

## APPENDIX

### A. STRATEGIES

#### A.1 Background traders

$R_{\min}$	0	0	0	0	0	0	500	0	0	2500	5000	5000
$R_{\max}$	125	125	250	500	1000	1000	1000	1500	2500	10000	10000	15000
$\eta$	0.8	1	1	1	0.8	1	0.4	0.6	1	1	1	1

Table 1: ZIR strategy combinations included in empirical game-theoretic analysis.

#### A.2 Market makers

$K$	100	100	100	100	100	100	100	100
$\omega$	64	128	256	256	512	512	1024	2048
$\xi$	100	100	50	100	50	100	100	100

Table 2: Market maker (MM) strategy combinations included in empirical game-theoretic analysis.

### B. EQUILIBRIA

#### B.1 Games without MM

Env	95% CI welfare	125*	125	250	500	1000*	1000	1000 <sup>‡</sup>	1500 <sup>†</sup>	2500
A1	[12935, 13058]	0	0	0	0.538	0	0.259	0.203	0	0
A1	[11894, 12022]	0	0	0	0	0.358	0.297	0.345	0	0
A2	[16430, 16571]	0	0	0	0.695	0	0.102	0.203	0	0
A4	[22298, 22469]	0	0	0	0.739	0.165	0	0.096	0	0
A12	[34734, 34972]	0	0.495	0	0	0.349	0.156	0	0	0
A12	[36549, 36786]	0.264	0	0.596	0	0.140	0	0	0	0
A12	[34744, 34986]	0	0	0	0.650	0.350	0	0	0	0
A60	[43202, 43484]	0	0	0.226	0.461	0	0.313	0	0	0
B1	[27275, 27499]	0	0	0	0	0.293	0.242	0.364	0.101	0
B2	[35515, 35781]	0	0	0	0.098	0.314	0.202	0.386	0	0
B4	[39039, 39321]	0	0	0	0	0.341	0.127	0.361	0.171	0
B12	[42492, 42778]	0	0	0	0.427	0.573	0	0	0	0
B120	[40933, 41219]	0	0	0	0.003	0.203	0.144	0.603	0	0.047
B120	[41325, 41613]	0	0	0	0	0	0.481	0.519	0	0
B120	[40923, 41209]	0	0	0	0	0.342	0	0.572	0.086	0
C1	[29323, 29544]	0	0	0	0.485	0.210	0.305	0	0	0
C2	[37400, 37665]	0	0	0	0.388	0.294	0.158	0.160	0	0
C4	[40784, 41067]	0	0	0	0.289	0.295	0.251	0.165	0	0
C12	[42859, 43143]	0	0	0	0.256	0.338	0.406	0	0	0
C120	[43164, 43451]	0.259	0.062	0.026		0	0.653	0	0	0

Table 3: Symmetric equilibria for games without market makers,  $N = 66$ , calculated from the 6-player DPR approximation. The numeric column headings give  $R_{\max}$  values for the ZIR strategies. All employ  $R_{\min} = 0$  with the exception of the double dagger (<sup>‡</sup>) value which uses  $R_{\min} = 500$ . All employ  $\eta = 1$ , except for the starred values which use  $\eta = 0.8$ , the dagger (<sup>†</sup>) value which uses  $\eta = 0.6$ , and the double dagger value which uses  $\eta = 0.4$ . Each row of the table describes one equilibrium found, with the 95% confidence interval of welfare and the mixture probabilities for strategies included.

Env	95% CI welfare	125*	125	250	500	1000*	1000	1000 <sup>‡</sup>	1500 <sup>†</sup>	2500
A1	[4072, 4149]	0	0	0	0	0	0.463	0.537	0	0
A2	[5058, 5148]	0	0	0	0	0	0.293	0.707	0	0
A4	[8494, 8592]	0	0	1	0	0	0	0	0	0
A4	[8186, 8289]	0	0	0	0.861	0	0.139	0	0	0
A12	[13567, 13711]	0	0	1	0	0	0	0	0	0
A60	[15712, 15885]	0	0	0.435	0	0	0.516	0	0.049	0
A60	[15824, 15996]	0	0	0	0.774	0	0.064	0	0.162	0
A60	[15543, 15715]	0	0	0	0.487	0.379	0	0.134	0	0
B1	[9521, 9661]	0	0	0	0	0.128	0.135	0.737	0	0
B2	[12710, 12874]	0	0	0	0.319	0.031	0	0.650	0	0
B4	[13736, 13909]	0	0	0	0	0.242	0.160	0.598	0	0
B12	[15585, 15760]	0	0	0	0	0	1	0	0	0
B12	[15541, 15714]	0	0	0	0	0.457	0.515	0	0	0.028
B12	[15589, 15761]	0	0	0	0.219	0.575	0.206	0	0	0
B120	[15571, 15745]	0	0	0	0	0	0.011	0.266	0.723	0
C1	[9762, 9900]	0	0	0	0	0.301	0.076	0.623	0	0
C1	[9941, 10079]	0	0	0	0.078	0.147	0.193	0.582	0	0
C2	[12597, 12758]	0	0	0	0	0.498	0	0.502	0	0
C2	[12976, 13139]	0	0	0	0.395	0	0	0.605	0	0
C4	[13880, 14054]	0	0	0	0	0.415	0	0.585	0	0
C12	[14601, 14775]	0	0	0	0	0	0.370	0.630	0	0
C12	[15706, 15879]	0	0	0	0	0.640	0.360	0	0	0
C120	[15959, 16135]	0	0	0	0	1	0	0	0	0
C120	[14924, 15099]	0	0	0	0.106	0	0.082	0.671	0.141	0

**Table 4: Symmetric equilibria for games without market makers,  $N = 25$ , calculated from the 5-player DPR approximation. Data presented is as for Table 3.**

## B.2 Games with MM

Env	95% CI surplus	95% CI welfare	125*	125	250	500	1000*	1000	1000 <sup>‡</sup>	1500 <sup>†</sup>	2500	var	128	256 <sub>50</sub>	256	512 <sub>50</sub>	512
A1	[13135, 13623]	[14808, 14929]	0.268	0	0.369	0.363	0	0	0	0	0	0	0	1	0	0	0
A2	[16687, 17318]	[18489, 18632]	0.323	0.106	0.147	0.424	0	0	0	0	0	0	0	1	0	0	0
A4	[22125, 22914]	[24434, 24606]	0	0	0.549	0.451	0	0	0	0	0	0	0	1	0	0	0
A12	[34291, 35361]	[37449, 37694]	0	0	0.554	0.440	0	0.006	0	0	0	0	0	1	0	0	0
A60	[40288, 41810]	[43155, 43443]	0	0	0	0	0.761	0.239	0	0	0	1	0	0	0	0	0
B1	[28828, 29301]	[29175, 29400]	0	0	0	0.316	0.270	0.151	0.263	0	0	0	0	0	0	0	1
B2	[36411, 36947]	[37088, 37354]	0	0	0	0.281	0.346	0.280	0.093	0	0	0	0	0	0	0	1
B2	[37122, 37725]	[37915, 38179]	0	0	0	0.496	0.379	0.125	0	0	0	0	0	0	0	1	0
B4	[40262, 41204]	[41120, 41403]	0	0	0	0	0.556	0.444	0	0	0	0	0	0	1	0	0
B12	[41683, 42846]	[43058, 43346]	0	0	0	0	0.711	0.289	0	0	0	0	0	0	1	0	0
B12	[41871, 42956]	[43014, 43300]	0	0.109	0	0	0	0.891	0	0	0	0	0	0	1	0	0
B120	[40342, 41595]	[42662, 42949]	0	0	0	0	0.307	0	0.274	0.419	0	0	0	0	1	0	0
B120	[41584, 42981]	[43552, 43840]	0	0.063	0.188	0	0	0	0	0.749	0	0	0	1	0	0	0
B120	[41719, 43029]	[43502, 43790]	0	0	0.206	0	0	0	0	0.794	0	0	0	0.784	0	0	0.216
B120	[41988, 43308]	[43497, 43785]	0	0	0.206	0	0	0	0	0.794	0	0	0	0.783	0	0.002	0.215
B120	[42074, 43463]	[43695, 43983]	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0
B120	[41354, 43023]	[43313, 43600]	0	0	0	0	0.776	0	0.103	0.121	0	0	1	0	0	0	0
C1	[29877, 30221]	[30733, 30953]	0	0	0.187	0.476	0.188	0.149	0	0	0	0	0	0	0	0	1
C2	[38207, 38896]	[40052, 40316]	0	0	0.387	0.613	0	0	0	0	0	0	0	1	0	0	0
C4	[39622, 40359]	[41253, 41534]	0	0	0	0	0.484	0.516	0	0	0	0	0	0.414	0.586	0	0
C12	[41096, 41881]	[43344, 43632]	0	0.205	0	0.014	0.781	0	0	0	0	0	0	0	1	0	0
C120	[41292, 42172]	[43290, 43577]	0	0	0.222	0	0	0.626	0.152	0	0	0	0	0.925	0.075	1	0

**Table 5: Role-symmetric equilibria for games with market makers,  $N = 66$ , calculated from the (6,1)-player DPR approximation. The numeric column headings give  $R_{\max}$  values for the ZIR strategies as in Table 3, followed by  $\omega$  values for the FUNDMM strategies. Column *var* indicates context-varying spread as calculated by the current quote, with  $\omega_t = ASK_t - BID_t$ . All MM strategies use  $\xi = 100$  except those with subscripts indicating  $\xi = 50$ . Each row of the table describes one equilibrium found, with the 95% confidence intervals of background-trader surplus and welfare and the mixture probabilities for strategies included.**

Env	95% CI surplus	95% CI welfare	125*	125	250	500	1000*	1000	1000 <sup>‡</sup>	1500 <sup>†</sup>	2500	var	128	256 <sub>50</sub>	256	512 <sub>50</sub>	512	
A1	[4574, 4834]	[5557, 5632]	0.403	0	0.597	0	0	0	0	0	0	0	0	0	0	0	0	1
A2	[6109, 6527]	[7042, 7132]	0	0.226	0.428	0.346	0	0	0	0	0	0	0	1	0	0	0	0
A4	[7934, 8304]	[8958, 9067]	0	0	0.286	0.714	0	0	0	0	0	0	0	0	0	0	1	0
A12	[12147, 12965]	[14342, 14492]	0	0.752	0	0.248	0	0	0	0	0	0	0	1	0	0	0	0
A60	[13847, 14822]	[16388, 16563]	0	0	0.507	0	0.454	0.039	0	0	0	0	0.022	0.978	0	0	0	0
B1	[11100, 11510]	[11886, 12025]	0	0	0.257	0.631	0	0	0.112	0	0	0	0	0	0	0	0.568	0.432
B2	[13005, 13460]	[14032, 14195]	0	0	0	0.696	0	0	0.304	0	0	0	0	0	0	0	0	1
B4	[13806, 14326]	[14682, 14857]	0	0	0	0.221	0	0.436	0.343	0	0	0	0	0	0	0	0	1
B12	[14855, 15430]	[16000, 16174]	0	0	0.270	0	0.730	0	0	0	0	0	0	0	0	0	1	0
B12	[14588, 15488]	[16226, 16404]	0	0	0.108	0	0.833	0	0	0	0.059	0	0	0	1	0	0	0
B120	[14750, 15719]	[16456, 16631]	0	0	0.390	0	0	0	0	0.610	0	0	0	0.995	0	0.005	0	0
C1	[10766, 11037]	[11612, 11749]	0	0	0	0.801	0.199	0	0	0	0	0	0	0	0	0	0	1
C1	[11092, 11403]	[12113, 12251]	0	0	0.261	0.739	0	0	0	0	0	0	0	0	0	0	1	0
C2	[13676, 14039]	[14828, 14996]	0	0	0.026	0.974	0	0	0	0	0	0	0	0	0	0	1	0
C4	[15044, 15594]	[16143, 16317]	0	0	0.219	0.781	0	0	0	0	0	0	0	0.238	0.762	0	0	0
C12	[14715, 15345]	[16293, 16468]	0	0	0	0	0.421	0.579	0	0	0	0	0	0	1	0	0	0
C120	[15009, 15695]	[16577, 16753]	0	0	0	0	0.395	0.563	0	0.042	0	0	0	1	0	0	0	0

**Table 6: Role-symmetric equilibria for games with market makers,  $N = 25$ , calculated from the (5,1)-player DPR approximation. Data presented is as for Table 5.**

### B.3 Surplus comparison, with and without MM

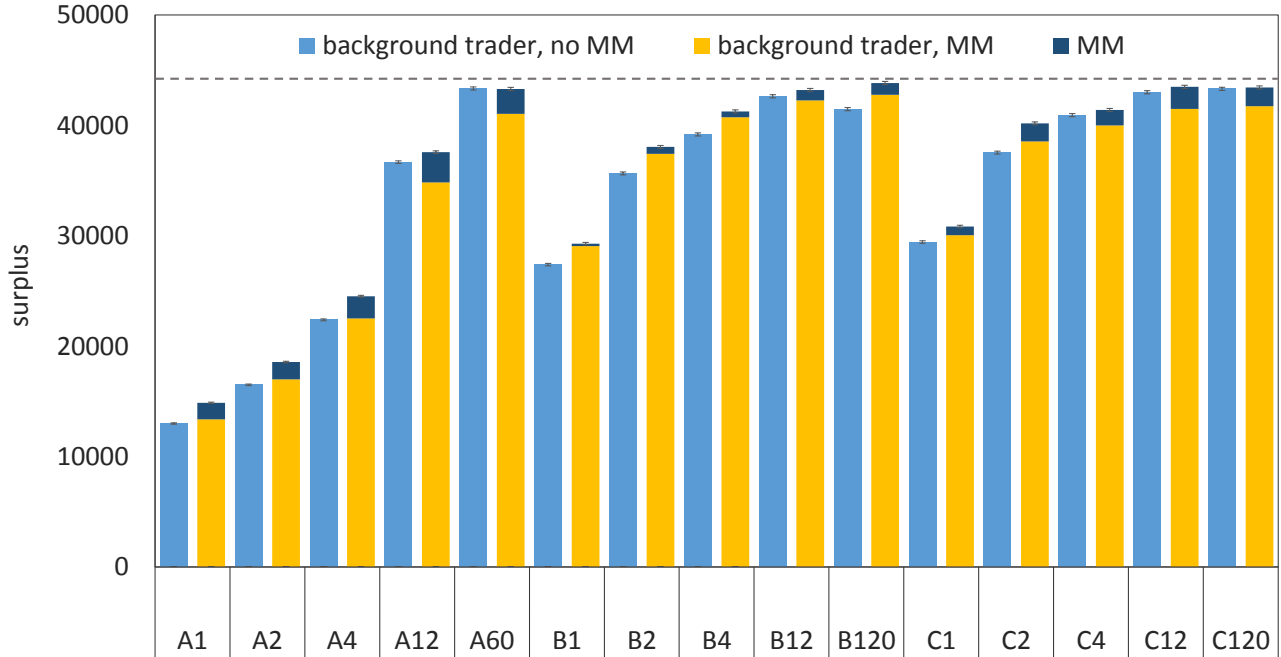


Figure 1: Comparison of background-trader surplus (with and without market makers) and MM profit for  $N = 66$ . The dotted line is the optimal social welfare available (44155). Error bars indicate the 95% confidence interval for total welfare in the maximum-welfare role-symmetric Nash equilibrium in each environment, with and without MM. Each bar is compiled from 10,000 samples.

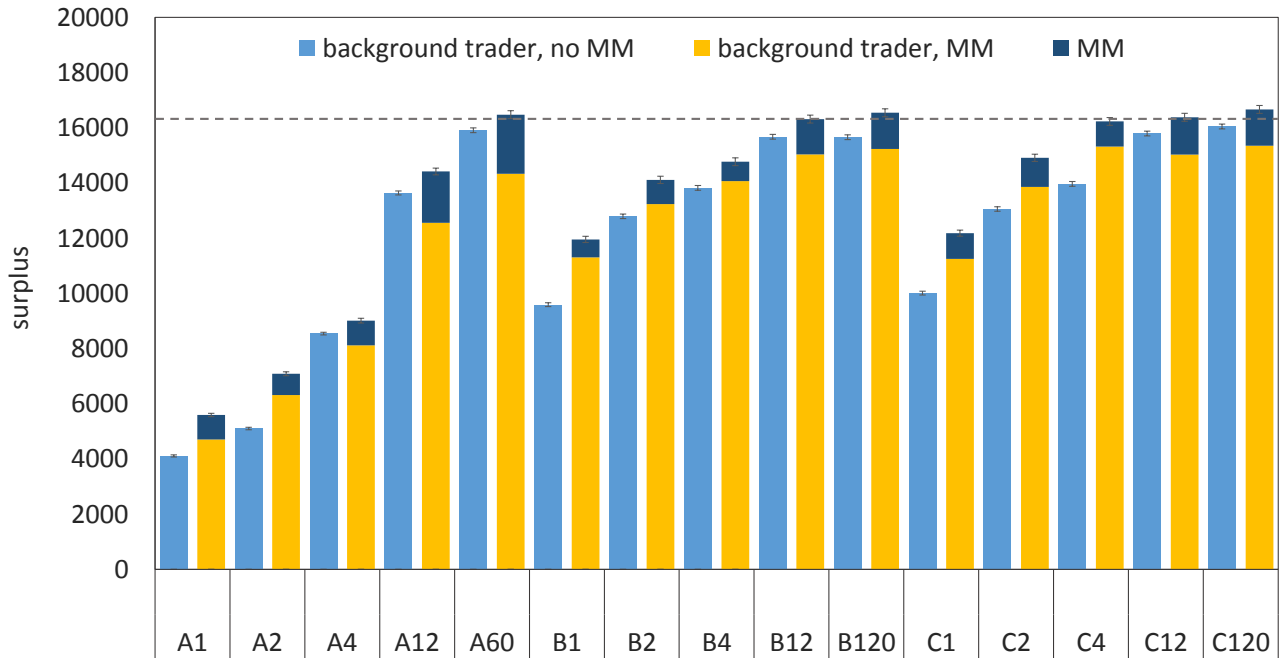


Figure 2: Comparison of background-trader surplus (with and without market makers) and MM profit for  $N = 25$ . The dotted line is the optimal social welfare available (16306). Error bars indicate the 95% confidence interval for total welfare in the maximum-welfare role-symmetric Nash equilibrium in each environment, with and without MM. Each bar is compiled from 10,000 samples.