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Ross School of Business Working Paper Series
Working Paper No. 1255
October 2014

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Firm Characteristics, Consumption Risk, and Firm-Level Risk Exposures

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This Draft: October 31, 2014

Abstract

Firm-level risk exposures and costs of equity are notoriously difficult to estimate. Using a novel approach mapping consumption risk exposures to firm characteristics, we combine the traditional portfolio-level approach to testing asset pricing models with firm-level information to measure firm-level risk exposures. First, at the portfolio level, we investigate the empirical performance of a simple two-factor consumption-based asset pricing model for the cross-section of equity returns. The priced factors in the model are innovations in the growth and volatility of aggregate consumption. Our empirical results show that this model can explain 66% of the cross-sectional variation in returns on a menu of 55 portfolios spanning size, value, momentum, asset growth, stock issuance, and accruals. Second, we use the estimated model to map point-in-time firm characteristics to consumption risk exposures. Through this measurement procedure, we uncover sizeable cross-sectional and time-series variation in firm consumption risk exposures. We verify that sorting on these ex ante consumption risk exposures produces portfolios with consistent ex post risk exposures and predicts cross-sectional variation in future firm equity returns.

*This paper has benefitted from the comments of Victoria Atanasov, Serhiy Kozak, Philippe Mueller, Sorin Sorescu, seminar participants at the Cheung Kong Graduate School of Business, Tsinghua University, and the Universities of Houston, Miami, and Washington, and participants at the 2014 ITAM Conference, 2014 European Finance Association Conference, and the 2014 SAFE Asset Pricing Workshop.
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1 Introduction

One of the principal uses of asset pricing models is in determining required rates of return for investments. Fama and French (1997) undertake estimation of costs of industry capital and encounter two principal difficulties. First, the authors note that the ratio of industry-specific variance to systematic variance is high, rendering estimates of risk exposures imprecise. Second, risk exposures are likely to be time-varying, exacerbating the measurement problem. The authors note that if these problems complicate industry-level estimation of risk exposures, the problems at the firm level are likely to be even more severe. Estimating firm-level risk exposures has been an empirical challenge in finance since early tests of asset pricing models. One of the first acknowledgements of this problem is found in Blume (1970), who notes the high ratio of idiosyncratic to systematic risk in firm returns as a source of estimation error. To alleviate this problem, he advocates the use of portfolios rather than individual firms in testing the Capital Asset Pricing Model, and this approach has become standard in the empirical asset pricing literature (see also Black, Jensen, and Scholes (1972) and Fama and MacBeth (1973)).

The problems encountered when estimating return factor risk exposures are likely to be compounded when estimating exposures based on macroeconomic variables such as consumption growth, as suggested by the models of Lucas (1978) and Breeden (1979). Because macroeconomic variables are generally measured at a low frequency, a long time series is required to obtain precise estimates at even the portfolio level. This need for a long time series is likely to be problematic when firms have a short listing history and when risk exposures are time-varying. Moreover, data points in macroeconomic variables that are particularly informative, such as those that occur during recessions, are observed only at very low frequencies. These issues suggest little hope for measuring the firm-level consumption risk exposure of recent asset pricing models such as Campbell and Cochrane (1999) or Bansal and Yaron (2004). This conclusion is disappointing given the recent empirical success of the consumption-based asset pricing framework.

We suggest a potential solution to connecting consumption-based risks to portfolio-level risk exposures. Using a novel approach of mapping consumption risk exposures to measurable firm characteristics known to be correlated with average returns, we combine the traditional portfolio-level approach to testing asset pricing models with firm-level information to estimate firm-level

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1While asset pricing tests traditionally employ portfolios as test assets, there is a lingering debate about whether firms or portfolios actually serve as the best test assets (see, for example, Ang, Liu, and Schwarz (2010) for a recent contribution). While we focus on the estimation of firm-level risk exposures, we do not directly contribute to this debate.

risk exposures. First, at the portfolio-level, we investigate the empirical performance of a simple two-factor consumption-based asset pricing model for the cross-section of equity returns. The model is tested on a cross-section of equity returns that span a broad set of firm characteristics. After verifying that the model performs well in pricing these assets, we then project the portfolio risk exposures onto the set of portfolio characteristics. Assuming that the mapping between risk exposures and characteristics at the portfolio level holds at the firm level, we use the regression coefficients to construct firm-level risk exposures. To verify our procedure, we then form portfolios sorted on risk exposures and verify that ex post risk exposures generated by our ex ante portfolio formation procedure are generally consistent with their ex ante rankings.

We find that a reduced-form model in the spirit of Bansal and Yaron (2004) can explain 66% of the cross-sectional variation in average returns of 55 portfolios. The priced factors are innovations in the level and the volatility of consumption growth, and the set of assets analyzed comprise size-, book-to-market-, momentum-, asset growth-, accruals-, and stock issuance-sorted portfolio returns. Using an expanding sample procedure, we map portfolio risk exposures to characteristics, and use this mapping to produce a time series of predicted firm-level risk exposures. We generate a set of nine portfolios sorted on consumption growth level innovation and volatility innovation risk exposures from 1983 through 2012. The resulting portfolios have average returns that increase across level innovation exposure quintiles and generally decrease across volatility exposure terciles, consistent with our portfolio-level evidence. Most importantly, the resulting portfolios have ex post exposures to consumption growth level and volatility innovations that are generally increasing across predicted terciles.3

The paper contributes to a growing list of papers that show that macroeconomic risks, in general, and consumption risks, in particular, are priced in the cross-section. Lettau and Ludvigson (2001) show that a conditional consumption CAPM with a measure of the consumption-wealth ratio as the conditioning variable can explain the cross-section of size- and book-to-market-sorted portfolio returns. Parker and Julliard (2005) demonstrate that the covariance of returns with future consumption growth explains returns on the same set of assets. Bansal, Dittmar, and Lundblad (2005) estimate consumption risk as the covariance of innovations in consumption growth with dividend growth, and show that the resulting risk measures explain size-, book-to-market-, and past 12-month return-sorted portfolios. The role of consumption of durable goods is investigated in Yogo (2006), who finds that adding a measure of durable consumption growth helps the basic consumption model explain size- and book-to-market-sorted portfolio returns. Jagannathan and Wang (2007) conjecture that growth in year-over-year fourth quarter consumption explains expected returns better than simple consumption growth, and find that exposure to this source of risk explains

3 These results are also related to the evidence that long run risk models perform poorly out of sample in Ferson, Nallareddy, and Xie (2013). While our results do not directly address their concerns, the rolling estimation results in less in-sample dependence than our full-sample results.

Our work is also related to a growing body of literature investigating the importance of exposures to aggregate volatility in explaining cross-sectional variation in expected returns. Ang, Hodrick, Xing, and Zhang (2006) demonstrate that exposures to innovations in the VIX have power for explaining cross-sectional variation in returns. Bansal, Kiku, Shaliastovich, and Yaron (2013) investigate the importance of integrated variance of industrial production growth for explaining differences in average returns across size- and book-to-market-sorted portfolios. Campbell, Giglio, Polk, and Turley (2013) examine the power of a VAR-based measure of volatility to explain cross-sectional differences in returns. Finally, Boguth and Kuehn (2013) estimate a model with time-varying volatility of consumption growth induced by a regime-switching model. They find that the model explains the cross-sectional variation in a wide set of portfolio returns. While our framework also features time-varying volatility in consumption, its principal goal is to estimate firm-level consumption risk exposures.

The remainder of the paper is organized as follows. In Section 2, we discuss the estimation of consumption innovation risks and the theoretical framework in which these risks are priced. We estimate risk exposures and analyze cross-sectional regressions of portfolio mean returns on risk measures in Section 3. Section 4 presents an analysis of utilizing portfolio characteristics and risk exposures to capture firm-level risk exposures. We make concluding remarks in Section 5.

2 Expected Returns and Consumption Moments

The canonical asset pricing model of Lucas (1978) states that an asset’s price is determined by its conditional covariance with a representative agent’s intertemporal marginal rate of substitution (IMRS),

\[ E_t [\exp (m_{t+1} + r_{i,t+1})] = 1 \] (1)

where \( m_{t+1} \) is the log IMRS, \( r_{i,t+1} \) is the log gross return on a risky asset \( i \), and the price of the asset is normalized to unity. Under the further assumption of conditional joint lognormality of the IMRS and the asset return, we can rewrite equation (1) as

\[ E_t [r_{i,t+1}] + \frac{1}{2} Var_t (r_{i,t+1}) = -E_t [m_{t+1}] - \frac{1}{2} Var_t (m_{t+1}) - Cov_t (m_{t+1}, r_{i,t+1}). \] (2)

Equation (2) emphasizes the fact that expected returns on assets in the cross-section are related to the covariation of innovations in the IMRS and the asset payoff.
A large number of formulations for investors’ utility yield a form for the IMRS that is log-linear in the moments of consumption growth. Two cases are of particular interest for our study. The first is power utility, in which the log pricing kernel

\[ m_{t+1} = \ln \delta - \gamma \Delta c_{t+1}, \]

with \( \gamma \) representing the agent’s relative risk aversion, \( \Delta c_{t+1} \) representing log growth in consumption, and \( \delta \) reflecting the agent’s time preference. The second is Epstein and Zin (1989) utility, in which the log pricing kernel is represented as

\[ m_{t+1} = \theta \ln \delta - \theta \psi \Delta c_{t+1} + (\theta - 1) r_{c,t+1}. \]

In this expression, \( \psi \) represents the intertemporal elasticity of substitution, which is separable from risk aversion, \( \gamma \), \( \theta = (1 - \gamma) / (1 - 1/\psi) \), and \( r_{c,t+1} \) is the log payoff of an asset that pays aggregate consumption as its dividend. Power utility is a special case where \( \gamma = 1/\psi \) and, consequently, \( \theta = 1 \).

Bansal and Yaron (2004) suggest parameterizing the log return on the consumption claim as a linear function of the state variables of the economy and consumption growth,

\[ r_{c,t+1} \approx \kappa_0 + \kappa_1 \mu_{t+1} + \kappa_2 \sigma^2_{c,t+1} + \Delta c_{t+1}, \]

(3)

where \( \mu_{t+1} \) is the conditional expectation of future consumption growth and \( \sigma^2_{c,t+1} \) is its conditional variance. We further assume that

\[ \Delta c_{t+1} = \mu_t + \sigma_t \eta_{t+1}, \]
\[ \mu_{t+1} = \mu_c + \rho \mu_t + \varphi \sigma_t \eta_{t+1}, \]
\[ \sigma^2_{t+1} = E_t [\sigma^2_{t+1}] + \sigma_w w_{t+1}, \]

where \( \eta_{t+1} \) and \( w_{t+1} \) are standard normal i.i.d. shocks. These dynamics are similar to those explored in Bansal and Yaron (2004), but we assume that the shock to consumption and its conditional mean are the same. As a result, consumption growth is an ARMA(1,1) dynamic process with time-varying volatility.

Under the assumption of log-linearity of the return on the consumption claim in the two state variables, the risk premium on an asset can be determined from equation (2) by

\[ E [r_{i,t+1} - r_{f,t}] = -Cov (m_{t+1} - E_t [m_{t+1}], r_{i,t+1} - E_t [r_{i,t+1}]) - \frac{1}{2} Var (r_{i,t+1}) \]
\[ = \pi_1 Cov (\sigma_t \eta_{t+1}, \eta_{t+1}) + \pi_2 Cov (w_{t+1}, \eta_{t+1}) - \frac{1}{2} Var (r_{i,t+1}), \]

(4)
where

\[
\begin{align*}
\pi_1 &= \theta \psi - (\theta - 1) (\kappa_1 \varphi - 1) \\
\pi_2 &= \kappa_2 (\theta - 1), 
\end{align*}
\]

and \( \eta_{i,t+1} = r_{i,t+1} - E_t [r_{i,t+1}] \), the shock to the asset return. This expression indicates that investors expect risk premia to compensate for shocks to first moment of consumption risk, \( \eta_{t+1} \), and second moment of consumption risk \( w_{t+1} \). Bansal and Yaron (2000) note that under power utility, \( \theta = 1 \), and therefore second moment risk will not be compensated in returns.

Converting back to arithmetic returns, the risk premium (4) can be expressed as

\[
E [R_{i,t+1} - R_f,t] = \lambda_1 \beta_{i,\eta} + \lambda_2 \beta_{i,w},
\]

where \( \beta_{i,\eta} = \text{Cov} (r_{i,t+1}, \eta_{t+1}) / \text{Var} (\eta_{t+1}) \) and \( \beta_{i,w} = \text{Cov} (r_{i,t+1}, w_{t+1}) / \text{Var} (w_{t+1}) \) are coefficients of regressing returns on the innovations \( \eta_{t+1} \) and \( w_{t+1} \). This expression suggests that cross-sectional variation in risk premia will be determined by assets’ return exposures to shocks to the first and second moments of consumption growth. Under power utility, \( \lambda_2 = 0 \) and only the conditional covariance of consumption growth levels with innovations in asset returns will bear risk premia. Under the further assumption of i.i.d. consumption growth, \( \beta_{i,\eta} \) can be more simply measured by regressing returns on consumption growth.

We close this section by noting that the expression of equation (5) is isomorphic to the risk premium expression in Bansal and Yaron (2004). However, the model does not necessarily have to be treated as a model of long run risk. The risk premium expression holds regardless of preference parameters and consumption mean dynamics. Thus, one does not have to assume a particular magnitude of elasticity of intertemporal substitution and persistence of the conditional mean of consumption growth. While these issues are important to investigate, they are not the focus of our analysis. Rather, we take a reduced form approach and allow the data to inform us about the magnitudes of prices of risk. Our intent is to highlight sources of cross-sectional variation in risk premia.

3 Cross-Sectional Analysis

3.1 Testing Portfolios

As discussed in the introduction, the goal of our paper is to identify a mapping between characteristics and risk exposures. In order to do so, we consider two aspects of a set of assets on which
to test the fit of the model. First, the set of assets should span a wide variety of characteristics in order to provide as much cross-sectional information as possible for our mapping. Second, the model also needs to fit cross-sectional variation in average returns well. Since average returns of characteristics-based portfolios are related to characteristics themselves, it is likely that a model that fares poorly in describing cross-sectional variation in average returns will generate a noisy map between average returns and risk exposures.

The immediate question that we face is on which characteristics to sort firms to form portfolios. Fama and French (1992) suggest that the cross-section of returns can be summarized by size and book-to-market, and advocate the use of portfolios sorted on these two variables in Fama and French (1993). The use of these portfolios in asset pricing tests, however, have come under recent criticism (Lewellen, Nagel, and Shanken (2010)) due to the ease of fitting a model to their two-factor structure. Harvey, Liu, and Zhu (2014) catalog 316 variables that have been found to have significant power to forecast cross-sectional variation in returns, and Green, Hand, and Zhang (2014) report over 330, and find 24 to be reliably statistically significant. Lewellen (2014) considers a set of 15 predictors, and finds that while 10 have significant $t$-statistics in Fama and MacBeth (1973) regressions, most of the variation in expected returns can be traced to log size, book-to-market, and past 12-month return. These papers suggest that the answer of which characteristics are relevant for testing asset pricing models remains unclear.

We opt to follow evidence in Lewellen (2014) that seven characteristics appear to have statistically significant $t$-statistics in Fama and MacBeth (1973) regressions, regardless of whether the tests are conducted on all stocks, all stocks but micro-caps, or only large stocks. The variables that we use are growth in assets (AG), log book-to-market ratio (BM), log market capitalization (MV), operating profit (OP), past 12-month returns (P12), stock issues (SI), and total accruals (TA). An additional advantage of these assets is that they include characteristics on which Fama and French (2013) suggest factors should be constructed, plus past 12-month return-, stock issue-, and accruals-sorted portfolios. We form portfolios on deciles of these variables with the exception of stock issues. Because the cross-sectional distribution of stock issuance and repurchase is not wide, to ensure that we always have firms in the portfolio we utilize a quintile sort for these assets. Details of the data construction and empirical studies documenting the predictive power of the variables are provided in the Appendix.

Summary statistics for the portfolio returns are presented in Table 1. Data are sampled at the quarterly frequency from the third quarter of 1953 through the fourth quarter of 2012. All portfolios are value-weighted and returns are deflated to real values using the PCE deflator. Mean returns

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4Lewellen (2014) uses return on assets as a measure of profitability. OP is similar, but uses operating income rather than income before extraordinary items, and scales by equity rather than total assets. Our choice of operating profit to equity rather than return on assets is in conformance to the measure of profitability used in Fama and French (2013).
exhibit patterns that are now familiar to readers of the empirical asset pricing literature; average returns increase in the book-to-market ratio, past 12-month return, and operating profitability, and decrease in market value, asset growth, total accruals, and stock issues. None of the average returns are perfectly monotonic in their characteristic deciles, but some characteristics appear to generate more nearly monotonic patterns than others. In particular, past 12-month returns appear to generate very nearly monotonic patterns in average returns, with only one deviation in the deciles; similarly, stock issuance quintiles deviate in monotonicity only in the middle quintile. The data suggest quite a large dispersion in average returns as well; the highest average return is on the tenth decile past 12-month return portfolio of 4.21%, and the lowest is on the first decile past 12-month return portfolio of -0.71%. The remaining sorts generate differences in returns of 1.08% for the difference in the bottom and top stock issuance quintile to 1.69% for the difference in the bottom and top market value decile.

3.2 Estimating Risk Exposures

The model presented in Section 2 suggests that the priced sources of risk are innovations in the mean and the volatility of consumption growth. The question of whether consumption has conditional moment variation is quite controversial. For example, Beeler and Campbell (2012) and Bansal, Kiku, and Yaron (2012) present conflicting evidence on this issue. We approach the issue as simply as possible. We assume that the innovations to consumption growth, $\eta_{t+1}$ are simply de-meaned consumption growth and that volatility of consumption growth can be characterized by the square of these innovations. The volatility innovation, $w_{t+1}$, is the residual from an AR(1) of these squared consumption growth innovations.

A number of studies suggest that there is low-frequency information in asset returns and consumption growth that is relevant for pricing. For example, Bansal, Dittmar, and Lundblad (2005) examine covariance of smoothed dividend growth and a moving average of consumption growth for cross-sectional variation in expected returns, and Hansen, Heaton, and Li (2008) and Bansal, Dittmar, and Kiku (2009) investigate the cointegration of consumption and dividends. With this evidence in mind, we estimate risk exposures via regressing cumulated portfolio returns on cumulated innovations in consumption growth and its volatility,

$$
\prod_{j=0}^{K-1} R_{i,t-j} = a_i + \beta_{i,\eta} \sum_{j=0}^{K-1} \hat{\eta}_{t-j} + \beta_{i,w} \sum_{j=0}^{K-1} \hat{w}_{t+1} + e_{i,t},
$$

for different windows $K$, where $R_{i,t-j}$ is the gross real return on portfolio $i$.

We find that the model seems to perform best in the cross-section for a window $K = 4$, and
present associated risk exposures for this window in Tables 2 and 3. Comparing top and bottom decile or quintile risk exposures in Table 2 suggests that the level risk exposure captures extreme portfolio return differences. For example, the differences in first and tenth decile asset growth, market value, stock issuance, and total accruals consumption innovation risk exposures are positive, while the differences in tenth and first decile book-to-market and past 12-month return consumption innovation risk exposures are also positive. However, the patterns are not consistent across deciles and, in the case of operating profit, the risk exposures tend to decrease across deciles, while average returns increase. The table suggests that the model might adequately, but not perfectly explain cross-sectional variation in portfolio average returns.

Exposures to innovations in consumption volatility are presented in Table 3. The majority of the portfolio exposures are negative, consistent with the idea that equity is a poor hedge against economic downturns, which are associated with increased volatility of consumption growth. The results also suggest that some sets of portfolios have differences in extreme decile exposures that are consistent with patterns in returns. For example, asset growth and operating profit portfolios have differences in extreme decile volatility exposures that are negatively correlated with the differences in average returns. Since an investor should be willing to pay a premium for an asset that pays off in bad economic times, this implied negative price of risk seems economically intuitive. Unfortunately, the pattern is reversed for size-sorted and past 12-month-sorted portfolios, and essentially flat for issue- and accruals-sorted portfolios. Further, the departures of monotonicity across deciles are perhaps even stronger for these risk exposures than the consumption growth risk exposures.

3.3 Cross-Sectional Regression Results

The standard approach to investigating whether risk exposures are related to average returns is the two-stage approach where returns are regressed on sources of risk and average returns are then regressed on the resulting risk exposure estimates. The first stage estimates are discussed in the previous section, we now examine cross-sectional regressions of the form

\[
\bar{R}_i - \bar{R}_f = \gamma_0 + \gamma_\eta \hat{\beta}_{i,\eta} + \gamma_w \hat{\beta}_{i,w} + u_i,
\]

(7)

where \(\bar{R}_i\) is the time series average of the return on portfolio \(i\), \(\bar{R}_f\) is the mean real quarterly compounded return on a Treasury Bill closest to one month to maturity from CRSP, and \(\hat{\beta}_{i,\eta}\) and \(\hat{\beta}_{i,w}\) are first stage estimates of univariate regressions of portfolio \(i\)'s return on the mean and volatility innovations, \(\eta_t\) and \(w_t\), respectively. In addition to the unrestricted model above, we also examine specifications where we consider the explanatory power of mean innovation and volatility

\(^5\)Results for risk exposures and cross-sectional regressions using windows \(K = 1\) through \(K = 8\) are available upon request from the authors.
innovation risks alone, restricting $\gamma_\eta = 0$ and $\gamma_w = 0$, respectively.

Results of the cross-sectional regressions are presented in Panel A of Table 4. For each of the three specifications, we present estimates of the intercept and slope coefficients, standard errors of the estimates, and adjusted regression $R^2$. The standard errors are corrected for estimation error in the first stage using the correction derived by Shanken (1992). In addition, we present in parentheses under the $R^2$ the 95% critical value of the model $R^2$ under the null that the risk measures are unrelated to the average returns. This critical value is motivated by the recommendations of Lewellen, Nagel, and Shanken (2010), who suggest that the cross-sectional $R^2$ may overstate the model fit. The critical value is calculated by generating 5000 random samples with 238 time series observations of two normally distributed variables with mean zero and standard deviation $\sigma_\eta$ and $\sigma_w$ to match sample standard deviations of the mean and volatility innovations. We regress returns on our sample assets on the random variables, and then perform second stage regressions of the mean returns on the resulting regression coefficients. Adjusted $R^2$ for the second stage regressions on the simulated risk measures are used to construct the null distribution of the adjusted $R^2$.

The first two rows of the table present parameter estimates and standard errors for the model with only growth rate innovation risk exposures priced, setting $\gamma_w = 0$. Consistent with our discussion in the previous section, there is evidence of a univariate relation between average returns and growth rate innovation risk exposures. The point estimate, 0.082, suggests that growth rate innovation risk exposures are positively related to average returns, which is consistent with the predictions of a consumption-based asset pricing model, and the point estimate is statistically significant at over five standard errors from zero. The model fares surprisingly well considering the well-documented poor performance of the simple consumption CAPM with an adjusted $R^2$ of 30.94%. While this adjusted $R^2$ does not exceed the 95% critical value implied in simulation, it does exceed the 90% critical value. Finally, the null hypothesis that the intercept is equal to zero, indicating a zero-beta rate close to the one-month T-Bill return, cannot be rejected. In summary, exposures to consumption growth innovations appear to have significant explanatory power for cross-sectional variation in returns.

In the next two rows of the table, we present the specification in which only volatility innovation risk is priced, $\gamma_\eta = 0$. In the previous section, we noted that some sets of extreme portfolios had volatility innovation risk exposures that correlated negatively with average returns, while others had positive correlations. These conflicting correlations have a large impact on the fit of a model with only priced volatility risk; the point estimate of 0.011 is positive and statistically insignificant, and volatility risk exposure by itself explains virtually none of the cross-sectional variation in average returns with an adjusted $R^2$ of -0.85%. There is limited evidence that volatility innovation risks exhibit explanatory power for average returns independent of growth innovation exposures.
We last turn to the unrestricted model in the final two rows of the table. The multiple regression results indicate that the model fares relatively well in explaining cross-sectional variation in average returns. The point estimate for the price of consumption growth exposure of 0.119 is positive and over six standard errors from zero. As expected, the point estimate for volatility innovation exposure of -0.054 is negative, indicating that investors are willing to pay a premium for assets that hedge against high volatility, and over three standard errors from zero. Moreover, the model has a reasonable cross-sectional fit of the average returns, with an adjusted $R^2$ of 41.14%, just slightly below the 95% critical value of 45.98%. Thus, our results indicate that a model with priced level and volatility consumption growth risks performs adequately at explaining cross-sectional variation in returns sorted across the seven characteristics used in this paper.

The exercise in the next section of the paper, in which we map consumption risk exposures to characteristics presumes that characteristics are good instruments for risk exposures. Our regression results suggest that predicted returns are approximately 65% correlated with observed average returns; since these returns are correlated with characteristics, the results imply a correlation between the characteristics and risk exposures. The question is whether this 65% correlation (41.14% adjusted $R^2$) is correlated enough. To get some insight into this question, it is useful to examine for which characteristic portfolios the model appears to provide good fit, and for which it provides poor fit. In Panel A of Figure 1, we plot actual average returns against those predicted by the multiple regression and report pricing errors in Panel B of Table 4. Our interpretation of the figure is that the model has particular difficulty with past 12 month return sorted portfolios and operating profit sorted portfolios. The model predicts expected returns that are 198 basis points per quarter higher for the first operating profit decile and 108 basis points per quarter higher for the second decile. The model also predicts average returns that are 109 basis points, 141 basis points, and 113 basis points higher for the first, second, and third decile past 12-month portfolios, respectively.

The evidence suggests that the model has a particularly difficult time fitting operating profit-sorted portfolios. Because our goal is to map risk exposures into characteristics as well as possible, we consider the fit of the model omitting these assets. Results of cross-sectional regressions for this reduced set of 55 portfolios are presented in Table 5. The results are qualitatively similar, but quantitatively improved relative to those in Table 4. Consumption growth exposure bears a positive price of risk with a point estimate of 0.113 that is nearly eight standard errors from zero. On its own, growth risk exposure explains a substantial portion of the variation in average returns, with an adjusted $R^2$ of 55.14%. Volatility risk exposure has a positive but insignificant price of risk and by itself explains very little cross-sectional variation in returns, with an adjusted $R^2$ of 1.18%. However, when both exposures are included in the regression, consumption growth risk retains a positive and significant price, with a point estimate of 0.146, and volatility innovation
risk bears a negative and statistically significant price. The two risk exposures combine to explain approximately two-thirds of the variation in average returns, with an adjusted $R^2$ of 64.46% (65.77% unadjusted).

A scatter plot of the model fit is presented in Panel B of Figure 1. As shown in the figure, and indicated by the improved model fit, pricing errors for most portfolios are considerably smaller in magnitude when operating profit portfolios are omitted. The model continues to have some difficulty with some of the past 12-month-sorted portfolio returns, predicting average returns that are 91, 146, and 108 basis points per quarter higher than those observed in the data for past 12-month-sorted portfolios in deciles one, two, and three, respectively. However, the model fares well in pricing the portfolios otherwise, with a mean absolute pricing error of 25 basis points per quarter, compared to 36 basis points when operating profit portfolios are included.

We proceed with the remainder of our analysis using only six characteristics; asset growth, book-to-market, market value, past 12-month return, net stock issues, and total accruals. In unreported results, we find that combinations of portfolios omitting a subset of these assets has little impact on model fit. Our purpose in this exercise is not to ignore the fact that the model has difficulty fitting operating profit-sorted portfolios. In fact, we believe that understanding this failure is an important question. However, since our goal is to map risk exposures into characteristics, we opt to use a set of characteristics for which the risk exposures in question best explain variation in average returns across the characteristic-sorted portfolios.

### 3.4 Alternative Models

In this section, we consider alternative models that might be used to map risk measures into characteristics. We examine alternative consumption-based pricing models and models that are based on returns as priced factors. Since our goal is to best map risk exposures into characteristics, rather than to advocate for any particular pricing model, we wish to ensure that we could not achieve superior results for the data that we consider with an alternative pricing model.

#### 3.4.1 Consumption-Based Pricing Models

The first alternative consumption-based model that we consider is a conditional consumption CAPM from Lettau and Ludvigson (2001). The authors propose using a measure of the consumption-wealth ratio as a conditioning variable, $cay_t$. The conditioning variable is the cointegrating residual from the trivariate cointegrating relation between per capita aggregate consumption, asset wealth, and labor income (measuring the dividend to human wealth). Following their example, we estimate
the following two-stage cross-sectional regression:

\[ R_{i,t+1} - R_f = a_i + \beta_{i,cay} cay_t + \beta_i \Delta c_{t+1} + \beta_{i,cay} \Delta c_{cay} \cdot \Delta c_{t+1} + e_{i,t+1} \] (8)

\[ \bar{R}_i - \bar{R}_f = \gamma_0 + \gamma_{cay} cay_t + \gamma \Delta c_{i,cay} + \gamma_{cay} \Delta c_{i,cay} \Delta c + u_i \] (9)

where \( \Delta c_{t+1} \) is the growth in per capita consumption of nondurables and services. Data for \( cay_t \) are obtained from Martin Lettau’s web page.\(^6\)

The second alternative is an unconditional consumption CAPM using a measure of ultimate consumption examined in Parker and Julliard (2005). Ultimate consumption growth is defined as the s-period forward growth in consumption of nondurables and services,

\[ g_{t+1,t+s+1} = \sum_{j=0}^{s} \Delta c_{t+j+1} \]

where, following the authors’ evidence, we set \( s = 11 \). We consider the case of their log-linearized model with a constant risk-free rate,

\[ R_{i,t+1} - R_f = a_i + \beta_{i,g} g_{t+1,t+s+1} + e_{i,t+1} \] (10)

\[ \bar{R}_i - \bar{R}_f = \gamma_0 + \gamma g \beta_{i,g} + u_i \] (11)

Consumption data are the same as those used earlier in the paper; however, due to the horizon \( s \), the return data are truncated in 2009.

A third alternative is investigated in Bansal, Dittmar, and Lundblad (2005), who suggest that the covariance of portfolio cash flows with a measure of the conditional mean of consumption growth explains cross-sectional variation in returns. Their measure of the conditional mean is a moving average of consumption growth,

\[ x_t = \frac{1}{K} \sum_{j=0}^{K-1} \Delta c_{t-j} \]

where the authors set \( K = 8 \). The model investigated is specified as

\[ \Delta d_{i,t+1} = a_i + \beta_{i,x} x_t + e_{i,t+1} \] (12)

\[ \bar{R}_i - \bar{R}_f = \gamma_0 + \gamma x \beta_{i,x} + u_i \] (13)

where \( \Delta d_{i,t+1} \) is the log growth in real dividends per share paid by portfolio \( i \). Portfolio dividends

\(^6\)http://faculty.haas.berkeley.edu/lettau/data.html. Thanks to Martin Lettau for making these data available.
per share are constructed through the recursion

\[ V_{i,t} = V_{i,t-1} \left( 1 + R_{i,t}^x \right) \]

\[ D_{i,t} = V_{i,t-1} \left( R_{i,t} - R_{i,t}^x \right), \]

where \( R_{i,t}^x \) is the ex-dividend portfolio return, \( D_{i,t} \) is the arithmetic dividend per share and \( V_0 = 100 \). Seasonalities are removed from dividends by summing over twelve months.

The final consumption-based alternative that we consider is the durable consumption model of Yogo (2006). In his framework, preferences are non-separable in consumption of nondurables, services, and durable goods. A log-linear approximation to the model results in the following specification:

\[ R_{i,t} - R_{f,t} = a_i + \beta_{i,nds} \Delta c_{nds,t+1} + \beta_{i,d} \Delta c_{d,t+1} + \beta_{i,m} R_{m,t+1} + e_{i,t+1} \]  

(14)

\[ \bar{R}_i - \bar{R}_f = \gamma_0 + \gamma_{nds} \beta_{i,nds} + \gamma_d \beta_{i,d} + \gamma_m \beta_{i,m} + u_i, \]  

(15)

where \( \Delta c_{nds,t+1} \) is the growth in log real per capita nondurable and services consumption, \( \Delta c_{nd,t+1} \) is growth in consumption of durable goods, and \( R_{m,t+1} \) is the return on the value-weighted market. The durable goods consumption data are taken from Motohiro Yogo’s website.\(^7\) These data are available through December, 2001.

The results of estimation of the four consumption-based alternative models are shown in Table 6. All four of the models fare reasonably well in terms of generating statistically significant coefficients on their hypothesized sources of priced risk. Ultimate consumption growth (point estimate=3.001, SE=1.347), consumption growth (point estimate=0.466, SE=0.113), durable consumption growth (point estimate=0.959, SE=0.332), and conditional mean of consumption growth (point estimate=0.166, SE=0.032) all bear positive prices of risk with point estimates more than two standard errors from zero. In terms of adjusted \( R^2 \) the four models display more heterogeneity in performance; the ultimate consumption model explains the least cross-sectional variation in returns with an adjusted \( R^2 \) of 13.82%, while the durable consumption model explains 52.72% of cross-sectional variation in returns. All four models perform better in terms of adjusted \( R^2 \) on a similar set of assets as those on which they were originally tested. On the size- and book-to-market portfolios, the conditional CCAPM, ultimate consumption model, and durable goods models explain 65.73%, 66.00%, and 54.12% of cross-sectional variation in returns respectively. The cash flow conditional consumption mean model explains 54.04% of the cross-sectional variation in returns sorted on size-, book-to-market, and past 12-month returns.

The conclusion that we draw from these results is that a model based on risks in innovations

\(^7\)https://sites.google.com/site/motohiroyogo/. Thanks to Moto Yogo for making these data available.
in the growth and volatility of nondurables and services consumption goes very far in explaining cross-sectional variation in returns. While the use of conditioning information, ultimate consumption growth, cash flows, and durable goods are all potentially important in understanding cross-sectional variation in returns, the moment innovations model dominates these models in terms of cross-sectional explanatory power. Our interpretation is that a model with innovations in growth and volatility of nondurables and services consumption represents a strong starting point for understanding cross-sectional variation in returns.

3.4.2 Return Factor Models

Our last analysis in this section is the performance of models with return-based rather than consumption-based sources of risk. We examine four return factor models. The first is the Fama and French (1993) three factor model, with the excess market return ($MRP$), the excess return on a small firm portfolio over a large firm portfolio ($SMB$), and the excess return of a high book-to-market ratio portfolio over a low book-to-market ratio portfolio ($HML$). The second augments the Fama and French (1993) three-factor model with the excess return of past winners over a portfolio of past losers ($UMD$), as proposed in Carhart (1997). The third factor model is a five-factor model from Fama and French (2013), augmenting the market, size, and book-to-market factors with the excess return on a high operating profitability portfolio over a low operating profitability portfolio ($RMW$), and the return on a portfolio of low asset growth in excess of the return on a portfolio of high asset growth ($CMA$). Finally, we examine the performance of the four-factor model of Hou, Xue, and Zhang (2014), motivated by the q-theory of investment, which utilizes market, size, return on equity, and investment portfolios. Data are aggregated to the quarterly frequency and converted to real using the PCE deflator.

As above, we examine two stage regressions where the first stage consists of univariate regressions of returns on the 55 portfolios in our sample on each of the three return risk factors. In the second stage, we regress average returns in excess of the risk free rate on the risk exposures from the second stage. We again use the Shanken (1992) correction to compute standard errors and construct the distribution of adjusted $R^2$. Results of these regressions are presented in Table 7. The first row of the table documents the performance of the Fama and French (1993) three factor model. The model explains a respectable 48.22% of the cross-sectional variation in average returns, but performs poorly along a number of important dimensions. While the price of size risk is positive and statistically significant, the prices of market and book-to-market risk are negative,

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8 We obtain data on the market portfolio, risk free rate, six portfolios sorted on size and book-to-market, and six portfolios sorted on size and momentum from Kenneth French’s website, \texttt{http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html}. Our thanks to Kenneth French for making these data available.

9 Thanks to Chen Xue and Lu Zhang for providing the data for these factors. As these data start in 1972, the results are not perfectly comparable to the results from the remaining models.
with the latter not statistically distinguishable from zero. The performance problems of this model are most likely due to the inclusion of momentum portfolios in the set of test assets, which pose a particular problem for the three factor model as documented in Fama and French (1996).

The conjecture that momentum is a key source of problems for the Fama and French (1993) model is confirmed by the second set of results, augmenting the model with a momentum factor. This model performs quite well, explaining 87.60% of cross-sectional variation in average returns. Moreover, the size, book-to-market, and momentum factor prices of risk are all positive and statistically significant, with only the market risk premium failing to exhibit significance. The only empirical weakness of the model is the large and significant intercept term, which implies a large zero-beta rate of return.

The third set of results in the table are for the five factor model proposed in Fama and French (2013). We construct these factors using the intersections of portfolios sorted on the medians of size, book-to-market ratio, operating profitability, and asset growth. As such, the size and book-to-market factors are different than those in the three-factor results. While the model explains 37.86% of the cross-sectional variation in returns, only two of the prices of risk, those of the market and book-to-market factor are statistically significant. Both risk prices, however, are negative. These results suggest that the four-factor model does not describe average returns on this set of portfolios well, perhaps again due to the inclusion of the momentum portfolios in the set of test assets.

The final set of results are for the investment-based factor model of Hou, Xue, and Zhang (2014). Again, the model performs quite respectably, generating a cross-sectional adjusted $R^2$ of 43.76%. Three of the prices of risk are statistically significant; the market risk premium, size, and return on equity. Only the investment factor is not statistically distinguishable from zero. Both the size and return on equity factors bear positive prices of risk, as expected. However, like the other four-factor model, the model implies a zero beta rate of return that is significantly higher than that of the risk free rate.

The goal of this section is simply to show that a consumption-based model based on economic theory with macroeconomic variables can perform comparably to, or better than a return-based factor model.¹⁰ The results seem to suggest that in order to outperform the consumption-based model with a return factor model, one needs to include a momentum return factor. We believe the choice of a return-based factor model for the computation of required rates of return rather than a model based on economic variables is a choice of the end user. The results here simply suggest that the consumption-based model represents a viable alternative.

¹⁰The four return factor model of Hou, Xue, and Zhang (2014) is in fact based on $q$-theory of corporate investment. We distinguish the model from the model in our paper simply by the use of returns as factors.
4 Firm Characteristics and Risk Exposures

The evidence in Section 3 suggests that exposures of asset returns to innovations in the mean and volatility of consumption growth go a long way in explaining cross-sectional variation in average returns on a set of assets formed on a broad set of characteristics. In this section, we speculate that the information in the relation between portfolio risk exposures and characteristics might be informative about firm level exposures to consumption risks. We briefly discuss theoretical links between characteristics and risk exposures, detail our procedure for measuring ex ante risk exposures, and present empirical evidence as to the time series of risk exposures and implied expected returns, as well as evidence suggesting that the ex ante predicted risk exposures align with ex post estimated risk exposures in the data.

4.1 Why are Characteristics and Risk Exposures Related?

One of the earliest suggestions that characteristics might proxy for risk measures is provided by Fama and French (1992), who argue that the fact that size and book-to-market capture much of the cross-sectional variation in average returns must be due to the characteristics capturing information in omitted risk measures. This intuition is formalized in the context of an investment-based asset pricing model in Zhang (2005). He shows that in the context of countercyclical risk premia and asymmetric adjustment costs of investment, assets in place can be riskier than growth options during economic downturns, leading to a larger unconditional risk premium for firms with a higher proportion of assets in place. As a result, the book-to-market ratio represents a proxy for the higher risk exposure of firms with more assets in place.

This point is made more explicit in Lin and Zhang (2013), who argue forcefully that characteristics and risk factor covariances represent two sides of the same coin. For example, the authors show that in a simple production-based model, the risk premium on an equity can be written as

\[
E_t \left[ r_{i,t+1}^S \right] - r_{f,t} = \beta_i^M \lambda_M = \frac{E_t \left[ \Pi_{i,t+1} \right]}{1 + a \left( I_{i,t}/K_{i,t} \right)} \beta_i^M = \left[ \frac{E_t \left[ \Pi_{i,t+1} \right]}{1 + a \left( I_{i,t}/K_{i,t} \right)} - r_f \right] / \lambda_M,
\]

where \( r_{i,t}^S \) is the return on firm \( i \)'s equity, \( \Pi_{i,t} \) is a profit function, \( I_{i,t} \) is investment, \( K_{i,t} \) is capital, \( a \) is an adjustment cost parameter, \( \beta_i^M \) is the coefficient from regressing the equity return on the stochastic discount factor, and \( \lambda_M \) is the price of stochastic discount factor risk. This expression makes it clear that the firm's risk exposure can be expressed as functions of profitability and investment intensity, as captured by the investment-to-capital ratio. In equilibrium, Lin and Zhang
(2013) note that the denominator $1 + a (I_i/K_i,1)$ will be the market-to-book ratio. Other characteristics can enter into these equations through more complicated adjustment costs, corporate taxes, and debt, as in Liu, Whited, and Zhang (2009).

Investment-based asset pricing has been linked to a number of firm characteristics in addition to the book-to-market ratio, including stock issues (Lyandres, Sun, and Zhang (2008) and Li, Livdan, and Zhang (2009)), accruals (Wu, Zhang, and Zhang (2010)), and momentum (Liu and Zhang (2014)). The implication is that these characteristics are related to a firm’s investment return and, consequently its equity exposure to risks in the stochastic discount factor. In our context, innovations in the stochastic discount factor are driven by innovations in consumption growth and its volatility. Hence, the investment-based asset pricing literature suggests a mapping between these consumption risk exposures and firm characteristics.

4.2 Relating Characteristics and Risk Exposures in the Cross Section

Since the portfolios we use in our analysis are created on the basis of characteristics, using these characteristics as instruments for portfolio formation generates average returns related to the characteristic deciles, and our risk exposures explain variation in these average returns, it follows that risk exposures and characteristics should be correlated. We first investigate this link by simply regressing our estimated consumption growth and volatility innovation betas on average portfolio characteristics,

$$
\hat{\beta}_{i,\eta} = a_{i,\eta} + b'_{i,\eta}\bar{x}_i + e_{i,\eta} \quad (16)
$$

$$
\hat{\beta}_{i,w} = a_{i,w} + b'_{i,w}\bar{x}_i + e_{i,w} \quad (17)
$$

where $\bar{x}_i$ is a vector of the time series average of portfolio $i$’s characteristics; size, book-to-market ratio, past 12-month return, asset growth, total accruals, and stock issuance. We construct portfolio characteristics by value-weighting the individual characteristics of each asset in the portfolio over 714 months from July, 1953 through December, 2012.

Regression results are shown in Table 8. The first set of results are for regressions of the consumption growth innovation risk exposure on average characteristics. As indicated by the results, there is approximately as strong a relation between the consumption growth innovation risk exposure and average characteristics and there is between the consumption growth innovation risk exposure and average returns. The adjusted $R^2$ indicates that 63.11% of the variation in consumption growth risk exposures can be traced to variation in characteristics. Of these characteristics, size, past 12 month return, and total accruals appear to be statistically significantly related to consumption growth risk exposures. There is little evidence for a statistical link between the remaining
three characteristics and these risk measures.

The mean characteristics perform similarly well in explaining portfolio return exposure to volatility innovations. The adjusted $R^2$ of 54.73% is substantially larger than results from univariate regressions of average returns on risk exposures. Because the regression is not forced to simultaneously fit an increasing relation between volatility innovation and characteristics and one characteristic-based sorted portfolio and a decreasing relation for another, the fit between characteristics and risk exposures is higher than that of risk exposures and average returns. Interestingly, two of the three variables that had no statistically significant explanatory power for consumption growth innovation betas have significant explanatory power for volatility innovation betas. Average asset growth is positively and significantly related to the volatility beta and the book-to-market ratio is negatively related to the volatility beta. Market value is also statistically significantly related to the volatility beta, and the remaining three variables are not statistically significant at conventional levels.

These results suggest that the characteristics we examine do have the potential to instrument for risk exposures. The mapping is not perfect, and some variables are statistically insignificant. However, the statistical significance is known to us only ex post, so we elect to include all characteristics in our procedure. The introduction of insignificant characteristics may, however, generate noise in the prediction of risk exposures, an issue to which we will turn later in the paper.

### 4.3 Measuring Firm Risk Exposures

We assume that the portfolio-level relation between risk exposures and characteristics holds at the firm level. That is, the relation between firm-level exposures and portfolio-level exposures and characteristics is given by

$$
\beta_{i,e,t} = a_t + b_t' x_{it} + u_{it}
$$

$$
\beta_{p,e,t} = \sum_{i=1}^{N} \omega_{i,t-1} (a_t + b_t' x_{it} + u_{it})
$$

$$
= a_t + b_t' x_{pt} + u_{pt},
$$

where $i$ indexes firms, $p$ represents portfolios, $\omega_i$ is the weight on asset $i$ in the portfolio, and $e = \eta, \omega$. Consequently, by estimating the coefficients $a_t$ and $b_t$ at the portfolio level, we can use the coefficients to retrieve firm-specific risk measures at the firm level. This translation between the portfolio and the firm level relations of risk exposures and characteristics is similar to the use of portfolio-level CAPM betas to measure firm-level betas in Fama and French (1992).

The specific procedure by which we apply portfolio-level estimates to the firm level proceeds as
follows:

1. Somewhat arbitrarily, we choose a time span $T = 120$, or 30 years of data from July, 1953 to June, 1982 to estimate initial risk exposures. We calculate demeaned consumption growth over this time period to obtain a consumption growth innovation, and a fit an AR(1) model to the squared innovations to obtain the volatility innovation. These innovations are summed over four quarters, and overlapping annual cumulative returns through June, 1982 are regressed on the summed innovations to obtain risk measures. We then regress the resulting risk exposures on the portfolio characteristics, $X_{it}$ for each month July, 1982 through September, 1982, and retain the regression parameter estimates. Note that this approach uses only data available at June, 1982 to estimate both consumption dynamics and relations between characteristics and risk exposures.

2. We roll forward one quarter, augmenting the consumption growth and return data by the new quarter’s observations, and re-estimate the model of consumption dynamics and accompanying risk exposures. We then regress characteristics for each month October, 1982 through December, 1982 on the risk exposures. Since our financial statement characteristics use the timing convention of Fama and French (1993), these characteristics are based on financial statement data known as of June, 1982. We continue this procedure, expanding the window over which the model of consumption dynamics and regression of returns on innovations is estimated until reaching the end of the sample.

3. We take the regression estimates from steps 1 and 2 and calculate betas at the firm level given the firm-level characteristics and the point estimates. We rank firms into terciles on the basis of the estimated growth innovation beta and volatility innovation beta. We form univariate-sort portfolios sorted just on tercile sorts and bivariate-sort portfolios based on the intersection of growth innovation beta and volatility innovation beta terciles.

In panel B of Table 8, we present the results of Fama and MacBeth (1973) style estimated coefficients and standard errors. The coefficients are the means of the coefficients estimated in the time series, and standard errors are calculated as the standard deviation of the coefficients scaled by the square root of the number of time series observations. The results indicate some differences in estimates of and significance of relations between risk exposures and characteristics. Considering first the relation between characteristics and consumption growth innovation exposures, we observe that all six characteristics have statistically significant explanatory power in the time series regressions, in contrast to the regressions on average characteristics. Second, while the average of the coefficients on market value and book-to-market ratio are remarkably similar to the coefficients in the regressions of risk measures on average characteristics, those of other characteristics are substantially different. In particular, the coefficient on past 12 month returns is roughly half that
of the earlier regressions, and the coefficients on asset growth and total accruals are approximately two thirds those of the earlier regressions. These results suggest that the relation between risk exposures and some of the characteristics may exhibit substantial time variation.

The differences in the relation between volatility innovation beta and characteristics between the two methods are even larger. Again, all six characteristics have significant explanatory power for volatility innovation betas in the Fama and MacBeth (1973) style regressions. The magnitude of the coefficients from these time series regressions range from 20% of the regressions on average characteristics for total accruals to 238% for stock issues. Only market value appears to have a relatively similar coefficient between the two estimation procedures. These differences may again be due to time variation in exposures, or the relation between risk exposures and characteristics. Alternatively, the differences may be indicative of measurement noise. The adjusted $R^2$ for the volatility coefficients range from approximately 20% to 60%, with a trend toward higher $R^2$ as the expanding sample incorporates more observations. In contrast, the adjusted $R^2$ for the growth innovation coefficients is more stably in the 50% to 70% range, with little evidence of trends in explanatory power.

4.4 Time Series of Ex Ante Risk Exposures and Expected Returns

As discussed above, we form tercile portfolios on the basis of predicted consumption innovation and volatility innovation betas, as predicted by the mapping between portfolio characteristics and portfolio risk exposures. The betas are depicted in Figure 2. Panel A depicts the ex ante exposures to consumption growth innovations for portfolios sorted on these exposures and Panel B depicts ex ante exposures to consumption growth volatility innovations for portfolios sorted on volatility innovation exposures. The plots show that the implied ex ante betas exhibit a large degree of high-frequency variation. It is not clear whether this variation captures true variation in risk exposures, the volatility of some of the characteristics used to form betas (in particular the past 12-month return), or noise in the mapping between portfolio and firm-level betas. However, some common driver seems to linking betas across terciles. The tercile one consumption innovation beta is 85% correlated with the tercile two beta and 73% correlated with the tercile three beta, while the tercile two beta is 93% correlated with the tercile three beta. Volatility betas are somewhat less correlated. The tercile one beta is 75% correlated with the tercile two volatility beta, but only 58% correlated with the tercile three volatility beta. The correlation between tercile two and three volatility betas is 94%.

There also appear to be interesting low-frequency trends in the betas, that are more apparent upon forming moving averages of the estimates. In Figure 3 we depict four quarter moving averages of the betas of each of the tercile portfolios, with the consumption growth innovation betas of the
consumption growth innovation beta-sorted portfolios in Panel A and the volatility innovation betas of the volatility beta-sorted portfolios in Panel B. Mean betas of the top tercile portfolio show a pronounced decline from the 1980s through the end of the 1990s, stabilizing in the 2000s, and tending to rise after the Great Recession of the late 2000s. The lowest tercile portfolio betas remain more stable in the 1980s, decline in the 1990s, and appear to increase in the 2000s. Further, the plots indicate a tendency of the betas to increase in recessions, although the increase does not perfectly map into the length of the recessions as dated by the NBER.

The volatility betas show somewhat less marked trends, and some of the trends differ across terciles as shown in Figure 3, Panel B. For example, through the 1980s, there is a generally increasing trend in the high volatility tercile betas, but a decreasing trend in the low volatility terciles. During the 1990s, the low volatility tercile betas exhibit a much more pronounced decrease than the high volatility tercile betas, while during the 2000s, the high tercile betas appear to generally decrease, while the low tercile betas are close to flat. Finally, during the Great Recession, there is a very large and pronounced decline in the high volatility tercile betas from approximately 11 to less than zero. While this decline happens across all terciles, it is sharpest in the high volatility beta quintile.

In Figure 4, we plot the risk premia implied by our point estimates of cross-sectional regressions and the ex ante betas for both sets of portfolio sorts. In order to highlight low-frequency variation in the risk premia, we again plot four quarter trailing moving averages. Not surprisingly, since the only variation in the plots is due to variation in betas, the patterns in risk premia mirror those shown in the betas. There is a tendency for the risk premia of the portfolios to fall over expansions and rise over recessions. The risk premium on the top mean exposure tercile portfolio falls over the 1990s from approximately 3.5% to 2.5%, while the risk premium on the bottom tercile portfolio falls from approximately 1.5% to 1.0%. Risk premia are higher in the 2000s, and increase sharply during the Great Recession, with the premium on the top tercile firms reaching 4.0% by the end of the recession and that of the bottom tercile firms reaching 2.0%.

Risk premia of the volatility beta portfolios exhibit complex co-movement, as shown in Panel B of Figure 4. We note several patterns that appear to us to be important in the data. First, while risk premia appear to increase in recessions and decrease in expansions, the pattern appears more pronounced for the high volatility tercile portfolios than the low volatility tercile portfolios. Second, the differences in risk premia appear to tend to converge in bad economic times and diverge in good economic times. This pattern is particularly pronounced in comparing the inter-recession period of the 1990s and 2000s to the Great Recession. Finally, risk premia on the low and high beta tercile portfolios (and for that matter, the second tercile portfolio) frequently cross; in some times the low volatility beta portfolio has a higher risk premium, and in others the high volatility beta portfolio has a higher risk premium. One clear implication of this result is that univariate sorting on volatility beta does not isolate the effects of volatility risk in portfolio returns from
growth innovation risk.

### 4.5 Returns on Portfolios Formed on Ex Ante Risk Exposures

Our final analysis examines the returns on portfolios formed on terciles of ex ante risk exposures. As discussed above, we form both tercile portfolios and intersections of portfolios on growth innovation betas and orthogonalized volatility innovation betas. The goal of this analysis is twofold. First, we want to examine whether there are average risk premia in the data that are consistent with our estimates of a positive price of risk for growth innovation exposure and negative price of risk for volatility innovation exposure. Second, we estimate ex post loadings of returns on growth and volatility innovations to ensure that our ex ante sorting procedure produces portfolios that are ex post consistent with our rankings.

Means of univariate sort portfolio returns are shown in Table 9. Returns are nominal and sampled at the monthly frequency over the period July, 1983 through December, 2012. Sorting on ex ante growth innovation betas produces a monotonically increasing pattern in average returns, ranging from 91 basis points per month for the first tercile to 134 basis points per month for the third tercile, or a 43 basis point premium across the terciles. As points of comparison, the premium on the market return in excess of the risk free rate over the same period is 55 basis points per month, while premia on the Fama and French (1993) SMB and HML portfolios are 6 and 34 basis points per month, respectively. The average return differential between top and bottom tercile growth innovation-sorted portfolios is statistically significant as calculated using Newey-West standard errors.

Ranking on volatility betas is somewhat more problematic. Sorting on these betas produces an essentially flat relation between volatility tercile and average return, contrasting with the decreasing relation implied by the risk premium estimates from our cross-sectional analysis. However, this result is perhaps not surprising given the plots of predicted risk premia from the previous section. In those plots, the risk premium on low volatility exposure firms sometimes exceeded that of high volatility exposure firms, and sometimes the pattern reversed. It is likely that the growth innovation exposures of these portfolios obscure any pattern between volatility tercile and average returns.

Ex post betas on the portfolios are measured using the same procedure as in Section 3; returns are cumulated first to the quarterly frequency and then to real values using the PCE deflator. Overlapping four quarter cumulative returns are then regressed on four quarter cumulated growth rate and volatility innovations. As shown in the table, the ex post growth rate betas of the ex ante growth rate beta portfolios increase monotonically across terciles, from 1.88 to 6.57. The ordering of the betas conforms to expectations, but the magnitude of the betas is considerably smaller than the ex ante betas, which average 23.86 and 10.77 for the third and first tercile, respectively. The
ex post growth innovation betas of the ex ante volatility innovation beta portfolios are also weakly increasing across terciles. Since the growth innovation risk premium is larger per unit of growth innovation beta, this may help explain why average returns on these portfolios do not vary with the volatility beta sort.

Volatility innovation betas are also increasing across terciles for both the growth innovation beta-sorted portfolios and the volatility innovation beta-sorted portfolios. The pattern for volatility innovation beta-sorted portfolios is encouraging, in that it suggests that ex ante rankings based on the mapping between characteristics and volatility risk exposures holds ex post for portfolios sorted on these risk measures. Like the growth innovation portfolios, however, the magnitudes of the ex post betas are considerably different than those of the ex ante betas. The table also indicates that growth innovation beta-sorted portfolios have ex post volatility innovation betas that are also increasing across terciles, and that the spread in these betas is actually somewhat larger than that of the volatility innovation beta-sorted terciles.

Mean returns for two way portfolio sorts are presented in Panel A of Table 10. Across growth innovation beta terciles, mean returns are generally increasing; for the second volatility beta tercile portfolios there is deviation from monotonicity in the pattern across growth innovation beta terciles in the second growth beta tercile. Extreme portfolio return spreads are all positive, with excess returns of third tercile growth innovation beta portfolios in excess of first tercile averaging 25, 21, and 89 basis points per month for the first, second, and third volatility innovation tercile, respectively. Patterns in the volatility beta tercile portfolio returns are more muddled. The relation between volatility beta tercile and average return are monotonic only for the third growth innovation beta tercile, but the difference in tercile one and tercile three average returns is negative, contrasting with the idea of a negative volatility innovation risk premium. The premium is strongest in the first tercile of growth innovation betas, where the difference in the 54 basis point premium of the low mean, high volatility portfolio relative to the low mean, low volatility portfolio of 91 basis points per month is -37 basis points per month.

Growth innovation betas for the portfolios are shown in Panel B of Table 10. Ex post growth innovation betas are increasing across terciles in the first volatility innovation tercile, and weakly increasing across the third volatility innovation tercile. The spread in growth innovation betas in the third volatility innovation beta tercile is the largest, with the third tercile growth innovation beta exceeding that of the first tercile by 9.19. This spread in betas may help explain why the third mean tercile growth innovation beta and third tercile volatility innovation beta portfolio has the highest average return, while the first tercile growth innovation beta, third tercile volatility innovation beta has such a low average return. The most problematic result that we find is in the second volatility innovation beta tercile, where the difference in the first and third tercile growth innovation betas is negative.
Like the growth innovation betas, patterns in volatility innovation betas, shown in Panel C of Table 10 also exhibit weakly increasing patterns across growth innovation beta terciles one and three, but decreasing in tercile 2. The largest spread in betas is in growth innovation beta tercile three, where the third volatility innovation beta tercile volatility innovation beta exceeds that of the first tercile by 4.16. In general, the volatility innovation betas are highly correlated with the growth innovation betas, with a cross-sectional correlation of 0.94.

We have only nine portfolios, and so performing a cross-sectional regression of average returns on portfolio risk exposures is only so informative, and standard errors are suspect. However, simply to clarify the multivariate relation between average returns and risk exposures, we conduct regressions of mean portfolio returns on ex post risk measures. We find that average returns are positively related to growth innovation betas and negatively related to volatility innovation betas, and the regression $R^2$ is 57%, or a 76% correlation between predicted and actual average returns.

5 Conclusion

Consumption-based asset pricing is an essential link between standard economic theory and finance, but has been difficult to implement practically due to the difficulty of measuring consumption risk exposures. We provide a framework for using information in portfolio-level consumption risk exposures and characteristics to predict firm-level exposures to consumption risks. Using a model that explains roughly two-thirds of cross-sectional variation in a set of 55 portfolios sorted on six firm characteristics, we implement a procedure to forecast firm-level exposures to consumption risks. We find that portfolios sorted on these consumption risks generally exhibit ex post exposures to consumption risks that are consistent with the ex ante predictions. Specifically, when we sort on ex ante growth and volatility innovation betas, portfolios formed on these betas have ex post risk exposures that increase across sorting terciles.

The procedure proposed in this paper has the potential to be applied to a wide range of uses for imputation of cost of capital. Implied risk premium estimates could be used for benchmarking mutual fund performance, evaluating the profitability of trading strategies, and estimating required rates of return. The advantage of using this framework is that it is firmly rooted in economic theory, specifically the consumption-based pricing models of Lucas (1978) and Breeden (1979). As such, it represents an economic-based alternative to risk adjustment and cost of capital estimation based on ad hoc return factors.
Appendix A  Construction of Test Portfolios

Our empirical tests utilize portfolios sorted on six characteristics: market value, book-to-market ratio, past 12-month return, asset growth, total accruals, and stock issuance. We describe the construction of these portfolios in this appendix. Accounting variables used to construct the characteristics are obtained from Compustat and data on returns, prices, and shares outstanding are obtained from CRSP. All accounting variables are matched to subsequent returns using the procedure in Fama and French (1993); it is assumed that the accounting variable known from the end of June of year $t$ through the end of June of year $t+1$ is the value as of fiscal year end financial statements ending in calendar year $t−1$.

**Market Value**

Banz (1981) and Fama and French (1992) document the ability of size to explain cross-sectional variation in returns. Firm size for July of year $t$ through June of year $t + 1$ is measured as the CRSP market value of the firm at the end of June of year $t$. Market value is the product of CRSP shares outstanding times CRSP price per share.

**Book to Market**

Fama and French (1992) show that the book-to-market ratio, together with market value dominate many other characteristics in describing cross-sectional variation in returns. The book-to-market ratio at June of year $t$ is the book value of equity in June divided by the market value of equity at December of year $t−1$. Following Fama and French (2008), we measure book value of equity as book equity plus balance sheet deferred taxes (Compustat item TXDB), investment tax credits (ITCB), and preferred stock. Book equity is total assets (AT) less total liabilities (LT). Preferred stock is, in order of preference, liquidating value (PSTKL), redemption value (PSTKRV), or carrying value (UPSTK).

**Past 12 Month Return**

Momentum strategies, defined as buying recent past winners and selling recent past losers is shown to be profitable in Jegadeesh and Titman (1993). The profit is maximized when the window over which past returns are calculated is 12 months and the holding period is three months. We calculate cumulative returns for firms over months $t−12$ through $t−2$ for portfolio formation in month $t$. Jegadeesh and Titman (1993) show that skipping one month enhances returns due to avoidance of one-month reversals. Past returns are recalculated each month.

**Asset Growth**

Cooper, Gulen, and Schill (2008) show that firms with low growth in total assets outperform
firms with high growth in total assets on average. In the same spirit as the authors, we calculate the growth in assets as the difference in log assets at June of year $t$ and June of year $t - 1$. Total assets are Compustat item AT.

**Total Accruals**

Firms with low total accruals have higher average returns than firms with high total accruals as shown in Sloan (1996). Accruals are measured as the change in working capital divided by average total assets. The change in working capital is measured as the first difference of current assets (ACT) minus the first difference of cash and equivalents (CHE) minus the first difference of current liabilities (LCT) plus the first differences of debt in current liabilities (DLC) plus depreciation expense (DP). Average total assets is the average of total assets (AT) at June of year $t$ and June of year $t - 1$.

**Stock Issues**

Pontiff and Woodgate (2008) show that sorting on net share issuance generates differences in average returns with firms with low net issuance outperforming firms with high net issuance. Following Fama and French (2008), we measure net stock issues as the log difference in split-adjusted CRSP shares outstanding from year $t - 1$ to year $t$. 
References


Green, Jeremiah, John R M Hand, and X Frank Zhang, 2014, The remarkable multidimensionality in the cross-section of expected u.s. stock returns, unpublished manuscript, University of North Carolina.


Harvey, Campbell R, Yan Liu, and Heqing Zhu, 2014, ...and the cross-section of expected returns, unpublished manuscript, Duke University.


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Tédognap, Roméo, 2013, Consumption volatility and the cross-section of stock returns, unpublished manuscript, Stockholm School of Economics.


Table 1: Average Returns

Table 1 depicts average returns on a set of 65 portfolios formed on the basis of six characteristics. Portfolios are formed on asset growth (AG), book-to-market ratio (BM), market value (MV), operating profitability (OP), past 12-month return (P12), net stock issues (SI), and total accruals (TA). We form value-weighted portfolios based on deciles of six characteristics and quintiles of net stock issues. Data are sampled at the quarterly frequency from September, 1953 through December, 2012. Returns are deflated to real using the PCE deflator from the NIPA tables at the Bureau of Economic Analysis.

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Table 2: Consumption Growth Innovation Risk Exposures

In Table 2, we present growth innovation risk exposures and standard errors from a regression of cumulative portfolio excess returns on cumulative consumption growth level and volatility innovations,

\[
\prod_{j=0}^{7} R_{i,t-j} = a_i + \beta_{i,\eta} \sum_{j=0}^{7} \tilde{\eta}_{t-j} + \beta_{i,w} \sum_{j=0}^{7} \tilde{w}_{t+1} + e_{i,t},
\]

where \( \tilde{\eta}_t \) is the innovation in the level of consumption growth and \( \tilde{w}_t \) is the innovation in consumption volatility. Consumption volatility is measured as the squared innovation in the level of consumption growth, \( \tilde{\eta}_t^2 \), and is assumed to follow an AR(1). Returns are on portfolios sorted on asset growth (AG), book-to-market ratio (BM), market value (MV), operating profitability (OP), past 12-month return (P12), stock issues (SI), and total accruals (TA). We present point estimates of growth risk exposures, \( \beta_{i,\eta} \) in Panel A and standard errors for the estimates, calculated using the Newey-West correction with eight lags, in Panel B. Data are sampled at the quarterly frequency over the period September, 1953 through December, 2012.

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Table 3: Volatility Innovation Risk Exposures

In Table 3, we present volatility innovation risk exposures and standard errors from a regression of cumulative portfolio excess returns on cumulative consumption growth level and volatility innovations,

$$\prod_{j=0}^{7} R_{i,t-j} = a_i + \beta_{i,\eta} \sum_{j=0}^{7} \hat{\eta}_{t-j} + \beta_{i,w} \sum_{j=0}^{7} \hat{w}_{t+1} + \epsilon_{i,t},$$

where $\hat{\eta}_t$ is the innovation in the level of consumption growth and $\hat{w}_t$ is the innovation in consumption volatility. Consumption volatility is measured as the squared innovation in the level of consumption growth, $\hat{\eta}_t^2$, and is assumed to follow an AR(1). Returns are on portfolios sorted on asset growth (AG), book-to-market ratio (BM), market value (MV), operating profitability (OP), past 12-month return (P12), stock issues (SI), and total accruals (TA). We present point estimates of growth risk exposures, $\beta_{i,\eta}$ in Panel A and standard errors for the estimates, calculated using the Newey-West correction with eight lags, in Panel B. Data are sampled at the quarterly frequency over the period September, 1953 through December, 2012.

### Panel A: Estimates

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Table 4: Cross-Sectional Regressions: 65 Portfolios

Table 5 presents estimates of cross-sectional regressions of average excess portfolio returns on risk measures,

\[ \bar{R}_i - \bar{R}_f = \gamma_0 + \gamma_{\eta}\beta_{i,\eta} + \gamma_{w}\beta_{i,w} + u_i, \]

where \( \bar{R}_i \) is the average real quarterly return on a set of 65 portfolios formed on asset growth, book-to-market ratio, market value, operating profitability, past 12-month return, net stock issues, and total accruals, and \( \bar{R}_f \) is the real quarterly compounded return on a Treasury Bill closest to one month to maturity. The independent variables \( \beta_{i,\eta} \) and \( \beta_{i,w} \) are slope coefficients from a first stage regression,

\[ \prod_{j=0}^{7} R_{i,t-j} = a_i + \beta_{i,\eta} \sum_{j=0}^{7} \hat{\eta}_{t-j} + \beta_{i,w} \sum_{j=0}^{7} \hat{w}_{t+1} + e_{i,t}, \]

where \( \hat{\eta}_{t+1} \) is the innovation in the level of consumption growth and \( \hat{w}_{t+1} \) is the innovation in consumption volatility measured as the AR(1) residual on the squared consumption growth innovation, \( \hat{\eta}_{t+1}^2 \). The table presents point estimates and adjusted \( R^2 \) for versions of the model with the restrictions \( \gamma_w = 0, \gamma_{\eta} = 0, \) and an unrestricted version. Standard errors corrected for first stage estimation bias following Shanken (1992) are presented in parentheses below the point estimates. Beneath the adjusted \( R^2 \), we present 95% critical values for adjusted \( R^2 \) from 5000 Monte Carlo simulations under the null that the independent variables have no explanatory power for the returns. Pricing errors are presented in Panel B of the table. Data are sampled at the quarterly frequency over the period September, 1953 through December, 2012.

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<td>(0.015)</td>
<td>(32.88)</td>
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| Coeff.                      | 2.040          | 0.011          | -0.85          |           |
| SE                          | (0.141)        | (0.016)        | (33.23)        |           |

| Coeff.                      | 0.044          | 0.119          | -0.054         | 41.14     |
| SE                          | (0.330)        | (0.019)        | (0.017)        | (45.98)   |

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<td>-0.11</td>
<td>0.07</td>
<td>-0.43</td>
<td>0.27</td>
</tr>
<tr>
<td>6</td>
<td>0.01</td>
<td>0.06</td>
<td>0.48</td>
<td>0.01</td>
<td>0.25</td>
<td>0.60</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>-0.46</td>
<td>-0.41</td>
<td>0.41</td>
<td>-0.26</td>
<td>0.54</td>
<td>-0.23</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>0.14</td>
<td>0.42</td>
<td>0.55</td>
<td>-0.16</td>
<td>0.64</td>
<td>-0.36</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>0.21</td>
<td>0.59</td>
<td>0.56</td>
<td>0.40</td>
<td>0.49</td>
<td>-0.58</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>0.11</td>
<td>0.77</td>
<td>-0.25</td>
<td>0.34</td>
<td>0.74</td>
<td>0.02</td>
<td></td>
</tr>
</tbody>
</table>

34
Table 5: Cross-Sectional Regressions: 55 Portfolios

Table 5 presents estimates of cross-sectional regressions of average excess portfolio returns on risk measures,

\[ \bar{R}_i - \bar{R}_f = \gamma_0 + \gamma_{\eta} \beta_{i,\eta} + \gamma_w \beta_{i,w} + u_i, \]

where \( \bar{R}_i \) is the average real quarterly return on a set of 55 portfolios formed on asset growth, book-to-market ratio, market value, past 12-month return, net stock issues, and total accruals, and \( R_f \) is the real quarterly compounded return on a Treasury Bill closest to one month to maturity. The independent variables \( \beta_{i,\eta} \) and \( \beta_{i,w} \) are slope coefficients from a first stage regression,

\[ \prod_{j=0}^{7} R_{i,t-j} = a_i + \beta_{i,\eta} \sum_{j=0}^{7} \hat{\eta}_{t-j} + \beta_{i,w} \sum_{j=0}^{7} \hat{w}_{t+1} + e_{i,t}, \]

where \( \hat{\eta}_{t+1} \) is the innovation in the level of consumption growth and \( \hat{w}_{t+1} \) is the innovation in consumption volatility measured as the AR(1) residual on the squared consumption growth innovation, \( \hat{\eta}_{t+1}^2 \). The table presents point estimates and adjusted \( R^2 \) for versions of the model with the restrictions \( \gamma_w = 0, \gamma_{\eta} = 0 \), and an unrestricted version. Standard errors corrected for first stage estimation bias following Shanken (1992) are presented in parentheses below the point estimates. Beneath the adjusted \( R^2 \), we present 95% critical values for adjusted \( R^2 \) from 5000 Monte Carlo simulations under the null that the independent variables have no explanatory power for the returns. Pricing errors are presented in Panel B of the table. Data are sampled at the quarterly frequency over the period September, 1953 through December, 2012.

### Panel A: Regression Results

<table>
<thead>
<tr>
<th></th>
<th>( \gamma_0 )</th>
<th>( \gamma_{\eta} )</th>
<th>( \gamma_w )</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coeff.</td>
<td>0.589</td>
<td>0.113</td>
<td>55.14</td>
<td></td>
</tr>
<tr>
<td>SE</td>
<td>(0.194)</td>
<td>(0.014)</td>
<td>(39.53)</td>
<td></td>
</tr>
<tr>
<td>Coeff.</td>
<td>2.180</td>
<td>0.024</td>
<td>1.18</td>
<td></td>
</tr>
<tr>
<td>SE</td>
<td>(1.207)</td>
<td>(0.284)</td>
<td>(40.81)</td>
<td></td>
</tr>
<tr>
<td>Coeff.</td>
<td>-0.188</td>
<td>0.146</td>
<td>-0.052</td>
<td>64.46</td>
</tr>
<tr>
<td>SE</td>
<td>(0.279)</td>
<td>(0.016)</td>
<td>(0.014)</td>
<td>(54.83)</td>
</tr>
</tbody>
</table>

### Panel B: Pricing Errors

<table>
<thead>
<tr>
<th>Decile</th>
<th>AG</th>
<th>BM</th>
<th>MV</th>
<th>P12</th>
<th>SI</th>
<th>TA</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.10</td>
<td>-0.12</td>
<td>-0.09</td>
<td>-0.91</td>
<td>-0.08</td>
<td>-0.34</td>
</tr>
<tr>
<td>2</td>
<td>0.55</td>
<td>0.32</td>
<td>-0.20</td>
<td>-1.46</td>
<td>-0.15</td>
<td>-0.22</td>
</tr>
<tr>
<td>3</td>
<td>0.47</td>
<td>0.45</td>
<td>0.22</td>
<td>-1.08</td>
<td>-0.30</td>
<td>-0.08</td>
</tr>
<tr>
<td>4</td>
<td>0.01</td>
<td>0.13</td>
<td>0.02</td>
<td>-0.13</td>
<td>-0.25</td>
<td>0.10</td>
</tr>
<tr>
<td>5</td>
<td>0.16</td>
<td>0.10</td>
<td>-0.03</td>
<td>0.10</td>
<td>-0.40</td>
<td>0.30</td>
</tr>
<tr>
<td>6</td>
<td>-0.03</td>
<td>-0.02</td>
<td>0.29</td>
<td>0.30</td>
<td>0.64</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>-0.60</td>
<td>-0.52</td>
<td>0.25</td>
<td>0.55</td>
<td>-0.26</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>0.06</td>
<td>0.30</td>
<td>0.49</td>
<td>0.57</td>
<td>-0.47</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>0.09</td>
<td>0.53</td>
<td>0.54</td>
<td>0.26</td>
<td>-0.70</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>0.04</td>
<td>0.57</td>
<td>-0.31</td>
<td>0.18</td>
<td>0.07</td>
<td></td>
</tr>
</tbody>
</table>
Table 6: Alternative Consumption Models

We investigate the performance of alternative consumption-based pricing models in Table 6. We consider four models. The first is a conditional consumption CCAPM from Lettau and Ludvigson (2001),

\[ R_{i,t+1} - R_{f,t} = a_i + \beta_{i,cay} cay_t + \beta_{i,dc} \Delta c_{t+1} + \beta_{i,cay} \Delta cay_t \cdot \Delta c_{t+1} + e_{i,t} \]

\[ \bar{R}_i - \bar{R}_f = \gamma_0 + \gamma_{cay} \beta_{i,cay} + \gamma_{dc} \beta_{i,dc} + \gamma_{cay} \Delta cay_t \beta_{i,cay} \Delta c + u_i, \]

where \( \Delta c_{t+1} \) is log real per capita growth in consumption of nondurables and services and \( cay_t \) is the cointegrating residual from a trivariate cointegrating relation between aggregate consumption, asset wealth, and labor income. The second alternative is the ultimate consumption model of Parker and Julliard (2005),

\[ R_{i,t+1} - R_{f,t} = a_i + \beta_{i,g} g_{t+1,t+s+1} + e_{i,t+1} \]

\[ \bar{R}_i - \bar{R}_f = \gamma_0 + \gamma_{g} \beta_{i,g} + u_i, \]

where \( g_{t,s} \) is cumulative growth in per capita consumption of nondurables and services over quarters \( t \) through \( t+s \), with \( s = 11 \). The third model is the consumption cash flow risk model of Bansal, Dittmar, and Lundblad (2005),

\[ \Delta d_{i,t+1} = a_i + \beta_{i,x} x_t + e_{i,t+1} \]

\[ \bar{R}_i - \bar{R}_f = \gamma_0 + \gamma_{x} \beta_{i,x} + u_i, \]

where \( \Delta d_{i,t+1} \) is the log growth in real dividends per share on portfolio \( i \) and \( x_t \) is the average growth in consumption over quarters \( t-7 \) through \( t \). The final model is the durable consumption model of Yogo (2006),

\[ R_{i,t+1} - R_{f,t} = a_i + \beta_{i,nds} \Delta c_{nds,t+1} + \beta_{i,d} \Delta c_d,t+1 + \beta_{i,m} R_{m,t+1} + e_{i,t+1} \]

\[ \bar{R}_i - \bar{R}_f = \gamma_0 + \gamma_{nds} \beta_{i,nds} + \gamma_{d} \beta_{i,d} + \gamma_{m} \beta_{i,m} + u_i, \]

where \( \Delta c_{nds,t+1} \) is growth in real per capita consumption of nondurables and services, \( \Delta c_d,t+1 \) is growth in real per capita consumption of durable goods, and \( R_{m,t+1} \) is the real return on the CRSP value-weighted index. We obtain data for \( cay_t \) and durable consumption growth from Martin Lettau's and Motshojo Yogo's websites, respectively. Shanken (1992)-corrected standard errors are presented below point estimates in parentheses. Data are sampled quarterly over the period September, 1953 through December, 2012 with the exception of the durable goods model, for which data ends in December, 2001.

<table>
<thead>
<tr>
<th>Model</th>
<th>Estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Conditional CCAPM</strong></td>
<td></td>
</tr>
<tr>
<td>Coeff</td>
<td>0.045</td>
</tr>
<tr>
<td>SE</td>
<td>(0.783)</td>
</tr>
<tr>
<td>( \gamma_0 )</td>
<td>0.618</td>
</tr>
<tr>
<td>( \gamma_{cay} )</td>
<td>0.466</td>
</tr>
<tr>
<td>( \gamma_{dc} )</td>
<td>0.005</td>
</tr>
<tr>
<td>( \gamma_{cay,dc} )</td>
<td>44.56</td>
</tr>
<tr>
<td>( \bar{R}^2 )</td>
<td>64.30</td>
</tr>
<tr>
<td><strong>Ultimate Consumption</strong></td>
<td></td>
</tr>
<tr>
<td>Coeff</td>
<td>0.659</td>
</tr>
<tr>
<td>SE</td>
<td>(0.603)</td>
</tr>
<tr>
<td>( \gamma_0 )</td>
<td>3.001</td>
</tr>
<tr>
<td>( \gamma_{g} )</td>
<td>13.82</td>
</tr>
<tr>
<td>( \bar{R}^2 )</td>
<td>44.29</td>
</tr>
<tr>
<td><strong>Cash Flow Consumption</strong></td>
<td></td>
</tr>
<tr>
<td>Coeff</td>
<td>1.698</td>
</tr>
<tr>
<td>SE</td>
<td>(0.102)</td>
</tr>
<tr>
<td>( \gamma_0 )</td>
<td>0.166</td>
</tr>
<tr>
<td>( \gamma_{x} )</td>
<td>39.36</td>
</tr>
<tr>
<td>( \bar{R}^2 )</td>
<td>47.72</td>
</tr>
<tr>
<td><strong>Durable Consumption</strong></td>
<td></td>
</tr>
<tr>
<td>Coeff</td>
<td>2.640</td>
</tr>
<tr>
<td>SE</td>
<td>(1.122)</td>
</tr>
<tr>
<td>( \gamma_0 )</td>
<td>-0.058</td>
</tr>
<tr>
<td>( \gamma_{nds} )</td>
<td>0.959</td>
</tr>
<tr>
<td>( \gamma_{d} )</td>
<td>-0.307</td>
</tr>
<tr>
<td>( \gamma_{m} )</td>
<td>52.72</td>
</tr>
<tr>
<td>( \bar{R}^2 )</td>
<td>60.24</td>
</tr>
</tbody>
</table>
Table 7: Return Factor Models

Table 7 presents results of cross-sectional regressions based on return factor models:

\[
R_{i,t+1} - R_{f,t} = a_i + \sum_{k=1}^{K} \beta_{i,k} r_{k,t+1} + e_{i,t+1}
\]

\[
\bar{R}_i - \bar{R}_f = \gamma_0 + \sum_{k=1}^{K} \gamma_k \hat{\beta}_{i,k} + u_i,
\]

where \(R_{i,t+1}\) is the return on one of 55 characteristics-sorted portfolios, \(R_{f,t}\) is the return on a one-month T-Bill, \(K\) represents the number of factors, and \(r_{k,t+1}\) is the excess return on factor \(k\). Four models are considered. The Fama and French (1993) three-factor model uses the excess return on the value-weighted market portfolio over the risk-free rate (\(MRP\)), the return on a small capitalization stock portfolio in excess of a large capitalization stock portfolio (\(SMB\)), and the excess return on a high book-to-market ratio portfolio over the return on a low book-to-market ratio portfolio (\(HML\)) as factors. The Carhart (1997) four-factor model augments these three factors with the return on a portfolio of past 12-month winners in excess of the return on a portfolio of past 12-month losers (\(WML\)). The Fama and French (2013) five-factor model retains a market, size, and book-to-market factor, and includes the excess return on a portfolio of high operating profitability firms over a portfolio of low operating profitability firms (\(RMW\)) and the return on a portfolio of firms with low asset growth in excess of the return on a portfolio of firms with high asset growth (\(CMA\)). The final model, from Hou, Xue, and Zhang (2014) includes a market and size factor, the excess return on a portfolio of high return on earnings firms over a portfolio of low return on earnings firms (\(ROE\)), and the return on low investment firms in excess of high investment firms (\(INV\)). The table presents point estimates with Shanken (1992)-corrected standard errors in parentheses below the point estimates. We also present adjusted \(R^2\) with 95% critical values from Monte Carlo simulations under the null that the independent variable has no explanatory power for returns. Data are sampled at the quarterly frequency over the period September, 1953 through December, 2012 and are deflated to real using the PCE deflator from the NIPA tables at the Bureau of Economic Analysis.

<table>
<thead>
<tr>
<th>(\gamma_0)</th>
<th>(\gamma_{MRP})</th>
<th>(\gamma_{SMB})</th>
<th>(\gamma_{HML})</th>
<th>(\gamma_{UMD})</th>
<th>(\gamma_{RMW})</th>
<th>(\gamma_{CMA})</th>
<th>(\gamma_{INV})</th>
<th>(\gamma_{ROE})</th>
<th>(R^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coeff.</td>
<td>7.347</td>
<td>-5.312</td>
<td>0.470</td>
<td>-0.310</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>48.22</td>
</tr>
<tr>
<td>SE</td>
<td>(0.990)</td>
<td>(0.960)</td>
<td>(0.208)</td>
<td>(0.409)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(64.62)</td>
</tr>
<tr>
<td>Coeff.</td>
<td>2.062</td>
<td>-0.129</td>
<td>0.397</td>
<td>0.946</td>
<td>2.433</td>
<td></td>
<td></td>
<td></td>
<td>87.60</td>
</tr>
<tr>
<td>SE</td>
<td>(0.604)</td>
<td>(0.589)</td>
<td>(0.088)</td>
<td>(0.201)</td>
<td>(0.163)</td>
<td></td>
<td></td>
<td></td>
<td>(69.68)</td>
</tr>
<tr>
<td>Coeff.</td>
<td>5.538</td>
<td>-3.610</td>
<td>0.820</td>
<td>-4.100</td>
<td>1.998</td>
<td>-2.944</td>
<td></td>
<td></td>
<td>37.86</td>
</tr>
<tr>
<td>SE</td>
<td>(1.224)</td>
<td>(1.193)</td>
<td>(2.152)</td>
<td>(2.260)</td>
<td>(3.699)</td>
<td>(3.180)</td>
<td></td>
<td></td>
<td>(72.97)</td>
</tr>
<tr>
<td>Coeff.</td>
<td>4.462</td>
<td>-2.634</td>
<td>0.894</td>
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<td>0.297</td>
<td>1.252</td>
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</tr>
<tr>
<td>SE</td>
<td>(1.173)</td>
<td>(1.139)</td>
<td>(0.260)</td>
<td></td>
<td>(0.293)</td>
<td>(0.356)</td>
<td></td>
<td></td>
<td>(69.29)</td>
</tr>
</tbody>
</table>
Table 8: Relation Between Characteristics and Risk Measures

Table 8 presents results of regressions of portfolio risk exposures on average portfolio characteristics. Risk exposures, \( \beta_{i,\eta} \) and \( \beta_{i,w} \) are regression coefficients from time series regressions of cumulative portfolio returns on cumulated innovations in consumption growth and volatility innovations, respectively:

\[
\prod_{j=0}^{7} R_{i,t-j} = a_i + \beta_{i,\eta} \sum_{j=0}^{7} \eta_{t-j} + \beta_{i,w} \sum_{j=0}^{7} \hat{w}_{t+1} + e_{i,t},
\]

The characteristics, asset growth (AG), book-to-market ratio (BM), market value (MV), past 12-month return (P12), net stock issues (SI), and total accruals (TA) are constructed at the firm level and value-weighted to calculate portfolio characteristics. In Panel A, the regressions are specified as

\[
\beta_{i,\eta} = g_0 + g_1 AG_i + g_2 BM_i + g_3 MV_i + g_4 P12_i + g_5 SI_i + g_6 TA_i + v_i
\]
\[
\beta_{i,w} = h_0 + h_1 AG_i + h_2 BM_i + h_3 MV_i + h_4 P12_i + h_5 SI_i + h_6 TA_i + z_i.
\]

In Panel B, we estimate Fama and MacBeth (1973) style expanding window regressions,

\[
\beta_{i,\eta,t} = g_{0t} + g_{1t} AG_{it} + g_{2t} BM_{it} + g_{3t} MV_{it} + g_{4t} P12_{it} + g_{5t} SI_{it} + g_{6t} TA_{it} + v_{it}
\]
\[
\beta_{i,w,t} = h_{0t} + h_{1t} AG_{it} + h_{2t} BM_{it} + h_{3t} MV_{it} + h_{4t} P12_{it} + h_{5t} SI_{it} + h_{6t} TA_{it} + z_{it},
\]

where \( \beta_{i,\eta,t} \) and \( \beta_{i,w,t} \) are estimates of risk exposures through quarter \( t \). Data are sampled at the quarterly frequency over the period September, 1953 through December, 2012.

Panel A: Average Characteristics

<table>
<thead>
<tr>
<th>Dep. Var.</th>
<th>AG</th>
<th>BM</th>
<th>MV</th>
<th>P12</th>
<th>SI</th>
<th>TA</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta_{i,\eta} ) Coeff.</td>
<td>7.52</td>
<td>-0.37</td>
<td>-2.12</td>
<td>17.22</td>
<td>-5.75</td>
<td>-36.61</td>
<td>63.11</td>
</tr>
<tr>
<td>SE</td>
<td>(6.44)</td>
<td>(1.24)</td>
<td>(0.31)</td>
<td>(2.67)</td>
<td>(8.12)</td>
<td>(12.45)</td>
<td></td>
</tr>
<tr>
<td>( \beta_{i,w} ) Coeff.</td>
<td>24.87</td>
<td>-4.66</td>
<td>-2.67</td>
<td>6.27</td>
<td>2.80</td>
<td>-26.22</td>
<td>54.73</td>
</tr>
<tr>
<td>SE</td>
<td>(7.92)</td>
<td>(1.53)</td>
<td>(0.38)</td>
<td>(3.27)</td>
<td>(10.00)</td>
<td>(15.33)</td>
<td></td>
</tr>
</tbody>
</table>

Panel B: Fama-MacBeth Regressions

<table>
<thead>
<tr>
<th>Dep. Var.</th>
<th>AG</th>
<th>BM</th>
<th>MV</th>
<th>P12</th>
<th>SI</th>
<th>TA</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta_{i,\eta} ) Coeff.</td>
<td>4.46</td>
<td>-0.33</td>
<td>-2.11</td>
<td>8.18</td>
<td>-6.10</td>
<td>-25.70</td>
<td>57.68</td>
</tr>
<tr>
<td>SE</td>
<td>(0.24)</td>
<td>(0.06)</td>
<td>(0.02)</td>
<td>(0.20)</td>
<td>(0.30)</td>
<td>(0.54)</td>
<td></td>
</tr>
<tr>
<td>( \beta_{i,w} ) Coeff.</td>
<td>17.95</td>
<td>-2.82</td>
<td>-2.22</td>
<td>9.28</td>
<td>6.69</td>
<td>-5.19</td>
<td>49.55</td>
</tr>
<tr>
<td>SE</td>
<td>(0.31)</td>
<td>(0.10)</td>
<td>(0.04)</td>
<td>(0.24)</td>
<td>(0.25)</td>
<td>(0.78)</td>
<td></td>
</tr>
</tbody>
</table>
Table 9: Innovation Risk-Sorted Portfolios: Univariate Sort

Table 10 presents mean returns and risk exposures for portfolios formed on predicted risk measures. Predicted risk measures are formed by regressing the risk exposures from expanding window regressions of the returns through time \( t \) on 55 portfolios sorted on six characteristics: asset growth (AG), book-to-market ratio (BM), market value (MV), past 12-month return (P12), net stock issues (SI), and total accruals (TA) on the innovation in the level and volatility of consumption growth on observed characteristics at time \( t \). The portfolio level coefficients are used to predict risk exposures at the firm level. Portfolios are formed by sorting firms into terciles on predicted growth innovation risk exposure and volatility innovation risk exposure. Panel A presents monthly average portfolio returns and ex post betas for growth innovation beta-sorted portfolios, where ex post betas are measured by regressing annual overlapping returns on cumulated innovations:

\[
\prod_{j=0}^{7} R_{i,t-j} = a_i + \beta_{i,\eta} \sum_{j=0}^{7} \hat{\eta}_{t-j} + \beta_{i,w} \sum_{j=0}^{7} \hat{w}_{t-j} + e_{i,t},
\]

where \( \eta_t \) is the innovation in consumption growth and \( w_t \) is the innovation in the squared innovation in consumption growth. Panel B presents similar results for volatility innovation beta-sorted portfolios. Data are sampled from July, 1983 through December, 2012.

Panel A: Growth Innovation Portfolios

<table>
<thead>
<tr>
<th>Tercile</th>
<th>Mean</th>
<th>( \beta_{i,\eta} )</th>
<th>( \beta_{i,w} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.91</td>
<td>1.88</td>
<td>-4.86</td>
</tr>
<tr>
<td>2</td>
<td>1.17</td>
<td>4.55</td>
<td>-0.99</td>
</tr>
<tr>
<td>3</td>
<td>1.34</td>
<td>4.69</td>
<td>0.30</td>
</tr>
</tbody>
</table>

Panel B: Volatility Innovation Portfolios

<table>
<thead>
<tr>
<th>Tercile</th>
<th>Mean</th>
<th>( \beta_{i,\eta} )</th>
<th>( \beta_{i,w} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.92</td>
<td>2.28</td>
<td>-4.71</td>
</tr>
<tr>
<td>2</td>
<td>1.10</td>
<td>3.76</td>
<td>-2.34</td>
</tr>
<tr>
<td>3</td>
<td>0.09</td>
<td>3.69</td>
<td>-1.11</td>
</tr>
</tbody>
</table>
Table 10: Innovation Risk-Sorted Portfolios: Bivariate Sort

Table 10 presents mean returns and risk exposures for portfolios formed on predicted risk measures. Predicted risk measures are formed by regressing the risk exposures from expanding window regressions of the returns through time \( t \) on 55 portfolios sorted on six characteristics: asset growth (AG), book-to-market ratio (BM), market value (MV), past 12-month return (P12), net stock issues (SI), and total accruals (TA) on the innovation in the level and volatility of consumption growth on observed characteristics at time \( t \). The portfolio level coefficients are used to predict risk exposures at the firm level. Portfolios are formed by sorting firms into terciles on predicted growth innovation risk exposure and volatility innovation risk exposure. Each portfolio represents intersections of growth innovation beta and volatility innovation beta terciles. In Panel A, we present monthly average portfolio returns. In Panels B and C, we present ex post growth and volatility innovation betas, where ex post betas are measured by regressing annual overlapping returns on cumulated innovations:

\[
\prod_{j=0}^{7} R_{i,t-j} = a_i + \beta_{i,\eta} \sum_{j=0}^{7} \hat{\eta}_{t-j} + \beta_{i,w} \sum_{j=0}^{7} \hat{w}_{t-j} + e_{i,t},
\]

where \( \eta_t \) is the innovation in consumption growth and \( w_t \) is the innovation in the squared innovation in consumption growth. Data are sampled from July, 1983 through December, 2012.

Panel A: Mean Returns

<table>
<thead>
<tr>
<th>( \beta_{i,\eta} ) Tercile</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta_{i,w} ) 1</td>
<td>1.91</td>
<td>1.13</td>
<td>1.16</td>
</tr>
<tr>
<td>2</td>
<td>1.00</td>
<td>1.23</td>
<td>1.22</td>
</tr>
<tr>
<td>3</td>
<td>0.54</td>
<td>1.06</td>
<td>1.44</td>
</tr>
</tbody>
</table>

Panel B: Consumption Innovation Risk Exposures

<table>
<thead>
<tr>
<th>( \beta_{i,\eta} ) Tercile</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta_{i,w} ) 1</td>
<td>1.94</td>
<td>4.55</td>
<td>6.73</td>
</tr>
<tr>
<td>2</td>
<td>3.16</td>
<td>4.31</td>
<td>0.58</td>
</tr>
<tr>
<td>3</td>
<td>0.07</td>
<td>-0.31</td>
<td>9.26</td>
</tr>
</tbody>
</table>

Panel C: Volatility Innovation Risk Exposures

<table>
<thead>
<tr>
<th>( \beta_{i,\eta} ) Tercile</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta_{i,w} ) 1</td>
<td>-5.14</td>
<td>-1.43</td>
<td>-0.80</td>
</tr>
<tr>
<td>2</td>
<td>-2.95</td>
<td>-1.49</td>
<td>-6.46</td>
</tr>
<tr>
<td>3</td>
<td>-3.92</td>
<td>-6.27</td>
<td>3.36</td>
</tr>
</tbody>
</table>
Figure 1 presents a scatterplot of average returns on predicted returns from the regression

$$
\bar{R}_i - \bar{R}_f = \gamma_0 + \gamma_\eta \beta_{i,\eta} + \gamma_w \beta_{i,w} + u_i,
$$

where $\beta_{i,\eta}$ and $\beta_{i,w}$ are slope coefficients from the time series regression

$$
\prod_{j=0}^{7} R_{i,t-j} = a_i + \beta_{i,\eta} \sum_{j=0}^{7} \hat{\eta}_{t-j} + \beta_{i,w} \sum_{j=0}^{7} \hat{w}_{t+1} + e_{i,t},
$$

where $\hat{\eta}_{t+1}$ is the innovation in the level of consumption growth and $\hat{w}_{t+1}$ is the innovation in consumption volatility measured as the AR(1) residual on the squared consumption growth innovation, $\hat{\eta}_{t+1}^2$. In Figure 1 (a), the asset menu is 65 portfolios sorted on the basis of asset growth (AG), book-to-market ratio (BM), market value (MV), operating profitability (OP), past 12-month return (P12), net stock issues (SI), and total accruals (TA). In Figure 1 (b), the asset menu is 55 portfolios, representing the previous set of assets except for operating profitability portfolios. Data are sampled at the quarterly frequency over the period September, 1953 through December, 2012.
Figure 2: Characteristic-Implied Betas

Figure 2 depicts consumption growth and volatility innovation betas implied by the portfolio-level relation between characteristics and risk exposures. Predicted risk measures are formed by regressing the risk exposures from expanding window regressions of the returns through time $t$ on 55 portfolios sorted on six characteristics: asset growth (AG), book-to-market ratio (BM), market value (MV), past 12-month return (P12), net stock issues (SI), and total accruals (TA) on the innovation in the level and volatility of consumption growth on observed characteristics at time $t$. The portfolio level coefficients are used to predict risk exposures at the firm level. Portfolios are formed by sorting firms into terciles on predicted growth innovation risk exposure and volatility innovation risk exposure. Figure 2 (a) presents implied consumption growth innovation betas for portfolios sorted on predicted consumption growth innovation beta. Figure 2 (b) presents implied consumption growth volatility innovation betas for portfolios sorted on predicted volatility innovation beta. Grey bars indicate NBER recessions.

(a) Growth Innovation Beta

(b) Volatility Innovation Beta
Figure 3: Smoothed Characteristic-Implied Betas

Figure 3 depicts consumption growth and volatility innovation betas implied by the portfolio-level relation between characteristics and risk exposures. Risk measures are smoothed over a four-quarter trailing moving average. Predicted risk measures are formed by regressing the risk exposures from expanding window regressions of the returns through time $t$ on 55 portfolios sorted on six characteristics: asset growth (AG), book-to-market ratio (BM), market value (MV), past 12-month return (P12), net stock issues (SI), and total accruals (TA) on the innovation in the level and volatility of consumption growth on observed characteristics at time $t$. The portfolio level coefficients are used to predict risk exposures at the firm level. Portfolios are formed by sorting firms into terciles on predicted growth innovation risk exposure and volatility innovation risk exposure. Figure 3 (a) presents implied consumption growth innovation betas for portfolios sorted on predicted consumption growth innovation beta. Figure 3 (b) presents implied consumption growth volatility innovation betas for portfolios sorted on predicted volatility innovation beta. Grey bars indicate NBER recessions.

(a) Growth Innovation Beta

(b) Volatility Innovation Beta
Figure 4: Implied Risk Premia

Figure 4 depicts risk premia implied by consumption growth and volatility innovation betas calculated from the portfolio-level relation between characteristics and risk exposures. Risk premia are calculated using the estimated prices of growth and volatility innovation risk from Table 4. Predicted risk measures are formed by regressing the risk exposures from expanding window regressions of the returns through time $t$ on 55 portfolios sorted on six characteristics: asset growth (AG), book-to-market ratio (BM), market value (MV), past 12-month return (P12), net stock issues (SI), and total accruals (TA) on the innovation in the level and volatility of consumption growth on observed characteristics at time $t$. The portfolio level coefficients are used to predict risk exposures at the firm level. Portfolios are formed by sorting firms into terciles on predicted growth innovation risk exposure and volatility innovation risk exposure. Figure 4 (a) presents implied risk premia for portfolios sorted on predicted consumption growth innovation beta. Figure 4 (b) presents implied risk premia for portfolios sorted on predicted volatility innovation beta. Grey bars indicate NBER recessions. Data are smoothed by a four-quarter trailing moving average.