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An enhanced XOR-based scheme for wireless packet retransmission problem

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SUMMARY

Solving wireless packet retransmission problem (WPRTP) using network coding (NC) is increasingly attracting research efforts. However, existing NC-based schemes for WPRTP are with high computational complexity resulting from computation on larger Galois field (GF(2^q)), or the solutions on GF(2) found by the schemes are less efficient. In this paper, combining the basic ideas in two existing schemes, denoted as ColorNC and CliqueNC, respectively, we present a new scheme named as ColorCliqueNC. The advantages of ColorCliqueNC include the following: (i) it is suitable for all kinds of WPRTP instances; (ii) it works on GF(2); thus, it is computationally efficient than the schemes working on larger Galois fields; and (iii) the solutions found by ColorCliqueNC usually have fewer packet retransmissions than those by ColorNC and CliqueNC despite that they all work on GF(2). Theoretical analysis indicates that ColorCliqueNC is superior to ColorNC and CliqueNC. Simulation results show that ColorCliqueNC generally outperforms ColorNC and CliqueNC. Compared with ColorNC, ColorCliqueNC can save up to 10% packet retransmissions. Copyright © 2013 John Wiley & Sons, Ltd.

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KEY WORDS: network coding; wireless packet retransmission problem; Galois field GF(2); wireless networks

ABBREVIATIONS

WPRTP	Wireless packet retransmission problem
P-WPRTP	Perfect WPRTP
IP-WPRTP	Imperfect WPRTP
NoNC	A straightforward scheme to solve WPRTP is to retransmit each requested packet once
ColorNC	The scheme proposed in [1]
CliqueNC	The scheme proposed in [9]
ColorCliqueNC	The scheme proposed in this paper, which combines the basic ideas in ColorNC and CliqueNC
Original-WPRTP	The given target WPRTP instance considered
Color-WPRTP	The new WPRTP instance created in ColorNC when solving the target WPRTP instance
Clique-WPRTP	The new WPRTP instance created in CliqueNC when solving the target WPRTP instance

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ColorClique-WPRTP	The new WPRTP instance created in ColorCliqueNC when solving the target WPRTP instance
Color-Graph	The graph model created in ColorNC when solving the target WPRTP instance
Clique-Graph	The graph model created in CliqueNC when solving the target WPRTP instance
ColorClique-Graph	The graph model created in ColorCliqueNC when solving the target WPRTP instance

1. INTRODUCTION

Packet retransmission is necessary for reliable communication over error-prone wireless links. One problem related to packet retransmission in wireless broadcast/multicast applications, named as wireless packet retransmission problem (WPRTP), is focused in this paper. A typical scenario of WPRTP is as follows [1]: One sender and several receivers in a wireless network form a subsystem, and all the receivers are in the radio range of the sender. The sender has a set of packets that are required to be transmitted to the receivers. However, each receiver has already obtained a subset of the packets (how these subsets of packets are obtained is out of the scope of this paper). Thus, the receivers request the sender to retransmit some packets. The set of the packets requested by a receiver is called as its *Want* set; meanwhile, the set of the packets already known to the receiver is called as its *Has* set.

A WPRTP instance can be formally described as a four-element tuple WPRTP(P, R, H, W), where $P=\{p_i|i\in\{1, 2, \ldots, |P|\}\}$ represents the set of the packets considered in the problem instance, $R=\{r_i|i\in\{1, 2, \ldots, |R|\}$ represents the set of the receivers, $H=\{H(r_i)|r_i\in R\}$ represents the set that contains the *Has* sets of the receivers, and $W=\{W(r_i)|r_i\in R\}$ represents the set that contains the *Want* sets of the receivers. A WPRTP instance is a perfect WPRTP (P-WPRTP) instance if for $\forall r_i\in R$, there is $P=H(r_i)\cup W(r_i)$, otherwise it is an imperfect WPRTP (IP-WPRTP) instance. A solution to a WPRTP instance is valid if each receiver can obtain all the packets in its *Want* set by decoding after receiving all the retransmitted packets in the solution. The objective of a WPRTP is to find a valid solution with minimum number of retransmitted packets. For ease of description, solutions mentioned in the later discussions are all assumed to be valid unless explicitly specified. A straightforward scheme to solve WPRTP is to retransmit each requested packet once. This scheme is denoted as NoNC in the following text.

By allowing mixing of packets at intermediate nodes, network coding (NC) [2] provides an interesting approach to many problems in networking realm, such as multicast [3], reliable communication [4], secure communication [5], and distributed storage [6]. Previous researches have already shown that NC can increase network throughput, enhance robustness, and improve fairness [7]. NC also provides a promising approach to WPRTP. By using NC, the sender can combine original packets into several coded packets and then transmits these coded packets instead of the original packets to the receivers. If properly designed, each receiver could obtain all the packets in its *Want* set by decoding from these coded packets. If the coded packets are fewer than the original packets requested by the receivers, communication overhead in the metric of packet retransmissions is reduced.

In recent years, some NC-based schemes have been proposed for WPRTP [1, 8–11]. The schemes in [1, 8, 9] search for solutions on Galois field (GF(2)), whereas the schemes in [10, 11] work on larger fields. The schemes in [8–10] are specially designed for P-WPRTP instances, whereas the schemes in [1, 11] are suitable for all WPRTP instances. As a summary, existing NC-based schemes for WPRTP are with high computational complexity resulting from computation on larger Galois fields (such as [10, 11]), or the solutions on GF(2) found by them are less efficient (such as [1, 8, 9]). Different from the NC-based schemes, NoNC is a traditional non-NC-based scheme.

In this paper, combining the basic ideas in the two schemes proposed in [1, 9], denoted as ColorNC and CliqueNC, respectively, we present a new scheme named as ColorCliqueNC. The advantages of this scheme include the following: (i) it is suitable for all WPRTP instances, including

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IP-WPRTP instances and P-WPRTP instances, and (ii) it works on GF(2); thus, it has lower computational complexity than the schemes on larger fields.

The rest of the paper is organized as follows. Section 2 gives an overview on a related work. Section 3 describes the basic idea and operation process of ColorCliqueNC. Section 4 presents some analysis results about several aspects of ColorCliqueNC as well as ColorNC and CliqueNC, which indicates that ColorCliqueNC outperforms the others. Section 5 evaluates the performance of ColorCliqueNC and some other typical schemes through numerical simulations. Section 6 concludes the paper.

2. RELATED WORK

The WPRTP has attracted a significant attention from the research community because of its theoretical significance and application potential in communication and networking realm. In the literature, WPRTP is named variously, such as index coding problem [12, 13] and local mixing problem [14]. Many existing works, such as [1, 12–14], focused on theoretical perspectives of the problem, whereas some other works [1, 8, 9] proposed some NC-based schemes for WPRTP. Several other works, such as [15, 16], although not focusing on WPRTP exclusively, proposed some schemes that can be used to solve WPRTP. Because ColorCliqueNC proposed in this paper combines the basic ideas in ColorNC [1] and CliqueNC [9], we only provide a more detailed description about ColorNC and CliqueNC here. For a detailed description about other related works, interested readers can refer to [11] and the references therein.

ColorNC [1] solves a WPRTP instance by transforming it to a graph coloring problem instance. It works as follows:

Step 1: The WPRTP instance is transformed to a new WPRTP instance by substituting each receiver that wants multiple packets with a set of new receivers meeting the following three criteria:

- The *Has* set of each new receiver is the same as that of the original receiver.
- The *Want* set of each receiver contains just one of the packets in the *Want* set of the original receiver.
- The union of the *Want* sets of these new receivers is the same as the *Want* set of the original receiver.

Step 2: An undirected graph model $G(V, E)$ is constructed for the new WPRTP instance with the following two criteria met:

- For each receiver in the new WPRTP instance, there is a corresponding vertex in $G(V, E)$.
- An edge exists between a pair of vertices in $G(V, E)$ if and only if either one of the following two conditions holds: (i) the two receivers have identical *Want* set; and (ii) the *Want* set of each of the two receivers is a subset of the *Has* set of the other.

Step 3: The complimentary graph of $G(V, E)$ is obtained. A solution to the graph coloring problem instance of the complimentary graph is obtained using some heuristic algorithm.

Step 4: The solution to the graph coloring problem instance is transformed to a solution to the original WPRTP instance as follows: The wanted packets of all the receivers that correspond to the set of vertices with the same color in the graph coloring solution are combined into one coded packet, and all such coded packets corresponding to the colors in the graph coloring solution make up a valid solution to the original WPRTP instance.

CliqueNC [9] also adopts graph theory to search for solutions to a P-WPRTP instance by transforming it to a clique cover problem instance. It works in the following steps.

Step 1: A graph model $G(V, E)$ is constructed for the P-WPRTP instance in the following two sub-steps: (i) create a vertex in $G(V, E)$ for each packet; and (ii) for any pair of packets, create an edge between the corresponding vertices in $G(V, E)$ if and only if no receiver whose *Want* set includes both packets.

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Step 2: A clique cover of $G(V, E)$ is obtained using some heuristic algorithm.
Step 3: A clique in the clique cover of $G(V, E)$ is mapped to a coded packet, and all coded packets mapped from the cliques in the clique cover make up a valid solution to the original P-WPRTP instance.

A simple approach to extend CliqueNC for solving IP-WPRTP instances is as follows: (i) transform the IP-WPRTP(P, R, H, W) instance into a P-WPRTP(P', R', H', W') instance by letting $P' = P$, $R' = R$, $H'(r_i) = H(r_i)$, $W'(r_i) = P - H(r_i)$; and (ii) obtain a valid solution to the P-WPRTP instance by using the original CliqueNC and use this solution as the final solution to the IP-WPRTP instance.

Solutions with fewer packet retransmissions may be found more easily on larger Galois fields. In [10], the number of packet retransmissions in optimal NC-based solutions on field GF(2^q) to P-WPRTP was analyzed, and a scheme that is optimal in the number of packet retransmissions was proposed on the basis of random network coding. Then in [11], upper and lower bounds on the number of packet retransmissions in optimal NC-based schemes for IP-WPRTP were analyzed, and then by exploiting the differences between the upper bound and lower bound in a divide and conquer based approach, a scheme named as IP-WPRTP-DC (divide and conquer based scheme for IP-WPRTP) was proposed for IP-WPRTP.

3. COLORCLIQUENC

In ColorCliqueNC, an original WPRTP instance is first transformed to a new WPRTP instance. Then a graph model is constructed for the new WPRTP instance. And then a solution to the clique cover problem instance on the graph model is obtained. Finally, the solution to the clique cover problem instance is transformed to a solution to the original WPRTP instance. On the one hand, similar to CliqueNC but different from ColorNC, it is a packet instead of a receiver that corresponds to a vertex in the graph model in ColorCliqueNC. On the other hand, edge creation criterion in ColorCliqueNC is similar to that in ColorNC but more different from that in CliqueNC.

In detail, ColorCliqueNC solves WPRTP instances in the following five steps:

Step 1: Transform the original WPRTP instance into a new WPRTP instance such that, for each packet $p_k \in \bigcup_{r_i \in R} W(r_i)$, there is a corresponding receiver c_k in the new WPRTP instance with $W(c_k) = \{p_k\}$ and $H'(c_k) = \{\cap H(r_i) | p_k \in W(r_i), \forall r_i \in R\}$.
Step 2: Construct a graph model $G(V, E)$ for the new WPRTP instance as follows: (i) for each receiver in the new WPRTP instance, create a corresponding vertex in $G(V, E)$ and (ii) create an edge between a pair of vertices in $G(V, E)$ if and only if the *Want* set of each of the two receivers is a subset of the *Has* set of the other.
Step 3: Find a clique cover of $G(V, E)$ using some heuristic algorithm.
Step 4: For each clique in the clique cover, construct a coded packet by XORing the packets corresponding to the vertices in the clique. All coded packets mapped from the cliques in the clique cover make up a valid solution to the original WPRTP instance.

The edge creation criterion in constructing the graph model for the new WPRTP instance in ColorCliqueNC assures the following property: If a pair of vertices is connected by an edge in $G(V, E)$ (here, we suppose that the two packets corresponding to the two vertices are p_1 and p_2, respectively), then packets p_1 and p_2 can be XORed together into a coded packet $p_1 \oplus p_2$. When receiving the assembled packet that contains $p_1 \oplus p_2$, each receiver who wants p_1 or p_2 could obtain the wanted packet by decoding. On the basis of this property, a clique in $G(V, E)$ is mapped to a coded packet, and all such coded packets corresponding to the cliques in the clique cover of the graph model in ColorCliqueNC make up a valid solution to the original WPRTP instance.

Table I provides a comparison among the operation processes of ColorNC, CliqueNC, and ColorCliqueNC for an example WPRTP instance. The WPRTP instance is as follows. $R = \{A, B, C\}$, $P = \{p_1, p_2, p_3\}$. The *Has* sets and the *Want* sets of the three receivers are $H(A) = \{p_1\}$, $W(A) = \{p_2, p_3\}$, $H(B) = \{p_2, p_3\}$, $W(B) = \{p_1\}$, $H(C) = \{\}$, and $W(C) = \{p_2\}$. As shown in the table, for this WPRTP instance, the numbers of coded packets in possible solutions determined by ColorNC, CliqueNC, and ColorCliqueNC are 2, 3, and 2, respectively. Although both ColorNC and ColorCliqueNC lead to two

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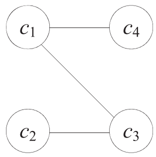
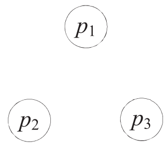
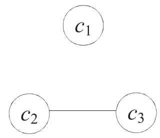
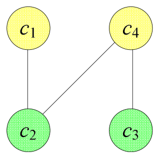
Table I. Operation processes of ColorNC, CliqueNC, and ColorCliqueNC for a WPRTP instance.

Scheme	ColorNC	CliqueNC	ColorCliqueNC
Original WPRTP Instance	$R=\{A, B, C\}$, $P=\{p_1, p_2, p_3\}$ $H(A)=\{p_1\}$, $W(A)=\{p_2, p_3\}$; $H(B)=\{p_2, p_3\}$, $W(B)=\{p_1\}$; $H(C)=\{\}$, $W(C)=\{p_2\}$		
New WPRTP instance	$R=\{c_1, c_2, c_3, c_4\}$ $P=\{p_1, p_2, p_3\}$ $H(c_1)=\{p_1\}$, $W(c_1)=\{p_2\}$; $H(c_2)=\{p_1\}$, $W(c_2)=\{p_3\}$; $H(c_3)=\{p_2, p_3\}$, $W(c_3)=\{p_1\}$; $H(c_4)=\{\}$, $W(c_4)=\{p_2\}$;	$R=\{A', B', C'\}$ $P=\{p_1, p_2, p_3\}$ $H(A')=\{p_1\}$, $W(A')=\{p_2, p_3\}$; $H(B')=\{p_2, p_3\}$, $W(B')=\{p_1\}$; $H(C')=\{\}$, $W(C')=\{p_1, p_2, p_3\}$	$R=\{c_1, c_2, c_3\}$ $P=\{p_1, p_2, p_3\}$ $H(c_1)=H(B)=\{p_2, p_3\}$,$W(c_1)=\{p_1\}$; $H(c_2)=H(A)\cap H(C)=\{\}$,$W(c_2)=\{p_2\}$; $H(c_3)=H(A)=\{p_1\}$,$W(c_3)=\{p_3\}$;
Graph model			
Complimentary graph			
Cliques/colors	$\{c_1, c_4\}$, $\{c_2, c_3\}$	$\{p_1\}$, $\{p_2\}$, $\{p_3\}$	$\{c_1\}$, $\{c_2, c_3\}$
Coded packets	$\{p_2\}$, $\{p_1 \oplus p_3\}$	$\{p_1\}$, $\{p_2\}$, $\{p_3\}$	$\{p_1\}$, $\{p_2 \oplus p_3\}$

packet retransmissions, the graph model in ColorCliqueNC has fewer vertices than that in ColorNC; thus, ColorCliqueNC will be more computationally efficient than ColorNC. With the facts of much fewer vertices in the graph model in ColorCliqueNC as well as the sub-optimality of the solutions determined by available heuristic algorithms for graph coloring problem, solutions found by ColorCliqueNC may require fewer packet retransmissions than those found by ColorNC. This argument is confirmed by simulation results in Section 5.

4. COMPARATIVE ANALYSIS ABOUT COLORCLIQUENC, COLORNC, AND CLIQUENC

In each of the three schemes ColorNC, CliqueNC, and ColorCliqueNC, a new WPRTP instance is constructed from the original WPRTP instance. In the following text, we use Original-WPRTP to represent the given target WPRTP instance; meanwhile, use Color-WPRTP, Clique-WPRTP, and ColorClique-WPRTP to represent the new WPRTP instances created in ColorNC, CliqueNC, and ColorCliqueNC, respectively. Additionally, in each of the three schemes, a graph model is created for the new WPRTP instance. In the following text, we use Color-Graph, Clique-Graph, and ColorClique-Graph to represent the graph models created in ColorNC, CliqueNC, and ColorCliqueNC, respectively.

In this section, to show the superiority of ColorCliqueNC to ColorNC and CliqueNC, we qualitatively and quantitatively analyze some properties of the three schemes. The inspected properties include the following:

- the sets of all valid solutions to Original-WPRTP, Color-WPRTP, and ColorClique-WPRTP; ■
- the numbers of receivers in Color-WPRTP and ColorClique-WPRTP; and
- the sets of edges in ColorClique-Graph and Clique-Graph.

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4.1. Relationships between valid solution sets of Original-WPRTP, Color-WPRTP, and ColorClique-WPRTP

About the relationships between the valid solution sets of Original-WPRTP, Color-WPRTP, and ColorClique-WPRTP, we have the following lemmas.

Lemma 1

Original-WPRTP and Color-WPRTP have exactly the same valid solution set.

Proof

It will be proved by showing that any valid solution to Original-WPRTP must be a valid solution to Color-WPRTP and vice versa.

According to ColorNC, a receiver in Original-WPRTP corresponds to a set of receivers in Color-WPRTP with three properties as described in Section 2. Here, we call these receivers in Color-WPRTP as the corresponding receivers of the original receiver in Original-WPRTP.

On the one hand, for any valid solution to Original-WPRTP, each receiver in Original-WPRTP must be able to obtain all the packets in its *Want* set. Noticing the following two facts, we know that this valid solution to Original-WPRTP is indeed a valid solution to Color-WPRTP:

- The *Has* set of each corresponding receiver in Color-WPRTP is the same with the original receiver in Original-WPRTP.
- The *Want* set of each corresponding receiver is a subset of the original receiver in Original-WPRTP.

On the other hand, with respect to any valid solution to Color-WPRTP, each receiver in Color-WPRTP must be able to obtain the packet in its *Want* set. Because the *Has* sets of the receivers in Color-WPRTP that correspond to the same receiver in Original-WPRTP are the same, the receiver in Original-WPRTP must also be able to obtain all the packets in the union of the *Want* sets of the corresponding receivers in Color-WPRTP. Hence, any valid solution to Color-WPRTP must also be a valid solution to Original-WPRTP.

As a conclusion, the lemma follows. ■

Lemma 2

The set of valid solutions to ColorClique-WPRTP is a subset of that of Original-WPRTP.

Proof

It will be proved by showing that any valid solution to ColorClique-WPRTP is a valid solution to Original-WPRTP, but there may exist some solutions that are valid to Original-WPRTP but are not valid to ColorClique-WPRTP.

Given any valid solution to ColorClique-WPRTP, we know that for each packet p, the receiver r in ColorClique-WPRTP whose *Want* set equals $\{p\}$ must be able to obtain packet p. Because the *Has* set of receiver r in ColorClique-WPRTP is a subset of each of the receivers whose *Want* set includes packet p in Original-WPRTP, these receivers in Original-WPRTP must all be able to obtain packet p. This situation applies to all the packets in ColorClique-WPRTP. Hence, this valid solution to ColorClique-WPRTP must also be a valid solution to Original-WPRTP.

However, there may be some solutions that are valid to Original-WPRTP but are not valid to ColorClique-WPRTP. Let us suppose an Original-WPRTP instance with receiver set $R=\{A, B, C, D\}$, packet set $P=\{p_1, p_2, p_3\}$, the *Has* sets, and the *Want* sets of the receivers are as follows: $H(A)=\{p_2\}$, $W(A)=\{p_3\}$, $H(B)=\{p_1\}$, $W(B)=\{p_3\}$, $H(C)=\{p_3\}$, $W(C)=\{p_2\}$, $H(D)=\{p_3\}$, and $W(D)=\{p_1\}$. For this Original-WPRTP instance, one can easily verify that $\{p_2 \oplus p_3, p_1 \oplus p_3\}$ is a valid solution. However, as explained in the following text, this solution is not valid to ColorClique-WPRTP. The corresponding ColorClique-WPRTP has three receivers (denoted as E, F, and G, respectively); the *Has* sets and *Want* sets of these receivers are as follows: $H(E)=\{p_3\}$, $W(E)=\{p_1\}$, $H(F)=\{p_1\}$, $W(F)=\{p_2\}$, $H(G)=H(A) \cap H(B)=\{p_2\} \cap \{p_1\}=\varnothing$, $W(G)=\{p_3\}$. It is

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easy to verify that $\{p_2 \oplus p_3, p_1 \oplus p_3\}$ is not a valid solution to ColorClique-WPRTP. The previous example instance is an IP-WPRTP instance. Now, suppose a P-WPRTP instance with receiver set $R=\{A, B, C\}$, packet set $P=\{p_1, p_2, p_3\}$, the *Has* sets, and the *Want* sets of the receivers are as follows: $H(A)=\{p_1\}$, $W(A)=\{p_2, p_3\}$, $H(B)=\{p_2\}$, $W(B)=\{p_1, p_3\}$, $H(C)=\{p_3\}$, and $W(C)=\{p_1, p_2\}$. The corresponding ColorClique-WPRTP is as follows: receiver set $R=\{r_1, r_2, r_3\}$, packet set $P=\{p_1, p_2, p_3\}$, the *Has* sets, and *Want* sets of these receivers are $H(r_1)=H(B)\cap H(C)=\{p_2\}\cap\{p_3\}=\{\}$, $W(r_1)=\{p_1\}$, $H(r_2)=\{\}$, $W(r_2)=\{p_2\}$, $H(r_3)=\{\}$, and $W(r_3)=\{p_3\}$. It is easy to verify that $\{p_2 \oplus p_3, p_1 \oplus p_3\}$ is a valid solution to this original-WPRTP, but it is not valid to the corresponding ColorClique-WPRTP. The previous two example WPRTP instances show that there may be some solutions that are valid to Original-WPRTP but are not valid to ColorClique-WPRTP.

As a conclusion, the lemma follows. ■

4.2. Number of receivers in ColorClique-WPRTP and Color-WPRTP

The number of receivers in ColorClique-WPRTP equals $\left|\bigcup_{r_i \in R} W(r_i)\right|$, whereas that in Color-WPRTP equals $\sum_{r_i \in R} |W(r_i)|$. Obviously, there is $|\cup_{r_i \in R} W(r_i)| \le \sum_{r_i \in R} |W(r_i)|$. Hence, the number of receivers in ColorClique-WPRTP is smaller than or equal to that in Color-WPRTP.

ColorCliqueNC and ColorNC both search for optimal valid solutions to WPRTP on GF(2). We call the set of all valid solutions on GF(2) to ColorClique-WPRTP as the solution search space of ColorCliqueNC and call the set of all valid solutions on GF(2) to Color-WPRTP as the solution search space of ColorNC. About the relationship between the solution search space of ColorCliqueNC and ColorNC, we have the following theorem.

Theorem 1

The solution search space of ColorCliqueNC is a subset of that of ColorNC, and the reverse may not hold.

Proof

Following the arguments similar to what are used in the proofs of Lemmas 1 and 2, it is easy to show that the two lemmas hold when only solutions on GF(2) are considered. In other words, Original-WPRTP and Color-WPRTP have exactly the same solution search space, and the solution search space of ColorClique-WPRTP is a subset of that of Original-WPRTP. Hence, we know that the solution search space of ColorClique-WPRTP is a subset of that of ColorNC, and the reverse may not hold. ■

Because the solution search space of ColorCliqueNC is a subset of that of ColorNC, the solution found by ColorCliqueNC may be worse than that by ColorNC. However, because searching for optimal NC-based valid solutions on GF(2) to WPRTP is a NP-complete problem [1], the solution found using some heuristic algorithm for a WPRTP instance is usually suboptimal. Hence, a larger size WPRTP instance may lead to lower quality solution with more coded packets. Thus, smaller WPRTP may partly compensate for the negative effect of the restriction on valid solution set in ColorCliqueNC. Hence, ColorCliqueNC is expected to obtain similar quality solutions with improved computational efficiency resulting from the fact that ColorClique-WPRTP is usually smaller than Color-WPRTP. Fortunately, simulation results in the following section show that the solutions found by ColorCliqueNC are usually better than those found by ColorNC.

To gain a deeper perception about the difference between the number of receivers in ColorClique-WPRTP and Color-WPRTP, a quantitative analysis is provided here.

Given a WPRTP(P, R, H, W) instance, we denote that $|P|=n$, $|R|=r$. With respect to any certain receiver r_i, we denote c_{Level} as the probability that a certain packet p_j is contained in $H(r_i)\cup W(r_i)$ and denote p_{Level} as the conditional probability that a certain packet p_j is contained in $W(r_i)$ on the condition that it falls into $H(r_i)\cup W(r_i)$. c_{Level} is called as packet consideration level, and p_{Level}

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is called as packet request level. A tuple (n, r, c_{Level}, p_{Level}) is called a WPRTP profile. Given a WPRTP profile (n, r, c_{Level}, p_{Level}), we have the following theorems about the number of receivers in Color-WPRTP and ColorClique-WPRTP.

Theorem 2

Given a WPRTP profile (n, r, c_{Level}, p_{Level}), the probability that the number of receivers in Color-WPRTP equals x, denoted as $p_{\text{COLOR}}(n, r, c_{\text{Level}}, p_{\text{Level}}, x)$, is given by Equation (1), and the expected number of receivers in Color-WPRTP, denoted as $N_{\text{COLOR}}(n, r, c_{\text{Level}}, p_{\text{Level}})$, is given by Equation (2).

$$p_{\text{COLOR}}(n, r, c_{\text{Level}}, p_{\text{Level}}, x) = \binom{nr}{x}\,(1 - c_{\text{Level}} \cdot p_{\text{Level}})^{nr-x} \cdot (c_{\text{Level}}\, p_{\text{Level}})^{x} \tag{1}$$

$$N_{\text{COLOR}}(n, r, c_{\text{Level}}, p_{\text{Level}}) = \sum_{x=1}^{nr}\left(x\binom{nr}{x}(1 - c_{\text{Level}}\, p_{\text{Level}})^{nr-x}(c_{\text{Level}}\, p_{\text{Level}})^{x}\right) \tag{2}$$

Proof

For each pair of (receiver r_i, packet p_j), there are totally three disjoint cases: (i) $p_j \in W(r_i)$, (ii) $p_j \in H(r_i)$, and (iii) $p_j \in P - H(r_i) - W(r_i)$. Denote the three cases as case 1, case 2, and case 3, respectively. We notice that the probability that a pair (r_i, p_j) falls in the union of case 1 and case 2, denoted as p(cases 1&2), equals c_{Level}. It is also obvious that the conditional probability that it falls in case 1 given that it falls in the union of case 1 and case 2, denoted as p(case 1|cases 1&2), equals p_{Level}. Hence, we have

$$p(\text{case } 1) = p(\text{cases } 1\&2)\,p(\text{case } 1|\text{cases } 1\&2) = c_{\text{Level}}\, p_{\text{Level}} \tag{3}$$

Thus, for each pair (r_i, p_j), the probability that $p_j \in W(r_i)$ equals $c_{\text{Level}} p_{\text{Level}}$.

The number of receivers in Color-WPRTP equals $\sum_{r_i \in R} |W(r_i)|$, which is equal to the number of (r_i, p_j) pairs that fall in case 1. There are nr pairs in total. According to the definition of $p_{\text{COLOR}}(n, r, c_{\text{Level}}, p_{\text{Level}}, x)$, we know that the value of $p_{\text{COLOR}}(n, r, c_{\text{Level}}, p_{\text{Level}}, x)$ is equal to the probability that, among these nr pairs, there are exactly x pairs falling in case 1. Hence, $p_{\text{COLOR}}(n, r, c_{\text{Level}}, p_{\text{Level}}, x)$ can be calculated as Equation (4):

$$\begin{aligned} p_{\text{COLOR}}(n, r, c_{\text{Level}}, p_{\text{Level}}, x) &= \binom{nr}{x}(1 - p(\text{case } 1))^{nr-x}(p(\text{case } 1))^{x} \\ &= \binom{nr}{x}(1 - c_{\text{Level}} p_{\text{Level}})^{nr-x}(c_{\text{Level}} p_{\text{Level}})^{x} \end{aligned} \tag{4}$$

Consequently, the expected number of receivers in Color-WPRTP can be calculated as Equation (5):

$$\begin{aligned} N_{\text{COLOR}}(n, r, c_{\text{Level}}, p_{\text{Level}}) &= \sum_{x=0}^{nr}(x p_{\text{COLOR}}(n, r, c_{\text{Level}}, p_{\text{Level}}, x)) \\ &= \sum_{x=0}^{nr}\left(x\binom{nr}{x}\,(1 - c_{\text{Level}} p_{\text{Level}})^{nr-x}(c_{\text{Level}}\, p_{\text{Level}})^{x}\right) \\ &= \sum_{x=1}^{nr}\left(x\binom{nr}{x}\,(1 - c_{\text{Level}} p_{\text{Level}})^{nr-x}(c_{\text{Level}}\, p_{\text{Level}})^{x}\right) \end{aligned} \tag{5}$$

As a conclusion, the theorem follows. ■

Theorem 3

Given a WPRTP profile (n, r, c_{Level}, p_{Level}), the probability that the number of receivers in ColorClique-WPRTP equals x, denoted as $p_{\text{COLORCLIQUE}}(n, r, c_{\text{Level}}, p_{\text{Level}}, x)$, is given by Equation (6), and the expected number of receivers in ColorClique-WPRTP, denoted as $N_{\text{COLORCLIQUE}}(n, r, c_{\text{Level}}, p_{\text{Level}})$, is given by Equation (7).

$$p_{\text{COLORCLIQUE}}(n, r, c_{\text{Level}}, p_{\text{Level}}, x) = \binom{n}{x}(1 - c_{\text{Level}}\, p_{\text{Level}})^{r(n-x)}\left(1 - (1 - c_{\text{Level}}\, p_{\text{Level}})^{r}\right)^{x} \tag{6}$$

$$N_{\text{COLORCLIQUE}}(n, r, c_{\text{Level}}, p_{\text{Level}}) = \sum_{i=1}^{n}\left(i\left(\binom{n}{i}(1 - c_{\text{Level}}\, p_{\text{Level}})^{r(n-i)}\left(1 - (1 - c_{\text{Level}}\, p_{\text{Level}})^{r}\right)^{i}\right)\right) \tag{7}$$

Proof

The number of receivers in ColorClique-WPRTP equals the number of packets wanted by at least one receiver, that is, $\left|\bigcup_{r_i \in R} W(r_i)\right|$, which is exactly the number of packets in the solution determined by NoNC. Hence, Theorem 2 in [10] can be used to obtain $p_{\text{COLORCLIQUE}}(n, r, c_{\text{Level}}, p_{\text{Level}}, x)$ and $N_{\text{COLORCLIQUE}}(n, r, c_{\text{Level}}, p_{\text{Level}})$.

According to the meanings of parameters in WPRTP profile (n, r, c_{Level}, p_{Level}) and those in WPRTP profile (n, r, p_{Level}) defined in [10], we know that $c_{\text{Level}}\, p_{\text{Level}}$ in WPRTP profile (n, r, c_{Level}, p_{Level}) corresponds to p_{Level} in WPRTP profile (n, r, p_{Level}). By replacing p_{Level} in Theorem 2 in [10] with $c_{\text{Level}}p_{\text{Level}}$, the equations in this theorem can be obtained easily. ■

We verify Theorems 2 and 3 through the Monte Carlo simulation. For WPRTP profile (n, r, c_{Level}, p_{Level}) with $n = 10$, $r = 10$, $c_{\text{Level}}p_{\text{Level}} = 0.2$, the simulation results shown in Figure 1 are obtained over 10,000 randomly generated WPRTP instances. In Figure 2, the curves labeled as 'Theory ColorClique-WPRTP' and 'Theory Color-WPRTP' represent theoretical results obtained using Equations (4) and (6), respectively. Bar charts labeled as 'Sim ColorClique-WPRTP' and 'Sim Color-WPRTP' represent the Monte Carlo simulation results. The fine fitness between simulation results and theoretical values proves the correctness of Equations (4) and (6). As a consequence, Equations (5) and (7) must also be correct.

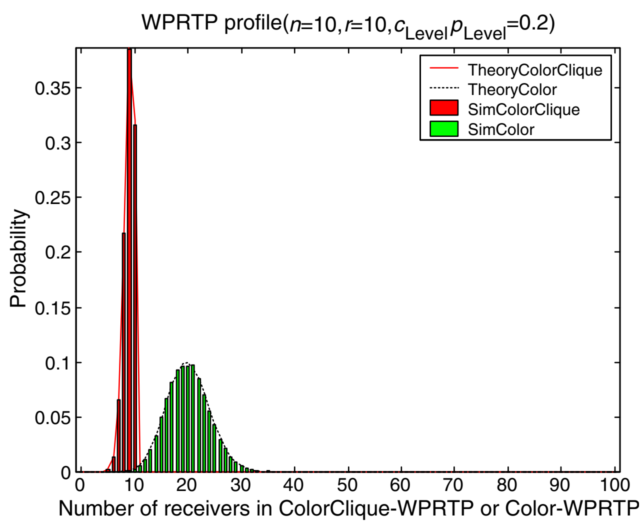

Figure 1. Verification of theoretical results in Theorems 2 and 3 with profile $n = 10$, $r = 10$, and $c_{\text{Level}}p_{\text{Level}} = 0.2$.

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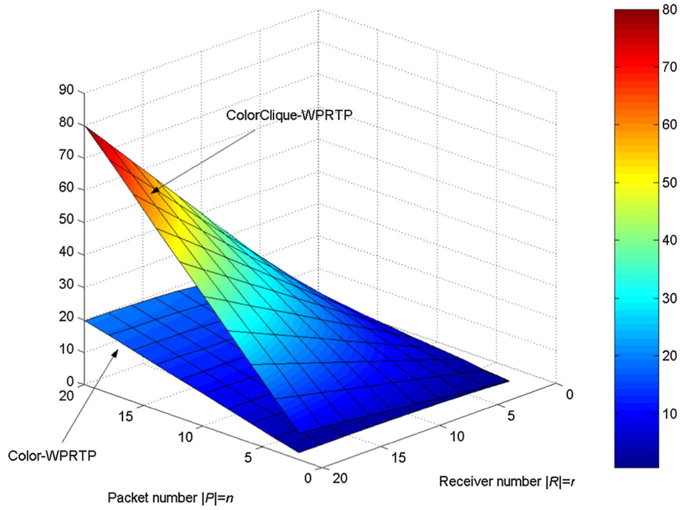
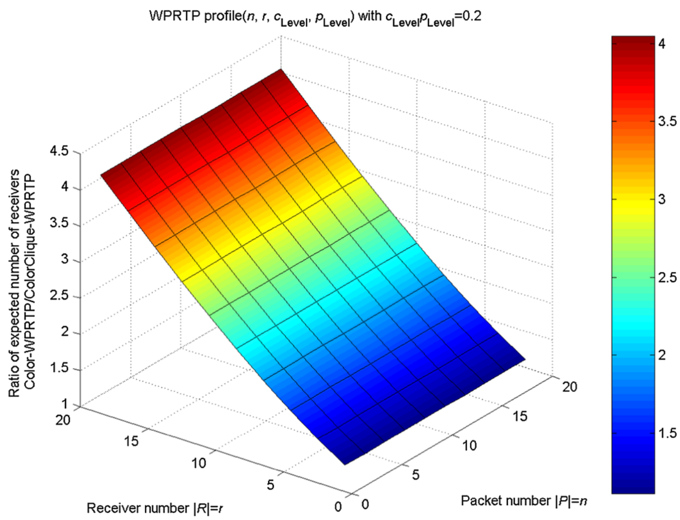


(a)Number of receivers



(b)Ratio of the number of receivers in Color-WPRTP to that in ColorClique-WPRTP.

Figure 2. Comparison between the number of receivers in Color-WPRTP and that in ColorClique-WPRTP.

Figure 2 shows numerical results about the expected number of receivers in Color-WPRTP and ColorClique-WPRTP according to Equations ((5)) and ((7)) with different WPRTP profiles where $c_{\mathrm{Level}}p_{\mathrm{Level}} = 0.2$. The results show that both n and r have great effects on the expected number of receivers; meanwhile, only r has a great effect on the ratio of expected number of receivers in Color-WPRTP to that in ColorClique-WPRTP. When $r = 2$, the ratio of the number of receivers in Color-WPRTP to that in ColorClique-WPRTP is about 2. As r increases, the ratio increases almost linearly. When $r = 20$, the ratio increases to about 4. When $r = 20$ and $n = 20$, the number of receivers in ColorClique-WPRTP is about 20, but that in Color-WPRTP is about 80. This implies that ColorClique-WPRTP will be more treatable than Color-WPRTP.

4.3. Edge sets of ColorClique-Graph and Clique-Graph

In this section, we make a comparative analysis about the edge set of Clique-Graph and ColorClique-Graph. By showing that the edge set of Clique-Graph is a subset of that of

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ColorClique-Graph, we can expect that ColorCliqueNC may lead to fewer packet retransmissions than CliqueNC.

Let $S(p_i) = \{r_j | p_i \in W(r_j),\ r_j \in R\}$, $T(p_i) = \{\cap H(r_j) | r_j \in S(p_i)\}$. With respect to the edge set of ColorClique-Graph and Clique-Graph, we have the following lemmas.

Lemma 3

For any P-WPRTP instance, there is an edge between a pair of vertices in ColorClique-Graph if and only if there is an edge between the corresponding pair of vertices in Clique-Graph.

Proof

Firstly, we prove that, for any P-WPRTP instance, if an edge exists between two vertices in Clique-Graph, then there must be an edge between the corresponding pair of vertices in ColorClique-Graph. For each packet $p_i \in \cup_{r_i \in R} W(r_i)$, we know that there is a corresponding vertex in Clique-Graph. If an edge exists between two vertices corresponding to packets p_i and p_j in Clique-Graph, there must be $S(p_j) \cap S(p_i) = \{\}$. This means that, for $\forall r_m \in S(p_i)$ and $\forall r_n \in S(p_j)$, there must be $p_i \in W(r_m)$, $p_j \notin W(r_m)$, $p_j \in W(r_n)$, and $p_i \notin W(r_n)$. In any P-WPRTP instance, there is $P = H(r_i) \cup W(r_i)$ for $\forall r_i \in R$. Hence, $p_j \notin W(r_m)$ indicates $p_j \in H(r_m)$, and $p_i \notin W(r_n)$ indicates $p_i \in H(r_n)$. Because $p_j \in H(r_m)$ and $p_i \in H(r_n)$ are true for $\forall r_m \in S(p_i)$ and $\forall r_n \in S(p_j)$, there must be $p_j \in T(p_i)$ and $p_i \in T(p_j)$. As a consequence, according to edge creation criterion in ColorCliqueNC, there must be an edge between the two vertices corresponding to packets p_i and p_j in ColorClique-Graph.

Secondly, we prove by contradiction that, if an edge exists between two vertices in ColorClique-Graph, there must be an edge between the two corresponding vertices in Clique-Graph. For any packet p_i belonging to the *Want* set of at least one receiver, there is a corresponding vertex in ColorClique-Graph. Suppose that an edge exists between two vertices corresponding to packets p_i and p_j in ColorClique-Graph but no edge exists between the two vertices that correspond to the same two packets in Clique-Graph. According to the edge creation criterion in constructing Clique-Graph, we know that the absence of the edge in Clique-Graph implies that there must be at least one receiver $r_k \in R$ with $W(r_k) \supseteq \{p_i, p_j\}$ in the corresponding Clique-WPRTP. Hence, $p_i \notin H(r_k)$ and $p_j \notin H(r_k)$. As a consequence, $p_j \notin T(p_i)$ and $p_i \notin T(p_j)$. Thus, according to the edge creation criterion in constructing ColorClique-Graph, there must be no edge between the two vertices corresponding to packets p_i and p_j in ColorClique-Graph. This contradicts with the assumption that an edge exists between the two vertices in ColorClique-Graph. Hence, we know that if an edge exists between two vertices in ColorClique-Graph, there must be an edge between the two corresponding vertices in Clique-Graph.

As a conclusion, the lemma follows. ■

In CliqueNC, as described in Section 2, an IP-WPRTP instance is first transformed to a corresponding P-WPRTP instance. In the following text, we use $S_{\mathrm{IP}}(p_i)$ and $T_{\mathrm{IP}}(p_i)$ to represent $S(p_i)$ and $T(p_i)$ relate to the IP-WPRTP instance, respectively. Similarly, we use $S_{\mathrm{P}}(p_i)$ and $T_{\mathrm{P}}(p_i)$ to represent $S(p_i)$ and $T(p_i)$ relate to the P-WPRTP instance, respectively.

Lemma 4

For any IP-WPRTP instance, if there is an edge between a pair of vertices in Clique-Graph, there must be an edge between the corresponding pair of vertices in ColorClique-Graph, but the reverse may not hold.

Proof

With respect to the IP-WPRTP(P, R, H, W) instance and the corresponding P-WPRTP(P', R', H', W') instance, because $W'(r_i) = P - H(r_i)$ holds for $\forall r_i \in R$, there must be $W(r_i) \subseteq W'(r_i)$. As a consequence, there must be $S_{\mathrm{IP}}(p_i) \subseteq S_{\mathrm{P}}(p_i)$ for $\forall p_i \in P$. Given that $H'(r_i) = H(r_i)$, there must be $T_{\mathrm{IP}}(p_i) \supseteq S_{\mathrm{P}}(p_i)$ for $\forall p_i \in P$.

As proved in Lemma 3, if an edge exists between two vertices corresponding to packets p_i and p_j in Clique-Graph, there must be $p_j \in T_{\mathrm{P}}(p_i)$ and $p_i \in T_{\mathrm{P}}(p_j)$. Because $T_{\mathrm{IP}}(p_i) \supseteq T_{\mathrm{P}}(p_i)$ and $T_{\mathrm{IP}}(p_j) \supseteq T_{\mathrm{P}}(p_j)$, we know that there must be $p_j \in T_{\mathrm{IP}}(p_i)$ and $p_i \in T_{\mathrm{IP}}(p_j)$. As a result, there must be an edge

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between the two vertices corresponding to packets p_i and p_j in ColorClique-Graph. Hence, we know that, if an edge exists between two vertices in Clique-Graph, there must be an edge between the two vertices in ColorClique-Graph.

Next, we show that the reverse may not hold by providing an example IP-WPRTP instance as shown in Figure 3. In the instance, receiver set $R = \{r_1, r_2, r_3\}$, packet set $P = \{p_1, p_2, p_3\}$, the *Want* sets, and the *Has* sets of the receivers are $W(r_1) = \{p_2\}$, $H(r_1) = \{\}$, $W(r_2) = \{p_3\}$, $H(r_2) = \{p_1, p_2\}$, $W(r_3) = \{p_1\}$, $H(r_3) = \{p_2, p_3\}$. We can easily verify that ColorClique-Graph and Clique-Graph for the instance are as shown in the figure. Notice that there is an edge between two vertices corresponding to packets p_1 and p_3 in ColorClique-Graph, but there is no edge between the two corresponding vertices in Clique-Graph. Thus, the example WPRTP instance indicates that the reverse does not hold.

As a conclusion, the lemma follows. ■

Theorem 4
The set of edges in Clique-Graph is a subset of that in ColorClique-Graph.

Proof
Directly applying Lemmas 3 and 4, this theorem can be obtained easily. ■

Theorem 4 implies that the solutions found by ColorCliqueNC will have fewer coded packets than those found by CliqueNC.

Theorem 5
The number of coded packets in a solution found by ColorCliqueNC for any WPRTP instance must be not larger than that of NoNC.

Proof
The number of coded packets in a solution found by ColorCliqueNC for any WPRTP instance is at most equal to the number of vertices in the corresponding ColorClique-Graph, which equals $\left|\bigcup_{r_i \in R} W(r_i)\right|$. The latter is exactly the number of requested packets, that is, the number of packets determined by NoNC. Hence, the theorem follows. ■

It is obvious that ColorClique-Graph and Clique-Graph have identical vertex set. Hence, according to Theorem 4, the number of cliques in a minimum clique cover of ColorClique-Graph

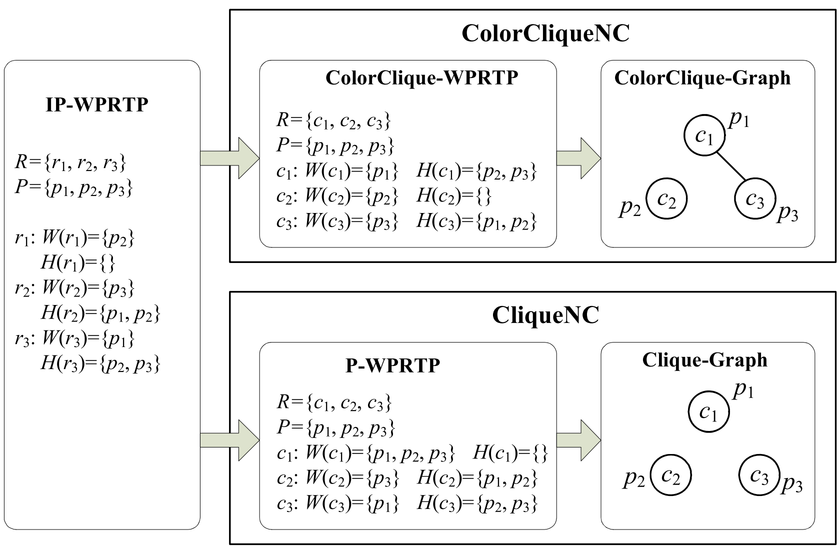

Figure 3. A WPRTP instance where an edge exists in ColorClique-Graph but does not exist in Clique-Graph.

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must not be larger than that of Clique-Graph. When using the same heuristic algorithm for solving the clique cover problem instances that emerged in solving a WPRTP instance, the number of coded packets in a solution found by ColorCliqueNC must be at most equal to that found by CliqueNC. ColorClique-WPRTP usually has few receivers than Color-WPRTP. Hence, ColorCliqueNC is computationally efficient than ColorNC. Simulation results in Section 5 even show that the solutions found by ColorCliqueNC usually have fewer coded packets than those found by ColorNC.

5. PERFORMANCE EVALUATION

The performance of ColorCliqueNC was evaluated and compared with ColorNC [1], CliqueNC [9], IP-WPRTP-DC [11], and NoNC through numerical simulation.

5.1. Performance metrics and simulation configuration

As in [11], two performance metrics are used in our simulations: (i) number of retransmitted packets and (ii) relative number of retransmitted packets. For NC-based schemes, the first metric represents the number of coded packets in a solution. For NoNC, this metric represents the number of requested packets in $\cup_{r_i \in R} W(r_i)$. Relative number of retransmitted packets of *scheme*1/*scheme*2 represents the ratio of the number of retransmitted packets of *scheme*1 to that of *scheme*2. This metric directly reveals the performance gain of a scheme over another.

Basic simulation parameters of WPRTP (P, R, H, W) include packet number $|P|$, receiver number $|R|$, packet consideration level c_{Level}, and packet request level p_{Level}. In simulation context, profile ($|P|$, $|R|$, c_{Level}, p_{Level}) is called as simulation configuration. For each simulation configuration, 100 instances are generated and treated using the tested schemes. A WPRTP instance is represented as a two-dimensional (2D) matrix M_P of size $|R| \times |P|$. Each element $a_{i,j}$ of M_P has three possible values: 0, 1, and 2. The value of $a_{i,j}$ indicates the relationship between packet p_j and receiver r_i: If $a_{i,j} = 0$, then $p_j \in W(r_i)$; if $a_{i,j} = 1$, then $p_j \in H(r_i)$; if $a_{i,j} = 2$, then $p \in P - W(r_i) - H(r_i)$. Each element $a_{i,j}$ is generated as follows: (i) randomly select a value x, which is uniformly distributed in range [0, 1]; (ii) if $x < c_{\text{Level}} p_{\text{Level}}$, then $a_{i,j} = 0$; if $c_{\text{Level}} p_{\text{Level}} \le x < c_{\text{Level}}$, then $a_{i,j} = 1$; if $x \ge c_{\text{Level}}$, then $a_{i,j} = 2$. The values of performance metrics are averaged over these WPRTP instances, and their 95% confidence intervals are obtained. In some of the following figures, confidence intervals are also shown. In the simulations, ColorNC, CliqueNC, and ColorCliqueNC work on GF(2), whereas IP-WPRTP-DC works on GF(2^8).

5.2. Numerical simulation results

5.2.1. Effects of packet request level and packet consideration level. A set of simulations was performed to test the effects of packet consideration level c_{Level} and packet request level p_{Level} on the performance of the schemes. Because $|P| = 10$ and $|R| = 10$ correspond to typical moderate size WPRTP instances, we set the simulation configurations as follows: $|P| = 10$, $|R| = 10$, c_{Level} increases from 0.1 to 0.9 with step size 0.1, and p_{Level} also increases from 0.1 to 0.9 with step size 0.1. The simulation results are shown in Figures 4 and 5.

Figure 4 shows the effects of p_{Level} and c_{Level} on the two performance metrics of all the tested schemes when $p_{\text{Level}} \in [0.1, 0.9]$ and $c_{\text{Level}} \in [0.1, 0.9]$. The smaller p_{Level} and c_{Level} are, the more the performance gains of NC-based schemes over NoNC. When $p_{\text{Level}} = 0.1$ and $c_{\text{Level}} = 0.9$, all the tested NC-based schemes have similar performance, and about 50% packet retransmissions are saved by using NC. ColorCliqueNC obtains fewer packet retransmissions than CliqueNC, which is consistent with Theorem 4 in the previous section.

Results in Figure 4 show that the difference in the performance of the schemes are more distinct when $p_{\text{Level}} = 0.6$. Hence, to gain a more clear view about the performance of the schemes, we provide the 2D curves of the performance metrics as c_{Level} increases from 0.1 to 0.9 when $p_{\text{Level}} = 0.6$ in Figure 5. When $c_{\text{Level}} \le 0.5$, there is no distinct performance difference between ColorNC and ColorCliqueNC. However, when $c_{\text{Level}} \ge 0.5$, ColorNC performs worse than ColorCliqueNC. As

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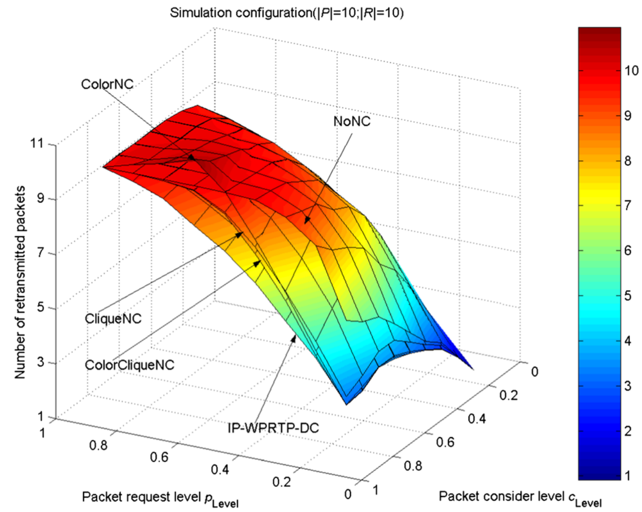
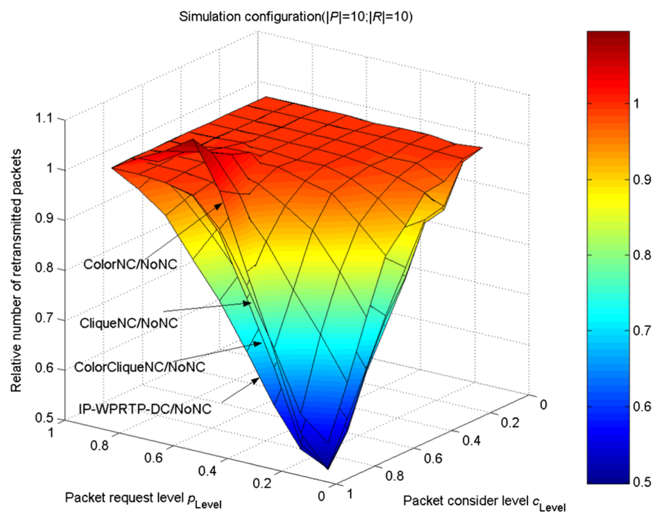


(a)Number of retransmitted packets



(b)Relative number of retransmitted packets

Figure 4. Effects of packet request level and packet consideration level.

c_{Level} increases, the performance difference between ColorNC and ColorCliqueNC becomes even larger. Compared with ColorNC, ColorCliqueNC saves about 10% retransmitted packets when $c_{\text{Level}} = 0.9$.

5.2.2. Effects of packet number and receiver number. To test the effects of packet number |*P*| and receiver number |*R*| on the performance of the schemes, another simulation set was performed. Simulation results in Figure 4 show that, when $c_{\text{Level}} = 0.8$ and $p_{\text{Level}} = 0.7$, performance differences of the tested schemes are more distinct. Hence, to gain a more clear view about the effects of packet number and receiver number on the performance of the schemes, we set the simulation configurations as follows: $c_{\text{Level}} = 0.8$, $p_{\text{Level}} = 0.7$, |*P*| increases from 3 to 30 with step size 3, and |*R*| increases from 2 to 10 with step size 1. Results of these simulations are shown in Figures 6 and 7.

Figure 6 shows the effects of |*P*| and |*R*| on the two performance metrics of all the tested schemes. The results show that, for small |*R*|, the performance gains of the NC-based schemes over NoNC show trivial differences. As |*R*| increases, the performance gains of the NC-based schemes diverge gradually. However, the performance metrics of ColorCliqueNC and CliqueNC become similar as

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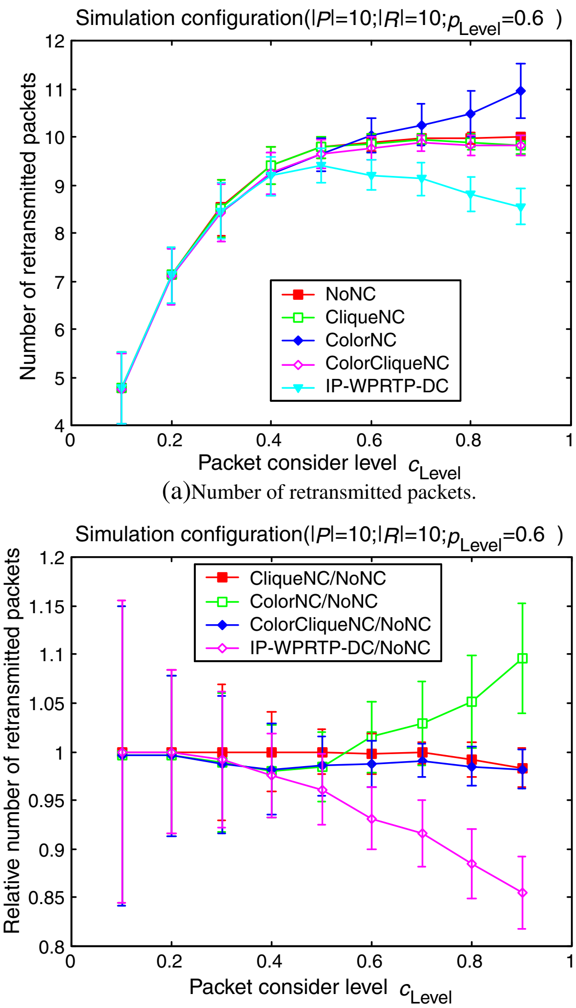

(a)Number of retransmitted packets.

(b)Relative number of retransmitted packets.

Figure 5. Effect of packet consideration level when $p_{\text{Level}} = 0.5$.

$|R|$ increases. This is reasonable when considering that, as $|R|$ increases, both ColorClique-Graph and Clique-Graph tend to become a collection of isolated nodes where each node corresponds to one packet.

The differences in the performance of the schemes when $|P| = 18$ are distinct and typical. To gain a more clear perception about the performance of the schemes, we provide the 2D curves of the performance metrics when $|P| = 18$ in Figure 7. When $|R| = 2$, the number of retransmitted packets in solutions determined by NoNC is about 14.2. However, those in solutions determined by the NC-based schemes are all about 12.5. Thus, about 11% packet transmissions are saved by using NC. As $|R|$ increases, the numbers of retransmitted packets of the schemes increase quickly. However, as the number of retransmitted packets approaches $|P|$, the increasing speed of the metric slows down. This metric of NoNC approaches $|P|$ when $|R| = 6$, whereas those of CliqueNC and ColorCliqueNC approach $|P|$ when $|R| = 10$. Contrastively, this metric of ColorNC increases continuously to even much greater than $|P|$. In these simulation configurations, ColorCliqueNC almost always outperforms CliqueNC and ColorNC.

We also notice that there is a more distinct performance difference between ColorCliqueNC and IP-WPRTP-DC. This mainly results from the fact that IP-WPRTP-DC works on GF(2^8), whereas ColorCliqueNC works on GF(2). Taking computational complexity into consideration, ColorCliqueNC may be more preferable than IP-WPRTP-DC in applications with restricted computing resource.

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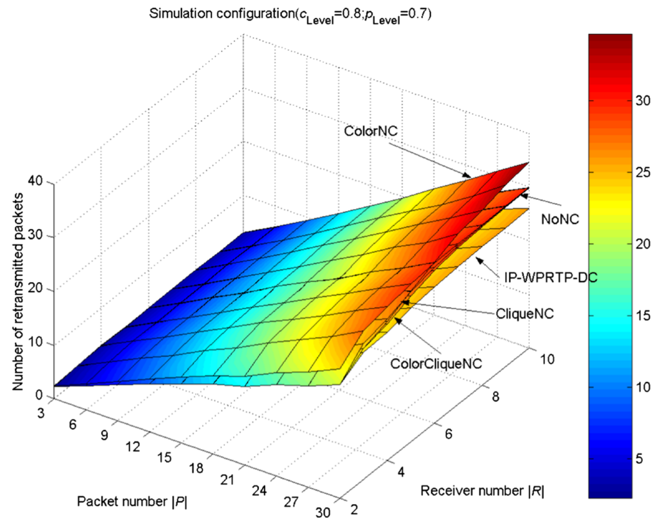
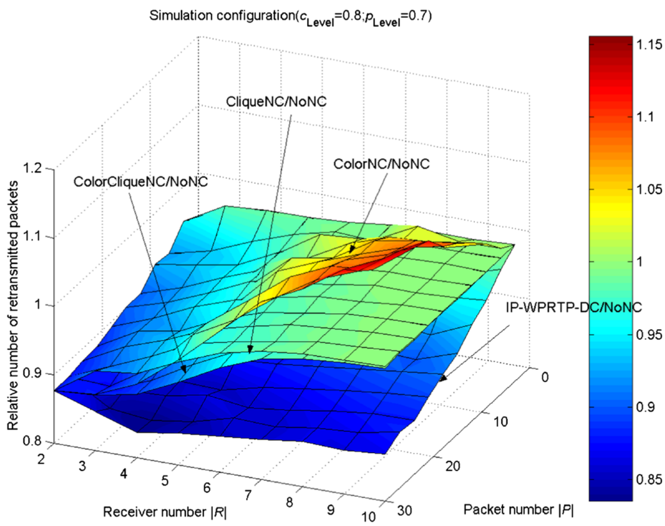


(a)Number of retransmitted packets



(b)Relative number of retransmitted packets

Figure 6. Effects of packet number and receiver number.

6. CONCLUSION

In this paper, combining the basic ideas in ColorNC and CliqueNC, we present ColorCliqueNC, which searches for solutions for WPRTP on GF(2). In ColorCliqueNC, an original WPRTP instance is first transformed to a new WPRTP instance. Then a graph model is constructed for the new WPRTP instance. Next, a solution to the clique cover problem instance on the graph model is obtained. Finally, the solution to the clique cover problem is transformed to a solution to the original WPRTP instance.

The advantages of ColorCliqueNC include the following: (i) it is suitable for all kinds of WPRTP instances; (ii) it works on GF(2); thus, it is computationally efficient than the schemes working on larger Galois fields; and (iii) solutions found by ColorCliqueNC usually have fewer coded packets than those by ColorNC and CliqueNC. Theoretical analysis indicates that ColorCliqueNC is superior to ColorNC and CliqueNC. Additionally, simulation results show that ColorCliqueNC generally outperforms ColorNC and CliqueNC.

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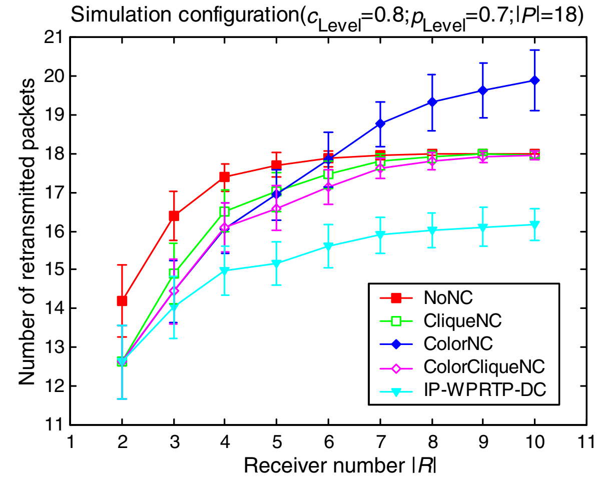
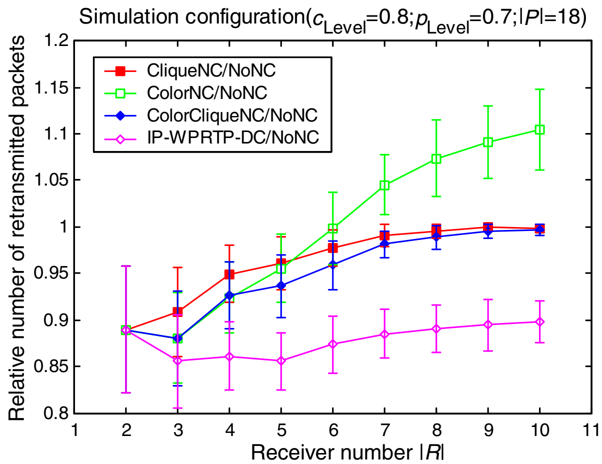


(a)Number of retransmitted packets.



(b)Relative number of retransmitted packets.

Figure 7. Effect of receiver number when $|P| = 18$.

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REFERENCES

1. Rouayheb SY, Chaudhry MAR, Sprintson A. On the minimum number of transmissions in single-hop wireless coding networks. *Proc. IEEE Information Theory Workshop*, IEEE Press. 2007: 120–125, DOI:10.1109/TVT.2008.927729.
2. Ahlswede R, Cai N, Li SYR, Yeung RW. Network information flow. *IEEE Transactions on Information Theory* 2000; **46**(4): 1204–1216, DOI: 10.1109/ITW.2007.4313060.
3. Wu YN., Kung SY. Reduced-complexity network coding for multicasting over ad hoc network. *Proc. IEEE International Conference on Acoustics, Speech, and Signal Processing*, Philadelphia, PA, USA, 2005: 18–23.
4. Chen SG, Wu M, Lu WF. Compressed error and erasure correcting codes via rank-metric codes in random network coding. *International Journal of Communication Systems*, 2012, **25**(11): 1398–1414. DOI: 10.1002/dac.1316.
5. Chen SG, Wu M, Lu WF. Counteracting malicious adversaries via secret and reliable coding mechanism in random network coding. *International Journal of Communication Systems*, 2013, **26**(5): 567–582. DOI: 10.1002/dac.1359.

6. Ruan ZQ, Sun XM, Liang W. Securing sensor data storage and query based on k-out-of-n coding. *International Journal of Communication Systems*, **26**(5): 549–566. DOI: 10.1002/dac.1358.
7. Fragouli C, Boudec JYL, Widmer J. Network coding: an instant primer. *ACM SIGCOMM Computer Communication Review* 2006; 63–68, DOI: 10.1145/1111322.1111337.
8. Nguyen D., Tran, T., Nguyen, BB Wireless broadcast using network coding. *IEEE Transactions on Vehicular Technology* 2009; **58**: 914-925, DOI: 10.1109/TVT.2008.927729.
9. Xu YL, Zhan C, Luo W, Li P. Network coding based reliable multicast for wireless ad hoc networks. *Journal of University of Science and Technology of China (in Chinese)*, 2008; **38**: 860–866. DOI: CNKI:SUN:ZKJD.0.2008-07-023.
10. Gao ZG, Xiang WD, Zhao YL, Zhao J, Cai SB., *et al*. Random network coding based optimal scheme for perfect wireless packet retransmission problems. *Wireless Communications and Mobile Computing*, 2013, **13**(6): 577–592. DOI:10.1002/WCM.1122.
11. Gao ZG, Yang M Wang JP, Nahrstedt K., *et al*. Network coding based schemes for imperfect wireless packet retransmission problems: a divide and conquer approach. *Wireless Personal Communications* 2012; **58**(2): 937–958, DOI: 10.1007/s11277-010-0102-9.
12. Bar-Yossef Z, Birk Y, Jayram TS, Ko T. Index coding with side information. *Proc*. *IEEE Symp*. *On Foundations of Computer Science* (*FOCS*'*06*), IEEE Press, 2006: 197–206. DOI: 10.1109/FOCS.2006.42.
13. Rouayheb SE, Sprintson A Georghiades C. On the index coding problem and its relation to network coding and matroid theory. *IEEE Transactions on Information Theory* 2010; **56**: 3187–3195, DOI: 10.1109/TIT.2010.2048502.
14. Wu Y., Padhye J., Chandra R., Padmanabhan V., Chou P. A. The local mixing problem. *Proc*. *Information Theory and Applications Workshop*, San Diego, USA, Feb. 2006.
15. Katti S, Rahul H, Hu WJ, Katabi D, Médard M, Crowcroft J. XORs in the air: practical wireless network coding. *ACM SIGCOMM Computer Communication Review* 2006; **36**: 243–254, DOI: 10.1145/1151659.1159942
16. Dong QF, Wu JM, Hu WJ, Crowcroft J. Practical network coding in wireless networks. *Proc*. *the 13th Annual International Conference on Mobile Computing and Networking* (*MobiCom*'*07*), 2007: 306–309, DOI: 10.1145/1287853.1287891.

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