## Supporting Information

# Constant pH Molecular Dynamics of Proteins in Explicit Solvent with Proton Tautomerism 

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## Supplementary Discussion on Modeling Interactions between Coupled Titrating Residues

Further validation of the $\mathrm{CPHMD}^{\mathrm{MS} \lambda \mathrm{D}}$ framework was performed on model dipeptide sequences Asp-Asp, Glu-Glu and Lys-Lys at zero ionic strength, where both residues were titrated simultaneously. The calculated $\mathrm{pK}_{\mathrm{a}}$ values are summarized in Table S 2 and Figure S1. For the aspartic acid dipeptide, we observed that the $\mathrm{pK}_{\mathrm{a}}$ values were 3.1 and 4.6 , with the N terminus Asp having a consistently lower $\mathrm{pK}_{\mathrm{a}}$ in all 3 simulations runs, suggesting that the two Asp residues are in a different electrostatic environment. An analysis of the hydrogen bonding contacts that each Asp side chain forms with the backbone of the dipeptide (Figure S2) indicated that the N -terminus Asp had 3 hydrogen bond donors within a $\sim 5 \AA$ radius, compared to the Cterminus Asp that had only 2 hydrogen bond donors. Thus, the increased presence of hydrogen bond donors around the N -terminus Asp facilitated the stabilization of its charged unprotonated state, explaining the decrease of its calculated $\mathrm{pK}_{\mathrm{a}}$ value. By contrast, the calculated $\mathrm{pK}_{\mathrm{a}}$ values for the glutamic acid dipeptide was 4.3 for both residues with no apparent $\mathrm{pK}_{\mathrm{a}}$ shift. Similarly, the $\mathrm{pK}_{\mathrm{a}}$ values for the lysine dipeptide was $\sim 10.4$ for both residues. The identical $\mathrm{pK}_{\mathrm{a}}$ for both N and C-terminus residues of both Glu-Glu and Lys-Lys dipeptides suggest that the electrostatic environment around each residue is similar. This is supported by the observation that no
hydrogen bonding capable backbone atom was present in a $\sim 5 \AA$ proximity from the titrating functional group, and so the backbone interactions that were responsible for creating an asymmetric environment in Asp-Asp is significantly reduced in both Glu-Glu and Lys-Lys dipeptides.

The calculated Hill coefficients of 0.7 suggest that anti-cooperative coupling is the dominant mode of interaction between the two adjacent titrating residues of these dipeptide systems. Prior work by Bashford and Karplus have demonstrated that when two residues titrate in the same pH region and have the same intrinsic (microscopic) $\mathrm{pK}_{\mathrm{a}}$, such as the Glu-Glu and Lys-Lys dipeptides in this analysis, the magnitude of their coupled interaction can raise/lower the apparent (macroscopic) $\mathrm{pK}_{\mathrm{a}}$ of the system. ${ }^{1}$ The existing HH-equation (i.e. eqn. 7) to which we fitted our data to calculate a $\mathrm{pK}_{\mathrm{a}}$ value is a rearranged form of the equation first proposed by Tanford and Roxby. ${ }^{2}$ When there is no coupling with other titrating residues (i.e. $n=1$ ), eqn. 7 reduces to a form that can be derived from a mean-field approximation. ${ }^{1}$ When there is coupling with other titrating residues (i.e. $\mathrm{n} \neq 1$ ), the convention is to add the Hill coefficients to describe the anti-cooperative proton binding behavior. However, prior work by Onufriev et. al. in their derivation of the decoupled site representation (DSR) framework has shown that this approach may not give the best fit to experimental macroscopic $\mathrm{pK}_{\mathrm{a}}$ values. ${ }^{3}$ Consequently, it is not unexpected that our analysis was unable to obtain the macroscopic $\mathrm{pK}_{\mathrm{a}}$ of the Glu-Glu and LysLys dipeptides, where one would expect to see two distinct $\mathrm{pK}_{\mathrm{a}}$ values. If one wishes to elucidate the coupled $\mathrm{pK}_{\mathrm{a}}$ behavior for these two dipeptides, the $\mathrm{pK}_{\mathrm{a}}$ values can be recalculated by fitting it to a modified version of the HH-equation (see Methods eqn. 8 for details), which can be derived from the DSR approach. ${ }^{3}$ In this revised fitting method, where we analyzed the net proton uptake without pre-assigning the identity of each residue, the apparent $\mathrm{pK}_{\mathrm{a}}$ values calculated cannot be
assigned to a specific titrating site (i.e. the calculated $\mathrm{pK}_{\mathrm{a}}$ values are not the microscopic $\mathrm{pK}_{\mathrm{a}}$ of specific residues). Using this approach, two clear and distinct $\mathrm{pK}_{\mathrm{a}}$ values emerge for Glu-Glu $(3.6,5.0)$ and Lys-Lys $(9.8,11.0)$, which is consistent with the perturbation of one protonated residue on the other.

## Derivation of Equation 7 from Mean Field Approximation

Here, we show how eqn. 7 of the main text can be derived from eqn. $1 b$ (i.e. mean field approximation) from Bashford and Karplus. ${ }^{1}$ We start with eqn. 1 b of ref 1:

$$
\begin{equation*}
\log \frac{\theta}{1-\theta}=p K a-p H \tag{1}
\end{equation*}
$$

where $\theta$ is the probability that the site is protonated:

$$
\begin{equation*}
\theta=\frac{N_{p}}{N_{p}+N_{u}} \tag{2}
\end{equation*}
$$

Therefore eqn. 1 b can be rewritten as:

$$
\begin{align*}
& \log \frac{\theta}{1-\theta}=\log \frac{\left(\frac{N_{p}}{N_{p}+N_{u}}\right.}{\left(\frac{N_{u}}{N_{p}+N_{u}}\right)}=p K a-p H  \tag{3}\\
& \log \frac{N_{p}}{N_{p}+N_{u}}=\log \frac{N_{u}}{N_{p}+N_{u}}+p K a-p H  \tag{4}\\
& \log \frac{N_{p}}{N_{p}+N_{u}}=\log \frac{N_{u}}{N_{p}+N_{u}}+\log \left(10^{p K a-p H}\right)  \tag{5}\\
& \frac{N_{p}}{N_{p}+N_{u}}=\frac{N_{u}}{N_{p}+N_{u}}\left(10^{p K a-p H}\right)  \tag{6}\\
& \frac{N_{u}}{N_{p}+N_{u}}+\frac{N_{u}}{N_{p}+N_{u}}\left(10^{p K a-p H}\right)=1 \tag{7}
\end{align*}
$$

$$
\begin{align*}
& \frac{N_{u}}{N_{p}+N_{u}}\left(1+10^{p K a-p H}\right)=1  \tag{8}\\
& \frac{N_{u}}{N_{p}+N_{u}}=\frac{1}{1+10^{p K a-p H}}=\frac{1}{1+10^{-(p H-p K a)}} \tag{9}
\end{align*}
$$

which is the same form as eqn. 7 for $\mathrm{n}=1$ :

$$
\begin{equation*}
S=\frac{N_{u}}{N_{p}+N_{u}}=\frac{1}{1+10^{-(p H-p K a)}} \tag{10}
\end{equation*}
$$

## Derivation of Equation 8 from Decoupled Site Representation

Here, we show how eqn. 8 of the main text can be derived from eqn. 15 (i.e. decoupled site representation) from Onufriev et. al. ${ }^{3}$ We start with eqn. 15 of ref 3:

$$
\begin{equation*}
\sum_{i}^{N}\left\langle x_{i}\right\rangle=\sum_{j}^{N} \frac{10^{p K a_{j}-p H}}{1+10^{p K a_{j}-p H}} \tag{11}
\end{equation*}
$$

Since $\mathrm{x}_{\mathrm{i}}$ represents the fraction of protonated states for each titrating residue $i$ :

$$
\begin{align*}
& \sum_{i}^{N}\left\langle x_{i}\right\rangle=\sum_{i}^{N}\left(1-S_{i}^{\text {unprot }}\right)  \tag{12}\\
& \sum_{i}^{N}\left(1-S_{i}^{\text {unprot }}\right)=\sum_{j}^{N} \frac{10^{p K a_{j}-p H}}{1+10^{p K a_{j}-p H}}  \tag{13}\\
& \sum_{i}^{N} S_{i}^{\text {unprot }}=\sum_{j}^{N}\left(1-\frac{10^{p K a_{j}-p H}}{1+10^{p K a_{j}-p H}}\right)=\sum_{j}^{N}\left(\frac{1}{1+10^{p K a_{j}-p H}}\right)=\sum_{j}^{N}\left(\frac{1}{1+10^{-\left(p H-p K a_{j}\right)}}\right) \tag{14}
\end{align*}
$$

The DSR framework maps a set of $i$ real sites to a set of $j$ non-interacting quasi-sites. Assuming a one-to-one mapping of real to quasi sites $(i=j)$, we obtain the following expression, which is the same expression as eqn. 8:

$$
\begin{equation*}
\sum_{i}^{N} S_{i}^{\text {unprot }}=\sum_{i}^{N}\left(\frac{1}{1+10^{-\left(p H-p K a_{i}\right)}}\right) \tag{15}
\end{equation*}
$$

Table S1: Sampling characteristics of 2-state titration simulations performed at $\mathrm{pH}=\mathrm{pK}_{\mathrm{a}}$.

| Residue | Fraction of Physical States | Transition (ns $\left.{ }^{\mathbf{- 1}}\right)$ |
| :---: | :---: | :---: |
| Asp | $0.74 \pm 0.04$ | $35 \pm 5$ |
| Glu | $0.75 \pm 0.02$ | $35 \pm 5$ |
| His- $\delta$ | $0.72 \pm 0.03$ | $60 \pm 10$ |
| His- $\varepsilon$ | $0.71 \pm 0.04$ | $64 \pm 14$ |
| Lys | $0.76 \pm 0.03$ | $50 \pm 8$ |

Table S2: Calculated $\mathrm{pK}_{\mathrm{a}}$ of various model dipeptide sequences. Values reported in the top table were calculated using equation 7 (identity of residue was pre-assigned), and those reported in the bottom table were calculated using equation 8 (identity of residue was not pre-assigned).

| Residue Identity Pre-Assigned |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Residue | Ref $\mathrm{pK}_{\mathrm{a}}$(of amino acid) | Site 1 |  | Site2 |  |
|  |  | $\mathrm{pK}_{\mathrm{a}}$ | n | $\mathrm{pK}_{\mathrm{a}}$ | n |
| Asp-Asp | 4.0 | $3.1 \pm 0.2$ | $0.7 \pm 0.1$ | $4.6 \pm 0.1$ | $0.7 \pm 0.0$ |
| Glu-Glu | 4.4 | $4.3 \pm 0.1$ | $0.7 \pm 0.0$ | $4.3 \pm 0.1$ | $0.7 \pm 0.0$ |
| Lys-Lys | 10.4 | $10.3 \pm 0.0$ | $0.7 \pm 0.0$ | $10.5 \pm 0.1$ | $0.7 \pm 0.0$ |
| Residue Identity Not Pre-Assigned |  |  |  |  |  |
| Residue | Ref $\mathrm{pK}_{\mathrm{a}}$(of amino acid) | Site1 ${ }^{\text {a }}$ |  | Site2 ${ }^{\text {a }}$ |  |
|  |  | pKa | n | pK ${ }_{\text {a }}$ | n |
| Glu-Glu | 4.4 | $3.6 \pm 0.0$ | - | $5.0 \pm 0.0$ | - |
| Lys-Lys | 10.4 | $9.8 \pm 0.1$ | - | $11.0 \pm 0.1$ | - |

${ }^{\mathrm{a}}$ Site 1 and Site $2 \mathrm{pK}_{\mathrm{a}}$ values are defined as the residue that produces the lower and higher "instantaneous" $\mathrm{pK}_{\mathrm{a}}$ value. When averaged across the entire trajectory, they would correspond to the two macroscopic $\mathrm{pK}_{\mathrm{a}}$ values recorded by experiments.

Table S3: $\mathrm{pK}_{\mathrm{a}}$ values of HEWL calculated using explicit solvent pH -REX CPHMD ${ }^{\mathrm{MS} \lambda \mathrm{D}}$ simulations show good convergence within a 20 ns trajectory.

| Residue | Exp pK $_{\mathbf{a}}$ | Explicit Solvent CPHMD ${ }^{\text {DSAD }} \mathbf{p K}_{\mathbf{a}}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\mathbf{0 - 5 n s}$ | $\mathbf{6 - 1 0 n s}$ | $\mathbf{1 1 - 1 5 n s}$ | $\mathbf{1 6 - 2 0 n s}$ |
| GLU-7 | $2.6 \pm 0.2$ | $3.3 \pm 0.2$ | $3.1 \pm 0.3$ | $2.9 \pm 0.1$ | $2.7 \pm 0.1$ |
| HIS-15 | $5.5 \pm 0.2$ | $5.6 \pm 0.2$ | $5.6 \pm 0.1$ | $5.8 \pm 0.1$ | $6.0 \pm 0.2$ |
| ASP-18 | $2.8 \pm 0.3$ | $1.9 \pm 0.0$ | $2.1 \pm 0.1$ | $2.0 \pm 0.2$ | $2.1 \pm 0.2$ |
| GLU-35 | $6.1 \pm 0.4$ | $7.3 \pm 0.3$ | $7.3 \pm 0.2$ | $6.8 \pm 0.4$ | $7.0 \pm 0.3$ |
| ASP-48 | $1.4 \pm 0.2$ | $1.3 \pm 0.0$ | $1.3 \pm 0.0$ | $1.3 \pm 0.0$ | $1.3 \pm 0.0$ |
| ASP-52 | $3.6 \pm 0.3$ | $4.6 \pm 0.2$ | $4.8 \pm 0.4$ | $4.5 \pm 0.3$ | $4.5 \pm 0.3$ |
| ASP-66 | $1.2 \pm 0.2$ | $1.4 \pm 0.0$ | $1.4 \pm 0.0$ | $1.4 \pm 0.1$ | $1.5 \pm 0.1$ |
| ASP-87 | $2.2 \pm 0.1$ | $1.3 \pm 0.0$ | $1.3 \pm 0.0$ | $1.3 \pm 0.0$ | $1.3 \pm 0.0$ |
| ASP-101 | $4.5 \pm 0.1$ | $5.5 \pm 0.4$ | $5.6 \pm 0.5$ | $5.6 \pm 0.6$ | $5.1 \pm 0.5$ |
| ASP-119 | $3.5 \pm 0.3$ | $1.9 \pm 0.1$ | $1.9 \pm 0.1$ | $1.7 \pm 0.1$ | $1.6 \pm 0.0$ |
| RMSE |  | $\mathbf{0 . 9 1}$ | $\mathbf{0 . 9 2}$ | $\mathbf{0 . 8 5}$ | $\mathbf{0 . 8 4}$ |

Figure S1: Titration curves of (a) Asp-Asp, (b) Glu-Glu and (c) Lys-Lys dipeptide sequences. The first two curves of each dipeptide correspond to the titration curve for the N and C -terminal residue respectively, which were fitted using eqn 7 . The third curve corresponds to the titration curve of the dipeptide fitted using eqn 8 . Colors represent the results from the triplicate runs.
a)



b)



c)




Figure S2: Average distances between the OD1/OD2 atoms on Asp and the N atoms on the $\mathrm{N}-\mathrm{H}$ groups of the peptide backbone for the (a) N-terminus and (c) C-terminus Asp residues in aspartic acid dipeptide as a function of pH .


Figure S3: Titration curves of Asp/Glu/His residues of HEWL from the last 5 ns segment of the trajectory. Colors represent the results from the triplicate runs.


Figure S4: Histogram of hydrogen bonding distances between the OD1/OD2 atom of Asp-119 with the amide backbone hydrogen ( HN ) of Gln-121 and Ala-121 indicates the presence of a persistent hydrogen bond at all pH environments.


Figure S5: Titration curves of Asp/Glu/His residues of (a) BBL and (b) NTL9. Colors represent the results from the triplicate runs.


b)







## References

(1) Bashford, D.; Karplus, M. J. Phys. Chem. 1991, 95, 9556.
(2) Tanford, C.; Roxby, R. Biochemistry 1972, 11, 2192.
(3) Onufriev, A.; Case, D. A.; Ullmann, G. M. Biochemistry 2001, 40, 3413.

