Behavioral Modeling of Network Traffic: Integrating Route Choice and Traffic Assignment

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Abstract Most of the current traffic modeling packages assume that drivers follow a shortest path to go from an origin to a destination. This approach has been adopted because of its simplicity and convenience. Unfortunately, in real life, drivers tend to use some other routes according to individual decision-making processes. These decisions depend on many factors such as travel time, number of intersections, quality of roads, traffic lights and other attributes. Therefore, the shortest path assumption cannot reflect drivers' behavior in real life and may lead to biased results.

The lack of a realistic model that can be used efficiently on computers is the real motivation for this project. In this paper, our objective is to develop a practical traffic model that reflects the route-choice process with greater accuracy. This new model would improve the existing driver's behavior model in dynamic network simulators.

1   Introduction

Network level traffic models and simulations generally assume drivers follow a shortest path to go from an origin to a destination. Shortest-path routing has been adopted over the years because of its simplicity, computational efficiency, and linkage with algorithms for generating equilibrium in static traffic assignment models. However, in real life, driver's routes are likely to deviate from optimal shortest paths in significant ways. Empirical research on route behavior shows that drivers use numerous criteria in formulating a route: travel time, number of intersections, quality of roads, traffic lights and other factors. Drivers' experiences, habits, cognitive limits and other behavioral considerations may also produce variations in route selection. Viewed in this light, one can see that the shortest path criterion is indeed an unrealistic abstraction of individual driver behavior, and when aggregated at the network level may result in an inaccurate representation of traffic.

The purpose of this paper is to present a behavior-based dynamic traffic assignment approach that combines an individual driver routing module with a

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dynamic traffic simulation module. Vehicles are routed in a manner that more accurately reflects the stochastic nature of individual route selection. Routing decisions are then aggregated in a dynamic simulation of network traffic flow. The result of our effort is a behavior-based traffic assignment that provides a more realistic traffic loading baseline.

Section two of this paper gives a review of micro-simulation assignment. Section three classifies the drivers on the road into two categories: familiar and unfamiliar. In section four, the unfamiliar-driver's behavior is studied in more detail. Discrete choice models are presented in the section five. In section six, a simple method for estimating the utility of a path is presented. The probit model is introduced in section seven. Sections eight and nine link the ideas discussed in previous sections to develop an applicable model that is superior to the driver's behavior models used currently in simulators. Finally, some numerical results are presented in section ten.

2 Review of Micro-Simulation Assignment

As a first step toward dynamic traffic assignment, researchers have developed network traffic simulators which assign vehicles, or packets of vehicles, to links in accordance with some vehicle routing logic. Early simulations of this type include CONTRAM [10], SATURN [8], and INTEGRATION [16]. Dynamic network traffic models generally employ some form of shortest route algorithm to determine the paths of vehicles entering the network. For example, in CON-TRAM, packets of vehicles are entered onto the network in accordance with specified departure times. Upon entry, the packets are assigned to a shortest path from their entry point to their destination. The packet then follows this path for a designated period of time. The assignment process is then repeated and the packet is reassigned to a new shortest path based on current link times.

A similar approach was used in the original version of INTEGRATION to simulate vehicles with route guidance systems. Guided vehicles have the ability to periodically reassign themselves in response to up-to-date traffic conditions. One of the benefits of this approach is that vehicles are routed according to prevailing traffic conditions including signal effects, queues and spillback.

SATURN departs from the responsive assignment approach of CONTRAM and INTEGRATION and uses an iterative equilibrium approach where link flows are determined through a modified capacity-restrained assignment procedure. The drawback of this approach is that the assignment does not take into account signal effects, queues and spillback. Also, the vehicles cannot respond to incidents or other unexpected traffic occurrences while en-route. The problem with all three approaches cited here is that the vehicle routes do not accurately capture the route-selection dynamics described in the empirical research.

To be fair, our descriptions of SATURN, CONTRAM and INTEGRATION captures their state of development at the time they were introduced, or shortly

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thereafter. Many improvements have been made to these models and their offspring in subsequent years. In this study, INTEGRATION is used to simulate the traffic in the networks of section ten.

3   Classes of Drivers on the Road

Drivers can be divided into different classes according to individual behavior, objective of the trip and knowledge of the area. For example, some drivers prefer a scenic route whilst others are under time pressure. The same driver may change his behavior depending on the goal of the trip: vacation, business, etc.

In order to simplify the study, drivers are classified into two different categories: familiar and unfamiliar. The familiar driver is the commuter who uses a route to go to his work and back every day. This type is the most important because it is associated with the majority of traffic on the road during peak hours. By unfamiliar driver we mean someone who does not know the area or has very little knowledge of the network and in most cases, uses a map to get to his destination. The percentage of these drivers is relatively small compared to the familiar drivers. The unfamiliar drivers are of special importance since they form the base for the study of familiar drivers. The following section describes the unfamiliar-driver's behavior and proposes a model to approximate it.

4   A Model for the Unfamiliar-Driver's Behavior

Unfamiliar drivers share many characteristics. For instance, they tend to plot the entire path of their trip before departing and stick to this predetermined route. Since this driver, in most cases, is using a map to design his trip, only a few paths are considered. The rest of the feasible paths are ignored because he assumes that they are inferior to the ones chosen. After finding the set of alternative paths, the driver evaluates each path and then chooses the best one. Obviously, different drivers have different evaluation techniques, hence, a different best path. The driver cannot evaluate exactly all the parameters of his trip. For example, the travel time depends on speed restrictions, congestion, number of available lanes, number of traffic lights and some other undetermined factors. The model has to reflect all of these factors to a reasonable degree.

The process of approximating the unfamiliar-driver's behavior has two stages. The first one is to generate the set of possible routes. A method is applied to narrow down the vast number of route possibilities, leaving only a few candidates to be considered. Once a set of options has been identified, it is necessary to measure the relevant attributes of these options and compare them. In the second stage, a discrete choice model is applied to compute the probability of

choosing an alternative based on its desirability. Section five outlines discrete choice models and how they can be used to compute these probabilities.

5 Discrete Choice Models

The hypothesis of discrete choice models is that an individual's preference towards each alternative can be represented by a utility. A utility is a function of the alternative's attributes and the decision maker's characteristics. Many of the attributes that influence an individual's utility cannot be observed and thus are treated as random. For example, the real travel time from origin to destination is usually not obtainable by drivers. The perceived travel time can be assumed to be a random variable with a given distribution. Therefore, choice models give only the probability with which the alternatives are chosen, not the choice itself.

Let A represent a given set of K alternatives and $\tilde{u}_k = v_k + \tilde{\epsilon}_k$ be the utility of alternative k.[1] Here, v_k is a constant and $\tilde{\epsilon}_k$ is a random variable with a given distribution. The decision maker tends to choose the alternative that yields the highest utility. The probability that alternative k will be chosen is then the probability that $\tilde{u}_k$ is higher than the utility of any other alternative. Therefore,

$$
\begin{aligned}
P_k &= P\{\tilde{u}_k \geq \tilde{u}_l, \forall l \in A/\{k\}\} \\
&= P\{v_k + \tilde{\epsilon}_k \geq \max_{\forall l \in A/\{k\}} \{v_l + \tilde{\epsilon}_l\}\}
\end{aligned}
$$

where,

$$
\sum_{k \in A} P_k = 1 \text{ and } P_k \geq 0, \forall k \in A
$$

The most commonly used driver-behavior model is the logit model which is developed using the previous equations.

In logit formulation, the utilities of different alternatives are assumed to be independent Gumbel-distributed random variables. Hence, the choice probability of a given alternative can be obtained by some simple calculations. The independence assumption is a major disadvantage since, in general, the paths considered by a driver overlap. Another discrete choice model is probit where a more realistic distribution for the random utilities is used. Also, the dependence between the random terms is taken into account which makes this model superior to the logit model. Later in this paper, the probit model is studied and then applied to some traffic networks.

Throughout the previous discussions, the utility was assumed known. In the following section, a simple method for estimating a route's utility is proposed.

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[1] The symbol ˜ is used to indicate that a variable is random or undeterministic.

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6 Utility Estimation

Many factors affect the driver's preference towards a particular path. For analytical simplicity, assume that travel time and number of turns dominate other factors. Also, it is assumed that the utility is linear in these attributes, i.e., $\bar{u} = -\bar{t} - \gamma n$. $\gamma > 0$ is a scalar obtainable from experimental studies. n is the number of turns encountered on the path and considered to be a deterministic attribute since the driver can obtain its value from the map. Note that $\bar{u} < 0$ and that a larger $\bar{u}$ value means a shorter travel time, a fewer turns or both. The only random term in the utility function is the travel time which cannot be estimated exactly.

Denote the perceived travel time on link c by $\bar{l}_c$. Let l_c be the mean of the random variable $\bar{l}_c$, i.e., $l_c = \mathrm{E}(\bar{l}_c)$. l_c can be chosen as the free-flow travel time on c. Since, in general, variance increases with larger travel times, it is reasonable to assume that $\mathrm{Var}(\bar{l}_c)$ is αl_c, where, $\alpha \geq 0$ is a scalar.[2] The perceived travel time on any path k is given by the relationship $\bar{t}_k = \sum_c \bar{l}_c I_{c,k}$. Here, I is the indicator function, i.e.,

$$I_{c,k} = \begin{cases} 1 & \text{if link } c \text{ is on path } k \\ 0 & \text{Otherwise} \end{cases}$$

One can compute the mean and variance of the random variable $\bar{t}_k$ in terms of the indicator function:

$$\mathrm{E}(\bar{t}_k) = \sum_c l_c I_{c,k}$$
$$\mathrm{Var}(\bar{t}_k) = \sum_c \mathrm{Var}(\bar{l}_c) I_{c,k}$$
$$= \alpha \mathrm{E}(\bar{t}_k)$$

The variance equality is justified since $\bar{l}_c$'s are independent random variables. Therefore,

$$\mathrm{E}(\bar{u}_k) = \mathrm{E}(-\bar{t}_k - \gamma n_k)$$
$$= v_k + \mathrm{E}(\bar{\epsilon}_k)$$
$$\mathrm{Var}(\bar{u}_k) = \alpha \mathrm{E}(\bar{t}_k)$$
$$= \mathrm{Var}(\bar{\epsilon}_k)$$

Given two paths k_1 and k_2, the covariance between the utility of k_1 and that of k_2 is:

$$\mathrm{Cov}(\bar{u}_{k_1}, \bar{u}_{k_2}) = \sum_c l_c I_{c,k_1} I_{c,k_2}$$
$$= \alpha t_{k_1,k_2}$$

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[2] Note that $\mathrm{Var}(\bar{l}_c)$ should be proportional to l_c^2. The linear relation is assumed to simplify the calculations. Although the variance has units of time×time implying α to have units of time, α is taken as a dimensionless quantity for numerical purposes.

where, t_{k_1, k_2} is the mean travel time on the common segment between k_1 and k_2.

After computing the expected values and variances, one can assume that the utility follows a suitable probabilistic distribution. Then, the probability of selecting a path can be computed accurately. Using a normally distributed utility leads to the probit model which is introduced in the following section.

7 Multinomial Probit Model

Following many other statistical models, it may be natural to assume that the random error term of each utility is normally distributed with mean zero, i.e., $\tilde{\epsilon} \sim N(0, \alpha \mathrm{E}(\bar{t}))$, hence, $\tilde{u} \sim N(-\mathrm{E}(\bar{t}) - \gamma n, \alpha \mathrm{E}(\bar{t}))$. This implies that the joint density function of these error terms is a multi-variate normal function. This is the underlying assumption of the probit model.

As in section five, the likelihood that a particular alternative will be chosen is given by the probability that its utility is the highest in the choice set. With the probit model, however, this probability cannot be expressed analytically since the cumulative normal distribution function does not have a closed form. The calculations of probit choice's probability, in the presence of more than two alternatives, are not straightforward.

The choice probability can be estimated by using either an analytical approximation or a Monte-Carlo simulation. A widely used analytical method is successive approximation. It is based on approximating the distribution of the maximum of two normally distributed random variables by another normal random variable. For example, to find $P_{K=\|A\|}$ one needs to approximate $\max\{\tilde{u}_1, \tilde{u}_2\}$ by a normally distributed random variable, call it $\tilde{\sigma}_1$. Then, approximate $\max\{\tilde{u}_3, \tilde{\sigma}_1\}$ by $\tilde{\sigma}_2$, and so on. In the final step, $\max\{\tilde{u}_1, \ldots, \tilde{u}_{K-1}\}$ is approximated by $\tilde{\sigma}_{K-2}$. P_K can be obtained from $\tilde{\sigma}_{K-2}$ by some integration methods. The successive approximation is fully described in appendix A.

The main advantage of the probit model is its ability to account for dependence between the utilities of different routes. On the other hand, computational costs for problems of the size encountered in network analysis are very high. Furthermore, the accuracy of this method deteriorates as the number of alternatives increases.

In the following section, the probit model is used in conjunction with the previously presented ideas to develop an algorithm that distributes unfamiliar drivers on the network according to the utility of each path.

8 A Route-Choice Algorithm for Unfamiliar Drivers

In traffic modeling, it is convenient to introduce the notion of origin-destination pairs, or simply O-D pairs. The network is assumed to have source nodes, or

origins, where the flow is generated, and sink nodes, or destinations, where the flow is absorbed. Each vehicle has a predetermined destination which cannot be changed during the trip. Hence, the only paths in the network that need to be considered are these connecting the O-D pairs.[3] This assumption simplifies the study and makes it more efficient on a computer.

Here is a description of the route-choice algorithm for unfamiliar drivers:

- Initialization. Set $i = 1$.[4] Using the free-flow travel time, compute the mean travel time, $t_k^i = E(\tilde{t}_k)$, for all paths k.

- General Step.

  1. Find the set of the K shortest paths connecting each O-D pair j.[5] Let this set be A^j.
  2. For each path $k \in A^j$, compute the mean of its utility, $u_k^i = -t_k^i - \gamma_k n_k$, and the variance, αt_k^i.
  3. From each set A^j, select the M paths with the highest average utilities.[6]
  4. For each O-D pair j, compute the probability of selecting path k, P_k^i, as described in appendix A.

The previous probabilities can be used in any dynamic traffic simulation package to distribute cars on the network in a realistic manner.

9 A Model for Familiar-Driver's Behavior

Any driver starts as an unfamiliar driver then, using his daily experience, adjusts his route choice to maximize his utility. Here is the algorithm used:

- Initialization. Set $P_k^0 = 0$ for all paths k. Set $i = 1$. Using the free-flow travel time, compute the mean travel time, $t_k^i = E(\tilde{t}_k)$, for all paths k.

- General Step.

  1. Perform the general step of the unfamiliar-driver algorithm described in section eight.
  2. If $| P_k^i - P_k^{i-1} | < \delta$ for all paths k, stop. P_k^i represents the distribution of familiar drivers over the network.
  3. Distribute drivers over different paths using P_k^i and then run a simulation to get the average travel time on each path $\overline{t_k}$.

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[3] From here on, the word path refers to a path connecting an O-D pair.

[4] i is a counter. It links the unfamiliar-driver algorithm to the familiar-driver algorithm presented in the following section.

[5] A practical range of K is 10 to 20. Shortest-path algorithms are described in [12].

[6] $M = 3$ is used in this study as recommended by [9] and [13].

4. Update $t_k^{i+1} = \omega t_k^i + (1 - \omega)\overline{t_k}$, $0 \leq \omega \leq 1$. ω is an exponential smoothing factor.

5. $i = i + 1$. Go to 1.

δ represents the maximum error in the computed probabilities.

A key question is whether this method converges to a steady state or not. More research should be done to find the necessary and sufficient conditions for convergence. In case of cycling, one may need to increase the number of paths considered or the value of δ.

10 Numerical Results

In order to test the algorithm of section eight, two traffic networks were used. The traffic on each of them was simulated using INTEGRATION. All drivers were considered to be unfamiliar drivers. The parameters γ and α were set to 0 5 and 1.5 respectively. These two values were chosen according to personal experience. The networks were loaded with cars during the first 26 minutes. The number next to each arc on the graph, represents the length of the link in kilometers in both directions.

The simulation was run twice for each network. In the first run, drivers followed the shortest path to travel from their origin to destination. A few arcs were used by the vehicles, this caused immense congestion in these areas, leaving the rest of the network with zero flow throughout the simulation. In the second run, the model described in section eight was used to assign drivers to the alternative routes in the network. The number of arcs used here was larger than had been used previously. The cars were distributed more realistically over the network and this resulted in less congestion.

Figure 1 shows the first network. The traffic flow from 4 to 8 is 840 cars/hour
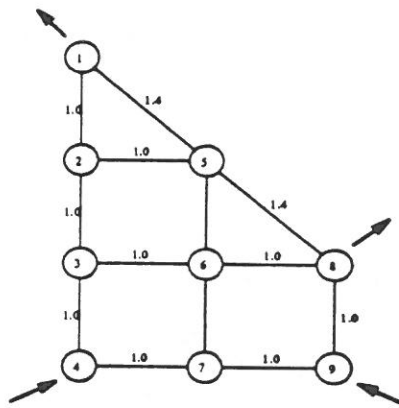
and from 9 to 1 is 851 cars/hour. The average travel time from 4 to 8, $\overline{T}_{4,8}$,

using shortest path is 3.88 minutes. Using the algorithm of section eight, $\overline{T}_{4,8}$ is 2.78 minutes. $\overline{T}_{9,1}$ is 4.15 minutes when shortest path is used and it is 3.44 minutes using the algorithm. The average travel times in the network are 4.02 and 2.98 minutes.

The second network is shown below. The traffic flow from 1 to 7 is 1020
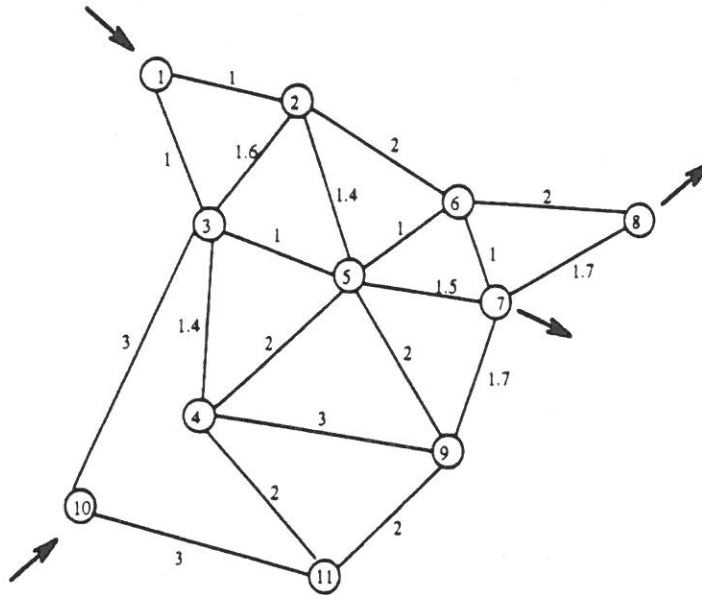
cars/hour and from 10 to 8 is 1000 cars/hour. $\overline{T}_{1,7}$ is 6.26 minutes when using the shortest path and 4.43 minutes when the algorithm is applied. $\overline{T}_{10,8}$ is 13.12 minutes and 8.67 minutes. The average travel times in the network are 9.71 and 6.56 minutes.

11 Conclusion

Two algorithms for modeling the behavior of unfamiliar and familiar drivers are introduced. Both of them generate a choice set and then use the probit model to distribute drivers over the network. A simple method for estimating the utility of each alternative is presented. Simulation results indicate that our model gives a more realistic distribution of the cars than the shortest path assumption, and therefore decreases the travel time in the network.

In order to improve the performance of the algorithms, more research should be done on the utility function and its estimation. A survey of the paths chosen by drivers in a real network would lead to a better understanding of their behavior. Considering more classes of drivers would improve the model.

12 Appendices

A Successive Approximation Method

In this section, the details of computing the probability of choosing a path are presented. It is convenient to use the notations introduced in section six: $t = E(\tilde{t})$ and $v = -E(\tilde{t}) - \gamma n$. The probit model is used, i.e., $\tilde{\epsilon} \sim N(0, \alpha E(\tilde{t}))$, and the utility is obtained as in section seven, i.e., $\tilde{u} \sim N(-E(\tilde{t}) - \gamma n, \alpha E(\tilde{t}))$. No other assumptions are made, hence, the results presented here are general.

Given two paths with utilities u_1 and u_2, one needs to approximate the probabiliy distribution of $\max\{u_1, u_2\}$ by a normal distribution. Let

$$
\begin{aligned}
\beta &= \sqrt{\alpha(t_1 + t_2 - t_{12})} \\
r &= \frac{v_1 - v_2}{\beta} \\
\nu &= v_1 \Phi(r) + v_2 \Phi(-r) + \beta\phi(r) \\
\mu &= (v_1^2 + \alpha t_1)\Phi(r) + (v_2^2 + \alpha t_2)\Phi(-r) + (v_1 + v_2)\beta\phi(r)
\end{aligned}
$$

where,

$$
\begin{aligned}
\phi(x) &= \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \\
\Phi(x) &= \int_{-\infty}^{x} \phi(y)dy
\end{aligned}
$$

then, the distribution of $\max\{u_1, u_2\}$ is approximated by $N(\nu, \mu - \nu^2)$. The covariance between $\max\{u_1, u_2\}$ and the utility u_3 of a third path is given by the following relationship:

$$
\text{Cov}(\max\{u_1, u_2\}, u_3) = \sqrt{\frac{\alpha}{t_3(\mu - \nu^2)}}(t_{13}\Phi(r) + t_{23}\Phi(-r))
$$

For example, given three paths, the probability of selecting path 3, P_3, is computed as follows:

$$
\begin{aligned}
P_3 &= P(u_3 \geq \max\{u_1, u_2\}) \\
&= \Phi\left(\frac{v_3 - \nu}{\sqrt{\alpha t_3 + (\mu - \nu^2) - 2\text{Cov}(\max\{u_1, u_2\}, u_3)\sqrt{\alpha t_3(\mu - \nu^2)}}}\right)
\end{aligned}
$$

The same procedure can be used to compute P_1 and P_2.

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