dependent structural characteristics, and were considered to be effective between two corresponding acceleration measurement locations.
- 3) The model parameters were estimated by minimizing the quadratic criteria of prediction errors using Gauss-Newton algorithm. The approach was verified

for simulated data in the presence of noise and then demonstrated on test data for frontal as well as side impacts. The data based model will be helpful not only in predicting but also in understanding test results.

CHAPTER 5

ADDRESSING PROBLEMS IN ESTIMATING MODEL PARAMETERS FROM THE CRASH DATA

5.1 Introduction

In the last chapter development of analytical models from crash data using system identification methods was demonstrated. An automobile crash is a complex event with extensive interaction of various components. The model parameters are time varying and test measurements are corrupted with noise. Because of these problems the development of the data based models from crash data is a challenging task. The purpose of this chapter is to address these challenges.

This chapter is organized as follows: In Section 5.2, complexities in the estimation of parameters from the automobile crash data are recognized. Sections 5.3, 5.4 and 5.5 present methods to address the complexities in the estimation of a model structure. In Section 5.6, applications of the methods proposed in Sections 5.3, 5.4 and 5.5 are demonstrated on the simulated as well as test data.

5.2 Complexities in estimation of parameters from crash data

From the review of the crash mechanism and the simple examples presented in Chapter 4, it is quite evident that a crash is a highly complex event. The complexities that makes estimation of model parameters from the test data difficult are listed below:

- 1 The test data, which are essentially accelerations measured at various locations, have a high noise content.
- 2 The crash is a transient event and hence the model parameters are not stationary and they change as the crash progresses.

- 3 Extensive interactions of various components are involved during the crash. Therefore, ideally a complex model with a large number of parameters is desired for the analysis.
- 4 Since the event of interest is transient, a quick tracking of the changing parameters is needed to minimize the error in prediction.
- 5 The estimation of a unique parameter set that minimizes the multi-error in a multi-dimension space is extremely difficult due to the presence of a number of local minima, Press et al. [1989].

The complexities involved in the crash event generate conflicting requirements on the estimation procedure. The crash being a complex event, a detailed model with a high number of unknown parameters is desired. However, an increase in the number of unknown parameters creates problems of uniqueness in the estimation. Similarly, the rapidly changing parameters demand an 'alert' estimation procedure whereas high noise content demands a 'sluggish' system, otherwise noise will 'misguide' the 'alert' estimator. Therefore, development of estimation procedures that compromise between these conflicting requirements are necessary.

The problems in the development of analytical models are addressed by using physical insights of the event and modifying the estimator to meet specific needs. In this chapter this approach is presented in the following steps:

- 1) Simplify the model structure.
- 2) Make use of physical insight.
- 3) Modify the estimator.

The details of these steps are presented in the next sections.

5.3 Simplify the model structure

The purpose of simplifying the model structure is to reduce the number of unknown parameters, i.e., to choose a model structure that represent the mechanisms

of interest with a minimum number of parameters. Consider the model structure considered in Equation (4.10) of Chapter 4. The equation error, e_k , is represented as the ARMA of white noise appended to the lumped parameter model. As the ARMA model is non-linear in parameters an iterative estimation method is required. The transfer function, or ARMA part of the model, is useful in understanding the structural vibration of the automobile at the accelerometer location during the crash event. If the interest is in estimating the stiffness and damping characteristics (i.e. K_i and C_i) then the transfer function part of the model can be dropped. This results in a linear model structure, even though the parameters are time varying. The estimation of parameters from the linear model structure is computationally easy and unambiguous. However, the concern is the equation error, e_k , which becomes a prediction error and is likely to be auto-correlated noise. The question is, would simplifying of the model structure affect the accuracy of estimation of the K_i and C_i parameters? This is examined below.

For a linear model structure expressed as,

$$Y_t = \Psi_t^T \theta_t + e_t ,$$

estimation of the parameters from n observations, using a least-squares method can be given by,

$$\theta_{ls} = \theta_{act} + \left[\frac{1}{n} \sum_{t=1}^n \Psi_t \Psi_t^T \right]^{-1} \frac{1}{n} \sum_{t=1}^n \Psi_t e_t$$

The estimated parameter, θ_{ls} , will tend to the actual parameter, θ_{act} , if the right most term on the right hand side is zero. This will be true if the input-output covariance matrix, $[1/n \sum_{t=1,n} \Psi_t \Psi_t^T]$ is non zero and,

- 1) the equation error e_t is uncorrelated white noise
- or
- 2) the error e_t is not correlated with the input output matrix Ψ .

It has been observed that for the crash data and the model structure considered in Chapter 4, the second condition will be met in most of the cases if there is no autoregressive part in matrix Ψ_t , Ljung [1987].

To track the time varying parameters, recursive estimation is used. For a linear model structure, the algorithm presented in Chapter 4 equation (4.10) becomes a recursive least squares (RLS) algorithm, which as per Ljung [1987] gives:

$$\begin{aligned} E_t &= Y_t - \Psi_t^T \theta_{t-1} \\ \Lambda_t &= \Lambda_{t-1} + \gamma_t (E_t E_t^T - \Lambda_{t-1}) \\ R_t &= R_{t-1} + \gamma_t (\Psi \Lambda_t^{-1} \Psi^T - R_{t-1}) \\ \theta_t &= \theta_{t-1} + \gamma_t (R_t^{-1} \Psi_t \Lambda_t^{-1}) E_t \end{aligned}$$

where

- E_t is the prediction error.
- Y_t is the output measurement
- Ψ_t is the input–output measurement
- θ_t is the parameter vector
- Λ_t is the error covariance matrix
- R_t is the input–output covariance matrix
- γ_t is the weighing function given by; $1/\gamma_t = (\lambda_t / \gamma_{t-1}) + 1$.

The forgetting factor, λ_t , which is the user's choice in the above algorithm, defines the weighing function, γ_t , such that it decays exponentially for older observations, i.e. for the estimation more importance is placed on recent observations. Decreasing the forgetting factor means increasing the importance of recent observations while increasing the forgetting factor means increasing the importance of older observations.

5.4 Make use of physical insights

The purpose of this section is to emphasize on the use of physical insights in modeling and estimation. The use of physical insight is a broad term referring to the knowledge based efforts in improving the modeling and estimation. Chapter 2 provides physical insights in the crash event. The data based models developed in Chapter 4 are physically meaningful and the lumped parameters are correlated with the physical characteristics of the structure. In this chapter it is desired to use physical insights in estimating the model parameters.

The mathematical expression for the model structure, i.e. Equation set (4.4) in Chapter 4, is essentially a set of equilibrium equations with a unknown set of parameters and equilibrium error. The parameters are estimated such that the equilibrium error is minimized. Minimization of multi-criteria in multi-dimensions is difficult because of multiple local minimas. Therefore use of physical insight in the estimation of parameters is highly recommended.

The knowledge of the system and parameter characteristics while estimating are mainly used in limiting the search space and thus reducing the number of minimas. This is accomplished in a number of ways. For example, initial values used in the estimator, i.e. parameters, covariance of parameters and covariance of noise etc., are selected based on the past knowledge of the system. In addition, known information such as values of the parameters and knowledge that the loading/unloading and tension/compression characteristics are different are also useful in improving the estimation.

5.5 Modify the estimator

The presence of noise in the measurements and the changing parameters demand conflicting requirements on the choice of the forgetting factor in the RLS

algorithm for a multi–input, multi–output system. A high forgetting factor slows the parameter correction mechanism and lowers the capability to track the change in the parameters. With a low forgetting factor the estimation is influenced by the noise, resulting in rapid change in the estimated parameter values. Therefore, a modification in the RLS is desired to identify the cause of change in the output (i.e. change in the parameters or noise) and then make a corresponding adjustment in the correction mechanism.

The duration of interest is from the beginning of the crash to the steady state conditions – approximately 0 to 100 ms (millisecond) in the frontal impact and 0 to 60 ms in the side impact. As the event of interest is short it is desirable to quickly track the change in parameters. The approach used here is to reuse the past observations with the modified correction mechanism after a change in the parameters is recognized.

Recursive parameter estimation techniques are used to identify time varying parameters. A survey of literature on recursive estimation methods was presented in Section 3.6 of Chapter 3. While developing a model for a crash event, the problem is not only rapidly changing parameters but also the presence of noise. Therefore, to track changes in parameters first it is essential to recognize the source of the change in output (i.e. noise or change in parameters). In the next section a modification to the RLS algorithm is developed to address this concerns.

5.5.1 Modified recursive least squares (MRLS)

The modified RLS (MRLS) can be written as ,

$$\begin{aligned} E_t &= Y_t - \Psi_t^T \theta_{t-1} \\ \Lambda_t &= \Lambda_{t-1} + \gamma_t (E_t E_t^T - \Lambda_{t-1}) \\ R_t &= R_{t-1} + \gamma_t (\Psi \Lambda_t^{-1} \Psi^T - R_{t-1}) \\ \theta_t &= \theta_{t-1} + \delta_t \gamma_t (R_t^{-1} \Psi_t \Lambda_t^{-1}) E_t \end{aligned}$$

The added term ' δ_t ' is a function of the moving average of the prediction error and modifies the parameter correction mechanism as follows :

$$\begin{aligned} \delta_t &= cr \quad \text{and } t = t-m & \text{for } |E_{mean_t}(i)| > \text{error limit}(i) \\ &= 1 \quad \text{and } t = t & \text{for } |E_{mean_t}(i)| \leq \text{error limit}(i) \end{aligned}$$

where,

$$E_{mean_t}(i) = \frac{1}{m} \sum_{j=t-m+1}^t E_j(i) \quad i = 1 \dots \text{number of outputs}$$

which is the moving average of the prediction error at time t , for the last m observations. The update in the parameter is constrained to the limit

$$\gamma_t (R^{-1}_t \Psi_t \Lambda_t^{-1}) E_t |\delta_t| \leq \theta_{limt}(i) \quad i=1 \dots \text{number of parameters.}$$

The choice of 'error limits', θ_{limt} , ' m ' and ' cr ' is based on the understanding of the particular problem and is discussed in the next section. A flow chart used for the computer program written for the MRLS is presented in Figure 5.1.

5.5.2 Intuitive explanation of the proposed algorithm

The above modifications are based on the need for improvement in the RLS estimation procedure in a crash environment. There is no theoretical approach available at this time to address the complexities discussed in Section 5.2. Therefore the approach makes use of the physical insight into the event. It is assumed that the variance of the prediction error (to define error limit) is known approximately and the maximum rate of change of the unknown parameters (to define θ_{limt}) is available.

Because of the choice of model structure, the prediction error is expected to be zero mean. Therefore, a change in the noise will not have a significant effect on E_{mean_t} . As a result it can be assumed that a change in the the moving average E_{mean_t} indicates a change in the the parameters. The E_{mean_t} is updated for every observation and compared with a pre-specified limit called the 'error limit'. Whenever E_{mean_t} exceeds the 'error limit', the change in the parameter is acknowledged and the added

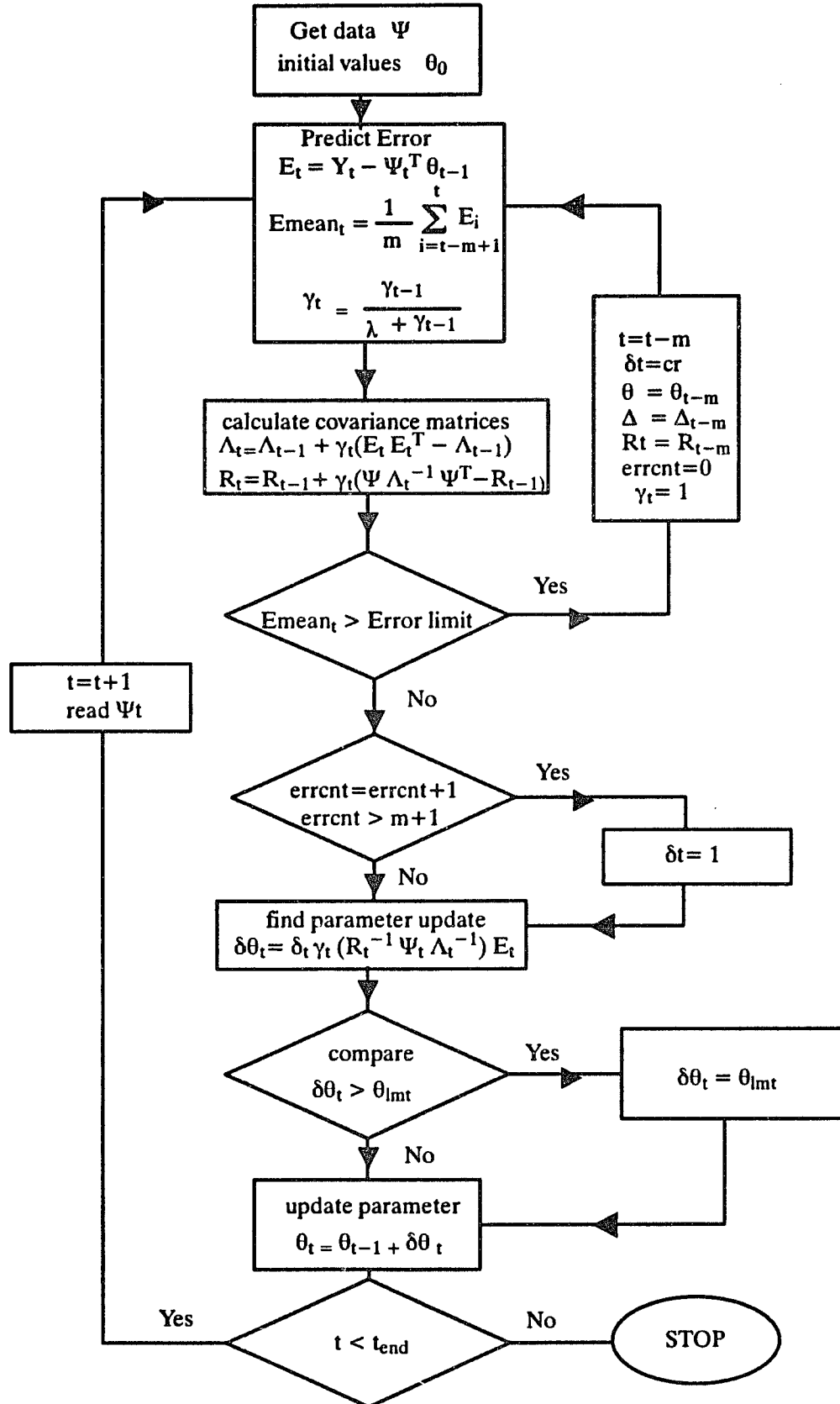


Fig. 5.1 Flow chart for MRLS algorithm

function δ_t is changed to 'cr' from its default value 1. The multiplier 'cr' is greater than 1 and thus increases the contribution of the recent data in estimation. Effectively, this means an 'alert' correction mechanism. The observation index, t , is reset backwards to $t=t-m$. This means that the past observations will be reused in the estimation with a more 'alert' correction mechanism.

The physical insight is included in the form of limits on E_{mean_t} , the error limit, and the limits on the increase in parameter per observation θ_{lim_t} . θ_{lim_t} is intended to counter-check the unrealistic changes in the parameters due to the introduction of multiplier, cr. This is necessary with multi-output systems. The details and guidelines for the selection of these parameters is discussed below.

The averaging number, 'm', is the number of observations for which the error is averaged to identify the change in the parameter. It will depend on the frequency content of the noise. A small value of 'm' will start tracking noise, whereas, a large value would make it difficult to track the change in the parameters. m can be approximated as follows,

$$m > \frac{1}{f_{\text{noise}} \cdot T_{\text{sample}}}$$

f noise : Lowest frequency of noise
T sample : Sampling interval

'Error limits' are the limits on the average value of prediction errors. The error limits will depend on the variance (S^2) of the prediction error and averaging number 'm'. For higher values of 'm' the error limit can be lowered. For example, assuming the prediction error has a normal distribution, we can say with 99 percent confidence that the parameters are not changing if

$$E_{\text{mean}_t} \leq \text{error limit} = \frac{S}{\sqrt{m}} \times 2.58$$

Ideally, error limit should be as low as possible.

'cr', the multiplier for the parameter correction part is intended to increase the sensitivity to the error. The choice of 'cr' will depend on the forgetting factor and noise in the system. It must be greater than 1. An increase in 'cr' has the same effect as decreasing the forgetting factor.

θ_{limt} is the maximum change in parameter per observation. The value depends on the knowledge of the system. The desired limit should be as high as possible.

When limited information about the system is available, the recommended procedure is to start with a low error limit and high θ_{limt} . Then, by increasing the error limit and decreasing θ_{limt} a satisfactory solution is achieved (i.e. no more decrease in prediction error). 'cr' should be greater than 1. The higher value of 'cr' improves computational efficiency by reducing the number of iterations, but also increases the possibility of magnifying the influence of the noise.

In the next section the proposed method is verified on the simulated data and then demonstrated on actual test data for simple crash models for front and side impact.

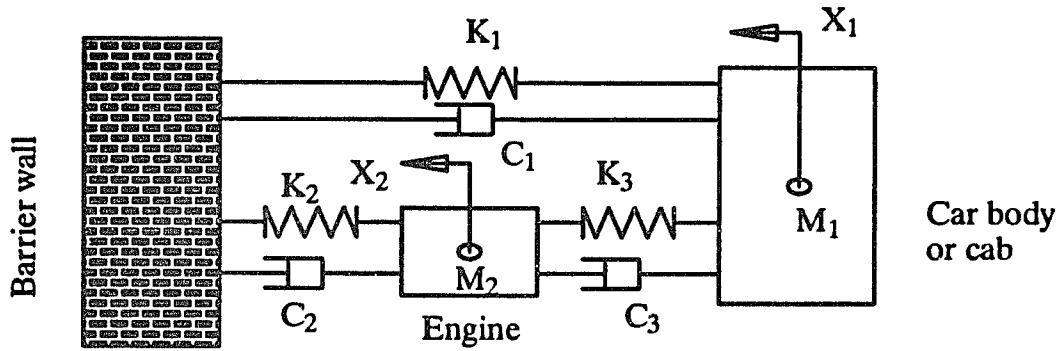
5.6 Applications

5.6.1 Verification of the MRLS

In this section the MRLS approach is verified and compared with the RLS for simulated data with added random noise. Two different models are considered.

Frontal impact

A lumped parameter model as shown in Figure 5.2 is considered. The car structure is divided into two parts: front end and occupant compartment or cab. It is



Mathematical expression for model

$$M_1 \cdot a_1 = K_1 \cdot X_1 + K_3 (X_1 - X_2) + C_1 \cdot V_1 + C_3 (V_1 - V_2)$$

$$M_2 \cdot a_2 = K_2 \cdot X_2 - K_3 (X_1 - X_2) + C_2 \cdot V_2 - C_3 (V_1 - V_2)$$

$$\text{Barrier Force} = -K_1 \cdot X_1 - K_2 \cdot X_2 - C_1 \cdot V_1 - C_2 \cdot V_2$$

Known data used in verification

$M_1 = 3 \text{ kg}$	$M_2 = 1 \text{ kg}$	$V \text{ initial} = 13.3 \text{ m/sec}$
$K_1 = ?$	$K_2 = ?$	$K_3 = 1000 \text{ N/m}$
$C_1 = 1 \text{ N/m/sec}$	$C_2 = 1 \text{ N/m/sec}$	$C_3 = 1 \text{ N/m/sec}$

Fig. 5.2 The model structure used for frontal impact

assumed that during the frontal crash only the front end is deformed while the cab acts as a rigid body. The model consists of two masses and three sets of springs and dampers. The two masses are the engine and the cab. The spring/damper K_1/C_1 represents the crush characteristics of the structure that is forward of the cab and does not interact with the engine. The spring/damper K_2/C_2 represents the crush characteristics of the structure that is forward of the engine and loads the engine during the crash. The spring/damper K_3/C_3 represents the crush characteristics of the structure that is between the engine and the car body and is loaded by the engine.

A set of nonlinear parameters are assumed and the simulation is carried out by numerical integration of the set of differential equations using 4th order Runge–Kutta method. The presence of noise is simulated by adding zero mean random noise in each of the acceleration measurements as shown in Figure 5.3.

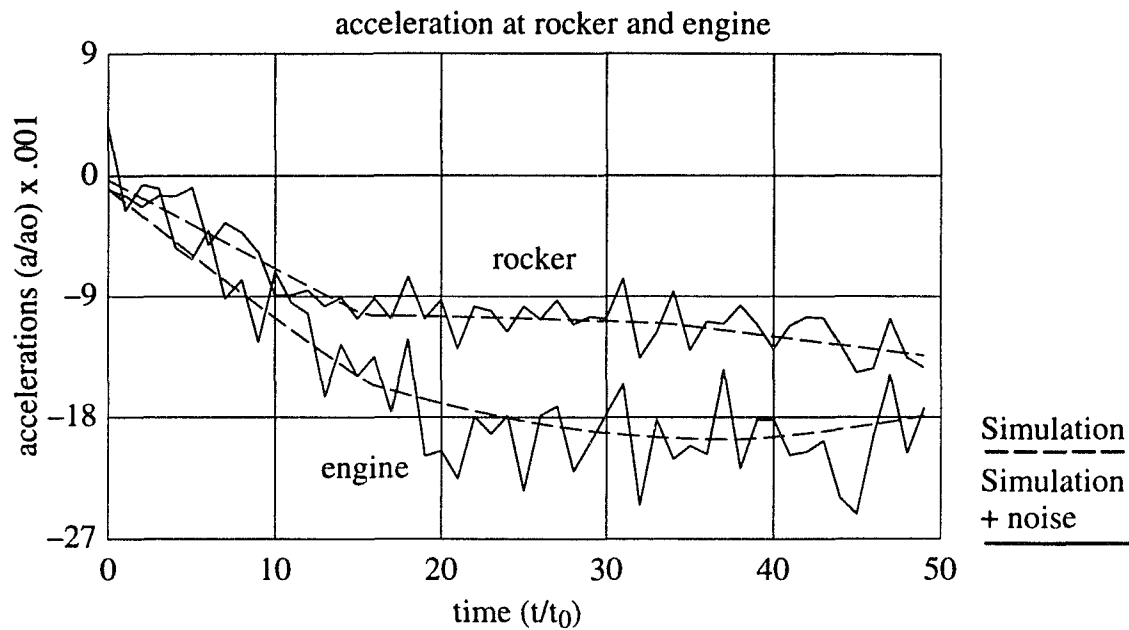


Fig. 5.3(a) Simulated data in frontal impact at rocker and engine

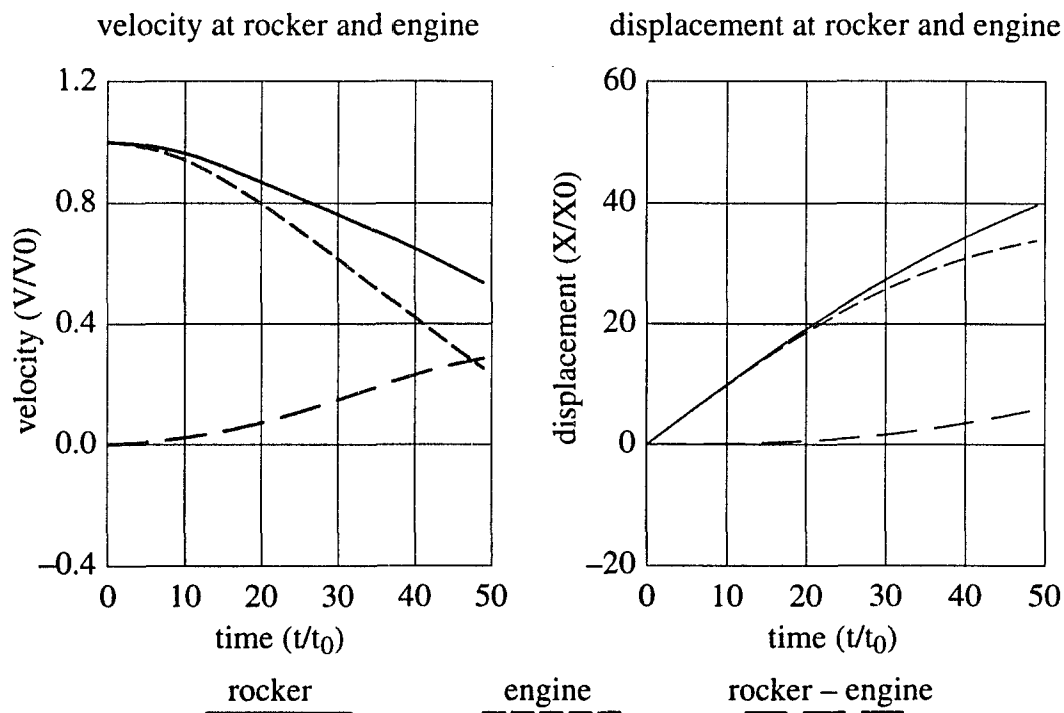


Fig. 5.3(b) Simulated data in frontal impact

Fig. 5.3(c) Simulated data in frontal impact

There are six parameters in the frontal impact model. For simplicity, it is assumed that only K_1 and K_2 are unknown parameters. The rest of the parameters are considered 'known' and are presented in Figure 5.2. The output measurements are modified to include the 'known' information. Then RLS and MRLS methods are applied on the simulated data to estimate the unknown parameters. The estimated parameters are compared with the actual parameters in Figure 5.4. The estimated outputs are compared with the simulated outputs in Figure 5.5. The mean and variance are compared in Table 5.1.

Side impact

The model used for side impact is presented in Figure 5.6. The model consists of three masses and two sets of springs and dampers. The three masses are the moving deformable barrier (MDB), the door and the test car. K_1/C_1 represents the crush characteristic of the structure between the MDB and the location of the acceleration measurement on the door inner. K_2/C_2 represents the crush characteristic of the structure between the door inner and the rocker of the non struck side of the test car.

The parameter values are assumed and the simulation is carried out by numerical integration of the set of differential equation using the 4th order Runge–Kutta method. The presence of noise is simulated by adding zero mean random noise in each of the acceleration measurements as shown in Figure 5.7.

There are four parameters in the side impact model. As in the frontal case it is assumed that only K_1 and K_2 are the unknown parameters. The output measurements are modified to include the 'known' information about the rest of the parameters. Then RLS and MRLS methods are applied on the simulated data to estimate the unknown parameters. The estimated parameters are compared with the actual parameters in Figure 5.8. The prediction errors are compared with the added noise in Figure 5.9. The mean and variance are compared in Table 5.2.

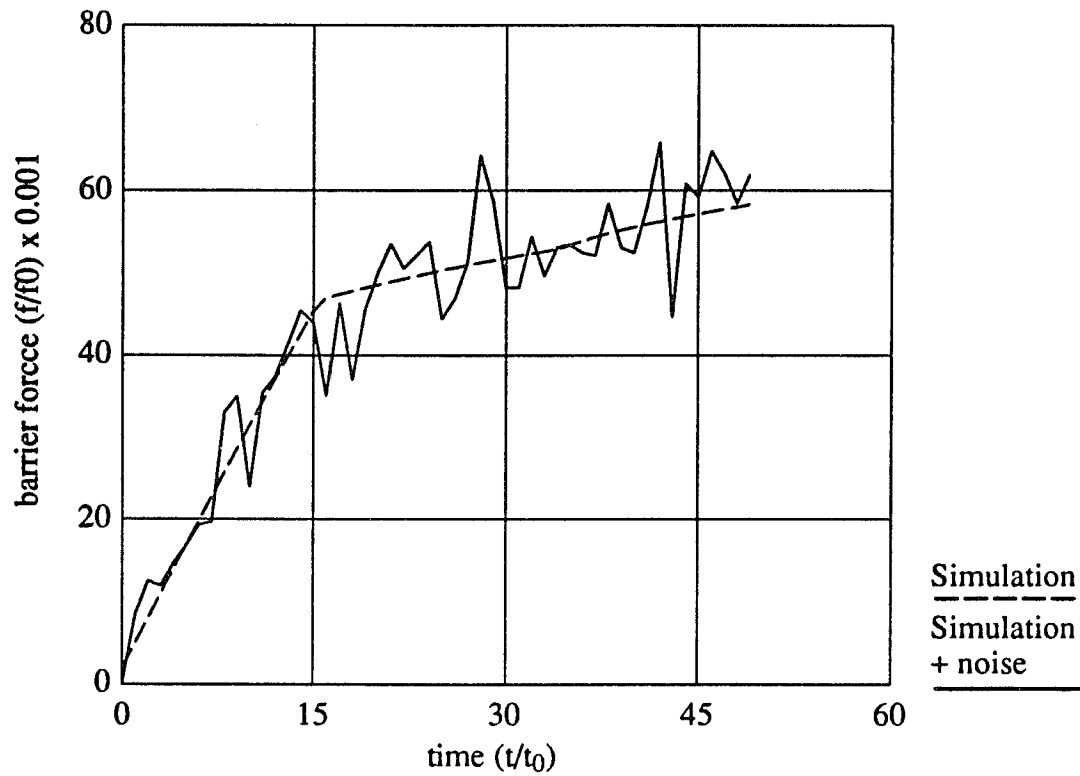


Fig. 5.3(d) Simulated barrier force in frontal impact

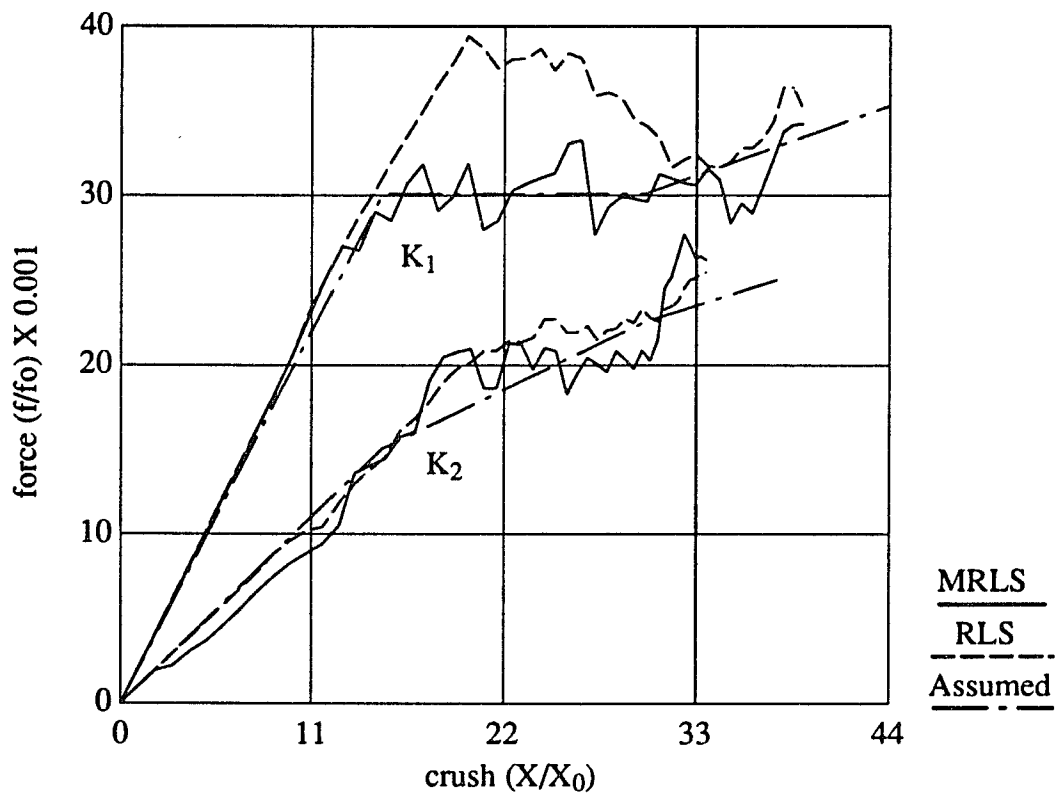


Fig. 5.4 Comparing assumed and estimated parameters K_1 and K_2 using RLS and MRLS.

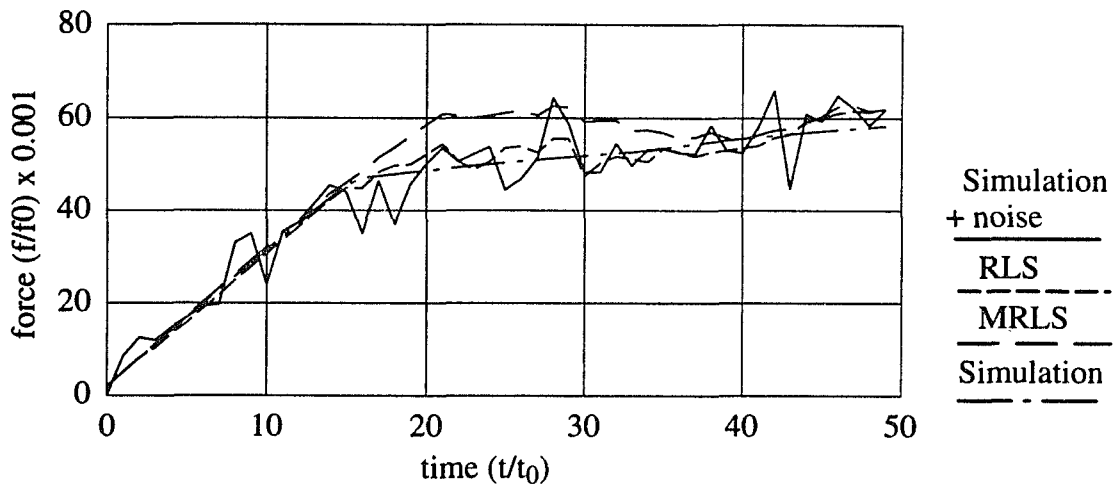


Fig. 5.5(a) Barrier force, comparison of RLS and MRLS estimates

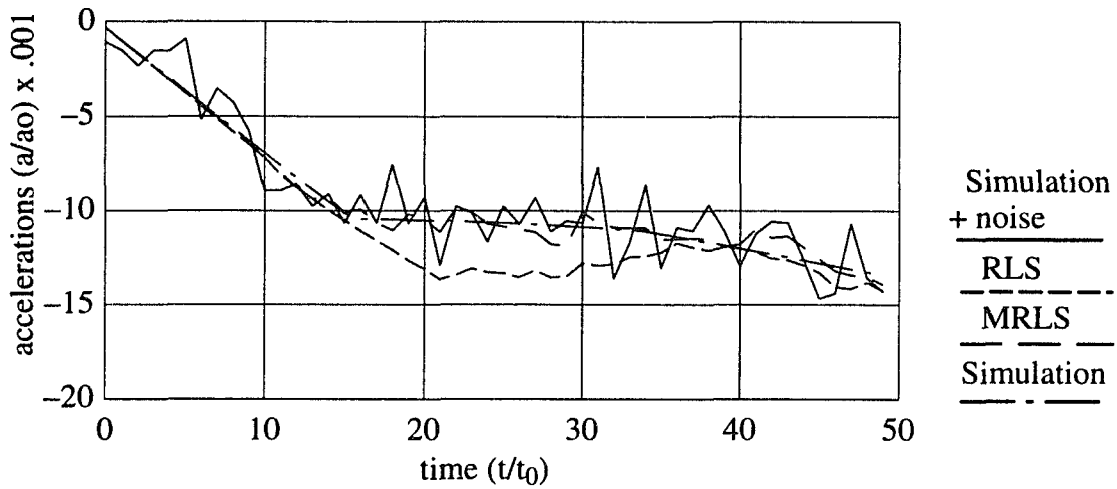


Fig. 5.5(b) Car acceleration, comparison of RLS and MRLS estimates

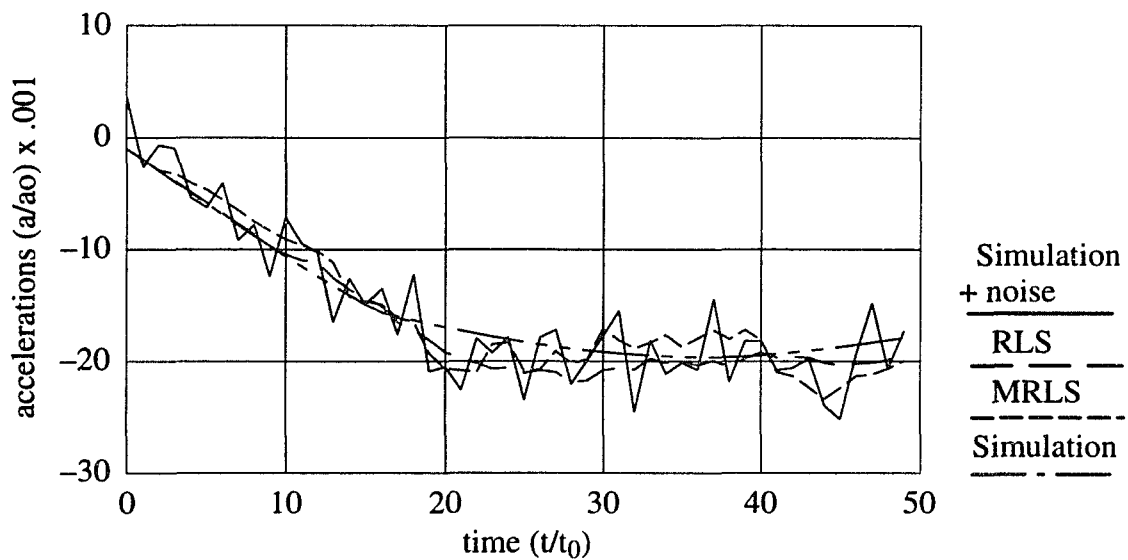


Fig. 5.5(c) Engine acceleration, comparison of RLS and MRLS

Table 5.1 Frontal impact model : mean and variance.

Simulated data	RLS		MRLS	
	mean .0751x10 ⁻³	variance 5.65x10 ⁻³	mean .0751x10 ⁻³	variance 5.65x10 ⁻⁹
Noise 1	0.9601E+01	0.3094E+04	0.9601E+01	0.3094E+04
Noise 2	-0.3692E+01	0.1370E+04	-0.3692E+01	0.1370E+04
Noise 3	0.5606E+01	0.4328E+04	0.5606E+01	0.4328E+04
Error 1	0.5101E+02	0.4554E+04	0.9883E+01	0.2915E+04
Error 2	0.7422E+01	0.1140E+04	-0.1798E+00	0.1084E+04
Error 3	-0.4691E+02	0.6323E+04	0.1813E+01	0.3902E+04
Input 1	-0.3676E+03	0.2198E+05	-0.3676E+03	0.2198E+05
Input 2	-0.2058E+03	0.8387E+04	-0.2058E+03	0.8387E+04
Input 3	0.5849E+03	0.4766E+05	0.5849E+03	0.4766E+05
Output 1	-0.4186E+03	0.2496E+05	-0.3775E+03	0.1859E+05
Output 2	-0.2132E+03	0.6772E+04	-0.2056E+03	0.7214E+04
Output 3	0.6318E+03	0.5725E+05	0.5831E+03	0.4768E+05
Test data	RLS		MRLS	
	mean .064x10 ⁻³	variance 4.1x10 ⁻⁹	mean .064x10 ⁻³	variance 4.1x10 ⁻⁹
Error 1	0.9549E+02	0.1458E+05	0.8390E+02	0.1583E+05
Error 2	-0.4852E+02	0.3805E+04	-0.1290E+02	0.1257E+04
Error 3	0.6758E+02	0.2116E+05	0.4355E+02	0.8355E+04
Input 1	-0.2412E+03	0.2237E+05	-0.2412E+03	0.2237E+05
Input 2	-0.7223E+02	0.6138E+04	-0.7223E+02	0.6138E+04
Input 3	0.4280E+03	0.6352E+05	0.4280E+03	0.6352E+05
Output 1	-0.3367E+03	0.2847E+05	-0.3251E+03	0.2767E+05
Output 2	-0.2371E+02	0.1860E+04	-0.5933E+02	0.5118E+04
Output 3	0.3604E+03	0.2476E+05	0.3844E+03	0.3848E+05

Note : 1: Car 2: Engine 3: Barrier force

Input: Simulated or measured data,

Output: Estimated data based on the identified parameters.

Error: The difference between the input and the output.

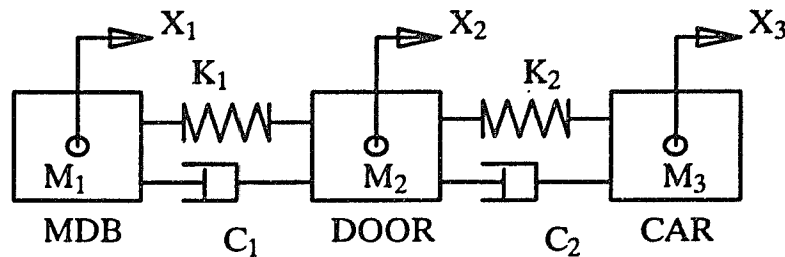
Closeness of mean and variance of corresponding input/output and noise/error indicates good estimates.

From the above two simulated models, it can be seen that the MRLS method is quicker in tracking changes in the parameter as compared with the RLS. The correlation analysis (auto-correlation of prediction error and cross correlation prediction error with the input) reveals that the prediction error for MRLS is not correlated whereas the prediction error for RLS is correlated in some cases. The estimator parameters used in the estimation are presented in Table 5.3. The results of correlation analysis for the simulated data are presented in Table 5.4.

5.6.2 Application to test data

Frontal impact

The acceleration measurement (normalized) from actual barrier tests are presented in Figure 5.10. For the frontal impact the C_1 , C_2 , C_3 and K_3 parameters are assumed to be 'known'. The K_1 and K_2 parameters are estimated from the measured data. Based on the past experience with the test data, it is known that the contributions of the assumed parameters in the overall behavior is not significant.



Mathematical expression for model

$$M_1 \cdot a_1 = -K_1(X_1 - X_2) - C_1(V_1 - V_2)$$

$$M_1 \cdot a_1 = K_1(X_1 - X_2) + C_1(V_1 - V_2) - K_2(X_2 - X_3) - C_2(V_2 - V_3)$$

$$M_3 \cdot a_3 = K_2(X_2 - X_3) + C_2(V_2 - V_3)$$

Known data used in verification

$$M_1 = 1.0 \text{ kg} \quad M_2 = 0.08 \text{ kg} \quad M_3 = 1.5 \text{ kg} \quad V_1 \text{ initial} = 14.0 \text{ m/sec}$$

$$K_1 = ? \quad K_2 = ? \quad C_1 = 2 \text{ N/m/sec} \quad C_2 = 2 \text{ N/m/sec}$$

Fig. 5.6 The model structure used for side impact

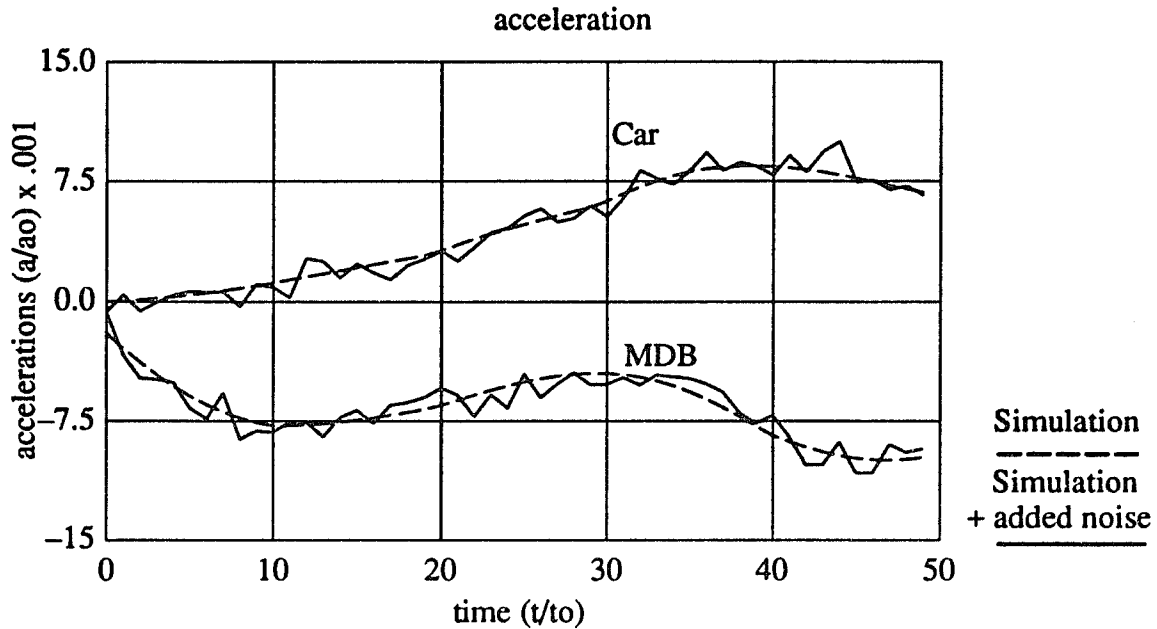


Fig. 5.7(a) Simulation : Accelerations at MDB and test car rocker.

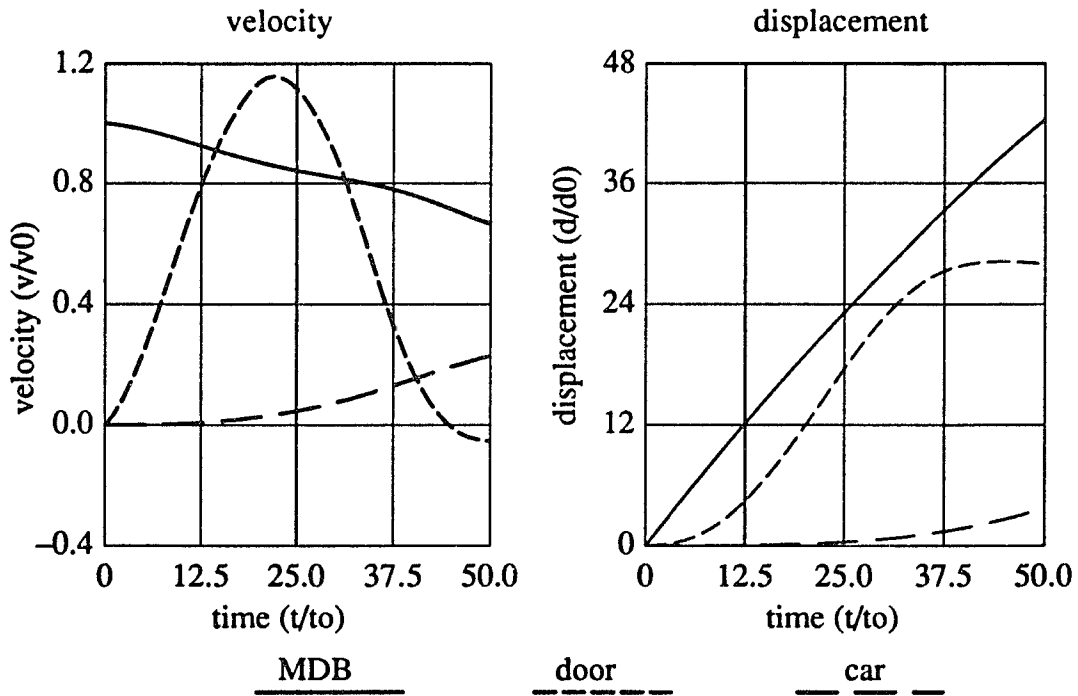


Fig. 5.7(b) Simulation : Measured velocity in side impact.

Fig. 5.7(c) Simulation : Measured displacement in side impact

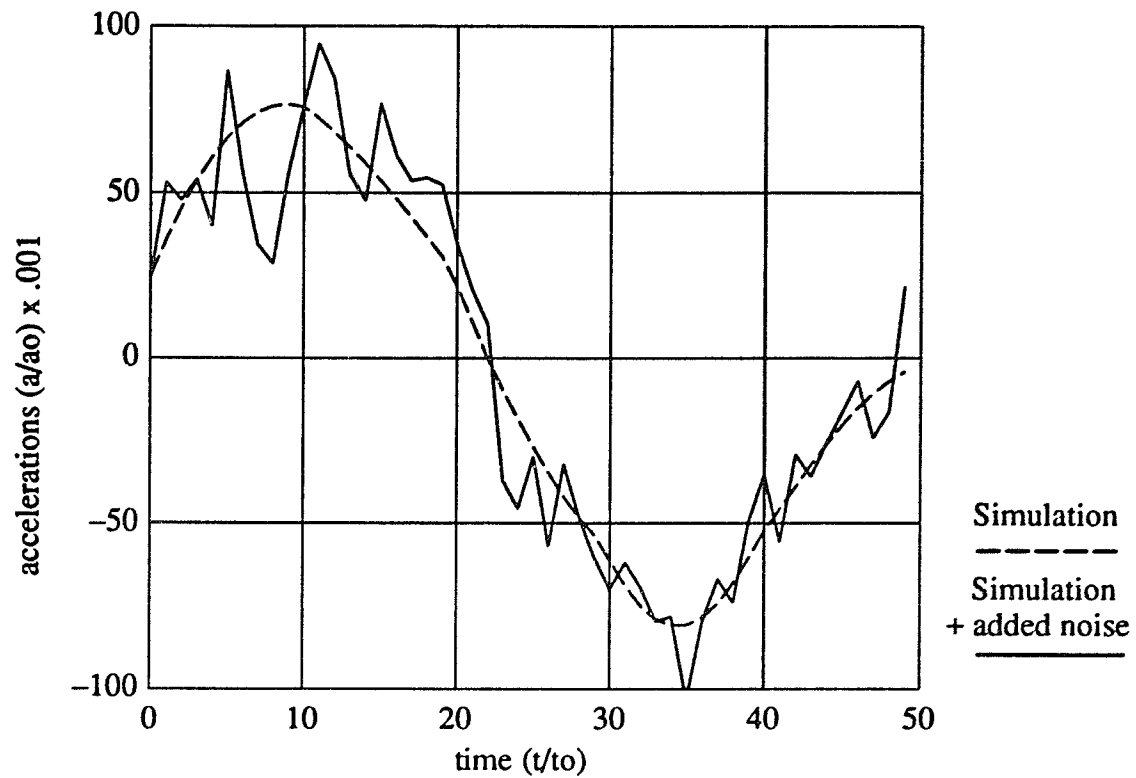


Fig. 5.7(d) Simulation : Acceleration at door

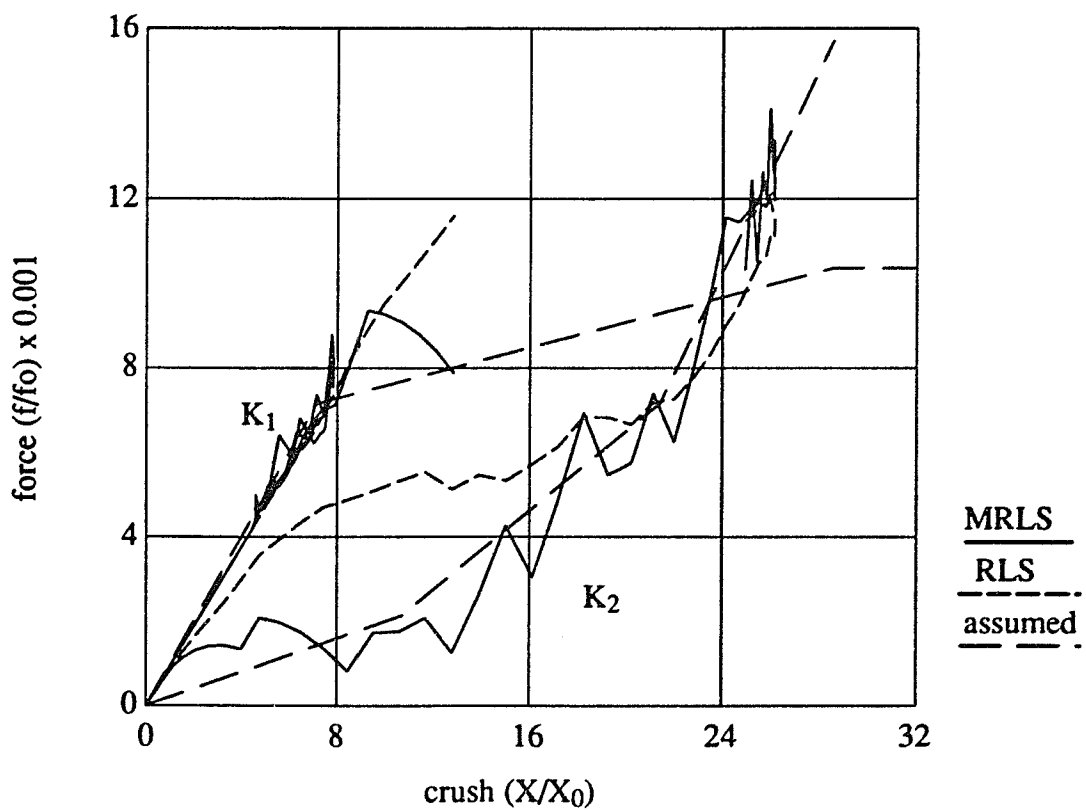


Fig. 5.8 Comparing assumed and estimated parameters using RLS and MRLS

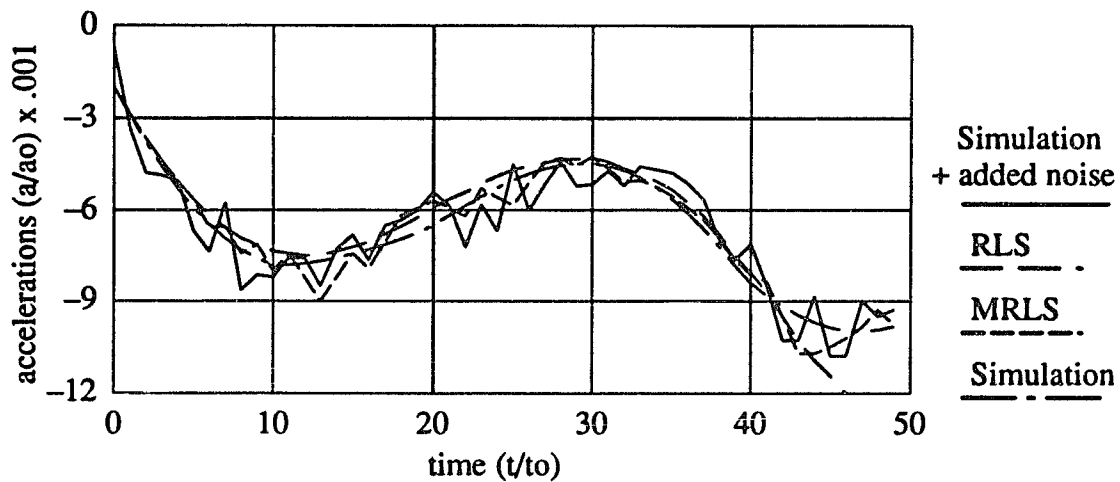


Fig. 5.9(a) MDB acceleration, comparison of RLS and MRLS estimates.

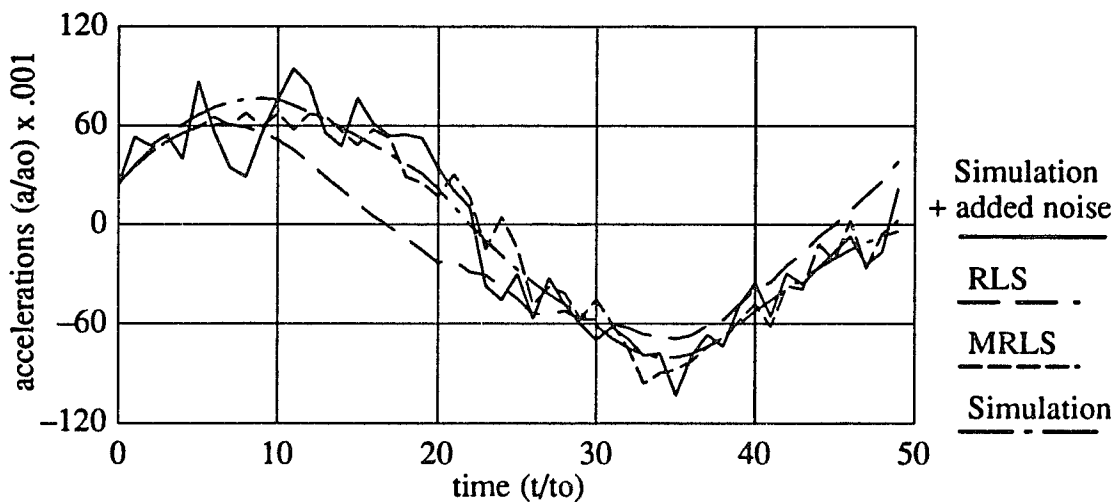


Fig. 5.9(b) Door acceleration, comparison of RLS and MRLS estimate.

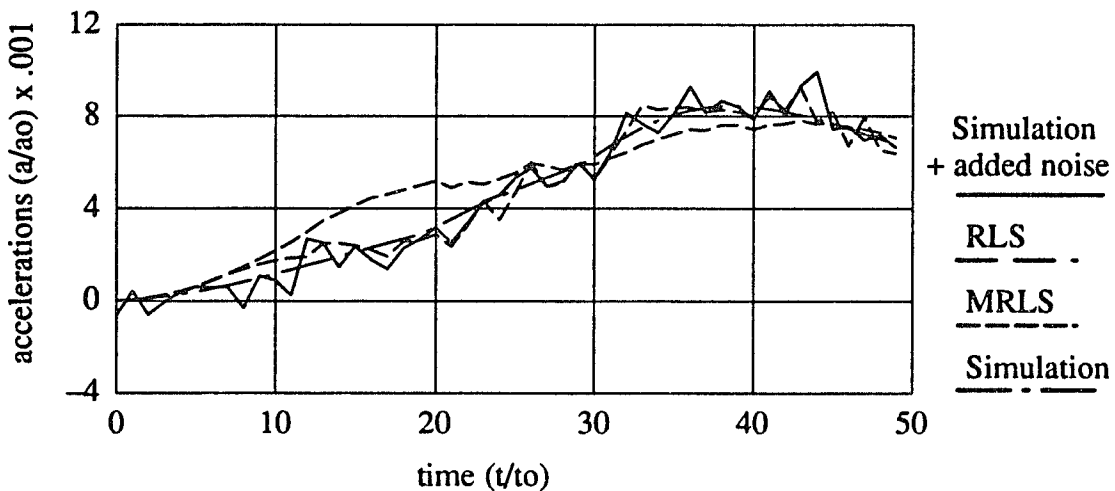


Fig. 5.9(c) Car acceleration, comparison of RLS and MRLS estimate

Table 5.2 Side impact : Mean and Variance

Simulated Data	RLS		MRLS	
	mean .0714x10 ⁻³	Variance 5.120x10 ⁻⁹	mean .0714x10 ⁻³	variance 5.120x10 ⁻⁹
Noise 1	-0.2294E+00	0.1188E+03	-0.2294E+00	0.1188E+03
Noise 2	-0.4173E+00	0.3337E+03	-0.4173E+00	0.3337E+03
Noise 3	-0.4757E+00	0.1872E+03	-0.4757E+00	0.1872E+03
Error 1	0.7636E+00	0.3127E+03	0.5571E-01	0.1047E+03
Error 2	0.6932E+01	0.1067E+04	0.4442E+00	0.3609E+03
Error 3	-0.8818E+01	0.7090E+03	-0.1622E+01	0.2067E+03
Input 1	-0.9326E+02	0.8127E+03	-0.9326E+02	0.8127E+03
Input 2	-0.1476E+01	0.3893E+04	-0.1476E+01	0.3893E+04
Input 3	0.9361E+02	0.4764E+04	0.9361E+02	0.4764E+04
Output 1	-0.9402E+02	0.1345E+04	-0.9331E+02	0.8566E+03
Output 2	-0.8408E+01	0.2430E+04	-0.1920E+01	0.3495E+04
Output 3	0.1024E+03	0.2902E+04	0.9523E+02	0.4060E+04
Test data	RLS		MRLS	
	mean .0746x10 ⁻³	Variance 5.56x10 ⁻⁹	mean .0746x10 ⁻³	variance 5.56x10 ⁻⁹
Error 1	-0.2186E+02	0.1590E+04	-0.1437E+02	0.1856E+04
Error 2	-0.1364E+01	0.1224E+04	-0.4280E+01	0.8965E+03
Error 3	0.2653E+02	0.7213E+04	0.2195E+02	0.7561E+04
Input 1	-0.1466E+03	0.3793E+04	-0.1466E+03	0.3793E+04
Input 2	0.1346E+02	0.1658E+04	0.1346E+02	0.1658E+04
Input 3	0.1365E+03	0.5882E+04	0.1365E+03	0.5882E+04
Output 1	-0.1248E+03	0.1786E+04	-0.1323E+03	0.2660E+04
Output 2	0.1482E+02	0.1143E+04	0.1774E+02	0.1210E+04
Output 3	0.1099E+03	0.4184E+04	0.1145E+03	0.4730E+04

Note : 1: MDB 2: Door 3: Car

Input: Simulated or measured data,

Output: Estimated data based on the identified parameters.

Error: The difference between the input and the output.

Closeness of mean and variance of corresponding input/output and noise/error indicates good estimates.

Table 5.3 The detail of the known parameters and the control parameters used in the MRLS estimation

Model	$\times 10^3$			$\times 10^{-3}$			C_1	C_2	C_3
	M_1	M_2	M_3	K_1	K_2	K_3			
Front Vrf	3	1	1	?	?	1000	1.0	1.0	1.0
Side Vrf	1.0	.08	1.5	?	?	—	2.0	2.0	—
Front Test	1.4	0.3	.000102	?	?	2000	1.0	0.2	1.0
Side Test	1.34	.08	1.52	?	?	—	0.3	14.0	—

M_1 , M_2 and M_3 are multiplier to accelerations

Model*	SD of added noise $\times 10^{-3}$			Initialize		error limits $\times 10^{-3}$			prm lmts $\times 10^{-3}$		cr $\times 10^{-3}$	m	λ
	eq ₁	eq ₂	eq ₃	start	end	eq ₁	eq ₂	eq ₃	K_1	K_2			
Front Vrf $\times .075$	20	40	80	0	14	40	30	70	30	30	50	5	0.5
Side Vrf $\times .0714$	10	200	10	0	9	15	20	15	50	50	20	4	0.8
Front Test $\times .064$	—	—	—	0	25	90	80	180	100	30	30	3	0.6
Side Test $\times .0746$	—	—	—	0	60	20	30	70	50	50	20	4	0.75

* Multiplier indicated are for eq1,eq2 and eq3 only in added noise and error limits columns

Note : error limit (approximate) = $\frac{M_i \text{ eq}_i (\text{SD of noise})}{\sqrt{m}}$ SD is standard deviation

Table 5.4 Cross correlation for simulated data used in verification, b : RLS, m : MRLS

Front Impact : Verification data

Input error	X_1	V_1	X_2	V_2	$X_1 - X_2$	$V_1 - V_2$
bar-b	-0.1311	0.0291	-0.1730	0.0170	0.1274	0.0019
bar-m	-0.0206	-0.0122	-0.0315	-0.0195	0.0456	0.0309
car-b	0.1384	-0.0587	0.1720	-0.0497	-0.0730	0.0356
car-m	-0.0552	0.0682	-0.0496	0.0684	-0.0816	-0.0686
eng-b	-0.0884	0.0756	-0.0874	0.0769	-0.0840	-0.0790
eng-m	0.0063	-0.0615	-0.0132	-0.0574	0.1194	0.0509

Side Impact : Verification data

Input error	$X_1 - X_2$	$X_2 - X_3$	$V_1 - V_2$	$V_2 - V_3$
car-b	-0.0696	-0.5679	0.4620	0.5890
car-m	0.1397	-0.3711	-0.065	0.0626
mdb-b	0.5892	-0.385	0.3638	0.5298
mdb-m	0.1954	-0.2032	0.0671	0.1408
door-b	-0.0985	0.4725	-0.5203	-0.6414
door-m	0.0668	0.0475	0.1022	0.0895

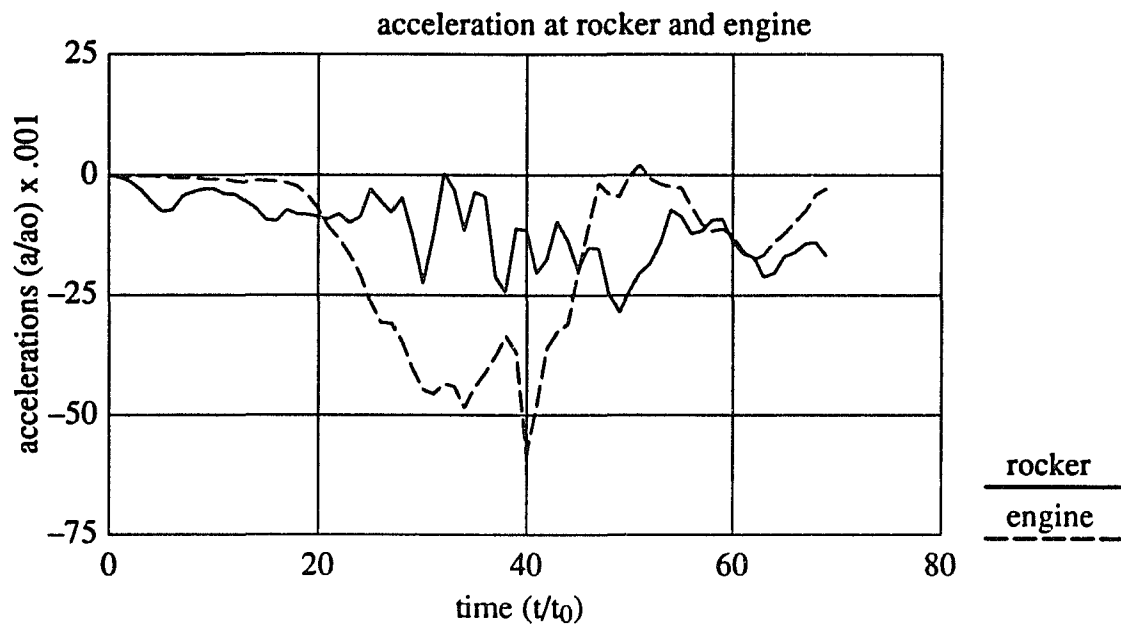


Fig. 5.10(a) Measured data in frontal impact at rocker and engine

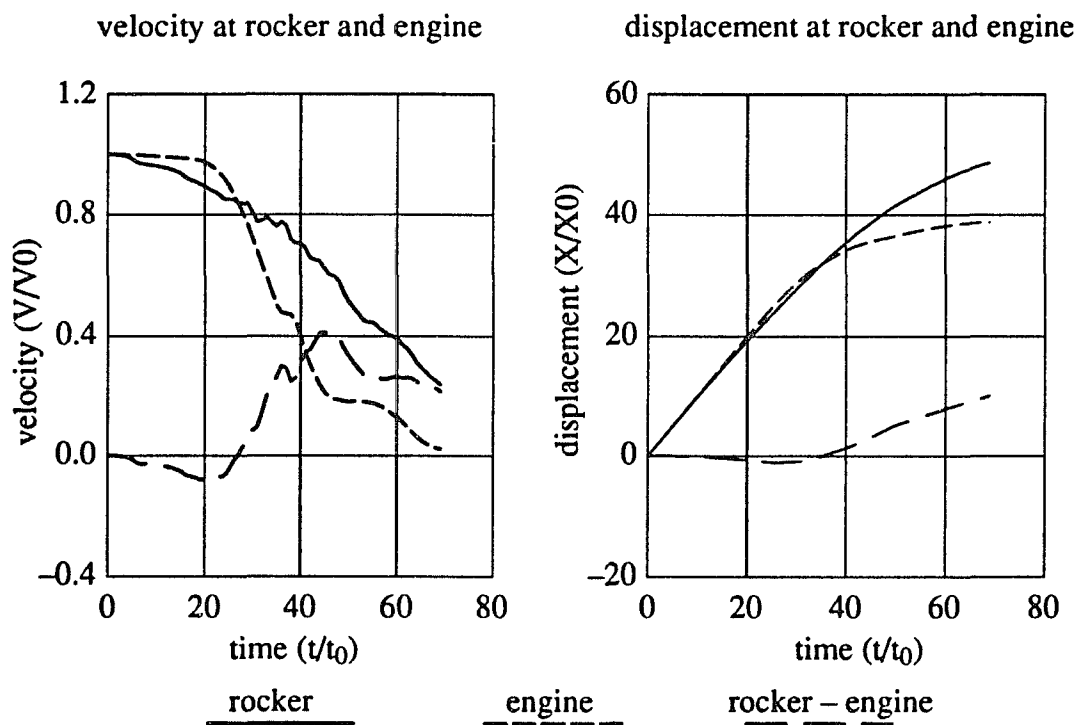


Fig. 5.10(b) Measured data in frontal impact

Fig. 5.10(c) Measured data in frontal impact

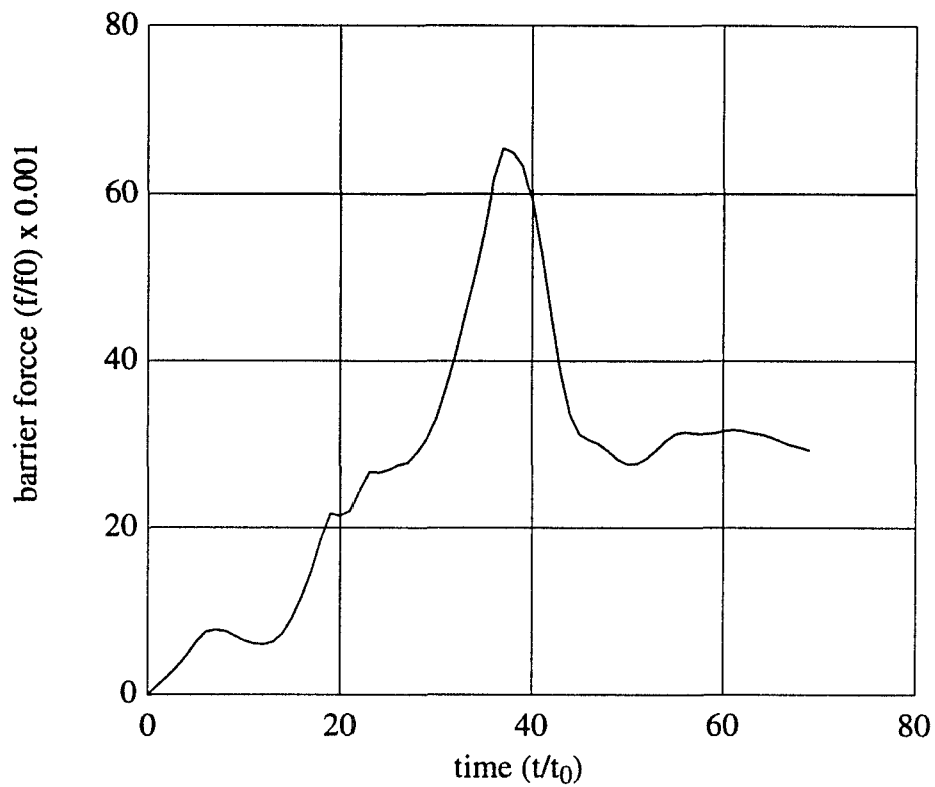


Fig. 5.10(d) Measured barrier force in frontal impact

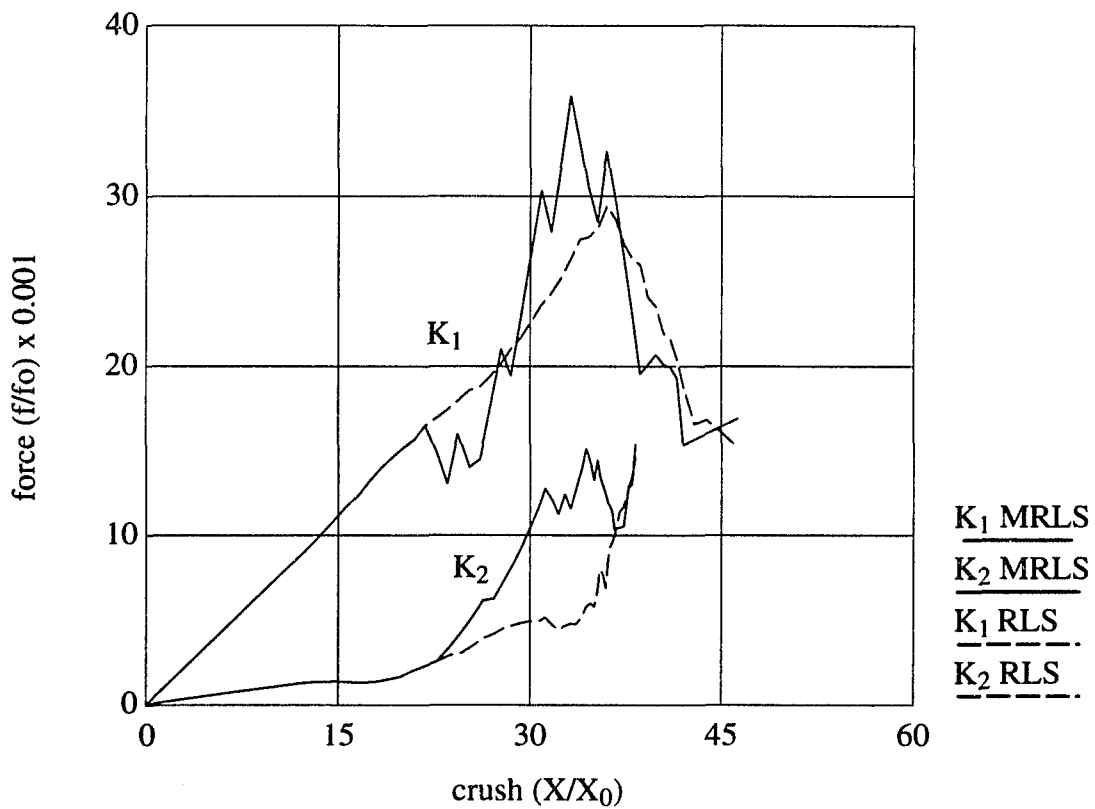


Fig. 5.11 Estimated parameters K₁ and K₂

The parameters K_1 and K_2 estimated using both RLS and MRLS techniques are compared in Figure 5.11. The estimated accelerations are compared in Figure 5.12. The mean and variance of prediction error are compared in Table 5.1. The correlation analysis for the prediction error is presented in Figures 5.13 through 5.15. The 99% confidence limits, based on Ljung [1987](also see Chapter 3 of this thesis), for both auto-correlation and cross correlation are also presented. The control parameters used are presented in Table 5.3.

Side impact

The acceleration measurement (normalized) from a side impact tests are presented in Figure 5.16. As in frontal impact it is assumed that C_1 and C_2 parameters are 'known'. The K_1 and K_2 are the unknown parameters to be estimated from the test data. Also based on the past experience with the test data, it is known that K_1 and K_2 are the most important parameters in the overall behavior of the car.

Parameters K_1 and K_2 estimated using both RLS and MRLS technique are compared in Figure 5.17. The mean and variance of prediction error are compared in Table 5.2. The estimated accelerations are compared in Figure 5.18. The correlation analysis for the prediction error is presented in Figures 5.19 through 5.21. The 99% confidence limits, based on Ljung [1987], for both auto-correlations and cross correlations are also presented. The control parameters used by the estimator are presented in Table 5.3.

5.6.3 Comments on the results

For the frontal impact as shown in Figure 5.11, the comparison of estimated parameter K_2 using RLS and MRLS estimator indicates that both the estimates are similar for crush up to $X/X_0 = 22$, then the MRLS estimates starts changing while the RLS estimates maintain their slope. Both the estimates become close again at

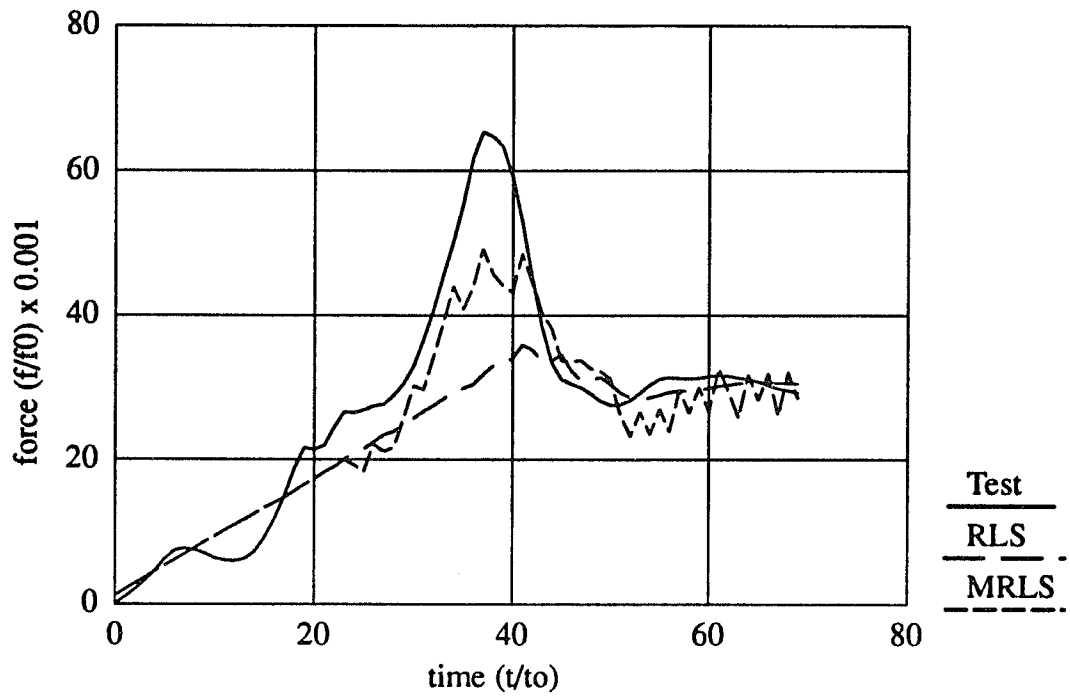


Fig. 5.12(a) Barrier force, Comparison of RLS and MRLS estimates

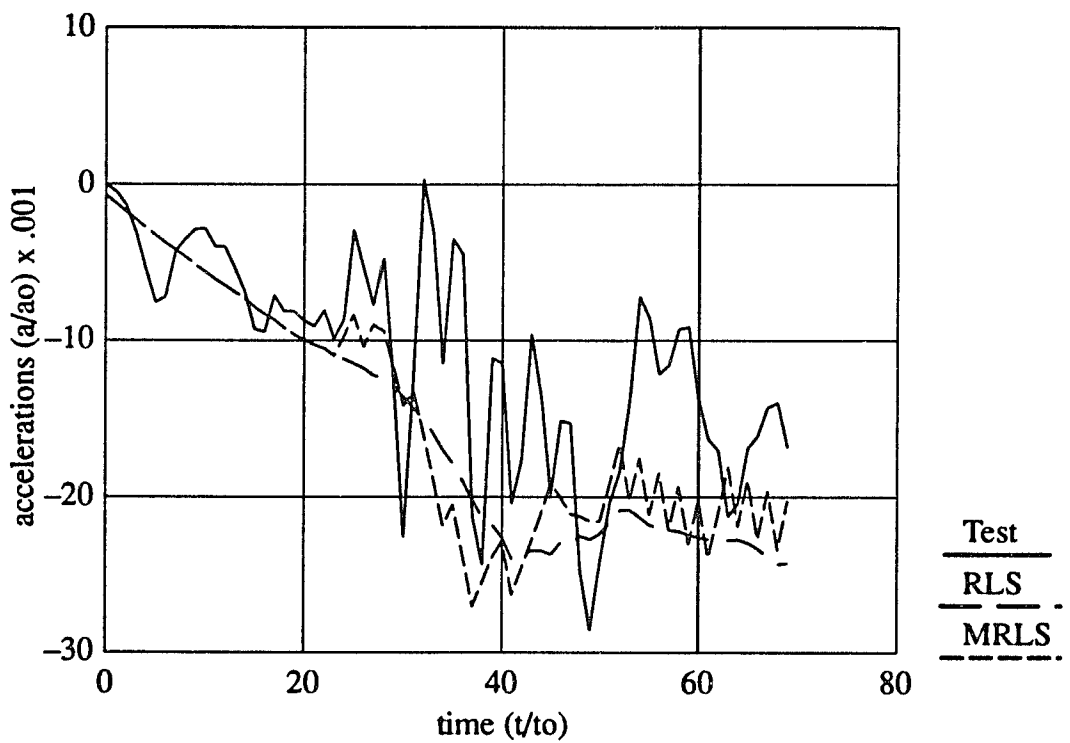


Fig. 5.12(b) Car acceleration, Comparison of RLS and MRLS estimates

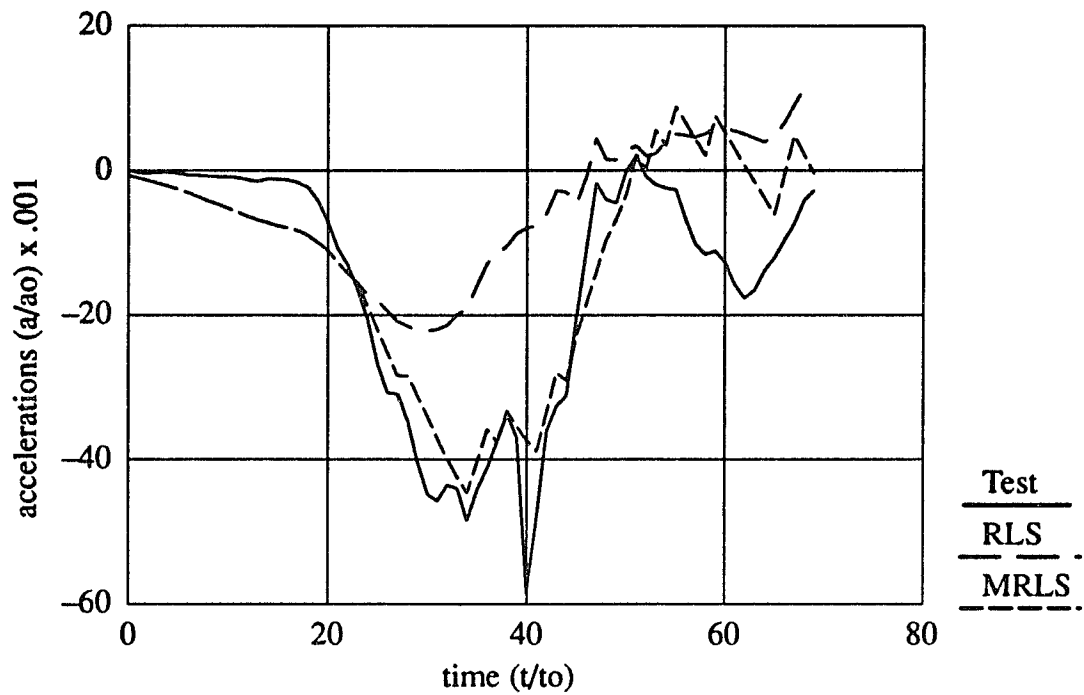


Fig. 5.12(c) Engine acceleration, comparison of RLS and MRLS

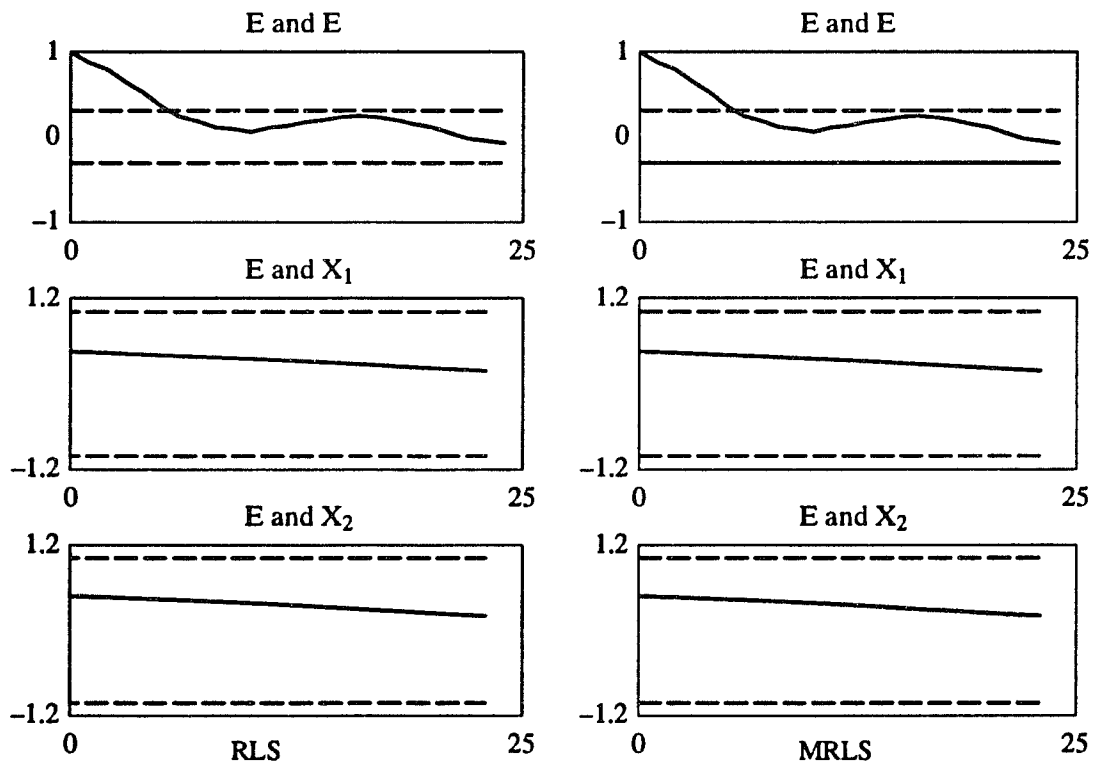


Fig. 5.13 Autocovariance of the residue and cross-covariance of the residue with the input for barrier. (E is the prediction error)

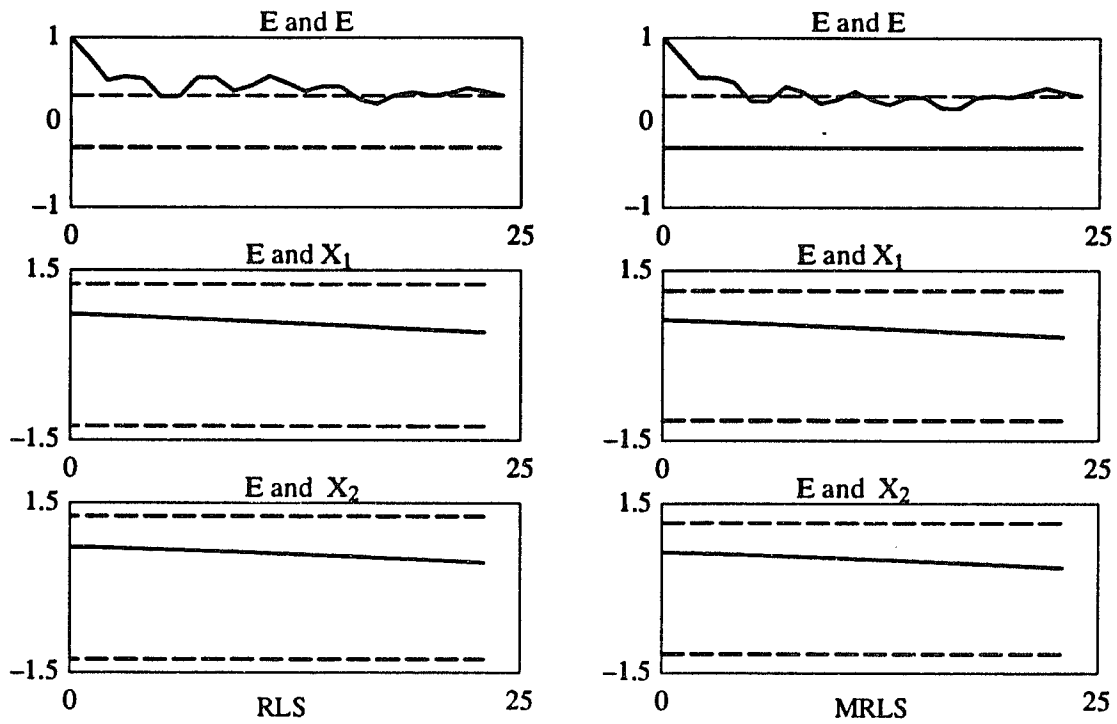


Fig. 5.14 Autocovariance of the residue and cross-covariance of the residue with the input for car. (E is the prediction error)

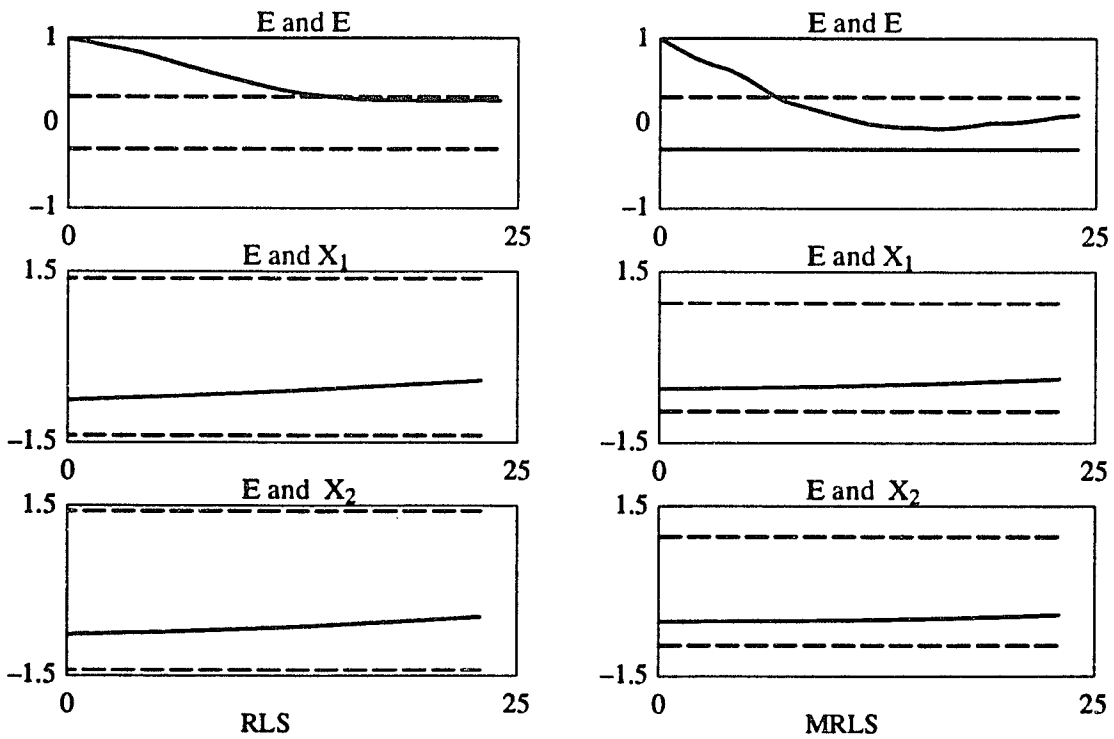


Fig. 5.15 Autocovariance of the residue and cross-covariance of the residue with the input for engine. (E is the prediction error)

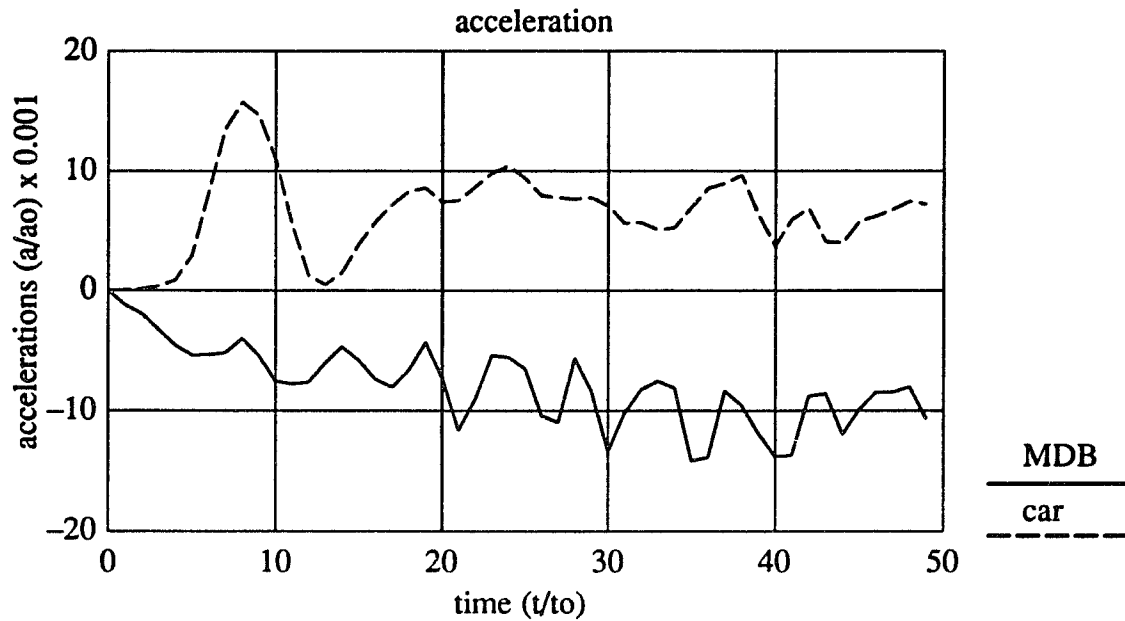


Fig. 5.16(a) Measured data in side impact on MDB and on car at the non struck side rocker.

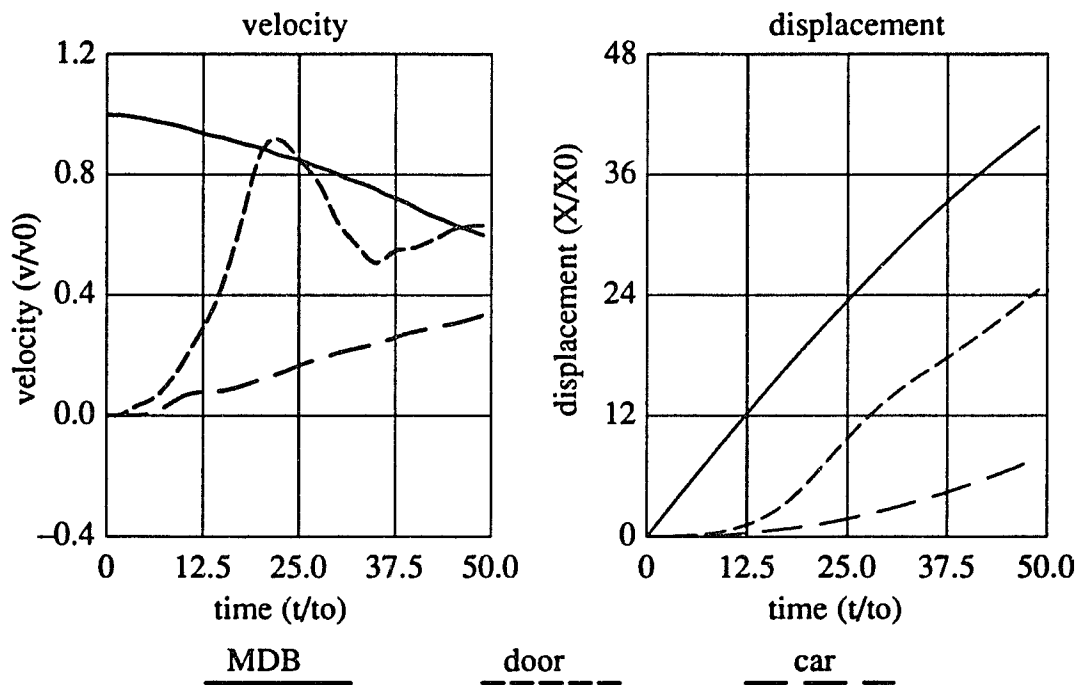


Fig. 5.16(b) Measured data in side impact

Fig. 5.16(c) Measured data in side impact

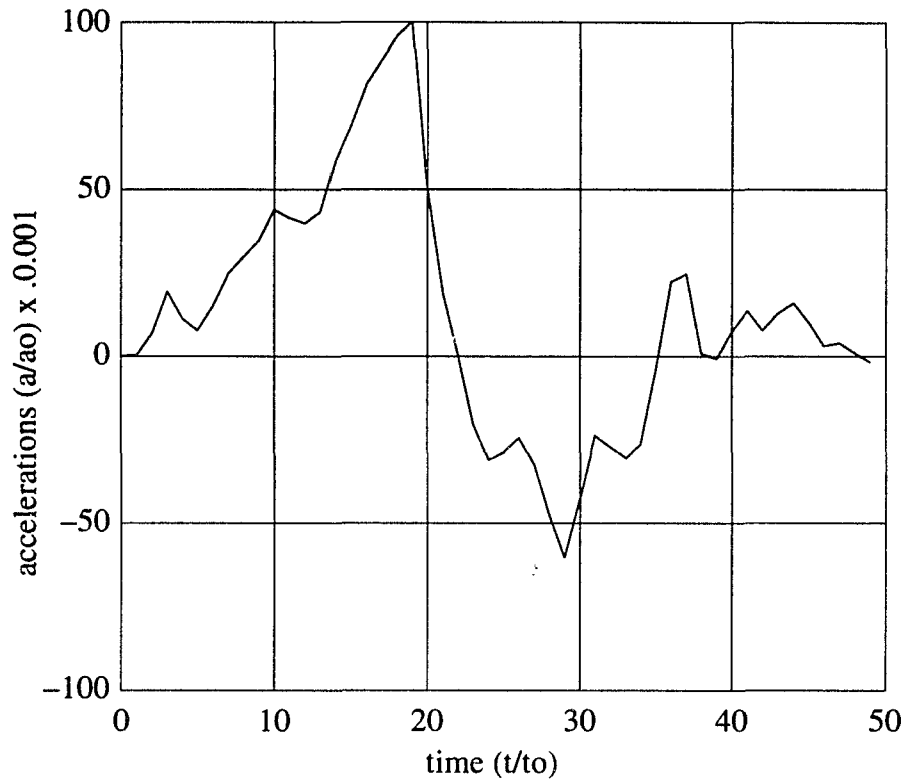


Fig. 5.16(d) Measured acceleration at door in side impact

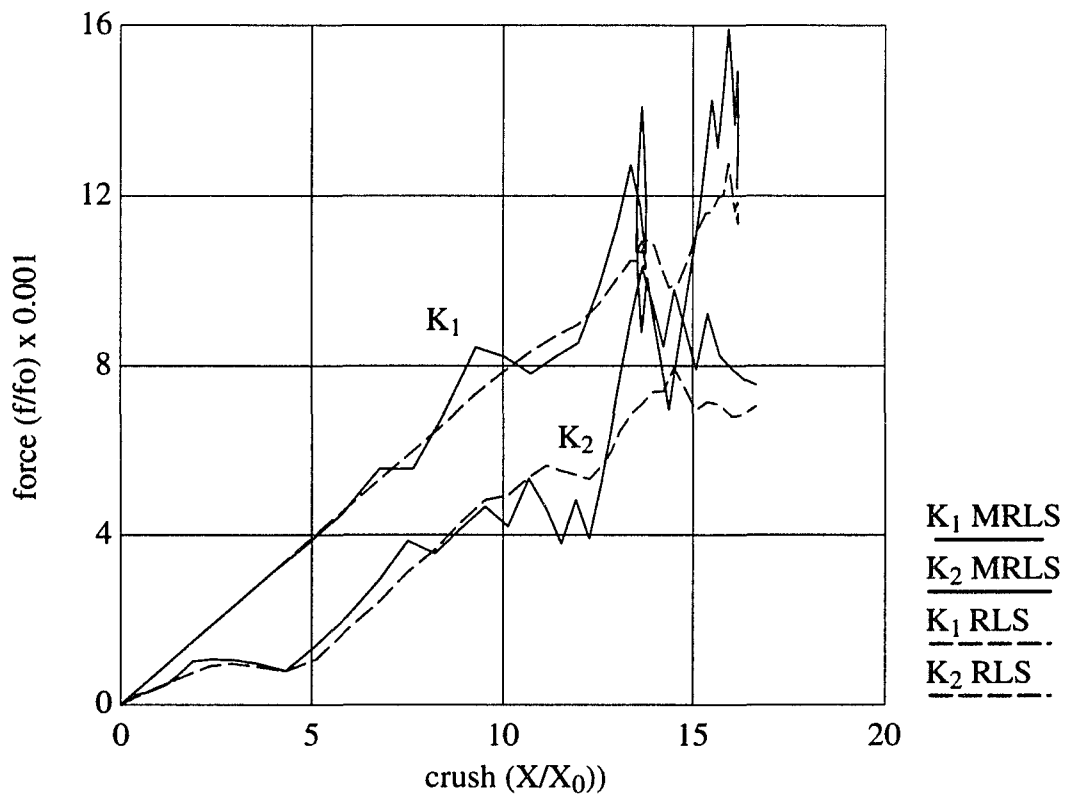


Fig. 5.17 Comparing estimated parameters using RLS and MRLS from test data

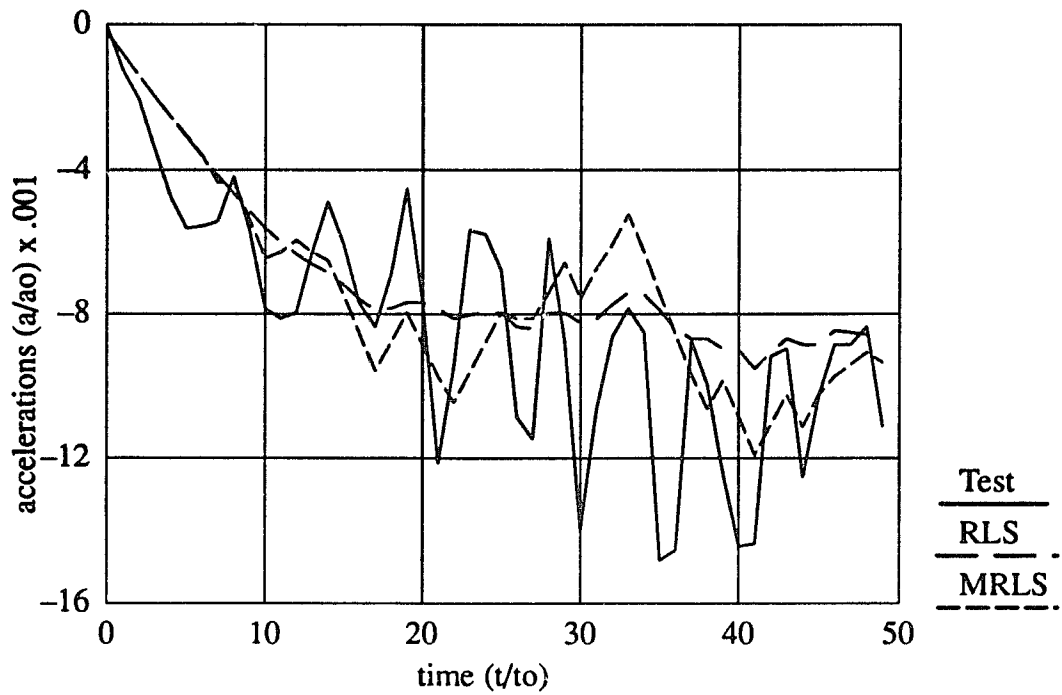


Fig. 5.18(a) MDB acceleration, comparison of RLS and MRLS estimates for test data.

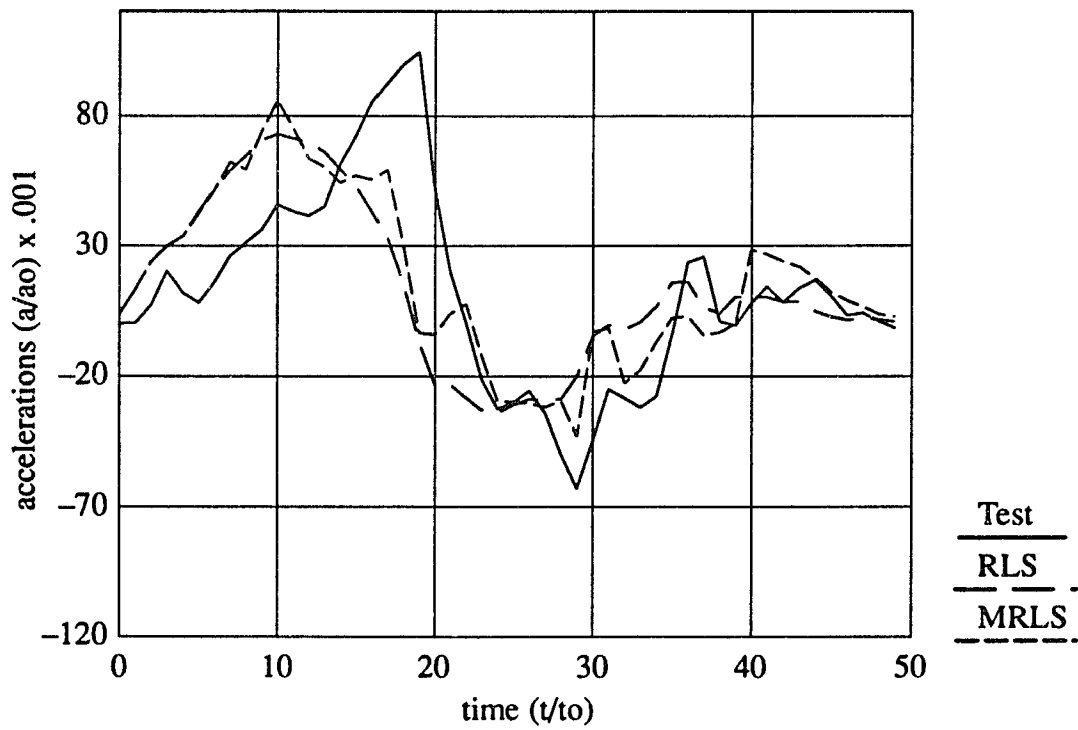


Fig. 5.18(b) Door acceleration, comparison of RLS and MRLS estimates for test data.

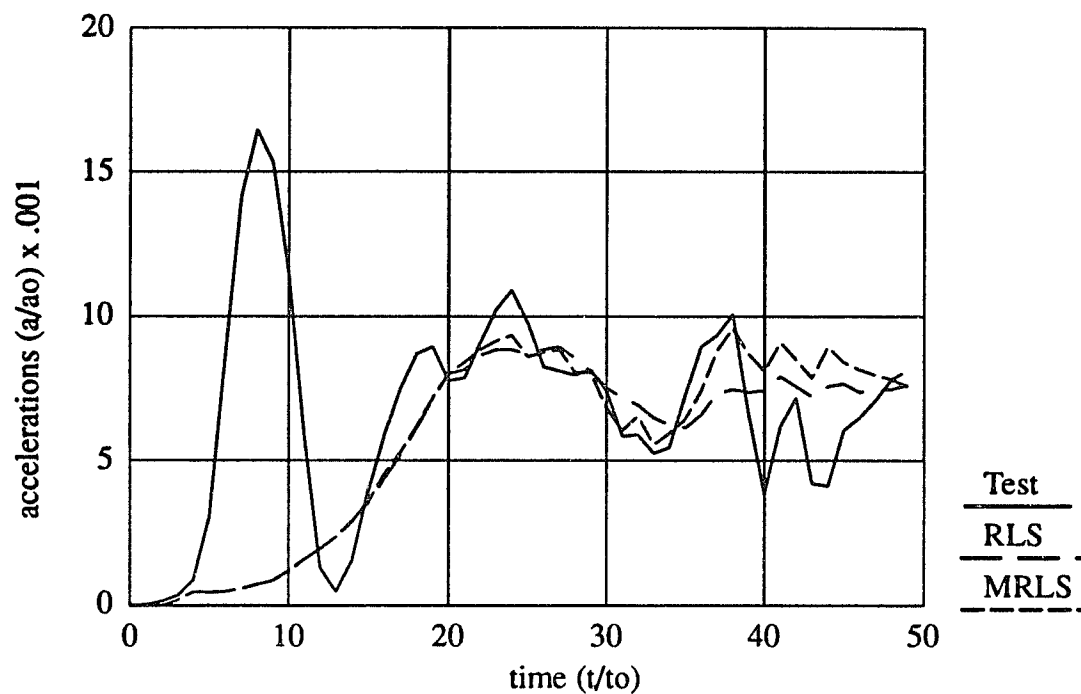


Fig. 5.18(c) Car acceleration, comparison of RLS and MRLS estimates for test data.

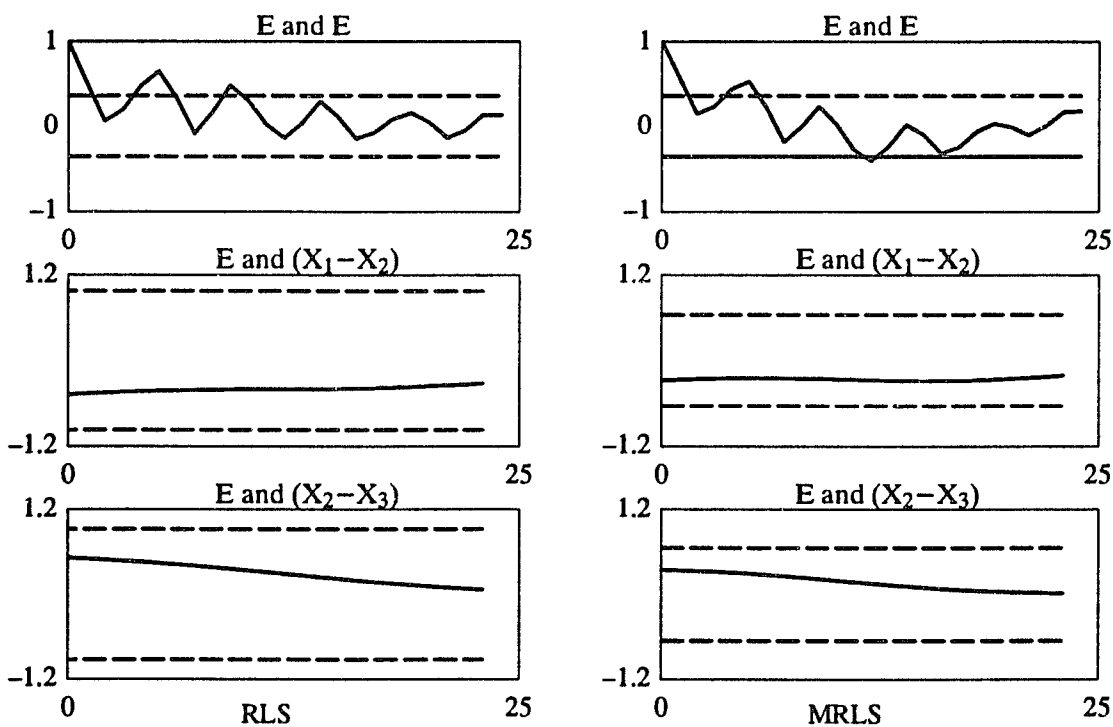


Fig. 5.19 Autocovariance of the residue and cross-covariance of the residue with the inputs for MDB, side impact. (E is prediction error)

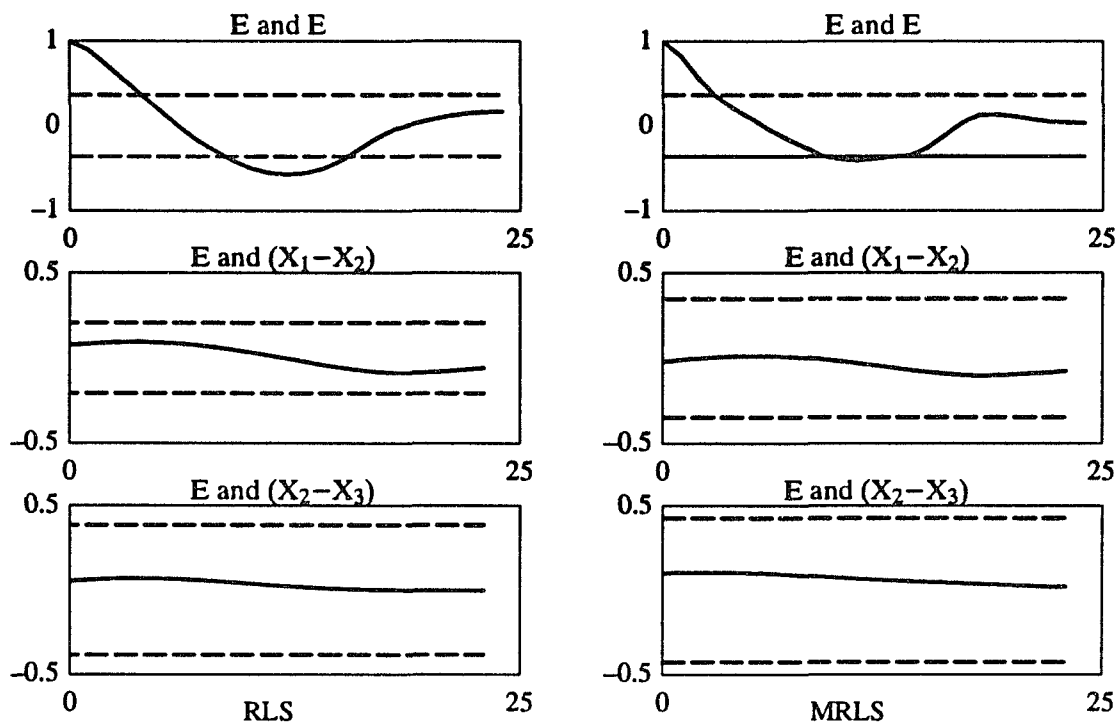


Fig. 5.20 Autocovariance of the residue and cross-covariance of the residue with the input for door, side impact. (E is prediction error)

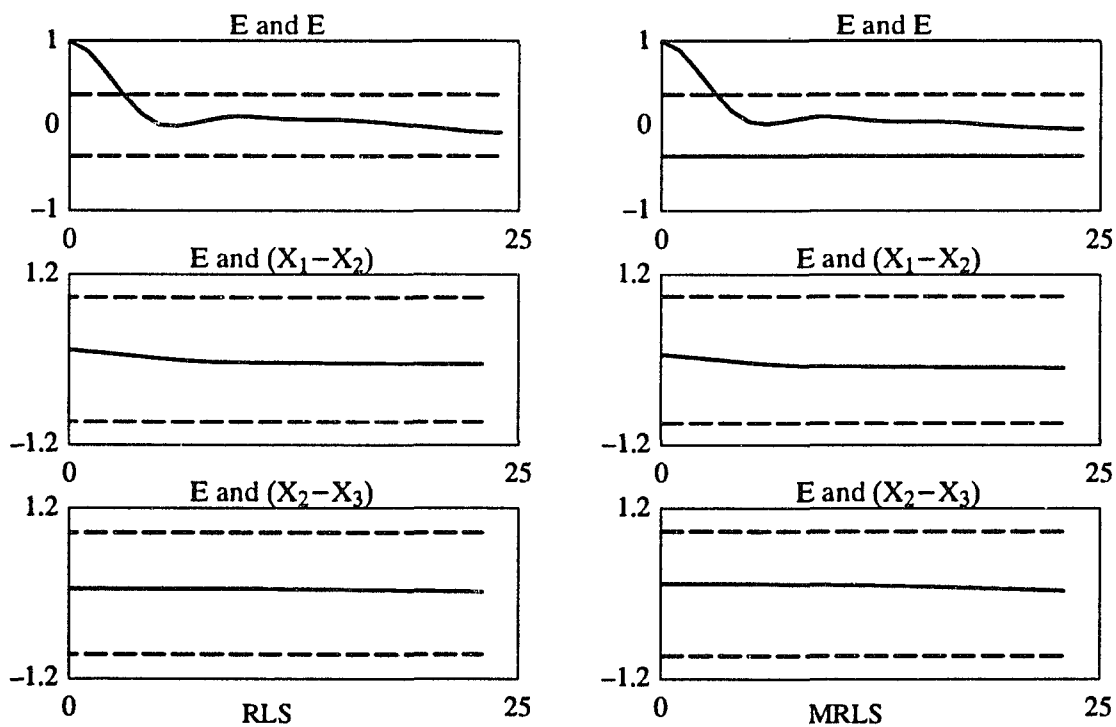


Fig. 5.21 Autocovariance of the residue and cross-covariance of the residue with the inputs for car, side impact. (E is prediction error)

approximately, $X/X_0 = 38$. The reason for this difference is because of the slow response of RLS in tracking the change in parameters. The parameters estimated by MRLS are considered to be closer to the actual crush characteristics of the system. This is based on a comparison of estimates of acceleration response presented in Figure 5.12. It can be seen that the error in acceleration estimates for MRLS is lower than for RLS, indicating that the MRLS estimate is better than the RLS estimate.

For the side impact, the parameters estimated using RLS and MRLS (see Figure 5.17) are quite close and so are the prediction errors. This is quite obvious as the parameters are almost constant (slope). This indicates that there is no advantage in using MRLS when the underlying parameters are constant or changing very slowly.

The auto-correlation of the prediction error for MRLS is lower than RLS in general (see Figures 5.13, 5.14, 5.15, 5.19, 5.20 and 5.21 and Table 5.4). This indicates improved estimates with the MRLS estimator.

5.7 Chapter summary

- 1) The estimation of structural characteristics from crash data is a challenging task, because of the presence of noise in measurements, time varying nature of the parameters and complexities of the event.
- 2) The difficulties in the estimation were addressed by simplifying the model structure, making use of physical insight and modifying the estimation algorithm.
- 3) Modified recursive least squares (MRLS) accomplished two objectives.
 - i) Improves the choice of forgetting factor: The cause of change in the prediction error was identified by monitoring the moving average of the prediction error. If a change in parameter was acknowledged the parameter corrective mechanism was altered to respond to the change in the parameters.

- ii) Quick response to change in parameters: Once a change in parameters was recognized the estimation was reset from a past observation, i.e. reusing the data with altered corrective mechanism to catch the change in parameters that might have been missed.
- 4) Application of MRLS on simulated data from a known parameter indicated a reduction in variance and auto-correlation of prediction error indicating improved estimation over the RLS.
- 5) Application of MRLS on actual test data indicated improvement in estimation for frontal impact parameters. There was no significant improvement for the side impact parameters, as the parameters were slowly varying.

CHAPTER 6

DEVELOPMENT OF ANALYTICAL MODELS FOR SIDE IMPACT FROM TEST MEASUREMENTS

6.1 Introduction

For a side impact, the major cause of injury is due to lateral impact on the occupant by the door interior. The severity of occupant injury depends on the manner in which the occupant is loaded and the force deflection characteristics of the interior. Therefore, in this chapter an analytical model to represent the occupant to structure interaction during a side impact is developed. The model structure is physically meaningful with nonlinear crush characteristics and mass being treated as lumped parameters. The lumped parameters are estimated directly from the test data using a Kalman filter (KF) estimator. The KF estimator, discussed in Chapter 3, is a minimum variance estimator with the parameter updating mechanism based on the knowledge of the measurement noise covariance matrix and process noise covariance matrix. Further, the KF estimator is modified to constrain the parameters in a physically meaningful space and to include the known information about the parameters.

The KF estimator is verified by estimating a known set of parameters from simulated data. Further, the estimation procedure is applied on a number of crash test data sets to estimate the unknown parameters. Simulation is carried out with the estimated parameters and the simulated response is compared with the test measurements.

This chapter is organized as follows: Section 6.2 describes injury causing mechanisms during a side impact. Section 6.3 presents a review of the current modeling approach. In Section 6.4 a lumped parameter model for the car to dummy interface

is developed and the model parameters are estimated using a KF estimator. The procedure is demonstrated on simulated as well as test data. In Section 6.5 the use of the estimated model in understanding the crash event is demonstrated and in Section 5.6 use of the estimated model in making design improvement is presented.

6.2 Side impact mechanisms

In the past few years concerns for the occupant safety in the side impact have increased. The recent amendment to FMVSS 214 (Federal Motor Vehicle Safety Standard) by (NHTSA [1990]) the National Highway Traffic Safety administration requires a full car dynamic side impact test. All cars sold in the US have to meet the required performance by 1997.

The detail of the side impact test procedure is available from the NHTSA [1990]. Essentially, as shown in Figure 6.1, the test car is rested stationary on the ground and is struck on the side by a specially designed moving deformable barrier (MDB) at a crabbed velocity of 33.5 mph (54.7km/hr). The test car has two occupant devices on the struck side to measure the injury. The occupant devices used in the test, called side impact dummies (SID), are designed to replicate 50th percentile male behavior during the lateral impact loading.

A schematic of the SID is presented in Figure 6.2. The dummy consists of a rib cage connected to a spine block. The spine block is a stiff steel structure connected to the lumbar spine at the bottom and the head at the top. The lumbar spine is made of rubber and is connected to the pelvic block. The abdomen cavity, located between the rib cage and the pelvic block, is essentially a rubber bag filled with air. The accelerometers are located on the rib, spine, and pelvis.

The acceleration measured at various locations on the dummy are previously correlated with the actual injury on the abbreviated injury scale (AIS). The AIS is an

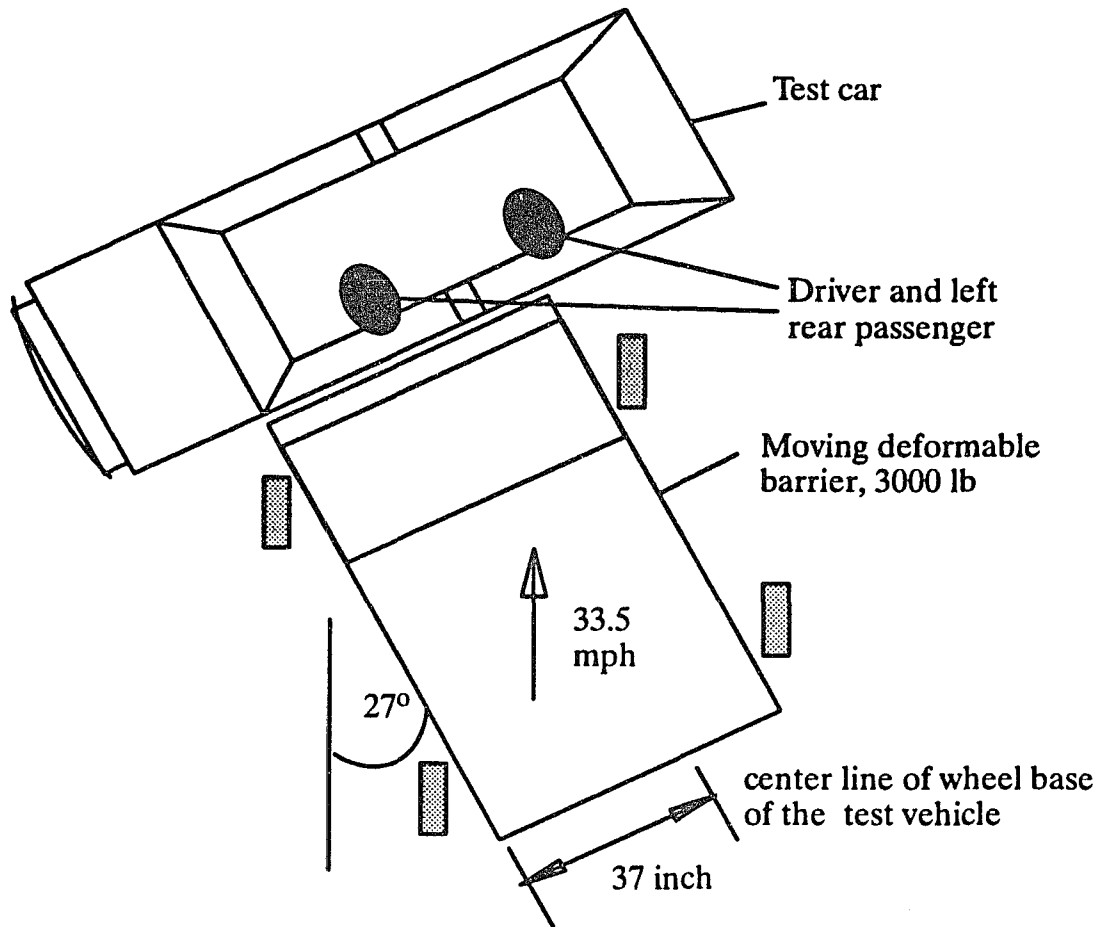


Fig. 6.1 Side impact test procedure

injury measurement based on the study of internal damage to cadavers and swine etc., estimated using standardized surgical procedures. Among the researchers in bio-mechanics, controversy over the acceptability of the NHTSA supported correlation of peak accelerations measurements with the actual injury is not resolved, see Viano et al. [1989]. For this chapter, it is accepted that the acceleration measurements are correlated with the actual occupant injury.

During a side impact the major part of the kinetic energy (KE) of the MDB is lost in the deformation of the test car and MDB, a very small part of the KE is lost in friction and the rest is retained as a combined KE of the test car and the MDB. For a typical mid-size car the approximate energy balance can be evaluated as follows:

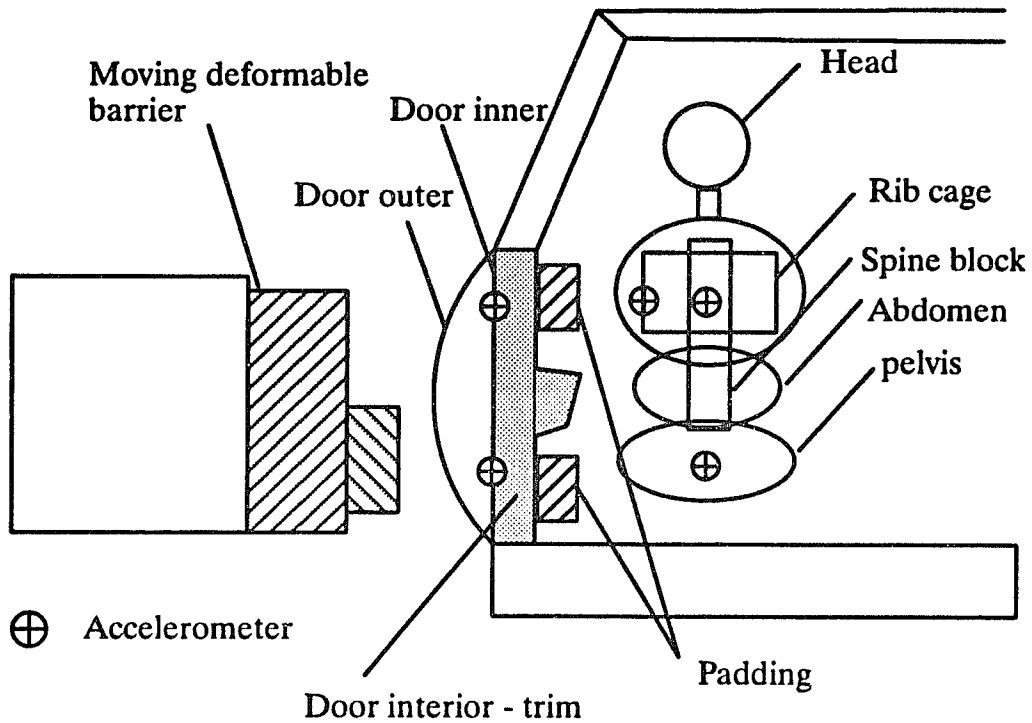


Fig. 6.2 Schematic of dummy and the car structure in side impact

Known data

Mass of the car = 1700 kg (assumed)

Mass of the MDB = 1362 kg

Mass of each Dummy = 84 kg

Initial velocity = 53.6 km/hr at a 27 degree crabbed angle.

Final velocity = 20 km/hr in lateral direction. (assumed)

Before Impact

KE, Total = 151 kNm (longitudinal 80%, lateral 20%)

After Impact

KE, Car+MDB	=	47.25 kNm	(31.4%)
KE, Dummy	=	2.59 kNm	(1.7%)
Deformation of (car+MDB)	=	100.16 kNm	(66.9%)

The above numbers are approximate and are based on the assumption that there is no friction loss and the final velocity in the longitudinal direction of the car is negligible. The final velocity in the lateral direction is based on past experience with typical mid-size cars.

In a side impact test of a typical mid-size car there is inboard deformation of about 300 to 400 mm in the cross car direction. Typical lateral clearance between the dummy and the door interior is in the range of 100 to 150 mm. Under these conditions a strike on the dummy by the interior is unavoidable. The injury measured on the dummy depends on the following car characteristics, Fukushima et al. [1991], Matsumoto and Tanaka [1991].

- 1) The force-deflection characteristics of the car interior.
- 2) Geometry of the car/door interior at the time of the interior contacting the dummy.
- 3) Severity of the impact, i.e. the difference in velocity between the car interior and the dummy at the time of contact.
- 4) Presence of any other parallel load transfer mechanism (e.g. seat, belt etc.) that can transfer the load from the car to the dummy.

A softer interior lowers the occupant injury by limiting the total force transferred to the dummy and hence limits the peak accelerations. The interior geometry defines the manner in which the dummy is loaded, i.e. the location of impact

and the sequence of loading has a different influences on the accelerations measurements at different dummy locations. A high differential velocity at the time of contact results in a higher impact load. It can be lowered, either by a stiffer side structure i.e. reducing the inboard velocity of the door, or by designing the car such that there is an alternate load path to the dummy, i.e. making the dummy start moving earlier.

In the next section a lumped parameter model is developed and then a technique to estimate the interface parameters directly from the available test data is presented.

6.3 Review of lumped parameter models used in the side impact

For the last two decades lumped parameter models have been extensively used in crash analysis. Lobdell's [1972] chest model was one of the earliest models developed to analyze the chest behavior under blunt impact loads. Viano [1987b,1989] modified Lobdell's model for lateral impact and evaluated the effect of energy absorbing materials for occupant protection during a side impact. Both models only include the thorax and are used in simulating idealized loading conditions. The blunt loading is different from the loading observed in the actual car crash. A number of papers describing occupant/structure combined models are also available. The model developed by Tomasaoni [1984] and Sakurai et al. [1989] consists of a structure with three degrees of freedom and an occupant with a single degree of freedom. Another model, with similar structure, but with three degrees of freedom of the occupant, was developed by Trella and Kaninthara [1985] at NHTSA and Bennett et al. [1989] at GM. Haswaga et al. [1989] at Toyota, improved upon the above model structure using detailed representation of the occupant.

Most of the structural models described above have the door modeled as a single component. It has been observed in actual tests that this is not true. The velocity

of the door in the upper portion and the lower portion is different. This is quite obvious from the test setup, shown in Figure 6.2, as the bumper face on the MDB is approximately 100mm forward of the rest and therefore the lower portion of the car is loaded earlier. Also, the construction of the dummy, presented in Figure 6.2, indicates that there are two load paths to the dummy, upper and lower. As a result the peak accelerations of the dummy will depend on the timing, location and magnitude of the strike forces acting on it. This understanding of the event indicates a need for a separate representation of the upper and lower door in the model.

In all the models discussed above with the exception of Trella and Kaninthara [1985], the structural properties are estimated from a quasi-static crush test of the individual components. Often it has been observed that the crush mode in the component level test is different than in the actual test. This is mainly because of improper boundary conditions and idealized loading in the component crush tests. Trella and Kaninthara [1985] estimated the structural properties from the crash test data by an inverse solution of the equations of equilibrium. This approach has some restrictions, it does not allow parallel load paths in the model, and the number of equations must be equal to number of unknown parameters. Also their procedure does not address measurement noise.

6.4 Model structure and estimation

6.4.1 Model Structure

The model structure selected for this chapter is presented in Figure 6.3. The model structure is selected to preserve the essence of the event while maintaining simplicity, so that the unknown parameter can be evaluated directly from the crash data. The modeling is based on the philosophy developed in Chapter 4. A lumped mass is assumed at the location of acceleration measurement and the structural members between two lumped masses are represented by one dimensional nonlinear springs.

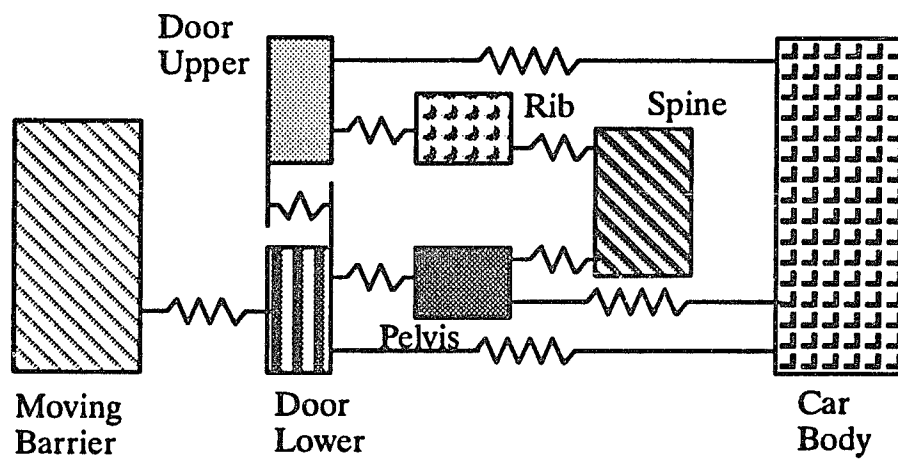


Fig. 6.3 Model structure for side impact

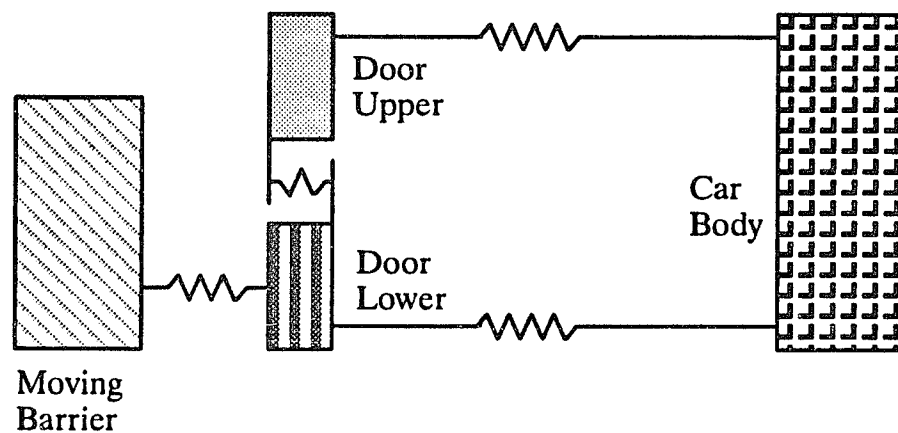


Fig. 6.4(a) Model for structure only

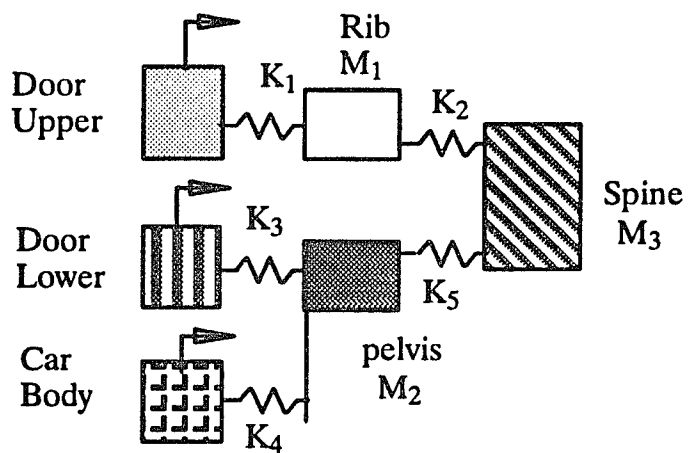


Fig. 6.4(b) Occupant to interior interface model

The resistance to crush (F) of the structural member is defined by $F=K(X)X$, where the stiffness parameter $K(X)$ is a non-linear function of crush X . As all the measured data is in the time domain and mathematical expression for the model is also in time domain the stiffness parameters $K(X)$ s are estimated as time varying parameters (i.e. $K(t)$) and then correlated with the corresponding crush-time history.

The model structure selected is based on an understanding of the physics of the system. The door is divided into upper and lower portions as discussed before. The dummy is represented by three masses. The loading of the dummy from the door interior is through two load paths - upper and lower. The upper portion of the dummy (the rib cage) is loaded by the upper door, and the lower portion of the dummy (the pelvis) is loaded by the lower door. It is assumed that the spine is loaded by the rib cage as well as pelvis. The load transfer between the pelvis and the rib cage through the abdominal cavity and the exterior jacket is considered insignificant.

The energy transfer during the side impact for a typical mid size car, as discussed in the previous section, indicates that approximately 69% of the KE goes into the deformation of the structure whereas less than 2% of the KE goes to the dummy. Based on these data it can be concluded that the door inboard crush-time history, largely depends on the car side structure crush strength rather than the presence or absence of a dummy. On the other hand, based on an understanding of the side impact mechanism discussed in the previous section, it can be concluded that the occupant injury is largely dependent on the door inboard deformation time history and the force-deflection characteristics of the car interior. Therefore the model structure considered in Figure 6.3 is further simplified into Figures 6.4(a) and 6.4(b). The car structure and the dummy/interior part of the model are separated as two different models. The underlying assumption is that the presence or absence of the dummy does not effect the door velocity.

For the dummy/interior model the loading of the occupant is from three sources, the door upper, the door lower and the car. All three loading mechanisms are forced displacement type (source of infinite energy). The forced displacement time history data for these three sources can be directly obtained from the test data. If the test data is not available, the simulated response from the separated structure model can be used.

The separation of the structure and the dummy, based on the insight of the system simplifies the estimation procedure. Also the effect of the structure and the interior in the occupant protection is separated. This is very helpful information while designing the car.

6.4.2 Estimation of parameters

The purpose here is to estimate the lumped parameters directly from the test data. The equilibrium equations for the model in Figure 6.4(b) can be written as follows,

$$\begin{aligned} M_{rib} a_{rib} &= K_1 (X_{du} - X_{rib}) - K_2 (X_{rib} - X_{spn}) + e_1 \\ M_{plv} a_{plv} &= K_3 (X_{dl} - X_{plv}) + K_4 (X_{car} - X_{plv}) - K_5 (X_{plv} - X_{spn}) + e_2 \\ M_{spn} a_{spn} &= K_2 (X_{rib} - X_{spn}) + K_5 (X_{plv} - X_{spn}) + e_3 \end{aligned} \quad (6.1)$$

Which can also be written as

$$\begin{bmatrix} M_{rib} a_{rib} \\ M_{plv} a_{plv} \\ M_{spn} a_{spn} \end{bmatrix} = \begin{bmatrix} (X_{du} - X_{rib}), & -(X_{rib} - X_{spn}) \\ & (X_{dl} - X_{plv}), (X_{car} - X_{plv}), & -(X_{plv} - X_{spn}) \\ & -(X_{rib} - X_{spn}) & (X_{plv} - X_{spn}) \end{bmatrix} \begin{bmatrix} K_1 \\ K_2 \\ K_3 \\ K_4 \\ K_5 \end{bmatrix} + \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}$$

Where

M : mass - assumed based on known data.

a : acceleration - measured.

X : displacement - evaluated from acceleration.

K_i : Stiffness parameter shown in Figure 6.4(b) - estimated

e_i : Equation error - needs to be minimized.

Subscripts are

rib : rib cage

plv : pelvic block

spn : spine block

Acceleration measurements on the car structure at the upper door, lower door, car floor and on the dummy at rib, spine and pelvis are available. The displacements can be evaluated from the measured acceleration data. The unknown parameters (K_1 to K_5) can be estimated by minimizing some function of the error vector, $e = [e_1 \ e_2 \ e_3]^T$. One popular approach is to use quadratic criterion i.e. $e^T \Lambda e$. Minimizing average of this criterion for a linear model structure gives the well known least squares estimator. A recursive version of least squares estimator, for a linear model structure given by, $Y_t = \Psi_t^T \theta_{t-1} + e_t$, can be presented as: (Ljung [1987]),

$$\begin{aligned}\theta_t &= \theta_{t-1} + L_t (Y_t - \Psi_t^T \theta_{t-1}) \\ L_t &= P_{t-1} \Psi_t (\gamma_t \Lambda_t + \Psi_t^T P_{t-1} \Psi_t)^{-1} \\ P_t &= (P_{t-1} - P_{t-1} \Psi_t [\gamma_t \Lambda_t + \Psi_t^T P_{t-1} \Psi_t]^{-1} \Psi_t^T P_{t-1}) (1/\gamma_t)\end{aligned}\tag{6.2}$$

Where

Y_t is the output measurement

Ψ_t is the input-output measurement

θ_t is the parameter vector

Λ_t is the error covariance matrix

γ_t is the weighing function given by ; $1/\gamma_t = (\lambda_t / \gamma_{t-1}) + 1$,
where γ_t is forgetting factor.

L_t is the gain matrix.

e_t is the prediction error.

Another popular criterion is variance of the error i.e. $e \cdot e^T$. Minimizing average of this criterion gives the well known Kalman filter estimator. The KF estimator is similar to the RLS estimator, except for difference in the parameter correction mechanism. The KF estimator in a form similar to RLS estimator can be presented as follows, per Ljung [1987]

$$\begin{aligned}\theta_t &= \theta_{t-1} + L_t (Y_t - \Psi_t^T \theta_{t-1}) \\ L_t &= P_{t-1} \Psi_t (R_{2t} + \Psi_t^T P_{t-1} \Psi_t)^{-1} \\ P_t &= P_{t-1} - P_{t-1} \Psi_t [R_{2t} + \Psi_t^T P_{t-1} \Psi_t]^{-1} \Psi_t^T P_{t-1} + R_{1t}\end{aligned}\tag{6.3}$$

where,

R_{1t} is the covariance matrix of the process noise and

R_{2t} is the covariance matrix of the measurement noise.

The major difference between the RLS and KF based estimation algorithms is in the mechanism used to alter the parameters - the gain matrix L_t . Instead of a single forgetting factor for all the parameters, the change in the gain matrix depends on the measurement noise covariance and parameter covariance matrix. This gives increased flexibility in implementing the 'known' physical insight of the system. In this chapter the KF estimator is used in the estimation of unknown parameters in the system.

The parameters sought here are physical parameters and some of their properties are known. The KF estimator is modified to include the known information. The known information used in this chapter is as follows:

- 1) The stiffness (K) is never negative.
- 2) Most of the deformation during the crash is plastic deformation, so the loading and unloading force deflection characteristics are different. The change from loading to unloading or from unloading to loading can be detected by monitoring the sign of the rate of crush.(see Figure 6.5)
- 3) Because of the nature of the model structure many of the stiffness components are active in compression only (e.g. the door interior to rib spring). As there is no attachment there will be no interacting force when they are separated and therefore the tensile strength will be zero. The tension or compression can be detected by monitoring the sign of crush.

The KF based estimation is modified to include the above physical insight in the estimation algorithm. The approach used here is to monitor the crush in each step before the estimation of parameters. If the sign for the crush or rate of crush is changed then depending on the assumed nature of that structural member a sudden change (discontinuity) in the parameter in the time domain is anticipated. As per the underlying assumption used in KF estimator the parameters are the state vector of a

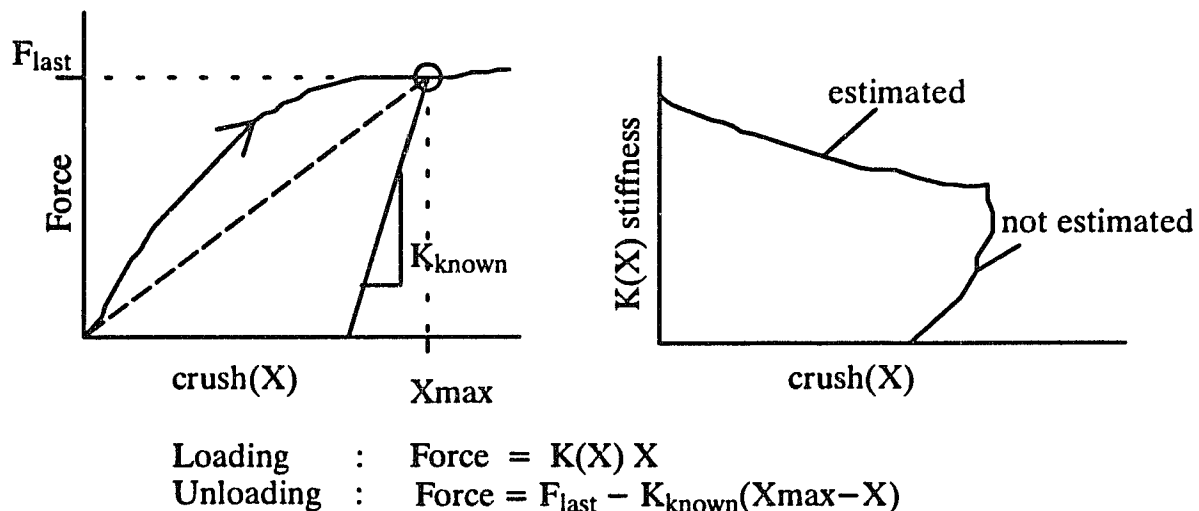


Fig. 6.5 Loading and unloading characteristic (schematic)

first order system and therefore the KF based estimator is not likely to track a sudden change in parameters. As a result a corrective action in the estimation procedure is necessary.

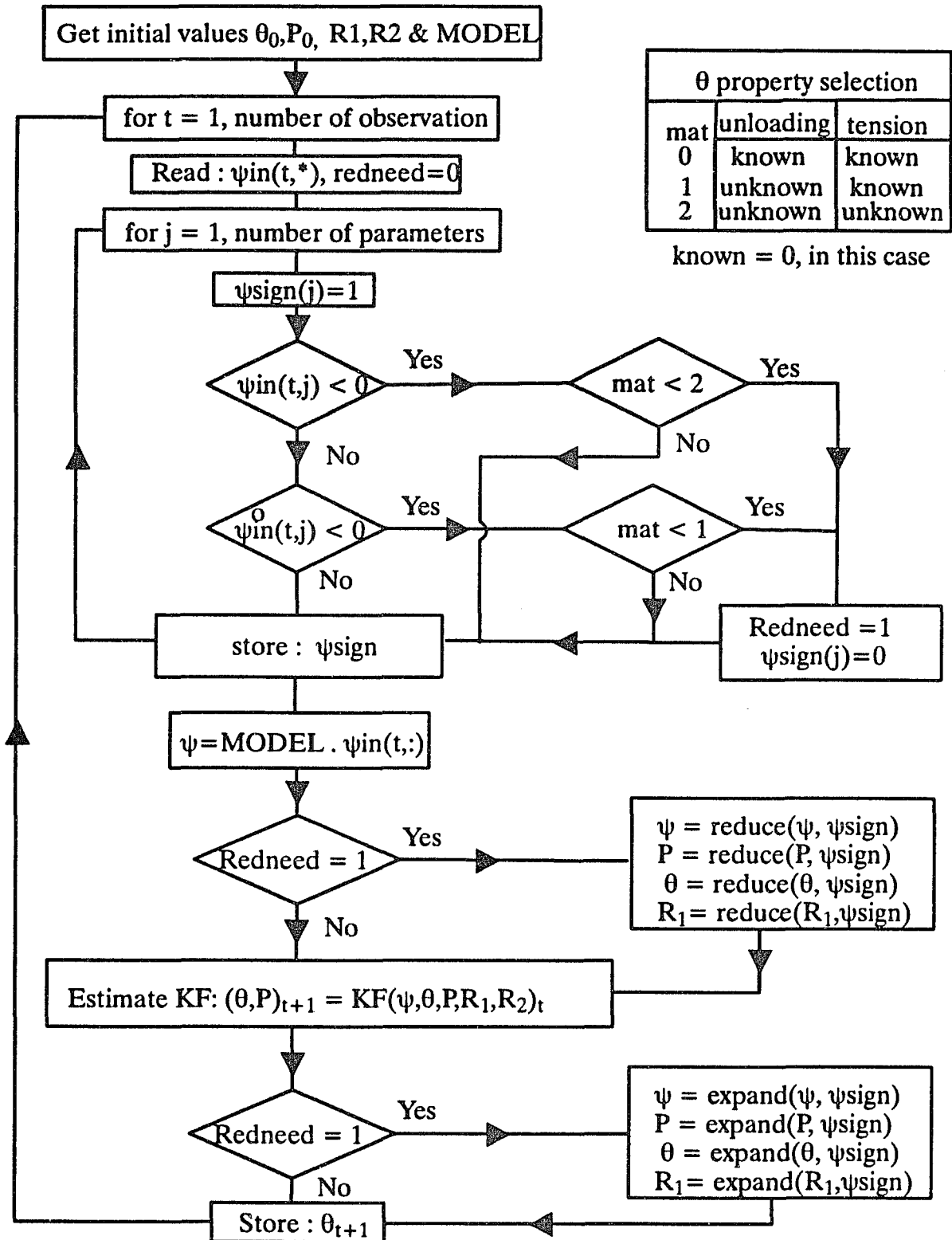
Now the question is what correction should be done. An ideal choice would be to start with some initial value and estimate parameters from that point of discontinuity as a new estimation problem. This is not practical because only a few data points are available. Therefore, an approximation is used. The parameter that becomes discontinuous is considered known, with a value based on past experience. Once a known parameter is detected then the output and the input are modified to reflect that known information and then that particular step is carried out to estimate only the unknown parameters, see Figure 6.5 to see how known information is used.

To obtain a physically meaningful stiffness, negative values are not allowed. The parameters are monitored after estimation for each step. If a parameter is found to be negative then it is declared to be known with zero value and the rest of the parameter are re-evaluated using the given set of data for that step. It is admitted that the above 'control' over the estimation procedure is based on the physical insight only and while implementing it the minimization of the equation error is compromised.

The algorithm is implemented in MATLAB [1991] and the flowchart is presented in Figure 6.6.

6.4.3 Verification of the KF in estimation of interface parameters

A known set of parameters, presented in Figure 6.9, was selected, and then the equation set (6.1) was simulated using the fourth order Runge-Kutta integration technique. The lumped masses used are assumed to be, $M_{rib} = 16.8$ kg, $M_{plv} = 39.2$ kg and $M_{spn} = 28$ kg. The loading is forced displacement and is presented in Figure 6.7.



Note : Reduce and expand are subroutines that reduces or expands a matrix size based on ψ_{sign} and known information. MODEL is matrix defining the model structure

Fig. 6.6 Flow chart for Modified KF algorithm.

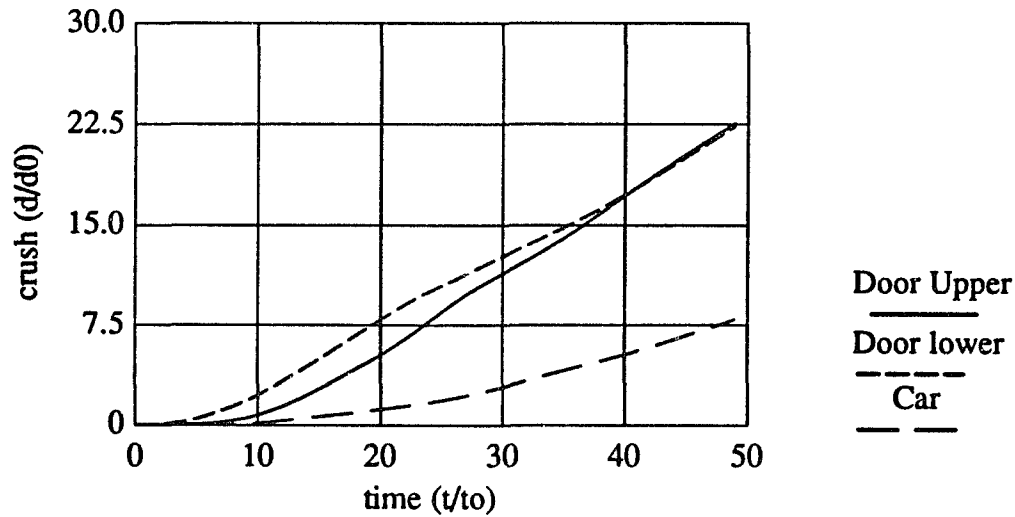


Fig. 6.7 Forced displacement time history

The simulated response is corrupted with normally distributed random noise (mean=0, variance= 11.37×10^{-9}) as shown in Figure 6.8. Then the parameters (K_1 to K_5 in Figure 6.4(b)) are estimated using the KF estimator. The covariance of the prediction error vector is presented in Table 6.1. The estimated stiffness parameters and the force-deflection data are compared with the assumed data in Figure 6.9.

Table 6.1 Verification : Covariance of prediction error

	$\times 5.68 \times 10^{-9}$		
	e_1	e_2	e_3
e_1	1.8478	0.0611	0.3487
e_2	0.0611	2.2947	-0.2540
e_3	0.3487	-0.2540	2.4089

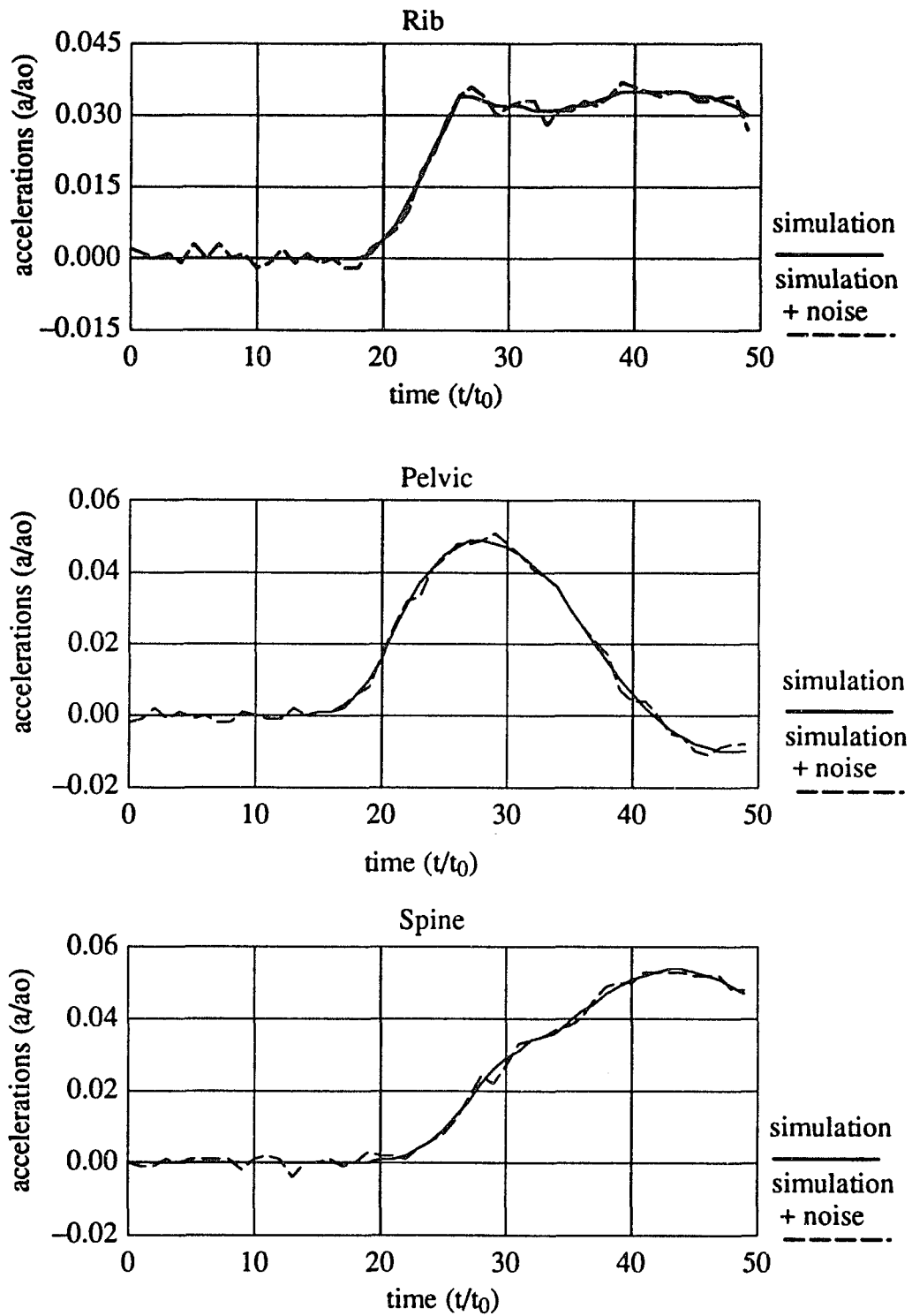


Fig 6.8 Verification : simulated response with and without noise
noise : mean = 0 and variance = 11.37×10^{-9}

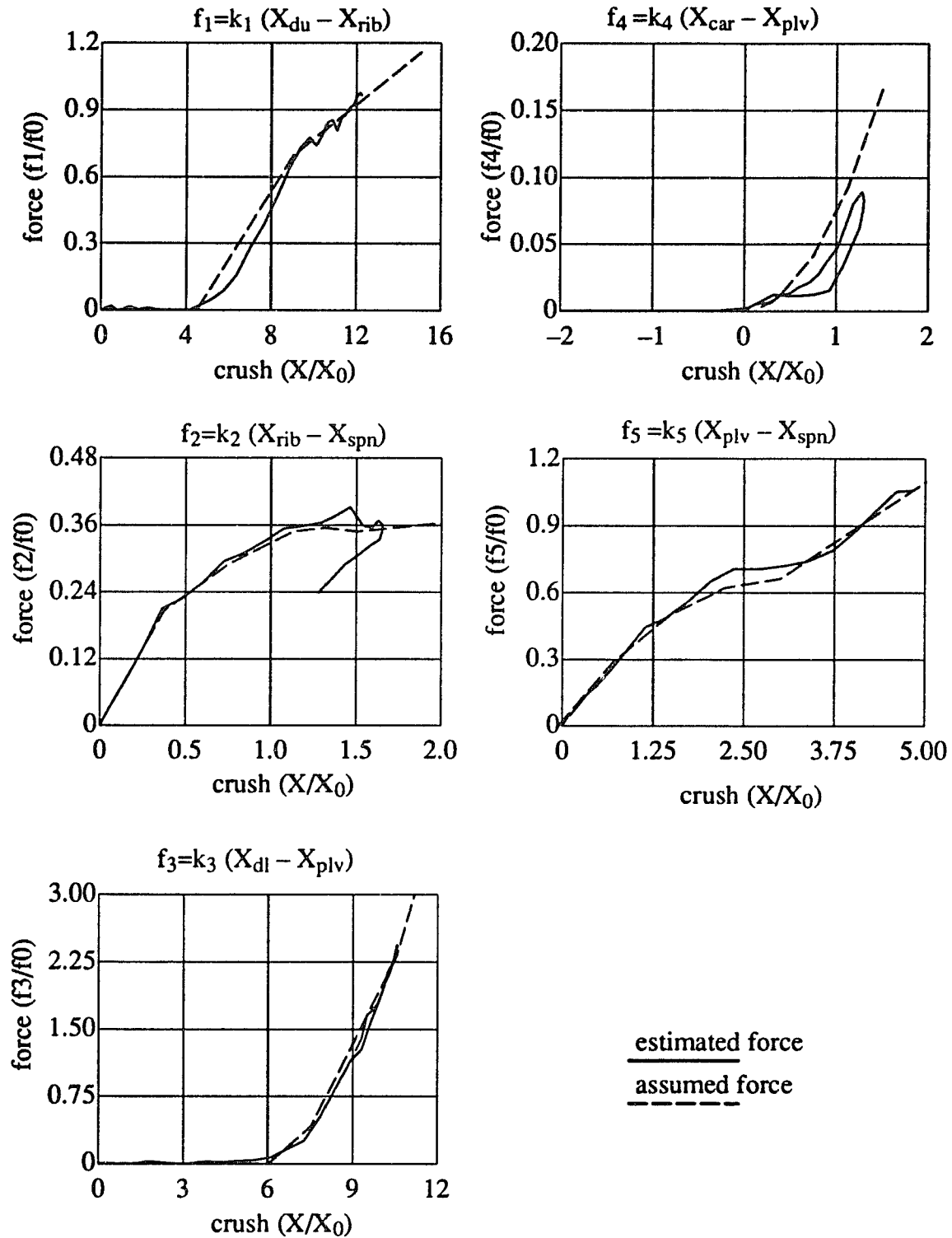


Fig. 6.9 Verification : comparing estimated force-deflection data with the assumed data.

6.4.4 Application to the test data

A number of identical cars have been tested under sponsorship of the Motor Vehicle Manufacturers Association (MVMA) to study repeatability and influence of padding on the SID and BIOSID dummy, is described in Wasko et al. [1991]. Four tests with a SID dummy were selected out of these test data. Two of the selected tests (test #1 and test #2) have padding added near the thorax and pelvis. The padding is 2 pound/cubic feet EPS foam with 5.65 units normalized thickness. The other two tests i.e. test#3 and test#4 were un-padded. The approximate location of the padding is presented in Figure 6.2. The measured accelerations of the dummy are presented in Figure 6.10, and the derived input used in the estimation are presented in Figure 6.11. Using these data, the unknown dummy parameters (K_1 to K_5 in Figure 6.4(b)) are estimated using the KF estimator. The covariance of the prediction error vector is presented in Table 6.2. The mass distribution for the dummy is assumed as follows :

$$M_{rib} = 16.8 \text{ kg}, M_{plv} = 39.2 \text{ kg}, M_{spn} = 28 \text{ kg}.$$

The estimated parameters for all four tests are compared in Figure 6.12. It can be seen that all five force deflection characteristics correlate well for similar cars. The stiffness between the door upper and the rib (K_1) for the tests #1 and #2 is different then for tests #3 and #4. The reason for the difference is the presence of padding. The force starts increasing for the padded case earlier i.e. at approximately 5.5 units of normalized crush. This correlates with the added padding in that area. Similar behavior is seen for the stiffness member between the door lower and the pelvis (K_3). The peak force for the padded interface is lower than the one for the un-padded case.

The dummy crush characteristics for the rib to spine for all four tests are close. The pelvis to spine stiffness are slightly different for the padded and un-padded cases. The pelvis to spine stiffness for the padded case drops earlier compared to the un-padded case. The mechanics involved behind this can be explained as follows: As

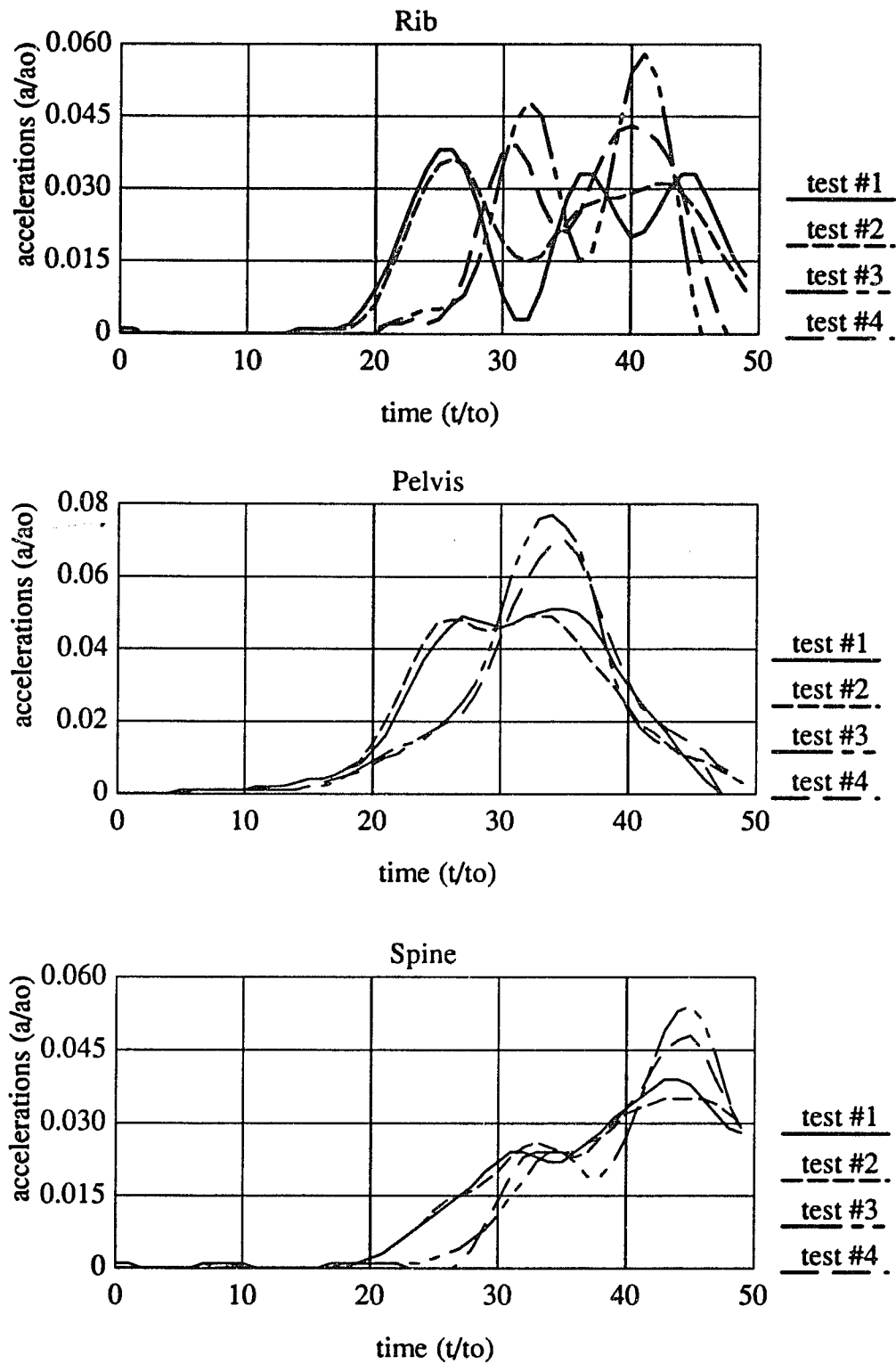


Fig. 6.10 Measured dummy data for tests #1, #2, #3 and #4

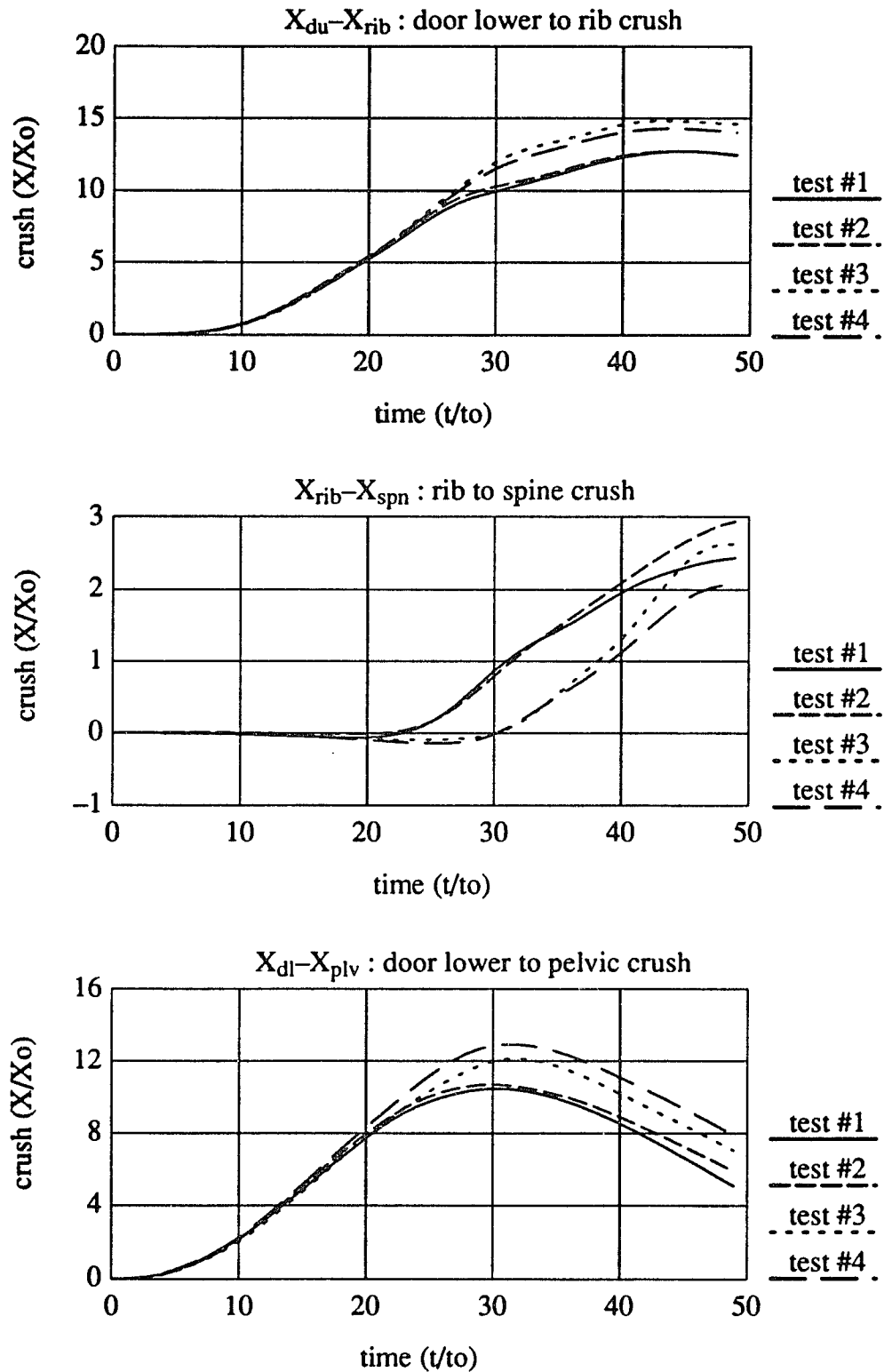


Fig. 6.11 Derived dummy input from the measured data
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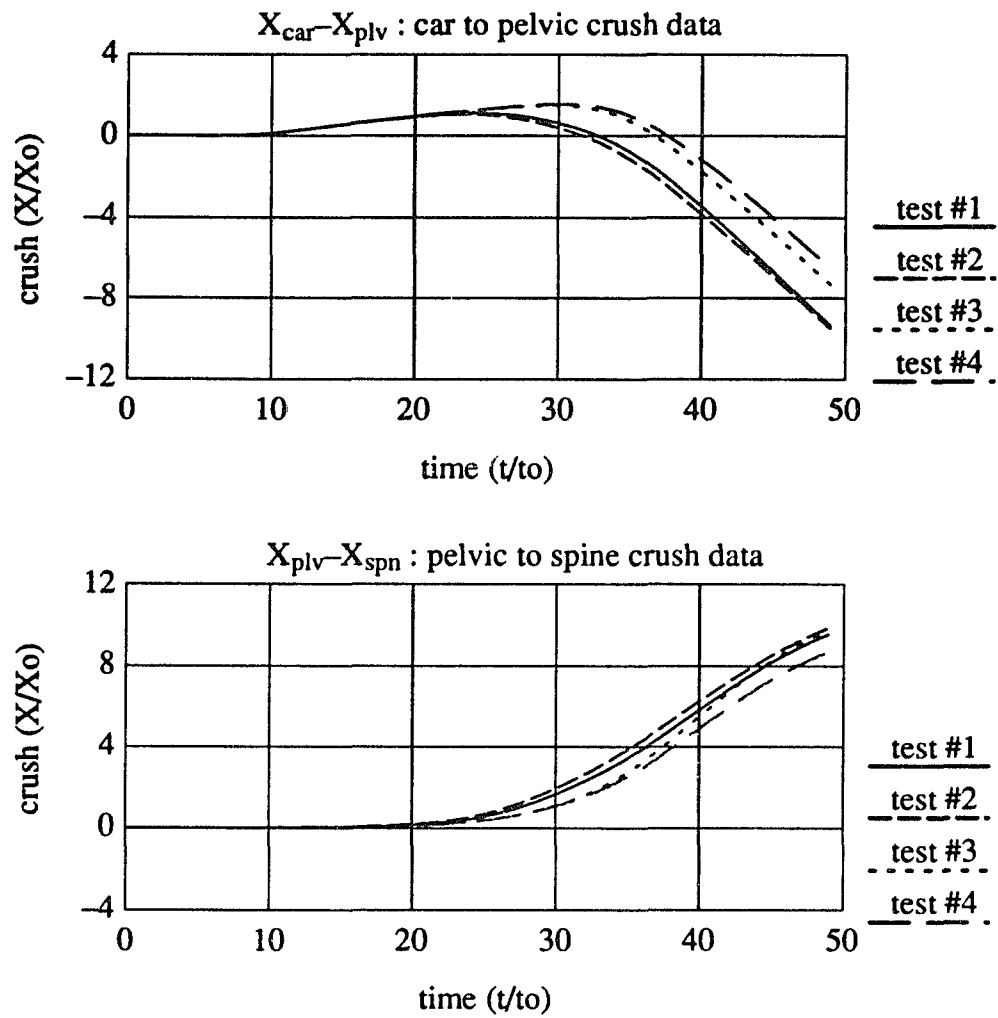


Fig. 6.11 Derived dummy input from the measured data
(continued from previous page)

Table 6.2 MVMA test data, covariance of prediction error

	$\times 5.68 \times 10^{-9}$			$\times 5.68 \times 10^{-9}$		
	e_1	e_2	e_3	e_1	e_2	e_3
	Test # 1			Test # 2		
e_1	15.3673	8.4327	0.8439	8.2671	7.0254	1.7392
e_2	8.4327	13.1813	-0.5957	7.0254	10.9763	1.4505
e_3	0.8439	-0.5957	6.3996	1.7392	1.4505	3.8008
	Test # 3			Test # 4		
e_1	13.5790	2.4976	-2.5932	10.5385	2.1044	-1.6198
e_2	2.4976	17.9818	12.2869	2.1044	12.4151	6.7379
e_3	-2.5932	12.2869	22.2811	-1.6198	6.7379	18.1498

the door hits the dummy, depending on the load distribution on the upper (rib) and lower (pelvis) load path, the dummy may rotate. The door interior geometry and crush characteristics for the padded and un-padded case are different because of added foam blocks. As a result the load paths to the dummy for the padded and un-padded case are different. This results in different dummy rotations. The model recognizes only one dimension, i.e. all the degrees of freedom are in the lateral direction. As a result the estimated translation between the pelvis and the spine based on the accelerometer data is influenced by the rotation of the dummy. If the dummy undergoes more anti-clockwise rotation for the padded case then there will be relative motion between the pelvic and spine without actual crush – this would indicate a drop in stiffness.

Simulation of the equation set (6.1) using estimated parameters from the above tests, is performed for one padded case (test #1) and one un-padded case (test #3). The displacement time history measured at door upper, door lower and the right (non struck side) rocker, in the test is used as forced displacement. The simulated response

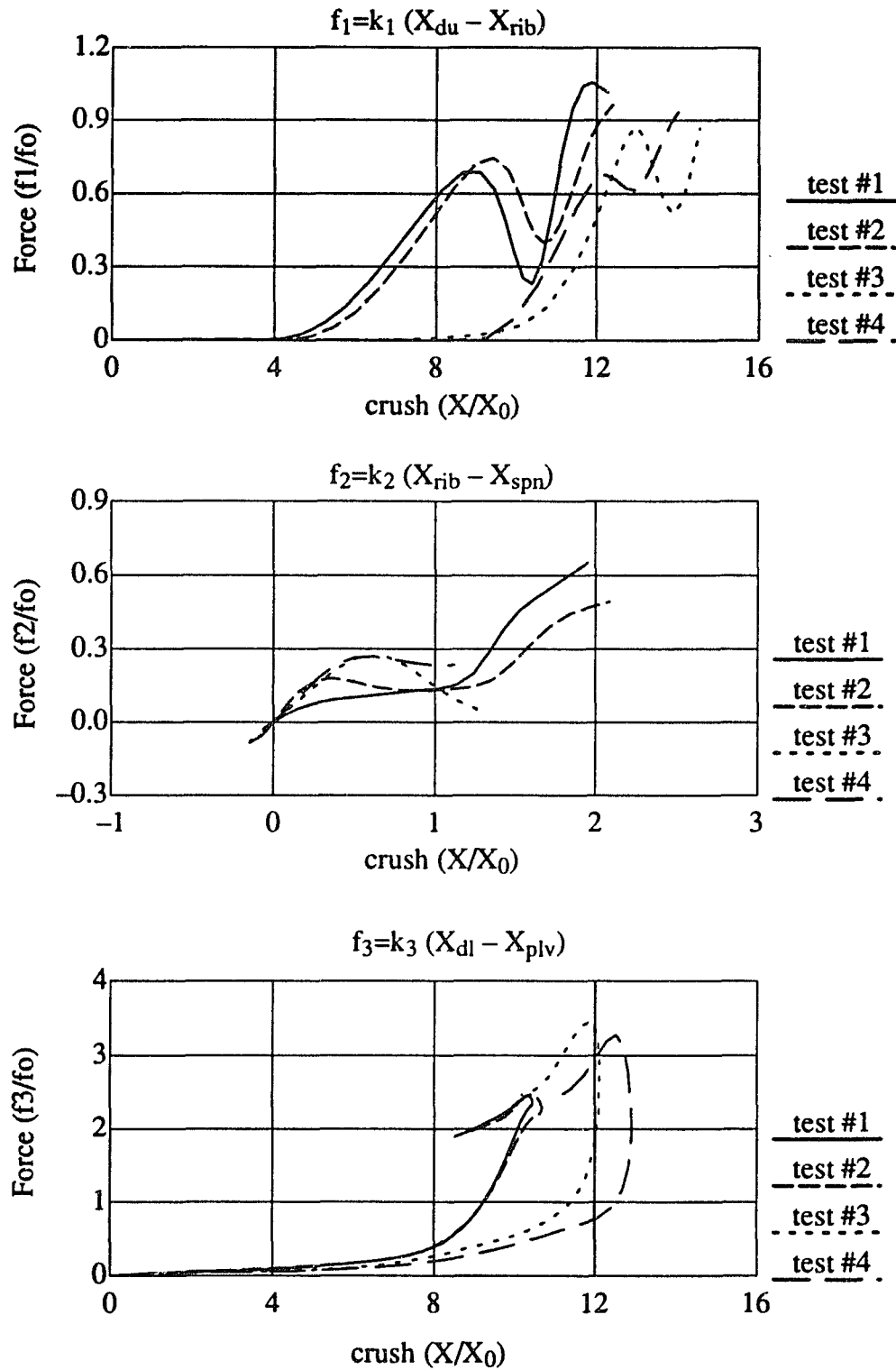


Fig. 6.12 Estimated force-deflection data for tests #1,#2,#3 and #4
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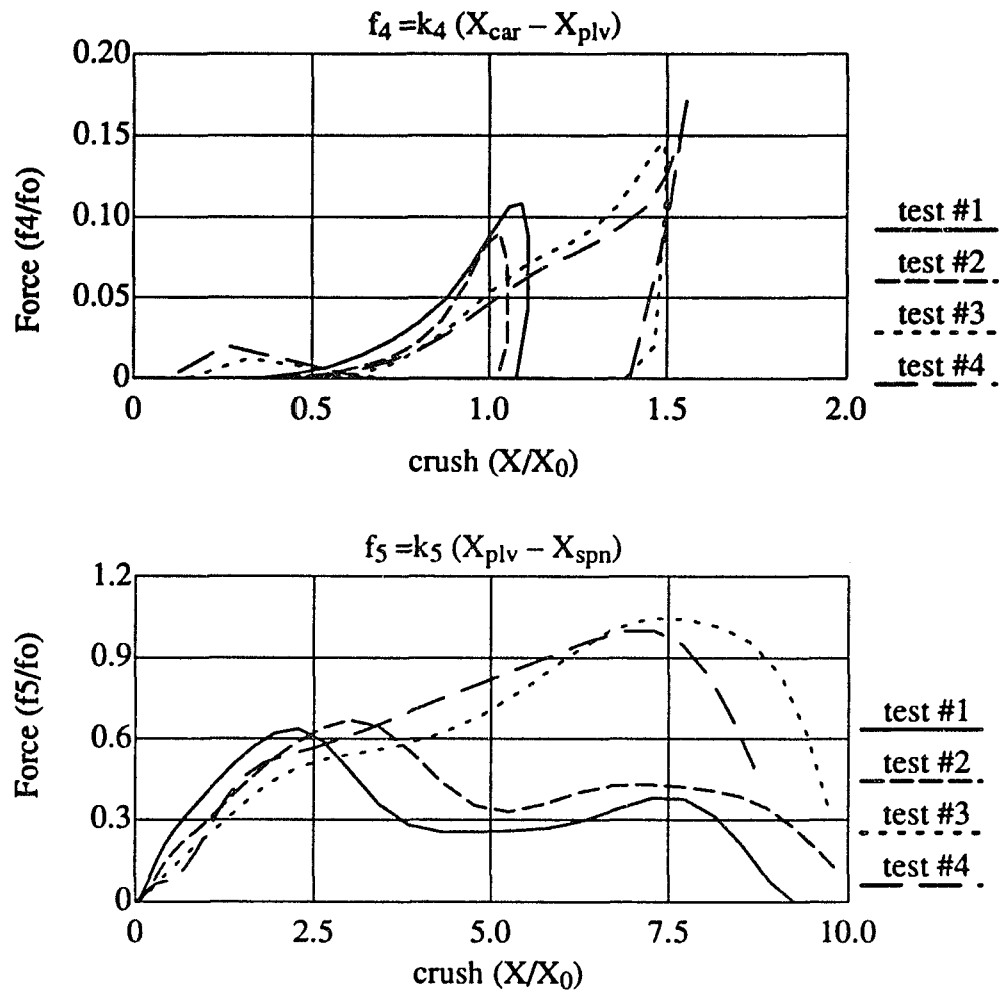


Fig. 6.12 Estimated force-deflection data for tests #1,#2,#3 and #4
(continued from previous page)

for the padded case is presented in Figure 6.13, and the un-padded case is presented in Figure 6.14.

Based on the correlation of simulated and test acceleration response, physical meaning of the estimated parameters for the test case and the closeness of assumed and estimated parameters for the simulated data, it is concluded that the selected model and estimation method proposed are adequate.

6.5 Use of the estimated model in understanding the crash event and design improvements.

Now the question is, how can the extracted information be used?

The structural properties estimated in this manner are in position characteristics, i.e. under actual boundary conditions (constraint and loading) and therefore give us a clear indication of the relative contribution of various components in the crash event, e.g. the estimated forces to the dummy through the upper load path and the lower load path are presented in Figure 6.15, and loads to the spine from the ribs and the pelvis are presented in Figure 6.16. It can be seen that the force through the lower load path is higher than the upper load path, and therefore the spine acceleration can be effectively lowered by adding padding (i.e. limiting force) in the lower portion of the door.

Along with an increased understanding of what is happening during the test, the model can be used in determining door interior stiffness and the desired door displacement time history to lower occupant peak accelerations. The change in the door displacement time history results in component level force deflection targets for the side structure. While designing the structure it is easier to realize force deflection targets as compared to dummy accelerations.

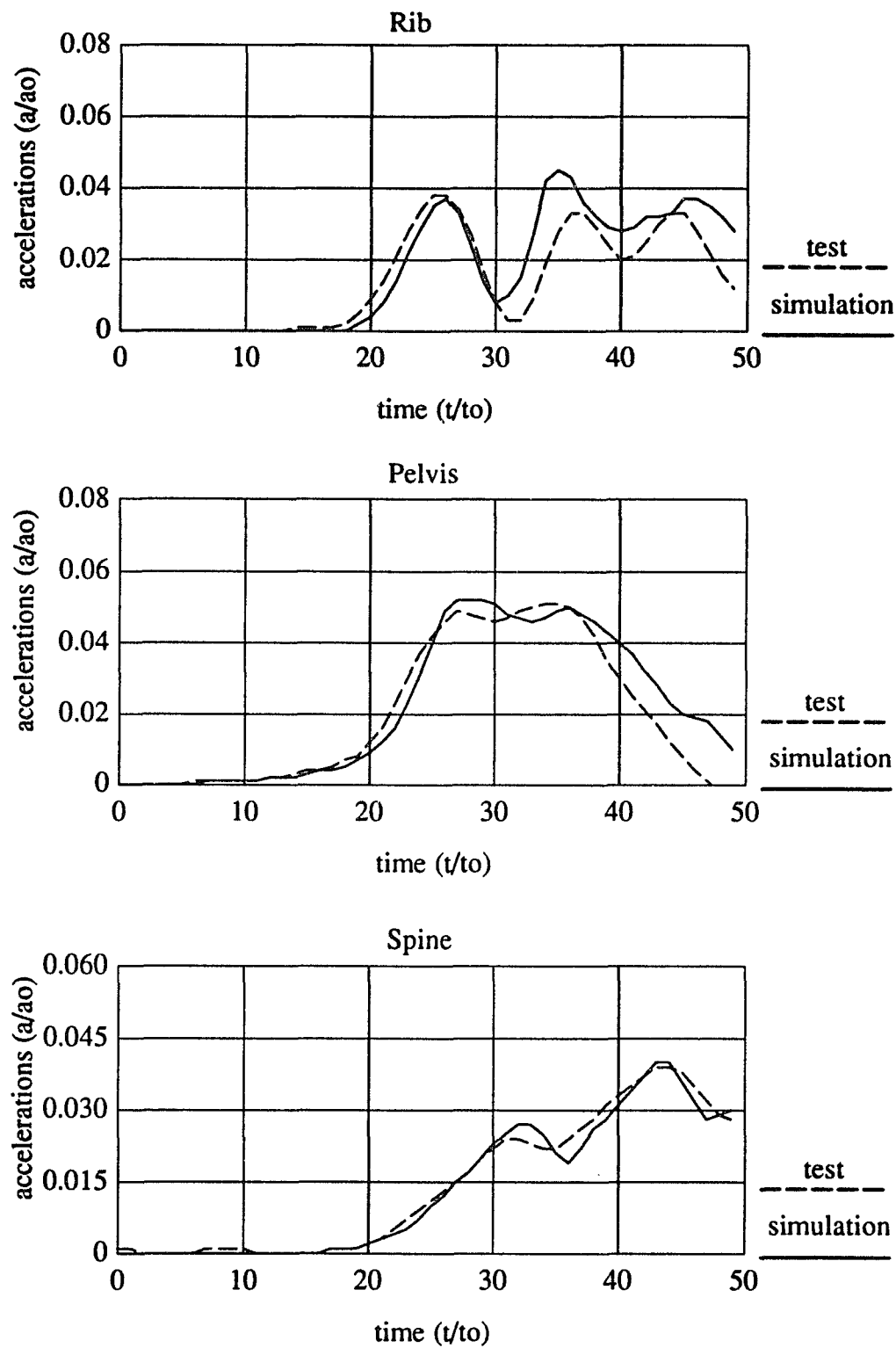


Fig. 6.13 Comparison of simulated response with measured data padded interior.

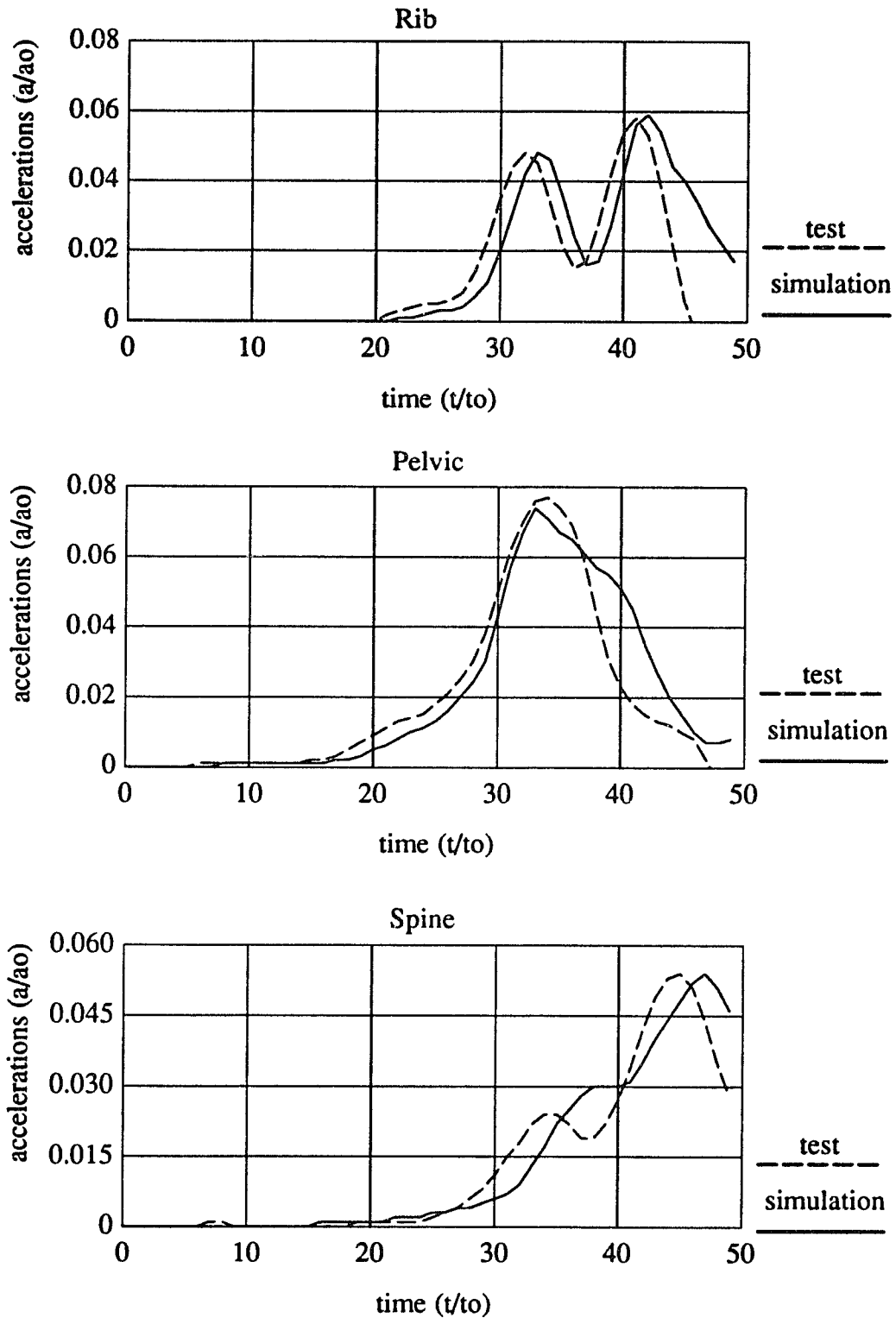


Fig. 6.14 Comparison of simulated response with measured data un-padded interior.

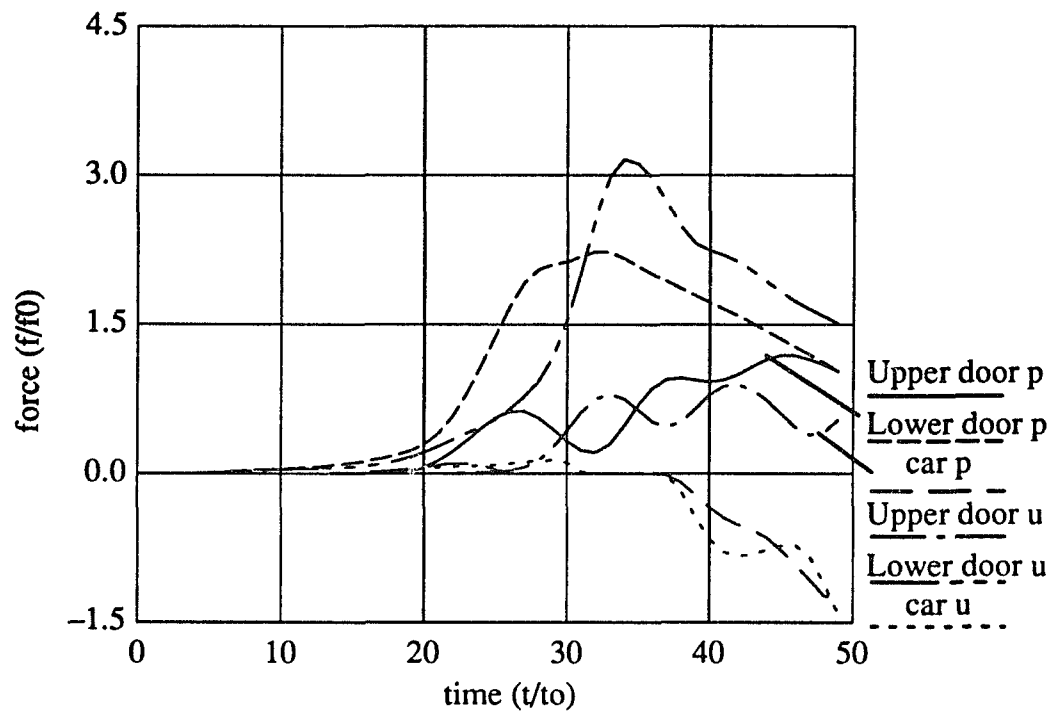


Fig. 6.15 Comparison of forces on the dummy from upper door lower door and car. p : padded, u : un-padded

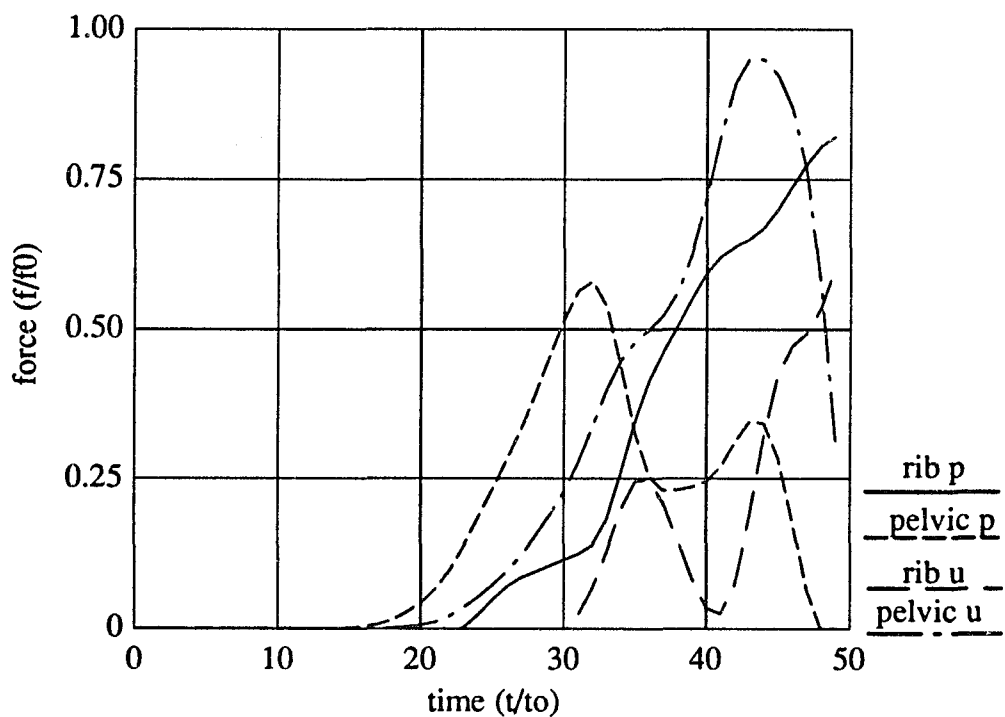


Fig 6.16 Comparison of forces on the spine from the rib and pelvis
p : padded, u : un-padded.

6.6 Application in design

In this section the application of the above estimation procedure in an actual design environment is presented.

Consider a car in the early phases of design. A car design intended for production, called 'case A' was tested to evaluate the side impact performance. The test results indicated need for improvements. As a result reinforcements were added on the side structure to increase the lateral stiffness and the door interior was redesigned to lower the force carrying capacity. The side impact test of the modified car called 'case B' indicated improvements in the occupant injuries (see Figure 6.21). The details of the physical changes in the modified car are presented in Figure 6.17(a) and 6.17(b). An approximation of the change in the structural stiffness due to these modifications was estimated from the test data by comparing the energy lost in deformation during the impact and is presented in Figure 6.17(c). Change in the door interior stiffness was estimated by evaluating the force-deflection characteristics of the door interiors on a static crusher. The results are compared in Figure 6.17(d).

It is desirable to optimize the use of the available data by estimating the contribution of each modification in improving the occupant protection from these two tests. That is we want to predict the performance for the following two test configurations,

Case C : car with only a softer interior

Case D : car with only a stiffer structure

The measured accelerations are presented in Figure 6.18 and the derived input data are presented in Figure 6.19. The mass distribution is assumed to be the same as in the previous case, i.e. $M_{rib} = 16.8$ kg, $M_{plv} = 39.2$ kg, $M_{spn} = 28$ kg. Using the model structure presented in Figure 6.4(b) and the available test data the parameters (K_1 to K_5) are estimated using KF estimator. While estimating the parameter for case A, it

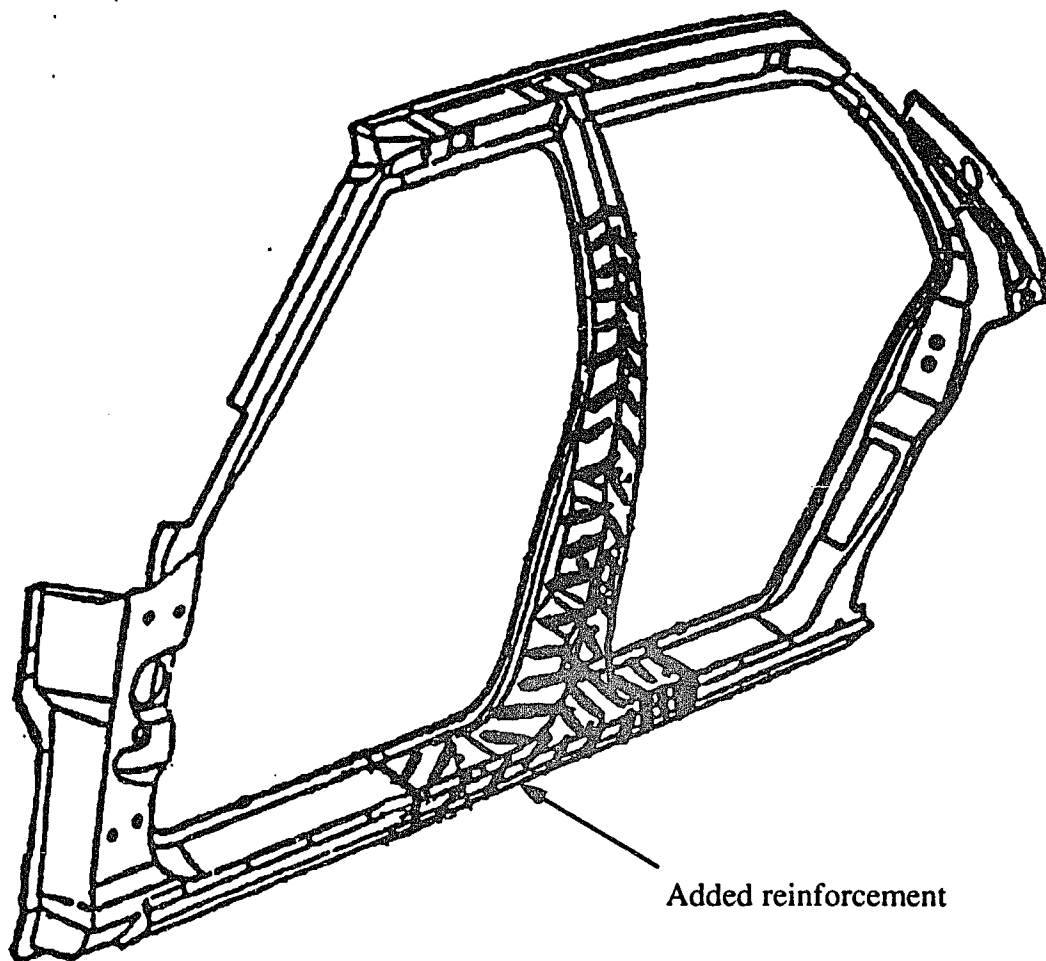
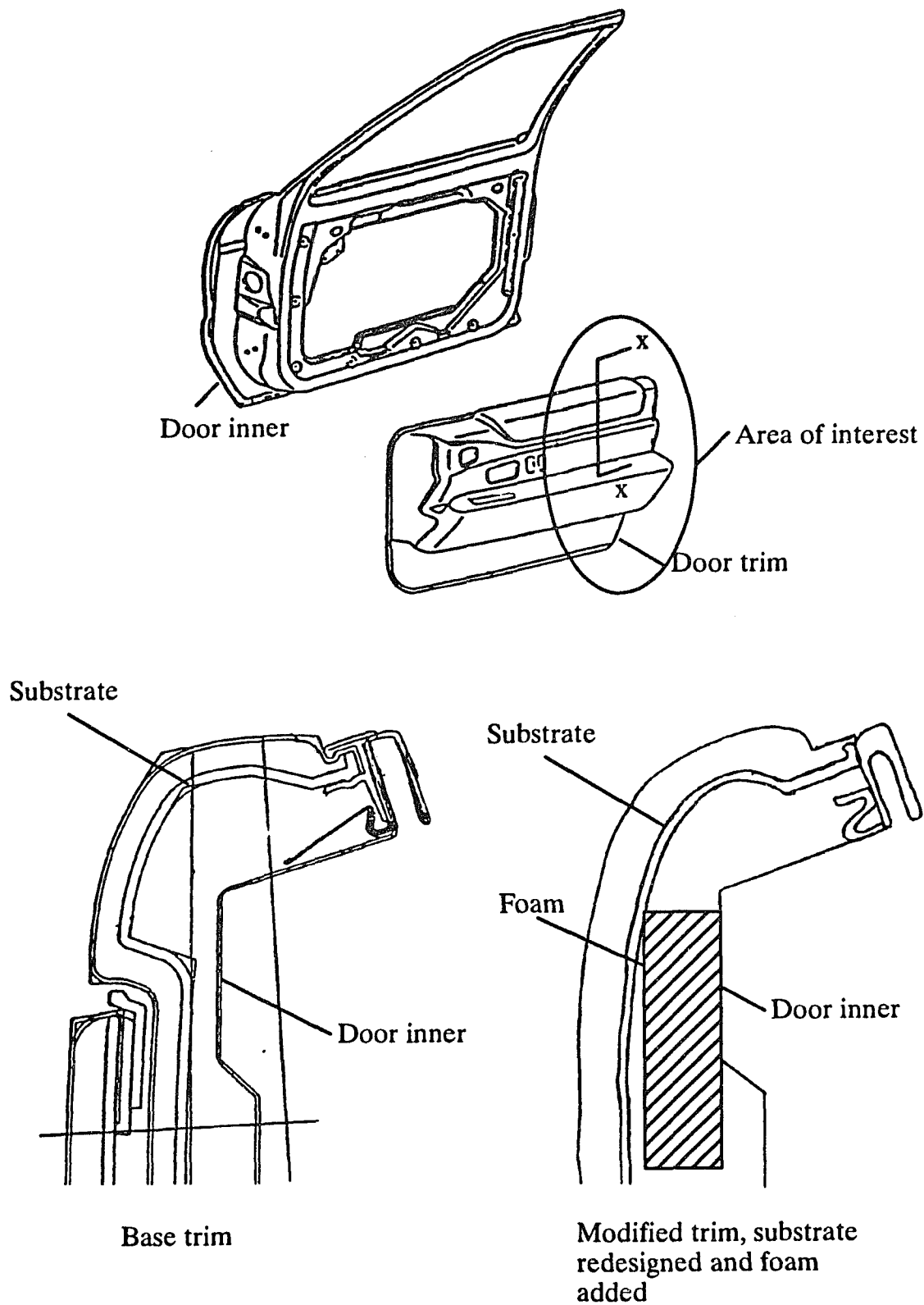


Figure 6.17(a) Detail of structural changes in the modified car



Section X – X through door trim

Figure 6.17(b) Detail of trim changes in the modified car

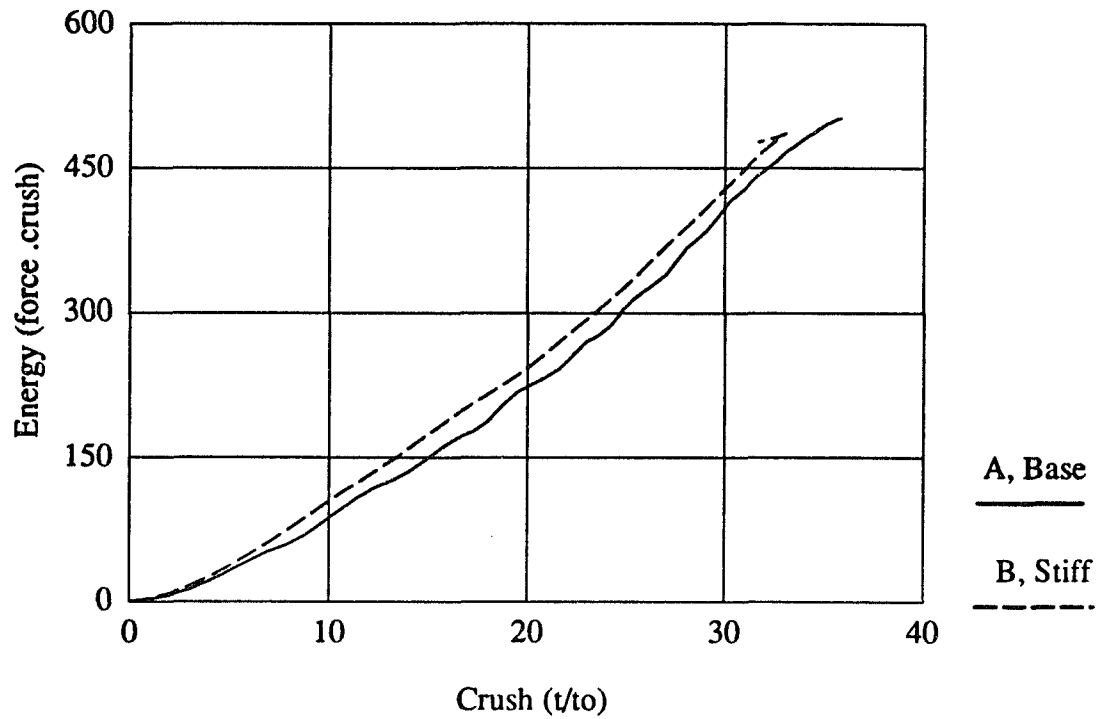


Fig. 6.17(c) Comparison of estimated structural stiffness of side structure for base car and modified car

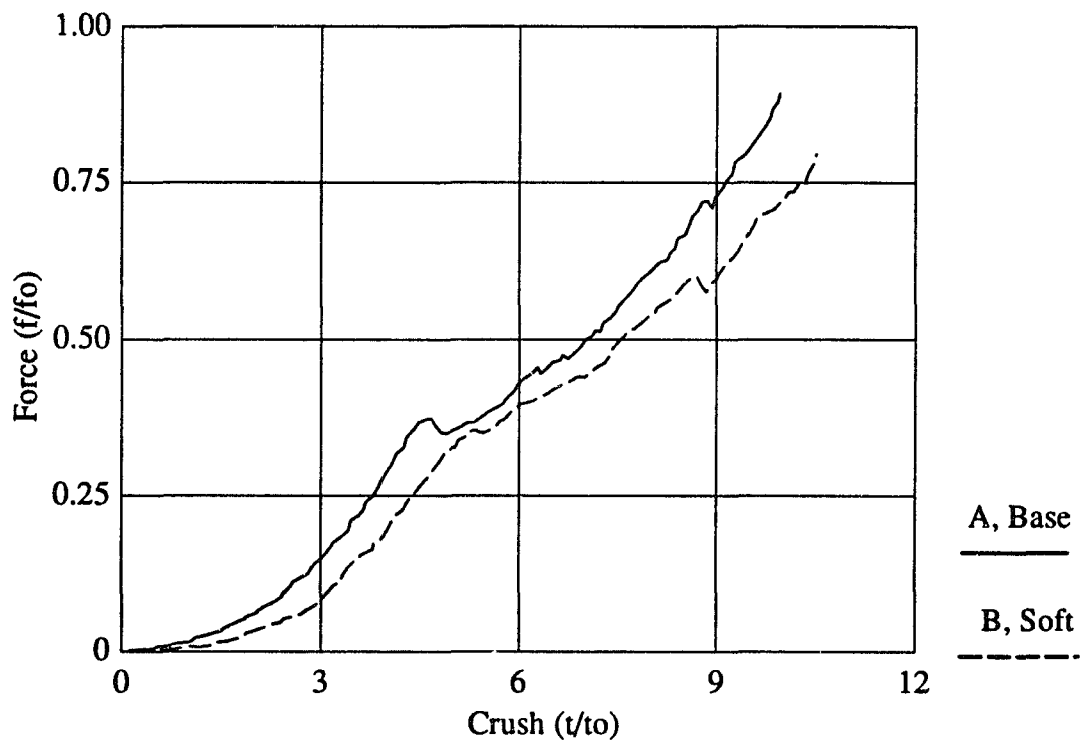


Fig. 6.17(d) Comparison of estimated interior stiffness of door for base car and modified car

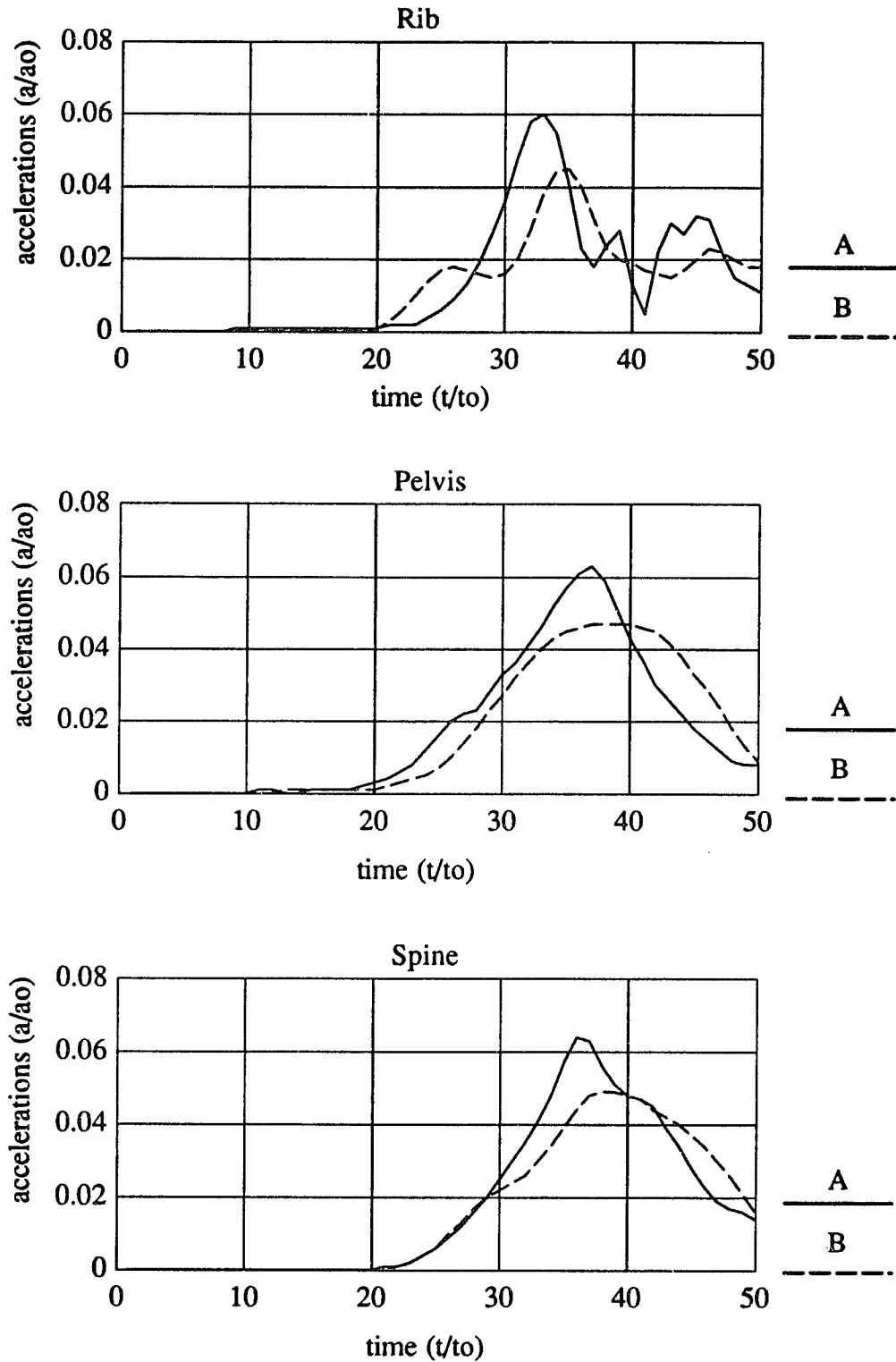


Fig. 6.18 Measured dummy accelerations for test cases A and B

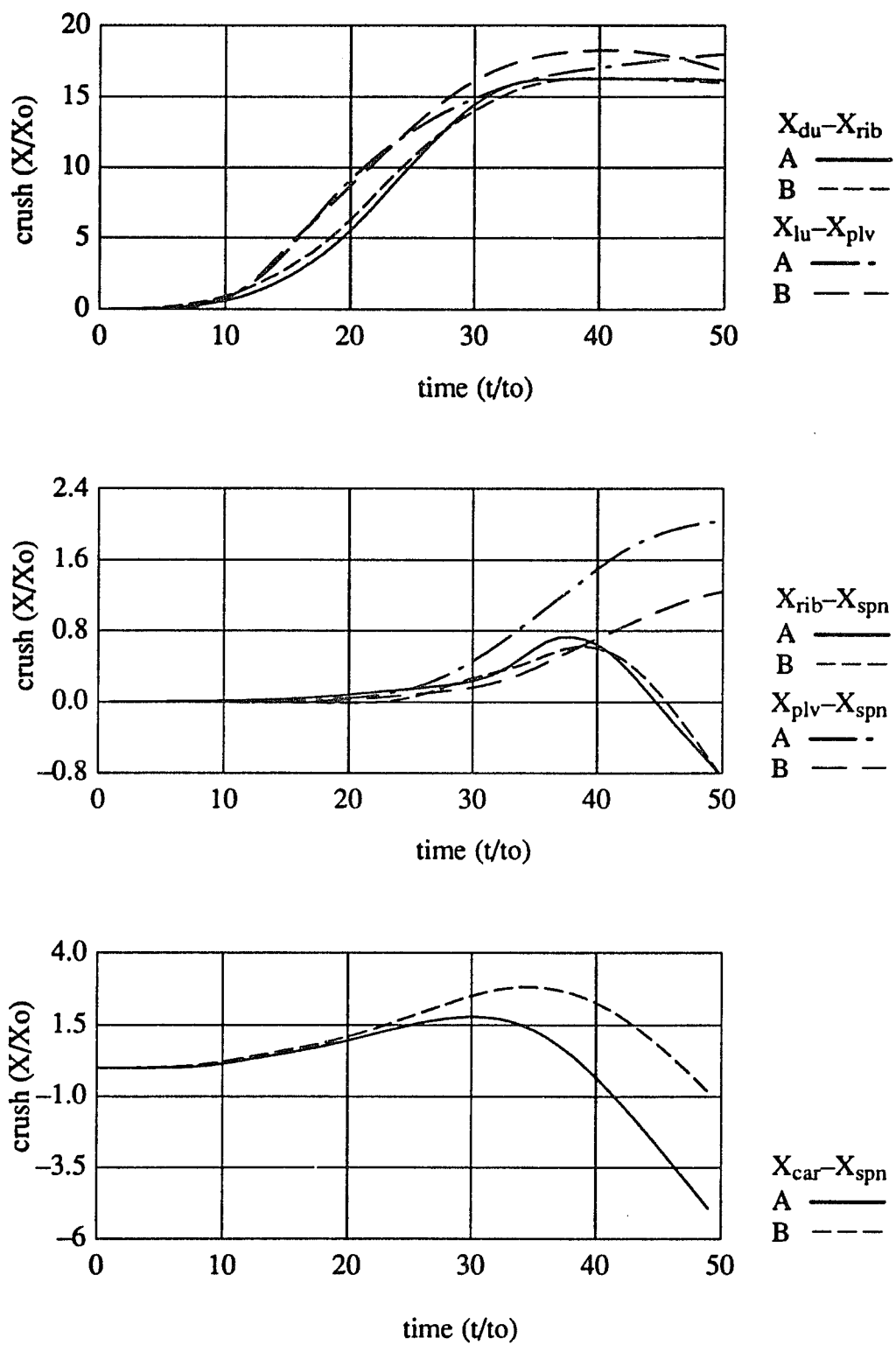


Fig. 6.19 Input used in estimation, derived from measured data for test cases A and B

is assumed that the rib to spine stiffness is known from test case B. The prediction error covariance matrix is presented in Table 6.3. The estimated parameters are presented in Figure 6.20. The softer interior stiffness (K_1 and K_3) for case B correlates with the known information about the car. Next, to check the accuracy of the estimated parameters, simulation is carried out using the estimated parameters. The simulated accelerations are compared with the test measurements in Figure 6.21.

Using the data from tests A and B, the performance of case C and D, is predicted as follows:

Case C : The forced displacements are from test A (representing base structure) while the interior parameters are from the test data B (representing softer interior).

Case D : The forced displacements are from test B (representing stiffer structure) while the interior parameters are from test data A (representing base interior).

The predicted dummy accelerations for case A,B,C and D are compared in Figure 6.22. The prediction indicates that, for a given car and given set of modifications, making the interior softer is more effective in protecting the occupant than making the structure stiffer.

Table 6.3 Covariance of prediction error

	$\times 5.68 \times 10^{-9}$					
	Test Case A			Test Case B		
	e_1	e_2	e_3	e_1	e_2	e_3
e_1	0.2525	0.2463	0.0813	0.1030	0.0812	0.1295
e_2	0.2463	0.9491	1.1173	0.0812	0.2297	0.5340
e_3	0.0813	1.1173	5.9103	0.1295	0.5340	6.6654

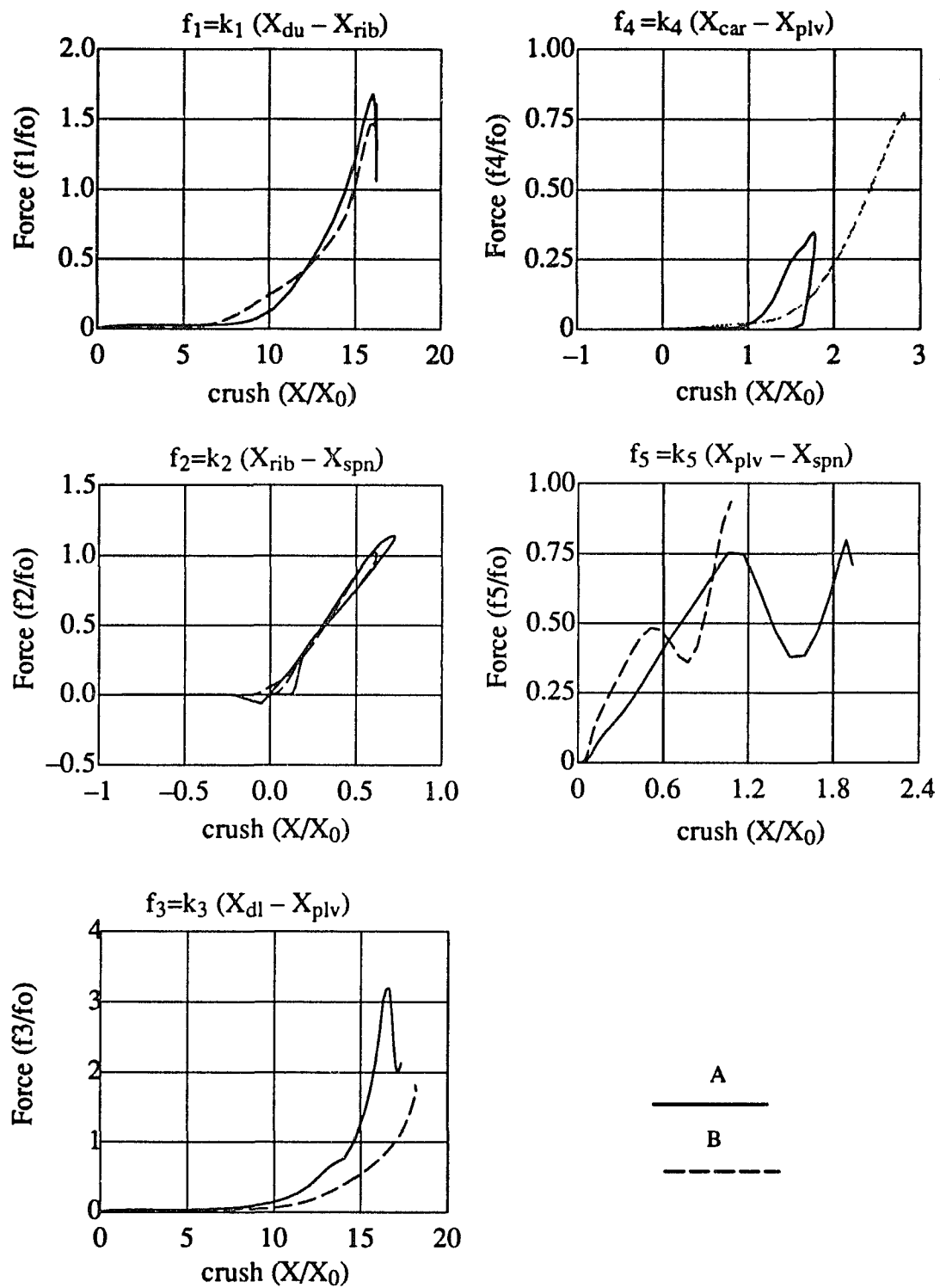


Fig. 6.20 Estimated parameters for test cases A and B

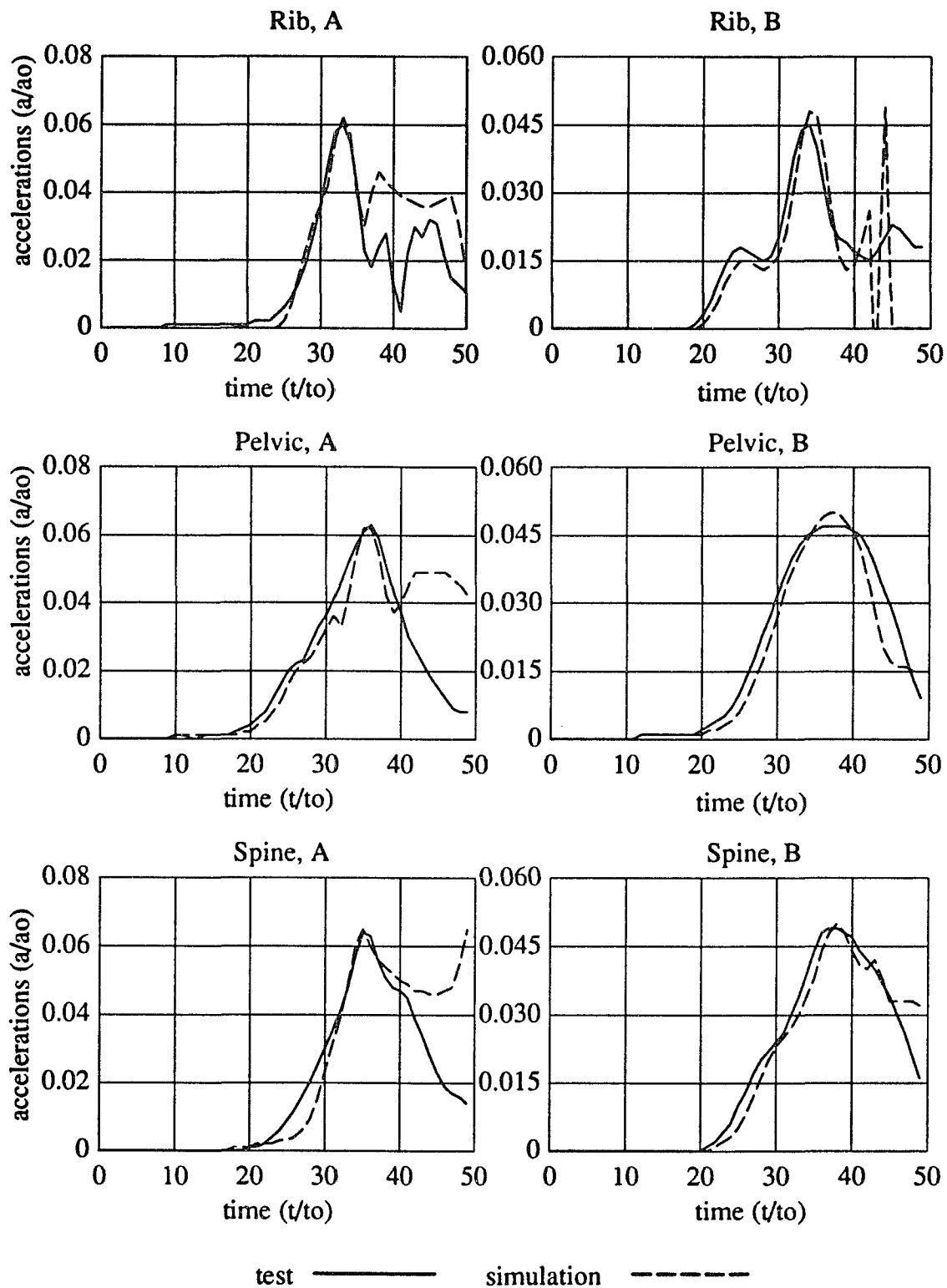


Fig. 6.21 Comparison of simulated acceleration response with measured test data
A : Base car, B: Base car with stiffer structure and softer interior.

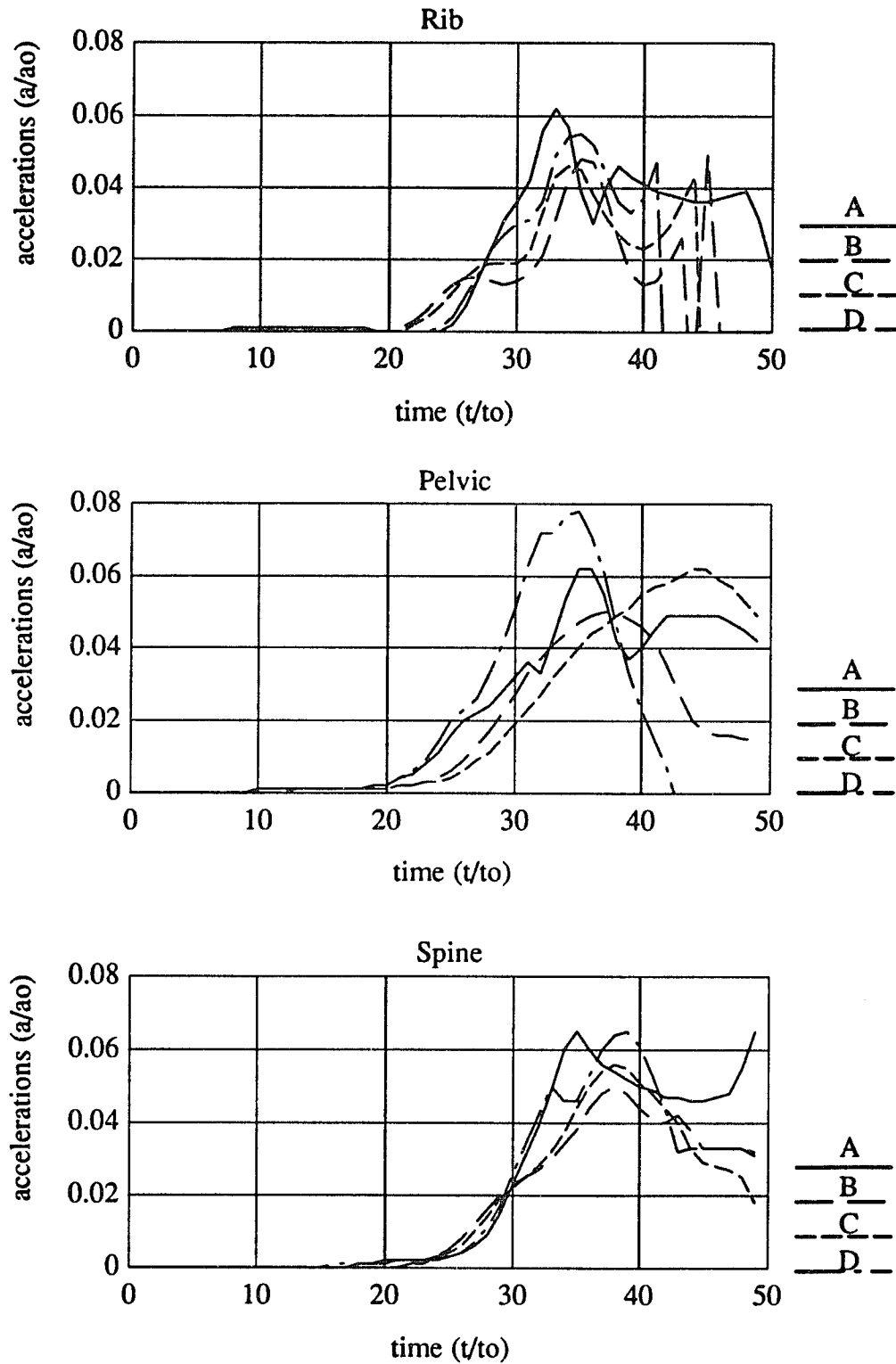


Fig. 6.22 Prediction in side impact for test case A,B,C and D, where
A : Base car B : Both interior and structure Modified
C : Only interior modified D : Only structure modified

It is desired to verify the above predictions with actual tests. No test data with configurations cases similar to C or D are available. However a test with base structure and softer interior (not with the same characteristics as case B, but similar) is available. The accelerations measured in this test are compared with the case D in Figure 6.23. The correlation is good, which indicates the usefulness of the proposed modeling technique.

6.7 Chapter summary

1. A physically meaningful lumped parameter model for the structure to occupant interface in side impact was developed. Study of the energy distribution indicated that the door velocity is not significantly affected by the interaction between the door and dummy and therefore the structure and dummy model can be separated.
2. The lumped parameters in the model were directly estimated from the crash test data and physical understanding of the system, using a Kalman filter estimator in a constrained environment. The constraint on the Kalman filter were based on physical insight into the system, e.g. knowledge that the loading/unloading, tension/compression characteristics of the structure were different.
3. The data based modeling technique was verified using simulated data with the added noise. For further verification, the model parameters were estimated from a number of actual test measurements, and then the model simulation using these estimated parameters was compared with actual test measurements. A good correlation with both the above verification efforts and the physical meaningfulness of the estimated parameters indicated the validity of the proposed estimation technique.

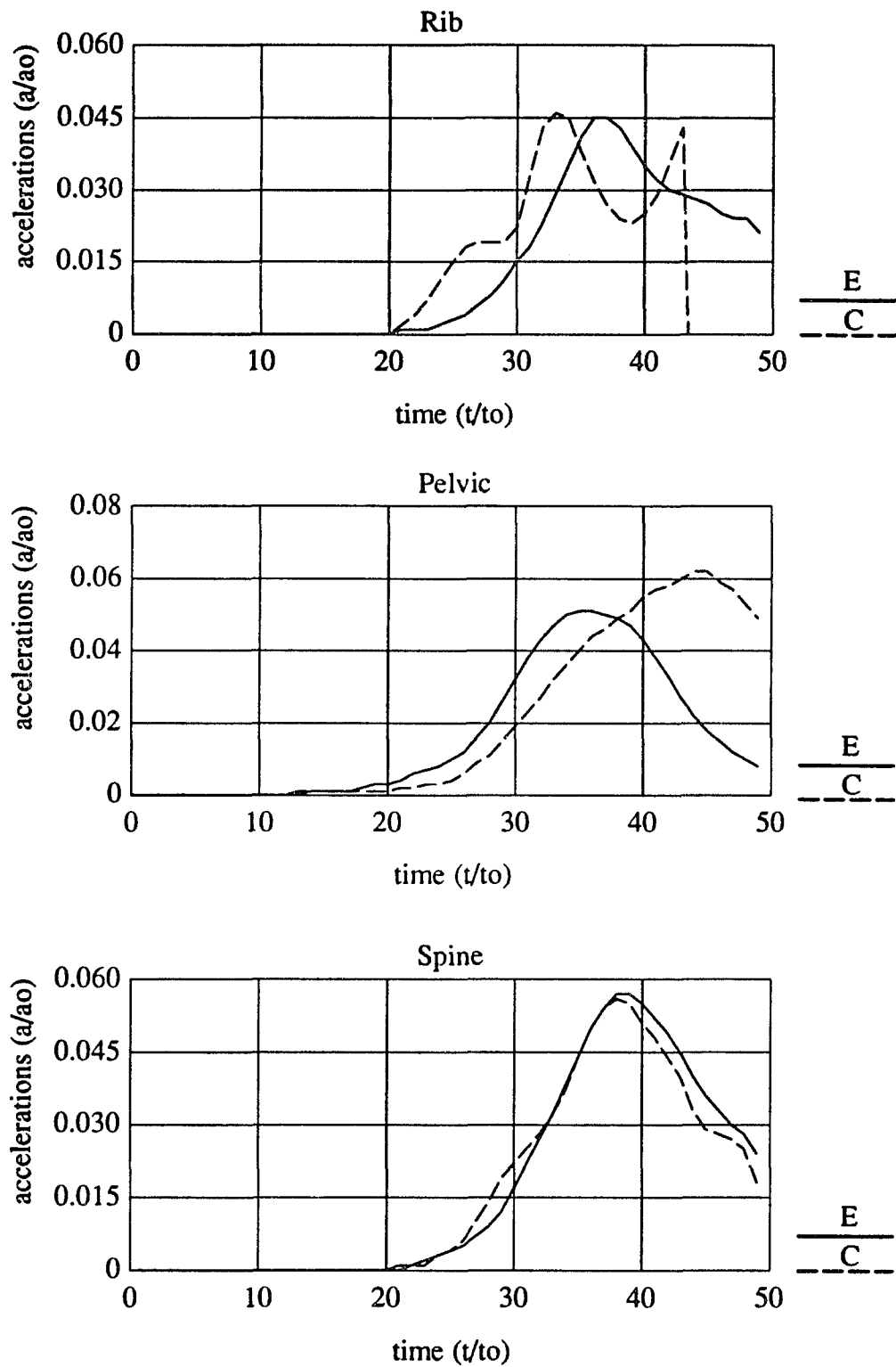


Fig. 6.23 Comparing prediction of car with modified interior with the test case E, i.e. a car with similar (but not the same) content.
 E : Test data C : Only interior modified - prediction

4. The data--based lumped parameter model were shown to be very useful in,
 - a) Understanding the crash event, i.e. the contributions of different components in overall crash performance.
 - b) Predicting the crash performance of the dummy under different design conditions.

CHAPTER 7

CONCLUSIONS AND RECOMMENDATIONS

7.1 Summary

Occupant protection during an automobile crash is an important design consideration. Extensive tests and analysis are performed during the early phases of design to improve desired crashworthiness properties in automobiles. The purpose of this thesis was to investigate methods to develop physically meaningful analytical models directly from crash test measurements, and then demonstrate their use in predicting crash performance, as well as in analyzing the crash test data.

To build a meaningful model it is important to understand the physics involved in the crash event. Therefore the major mechanisms involved in the crash event were identified and a comprehensive overview of testing and analysis techniques currently used in crashworthiness engineering were presented. Based on the overview of the current status of crashworthiness engineering, the need for physically meaningful analytical models from the readily available test data was established. These types of models are also called data based models.

The data based models proposed for crash simulation were made up of two parts, i.e. lumped parameters representing the rigid body response of the system and a transfer function model representing the vibration response of the system. To use the data based models in developing design directions the lumped parameters in the models were correlated to the physical characteristics of the automobile structure. The masses are assumed to be lumped at the location of the acceleration measurement and the stiffness and damping characteristics were assumed to be acting along the line

joining the two accelerometers or lumped masses. The parameters were estimated by minimizing quadratic criterion of one step ahead prediction error using a least squares or Gauss–Newton method. The time varying nature of the parameters was addressed using a recursive parameter estimation approach.

Modeling an automobile crash event is quite complicated due to extensive interactions of various components. In addition the estimation of the model parameters is not easy due to non-linear structural characteristics, presence of noise in the measurements and a large number of unknown parameters. These problems in development of the data based models were addressed by simplifying the model structure, using the physical insights of the event and by modifying the estimators. The purpose of simplifying the model structure was to improve estimation by reducing the unknown parameters. The physical insights, which essentially are the understanding of the crash event, are discussed in Chapter 2. This knowledge was useful in modeling and in selecting initial values of parameters, noise characteristics etc., for the estimator. The parameter estimation algorithms were modified based on the understanding of the complexities in the event. Well known recursive least squares (RLS) is modified to address the problems in selecting a forgetting factor to estimate changing parameters and in the presence of noise the improved RLS algorithm is called modified recursive least squares (MRLS).

The understanding of crashworthiness engineering, the generalized approach to develop physically meaningful data based models and the estimation tools discussed above, together provide a comprehensive package that is very helpful in using a data based model in crashworthiness analysis. To demonstrate use of the proposed approach in design, a data based model for a side impact analysis was developed from the crash data. The purpose of the model was to correlate the structural measurements with the occupant injury. The model parameters were estimated using a KF estimator

with the modifications to include the known information. Use of this model in predicting the test performance and analyzing the data was also demonstrated.

Finally, the limitations of the approach must be recognized. First, data from at least one test are necessary to develop data based models discussed in this thesis. This may not be possible in the early phases of design. Second, the data based models are intended to compliment the finite element (FE) models. This is because the FE models are 'ground up' models and hence are capable of representing geometrical and material details of the automobiles.

7.2 Conclusions

1. A data based analytical model with physically meaningful parameters can be developed for crash simulation from the readily available crash test data.
2. The unknown parameters in the model can be estimated by minimizing the one step ahead prediction error.
3. Recursive estimation methods can be used to address the time varying nature of the parameters.
4. Complexities in the crash event, changing parameters and presence of noise are important problems that need to be addressed in the development of data based models.
5. The physical insights such as initial values, approximation of covariance of noise and knowledge of the structural characteristics (e.g. difference in loading/unloading and tension/compression characteristics etc.) are helpful information in estimating the model parameters.
6. The modified recursive least squares (MRLS) estimator, which is a modification of the recursive least squares (RLS) estimator is helpful in estimating time varying parameters in the presence of noise.

7. A side impact model developed using a KF estimator is found to be useful in understanding the crash performance as well as in crash prediction.

7.3 Future work recommended

- 1) Develop models with increased complexities – with greater numbers of parameters and more detailed representations of the automobile components. Then estimate parameters using physical insight into the event.
- 2) Apply similar identification procedure on other systems, e.g. ‘in test’ dummy parameters.
- 3) The effect of sampling was checked on a very simple model where the parameters are not changing very rapidly – further investigation on more complex models is recommended.
- 4) Pre-filtering of the test data was avoided – however as the major parameters of interest are not affected by high frequencies, estimation from filtered data can be considered.
- 5) The loading and unloading characteristics of the structural parameters are different. Currently they are addressed by assuming the unloading part to be known and constant. Estimation of the parameters in the unloading part can be considered.
- 6) To reduce the number of unknown parameters the contribution of damping in some of the cases was not included. Once the stiffness parameters are identified, damping parameters can be estimated by using the known stiffness parameters.

The following are specific to the GM environment :

- 7) The covariance of measurement noise for the KF estimator was assumed to be constant and selected on the basis that the residue was minimized. The test data from a number of tests are readily available at GM. A time varying variance of measurement noise can be developed from these data. This will be very useful information in any future estimation analysis.
- 8) The transfer function part of the model represents the vibratory response in the measurements. Comparison of the transfer function part of the two identical tests for similar cars is expected to be helpful in identifying structural dissimilarities and hence the source of discrepancies between the two tests.

Appendix

Normalizing of test data :

The test data used in this thesis has been normalized. The normalization is carried out as follows.

$$\begin{aligned}d_n &= d_m/d_o & (\text{displacement}) \\v_n &= v_m/v_o & (\text{velocity}) \\a_n &= a_m/a_o & (\text{acceleration}) \\k_n &= k_m/k_o & (\text{stiffness, defined as, force} = k_n \times d_n)\end{aligned}$$

Where subscript

$$\begin{aligned}n &: \text{normalized} \\m &: \text{measured} \\o &: \text{normalizing factor}\end{aligned}$$

The relationship among d_n, v_n, k_n and a_n can be established as follows,

$$\begin{aligned}V_n &= \Delta d_n / \Delta t_n = d_{n1} - d_{n2} / (t_{n1} - t_{n2}) \\&= d_{m1} - d_{m2} / (t_{m1} - t_{m2}) (t_o / d_o) \\&= v_m (t_o / d_o)\end{aligned}$$

$$\text{Therefore, } v_o = d_o / t_o$$

$$\text{Similarly } a_o = v_o / t_o = d_o / t_o^2$$

$$K_n = K_m / K_o = (\text{mass} \times a_m / d_m) / K_o = (\text{mass} \times a_n / d_n) / K_o (a_o / d_o)$$

$$K_o = (a_o / d_o) = 1 / t_o^2$$

$$\text{and } C_o = 1 / t_o$$

Where

v_o = initial velocity of impact.

t_o = time between two consecutive samples or sampling interval.

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