A Prospect Theory-Based Real Option Analogy for Evaluating Flexible Systems and Architectures in Naval Ship Design

by

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A dissertation submitted in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy
(Naval Architecture and Marine Engineering)
in the University of Michigan
2014

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To America.

ACKNOWLEDGEMENTS

I am very thankful for the continuous help and direction of my advisor, Dr. David Singer, throughout the years creating this dissertation. I hope this research is a reflection of his unique vision and approach to many of the most difficult problems in Naval engineering.

Dr. Matthew Collette provided invaluable advice and direction on structural design and optimization which greatly improved the quality of one of my case studies. Thanks are also given to my research peers for their feedback and friendship. Special thanks to Jason Strickland, Doug Rigterink, and Morgan Parker for their repeated help and discussions.

This research would not have been possible without the financial support of Kelly Cooper and the Office of Naval Research. I am thankful not only for their funding, but for their long term vision and patience to allow me to pursue a degree in Financial Engineering, which I believe greatly improved the quality of this dissertation.

No Ph.D. is without its trying times, where the student is tempted to leave early for the golden shores of "industry." I am extremely grateful to my uncle, Dr. Doyle Knight, for helping me to remain resolute through my own trying times.

No man is an island, said John Donne. I am fortunate to have two wonderful parents, Lisa and Duane Knight, who have always pushed me to fulfill my highest potential and surrounded me with encouragement and motivation. And standing next to me through every page has been my loving wife, Cynthian. With you, I know I can accomplish anything.

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ABSTRACT

A Prospect Theory-Based Real Option Analogy for Evaluating Flexible Systems and Architectures in Naval Ship Design

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A constant trend in U.S. Navy design and acquisition programs has been the emphasis on flexible systems and architectures. Modularity and design-for-upgradability are two examples of this trend. Given the increasing importance of flexibility in Naval design, the methods used for valuing Naval assets should adequately capture the impact of such flexibility. Current static budgetary techniques and net present value (NPV) analysis underestimate the value of the embedded "optionality" of flexible design features. The use of real options analysis (ROA) has been proposed to correct this underestimation, however the theory is not universally applicable to the naval domain because of key assumptions made by a real options approach. For instance, ROA assumes that assets generate cash flows, which have a measurable value based on their volatility and the prevailing "market price of risk." Naval assets, however, do not generate cash flows, nor are they traded on a market. Furthermore, traditional ROA does not allow for the possibility of the option's value being interdependent with the decisions of other agents in one's environment.

These deficiencies leave designers and decision makers to rely on their intuition and engineering experience when evaluating flexible systems and architectures. A quantitative evaluation framework would add valuable analytical rigor to increasingly complex designs and demanding mission requirements.

This research presents a novel framework for evaluating flexible Naval assets, called prospect theory-based real options analysis (PB-ROA). The framework abstracts the principles of ROA to suit a wide variety of naval applications. Since naval assets do not generate cash flows, utility theory provides the alternative measure of value within PB-ROA. However, without a market where the assets are traded, a new source for data on prevailing risk tolerances in needed to properly adjust the option's value according to uncertainty. Where some prior research relies solely on utility curves to determine risk aversion, PB-ROA uses a unique mechanism inspired by Prospect Theory to derive the risk-adjusted probability measure from the decision maker's marginal utility curve(s). This enables PB-ROA to include the impact of loss aversion where previously it has been ignored. Game theory is also incorporated into PB-ROA to address the unique characteristic of some naval options which may be leveraged to influence the behavior of other agents in the Navy's environment. With game theory, PB-ROA lends a new perspective on the value of "game changing" options, which do not simply react to changes in the environment, but exert a feedback effect on it as well.

Relevant literature is reviewed, the theory supporting the framework is developed, and case studies are presented demonstrating the valuable insight which the framework may generate.

CHAPTER I

Introduction

Naval vessels are required to change, adapt, or upgrade during their service life. The acquisition and operating environments for the U.S. Navy are rapidly changing, posing new challenges for the ship designer. The projected service lives of naval vessels continue to lengthen. Budgets are contracting. Threats require increasingly mobile and adaptable response capabilities. When aggregated, these factors demand that naval ships be increasingly flexible. Flexibility is broadly defined here as the ability to change. Flexibility applies at many levels of naval design ranging from an individual subsystem, to a system of systems, a ship architecture, and even entire fleet architectures. Modularity and Design-for-Upgradability are two manifestations of flexibility in naval design, and there are many others. Flexible systems and architectures can be used to help a ship or fleet shift operations, upgrade technology or machinery, adopt new or multiple missions, and actively manage risks.

While designers have an intuitive appreciation for the value of flexibility, decisions are currently made based largely on experience, conjecture, or iteration from a previous ship design. However, engineering experience and judgement are less useful as system complexity increases, or the bounds of current practice are pushed or exceeded. To date, there are no widely-used rigorous, analytical methods for evaluating candidate flexible systems or architectures in naval design [38, 94]. As noted by Gregor (2003), "There is no way in the current system to value adding flexibility to the design, since under certainty, flexibility has no value" [47]. A quantitative, defensible framework is needed to evaluate flexible systems and architectures in early-stage naval ship design. [68]

The purpose of this research is to demonstrate the ability to quantitatively inform the early-stage decision making for naval ship design and acquisition through the intersection of real options theory, utility theory, prospect theory, and game theory. Utility theory will allow the construction of relevant value measures for naval applications, and real options theory will provide the machinery to assess a system's value given one or more sources of risk and uncertainty. Since naval assets do not generate cash flows, or exist within any traditional commercial market, prospect theory will provide an alternative method for accounting for risk aversion, which is composed of aversion to not just uncertainty, but also loss. Game theory will generalize the framework to naval applications involving interactive decision making and decision-dependent environmental development.

CHAPTER II

Background

2.1 Motivation

> *Naval ship design and acquisition is an option-laden environment. Therefore if a naval version of the real options analogy were developed, it would add considerable insight.* - Dr. Philip Koenig, NAVSEA (2009)

> *The determination of the value of an item must not be based on its price, but rather on the utility it yields.* - Daniel Bernoulli (1738)

The U.S. Navy has a need for a framework to evaluate the total life cycle performance of flexible systems and architectures. Traditional metrics like cost and mission effectiveness alone do not paint a complete picture of an asset's performance when design requirements and/or mission requirements may change. The flexibility of an asset is also an important measure of performance. However, the accepted practice is still to optimize a design for a static set of requirements, as epitomized by the design spiral [94][82]. How one values non-traditional attributes like flexibility will impact future Naval design considerations.

Static budgetary techniques and net present value (NPV) analysis underestimate the value of managerial and operational flexibility [115], and the embedded "optionality" of design features such as modular systems and design-for-upgradability. Currently, decisions made concerning systems with a high degree of optionality are largely based on anecdotal evidence [94], or engineering experience. This is because designers intuitively understand the value of flexibility, and the need to hedge against uncertainty. This is accomplished through a variety of means, such as modularity, structural and growth margins, and design structure matrices (DSM) to isolate dependencies between systems [23][40], among others. However, a rigorous, mathematical framework for performing such flexibility evaluations would add considerable value to the U.S. Navy, particularly as designs are pushed to be increasingly adaptable, have longer service lives, and exceed the limits of current engineering experience. The use of real options analysis (ROA) has been proposed for such a framework [68], however the theory is not universally applicable to the naval domain because of key assumptions made by a real options approach.

"Volatility" is a key input parameter for option valuation, and is a measure of the risk of an investment. Volatility is traditionally assumed to be the standard deviation of changes in the value of the underlying asset. For financial options, this is typically the standard deviation of returns on a stock price, interest rate, etc. For real options in a commercial market, this is typically the standard deviation of changes in cash flow generated by the investment. Risk is considered to increase with volatility. The problem with applying such an assumption to the naval domain is that naval assets do not generate cash flows. Therefore, an alternative risk metric is necessary for valuing real options for the U.S. Navy.

Another key metric in real options analysis is "value." For commercial applications, the value of an investment is typically expressed in terms of some currency (e.g., dollars). However, currency may not be the relevant measure of utility for an asset in a naval context. Furthermore, stock options, and many real options, are priced relative to a "risk-free" asset. This is typically assumed to be some U.S. government bond for which the rate of return is known at the time of purchase and carries negligible risk. It is unclear if such an asset exists for naval option applications.

Pricing financial derivatives, like options, also assumes that the world is "risk-neutral." This does *not* imply that investors are truly risk-neutral. However, the use of a risk-neutral measure when pricing the derivative is necessary to ensure that no arbitrage opportunities are introduced into the market. In the process of ensuring no-arbitrage, this adjusts the derivative's price in accordance with investors' risk appetites, known as the "market price of risk." While arbitrage opportunities may not exist in a real options market for practical reasons, pricing must still be done in the risk-neutral space in order to fully account for investors' risk appetites. Who determines the price of risk? The answer is, the Market. However, a market does not exist for naval options, and therefore cannot be relied upon to determine the risk-neutral measure necessary for valuation. This must be addressed by whatever framework is used.

Furthermore, most real options analysis assumes that the payoff to the option owner is exclusive. The market is tacitly assumed to be composed of many rivals, each sufficiently small that no action taken by any one agent will have a measurable effect on market behavior. This assumption is invalid in many naval applications involving interactive decision making between rational participants where the purchase, or creation of an option may influence other participants' strategies and hence change the operating environment, or market behavior. One unique aspect of naval assets, to include option-like naval assets, is their ability to direct, or at least attempt to direct the behavior of other agents in their environment toward a desirable outcome.

Another way to express this distinction is that financial options, and traditional real options, tacitly submit to changes in the market. In contrast, some naval options may be leveraged to induce change in the market (i.e., their operating environment). Such *game changing* options cannot be evaluated by traditional ROA, but promising avenues of research exist in the field of options game theory.

The preceding paragraphs attempt to explain why a traditional real options approach to evaluating flexibility will not be suitable for many naval design applications. A new framework is required in which markets cannot be discussed in any traditional sense, because naval assets are neither traded, nor generate cash flows. The new framework must not be constrained to the same definitions of volatility and value used in traditional ROA. And the new framework must account for the interdependence of agent decision making, and the opportunities for option feedback on the operating environment.

This doctoral research dissertation presents a framework analogous to real options based on prospect theory and game theory for naval applications which addresses some of the limitations of traditional ROA. Most importantly, this dissertation shows how marginal utility curves can be used as weighting factors in expected value calculations to allow naval option valuation to be performed with real probabilities, instead of risk-neutral probabilities. The significance of this is that there is no need for a market to provide the risk-neutral measure to value naval options, when the payoffs are associated with a utility, instead of a currency. Game theory allows the framework to build on this prospect theory-based approach when the option(s) have a feedback affect on the environment. The theory is developed and a naval application of the theory is presented through several case studies.

2.2 Overview of Framework

This section summarizes the major steps in the novel framework and ties these steps to broad areas of study to be discussed in sections 2.3 and 2.4. Each step in the framework will also be discussed in greater detail in Chapter III. Figure 2.1 gives a high-level overview of the different theoretical disciplines that are combined to form the framework presented in this research. The figure shows that in order to transition traditional ROA from its current realm of commercial applicability to the Naval realm, one first needs utility theory in order to express value beyond just currency because Naval assets do not generate cash flows. Then one needs prospect theory because it provides a mechanism for risk adjustment that includes loss aversion, which utility theory alone cannot provide. Finally, the method integrates game theory to allow the analysis of naval options that have a feedback effect on their environment.

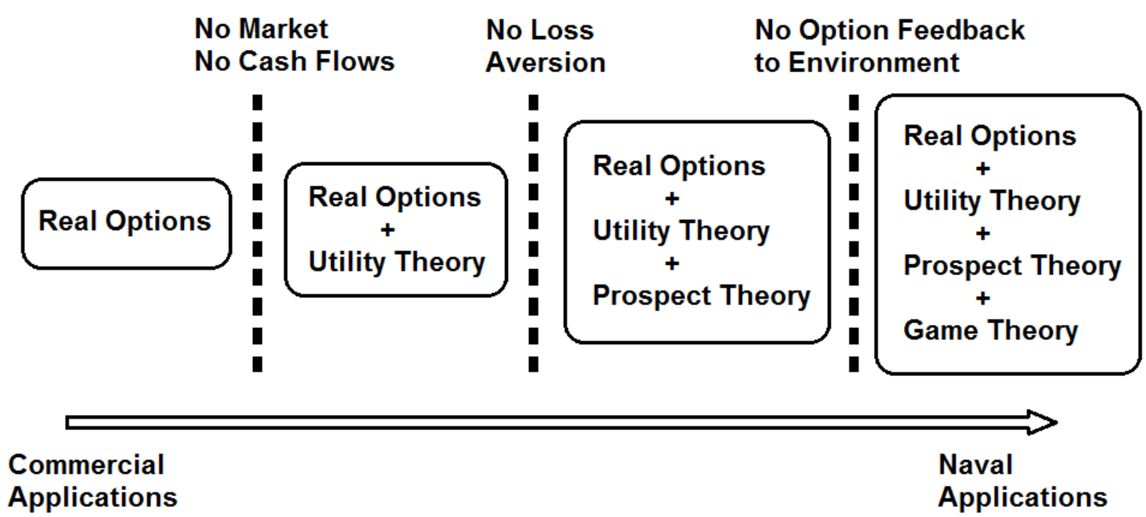

Figure 2.1: High-level overview of the PB-ROA framework

The major steps in the prospect theory-based real options analysis framework (PB-ROA) are:

1. Identify the relevant risk factors, $X_i(t)$, which the asset is exposed to. Risk factors are random variables, and will have associated density functions, whether continuous or discrete.

2. Identify the asset's relevant design features, d_j. Design features are physical attributes which dictate performance.

3. Determine the complexity of the asset's design features.

4. Combine design features, risk factors, and the complexity metric to generate the asset's utility curve (or surface), reflecting the agent's preferences over future outcomes. This step draws on insights from utility theory.

5. If the asset under consideration **is not** effected by interdependent decision making of more than one agent, then the density function of the risk factor(s) is transformed to a "risk-adjusted" measure by weighting the probability density of every possible outcome with its marginal utility. While straight forward for utility functions with only one input, this process is more complicated for utility functions with more than one input. This step draws on insights from prospect theory.

6. Then, valuation of the asset may be preformed using the risk-adjusted measure, accounting for both risk aversion and loss aversion. This step draws on standard techniques from both financial options and real options theory.

7. If the asset under consideration **is** effected by interdependent decision making of more than one agent, then a game analysis is performed, comparing the Nash equilibriums of the games' structures before and after the asset is introduced. This step draws on standard practices from game theory.

8. Then, the value of the asset equals the change in the utilities of the Nash equilibria between the two games.

While each step is necessary for the framework, the contributions of the research lie mainly in steps 5-8, and to a lesser extent in step 4. These steps will be discussed in greater detail in chapter III.

2.3 Background

This section aims to provide an overview of the material necessary to understand the basic theory and procedures behind options analysis, utility theory, prospect theory, and game theory. The theory and procedures described in this section will carry over into specific applications in the related works section and the development of the prospect theory-based real option analysis framework (PB-ROA) of this research.

2.3.1 Options

An important starting point is to understand just what an option is. To answer this, let us first limit our discussion to financial options, as they are the predecessors of real options. A financial option is a contract that grants the holder the right, but not the obligation, to take certain actions on an underlying asset, within a certain time period, at an agreed upon price. It is a type of *derivative*, meaning that its value derives from the state of the underlying asset (and potentially other independent variables) such as a stock price (e.g. Google), an index (e.g. the S&P 500), an interest rate (e.g. LIBOR), a commodity (e.g. the price of crude oil), or any number of other financial instruments. The element of choice is what distinguishes options from other derivatives like forwards and futures. This element of choice lends *flexibility* to the contract that is potentially very valuable.

There are many different types of options. A *call* option, for example, gives the holder the right to purchase the underlying asset, at an agreed upon price, during some time period in the future. A *put* option gives the holder the right to sell that underlying. *European* options limit the time period when the holder may exercise their right(s) to a single point in the future (e.g. 5pm, 21st Dec., 2013). *American* options allow the holder to exercise their right(s) at any time before the stated termination time. European and American calls and puts are the most common option types. Other, more complicated types of options are generally termed *exotic* options. *Real options* are an extension of financial options theory to contracts on physical goods, or capital management of projects. The principles remain the same, however the machinery used to evaluate them may differ.

It is common to speak of the *payoff* associated with holding an option. Consider momentarily a European call option written on the underlying asset, S, who's future value is uncertain. This option has contracted for a strike price, K, and time of maturity, T. Given this information, the payoff, Φ, to the holder of the option at the time of maturity will be, $\Phi(T) = \max\left[S(T) - K, 0\right]$, because the element of choice means that the holder will only exercise the option if it is beneficial to do so. Similarly, the payoff from a European put option would be, $\Phi(T) = \max\left[K - S(T), 0\right]$. These payoffs are shown diagrammatically in figure 2.2.

Much of option theory deals with how to determine the fair market price for an option contract. The cornerstone of pricing financial options is the concept of *no arbitrage*. Since financial options are liquid, traded instruments, it is important that they be priced in such a way that prevents other agents in the market from having a "free lunch." Since options are derivatives, their value at any given time can be replicated be a combination of other assets. For example, an option on Google stock can be replicated by actively trading in shares of Google stock and U.S. Treasury bonds. If the option is mis-priced, the agent can simultaneously trade between the stock option and the synthetic option to earn a risk-free return, violating market principles.

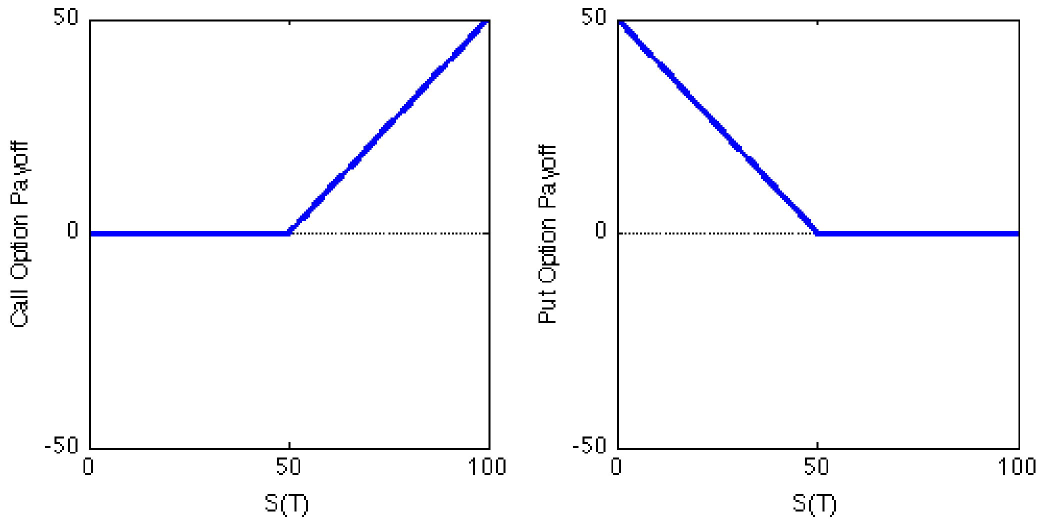

Figure 2.2: Payoffs for a European call option (left) and put option (right) with strike price of 50.

This leads to the concept of a *replicating portfolio*, one of the most commonly used approaches to pricing financial options. Replicating portfolio pricing follows a very simple line of argument; If the payoff of the option in any future state can be replicated by a portfolio of other traded assets, then the price of the option is equal to the value of the replicating portfolio (see Proposition 2.9 of [15]).

A key consideration when pricing an option is how to model the dynamics of the underlying asset. There are discrete time, as well as continuous time models which are frequently used. Of the discrete time models, recombining binomial trees [30] and trinomial trees [22] are common. An advantage of discrete time models is the existence of simple solutions for the option price. In continuous time, the existence of a closed-form solution is not guaranteed. However, under certain limiting assumptions, closed-form solutions may be found. Perhaps the most famous example of one such continuous time model is the Black-Scholes-Merton pricing formula[17] for a European call option, for which Myron Scholes and Robert Merton won the 1997

Nobel Prize in Economics, where the dynamics of the underlying asset are assumed to follow a geometric Brownian motion with constant drift and diffusion parameters, and the risk-free interest rate is assumed to be deterministic and constant. The Black '76 model is another notable continuous time formulation with a closed-form solution [16]. It is an extension of the Black-Scholes-Merton model for options on bonds, interest rate caps, and swaptions. When models do not allow for closed-form solutions, Monte Carlo methods may be used [21][3][20].

Whatever the particular method being used, the pricing is always being performed in what is termed a *risk-neutral* framework. Due to the natural differences that arise between the risk appetites of market agents, the current price of a derivative cannot be simply calculated by its expected value. Each agent's expected value would differ based on their risk appetite. Instead, expectations are calculated using an *equivalent martingale measure*, under which the dynamics of the underlying asset, after discounting at the risk-free rate, are a martingale (see section 10.5 of [15], or sections 9.2 and 11.6 of [60]). This may be best demonstrated through the example to follow. But the crux of risk-neutral pricing is that it requires a market. The market is what determines the risk-neutral measure. One goal of this research is to present an approach enabling the use of option-like valuation techniques in the naval engineering domain where there is no market (i.e.- assets are not traded) to provide the risk-neutral measure, and assets do not generate cash flows.

2.3.1.1 Binomial Tree Option Pricing Example

To illustrate some of the basic principles of real options analysis, consider the example in figure 2.3, first presented in [67]. The example is overly simplified and unrealistic, but helps to demonstrate basic option theory.

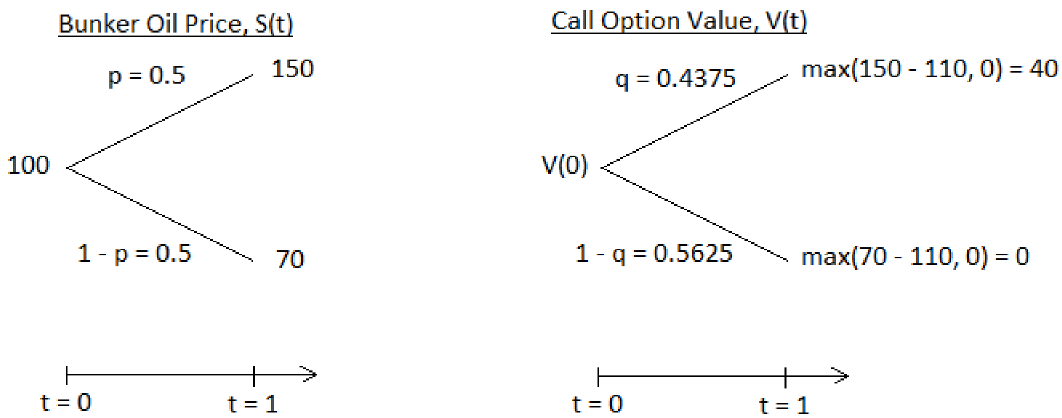

Figure 2.3: Basic binomial tree model for option valuation

A ship operator knows that 1000 barrels of bunker oil will be needed in one year's time. The current price of bunker oil is $S_0 = 100$ USD per barrel, and experts believe that in one year's time the price of oil will either increase 50% with probability $p = \frac{1}{2}$, or decrease 30% with probability $1 - p = \frac{1}{2}$. The ship operator has three choices for how to hedge against undesired movements in the price of bunker oil. One choice is to buy the oil now, for 100 USD per barrel, and store it for one year. This choice is costly and may not be available to all ship operators. The second choice would be to purchase a forward (or futures) agreement on oil, for say 110 USD per barrel. Then, no matter how the price of bunker oil moves, the ship operator will pay 110 USD. This strategy pays well in the event that prices rise. However, in the event prices fall to 70 USD per barrel, the ship operator will be overpaying for the oil. An alternative would be to purchase a call option on bunker oil with a strike price of, say, $K =$ 110 USD. The call option gives the ship operator the right, but not the obligation, to purchase bunker oil for 110 USD per barrel. In other words, the option's payoff can be written, $v(t = 1) = max(S(t = 1) - K, 0)$. In the event prices rise, the ship operator will exercise the option and purchase the oil for the contracted price of 110 USD. In the event prices fall, the ship operator will not exercise the option. Instead, the bunker oil will be purchased according to the going market price of 70 USD per barrel. The binomial tree model for option pricing can be used to determine what such an option would be worth.

One of the things that makes options analysis so appealing is that one does not need to know the real probability distribution of future bunker oil prices in order to price the option, which in practice may be difficult to calculate or reliant on subjective estimates. The option is priced using a risk-neutral probability distribution which can be calibrated directly from the price movements of the bunker oil, and the prevailing risk-free interest rate. For this example, assume that the risk-free rate is $r = 5\%$. Then, under the binomial tree model, the risk-neutral probability of an increase in bunker oil prices is given by:

$$q = \frac{(1+r)S_0 - S^-}{S^+ - S^-} = \frac{(1+0.05)100 - 70}{150 - 70} = 0.4375 \quad (2.1)$$

The current value of the call option is the expected value of payoffs from the option taken according to the risk-neutral probability measure, discounted at the risk-free rate.

$$V_0 = 1000 \cdot \frac{1}{1+r}\left[qV^+ + (1-q)V^-\right] \quad (2.2)$$

$$V_0 = 1000 \cdot \frac{1}{1+r}\left[q \cdot \max(S^+ - K, 0) + (1-q)\max(S^- - K, 0)\right] \quad (2.3)$$

$$V_0 = 1000 \cdot \frac{1}{1+0.05}\left[0.4375\max(150-110, 0) + 0.5625\max(70-110, 0)\right] \quad (2.4)$$

$$V_0 = 16,667 \text{ USD} \quad (2.5)$$

So the call option is worth 16,667 USD. One alternative approach to the binomial tree used in this example is Monte Carlo simulation. Instead of building a tree with probabilities assigned to each branch, many sample paths are generated for the underlying asset (in this case, bunker oil prices), and an average of the option's payoff across all sample paths is taken. However, all of the same principles still hold which were demonstrated using the binomial tree, to include the use of a risk-neutral probability distribution and discounting according to the risk-free rate of interest.

2.3.2 Utility Theory

In finance, and options theory, it is standard to express the value of an asset in terms of some currency (e.g. a share of XYZ stock is worth \$100). The expected value criterion for comparing between two risky assets suggests that one should invest in the asset with the higher expected monetary value. Consider, for example, the scenario where an agent is presented with the choice between taking \$1 with certainty, or entering into a gamble with a 1-in-49 chance of winning \$50. If the agent's objective is to maximize expected value, then the agent will choose the gamble, since it has the higher expected value of wealth, $E[w] = (1/49) \cdot \$50 = \$1.02$. However, in practice, many agents will choose the certain \$1 over the gamble.

A fundamental premise of the expected utility hypothesis is that expected value (in the monetary sense) is insufficient to capture true agent preferences. In the example above, more risk-averse agents will choose the certain \$1, while other more risk-tolerant agents will choose the gamble. Utility theory is a common approach to explaining and quantifying risk aversion.

Another added insight from utility theory is the oft-observed *diminishing marginal utility of wealth* (or any asset), also known as Gossen's First Law [45]. Simply put, ten units of a good *may not* have the same utility as ten times the utility of one such good. Even goods with an infinite expected value (in the monetary sense) will likely have finite utility. This is demonstrated by the St. Petersburg Paradox, involving a gamble with infinite expected value. In the St. Petersburg Paradox, a player is offered a gamble where a fair coin is repeatedly flipped until a tail appears. The pot starts at $1, and is doubled for each head that appears, until the appearance of a tail ends the game. Thus, the expected value of the gamble is:

$$
\begin{aligned}
E &= 1 \cdot \frac{1}{2} + 2 \cdot \frac{1}{4} + 4 \cdot \frac{1}{8} + \ldots \\
E &= \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \ldots \\
E &= \infty
\end{aligned}
$$

The paradox is that since the expected value of the gamble is infinite, the players should be willing to pay any price to participate. However, this does not fit with the real world, in which people often make very modest bids to play the game. Daniel Bernoulli used marginal utility theory to partially explain this paradox [12]. Because of the diminishing marginal utility of wealth, the expected utility of this gamble is in fact finite, and therefore agents have threshold prices over which they will not accept the gamble. Importantly, each agent's threshold price may differ, perhaps substantially, because of their differing risk appetites and initial wealth (i.e., resources).

Recall from the background on option theory that the equivalent martingale measure necessary for pricing options is derived from the market. In certain circumstances (e.g. incomplete markets, see ch. 15 of [15]), multiple equivalent martingale measures may exist, resulting in multiple prices for the option, or a range of prices. In order to settle on a single optimal price, several authors have proposed using utility theory to incorporate more information about agent-specific risk tolerances [84][59]. Thus, there already exists an intimate relationship between options theory and utility theory, which will be expanded upon in the section on related work and in this research.

It is worth mentioning here that the utility functions to be used throughout this research are *cardinal* utility functions. Cardinal utility functions are assumed to be quantifiable, and directly comparable on a scale. For instance, consider three assets with utilities of two "utils," four "utils," and six "utils," respectively. Then, using cardinal utility one may say that the third asset is preferable to the second, by the same amount that the second is preferable to the first. If a zero-utility point may be established, then even stronger statements may be made. Such statements would not be permissible if using an *ordinal* utility function, which may only be used to rank preferences, but not directly compare between alternatives (an interesting discussion of the similarities between cardinal and ordinal dimensions of utility may be found in [119]). Furthermore, the utilities used in this research will be von Neumann-Morgenstern (VNM) utilities, as described in chapter 3 of [121], and obeying the axioms laid out in section 3.6 of that work. Assuming cardinal utility functions facilitates the systematic approach to risk-adjustment in step 5 of figure 3.1 which uses the marginal utility curve as a weighting function.

However, as a descriptive model of economic behavior, expected utility theory has many limitations. While it may explain risk aversion, and the diminishing marginal utility of wealth, it does not fully capture loss aversion. Where risk aversion captures most individuals' preference for certainty over uncertainty, loss aversion captures most individuals' disproportionate aversion to negative outcomes. Prospect theory builds on expected utility theory to better address loss aversion, and other limitations.

2.3.3 Prospect Theory

While expected utility theory says that individuals are expected utility maximizers, empirical evidence has shown that this is not always the case. Prospect theory, developed by Daniel Kahneman and Amos Tversky, corrects many of the descriptive shortcomings of utility theory. As detailed in [63], "an essential feature of prospect theory is that the carriers of value are changes in wealth or welfare, rather than final states." (p. 277) This is to say that individuals do not tend to think in terms of absolute wealth or resources, but rather relative to an initial reference point. Furthermore, the observation that people tend to overweight unlikely outcomes when the outcome is negative, led to the conclusion that individuals do not weight the utilities of uncertain outcomes by their probabilities, but by *decision weights* which may be non-linear, and may not always satisfy the requirements of a probability density.

Consider a gamble consisting of two possible outcomes paying x and y, with real probabilities p and q, respectively, and paying 0 with probability $1-p-q$. The expected utility of this gamble would simply be:

$$U(x,p;y,q) = pu(x) + qu(y) \tag{2.6}$$

where $u(x)$ and $u(y)$ are the the utility of payments x and y, respectively, and $U(x,p;y,p)$ is the expected utility of the gamble. Prospect theory defines a value function for the payoffs, $v(x)$ and $v(y)$, relative to some reference point, and decision weights $\pi(p)$ and $\pi(q)$. Then, in general, the expected value is:

$$V(x,p;y,q) = \pi(p)v(x) + \pi(q)v(y) \tag{2.7}$$

According to Kahneman and Tversky, in practice the decision weighting function might look something like that shown in figure 2.4. Kahneman and Tversky expand on the psychological foundations of prospect theory in [64]. Barberis et al. [7] use a prospect theory-based approach to asset pricing and replicate some historical observations from the stock market. What is significant for the purposes of this research is the similarity between the risk-neutral probability measure used in options theory and the decision weights used in prospect theory; namely that both are non-linear

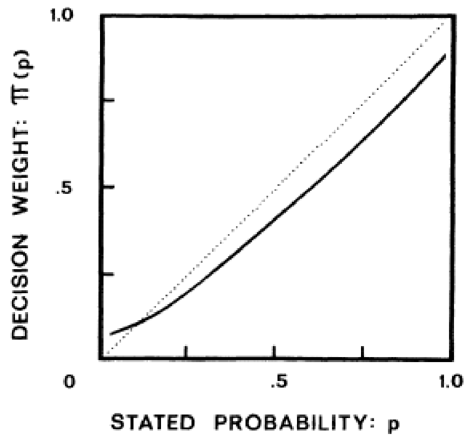

Figure 2.4: Hypothetical weighting function, taken from [63], p. 283

transformations of the real probability measure that reflect loss aversion. This research presents a systematic method for constructing the decision weight function, to be called the risk-adjusted measure, from an agent's marginal utility curve. This will account for loss aversion in naval decision making, and will be discussed in chapter III.

2.3.4 Game Theory

Game theory is "the study of mathematical models of conflict and cooperation between intelligent rational decision-makers." (p. 1, [85])

Anyone who has ever played chess knows that your opponent's decisions impact yours, and the converse is also true. Game theory is a very broad field originating from mathematical economics that enables a quantitative, systematic analysis of such scenarios involving multiple agents with interrelated decision-making. There are nearly as many types of games as there are people who have studied them. Games may be cooperative or competitive, symmetric or asymmetric, simultaneous or sequential, with perfect or imperfect information... the list continues. The general nature of game theory has encouraged a range of applications of it to fields as diverse as economics, biology, psychology, and many other fields of both social and natural sciences.

Some important concepts in game theory are those of a solution, dominant strategies, and Nash equilibrium. A solution is a set of strategies. While strategies yield utilities for the players, the utilities themselves are not the solution, but the strategies. A strategy x dominates another strategy y if the payoffs resulting from x are preferred to those of y in all possible outcomes. An equilibrium strategy is a stable one, where none of the players wish to change their strategies. It is important to note that an equilibrium may not be optimal, in the sense that it yields the highest possible utility (see the "prisoner's dilemma," for example). An equilibrium may either be a *pure strategy* equilibrium, or a *mixed strategy* equilibrium. Mixed strategies involves applying probability distributions to an agent's choice of strategy to randomize it. Pure strategies may not exist for certain games, and mixed strategies can be particularly useful in repeated games. Von Neumann and Morgenstern showed the existence of mixed equilibrium for finite, zero-sum games in [121]. John Forbes Nash extended this concept of equilibrium, now called the Nash Equilibrium, and proved that there is at least one Nash equilibrium for all finite non-cooperative games, to include non zero-sum games [86]. Non-cooperation simply means that players are not allowed to communicate or form coalitions; they act independently. This proof is significant as it guarantees that every finite game has at least one solution.

Material on game theory is expansive. This is but a cursory overview of the most basic concepts in game theory. For more in-depth discussion, the interested reader is referred to [85]. The digital video lecture series on game theory entitled, "Games People Play; Game Theory in Life, Business, and Beyond," is another, more interactive reference for learning more on this subject [111].

2.4 Related Work

This section summarizes a wide range of related work. Each of the works cited in this section is relevant to the framework presented in this dissertation as an application of options theory to a related field, as in section 2.4.2, or a modification of options theory to suit an engineering system, as in section 2.4.3, or because of similar problem structures as sections 2.4.4 and 2.4.5. While each of these papers is notable for their own achievements, nearly all of them focus on the economic, or monetary, value of flexibility. This research will be distinct in its attempt to express value and inform decision making in non-monetary terms.

2.4.1 Real Options and the U.S. Navy

Gregor [47] values flexible designs for U.S. Naval vessels using a multi-attribute utility (MAU) score combined with a novel willingness-to-pay (WTP) framework. The MAU gives a combined utility score for a candidate ship design based on cost and mission overall measure of effectiveness (OMOE). Such treatment is unique in the real options literature, which is applied almost exclusively to economic projects with value measured in some currency. The WTP framework then translates this utility score into a price which the Navy should be willing to pay for such capability. It is not an application of real options analysis, strictly speaking. It is more similar to an expected utility maximization problem. However, it is a novel publication of options-thinking in a naval context.

Koenig et al. studied alternative fleet architectures to address the rising cost and lengthening service lives of naval vessels [70][69]. Koenig recommends the development of a naval real options analogy to aid in decision making [68]. Some difficulties of a real options approach are identified, such as the lack of cash flows and observable asset prices (i.e.- a public market) for government projects.

Page [91] applies techniques inspired by real options analysis to evaluate the total lifecycle cost savings of flexible architectures for the U.S. Navy, with specific application to a proposed new concept architecture, the Scalable, Common, Affordable, Modular Platform (SCAMP). Page's work is significant for noting that the established real options and NPV techniques cannot assist in valuing non-economic assets, like government projects, that do not generate cash flows, and that an alternative valuation method is possible using utility theory, much like Gregor. However, the application explored still focuses on cost-saving initiatives and the budgetary impacts of flexible designs. While probability distributions are applied to key risk factors, the probability distributions are not risk-adjusted as in traditional real options.

Where this related work has either focused on cost savings or has been a simplified options-thinking approach, this research aims to abstract the use of real options beyond cost saving for the Navy and answer the open question of how to adjust probabilistic calculations for both risk and loss aversion so that applications of the framework may be more than simple utility maximization problems.

2.4.2 Maritime Real Options Applications

Bjerksund and Ekern [14] studied how to evaluate contingent claims written on assets in the shipping industry with mean-reverting dynamics. Specifically, they assume that spot freight rate follows an Ornstein-Uhlenbeck process. A methodology is presented for valuing options on time charter contracts for such assets. A closed form solution is derived for the present value of a European call option with a lump sum exercise price. Valuation is performed in a risk-adjusted framework where the market price of risk is determined by a traded "twin asset."

Tvedt [117] studied the valuation of operational flexibility for Very Large Crude Carriers (VLCCs). He models the decision to lay up tonnage in poor market conditions as well as the decision to scrap as real options. In this study, the underlying asset which is modeled is the time charter equivalent freight rate. Ornstein-Uhlenbeck and geometric mean reversion processes are considered. Tvedt concludes that the choice of asset dynamics can have a considerable impact on the option valuation in situations with high managerial flexibility, and is therefore very important. The real options in this study are American, and valued using simulation in a risk-neutral framework (ie - the values are not risk-adjusted). However, payoffs from the exercise of either option are deterministic. Moreover, the option to lay up is reversible.

Buxton and Stephenson [24] investigated the impact of "design for upgradability" on the net present value of a container ship. They essentially value an option to expand the capacity of a container ship at a pre-specified future date through "jumboisation," a fairly common process in the marine industry. They do not use the term real options, however their approach is in the spirit of real options valuation. They use a spreadsheet-based simulation methodology which allows for basic probability distributions to be applied to certain input parameters; they apply a Gaussian distribution to freight rate.

Bendall and Stent [10] model the managerial flexibility to choose between the most valuable of three different shipping strategies as a real option. Specifically, the ship operator must decide which ports to service as well as how many ships to operate given prevailing market conditions. The option studied is American and valued using a multinomial tree, and is complicated by the introduction of multiple sources of uncertainty whose correlations are modeled using Gaussian copula functions due to their assumed triangular marginal probability distributions. The option affects fleet composition, but the design of each ship not impacted.

Tsolakis [116] devotes the tenth chapter of his doctoral dissertation to identifying several real options in the shipping industry and methods for valuing them. Options discussed are options to expand, timing and defer options, options to choose between competing assets, and switching options. He investigates both European and American options, and finds that ROA is highly beneficial for evaluating investment decisions in shipping. Dahr [32] focuses his doctoral dissertation on real options within the liquified natural gas (LNG) tanker market, such as the option to extend a time charter contract. Valuation is performed using a Black-Scholes framework, and the options he demonstrates are either operational or contractual and do not influence ship design. Moreover, because the options are contractual in nature, their payoffs are completely known at the time of exercise.

Dikos [35] and Dikos and Thomakos [36] use the mark up value from real options to explain aggregate investment activity in the tanker markets. Count data models, assuming a generalized Poisson process, and historical data are used to validate the real options hypothesis that the value of an investment in new tonnage must exceed the cost, plus a premium which represents the option to delay before a project is undertaken.

While the valuation methodologies used in each of these works may not be directly transferable to the Naval domain, they are nonetheless significant for the framework presented in this disseration as they highlight the many forms of managerial flexibility in ship construction and operation that are so commonly ignored by static valuation techniques like net present value (NPV). While there are many differences between commercial and Naval marine assets, they share many of the same operational flexibilities.

2.4.3 Real Options and Complex Engineering Systems

Wang and de Neufville distinquish between options “on” projects and options “in” projects [123][124]. The former is the realm of traditional financial and real options, and the latter is the realm of naval options where the option may derive value from physical design characteristics. This work is important to this research as it was the first options research to highlight the option-like characteristics of many engineered systems, and the designer’s ability to make physical changes to those systems to maximize flexibility.

Baldwin and Clark [6] use ROA to investigate the value of modularity in complex engineering systems. The focus is on the economic value of modularity in the design of computers, but the framework may be abstracted to other engineering systems. However, for simplification they assume (as with many other authors) that the firm is risk-neutral, and that design intervals are short enough to neglect the time value of money. They conclude that modularity is a powerful tool to keep firms competitive, and that ROA may be used to demonstrate such value. Similarly, Engel and Browning [39] use ROA to value flexibility in adaptable engineering systems. In an extra step beyond Baldwin and Clark, they define a “System Adaptability Factor” (SAF) which is used to effectively penalize options for considerations such as maintainability, number of interfaces, etc. This research is significant for its specific treatment of modularity, as a design attribute, which is also focus for modern naval vessels like the Littoral Combat Ship (LCS), and offers a good starting point for Naval modular systems analysis.

Bowe and Lee [19] show that a static cash flow evaluation underestimates the value of flexibility in the case of a high-speed railway. More importantly, they demonstrate the interactions between multiple real options in the same system, further demonstrating that option values may be non-additive. Thompson et al. [114] value real options in a natural gas storage facility. Their work is notable for incorporating operational factors, such as delivery and injection rates, into an analysis which typically only considers economic data. The ROA derives an optimal operating strategy. Hassan et al. [53] use ROA to improve the value-at-risk characteristics for a proposed satellite fleet under market uncertainty. Martin [80] shows how ROA can better inform decision making for future design features in aerospace systems, such as the retrofitting of winglets to Southwest Airline's fleet of Boeing 737s to improve fuel efficiency. Martin demonstrates that ROA can be used to front load more useful information into the early stages of design for complex engineering systems.

Each of the papers cited in this section are notable for their own achievements. Most importantly, they offer examples of ROA applied to complex engineering systems where the ROA incorporated elements of design, and/or operation. However, they each focus on the economic, or monetary, value of flexibility. This research will be distinct in its attempt to express value and inform decision making in non-monetary terms.

2.4.4 Utility Theory and Pricing Options in Incomplete Markets

In finance, a *complete* market is one where all contingent claims can be priced. In contrast, in *incomplete* markets, not all contingent claims (e.g. options) may be priced. The simplest explanation for this is because some contingent claims in incomplete markets cannot be replicated. Recall that the payoff from a stock option may be replicated by dynamically trading in the underlying stock and the risk-free asset (e.g. a bond). Hence, the price of the option should be equal to the price of the "replicating portfolio," in order to ensure no arbitrage opportunities. Generally speaking, assets in incomplete markets will have a *range* of arbitrage-free prices. Depending on the specific instrument, these ranges can be quite broad. Since naval assets do not generate cash flows and are not traded on a market, their value cannot be replicated by other instruments. In short, naval assets are analogous to assets in incomplete markets. So, how does one uniquely price such assets? The following works were chosen for the insight they bring to problems with structures similar to those in the Naval domain; specifically, the valuation of assets in incomplete markets.

One valuation approach to is incorporate agent-specific information about risk appetite by means of a utility function. Then, using information about the agent's initial resources and risk attitude, a unique price may be determined from the previous range of prices. Two general frameworks exist for this purpose; a utility-based framework, and a marginal-utility-based framework.

Henderson [55] uses a utility-based framework to optimally price and hedge an agent's position in an option on a non-traded asset using a similar, traded asset. Results are compared for constant absolute risk aversion (CARA) and constant relative risk aversion (CRRA) utility functions. The optimal price is determined by solving the dual of a utility maximization problem. Monoyios [84] showed how this utility-based framework leads to superior hedging performance compared to calculations that approximate the incomplete market as a complete one. While these papers demonstrate how to use additional risk information from agent utility preferences, it still requires a similar *traded* asset, which may not always be present in naval applications.

Davis [33] uses the agent's marginal utility function to determine the fair price of contingent claims in incomplete markets. The advantage of this method over the utility-based method described above is simplicity. It does not require any similar, traded assets. Nor does it require solving the dual of an optimization problem. While Davis does not explicitly discuss it, he is essentially defining a new equivalent martingale measure using marginal utility information. Such a process was suggested in the doctoral dissertation by Beja (see p. 29-31, [8]), and later made even more explicit by Nau and McCardle (see p. 207, [87]). In this approach, the risk-neutral probability measure to be used for calculating option prices is given by:

$$q(\theta) \propto p(\theta)u'(w(\theta)) \tag{2.8}$$

where $p(\theta)$ is the true (objective) probability for state θ, $w(\theta)$, is the wealth resulting from state θ, and $u'(w)$ is the marginal utility for wealth evaluated at w. The measure q must be renormalized such that it integrates to one, and thus satisfies the conditions of a probability distribution. While this equation considers wealth, it is possible to abstract wealth to mean "capability," and define it meaningfully for a naval context. Hugonnier et al. later proved that, under certain conditions, a risk-neutral measure defined in such a way is in fact unique, which leads to a unique price for the option [59]. This research pulls heavily from the work of Beja [8], Nau and McCardle [87], Davis [33], and Hugonnier et al. [59].

2.4.5 Options and Game Theory

A typical financial option does not consider the interdependencies of agent decision making. This is because such interdependencies either do not exist, or are considered negligible according to the efficient market hypothesis [41]. However, the impact of such interdependencies may be significant in certain real options applications, and particularly for the U.S. Navy. To address such interdependencies, a small group of authors have begun to study what may generally be termed "game options." Such game options may arise situation like research and development, price wars, and first mover advantage.

Yuri Kifer introduces a financial security he calls a game option, that may be terminated by either the buyer or the seller [65]. Kifer uses the theory of optimal stopping games to calculate the price of such an option. It is significant because it shows how the threat of cancellation by the seller may encourage the buyer to exercise the option earlier than what was previously considered optimal. Smit and Ankum [107] consider an analogous real call option on production facilities. In a strictly options analysis, there is value to delaying the investment to see how market demand evolves. However, in the face of competition from other agents, Smit and Ankum note that the firm may be forced to invest early in order to protect their own returns.

Lukas et al. [79] use an option game-theoretic approach when investigating option value in mergers and earnout applications. Uncertainty is included in the game to impact optimal timing of the options exercise. Villani uses option game theory to inform decision making for firms investing in R&D in competitive environments. The inclusion of game theory allows one to consider both the positive (additional market share) and negative (additional information for the follower) aspects of first mover advantage.

Smit [106] uses game option theory to analyze the value of early investment in infrastructure to enable future expansion, focusing on European airports. Smit and Trigeorgis (2006, [108]) consider the impact of coordination and collaboration between agents on strategic option value. Smit and Trigeorgis (2007, [109]) also give a general review of how to use game option theory to value strategic options in competitive environments with interdependent decision making of agents.

These works greatly inform this research, and show that there is a significant intersection between real options and game theory. However, where these works have largely considered the impact of game-like decision interdependencies on the value of an option, this research will go one step further and investigate how the introduction of new options into an existing game can change the structure of the game or its equilibrium(s). Such an analysis framework would be useful for the Navy when considering the acquisition of assets with option-like characteristics which may be strategically deployed to change, or influence, their operating environment. **Where others have investigated the impact of the environment on the option, this research investigates the impact of the option on the environment.**

CHAPTER III

Prospect Theory-based Real Options Analysis

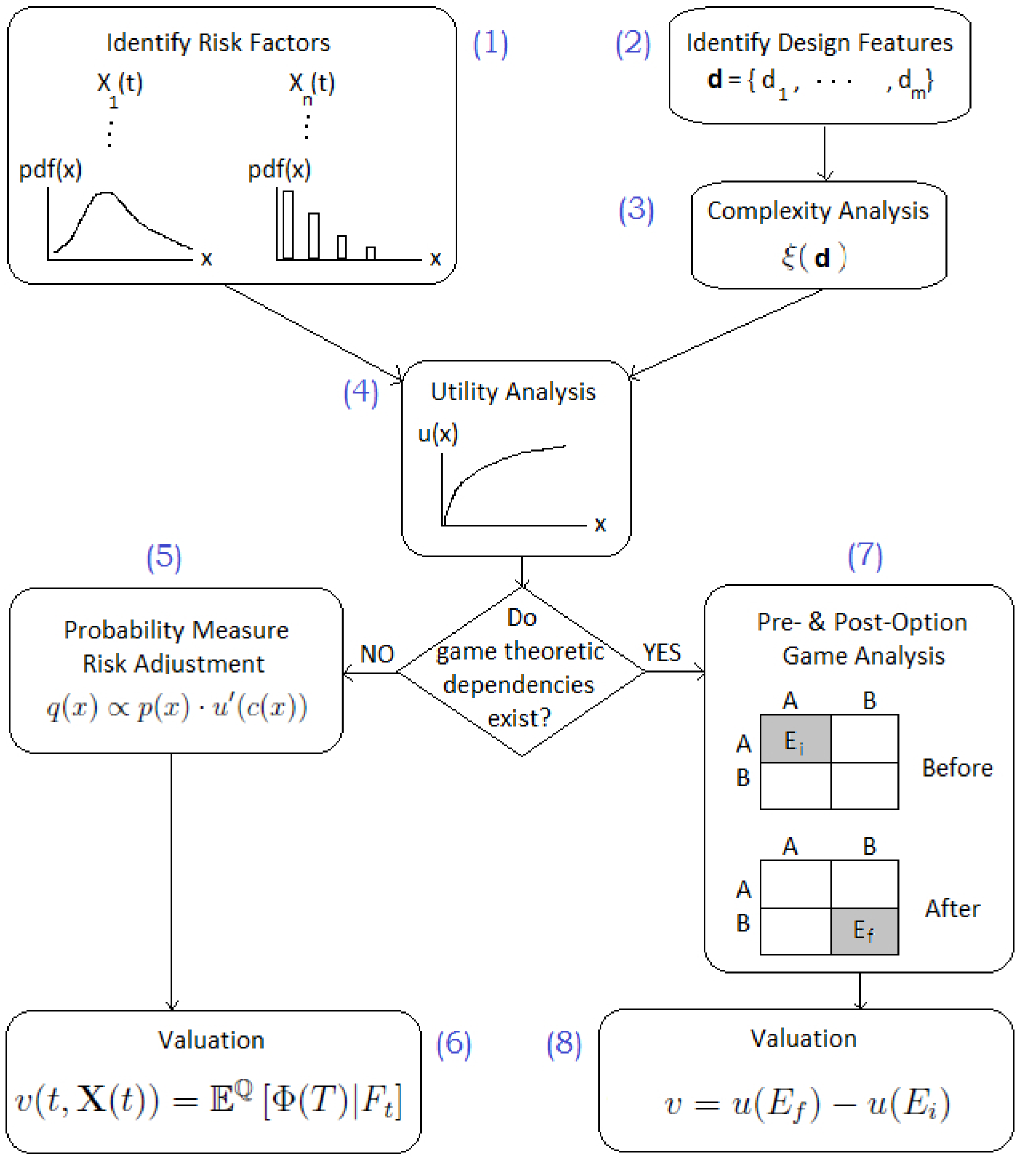

Figure 3.1: Process flowchart for PB-ROA framework

Figure 3.1 shows the flowchart for the eight steps of the presented framework, as discussed in section 2.2. To build a new framework for valuing real options for the U.S. Navy, let us begin by identifying the set of *risk factors* that a particular naval option is exposed to. These risk factors are random variables, and may also have a time component.

$$\mathbf{X}(t) = \{X_1(t), \ldots, X_n(t)\}$$

Let $\mathbf{X}(t)$ be the set of all risk factors which an option is exposed to. The individual risk factors, $X_i(t)$, may be either continuous or discrete. These risk factors may also be thought of as states of the world, and will effect the utility of a design. The set of relevant risk factors will depend on the application. For instance, technology readiness level (TRL) may be a relevant risk factor for an option involving new technological innovations. Likewise, the prices of different types of fuel may be relevant risk factors for a duel-fuel engine design. In practice, some risk factors, like fuel prices, will be easier to model than others, like TRL. There may be risk factors that simply do not lend themselves to quantitative modeling. This research addresses such issues on an application-by-application basis. This research attempts to create an option valuation framework that is independent of the models chosen for individual risk factors. While critically important for the future adoption of this research, the quantitative modeling of specific risk factors is considered to be a distinct avenue of research to that being presented here.

Let $p\left(t, \mathbf{X}(t)\right)$ denote the joint probability density function associated with the set of risk factors. This will often be referred to simply as the *real probability measure*, $\mathbb{P}$.

Next, in this research a system design may be broken down into a set of *design features* . For the purposes of this research, **design features are defined to be those physical attributes of a system which contribute to, or dictate, its performance.** This definition is purposely abstract, as this research is not specifically about naval ship design, but about how to value flexibility in a naval design. The specific definition of a design feature will likely change depending on the system or application under consideration. But, for example, a design feature may be as granular as the web and flange dimensions of a stiffener, or as large as a davit for launching autonomous vehicles. Let $\mathbf{d}$ be the set of all design features which influence the system's performance.

$$\mathbf{d} = \{d_1, \ldots, d_m\}$$

From the sets of risk factors and design features, this framework defines a system's *capability.* A system may have one, or many capabilities. For instance, the capability of a pump design, who's sole function is water delivery, could be measured in gallons per minute (gpm). However, the latest variant Aegis Combat System on the DDG-51 Arleigh Burke class guided missile destroyer (a system of systems) has multiple capabilities, such as radar tracking and guidance for cruise missiles (offensive capability), and ballistic missile defense (defensive capability). It's offensive and defensive capabilities may even be measured differently.

$$\mathbf{c}(\mathbf{d}, \mathbf{X}(t)) = \{c_1(\mathbf{d}, \mathbf{X}(t)), \ldots, c_r(\mathbf{d}, \mathbf{X}(t))\}$$

Let $\mathbf{c}(\mathbf{d}, \mathbf{X}(t))$ be the system's set of capabilities. The capabilities will depend not only on the design features, $\mathbf{d}$, but also on the state of the risk factors, $\mathbf{X}(t)$. For example, a radar system's capabilities may depend on the weather conditions (the risk factor).

From the set of design features this research also defines a *complexity metric* for the system, $\xi(\mathbf{d})$. This research assumes complexity is a known quantity, resulting directly from the set of design features. In other words, complexity is not random. Complexity will be important when comparing the utilities of alternative designs. Complexity will be discussed in greater detail in specific applications and case studies, but Rigterink et al. [97] offer one example of such a complexity metric for the design of stiffened panels. The construction complexity of a panel design is assessed based on a number of factors such as welding access, bracketing, and steps in plate thickness. It is believed that lower construction complexity leads to lower through life costs.

This research also assumes that a utility function may be quantified for each system under the option analysis. Utility is a measure of the total value of a system, and is a function of the system's design complexity and capabilities. Utility may also have a time component.

$$u = u\left(t, \xi, \mathbf{c}\right)$$

Let $u\left(t, \xi, \mathbf{c}\right)$ be the system's utility function, where t is time, ξ is the complexity metric, and $\mathbf{c}$ is the capability set. Although the system capability, $\mathbf{c}$, may be a vector, utility is a scalar function. Utility can have different units depending on the application, or may be unitless. This author believes that the elasticity of a measure like utility has many advantages over rigidly defined measures such as currency, which will allow more meaningful analysis of flexible systems for the Navy, where the cost of such systems is only one consideration. Utility will often be denoted simply by $u(c)$. When constructing a utility function for a naval application, this research requires that all such functions must abide by the following short list of naval utility axioms, in addition to the von Neumann-Morgenstern (VNM) axioms laid out in section 3.6 of [121]:

1. Utility is dependent on the state of the risk factors, via the capability function.

2. Higher capability is preferred over lower capability, given equal complexity.

3. Lower complexity is preferred to higher complexity, given equal capability.

With risk factors modeled, design features identified, capability and complexity quantified, and the utility function defined, it is finally possible to value an option. A European-style naval option has present value, v, of:

$$v(t, \mathbf{X}(t)) = \mathbb{E}^{\mathbb{Q}}\left[\Phi(\mathbf{X}(T))|F_t\right] \tag{3.1}$$

where $\Phi(\mathbf{X}(T))$ is the payoff of the option at terminal time T, given the prevailing state of the risk factors, $\mathbf{X}(T)$, and $\mathbb{E}^{\mathbb{Q}}$ signifies that the expectation is being calculated using the risk-adjusted probability measure, $\mathbb{Q}$. F_t is the filtration[1] on the risk-adjusted probability space, $\mathbb{Q}$, up to time t.

Similarly, the present value of an American-style naval option with payoff function, $\Phi(\mathbf{X}(\tau))$, is:

$$v(t, \mathbf{X}(t)) = \sup_{\tau \in [t,T]} \mathbb{E}^{\mathbb{Q}} \left[\Phi(\tau) | F_t \right] \tag{3.2}$$

where τ is the optimal stopping time for the option.

At first glance, these equations seem the same as the typical equations found in financial option valuation literature. However, there are two highly important differences. The **first** is that any discounting due to the time-value of the naval asset must be taken into consideration through the utility function. In other words, if a fixed quantity of a naval asset is worth more today than it will be in the future (as with money), then the utility function for the asset must perform the discounting. **Secondly**, and most importantly, the probability measure, $\mathbb{Q}$, which is used for the valuation is not the same risk-neutral measure as in standard options analyses. Typically, the risk-neutral measure is provided by the market thanks to a no-arbitrage argument. For naval options analysis, this research advocates that a risk-adjusted measure, also denoted by $\mathbb{Q}$, should be used and is formulated by re-normalizing the product of the real probability measure, $\mathbb{P}$, and the marginal utility. For a system with a scalar capability function (i.e.- the vector $\mathbf{c}$ contains only one element), the

[1] In simple terms, a filtration collects all of the information from time zero to the present, allowing conditional probability calculations to be performed.

risk-adjusted probability measure is given by,

$$q(x) \propto p(x) \cdot u'(c(x)) \tag{3.3}$$

where $q(x)$ is the risk-adjusted probability density for event x, $p(x)$ is the real probability density for event x, and $u'(c(x))$ is the marginal utility for event x. The marginal utility is,

$$u'(c(x)) = \left. \frac{\partial u}{\partial c} \right|_{c=c(x)} \tag{3.4}$$

For a system with a vector capability function, like the Aegis example, the marginal utility weight is a function of the magnitude of the gradient of the utility function.

$$||\nabla u(\mathbf{c})|| = \left. \left[\left(\frac{\partial u}{\partial c_1} \right)^2 + \ldots + \left(\frac{\partial u}{\partial c_r} \right)^2 \right]^{1/2} \right|_{\mathbf{c}=\mathbf{c}(x)} \tag{3.5}$$

However, for our purposes, rescaling of capability measures may be necessary for applications where the magnitudes of individual capabilities differ greatly. For example, consider an asset with two capabilities. The first, being related to deck space, is measured in square feet. The second, related to transit speed, is measured in knots. Deck areas are commonly on the order of 10^4 square feet, or more, for Navy assets. But speed is commonly below 40 knots. Due to the drastic difference in scale, the marginal utility weight in equation (3.5) would be dominated by the speed capability, as the marginal utility *per square foot* of deck area is negligible by comparison. For this reason each component of the gradient is normalized according to its maximum value.

$$u'(\mathbf{c}(x)) = \left. \left[\left(\frac{1}{dc_1^*} \frac{\partial u}{\partial c_1} \right)^2 + \ldots + \left(\frac{1}{dc_r^*} \frac{\partial u}{\partial c_r} \right)^2 \right]^{1/2} \right|_{\mathbf{c}=\mathbf{c}(x)} \tag{3.6}$$

Where

$$dc_i^* = \sup \frac{\partial u}{\partial c_i} \tag{3.7}$$

Despite the complications of vector capability functions, the general approach of using the marginal utility function to formulate a risk-adjusted measure for option valuation is supported by research in [33] [87] [59] and [8], but is also unique among the literature on valuing real options for naval applications. It is similar to the use of non-linear weighting functions in prospect theory [63]. Hence, the framework presented in this research is referred to as the prospect theory-based real options analysis framework (PB-ROA). Also in accordance with prospect theory, the payoff function, $\Phi(\cdot)$, is measured in the same units as the utility function, but the payoff is expressed *relative* to a reference point. The reference point will be specific to each application.

3.1 Constraints on Allowable Utility Functions

The reader will recall from the background discussion of options that *no-arbitrage* is an important concept in options analysis. In financial markets, and some real options markets, it is important to price options such that there are no arbitrage opportunities present. The existence of arbitrage offers the chance of positive gains, without any downside risk or capital expenditure. In short, arbitrage is a "free lunch." In such markets, when an arbitrage opportunity exists, it is because it is possible to actively trade a portfolio of other assets that will replicate the payoff from the option (recall that stock options are derivatives on the underlying stock price).

Since naval assets do not generate cash flows, and are not traded on a liquid market, it is not necessary to abide by this strict principle of no-arbitrage valuation for naval options. However, an important analog to no-arbitrage does exist, and cannot be ignored for the purposes of this research. This analog is *coherence*.

> "It has been shown that rational (coherent) behavior under uncertainty requires the existence of a supporting [risk-adjusted] probability distribution under which every acceptable transaction has non-negative expected value. If the additional assumption is made that beliefs and preferences are completely ordered - i.e., that between any two alternatives an agent can always assert a direction of weak preference - then it follows that the [risk-adjusted] distribution must be unique." - Nau, et al. (p. 210, [87])

In other words, there exist constraints on the allowable utility function. One constraint is that it cannot lead the agent to make decisions with negative expected payoff under the risk-adjusted measure (an assumption of minimal consequence). Another constraint is that for the value of the option to be unique, the utility function must be completely ordered (potentially less benign in some applications). However, even if the utility function is not completely ordered, the result will simply be a range of values for the option.

$$q(x) \propto p(x) \cdot u'(c(x))$$

Recall equation (3.3), show again above. One practical constraint the above discussion of ordering places on the utility function is that it cannot have a negative first derivative. That is, the marginal utility cannot be negative. This is a perfectly benign assumption for financial decision making, where it may reasonably be assumed that more wealth is always preferred over less wealth, and the utility function for wealth is thus fully ordered, as shown in the figure 3.2.

However, it may be possible to conceive of naval applications where more of a given asset may be preferred over less of that asset, only up to a certain amount. In other words, there may be an optimal amount of a particular asset. Currently, PB-ROA does not allow non-fully-ordered utility functions, such as the one shown in figure 3.3. At a fundamental level, such curves would lead to negative values in the risk-adjusted probability which is not possible. The true limitations that this constraint imposes in practice remain to be seen as it is yet unclear how many naval applications have

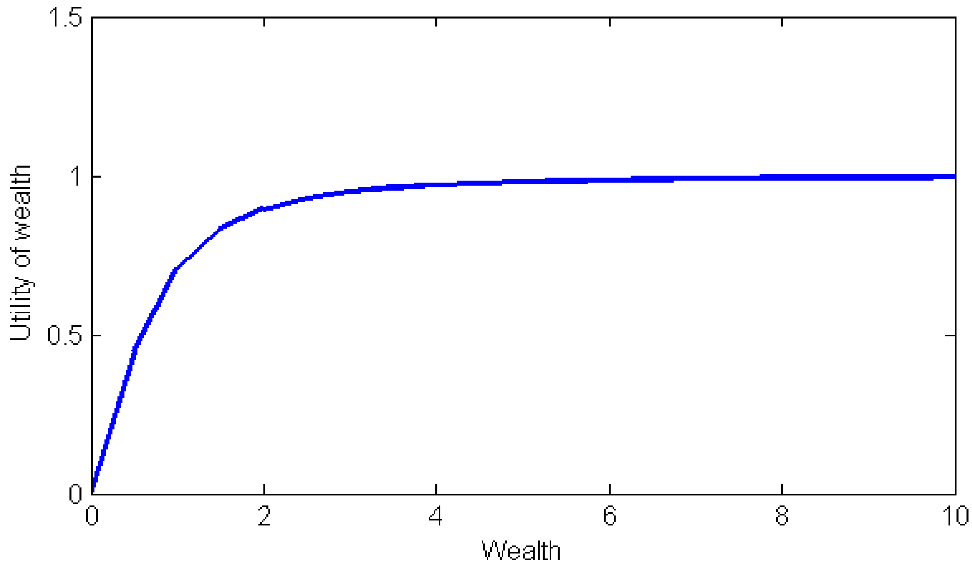

Figure 3.2: Non-decreasing utility of wealth

utility curves with negative marginal regions. However, in the unlikely event that a particular application exhibited a marginal utility curve with negative regions, it may be possible to simply limit the analysis to those regions with positive marginal utility, or otherwise transform for the problem. For example, if it were under the decision makers control, there would be little reason to extend the analysis of the curve in figure 3.3 beyond its apex.

3.2 Naval Options with Interdependent Decision Making: Games

The new framework discussed so far has only considered options with a one-way dependence on their environment - what this research terms *reactionary options*. Once a reactionary option is purchased, or created, the owner observes what changes occur in the environment as time passes, and then decides (at the appropriate time) whether or not to exercise the option. The assumption with reactionary options is that the existence of the option has no feedback effect on developments in the envi-

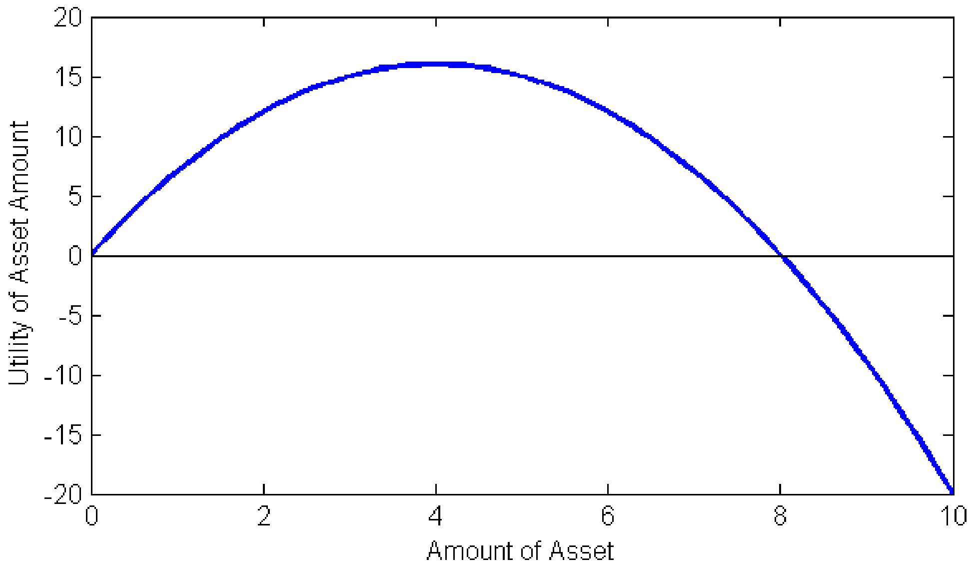

Figure 3.3: Example utility function which is not fully ordered

ronment. The dependence is one-way. This is the standard assumption with financial options, as well as virtually all real options analysis, where no single agent is capable of moving the market, in accordance with the efficient market hypothesis.

If, on the other hand, the value of the option relies on the interdependent decision making of multiple agents, then the preceding framework will not yield an accurate option value. A modified, game-theoretic approach is required. Such is the case with many naval options.

> **Proposition:** If a naval option exists in the presence of interdependent decision making of multiple agents, then the value of the option is equal to the change in the value of the Nash equilibriums of the games before and after the introduction of the option.

This may be illustrated through the following theoretical example. Consider that the status quo (before the introduction of the option to the environment) is reflected by the game in figure 3.4. This is, of course, very nearly the famous prisoner's dilemma

Player 1 \ Player 2	A	B
A	(−1, −1)	(−3, 0)
B	(−1.5, −3)	(−2, −2)

Figure 3.4: Status quo game with suboptimal Nash equilibrium

game. If both players were to choose strategy A, then the payoff to both would be -1. However, Player 2 has an incentive to change to strategy B, if they believe the other will play A. This leads both players to finally play strategy B, which is the Nash equilibrium of this game despite being suboptimal.

Now suppose that Player 1, in the process of devising a way to be able to safely play strategy A, is considering purchasing an option which would give her some form of leverage over Player 2 in the event that Player 2 did not also play strategy A. The option works as follows. Player 1 will begin by playing strategy A. If Player 2 also plays strategy A, then Player 1 will not exercise the option, which will result in a payoff of -1 for both players. However, if Player 2 chooses strategy B, then Player 1 will exercise the option, resulting in payoffs of -3, and -1.5 for Players 1 and 2, respectively. This is shown in figure 3.5.

Player 1 \ Player 2	A	B
A	(−1, −1)	(−3, −1.5)
B	(−1.5, −3)	(−2, −2)

Figure 3.5: Game with option. Game now has optimal Nash equilibrium

This game, after the introduction of Player 1's option, now has an optimal Nash equilibrium. According to the framework presented in this disseration, the value of the option in this example, v, is $v = -1 - (-2) = 1$.

The critical reader may argue that this research is simply changing the structure of the game. This is true. However, it is exactly this *game changing* attribute of many naval options which one is unable to value using standard real options analysis. One of the original contributions of PB-ROA is this perspective on naval options that may be game changing.

3.3 Elicitation of Utility Curves

One of the practical hurdles to institutionalizing the PB-ROA framework is how one determines the shape of the utility curve(s). The subject of utility elicitation is not the focus of this dissertation, however a brief treatment is merited since utility is a core concept of PB-ROA.

In the naval options thesis by Page [91], which makes use of multi attribute utility functions, an analytical hierarchy process (AHP) is used to rank design alternatives. AHP is commonly used in the U.S. Navy as a decision making aid to weigh the relative importance of many factors at once. Attributes of designs (ex: maintainability), and their relation to objectives (ex: reduced cost) are weighted in pairwise comparisons against other attributes and objectives. Then, design alternatives are ranked according to the weighted sum of their performance in each attribute-objective category. AHP may be a useful tool for mapping design features to sets of capabilities, and then weighing those capabilities against each other in a pairwise manner to determine an overall utility score. However, care should be taken because many AHP applications assume constant objective weights over the entire design space [125], which may lead to a linear utility surface implying that decision makers are risk-neutral. Since this is not true, modifications to the AHP process may be necessary.

A common method to elicit utility, from the area of experimental economics, is to present test subjects with a series of gambles, and systematically determine the subjects' certainty equivalent for that gamble. This means that subjects are presented with a gamble, for example a 50% chance at gaining $1 and 50% chance of nothing. Then, the subjects are offered certain payments (i.e. guaranteed payments), for example a certain payment of $0.25. This certain amount can be varied experimentally to determine the threshold at which the subject is indifferent between taking the certain payment, and taking the gamble. If a parametric form is assumed for the shape of the utility function (such as a power function), then a least squares regression from subject data can be used to estimate the parameters. When using this approach assuming prospect theory, however, things are more complicated. Since prospect theory states that people's decisions are made with a combination of utility and non-linear weighting of the probability of risky outcomes, the process of certainty equivalent testing is slightly more involved. Abdellaoui [1] and Abdellaoui et al. [2] give excellent overviews of how the process works under a prospect theory assumption. Their process involves separate testing of the gains and losses regions. What is significant about their work, however, is that they propose ways to elicit the utility curves without resorting to parametric assumptions.

It is highly likely that accurately eliciting utility curves for the U.S. Navy would require integration of the above approaches (or some hybrid method) with war game simulation. Since war games simulation is outside the scope of this dissertation the subject is left for future work. However, it is likely that determining the non-linear shapes of utility curves would be incomplete without some integration of information from war games simulation.

3.4 Test Case: Hospital Variant of a High Speed Connector

This section presents a simplified example intended to demonstrate how the PB-ROA framework for valuing naval options described in the opening section of this chapter can be used to generate useful insight for decision makers in the design of a vessel. The vessel considered in this example is a theoretical high speed connector (HSC) vessel.

The proposed primary role for this vessel is as a high speed connector, transporting personnel and material of both combat and non-combat natures quickly between and within theatres. It is specifically designed for agile maneuvering in the littorals, compatibility with austere ports, and rapid reconfiguration of its large and open cargo area. The concept of operations for this HSC includes both wartime and peacetime roles for personnel and material transport, such as humanitarian aid and disaster relief (HADR).

In addition to its primary role as a high speed connector, a secondary mission as a fast response medical support ship is being proposed. For this mission the HSC would temporarily install portable medical facilities in its cargo area. The important design consideration is how to best implement such a hospital variant, as the necessary medical equipment will have considerable space, weight, and power generation impacts on the HSC.

For the purposes of this example, let us consider that two design alternatives are being proposed. Let the first alternative ("variant 1") be an unaltered HSC, where all medical facilities and supporting equipment are installed modularly on an on-demand basis. This design accommodates 150 beds for patients, after allocating space for the additional electrical power generators required by the mission. Let the second alternative ("variant 2") be an altered HSC, with permanent extra electrical power generation capacity built into the vessel. The extra power capacity is unnecessary for the connector mission, but can provide the necessary power for the medical mission. While such preinstalled power capacity increases the complexity of the design, it comes with the benefit of more space for beds for patients, 200 beds. For the purposes of this example, the only significant capability difference between variants is the number of beds.

Which design alternative should the decision maker choose? For this example, the relevant risk factor, $X(t)$, is the number of HSC's that will be configured for the medical mission at any given time. If we let n be the existing HSC fleet size (of variant 1 type), and m be the number of new HSC's being acquired, then it is possible to model the risk factor using the binomial distribution,

$$p(x;\alpha) = \binom{n+m}{x} \alpha^x (1-\alpha)^{n+m-x} \tag{3.8}$$

where α is the probability of any one vessel being configured for the medical mission. This is the real probability measure, $\mathbb{P}$. For this example it is also assumed that:

1. Variant 1 has capacity of $c_1 = 150$ beds, and has complexity metric, $\xi_1 = 1$.
2. Variant 2 has capacity of $c_2 = 200$ beds, and has complexity metric, $\xi_2 = 1.05$.
3. The U.S. Navy's utility functions for each design alternative, in this theatre, are captured by;

$$u_1(c(x)) = \frac{1}{\xi_1} - e^{-ac_1x} \tag{3.9}$$

$$u_2(c(x)) = \frac{c_2}{c_1}\left(\frac{1}{\xi_2} - e^{-ac_2x}\right) \tag{3.10}$$

where $a = 1.7 \times 10^{-3}$, and x is the volume of beds provided by the fleet of HSCs. These are exponential utility curves, and are frequently used in the economics literature. The parameter a controls the steepness of the utility curve, and is chosen somewhat arbitrarily for this example. This value of a means that 15 vessels will provide approximately 98% of the maximum possible utility of HSC medical capabilities. **The U.S. Navy's true utility curve may be different. What is important for this example is that the utility curves capture the diminishing marginal utility of bed capacity.**

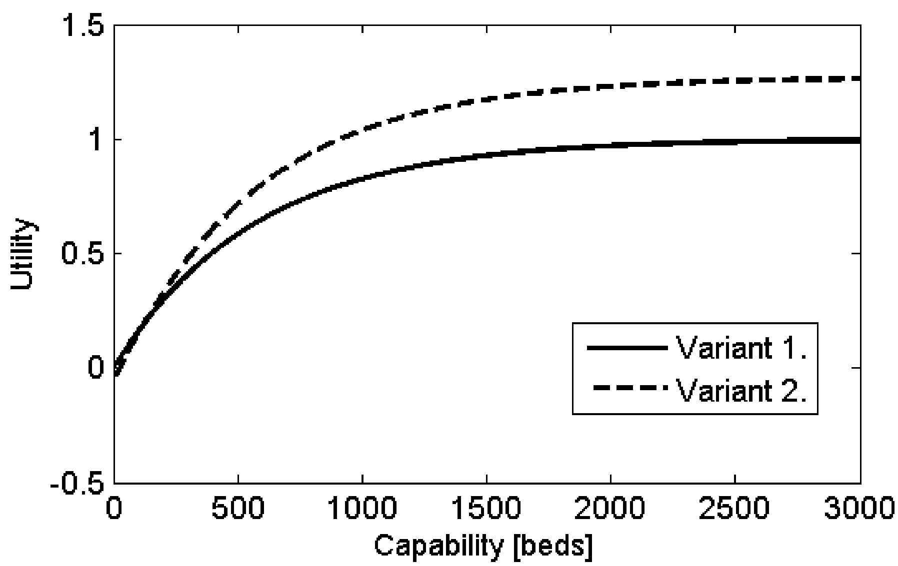

Figure 3.6: Utility function for HSC hospital design alternatives

As can be seen by figure 3.6, variant 2 has the greater maximum utility. However, it also has negative utility when not in use, reflecting the increased cost and maintenance of a more complex system that is unused. The risk-adjusted measure, $\mathbb{Q}$, for each variant is then found by,

$$q_1(x) \propto \binom{n+m}{x} \alpha^x (1-\alpha)^{n+m-x} * a * e^{-ac_1 x} \tag{3.11}$$

$$q_2(x) \propto \binom{n+m}{x} \alpha^x (1-\alpha)^{n+m-x} * \frac{ac_2(x)}{c_1(x)} * e^{-ac_2 x} \tag{3.12}$$

where it is necessary to re-normalize each q_i such that it integrates to one. Finally, it is possible to calculate the value (the expected utility), under the risk-adjusted measure, of the expanded fleet of HSCs for each of the proposed variants.

$$v_i = \mathbb{E}^{\mathbb{Q}}\left[v_i(c_i(X(t)))\right] \tag{3.13}$$

where the value function is given by

$$\begin{aligned} v_i(x) &= u_i(c_i(x)) - U_{ref,i} && (3.14) \\ U_{ref,i} &= \mathbb{E}^{\mathbb{Q}_i}\left[u_1(c_1(X(t)))\right] && (3.15) \end{aligned}$$

The reference point is, in other words, the $\mathbb{Q}$-expected utility of the starting fleet of vessels. Such calculations can be performed for all values of $\alpha \in [0, 1]$, and it is possible to find the critical probability, α^*, at which the decision of which variant to choose changes. This is shown in figure 3.7, for the case of $n = 0$, and $m = 1$, where the critical point is approximately $\alpha^* = 50\%$. This means that if it is believed that

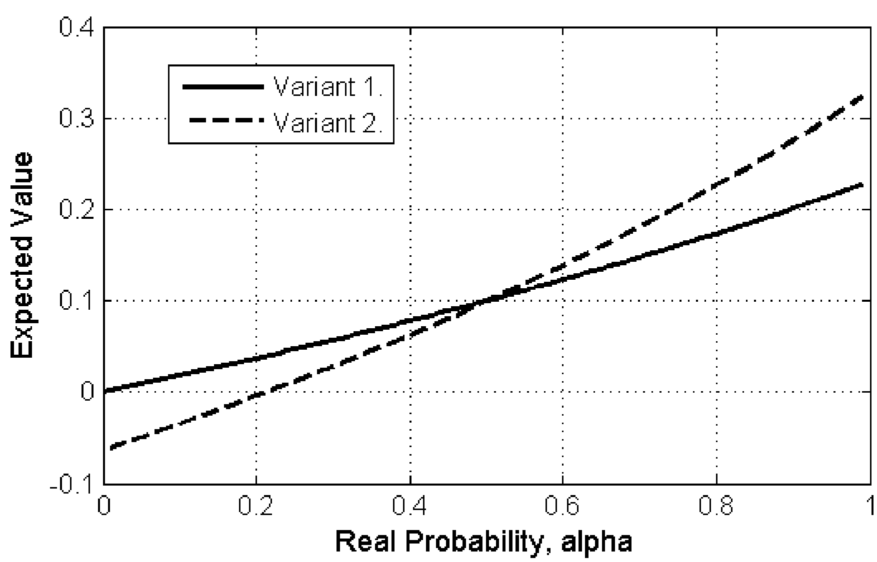

Figure 3.7: Expected utility of HSC fleet for the medical mission; n=0, m=1.

the probability of an HSC being configured for the medical mission at any given time is less than 50%, then variant 1 should be chosen (the variant without the preinstalled generators). However, if it is believed to be greater than 50%, then variant 2 should be chosen.

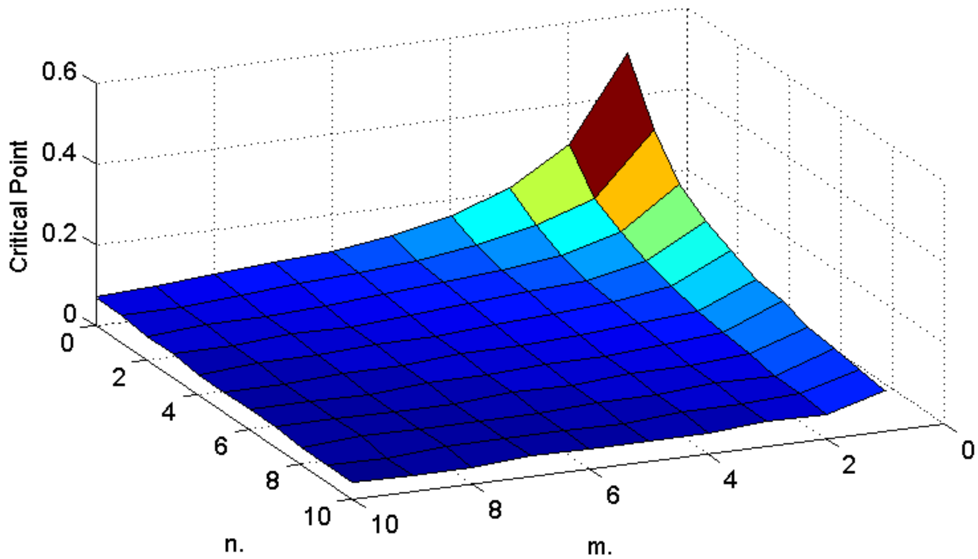

Figure 3.8: Variation in probability decision threshold, α^*, with fleet size using PB-ROA

How the value of α^* changes as a function of total fleet size is shown in figure 3.8. Of course, this surface will change based on the shape of the utility curves. The results of this demonstration do not apply to any actual HSC program. The results of this highly simplified example are intended solely for demonstration purposes. However, it is possible to use this example to make important observations about the PB-ROA valuation framework. For instance, within the PB-ROA framework, it is possible to analyze how one's decisions regarding flexible assets might vary with initial resources, the number of assets being proposed to acquire, and assumptions about operating conditions.

Since one of the original contributions of this research is the inclusion of loss aversion through prospect theory, then it is interesting to compare the results from PB-ROA to what would be had from a traditional expected utility approach. One major difference between the appoaches is that expected utility methods use the real probability measure, $\mathbb{P}$, instead of the risk-adjusted measure. In an expected utility approach, all risk aversion is assumed to be captured in the shape of the utility curve. A cross section of the surface in figure 3.8 is taken for the case of $n = 0$ (current HSC fleet size equal to zero). Figure 3.9 shows how the critical value of α varies depending on the number of HSCs acquired. Both methods result in the same general trend; the critical value decreases exponentially as the fleet size increases. However, the threshold value is always higher for PB-ROA by a magnitude of 3-10%. The difference between the curves reflects the *loss premium* that decision makers require. Both PB-ROA and expected utility approaches capture aversion to uncertainty. But only PB-ROA, by its use of prospect theory to adjust the probability measure, captures the added concern of loss aversion.

To simplify future analyses, the critical reader may suggest that some margin simply be added to the results of an expeced utility analysis to account for loss aversion. Since PB-ROA is a more complicated analysis framework than expected utility, such an approach might be appealing if it were possible. However, PB-ROA does not always result in the same trend as expected utility analyses, as it did in this example. In cases with severe loss aversion, PB-ROA and expected utility may yield divergent results. To illustrate this possibility, the analysis of the hospital variant of the HSC will be repeated with a different assumption about how vessels are deployed.

In the preceeding results, it was assumed that vessels may be deployed on the medical mission on an individual basis. In this way, the relevant risk factor was the number of vessels needed at any given time, which was modeled by a binomial distribution. Let

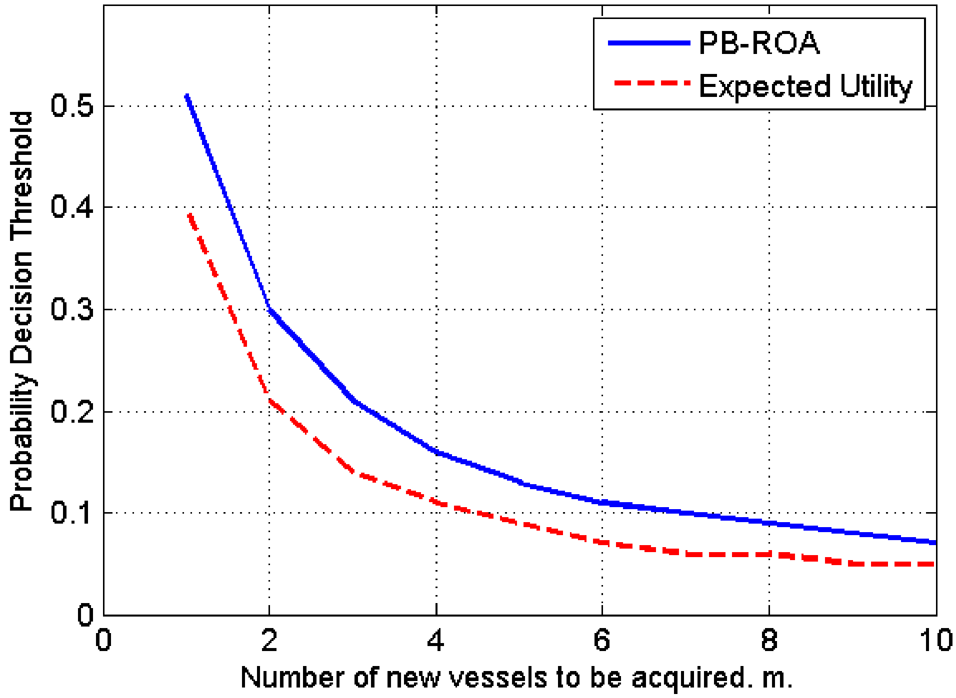

Figure 3.9: Comparison of the variation in probability decision threshold, α^*, with fleet size between PB-ROA and an Expected Utility approach, for $n = 0$.

this assumption be known as the “variable deployment assumption.” For the sake of illustration, let us now assume that vessels must be deployed together, regardless of the fleet size. This assumption will be known as the “all-or-nothing assumption.” Under the all-or-nothing assumption, the relevant risk factor is reduced to a Bernoulli random variable; either the fleet is deployed on the medical mission, or it is not.

If the real probability of deployment is p, then to get the risk-adjusted measure, first define intermediate variables for each variant.

$$t_1^u = p \cdot u_1'(c_1(n+m)) = p \cdot a \cdot e^{-ac_1(n+m)} \quad (3.16)$$

$$t_1^d = (1-p) \cdot u_1'(0) = (1-p) \cdot a \cdot e^0 \quad (3.17)$$

and

$$t_2^u = p \cdot u_2'(c_2(n+m)) = p \cdot a\frac{c_2}{c_1} \cdot e^{-ac_2(n+m)} \tag{3.18}$$

$$t_2^d = (1-p) \cdot u_2'(0) = (1-p) \cdot a\frac{c_2}{c_1} \cdot e^0 \tag{3.19}$$

Where the superscript u denotes when the medical capabilities are utilized, and d denotes de-utilized. Then the $\mathbb{Q}$-measures for each variant become:

$$q_1 = \frac{t_1^u}{t_1^u + t_1^d} \tag{3.20}$$

$$q_2 = \frac{t_2^u}{t_2^u + t_2^d} \tag{3.21}$$

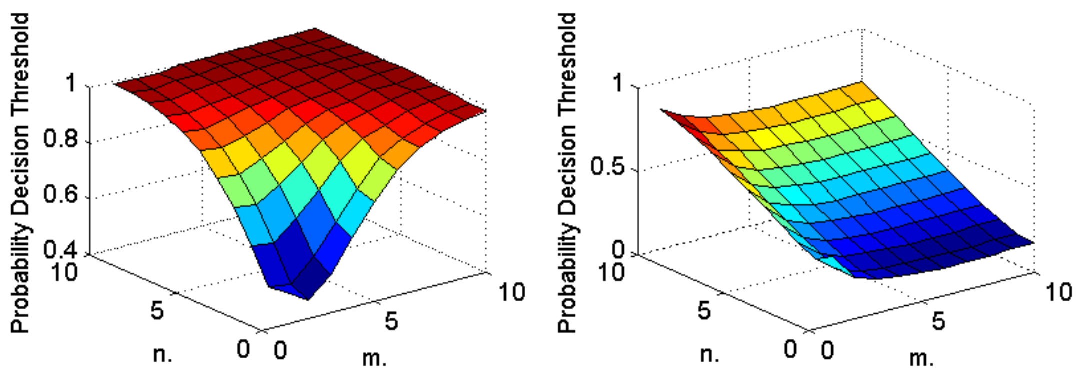

Figure 3.10: Variation in probability decision threshold using PB-ROA (left), and Expected Utility Theory (right), under the "all-or-nothing" assumption.

Under the all-or-nothing auumption, the decision threshold approaches 100% asymptotically in both n and m, as shown in figure 3.10 (left). What is significant to note is that this positive asymptotic relationship does not express itself in a standard expected utility analysis. The probability decision threshold according to an expected utility analysis is shown in figure 3.10 (right). In this case, the threshold increases approximately linearly in n, and is slightly quadratic in m.

To highlight the differences between the standard expected utility method and the PB-ROA framework presented in this research in more detail, figure 3.11 shows the cross section of the surfaces in figure 3.10, for the case of $n = 0$. The two approaches no longer result in the same trend, as they did under the variable deployment assumption. In fact, the results are completely divergent.

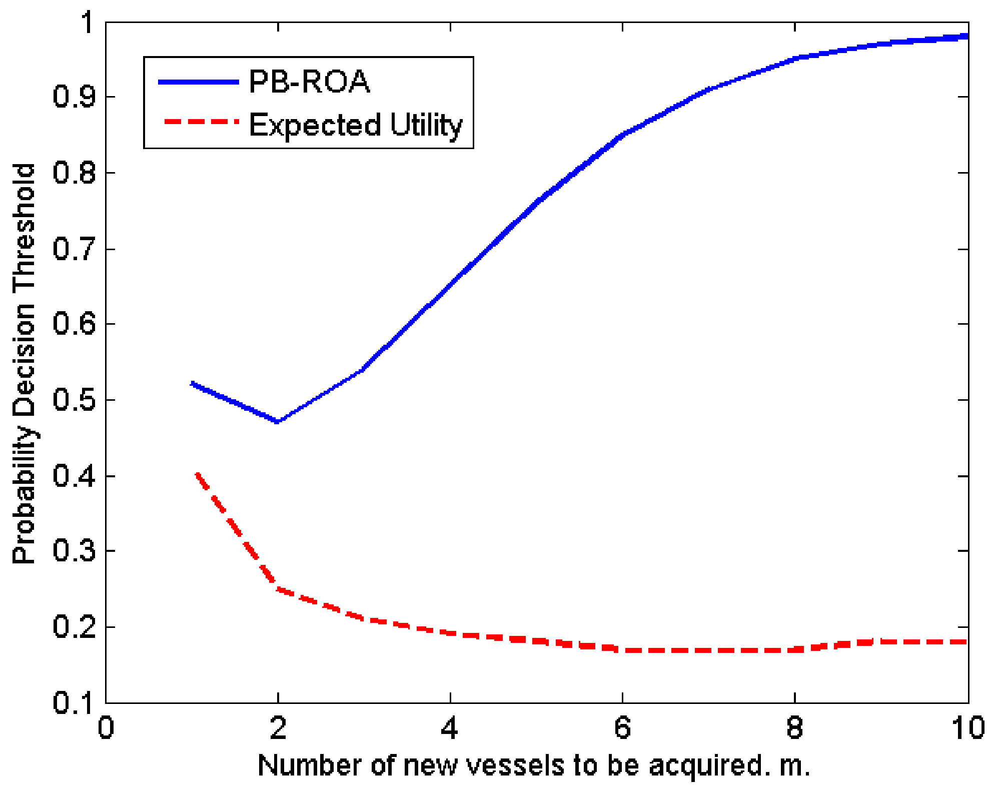

Figure 3.11: Variation in probability decision threshold over m for PB-ROA and Expected Utility methods, $n = 0$.

The explanation for this is that the all-or-nothing assumption has greater perceived loss than the variable deployment assumption. In this example, loss is the perceived loss of underutilizing an asset. The perceived loss is even greater when underutilizing the upgraded variant. Under the variable deployment assumption, any upgraded variants in the fleet could be deployed first in order to speed up recoupment of the "investment" in the upgrades. For example, if two ships are required in 2015 on the medical mission, and the fleet consists of four variant 1 vessels, and two variant 2 vessels, then the variant 2 vessels could be deployed to exploit their upgrades. If three vessels were required, they could be augmented with another variant 1 vessel. In other words, it is possible to increase utilization of the upgraded variants under the variable deployment assumption, which will mitigate some of the effects of loss aversion. However, under the all-or-nothing assumption, such mitigation is impossible. If the fleet is not deployed on the medical mission, the perceived loss is high because *all* of the vessels are underutilized simultaneously. It is not the point of this dissertation to argue the validity or fidelity of either the variable or all-or-nothing assumptions. But they are important from a research perspective because they highlight a significant difference between the PB-ROA framework and existing expected utility methods. One reason PB-ROA has such potential benefit as a decision tool, is because the implications of loss aversion may not be known *a priori.*

It is plain to see that these two methods may lead to drastically different conclusions. This is especially true for applications where loss aversion is prevalent. Finally, to highlight the similarity with prospect theory, figure 3.12 compares the shapes of the weighting function from prospect theory with the PB-ROA framework. The differences at the endpoints arise from the constraint in PB-ROA that q must be a true probability measure.

3.5 Contribution

To summarize the contents of this chapter, the following list of original contributions is offered:

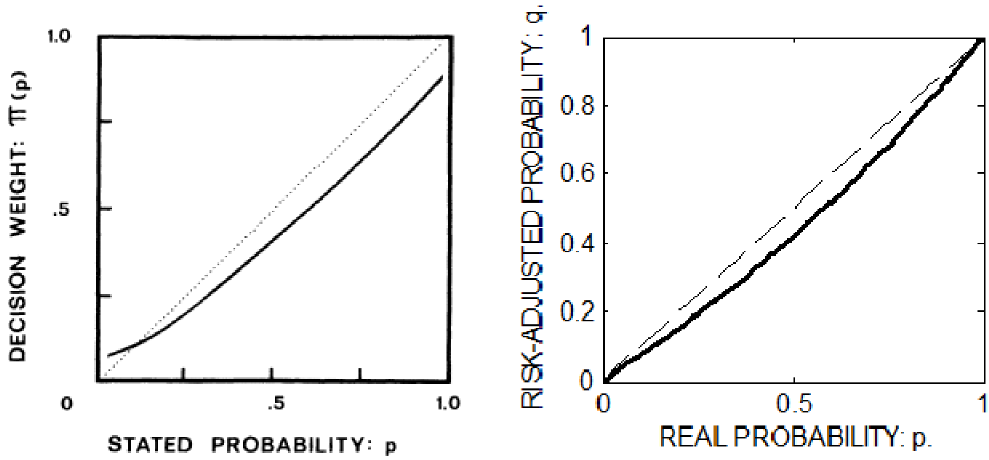

Figure 3.12: A hypothetical decision weight in prospect theory, from [63] (left) and the risk-adjusted measure, q, for variant 2 (n=0, m=1) from PB-ROA (right).

1. An analytical framework for valuing flexible systems and architectures in the absence of a market and cash flows, rooted in utility, and suited for a wide range of Naval applications.

2. A systematic method for risk adjustment which also includes loss aversion, based on marginal utility curves. With this, it is no longer necessary to assume that agents are risk-neutral, as is currently common.

3. A game theoretic perspective on Naval options which includes the possibility that the introduction of the option may change the environment, resulting in a different game structure and, possibly, equilibrium.

CHAPTER IV

Case Study: An Option to Extend Service Life

4.1 Introduction

In chapter II the need for a repeatable framework was discussed that determines the value of naval options from sets of capabilities - not monetary cost or cash flows. The theory powering the prospect theory-based real option analysis (PB-ROA) framework, which forms the backbone of this dissertation, was presented in chapter III. This chapter offers an in depth case study demonstrating the principles of the PB-ROA framework and the types of insights it can yield for naval decision makers in early stage design.

The case study presented in this chapter adds to the burgeoning body of research on real options "in" projects, as defined in the work of Wang and DeNeufville [123, 124]. Real options "in" projects differ substantially from options "on" projects because they cause physical changes to be made to the design of an engineering system to achieve greater flexibility. Many naval real options fall into this category of real options that result from physical changes to the design of the asset. If one views Naval systems design as a process of negotiation between *capability* and *complexity* then it becomes apparent that the flexibility granted from a real option will inevitably impact the design, and hence the complexity, of the greater system. Within PB-ROA, the balance between capability and complexity for an asset is what gives it its utility. One may then measure risk by the variation in utility of an asset.

Because many naval real options exist *in* an interdependent system, it becomes even more important to integrate their analysis with early stage design efforts where the freedom to exploit physical design change opportunities is greatest. This is true for modular systems and architectures which may offer substantial flexibility but require considerable structural modifications. Capt. N. H. Doerry (USN) notes,

> "While many [modular adaptable ship] technologies have been available for many years, and in many cases have been installed onboard ships in a ad hoc manner, a design methodology does not currently exist to establish a sound technical basis for determining how much of what type of modularity to install on a ship." [38]

The author contends that this problem is not limited to modular technologies, but also applies to general design features which enable flexibility. For instance, under the ship-as-a-truck paradigm [37], the performance of the ship structure itself becomes critically important because every other system is housed by the ship structure and hence dependent on it. As Collette notes,

> "[F]or naval structures, the structural system is typically supporting an investment of weapons, sensors, machinery, and other vessel systems worth many times the value of the structure itself but effectively permanently tied to the structure." [27]

Two potential measures of a ship's structural performance are availability and cargo capacity[1]. The degree of flexibility provided by a ship structure is dictated (at least in part) by the combination of its ability to carry the demanded cargo and be available, where structural availability is related to fatigue and cracking of structural

[1]Basically, how much the ship can carry before it exceeds its design displacement or becomes unstable.

members that may prevent the ship from going to sea or otherwise completing its mission, as studied by Hess [56].

To demonstrate the principles of PB-ROA and the types of insights possible with its use, the value of a real option to extend the service life (ESL option) of a ship is investigated. The ship is a high speed military catamaran with aluminum structure, making fatigue and cracking a critical issue. In this study, the ESL option may be "purchased" by making enhancements to the structural design of the strength decks which reduce the expected number of cracks over its lifetime. The resulting increase in structural weight is considered by PB-ROA to expose partitions in the design space and the conditions under which one candidate structural design can be said to maximize flexibility (option value) over another candidate design.

4.2 Background and Related Work

While steel structures will also suffer from fatigue cracking under repeated stress, the problem is generally considered to be more severe for aluminum structures. Because fracture mechanics are highly sophisticated and subject to stochastic unknowns, the industry standard practice for many years has been to establish rules which designs must satisfy. An example of such rules is the "Guide for Building and Classing High Speed Naval Craft" formulated by the American Bureau of Shipping [4]. While some rules may, at times, appear to be arbitrary, their intention is to provide a set of best practices for safe vessel designs.

One limitation of these rules is that the rules alone cannot give a designer or decision maker any indication of the added value of exceeding any of the requirements put forth in the rules. In some applications, there may be a benefit to exceeding the requirements set by the rules. Such analysis, however, takes a shift from requirements-thinking to performance-thinking, which is not a typical approach to structural design. In his doctoral dissertation, Hess [56] develops a reliability-based, operational performance analysis framework for naval ship structures. Hess defines three new performance metrics for structures; capability, dependability, and availability. Operational capability of the structure relates to the probability of countering a threat, or performing the mission. Operational dependability is the probability that the system can complete its mission once it has successfully started. Operational availability is the probability that the system will be fully functional when needed. Such performance metrics for structures have begun to be used to examine tradeoffs from a full lifecycle perspective by [27, 98, 113] and others.

A critical tool for enabling the shift to performance-thinking for structures is stochastic fatigue analysis. Fatigue analysis is used to evaluate when a structural element will crack or fail under stress. Fatigue analysis is a large area of academic research. A thorough literature review on the subject is outside the scope of this dissertation. However, the interested reader is referred to review of the state of the art in fatigue analysis by Fricke for a summary of the field [43]. For this dissertation it suffices to explain that fracture of a structure is typically divided into three phases; the crack initiation phase, the crack propagation phase, and failure of the structure. In this case study, only the crack initiation phase will be considered - the number of stress cycles applied to a structural detail before a crack first forms. While there are many methods for determining the time to crack initiation, this case study relies on a *nominal stress* approach. Under a nominal stress analysis, the standard $S - N$ curve for a material is translated based on classes of basic joints which are cataloged in several standards and guidelines, such as the International Institute of Welding [57], and others [18].

4.3 Case Study Problem Formulation

The platform under consideration in this case study is a fictional military high speed aluminum catamaran. It's primary mission is as a fast intratheatre transport for troops and materiel. Conceptually, it is inspired by the Spearhead-class Joint High Speed Vessel (JHSV) [100]. However, the case study is purely the perspective of the author and is not reflective of the actual JHSV program.

The typical service life for this class of vessel is assumed to be twenty years. However, decision makers intuitively understand there is value in having the flexibility to extend the service life of vessels beyond twenty years. In the early design stages for this high-speed catamaran, the U.S. Navy is considering investing in a real option that would enable them to extend service life (ESL option) by an additional five years, if desired. Purchasing this flexibility *may* come at the cost of structural modifications to the vessel and increases in complexity.

A critical limiting factor when considering the service life extension of a high-speed aluminum catamaran is cracking of the structure. This concern is elevated by the desire to minimize the weight of the structure for high-speed catamarans which are highly sensitive to exceeding design displacement. Exceeding design displacement results in dramatic increases in hydrodynamic resistance which can cause a vessel to fail to meet such requirements as speed. Likewise, minimizing structural weight allows for increased cargo capacity. Even within the base service life, however, fatigue cracking is also an important consideration because it impacts the vessel's availability. Cracks must be fixed, which often requires time in ports with the facilities capable of performing such repairs. The time spent in port, as well as the time spent in transit to and from the port, remove the vessel from performing its mission.

This illustrates a potential conflict between two important capabilities of this high-speed catamaran; *availability* and *cargo capacity* . Designs which maximize availability (minimize cracking) will tend to be heavier, thus carrying less cargo. Designs which maximize cargo capacity will tend to be lighter (for a constant displacement), thus exhibiting more cracking. This case study examines the tradeoff between cargo capacity and expected availability for a high-speed aluminum catamaran. More importantly, we evaluate how early stage structural design decisions impact the value of the option to extend service life. The case study illustrates how PB-ROA can be used to generate insight on the conditions in which additional section modulus for the midship section of a vessel, above and beyond that required by regulatory institutions like the American Bureau of Shipping (ABS), may be beneficial to maximize the overall flexibility of the vessel.

The case study uses a simplified structural model in place of a complete midship section for the catamaran. The structure used models two parallel strength decks in the catamaran, as stylized in figure 4.1. It is transversely framed, and represents the large open decks of a roll-on-roll-off (RO-RO) ferry with longitudinal stiffeners, transverse frames, and longitudinal and transverse girders welded to flat plate. This case study also requires many other input models for the PB-ROA framework, as shown in figure 4.2. None of the models used in this case study are perfect. In fact, many are highly simplified. However, it should be stressed that all models are interchangeable. The PB-ROA framework, and the types of insights will remain the same regardless of the models.

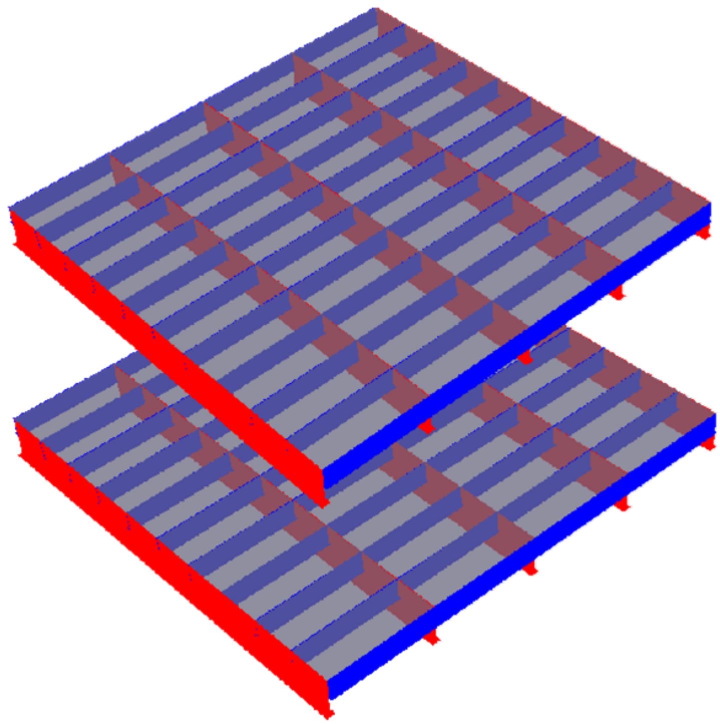

Figure 4.1: Stylized representation of the stiffened panel structure.

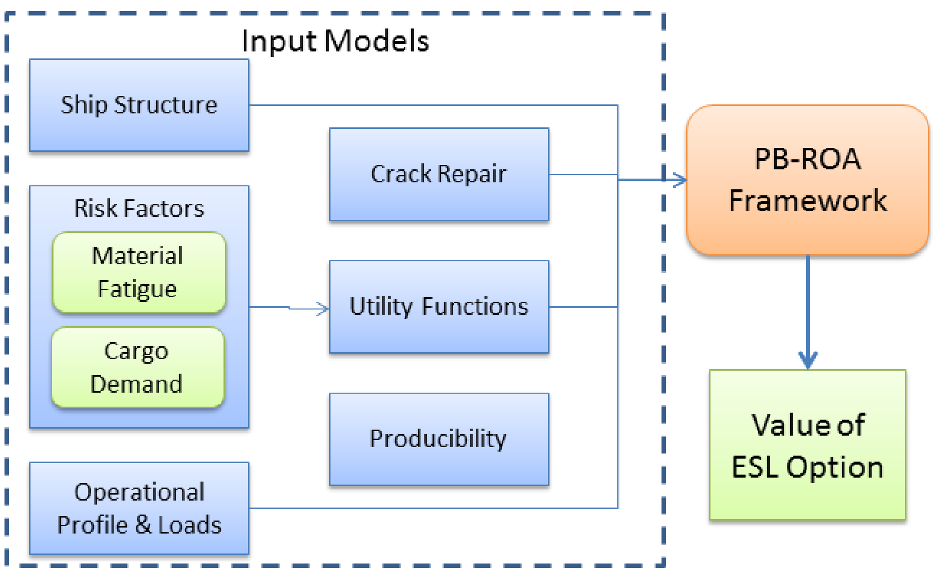

Figure 4.2: Many input models to the PB-ROA framework for this study.

4.3.1 Risk Factors

The first step in the PB-ROA framework is always to identify and quantify the relevant risk factors for the application. For the purposes of this illustrative case study, those risk factors, X_i, are:

$$X_1(j) = \text{Time to crack initiation of welding detail } j$$

$$X_2(t) = \text{Weight demand for cargo in year } t$$

The time to crack initiation for a each welding detail is relevant because the number of cracks in any given year will directly effect the vessel's availability. For this study, a welding detail is defined to be the intersection of any longitudinal and traverse members of the grillages, to include intersections with girders. Following the work of Collette [26], Temple and Collette [113], and others, a lognormal distribution is used to model the stochastic nature of crack initiation. The general shape of a lognormal distribution with mean 0.5 and standard deviation 0.3 is shown in figure 4.3. The lognormal is an asymmetric bell-shaped curve that is always positive. Under a conventional S-N fatigue life approach, with a Palmgren-Miner cumulative damage rule included, the number of cycles to crack initiation of a welding detail is:

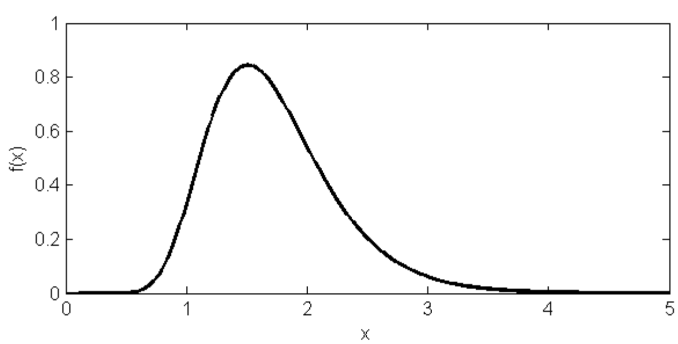

Figure 4.3: Probability density function for a lognormal distribution with mean 0.5 and standard deviation 0.3.

$$N = \frac{D_{cr} A}{(k_f \Delta\sigma)^m} \tag{4.1}$$

Where N is the number of cycles to crack initiation under an oscillating stress range $\Delta\sigma$, D_{cr} is the cumulative damage rule, k_f is the stress intensity factor for the weld detail, and A and m are experimentally determined parameters related to the material. $\Delta\sigma$ and m are assumed constant. By acknowledging the stochastic nature of the parameters A, D_{cr}, and k_f the number of cycles to crack initiation also becomes stochastic. Common practice is to model A, D_{cr}, and k_f each with lognormal distributions which results in N also being lognormal. The probability density function for $X_1(j)$ is then:

$$f_1(x_1) = \frac{1}{\zeta\sqrt{2\pi}x_1} \exp\left(-\frac{(\ln(x_1) - \lambda)^2}{2\zeta^2}\right) \tag{4.2}$$

Where x_1 is the number of cycles to crack initiation with mean λ, and standard deviation ζ. The values for the mean and standard deviation are given by:

$$\lambda = \lambda_{D_{cr}} + \lambda_A + m(\lambda_{k_f} + \ln \Delta\sigma) \tag{4.3}$$

$$\zeta = \sqrt{\zeta^2_{D_{cr}} + \zeta^2_A + \zeta^2_{k_f}} \tag{4.4}$$

The weight demand is the second risk factor in this study because of the sensitivity of high speed catamarans to displacement excesses. If the demand for cargo weight is greater than the design's allowance, then some fraction of the cargo must be transported by other means; a second catamaran, increased sorties, or some other method of transport. This could have drastic implications for mission effectiveness. For this study, a triangular distribution is chosen to model the stochastic demand for cargo weight in each year of the vessel's service life. Triangular distributions are commonly used for applications with sparse data to calibrate other distributions. It is also chosen to illustrate that PB-ROA is not constrained to using only lognormal distributions as is sometimes misunderstood in other applications of real options that use a Black-Scholes formulation. PB-ROA can accommodate *any* shape of distribution. The probability density function for the triangular distribution for weight demand, $X_2(t)$ is given by:

$$f_2(x_2) = \begin{cases} 0 & \text{for } x_2 < a, \\ \frac{2(x_2-a)}{(b-a)(c-a)} & \text{for } a \le x_2 \le c, \\ \frac{2(b-x_2)}{(b-a)(b-c)} & \text{for } c \le x_2 \le b, \\ 0 & \text{for } b < x_2. \end{cases} \tag{4.5}$$

Where a, b, and c are left limit, right limit, and mode of the triangle, respectively. The shape of this probability density function is shown in figure 4.4 for parameters $a = 0$, $b = 4$, and $c = 3$.

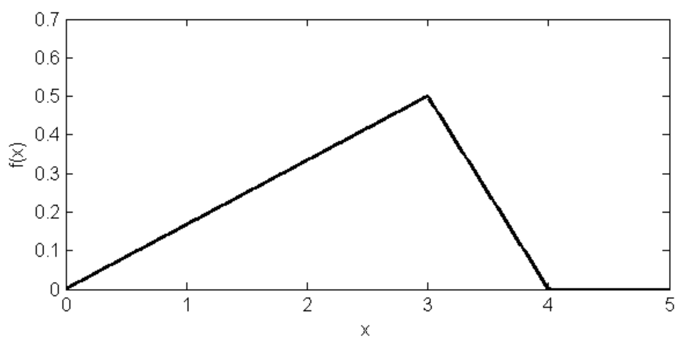

Figure 4.4: Probability density function for a triangular distribution with $a = 0$, $b = 4$, and $c = 3$.

It is assumed that the time to crack initiation for each detail is independent and identically distributed; $X_1(j)$ are i.i.d. $\forall j$. Likewise, the demand for cargo weight each year is assumed to be i.i.d. Finally, the simplifying assumption is made that X_1 and X_2 are independent.

While one may question the fidelity of the assumption of independence between cargo weight and cracking, the impact on the PB-ROA framework is purely numerical. That is, one could model the correlation between these two variables using Copula functions and solve for the option value through Monte Carlo simulation [25, 75, 10]. The result will differ from that to be presented here, of course. But the PB-ROA framework is unchanged. Only a different set of numerical tools must be employed.

4.3.2 Design Features

For this study, the relevant design features are those related to the structural design of two strength deck grillages near the midship of the catamaran. A grillage is defined as a collection of longitudinal and transverse beams welded to flat plate in a grid assembly, which is stylized in figure 4.5. Longitudinally, there are stiffeners and girders. Transversely, there are frames and girders.

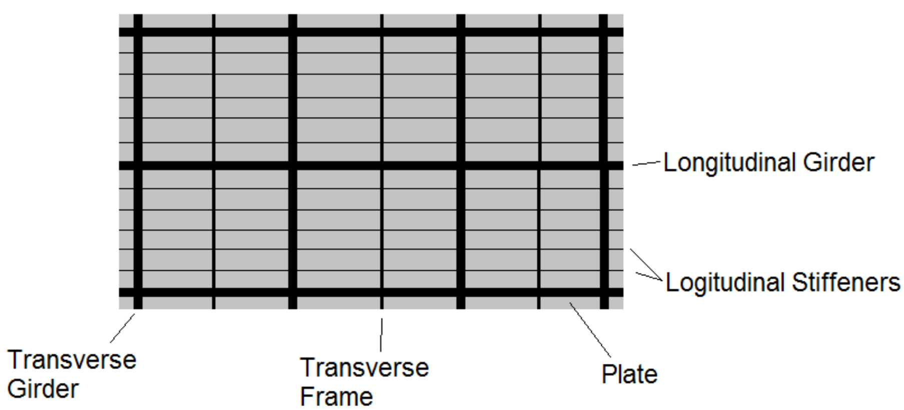

Figure 4.5: Simplified plan view of a grillage.

Each beam member is assumed to be a t-beam. As shown in figure 4.6, each beam member may be discretized into a flange and web. The flange and web each have associated heights (or widths) and thicknesses. The plate also has an associated thickness. In this case study, the simplifying assumption is made that beams and plates are all made of the same aluminum alloy; AL 5083-H116. 5083 aluminum is a common marine-grade aluminum for structural applications. It is possible to extend this case study by allowing for different beam and plate materials. This is left for future work.

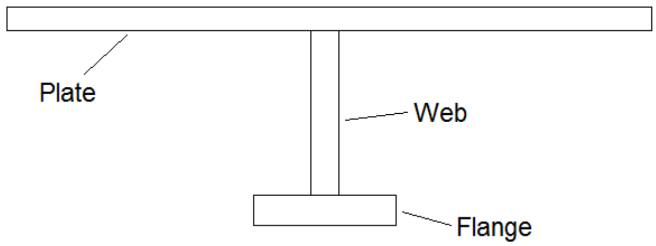

Figure 4.6: Simplified cross section of a stiffened panel.

Given this setup, the list of design features is:

d_1 = Thickness of deck plate.

d_2 = Number of longitudinal girders[2]

d_3 = Thickness of flange of longitudinal girders

d_4 = Width of flange of longitudinal girders

d_5 = Thickness of web of longitudinal girders

d_6 = Height of web of longitudinal girders

d_7 = Number of longitudinal stiffeners between girders[3]

d_8 = Thickness of flange of longitudinal stiffeners

d_9 = Width of flange of longitudinal stiffeners

d_{10} = Thickness of web of longitudinal stiffeners

d_{11} = Height of web of longitudinal stiffeners

d_{12} = Number of transverse girders[4]

d_{13} = Thickness of flange of transverse girders

d_{14} = Width of flange of transverse girders

[2]Total deck width is 20 meters.
[3]Stiffeners are evenly spaced between girders.
[4]Total deck length is 40 meters.

d_{15} = Thickness of web of transverse girders

d_{16} = Height of web of transverse girders

d_{17} = Number of transverse frames between girders[5]

d_{18} = Thickness of flange of transverse frames

d_{19} = Width of flange of transverse frames

d_{20} = Thickness of web of transverse frames

d_{21} = Height of web of transverse frames

4.3.3 Design Complexity

As a method quantifying the complexity of a particular structural design, this study leverages the work of Rigterink et al. in rating the producibility of stiffened panel designs [97, 98]. Their producibility metric is actually the inverse of complexity, where designs with high producibility metrics are more preferred. The metric is a scalar value between zero and one, and is useful for comparing between multiple candidate designs when their performance is otherwise similar. For a detailed formulation of the metric, the interested reader is referred to [97, 98]. For this dissertation it suffices to highlight the various components of the metric.

In determining the producibility of a stiffened panel, this metric weighs the following factors:

1. X-direction and Y-direction access: Weighs the space for a shipyard worker to reach between beams for welding. Higher access is preferred.
2. Ratio of stiffener spacing to plate thickness: Relates to distortions resulting from the welding process.

[5]Frames are evenly spaced between girders.

3. Effective panel aspect ratio: This is a predictor of residual stresses and distortion.

4. Sum of joint thicknesses: Weighs the ease of joining two plates together.

5. Joint thickness Ratios: Weighs the ease of joining two plates together of different thicknesses.

6. Outfit potential: Weighs the difficult of running pipes, cable, and ventilation through or around a stiffened panel.

4.3.4 Capabilities and Utility Functions

The two capabilities included in this case study are availability and cargo fraction. Availability is derived from the number of new cracks each year. This study makes the simplifying assumption that each new crack removes the vessel from service for two days. This assumption is discussed in greater detail in section 4.3.6. Since the relevant risk factor was actually the time to crack initiation for each weld detail, then several steps are necessary to translate that risk factor into availability. Recall the probability density function for the number of stress cycles until crack initiation:

$$f_1(x_1) = \frac{1}{\zeta\sqrt{2\pi}x_1}\exp\left(-\frac{(\ln(x_1)-\lambda)^2}{2\zeta^2}\right) \tag{4.6}$$

The associated cumulative density function is:

$$F_1(x_1) = 0.5 + 0.5\text{erf}\left[\frac{\ln(x_1)-\lambda}{\sqrt{2}\zeta}\right] \tag{4.7}$$

From this it is possible to define the probability of a crack occurring for any given year, p_t. If the structure experiences s stress cycles each year, then p_t is given by:

$$p_t = F_1(st) - F_1(s(t-1)) \tag{4.8}$$

Given that there are n weld details in total in the structure, the number of cracks which will occur in a given year is able to be modeled by a binomial distribution with the associated probability mass function given by:

$$p_t^{binom}(k) = \binom{n}{k}(p_t)^k(1-p_t)^{n-k} \tag{4.9}$$

Where p_t is the probability of a crack occurring in year t for a single weld detail, n is the total number of weld details, and $p_t^{binom}(k)$ is the probability of k cracks occurring in year t throughout the structure. Since the number of weld details is quite large, this binomial distribution is well-approximated by a Gaussian, or Normal distribution. In this way it is possible to define an intermediate risk factor, $Y_1(t)$, which will be used to denote the total number of cracks occurring in year t.

$$Y_1(t) = N[np_t, \sqrt{np_t(1-p_t)}] \tag{4.10}$$

The only care that should be taken if using this approximation for simulation is to prevent simulating negative values. While the normal distribution may take negative values, the number of cracks is obviously non-negative. With the intermediate risk factor, $Y_1(t)$ it is finally possible to define an equation for availability.

$$c_1(t) = \frac{365 - 2Y_1(t)}{365} \tag{4.11}$$

Where $c_1(t)$ is the fraction of time in year t that the vessel is available, and $Y_1(t)$ is the number of cracks that occur in that year.

A design's cargo fraction is measured to be the percentage of the demand for cargo weight which the vessel can accommodate. If the demand for cargo is ever greater than the design's maximum cargo capacity, then only a fraction of the demand can be met. This assumes that the cargo is infinitely divisible, which of course may not be true in practice. Discrete cargo fractions is a possible extension for future work. Cargo fraction is thus given by:

$$c_2(t) = \min\left[\left(\frac{c_{cargo}}{X_2(t)}\right), 1\right] \tag{4.12}$$

Where $c_2(t)$ is the cargo fraction for time t, c_{cargo} is the maximum cargo capacity for the design, and $X_2(t)$ is the demand weight for cargo. The cargo fraction will change through time as the cargo demand risk factor changes.

The next step in the PB-ROA framework is to translate these capabilities into a utility score. For this study, power functions are use to represent the utility of individual capabilities.

$$u_{avail} = \max\left(0, \left(\frac{c_1(t) - c_1^{min}}{1 - c_1^{min}}\right)^{1/2}\right) \tag{4.13}$$

$$u_{cargo} = \max\left(0, \left(\frac{c_2(t) - c_2^{min}}{1 - c_2^{min}}\right)^{2/3}\right) \tag{4.14}$$

$$u_{prod} = \xi \tag{4.15}$$

Where u_{avail} is the utility function for availability, u_{cargo} is the utility function for cargo fraction, and c_1^{min} and c_2^{min} are constants related to the constraints for minimum allowable values for availability and cargo fraction, respectively. These constraints reflect thresholds in the vessel's performance below which the platform can no longer reasonably perform the mission. For illustrative purposes a constraint of 85% is used for availability and 90% for cargo fraction. Values of $c_1^{min} = 0.84$ and $c_2^{min} = 0.89$ are used for calculations, however, to prevent the marginal utility from being infinite for designs lying on the constraint boundary. ξ is the producibility score for the grillage design. u_{prod} is the utility of producibility for the structure. For this, a linear function is used. Because risk-aversion is (partially) captured by the magnitude of

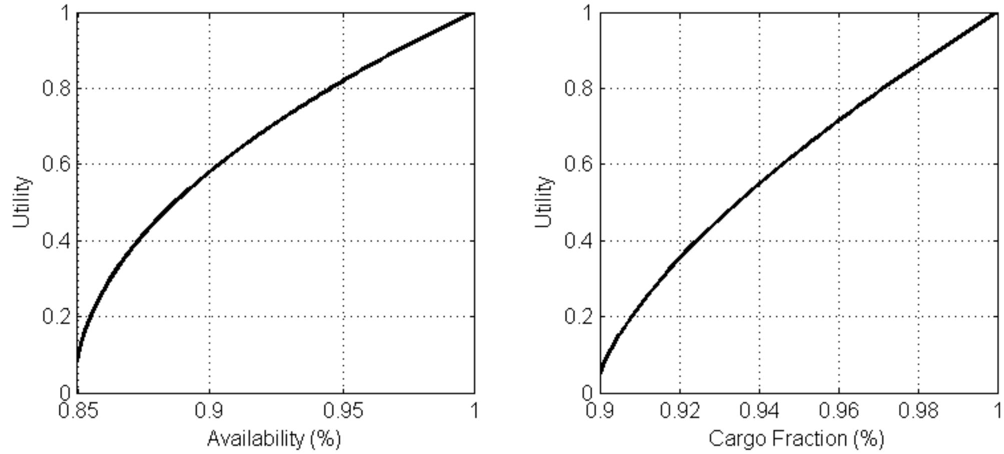

Figure 4.7: Utility curves for availability (left) and cargo fraction (right).

the curvature (second derivative) of the utility curve, these functions are stating that the decision maker(s) is most risk-averse in availability, then cargo fraction, and risk-neutral in producibility.

These individual utility curves are combined into one utility function for the vessel through a simple multiplication.

$$u = u_{avail} \cdot u_{cargo} \cdot u_{prod} \tag{4.16}$$

4.3.5 Multiobjective Structural Optimization

Recall that the purpose of this case study is to evaluate the worth of an option to extend the service life (ESP option) of a high speed aluminum military catamaran from twenty to 25 years. The utility of the vessel may vary greatly in that five year period based on its structural design. It has already been discussed how there may be a tradeoff between availability and cargo fraction, which are both desirable capabilities. Heavier structures will generally exhibit less cracking but place a lower limit on cargo capacity. Similarly, lighter structures may allow for greater cargo at the expense of more cracking, especially later in the service life. In order to evaluate the ESL option, it is first necessary to quantify this tradeoff. In this section, the results of a two-objective optimization problem are presented for the simplified structure minimizing both weight and *expected* cumulative cracking subject to many constraints.

The variables for the optimization are the design features enumerated in section 4.3.2. The constraints are various strength constraints derived from the American Bureau of Shipping Guide for Building and Classing High Speed Naval Craft [4]. Appendix A contains a detailed mathematical formulation of the optimization problem. Only the results are presented here. The Pareto front is found using the non-dominated sorting genetic algorithm (NSGA-II) by Deb et al. [34]. The front is shown in figure 4.8.

Figure 4.8 shows that the weight[6] of the two decks structures ranges from approximately 98 tons to 118 tons. For this range of weights, the cumulative expected number of cracks over the first 20 years of the vessel drops from 100 to essentially zero. Another way to state this is that by paying a "cost" of 20% increase in weight, one could achieve *almost* perfect availability. One can never achieve truly perfect availability because under the lognormal pdf for cracking there is always a positive probability of at least one crack occurring. What is especially interesting from a design perspective is that the tradeoff between weight and cracking is not linear. Note that for a roughly 10% increase in weight (relative to the minimum weight design), over 90% of cracking is avoided.

[6]The optimizer only calculates weight of the structural beams and plating. It does not include outfitting, auxiliary systems, etc.

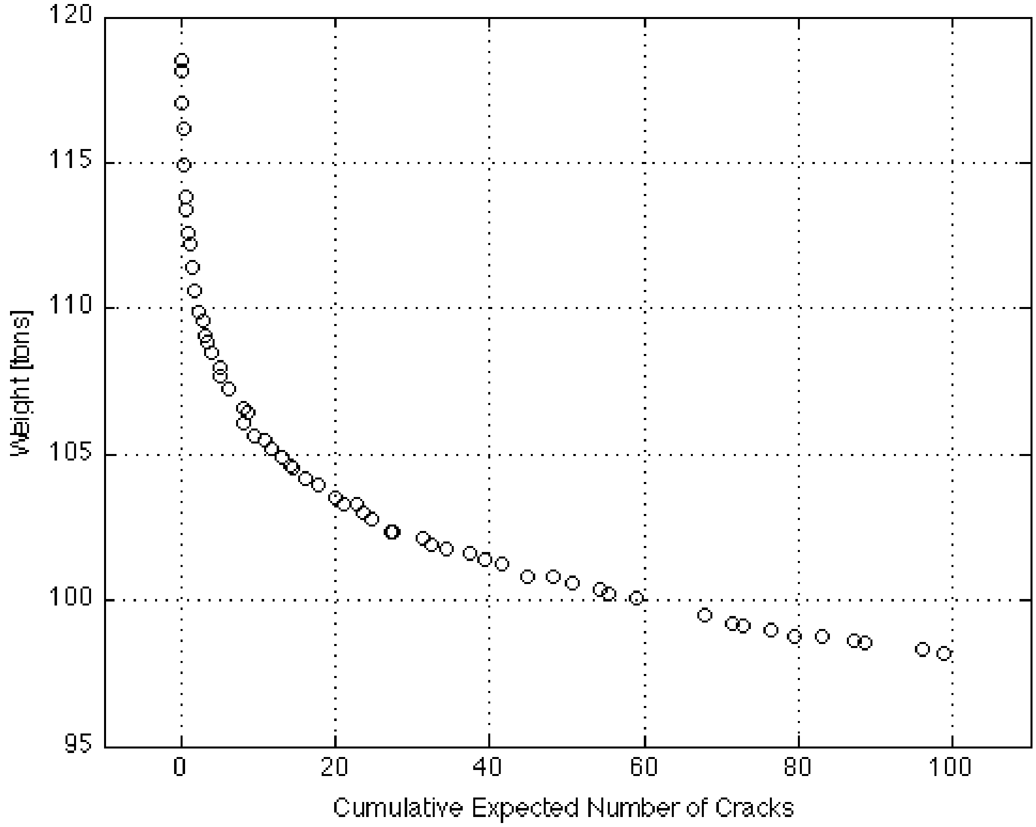

Figure 4.8: Pareto front for optimization of catamaran structure.

4.3.6 Modeling Difficulties and Assumptions

As shown in figure 4.2, there are many input models for the PB-ROA calculations in this case study. This section discusses the difficulties related to modeling some key inputs, and assumptions made in this case study to mitigate those difficulties.

Operations and Loads This study makes the simplifying assumption that the variation in the magnitude of loads applied to the structure is captured in the random variable k_f in equation (4.1). Moreover, the study assumes the loads occur at a steady frequency, which allows for simple translation from the frequency domain to the time domain. However, what equation (4.1) cannot capture are the effects of operations and load sequence on the fatigue life of the structure. Having an operations model is important because load frequency may not be constant, impacting the fatigue life calculations. And it is known that the sequence of loads a structure experiences has a significant effect on its fatigue life. For example, a very stressful event experienced early in the structure's life will have a very different impact than one experienced late in life. For an overview of such considerations, see the summary by Fricke [43].

Maintainability It is well known that different ship types are harder to maintain than others. This case study does not include a model for maintainability. The reality of ship maintenance programs is that they are highly influenced by aspects of the vessel's design like access and arrangements. An extreme example of this is the maintenance of submarines where access is limited because the integrity of pressure hull cannot be sacrificed, among other things. The producibility metric used to quantify the complexity of candidate structural designs in this study incorporates access between stiffeners as that impacts maintenance and construction. However, beyond the use of the producibility metric, no maintenance model is included in this study. In its place, the assumption is made that each new crack removes the vessel from service for two days.

Time to Repair Closely related to maintainability is the time to repair cracks in the structure. While this study assumes that each new crack requires two days to fix, the reality is that the time to repair is non-linear with respect to the number of cracks. For instance, multiple cracks may be repaired in the same visit to port, splitting the transit time between them. Some minor or easily accessible cracks may be repairable at sea, expediting the repair process. Beyond these operational aspects, the time to repair is itself stochastic. The PB-ROA framework is not limited to two risk factors, as demonstrated in this study. Future work could expand this study by including a third risk factor for the time to repair with an associated pdf.

Inspection While this study uses the time to crack initiation as a risk factor, one might reasonably argue that a more appropriate choice would be the time to crack *discovery.* The time to crack discovery, however, will depend on inspection practices. Many fatigue details may be covered with thermal or acoustic insulation, for example, making is impossible to spot cracks without first removing the insulation. The difficulty with inspection from a modeling perspective is how one might calibrate it. For the time to crack initiation, there is a wide body of literature and experimental data for different materials and joint configurations with which to calibrate the models. Inspection is more difficult to quantify. For this reason, it is omitted from this study.

Fleet Aspects This case study analyzes the vessel in isolation. However, in reality the vessel might be deployed as part of a fleet, where it is one component of achieving mission success. Other vessels or naval assets might be relying on the high speed catamaran in order to perform their mission(s). For this reason the availability of the catamaran could have far reaching implications for other aspects of the fleet. This is outside the scope of this thesis, but is an interesting area for future work.

The Time-Value of Capability When evaluating the performance, or worth, of an asset over long periods of time or at points of time in the distant future, one must address the impact of time on decision making. This is very clear in financial applications where there is a time-value of money. If an investor has access to an instrument with a riskless annual rate of return[7], r, then the present value, v, of \$1 to be received at some time in the future T is given by:

$$v = \$1 \cdot e^{-r \cdot T} \tag{4.17}$$

Figure 4.9 shows the diminishing present value of \$1 over increasing time horizons

[7]Continuously compounded.

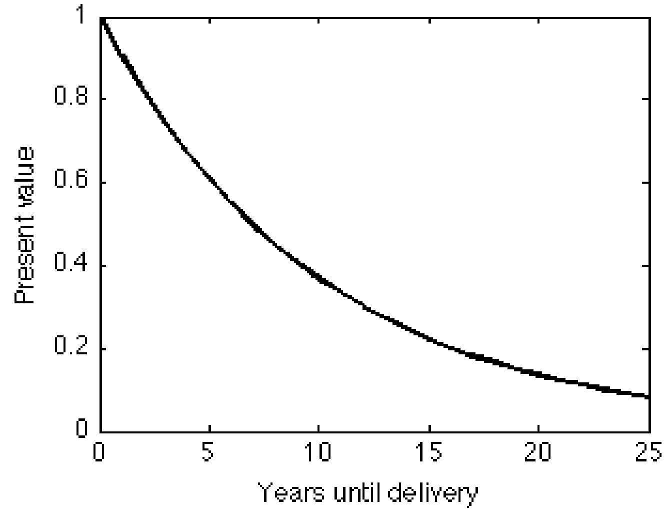

Figure 4.9: Present value of \$1 to be received in T years, for $r = 10\%$.

for the case of $r = 10\%$. This simply illustrates how the value of \$1 to be received in 10 years is much less than the value of \$1 to be received in 1 year.

A similar concept for Naval applications is the time-value of capability. For example, the value of availability for a ship may be greater now than for a point in the future if plans are underway to construct a replacement vessel. Conversely, the value of availability for a ship may be greater in the future than it is currently if its sister ship is known to be retiring, leaving it with the full mission load. Similar time-value arguments may also apply to cargo fraction. In practice, determining the time-value of a capability may require information from a Navy future force structure analysis, or war games analysis. This is outside the scope of this thesis work, but is important to note for real Navy applications because these considerations will impact the shapes of the utility curves in figure 4.7. In this study, a constant time-value of capability is assumed.

Decision to Extend Service Life In order to address the value of the ESL option, one must be able to quantify the probability of the option being exercised. For a financial European call option on a stock, for instance, this relates to the probability of the stock price being above the strike price at the time of maturity. The financial literature contains many methods for modeling this probability. The decision to extend the service life of a vessel is much more complex. That decision will be influenced by many intangible factors like political negotiations, budget constraints or surpluses, emergence of new (potentially unforeseen) technology, the global military threat environment, and much more. Rather than attempt to model this complex decision precisely, this study side steps the modeling issue by treating the probability of exercising the ESL option as a parameter which may be varied. In this way, rather than attempt to express an exact, singular value for the ESL option this study examines how the value of the ESL option changes with respect to the probability of exercise, and under what conditions the ESL option may desirable, or undesirable. This study also makes the simplifying assumption that the decision to exercise the ESL option is independent of the other risk factors; number of cracks and cargo demand.

4.4 PB-ROA Analysis

With the risk factors, capabilities, utility curves, and Pareto front for the optimized structures all quantified, it is possible to move forward with determining the value of the ESL option. Since the ESL option is only able to be exercised at the twenty year mark, it is a European option with value expressed in the general form:

$$v(t, \mathbf{X}(t)) = \mathbb{E}^{\mathbb{Q}} \left[\Phi(\mathbf{X}(T)) | F_t \right] \tag{4.18}$$

where $\Phi(\mathbf{X}(T))$ is the payoff of the option at time T, given the prevailing state of the risk factors, $\mathbf{X}(T)$, and $\mathbb{E}^{\mathbb{Q}}$ signifies that the expectation is being calculated using the risk-adjusted probability measure, $\mathbb{Q}$. F_t is the filtration[8] on the risk-adjusted probability space, $\mathbb{Q}$, up to time t.

The $\mathbb{Q}$-measure is calculated by re-normalizing the product of the joint probability distribution of all risk factors with the marginal utility function.

$$q(x) \propto p(x) \cdot u'(c(x)) \tag{4.19}$$

The probability of exercising becomes like a third risk factor in this case study, where the probability of exercise is a Bernoulli random variable, E, with parameter α. It has already been explained that all risk factors are assumed to be independent. This way, our expression for $p(x)$ becomes:

$$p(e, y_1, x_2) = f_E(e) \cdot f_1(y_1) \cdot f_2(x_2) \tag{4.20}$$

Where

$$f_E(e) = \begin{cases} (1-\alpha) & \text{for } e = 0, \\ \alpha & \text{for } e = 1. \end{cases} \tag{4.21}$$

$$f_1(y_1) = \frac{1}{\sigma_Y \sqrt{2\pi} y_1} \exp\left(-\frac{(y_1 - \mu_Y)^2}{2\sigma_Y^2}\right) \tag{4.22}$$

$$f_2(x_2) = \begin{cases} 0 & \text{for } x_2 < a, \\ \frac{2(x_2-a)}{(b-a)(c-a)} & \text{for } a \le x_2 \le c, \\ \frac{2(b-x_2)}{(b-a)(b-c)} & \text{for } c \le x_2 \le b, \\ 0 & \text{for } b < x_2. \end{cases} \tag{4.23}$$

[8] In simple terms, a filtration collects all of the information from time zero to the present, allowing conditional probability calculations to be performed.

The marginal utility function for this case study is proportional to the magnitude of the gradient of the utility function.

$$||\nabla u(\mathbf{c})|| = \left[\left(\frac{\partial u}{\partial c_1} \right)^2 + \left(\frac{\partial u}{\partial c_2} \right)^2 \right]^{1/2} \Bigg|_{\mathbf{c}=\mathbf{c}(y_1,x_2)} \quad (4.24)$$

The individual marginal utility functions, $\partial u/\partial c_1$ and $\partial u/\partial c_2$ may differ greatly in magnitude. This is partly because the first capability (availability) spans a range $c_1 \in [0.85, 1.0]$ which is larger than the range for the second capability (cargo fraction) $c_2 \in [0.9, 1.0]$. This is true generally of multi-attribute utility functions where the various inputs may span different ranges, or even be measured in different units. To prevent either capability from dominating the marginal utility calculation, each individual marginal utility is normalized according to its maximum value. Let

$$dc_1^* = \sup \frac{\partial u}{\partial c_1} = \frac{1}{2} \left(\frac{0.01}{0.16} \right)^{-1/2} \quad (4.25)$$

$$dc_2^* = \sup \frac{\partial u}{\partial c_2} = \frac{2}{3} \left(\frac{0.01}{0.11} \right)^{-1/3} \quad (4.26)$$

Then

$$u'(c(y_1, x_2)) = \left[\left(\frac{1}{dc_1^*} \frac{\partial u}{\partial c_1} \right)^2 + \left(\frac{1}{dc_2^*} \frac{\partial u}{\partial c_2} \right)^2 \right]^{1/2} \Bigg|_{\mathbf{c}=\mathbf{c}(y_1,x_2)} \quad (4.27)$$

Where

$$\frac{\partial u}{\partial c_1} = \frac{1}{2} \left(\frac{c_1(y_1) - c_1^{min}}{1 - c_1^{min}} \right)^{-1/2} \quad (4.28)$$

$$\frac{\partial u}{\partial c_2} = \frac{2}{3} \left(\frac{c_2(x_2) - c_2^{min}}{1 - c_2^{min}} \right)^{-1/3} \quad (4.29)$$

According to this notation, y_1 is a realization of the random variable $Y_1(t)$ which is the number of cracks which will occur in year t. Likewise, x_2 is a realization of the random variable $X_2(t)$ which is the demand for cargo weight in year t. Recall that each of these steps were important in order to derive the risk-adjusted probability measure, $\mathbb{Q}$, which is finally expressed by:

$$q(e, y_1, x_2) = \frac{1}{C_q} \left[f_E(e) \cdot f_1(y_1) \cdot f_2(x_2) \cdot u'(c(y_1, x_2)) \right] \tag{4.30}$$

Where the normalizing constant C_q is necessary in order to ensure that q obeys the laws of a probability measure (i.e. integrates to one), and is calculated by:

$$C_q = \int_0^n \int_a^b f_1(y_1) \cdot f_2(x_2) \cdot u'(c(y_1, x_2)) \mathrm{d}x_2 \mathrm{d}y_1 \tag{4.31}$$

Where n is the number of weld details in the structure, a is the minimum cargo demand for the triangular distribution, and b is the maximum cargo demand.

If exercised, the ESL option will pay a utility $u(t)$ each year. For this reason, the payoff function for the ESL option, $\Phi(\cdot)$, is the sum of the yearly utilities over the five year extension period.

$$\Phi(E, Y_1(T), X_2(T)) = E \left(\sum_{t=21}^{25} u(c_1(Y_1(t)), c_2(X_2(t)), \xi) \right) - K \tag{4.32}$$

Where E is the Bernoulli random variable for whether or not the option is exercised ($E \in \{0, 1\}$), and K is the strike price of the ESL option. A significant difference between a typical European call option on a financial asset and this Naval ESL option is how the strike price is defined. In a financial setting, the strike price for a European call option would be a contractual price you agree to pay for the underlying asset. This price is paid at the time the option is exercised (if it is exercised) so the option holder effectively collects the difference between the prevailing asset price and the strike price. However, for the ESL option, the strike price is paid up front, regardless of whether or not the ESL option is ever exercised. This is because "purchasing" the ESL option requires making physical changes to the structural design of the vessel in the design stage. The effects of those structural changes are carried through the entire life of the vessel regardless of whether the ESL option is ever exercised.

The strike price for the ESL option is defined relative to the utility of the minimum weight design. It is the difference between the utility of the minimum weight design, and the new candidate design. For example, consider the "min-weight" and "alternative" designs shown on the Pareto front in figure 4.10.

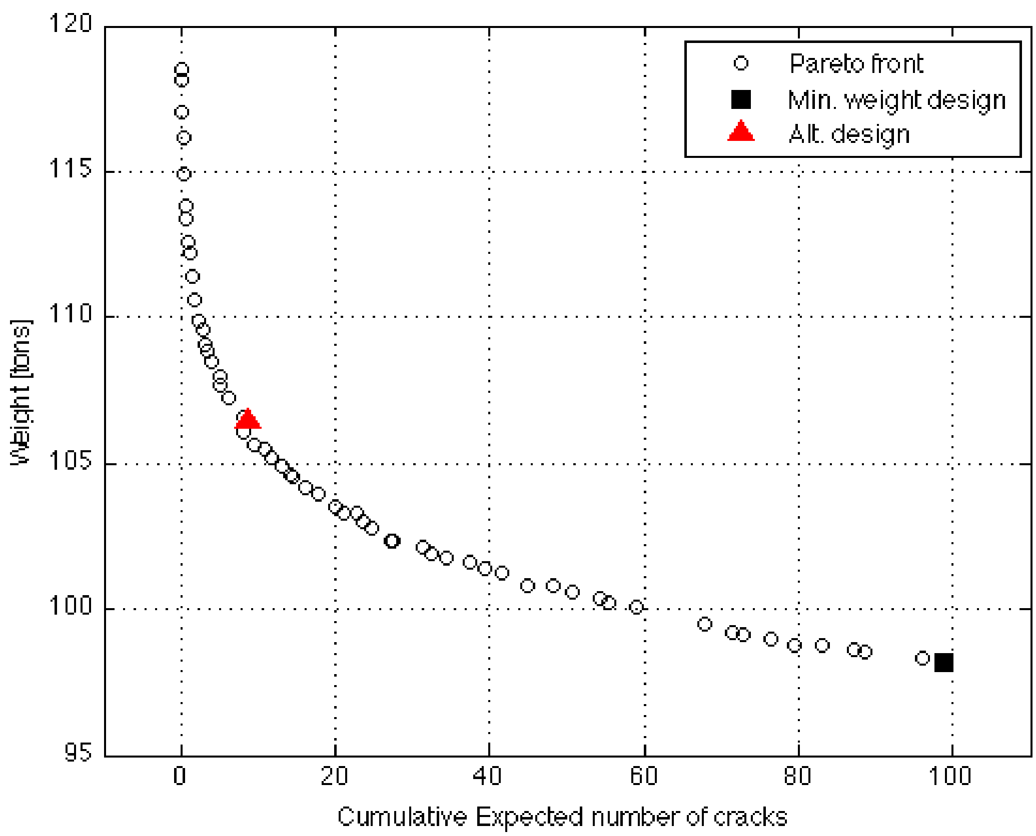

Figure 4.10: Position of the "Alternative" design relative to the min-weight design on the Pareto front

The alternative design is heavier but also has much less expected cracking. However, the impact of cracking is most significant later in the vessel's service life. In the early years, even very light structures are not expected to show much cracking. In the early years, the heavier structure will suffer from a lower cargo capacity, relative to the min-weight design. For this reason, the min-weight design has a higher expected utility in the first several years of its service life. However, the expected utility of the alternative design will eventually be greater than the min-weight design in later years when cracking is prevalent. This is captured in figure 4.11. The strike price for the option is analogous to the shaded area on the left-hand side of figure 4.11.

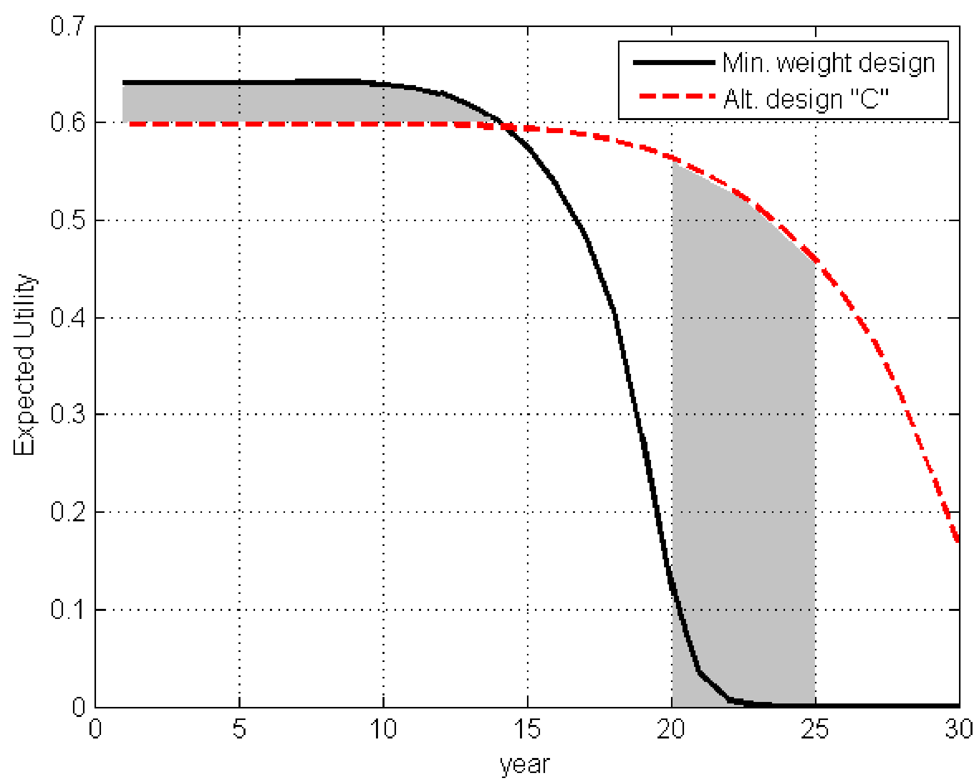

Figure 4.11: Difference in expected utilities of the "alternative" and min-weight designs over time.

Mathematically, the strike price for the ESL option is derived with respect to a crossing time, τ, where the min-weight design ceases to have higher utility.

$$\tau = \sup\{t \in [0, T] : \mathbb{E}^{\mathbb{Q}}[u_0(t)] > \mathbb{E}^{\mathbb{Q}}[u(t)]\} \tag{4.33}$$

Where $u_0(t)$ is the utility of the min-weight design, and $u(t)$ is the utility of the design in question for the ESL option. It is possible that τ does not exist for some designs. This is the case for designs which always have a higher expected utility than the min-weight design. This is possible because producibility also factors into the utility score. If an alternative design has significantly higher producibility than the min-weight design, it may have a higher expected utility at each time step despite having heavier structure. So the strike price is then expressed by:

$$K = \begin{cases} \sum_{t=1}^{\tau}(u_0(t) - u(t)) & \text{if } \tau \text{ exists,} \\ 0 & \text{otherwise.} \end{cases} \tag{4.34}$$

With the strike price mathematically defined, the payoff function for the case where τ exists is given by:

$$\begin{aligned} \Phi(E, Y_1(T), X_2(T)) = E &\left(\sum_{t=21}^{25} u(c_1(Y_1(t)), c_2(X_2(t)), \xi) \right) \\ &- \sum_{t=1}^{\tau} \left(u_0(c_1(Y_1(t)), c_2(X_2(t)), \xi) - u(c_1(Y_1(t)), c_2(X_2(t)), \xi) \right) \end{aligned} \tag{4.35}$$

If the crossing time τ does not exist, then the equation for the payoff reduces to:

$$\Phi(E, Y_1(T), X_2(T)) = E \left(\sum_{t=21}^{25} u(c_1(Y_1(t)), c_2(X_2(t)), \xi) \right) \tag{4.36}$$

Inserting equation (4.35) into the European option value equation (4.18), one is left with:

$$v(t) = \alpha \sum_{j=21}^{25} \mathbb{E}^{\mathbb{Q}}\Big[u(j)\Big] - \sum_{k=1}^{\tau} \left(\mathbb{E}^{\mathbb{Q}}\Big[u_0(k)\Big] - \mathbb{E}^{\mathbb{Q}}\Big[u(k)\Big] \right) \tag{4.37}$$

Where $u(t)$ is shorthand notation for $u\Big(c_1(Y_1(t)), c_2(X_2(t)), \xi\Big)$.

The value of the ESL option is a function of the parameter α which is the probability of the option being exercised.

4.5 Results and Interpretation

Recall the alternative design, whose position on the Pareto front relative to the min-weight design is shown in figure 4.10. The value of the ESL option on that design, as a function of α, is shown in figure 4.12.

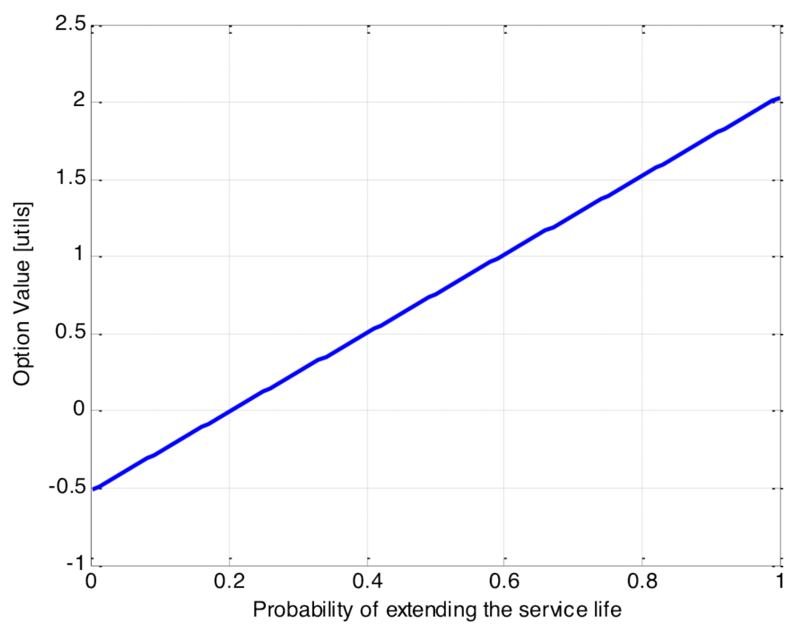

Figure 4.12: Value of the ESL option for the alternative design as a function of α, the probability of exercising the option.

What is important about this plot is the point where the ESL option's value becomes positive at roughly $\alpha = 20\%$. This means that if the probability of extending the service life of this vessel is less than 20%, the ESL option is not desirable *for this design.* Said another way, the option to extend the life of the alternative structure is valuable if the probability of extending the life is above 20%. The PB-ROA analysis has uncovered a limit in the design space above which the decision maker should give serious thought to investing in the ESL option. In a quantitative, repeatable framework, PB-ROA is able to analyze the conditions under which the ESL option on the alternative structure is valuable. But what is even more interesting is to repeat the option value calculations for every point on the Pareto front to see which designs yield the highest option value, and under what conditions. The results are quite interesting from a design perspective.

Consider two new alternative designs, shown in figure 4.13. The two new alternative designs shall be referred to as alternatives "A" and "B." Alternative A weighs approximately 103 tons, and will experience 20 cracks over its first twenty years of service, in expectation. Alternative B weighs approximately 105 tons, and will only experience about 9 cracks over its first twenty years of service, in expectation. Alternative C is the previously discussed design from figure 4.10.

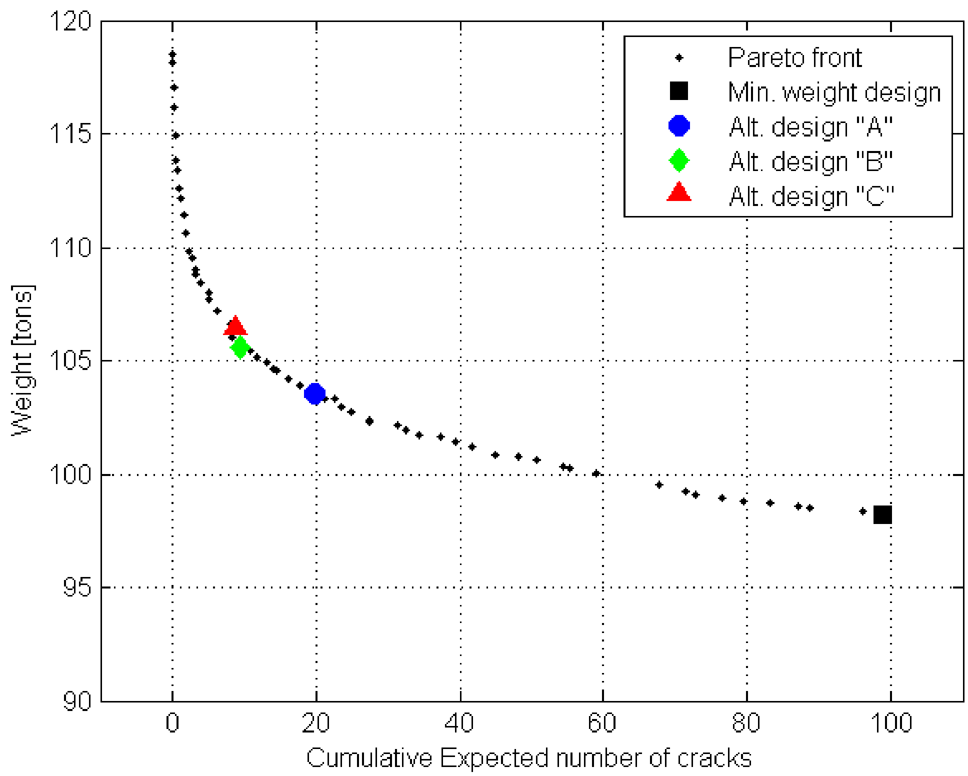

Figure 4.13: Pareto front highlighting alternative designs "A" and "B".

Figure 4.14 shows how the expected utility, under the risk-adjusted measure, changes over time for each of these designs relative to the min-weight design. While cracking becomes a significant influence on utility at around the 10 year mark for the min-weight design, it is not expected to be an issue for alternatives A or B until roughly the 15 year mark. Design A, which is the lighter of the two alternatives here has the highest expected utility until roughly year 16. After that time, one expects design B to yield the highest utility because it is the least susceptible to cracking. What is particularly interesting, however, is that design A *always* has a higher expected utility than the min-weight design - even in the first year. Since design A has heavier structure, limiting its cargo capacity, one would expect its utility to be lower than the min-weight design's initially. However, recall that the producibility of the structure also factors into the structure's utility score. The producibility score

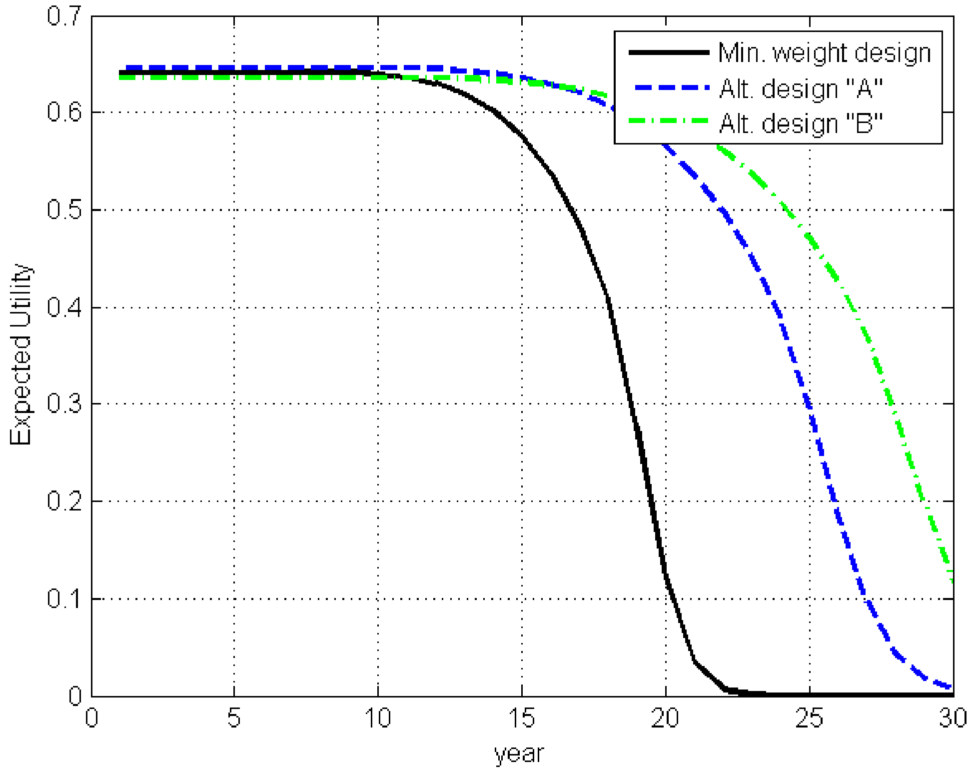

Figure 4.14: Expected yearly utility of alternative designs "A" and "B".

for every design along the Pareto front is shown in figure 4.15, with the alternative designs highlighted.

Figure 4.15 shows that both alternative designs A and B have (slightly) higher producibility scores than the min-weight design. It is because of its higher producibility score that design A has a higher expected utility than the min-weight design, despite having a lower cargo capacity. When calculating the value of the ESL option according to equation (4.37), the strike price for design A is zero. This makes design alternative A analogous to an arbitrage opportunity in finance. Relative to the min-weight design, design A has *some* upside potential and *zero* downside potential[9].

[9]The author acknowledges that this study does not consider the implications of acquisition or construction cost. This statement is based solely on the relative utilities of the designs as defined in the study.

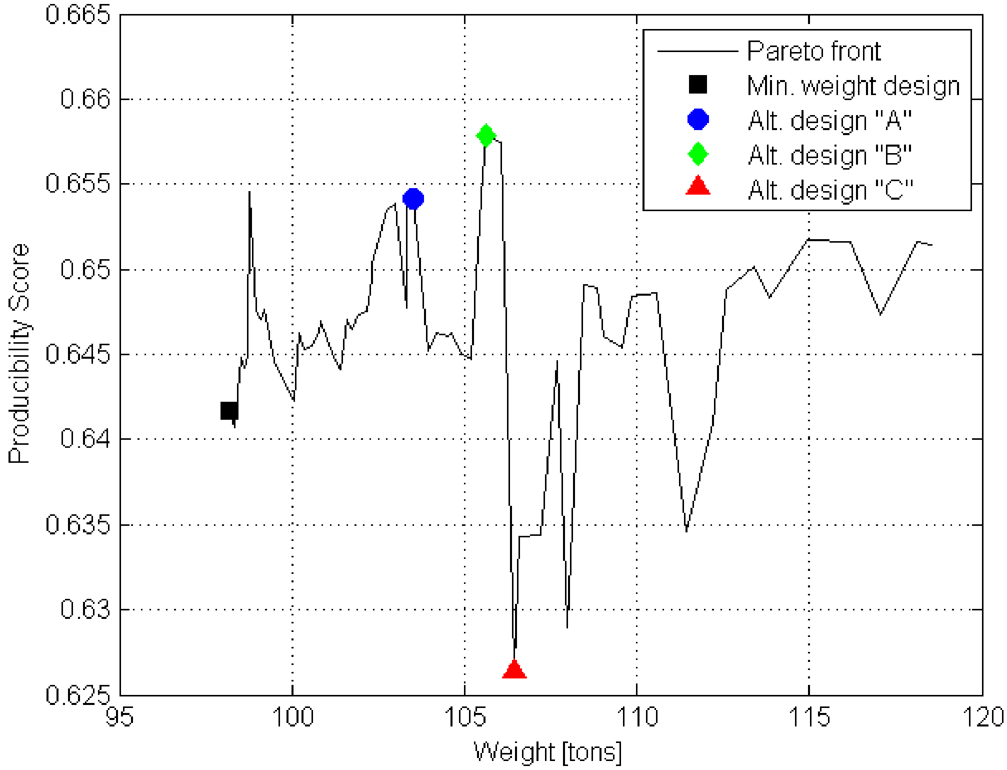

Figure 4.15: Producibility score for each design on the Pareto front, as a function of weight.

Another way of stating this is that the min-weight design is optimal from a weight perspective, but suboptimal from an expected utility perspective. If taking a full life cycle point of view on design, the decision maker(s) should significantly discount the min-weight design.

The maximum value of the ESL option for all designs on the Pareto front is plotted in figure 4.16. What is significant is that there is a threshold at approximately $\alpha = 10\%$ where the design with the highest ESL option value switches from design A to design B. Of all the candidate designs on the Pareto front, only designs A and B are optimal, and whether the decision maker prefers A or B depends on the likelihood of extending the service life of the vessel. If the likelihood is low (less than 10%), then design A will maximize the value of the ESL option. However, if it is believed that the likelihood of extending the life of this vessel is large (greater than 10%), then design B will maximize the value of the flexibility provided by the ESL option. This is analogous to a threshold policy in the industrial and operations engineering world.

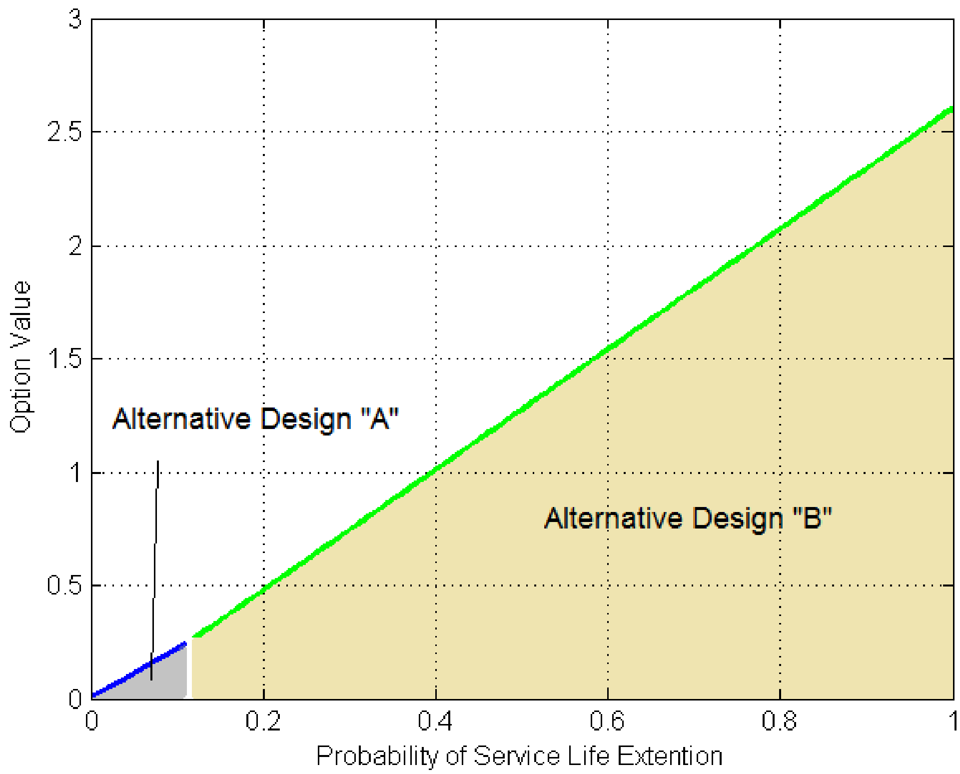

Figure 4.16: Value of ESL option as a function of α, and associated decision threshold policies.

In an early stage design scenario this outcome would be highly insightful and beneficial. It would be problematic if the "optimal" design continually changed with respect to α, because pinpointing the likelihood of extending the service life is difficult at best, and impossible at worst. But instead this study resulted in two stable design regions with a demarkation of where one design becomes favorable over the other. This would allow decision makers to narrow the design space to these two areas, or focus attention on determining beliefs about the likelihood of service life extension for this vessel to then narrow design efforts even further.

It is worth noting that while PB-ROA yields a numeric value for the expected payoff from the ESL option (measured in utility), the real insight generated by the framework is in understanding the conditions in which an option is favorable, or unfavorable, and how such conditions can be used to guide design decisions. In this case, PB-ROA revealed that the min-weight design is utility-dominated by another design and hence should probably be discarded. Furthermore, PB-ROA revealed that from an entire Pareto front of candidate structural designs, two designs were robust to changing beliefs about a key risk factor (α). And perhaps most importantly, this information is already adjusted for the risk tolerances of Navy decision makers captured in the shapes of the utility curves.

4.6 Perturbations of the Model

Since the results of this case study depend on the assumptions made in modeling the many inputs to the PB-ROA framework, this section evaluates how the results change in two fundamental perturbations. The first is the change that occurs if the Navy is assumed to be risk-seeking with respect to availability rather than risk-averse. The second perturbation is to apply an exponential decay for the time-value of availability, and seeing how ones decision might change if one weights near-term utility higher than far-term utility.

4.6.1 Increasing Marginal Utility of Availability

As shown by the utility curve for availability in figure 4.7, the results for this case study assumed a decreasing marginal utility of availability. This is equivalent to stating that the Navy is risk-averse with respect to availability. One may argue that a concave shape for availability does not accurately reflect Navy preferences. If so, PB-ROA can also handle the opposite assumption - that the Navy is risk-seeking with respect to availability. Consider, for example the function:

$$u_{avail} = \begin{cases} 0 & \text{if } c_1 < c_1^{min}, \\ \left(\frac{c_1(t) - c_1^{min}}{1 - c_1^{min}}\right)^3 & \text{otherwise.} \end{cases} \tag{4.38}$$

Where c_1 is availability, and c_1^{min} is the minimum allowable availability. This function has increasing marginal utility of availability, meaning that it is ever more desirable to increase the availability of the vessel. This is also known as risk-seeking preferences. It is shown in figure 4.17, and compared to the shape of the previously assumed risk-averse curve.

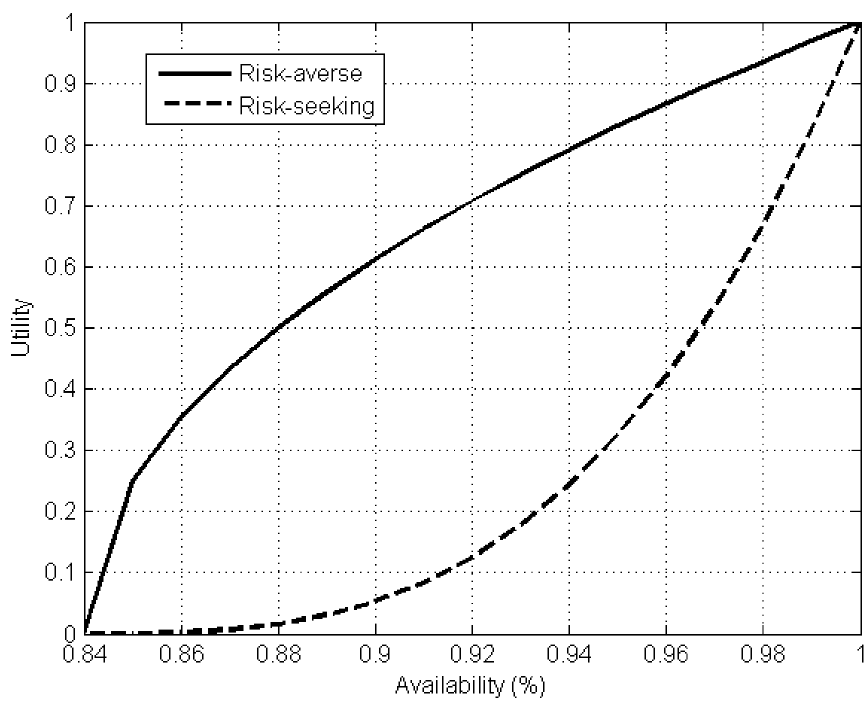

Figure 4.17: Comparison of risk-seeking and risk-averse utility curves for availability.

The risk-seeking assumption for availability results in very different design thresholds. Instead of two stable designs, there are now four which maximize option value under varying beliefs about α, as shown in figure 4.18.

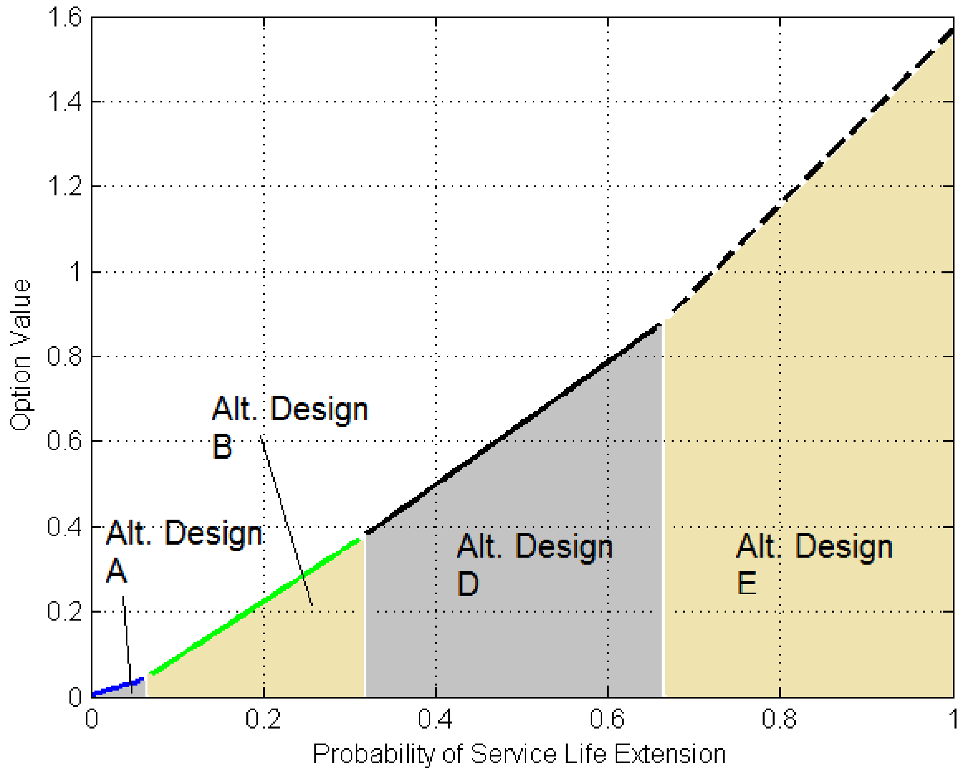

Figure 4.18: Value of ESL option as a function of α, and associated decision threshold policies under a risk-seeking assumption for availability.

The position of each of these designs on the Pareto front is shown in figure 4.19.

The fundamental interpretation of these results is that under a risk-seeking assumption for availability, the Navy will more readily consider designs with heavier structures that resist cracking. Recall that under the risk-averse assumption for availability, there was a threshold of approximately $\alpha = 10\%$ where design B (the heavier design) maximized the ESL option's value over design A. Under the new risk-seeking

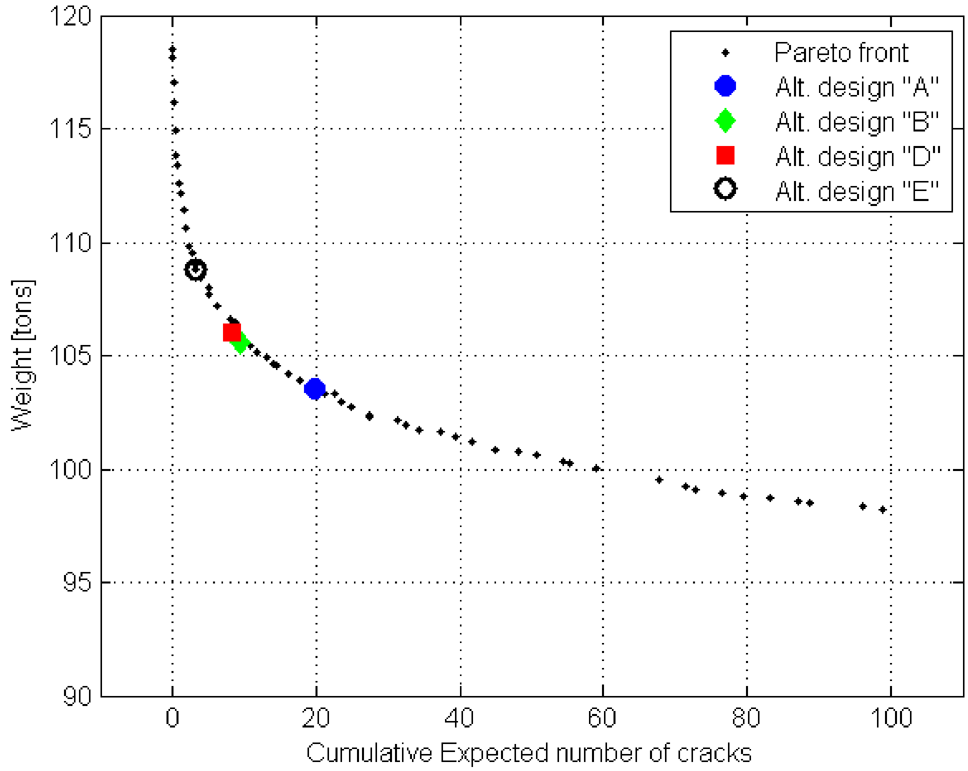

Figure 4.19: Pareto front highlighting alternative designs “A”, “B”, “D”, and “E”.

assumption, this threshold has dropped to $\alpha = 7\%$, expanding the set of conditions under which the Navy would prefer the heavier, crack-resistant design.

Furthermore, under the risk-seeking assumption for availability, threshold policies for two more designs are introduced, which are even heavier and more crack-resistant than design B. With higher fidelity structural modeling, it may be than designs B and D could be merged into a single design since they are on a similar position of the Pareto front. However, the general result remains that by changing from a risk-averse model to a risk-seeking model for availability, there is increased preference for heavier structures under a wide set of conditions.

4.6.2 Time Value of Utility

The initial results assumed that decision makers weight utility equally across all time horizons. Since this assumption is arguable for many applications, this section examines the changes that occur if a time value of utility is applied. For demonstrative purposes, a simple exponential decay is applied to the payoff and strike prices.

$$\Phi(E, Y_1(T), X_2(T)) = E\left(\sum_{t=21}^{25} e^{-rt} u(c_1(Y_1(t)), c_2(X_2(t)), \xi)\right) - \sum_{t=1}^{\tau} e^{-rt}\Big(u_0(c_1(Y_1(t)), c_2(X_2(t)), \xi) - u(c_1(Y_1(t)), c_2(X_2(t)), \xi)\Big) \quad (4.39)$$

Where r is the time rate of decay for utility for this vessel[10]. This applies greater weight to utility which is realized early in the vessel's life, and less weight to utility that is realized in the distant future. For this reason, the threshold value increases for which design B yields the highest value for the ESL option, as shown in figure 4.20. Previously the threshold occurred at approximately $\alpha = 10\%$. However, under a decreasing time value of utility, that threshold gets pushed up to roughly $\alpha = 30\%$. Since all designs are being discounted equally, the optimal designs do not change, just the threshold for optimality. Many different assumptions could be made about the time value of utility; constant, decreasing, increasing, varying. This section serves to demonstrate that the results from PB-ROA will certainly change, but also that the framework can handle such assumptions.

4.7 Conclusion and Contributions

This chapter used the prospect theory-based real options analysis framework presented in this thesis to examine the value of a real option to extend the service life of a high speed military aluminum catamaran. The requirements for this vessel call for

[10] Analogous to the risk free rate of return in finance.

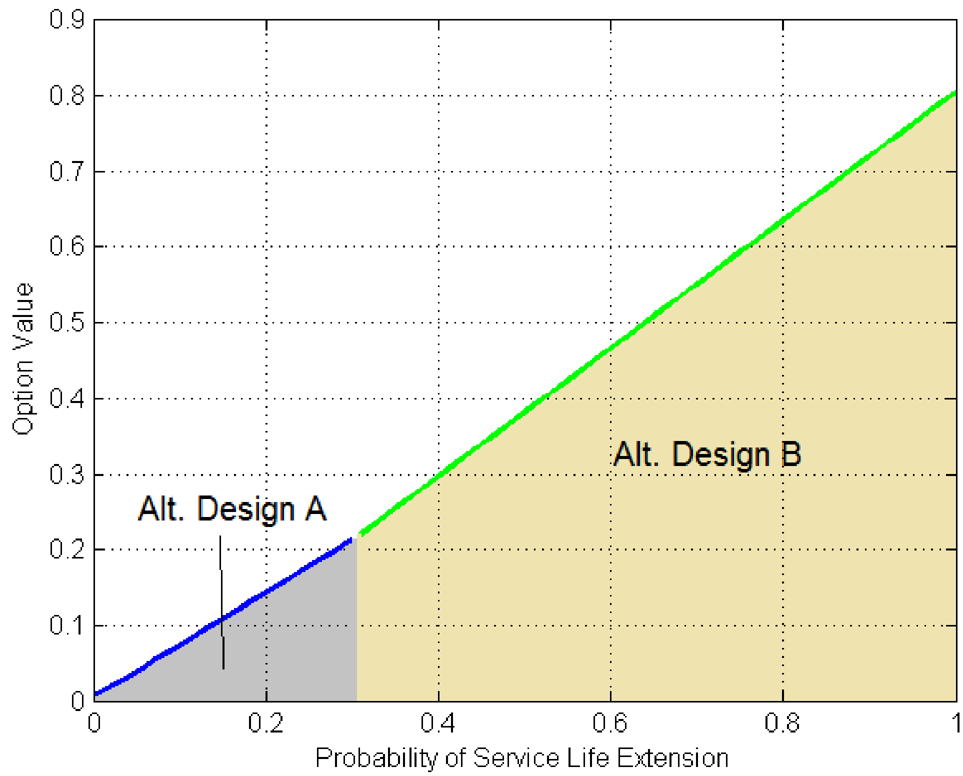

Figure 4.20: Value of ESL option as a function of α, and associated decision threshold policies under a decreasing time value of utility.

a twenty year service life so purchasing the option to extend service life (ESL option) by another five years required making physical changes to the structural design. A simplified structural synthesis model was developed and a multi objective optimization was performed for weight and expected cumulative cracking using a well known genetic algorithm (GA). The optimization quantified the tradeoff between weight and expected cracking for a range of structural designs. Then, the PB-ROA framework was used to value the ESL option for each design on the front taking into consideration each structures performance relative to cargo capacity and availability resulting from cracking, as well as the complexity of the stiffened panels forming the structure.

PB-ROA revealed useful insight into the structural design for this high speed aluminum catamaran. It revealed stable regions of the design space where the maximum value of the ESL option was achieved using the same structural designs. PB-ROA exposed thresholds in certain conditions that would cause the design with the highest associated ESL option value to shift. Specifically, it showed which two designs resulted in the greatest option value depending on the likelihood of service like extension. Such insight could be extremely valuable to guide efforts and resources in early stage design and offer a quantitative support tool for design decisioning pertaining to flexibility.

This chapter also demonstrates how PB-ROA can be used to incorporate structural performance as a component of system value where previously it was considered separately, or strictly from a requirements fulfillment perspective. The case study shows how the PB-ROA framework can accommodate multiple risk factors with arbitrary probability density functions (pdf's), multiple capability measures for an asset, and varying degrees of modeling fidelity, making it a potentially powerful approach to evaluating flexible systems in a wide variety of contexts.

Finally, this chapter also adds to the body of research on real options "in" projects, which are substantially different than real options "on" projects as described by Wang and DeNeufville [123, 124].

CHAPTER V

Case Study: The Government Budgeting Game

5.1 Introduction

One of the differences between traditional real options and naval options is the potential for naval options to have a feedback effect on the environment, or change the behavior of other players. In finance this is almost always ignored because of the Efficient Market Hypothesis [41]. One result of the Efficient Market Hypothesis is that no individual player is large enough, or influential enough, to "move the market" on their own. Financial analysis is greatly simplified by assuming that each player is a "price taker," who must accept the current price being determined by the Market. For most financial analysis, to include options analysis, this is an acceptable assumption and closely models what actually occurs in the markets. In less mature markets, or illiquid markets, the Efficient Market Hypothesis begins to break down and there is also a growing field of study that merges game theory with options theory to analyze situations with interdependent decision making, which will be discussed in the following sections.

What is significant for this research is that the Navy is a sufficiently large player in their "market" that there are many conceivable scenarios where part of a naval option's value would be in its ability to influence the decisions of other players, potentially to the Navy's advantage. Traditional real options analysis was not designed to handle such scenarios, and therefor the Navy requires a framework that goes beyond what traditional real options analysis can offer. The prospect theory-based real options analysis (PB-ROA) framework is designed to handle these scenarios.

This chapter presents an academic example to illustrate the steps of the PB-ROA framework, and how it can generate useful insights for such game-like naval options. The example builds on the option to extend the service life (ESL option) of the high speed aluminum catamaran in chapter IV. This example examines how the ESL option might influence the decisions of government budget makers. So if the previous case study evaluated the ESL option from the perspective of a single asset, then this case study evaluates it from an institutional perspective.

The remainder of this chapter is organized as follows. Section 5.2 reviews the basics of game theory, covering many famous games and their relevance. Section 5.3 is a literature review of the how game theory has been used in the Naval and marine domains. Section 5.4 details how game theory is used in the PB-ROA framework. Then, section 5.5 describes the case study and its game structure. Sections 5.5.1 and 5.6 show the outcome of the analysis and discuss the results.

5.2 Fundamentals of Game Theory

5.2.1 What is a Game?

Game theory is "the study of mathematical models of conflict and cooperation between intelligent rational decision-makers." (p. 1, [85])

In the simplest sense, a game is a mathematical structure describing the strategies and corresponding rewards for rational agents in scenarios where their decision making is interdependent. When decisions are interdependent, it is implied that one's preferred strategy may change when conditioned on the choice(s) of the other agent(s). The game of chess is an example of interdependent decision making. In chess, a player's decisions on strategy will be influenced by their beliefs about the future strategies of their opponent, and vice versa. There are nearly as many types of games as there are people who have studied them. Games may be cooperative or competitive, symmetric or asymmetric, simultaneous or sequential, with perfect or imperfect information... the list continues. The general nature of game theory has encouraged a range of applications of it to fields as diverse as economics, biology, psychology, and many other fields of both social and natural sciences.

5.2.2 Solution Concepts in Game Theory: Nash Equilibrium

Some important concepts in game theory are those of a solution concept, dominant strategies, and Nash equilibrium. A solution concept is a set of strategies. While strategies yield utilities for the players, the utilities themselves are not the solution, but the strategies. Consider the general two person game in figure 5.1, where each player has two pure strategies.

		Player 2	
		θ	ϕ
Player 1	α	A, a	B, b
	β	C, c	D, d

Figure 5.1: General two person, two strategy game structure

In this game, player 1 has two pure strategies to choose from, α and β. Likewise, player two has two strategies, ϕ and θ. Either player may also form an infinite number of mixed strategies. For example, player 1 could choose to play strategy α with probability, p, and β with probability $(1-p)$. This is a linear combination of their pure strategies where the weights sum to one. Note that pure strategies may be considered as special cases of mixed strategies where $p = 1$, or $p = 0$, in this example. (A, B, C, D) and (a, b, c, d) are the payoffs for players 1 and 2, respectively, for each possible outcome of the game.

A strategy x dominates another strategy y if the payoffs resulting from x are preferred to those of y. For instance, strategy α would *strongly* dominate β for player 1 if $A > C$ and $B > D$, strictly. Strategy α would *weakly* dominate β for player 1 if $A > C$ and $B = D$.

Solution concepts are sets of equilibrium strategies. An equilibrium strategy is a stable one, where none of the players wish to change their strategies. It is important to note that an equilibrium may not be optimal, in the sense that it yields the highest possible utility (see the "prisoner's dilemma," for example). An equilibrium may either be a *pure strategy* equilibrium, or a *mixed strategy* equilibrium. Von Neumann and Morgenstern showed the existence of mixed equilibrium for finite, zero-sum games in [121]. John Forbes Nash extended this concept of equilibrium, now called the Nash Equilibrium, and proved that there is at least one Nash equilibrium for all finite non-cooperative games, to include non zero-sum games [86]. Non-cooperation simply means that players are not allowed to communicate or form coalitions; they act independently. This proof is significant as it guarantees that every finite game has at least one solution. For a solution concept to be stable, it must be a Nash equilibrium. A Nash equilibrium exists when no players has an incentive to unilaterally change their strategy. This rather vague definition will be made clear in the following sections.

5.2.3 Refinement of Solutions

Since Nash equilibrium are the solution(s) to games, and it is often the case that games have multiple Nash equilibria, it is desirable if one may *refine* the set of Nash equilibria to perhaps yield a unique solution to the game. This desire has led to important concepts like focal points and stable equilibria.

A focal point may exist if players are inclined toward a particular solution, or give more attention or consideration to one solution over another in the absence of communication. Another way to describe this phenomenon is that one solution may be more salient to the players than others. The idea was introduce by Nobel Prize winning economist, Thomas Schelling [101]. For instance, in the Battle of the Sexes game, (see section 5.2.4.4), going to the football game could be a focal point if that was where the couple had their first date, and is therefor salient over shopping. Another clear example is by Mehta et al [83]. In this example, people were asked to name a mountain (any mountain in the world). As long as the other player, with whom they were not able to communicate with, named the same mountain they would receive a positive payoff and the payoff was the same no matter which mountain was named. This is a classic coordination game. While every mountain in the world is a potential solution, 89% of participants in their experiment named Mount Everest. In this experiment, Mount Everest was the focal point.

Risk dominance and payoff dominance are two other common types of refinement for games with multiple Nash equilibria. This concept is best illustrated in the classic Stag Hunt game (see section 5.2.4.3). In this game, there are two pure strategy Nash equilibrium. The strategy where both players coordinate to hunt a stag is the *payoff* *dominant* equilibrium because catching a stag is more valuable than a rabbit. The strategy where both players coordinate to hunt rabbits is the *risk dominant* equilibrium because catching rabbits is less risky than stags (presumably your probability of success is higher with rabbits than stags). This creates a tradeoff between the two Nash equilibria where one is desired because it is worth more, but the other is desired because it is safer [112][102]. John Harsanyi and Reinhard Selten pioneered this area of refinement which considers the tradeoff between risk and reward to select a unique solution for noncooperative games, or at least narrow the set of plausible solutions [52][50][51]. Depending on the relative probabilities of success for stags and rabbits, and the hunters risk tolerances, one of the two Nash Equilibria will usually dominate the other.

Another method of refinement when multiple Nash equilibria exist is to apply the concept of *stable equilibrium* first introduced by Kohlberg and Mertens [71], and later reformulated by Govidan and Mertens [46]. In order for a Nash equilibrium to also be a stable equilibrium, it must satisfy four criteria: backwards induction, invariance, admissibility, and iterated domination. To quote from Kohlberg and Mertens, each of these criteria mean the following.

> "*Backwards Induction* : A solution of a tree contains a backwards induction (e.g. sequential or perfect) equilibrium of the tree.
> *Invariance* : A solution of a game is also a solution of any equivalent game (i.e., having the same reduced normal form)
> *Admissibility* : The players' strategies are undominated at any point in a solution.
> *Iterated-Dominance* : A solution of a game G contains a solution of any game G' obtained from G by deletion of a dominated strategy."
> (See [71], p. 1020)

Stable equilibrium is an important concept for determining the unique solution proposed for the Battle of the Sexes game with burning money, discussed in section

5.2.4.4.

5.2.4 Famous Games

The following is a short selection of famous games from the game theory literature. Each are significant for their unique insights or structure, which will be discussed. It is common for real world applications of game theory to be composed of smaller "subgames" and for each subgame to be directly or closely related to some of the following famous games. They are also presented to give the reader who is perhaps unfamiliar with game theory tangible examples of the principles involved.

5.2.4.1 Prisoners Dilemma

The typical setup for the prisoner's dilemma game is the following. You and a friend have committed a crime together and been caught. The police take each of you into separate rooms for interrogation because their goal is to get at least one of you to betray the other by giving up details of the crime. This also makes the game non-cooperative. So each player (you and the accomplice) has two choices, to remain silent (strategy A) or to betray the partner (strategy B). This crime carries a maximum sentence of three years. If both players betray each other then each will spend two years in prison. Hence, the payoffs for (B, B) are (-2,-2). If both players remain silent, then each will spend 1 year in prison (presumably the police don't have enough evidence to get the 3 year sentence without a confession). Hence, the payoffs for (A, A) are (-1, -1). If, however, one player betrays the other player who has chosen to remain silent, then the silent player will go to prison for 3 years, and the betrayer will walk free. The normal form of this game is shown in figure 5.2.

A neutral third party might suggest that each player should remain silent. After all, this outcome is "fair" because it yields equal payoffs for both players, and it results in

		Player 2	
		A	B
Player 1	A	-1, -1	-3, 0
	B	0, -3	-2, -2

Figure 5.2: Prisoner's Dilemma Game

less prison time compared to both players betraying each other. However, (A, A) is not a Nash equilibrium, and therefor cannot be a solution concept for this game. The reason is that beginning from (A, A), both players have an incentive to betray the other. For example, once Player 1 believes that Player 2 will remain silent, there is an incentive for Player 1 to betray because this results in a higher payoff for theirself. As rational, utility maximizers [121] that player will betray, leading to outcome (B, A). Of course, starting from position (B, A), player 2 has an incentive to change their strategy to also betray, leading to outcome (B, B). Only (B, B) is a Nash equilibrium for this game because it is the only position from which neither player would benefit from unilaterally changing their strategy [86]. What is distressing is that (B, B) has suboptimal payoffs. Both players would be better off if they could somehow coordinate to agree on strategy (A, A).

5.2.4.2 Chicken

The game of chicken, also known as the hawk-dove game, is an *anti-coordination* game, meaning that the highest paying outcomes occur when the players choose opposing strategies. In the most general description, it is used to model scenarios where players must compete over a limited, and indivisible resource [62][96]. The game is also commonly used to study appeasement and escalation in conflict, as well as brinksmanship [110]. The name is a cultural reference to the game in which to people drive their cars at high speed towards each other, and the first person to swerve is the loser, or the coward. So each player can be said to have the same two strategies to choose from: swerve (A), or go straight (B). If both players swerve, (A, A), then the outcome is a wash. Neither player wins. If both players go straight, (B, B), then there is a crash. Mathematically this is shown by large negative payoffs for both players. If one player swerves while the other goes straight, (A, B) or (B, A), then the swerving player loses. The normal form of this game is shown in figure 5.3.

		Player 2	
		A	B
Player 1	A	0, 0	-1, 1
	B	1, -1	-10, -10

Figure 5.3: Chicken Game

All anti-coordination games have three Nash equilibria. There are two pure strategy Nash equilibria, (A, B) and (B, A). And there is a mixed strategy Nash equilibrium with symmetric payoffs where each player swerves with probability 9/10 and goes straight with probability 1/10.

Since there are three Nash equilibria one of the things that makes this game interesting is devising variations of the basic game which can bias the outcome more definitively in one player's favor. For example, player 1 could openly state their intent to go straight before the game is played [66]. While their statement is non-binding, the common knowledge of this statement could make (B, A) a focal point (depending on the psychologies of both players). Another common variation is to include a binding commitment to go straight on the behalf of one player, and to make this commitment common knowledge before the game. A colorful example would be for player 1 to hold his hands out the window of the car while driving, where the player 2 can see them. By doing so, player 1 is signaling a commitment to go straight, in which case player 2's best response is to swerve, securing player 1 the victory. Such binding commitments are interesting because game theory is able to mathematically demonstrate the counter-intuitive possibility that it may be in one's favor to voluntarily limit their choices in some scenarios [62].

5.2.4.3 Stag Hunt

The stag hunt game is another model of social coordination very similar to the prisoner's dilemma game. As shown in figure 5.4, the main difference between it and the prisoner's dilemma is that there are two pure strategy Nash equilibria; (A, A) as well as (B, B). The classic description, attributed to Jean-Jacques Rousseau, is of two hunters who are faced with the choice between hunting a stag or a hare. Hunters can catch hares independently, but they require each others' help to capture a stag. Stags are also much more valuable than hares. Following this description, strategy A is to hunt stags, and B is to hunt hares. The game is significant because it illustrates the tradeoff between safety and social cooperation in decision making [105]. Of the two Nash equilibria, strategy (A, A) is *payoff dominant* and Pareto efficient, because it yields higher expected payoffs for both players. The strategy (B, B) is *risk dominant.*

		Player 2	
		A	B
Player 1	A	4, 4	0, 2
	B	2, 0	1, 1

Figure 5.4: Stag Hunt Game

Because human beings are generally risk averse, some players may choose strategy B, even though (A, A) has higher expected payoff. There may be many reasons for this. For instance, if the probability of success given strategy (A, A) is less that that for (B, B), the risk averse hunter may be lead to “settle” for the less valuable hare. Incomplete information may be another reason [51]. For instance, player 1 may be unsure about player 2’s hunting skills, which will of course effect the probability of success of hunting stags. In this discussion, strategy (B, B) is the *risk dominant* solution because it is perceived to be less risky [112]. Such concepts are important for the application of game theory to the naval domain where decision makers may be particularly risk averse.

5.2.4.4 Battle of the Sexes

The commonly termed Battle of the Sexes game is another famous coordination game. Just like Stag Hunt, the players benefit by coordinating strategies. However, the payoffs are asymmetric, among other unique properties. The typically description is of a husband and wife who have forgotten where they agreed to meet that night, and must decide between shopping and football, in the absence of communication. The husband prefers football, but only when in the company of his wife. Similarly, the wife prefers shopping, but also wants the other’s company. The normal form of such a game is shown in figure 5.5, where the wife is player 1, so player 2 is the husband. Strategy A is to go shopping, whereas B is to go to the football game.

		Player 2	
		A	B
Player 1	A	3, 1	0, 0
	B	0, 0	1, 3

Figure 5.5: Battle of the Sexes Game

This game has three Nash equilibria. One is choose (A, A) in which case the wife gets her preferred outcome. Another is to choose (B, B) in which case the husband gets his preferred outcome. Since one player is getting their preferred outcome while the other settles for their less-preferred outcome, such equilibria might be described as unfair. The third equilibrium is a mixed strategy. For the example in figure 5.5, both players would play their preferred strategy with probability $p = 3/4$, resulting in an expected payoff of 0.75 for both players. While one might describe this outcome as being fair, it actually results in a *lower* payoff for both players than one could achieve by agreeing to always choose their less-preferred strategy. This is one of the things that make Battle of the Sexes so interesting.

A nearly infinite number of variations of the battle of the sexes game have been proposed which may resolve the problem of multiple equilibria in the original formulation to a unique solution. A common variation includes allowing communication between players before the game is played [28]. Another variation of the game, which is significant for the formulation of the following case study, called the battle of the sexes game with burning money. The example comes from the work of van Damme [118], Osborne [90], and Ben-Porath and Dekel [9].

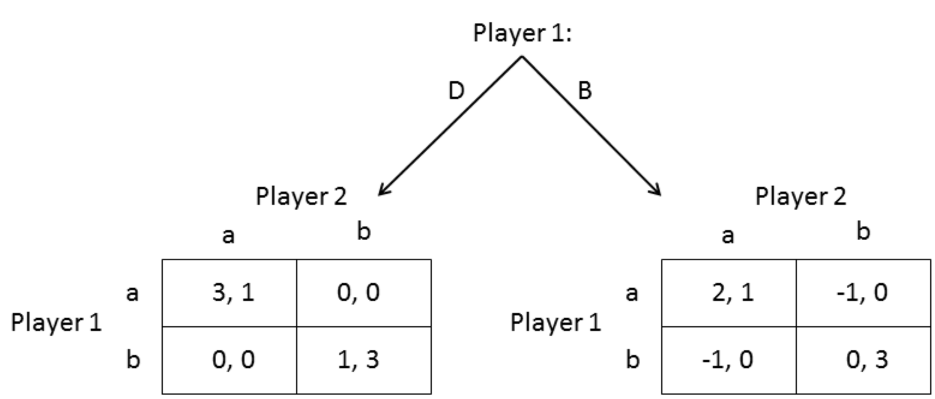

Figure 5.6: Battle of the Sexes Game with Burning Money

In the burning money variation, shown in figure 5.6, the structure is changed slightly allowing the wife the option to burn some of her money. Should she choose to burn money (strategy B), her payoffs are uniformly lowered in all scenarios of the game. If she does not burn money (strategy D), the game is exactly the same as before. The option to burn the money is common knowledge, and the husband is able to observe the wife's choice (burn, or not burn) before making his decision to play strategy a or b. While burning money does not actually affect any of the husband's payoffs, it results in a unique solution in which the wife always gets her preferred outcome.

The outcome of this game is somewhat controversial, as it relies on the concept of *forward induction* which means that the wife, by not burning any money, is sending a signal to the husband that she expects a better outcome than any attainable if she had burned the money. She is signaling that she expects the husband to then choose a. By not burning the money, which is a visible decision to her husband, the wife is signaling her intent to then play a. Given this information, the husbands best response is to also play a. After all, if the wife is signaling her true intent, then the husband does not stand to benefit by unilaterally deviating from choosing a. The unique solution arises through the iterated elimination of weakly dominated strategies. The normal form of the burning money game is shown in figure 5.7.

		Player 2			
		aa	ab	ba	bb
Player 1	Da	3, 1	3, 1	0, 0	0, 0
	Db	0, 0	0, 0	1, 3	1, 3
	Ba	2, 1	-1, 0	2, 1	-1, 0
	Bb	-1, 0	0, 3	-1, 0	0, 3

Figure 5.7: Battle of the Sexes Game with Burning Money - Normal Form

For player 1, strategy Ba signifies that they are choosing to first burn money, and then play a. Similarly, Db signifies that they are choosing to not burn money, and then play b. For player 2, strategy aa means that they will choose a regardless of player 1's decision to burn money or not. Similarly, ba means that they will choose b if player 1 does not burn money, but will choose a if player 1 does burn money.

By the iterated elimination of weakly dominated strategies: Da dominates Bb, then ba dominates bb, then aa dominates ab, then Ba dominates Db, then aa dominates ba, then Da dominates Ba, and finally the only solution left is (Da, aa). While mathematically sound and in accordance with Mertens stability [71, 46], some authors have questioned the validity of the threat to burn money, and whether forward induction is a logical criteria for refinement of equilibria [99, 89]. Others have experimentally tested the descriptive power of forward induction, and found mixed results for games with slight variations [58, 29].

5.3 Game Theory Literature Review

Aside from it's many applications in analysis of conflict, war, and military strategy [61, 73, 54, 11, 5, 31], game theory has also been used extensively in complex system design and real options analysis. What follows is a brief review of related work from the academic literature on game theory, design, and real options. It is not meant to be exhaustive, but to give the reader a taste of some of the applications of game theory related to naval ship design.

Rao et al. studied how game theory can lend a structure to negotiating between conflicting objectives in a multiobjective optimization structural design [95]. Their specific application was to large space structures where the designer is faced with competing desires to minimize weight, yet also control the vibratory characteristics of the structure. In such conflicting design scenarios, optimization will yield a Pareto front of undominated designs. Mathematically speaking, one cannot say any one design on front is the unique optimum. Roa et al.'s contribution was to show how game theory could lend a structured approach to negotiating between objective functions to reduce the Pareto front to a single point design, assuming cooperation between players.

In practice, different designers or design disciplines may not be in perfect cooperation - whether by choice or necessity. This can create added uncertainty in the design process. Hacker and Lewis include a case study where the weights team of an aerospace design doesn't know the exact objective function of the aerodynamics team, and vice versa [49]. In their study, each discipline had control to choose certain variables which impacted the other disciplines. So each disciplines decisions are interdependent. They demonstrated how game theory can be used to mitigate this kind of uncertainty by helping design teams to formulate strategies which are optimal to the others' rational reaction sets (RSS) - essentially playing Nash equilibrium sets.

Where Hacker and Lewis were choosing point values for the variables in their study [49], Panchal et al. negotiate over ranges of values for each variable [92, 93]. In doing so they show how game theory can help to negotiate variables in non-cooperative, set-based design environments [78, 13, 104]. A perennial question when implementing a set-based design framework is *how* to reduce the variable sets to eventually converge on a final design. The decision making process for convergence is critically important. While many authors implicitly or explicitly assume that decision making is centralized (ex: guided by a "project manager" or "lead architect") [81], Panchal et al. are unique in their treatment of the decision making process as decentralized. Liang et al. apply a similar game theory structure specifically to the ship design problem [77].

These works all considered how the interactions of decision makers in the design process can be modeled by game theory. In this way they all might be described as being intra-organizational research. This dissertation will expand the use of game theory to generate insights outside of the design organization. Ultimately, this research is motivated by the need to understand how certain real options may change the relationships between interdependent decision makers, possibly from many different organizations both friendly and confrontational.

A typical financial option does not consider the interdependencies of agent decision making. This is because such interdependencies either do not exist, or are considered negligible according to the efficient market hypothesis [41]. One notable exception is Kühn [72] who studied how to price game contingent claims - financial derivatives that could be exercised (i.e. terminated) by both the buyer and the seller. However, the impact of such interdependencies may be significant in certain real options applications, and particularly for the U.S. Navy. To address such interdependencies, a small group of authors have begun to study what may generally be termed "game options." Such game options may arise in situations like research and development, price wars, and first mover advantage.

Yuri Kifer introduces a financial security he calls a game option, that may be terminated by either the buyer or the seller [65]. Kifer uses the theory of optimal stopping games to calculate the price of such an option. It is significant because it shows how the threat of cancellation by the seller may encourage the buyer to exercise the option earlier than what was previously considered optimal. Smit and Ankum [107] consider an analogous real call option on production facilities. According to options analysis, there is value to delaying the investment to see how market demand evolves.

However, in the face of competition from other agents, Smit and Ankum note that the firm may be forced to invest early in order to protect their own returns.

Lukas et al. [79] use a game option approach when investigating option value in mergers and earnout applications. Uncertainty is included in the game to impact optimal timing of the options exercise. Villani uses option game theory to inform decision making for firms investing in R&D in competitive environments [120]. The inclusion of game theory allows one to consider both the positive (additional market share) and negative (additional information for the follower) aspects of first mover advantage.

Grenadier [48] gives a general solution approach, suitable for a wide variety of applications, for this issue of finding equilibrium investment strategies for firms in environments where competition may erode the value of an option to delay.

Smit [106] uses game option theory to analyze the value of early investment in infrastructure to enable future expansion, focusing on European airports. Smit and Trigeorgis (2006, [108]) consider the impact of coordination and collaboration between agents on strategic option value. Smit and Trigeorgis (2007, [109]) also give a general review of how to use game option theory to value strategic options in competitive environments with interdependent decision making of agents. Other applications of game option theory includes optimal bidding strategy at electricity market auctions [76], and the tradeoff between outsourcing and vertical integration and its impact on supplier pricing [122].

These works greatly inform this research, and show that there is a significant intersection between real options and game theory. However, where these works have largely considered the impact of game-like decision interdependencies on the value of an option, this research will go one step further and investigate how the introduction of new options into an existing game can change the structure of the game or its equilibrium(s). Such an analysis framework would be useful for the Navy when considering the acquisition of assets with option-like characteristics which may be strategically deployed to change, or influence, their operating environment. **Where others have investigated the impact of the environment on the option, this research investigates the impact of the option on the environment.**

To the author's knowledge, the most similar concept discussed in the literature on game options is of the impact of *outside options* on negotiation between players in a game. If negotiation takes the form of a game between a negotiator and a candidate, and the negotiator may choose between multiple candidates to negotiate with, then the candidates are outside options with respect to each other. According to Li et al. (p. 31, [74]),

> "The outside options contribute to the environment of the negotiation with a candidate."

Li et al. give the example of the Navy's need to match open billets with sailors. If there is a three month period when sailors are listed as available for rotation, then Navy command isn't just negotiating with the current sailors applying for an open billet. Command also has the outside option of the sailors who *may* become available (and eligible) for the billet before the assignment deadline arrives. To complicate matters, there may be uncertainty as to the number of outside options that will be realized, and their values. The strategy for the negotiator is different depending on whether or not she considers the outside options. Li et al. show that including the outside options in the decision making process leads to higher utility for the negotiator. Basically, this is because the outside options make it optimal for the negotiator to "sit and wait" while uncertainty is resolved, ensuring better outcomes. In fact, Sim proved that a "sit and wait" strategy is dominant in such bilateral negotiations with potential outside options [103]. This research adds to the current state of the art by expanding the scope to more than outside options. In theory, the PB-ROA framework presented in this dissertation can handle *any* type of option.

5.4 Game Theory within the PB-ROA Framework

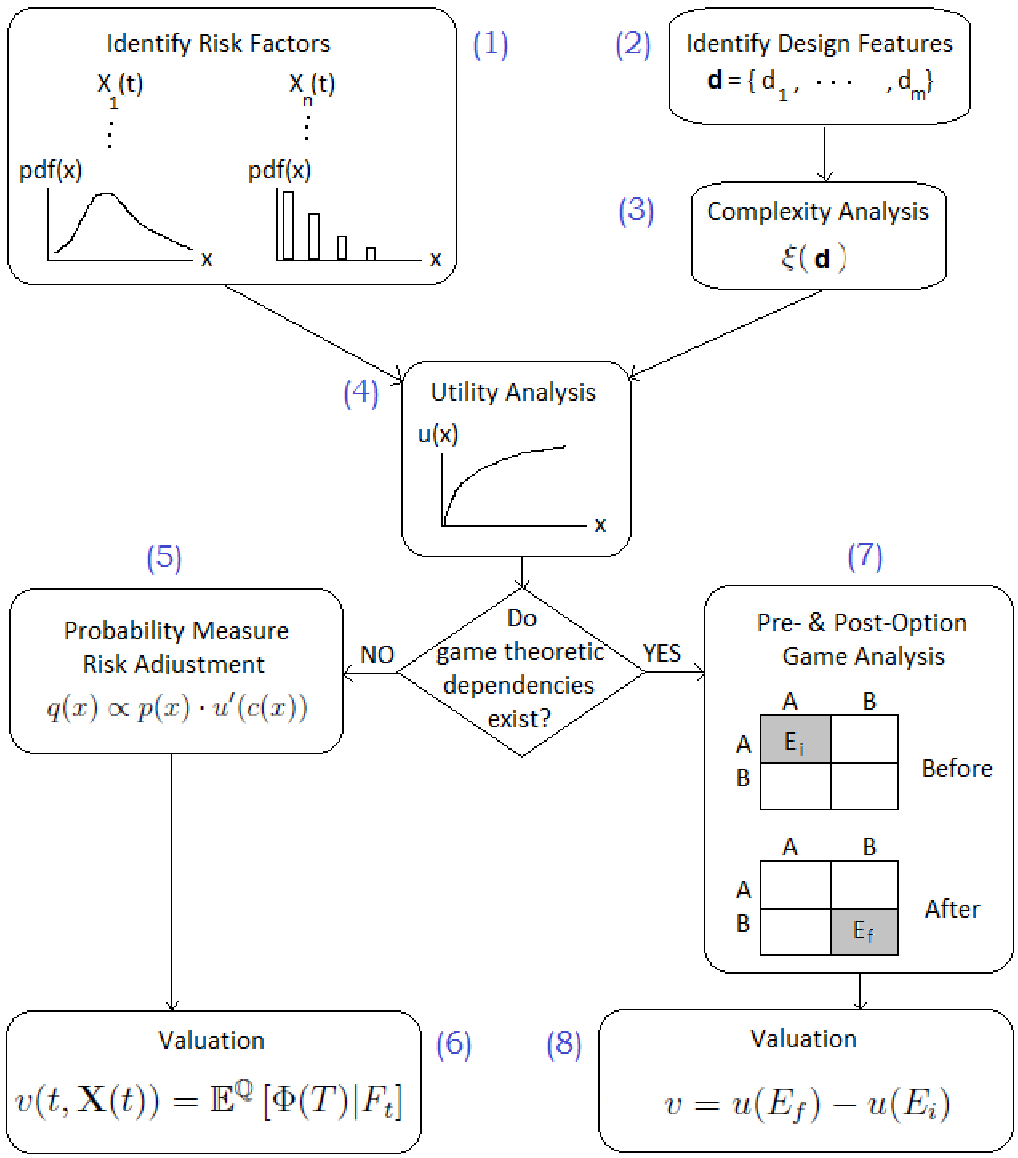

Figure 5.8: PB-ROA framework flowchart

To review, this research advocates using game theory for those options which have a feedback effect on their environment. As shown in the PB-ROA framework flowchart, some naval options may have game theoretic dependencies. A rule of thumb for determining if this is true is to ask, "does the existence of the option change the behavior of any other players in my environment?" If the answer is yes, then most likely there are game theoretic dependencies to be addressed.

In this framework, real options are viewed as being potentially *game changing* . This is both literal and figurative in its meaning. Figuratively, a real option may be game changing because it offers a radical departure from previously established norms. For example, modular ship design and construction practices were game changing compared to the traditional “keel-up” method of building ships. Literally, a real option may be game changing if it alters the structure, strategies, or payoffs of decisions in a multi-player interdependent context. Designers may intuitively understand the value of a real option that changes the state-of-the-art in their field. This research offers a quantitative method for understanding the the value of many such options.

In general, this research states that the value of the option is equal to the change in the value of the Nash equilibriums of the games before and after the introduction of the option.

$$v = u(E_f) - u(E_i) \tag{5.1}$$

where E_i is the Nash equilibrium of the initial game, E_f is the Nash equilibrium of the final game after the option is introduced, $u(E)$ is the utility payoff of a Nash equilibrium, and v is the option value. This proposition is simple in theory, but in practice it can be more complicated. This is because, as discussed previously, many games have multiple Nash equilibria. For this reason, we address each possible scenario. In each of the following scenarios it is important to apply all refinements first.

One-to-One Equilibria This is the simplest possible scenario. One-to-one means that there is a unique Nash equilibrium in both the before and after games. In this scenario, the value of the option is straight-forward to calculate, being the scalar difference between the utility payoffs from the Nash equilibria of the before and after games.

$$v = u(E_f) - u(E_i) \tag{5.2}$$

Many-to-One Equilibria In this scenario the original game (without the real option) has multiple Nash equilibria. However, after the real option is considered, the structure of the game is changed such that there is a unique Nash equilibrium. To determine the value of the option in this scenario we begin by considering the best possible outcome from the set of Nash equilibria for the initial game, $\max[u(E_i)]$. Then, the value of the option is *at least* equal to the difference between the utility of the final equilibrium and this best case starting point.

$$v \geq u(E_f) - \max[u(E_i)] \tag{5.3}$$

In this scenario PB-ROA is only able to provide a lower bound on the option's value. This is because there is added value from the decreased uncertainty the option is providing. However, it is difficult (perhaps impossible) to exactly quantify this added value. Uncertainty in game theory cannot easily be quantified in the way that it can be in finance. In finance, uncertainty is reflected in the standard deviation of returns on the price of the underlying asset, which can be calibrated to historical data or forecasted using some other analysis tools. However, there is no such thing as standard deviation between Nash equilibria. Therefor we cannot quantify the exact value of mitigating such uncertainty. However, since we know that mitigating such risk *does* have positive value, PB-ROA can at least give a lower bound. Of course this proposition relies on the assumption that decision makers for the U.S. Navy are risk averse. If that assumption is ever not true, then equation 5.3 would be misleading.

One-to-Many Equilibria This scenario is the opposite of the previous scenario. In the one-to-many scenario, adding the real option creates multiple Nash equilibria where previously there was only one. In this scenario, we consider the best case outcome, this time for the after game, $\max[u(E_f)]$. Then, the value of the option is *no more* than the difference between this best outcome and the starting Nash utility. Because it is assumed that the Navy is risk averse, then adding uncertainty would have negative value.

$$v \leq \max[u(E_f)] - u(E_i) \tag{5.4}$$

Many-to-Many Equilibria This is the most difficult scenario to analyze, there are multiple Nash equilibria in both the before and after games. This research does not propose an equation valuing real options of this type. In this scenario, PB-ROA may lend qualitative insights by showing how the structure and payoffs change after the option is introduced.

Ultimately, the exact number given to the value of an option may be of less importance for these types of game changing options. What is important is having a consistent framework to evaluate how these options alter the behavior of other players or otherwise have feedback on the Navy's environment. With the incorporation of game theory, PB-ROA can provide such a consistent framework. Critics may suggest that this statement is reverting back to the heuristic, anecdotal type analyses used for naval options in the past which this research is supposed to supplement. I disagree. Game theory provides a highly quantitative tool for assessing option value, even if such value is not expressed as a single number as it is in finance.

5.5 Applied Example

The previous chapter presented a case study in which the value of extending the service life of a high-speed aluminum military catamaran was analyzed. The extended service life (ESL) option was evaluated from the perspective of a single ship. Any influence that the ESL option may have on other players in the Naval environment was ignored. In this chapter we continue the analysis of the ESL option, this time examining how it may change the behavior of other players.

For this academic example, we consider the general process of negotiating budgets between the U.S. Navy and another government entity, and what impact the ESL option may have on those negotiations. To set the scene for this case study, consider the following:

- Development, and sometimes acquisition, of "next generation" or replacement platforms typically occurs well in advance of the existing platform's retirement.
- The U.S. Navy must agree with certain other government entities on a budget. The Navy cannot unilaterally alter a pre-existing budget agreement.
- There are many cases where *both* the U.S. Navy *and* other government entities have an interest in keeping next generation and replacement programs alive before they have reached the acquisition phase.
- Historically, the U.S. Navy (and Department of Defense generally) has seen periods of contracting budget pressures.

The main question at hand is whether the ESL option can be used to leverage negotiations in a contracting budget environment. Intuitively, one knows that the ESL option could provide Navy operations a hedge against negative developments in the replacement program, whether technological or budgetary. However, could it also be possible for the ESL option to influence budget negotiations in such a way that the replacement program is protected from budget contractions?

While such a notion may seem counterintuitive at first, similar events have occurred in the past. While they are not exactly the same as the example presented in the following sections, one merits a brief mention. It is the repeated budgetary disagreements between the DOD and Congress over the continued funding of the alternate engine program for the F-35 joint strike fighter. The DOD and presidential administrations repeatedly submitted budgets de-funding the alternate engine program only to have Congress pass bills calling for its further funding [88, 44]. This dissertation does not speculate as to whether or not the DOD was leveraging the cancellation of the replacement engine program to achieve other objectives in their budget negotiations. It is simply an example of a program which received additional funding from Congress after the DOD recommended its cancellation, at least partly because Congress had a vested interest in its continuation.

In the analysis that follows, many simplifying assumptions have been made. The intent of this example is to give an illustration of how the PB-ROA framework can lend insight on the value of naval options from an institutional perspective where they have the ability to influence the behavior of other agents in the Navy's environment. The process of Navy budgeting is more complicated than the game analysis that follows, where only two players are considered; the U.S. Navy, and another government entity with a role in budgeting.

One could argue that the government should not be modeled as a single player, as it is in this case study. Within Congress, for instance, which plays a role in Navy budgeting, there are many distinct committees composed of a select few representatives. Furthermore, more than one voice may be reflected in any Congressional decision, complicating the notion that government entities may be of "one mind" on an issue. Navy budgeting in reality also occurs in stages, and the structure of the games may be quite different in each stage. For instance, one could argue that under normal circumstances the stage of negotiations between Navy and Congress should be modeled by a zero sum game, reflecting the process of allocating a fixed quantity of dollars between a portfolio of programs or assets. The stage of negotiation where increases or decreases to the total budget are made might occur between the Navy and the Office of Management and Budget (OMB). OMB is part of the Executive branch, and this stage would occur before reaching talks with Congress. In addition, there may also be shipyard considerations. If owning the option to extend the service life of a class of vessels could result in a decrease in shipyard production rates, then the shipyards might be additional players in the negotiation game. Addressing any of these complications is an exciting area for future work with PB-ROA. However, such efforts are outside the scope of this dissertation.

5.5.1 Game Structure: the Government Budgeting Game

The following game is refered to as the government budgeting game, and begins by modeling the game before the introduction of the extended service life (ESL) option. In all that follows the U.S. Navy is player 1 and player 2 is another government entity with a role in budgeting. Assuming that the current environment is one of contracting budget pressure, the Navy may choose between proposing to fully fund all programs, or make percentage cuts across the board. This assumes that all current programs are necessary to achieve the Navy's various missions so nothing can be outright cancelled. One of these programs facing potential cuts is the high speed catamaran replacement platform. Likewise the government entity may choose between continuing previous levels of funding, or forcing cuts to the Navy's budget. With this in mind, the following strategies are defined for both players:

F = fully fund all current Navy programs.

C = make percentage cuts across all Navy programs.

Since the Navy and government entity *must* eventually agree on a budget, the negotiation process may be modeled using a coordination game almost identical to the battle-of-the-sexes game in section 5.2.4.4, shown in figure 5.9. This game has three Nash equilibria. The two pure strategy Nash equilibria are (F, F) and (C, C). The mixed equilibrium is ignored because this is not a repeated game, and mixed strategies are not relevant for this application.[1]

		Government Entity	
		F	C
Navy	F	3, 2	0, 0
	C	0, 0	1, 3

Figure 5.9: Government budgeting game - without the ESL option

This model of course assumes that the game is non-cooperative and therefor players cannot communicate. While communication does indeed occur in government budget negotiations, it is nonetheless a reasonable model of the conditions *before* the first budget proposal is made. The unfortunate thing about this game from the Navy's perspective is the high degree of uncertainty over the final outcome. It is possible that they will be forced to make cuts.

Now consider the environment if the Navy had invested in the ESL option for the

[1] It is too unrealistic to suggest that either player in this example would make budget decisions randomly.

existing fleet of high speed catamarans. A fundamental difference resulting from the ESL option is that the Navy could conceivably cancel the replacement program. It can do so because it owns the option to extend the service life of the existing vessels, thereby still fulfilling its mission requirements. Without the ESL option, the Navy cannot cancel the replacement program (without incurring a prohibitively high cost penalty) because the existing fleet does not have enough lifecycle remaining to fulfill the Navy's mission requirements.

With this flexibility the Navy may restructure the government budgeting game to somewhat resemble the burning money game from figure 5.6. The new government budgeting game could be modeled by the structure in figure 5.10.

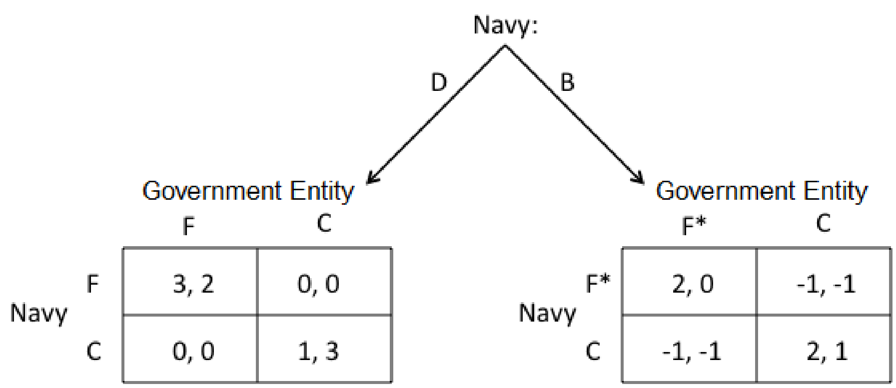

Figure 5.10: Government budgeting game - with the ESL option

In this model, the strategies mean the following:

D = don't cancel the replacement program ("don't burn").

B = cancel the replacement program ("burn").

F = fully fund all current Navy programs.

C = make percentage cuts across all Navy programs.

F* = fully fund all remaining Navy programs after the replacement program is cancelled.

As it has been mentioned previously, the relative magnitudes of the payoffs in each cell of these game matrices do not matter for finding the Nash equilibria. One cannot even necessarily directly compare the values between players. In other words, 2 utils does not necessarily have the same value for the Navy as it does for the government entity. What does matter is the order of preferences created by each payoff for each player. So the structure chosen for this case study gives the following preferences over outcomes for the Navy, in order from most preferred to least preferred:

1. Not cancel the replacement program, and have the government entity agree to fully fund all Naval programs.

2. Exercise the option to extend the service life of the existing catamarans, cancel the replacement program, and have the government entity agree to fully fund all remaining Naval programs.

3. Exercise the option to extend the service life of the existing catamarans, cancel the replacement program, and accept small cuts to remaining programs.

4. Not cancel the replacement program, but accept large cuts to all programs.[2]

5. Keep the replacement program, but disagree with the government entity over budgeting.

6. Cut the replacement program and still disagree with the government entity over budgeting.

For the government entity the order of preferences over outcomes, from most preferred to least preferred is:

[2]The assumption is that under-funding many programs at the same time will prevent satisfactory mission performance. The model states that the Navy would prefer to cut the replacement program than have all programs operate under-funded.

1. Keep the replacement program alive, but force budget cuts.[3]

2. Fully fund all Naval programs, to include the replacement program.[4]

3. Make further budget cuts after the Navy cancels the replacement program.[5]

4. Fully fund all remaining programs after the Navy cancels the replacement program.

5. Keep the replacement program but disagree on a budget.

6. Cancel the replacement program and still disagree on a budget.

The normal form of this game is shown in figure 5.11. This game has four pure strategy Nash equilibria: $(DF, FF*)$, (DF, FC), $(BF*, CF*)$, and (BC, CC). Since the ESL option has transitioned from a game with two equilibria, to a game of four equilibria, one might wonder what the value really is. But note that the strategies $(BF*, CF*)$, and (BC, CC) are payoff dominated with respect to $(DF, FF*)$, and (DF, FC). Since risk dominance is not a factor in this game, and both players are rational utility maximizers, then the only equilibria that could actually occur are $(DF, FF*)$, and (DF, FC). Next, note that the outcome is really the same for both of these Nash equilibria. So we see that the ESL option has actually changed the outcome of the government budgeting game to be in their favor.

5.5.2 Perturbations of the Government Budgeting Game

Since the results of this case study are entirely dependent on the payoff structure of the game matrix, perturbations of the game are discussed to show where these

[3]This is the government entity's most preferred outcome because it allows them to trim the budget while also keeping all their constituents and/or companies involved with Naval programs sufficiently satisfied.

[4]This is ranked second because as much as the government entity wants to cut budgets, in this game they prioritize keeping constituents and/or companies satisfied.

[5]Since constituents will already be angry about the program cancellation, the government entity can at least get some small benefit from budget savings.

		Government Entity			
		FF*	FC	CF*	CC
Navy	DF	3, 2	3, 2	0, 0	0, 0
	DC	0, 0	0, 0	1, 3	1, 3
	BF*	2, 0	-1, -1	2, 0	-1, -1
	BC	-1, -1	2, 1	-1, -1	2, 1

Figure 5.11: Normal form of the government budgeting game - with the ESL option

results might break down.

5.5.2.1 Naval aversion to replacement program cancellation

The game above assumes that the Navy would prefer to cancel the replacement program than take significant cuts across all other programs. This assumption is based on the premise that under-funding a large number of programs would prohibit the Navy from satisfactorily fulfilling its mission. What if this were not true? This perturbation of the government budgeting game considers the opposite scenario where the U.S. Navy prefers to keep the replacement catamaran program alive, even if it means making budget cuts to other programs. A similar perturbation, resulting in equivalent outcomes, is if the government entity does not believe that the Navy would prefer to cancel the replacement catamaran before taking cuts to other programs. In either case, the payoff matrix for this new game is shown in figure 5.12.

From the Navy's perspective, strategies $BF*$ and BC are now weakly dominated by the mixed strategy of playing DF with probability $p = 0.5$ and DC with probability $1 - p = 0.5$ By removing the weakly dominated strategies, the resulting payoff matrix reduces to a version of the battle-of-the-sexes game, which is exactly the game the Navy was playing before the introduction of the ESL option. So, under this pertur-

		Government Entity			
		FF*	FC	CF*	CC
Navy	DF	3, 2	3, 2	0, 0	0, 0
	DC	0, 0	0, 0	2, 3	2, 3
	BF*	1, 0	-1, -1	1, 0	-1, -1
	BC	-1, -1	1, 1	-1, -1	1, 1

Figure 5.12: Perturbation of the government budgeting game where the Navy prefers keeping the replacement catamaran program over taking cuts to other programs. Changes highlighted in red.

bation, the ESL option has no value[6]

From this it can be concluded that for the ESL option to have the power to change the government entity's behavior, one of two things must happen. Either the U.S. Navy must make it common knowledge that canceling the replacement catamaran program is necessary if other programs are to perform their mission satisfactorily under a constricted budget. Or, the government entity must at least be made to *believe* that canceling the replacement program is the Navy's preferred response to a constricted budget. If neither of these conditions is true, then the ESL option will not change the government entity's behavior.

5.5.2.2 Change in Navy preferences if the replacement program is cancelled

The original game assumes that, if the replacement catamaran program is cancelled, the Navy is indifferent between the government entity increasing or decreasing funding. This is based on the premise that once the replacement program is cancelled, their is sufficient money in the budget to adequately (if not fully) fund all

[6]No value in the strict sense that it does not change the behavior of the government entity.

the remaining Naval programs. This perturbation of the government budgeting game examines the effect of altering this assumption.

Consider, that the payoff to the Navy in cells (BC, FC) and (BC, CC) is now α, as shown in figure 5.13.

		Government Entity			
		FF*	FC	CF*	CC
Navy	DF	3, 2	3, 2	0, 0	0, 0
	DC	0, 0	0, 0	1, 3	1, 3
	BF*	2, 0	-1, -1	2, 0	-1, -1
	BC	-1, -1	α, 1	-1, -1	α, 1

Figure 5.13: Perturbation of the government budgeting game where Navy preferences change in the subgame where the replacement program has been cancelled. Changes highlighted in red.

For $1 < \alpha < 3$, there are the same four Nash equilibria as in the original game: $(DF, FF*)$, (DF, FC), $(BF*, CF*)$, and (BC, CC). Moreover, $(DF, FF*)$ and (DF, FC) are still payoff dominant. This means that for any value $1 < \alpha < 3$, the ESL option will still change the structure of the negotiation game in the Navy's favor. However,

- If, $\alpha \leq 1$, then (DC, CC) becomes a Nash equilibrium. This undermines the ESL option's value because (DC, CC) is also non-dominated, which means that the outcome of the negotiations is once again very uncertain.

- If, $\alpha \geq 3$, there are still the same four Nash equilibria as in the original game, but payoff dominance can no longer be used to eliminate (BC, CC). This also means that the outcome of the negotiations is once again very uncertain.

The first point could result from a scenario where the government entity proposes even stricter budget cuts if the replacement program is cancelled - a *severe* budget cut environment. In this scenario the Navy is better off keeping the replacement program since steep cuts will be made regardless. The second point could result if the replacement program is actually detrimental to Naval operations. While difficult to conceive of such a program, it is nonetheless theoretically possible.

From this it can be concluded that there are important lower and upper bounds on the Navy's payoffs if the program is cancelled, in order for the ESL option to influence the government entity's behavior. If it is common knowledge that the payoffs exceed either of these bounds, the ESL option will once again become worthless (in the game theoretic sense).

5.5.2.3 Stronger governmental consequences to budget cuts

A third possible perturbation of the government budgeting game is to decrease the government entity's payoff for making budget cuts. Because many private companies and state governments also have vested interests in many Naval programs, it may be the case that by forcing budget cuts the government entity will upset too many constituents. This type of scenario is shown by the game in figure 5.14.

In this scenario the game has reduced to a version of the famous stag hunt game. Except this version does not involve a risk-dominant solution. There are two Nash equilibria, (F, F) and (C, C). But since (F, F) is payoff dominant it can be said that both players will choose to fully fund all Naval programs. For this trivial solution, the ESL option is irrelevant, since the interests of both the government entity and the Navy are already aligned. The government entity may still want to decrease the budget, but the pain in doing so outweighs that desire.

		Government Entity	
		A	B
Navy	A	3, 2	0, 0
	B	0, 0	1, 1

Figure 5.14: Perturbation of the government budgeting game where the government entity prefers to keep the replacement catamaran program. Changes highlighted in red.

5.6 Results and Discussion

This chapter has argued how real options in (and on) U.S. Navy assets can be *game changing*. Since the Naval market does not conform to any efficient market concept, then it is possible that the Navy may exert a feedback effect on their environment, or change the behavior of other players by leveraging certain real options. For this reason, this research has argued that no approach to evaluating Naval options would be complete without a game theory component capable of analyzing the interdependent decision making of many Naval situations, and how introducing a real option may change the structure of those interactions.

The applied example in this chapter considered the possible influences on government budget negotiations that an option to extend service life (ESL) on a class of vessels may have. In this example the class of vessels was the same high speed aluminum catamarans studied in the previous case study, but the results of this example are independent of the exact class. Intuitively, one understands that owning the option to extend the service life of an existing vessel can hedge against negative developments in the replacement vessel - whether technological, political, or other. What is significant about this example is that it demonstrates how, under certain conditions, that same ESL option can be leveraged in budget negotiations to prevent the cancellation of the replacement program. Moreover, the case study illustrates how the prospect theory-based real options framework put forth in this dissertation can generate insight into the value of naval options from *both* a vessel perspective *and* an institutional perspective. The previous case study showed how the structural modifications needed in order to "purchase" the ESL option have value from a vessel perspective because those modifications result in less cracking of the aluminum and can prolong operational viability. This case study showed how that ESL option may also have value from an institutional perspective by providing leverage in government budget negotiations regarding the replacement catamaran program.

CHAPTER VI

Conclusion

Naval vessels are required to change, adapt, or upgrade during their service life. The acquisition and operating environments for the U.S. Navy are rapidly changing, posing new challenges for the ship designer. The projected service lives of naval vessels continue to lengthen. Budgets are contracting. Threats require increasingly mobile and adaptable response capabilities. When aggregated, these factors demand that naval ships be highly flexible. Modularity and Design-for-Upgradability are two manifestations of flexibility in naval design, and there are many others. Flexible systems and architectures can be used to help a ship or fleet shift operations, upgrade technology or machinery, adopt new or multiple missions, and actively manage risks.

While designers have an intuitive appreciation for the value of flexibility, decisions are currently made based largely on experience, conjecture, or iteration from a previous ship design. However, engineering experience and judgement are less useful as system complexity increases, or the bounds of current practice are pushed or exceeded. To date, there are no widely-used rigorous, analytical methods for evaluating candidate flexible systems or architectures in naval design [38, 47, 94]. This research has presented a quantitative, repeatable, and defensible framework for evaluating flexible systems and architectures in early-stage naval ship design.

The new framework, called prospect theory-based real options analysis (PB-ROA), is built on a combination of real options analysis, utility theory, prospect theory, and game theory. The intersection of these four academic disciplines was necessary to transition traditional real options analysis (ROA) from a state of commercial applicability to the naval realm. Utility theory allows the PB-ROA framework to assess the value of naval assets which, unlike commercial assets, do not generate cash flows. Prospect theory augments the use of utility to include loss aversion and, more importantly, provides a structured method for calculating the risk-adjusted probability measure in the absence of a commercial market, as was previously necessary under a traditional ROA approach (and frequently ignored). Game theory allows the PB-ROA framework to analyze the very unique property of many naval options that have a feedback effect on their environment - game changing options.

The following is a list of original contributions to theory made in this dissertation:

1. An analytical framework for valuing flexible systems and architectures in the absence of a market and cash flows, rooted in utility, and suited for a wide range of Naval applications.

2. A systematic method for risk adjustment which also includes loss aversion, based on marginal utility curves. With this, it is no longer necessary to assume that agents are risk-neutral, as is currently common.

3. A game theoretic perspective on Naval options which includes the possibility that the introduction of the option may change the environment, resulting in a different game structure and, possibly, equilibrium.

Beyond the theoretical contributions of the prospect theory-based real options analysis framework, this research also demonstrated how the framework could generate unique and useful insights to early stage naval design and decision making.

A case study examined the impact that structural modifications may have on the operational flexibility of a high speed aluminum catamaran. PB-ROA was used to evaluate the tradeoff between cargo capacity and structural availability in the face of stochastic demand for cargo and fatigue cracking of the ship's structure from repeated stress. The real option in this case study was the option to extend the service life (ESL option) of the catamaran from twenty to twenty-five years. PB-ROA exposed partitions in the design space which optimized the value of the ESL option, and the conditions under which certain candidate structural designs resulted in maximum option value. Specifically, it showed which two designs resulted in the greatest option value depending on the likelihood of service like extension. Such insight could be extremely valuable to guide efforts and resources in early stage design and offer a quantitative support tool for design decisions pertaining to flexibility. This study also demonstrated how PB-ROA can be used to incorporate structural performance as a component of system value where previously it was considered separately, or strictly from a requirements fulfillment perspective.

A second example examined how the flexibility provided by the ESL option on an existing class of vessel might influence the behavior of multiple agents in a budget negotiation. A game, called the government budgeting game, was presented. It is a modification of the famous battle-of-the-sexes game with the option to burn money. In the government budgeting game, the Navy has the option to cancel the replacement, or next-generation, program for the class of vessel. This option is enabled because of the ESL option on the existing class. While it is intuitively understood that the ESL option on the existing class may provide a hedge in the event of negative developments in the replacement program, PB-ROA shows an interesting counter-intuitive result.

PB-ROA exposes conditions under which the ESL option on the existing platform could prevent the cancellation of the replacement program in a budget negotiation.

The first case study demonstrates how PB-ROA may generate useful insight from an *asset perspective* . The second academic example generates insight from an *institutional* perspective. It is significant that one framework can generate insights from both perspectives.

An area for future work is in examining the effect of loss aversion on the outputs of the PB-ROA framework when compared to a straight-forward expect-utility approach. The test case involving the hospital variant of the joint high speed vessel (JHSV) gave one example where the two methods resulted in diverging solutions. However, there is much more to examine in this area to understand the conditions where loss aversion is a significant concern, or areas where it may be ignored for expediency.

Another area for future work would be to implement PB-ROA on problems with correlated risk factors. For the ESL option in this dissertation, the risk factors were all considered to be independent. To model the correlation between them would most likely require Monte Carlo simulation to solve for the value of the option. The effect of correlation, and efficient computational methods (especially as the number of risk factors increases) is an interesting area of future work. Along these same lines, new case studies with many-to-one or many-to-many mappings from risk factors to capabilities should be performed. All of the case studies in this dissertation had only one-to-one mappings. For example, in the case study on the ESL option in chapter IV there was a one-to-one relationship between the time to crack initiation (risk factor) and availability (capability), as well as between the demand for cargo weight (risk factor) and ship cargo fraction (capability). Future case studies should demonstrate PB-ROA on problems with more complex mappings between risk factors and capabilities.

Finally, in order to bring PB-ROA closer to institutional use, further effort is required in the area of utility elicitation for Naval assets. The work of Abdellaoui [1] and Abdellaoui et al [2] form an interesting theoretical basis for this, in the author's opinion. Integration of their work with some form of war game simulation tools could move the research presented in this dissertation closer to integration in the U.S. Navy's ship design process.

APPENDICES

APPENDIX A

Multi-Objective Optimization of a Simplified Catamaran Structure

This appendix offers a detailed description of the optimization problem formulation and solution process for the simplified twin deck structure of the high speed aluminum military catamaran discussed in chapter IV.

As previously discussed, the case study lacks a complete ship synthesis model for the catamaran. Instead, a simplified structural model of two decks, composed of grillages of stiffened panel assemblies is used. A stylized representation of this structure subject to a longitudinal moment load is shown figure A.1. This has little impact on the case study, since its purpose is to demonstrate the PB-ROA framework and not to offer advice or solutions for a specific vessel. However, the lack of a complete ship synthesis model does require special consideration for the optimization problem formulation. The end goal of the structural optimization is to uncover the Pareto front of non-dominated candidate structural designs for two objectives; structural weight, and cumulative expected number of cracks over the twenty year service life. To achieve that end goal, the study uses a four step process.

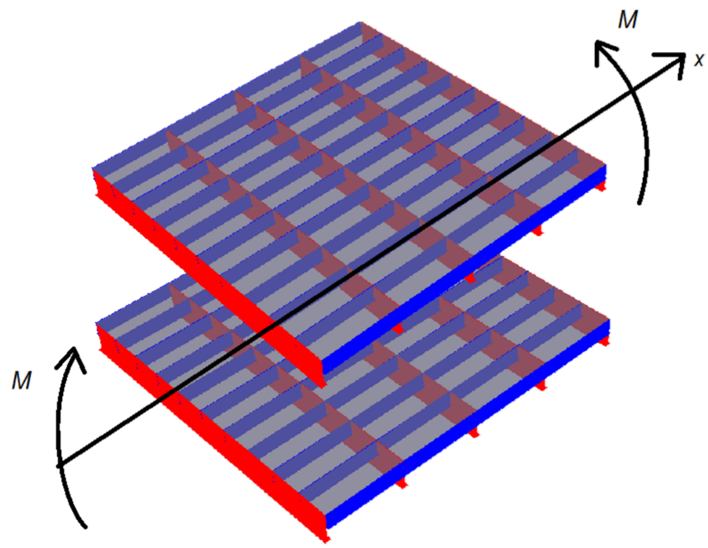

Figure A.1: Stylized representation of the twin deck structural model.

1. Find the stress range, $\Delta\sigma$, that results in a 5% chance of cracking in 20 years, for a single weld detail.

2. Perform a single objective optimization on structural weight. Calculate the section modulus of the min-weight structure, SM_{min}.

3. Use $\Delta\sigma$ and SM_{min} to calibrate the design moment load, $\Delta M = \Delta\sigma \cdot SM_{min}$.

4. Perform a two-objective optimization minimizing both weight and expected cumulative cracking, subject to the design moment load, ΔM.

These steps are necessary because without a full ship synthesis model, an alternative method for determining the load(s) on the structure is needed. One output from a full ship synthesis model would be a design longitudinal moment amidships. It is assumed that this value would remain approximately constant across a range of structural designs. This is because the principle dimensions and displacement of the vessel are assumed to be kept constant. (Recall that high speed catamarans are highly sensitive to changes in displacement.)

In place of a full ship synthesis model, this study calibrates the design longitudinal moment from the strength characteristics of the minimum weight design. The design load is the maximum moment the min-weight structure could experience, and not have more than a 5% probability of crack initiation at the 20 year mark for each weld detail. Another way of saying this is that the longitudinal design moment is chosen such that the min-weight design lies on the threshold of acceptability with respect to cracking. The remainder of this appendix will describe in detail the above four steps to complete the multi-objective structural optimization.

There are twenty-one variables for the structural model. They are shown in table A.1. The parameters of the model are given in table A.2. Each of the optimizations described is solved using the non-dominated sorting genetic algorithm (NSGA-II) [34], with a population size 120 and 300 generations. This was sufficient to reach convergence in all problems.

The first step is to set up a single objective optimization problem to solve for the range of stress that would result in a 5% probability of crack initiation, per welding detail, by 20 years. The choice of 5% may be varied in future work, but results in a good tradeoff in the design space for demonstrative purposes in this case study. Recall that the number of stress cycles until crack initiation, for a single weld detail, is given by.

$$N = \frac{D_{cr} A}{(k_f \Delta\sigma)^m} \tag{A.1}$$

Where N is the number of cycles to crack initiation under an oscillating stress range $\Delta\sigma$, D_{cr} is the cumulative damage rule, k_f is the stress intensity factor for the weld detail, and A and m are experimentally determined parameters related to the ma-

Table A.1: Design variables for the structural model.

Variable	Lower Bound	Upper Bound	Desciption
d_1	5 mm	75 mm	Thickness of deck plate.
d_2	2	9	Number of longitudinal girders
d_3	5 mm	75 mm	Thickness of flange of longitudinal girders
d_4	25 mm	750 mm	Width of flange of longitudinal girders
d_5	5 mm	50 mm	Thickness of web of longitudinal girders
d_6	100 mm	1500 mm	Height of web of longitudinal girders
d_7	2	25	Number of longitudinal stiffeners between girders
d_8	5 mm	25 mm	Thickness of flange of longitudinal stiffeners
d_9	25 mm	300 mm	Width of flange of longitudinal stiffeners
d_{10}	5 mm	15 mm	Thickness of web of longitudinal stiffeners
d_{11}	50 mm	500 mm	Height of web of longitudinal stiffeners
d_{12}	3	7	Number of transverse girders
d_{13}	5 mm	75 mm	Thickness of flange of transverse girders
d_{14}	25 mm	750 mm	Width of flange of transverse girders
d_{15}	5 mm	75 mm	Thickness of web of transverse girders
d_{16}	100 mm	1500 mm	Height of web of transverse girders
d_{17}	1	9	Number of transverse frames between girders
d_{18}	5 mm	50 mm	Thickness of flange of transverse frames
d_{19}	25 mm	500 mm	Width of flange of transverse frames
d_{20}	5 mm	30 mm	Thickness of web of transverse frames
d_{21}	90 mm	1000 mm	Height of web of transverse frames

Table A.2: Design parameters for the structural model.

Parameter	Value	Desciption
N_{cycle}	$3 \cdot 10^7$	Number of stress cycles in 20 years
ρ	2.66 g/cc	Density of AL 5083-H116
E	70.3 GPa	Modulus of elasticity of AL 5083-H116
ν	0.33	Poisson ratio for AL 5083-H116
σ_{yield}	228 MPa	Yield stress of AL 5083-H116
σ_a	$0.75\sigma_{\text{yield}}$	Allowable stress
p	15 kPa	Design deck pressure
L	40 m	Length of deck structure
B	20 m	Beam of deck structure
h	5 m	Height between decks
k_0	132420000	Regression coefficient for required longitudinal section modulus
k_1	-7750000	Regression coefficient for required longitudinal section modulus
k_2	180000	Regression coefficient for required longitudinal section modulus
k_3	837	Regression coefficient for transverse bending moment
k_4	-11377	Regression coefficient for transverse bending moment
λ_A	3.31E11	Mean value for fatigue parameter A
ζ_A	1E10	Standard deviation of fatigue parameter A
λ_{kf}	1.0	Mean value for stress intensity factor, k_f
ζ_{kf}	0.1	Standard deviation for stress intensity factor, k_f
λ_{Dcr}	1.0	Mean value for cumulative damage coefficient, D_{cr}
ζ_{Dcr}	0.3	Standard deviation for cumulative damage coefficient, D_{cr}
m	3	Coefficient for fatigue calculations

terial. $\Delta\sigma$ and m are assumed constant. By acknowledging the stochastic nature of the parameters A, D_{cr}, and k_f the number of cycles to crack initiation also becomes stochastic. Common practice is to model A, D_{cr}, and k_f each with lognormal distributions which results in N also being lognormal. The probability density function for the time to crack initiation is then:

$$f(x_1) = \frac{1}{\zeta\sqrt{2\pi}x_1}\exp\left(-\frac{(\ln(x_1)-\lambda)^2}{2\zeta^2}\right) \tag{A.2}$$

Where x_1 is the number of cycles to crack initiation with mean λ, and standard deviation ζ. The values for the mean and standard deviation are given by:

$$\lambda = \lambda_{D_{cr}} + \lambda_A + m(\lambda_{k_f} + \ln\Delta\sigma) \tag{A.3}$$

$$\zeta = \sqrt{\zeta^2_{D_{cr}} + \zeta^2_A + \zeta^2_{k_f}} \tag{A.4}$$

The cumulative density function for the lognormal distribution is given by:

$$F(x_1) = 0.5 + 0.5\text{erf}\left[\frac{\ln x_1 - \lambda}{\sqrt{2}\zeta}\right] \tag{A.5}$$

To solve for $\Delta\sigma$, a single objective optimization is solved, for $x_1 = 3\cdot 10^7$.

$$\begin{aligned} \underset{\Delta\sigma}{\text{minimize}} \quad & |0.05 - F(x_1;\Delta\sigma)| \\ \text{subject to} \quad & \Delta\sigma \geq 0 \\ & x_1 = 3\cdot 10^7 \end{aligned}$$

The second step is to solve another single objective optimization problem minimizing the weight of the twin deck structure. Expressed in terms of the design variables, the weight function is given by:

$$W = 2\rho\Bigg[LBd_1 + d_2(d_3d_4 + d_5d_6)L + (d_2+1)d_7(d_8d_9 + d_{10}d_{11})L + d_{12}(d_{13}d_{14} + d_{15}d_{16})B + (d_{12}+1)d_7(d_{18}d_{19} + d_{20}d_{21})B\Bigg] \quad (A.6)$$

In total, there are eighteen constraints (aside from the variable bounds) that must be met in the structural optimization. Let $\{c_1, \ldots, c_{18}\}$ denote this set of constraints. They are summarized in table A.3. The single objective optimization is then,

$$\begin{aligned} \underset{\{d_1,\ldots,d_{21}\}}{\text{minimize}} \quad & W \\ \text{subject to} \quad & \{c_1, \ldots, c_{18}\} \end{aligned}$$

The section modulus of the minimum weight design, SM_{min}, is then calculated and used along with $\Delta\sigma$ in the third step to calculate the "design bending moment,"

$$\Delta M = SM_{\text{min}} \cdot \Delta\sigma \quad (A.7)$$

In the last step, a two objective optimization of the structure is performed minimizing weight and expected cumulative cracking, using the design bending moment for the global oscillating load. The expected cumulative number of cracks is given by:

$$K = n \cdot F(3 \cdot 10^7) \quad (A.8)$$

Where n is the total number of weld details in the structure, which under the model being used is given by the equation:

$$n = 2\Big(d_2 + (d_2+1)d_7\Big)\Big(d_{12} + (d_{12}+1)d_{17}\Big) \quad (A.9)$$

Recall that a weld detail in this study is defined to be the intersection between any two perpendicular members of a grillage. In this way, the two objective optimization becomes:

$$\begin{aligned} \underset{\{d_1,\ldots,d_{21}\}}{\text{minimize}} \quad & W \\ & K \\ \text{subject to} \quad & \{c_1, \ldots, c_{18}\} \end{aligned}$$

K will vary across candidate designs with structural weight because the resulting stress in each design will be different based on its own section modulus.

A.1 Constraints

This section summarizes the many constraints applied to the structural optimization problems. There are eighteen constraints total, aside from the variable bounds. In some cases these constraints are derived from rules set by the American Bureau of Shipping (ABS) for high speed military craft [4]. In other cases they are rules of thumb, or heuristics which capture designer intent to prevent the optimizer from choosing designs which may be infeasible for practical reasons, like the size of girders relative to stiffeners. Table A.3 list all of the constraints with a brief description of their purpose. The rest of this section gives references to the ABS rules on which some constraints are based.

References and Justification for Constraints

1. This constraint for global longitudinal strength is adapted from ABS HSNC Rule 3-2-1/1.1.1 (see [4]). A quadratic regression of the rule was multiplied by 0.5 to derive this constraint. The regression is shown in figure A.2. It is intended to reflect global bending concerns in the optimization. The regression uses: $L = 40m$, $B = 20m$, speed of 40 knots, and demihull beam of 4 m.

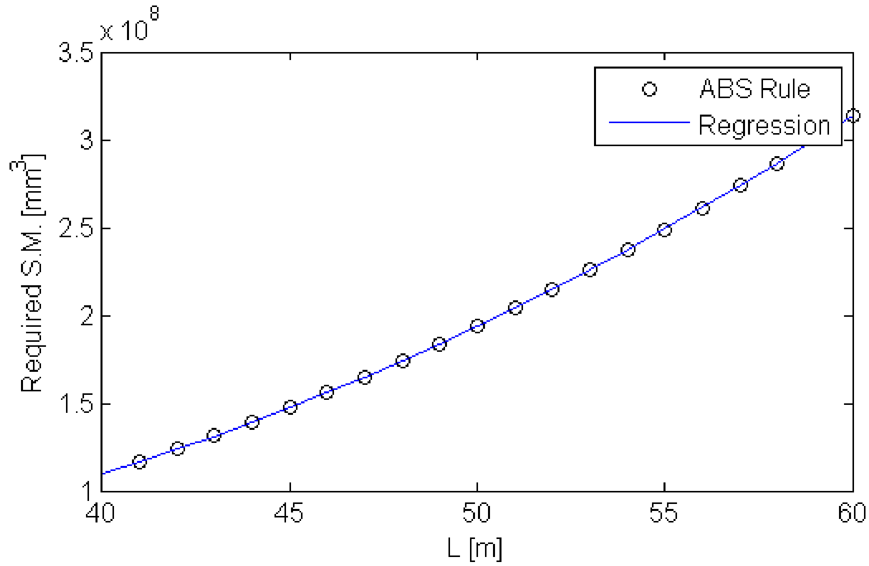

Figure A.**2:** Quadratic regression of ABS HSNC Rule 3-2-1/1.1.1.

2. The Faulkner stress for a grillage is the minimum between its buckling and tripping stresses. It is evaluated using the formulation proposed by Faulkner et al. [42]. It reflects secondary stength concerns in the optimization.

3. ABS HSNC Rule 3-2-4/1.3.1 applies to the section modulus of individual girders and stiffeners in the grillage to account for secondary strength concerns in the optimization. Here, p is the design pressure on the decks, s is the space between the members under consideration, and l is the distance between perpendicular supports.

4. See above.

5. This constraint for global transverse strength is adapted from ABS HSNC Rule 3-2-1/3.3 (see [4]), which determines a design bending moment for multihulls. A linear regression of the rule was used. The regression is shown in figure A.3. It is intended to reflect global bending concerns unique to multihulls in the optimization.

6. See number 3.

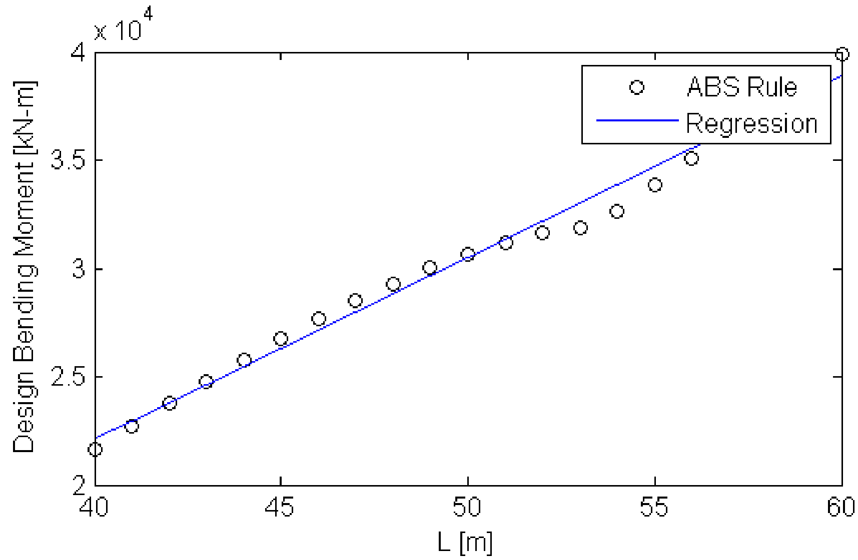

Figure A.3: Linear regression of ABS HSNC Rule 3-2-1/3.3.

7. See number 3.

8. ABS HSNC Rule 3-2-4/1.5.6 applies to the local buckling of the members of grillages.

9. ABS HSNC Rule 3-2-4/1.5.6 applies to the local buckling of the members of grillages.

10. ABS HSNC Rule 3-2-4/1.5.6 applies to the local buckling of the members of grillages.

11. ABS HSNC Rule 3-2-4/1.5.6 applies to the local buckling of the members of grillages.

12. The rest of the constraints are hueristics to guide the optimizer toward designs which are feasible and practical.

Table A.3: Constraints for the structural optimization problems.

	Constraint	Description
1	$SM_L \geq k_2L^2 + k_1L + k_0$	Primary Longitudinal Strength, Section Modulus
2	$\sigma_{\text{Faulkner}} \geq \sigma_a$	Secondary Longitudinal Strength, Compressive stress
3	$SM_{\text{L, Girder}} \geq \frac{83.3psl^2}{\sigma_a}$	Secondary Longitudinal Strength, Section Modulus of Longitudinal Girders
4	$SM_{\text{L, Stiffener}} \geq \frac{83.3psl^2}{\sigma_a}$	Secondary Longitudinal Strength, Section Modulus of Longitudinal Stiffeners
5	$SM_T \geq \sigma_a\left[k_3L + k_4\right]$	Primary Transverse Strength, Section Modulus
6	$SM_{\text{T, Girder}} \geq \frac{83.3psl^2}{\sigma_a}$	Secondary Transverse Strength, Section Modulus of Transverse Girders
7	$SM_{\text{T, Frame}} \geq \frac{83.3psl^2}{\sigma_a}$	Secondary Transverse Strength, Section Modulus of Transverse Frames
8	$\frac{d_{14}}{d_{13}} \leq 0.5\sqrt{\frac{E}{\sigma_y}}$	Tertiary Strength, Buckling of Flange of Transverse Girders
9	$\frac{d_{16}}{d_{15}} \leq 0.5\sqrt{\frac{E}{\sigma_y}}$	Tertiary Strength, Buckling of Web of Transverse Girders
10	$\frac{d_{19}}{d_{18}} \leq 0.5\sqrt{\frac{E}{\sigma_y}}$	Tertiary Strength, Buckling of Flange of Transverse Frames
11	$\frac{d_{21}}{d_{20}} \leq 0.5\sqrt{\frac{E}{\sigma_y}}$	Tertiary Strength, Buckling of Web of Transverse Frames
12	$d_{21} \geq 2d_{11}$	Relative height of transverse frames and longitudinal stiffeners
13	$d_{11} \leq 5d_9$	Aspect ratio of longitudinal stiffeners
14	$d_{21} \leq 2.5d_{19}$	Aspect ratio of transverse frames
15	$d_6 \leq 4d_4$	Aspect ratio of longitudinal girders
16	$d_{16} \leq 3.5d_{14}$	Aspect ratio of transverse girders
17	$d_{16} \geq 1.5d_6$	Relative height of transverse and longitudinal girders
18	$d_6 \geq 2d_{11}$	Relative height of longitudinal girders and stiffeners

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