TENSILE STRENGTH AND SHEAR STRENGTH OF ICE

June 4, 1948 John M. Horeth

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by

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INTRODUCTION

The following report is a presentation of experiments conducted for the purpose of determinning tensile and shear strength of natural and "artificial" ice, in a cold room of the Engineering School of the University of Michigan, Ann Arbor, Michigan and aboard the U. S. Coast Guard Cutter Mackinaw. The experiments were carried out at the request of the U. S. Coast Guard. Extensive research into the literature showed that even though much is known about the physical properties of individual ice crystals, (N. E. Dorsey[1/]), only little is known about the strength of ice in tension and shear. The only extensive data available is that of E. Brown[2/] and John N. Finlayson[3/] who determined the tensile strength and shear strength of natural ice respectively by means of experiments similar to those of the writer.

The writer wishes to express his appreciation to Dr. James T. Wilson of the University of Michigan for giving the writer the opportunity to conduct the experiments and for his kind help and encouragment. The writer also wishes to thank Mr. Richard Strong of the University of Michigan for his information on available literature.

SUMMARY

Experiments were carried out in a cold room of the Engineering School of the University of Michigan, Ann Arbor, Michigan, and aboard the U. S. Coast Guard Cutter Mackinaw to determine the tensile and shear strength of ice when subjected to loads acting parallel to the direction of the crystal axes of the ice. Ice specimens for these two sets of tests were obtained by freezing water in a box whose inside walls were lined with excelsior or sponge rubber padding which absorbed the expansive forces of the water upon freezing. Other ice specimens were obtained from ice cakes picked up from the Mackinac Straits region of Lake Michigan. All ice cakes, both at Ann Arbor and on the

Coast Guard Cutter Mackinaw were cut into specimens by means of an eight point carpenter's rip saw. A miter box was used for sawing up ice into shear test specimens. Testing of ice beams for tensile strength was accomplished by subjecting the beams to bending forces in a simple test apparatus. The apparatus consisted of two I-beams connected by four tie rods; one I-beam being mounted on a saw horse for steadiness, and also for placing the system in an upright position. A hydraulic jack was placed and secured on the lower I-beam. A crosshead was fixed to the ram of the jack and the ice beam layed upon it. The ram was then raised until the ice beams rested against two wooden blocks fastened to the under side of the upper I-beam and appropriately spaced. The ice beam was then centered and tested.

For shear test experiments the hydraulic jack was removed and a shear box fastened to the lower I-beam in its place. The ice specimen, with its crystals vertical, was inserted into this box, a piston placed on top of it and a hydraulic jack set on top of the piston. The system was centered and the ice specimen tested, the ram of the jack acting against the underside of the upper I-beam.

From these experiments it was found that the tensile strength of ice increases with decreasing temperatures below 32°F. At 32° F the average tensile strength of ice was found to be 180 p.s.i. The average shear strength of ice was 98 p.s.i. Experimental evidence indicates that this average value does not vary with temperature.

Studies of thin sections of Lake Michigan ice revealed that the crystals of the ice were approximately perpendicular to the surface of refrigeration and that the size of the crystals increased with depth. The largest crystals measured were 1.5" in diameter and the smallest 0.1" in diameter. Comparison studies of thin sections of the artificial ice disclosed similar crystal structure.

Methods of Freezing and Preparation of the Test Specimens.

Boxes Employed in the Freezing of the Water: As the freezing of ice in ordinary containers, such as metal trays, would cause the ice to bulge upward and crack in various places, due to a three percent linear expansion of the ice upon freezing, it was necessary to find a method of freezing either tap or distilled water so that the ice obtained from it would resemble as closely as possible natural lake ice both in crystal structure and strength.

The problem was readily solved by constructing a wooden box out of 1 1/8" thick maple and whose inside dimensions were 50" x 26" x 7". The pieces were both glued and screwed together. In order to absorb the expansion of the ice, the inside walls of the box were lined with excelsior three inches thick, see Figure 1, page 4. To make the whole set-up water tight, a thin rubber sheet was fastened to the edges of the box and allowed to completely cover the bottom and excelsior lined walls of the box. This method proved highly satisfactory, for no leakage could be detected and the excelsior took up the expansion of the ice quite readily.

The ice cakes obtained by freezing water in this box were marked by smooth flat surfaces and no flaws or cracks could be detected.

To facilitate easy removal of the ice cakes from the box, which, on the average, were four inches thick, handle bars were introduced into the water along the long edges of the box and allowed to remain in the water while it was freezing as is shown in Figure 3, page 5.

During later stages of the experiments it was decided to change to smaller specimens. Another box was therefore constructed, again from 1 1/8" thick maple. The inside dimensions of this box were 20" x 10" x 4". In this case, however, the long inside walls were lined with ½" thick sponge rubber padding, while each end wall was lined with two ½" thick rubber paddings, see Figure 2, page 4. Again a thin rubber sheet was allowed to cover both the

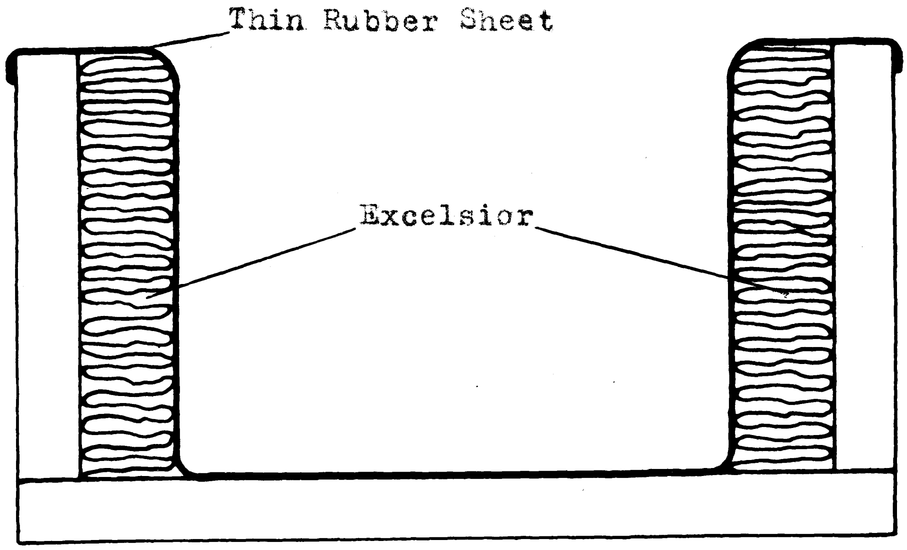


FIGURE 1. CROSS-SECTION OF LARGE BOX

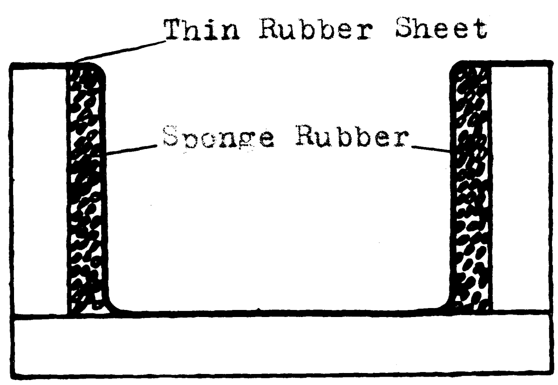


FIGURE 2. CROSS-SECTION OF SMALL BOX

bottom and the rubber padding, the rubber sheet being fastened to the box by means of thumb tacks. To remove the ice cake, it was only necessary to remove the thumb tacks, than the rubber paddings and lift the cake out of the box.

FIGURE 3. BOX WITH HANDLE BARS IN PLACE

Preparation of the Specimens: To obtain specimens for bending tests, the cakes obtained from the large box were sawed up into beams measuring 36" long, 3" wide, and averaging 3.5" in thickness. In every case the crystals were perpendicular to the surface of refrigeration. Both the top and bottom surfaces of the beam represented the original surfaces, the beam being smoothed only when necessary. To saw out beams from the cakes obtained from the smaller box, a miter box was constructed and the beams thus obtained measured 17" long, 2½" wide, and 2½" thick. In this case only the top surface was left unaltered, the bottom surface representing a sawed out surface.

All shear test specimens were sawed out in a miter box specifically constructed for this purpose. They were 5" long, $2\frac{1}{2}$" wide, and $2\frac{1}{2}$" thick.

In every case, an eight point carpenter's rip saw was employed. Few difficulties were encountered as far as splintering or chipping of the ice was concerned, and the sawed surfaces were very smooth.

Determination of the Tensile Strength of Ice.

Description of Test Apparatus: To subject the ice specimens to bending tests, the apparatus shown in Figure 4 was constructed. Two four inch I-beams

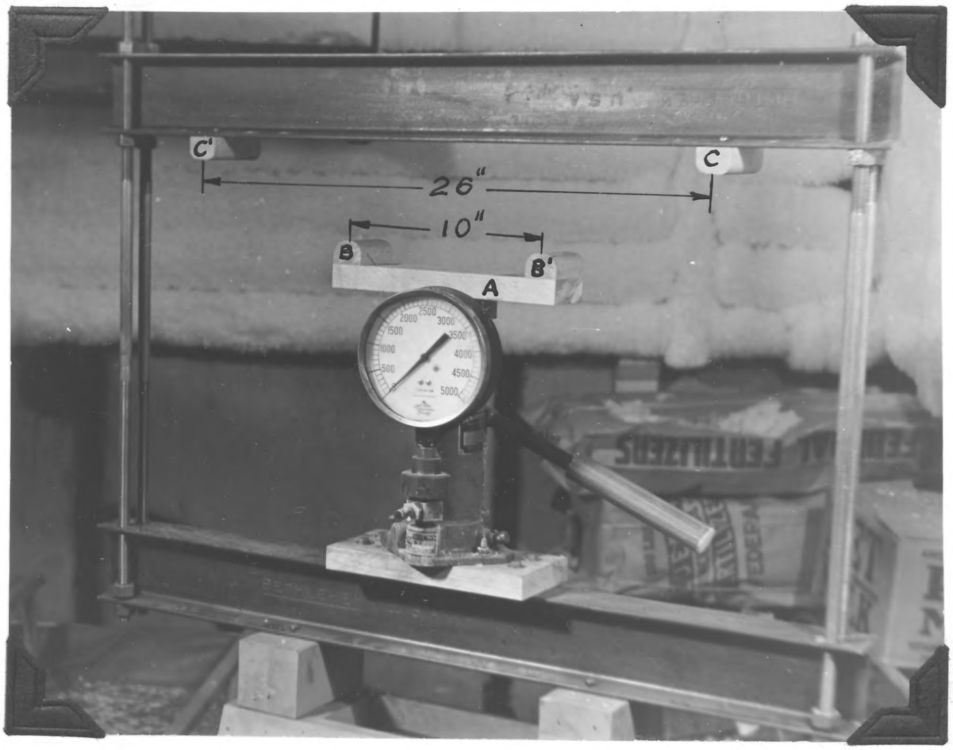


FIGURE 4. TEST APPARATUS FOR BENDING TESTS.

were connected by means of four 5/8" tie rods which were threaded most of the way to facilitate easy parallel alignment of the I-beams. One I-beam was then fastened to a support so that the system would be in the position as shown in Figure 4. A hydraulic jack, capable of delivering forces up to 5000 lbs. was securely fastened to the lower I-beam at its center. For later experiments, when smaller specimens were tested, a jack capable of delivering up to 2000 lbs of force was used.

To subject test beams to tension forces, cross-head A, see Figure 4, was fitted over the ram of the jack and equipped with wooden blocks B and B' whose top surfaces were rounded so as to give a dull single line support. Blocks C and C', being of the same size and shape as the blocks B and B', were mounted on the under side of the upper I-beam and spaced as shown. All blocks were centered relatively to the ram of the jack.

Testing Procedure: The ice beams were tested by placing the beam, with its crystals vertical, on the blocks B and B' of the cross-head A, see Figure 5. The cross-head was then raised until the upper surface of the beam

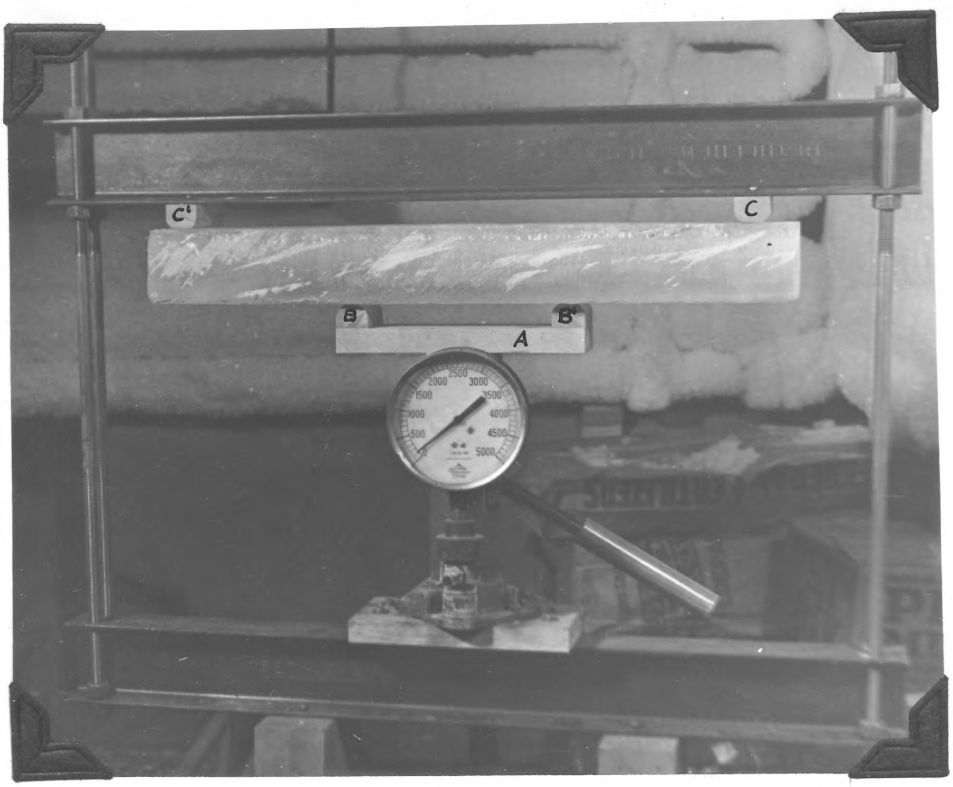


FIGURE 5. TEST APPARATUS WITH ICE BEAM IN PLACE BEFORE TESTING.

would rest against blocks C and C'. The beam was then centered with respect to the ram, making sure that the long edges of the beam would be perpendicular to the blocks. Once centered, pressure was gradually increased upon the beam until the beam would fail as shown in Figure 6. The time interval required was approximately 3 minutes. Before testing, the beam was carefully measured for width and thickness. The average dimensions of the large beams were 30" x $3\frac{1}{2}$" x 3". The average dimensions of the small beams were 17" x $2\frac{1}{2}$" x $2\frac{1}{2}$". The thickness of the Lake Michigan ice beams tested varied according to the lake ice thickness. In some cases, the thickness, 10" to 15", represented a

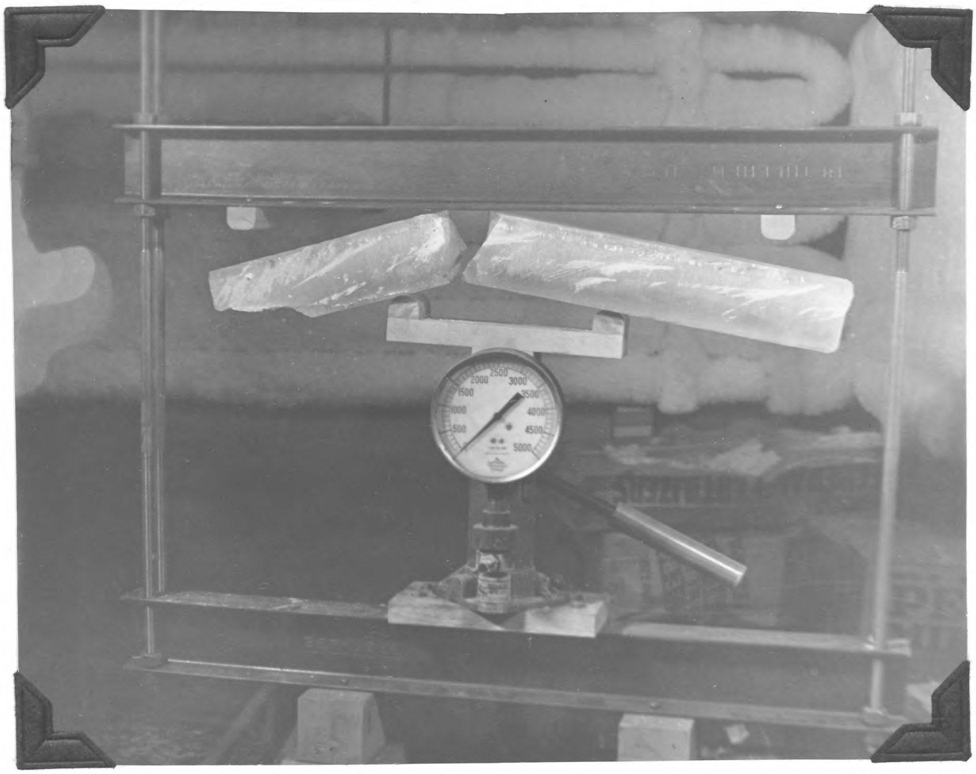


FIGURE 6. TEST APPARATUS WITH ICE BEAM AFTER TESTING.

complete vertical section with about an inch of cloudy ice as the top surface. In other cases, a complete vertical section was cut in half, each specimen then being approximately 6" thick. The length and width of each specimen was approximately 30" and 5" respectively.

Temperature Variations Imposed Upon the Specimens: The only condition imposed upon the specimens was that of temperature variation. Two sets of experiments were made at Ann Arbor; one set of beams was tested at -9° F, the other at 32° F. Since it would have been rather difficult to maintain the temperature of the cold room at 32° F, the specimens were taken into a room at approximately 70° F and allowed to remain there until the surfaces of the ice beam just began to melt. Temperature measurements on separate test specimens showed that when this condition was reached, the center of the beam would then be at 32° F. The time required for the specimen to warm up to 32° F was approximately 30 minutes. Once the specimen had reached this condition, it was taken back into the cold room, measured and tested at once.

The temperatures of the specimens tested aboard the Mackinaw were the same as the air temperature which varied from 25° F to 10° F during the time of testing.

Results of Bending Tests: Results of the bending tests are given in Tables I and II. Table I contains the results of ice tested at Ann Arbor while Table II gives the results of experiments on Lake Michigan ice. Table III presents the results of similar bending tests of St. Lawrence River ice carried out by E. Brown[2/]. The data of Tables I, II and III are shown plotted in Figure 7, page 11.

Table I.

Results of Bending Tests of "Artificial" Ice.

	32° F	-9° F
	126 p.s.i.	226 p.s.i.
	133 "	226 "
	142 "	231 "
	154 "	248 "
	167 "	256 "
	168 "	261 "
	190 "	273 "
	191 "	286 "
	204 "	296 "
	240 "	
	266 "	
Average:	180 p.s.i.	255.5 p.s.i.

Table II.

Results of Bending Tests of Lake Michigan Ice.

20° F	15° F	12° F
150 p.s.i.	220 p.s.i.	183 p.s.i.
181 "	241 "	
	260 "	

Table III.

Results of Bending Tests of St. Lawrence River Ice.

	29° F	15° F
	126 p.s.i.	177 p.s.i.
	133 "	178 "
	134 "	191 "
	143 "	201 "
	158 "	202 "
	177 "	203 "
	193 "	205 "
	197 "	209 "
	200 "	222 "
	210 "	230 "
	229 "	237 "
	306 "	311 "
Average:	184 p.s.i.	214 p.s.i.

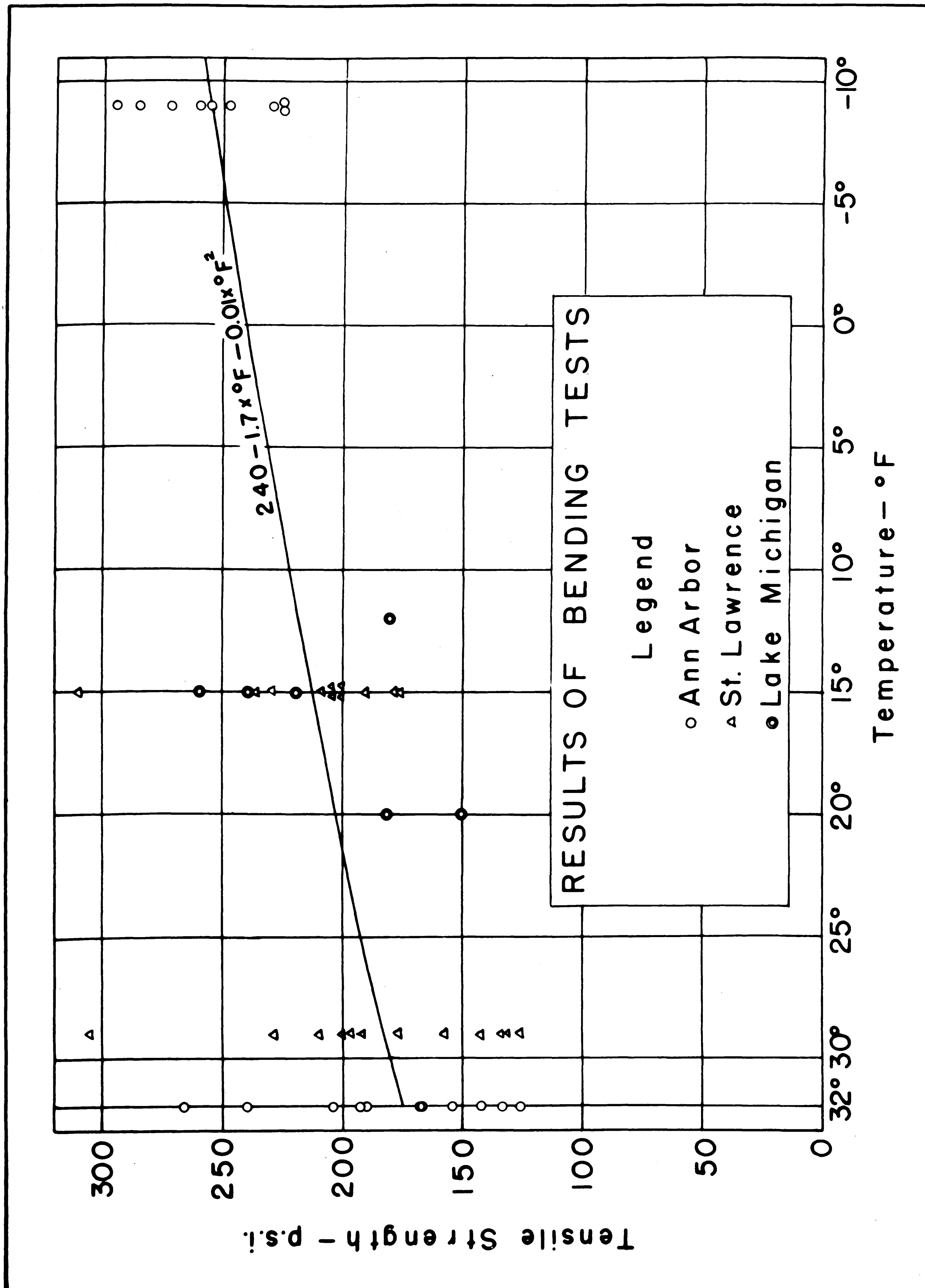
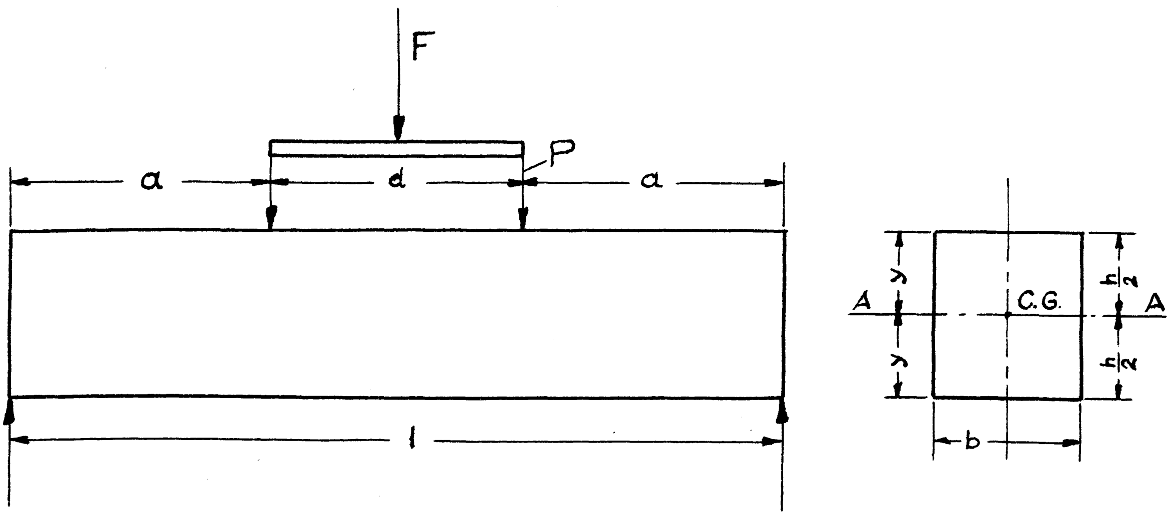

RESULTS OF BENDING TESTS
Legend
o Ann Arbor
△ St. Lawrence
⊙ Lake Michigan
240 − 1.7 × °F − 0.01 × °F²
Tensile Strength − p.s.i.
300
250
200
150
100
50
0
Temperature − °F
32°
30°
25°
20°
15°
10°
5°
0°
−5°
−10°

Figure 7.

To calculate the various values, the following formula was used:



$$M = \frac{f\,J}{y} \text{ pound inches}$$

where M = maximum moment which can be carried by a beam with J inches[4] of moment of inertia and y inches greatest distance from center of gravity to outer fiber, without exceeding an intensity of stress of f p.s.i. in the outer fiber.

$$f = \frac{M_{max}\,y}{J} \text{ p.s.i.}$$

$$M_{max} = Pa$$

$$= \tfrac{1}{2}F\,\frac{l-d}{2} \text{ pound inches}$$

$$J_{AA} = \frac{bh^3}{12}$$

$$y = \frac{h}{2}$$

(1) $$f = \frac{3}{2}\,F\,\frac{l-d}{bh^2} \text{ p.s.i.}$$

<u>Conclusions of Bending Tests</u>: By averaging the results at the different temperatures, it was found that the average tensile strength of an ice beam increases with decreasing temperature according to the following relation:

(2) $$f = 240 - 1.7\,T - 0.01\,T^2$$ where

f = tensile strength in p.s.i.
T = temperature in degrees Fahrenheit

Results of bending tests of the Lake Michigan ice were not used in deriving the above expression. The equation indicates that the slope of the curve plotted in Figure 7 will diminish at low temperatures indicating that a practical limiting tensile strength value of ice at very low temperatures is reached. From Figure 7 it may also be noted that at lower temperatures the tensile strength of ice is much more consistent for individual specimens than at temperatures near the freezing point.

Since all the specimens failed between the blocks B and B' of cross-head A, the values as given in Tables I and II are taken to represent the tensile strength of ice beams with the crystal axes of the beam parallel to the direction of force.

No experiments were made with the crystal axes perpendicular to the direction of force application. Experimental results obtained by Brown show no differences in tensile strength of an ice beam with its crystal axes perpendicular to the force. This is to be expected since in either case the crystals are pulled apart point for point. It may be noted here that the tensile strength of the lake ice falls well within the limits of that of "artificial" ice. The word "artificial" is used rather loosely as the ice tested at Ann Arbor was frozen under conditions which closely resemble natural environments. In particular, the ordinary confining forces of tray ice were eliminated, as has already been shown.

It would therefore be quite reasonable to assume that the curve represented by equation (2) in Figure 7 gives the average tensile strength of natural ice at any one temperature value below 32° F.

Determination of the Shear Strength of Natural Ice.

Description of Test Apparatus: The test apparatus described in the previous section was adopted for shear tests by replacing the hydraulic jack with the shear box shown in Figure 8, page 14. Parts A and A' were separated 3". The shear box and the rectangular piston E were faced with quarter inch thick brass plates D, C, and F, as shown. The edges of plates D served as

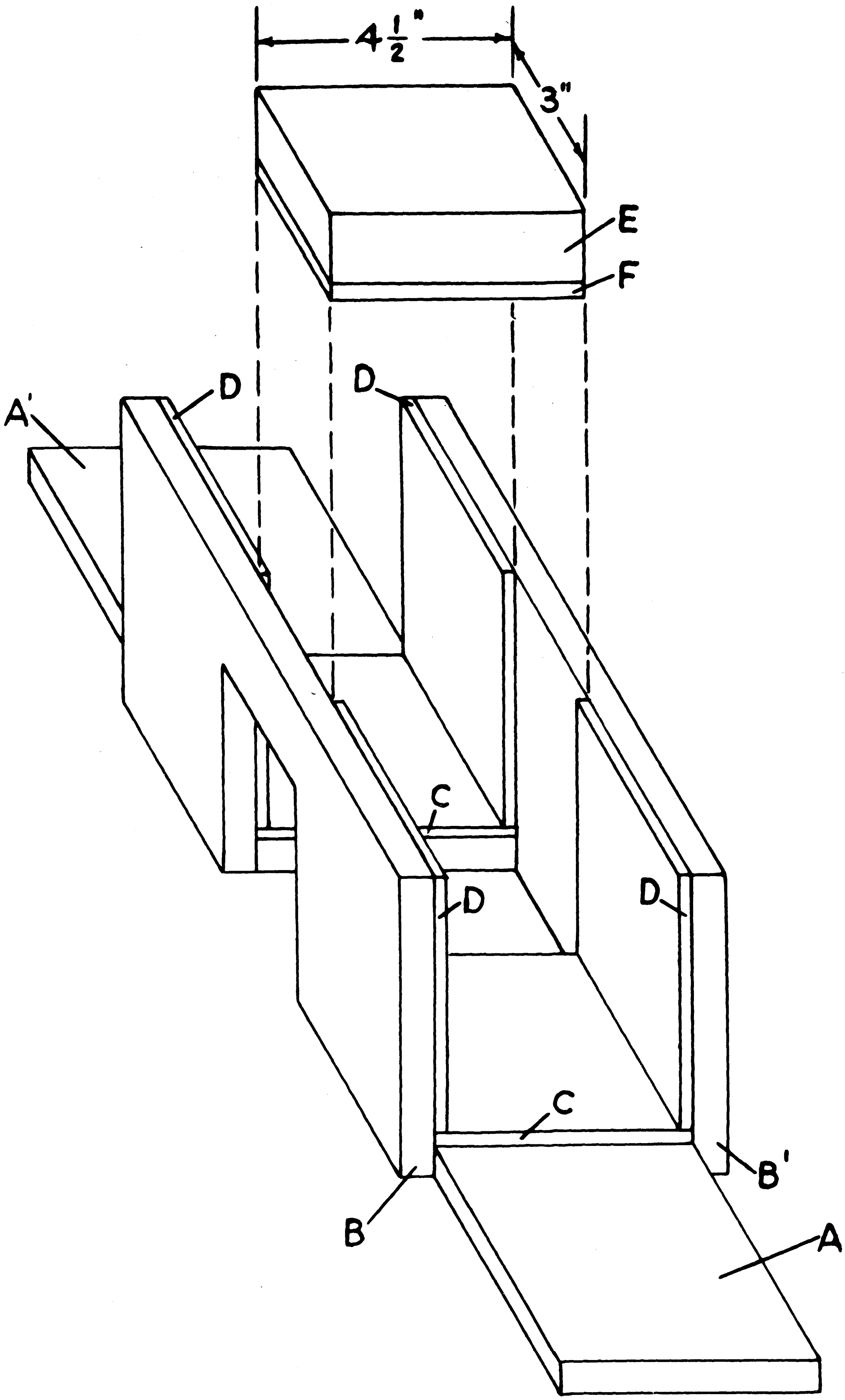


FIGURE 8. DIAGRAM OF SHEAR BOX

guides for the piston E. Approximately 0.003" clearance was allowed between the edges of plates D and the piston E. Side wall B was slotted for observation of the ice specimen during testing.

Method of Testing Ice Specimens: To determine the shear strength of ice, the ice specimens, whose preparation has already been described, was placed in the shear box with its crystals vertical and centered with relation to the opening between parts A and A'. Piston E was then inserted with its brass faced side resting on the specimen. A hydraulic jack capable of delivering up to 2000 lbs of force was then placed on top of the piston, the ram pumped out until it barely touched the upper I-beam, see Figure 9. The ram was

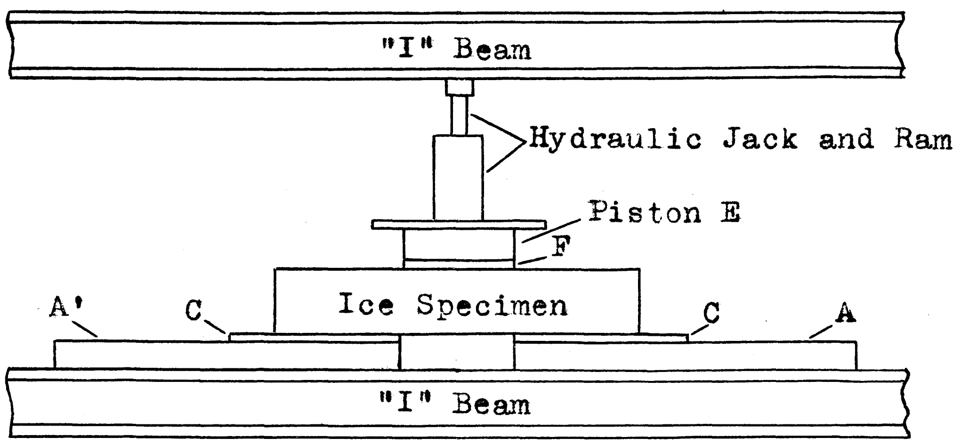


Figure 9. Diagramatic cross-sectional view illustrating the method of testing the ice specimen.

then centered so that it would be perpendicular with respect to the piston and its longitudinal center line would coincide with the vertical center line of the piston. Once the system was adjusted, the specimen was subjected to increasing forces until it failed.

Temperature Variations Imposed Upon the Specimens: As in the bending tests, the only condition imposed upon the specimens was that of temperature variation. At Ann Arbor, specimens were tested at temperatures of 32° F, + 2° F, and - 10° F. To warm specimens up to 32° F, the procedure already outlined was followed. Also, the temperatures of the Lake Michigan ice specimens were again at air temperature and coincided with those of the ice beams.

<u>Results of Shear Test Experiments</u>: Results of the shear test experiments are given in Tables IV and V. Table IV gives the results as obtained from experiments carried out at Ann Arbor. Table V gives the result of experiments on Lake Michigan ice. In addition, Table VI gives the average shear strength values of river ice with its crystals parallel to the direction of application of force at various temperatures as obtained by John N. Finlayson[3/].

Table IV.

Results of Shear Tests of "Artificial" Ice.

	32° F	2° F	-10° F
	65.5 p.s.i.	58 p.s.i.	65 p.s.i.
	85.5 "	59 "	70 "
	88 "	70 "	80 "
	89.5 "	77.5 "	81.6 "
	100 "	88 "	113 "
	102 "	91.5 "	122 "
	103 "	104 "	135 "
	115 "	112 "	153 "
	126 "	116 "	160 "
	161 "	120 "	173 "
Average:	94 p.s.i.	90 p.s.i.	115 p.s.i.

Table V.

Results of Shear Tests of Lake Michigan Ice.

	20° F	15° F	12° F
	*39 p.s.i.	*49 p.s.i.	64 p.s.i.
	65 "	74 "	66 "
	88 "	80 "	92 "
		*81 "	
		82 "	
		86 "	
		103 "	
Average:	64 p.s.i.	79 p.s.i.	74 p.s.i.

*Values starred represent the shear strength of specimens which were taken for the very top of the lake ice and contained about an inch of cloudy ice with no apparent crystal structure and with quite a few air holes.

Table VI.

Average Shear Strength of River Ice Tested by Finlayson

Temperature	Shear Modulus
29° F	101 p.s.i.
28° F	102 "
26° F	98 "
8° F	99 "
5° F	91 "
4° F	94 "
-3° F	101 "

The average shear strength values of Tables IV, V and VI are plotted in Figure 10, page 18.

The following formula was used for calcultaing the various values:

(3) $$S = \frac{3}{4} \frac{F}{bh} \text{ p.s.i.}$$ where

F = pressure indicated on jack gage
b = width of specimen
h = thickness of specimen

Conclusions Reached From the Experiments: The average shear strength values of ice tested at different temperatures did not show any definite relationship between the shear strength and a decrease or increase of temperature. Disregarding the values of the Lake Michigan ice, because of the cloudy ice in some specimens, Figure 10, page 18, shows that the average shear strength of natural ice is 98 p.s.i. at any temperature below the freezing point. This conclusion is in close agreement with that of John N. Finlayson who states that the shear strength of ice is independent of temperature. Table IV brings out the fact that the shear strength of individual specimens near the freezing point fall within a much closer range of the average shear strength value than do those at lower temperatures. However, the writer feels that if the experiments were carried out to a higher degree of accuarcy, a variation of shear strength with a varying temperature would be found; the strength of ice in shear being higher at lower temperatures.

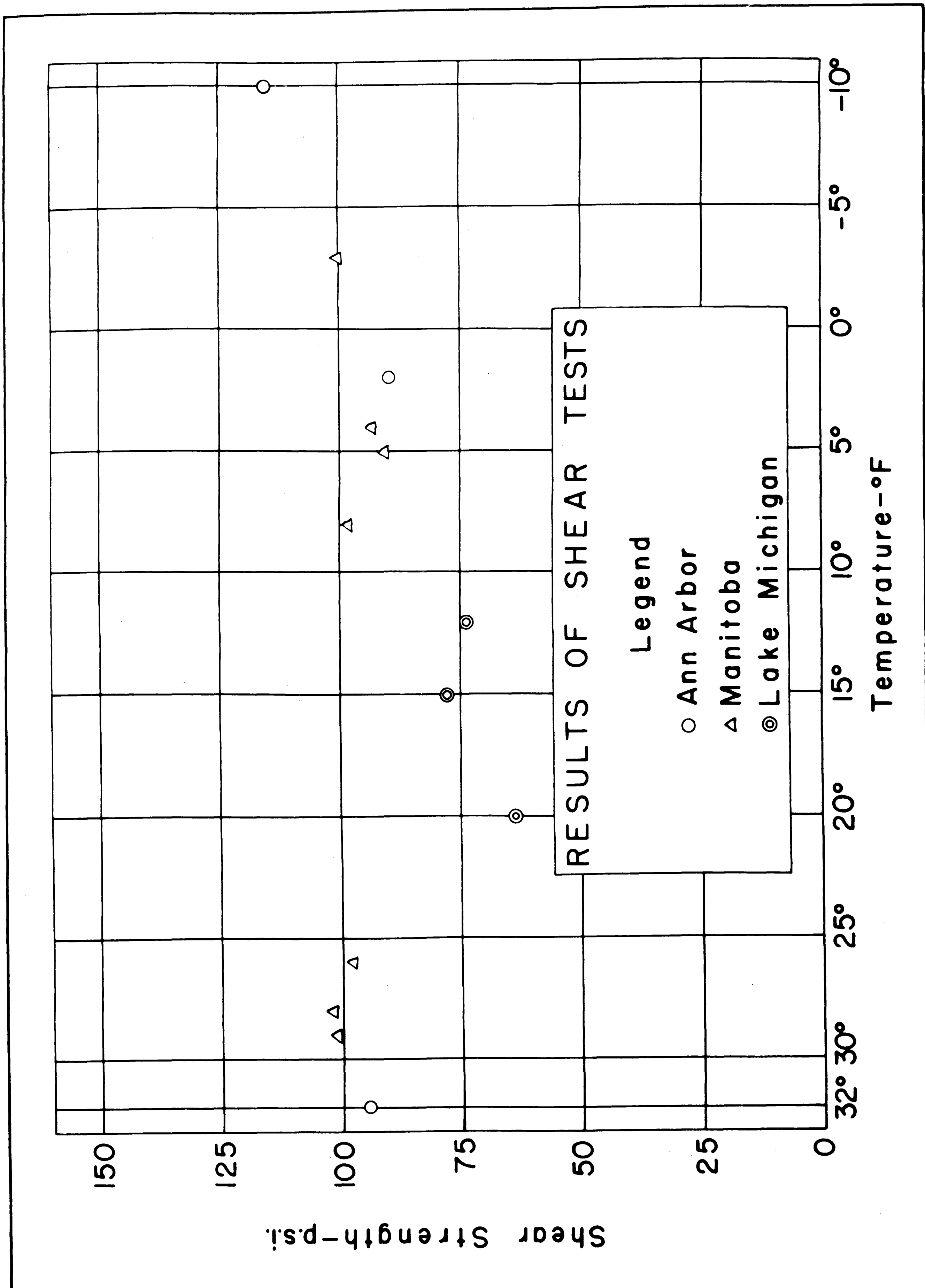

RESULTS OF SHEAR TESTS
Legend
○ Ann Arbor
△ Manitoba
◎ Lake Michigan
Temperature-°F
Shear Strength-p.s.i.
0
25
50
75
100
125
150
32°
30°
25°
20°
15°
10°
5°
0°
-5°
-10°

Figure 10.

Relationship Between Shear Stress and Tensile Stress: The shear stress and the tensile stress to which a fiber of the ice is subjected is as follows:

$$\text{From (1)} \quad F = \frac{2}{3} f \frac{bh^2}{1 - d} \text{ lbs.}$$

$$\text{From (2)} \quad F = \frac{4}{3} S\, bh \text{ lbs.}$$

$$\therefore \quad f = KS \qquad \text{where}$$

$$K = \frac{2(1-d)}{h}$$

If we take f = 184 p.s.i. at 29° F, Table III, and S = 98 p.s.i., we find that K = 1.88. Thus, if $\frac{2(1 - d)}{h} > K = 1.88$, the beam, when subjected to bending forces, should fail in tension within the blocks B and B' of Figure 6. This is evident from the fact that the tensile stress of a fiber increases 1.88 times as fast as does the shear stress. Therefore, in conducting the bending tests, blocks B, B', and C, C' were so spaces that for a given thickness of the ice beam $\frac{2(1 - d)}{h}$ would always be > K.

Crystal Size and Form of Natural Ice.

Method of studying Natural Ice: To study the crystal size and form of natural ice, blocks of the Lake Michigan ice were cut into 5/8" thick slabs parallel to the surface of refrigeration. Each specimen was then placed on a stand between two ten-inch polaroid disks with their transmission directions at right angles. Each specimen was photographed after slight melting of the ice surfaces had rendered it perfectly transparent. A light box was used for a uniform light source. The negatives were mounted as lantern slides. The slides, with a scale, were projected on a screen and the size and distribution of the crystals determined.

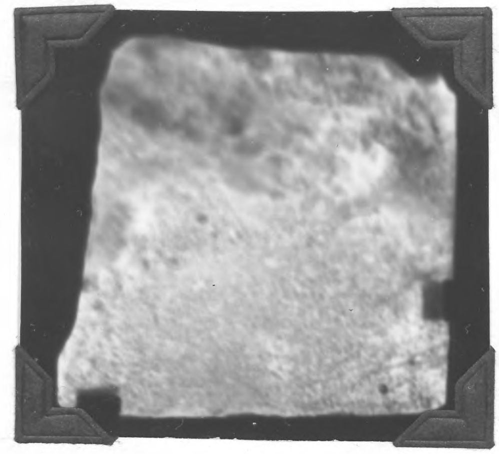
Comments on the Crystal Size, Form, and Distribution of Natural Ice: The study of sections comprising a complete vertical section of the lake ice revealed that the size of the crystals increased with depth. In every case the first inch of the upper-most surface of the ice was found to consist of cloudy ice with numerous air holes and no macroscopic crystals. Below this layer small crystals were visible. The size of the crystals increased downward while the number decreased. A typical section is reproduced in Table VII, while Figure 11 gives the corresponding pictures.

It may be noted here that as the crystals became larger intergrowth of the crystals became very common.

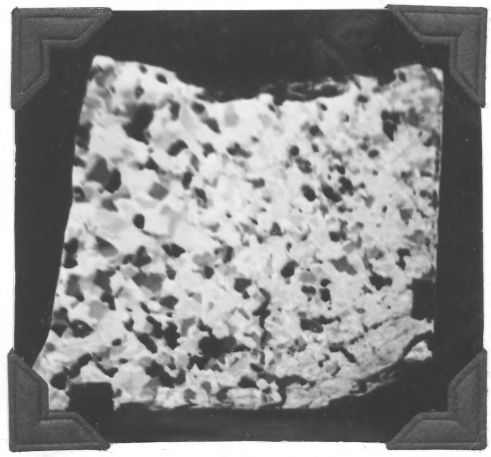
As for crystal form, Dorsey states that in all probability the crystals of natural ice belong to the bipyramidal class of the hexagonal system. While no such definite form was observed in any section, suggestions of the hexagonal form were frequently found. Furthermore, studies revealed that the crystal walls were practically perpendicular to the surface of the ice. In most cases the major crystal axes were within 8° of the vertical, and in only rare instances were they off by 12° or more from the vertical. The larger crystals were found to be continuous in depth while the smaller were found between the larger ones and wedged out over a relatively short interval. In some cases it was also found that small crystals had formed in the larger ones.

Table VII.

No. of Section	Position of Section	Size of Crystals in Diameter
1.	Very top specimen.	Cloudy ice. No visible crystals.
2.	1" from the top.	Average diameter = .2".
3.	2" from the top.	Average diameter of a few xls = .8" Average dia. of all other xls = .4"
4.	6" from the top.	Large xls = 1.2" average diameter Medium xls = .8" " " Small xls = .5" " " Very small xls = .15" av. dia.
5.	8" from the top - bottom specimen	Average diameters same as in preceeding section.



Section 1.



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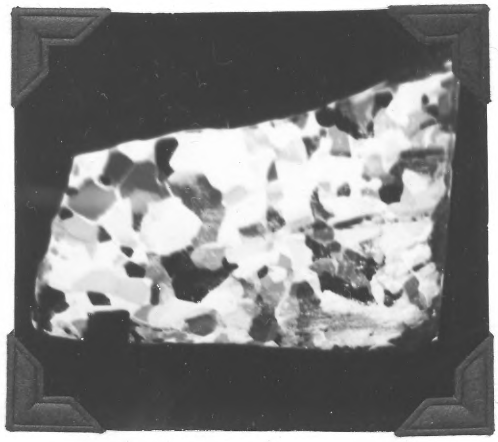
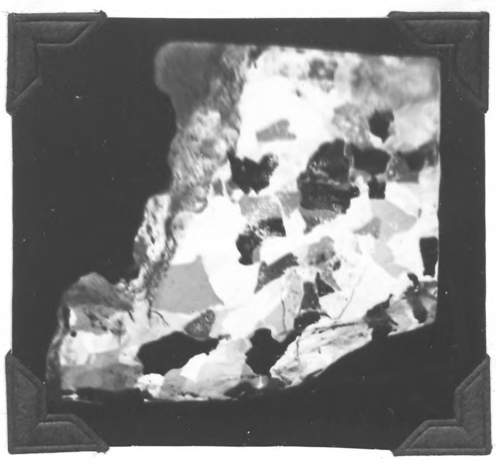


Section 3.

Section 4.



Section 5.

0 1" 2"

Scale

FIGURE 11. SLIDES OF THIN SECTIONS GIVEN IN TABLE VII.

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