Investigating Opportunities to Develop the Practice of Responding to Students’ Contributions to Mathematical Discourse in Embedded Rehearsal

by

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CHAPTER 1
INTRODUCTION

The mathematics education community has widely adopted a complex vision for student learning, “mathematical proficiency,” in which deep conceptual understanding is intertwined with procedural fluency, strategic competence, adaptive reasoning, and productive disposition (Kilpatrick, Swafford, & Findell, 2001). Most recently, this vision has been operationalized into a set of mathematical practice and content standards in the Common Core State Standards. These standards are intended to “describe the ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise” (NGA Center & CCSSO, 2010). They represent a growing recognition that in order to both develop and demonstrate mathematical proficiency students should regularly participate in the reflective and communicative practices central to the discipline (Hiebert et al., 1997). The Practice Standards identify a set of eight of these, including making sense of problems and persevering in solving them; reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision; looking for and making use of structure; and looking for and expressing regularity in repeated reasoning (NGA Center & CCSSO, 2010).
Not only do these practices identify goals for independent student activity, but also they point toward how students are to engage with each other, with content, and with the teacher inside of classroom instructional interactions. As such, these practice goals are both intellectually and socially ambitious. They imply a system of instruction radically different from the traditional, prevailing system in which students are passive recipients of teacher-dispensed knowledge. Instead, students are expected to actively engage with each other in cognitively demanding collective pursuits representative of the authentic work of mathematical practitioners. They are responsible for constructing and improving their own strategies, sharing their reasoning to make it understandable to their peers, making sense of and strategically utilizing the ideas of others, and collectively determining what makes mathematical sense.

In turn, the work of teaching and the role of the teacher in instruction shift significantly. Teachers must be able to design or select rich tasks that provide all students with equitable opportunities to problematize mathematical situations, to develop their own solution methods using knowledge and tools they already possess, and to reflect on their process through communicating with their peers about those methods. Drawing on their own deep understanding of the subject and student thinking, teachers must sequence tasks to provide a coherent, cumulative trajectory of educative experiences for students. They must orchestrate classroom activities to maintain a focus on their mathematical goals, while simultaneously supporting students’ intellectual autonomy and guiding interactions over time to engender a classroom culture focused on the public discussion of methods. This requires that teachers recognize multiple solutions as an opportunity to engage the class in mathematical reasoning and inquiry, instead of as dangerous territory
to be avoided. They must do this with sensitivity to relational dynamics and issues of equity. Each time a student contributes, teachers must attend to how the rest of the class responds, determining whether or not to intervene to scaffold respectful engagement with each other’s ideas. In sum, teaching that aims to develop students’ deep mathematical understanding by engaging them in the practices of the discipline entails constantly assessing, and responsively adapting instruction to, students’ improvisational performances. This kind of teaching is challenging, especially for beginners.

Along with these recent advances in conceptualizing and articulating instructional goals, the teacher education community is working to figure out how to engender the changes in mathematics teaching required to realize these ambitious aims at a meaningful scale. To a large extent, we face a “problem of enactment” (Kennedy, 1999): a majority of teacher education graduates who struggle to apply content knowledge and learning theory principles studied in their coursework when they move into teaching positions and face the complexities of classroom interactions and the pull of pervasive traditional instructional culture. This problem has contributed to increasing criticism of formal teacher preparation, with some going so far as to suggest that beginners can only learn through on-the-job training (e.g. Wasburn-Moses, 2013).

To better prepare beginning teachers capable of managing the complexities of ambitious instruction, members of the teacher education community are increasingly calling for teacher education that squarely situates pre-service teacher learning in practice (Ball & Cohen 1999; Ball & Forzani, 2009; Grossman, Hammerness, and McDonald, 2009; Lampert, et al., 2010; National Council for the Accreditation of Teacher Education, 2010). Many proponents of such a practice-oriented approach recognize that it demands
significant re-envisioning and re-structuring. A recent report published by the National Council for the Accreditation of Teacher Education (2010), for example, characterized the changes required this way:

The education of teachers in the United States needs to be turned upside down. To prepare effective teachers for 21st century classrooms, teacher education must shift away from a norm, which emphasizes academic preparation and course work loosely linked to school-based experiences. Rather, it must move to programs that are fully grounded in clinical practice and interwoven with academic content and professional courses. (p. ii)

Turning teacher education “upside down” toward a more practice-focused approach presents a suite of design challenges for reformers. These include issues regarding the development of practice-oriented curricula, practice-oriented pedagogies, and an organizing infrastructure in which pedagogies and curriculum fit together into a system that provides a coherent, transformational experience for beginning teachers. How do we explicate our current understandings of ambitious instructional interactions with sufficient clarity and specificity to direct the development of practice-oriented curricula and professional performance standards? Given limited time and resources, what high-leverage activities will provide teaching candidates with sufficient inoculation to withstand traditional forces that exist in most schools and work against reform? How should a practice curriculum be integrated with opportunities to develop candidates’ deep understanding of the subject and student thinking? Given the paucity of existing examples of ambitious practice in schools, in what settings should novices’ clinical experiences be situated? To address these questions, we as teacher educators must be involved in naming and categorizing features of the work, prioritizing elements worthy of emphasis, and ordering work on elements of practice based on hypotheses about novice
learning trajectories. We must also identify and/or engineer the learning environments in which novices’ authentic engagement in ambitious practice-oriented work could occur.

In addition to these curricular challenges, we must find ways to make the complex practice of ambitious mathematics teaching do-able for novices. The practice must not only be made more explicit, but also must be approached in ways that initially reduce its complexity without compromising the integrity of the work. Because ambitious teaching entails responding appropriately to students in the midst of instructional interactions, beginning teachers must develop adaptive competence (Bransford, Brown, & Cocking, 1999; Hatano & Inagaki, 1986), or the capacity to make judgments and respond effectively within professionally justifiable constraints. This means that we must develop a practice of teacher education that simultaneously scaffolds beginners in learning a well-specified set of generative, flexible routines and provides opportunities to develop professional judgment.

To confront these challenges of system design, some proponents of practice-based teacher education are in the midst of conducting design experiments to develop, implement, and evolve new forms of teacher preparation. This dissertation focuses on one such design enterprise, that of the Learning in, from, and for Teaching Practice (LTP) Project (see Learning in, from, and for Teaching Practice Project, n.d.; Lampert et al., 2013). The core organizing structures in this design include:

• a shared “working” vision of ambitious mathematics teaching with an accompanying language of guiding principles and practices under continuous development,
• Cycles of Enactment and Investigation (CEIs) that provide an organizing framework for a coherent suite of complementary pedagogies that engage novices collectively in the observation, analysis, preparation, rehearsal, and documented enactment of ambitious teaching

• a starter set of well-specified instructional activities (IAs) that serve as the units for deliberate practice and that instantiate the working vision of ambitious instructional interactions, attending to both task structure and social participation structure, and

• the keystone pedagogy of rehearsal within the Cycle that serves as a practice field in which novices and more experienced, knowledgeable teacher educators co-participate in the simulation and analysis of instructional interactions in preparation for facilitating IAs with children.

Taken together these design components constitute an organizing infrastructure for engaging novices in the collective “deliberate practice” (Ericcson, Krampe, & Tesch-Römer, 1993) of ambitious teaching, including notably the interactive responsive aspects of the work.

My investigative focus for this dissertation is the potential of this evolving practice-oriented approach to support novices’ development of one particular component of the interactive work of ambitious teaching: the practice of responding to students’ contributions to mathematical discourse. When students regularly participate in the reflective and communicative practices central to the discipline (Hiebert et al., 1997), the teacher’s work fundamentally entails responding improvisationally to what students do and say to further mathematical goals. As I will elaborate later on, managing the
contingencies and ambiguities of such “collaboratively emergent” (Sawyer, 2004) instruction is especially challenging for beginning teachers. Therefore, any practice-oriented approach must aim to prepare beginning teachers to engage in this interactive practice competently. Within the LTP design, rehearsal is the pedagogy designed specifically to provide beginners with opportunities to learn to elicit, and then tailor their responses to, students’ contributions through instructional simulation and analysis. So it is to rehearsal that I turn my attention.

The pedagogy of rehearsal, embedded in this broader design, has already received significant analytic attention by the LTP project itself. The team, of which I am member, conducted a thorough examination of the interactions between novices and teacher educators inside of rehearsals to characterize in detail consistencies in the substance of exchanges and interaction patterns across the three study sites (Lampert et al., 2013). For example, we found that multiple aspects of ambitious teaching (both practices and principles) were consistently worked on simultaneously, and the teacher educator intervened in the midst of this work in a range of ways across any given rehearsal. By attending to such consistencies, we were able to clearly specify rehearsal as a distinctive pedagogy and begin to differentiate the particular opportunities to prepare novices for the interactive work of teaching made available in rehearsal from those possible in other activity settings.

It is my intention to contribute to this existing research enterprise, using a set of three sequential case studies of the interactive work occurring in rehearsal deeply situated in the layers of context that contributed to that work. Through detailed narrative descriptions, these cases will provide elaborated images of how the components of the
LTP design can synergistically support work on the focal practice of responding to students’ contributions to mathematical discourse. Yet, even more importantly, by examining this work across a month-long implementation period, I begin to characterize how it changes and to what that change might be attributed. When learning is understood as a developmental process evidenced by transformations both in individual participation within a community of practice and in the practices of the community itself (Rogoff, Baker-Sennet, Lacasa, & Goldsmith, 1995), investigating change over time is essential for characterizing opportunities to learn. Given this aim, the extended implementation of the LTP design in the University of Michigan context is particularly promising. Here work was tailored to novices who were coached daily in rehearsals in preparation for immediate enactment with children for four consecutive weeks. Over the course of these four weeks, novice teachers rehearsed and taught four different instructional activities, deliberately sequenced to scaffold the interactive complexity of instruction. In addition, on a daily basis teacher educators observed novices teaching children and were able to adaptively facilitate rehearsals in response.

The remainder of the dissertation is organized into the following chapters. In Chapter 2, I explain the importance of the practice of responding to students’ contributions to mathematical discourse in ambitious teaching and why it should be a central component of any practiced-oriented curriculum. Then, I conceptualize what is entailed in the practice, integrating Sawyer’s (2004) framing of teaching as disciplined improvisation with Schon’s (1983) conception of reflection-in-action as a transactional process. This framing delineates phases entailed in making sense of an interactive situation in the midst of performance in order to craft an effective response and
emphasizes the mediating role of structuring resources in enabling and constraining the process. Then I consider how this kind of improvisational practice could be learned in the context of teacher education, particularly in the context of the LTP design. I conclude the chapter by applying this conceptual framing to my investigative focus in order to specify a set of research questions that structure the development of my case narratives.

In Chapter 3, I begin by situating the particular implementation of the LTP practice-oriented approach under study within its particular programmatic setting – the first semester of the University of Michigan Elementary Master of Arts with Certification program (ELMAC) in the Summer Learning Institute (SLI) in 2011. Then, I explain the study design, providing a rationale for my selection and use of a set of sequential narrative case studies to address the research questions and contribute to the broader design research enterprise of the LTP project. Finally, I identify data sources used and outline my methods of analysis, linking these back to the aims of the dissertation.

In Chapter 4, I provide a detailed description of the three instructional activities being rehearsed in the case studies. This includes the suite of instructional goals for both students and for interns that were translated into each activity’s elaborated participation structure. This is critical background information for understanding the purposes driving activity within rehearsal. In this chapter, I write as a teacher educator intimately involved in the implementation of the LTP design in the University of Michigan context, drawing primarily on my own experiences and perspective.

Chapters 5-7 consist of three detailed case narratives. For each case, I examine an episode or set of related episodes of work occurring within a rehearsal on the core practice of responding to students’ mathematical contributions. These episodes have been
drawn from three rehearsals that together span the four weeks of the Summer Learning Institute – one drawn from week one, one from week two, and one from week four. In these chapters, I write as a researcher, standing back from the practice and analyzing it using a large corpus of multi-media data. Each case narrative comprises the following sections, intended to situate the focal episode, or episodes, of work responding to students’ contributions to mathematical discourse within the set of embedded levels of context identified in the study design. After providing an overview of the case, I move into situating the focal episode/s in relation to the repeated Cycles and the rehearsal in which it occurs. Then, I provide an analytic narrative for each episode, drawing on my conceptual framing of the transactional process of improvising and identifying the structuring resources influencing the interactive work. I conclude each case with a discussion of how these structuring resources contribute to the kinds of opportunities to learn disciplined improvisation made available in embedded rehearsals.

In the culminating chapter, I conclude with a discussion of the contributions of this dissertation research. This includes a review of the key findings looking across the set of case studies, analyzing changes in the collective work on the practice of responding to students’ contributions to mathematical discourse that occurred over time. I end with limitations of this study and suggest directions for future research and design efforts to support practice-based teacher education reforms.
 CHAPTER 2
CONCEPTUAL FRAMEWORK

Overview

In this chapter, I begin by characterizing the kind of teaching implied by ambitious mathematics learning goals for students, and called for by current visions of instructional reform, as requiring a high level of responsiveness to students. This kind of ambitious teaching demands a capacity to rapidly interpret student contributions, make judgments about what to do, and take action effectively in the midst of instructional interactions. In ambitious mathematics teaching, a central practice then is responding to student contributions to mathematical discourse, work that is particularly difficult for beginners. And therefore, it is of pressing concern for those preparing beginners capable of, and committed to, developing ambitious teaching practice to find ways to scaffold beginners’ development of this central practice.

A starting point is to clearly conceptualize what is entailed in effectively responding to student mathematical contributions. For this, I build on Sawyer’s (2004) conception of teaching as disciplined improvisation, which serves as an organizing metaphor that emphasizes the role of structuring resources in enabling and constraining a practitioner’s capacity for responsiveness in the midst of instructional interactions. Then, I overlay Schon’s (1983) notion of reflection-in-action to characterize disciplined improvisation as a transactional process that entails appreciating key situational
elements, interpreting those elements to frame the situation, acting experimentally and attending to “backtalk” from the materials of the situation, and assessing the set of consequences that emerge as a result of one’s experimental actions. To further develop the concept, I identify three categories of *structuring resources* to provide an analytic framework for considering the types of structures involved in disciplining improvisation. Finally, I consider the implications of this conceptualization of teaching for the design of practice-based teacher education and characterize embedded rehearsal as a particularly promising pedagogy for developing beginners’ capacities for disciplined improvisation.

**The Core Practice of Responding to Students’ Contributions to Mathematical Discourse in Ambitious Teaching**

In this section, I develop a case for recognizing the practice of *responding to students’ contributions to mathematical discourse* as a core component of ambitious teaching. Using the term “ambitious” enables me to distinguish the particular instructional system advocated by reformers and operationalized in the Common Core Standards, in which students mathematical activity is “congruent with ideas about what it means to do math in the discipline” (Lampert, 1990, p. 33), from other instructional systems in which knowing and doing math are understood differently. For example, Lampert (1990) characterized the nature of mathematical activity commonly occurring in schools this way:

*doing* mathematics means following the rules laid down by the teacher; *knowing* mathematics means remembering and applying the correct rule when the teacher asks a question; and mathematical *truth is determined* when the answer is ratified by the teacher. (p. 32)
Alternatively, in ambitious instruction, the teacher’s role fundamentally involves teaching mathematical content in a context where students are regularly engaging in mathematical ways of knowing.

I use the phrase “responding to students’ contributions to mathematical discourse” deliberately, to carefully delimit and characterize the practice at the center of my dissertation study. In some respects, everything an experienced teacher does is in response to students. She considers what she has learned about her students’ current level of understanding based on previous days of instruction and plans accordingly in response in order to teach particular content. However, I am interested in the more moment-to-moment responsiveness necessary when students regularly make public their mathematical reasoning, and when extended deliberation about what to do is not an option. Therefore, I restrict the practice to that of responding only inside of interactive instructional situations in which students are contributing to mathematical discourse that is actively being orchestrated by the teacher aiming to develop students’ deep conceptual understanding of mathematical content.

On the other hand, by using “mathematical discourse” I am bounding a broader practice territory than if I were to refer to “mathematical discussion.” The latter term has been used to specifically identify the final “discuss and summarize” phase in a problem-solving activity, in which the teacher facilitates an extended, whole-class discussion of student strategies for solving an open-ended complex task (Stein, et al., 2008). Mathematical discourse, in contrast, is a central component of any instructional activity in which students are expected to actively participate in collective sense-making of
mathematical ideas and concepts (Lampert, 1990). Discussion also suggests a limited attention to talk, whereas discourse has been used more broadly to include both verbal and non-verbal forms of communication. Additionally, I refer to students’ contributions to, rather than during, mathematical discourse in recognition that their contributions are integral components of that discourse, a point that is emphasized later in the chapter when I discuss Sawyer’s (2004) notion of the “collaborative emergence” of instruction.

As noted in the introduction, the ambitious mathematics content and practice goals advocated by reformers imply students’ active participation in mathematical discourse communities. Rather than taking passive roles, students are responsible for constructing and improving their own strategies, sharing their reasoning to make it understandable to their peers, making sense of and strategically utilizing the ideas of others, and collectively determining what makes mathematical sense (Hiebert et al., 1997). These practices, central to the character of discourse and ways of knowing within the discipline (Lampert, 1990), have recently been translated into a set of mathematical practice standards in the Common Core State Standards. To engender this kind of ambitious student participation, teachers’ work fundamentally involves structuring classroom interactions to teach students how to engage respectfully in “a common discourse of mathematical reasoning” (Lampert, 2001, p. 175) while “supporting

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1 Lampert (1990) argued that mathematical discourse is the product of a discourse community in which students participate collectively in ways of knowing and doing mathematics central to the discipline, as articulated by Lakatos and Polya:

Mathematical discourse is about figuring out what is true, once the members of the discourse community agree on their definitions and assumptions. These definitions and assumptions are not given, but are negotiated in the process of determining what is true. Students learn about how the truth of a mathematical assertion gets established in mathematical discourse as they zig-zag between their own observations and generalizations—their own proofs and refutations—revealing and testing their own definitions and assumptions as they go along. At the same time, they are introduced to the tools and conventions used in the discipline, which have been refined over the centuries to enable the solution of theoretical and practical problems. (p. 42)
students’ productive disciplinary engagement” (Stein et al., 1998, p. 314) to further students’ deep conceptual understanding of mathematical content.

However, when students’ own improvisational contributions come to drive instructional dialogue, teachers’ work involves much more interactional complexity than in lectured-based forms of instruction. The suite of improvisational demands facing the teacher attempting to effectively respond to these contributions in the midst of the “hot action” (Eraut, 1985) of instruction are great. Student errors and misconceptions are deliberately surfaced along with novel strategies and unanticipated reasoning. Hence, teachers are confronted with more ambiguous information and unpredictability, producing high interpretation demands and in turn high knowledge demands. Ma (2010), for example, found in her cross-cultural study of U.S. and Chinese teachers that in order to respond ambitiously to students’ non-traditional mathematical ideas, teachers must have “profound” mathematical understanding, not just procedural understanding. Teachers must know their subject domain and their students in ways that enable that knowledge to be marshaled for use in the classroom to quickly interpret ambiguous, often unanticipated, student contributions. They need to determine what each contribution suggests about student understanding and assess its potential as a resource for developing mathematical concepts.

Responding ambitiously to student contributions also requires the teacher to rely more heavily on professional judgment. Teachers face a myriad of decision points as they attempt to structure mathematical discourse toward ambitious learning goals. Lampert (2001), for example, identified a broad range of choices she faced in responding to student contributions in the midst of her work leading a whole class discussion: whether
or not to stay with the same student to elicit further explanation; when to move on to
other students to request alternative explanations and counter-speculations or to elicit a
different strategy; when to offer her own interpretations, to rephrase using more precise
language, to make use of a representation on the board, or to step back and provide meta-
commentary on the nature of the task or process. These choices entail dealing with
multiple, often competing, goals and managing ever-present dilemmas, such as
maintaining a productive balance between allowing children to exercise authority for
mathematical ideas while staying accountable to the discipline (Ball, 1993). In order to
make informed judgments about how to respond and toward what ends, teachers need to
have clearly established instructional agendas\(^2\) (Leinhardt & Greeno, 1986) and to rely on
well-developed professional values and commitments (e.g. Weick & McDaniels, 1989).\(^3\)

Not only is the practice of responding to students’ contributions a prominent and
demanding component of ambitious teaching, but also how this central practice is enacted
contributes significantly to students’ opportunities to learn. Kazemi & Stipek (1998), for
example, demonstrated that teachers’ patterns of response to student contributions
significantly impact the conceptual press of classroom discourse. Borasi (1994)
illustrated that when a teacher responds to “contradictions, tentative hypotheses,
contrasting results, or results that do not make sense” (p. 170) by capitalizing on them as
springboards for inquiry, students have opportunities to engage in problem-solving, to
monitor and justify their own mathematical activity, and to reflect on the nature of

\(^2\) Leinhardt and Greeno (1986) define agendas as the “operational plan” that guides the conduct of a lesson,
including “activity structures and operational routines that are specific versions of schemata in the teacher's
general knowledge base” as well as “decision elements that permit continuous updating and revision of the
agenda itself” (p. 76).

\(^3\) In this chapter, Weick and McDaniels (1989) theorize that in situations where outcome preferences
(instructional agendas) are established but cause-effect relationships are uncertain (whether or not a
teaching move or series of moves will result in student learning) the professional must rely on judgment
strategies informed by professional values and commitments.
mathematics as a discipline, to name just few. Stein et al. (1998) illustrated that
orchestrating rich mathematical discussions requires much more than motivating and
eliciting a range of student contributions and facilitating superficial show-and-tell
formats. Instead, teachers must knowledgably interpret and manage students’
contributions to mathematical discourse in ways that support students in critically
evaluating the merits of invented problem-solving approaches and making connections
across strategies and concepts toward mathematical goals.

Furthermore, we know that being responsive is particularly challenging for
beginners, who do not yet possess the knowledge, professional repertoire, and experience
on which to draw to anticipate, interpret, and respond ambitiously to student
contributions. Borko and Livingston (1989), for example, found that novices encountered
problems when unexpected student questions and misconceptions required them to
deviate from their prepared scripts for a lesson. These difficulties significantly affected
the beginners’ teaching, frequently resulting in their decisions “to curtail student
questioning in subsequent class sessions” (p. 488). Arguably, such field-based
experiences could be “mis-educative,” to borrow a term from John Dewey (1938),
contributing to beginners developing instructional habits counter to those required for
ambitious teaching.

For all of these reasons, supporting beginners to develop the core practice of
responding ambitiously to students’ contributions to mathematical discourse is an
important task for teacher education. Teacher education has traditionally done a better job
preparing beginners for the pre-and post-active aspects of teaching that do not require the
kind of in-the-moment interpretations, judgments, and deliberate actions entailed in
ambitious instructional interactions. Lesson planning, task design, and analyses of student work, for example, are more familiar components of traditional methods courses (Grossman & Ronfeldt, 2008). Practice-oriented approaches to teacher preparation, like that of the Learning in, from, and for Teaching Practice (LTP) Project, aim to develop beginners’ interactive capacities as well.

A Conception of Responding to Students’ Contributions to Mathematical Discourse as Disciplined Improvisational Performance

An important part of figuring out how to prepare beginning teachers to respond effectively to student contributions is conceptualizing what it is that they need to be able to do. Here Sawyer’s (2004) metaphor of teaching as disciplined improvisation⁴ is useful for understanding the interactive aspects of the work entailed in facilitating classroom mathematical discourse. Sawyer is not alone in framing the work of teaching as improvisational, nor was he the first to propose that teachers’ improvisational capacities depend on a repertoire of routines and knowledge resources. His argument draws heavily on previous work. For example, Borko and Livingston (1989)⁵, whom he references four times, integrated two frameworks to explain the different patterns in the teaching of experts versus beginners. These two frameworks included characterizing teaching as a complex cognitive skill dependent on well-developed accessible schema and secondly framing teaching as improvisational performance. Erickson (1982) also recognized that instructional discourse is characterized by improvisation within task and participation

⁴ Adaptive expertise is an associated concept frequently identified as crucial for enabling teachers to be responsive to their students. I see adaptive expertise and discipline improvisation related in the following way: a teacher who is able to engage effectively in disciplined improvisation is demonstrating her adaptive expertise.

⁵ They in turn reference Yinger (1987) for proposing improvisational performance as a metaphor for teaching.
structures. Others have made similar arguments using different terms. Tharp and Gallimore (1985), for example, framed teaching as the practice of responsively assisting student performances through “instructional conversations.” However, it is Sawyer’s juxtaposition of seemingly incongruous terms, that leads me to build upon his language of “disciplined improvisation” in order to emphasize the relationship between structure and agency in improvisational performance.

I now turn to expanding upon this metaphor. I begin by summarizing Sawyer’s argument while contributing my own emphasis to certain key ideas. Then, I overlay Schon’s (1983) notion of reflection-in-action to characterize disciplined improvisation as a transactional process that entails appreciating key situational elements, interpreting those elements to frame the situation, acting experimentally while attending to “backtalk” from the materials of the situation, and assessing the set of consequences that emerge as a result of one’s experimental actions. To further develop the concept, I identify three categories of structuring resources to provide an analytic framework for considering the types of structures involved in disciplining improvisation. While Sawyer uses the term “structures,” I have elected to refer to the set of structures collectively as “structuring resources” to emphasize the idea that activity is not determined by structure, but instead is mediated by structures. Agentic actors make choices about when and how to employ structures as resources or tools for dealing with problematic situations that arise in the midst of goal-oriented activity.

Sawyer begins with an insight drawn from this research base that “the most effective classroom interaction balances structure and script with flexibility and
improvisation” (p. 13). Expert teachers flexibly utilize a repertoire of exchange routines6 (Leinhardt & Steele, 2005) and activity structures7 (Leinhardt & Greeno, 1986) to scaffold student participation and classroom discourse. Sawyer characterizes teaching and learning activities as “improvisation” preceded by the modifier “disciplined” to capture this seeming incongruity. I now take up and expand upon this paradoxical conception of teaching to unpack some of what is entailed in the practice of responding to students’ contributions to mathematical discourse.

**Teaching as Improvisational Performance**

Teaching can be understood as improvisational, according to Sawyer (2004), in two complementary senses:

> Conceiving of teaching as improvisation emphasizes the interactional and responsive creativity of a teacher working together with a unique group of students. In particular, effective classroom discussion is improvisational, because the flow of the class is unpredictable and emerges from the actions of all participants, both teachers and students.” (italics added for emphasis, pp. 12-13)

While he was primarily focused on the improvisational demands entailed for the teacher, he is characterizing instructional interactions, and therefore the activity of both students and teacher, as improvisational. Both are involved in the creative, collaborative construction of emergent instructional discourse in which the teacher motivates and

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6 Leinhardt & Steele (2005) define routines as “small, socially shared, scripted pieces of behavior” that “evolve over time and are jointly built by teachers and students” (pp. 91-92). Exchange routines are one category of these routines that structure participant exchanges in the classroom. For example, they identify a range of exchange routines employed by Magdalene Lampert in the midst of her teaching, including call-on and revise routines. As a set, they argue, these exchange routines “support and structure the instructional dialogues, which in turn helps students understand their roles in the classroom and in the construction of both personal and public knowledge. The effect of these exchange routines was to inform students indirectly of what was expected and valued in her classroom as well as to inform them of the nature of the epistemology of the discipline and their stance as learners.” p. 145

7 I am using the term “activity structures” as defined by Leinhardt & Greeno (1986) to refer to the “main segments of a lesson” (p. 76), including activities like “homework correction, lesson presentation, and guided practice” (p. 77).
elicits student contributions, “students ‘act back’ and responses must be tailored to their action” (Lampert, 2005, p. 36).

This view of instruction as “collaboratively emergent” corresponds with insights from socio-cultural perspectives on learning as involving social processes of co-construction through interaction. In addition, this characterization is particularly fitting to the kind of instruction called for by ambitious learning goals for students, in which “the teacher creates a dialogue with the students, giving them freedom to creatively construct their own knowledge, while providing the elements of structure that effectively scaffold that co-constructive process” (Sawyer, 2004, p. 13).

**Improvizational Performance as Disciplined**

For Sawyer (2004), “creative teaching is disciplined improvisation because it always occurs within broad structures and frameworks” (p. 13, italics in original). Expert teachers design and adaptively employ activity structures and routines to support their orchestration of instructional discourse and to guide student participation (Berliner, 1987; Leinhardt & Greeno, 1986). They also make use of structuring resources to guide their own thinking and doing. For example, they draw on pedagogical frameworks (Simon, 1995), or general theories regarding the relationship between teaching and learning, to identify instructional goals and select the tasks that guide their work with students. These structuring resources both enable and constrain improvizational performance, making it more manageable. This explains the seeming contradiction in the finding that the practice of expert teachers is characterized by greater reliance on interaction routines and more improvisation than novice practice (for example, see Borko & Livingston, 1989). Experienced teachers have a larger repertoire of accessible structuring resources to
marshal in the midst of instructional interactions. So while their practice is not scripted, it
is also not random or extemporaneous.

Improvisation requires significant preparation. Pointing to the preparation
practices of improv theater groups, Sawyer emphasizes that the structuring resources that
discipline improvisational performances are pre-developed, repeatedly rehearsed, and
nested within a coherent system or approach. Beginning improv actors, for example,
learn interaction norms such as the “yes-and” technique (in which one actor accepts and
builds onto the preceding move of another) through repeated rehearsals with their theater
troupe. Such routines discipline individuals’ participation in collaborative
improvisational performances. In rehearsal, these routines are practiced inside of the
troupe’s repertoire of activity structures designed to enact a range of plot frameworks.
For example, one troupe studied by Sawyer uses a sitcom activity structure, characterized
by three acts separated by commercials, that can be used with different plot frameworks
like “false crisis” and “misunderstood overhearing.” It is through this extensive collective
preparation enacting the troupe’s repertoire of activity structures that beginners come to
know the “collectively established distinctions and standards of excellence” (Yanow &
Tsoukas, 2009, p. 1345) that enable individual participation in improvisational
performances.

**Using Schon’s Reflection-in-Action to Further Develop the Concept of Disciplined Improvisation**

From Sawyer (2004), we understand the interactive aspects of teaching as
involving the teacher’s deliberate use of structuring resources to discipline
collaboratively emergent instructional interactions. However, he does not elaborate on the
process of disciplined improvisation to delineate a more nuanced representation of how it plays out for the practitioner engaged in activity. For this, I turn to Schon’s (1983) conception of reflection-in-action to contribute a clearer image of what an experienced individual practitioner is doing in the midst of improvisational practice. He outlines an embodied transactional process that entails appreciating key situational elements, interpreting those elements to frame the situation, acting experimentally while attending to “backtalk” from the materials of the situation, and assessing the set of consequences that emerge as a result of one’s experimental actions. By viewing the practice of responding to students’ contributions to mathematical discourse through this lens, I am contributing a way of sub-dividing the practice into its constituent, albeit inseparable, phases to better inform an understanding of what it is exactly that novices need to learn to do and for what purposes structuring resources may be employed in the process.

**Disciplined Improvisation as a Transactional Process**

Schon (1983) frames the activities of a reflective practitioner in terms of a transactional process that “spirals through stages of appreciation, action, and reappreciation” (p. 131-132, italics added). The process is initiated when the practitioner is confronted with a puzzling situation, characterized by ambiguity and uncertainty. To make sense of the situation, the practitioner begins by “attend[ing] to the peculiarities of the situation at hand” to understand its particular features. His repertoire of past

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8 Schon’s conception of this transactional process draws heavily from John Dewey’s (1938) conception of the experiential learning process:

The formation of purposes is, then, a rather complex intellectual operation. It involves: (1) observation of surrounding conditions; (2) knowledge of what has happened in similar situations in the past, a knowledge obtained partly by recollection, and partly by the information, advice, and warning of those who have had a wider experience; and (3) judgment, which puts together what is observed and what is recalled to see what they signify. A purpose differs from an original impulse and desire through its translation into a plan and method of action based upon foresight of the consequences of action under given observed conditions in a certain way. (p. 69)
experiences is not ignored but rather serves as a reference for making sense of the current situation through metaphor:

When a practitioner makes sense of a situation he perceives to be unique, he sees it as something already present in his repertoire. To see this site as that one is not to subsume the first under a familiar category or rule. It is, rather, to see the unfamiliar, unique situation as both similar to and different from the familiar one, without at first being able to say similar or different with respect to what. The familiar situation functions as a precedent, or a metaphor, or—in Thomas Kuhn’s phrase—an exemplar for the unfamiliar one…. Seeing this situation as that one, one may also do in this situation as in that one. (pp. 138-139, italics in original)

A reflective practitioner draws on past experience to generate hypotheses about how the current situation is similar to and different from it, enabling him to begin to frame the situation and make judgments about how to act in response.

Once the practitioner has developed a tentative interpretation of the situation, Schon (1983) theorizes, he engages in experimentation serving three purposes simultaneously: to explore and develop an understanding of the situation, to act deliberately to change the situation in desirable ways, and to test developing interpretations or hypotheses about the situation. He accomplishes this by trying something: “The situation talks back, the practitioner listens, and as he appreciates what he hears, he reframes the situation once again.” The transaction continues until the practitioner judges that the set of intended and unintended effects of his experimenting are “on the whole satisfactory, or by the discovery of new features which give the situation new meaning and change the nature of the questions to be explored” (p. 151).

Like Sawyer, Schon also identifies a set of structuring resources, what he calls “constants,” that a reflective practitioner draws on in this transaction with a situation. These include, but are not limited to, the following (p. 270):
• the media, languages, and repertoires that practitioners use to describe reality and conduct experiments;
• the appreciative systems they bring to problem setting, to the evaluation of inquiry, and to reflective conversation;
• the overarching theories by which they make sense of phenomena;
• the role frames within which they set their tasks and through which they bound their institutional settings.

These resources enable the reflective practitioner to apply an interpretive framework to emerging practice situations to make sense of what is going on and to figure out what to do, drawing on a repertoire of strategies and conceptual tools developed through prior experience. They also provide a means for evaluating a course of activity in light of the set of consequences, both actual and anticipated, that follow from it.

### Identifying Three Analytic Categories of Structuring Resources

So far, by integrating the concepts of disciplined improvisation and reflection-in-action, I have developed an elaborated image of what is entailed in the practice of responding to students’ contributions to mathematical discourse. Central to this conceptualization is the teacher’s reliance on a pre-established suite of structuring resources that are marshaled in the midst of activity to support improvisational performance and are developed through their use across a range of instructional situations. Not only do these resources mediate what the teacher does in response to a situation but also how the teacher attends to and makes sense of the situation prior to taking action. Structuring resources also serve as a means for the teacher to retrospectively evaluate her performance based on the consequences that emerge in the midst of improvisation.

Given their importance, it is useful to consider the kinds, or categories, of structuring resources involved to further inform an understanding of what teachers need
to know and be able to do in order to effectively respond to students. For this reason, in what follows I propose three analytic categories of structuring resources, providing examples drawn from a variety of sources to briefly illustrate each. These include (1) participation structures that discipline both individual participation and collective interaction; (2) conceptual structures that discipline cognition, including reflection-in-action that occurs in the midst of practice; and (3) material artifacts, or reifications (Wenger, 1998), that are not only the products of practice but also resources that discipline both action and cognition as they are utilized. Together these serve to enable and constrain the thinking and action that co-occurs in the midst of interactional, improvisational performances.

**Participation Structures**

Participation structures are resources that structure, or discipline, individual and collective activities, including dialogue, across a range of scales. In teaching, these include everything from small exchange routines (Leinhardt and Steele, 2005), to activity structures (Leinhardt & Greeno, 1986), to general cultural participation norms that develop in a classroom community over time. As a set, these structures entail expectations for who does what, with whom, when – establishing individual roles and responsibilities within the classroom. They make improvisational interactions more predictable, providing the teacher with a means for maintaining some control over student participation and the mathematical territory explored through discursive activity.

**Conceptual Structures**

The structuring resources in this category discipline cognition. Borko and Livingston (1989) argued that it is a teacher’s “extensive, network of interconnected,
easily accessible cognitive schemata” that enables successful improvisational
performance. Referencing Shavelson (1985), they identify three types of schemas or
“abstract knowledge structure[s] that summarize information about many particular cases
and the relationships among them” (p. 475): scripts, organizing structures for
understanding familiar patterned experiences in terms of temporal relationships; scenes,
mental representations of spatial relationships among people and objects in particular
activities; and propositional structures, representations of factual knowledge of teaching-
learning components and their relationships. Pedagogical content knowledge,
professional values and commitments, and aims for student learning all fall into this
category. Together, these schemata form a conceptual net comprising different levels of
generality that “provides a basis for performance in a complex cognitive task involving
integration of high-level goals and actions with their lower level components” (Leinhardt
& Greeno, 1986, p. 75).

**Material Artifacts**

Material artifacts, or reifications in Wenger’s (1998) terms, are both the products
of engagement in practice and resources that discipline that engagement. Nicolini (2009)
summarizes how they function to structure cognition and activity and link current
practices to a coherent cultural system through time:

Tools and artefacts carry the script their designers embedded in them, and for this
reason they convey a particular culture of action. As a result, cultural artefacts
constitute a means of transmission of social knowledge by carrying, inscribed
within them, objectified norms of cognition, the assumptions on how work should
be carried out, and purposes of use. These all participate as formative elements in
the practice itself (Miettinen 1999). (p. 1406)

Examples of important material artifacts in teaching would include things such as
curriculum frameworks, lesson plans, visual representations, and terminology.
Developing Individual and Collective Capacities for Disciplined Improvisation

Having established a beginning framework for understanding what is entailed in the work of responding to students’ contributions to mathematical discourse, I now turn to linking this to a conceptualization of how this complex practice might be learned by novices in the context of formal teacher education programs. For this, I rely heavily on the project’s own articulation of the role of deliberate practice (Ericcson, Krampe, & Tesch-Römer, 1993) in developing practitioners’ adaptive competence (Hatano & Inagaki, 1986, and Bransford et al., 1999) in a complex domain. To begin, I revisit key insights regarding the learning of improvisational practices touched on or alluded to above in my elaboration of what is entailed in disciplined improvisation. I will use these ideas as a starting point for developing the outlines of a theory of learning the responsive interactional aspects of teaching.

Developing Responsive Practice through Participation

From Sawyer’s (2004) improv theater examples, we see that a new troupe member’s learning entails significant preparation with the troupe. The new actor enters an existing community of practice (Wenger, 1998) with established ways of doing improvisational theater, embodied by experienced troupe members who are carriers of the group’s collectively established practice. Their practice is disciplined by shared fundamental principles and commitments guiding interaction, outcome preferences, and shared standards of what constitutes quality performance that have developed over time through a history of joint activity and communication. These are instantiated in a nested, multi-level participation structure consisting of exchange routines embedded within a set of activity structures, each of which can be employed to address a range of plot
frameworks and that comprise the troupe’s existing repertoire. Participation over time in collective performances, both rehearsals and live, is therefore central to the actor’s learning.

As Wenger (1998) points out in his explication of a social theory of learning, a focus on participation means attending to both “a process of taking part and also to the relations with others that reflect this process” (p. 55). By taking part in the work, not only does a beginning practitioner accrue a repertoire of past experiences on which to draw for interpreting new situations (Schon, 1983), but also he/she has opportunities to confront and learn to manage problematic situations. When he confronts these situations publicly, in the midst of joint activity, other members within his community of practice can assist, drawing on the structuring resources available within the community. Over time, through this interaction in and around the work, a novice has opportunities to internalize the practices of the community while contributing to the continued development of those practices. As Rogoff et al. (1995) state, “When individuals participate in shared endeavors, not only does individual development occur, but the process transforms (develops) the practices of the community” (pp. 45-46).

This reciprocity of development is particularly rich in settings where interaction is characterized by high exchange of information and the collective articulation and negotiation of the practice regularly occurs. Here is how the LTP Project articulated this set of ideas:

Deliberate practice is not just repeated doing but cycles of repetition with feedback, where the feedback can bring conceptual elements to bear on particular problems. In analyzing how expertise at complex work develops, Dreyfus and Dreyfus (1986) point out, “As the novice gains experience actually coping with real situations . . . he or she begins to note, or an instructor points out, perspicuous examples of meaningful additional aspects of the situation or
domain” (p. 177, italics added). Deliberate practice in the company of others (peers, more experienced teachers, and TE) helps the learner develop an organized system for knowing when, why, and how aspects of their competency are relevant to any particular situation. This organized system becomes the conceptual framework that guides adaptation and innovation in situations of uncertainty (Hatano & Inagaki, 1986). Who the novice interacts with under what circumstances while learning for practice shapes what can be learned and also determines whether he or she will have the motivation to use what is learned. (Lampert et al., 2013, p.228)

Through deliberate practice in the company of others, a novice practitioner is introduced to and has opportunities to experiment with and internalize over time the structuring resources of a community of practice that discipline, and therefore enable, improvisational interactive performance.

As a novice gains experience in such a supportive context, he/she learns what information is worthy of attention, develops a perspective with which to interpret novel situations, shifts from a detached analytic stance to one of deep emotional involvement, and takes responsibility for his/her own successes and failures (Dreyfus & Dreyfus, 1986). With this development of proficiency in the domain, the novice moves away from rule-following behavior toward a greater capacity to rely on judgment. All of these changes enable the novice to engage with greater competence in improvisational performances.

**Structuring Participation for Deliberate Practice**

The Learning in, from, and for Teaching Practice (LTP) Project’s practice-oriented design for teacher preparation is intended to serve as an infrastructure for organizing novices’ participation in a community of practice whose joint work is learning to do ambitious teaching and developing the reflective and communicative practices that
support continued learning in and from teaching. As noted in the introduction, the core organizing structures in this design include:

- a shared “working” vision of ambitious teaching with an accompanying language of guiding principles and practices under continuous development,
- Cycles of Enactment and Investigation (CEIs) that provide an organizing framework for a coherent suite of complementary pedagogies that engage novices collectively in the observation, analysis, preparation, rehearsal, and enactment of ambitious teaching,
- a “starter” set of well-specified instructional activities (IAs) that serve as the unit for deliberate practice and that instantiate the working vision of ambitious instructional interactions, attending to both task structure and social participation structure, and
- the keystone pedagogy of Rehearsal within the Cycle that serves as a “practice field” in which novices and more experienced, knowledgeable teacher educators co-participate in the simulation and analysis of instructional interactions in preparation for facilitating IAs with children.

Each of these components and the rationale for the design is explained in detail in Lampert et al. (2012). Together these components of the design are intended to provide the “deliberate practice” identified by Ericsson and his colleagues (1993) as responsible for continued development of expertise in a complex domain. They constitute an approach to practice-oriented teacher preparation that the project describes on its website in this way (Learning in, from, and for Teaching Practice Project, n.d.):

We are breaking down current boundaries between coursework and fieldwork and between learning content and learning methods of teaching that content. We are
building a different kind of organization for professional preparation in which novices move back and forth between engaging with actual, particular enactments of teaching and the investigation of the skills, knowledge, and principles involved in those enactments. Focal skills and principles are selected based on current research on student learning and on professional standards. They are taught to novices in relation to actual problems of practice that arise in teaching in particular classrooms, with the goal of student learning always in the forefront. Knowledge of content is also developed in relation to problems of practice by designing the situations in which novices teach so that the full spectrum of elementary school mathematics is examined during their program. (About the Project section, para. 2; italics added)

But how do the components of the LTP design come together to provide such integrated opportunities to learn?

Within the LTP design, rehearsal serves as a primary activity setting in which novices engage in the kind of “back and forth” movement between doing teaching and investigating it identified above. It is the pedagogy designed specifically to provide beginners with opportunities to learn to tailor their responses to students’ contributions through instructional simulation and analysis. It serves as a “practice field” where novices can experiment with teaching, confront problematic instructional situations, receive feedback from a teacher educator, and participate in collective inquiry. So it is to rehearsal that I turn my attention. In the section that follows, I describe this keystone pedagogy in greater detail, explaining how it functions within the broader design to operationalize the vision of practice-oriented professional preparation proposed by LTP.

Embedded Rehearsal as a Setting for Developing the Practice of Responding to Students’ Contributions to Mathematical Discourse

Rehearsal is a distinctive pedagogy, in large part, because of its relationship to the broader practice-oriented design (summarized above) in which it is situated. As I will illustrate with my sequential case studies, recognizing these relationships between
rehearsal and the broader design is crucial for understanding the kinds of opportunities made available within rehearsal to collectively work on the practice of responding to students’ contributions to mathematical discourse. For this reason, I refer to rehearsal as *embedded* to signal its relationship to the other pedagogical activities occurring across the Cycle (see Figure 2.1).

Figure 2.1: *Rehearsal Embedded within the Cycle of Enactment & Investigation.*

In this practice-oriented design, rehearsal is always conducted within a Cycle of Enactment and Investigation. This means that rehearsal is prefaced by collective analysis of the instructional activity (IA) to be rehearsed and followed by novices’ enactments of the same IA with children. These enactments then become a resource for collective analysis, in which novices’ personal experiences teaching the IA and the problems of practice they faced have relevance for the entire group. The set of practices and principles identified in the curriculum, one key material artifact of the design process, provides a language for describing and analyzing the teaching. These include the following (Lampert, et al., 2013):

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9 The proximity of enactment to rehearsal may vary depending on different programmatic contexts.
preparing for instruction in addition to the interactive work of launching an activity, managing materials and space, managing time and pacing, using body and voice, managing student engagement, positioning students as competent, eliciting and responding to student contributions, representing student thinking verbally and on the board, orienting students to one another, assessing student understanding, and managing transitions. The principles that guide the implementation of these practices in everyday teaching are conceived with the aim of maximizing students’ access to learning important mathematics with meaning and using that mathematics to solve problems. These principles include treating students as sense-makers, designing instruction for all children to have equitable access to rigorous academic work in school, referring to clear instructional goals to guide interaction, being responsive to the requirements of the school environment while wrestling with the need to improve schools as institutions in a democracy, and attending to students as individuals and learners. (p. 228)

The instructional unit being rehearsed is always a well-specified instructional activity (addressed in depth in Chapter 4), either in its entirety or portions of it. Each IA instantiates the vision of ambitious teaching articulated by the curriculum of practices and principles, specifying both how students are to participate together in mathematical practices to engage with important content and the role of the teacher in facilitating this participation. Rehearsing this instructional unit enables practices to be learned in relation to each other and in relation to how they could further mathematical goals. Hence, opportunities to develop novices’ judgment are made available in rehearsal, in a way that is not possible when discrete skills or strategies are practiced to “get the mechanics” sans purpose. Instructional activities also serve as an important resource for novices, providing a participation structure that guides their early experimental attempts to teach ambitiously. “The routine components of these activities,” we argued in our previous work, serve “as a stable and rehearsable backdrop for the dynamic work of responding to student thinking” (Lampert et al., 2013, p.133), thereby scaffolding complex improvisational performance.
The participation structure of rehearsal also differentiates it from other pedagogies of enactment (Grossman et al., 2009). It involves the deliberate back-and-forth interplay of engaging novices in teaching through simulations and interacting around that teaching. This stepping in and out of simulation is represented in the following diagram (Figure 2.2) that provides a visual image of the interactive participation structure of rehearsal in terms of two categories, the interaction that occurs inside of simulated instruction and the interaction that occurs outside of simulated instruction when simulation is paused.

Figure 2.2: Alternating Two-phase Structure of Embedded Rehearsals

This alternating two-phase structure affords distinctive learning opportunities for groups of novices working with a teacher educator to publicly and collaboratively confront and investigate problems of practice.

During simulation phases of activity, a novice teacher rehearses an instructional activity, stepping into the teacher role and trying on the professional persona of someone responsible for developing instructionally productive relationships with students. Other novices in the group take on the roles of cooperative students, providing them with opportunities to experience the activity from the student perspective. The teacher educator also participates as a student, “acting back” in response to what the rehearsing novice teacher tries. In this role the teacher educator has opportunities to deliberately

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10 The norm for other novices to play cooperative students shifts over time as they gain insight into the ways in which real students might respond and what kinds of responses would be useful in the collective work occurring within a rehearsal.
introduce particular demands into the simulated instructional situations. For example, the
teacher educator may deliberately contribute an incorrect solution or ambiguous
explanation to which the rehearsing novice must respond.

As a result, rehearsing novices gain valuable experience improvising in a setting
where mistakes do not jeopardize actual student learning. As Twelker (1967) pointed out
over four decades ago:

In brief, instructional simulation forces the student teacher to focus on a situation
and devise different modes of responding. Simulation offers the student an
opportunity to build his own strategies of searching for cues that signal a decision
making process on his part, to test hypotheses he has about how to respond to
these problems, and to change his behavior in view of the feedback he receives.
(p. 200)

The extent to which the simulated interaction approximates real classroom interactions
can be controlled by the teacher educator through a variety of means including, but not
limited to, determining the scope of what is rehearsed, controlling the length of simulated
instructional exchanges, setting guidelines for how other novices are to participate as
students, and deliberately calibrating her own “acting back” in the role of student.

While all of this activity occurs inside of simulation phases, there are regular
pauses or interruptions in the simulation. Pauses create space in which to scaffold the
transactional process of improvisational performance at various stages as it is unfolding.
This may include the teacher educator stopping the action to engage novices in
interpreting a student error or anticipating how a student might respond in a particular
situation. The teacher educator might initiate this stepping out to provide specific
feedback for immediate use, to facilitate collective problem-solving, or to provide time
for the rehearsing novice to consider the consequences of a course of action. Novices, on
the other hand, might initiate a pause in the simulation to ask a question, request
feedback, or suggest an alternative approach. Such instances of stepping outside of the simulation create opportunities to develop a shared understanding of key practices and principles of ambitious teaching outlined in the curriculum as they relate to the instructional situations experienced in the simulation.

**Investigating Collective Work on the Practice of Responding to Students’ Contributions to Mathematical Discourse in Embedded Rehearsals**

It is within this organized infrastructure, specifically designed to create repeated opportunities for novices to rehearse, refine, and enact the practice of responding to student contributions, that I situate my investigation. In particular I zoom in on the pedagogy of rehearsal within this system to address the following central question:

*How can embedded rehearsal contribute to opportunities to develop novices’ capacities for disciplined improvisation when work is focused on responding to students’ contributions to mathematical discourse over time?*

As noted earlier, rehearsal serves as the primary setting in which novices and teacher educators interact within and around simulated instructional interactions. As such it is a rich setting for investigating the nature and sources of assistance available to novices as they experiment with, revise, and refine approximations of ambitious teaching in response to simulated students.

However, as illustrated above, rehearsal is not a stand-alone pedagogy. It is embedded within a broader intentional design, a system of interrelated activity settings comprising complementary phases of work occurring across repeated Cycles of Enactment and Investigation. Therefore, the work that occurs inside rehearsal cannot be fully understood without situating particular episodes of work within the broader design context and the trajectory of experiences enabled by the design-as-implemented for
participants over time. For these reasons, the following sub-questions further framed my analyses:

- **Research Question 1**: How can the participation structure of rehearsal contribute to the interactive work that occurs?

- **Research Question 2**: How can the participation structure of the instructional activity being rehearsed contribute to the interactive work that occurs?

- **Research Question 3**: How can the broader organizational design of tight, repeated Cycles of Enactment and Investigation occurring over time contribute to the work that occurs? Specifically, what material, conceptual, and social resources can be made available as a result of working across tight, repeated Cycles? And how can the work change over time?

A focus on the participation structure of rehearsal allows me to attend to the particular roles assumed by the teacher educator and novice teachers and the interaction patterns that arose and became routine within the rehearsal activity setting. A focus on the instructional activity being rehearsed provides a means for characterizing the content of the interactive work involved in responding to students’ contributions to mathematical discourse in terms of instructional situations. Finally, by focusing on the contribution of the broader design, I am able to analyze the work occurring in rehearsal in relation to its embeddedness within the Cycle on which it depends. Together the set of sub-questions provides a framework for holistically analyzing the set of structuring factors at play in rehearsal that enable and constrain the collaboratively emergent work that occurs over time.
CHAPTER 3
RESEARCH DESIGN & METHODS

Overview

To investigate the research questions identified in the previous chapter, I am analyzing interaction processes within rehearsal in detail, and constructing a set of three sequential case studies representative of the range of work that occurs over the course of one semester of intensive deliberate practice. To develop these cases, I am utilizing video records of rehearsal, segmenting interaction into meaningful ethnographic chunks (Jordan & Henderson, 1995) representing episodes of interactive work on responding to student contributions to mathematical discourse. I am also drawing on course documents and materials and knowledge drawn from my own experience as one of the teacher educators elaborating and enacting the design at the University of Michigan (UM) to help me situate each case within its rich social, temporal, and spatial context.

In this chapter, I begin by situating the particular implementation of the LTP practice-based system under study within its particular programmatic setting – the first semester of the UM Elementary Master of Arts with Certification program (ELMAC) in the Summer Learning Institute (SLI) in 2011. Then, I explain the study design, providing a rationale for my selection and use of a set of sequential narrative case studies to address the research questions and contribute to the broader design research enterprise of the LTP
project. Finally, I identify data sources used and outline my methods of analysis, linking these back to the aims of the dissertation.

**The LTP Practice-Based Design in Context**

Design research, according to the Design-Based Research Collaborative (2003, p. 5) “must account for how a design functions in authentic settings” and these accounts should rely on “methods that can document and connect processes of enactment to outcomes of interest” (cited in Schoenfeld, 2006, p. 200). Therefore, studying any practice-based design for professional learning requires situating particular implementations in their particular contexts, focusing on the interactional processes that emerge over time in response to evolving conditions, unexpected events, and new understandings of the practice and the design that develop through implementation. In this way, design research can contribute to a “greater understanding of a learning ecology” (Cobb et al., 2003, p. 9, cited in Schoenfeld, 2006, p. 200).

To study the LTP practice-based design in context, I elected to focus on implementation at one site, the University of Michigan, in the last year in which data was collected for the project, 2011. At Michigan the LTP design has been intentionally employed, since the summer of 2008, as the organizing infrastructure for the first semester practice-based experiences of participants in the Elementary Master of Arts with Certification program (ELMAC). In this context, the implementation of the LTP design was characterized by

- daily repeated Cycles of Enactment & Investigation (CEI),
- timely embedded rehearsals that immediately preceded interns’ enactments with children,
• a set of instructional activities (IAs) selected, elaborated, and sequenced with the learning of both particular novice teachers and a particular population of students learning in mind (see Chapter 4 for a detailed description), and

• frequent opportunities for math teacher educators to observe, discuss, and formatively assess interns’ teaching performances to inform subsequent work.

These characteristics are noteworthy, because they differentiate this implementation of the design from other possible ways of implementing the same design in different contexts. The 2011 enactment represents the most robust implementation of the design at Michigan, building on lessons learned from previous years’ enactments. Additionally, I am intimately familiar with this particular context, having served as one of the teacher educators responsible for translating the LTP design into a system adapted for use in the ELMAC context.

The Elementary Master of Arts with Certification Program (ELMAC)

ELMAC is a full-time Master’s program designed to provide a cohort of 22-25 participants with an opportunity to earn both a master of arts in educational studies degree and a recommendation for Michigan teacher certification in one intensive twelve-month period. Participants in the program, referred to as interns\(^{11}\), represent a mix of those having just completed undergraduate degrees and those changing careers, ranging in age accordingly from 22 to 55. Although interns enter the program with a variety of work and life experiences, all have had significant experiences working with children (“Elementary Teacher Education,” 2013). This experience may have been in school

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\(^{11}\) To differentiate between teacher education students and elementary school students, I have elected to refer to the former as either “interns” or “novices” only and the later as “students,” “children,” or “third-graders.” Therefore, whenever the term “students” is employed, it is referring to elementary students.
settings as tutors or teacher’s aides or in other settings as camp counselors, coaches, or volunteers.

ELMAC interns engage in a set of integrated academic courses and structured field-based experiences, beginning in late June and continuing through mid-June of the following year. As part of this integrated design, traditionally separate math content and methods courses have been combined into two semesters of integrated work on both content and methods. The first of these integrated courses addresses early elementary (grades K-3) math content and teaching methods while the second semester course focuses on teaching upper elementary math content (grades 4-6). This progression deliberately takes advantage of the first semester course coinciding with interns’ first field-based experience working with early elementary students set within the Ann Arbor School District’s Summer Learning Institute (SLI). During the 7-week summer semester, ELMAC interns engage in two weeks of intensive course-work at the University that includes preparation for working in the SLI. They spend the next four weeks, spanning from late June through the beginning of August, working with course instructors in the SLI context. After this, interns have one final week back at the University, completing final projects and semester exams.

**Co-Instructors for the Integrated Mathematics Content and Methods Course**

The 2011 summer semester integrated mathematics content and methods course was co-facilitated by two members of the Learning in, from, and for Teaching Practice (LTP) Project team, myself and Dr. Hala Ghousseini. At the time I was serving as a Graduate Student Research Assistant for the project and a doctoral candidate in the Foundations, Administration, Research Methods, and Policy Program at the University of
Michigan, School of Education. I have seven years of teaching experience – two years in a private middle school and five years in a public high school where I taught algebra, geometry, and trigonometry/pre-calculus. The 2011 summer was my third year co-teaching the course with Dr. Ghousseini.

Dr. Ghousseini, a graduate of the University of Michigan and currently an assistant professor in the department of Curriculum and Instruction at the University of Wisconsin-Madison, was serving as a post-doctoral fellow for the LTP Project at the time. She has nine years of experience teaching elementary and middle school mathematics. Prior to joining the Elementary Master of Arts with Certification (ELMAC) program at the University of Michigan, she worked as a field instructor and taught secondary mathematics methods courses.

**The Summer Learning Institute (SLI)**

This summer school program brings rising first- and third-grade children, many from low-income households, from across the district together for four weeks of half-day remedial instruction in math and literacy. The SLI represents a close collaboration between the Ann Arbor Public Schools and the University of Michigan’s ELMAC program. The name, Summer Learning Institute, was selected to signify the original designers’ aim to deliberately attend to the learning of all participants – elementary school students, classroom teachers, pre-service teaching interns, and teacher educators – through a community-wide commitment to developing and enacting instruction responsive to the needs of the participating elementary students. To support this goal, instructional activities were utilized not only as the centerpiece of the practice-based curriculum for interns, but also as an organizing component of teacher professional
and the SLI’s curriculum for rising third-graders. This collaborative work contributed to the development of a shared language and understanding of ambitious teaching across teachers, district administrators, and teacher educators.

**Situating the LTP Design in the 2011 Summer Learning Institute Context**

In order to maximize the relevance of the early-elementary math content addressed in the course, interns worked exclusively with the cohort of rising third-graders. Many of these students were performing below grade-level expectations in mathematics, and therefore, working with this group of children required understanding a broad conceptual terrain, representative of content goals spanning grades K-3. In 2011, this cohort of rising third-graders comprised roughly 170 children. They were organized into ten classes of 15-18 children. Ten district teachers were recruited to work with these students and assigned classrooms according to content expertise and interest; five were assigned to teach math and five to teach literacy. Each half-day of instruction at the Institute was divided into two hour-and-fifty-minute learning sessions. During the first session, half of the students in each grade participated in math instruction while the other half participated in literacy instruction. After a short snack/recess break, students switched classes and those in literacy during the first session moved to math and vice versa for the next hour and fifty minutes.

To coordinate with this schedule for students, the 2011 cohort of 22 interns was organized into 10 teaching teams comprising eight pairs and two groups of three interns. During the first instructional session, five of these teaching teams (11 interns) would participate in a math methods clinical session, while the other five teams (11 interns)

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12 In math, this goal was furthered by the UM teacher educators provision of professional development for participating teachers prior to the start of the SLI each year.
would participate in a literacy methods clinical session. Like the children, after the break, intern teaching teams would switch places, moving from their math methods clinical to their literacy methods clinical, or vice versa (see Table 3.1 at the end of this section).

Math clinical sessions were facilitated by a pair of teacher educators, who worked together to plan and implement the sessions based on their daily observations of interns’ enactments and collaborative assessments of their current needs. Each clinical session was organized to incorporate the rehearsal and enactment phases of the Cycle, along with some time for reflection on and analysis of interns’ teaching experiences prior to rehearsal and after enactment. The first 50 minutes of each clinical session were reserved for interns’ teaching preparation (Prep Period). This time was divided into two distinct phases, a 10-20 minute whole-group meeting and a 30-40 minute rehearsal period. Whole-group time was used for a variety of purposes that contributed to preparatory work. For example, teacher educators might elicit information about interns’ teaching experiences the day before, model aspects of the instructional activity being rehearsed that day, facilitate a review of the instructional goals for the IA, and/or field interns’ general instructional questions or concerns. This time provided one opportunity for collectively grappling with problems of practice that interns confronted in their teaching on previous days. Then in the time remaining, the teacher educators would divide the group in half and move into separate spaces to rehearse with small groups of five or six interns. During this rehearsal period, typically two to three interns in each

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13 In order to secure funding to support two teacher educators, the program had to use existing positions and moneys creatively. In this case, one teacher educator (me) was designated as the math methods course instructor while the other (Hala) was designated as the field practicum instructor, although we both assumed similar roles during the clinical sessions.
small group would take on the teaching role, such that every individual had a turn every 2-3 days on a rotating basis.

After rehearsal, teaching teams from both small groups transitioned into their assigned third-grade math classrooms, where instruction was in progress. Interns’ arrival coincided with the regular classroom teacher’s initiation of a short 15-20 minute mini-lesson exploring a key mathematical idea, representational tool, or strategy. Each mini-lesson was conceptually related to the instructional activity that interns would be teaching that day and provided interns with an opportunity to observe and take notes on students’ participation and mathematical contributions. After the mini-lesson, students transitioned into rotations. Rotations consisted of two 15-minute periods. During the first rotation, half of the class worked with the regular classroom teacher and the other half joined an intern who facilitated the rehearsed IA. For the second rotation, children switched places and a second intern facilitated the IA. This meant that each intern generally taught math for one fifteen-minute rotation per day. Dividing the class in half not only made it possible for two interns to teach the rehearsed IA on any given day, but also it served as a way of reducing the interactional demands facing the novices.

Each of the two math teacher educators followed a teaching team into their classroom placements to observe and video-record the enactments of those interns who had just rehearsed in their small group. This enabled the teacher educators to provide feedback to observed interns immediately following their enactments and to get a sense of what areas of practice to focus on in subsequent rehearsals. The video records also

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14 For teaching teams consisting of three interns, a given intern would teach two out of every three days. Also, in the first week of the SLI when interns were facilitating games, children were divided into smaller groups of 2-4 so that each intern could work closely with a few students during both rotations on a given day.
served as a resource from which teacher educators occasionally pulled clips to illustrate pedagogical concepts and initiate analytic discussions of teaching with the whole group at opportune times.

Before each of the two Clinical Sessions, the teacher educators would briefly convene for fifteen to twenty minutes to collaboratively identify issues to be addressed during the whole-group meeting and small-group rehearsal periods for the day. Their preparation for the second Clinical Session could be informed by what they had observed interns doing in classrooms with children during the first Clinical Session. This provided teacher educators with opportunities to reflect on their own practice and revise their plans between Sessions accordingly.

In the afternoons, interns remained at the school where they took a lunch break and had time to meet with the math and literacy teachers in whose classrooms they participated during the morning sessions. These meetings served as a time for interns to debrief, plan, and problem-solve with the regular classroom teachers. After lunch on occasion, teacher educators also facilitated half-hour small group workshops on specific problems of practice (“PoP”) identified by interns as of pressing concern based on their recent experiences in classrooms, using video records of their teaching as a resource. For example, an intern may have observed an unanticipated student error that was difficult to interpret or experienced an inappropriate student behavior for which they did not have a sufficiently developed repertoire to manage.

In addition, Monday through Thursday afternoons from 2:15pm to 4:15pm interns alternated between literacy (M/W) and math (Tu/Th) content and methods coursework to complement and build on their clinical experiences in the mornings. In math, these
afternoon sessions involved a range of work designed to develop interns’ understanding of mathematics, student thinking, and teaching. These activities included, but were not limited to, engaging in activities designed for interns’ own learning of math; anticipating and analyzing common student invented strategies and errors; discussing key course readings; analyzing the work of teaching using video clips drawn from interns’ own enactments of IAs, models presented by a teacher educator, and/or other records of practice; and preparing for and rehearsing upcoming IAs. Given the importance of developing novices’ profound understanding of the mathematical content they must teach, these sessions were sequenced to provide interns with a firm conceptual foundation of the whole number and operation concepts central to early elementary mathematics (grades K-3). While this work focused on the mathematical content and activities required for interns’ work with SLI students in particular, it also included work on multiplication and division concepts in order to more fully address early elementary number and operations content.

Table 3.1: ELMAC Interns’ Daily Schedule, Summer Learning Institute 2011.

<table>
<thead>
<tr>
<th>Learning Session 1</th>
<th>Group A (5 teaching teams = 11 interns)</th>
<th>Group B (5 teaching teams = 11 interns)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8:30am – 10:20am</td>
<td><strong>Math Clinical Session 1:</strong></td>
<td><strong>Literacy Clinical Session 1:</strong></td>
</tr>
<tr>
<td></td>
<td>• 50-min. “Prep Period”</td>
<td>(similar to math, with some teaching</td>
</tr>
<tr>
<td></td>
<td>• Whole-group meeting</td>
<td>teams working in 1st-grade classrooms)</td>
</tr>
<tr>
<td></td>
<td>• Small-group rehearsals</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Transition to 3rd-grade classrooms</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Observe student participation</td>
<td></td>
</tr>
<tr>
<td></td>
<td>during teacher-led mini-lesson</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Intern facilitates rehearsed IA</td>
<td></td>
</tr>
<tr>
<td></td>
<td>during first 15-min. rotation</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Second intern facilitates IA</td>
<td></td>
</tr>
<tr>
<td></td>
<td>during 2nd 15-min. rotation</td>
<td></td>
</tr>
<tr>
<td>Break</td>
<td>• Immediate debriefing of IA enactment</td>
<td></td>
</tr>
<tr>
<td></td>
<td>with a teacher educator</td>
<td></td>
</tr>
</tbody>
</table>
**Literacy Clinical Session 2:**
(similar to math, with some teaching teams working in 1st-grade classrooms)

**Math Clinical Session 2:**
- 50-min. “Prep Period”
  - Whole-group meeting
  - Small-group rehearsals
- Transition to 3rd-grade classrooms
- Observe student participation during teacher-led mini-lesson
- Intern facilitates rehearsed IA during first 15-min. rotation
- Second intern facilitates IA during 2nd 15-min. rotation

**Lunch Break**
- Immediate debriefing of IA enactment with a teacher educator
- Debriefing and planning with classroom teachers in teams

**“PoP” Workshop**
- Small-group discussion/analysis of problems of practice identified by interns, centered around representative video clips drawn from their classroom experiences

**Methods Course**
2:15pm – 4:15pm

**Math Content & Methods Course:**
(Tuesdays & Thursdays)
- Engaging in activities designed for interns’ own learning of math
- Anticipating and analyzing common student invented strategies and errors
- Discussing key course readings
- Analyzing the work of teaching using video clips drawn from interns’ own enactments of IAs, models presented by a teacher educators, and/or other records of practice
- Preparing for and rehearsing upcoming IAs

**Literacy Content & Methods Course:**
(Mondays & Wednesdays)

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**Study Design**

Investigating deliberately structured opportunities to learn disciplined improvisation in embedded rehearsals over time entails a focus on a situated interactive process. A process is characterized by “its evolving nature and its varying forms, rhythms, and pacing all related to some purpose” (Strauss & Corbin, 1998, p. 165) and therefore requires a method of inquiry able to capture change and stability over time and
the complex suite of inter-related situational factors that contribute. The case study, with its narrative qualities, provides a means for capturing this kind of phenomena that is inseparable from its contextual conditions (Yin, 2009, p. 18). Case studies simultaneously attend to the social, physical, and temporal contexts in which interactivity is situated, maintaining a holistic representation of events as they occurred.

However, given the length of the implementation period under investigation and my aim to characterize in rich detail the interactive work occurring within rehearsal across time, I needed to make choices about what to include and exclude for it to be manageable. For this reason, I elected to use a set of three sequential cases with embedded units of analysis (de Vaus, 2001, p. 228). This set was representative of work occurring at the beginning, middle, and end of the SLI. The temporal sequencing of the three cases provided an opportunity to examine change and stability over time in the conduct and substance of interactive work within rehearsal. An important analytic advantage of comparing across cases, Strauss and Corbin (1998) argue, is that the analyst is “more likely to recognize both sameness and variation in categories and to see how what applied in one case also might be relevant in the next case and where the two cases differ” (p. 89).

The embedded character of the study design provided a means for organizing analyses around nested levels of context based on the central structuring components of the LTP practice-based design for novice learning. Figure 3.1 provides a visual model of this multiple-case study design, representing the embedded levels of analysis attended to in each of the three cases.
The foundational unit of analysis for each case is an episode, or set of inter-related episodes, of work on responding to student contributions to mathematical discourse. To explore how this work was enabled and constrained by the organizing infrastructure of the LTP design, the episodes are contextualized within the activity setting of rehearsal, which in turn is understood as embedded within tight, repeated Cycles of Enactment and Investigation occurring over the course of the SLI. Additionally, the instructional activity (IA) being rehearsed is recognized as a crucial factor contributing to the substance of episodes of interactive work.

**Data Sources & Sampling Rationale**

Given this study design, I needed data in a form that would allow me to (1) examine rehearsal interactions in detail in order to identify representative episodes of work on responding to students’ contributions for each case and (2) situate those episodes within the embedded levels of context outlined above.
Rehearsal Video Records in Which I am the Teacher Educator

To satisfy the first requirement, I turned to video records of rehearsal collected as part of the LTP design research project. Preliminary analyses clearly indicated that while talk was a component of these interactions, nonverbal acts also played a central role. For example, the teacher educator sometimes deliberately acts like a student who is demonstrating disengagement through her body language, looking around or lying on the floor. In turn, the rehearsing novice teacher may respond to such acts-as-student in nonverbal ways, gesturing with hand movements or using eye contact as a means for modeling appropriate student participation or attempting to re-engage the student. Teacher educator feedback is often focused on the novice’s body positioning, movements, or tone of voice, suggesting that what the teacher educator is attending to is more than just novice talk. Finally, a teacher educator may pause simulation and give a verbal direction to which the intern responds by stepping back into the teaching role to try out a suggested instructional move. For these reasons rehearsal transcripts were insufficient for understanding exchanges between the teacher educator and novices and therefore the nature of work occurring in rehearsal.

I began with the set of rehearsal videos collected during the summer of 2011 in this setting, comprising four rehearsals (2 per teacher educator) per day for 15 days with a few exceptions, for a total of 58 rehearsal videos. As noted earlier, because this is the third and last summer in which data was collected in this setting, it represents the most informed, robust enactment of the LTP design. Additionally, the process of video recording rehearsals had been refined over the three years of the project to ensure that the

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On one day, I was the only teacher educator conducting rehearsal during the first session and another day the camera failed to record my rehearsal yielding the total of 58 videos rather than 60.
frame consistently captured as much of the talk and activity occurring in rehearsal as possible in most cases.

However, in order to conduct a fine-grained analysis of rehearsal interactions, I needed to further limit my dataset to a manageable subset of these rehearsal videos. As mentioned earlier, in this setting the two teacher educators facilitated two rehearsal sessions daily. Having observed interns’ enactments in the first session, the teacher educator is more likely to explicitly reference what she observed and how that is guiding her interventions in rehearsal in the second session’s rehearsal. For this reason, I first limited my subsample to include only those rehearsal videos from the second session, since I am interested in understanding how the tight, repeated Cycles in which rehearsals are embedded contribute to the work that occurs. To further limit my subsample, I elected to analyze only those videos in which I am the teacher educator. By using only my own rehearsal videos, I could maintain a consistent emic perspective throughout my analyses, a perspective that enabled me to draw on my own knowledge and experience of both the design-as-intended and the design-as-enacted in the particular context under study. This approach also enabled me to craft more coherent narratives about the events under investigation than would be possible had I used rehearsals facilitated by another teacher educator. This yielded a video data set of 14 rehearsal sessions, ranging in length from 23-36 minutes with an average time of 30 minutes, for an approximate total of 420 minutes of rehearsal video recordings.

Video data offers two other advantages (Jordan and Henderson, 1995) that are particularly applicable to my research study. First, video enables the study of what “really” happened in contrast to accounts or recollections of what happened. Since I am
the teacher educator in the rehearsals that I am analyzing, this advantage is particularly relevant given my intimacy with the work of the LTP project and the associated risk of observer bias. Using video records provides a means to look at interactions in which I am involved with evidence that others could inspect. Second, the ability to repeatedly view the same record enables a researcher to change focus in order to capture more of the detail, and therefore the complexity, of interaction. In my case, this affordance allowed me to attend to various constituent components of activity in different viewings and consider the influence of various structuring resources on interaction.

**Analyses**

Data analyses consisted of two stages: (1) identifying the core units of analysis – episodes of work on responding to students’ contributions to mathematical discourse – and categorizing them based on the activity structure of rehearsal, and (2) selecting focal episodes and constructing the set of three embedded case narratives.

**Stage 1: Identifying and Categorizing the Core Units of Analysis**

In order to identify episodes of work responding to students’ contributions to mathematical discourse across the set of 14 rehearsals, I drew on analytic insights from both discourse analysis and interaction analysis (Jordan and Henderson, 2010). Discourse analysis recognizes that communication is characterized by norms of turn-taking, in which certain participant moves lead to expected responses, which may in turn extend or truncate exchanges (e.g. Wells, 1996, and Nassaji and Wells, 2000). Its methods of analysis situate individual “talk moves” such that they are understood in terms of the following: their function within the sequence (e.g. does the move create a demand for another to respond or is it in response to the demand of another?); the roles assumed by
participants given what moves they make in the sequence (e.g. who is doing what and what is the significance of who does what in terms of participant identity/position?); and interaction patterns, or routine exchange structures, in which the moves occur.

Building on these ideas to encompass activity as well as talk, I borrowed the concept of “ethnographic chunks” (Jordan and Henderson, 2010, p. 57) from interaction analysis to segment rehearsal interactions into coherent sequences of interaction that would have meaning to the participants involved. I have elected to refer to these “chunks” as episodes of work, with “work” emphasizing the purpose-driven nature of the exchanges. The substantive focus of a given episode was an instructional situation in which the purpose of the work at the most general level centered on figuring out how to respond to students’ contributions to mathematical discourse. Instructional situations were emergent in the sense that they were co-constructed by the participants in the midst of interaction. Given the alternating two-phase structure of rehearsal, this occurred either through simulation, collaborative narrative construction, or a combination of the two. When work on an instructional situation is collective, its meaning and significance is also co-constructed through interaction. As an emergent phenomenon, an instructional situation can also be understood as ambiguous, open to multiple interpretations such that a range of problems of practice might be identified. The work is contingent on what aspects of the situation are perceived as surprising, challenging, or promising for participants and their improvisational contributions to the developing exchange about the situation.

Because interaction flows without pause, segmenting it after the fact by identifying coherent exchanges was not completely straight-forward, and I had to rely on
linguistic cues and shifts in activity to determine episode boundaries. This involved attending to how participants themselves framed the boundaries of a given situation and when interactive work shifted focus. In the most straight-forward instances, participants talked about the demands they were facing or named the problem being addressed. But often episode boundaries were blurry, especially in cases where participants did not explicitly talk about simulated exchanges by pausing the simulation for analysis or discussion. Yet, what this method of segmenting interaction lacks in precision, it makes up for in terms of how it corresponds to what is arguably experienced as meaningful “situations”\textsuperscript{16} for participants.

To identify these episodes and maintain their chronological relationship to one another across the set of 14 rehearsals, I used StudioCode, a video analysis software package. StudioCode allows the analyst to segment video and to associate narrative descriptions and comments directly to these segments, rather than to supporting text documents such as transcripts. These are graphically represented on a “timeline” that preserves chronology and hence enables the analysis of interaction patterns over time and at different levels. StudioCode also facilitates multiple viewings of the same episodes, to produce layers of analysis that capture comprehensive descriptions of what occurred.

Only those episodes in which participants were engaged in grappling specifically with how to respond to student contributions to mathematical discourse were included as units of analysis. This set encompassed situations that involved responding to one or more student contributions and may have entailed one or more student errors or

\textsuperscript{16} This resonates with John Dewey’s (1938) definition of a situation: “what is designated by the word ‘situation’ is not a single object or event or set of objects and events. For we never experience nor form judgments about objects and events in isolation, but only in connection with a contextual whole. This latter is what is called a ‘situation’” (cited in Cole, 1995, p. 66).
Episodes were not included if the substantive focus of interaction centered on the practice of eliciting student contributions to discourse. Nor did I include episodes involving work on responding to student comments or behavior outside of mathematical discourse. Segmenting the video data in this way yielded 73 episodes of work on the focal practice of responding to students’ contributions to mathematical discourse. As a set, these episodes represent just over half of (~53%) the total time spent in rehearsal, evidence that the practice was indeed a central focus of collective work in this setting.

Next, I sought to overlay the two-phase activity structure of rehearsal onto these 73 episodes to begin to characterize its influence on the nature of interaction occurring within the episodes. As a reminder, activity occurring within rehearsal is characterized by a back and forth movement across phases of simulation and pauses (refer back to Ch. 2). My assumption was that this two-phase structure has implications for how interns and the teacher educator collectively engage in work on responding to student contributions to mathematical discourse, yielding different kinds of opportunities to learn. It is the back and forth movement between simulating instructional interactions and collective analysis of those interactions with opportunities to bring in past experiences, questions, etc. that enables distinctively integrative work on practice. As noted in Chapter 2, instantiations of practice can be explicitly linked to a conceptual framework for understanding the work of ambitious teaching. This conceptual understanding, in turn, enables the adaptive

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17 Originally, I had planned to investigate only those situations that entailed managing multiple student solutions. However, as I became more familiar with the set of rehearsals, it became clear that work progressed across the course of the SLI, starting with situations that required responding to one student at a time and moving toward more complex situations in which responding to multiple student contributions (often involving multiple student errors) was required. In order to capture this important shift in the work, I redefined my units of analysis more broadly.
capacities crucial for effectively responding to students’ contributions to mathematical discourse in the midst of collaboratively emergent interactions.

To use this two-phase structure to categorize the episodes, I took another pass through the rehearsal videos with StudioCode, creating two more analytic rows in each timeline, one representing simulation phases in the rehearsal and the other representing pause phases. Figure 2 below illustrates how this was done, using the StudioCode timeline for the third rehearsal (R3) as an example with portions enlarged to clearly show details. Attending first to the enlarged portion labeled as “A,” we see that row 1 (“R3_Responding Work”) captures the primary units of analysis, i.e. video segments identified as episodes of work responding to student contributions to mathematical discourse. In this rehearsal, I have identified 7 episodes. Row 2 (“R3_Simulation”) and 3 (“R3_Pause”) capture simulation and pause phases, respectively.

Layering the three rows in a single timeline enabled me to then differentiate the episodes based on each episode’s occurrence within or across simulation and/or pause phases. This yielded four categories: pause-only episodes, simulation-only episodes, simple “spanning” episodes that transition across phases 1-2 times, and complex “spanning” episodes that transition across phases 3 or more times. Part “B” in Figure 2 illustrates each of these categories.
Stage 1 analyses yielded a total of 73 episodes of interactive work on responding to student contributions to mathematical discourse across the 14 rehearsals. The 73 episodes ranged in length from approximately 30 seconds to 16 minutes. Of these 73 episodes, 7 occurred entirely within a pause phase of the rehearsal, 8 occurred solely within a simulation phase, and the remaining 58 episodes spanned simulation and pause phases. Of these 58 “spanning” episodes, 17 were categorized as simple “spanning” episodes, transitioning across phases only 1 or 2 times; 41 were categorized as complex “spanning” episodes, shifting across simulation/pause phases 3 or more times. See Figure 3.3 for a representation of the distribution of episodes across the 14 rehearsals by category. This visual clearly shows that the distribution of episodes by category is not uniform across time, changing over the four weeks at the SLI, with episodes generally increasing in length and involving more complex spanning structures. Therefore, I was able to assume that my unit of analysis was indeed useful for capturing change over time in collective work on the practice of responding to students’ contributions to mathematical discourse.
Figure 3.3: Episodes by Phase Structure Categories Across the Set of 14 Rehearsals
Stage 2: Selecting Focal Episodes and Constructing Embedded Case Narratives

To get a sense of the range of work occurring across the set of 73 episodes and to develop informed criteria for selecting focal episodes for the case studies, I began by creating brief summary descriptions of each episode. These included partial transcriptions, information about how both the episode itself and transitions across phases were initiated and by whom, characterizations of individuals’ participation, and the nature/substance of the instructional situation at the center of each episode. StudioCode enabled me to link these descriptions directly to the video episodes of work in the set of rehearsals. Looking across these descriptions provided a means for identifying preliminary suppositions about the range of work occurring across the set of episodes.

This initial effort to describe each episode informed the selection criteria for identifying the three cases. To create some consistency across the cases, I chose episodes in which the focal instructional situation involved one or more student errors in response to a mathematical problem posed. Then, I sought to identify episodes that would be representative of work occurring at the beginning, middle, and end of the SLI. Because I also wanted to investigate the relationship between the work occurring and the particular instructional activity being rehearsed, I selected episodes from rehearsals of three different IAs: a games IA rehearsal from the first week, a quick images IA rehearsal from the second week, and a strings IA rehearsal from the fourth week. Then, each case was selected based on its particular affordances for exploring how work changes over time. The beginning case, for example, exemplifies one means by which interns could bring puzzling classroom experiences into rehearsal by initiating work in a pause phase of rehearsal. The set of episodes composing the second case represent how the same kind of
instructional situation can be repeatedly simulated and worked on within a single rehearsal. And, the last case focuses on a single episode where the complexity of the instructional situation being worked on results in an extended interaction among participants.

Each case narrative comprises the following sections, intended to situate the focal episode, or episodes, of work responding to students’ contributions to mathematical discourse within the set of embedded levels identified in the study design. In a separate chapter preceding the case narratives, I begin with detailed descriptions of the particular instructional activities being rehearsed. This includes the suite of instructional goals for both students and for interns informing their design and how these goals were translated into each activity’s participation structure. This is critical background information for understanding the purposes driving activity within rehearsal. I move into situating the focal episode/s in relation to the repeated Cycles and the rehearsal in which it occurs. To contextualize the focal episodes selected for the three case studies, I turned to other data sources, including course documents (such as syllabi, IA protocols and guidance, lessons plans), field notes, personal reflections, LTP team meeting notes, etc. Then, I provide an analytic narrative of the episode/s, drawing on my conceptual framing of the transactional process of improvising and identifying the structuring resources influencing the interactive work. I conclude each case with a discussion of how these structuring resources contribute to the kinds of opportunities to learn disciplined improvisation made available in embedded rehearsals.

The three sequential cases are not intended to capture the entire range of the work that occurred across the four weeks, nor would that be reasonable to expect. Instead, my
goal is to use rich descriptions to support a holistic understanding of episodes of work as embedded in multiple levels of activity and design that structure (enable and constrain) interaction and hence novices’ opportunities to develop capacities for disciplined improvisation. This understanding, in turn, can contribute to developing analytic themes and categories that I can then apply to the entire set of episodes to trace change over time at a more comprehensive level of analysis. Sequential descriptive case studies in this sense are exploratory, useful for developing some of the parameters for future explanatory analyses.

One final point about using narrative case studies to better understand design experiments is worth emphasizing here. The central engine of design research is what Collins, Joseph, and Bielaczyc (2004) have referred to as “progressive refinement” (p. 18), the process of implementing a version of the design, observing how it works, and revising and refining it, often in the midst of its implementation. This is one of the primary means by which designers come to understand the nuances of the problem the design is intended to address and how the design might be changed to better accommodate those nuances. This means that a case narrative that does not attempt to capture some of the insights garnered through the implementation process, in addition to analyzing the case retrospectively from a fixed time point, is neglecting some of the most important information about the evolutionary design process. These “lessons learned” are useful for others attempting to implement a version of the design in a different setting under different circumstances, because they link particular conditions to particular adaptations in the design-as-implemented.
Therefore, I opted to risk muddying my retrospective analyses of the process by integrating some of these insights within the narratives for each case. This approach takes advantage of my emic perspective as one of the teacher educators engaged directly in the process of “progressive refinement” in this context. In particular, I am able to contribute information about how the design was fitted to emergent conditions in the midst of implementation, of which an outsider would likely not be aware. However, I have the additional responsibility to mindfully maintain a critical stance and to take care not to make evaluative judgments about the effectiveness of these adaptations but rather to describe them as part of the structure of intentionality in which episodes of work were embedded.
CHAPTER 4:
USING INSTRUCTIONAL ACTIVITIES TO TARGET LEARNING GOALS
FOR BOTH NOVICE TEACHERS AND CHILDREN

Introduction

If novices are to work with children regularly as part of their professional training, teacher educators must consider simultaneously instructional goals for both the novice teachers and the children with whom they are working. In this chapter, I write as a teacher educator who, given the organization of the Summer Learning Institute, had to manage these dual responsibilities for the learning of interns and third-graders on a daily basis. I draw on my own experiences to provide a detailed account of some of the entailments of this work. With my teaching partner, Hala Ghousseini, we had to envision learning trajectories for both sets of learners and find ways to coordinate those trajectories so that interns’ daily interactions with children would be instructionally productive. We needed to provide opportunities for interns to develop the profound understanding of content necessary to responsively teach that content to their students. In addition, for students to engage in the kind of mathematical discourse and ways of knowing that are articulated in the Common Core mathematical practice standards, interns first would need to learn how to elicit student contributions to discourse. Only after interns developed a sufficient beginning repertoire for eliciting student contributions and students began to talk, would interns confront ambiguous and unanticipated student
contributions in the midst of teaching and have opportunities to learn in and from those experiences: how to listen to, appreciate, and interpret what students say; how to respond in ways that aim to further their mathematical goals while maintaining students’ productive engagement in mathematical discourse; and how to evaluate their responses given the resulting instructional consequences.

A primary means by which we sought to target our goals for both sets of learners in coordination was through the selection, sequencing, and elaboration of the set of instructional activities (IAs). Each IA provided an established task and participation structure for the collaboratively emergent interactions resulting when students are expected to reason publicly and to collectively engage in mathematical discourse. The process of translating our aims for children and novice teachers into well-specified IAs enabled us to control some of the factors contributing to the interactional complexities of responsive, improvisational teaching for the purposes of interns’ development. At the same time, the IAs served as rehearsable units of teaching for targeting important mathematical content for students. In the particular context of the SLI, this content included a bounded mathematical terrain aiming towards preparing students to add and subtract pairs of two-digit numbers with understanding by the end of the SLI (see Figure 4.1 below).
In this chapter, I provide an in-depth description of our specific aims for the interns and children in the 2011 Summer Learning Institute and explicate how each of the instructional activities that we utilized were designed to support those aims. This information is crucial for understanding the episodes of work in rehearsal examined across the three case studies that follow this chapter. The nature of the instructional situations that the interns experienced and the demands they faced in responding were directly related to the instructional activity they were working on and the associated instructional goals they were aiming to support through that activity’s structure. Teaching the particular IA to their particular students to target a particular set of content and practice goals situated the interns in a particular problem space. Therefore, an awareness of this set of aims is key to understanding the interactive work that occurred in the rehearsals highlighted in each case.

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18 This representation served as a material resource, or tool, for interns and teacher educators, helping us to visualize and discuss how key mathematical concepts were related, in contrast to a list of discrete concepts and facts.
I also intend for this detailed description to provide a clear image of how instructional activities could be used as a key structuring resource by teacher educators attempting to situate novice teachers’ learning opportunities in practice and the associated entailments of this work. The process of translating multiple aims into activity structures required balancing multiple considerations and making compromises, given a set of affordances and constraints to which we attended in our particular context. Some of these included practical considerations such as the availability and cost-effectiveness of materials; considerations of our past experiences and lessons learned; and considerations of how the interns’ work with children would fit into, and be influenced by, the work of the teacher in whose classroom they were working. Over the course of several years of implementation, we experimented with different ways of ordering the set of instructional activities and developed routines for rehearsing them with interns. We observed patterns in the kinds of student errors and misconceptions likely to arise for students attending the SLI, and experimented with when and how to bring these into rehearsal.

The process also required making value judgments related to central dilemmas entailed in assisting novice performances, such as how much scaffolding to offer, and in what forms, across different points in the summer. In particular, “How much specification of the instructional activity is productive for supporting interns’ development of ambitious practice?” was, and still is, a driving question. In the context of the Summer Learning Institute, how we addressed this issue was fundamentally influenced by our responsibility to sufficiently prepare our interns to work successfully with the children attending the Institute. Many of these children were already behind their peers in math, and their learning was a paramount concern. Given this context, we could
not allow novices free range to experiment when the potential consequences involved missed learning opportunities for these at-risk children. I include some of our key decision points in what follows to illustrate the role of this suite of considerations in the design and implementation process. My intention is not only to describe the structure of the design, but also to acknowledge the uncertainty involved and to address preemptively a few potential concerns about certain details of the activity that may be misunderstood by outsiders unfamiliar with our context.

Instructional Activities: A Central Component in the LTP Design for a Practice-Oriented Approach to Teacher Preparation

As noted in the introduction, instructional activities (IAs) are a core component of the LTP design for a practice-oriented approach to teacher preparation. In this section, I summarize the project’s conception of instructional activities and the rationale underlying the development and use of a spare set of instructional activities, before moving into more detail about our particular elaboration and use of IAs in the Summer Learning Institute at Michigan.

As conceived by the LTP project team, an instructional activity is a coherent instructional unit with a regular participation structure for organizing classroom interactions in ways that instantiate the field’s current understanding of how children productively engage with mathematics, with each other, and with the teacher to develop deep conceptual understanding and fluency in mathematics. Each IA can be adapted to target a range of content goals across grade levels with diverse student learners, and therefore has wide applicability. This is an important consideration given the need to prepare pre-service teachers to work across grade-levels. Our intention is to identify a
generative set of these activities that can “serve as a productive starting place for novice teachers, enabling them to develop broadly applicable skills and knowledge” (p. 136, Lampert et al., 2010). By specifying both how students are to participate together to engage with important mathematical content and the role of the teacher in facilitating this participation, IAs scaffold the interactional complexity of collaboratively emergent, improvisational instructional discourse. As such, they can serve as a valuable structuring resource for novices, disciplining their early experimental attempts at ambitious teaching. The regular structures of these activities, the project team hypothesizes, could serve “as a stable and rehearsable backdrop for the dynamic work of responding to student thinking” (Lampert et al., 2010, p.133).

Each instructional activity is characterized by a particular macro-structure of phases with room for responsiveness to student performances. Each phase is distinguishable by its own set of general instructional purposes that taken together contribute to an overall flow of the activity. Within these phases of an activity, we also specify common discourse routines, a form of micro-structure, for eliciting complex student performances (Learning in, from, and for Teaching Practice):

These routines are not “standard operating procedures” that provide mechanical solutions to the problems of practice (Feldman & Pentland, 2003). Rather, they are well-designed procedures that have been proven in practice, that take account of the complexity of the goals that need to be accomplished, and that allow the practitioner to temporarily hold some things constant while working on others. The use of such routine procedures involves not only acquiring the capacity to do the steps in the routine in an actual working environment but also the [sic] learning professional norms or principles that would enable the practitioner to make appropriate judgments about when and where it is appropriate to use the routines (Weick & McDaniels, 1989). (pp. 130-131)

Rehearsing IAs enables practices to be learned in relation to each other and in relation to how they could further mathematical goals. Hence, opportunities to develop novices’
judgment are made available in rehearsal, in a way that is not possible when discrete skills or strategies are practiced to “get the mechanics” sans purpose. The descriptions of the three instructional activities that follow will serve to illustrate these two levels of participation structure.

To provide a sense of how an intern is expected to facilitate each instructional activity, I describe in more detail each activity’s participation structure. Zooming in to this level of detail is required to see how our curriculum of principles and practices are integrated within the elaborated specifications of each IA. These descriptions also provide me with a chance to highlight where opportunities to work on responding to students’ contributions to mathematical discourse might arise in rehearsal. Each description is organized into sub-sections that signify the phases, or macro-structure, of the given activity. Within each sub-section, I emphasize key specifications by providing some of the rationale underlying their inclusion. In some instances, I suggest alternatives, in part to acknowledge our recognition that there may be other ways to achieve similar objectives and to position what we did in the larger context of a dynamic design experiment, under continuous reflective development. The descriptions that follow represent the cumulative insights garnered across three years of implementing versions of the same instructional activities in the Summer Learning Institute context.

**Translating Aims for Students and Interns into an Activity Structure in Week 1: A Turn-Taking Game with Ten-Frames**

**Aims for Rising Third Graders**

A suite of proximal content and process goals for the particular children attending the Summer Learning Institute informed our selection of the ten-frame game as the first instructional activity. We wanted an activity that could serve as a review of basic addition
facts with sums of ten or less, with a particular emphasis on pairs of numbers that sum to ten. Additionally, we sought an activity that could provide children with opportunities to visually see ten as composed of sums of smaller numbers and to notice and articulate examples of the commutative property of addition and the inverse relationship between addition and subtraction. These proximal goals for week one were situated within a broader trajectory of work aiming towards preparing students to add and subtract pairs of two-digit numbers with understanding by the end of the SLI.

The instructional activity was selected with not only content goals in mind, but also a set of participation goals for individual children and for the class as a community. These goals were intended to support the development of a classroom culture in which students would regularly engage in the kind of mathematical practices identified in the Common Core. Our aim was to design an activity structure that would provide all students with opportunities to originate and articulate their own strategies – positioning them as strategic problem-solvers responsible for, and capable of, doing intellectual work. This would require an activity in which multiple strategies were not only possible, but also made available and valued through public discourse. In order to cultivate a classroom culture in which students are responsible for determining what strategies and solutions make sense, they needed to learn to listen to and communicate with one another about their strategies and mathematical reasoning. From previous experience, we knew that most of the children attending the Summer Learning Institute were not accustomed to engaging with math and each other in these ways. So, we sought an activity structure that would scaffold the development of these practices as participation norms.
Practice Aims for Novice Teachers

In this first week, we hoped our interns would have repeated opportunities to strategically elicit, and carefully listen to, student’s mathematical thinking. Basic eliciting skills are required in order for interns to gather sufficient information to interpret student thinking and respond appropriately. As elegantly stated by Lampert (1990), “It is the strategies used for figuring out, rather than the answers, that are the site of the mathematical argument, and it is these strategies that reveal the assumptions a student is making about how mathematics works” (p. 40). Our aim was for interns to have opportunities to learn about their students’ degree of fluency and flexibility with basic addition facts and experience some of the challenges inherent in supporting students to clearly articulate their thinking. We also wanted interns to develop habits of valuing students’ reasoning and strategies as evidence of their understanding, as opposed to attending solely to correct/incorrect answers. Developing the skills and judgment associated with eliciting student thinking is a prerequisite for establishing a classroom culture in which students have mathematical authority and engage in the kind of mathematical practices at the center of educational reform. In addition to the practice of eliciting student thinking, we wanted interns to have repeated opportunities to respond to individual students in ways that were guided by their math goals and the principle of treating students as sense-makers. Part of this work of responding would involve interns in supporting students’ early performances explaining their thinking, so we wanted to provide both the interns and the students with a representational tool to use as a resource.

From prior experience working with novices, we knew that many of them are uncomfortable taking on the new role of teacher. While most had prior experience
interacting with children as baby-sitters or camp counselors, they needed support in transitioning from roles as playmates and caretakers. For this reason we sought an activity structure that would scaffold them in trying out a new way of relating to children to begin cultivating their identities as ambitious teachers. As a group our novices tended to be self-conscious at first, focusing intently on what they are saying and doing. While this is understandable, it can interfere with their ability to really hear what students are saying. For this reason, we sought to minimize the cognitive load on interns in the first week, seeking an activity structure that would limit interactional complexity to help them attend to what students were saying and doing.

**The Instructional Activity of a Turn-Taking Game with Ten-Frames**

The game, which we named “Make Ten to Win19,” was the first of four instructional activities that interns taught over the course of the four weeks of the Summer Learning Institute. To provide opportunities for students to review basic addition facts and think flexibly about sums of ten, the game utilized ten-frame models.

**Why ten-frames?** A ten-frame is a 2 by 5 rectangular array on which a number from one to ten may be represented by dots or other markers arranged in the squares of the array grid (Van de Walle, et al., 2010).

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19 The game was inspired by activities described in Van de Walle and a range of other resources available on the web.
Due in part to this structure, which emphasizes five and ten as relational anchors for other numbers, the ten-frame is a mathematical model with rich mathematical affordances.\textsuperscript{20} It is used to support children with a range of learning styles in developing mental images for numbers, visualizing arithmetic relationships, and experiencing the organization of the number system in relation to ten. (p. 312-313, Losq, 2005). The ten-frame can serve as a concrete referent to which both students and teachers can refer to represent strategies and support students’ sense-making. As such, it may be utilized in ambitious mathematics teaching, where equitable access to learning opportunities is a core commitment, to target core number and operation concepts in the early elementary grades. For these reasons, ten-frame models were to be used throughout the summer curriculum. So the game, as the first instructional activity interns would teach, was intended to familiarize students with the structure and affordances of ten-frames.

Play proceeds with each student being presented individually with a ten-frame card, either dealt or flashed quickly by the intern depending on the student’s facility

\textsuperscript{20} Losq identifies the ten-frame as a mathematical model with both rich \textit{descriptive} and \textit{predictive} power, drawing on the language from the NCTM Standards, in which students are expected to “develop the idea that a mathematical model has both descriptive and predictive power” (from p.162 of standards, p. 312-313 in Losq, 2005).
determining the number of dots on the card quickly. This turn-taking, a key aspect of the participation structure of the game, meant that only one student at a time was expected to contribute, thereby reducing the interactional complexity interns would face. During a student’s turn, she is expected to identify the number of dots represented on her ten-frame card and the number of dots still needed to fill the frame, or “make ten.” A student with a ten-frame card representing the number “8,” for instance, would report that her card had eight dots and that she needs two more to make ten. Other students are expected to listen carefully, because on occasion the intern will ask someone else to either restate the shared strategy or offer an alternative strategy for determining the number of dots or spaces for a bonus point. This work is a form of orienting students to each other, a focal practice identified in the practice-oriented curriculum for interns.

Through repeated practice with the game, and exposure to each other’s strategies, students are expected to move beyond counting dots singly toward more efficient strategies for recognizing the number of dots and spaces on the card, using five and ten as relational anchors for other numbers. For example, on the card shown above with eight dots, a child might see eight as composed of five and three more or, alternatively, as two fewer than ten.

**Phase 1: Facilitating a transition and launching the activity.** To lead the game, the facilitating intern gathers a group of two to four students around a small table. After quickly reviewing expectations for student participation, the intern introduces the ten-frame to students by asking them to share what they notice about a ten-frame card with a full top row (representing the number five). Building on student contributions, the intern emphasizes the structure of the ten-frame, visually and verbally highlighting the two rows
of five. The introduction is intended to provide access for each student to engage productively in the game moving forward. Then, the intern uses the five-card to engage students in a practice round to introduce the rules of the game in an interactive fashion.

**Phase 2: Facilitating turn-taking, eliciting individual student thinking, and guiding students to listen to each other.** The facilitating intern strategically elicits the student’s strategies for determining the number of dots and spaces on the card, using questions like “How did you figure that out?” or “Show us what you did to figure that out.” These prompts are intended to position students as capable, strategic thinkers, emphasizing the importance of strategies and reasoning, as opposed to the correctness of answers.

Other students are expected to listen carefully, because on occasion the intern will ask someone else to either restate the shared strategy or offer an alternative strategy for determining the number of dots or spaces for a bonus point. Bonus point questions are to be utilized strategically to target the math goals outlined above. For example, if a student explains that he determined that there were eight dots on his card by counting by ones, the intern might choose to elicit another strategy for a bonus point in order to get a more efficient conceptual subitizing strategy on the table. Alternatively, if a student has just shared a strategy that makes use of the five-structure, the intern could have a student, who has been counting by ones, restate the subitizing strategy for a bonus point. Bonus points, therefore, are intended to provide opportunities for students to have access to more efficient strategies and to position student contributions as resources for collective learning. These different scenarios can be simulated in rehearsal, providing interns with
opportunities to practice when to ask follow-up bonus questions to target their math goals.

Finally, the student whose turn it is must correctly summarize the information in the form of a statement like “Six and four more make ten” to earn five points for her team. The objective is for the team of students to earn more points than the teacher, who begins the game with a cache of five automatic points per student at the table. Therefore, earning the bonus points is key for the students to beat the teacher, an aspect of the structure of the game intended to provide students with motivational reinforcement to listen to each other’s strategies and be prepared to share their own.

**Phase 3: Facilitating the group in determining the score and closing the activity.** Before the 15-minute rotation period is up, interns must bring the game to a close by tallying the final scores for the students and the teacher. Discussing strategies for how to figure out the final scores provides another opportunity to work on using fives and tens to count. This is enabled in part by the way in which the intern records points.

**Translating Aims for Students and Interns into an Activity Structure in Week 2:**

**Quick Images with Ten-Frames**

**Aims for Rising Third-Graders**

In order to be prepared for double-digit addition and subtraction, a content goal on the horizon, students need be able to make use of fundamental properties of addition, relationships across single-digit addition facts, and place value understanding. Going into the second week of the SLI, the third-graders are now familiar with ten-frames and have experience using subitizing strategies to determine quickly the number of dots and spaces on a single frame. They have worked extensively to develop fluency with addition pairs
that sum to ten (i.e. $4 + 6$, $8 + 2$, etc.) with the regular classroom teacher and with interns in the “Make Ten to Win” game. Content goals for this second week included engaging students in developing and using an understanding of teen numbers as composed of one group of ten and some ones (e.g. $15 = 10 + 5$). This understanding, along with fluency in pairs that sum to ten, is a prerequisite for students inventing and using make-ten strategies for teen sums, a central aim for the week. Make-ten strategies include strategies like the following:

Table 4.1. Two Examples of Strategies Involving Making a Ten to Add.

<table>
<thead>
<tr>
<th>Breaking apart one addend in order to make the other addend a ten</th>
<th>Adding onto an addend to make ten, and compensating at the end by subtracting</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Diagram" /> 9 + 6 = 1 5 10 + 5 = 15</td>
<td><img src="image2" alt="Diagram" /> +1 9 + 6 = 10 + 6 = 16 15 -1</td>
</tr>
</tbody>
</table>

As Murata (2004) notes, these strategies are particularly useful because they generalize across multi-digit addition problems. For example, using the strategy on the left, a student might solve “39 + 26” by thinking about it as “40 + 25.” Returning to the concept map created for the SLI (see Figure 4.1 above), developing make-ten strategies can be understood as representing work at the intersection of relational understandings of basic facts and place value understandings, since these strategies involve composing and decomposing numbers to make use of “friendly” tens.

Likewise, we endeavored to build on the discourse goals for student participation targeted in the first week with the game. The game provided students with guided opportunities to invent and articulate subitizing strategies and to listen to the strategies of others in order to consider what would count as a different strategy. With the game, we
hoped to set the stage for students coming to understand their peers’ contributions as valuable learning resources. However, it did not provide students with opportunities to truly collaborate to evaluate different solutions and collectively determine correctness. Only the student whose turn it was offered a solution. To contribute to the development of a classroom culture in which students are willing to offer multiple solution conjectures and engage collectively in determining which solutions and strategies make sense in which situations, we wanted students to have opportunities to practice revising their answers publicly when confronted with good reasons to do so. “When a student is in charge of revising his or her own thinking and expected to do so publicly,” Lampert (1990) explains, “the authority for determining what is valid knowledge is shifted from the teacher to the student and the community in which the revision is asserted” (p. 52). To target this goal for students, we sought an activity structure in which publicly revising could be more readily normalized, without too much emotional or social risk to an individual, many of whom had low confidence in their math abilities already.

**Practice Aims for Novice Teachers**

By this point, we have worked with interns to develop an understanding of the centrality of place value in operating in a base-10 number system. They have read about and discussed the value of student-invented strategies and the importance of developing students’ relational understanding of basic facts. With this work as background, we sought an instructional activity that would provide interns with an opportunity to put to use, to continue to develop, and to experience the instructional relevance of these mathematical concepts and principles of teaching and learning. More specifically, for this second week, we sought to increase the complexity of instructional situations that interns
would experience in both rehearsal and enactments. To do this, we needed an instructional activity with a participation structure that would allow interns to work successfully with larger groups of students and to elicit and respond to a set of student mathematical contributions as opposed to one student solution at a time.

From our repeated opportunities observing interns’ teaching in both rehearsal and enactments with children this year and in previous years, we had identified some specific aspects of practice we hoped interns would continue to develop and refine. For example, during the first week of the SLI, we observed many interns over-using a very limited set of management moves. These included repeatedly asking students to sit up or to use a “quiet thumb” if they wanted to speak, even in situations where students were engaged and not interfering with the learning process. This over-managing was problematic because it was slowing down the pace of instruction and, in some cases, resulting in students losing their initial enthusiasm to participate. Therefore, we wanted interns to become more thoughtful about the reasons for employing particular management strategies with the goal of maintaining a safe, inclusive learning environment increasingly informing their judgments about what to do, and not do, in particular situations. As part of this work, we sought to expand interns’ repertoire of management strategies so that they could tailor their responses to students more effectively.

Whereas with the game we focused heavily on eliciting individual student thinking in order to interpret and informally assess student understanding, we now wanted interns to practice eliciting, interpreting, and responding to a set of student contributions. For this reason, orienting students to each other’s contributions while targeting particular mathematical goals was afforded greater emphasis this week. This
work, we envisioned, would include experimenting with strategies such as asking a student to paraphrase another students’ contribution when it involved an important invented strategy or pressing students to consider why another’s strategy might be useful or efficient. Our aim was also for interns to work on positioning different student answers as conjectures, responding to each in a non-evaluative manner, and guiding students to collectively and respectfully identify effective strategies and to determine correctness. Accurately representing students’ thinking and strategies remained a goal, with more attention given to the precision with which this work was done.

**The Instructional Activity of Quick Images**

In general, a quick images activity is designed to engage groups of students in determining the total number of items in a collection when flashed quickly, using perceptual and conceptual subitizing (Clements, 1999) and other strategies. The activity supports children in visualizing quantities and developing mental images of those quantities, which in turn support the development of number sense and flexibility manipulating and computing with numbers. It is fairly fast-paced and engaging, drawing students’ attention each time an image is flashed. These features of the activity are especially note-worthy for our novices, now attempting to engage groups of six to eight students. Because the images are flashed quickly, students may perceive different totals, and depending on how the items are arranged, they may determine the total using a range of different strategies. Thus, quick images is an activity in which student answers can naturally be positioned as conjectures and multiple student solutions and strategies can be made available. Given these affordances, it is an appropriate selection for developing classroom norms of conjecturing, revising, and evaluating strategies and solutions.
The particular quick images activity in the second week of the SLI was designed to support students in using knowledge of addition facts involving sums of ten plus another number to generate strategies for adding nine or eight to a number. To target this mathematical territory, students were shown flashcards with pairs of ten-frames, organized into three sets, representing a trajectory of work intended to take four to five days. As noted in the description of the game, single ten-frames can be used to support children with a range of learning styles in developing mental images for quantities up to ten and visualizing arithmetic relationships, both in relation to five and ten structures.

Pairs of ten-frames have additional affordances. The first set of flashcards, for example, had a full ten-frame on the left and a partially-filled frame on the right (see Figure 4.3). These cards provide students with repeated opportunities to visualize, with concrete referents, teen numbers as composed of one unit of ten and some ones. The second and third sets of cards represent quantities of nine plus some and eight plus some, respectively. When deliberately sequenced, students can use these visual models to invent make-ten strategies, mentally manipulating the dots in the image to fill the first ten-frame and determine the total efficiently.

However, rising third-graders are also expected to connect concrete models of addition facts with their symbolic forms. To support this, the concrete representation on each flashcard is related to its symbolic form on a poster organized to emphasize particular patterns and relationships across the set of addition facts (see Figure 4.3 below). Once the central column is completed, for example, students can explore patterns associated with adding a “friendly ten” and another number. Alternatively, looking across a row, students can compare how sums with different addends are related. For example,
they can notice that 10 + 5 and 9 + 6 are equivalent, reinforcing make-ten strategies discussed in relation to the visual work with the flashcards.

Figure 4.3. *Materials for the Quick Images Activity.*

<table>
<thead>
<tr>
<th>Flashcards</th>
<th>Poster</th>
</tr>
</thead>
<tbody>
<tr>
<td>Set 1, Example: 10 + 5 = 15</td>
<td>9 + ___ = 10 + 1 = 8 + ___ =</td>
</tr>
<tr>
<td><img src="image_url" alt="Flashcards" /></td>
<td>9 + ___ = 10 + 2 = 8 + ___ =</td>
</tr>
<tr>
<td><img src="image_url" alt="Flashcards" /></td>
<td>9 + ___ = 10 + 3 = 8 + ___ =</td>
</tr>
<tr>
<td><img src="image_url" alt="Flashcards" /></td>
<td>9 + ___ = 10 + 4 = 8 + ___ =</td>
</tr>
<tr>
<td><img src="image_url" alt="Flashcards" /></td>
<td>9 + ___ = 10 + 5 = 8 + ___ =</td>
</tr>
<tr>
<td><img src="image_url" alt="Flashcards" /></td>
<td>9 + ___ = 10 + 6 = 8 + ___ =</td>
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<tr>
<td><img src="image_url" alt="Flashcards" /></td>
<td>9 + ___ = 10 + 7 = 8 + ___ =</td>
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<tr>
<td><img src="image_url" alt="Flashcards" /></td>
<td>9 + ___ = 10 + 8 = 8 + ___ =</td>
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</table>

**Why flashcards?** While flashcards are one form of presentation, other presentation tools could also be employed. A document camera, for example, could be used to project the ten-frame pairs. Manipulatives could allow the teacher facilitating the activity to actually move dots around, flexibly adjusting which total to present to students based on intuitive, in-the-moment judgments about what should come next. However, given that we were working with a group of novices, several considerations come into play that led to this choice. Most interns have not yet developed the capacity to effectively and spontaneously improvise on their own, so the flexibility afforded in this alternative scenario could hamper interns’ successful enactment with children. Pre-made flashcards support the intern in maintaining the pace of the activity. More practically, card-stock flashcards are easy to mass-produce, inexpensive, and portable. Interns can take them home, practice on their own, and bring them into rehearsal. Finally, with these materials, interns are able to set up the activity in an empty corner or the back of the
classroom, when space is a limiting factor and other projecting tools are not readily available.

Why specify card order? The order of the cards was specified based on several considerations. First, the cards are sequenced deliberately to avoid going in order down the column. This is to prevent students from anticipating the total using the poster, and as a result becoming less engaged with the mathematical focus of the activity. This was an issue that arose for interns the first year we experimented with this instructional activity. Secondly, the first two cards, representing “10 + 2 = 12” and “10 + 5 = 15” were selected in order to provide initial access to students still defaulting to counting-on strategies to find the total (e.g. “10, 11, 12”) and to provide an early opportunity to review the strategic significance of a full row of five, respectively. In our setting, in which we were responsible for ensuring that our interns were sufficiently prepared to support student learning, we chose to include these specifications and explain the underlying rationale. One could imagine an alternative scenario, in which we suggested some of the considerations identified above as guidelines, and left the work of determining an instructionally appropriate order to the interns. Experimenting with such possibilities is one key affordance available when working with new groups of beginners across several years of implementation.

Leading Quick Images on the First Day

Phase 1: Facilitating a transition and launching the activity. To lead the quick images activity on the first day, the intern helps facilitate the transition from the mini-lesson, led by the regular classroom teacher, to the rotations period, where he/she works with half of the class while the teacher works with the other half. Identifying which
students will be joining his group, the intern directs those students to walk over quietly and arranges them strategically in chairs, making sure that students are positioned to see the flashcards and the poster (see Figure 4.4 below for one possible arrangement). This work facilitating the transition is a new responsibility for interns, who had previously relied on the teacher in whose classroom they were working to orchestrate the transition.

Figure 4.4. One Arrangement for Facilitating Quick Images.

Once students are seated, the intern briefly explains general expectations for their participation, including concise reminders such as being respectful and using a “quiet thumb” to show readiness to contribute. He/she also states the objective of the activity, for students to determine the total number of dots shown on each card and to explore strategies for determining that total quickly. This provides students with a sense of how they are expected to participate in the particular activity structure of quick images.

Phase 2: Guiding students in visualizing quantities and sharing strategies.

Gathering students’ attention, perhaps with a count down (“3-2-1!”) or similar technique, the intern then flashes the first card. He pauses to give students time to process what they saw and mentally calculate the total, and then elicits students’ conjectures. Eliciting multiple student conjectures is a routine component across instructional activities. Within quick images, the intern uses variations of the questions “How many did you see?” and
“Did anyone see anything different?” for this purpose. From our past experiences working with interns in the SLI context, we know that after eliciting one correct solution many often default to a question like, “Did everyone see [correct answer]?” This question may have the effect of stifling other students from sharing different answers and strategies. Repeatedly rehearsing the alternative, “Did anyone see anything different?” is intended to provide interns with the opportunity to develop a new default routine, or habit – one of deliberately gathering multiple student solutions rather than avoiding them. This in turn opens up opportunities to experience and work on instructional situations that demand responding to and managing multiple student solutions. As a result, questions arise for interns such as who to call on, in what order, and why.

If the intern elicits different totals with this pair of questions, he asks each student with a different answer “How did you figure that out?” This question is established as part of the elicitation routine that is generalizable across ambitious instructional activities, in which student strategies and reasoning are emphasized. Within quick images, this question also provides each student with an opportunity to potentially explain a correct strategy for determining the total, even if they perceived something not on the cards. For example, if a student sees ten and six, he can correctly identify the total as sixteen even if the card actually showed ten and seven. Once students have explained how they saw their totals, the intern leads the group in checking the card to determine the total actually represented. This creates opportunities for the intern to encourage students to revise in situations where the student error is likely one of misperception rather than misunderstanding.
As mentioned above, there is a progression of work across several days. On the first day of the activity, the intern works with a set of cards representing sums of ten plus another number. Each of these cards has a full ten-frame on the left and a partially-filled frame on the right.

**Figure 4.5: Examples of Flashcards Representing Sums of Ten Plus a Number.**

![Figure 4.5: Examples of Flashcards Representing Sums of Ten Plus a Number.](image)

As a set, the cards provide students with repeated opportunities to visualize, with concrete referents, teen quantities as composed of one group of ten and some ones. This is crucial because many of the students attending the Summer Learning Institute have not developed a secure understanding of place-value concepts in relation to two-digit numbers. Instruction emphasizing that “15” comprises one full group of ten and five ones supports these students in developing this understanding in preparation for adding and subtracting two-digit numbers.

This instruction occurs in response to student contributions that afford opportunities to emphasize relevant strategies and important mathematical concepts. The activity supports this improvisational work of responding, placing a tool for representing student thinking (flashcard) in the intern’s hands. In rehearsal we work on how to make use of this tool to respond ambitiously to students: to visually represent their strategies, to
support them as they try to articulate their strategies in terms of five and ten structures, to centrally focus their attention, and to facilitate checking and revising routines. In these ways, the flashcards assist the novice as he/she endeavors to respond to students in ways that scaffold students’ individual and collective mathematical performances inside the activity.

**Phase 3: Relating the visual representation to its symbolic form.** To support students in exploring patterns across this set of sums, the intern records each sum in symbolic form on a poster organized in three columns, introduced above. He places each flashcard adjacent to the appropriate cell in the middle column, demonstrating to students the relationship between the dots on the card and the symbolic form by “mapping” between each ten-frame and the corresponding addend. This entails using the representations to emphasize how a full ten appears in the total as the “1” digit in the tens place in the number 15 (see below).

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<table>
<thead>
<tr>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>9 + ___ = ___</td>
<td>10 + 1 = ___</td>
<td>8 + ___ = ___</td>
</tr>
<tr>
<td>9 + ___ = ___</td>
<td>10 + 2 = 12</td>
<td>8 + ___ = ___</td>
</tr>
<tr>
<td>9 + ___ = ___</td>
<td>10 + 3 = ___</td>
<td>8 + ___ = ___</td>
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<tr>
<td>9 + ___ = ___</td>
<td>10 + 4 = ___</td>
<td>8 + ___ = ___</td>
</tr>
<tr>
<td></td>
<td>10 + 5 = 15</td>
<td>8 + ___ = ___</td>
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<tr>
<td></td>
<td>10 + 6 = ___</td>
<td>8 + ___ = ___</td>
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<tr>
<td></td>
<td>10 + 7 = ___</td>
<td>8 + ___ = ___</td>
</tr>
<tr>
<td>9 + ___ = ___</td>
<td>10 + 8 = ___</td>
<td>8 + ___ = ___</td>
</tr>
</tbody>
</table>

In rehearsal, we attend to this representational work and responsively guide interns in using their hand gestures precisely to illustrate this important concept. After flashing at least five or six cards from this set, the intern may exercise judgment and move away
from the cards, engaging students in filling out the remainder of the center column using just the symbolic expressions. This can serve as a formative assessment to determine which students are able to add ten and a number quickly without referencing a visual image and how they are thinking about these sums.

**Phase 4: Facilitating pattern exploration.** Once the middle column is completed, the intern then engages students in looking for patterns across the column. This phase of exploring patterns is not included in the macro-structure of all quick images. However, because we endeavored to guide students toward inventing make-ten strategies, we wanted to provide opportunities for students to explicitly talk about (in their own terms) why adding ten to a number is “friendly.” The goal for this phase of the activity is to provide students with an opportunity to discover and articulate what happens when one group of ten and some ones are added and how that is represented in the symbolic form of the total. Students, for example, might notice that all of the answers have a 1-digit in the tens place, or that the second addend matches the digit in the ones place in each total (see below). The intern responds to these patterns in ways that explicitly reinforce place value concepts and provide access to other students. This work involves using language deliberately to emphasize, for example, “one full group of ten,” and directing multiple students to restate a given pattern or explain why the pattern works. In turn this work can be enhanced when the intern utilizes the poster to visually represent each pattern shared in order to support both the student in clearly articulating a pattern and other students in making sense of the pattern themselves.
Facilitating Quick Images on Subsequent Days

Once the middle-column work with sums involving ten is completed, interns move on to using the second set of flashcards, involving sums with nine, and eventually to the third set of flashcards involving sums with eight.

Figure 4.6: Examples of Flashcards Representing Sums of Nine Plus a Number.

For each set of flashcards, interns employ the same sequence of questions to elicit a set of student conjectures for the total number of dots shown and strategies for determining the total: “How many did you see?” “Did anyone see something different?” “How did you figure that out?” However, strategies that involve making ten are deliberately emphasized.
and students are actively encouraged to experiment with these strategies once they have been identified and explained by classmates.

To support students in making sense of and appropriation of these strategies, the intern uses precise hand gestures to represent strategies like moving a dot from the right frame to fill the ten-frame on the left, or “make a ten.” For example, the diagram below illustrates how an intern might use the flashcard shown above \((9 + 6)\) to represent the strategy of moving a dot over to complete the ten-frame on the left. Hand gestures like representing the moving of the dot, circling the full ten-frame, and pointing to the remaining five dots in the right-hand frame can be accompanied by precise language that reinforces the “one full group of ten” that is created. This kind of physical activity is something that beginners need to practice before doing it in front of students, because doing it inaccurately would interfere with achieving its purpose. It is an example of an aspect of performance that we do not want novices to have to pay attention to while they are teaching, so that they can concentrate on other aspects. The physical work here needs to become a kind of “muscle memory” before taking this into the classroom.

<table>
<thead>
<tr>
<th>“Moving a dot...”</th>
<th>...to make one full group of ten...</th>
<th>...and leaving five ones.”</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Diagram" /></td>
<td><img src="image2" alt="Diagram" /></td>
<td><img src="image3" alt="Diagram" /></td>
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</table>

When working together on responding to a range of student contributions in rehearsal, we have opportunities to both experiment with and consider when and how to differentially respond in order to target the particular mathematical goals for the activity.
The order of the second and third sets of flashcards, like the first set, is very intentional. The second set, for example, begins again with a card considered to provide wide access, 9 + 2. Students may or may not employ a “make ten” strategy with this card, but they are likely able to determine the total, using other strategies such as counting on (9, 10, 11). The next card, 9 + 6, however, naturally lends itself to students noticing the potential of moving a dot to fill the first frame. Looking at the illustration above, you can see how the single dot in the bottom row of the frame on the right matches up with the empty space in the frame on the left. This card has proven especially useful for interns guiding students to discover this strategy for making a ten. In rehearsal, working with this card usually leads to an intern asking a version of the question, “But what if no one shares this strategy?” This question points to a central dilemma in ambitious teaching that we can then explicitly discuss in the context of this particular work: how to maintain an appropriate balance between targeting your mathematical goals by guiding student performance and leaving the work of constructing mathematical strategies and meaning to students. When this arises, we talk about possible teaching moves that could enable striking an appropriate balance, such as the following: “Last year, I had a student who said that she made a ten in her head for this one. How might she have made a ten? How do you think that might have helped her figure out the total?”

After the intern facilitates the group in collectively determining the total shown on each card and considering strategies with emphasis on those involving making a ten, the intern represents the sum on the poster. With the second and third sets of cards, this work is more involved than for the first set of cards. This is because one purpose for the work now includes emphasizing the relationship between the given sum and its equivalent sum
in the middle column. For example, we now want to highlight that “9 + 6” yields the same total as “10 + 5.” Again, this work requires a level of precision in gestures and language that may not be appreciated by a casual observer of the activity.

Once the first column, representing sums with nine, is completed, interns again facilitate the group of students in looking for and explaining patterns across the column. This time patterns of particular interest, given the mathematical goals for the activity, would include contributions that point to the equivalence of the first column sums with the second column sums and the fact that the ones-digit in the solution is one less than the number added to the nine. Responding to such contributions in ways that would engage students in making sense of these patterns and how they relate to make-ten strategies is worked on in rehearsal.

Translating Aims for Students and Interns into an Activity Structure in Weeks 3 and 4: Strings of Two-Digit Addition Problems

Aims for Rising Third-Graders

By the end of the third week of the SLI, students have engaged in a variety of activities to review, and develop greater fluency with, their basic addition facts. They
have participated in choral counts designed to get them exploring place value patterns associated with repeatedly adding ten to various numbers and to think of groups of ten as units. Classroom teachers also have worked with the students to represent two-digit numbers in various forms emphasizing their place-value composition. Students were introduced to and then asked to produce and explain representations of two- and three-digit numbers using ten-frames and expanded form in various activities across the third week. For example, students might now be able to represent the number 46 in the following ways:

In this last week and a half of the SLI, we wanted students to use these understandings to develop and use intuitive strategies for adding pairs of two-digit numbers with understanding. Many of these students began the SLI either with no strategies for adding two-digit numbers other than counting by ones or with procedural familiarity “stacking” numbers to add using the traditional algorithm with little or no conceptual understanding of why the strategy worked. Therefore, although the work done in previous weeks was understood to serve as preparation for adding and subtracting pairs of two-digit numbers, work on these mathematical content goals would be the most challenging for students so far.
Again, we also attended to participation goals for students in this last week. By this point in the summer, students have become accustomed to publicly revising. They have repeatedly been prompted to listen to and paraphrase classmates’ contributions. We hoped to build on these developing participation norms to engage students at the end of the summer in comparing strategies to determine whether or not they are mathematically similar or different and to evaluate their efficiency.

**Practice Aims for Novice Teachers**

For the final week of the Summer Learning Institute, we aimed to create opportunities for novices to manage increasingly complex instructional situations with greater improvisational demands. In addition, we also sought to give interns increasing responsibility to prepare for instruction with their teaching partners and adapt instruction to fit the needs of their particular students with less scaffolding from teacher educators.

**The Instructional Activity of Strings of Computation Problems**

In general a computation string is designed to engage students in focused work exploring computational strategies in a given operation, often to support students mental math capacities. A set of computation problems form a string when they are deliberately sequenced such that the reasoning used to solve early problems can be used for subsequent problems, which generally increase in difficulty. The first problem in the string is carefully selected to provide broad initial access to the range of students in the class. Introducing and supporting the development of increasingly efficient strategies is one common goal for computational strings. Frequently, a representational tool is utilized in conjunction with the computation problems to support students’ visualization of a given strategy or set of strategies. In these ways, the activity can be used to target
particular strategies and operation concepts while leaving space for students to approach computations using what they already know and understand.

The computation strings being used in the last week and a half at the Summer Learning Institute were specifically designed to support students in using their developing understanding of place value to make use of addition strategies that involve decomposing one addend into groups of ten or multiples of ten and ones to take “friendly” jumps on an open number line.

**Why the open number line?** The open number line is a model for representing linearly the relative position and magnitude of numbers (both whole and fractional), including their proximity to landmark numbers, and for visualizing the operations of addition and subtraction. Because it is empty, beginning without regular markings like those found on structured number lines, students can partition the linear space as necessary given the problem context and their invented strategies for solving the problem, representing only those numbers utilized. The open number line model has been shown to allow more room for children’s invented strategies while also promoting greater active cognitive engagement than other models because it is not pre-structured like a hundred chart or structured number line (p. 445, Klein, Beishuizen, Treffers, 1998). Therefore, the open number line model is useful for supporting students in moving away from tedious addition strategies like counting by ones, toward more efficient strategies including keeping one number whole and jumping to the closest “landmark” multiple of ten, or jumping by tens and multiples of ten (see Table 4.2 for examples).
Table 4.2. Two Strategies for Adding 26 + 12 on the Open Number Line.

<table>
<thead>
<tr>
<th>Jumping to the closest “landmark” multiple of ten</th>
<th>Jumping by units of tens or multiples of ten</th>
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These jumping strategies, which can be visually represented on an open number line draw on and build students understanding of two-digit numbers in terms of place value units of tens and ones. This understanding serves as a conceptual foundation that enables students to devise computation strategies flexibly, depending on the problem context and numbers involved.

**Leading Addition Strings Across Several Days**

While we anticipated that this work would progress across the final week and a half of the SLI, with the addition strings becoming increasingly more challenging, the exact set of computational problems was not specified in advance except for the first day. On subsequent days of instruction, intern teaching teams were given options of possible strings to choose from, based on their assessment of their students and were also left the flexibility to create their own (although none of the teams elected to do so). Similar to the other instructional activities, a computation string begins with a launching phase, in which the intern facilitates a transition into the activity, sets expectations for student participation, and introduces key representational tools and/or materials that will be utilized in the activity. In this case, the open number line is introduced. On the first day of the activity, in particular, the intern must spend more time supporting students in understanding how to use it as a tool to show their thinking. Then, the intern poses the
first problem in the string, writing it on the board in the upper right corner to leave sufficient space for representing student strategies. The intern provides students with time to devise a strategy for determining the total. While students work on a given problem in the string, the intern circulates to carefully observe student work, to support students who are having difficulty getting started, to maintain student engagement, and to begin to make decisions about which strategies to work on, and in what order, with the whole group.

Once students are finished working, the intern then facilitates students in sharing a range of strategies for adding the pair of numbers in the problem. The intern does this strategically, in order to guide students toward increasingly efficient strategies involving jumps of ten and multiples of ten. As one student explains his/her strategy, the intern attends to the participation of other students, orienting them to the speaker using a range of prompts to support their active engagement. The intern also guides students in considering how different strategies are related. This work is especially important across the set of problems posed, since a given string of computations is designed so that later problems build on prior ones.

**Toward Designing Learning Trajectories for Novice Teachers**

In this chapter, I described in detail our set of goals for interns and the children with whom they were working during the Summer Learning Institute and how we sought to target those goals in coordination by utilizing a set of well-specified instructional activities. The description provides an image of some of the affordances available to teacher educators in selecting, elaborating, and sequencing instructional activities as one key resource for structuring a practice-oriented approach to novice teacher preparation. In
what follows I summarize some of these affordances, using examples drawn from this chapter.

**Affordances of Selecting & Elaborating Instructional Activities**

Besides serving as a primary means through which we have been able to target learning goals for both interns and student, selecting and elaborating instructional activities offers other important affordances. Their participation structures can serve to operationalize guiding professional commitments to ambitious instruction, including the set of principles and practices identified by the practice-oriented curriculum. For example, the elicitation question sequence specified in both the quick images and strings activities provides a means for interns to gather multiple student solutions, to respond to each in a non-evaluative manner positioning them as conjectures, and to make available a range of strategies in preparation for facilitating collective reasoning to determine correctness. These practices, in turn, operationalize commitments to treating students as sense-makers and positioning them to exercise mathematical authority. In this way, instructional activities can be used to scaffold interns’ early enactments of ambitious teaching, while making it possible to draw explicit connections back to the principled conceptual framework underlying their design.

In addition, through the selection and elaboration of instructional activities for dual use with both sets of learners in mind, teacher educators can develop a clearer sense of what constitutes quality enactments. They can translate a set of principled commitments identified in a practice-oriented curriculum into performance standards specific to the particular contexts in which novices are working. In the SLI context, for example, we had to carefully think through how each IA could be utilized to target
particular mathematical goals for the third-grade students in attendance. We also had to figure out how to utilize those same IAs to target practice goals for interns. Where would there be opportunities, for instance, to work on using representations in response to student contributions, and what about the practice might be worthy of focus with a given IA? Through the elaboration of the set of instructional activities, teacher educators can responsively scaffold novices’ performances based on their developing competencies and the contexts in which they are working. Some specifications of the activities can serve to reduce cognitive load so that novices can focus on other more important aspects of teaching the IA. For example, rules of the game dictate the order in which to call on students and when to elicit other student contributions, allowing interns to attend carefully to individual student contributions. Other specifications can serve to ensure that novices experience some level of success in the classroom contexts in which they are working. Using flashcards to facilitate quick images, for example, can enable them to easily set up any where in the room. In addition, some level of specification can reduce the variability across novices’ enactments so that individuals have sufficiently similar shared experiences to contribute to collective work. Everyone can experiment with the same basic protocol and revisions or refinements to this protocol can then be collectively negotiated by pooling interns’ teaching experiences.

Affordances of Sequencing Instructional Activities

By strategically sequencing the set of instructional activities that interns teach across the four weeks of the Summer Learning Institute, teacher educators can structure a learning trajectory for novices working on the practice of responding to students’ contributions to mathematical discourse. Instructional activities can serve as an important
resource, affording some level of control over the nature and complexity of the mathematical and interactional demands facing interns at different points in their development. Moving from one instructional activity to the next, additional teaching responsibilities can be added for novices as they acquire experience and skill. For example, moving from teaching the game in the first week to teaching quick images in the second, interns began to work with larger groups of students and took on new responsibilities such as transitioning students from whole class work into the rotations period.

Enacting a deliberate progression of instructional activities across a given time frame also provides a means for increasing the complexity of the work of responding to student contributions to mathematical discourse by affording new kinds of problematic instructional situations to be managed. For example, in facilitating the game interns can confront one student solution at a time and minimal student errors. The ten-frame cards serve as the primary tool for representing student strategies as part of the work of responding to target the mathematical goals for the activity. When interns rehearse and teach the next activity, quick images, they can begin to confront multiple student solutions, including errors of misperception. Like in the game, the quick images activity constrains the representational tools available to interns responding to student contributions to using the ten-frames on the flash card and the number sentences on the poster. In contrast by the end of the summer, interns teaching the strings activity face multiple student solutions and strategies. This activity requires interns to use their developing professional judgment to make choices about both how to represent a range of student solution strategies and how to support students in making important mathematical
connections across them. As instructional situations increase in complexity, so too do the improvisational demands on novices.

The case studies in the chapters that follow will illustrate these affordances and some of the ways in which the group worked collectively across time inside rehearsal to learn to manage the associated demands of responding to student mathematical contributions in increasingly complex situations.
CHAPTER 5

CASE STUDY 1: REHEARSING THE TEACHING OF A GAME

Introduction

Novice teachers confront a range of puzzling situations when they attempt to create opportunities for children to meaningfully contribute to classroom mathematical discourse. Children do and say unexpected things. Novices have not yet had sufficient experience to anticipate typical student strategies, errors, or misconceptions. They do not have a well-developed, principled conceptual framework with which to interpret student contributions. Nor have they developed a robust repertoire of strategies for responding to the unexpected things students do and say. Additionally, beginners may harbor misconceptions about the work of teaching and the role of the teacher based on their extended “apprenticeship of observation” (Lortie, 1975). These misconceptions may be in opposition to ambitious goals for student learning. Likewise, they may hold “beliefs about how to do mathematics and what it means to know it in school” (p. 32, Lampert, 1990) that conflict with modes of knowing and doing mathematics in the discipline. For example, beginning teachers may narrowly define their role as helping students get the right answer in the short term, rather than helping students develop problem-solving capacities. Therefore, a host of questions arise for novices related to anticipating,
eliciting, interpreting, and ultimately responding to individual student contributions when they begin to engage with students around the doing of mathematics.

This first case exemplifies some of these early challenges that a novice confronts when attempting to respond improvisationally to students’ contributions. The two focal episodes of work that constitute this case are drawn from the third rehearsal in the first week of the Summer Learning Institute. Both exchanges are brief, lasting approximately three minutes each. The first is categorized as a “simple spanning” episode, encompassing a simulation phase that transitions into a pause, initiated by a novice who asks a question. The second episode occurs completely within a pause phase of rehearsal. Thus, the episodes are representative of the shorter, simpler episode structures that characterize work on the practice of responding in the beginning of the Summer Learning Institute. They are complementary in that they each center around a problematic situation introduced by a novice, drawn from their teaching experiences from the day before, that gets positioned as an object for collective reflection and analysis outside of simulation.

Yet, the problems are distinct as is the way in which the teacher educator facilitates each exchange. The first episode (R3.1.9) is initiated by a novice who respectfully interrupts the simulation to ask whether or not it is acceptable for a student to blurt out an answer at that particular moment in the launch of the activity. The teacher educator in this exchange offers an interpretation of the student behavior and then the group brainstorms possible responses within this proposed framing of the situation. The second episode (R3.4.12) is also initiated by a novice, who takes advantage of a transition

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21 The designation “3.1.9” refers to the episode as occurring in Rehearsal 3, the first episode in this rehearsal (out of 7), and the ninth episode in the dataset as a whole (out of 73). See Figure 3.3 in Chapter 3 for a visual representation of the entire dataset comprising 14 rehearsals and 73 total episodes.
to a second rehearsing intern to ask for guidance about how to respond to a student “who doesn’t know the answer and is struggling.” In this episode, the teacher educator publicly guides the intern in recounting the situation and re-appreciating key details that had gone unnoticed. This leads to a re-interpretation of the situation and generating an actionable hypothesis for testing. Therefore, as a pair, the two episodes provide some sense of the range of problems confronting novices early on and how they can be worked on in the setting of embedded rehearsal.

Figure 5.1: Focal Episodes for Case 1 as Situated within Rehearsal 3.

The case as a whole illustrates how embedded rehearsal serves as an activity setting in which novices’ prior experience enacting a well-specified instructional activity with fidelity, and the resulting problems of practice they confront, can serve as a resource for collective preparation. In the language of the LTP project, the case exemplifies learning from practice in practice for practice. This opportunity is made possible because participation in repeated Cycles of Enactment and Investigation provide novices with relevant teaching experiences (from practice), pauses in rehearsal provide space for them to make those experiences public (in simulated practice), and the immediacy of enactments with children motivates high levels of collective engagement (for upcoming practice). In both episodes, the act of simulating instruction triggers participating novices’ to make public problematic situations from their previous experiences teaching the same instructional activity. Rehearsal embedded within repeated Cycles where
enactment and analysis can co-occur serves as a setting in which the structuring resources of principles, commitments, and aims that discipline professional judgment can be identified and linked directly to practical situations of high relevance to novices.

Horn (2010) has demonstrated how teachers’ personal, detailed accounts of specific classroom situations, or “teaching replays,” can function both as a means for rendering problematic situations for consultation and as a resource for developing general knowledge for teaching within a community of practice (p. 245). This occurs when narrated replays are embedded in re-visioning routines. For Horn, a re-visioning routine consists of a three-part sequence in which one participant renders a classroom situation, another participant prompts the first to elaborate or provide more information, and then someone offers a re-interpretation of the situation given the elaborated scenario. In the episodes that follow, we see how rehearsal serves as a setting in which there is space for a novice’s account of an experienced problematic situation and guidance by the teacher educator to support this kind of re-visioning routine. Therefore, the case illustrates the potential of rehearsal to provide novices with opportunities to learn to productively participate in discourse practices central to the development of informal professional learning communities in school settings. However, in this case we see that this re-visioning routine, in the context of rehearsal, does not end when a new interpretation of the puzzling situation has been developed. The interpretation then must be translated into an actionable response that the novices can try out if they confront a similar situation in their teaching immediately following rehearsal. In Schon’s (1983) terms, “the hypothesis must lend itself to embodiment in a move” (p. 152).
The case also illustrates the teacher educator’s active role in facilitating this work. She uses judgment to determine when and how to make use of novices’ prior experiences for the benefit of the group. In this case, the teacher educator pursues different approaches in the two episodes, approaches disciplined by her knowledge of interns’ needs in the first week of the Summer Learning Institute, and responsive to the particular situation. The teacher educator represents the most expert practitioner in the group and therefore, within the joint activity in rehearsal, has the opportunity to “demonstrate the nature of expertise to those who seek to acquire it” (p. 41, Lampert, 1990). The teacher educator can model the kind of experimentation and reflection entailed in the transactional process of responding to student contributions. This expertise is not about following rules, but instead about marshalling structuring resources (including knowledge, repertoires, principled commitments) for appreciating situations and making disciplined judgments about what to do. She makes public the set of considerations and information to which she is attending, modeling an approach to making sense of student contributions.

Publicly framing the problems to be addressed by the group is especially important early in work with novices. They do not yet have a repertoire of experiences or a robust set of structuring resources on which to draw to support the transactional process theorized by Schon (1983) and summarized in Chapter 2. Nor has the group had an opportunity to develop a level of shared understanding regarding the aims of ambitious teaching and the role of the teacher in forwarding those aims. As Schon points out, “when ends are confused and conflicting, there is as yet no ‘problem’ to be solved” (p. 41). This case provides evidence that rehearsal can serve as a setting in which groups of interns and
a teacher educator can collaborate to identify and frame the problems to be collectively addressed. In this way, the group can “organize and clarify both the ends to be achieved and the possible means of achieving them” (p. 41) and develop a shared understanding of the domain of ambitious teaching and the parameters in which technical expertise may be expressed (Schon, 1983). In the following section, I situate these episodes in their embedded levels of context prior to moving into their description and analysis.

Embedding the Third Rehearsal in the Design Context and First Week of the Summer Learning Institute

Situating the Rehearsal within a Trajectory of Repeated Cycles

It is Friday morning, July 8, the end of the first week at the 2011 Summer Learning Institute. It is the third and final day interns are teaching the turn-taking game with ten-frames (see Chapter 4 for a detailed description of this instructional activity). Hala Ghousseini, the co-instructor for the course, and I have each facilitated two rehearsals a day with groups of interns for two days prior, so this is interns’ third day of rehearsal in the Summer Learning Institute. Part of what interns must learn at this point is how to participate productively in this setting. They are still getting accustomed to making public their first, often awkward, attempts approximating ambitious teaching. As the teacher educators, we must take care not to overwhelm interns with too much information, not to demoralize them with too many interruptions or simulated complications. We need to establish some level of trust and for interns to develop an appreciation for the usefulness of rehearsal so that they are motivated to take risks, to mess up in front of their peers, and to accept coaching. We need to encourage broad participation and work to keep all interns engaged even though only two interns in the
group will rehearse on this day, so that rehearsal serves as preparation for each of them. For all these reasons, our early interventions must be particularly sensitive, and we must find ways to reduce and control the kinds of demands entailed in improvisational performance.

One way in which this was accomplished, as noted in Chapter 4, was through the selection, sequencing, and elaboration of the set of instructional activities. The game was chosen deliberately by the teacher educators (myself and Hala Ghousseini) as the first instructional activity novices would teach because of its turn-taking structure, constraining mathematical discourse so that the teacher could focus on one student’s reasoning at a time. Another way in which we elected to control the level of complexity confronting interns rehearsing in the teaching role was to establish ground rules for how other interns should participate as simulated students in early rehearsals. We witnessed in our first year experimenting with rehearsals that most novices did not have sufficient knowledge of student thinking to accurately simulate reasonable student contributions early in the semester. We also recognized that we needed to limit the demands created when interns act as problematic students, in order to maintain some level of control over the nature and number of complications facing the rehearsing novice. So at this point in the summer, novices have been instructed to only act cooperatively. The teacher educator alone simulates problematic student behavior or mathematical reasoning. The brevity of the interactive episodes occurring in this first week, in comparison to episodes in the last week, is in part due to these very real considerations and constraints.

**Situating the Rehearsal within the Second Prep Period of the Morning**

For the focal episodes analyzed in this case, we join a small group about seven
minutes into the second rehearsal period of the day. As described in Chapter 3 (see Table 3.1 for a summary of the interns’ daily schedule), the two teacher educators work together to lead two Prep Periods each morning at the Summer Learning Institute. The rehearsal analyzed for this case is drawn from the second Prep Period of the day. Therefore, we have already facilitated one complete Prep Period, including a whole-group meeting and small-group rehearsals in which we divide the group in half to rehearse separately. We have also observed the first group of interns teaching children and debriefed our observations immediately prior to this second Prep Period to inform our work with this new group of interns.

By this third day working with the game, most interns have elected to tweak the structure of the game in response to their particular students’ facility identifying the number of dots on the card and determining how many more dots are needed to fill the frame. Almost all of the interns have adapted the game to encourage conceptual subitizing and to increase the challenge, flashing the cards for individuals rather than giving each student their own card to study and leading their small group of students in writing each representative number sentence in their notebooks. That is the version the group rehearses on this day and that interns will facilitate with children immediately after rehearsal.

**Whole-group meeting.** To launch this second Prep Period, we have convened as a whole group, prior to splitting into two groups to rehearse. I begin by asking assembled interns to share some of their experiences experimenting with these adaptations to the game the day before with groups of children. This debriefing provides an opportunity to encourage and normalize the sharing of problematic situations from practice and provides
more information about interns’ experiences that can be used by the teacher educators facilitating rehearsal. On this day interns identify a range of issues. One intern shares that her students are now “getting ahead of [her],” anticipating her routine questions and answering them in advance. Another shares that she thought she was being “mean” in reinforcing participation expectations, but upon watching the video-recording of her teaching noticed that her tone came across as up-beat instead. Yet another intern describes her problem of losing momentum when asking students to listen to each other in order to revoice a classmate’s strategy or offer a different strategy.

Most of these contributions relate in some way to concerns about how students should be expected to participate. As a group we talk briefly about some of the considerations to take into account when these kinds of issues arise and identify some possible strategies for responding in each case. Hala and I contribute a few issues that we observed in interns’ enactments earlier that morning. We also provide related commentary to frame the issues that are shared and guide interns’ thinking. For example, Hala encourages interns to start making distinctions between behavior management problems and situations in which kids are just acting normally:

There’s a level of kids being excited. …That’s normal. …I feel that sometimes we raise the bar too high. …It’s hard for a child, like think now logically, to be sitting like this all the time (modeling a stiff, still posture). Some people were really being too hard on themselves, saying “Oh my kids were really awful.” …There’s a fine line between when it is a behavior management problem and when it is just kids wanting to move.

This is a common theme for our work both this week and next, as interns are learning how to interpret the range of student participation and respond in situationally appropriate, instructionally productive ways. In another example of the framing that we
do during this time, Hala invokes our guiding principle of treating students as sense-makers as one resource in these situations:

    Explaining to children why I am asking you to do this, in relation to a reason, is treating them as sense-makers, not just as people who follow your directions. And relating it to the work of the team and the community so they know what it is they are doing wrong.

Then, I suggest how to end the game on this last day of the activity with a quick review to gather information about what students know now about pairs of numbers that add to ten. I model a version of how to conduct the review, and Hala and I discuss what mathematical terrain could be addressed. For example, we point out that part of doing math includes figuring out if we have exhausted the set of pairs, making conjectures and providing justification for how we know. This could include questioning students about whether or not eight plus two (8+2) and two plus eight (2+8) are the same or different, addressing the commutative property of addition. An intern suggests linking addition facts to associated subtraction facts, something his mentor teacher was working on with students. We brainstorm how subtraction and addition might be related in the activity today. These regular discussions of the mathematical potential of the instructional activity being rehearsed represent one way in which we develop interns understanding of particular mathematical terrain and how concepts are related within it. This collective work also provides an opportunity to develop shared beliefs about what it means to know and do mathematics in ways aligned with ambitious learning goals for students.

    Small-group rehearsals. Hala and I have divided the whole group in two, and I am working with a small group of seven interns. At my direction, the group has gathered around a small table, each one sitting in a child-sized chair or on the floor. Kelly has already rehearsed facilitating students’ transition from whole class activity to the rotation
she is leading, directing them where to sit and reviewing expectations for student participation. During this time I acted as a student who was drawing in her notebook and not listening. After Kelly continued without noticing this behavior, I intervened to draw the group’s attention to this behavior and point out the need to provide students with clear directions for managing their materials. The practice of setting and maintaining one’s expectations for student participation has been an early focus of collective work in rehearsal this week in order for interns to begin to establish instructionally productive relationships with students. This identified focal practice is one structuring resource disciplining interns’ attention and appreciation of key situational elements, contributing to the kinds of situations that get worked on in rehearsal early in the Summer Learning Institute.

The two episodes of work on responding to students’ contributions to mathematical discourse that make up this case study of an early rehearsal are situated within this broader context. In what follows, I move back and forth between narration and analysis for each episode, linking detailed descriptions of episode segments to ideas outlined in the conceptual framework in Chapter 2 in order to make sense of interactions at the level of specific exchanges. I distinguish the analytic portions from sections of description by identifying them with subheadings starting with the phrases “Stepping Back” and “Episode Description” respectively.
Episode 3.1.9:
Working on Responding to a Single Student who has Blurted Out an Answer in the Ten-Frame Game Instructional Activity

Episode 3.1.9 Summary

This first episode (R3.1.9) of work on responding to students’ contributions to mathematical discourse is initiated by a novice who respectfully interrupts the simulation to ask whether or not it is acceptable for students to blurt out an answer at a particular moment in the launch of the activity. The teacher educator in this exchange offers an interpretation of the student behavior and then the group brainstorms possible responses within this proposed framing of the situation.

Episode 3.1.9 Description:
Framing the Problem to Be Managed

The episode is initiated just after Kelly completes recording her beginning score for the game and guiding the group of simulated students in determining her total. Sam interjects to ask a question about a problematic situation he is anticipating at that point in the activity, based on his experience teaching the day before:

1  Sam: I have a question. Sorry. Is it a big deal if they just jump in right there when you’re writing it down and like say “Fifteen!”
2  Jennifer: That’s what they did in my group too.
3  Kelly: Mine too.

Sam’s question is one of how to interpret a particular student contribution in terms of whether or not it is an acceptable form of participation in the mathematical discourse at this point in the activity. He is asking how he should frame, or understand the significance of, this situation in order to determine what would constitute an appropriate teaching response: Is it a big deal or not that a student has blurted out an answer without being called on, and, in either case, how should I respond? Others in the
group express immediate recognition of this issue, voicing publicly that they too faced a similar situation in their teaching of the game on previous days. In response to Sam’s question, I start to make a suggestion for how he could respond, but stop and instead offer a framing of the situation he has posed in terms of a dilemma:

4    Heather: So you could say…(restarting) Here’s where, this is where you want to sort of balance (gesturing with my hands making a back and forth motion). They’re excited to participate. They’re anticipating what question you’re going to ask.

5    Sam: Right.

6    Heather: But you want to give other students think time, so you could say, um… (trailing off and pausing for a few seconds, looking down in apparent concentration)

Here I offer a positive interpretation of the student behavior as evidence that “they’re excited to participate”, while identifying a second teaching consideration of providing other students with time to think. After articulating this framing of the situation, and suggesting that responding would require balancing these twin aims, I pause to think about how to respond appropriately. Trailing off to consider what this framing means for responding in this situation creates space for several interns to jump in with their own suggestions for how a teacher might respond:

7    Jayma: I actually might just say, “Ok, let’s find out.”

8    Laura: “If you think you know, let’s check.” (Simultaneously, Jennifer is suggesting a response that is inaudible.)

9    Heather: (nodding as each intern speaks and repeating pieces) “Let’s check and figure it out.” Yeah. That’s good, because you could also say, “Remember, use your quiet thumb,” but sometimes you don’t want to go there because if kids are engaged and participating, you have to use your judgment about when to remind them of your expectations, (turning back to Sam now) and that may be a case where you don’t want to say “Now, Sam, you need to use your quiet thumb” (several interns chuckle). I like the ideas of, yeah, “Let’s check and see. Let’s all count together and double-check.”

10   Sam: Yeah, and that’s kinda’ how I dealt with it. (Several interns nod in agreement, with some inaudible comments.)
Jayma: (becoming audible) …a lot of places saying, “Hmmm. Let’s think about that.” Like when students say, “I think you want us to win,” “Hmmm, let’s think about that.”

Heather: Or “interesting” when you want to say, “Very good! Great job! Excellent.” “Interesting. Let’s keep going.” (The group chuckles. Then, I direct Kelly, the rehearsing novice, to resume the simulation.)

Stepping Back to Examine Episode 3.1.9 in its entirety

In this episode, an intern requests support in interpreting a particular instructional situation, an early phase in the transactional process of reflection-in-action elaborated by Schon (1983). The situation has been partially simulated by the rehearsing novice. Layering onto the simulated activity, Sam narrates additional details (a student who “jumps in” with an answer) to set the scene, molding the simulated situation to fit the particular circumstances he confronted in his teaching the day before. In doing this, he is communicating to the teacher educator, and the assembled group of interns, the key situational elements that he attended to in his experience. This move represents one way in which participating novices can influence the collective work on responding to students that occurs in embedded rehearsal, contributing to the collaborative emergence of particular problematic instructional situations for collective analysis. In the rehearsal setting, problematic situations can be hybrid constructions of simulation and description because of the alternating structure of rehearsal across phases of simulations and pauses.

The intern, with a limited repertoire of past experiences and underdeveloped conceptual resources on which to draw to interpret the situation, is open to the interpretation of a more knowledgeable other. The teacher educator has an opportunity here to suggest a way of framing the situation as a means for supporting the intern in retrospectively figuring out how to respond. In framing the situation, the teacher educator
sets the problem to be addressed in terms of balancing two potentially competing aims: encouraging and maintaining active student engagement and setting and maintaining expectations for participation that enable equitable opportunities for students to think and contribute. This framing serves as a structuring resource for understanding what is going on that disciplines, or constrains, the set of options for responding in the situation to moves that would yield this balance. The framing of the situation also implies criteria for judging the appropriateness of possible responses in terms of the consequences for students. Interns’ suggestions for how to respond attend to this framing. At the same time, their suggestions involve positioning the blurted out answer as a conjecture to be verified, or “checked,” by the group of students. This kind of response supports the development of classroom norms in which mathematical authority for determining correctness does not reside with the teacher (who avoids evaluating the answer) or with students who are quick to answer and instead shifts mathematical authority to collective negotiation. This implicit link to the broader aims of promoting ways of knowing and doing mathematics in the discipline may have contributed to the teacher educator’s enthusiasm for the interns’ suggestions. A different framing would imply different criteria for judging the appropriateness of these suggestions.

The work that occurs on responding can be characterized as shared problem-solving. Once the problem is framed by the teacher educator, the group engages in coming up with a possible response. The problematic situation is worked on through a collective discussion occurring outside of the simulation, rather than through it being simulated for the rehearsing novice to deal with in the moment. This is one option for working on authentic problems of practice that scaffolds the complexity of
improvisational performance for the rehearsing novice by reducing the set of *in situ* demands. The activity setting of rehearsal provides opportunities for novices to introduce problematic situations without doing it in a way that could overwhelm the person rehearsing. We will see how this work shifts over time, in later rehearsals examined in Case 2 and Case 3, as interns become more capable of dealing with improvisational demands in the role of teacher and the group of interns learns to bring prior experience into simulation by acting authentically as students.

**Episode 3.4.12:**
*Working on Responding to a Single Student Who Has Contributed an Error in the Ten-Frame Game Instructional Activity*

**Episode 3.4.12 Summary**

In this episode, the focal situation is again contributed by a novice, recounting her experience the day before. She does not understand why students are incorrectly answering the question “How many more do you need to make ten?” and asks for advice on how to respond. She begins with a general framing of the situation as one in which students “don’t know the answer, and they’re really struggling.” She has recognized that her repertoire for responding to her students’ incorrect answers is severely limited and offers a suggestion for an alternative response, given her framing, for the teacher educator to evaluate. This episode illustrates how problematic situations drawn from an intern’s prior experience can be publicly replayed in rehearsal, creating opportunities for the teacher educator to support reflection on the transactional process of responding to a student’s contribution for the benefit of all participating novices.
Episode 3.4.12 Description, Part 1: Positioning an Intern’s Experience as a Resource for Public Investigation

Kelly has just finished her turn rehearsing, and I am facilitating the transition to another intern, who is organizing her materials in preparation for taking on the teaching role. As evidenced in the previous episode, the other interns in the group have been actively engaged, not only taking their roles as simulated students seriously but also attending carefully to my feedback, asking questions, and contributing suggestions for possible teaching moves. Taking advantage of the pause in simulation, Laura initiates an exchange with me, asking a question about a problematic situation drawn from her own experience teaching the game the day before:

Laura: I don't know if we can practice this now or anything, but if students don't know the answer, and they're really struggling, you know? I found, I was watching my video, and I found myself keep saying over "How many more do you need to make ten?" and then... Do you just ask, see if they want to ask a friend for help or?

(She trails off. Other interns organize their decks of ten-frame cards while looking up at Laura, heads turning to me as I begin to respond.)

In her question, Laura first expresses uncertainty about how to bring her experience into the public rehearsal setting. The teacher educator is assumed to be responsible for determining how to take up Laura’s contribution. Laura then recounts a situation in which she got stuck, a place where the instructional guidance for the well-specified activity that she had carefully rehearsed was insufficient for knowing how to respond. She characterizes the situation by naming the problem she faced in general terms – “students [who] don’t know the answer, and they’re really struggling.” She continues by describing her response as simply repeating the same prompt of “How many do you need to make ten?” over again, not knowing what else to try. Laura identifies her video recording as a
resource for recalling what occurred and noticing the problem with her limited repertoire. She then suggests a possible response, to “see if they want to ask a friend for help,” and asks the teacher educator to evaluate this idea as a general rule of thumb. We might think of her question this way: “As a rule, when students don’t know the answer, and they’re really struggling, can we see if they want to ask a friend for help.”

In recounting this situation, Laura makes public the sense she has made of her experience, providing information about what she has attended to and how she is framing what happened. She has not offered any details about what her students are struggling with nor provided a rationale for why her students might be having difficulty. I first respond in a way that treats Laura’s suggested response as one reasonable possibility given her general account, taking up her suggestion as a starting point.

2 Heather: Yeah. That would be one strategy. “Pick somebody to help you out on this one, and listen carefully to how they figured it out,” or something like that. You could also ask them to count [the spaces]. If they have no other strategy, counting is a way into getting the answer.

I confirm Laura’s suggestion, naming it as “one strategy” among other possibilities, thereby pushing back on the novice’s search for a general rule. I then model specific language for how one could enact Laura’s strategy, again tagging it with “or something like that” implying that there could be other ways to phrase the directive.

After addressing Laura’s suggestion, I then offer another strategy for how one could respond that is more particular to the activity: directing students to count the empty spaces on their ten-frame card. With this suggestion, I direct interns’ attention to a representational resource in the activity, the ten-frame model. I reason that this kind of response would provide students with a “way into getting the answer” themselves. As
such, the suggestion embodies a commitment to leaving the intellectual work for figuring out a solution to the students, introducing one criteria for evaluating the suggested response in terms of its possible consequences for students.

**Stepping Back to Examine Part 1 of Episode 3.4.12**

We see evidence in this opening exchange of the relationship between the broader design and the kind of work made possible in rehearsal. Because novices teach with the same instructional activity across several Cycles of Enactment and Investigation (in this case this is the third day they will be teaching a version of the game) they obtain relevant teaching experience from which to draw in subsequent preparation. When pauses are built into the structure of rehearsal, in this case slack time created when the teaching role is handed off from one intern to another, novices have opportunities to contribute situations from their own experience as potential foci for the collective work that occurs in rehearsal. Because everyone is teaching the same instructional activity with the same instructional goals to groups of students in the same grade, one intern’s experience is representative of what could happen for the others. It has high relevance for the group and can be utilized for collective preparation. Presumably, the active participation that we see here is also attributable to the immediacy with which interns are going to teach children.

However, interns may not know how to make use of their personal experience for their own and other’s continued development as ambitious teachers. In this episode, the responsibility for this is squarely on the shoulders of the teacher educator. The teacher educator is asked to determine whether and how to take up the novice’s experience and must judge its potential value for the group’s preparation activities. Therefore, what gets
worked on in this episode is an emergent joint accomplishment; the intern provides the focus of the exchange, a problematic situation from her own experience, while the teacher educator exercises professional judgment to determine how to make use of it in rehearsal. In taking up novice-initiated problems of practice, the teacher educator can signal to the assembled interns that such questions are worth pursuing in the public space of rehearsal. Over time, this practice of positioning novices’ experiences as valuable can normalize making teaching problems public and support the development of a shared sense of the work entailed in ambitious teaching.

We also see in this exchange how an intern’s own video record of practice can serve as a valuable resource for him/her to participate in this way in rehearsal. These records, deliberately collected during the enactment phase of the Cycle, enable novices to more accurately recall situations from personal experience and notice potential problems. This form of assistance may be crucial for novices, whose initial performance anxieties may interfere with their memories of classroom events.

As we continue to look in on this episode of work, the teacher educator continues to elicit more of these details from the novice. With her questions, the teacher educator supports the novice in making her experience public and simultaneously guides her in re-appreciating and re-framing the situation.

**Episode 3.4.12 Description, Part 2: Guiding the Re-Appreciation of a Puzzling Situation from a Novice’s Experience**

In the next part of the exchange, I extend the interaction by pressing Laura for more information about the situation through a series of questions, leading her to elaborate her account, re-appreciating details about her “struggling” students’ specific contributions. Based on classroom observations of interns’ teaching the day before and
interns comments during the whole-group debriefing at the beginning of rehearsal, I know that most students by now could quickly determine the number of dots and those missing, often answering before the novice had time to prompt them. Given this context, Laura’s problem of “struggling” students is puzzling to me at that moment. As the exchange continues, the other interns listen attentively, looking back and forth between Laura and me.

2 Heather: How many students like that do you have?

3 (cont.)

Laura: There’s at least two and sometimes three (inaudible).

4 Heather: And are they having trouble with the “How many more do you need?”

5 Laura: Mmm hmm.

6 Heather: Or counting the spaces. But they don't have any trouble figuring out the number of dots?

7 Laura: They haven't been.

8 Heather: That's interesting… (I pause to consider this information. Lindsay puts down her deck of cards and leans in, looking at Laura.)

9 Laura: Or they would, I would say “How many more do you need to make ten?” and they would say, [when there were] four dots on the card, and they would say “One,” and I would say, “Show me how you saw that” like getting to five… I don't know, but they just got more confused. (She shakes her head suggesting that she doesn’t know what to do. Several interns turn and look at me.)

I pose questions that guide Laura’s reflection on her experience, eliciting more information from Laura about her students. The problem of “struggling students” is bounded and refined, first by establishing that only two or three students are “struggling” (turns 2 & 3) and then by establishing that those students only have difficulty answering the particular question of “How many more do you need to make ten?” (turns 4-7). What students are able to do is now part of Laura’s account and is as relevant as information about what students are “struggling” with. Next, I label students’ incorrect responses to the question, as an “interesting” discrepancy (turn 8). The move both marks this
information as important, and, with the pause that follows, creates space for Laura and the group to consider its significance. Simultaneously, it buys me time to consider the situation in light of Laura’s elaborated account.

**Stepping Back to Examine Part 2 of Episode 3.4.12**

Here, the teacher educator is acting as a more experienced practitioner who, unlike the novice in search of a general rule for what to do when students are struggling, is seeking to understand the particulars of the situation in order to craft an appropriate response. By asking a series of questions, the teacher educator guides the novice to articulate and reconsider additional details from her experience, information that the teacher educator understands as important for diagnosing the source of the student error. With the scaffolding provided by these questions, the intern develops a refined account of the situation that takes into consideration additional evidence of what her students seem to be able to do, not just what they are struggling with.

This questioning also serves as a form of meta-cognitive modeling, demonstrating the kinds of questions to ask oneself when confronted with a puzzling situation in which students appear to be having difficulties. The teacher educator’s approach is to look for the reasons underlying the student’s error in order to use judgment, rather than a rule, in determining how to respond. Like in the previous episode, the teacher educator acts in accord with a conception of teaching as entailing judgment regarding what to do given the particulars of the situation. This framing of the work begins when the teacher educator positions the responding move suggested by the novice as one option among possibilities, and continues as she acts on this framing to gather information that would enable a diagnosis of the reasons underlying the students’ struggle.
Episode 3.4.12 Description, Part 3:
Re-framing the Situation, Generating a Hypothesis, and Translating It into a “Testable” Response

Only after Laura has provided an elaborated account that attends more carefully to the details of her students’ contributions, do I suggest a possible reason for the particular student error Laura confronted. The crucial information in making this interpretation comes from her describing the specific example of a student who has four dots and answers that he needs one more when asked how many more dots are needed to make ten.

9  Laura (NT): Or they would, I would say “How many more do you need to make ten?” and they would say, [when there were] four dots on the card, and they would say “One,” and I would say, “Show me how you saw that” like getting to five... I don't know, but they just got more confused. (She shakes her head suggesting that she doesn’t know what to do. Several interns to turn to me.)

10  Heather (TE): I wonder if they're not understanding what it, like (shifting gears), maybe switching to “How many empty spaces are there?”

11  Laura (NT): (nodding that she understands) Ok. I’ll try that.

12  Heather (TE): And try, see if that cues them into what they are supposed to attend to, 'cause [with] “How many more to make ten?” they might not fully understand what it is they're trying to figure out. And so directing them to look at the empty spaces, “How many empty spaces are there?” (Laura nods her head again.) Try it and see what happens.

(Then, the teacher educator shifts her attention back to Jayma, who was preparing to rehearse before Laura asked her original question, and directs the scene “Ok, we’re the students now and we’re wandering around. You’ve got to get us over [here].” The teacher educator and other interns stand up and, acting now as students, start wandering around.)

In her example, Laura now describes utilizing a follow-up elicitation question, “Show me how you saw that.” This question is a variation of “How did you figure that out?” – a
routine for eliciting student’s strategies specified in the participation structure of the game. Laura’s variation of the question makes use of the ten-frame card as a tool for supporting a student in communicating his thinking. She suggests that what the student was doing was “getting to five.” In her original framing of the situation, Laura stated that her only response to students with incorrect answers had been to repeat the same question, “How many more do you need to make ten?” However, as a result of my prompting, she is now recognizing additional aspects of what she tried in the moment to make sense of the student error.

With this new information supplied by the novice, I offer an alternative interpretation of the situation, making sense of the puzzling discrepancy: students “might not fully understand what it is they’re trying to figure out” because they are not understanding the particular question, “How many more to make ten?” Only after Laura has sufficiently elaborated her account to include key details about student performances am I able to make a reasonable conjecture about what to do in response. I position my interpretation as a tentative hypothesis through my use of language like “I wonder,” “maybe,” and “might” (turn 9). Given this interpretation, I suggest responding to the student error by asking a different question that could help students attend to the empty spaces on the ten-frame card, “How many empty spaces are there?” Because Laura and the other novices in the rehearsal are preparing to teach a version of the same activity from the day before, they may have the opportunity to “try it and see what happens.”

**Stepping Back to Examine Part 3 of Episode 3.4.12**

The intern’s enactment of the elicitation routine provides critical information necessary for the teacher educator to exercise professional judgment to interpret the
puzzling student contribution and to suggest an appropriate response. Because Laura
directs the student to “show me how you got that,” she finds out that the student making
the error was “getting to five.” She does not yet recognize the value of this information
that her prompt has yielded about her student’s thinking. However, because she taught
the instructional activity with fidelity, enacting the key components as specified, she has
acted in a way that embodies a commitment to uncovering student reasoning before she
understood her action’s significance in these terms. With the help of a more experienced
practitioner who knows to attend to that information and has the knowledge and
experience to interpret it, the intern and her peers (if they are engaged in the exchange)
have an opportunity to experience the value of the elicitation routine as a resource. It
serves as an enactment tool for gathering information needed to make sense of a student’s
error that in turn informs judgment in crafting an appropriate response.

This is a qualitatively different kind of learning opportunity than having the value
of the routine explained to them in general terms. Instead, the value of the elicitation
routine is experienced first hand as crucial for interpreting and then responding to a
particular authentic demand. Its usefulness is discovered through its use. This kind of
opportunity is made possible when interns are able to repeatedly and deliberately practice
the routine components of specified instructional activities, so that they can enact them
with sufficient fidelity. This deliberate practice is afforded through repeated Cycles of
Enactment and Investigation with the same IA across multiple days and the co-
participation of novices and a teacher educator acting as a more experienced reflective
practitioner in the rehearsal setting.
To a casual observer, the episode could be superficially interpreted as one in which the teacher educator merely recommends a course of action in response to the intern’s question. After all, the teacher educator does ultimately suggest an alternative question for the novice to ask. This is in contrast with the previous episode in which the teacher educator opens the floor for intern suggestions. However, when carefully observed through the lens of the conceptual framing presented in Chapter 2, we see that there is much more going on in this brief encounter. And, the nature of the work is fundamentally different from what would be likely in a similar exchange outside of the rehearsal setting. The teacher educator intervenes to support a novice in narrating her experience and guiding her through a public process of re-appreciating and re-framing the situation to retrospectively make sense of the student error to which the novice did not know how to respond. This re-interpretation of the situation enables the teacher educator to generate an actionable hypothesis that interns may test when they go teach the same activity to children immediately following rehearsal. Although the exchange is primarily between the teacher educator and a single intern, because it occurs within the public space of rehearsal, it can serve as a model of the approach to problematic situations characteristic of an experienced reflective practitioner, drawing on a range of structuring resources.

**Case Summary and Discussion**

The data explored in this case are evidence that, by the third day of the Summer Learning Institute, novices are motivated to make public problematic situations from their own teaching experience in which they were unsure how to respond to particular student contributions to mathematical discourse. We see that daily video records of their teaching
and simulations recreating similar experience serve as resources that enable interns to remember practice situations and render the details of their experience with some accuracy. The case demonstrates that novices have opportunities to initiate this work during pauses built into rehearsal, pauses that enable the kind of “re-visioning” routines identified by Horn (2010) in the activity of teacher communities in which professional learning occurs. Such pauses provide space for weaving together investigations of previous teaching experiences with preparation for immediate enactment with children. The novice’s previous experience can be utilized as a resource for collective preparation, because interns are preparing to immediately teach a version of the same instructional activity to the same children. For this reason, other interns are likely to be engaged in the exchange. The teacher educator facilitating rehearsal uses her judgment to determine how to make use of novices’ experiences for this collective work.

Although the instructional activity was well-specified, the analysis here suggests that the routine questions built into its participation structure were not enough for novices to be prepared to respond to students who were blurtting out answers or struggling to understand what was being asked of them. When the practice of responding is understood as involving a transactional process, we recognize that interns must be able to appreciate key situational elements and interpret that information to frame each problematic situation. However, the act of appropriately framing a practice situation requires a repertoire of experiences. As Schon (1983) argued, past experience is a key resource enabling a reflective practitioner to frame a situation, drawing on a repertoire of experience to identify a familiar situation “as an exemplar for the unfamiliar one.” For a novice to ambitious teaching, the only accessible repertoire comes from his/her own
idiosyncratic apprenticeship of observation unless a more experienced teacher, with a history of participation in ambitious practice, is available to guide interpretation. When a teacher educator can act as a more experienced practitioner of ambitious teaching in the rehearsal context, she has the opportunity to guide novices’ framing and interpretation of instructional situations. The interpretation then must be translated into an actionable response that the novices can try out if they confront a similar situation in their teaching immediately following rehearsal. As mentioned in the introduction, in Schon’s (1983) terms, “the hypothesis must lend itself to embodiment in a move” (p. 152).

This process was highly scaffolded in the second episode examined in this case. The teacher educator supported the retrospective transactional process – asking questions that guided the novice in re-appreciating what happened, marking particular situational features as significant, and using her professional judgment to craft an appropriate response to the particular student error. In this way the teacher educator structured an ambitious approach for responding to student errors, one that emphasizes the importance of surfacing and interpreting information about student reasoning. This approach is possible for the teacher educator who has the repertoire, expertise, and professional commitment to treating students as sense-makers that disciplines and enables interpretation of non-routine information and provides standards for evaluating consequences of a course of action on students’ opportunities to learn. Whereas the novice, who has not yet appropriated these resources, originally approached the puzzling situation by attempting to frame it with the general category of responding to “struggling students” with incorrect answers. She had framed the situation a routine problem in which a standard operating procedure might be applied (Weick & McDaniel, 1989). The
teacher educator publicly supports the re-framing of the situation as one requiring judgment. She makes public the set of considerations and information to which she is attending, modeling an approach to making sense of student contributions and making available the set of structuring resources on which she is drawing to discipline her judgment.
CHAPTER 6

CASE STUDY 2: REHEARSING THE TEACHING OF A QUICK IMAGES ACTIVITY

Introduction

As novices work to elicit student participation in mathematical discourse, they are confronted with, and are in part the creators of, problems and opportunities associated with responding to students’ contributions. While responding to children’s mathematical contributions individually is demanding work for a beginner, as we saw in the previous case, those demands are compounded in situations that involve surfacing multiple student solutions and strategies. To move toward managing the kind of complex situations that typify ambitious instructional interactions in whole-class settings, interns need repeated opportunities to practice responding in increasingly complex situations, in which they first are able to manage with assistance and then transition to being able to manage on their own. To cultivate the kind of adaptive competence necessary for engaging in responsive teaching, this repeated practice should entail opportunities to experiment with variations across similar situations, to revise and refine performance in response to principled feedback, and to develop a flexible repertoire of strategies grounded in conceptual understandings of their instructional significance for student learning and participation.
The next case, drawn from the first rehearsal in the second week\(^{22}\) of the Summer Learning Institute, represents the kind of deliberate, repeated practice possible in embedded rehearsals. It comprises three related episodes of interactive work on the practice of responding to students’ contributions to mathematical discourse (see Figure 6.1 below).

Figure 6.1: *Focal Episodes for Case 2 as Situated within Rehearsal 4.*

Examining this set of episodes makes it possible to capture some of the patterning across exchanges that contributes to experiencing a whole rehearsal as coherent – in a way that looking at episodes in isolation does not. In each of the three episodes, I act as a student to deliberately create a situation in which the novice rehearsing in the teacher role must manage two different student solutions.

This case, therefore, illustrates a second crucial role for the teacher educator in rehearsal in addition to serving as a more experienced practitioner – that of acting as a student in simulation. By acting as a student, the teacher educator is able to purposefully “serve up” situational demands that must be managed by the rehearsing novice in the midst of his/her performance. In this case, for example, I was able to repeatedly focus work on the problem of how to manage two student solutions. This means that the teacher educator can deliberately target particular practice terrain in rehearsal. When I act as a

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\(^{22}\) This is the fourth rehearsal in the data set that includes 14 total rehearsals. The three episodes represent the 17\(^{th}\), 19\(^{th}\), and 21\(^{st}\) episodes in the data set that includes 74 total episodes. In this particular rehearsal, there are 9 coded episodes of work on the practice of responding to student contributions to mathematical discourse. See Chapter 3 for more information about, and a visual representation of, the data set in its entirety.
student to repeatedly simulate similar kinds of situations, collective work includes opportunities to gradually refine a shared approach to address multiple considerations across a progression of work rather than all at once. In the role of student, a teacher educator is also able to act out the potential consequences of an intern’s experimental attempt at managing a given situation. Schon (1983) referred to this type of feedback from the materials of the situation as “backtalk,” conceptualizing the reflective practitioner in a conversation of sorts with the situation. This conversation is an ongoing, goal-oriented process of interpreting, acting to experimentally test hypotheses, observing the consequences of one’s own experimental actions, evaluating performance in relation to those consequences, and adjusting accordingly to stay in motion to achieve one’s goals. When students’ actions and reactions are simulated in instructional exchanges, interns have opportunities to engage in this full transactional process.

The case also illustrates a particular interaction pattern in rehearsal, the action replay, which provides rehearsing interns with opportunities to revise and refine their experimental performances in light of responsive feedback. Unlike in the previous case in which I assisted novices’ reflection on their performances from the day before, the action replay enables the teacher educator to responsively assist a novice in the midst of performance. This means that the teacher educator, in the rehearsal setting, has opportunities to support novices as they are engaging in the transactional process characterizing the work of improvisationally responding to students. This support, in the form of responsive feedback, can provide guidance to the rehearsing novice regarding what considerations should come into play to discipline judgment and what else needs to be accomplished. Teacher educator feedback can also serve as a form of cognitive
structuring for all participants. Cognitive structures organize how learners make sense of the world. They are linguistic resources for interpreting, categorizing, and sequencing experience (Tharp & Gallimore, 1988). Pauses in the flow of simulation provide opportunities to link simulated instruction with conceptual resources for developing an understanding of the purposes underlying the work. When interns develop an understanding of these purposes in tandem with a repertoire of skills, they can begin to adaptively respond in similar situations. We will see evidence of interns improvising minor variations on a theme across the set of episodes examined for this case.

This case also represents an intermediate stage in a deliberate progression of work on responsive practice. By moving to a different instructional activity, quick images, our work shifts from a focus on responding to individual student contributions one at a time in the context of the game to responding to and managing a pair of different student solutions. The practice, and challenge, of responding ambitiously to student contributions to mathematical discourse is preserved as a central focus for interns’ learning and development. However, teaching this new instructional activity brings additional layers of complications and demands for interns to manage. By maintaining a focus on the work of responding to students’ contributions to mathematical discourse over time across different instructional activities, opportunities to develop more nuanced understandings of the work are made available as these new complications arise and are grappled with in the public space of rehearsal.

We will see in this case how the participation structure of quick images contributes to the repeated opportunities to work authentically on the same kind of problematic situation involving a student error of misperception. The participation
structure of the quick images activity serves as a valuable resource for both interns and the teacher educator working together to simulate an ambitious approach to responding to two student solutions. Expectations for how students are to engage with each other and mathematical ideas as well as how the teacher is to facilitate that engagement are built into the activity. The teacher, for example, is expected to lead the group of children in checking the set of proposed solutions and providing students with opportunities to revise their answers. This pre-specified approach provides the teacher educator with a framework for evaluating an intern’s performance and determining when to intervene.

Another affordance of the quick images activity structure is its repetitiveness. Each time a card is flashed, a routine instructional sequence ensues. This enables repeated opportunities to work on the same kinds of situations over and over, repeatedly practicing the pre-specified approach to responding to multiple answers that leaves the work of determining correctness to students and provides opportunities for students to revise with the teacher responsively assisting these practices. In this way, the activity is particularly high-leverage for establishing routine components of practice and for experimenting with variations on a theme across similar, but distinct, situations.

With an understanding of the design of the quick images activity and its aims as background (see Chapter 4), let us examine the work that occurs on the first day interns are preparing to teach the activity to children. Three episodes comprise this case. In the first two episodes, the rehearsing intern is confronted with two different student answers as a result of the teacher educator deliberately acting as a student who has perceived an incorrect number of dots on the flashcard. Each time he has an opportunity to experimentally respond and each time the teacher educator pauses simulation to provide
public feedback on his improvisational performance. By rewinding the action, the intern has an opportunity to put that feedback to immediate use and refine his approach to the situation. In the third episode, a second intern confronts the same kind of situation, and we see what she has taken up from the first intern’s public rehearsal. In the following section, I situate these episodes in their embedded levels of context prior to moving into their description and analysis.

Embedding the Fourth Rehearsal in the Design Context and Second Week of the Summer Learning Institute

Situating the Rehearsal within a Trajectory of Repeated Cycles

It is Monday of the second week of the Summer Learning Institute, July 11th. Although we are preparing to teach the quick images activity for the first time, interns are getting accustomed to participating in public rehearsals by this point in the summer and have some familiarity with the activity. Hala Ghousseini, the co-instructor for the course, and I have facilitated rehearsals of a version of quick images a few times with small groups of interns in the afternoon methods class. Additionally, because we work with each instructional activity across each stage of the Cycle (refer back to Chapter 2), interns have participated as students in a quick images activity facilitated by me at the beginning of the semester in the university setting. This early experience served as an object for collective reflection in which interns analyzed both their own participation as students and what I did as the teacher to facilitate that participation. They have worked on understanding the centrality of place value in operating in a base-ten number system. They have read about and discussed the value of student-invented strategies and the importance of developing students’ relational understanding of basic facts. With this
work as background, today’s rehearsal provides interns with an opportunity to put to use, to continue to develop, and to experience the instructional relevance of these mathematical concepts and ideas about student learning.

**Situating the Rehearsal within the Second Prep Period of the Morning**

As described in Chapter 3 (see Table 3.1 for a summary of the interns’ daily schedule), the two teacher educators, myself and Dr. Ghousseini, work together to lead two Prep Periods each morning at the Summer Learning Institute. The rehearsal analyzed for this case is drawn from the second Prep Period of the day. Therefore, we have already facilitated one complete Prep Period, including a whole-group meeting and small-group rehearsals in which we divide the group in half to rehearse separately. We have also observed the first group of interns teaching children and debriefed our observations immediately prior to this second Prep Period to inform our work with this new group of interns.

**Whole-group meeting.** We begin this portion of the Prep Period by eliciting interns’ participation in articulating the mathematical content and practice goals for today’s quick images activity. This work to identify goals, which has become a regular component of whole-group meeting time, represents a structural innovation that arose in response to observations of interns in previous years. We recognized that interns who understood and took ownership for student learning goals were more prepared to respond adaptively to students in ways that targeted key mathematical concepts and practices. This routine also represents one way in which we, as a community, can negotiate a shared understanding of what is involved in knowing and doing math. Several interns contribute, and we offer support by paraphrasing key ideas and naming a few additional
considerations. Then, we transition to alternately explaining and modeling the phases and key details of the activity structure of quick images in relation to these goals for children.

After this quick run-through of the activity, I explain to the assembled interns how we will continue to work on the focal practice of establishing and maintaining expectations this week:

This week we’re going to work on our judgment about when to reinforce and maintain our expectations and when to let kids participate as kids and not feel like, all the time, we have to make them sit up straight and every single time remember their quiet thumb. So, in order to make judgments, to maintain that right balance of letting kids be kids and participate enthusiastically and maintaining our expectations, we have to make judgments based on good reasons. Ok?"

Prior to the rehearsal, Hala and I had identified this as a concern based on our observations of interns’ teaching in the first week. For example, in the previous case, Sam’s question regarding how to interpret a situation in which a student blurts out an answer, is representative of interns’ difficulty in determining when to reinforce expectations and when to “let kids be kids.”

To begin this work, we then act out a couple of quick scenes, playing students sitting next to each other. In the first scenario, Hala deliberately bumps my chair and in the second scenario, she bounces up and down in her chair, excited to see the next flashcard. After each scene, we engage the group in considering how a teacher might respond and why. Finally, we field a few clarification questions about the general sequence of the quick images activity and divide the group in half to rehearse.

Small-group rehearsals. After gathering six interns to join my rehearsal group, I direct Sam to take on the teaching role first. I will be following Sam and his teaching partner, Jennifer, into their classroom today to observe and video-record their teaching.
Sam stands facing me and his peers. We are acting as students and sitting in chairs arranged in two rows facing him and the white board. On the white board to Sam’s left, the poster is taped where he will record symbolically the number sentences represented by the dots arranged in the ten-frame pairs on each card.

He has just rehearsed transitioning students into the activity, reminding students of expectations for participation, and facilitating student engagement with the first flash card. I had paused the rehearsal briefly in the midst of Sam’s work with the first card to review the order of elicitation questions to be posed with each card, establishing this order as something to be held constant.

The three episodes of work on responding to students’ contributions to mathematical discourse that make up this case study of a rehearsal in the second week of the SLI are situated within this broader context. In what follows after providing a short summary, I move back and forth between narration and analysis for each episode, linking detailed descriptions (including transcripts) of episode segments to ideas outlined in the conceptual framework in Chapter 2 in order to make sense of the interactions at the level
of specific exchanges. I distinguish the analytic portions from sections of description by identifying them with subheadings starting with the phrases “Stepping Back” and “Episode Description” respectively.

**Episode 4.2.17:**
**Working on Responding to Two Different Student Solutions in the Quick Images Activity for the First Time**

**Episode 4.2.17 Summary**

In this first episode of three, the intern rehearsing in the teaching role, Sam, is confronted with two different student answers when he engages in the practice of eliciting multiple student solutions using the question “Did anyone get a different answer?” as a resource. The teacher educator, myself (Heather in the transcripts), deliberately creates this situation by acting as a student in the simulation who has perceived an incorrect number of dots on the flashcard and shares her incorrect total while another intern also acting as a student, Jennifer, shares the correct total. As a result, Sam has an opportunity to experimentally respond, and in doing so neglects to provide students with an opportunity to collectively determine which answer is correct. The teacher educator, perceiving this problem in his response, pauses the simulation. She provides public feedback on his improvisational performance and directs him to try again. By rewinding the action, Sam has an opportunity to put that feedback to immediate use and refine his approach to responding to two student solutions.
Episode 4.2.17 Description, Part 1: “Acting the Student” to Deliberately Simulate a Problematic/Promising Situation

When Sam moves on to the second flashcard, we shift from a focus on the elicitation question sequence to working on how to respond when eliciting student thinking results in two different answers. The next card in the sequence is this:

Sam flashes it quickly, conceals it, and then gathers student ideas about how many total dots were on the card.

Sam: Quiet thumb if you think you know. Heather?
Heather: Fourteen.
Sam: Heather saw fourteen. Did anyone see anything different? (pausing and looking around at the group of students). Jennifer?
Jennifer: I saw fifteen.
Sam: Fifteen. Did anyone see anything else? (pausing again briefly and looking around). And how did you see fourteen, Heather?
Heather: I saw a full ten-frame and then I think I saw four, and ten plus four is fourteen.

The type of student error that I am simulating here is one that students make frequently in this activity since the cards are purposefully flashed quickly to encourage conceptual subitizing. It is an error of misperception. I thought that I saw ten and four but correctly calculated the total for this pair of numbers. Students who make this type of error may not be utilizing the row of five on the right to subitize and see the quantity quickly and may be trying to count dots individually.

Stepping Back to Examine Part 1 of Episode 4.2.17

As a result of asking the question, “Did anyone see anything different?” the rehearsing intern is now confronted with a simulated situation in which he must respond to a pair of student solutions – one that is correct and one that is incorrect. Enacting the
instructional activity as specified, he has gotten himself into a problem space particular to ambitious teaching. He must interpret the error (deliberately simulated by the teacher educator) in a manner that assumes the student is making sense and consider possible reasons underlying the error. In the terms of the practice-oriented curriculum, he must apply the principle of treating the student as a sense-maker. Does it suggest that the student holds a mathematical misconception or is not recognizing how to make use of the structure of the ten-frame to see a full row as five? Perhaps, the novice has just flashed the card too fast. His knowledge of the mathematics involved in conceptual subitizing may inform how he interprets the student error in this context.

How he interprets the error will inform how he responds. Because this is occurring in the context of rehearsal, the novice has an opportunity to try something, testing his interpretation by acting on it and seeing what follows. His response, made public in this setting, can then become an object of collective focus. The teacher educator must continuously exercise professional judgment in determining how to respond based on her on-going assessment of the situation in relation to her goals for both the rehearsing novice and the group. The aspects of practice that get worked on are contingent upon how the rehearsing novice responds to the simulated situation in the teaching role. What he does, or does not do, will influence if and how the teacher educator intervenes and which aspects of practice will become a focus for public reflection. For example, does his response suggest that he is indeed treating the student as a sense-maker, looking for the reasons behind the error? In this way, the work of responding to student contributions in the rehearsal setting can be both responsive to interns’ own experimental performances and attentive to principles of ambitious teaching.
Also significant here is that by deliberately acting as a student, the teacher educator is able to simulate a situation that demands not only interpreting a student’s mathematical contribution, but also experimentally acting on that interpretation within an authentic instructional situation. When interpreting a student error occurs in the midst of simulation, the rehearsing intern must make sense of the error using elicitation questions to gather information about what the student is thinking while simultaneously figuring out on the spot how to respond and manage other student participation. He acts experimentally, testing his initial interpretation of the situation by responding and then paying attention to how simulated students in turn react to what he has tried. Obviously, the analytic skills cultivated in a separate error analysis activity such as analyzing student written work, while relevant, are insufficient for knowing what to do in such a complex instructional situation. Here, inside of simulated instructional interactions, Sam’s interpretation and analysis must be translated into action. In rehearsal, the intern in the role of teacher has opportunities to engage in the transactional process of disciplined improvisation in its entirety. And, because this performance is public and situated in a setting where pausing his performance is possible, there are opportunities to intervene and collectively investigate the work of teaching at each phase in the process.

**Episode 4.2.17 Description, Part 2:**
*Identifying an Intern’s Problematic Response to Students’ Contributions in Relation to a Conceptual Framework for Ambitious Teaching*

Let us return to the description of the episode and see how Sam, in the teaching role, responds to the two student solutions.

Sam: Ok. And Jennifer, you saw fifteen. How did you see fifteen?
Jennifer: Um, I saw it the same way Heather did, but I saw a full top row, and that’s how I got my answer.
Sam: And a full top row is?
Jennifer: Five.
Sam: And you know that…(prompting her to continue)?
Jennifer: Oh, five and ten is fifteen.
Sam: So let’s see.

He holds up the card again at chest level where the group of students seated in front of him can look at it for a brief moment.

Then, he moves toward the poster, placing the card next to the line reading “10 + 5 = ___”, moving to the next phase of the activity that involves “mapping” between the visual representation on the card and the symbolic representation of 10 + 5 = 15 on the poster. I pause the rehearsal here, recognizing this point as an opportunity to assist a more ambitious form of responding to two solutions.

**Stepping Back to Examine Part 2 of Episode 4.2.17**

By this point in the summer most interns are deliberately trying to avoid automatically evaluating student solutions or explaining the correct answer themselves. I have observed them in their daily teaching begin to self-correct when they slip into using comments like “Great!” in response to student contributions. They might even explain, if asked, that they are attempting to “leave the intellectual work to the students.” I see evidence of this in Sam’s response. He has initially avoided signaling which student is correct, providing both Jennifer and me (who are acting as students) with opportunities to explain how we arrived at our totals. However, he has not provided the group of students with an opportunity to collectively determine which answer is correct and has missed an opportunity to reinforce revising as a classroom norm. By moving on to representing the

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23 This phrase is one way that we, as a community in the SLI, talk about maintaining the cognitive demand on a student by holding them accountable for producing mathematical methods and reasons.
correct addition sentence on the poster, Sam is in effect assuming the responsibility for determining correctness by not providing students with the opportunity to publicly come to a consensus drawing on the logic and structure of mathematics. I recognize this missed opportunity, in large part, because Hala and I purposefully elaborated the participation structure of the activity to entail this mode of student participation. It is an aspect of how we believe students should actively engage with each other in the activity.

Sam may also assume that once a correct strategy has been shared, other students will realize their mistake. This is a pervasive expectation among our interns. This assumption is instructionally problematic for several reasons. Interns are prevented from uncovering the sources of student misconceptions and errors, because they avoid eliciting the reasoning behind those errors. An important opportunity to gain valuable information about student thinking is missed. In turn, students also miss the opportunity to articulate reasoning underlying an error or misconception and making sense of it. They are denied the opportunity to revise their thinking, and the intern misses a chance to reinforce revising as a classroom norm. Over time, students who arrive at correct answers quickly are likely to get positioned as the ones with mathematical authority. This in turn has the potential to exacerbate differential student participation. I am on the look out for evidence of this pattern of responding at this early point in the SLI, having learned through experience that our interns generally need repeated opportunities to develop new habits of response to combat this persistent tendency.
To provide Sam with an opportunity to practice leaving the work of determining correctness to the students, I pause rehearsal at this point to offer feedback on what he has done and what I would like him to try. I begin with positive evaluative feedback. This includes identifying specifically the work he has done well, to provide two students with opportunities to explain their thinking, and providing a rationale for why it is commendable in terms of targeting his instructional goals. Then, I provide directive feedback, explaining what needs to happen next in order to provide students with the opportunity to figure out for themselves which of the two answers is correct.

Heather: Ok, freeze just for a second. That was nice. You let me explain how I saw fourteen. You let Jennifer explain how she saw fifteen. We’re both working on [adding] ten and some number. And we both, based on what we saw, got it right. And so now, that was a nice way to let us work on the instructional goal [unintelligible]. And now you’re going to say, “Ok, we’ve got two different answers. Let’s check.” So now we have, as a group, to figure out “Was is fourteen or was it fifteen?” Does that make sense?

Sam continues the exchange by asking a clarification question, “Ok, before we even go here?” gesturing toward the poster. This provides me with an opportunity to further elaborate a rationale for my feedback: “Before you go there, because you want to give me a chance to revise.” I continue by explaining to the group that we want to establish revising as a norm and suggest that Sam might say something like “Let’s check it. Heather, what do you see now?”

Stepping Back to Examine Part 3 of Episode 4.2.17

In this exchange, we see how the teacher educator can articulate core components of an ambitious response along with a rationale for why those components matter by
providing two kinds of feedback, evaluative and directive, in coordination. Together these serve to outline performance standards for the work of responding when two student solutions are on the table. These standards for what to do have been explicitly linked by the teacher educator to core commitments, or why to do it. These commitments include leaving the intellectual work of explanation to individual students (e.g. “You let me explain…You let Jennifer explain.”), deliberately targeting mathematical goals (“…that was a nice way to let us work on the instructional goal”), and facilitating collective sense-making to determine correctness (“So now we have, as a group, to figure out…”). When made public, these principled performance standards are also now available for collective negotiation. I attempt to make space for negotiation by ending with the question, “Does that make sense?” simultaneously posed to Sam and the other interns participating in rehearsal.

Although I am responding to Sam’s performance, I am not inventing these performance standards that guide my feedback on the spot. My feedback is disciplined by both the structure of the quick images activity and our guiding principles. Expectations for how students are to participate in the activity and what the teacher is expected to do to facilitate that participation are built into its structure. For example, we have specified that the teacher engage students in checking answers by returning to the card to collectively make sense of strategies shared by students. These specifications are designed to instantiate our principles. When I commend Sam for providing students with opportunities to explain their thinking and suggest that he now facilitate the group in determining “Was it fourteen or was it fifteen?” I am attending to our commitment to position students as sense-makers. The teacher educator’s interventions must be
disciplined by professional knowledge and commitments regarding the kind of teaching understood to promote ambitious learning goals for students rather than based on personal idiosyncratic beliefs or style preferences. The activity structure, a product of collective negotiation among a team of teacher educators, supports the teacher educator in making determinations about when to intervene to reinforce non-negotiable aspects of the activity’s participation structure and when to let other things slide. The activity structure, as an artifact of this negotiation process of professional standards, can also serve as an object for continued professional reflection and development.

This is important because there are trade-offs between pausing simulation and maintaining the press on the rehearsing novice to manage to stay in motion. If the action is paused too often, interns will not get to experience the authentic pressures and pace of instruction. They may not have sufficient opportunity to practice thinking on their feet. At the same time, we know from research on the effects of feedback (and I experienced in my first year facilitating rehearsals) that providing too much information can hinder performance rather than assist it (see for example Kanfer & Ackerman, 1989). Excessive feedback has the potential to overwhelm and could undermine an intern’s sense that ambitious teaching is do-able. For these reasons, a teacher educator’s interventions must be well considered.

However, pausing the simulation provides the teacher educator with valuable opportunities to responsively guide a novice in the midst of performance, enabling the novice to maintain an ambitious instructional trajectory that he could not sustain on his own. I am able to position his work as an object of collective reflection and introduce performance standards that can guide his attempt to approximate ambitious teaching
moving forward. If this feedback were withheld until the end, Sam would not have the opportunity to put it to use immediately. I would not have the opportunity to observe how he is interpreting my feedback and translating it into an actionable response. Furthermore, if my objective is for Sam to practice, and for other interns to experience and investigate, an ambitious form of teaching, I must support him in maintaining an ambitious trajectory as he continues. Otherwise, his rehearsing could reinforce bad habits and serve as mis-educative experience. He can now rewind the action and try putting my directive feedback, to facilitate the collective checking of two solutions, to immediate use in rehearsal.

**Episode 4.2.17 Description, Part 4: The Action Replay: Rewinding to Revise and Refine Performance**

Let’s return to the rehearsal to see how Sam takes up this feedback as he steps back into the teaching role:

Sam: Ok, so here is the card (showing the card again so that all students can see it). Let’s check.

Heather: Good.

Sam: Ok, here we go. There’s a full ten-frame here (circling the ten-frame on the left with his index finger). And a full ten-frame is how many?

Heather: Ten.

Sam: Ten. And another ten-frame with five dots (then revising his language in a way that avoids giving away how many dots are in the second ten-frame), half full is how many, Heather?

Heather: Oh, I thought I saw four before. But now that I’m looking at it, I agree with Jennifer. There’s five there.

Sam: And I did flash the card pretty fast, maybe a little too fast and you thought you saw four.
In rewinding the action to try an alternate way of responding, Sam does several things differently. He begins by employing the “Let’s check” language that I had suggested. However, he must then improvise a way to facilitate this collective checking, something that I did not specify in my feedback. To do this, Sam asks a series of questions that draws attention to the five and ten structures of the ten-frames. His self-correction from the language of “five dots” to “half full” suggests that he is doing this work mindfully. Directing me (acting as a student) to attend to the full row and noting that it is a “half full” frame is particularly responsive given the nature of my error. We want students to be able to use the five-structure of a full row to see quantities quickly, and Sam is directly targeting that mathematical terrain. His guiding questions result in me (acting as a student) revising my answer to agree with Jennifer. In response, Sam continues to improvise, sharing the responsibility for my misperception by commenting, “And I did flash the card pretty fast, maybe a little too fast.”

**Stepping Back to Examine Part 4 of Episode 4.2.17**

The action replay routine, described above, represents an important way in which structuring resources for disciplining improvisational performance can be introduced, investigated, and applied by a community of learners in rehearsal. When a rehearsing novice confronts simulated situations publicly, in the midst of joint activity, other members within his community of practice can assist, drawing on the structuring resources available within the community. Generally, at first this assistance is provided by the teacher educator who serves as a more experienced ambitious practitioner, who makes sense of novices’ early performances through a particular conceptual framework. Through her feedback, she can bring professional commitments and guiding principles to
bear on particular problems. Over time, through this interaction in and around the work, a
novice has opportunities to internalize the practices of the community while contributing
to the continued development of those practices. As a novice gains experience in such a
supportive context, he/she learns what information is worthy of attention, develops a
perspective with which to interpret novel situations, shifts from a detached analytic
stance to one of deep emotional involvement, and begins to take responsibility for his/her
own successes and failures (Dreyfus & Dreyfus, 1986). These changes signal a novice’s
trajectory toward developing reflective practice and adaptive competence.

What Sam has done is not perfect. One could argue that his questions were a bit
leading or point out that he could have done more to engage other students. However, I
understand his performance as promising, a reasonable approximation of ambitious
teaching for an intern preparing to teach for only the fourth time during the SLI. We will
have other opportunities within the same rehearsal and across the week to continue to
refine an ambitious form of responding to student contributions in the context of the
quick images activity. Because rehearsal is a space in which particular instructional
situations can be deliberately simulated, the teacher educator can create repeated
opportunities to work on the same thing within a single rehearsal by acting as a student.
The structure of the quick images instructional activity supports this repeated practice,
because each time the intern flashes a card, a new but similar instructional situation can
be simulated. This enables the teacher educator to repeatedly pose the same problem of
practice for the novice to work on – namely responding to two different answers. To see
an example, we will skip ahead to Episode R4.4.19.
Episode 4.4.19:
Working on Responding to Two Different Student Solutions
in the Quick Images Activity for the Second Time

Episode 4.4.19 Summary

In this second episode of work, I again act as a student with an incorrect total, creating a second opportunity for Sam to practice responding to two student solutions. Sam demonstrates what he has taken up from my previous feedback, incorporating an improvised variation on the group checking routine while also adaptively refining his approach to the situation including a few of his own small innovations.

Episode 4.4.19 Description, Part 1:
“Acting the Student” to Recreate a Similar Problematic/Promising Situation for the Same Novice in the Teaching Role

Sam has just flashed the fourth card (10 + 6) and is in the process of eliciting a set of student solutions, smoothly employing the elicitation question sequence built into the activity structure:

Sam: Quiet thumb if you have an answer (looking around). Rick?
Rick: Sixteen.
Sam: You saw sixteen. Did anyone see anything different? Heather?
Heather: (raising my thumb in front of my chest conspicuously)
Seventeen!
Sam: Seventeen. Ok, any other answers? (looking around). Rick, how did you see that, sixteen?
Rick: I saw a full ten-frame and then the top row of the other one was full and there was one down on the bottom so I knew that was six, and ten and six is sixteen.
Sam: And Heather, you saw seventeen. How did you see seventeen?
Heather: I saw a full ten and then on the other side I saw a bunch of dots, and I knew it was more than five ‘cause there were some on the bottom, but I wasn’t sure. I thought it was seven, and ten and seven is seventeen.
As in the previous episode, as a result of asking for different answers, Sam must again manage a situation involving two student solutions. Again, he provides both students with an opportunity to explain how they arrived at their total, a component of the work of responding that I identified and reinforced with my earlier evaluative feedback.

Sam: Ok. Well, let’s see. Here’s the card that was flashed (holding the card at chest level so that all students can see). Can everyone see it? (looking around) Ok, Rick, can you walk us through? You said you saw ten (circling the full ten-frame with his finger).

Rick: I saw the full ten-frame on that one and so I knew that was ten and then the full top on the other one so I knew that’s five and then I saw one down on the bottom. So I knew five and one is six and then ten and six is sixteen. (Sam underlines the row of five with his finger and then points to Rick’s explanation.)

Sam: Heather, do you see that, how there are sixteen?
Heather: Yeah.
Sam: Not seventeen?
Heather: I see now.
Sam: So, but your strategy was correct, if there was another dot here, you were absolutely correct. I was flashing them fast.

**Stepping Back to Examine Part 1 of Episode 4.4.19**

What Sam has done here indicates what he has taken away from his interaction with the teacher educator in the previous episode. Recreating a similar instructional situation for an intern to confront provides an opportunity for the teacher educator to conduct an informal performance assessment, providing information about how the intern interpreted prior feedback and what if anything he has taken from that experience. If the rehearsing intern enacts the teacher educator’s suggestions, his actions also provide a
publicly embodied translation, or model, of the performance standards that were articulated in the previous episode.

Sam is applying feedback from earlier but not as an inflexible script, enacting the core components of an ambitious pattern of managing two student solutions. He again facilitates group checking, using different language from “Let’s check” but toward the same purpose when he says, “Well, let’s see. Here is the card….” He is arguably doing more to attend to other students this time – asking the group if everyone can see the card and using inclusive language (“Rick, can you walk us through?”). This time, rather than asking me (acting as a student) questions about what is shown on the card, Sam directs Rick to walk through his strategy again with the card visible to the group. As Rick explains how he arrived at sixteen, Sam represents Rick’s strategy on the card using precise hand gestures, supporting other students to make sense of Rick’s strategy. Then, he checks back in with me, to see if I am ready to revise.

Sam’s performance here has incorporated central components of an ambitious approach to responding: eliciting a set of student solutions and the student thinking behind each solution, and facilitating the group in collectively checking solutions to come to a consensus. He has also refined his management of two solutions, doing more to engage broader participation.

**Episode 4.4.19 Description, Part 2: Attending to Additional Considerations to Further Refine Performance**

When I intervene next, as this episode continues, I focus my feedback on ways Sam might further refine his performance, moving beyond the central sequence to layering on moves that could reinforce the norm of revising:
Heather: *(stepping out of the student role to make a suggestion)* You could even commend me for, “In math, when we, when we have good reason to, we revise our answer. Heather just revised her answer when she got to look at the card carefully.”

Sam: Ok.

Heather: “Good job revising your answer,” to reinforce that as a norm.

This time we do not rewind the action, instead continuing the simulation and maintaining the pace of the activity. Sam completes his work with this card, mapping between the visual representation of the quantities on the ten-frames and their symbolic forms on the poster.

**Stepping Back to Examine Part 2 of Episode 4.4.19**

Through repeated practice responding in similar situations, rehearsing interns have opportunities to embody an increasingly refined approach to managing two student solutions in the quick images activity setting. Guided by a teacher educator, work can deliberately progress from establishing general performance standards for an ambitious approach to responding toward refining and adapting that approach in order to attend to additional considerations. For example, my feedback in this portion of the episode, more suggestive than directive this time, is positioned as something that one “could” do in this situation, rather than a requirement. Our work here could be understood as expanding upon a repertoire of moves for reinforcing norms of revising. While the approach is recognizable in its consistency of purpose, to engage students in particular mathematical practices such as revising, how an intern works toward those purposes may vary given the particular situation. As novices practice responding across similar situations, variations on the approach arise and small innovations are invented in the midst of performance. Sam’s comment of “Well, let’s see” in response to having elicited two different solutions, represents an alternative to the language of “Let’s check” that
accomplishes the same purpose. Such variations and innovations may contribute to the continued development of the practice if they are incorporated into the collective repertoire.

But what are other interns taking from all of this? My opportunities to assess how they are interpreting my feedback and Sam’s embodied performance are limited. So far other interns have been actively participating as students in the simulation and engaged in making sense of my feedback to Sam. For example, to conclude Sam’s turn in the teaching role, I comment on his work representing student strategies as part of how he responded to simulated students, pointing out his use of precise hand gestures that emphasized the five- and ten-structures on the ten-frames. Rick then asks for clarification regarding what we are trying to accomplish by representing student strategies, providing me with an opportunity to articulate a rationale. In this way, I am supporting the group in developing an understanding of the instructional value of this representational work. However, I have only observed and assisted Sam’s teaching performance so far. For insight into what at least one other intern has taken up from our work in rehearsal so far, let’s see what the next intern to assume the teaching role does when confronted with a similar situation.

**Episode 4.6.21**

*Working on Responding to Two Different Student Solutions in the Quick Images Activity for the Third Time*

**Episode 4.6.21 Summary**

The last episode analyzed for this case represents the third time in the rehearsal that we collectively simulate a situation that demands responding to two student solutions. Like in the previous two episodes, the teacher educator again acts as a student
who shares an incorrect total for the number of dots on the flashcard. This time, however, a second intern, Jennifer, has a chance to try out a response in the teaching role. When she adaptively enact the core components of the ambitious approach refined in the previous episodes of work, the teacher educator uses it as an opportunity to position the previous intern’s performances as a valuable resource for collective learning.

**Episode 4.6.21 Description, Part 1:**
“Acting the Student” to Recreate a Similar Problematic/Promising Situation for a Different Novice in the Teaching Role

Jennifer, Sam’s teaching partner rehearses next. After rehearsing the launch phase of the activity at my direction, she picks up where Sam left off, flashing the next card that visually represents the sum “10 + 4.”

Here, I re-create a similar demand on Jennifer to manage two student solutions by again acting as a student who has misperceived the total. Like Sam, Jennifer asks for different answers, eliciting fourteen from Laura and fifteen from me. Also like Sam, she presses both of us to explain how we arrived at our totals. She then says, “Ok. Let’s check. Let’s check and see.” First she establishes that everyone saw a full ten-frame on the left. She reinforces that a full frame is ten and checks in to see if I am ready to revise:

Jennifer: So we have the full ten-frame that everybody saw and just knew it was ten (carefully circling that ten-frame with her finger) and then, Heather, how many dots do you see now?”

Heather: Oh, just four. So that’s fourteen.

Jennifer: Good revising. Good revising, changing from fifteen to fourteen when we double-checked.
Stepping Back to Examine Part 1 of Episode 4.6.21

Having a different intern rehearse provides an opportunity for me to see what one other intern is taking away from the two coaching exchanges with Sam. It also provides another opportunity for us to continue to work together on responding to students’ mathematical contributions, building onto what Sam has accomplished with support so far. We see evidence here that Sam’s experimental performances, assisted by my coaching, have served as a model for what Jennifer does when it is her turn to rehearse. She has attended to and enacted the central sequence that we developed and named during Sam’s turn: eliciting a set of student solutions and the student thinking behind each solution, facilitating the group in collectively checking solutions to come to a consensus, and providing opportunities for students with errors to publicly revise. And, she has experimented with a variation of the move I suggested for reinforcing revising norms when she says, “Good revising. Good revising, changing from fifteen to fourteen when we double-checked.”

Episode 4.6.21 Description, Part 2: Positioning an Intern’s Public Rehearsal as a Resource for Developing Practice

The simulation continues without pause, and Jennifer moves on to the next phase of the activity, mapping between the visual representation of ten and four on the card to its symbolic equivalent on the poster. Only after she has had an opportunity to complete the full sequence of work with this flashcard, and for the group to experience a simulated approximation of the pacing of the activity, do I intervene:

Heather: Ok, freeze just for a second. I saw a definite connection between Sam’s rehearsal and Jennifer’s right there, because we got to play it out and work it through, that sequence of what to do when there are two different answers. And now Jennifer got to try out what we worked on with Sam, and it went smoothly. So any questions about that piece? (pausing to look
around, and then turning to direct the next comment directly to Sam)
Thanks for going first.

Group: (laughing)
Jennifer: It really helped!

Here I seize an opportunity to make Jennifer’s “take-up” explicit and public.

**Stepping Back to Examine Part 2 of Episode 4.6.21**

In this portion of the episode, the teacher educator has an opportunity to reinforce a norm for rehearsal, trying something and then revising one’s performance publicly, by pointing to how an intern’s experimenting allowed the group to “play it out and work it through.” I highlight the value of this work for others by relating Sam’s rehearsal to what Jennifer has just accomplished. Sam’s experimental attempts and revised and refined practice has been a resource for Jennifer, assisting her performance by serving as a model. I also name what got worked on, “that sequence of what to do when there are two different answers,” rendering it as an object that can be discussed and analyzed moving forward.

As in previous episodes, my commentary as the teacher educator provides cognitive structures that can serve as resources for interpreting, categorizing, and sequencing shared experiences in rehearsal moving forward. Over time, as members of this community jointly engage in regular preparation for teaching in the rehearsal context, they have repeated opportunities to negotiate and develop a shared approach to making sense of instructional situations and responding to students contributions to mathematical discourse. Through deliberate practice in the company of others, novice teachers are introduced to and have opportunities to experiment with and internalize over time the structuring resources of a developing professional community committed to ambitious teaching. As those resources are collectively developed and appropriated, they can serve
to discipline, and therefore enable, the improvisational interactive performances of individual beginning teachers, responding to students’ contributions to mathematical discourse.

**Case Summary & Discussion**

The episodes of work described in this case involved making public and elaborating an approach for how we, as a professional community, respond to multiple student solutions. This approach is disciplined by commitments to positioning students as competent sense-makers and engendering broad participation in reflective and communicative mathematical practices identified in the Common Core State Standards as crucial for developing conceptual understanding (Hiebert, et al., 1997). In the exchanges, a generalized sequence for how to manage two student solutions was identified and elaborated upon, serving as a framework for action. Once this framework was established, rehearsing novices could work within its structure to begin to flexibly respond in simulated situations. Previously specified by the instructional activity structure developed by our team of teacher educators, this sequence includes: eliciting a set of student solutions and the student thinking behind each solution, facilitating the group in collectively checking solutions to come to a consensus, and providing opportunities for students with errors to publicly revise.

In order to support the development of adaptive expertise in a complex domain, we know that learners need repeated opportunities to practice and must be motivated to improve (Ericsson, Krampe, & Tesch-Römer, 1993). This repeated practice should entail opportunities to work on increasingly complex problems of practice and include feedback that supports not only skill development but also conceptual understanding of the task.
Embedded rehearsal enables the teacher educator to repeatedly simulate the same kind of instructional situation for rehearsing novices to experimentally attempt to manage. Based on repeated observations of novices’ teaching in the enactment phase of the Cycle, the teacher educator can tailor this work in response to novice development.

By acting as a student in rehearsal, a teacher educator can deliberately introduce authentic student errors that contribute to the complexity of simulated instructional situations, to novices who have little experience with how students might react to a particular mathematical challenge. In attempting to cope in these situations, rehearsing interns have opportunities to experiment with, and refine an approach for, managing different student solutions in the context of a well-defined instructional activity. The teacher educator, pausing simulation judiciously, has opportunities to offer responsive support to enable the intern to maintain an instructional trajectory that approximates ambitious teaching. Because this is occurring inside of an embedded rehearsal, these early assisted attempts are public and can be positioned as representative of how the work should be conducted by everyone. This means that other participating novices have opportunities to learn as well, not only observing the enacted approximation but also participating in the analysis of it.
CHAPTER 7

CASE STUDY 3: REHEARSING THE TEACHING OF AN ADDITION STRING

Introduction

When facilitating whole-class instruction, a teacher must attend to, and manage productively, relational dynamics at multiple levels, grappling with overlapping problems of practice simultaneously. The work is particularly challenging inside of activity structures in which students are expected to select, represent, and justify their own solution strategies, and to make sense of and respectfully evaluate the strategies of their peers. The teacher must surface a set of student strategies representative of the range of understanding in the class. She must do this with sensitivity to relational dynamics and issues of equity, drawing on what she knows about individual students as learners. To create a space in which multiple strategies may be considered, she must see multiple solutions as an opportunity to engage the class in mathematical reasoning and inquiry, practices central to authentic mathematical work, instead of as dangerous territory to be avoided. She must take care to elicit student ideas without evaluating them. She must make sense of each strategy while simultaneously scaffolding individual students to articulate their thinking clearly. Each time a student contributes, she must attend to how the rest of the class responds, determining whether or not to intervene to scaffold respectful engagement with each other’s ideas. Orchestrating this kind of activity entails
engaging all students, guiding them to participate appropriately, and maintaining a focus on one’s mathematical goals for the activity.

In this final case, I zoom into a single, extended episode of work on responding to students’ contributions to mathematical discourse that occurred during a rehearsal in the fourth and final week of the Summer Learning Institute. This episode, approximately sixteen minutes in length, is much longer than those examined in the previous case studies. This is a result of the complexity of the situation as framed by the teacher educator, and the way in which the group interactively simulates it and works through it, moving in and out of simulation as the complex situation evolves (see Figure 7.1). For this case, I describe and analyze only a portion of the episode, lasting just under ten minutes, in which the rehearsing novice has an opportunity to experimentally respond to a set of student solution strategies, including three student errors.

Figure 7.1: Focal Episodes for Case 3 as Situated within Rehearsal 13.

Skipping ahead to this episode will provide a glimpse of what is possible when groups of novices and teacher educators engage over a four-week period in repeated Cycles of Enactment and Investigation. We will see evidence that, in a relatively short amount of time, they have come to function as a professional learning community, in which interns have assumed ownership of their own learning and that of group. Together in this context, novices and the teacher educator can put to use their accumulated experiences in both classrooms and rehearsals to co-construct a complex instructional
situation. This situation approximates the “collaboratively emergent” nature of actual whole-class interactions described by Sawyer (2004) in Chapter 2. Not only does this final case illustrate the kind of instructional complexity that can be simulated, but also it demonstrates some of the shifts over time in interns’ participation in the collective work on responding to students’ contributions to mathematical discourse in embedded rehearsal. By acting authentically as students, drawing on their experiences with children, participating novices contribute significantly to the complexity of the situation. We see how this participation provides novices with a means of tailoring rehearsal to their own learning needs, by simulating the kinds of problems of practice they hope to work on. In addition, this form of participation is necessary to simulate increasingly complex situations involving managing multiple student ideas, including several errors or misconceptions, simultaneously. The intern rehearsing in the teacher role also demonstrates increased capacity to effectively respond to a range of potentially problematic student contributions. She draws on her developing repertoire and commitment to the shared principles negotiated over time within the community of practice that has emerged as a result of daily collaborative preparation. In the following section, I situate this episode in its embedded levels of context prior to moving into its description and analysis.

**Embedding the Thirteenth Rehearsal in the Design Context and Fourth Week of the Summer Learning Institute**

**Situating the Rehearsal within a Trajectory of Repeated Cycles**

It is Tuesday morning, July 26, and the Summer Learning Institute is winding down. Interns have been teaching groups of third graders now for over three weeks –
facilitating games, quick images, and choral counting. Today is the third day interns are leading the fourth and final instructional activity, addition strings, with groups of 7-9 children in their classrooms. Our work together in rehearsal has shifted focus, moving away from issues of individual student participation and management toward the practices of orienting students to each others’ reasoning, using multiple representations strategically to support student engagement with mathematical ideas, and coordinating groups of students in making sense of strategies shared by classmates. The strings activity, in particular, places greater demands on the teacher to respond improvisationally using these practices. However, its activity structure is designed to support this increasingly complex work. The circulating phase of the activity that follows the posing of a problem, for example, provides the teacher with an opportunity to preview student strategies and solutions and determine the order in which to elicit student contributions (see Chapter 4 for more details). Yet, despite this shift in focus toward these practices as the complexity of instructional situations has increased, responding to students’ contributions to mathematical discourse remains a central concern. In fact, as evidenced by the patternning of episodes across the entire data set (see Chapter 3, Figure 3.3) – with episodes generally increasing in length, involving more complex spanning structures, and comprising a greater proportion of rehearsal time – collective work in rehearsal has come to be dominated by this problem.

Over the past two weeks, Hala, the co-instructor for the course, and I have also gradually shifted the structure of the rehearsal period in response to interns’ evolving concerns and development. During the first two weeks of the SLI, we spent the first 15-20 minutes modeling components of the instructional activity, facilitating discussion of
the instructional purposes for the instructional activity, and sharing insights from classroom observations the day before. In the last week, we have given interns more time to talk with their teaching partners prior to jumping into simulation. During this time, they debrief their teaching enactments from the day before, analyzing student work, identifying problems of practice that they would like to bring into rehearsal, and brainstorming possible courses of action for dealing with those problems.

Situating the Rehearsal within the Second Prep Period of the Morning

As described in Chapter 3, the two teacher educators, myself and Dr. Hala Ghousseini, work together to lead two Prep Periods each morning at the Summer Learning Institute. The rehearsal analyzed for this case is drawn from the second Prep Period of the day. Therefore, we have already facilitated one complete Prep Period, including a whole-group meeting and small-group rehearsals in which we divide the group in half to rehearse separately. We have also observed the first group of interns teaching children and debriefed our observations immediately prior to this second Prep Period to inform our work with this new group of interns.

Whole-group meeting. On this morning, we direct interns to work in their teaching teams to choose an appropriate string among a set of options and to discuss student errors they experienced the day before and how they plan to address those errors. We circulate, offering suggestions and brainstorming with teaching teams. Sam and Jennifer, for example, are looking at their students’ written work collected from the day before that they have organized in a binder. They alternate between negotiating a shared interpretation of a given student mistake and discussing how they could utilize various representations to support the student in making sense of the error. In the midst of this
collaboration, they invite me over to talk with them about options for representing the decomposition of the addends in a problem to reinforce place value and how they might connect that representation to strategies using jumps on the open number line. They were particularly interested in the work of a student who had started with the first addend and then taken jumps totaling that same addend, rather than the second addend. After discussing this error, Sam asks for feedback on his teaching from the day before. I retrieve my observation notes and quickly review them. Part of my feedback to Sam includes the following:

Students were really interested, actually, in sharing their thinking in your group, and it’s just a matter of keeping it under control. And taking that, uh, ‘authoritative Sam’ presence to help orchestrate them in participating in appropriate ways.

We will see how he brings both of these issues – responding to the particular student error and orchestrating student participation – into today’s simulation by strategically acting as a student. This provides him with opportunities to pose similar problems of practice for the rehearsing novice, giving him an opportunity to see how a peer would manage in similar situations.

**Small-group rehearsals.** After about 25 minutes, we divide the group in half for rehearsing with three teaching teams (6 interns working in pairs) staying with me. As interns settle in to rehearse, I articulate two foci for the day’s work in rehearsal. These goals have been informed by my conversations with teaching teams, observations of interns teaching the same string to groups of children earlier this morning, and debriefing with the co-instructor after our observations. In this way, we can responsively tailor the learning agenda for rehearsal to interns’ immediate needs as they attempt to support
particular learning goals for their students. I explicitly reference those sources as I name each foci:

- “One of the interns this morning, for one of the problems had four different answers, for the first problem: 54, 64, 27, and 81. And when you have that many different answers, trying to coordinate all of them and help students figure out if they need to revise is tricky. So, I think we need to practice that in rehearsal today.”
- Responding to an intern’s comment that students were struggling to use the open number line this week after the long weekend, I identify this goal: “Something that we’re going to practice in rehearsal is making sure that we’re giving meaning to the addition problem. So, we have to really focus on, everything that we do on an open number line, or everything that we do in decomposing the numbers into tens and ones, we have to show how it’s related to adding two numbers together. Because, a lot of the stuff that I saw yesterday, students didn’t have a sense of how the addition problem and the representation of jumps were related. There was a disconnect between the problem and the representation. So we’ll work on that.”

After naming these objectives for rehearsal, I direct Laura to take on the teaching role. As she comes to the front to rehearse, I continue to set the scene: “I am going to be a student who knows how to add, and I have a strategy for adding, but I don’t know how to use the open number line.” By identifying this complication in advance, I can prime interns to carefully attend to what I do acting as a student. In my classroom observations the past couple of days, I had seen multiple examples of students just like this, and interns who, in my opinion, were not responding effectively. This kind of support can guide their appreciation of situational details that could provide important information about student thinking, useful for interpreting the particular difficulties students are having. I also set the parameters of the problem space we will work on today, framing the scope this way: “And we’re going to try to work on how to deal with multiple student answers, and what you can do to help us figure out which answer is right.” As will be evidenced in the focal episode analyzed for this case, instructional situations of this scope are significantly more
complex than those worked on earlier in the summer and described in the previous two case studies.

In what follows after providing a short summary, I move back and forth between narration and analysis for the episode, linking detailed descriptions of episode segments to ideas outlined in the conceptual framework in Chapter 2 in order to make sense of the interactions at the level of specific exchanges. I distinguish the analytic portions from sections of description by identifying them with subheadings starting with the phrases “Stepping Back” and “Episode Description” respectively.

**Episode 13.1.69:**
**Working on Responding to Multiple Student Solutions Including Several Complicating Student Errors**

**Episode 13.1.69 Summary**

In this episode the rehearsing novice is confronted with four different student solutions to which she must respond. She has a range of options for how to represent this set of solutions and in what order to elicit student reasoning. She must simultaneously support individual students in making sense of their errors and keep the group of students engaged productively, orienting them to each others’ contributions so that as a group they can evaluate and compare the range of strategies shared. In the collective work on this complex situation, multiple problems of practice are identified and addressed at a range of scales across the simulation. One problem for children that gets simulated, for example, is that many of them are coming into the activity with prior knowledge of the standard algorithm procedure (or “stacking” method) for adding two digit numbers. They use the procedure without place value understanding and can therefore arrive at correct answers, especially when no ‘carrying’ or regrouping is required. Because they do not
understand what they are doing, they struggle to make sense of representations on the open number line. Interns, in turn, are uncertain as to the reasons why these students are having difficulty, and they are not sure how to build connections between students’ current knowledge and work on the open number line.

**Episode 13.1.69 Description, Part 1:**
**Getting Into A Complex Problem Space**

Laura, now in the teaching role, launches the activity and poses the first problem in the string, $31 + 23$, writing it in the top right corner of the white board.

![31 + 23 =]  

She directs the rest of us, who are acting as students, to work independently to solve the problem and to try to represent our strategies on an open number line. As we get to work, she circulates to see what students are doing. On previous days rehearsing strings, we had talked about and practiced this work of circulating while students work independently – emphasizing its importance for helping students get started, gathering information about what students are doing, and making decisions about the order in which to selectively elicit a range of strategies during the whole-group phase of instruction that follows. Acting as a student, I quickly write down an answer of 54, without representing my strategy on the open number line, and then sit there looking around at my neighbors.
Laura walks over to me. Rather than talking me through a procedure for showing addition on the open number line, she asks a series of questions to get me started: “Where are you going to start? What are you going to jump by?” and when I look puzzled, “What numbers are you adding in this problem?” After I respond correctly, she directs me to “try to figure out how can you add twenty-three on the number line by making jumps,” and continues to circulate.

Interns, acting as students, begin to look up from their papers. Jennifer pretends to write on Sam’s arm with her pencil. Noticing that most students seem to be finished, Laura transitions from independent work time to the whole-group discussion phase: “Ok, if you have an answer to this problem, show me a quiet thumb.” Several interns use a thumbs-up signal with their hand at their chest, and Laura collects a set of answers, asking a version of “Does anyone have a different answer?” three times. Sam offers sixty-two and Jennifer and I both say fifty-four. Whispering to Claire sitting next to me, I direct her to change her answer to eighty-one, adding a third solution into the mix. Laura writes each of these answers on the board as they are shared, responding each time in a non-evaluative tone.

\[
31 + 23 = \begin{array}{c} 62 \\ 54 \\ 81 \end{array}
\]
Once she has elicited the set, she exclaims, “My! Let’s see if we can look at our number line to prove some of those answers, to figure out which one we think it is.”

**Stepping Back to Examine Part 1 of Episode 13.1.69**

The simulated instructional situation that is taking shape in this episode is dependent, in part, on what the rehearsing novice is able to do at this point late in the summer. Laura smoothly gathers multiple student answers using a variation of the question “Does anyone have a different answer?” three times. Each time she responds in a non-evaluative way to avoid signaling which answer is correct, writing each proposed answer on the board. Communicating interest in the set of answers with her tone of voice and expression, she then frames the work that needs to happen next as collective work “to figure out which one we think it is.”

Not all teachers would find themselves in such a complex situation. In many cases, teachers have learned through personal experience to avoid this kind of instructionally productive predicament, because they do not know how to manage the complexity of a situation in which multiple student solutions have been elicited. In contrast, Laura and the other interns have publically and repeatedly rehearsed a routine for eliciting multiple student ideas in previous rehearsals. They have discussed the instructional goals for student participation underlying the routine and how it represents a commitment to both treating students as sense-makers and maintaining the cognitive demand of the task. Now we see evidence of Laura purposefully enacting the routine of gathering multiple student ideas. While her performance is structured by this routine and the commitments underlying its use, it is not determined by the routine. She must actively
make judgments about who to call on and in what order, keeping in mind the whole group phase of work that will follow and her mathematical goals for the activity.

Through this exchange, I have an opportunity to evaluate Laura’s performance in relation to the focal principles and practices identified by the practice-oriented curricular framework under continual development. For example, is she interacting with me (acting as a student) in a way that reduces or maintains the cognitive demand? In this case, Laura appears to be trying to leave the mathematical work to the student. However, she has not yet engaged with me in a way that could yield insight regarding how I arrived at my correct solution – a solution that did not require an understanding of how to represent the addition problem on the open number line. So, we are left to wonder, what might a student like this already know about adding two-digit numbers together that would enable her to produce an answer with no visible strategy? Also, we see that she has elicited a set of answers and framed the work to come as figuring out together what makes sense. Does she maintain this ambitious trajectory moving forward? Is she able to engage the group of students in collectively making sense of the set of answers, leaving the work of determining correctness to the students, while guiding individual students toward understanding addition in relation to place value? Let’s return to the description of this episode to see how the group simulation continues to evolve.

**Episode 13.1.69 Description, Part 2: Simulating a Complex Situation that Involves Managing Multiple Student Solutions**

With three different answers on the table and at least one student who does not seem to know how to use the open number line, Laura has a lot to manage. She has just paused, briefly stepping out of the teacher role to explain that she wants to open the group discussion by eliciting and representing the strategy from a student who has used
jumps of ten on the number line. In so doing, she is making public one consideration that is guiding her performance moving forward. Jennifer volunteers to act like a student with this strategy, and Laura shifts back into the teacher role, “Ok, and Jennifer, what was your strategy? What did you jump by on this number line?” Jennifer explains that she “made two jumps of ten and then one jump of three.” Laura continues by elaborating on where Jennifer’s jumps came from saying, “Ok, ok, so you, you knew that you were adding the twenty-three, and you broke that twenty-three into what, into two groups of ten and one group of three?” As she speaks, she simultaneously represents the place value decomposition of 23 on the board:

Then, Laura turns to me and asks, “So, Heather, what, what was the first jump that Jennifer took?” I look at Laura with a puzzled expression and silently shake my head, signaling that I don’t know. I could have answered here, but by acting confused I am staying in the role of the student who does not understand the number line. In so doing I maintain the press for Laura to respond improvisationally in the midst of performance. Laura uses this as an opportunity to orient students to each other, one of our focal practices. She directs me to ask Jennifer: “Do you want to ask her? Ask her what her first
jump was.” I do as directed. Jennifer turns to me and explains that she made a jump of “plus ten.” I repeat this, and Laura continues, representing the jump on the number line as “+10.”

She then engages Sam, who is acting as a student with the incorrect answer of 62: “If we make one jump of ten, Sam, where are we going to land? We start at 31 (pointing to it) and make a jump of ten (motioning the jump by tracing it with her finger). Where are we going to land?” Sam answers correctly, forty-one, and Laura repeats this line of questioning for the next jump. This time she calls on Claire, who had the answer of 81: “Where are we going to land over here?” Laura then summarizes the rest of Jennifer’s strategy saying, “And Jennifer told us she made three more little jumps of one,” drawing three little jumps on the board.
In response, Jennifer emphatically states “No!” Other interns laugh – signaling, perhaps, their recognition of her authentic student reaction. Seemingly un-phased, Laura asks for clarification, and Jennifer explains that she had in fact made one jump of three. Laura repeats this as she revises her representation on the board, erasing the three little jumps and replacing them with one proportionally bigger jump of three:

As Laura turns back to face the class, Sam blurts out, “I didn’t do it that way.” Without hesitation, Laura replies, “Ok, well let’s think about Jennifer’s strategy right now,” making use of a re-focusing move that we had discussed in a previous rehearsal.
“Andrea,” Laura continues, “if we start at 51 and make a jump of 3, where are we going to land?”

Only after Andrea answers 54, completing Jennifer’s solution strategy, does Laura return to Sam to elicit his different strategy: “Sam, how did, what was your strategy?” Sam explains how he “made more jumps” of ten, jumping ten to 41 and then to 51 and then to 61. As he describes his first two jumps, Laura traces them on the number line already on the board with her finger.

But when he says that he jumped to 61, Laura pauses: “Why did you jump to 61?” “Because of thirty-one. I made three jumps of ten and one more,” Sam replies with confidence. Laura responds in a way that provides Sam with an opportunity to reflect on his own work in relation to the original addition problem. “You said you started at 31. And then what are you adding to 31?” she asks him. “I added, uh,” pausing to look down at his paper, “thirty-one,” he replies with some hesitation now in his voice. Laura replies by pointing back to the original problem and noting, “we’re actually adding 23.” Sam, acting as if he is realizing his error, exclaims, “Ohhhhh! I see. Ok.”
Stepping Back to Examine Part 2 of Episode 13.1.69

It is worth pausing in the narrative here to provide a bit more background on the significance of Sam’s participation in this part of the episode. By acting as a student, he has re-created two different problematic situations that he confronted the day before. First, when he interrupts Laura’s work processing Jennifer’s strategy, he is creating a situation that requires Laura to re-direct him to maintain the group’s focus. In prior rehearsals, we had worked on how to re-direct students and stay on point, as part of the work of orchestrating group discourse to target your math goals. As part of this work, we had also discussed circling back to a student like the one Sam is portraying, so that he still has an opportunity to participate productively. Laura models a version of this strategy as she manages Sam’s participation, first responding to re-direct Sam back to the topic at hand, Jennifer’s strategy, and then coming back to him to provide an opportunity for him to share his own strategy. The consequence of Laura’s approach to this simulated situation is that she is able to maintain a focus on Jennifer’s strategy, while also providing Sam with an opportunity to participate appropriately.

The second problem of practice that Sam is able to bring to rehearsal by acting as a student came from his earlier examination, with his teaching partner, of their students’ work from the day before. One student had started on the number line with the first addend in a problem and then taken jumps totaling that same addend, rather than using the second addend. By acting like a student who has made this same error, Sam creates an opportunity for himself and the assembled group to see how someone might respond to a student making this error. Laura uses elicitation questions that press Sam to articulate his
strategy. The consequence of her line of questioning is that Sam (acting as a student) is supported in reflecting on what he has done and comes to recognize his error.

Although it is not his turn to rehearse in the teaching role, he has created an opportunity to work publicly on two different problems of practice. He does this by re-creating similar complications to ones he confronted in practice by acting as a student. His improvisational acts as students are disciplined by his developing understanding of student thinking and behavior as well as his understanding of how to support his own development in the rehearsal setting. His experience is translated into an instructional situation with the potential to contribute to other’s understanding of the kinds of errors students might make in the activity. In this way rehearsal can serve as a setting in which individual experience is pooled for the benefit of collective preparation. Not only does one of his peers have the opportunity to try to manage these complications in the role of teacher, but also Sam and other participating interns have the opportunity to observe her improvised response. The rehearsing intern, drawing on a developing repertoire of pre-rehearsed moves, models an ambitious approach for managing each of these issues Sam has created. And, it is Sam in the role of student, rather than the teacher educator, who acts out the consequences of Laura’s action.

This is representative of a significant shift in intern participation in rehearsal made possible through repeated Cycles of Enactment and Investigation across four weeks, and indicative of novice learning. It is a different way of working on problems of practice than what we observed in Case 1, for example, through re-visioning routines. Rather than asking a question directed to the teacher educator about how to respond during a pause in simulation, an intern poses a complication (by acting as a student) to a
peer (acting as the teacher), who is responsible for trying out a possible way of responding. The authority for crafting the response has shifted from the teacher educator to the rehearsing intern. While at the same time, the responsibility for simulating problematic instructional situations to be worked on is shared by the teacher educator and participating interns.

All of this happens without pausing the simulation. Laura has maintained an ambitious trajectory, moving from eliciting multiple student solutions into managing the evolving situation and responding smoothly to a range of potentially problematic student contributions. Especially noteworthy in her improvisational performance, she has worked purposefully to engage each student with an error in making sense of one correct strategy belonging to Jennifer, carefully drawing students’ attention to visual representations on the board. This component of the work of responding to a student contribution, we call orienting students to each other’s reasoning. Laura and other interns have been deliberately experimenting with a repertoire of moves for orienting students to simultaneously broaden student participation and emphasize important mathematical concepts and strategies. When I do pause the simulation next, it functions as an opportunity to guide the collective appreciation of what has transpired so far.

**Episode 13.1.69 Description, Part 3: Maintaining an Ambitious Trajectory for Managing Multiple Student Solutions**

At this point in the rehearsal, I pause the simulation, summarizing what has been accomplished and suggesting what needs to be accomplished next, providing a meta-commentary on the simulation:

Laura has helped one student with an incorrect answer figure out the problem. And the way that you did it was, you found out what he did, right? So now you’ve
interacted with him and helped him revise. We still have eighty-one up there, so we’ve got to make sense of that one too.

I look back and forth between Laura and the rest of the interns as I speak.

Laura responds with a hint of skepticism in her voice: “So I have to ask each student?” The other interns look at Laura and then at me. “Well,” I reply, “you need to at least come back and see if Claire has revised. She probably hasn’t. So, she’s still thinking her eighty-one is right.” Laura nods that she understands, and steps back into the role of teacher. First, she creates an opportunity for Sam to publicly revise his answer asking, “So Sam, do you want to revise your answer?” He replies, “I do.” She presses him a bit, “You don’t think it’s sixty-two any more?” “No, I think it’s 54 now,” he continues. Then, she moves on to elicit Claire’s strategy that resulted in an answer of 81, establishing that Claire took five jumps of ten, two and then three, for each digit in the number 23. “And now let’s see,” Laura says. “If we have two tens, that gives us our two groups of ten. And what do we need to make twenty-three? Do we need three more groups of ten or do we need three ones?” Claire responds by repeating “three ones” and Laura finishes by pointing to the jump of “3” she has already represented on the board, saying, “And so that’s our little jump of three right here.”

I intervene at this point, saying “So, something else that could happen…,” prepared to explain what I had seen a student do that morning who got 81 as an answer. Before I can do this, however, Laura interrupts to critique her own performance: “I just like fed it to her though.” Smiling gently, I acknowledge this self-assessment with “sort of” and offer a suggestion for how she could start again, this time representing Claire’s strategy on a new number line. Laura resumes the simulation by rewinding the action without prompting. She asks me, “Heather, where will Claire land if she makes two
jumps of ten?” Not only is Laura revising her performance as teacher based in part on her own self-assessment, but Claire also revises her performance as student. When Laura asks her this time if she made “three more jumps of ten,” Claire deliberately responds, “I made three more jumps.” Again, I pause the rehearsal to draw attention to this change, because it is significant for making sense of the mistake behind the answer of 81: “So probably what she did is made three more jumps. She doesn’t know what they are, but because she made three more jumps, she went ‘sixty-one, seventy-one, eighty-one.’”

Claire’s revised response to the same question is now more aligned with what I had seen a student do that morning with an answer of 81. He had shown his work this way, not labeling the size of any of his jumps, but showing two big jumps and three smaller ones:

![Diagram](image)

I intervene again to point this out, “So probably what she did is made three more jumps. She doesn’t know what they are, but because she made three more jumps, she went, ‘sixty-one, seventy-one, eighty-one.”’ I suggest that Laura draw the last three jumps as little jumps, like the boy had done that morning, and then lead the group in counting on by ones from 51. Laura tries this out, and Claire revises her answer. I intervene again, explicitly referencing my observation saying, “Nice. That’s exactly what happened this morning. Someone said eighty-one for that reason. And showing the little jumps of one enabled the little boy to say, ‘Oh! It’s not eighty-one. It’s fifty-four.’”
Before stepping back into the teacher role to resume simulation, Laura asks, “So I just do the next problem? Or do we still need to reinforce the groups of ten, or?” I take it as an opportunity to remind her about the student who I am portraying:

I wouldn’t get too bogged down in that. However, there’s still me, and I’m the student who doesn’t know how to use the open number line. I got fifty-four, which is the right answer. So, you need to know how I got the right answer without using the open number line.

Laura pushes back a little asking,

So I do still ask you? Because I have a student, who also knows how to get the answer but then uses the number line incorrectly, and Hala was saying that perhaps, like, I could coach them individually and I would, um, use someone’s strategy that was correct and say like ‘Heather, did you see why we jumped two groups of ten?’ Or should I actually ask you how you added?

In my reply, I direct her to “use your judgment on this,” and then identify one reason why she might want to ask me how I solved the problem. “Working with me,” I suggest “would help the group [of students to see] the number of tens in each of the numbers,” relating to one goal for the activity – to support students’ understanding the role of place value in double-digit addition. Then, I continue by explaining how I (acting as a student) actually solved the problem, using the same language of ‘stacking’ that I heard children using: “So the way that I solved the problem is I stacked them, and I added the 3 and the 2 and the 1 and the 3.” Laura steps back into the teacher role and elicits details on my ‘stacking’ strategy with a series of questions.

**Stepping Back to Examine Part 3 of Episode 13.1.69**

In this part of the extended episode of work on responding to multiple student solution strategies, the teacher educator pauses the simulation several times to narrate what has happened so far, what the rehearsing novice has accomplished, and what has not yet been accomplished. The first time I pause rehearsal, for example, I explicitly remind
interns of our commitment to providing students with opportunities to exercise mathematical authority when I say, “We still have eighty-one up there, so we’ve got to make sense of that one too.” My use of “we” signals that as a group acting as students, it is our collective responsibility to make sense of the set of answers proposed, not just a correct solution. At that point, I also am anticipating that Laura might move on to the next problem in the string. From my experience in previous years of the Summer Learning Institute, I know that a common misconception among our interns is expecting that it would be sufficient for students with misunderstandings or errors to engage with one correct strategy. If Laura were to move on to the next problem in the string here, she would miss an opportunity to gather information about how students with incorrect answers had determined the total. She would miss an opportunity to provide those students with a chance to publicly revise, and for the group to negotiate some consensus. So, when I point out that the student Claire is portraying likely still thinks her incorrect answer is correct, I am suggesting an interpretation that problematizes the current situation and disrupts Laura’s likely expectation that students “get it” now.

Later in this episode Laura again seeks to move onto the next problem in the string before doing anything to elicit the reasoning of the student, played by me, who is arriving at a correct answer without using the open number line. Although Laura has successfully supported two students with incorrect answers to revise in the context of whole-group discourse, this is the second time that she has been prepared to move on without having information about how the student I am portraying is figuring out the total. This is concerning because it is not clear if she is attending to this student. Again, I pause the simulation to remind her about this student. In response, Laura explains how
she planned to privately address the student’s difficulty with the open number line individually. She makes her thinking public here, including her reason for approaching the student individually in terms of what was suggested to her by another teacher educator. This provides me with an opportunity to frame the other teacher educator’s suggestions as one possible response in a situation that requires professional judgment. Then, I suggest another consideration, how a different a response might enable her to further her mathematical goals for other students, as a resource in disciplining her judgment.

This ability to pause in the midst of simulation is an invaluable characteristic of the rehearsal participation structure if we aim to support beginners as they engage in the transactional process conceptualized by Schon (1983) and outlined in Chapter 2. The teacher educator can intervene to guide novices’ interpretation of a collaboratively emergent, complex situation in the midst of improvisational performance. In this particular exchange, we see that by intervening, the teacher educator can disrupt the rehearsing intern’s certainty that facilitating the group in making sense of one correct strategy is sufficient. Structuring resources, such as some of the considerations that should discipline judgment, can be explicitly identified and utilized to appreciate and interpret key aspects of the evolving situation. In addition, interns have opportunities during pauses to make their pedagogical reasoning available both as a resource for the teacher educator to gain insight into how they are appreciating problematic instructional situations and as an object for collective inquiry.

For rehearsing interns, these pauses provide opportunities to take stock, to look back and evaluate one’s performance so far, and to consider the consequences of what
has occurred to inform one’s judgment regarding how to proceed. For example, during one pause, Laura publicly evaluates her own performance in the teaching role. She characterizes her interaction with Claire as being too leading when she says, “I just like fed it to her though.” In doing this she publicly utilizes a shared performance criteria to self-evaluate. Likewise, interns acting as students have opportunities to reflect on their performances as well. In this episode, we see evidence that Claire has figured out a possible logic behind the particular student error that I have directed her to act out when she revises her explanation as student. And, I am able to pause the rehearsal to draw the group’s attention to this change, because it is significant for making sense of the mistake behind the answer of 81. Participating as students provides interns with opportunities to learn about student thinking that differ significantly from those made available through analysis of student work. In rehearsal, they actually embody the student’s reasoning in the context of instructional interaction, with opportunities to try to contribute and respond in ways that would authentically represent the student’s line of reasoning.

**Case Summary & Discussion**

The episode analyzed for this case is representative of the kind of complex instructional situations that can be collaboratively co-produced in rehearsal when a group of novices and teacher educators have the opportunity to engage in joint work daily for four weeks. As novices gain experience, knowledge, and skill their participation in rehearsal can shift across this period. Those stepping into the teaching role have more resources on which to draw to improvisationally respond in the moment to demands and complications created when students engage in mathematical discourse, enabling the group to work on increasingly complex situations. Laura, for example, was able to stay in
motion despite several student responses that could have tripped her up by adaptively drawing on her developing repertoire of previously rehearsed moves and routines. In addition, we see some evidence in this episode that she has appropriated a criterion for evaluating her own performance, a resource that enables her to replay and revise her interaction with a simulated student, to avoid “feeding” her the answer.

In the episode analyzed for this last case, interns participating as students also have greater capacity to translate their developing understanding of student thinking and behavior into their roles as simulated students. In this context, learning to act authentically as a student participating in math discourse is part of what it means to learn how to learn in and from practice. This kind of participation is different from other ways of learning about student thinking. Novices must put to use knowledge of students for purposes of preparing for teaching. In the process, they have opportunities to develop deeper understandings of student reasoning as we saw when Claire revised her improvisational performance as a student. When novices are able to participate authentically as students in rehearsal, the group can simulate increasingly complex situations, and as a result work on problems of practice that would be difficult for the teacher educator on her own to pose by acting as a student.

As novices develop these capacities, the teacher educator can frame instructional situations of increasing complexity, setting parameters around a set of problems of practice to be managed simultaneously, rather than drawing attention to one at a time. In this case, for example, I frame a broader scope for the situation to be worked on when I delineate its boundaries as how to manage a set of multiple different solutions. Novices can also assume greater responsibilities for their collective preparation to teach
immediately following the rehearsal period by anticipating student errors and misconceptions and guiding the learning agenda.

As these changes occur over time, the group develops its capacity to engage jointly in reflective practice. Together they share responsibilities for identifying and simulating the problematic situations that demand disciplined improvisation and that become the objects for collective reflection. They work together to interpret instructional situations as those situations develop in interactive simulation. Sometimes this interpretation occurs in the midst of the action, or in transaction with the materials of the situations in Schon’s (1983) terms, without pausing simulation. For example, when an intern in the teaching role crafts a response to a student error and another intern acts out how that student might react, both must reflect-in-action to appreciate the evolving situation and make judgments about what would constitute an authentic response in their respective roles.

At other times, the group steps out of the flow of simulation to reflect on practice, identifying and inquiring into the considerations and commitments that should guide their interactive performance. In this particular case, pauses in simulation initiated by the teacher educator provide opportunities for participants to reflect on and articulate some of these considerations. Laura, for example, during one such pause publicly identifies a commitment to maintain the cognitive demand on students and avoid “feeding” them answers with overly leading prompts. In naming this criteria, she holds herself accountable to the professional standards established by her local practice community. Whether reflecting in the midst of simulation or in pauses, participants draw on a repertoire of strategies, knowledge resources, and conceptual tools developed through
prior experience in small-group rehearsals and in their teaching to contribute to making sense of collaboratively emergent instructional discourse. As they make use of these structuring resources and reflect on their use in the company of others who share similar commitments, they develop as ambitious teachers while contributing to the continued development of the practice of ambitious mathematics teaching.
CHAPTER 8
DISCUSSION

Overview

In this dissertation, I investigated the kinds of opportunities to learn to do the improvisational, responsive work of ambitious teaching that can be made available in embedded rehearsals when beginners work on the core practice of responding to students’ contributions to mathematical discourse. Using in-depth, sequential case studies of a spare set of episodes of work on this practice at three time points spanning four weeks of rehearsals, I was able to richly characterize these opportunities to learn using the conceptual framework elaborated in Chapter 2. To do this, I integrated Sawyer’s (2004) framing of teaching as disciplined improvisation with Schon’s (1983) conception of reflective practice as a transactional process and identified three categories of structuring resources that discipline this transactional process. In what follows, I summarize the primary contributions of this dissertation research, including the conceptual framework and the findings across the set of cases in relation to my research questions. Then, I consider the dissertation’s limitations and future directions for research.

Understanding Responsiveness in Ambitious Mathematics Teaching as a Disciplined, Improvisational Transaction with Students

In Chapter 2, I elaborated a conceptual framework for understanding the practice of responding to students’ contributions to mathematical discourse and how that practice
might be learned in the context of formal teacher education. This framing was composed of the following integrated big ideas:

• *The practice of responding to student mathematical contributions entails improvisation when enacted ambitiously, because instructional interactions are “collaboratively emergent.”* They are the product of improvisational performances on the part of both students and the teacher (Sawyer, 2004): the teacher acts to motivate and elicit broad student engagement; students “act back,” making their mathematical reasoning public in the midst of discourse; and the teacher, in turn, must craft responses in the moment to manage instructional situations and associated problems of practice as they evolve to target ambitious mathematical goals for students.

• *The improvisational performances of experienced ambitious teachers are not random or completely extemporaneous, but instead are “disciplined” by a set of mediating resources that are marshaled in the midst of interactivity (Sawyer, 2004).* These structuring resources both enable and constrain performance and include a range of participation structures, conceptual structures, and material artifacts.

• *Furthermore, when an experienced teacher responds improvisationally to students’ contributions to mathematical discourse to accomplish ambitious aims she engages in a transactional process.* This process comprises phases of appreciating and interpreting the evolving instructional situation in the midst of performance, acting experimentally while attending to “backtalk” from the materials of the situation (i.e. students’ contributions), evaluating the set of consequences that result, and re-appreciating the situation (Schon, 1983).
• The structuring resources that mediate each of these phases are developed and made available over time by members of a practice community participating in joint goal-oriented activity. Through their participation in practice, individual practitioners gain valuable experience while simultaneously the community negotiates the professional commitments, values, and standards of expertise to which members’ practice is held accountable.

• When novices engage with more experienced teachers over time in joint work, they have opportunities not only to be introduced to and appropriate the structuring resources previously developed by the practice community, but also to contribute to the continued development of those resources. Experienced ambitious teachers serve as carriers of the practice, bringing resources and insights from their own experiences and personal histories of participation in the practice community to joint work. Novices, when motivated to become competent members of a practice community, learn the practice by engaging in it in the company of more experienced members.

• Opportunities for novices to develop capacities for adaptive, improvisational performance are especially rich in organizational settings designed to foster deliberate practice, including performance feedback, and regular collective inquiry into the problematic instructional situations that arise for them. Through deliberate practice, novices are afforded structured opportunities to step back and analyze their teaching with feedback from more experienced teacher educators and each other. Together, by engaging in collaborative inquiry into problematic situations that arise in the midst of interactive performance, experienced teachers and novices make their shared work explicit, linking performative aspects of practice to the conceptual
resources that enable adaptation and improvisation across a range of instructional situations.

Taken together, these ideas integrate and build upon key insights drawn from a corpus of research related to the development of adaptive expertise in teaching grounded in a socio-cultural perspective on learning a responsive, improvisational practice. They provide a conceptual framework for making sense of episodes of work in the context of embedded rehearsal, an activity setting designed to provide opportunities for teacher educators to guide deliberate practice and collective inquiry. As noted earlier, by viewing the practice of responding to students’ contributions to mathematical discourse through this lens, I have contributed a way of sub-dividing the practice into its constituent, albeit inseparable, phases to better inform an understanding of what it is exactly that novices need to learn to do and for what purposes structuring resources may be employed in the process. This framing proved useful for interpreting the aspects of responsive practice being worked on in rehearsal, the respective roles of the teacher educator and novice teachers, and the structuring resources mediating that work. Attending to these themes enabled me to construct more nuanced narrative representations of the interactive work occurring inside rehearsals and begin to characterize the opportunities to learn to respond to student contributions to mathematical discourse afforded by that work. In the sections that follow, I revisit the sub-questions guiding my narrative analyses to summarize key themes distilled from looking across the three case studies.

**Affordances of the Two-Phase Alternating Structure of Embedded Rehearsal**

In order to investigate the contribution of rehearsal’s participation structure, I used interaction analysis to render focal episodes of work on the practice of responding to
students’ contributions to mathematical discourse in enough detail to examine rehearsals at the level of participant exchanges. This method of analysis represents a novel approach for examining teacher education pedagogy. As described in Chapter 2, the participation structure of rehearsal is characterized by the back-and-forth alternation across two distinct phases, simulations and pauses. By zooming into rehearsal interactions at the level of participant exchanges, I have been able to characterize processes involved in the collaborative construction of opportunities to work on the practice of responding to students’ contributions to mathematical discourse in the activity setting of rehearsal. The set of case studies demonstrate that, as a result of this two-phase structure, rehearsal enables distinctive opportunities to work on this crucial aspect of ambitious teaching.

**Affordances of Simulating Instruction**

Simulation phases of rehearsal provide novices with regular opportunities to participate in enacting instructional interactions in two different roles, acting as either the teacher or a student. In the teacher role, novices are afforded repeated opportunities to experiment with, revise, and refine responses to simulated students participating in mathematical discourse. In both the second and third case studies, for example, interns rehearsing in the teacher role confronted a range of interactional demands that arose in the midst of simulated instruction. Sam had to manage two student solutions as he facilitated a quick images activity in a rehearsal in the second week of the Summer Learning Institute. Likewise, Laura needed to cope with four different student strategies, including two student errors, among other complications as she led a computational string activity in week four. In each case, they had opportunities to engage with developing situations transactionally, acting experimentally on their interpretations of those
situations to further their mathematical goals, and experiencing the resulting consequences of their actions for student participation moving forward.

The opportunity to evaluate one’s own performance in situ by observing feedback in the form of “backtalk” from the materials of the situation, in Schon’s (1983) terms, is a particularly noteworthy affordance of rehearsal. Data examined for this dissertation demonstrate how these opportunities can be deliberately simulated by the teacher educator acting as a student. In this role, she can purposefully “serve up” situational demands that must be managed by the rehearsing novice in the midst of his/her performance to target particular practice terrain in rehearsal. The set of episodes analyzed in Case 2 exemplify some of the affordances when the teacher educator participates in this way. In this case, for example, the teacher educator was able to repeatedly focus work on the problem of managing two student solutions in quick images by acting as a student who shares an incorrect total three different times. As a result, rehearsing interns had an opportunity to repeatedly practice and refine an ambitious approach to responding to two student solutions. This kind of authentic repeated practice can support novices in developing the capacities to cope with the ongoing demands and pacing of collaboratively emergent instruction in which they are required to continually adapt and improvise.

On the other hand, when novices participate as simulated students, they have opportunities to put to use and continue to develop knowledge of student thinking and behavior to experimentally act out authentic student contributions. We saw evidence of this in Claire’s participation as a student in Case 3, in which she revised her performance to reflect her developing sense of the reasoning behind the student error that she was
simulating. By acting as students in simulation, novices gain experience anticipating not only student ways of reasoning and making sense of mathematics but also how students might express their thinking in classroom discourse. Case analyses suggest that acting as a student also enables novices to bring puzzling or problematic student contributions into rehearsal in order to work through with the group how to respond. Sam, for example, participated in this way in the last case study: first when he posed a particular mathematical error that he had confronted in his own teaching the day before, and then when he acted like a student who wanted to share his own strategy when asked a question about another student’s strategy. Laura, his peer in the teaching role, got to practice responding to these challenges improvisationally, modeling for Sam and other participating interns an ambitious approach. Participating as a student can provide an important means for novices to exercise responsibility for their own learning and that of their peers in rehearsal by contributing to the learning agenda.

**Affordances of Pausing Simulation**

When simulation phases are interspersed with pauses in the action, other distinctive learning opportunities can be made available. The data explored in Case 1 illustrate how pauses create space for novices to bring in relevant personal teaching experiences as resources for collective preparation. We saw evidence of this in two different episodes of work on responding to student contributions to mathematical discourse. In the first instance, Sam asked a question about appropriate student participation, the teacher educator offered an interpretation of the student behavior, and then the group brainstormed possible responses within this proposed framing of the situation. In the second episode analyzed for Case 1, Laura initiated work on a different
problem of practice drawn from her teaching experience the day before when she asked for advice on how to respond to “students who are really struggling.” This request led to an exchange in which the teacher educator guided Laura through a process of reflecting on and re-appreciating the situation, or what Horn (2010) has termed a “re-visioning” routine, yielding a new interpretation of the student’s error. The exchange concluded with the teacher educator suggesting a strategy for responding to the student that Laura and her peers can experiment with in their upcoming teaching.

Both of these examples from Case 1 evidence the crucial role of the teacher educator in responsively guiding collective work, using judgment to determine how to take up novices’ experiences for the benefit of group learning and framing the problems to which they attend. Data analyzed across the three case studies further indicate that problem framing is a salient aspect of the teacher educator’s work in rehearsal. In Case 3, for instance, the teacher educator intervenes several times throughout the simulation of a complex instructional situation involving managing multiple student solutions. In these interventions, she narrates what has been accomplished and suggests what needs to be accomplished next, providing a meta-commentary on the simulation. This commentary can guide collective interpretation of the situation while also supporting the novice in the teaching role to maintain an ambitious trajectory moving forward. We see evidence of these advantages in Laura’s performance, as she is guided to systematically address each student error rather than move on to the next problem in the string.

Pauses in simulation also provide valuable opportunities for the group to step out of the flow of simulation to reflect on practice, identifying and inquiring into the commitments and aims that should guide their interactive performance. The teacher
educator can initiate pauses to provide feedback on performance, identifying evaluation criteria and providing a principled rationale for aspects of the work. In the second case analyzed for this dissertation, the teacher educator intervenes in this way when she offers both evaluative feedback and directive feedback to Sam as he confronts two student solutions. He rewinds the action to put to use this feedback. Such action replays are another affordance of the two-phase alternating structure of rehearsal. In Case 3, we see how pauses initiated by the teacher educator can provide opportunities for the novices themselves to reflect on and articulate some of these considerations. During one such pause, Laura publicly identifies a commitment to maintain the cognitive demand on students and avoid “feeding” them answers with overly leading prompts. In naming this criteria, she holds herself accountable to the professional standards established by her local practice community. When interns develop an understanding of professional standards and commitments in tandem with a repertoire of skills, they can begin to adaptively respond in similar situations.

Changes Over the Four Weeks in Work on Responding to Students’ Contributions to Mathematical Contributions

Changes in the Complexity of Focal Instructional Situations

In Chapter 4, I described how instructional activities served as important resources in our work designing and facilitating practice-centered learning opportunities for novices in the Summer Learning Institute context. By strategically sequencing a set of instructional activities, I explained, we aimed to design a practice-oriented learning trajectory for the interns with whom we worked. At the same time, instructional activities provided us with a tool for targeting mathematical goals for the students with whom our
interns were working. Moving from one instructional activity to the next, I argued, provides a means for controlling the nature and complexity of the mathematical and interactional demands facing interns at different points in their development. The data explored across the set of case studies demonstrate these affordances. We see some evidence that, as interns gained experience and became more proficient with aspects of the work of responding to students’ contributions to mathematical discourse, they indeed faced increasing improvisational demands by moving through the deliberately sequenced progression of instructional activities. In particular, we observed the instructional situations that served as the substance of interactive work becoming increasingly complex as a result of changes in the participation structures of instructional activities. In Case 1, the two problems of practice identified and worked on both involved responding to a single student’s contribution. The turn-taking structure of the game enabled this narrow focus on one student contribution at a time. In Case 2 and Case 3, problematic instructional situations at the center of episodes of collective work involved managing two and then four student solutions respectively. In addition, as the mathematics targeted by each successive activity became more challenging, the anticipated student errors simulated in rehearsal created greater knowledge demands for novices.

Although the instructional situations simulated in rehearsal became increasingly complex over the four weeks, the problem of how to respond to students’ contributions to mathematical discourse to further ambitious mathematical content and practice goals for students remained a central focus. Returning to the entire data set of 73 episodes, the change over time in the length and complexity of episodes can be attributed in part to the increasing complexity in the nature of the instructional situations being addressed by the
group in rehearsal. We also see that over the four weeks increasingly more time was spent in rehearsal working on this problem. When the same problem is addressed over time across increasingly complex instructional situations, novices have opportunities to develop increasingly nuanced understandings of the problem and how to manage it. This is one means by which practice-oriented teacher education can initially reduce the complexity of ambitious teaching without compromising the integrity of the work, an aim identified in the introductory chapter.

Changes in Participation in Rehearsals Over Four Weeks

Looking across the set of case studies, there is also some evidence that interns’ participation in collective work on the practice of responding to students’ contributions to mathematical discourse changed across the four weeks. As novices gained experience, they brought additional resources to the work and assumed greater personal responsibilities for contributing to the learning agenda. Those stepping into the teaching role have more resources on which to draw to improvisationally respond in the moment to demands and complications created when students engage in mathematical discourse, enabling the group to work on increasingly complex situations. Interns participating as students also have greater capacity to translate their developing understanding of student thinking and behavior into their roles as simulated students. Novices can also assume greater responsibilities for their collective preparation to teach immediately following the rehearsal period by anticipating student errors and misconceptions. By the end of the summer, we see evidence that the group has learned to collaboratively construct the problematic instructional situations that serve as the focus for collective inquiry.
As these changes occur over time, the group develops its capacity to engage jointly in reflective practice. Together they share responsibilities for identifying and simulating the problematic situations that demand disciplined improvisation and that become the objects for collective reflection. They work together to interpret instructional situations as those situations develop in interactive simulation. Whether reflecting in the midst of simulation or in pauses, participants draw on a repertoire of strategies, knowledge resources, and conceptual tools developed through prior experience in small-group rehearsals and in their teaching to contribute to making sense of collaboratively emergent instructional discourse. As they make use of these structuring resources and reflect on their use in the company of others who share similar commitments, they develop as ambitious teachers while contributing to the continued development of the practice of ambitious mathematics teaching.

Directions for Future Research

In this dissertation I developed a set of three exploratory descriptive case studies. By focusing on six episodes of work on the practice of responding to students’ contributions to mathematical discourse across three rehearsals drawn from the beginning, middle, and end of the four-week period, these case studies contributed rich descriptions to support a holistic understanding of episodes of work as embedded in multiple levels of activity and design that structure (enable and constrain) interaction and hence novices’ opportunities to develop capacities for disciplined improvisation. However, they represent only a small proportion of the work conducted across the four weeks of rehearsal on the central practice. The entire data set of 73 episodes is worth
returning to in order to trace opportunities to learn and changes in participation over time at a more comprehensive level of analysis.

For example, the data set could be used to examine more systematically changes in individual interns’ participation in rehearsal. Laura and Sam featured prominently in two cases, and we see some evidence of their learning across the four weeks. However, the dissertation does not take individual interns as the unit of analysis. Analyzing individual learning trajectories would enable a more nuanced portrayal of how rehearsal’s impact may vary across novices. To get a fuller picture of the impact of rehearsal, we also need to follow novices out of rehearsal and investigate what they take with them into their work with children in new contexts, both proximally during the Summer Learning Institute and more distally as they assume teaching positions in their first years after completing their certification requirements. This could include examining the contextual factors that help maintain or impede habits of learning in and from practice. We have anecdotal evidence, for example, from our observations of interns in their field placements in the final semester of the ELMAC program that many continue to maintain an openness to observation and feedback and using video records of their teaching for self-reflection. Additionally, comparative investigations of the impact of rehearsal in other contexts on novice learning would be useful to further explore the potential of rehearsal for developing novices’ improvisational capacities and the role of the teacher educator in supporting that development.
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