

Auxiliary Figure 1 – In this Figure, we show examples of synthetic signals used in three tests designed to model the expected error associated with Fourier analysis of non-stationary ULF wave signals. Panels 1 and 2 are for the first test. In this case, a stationary 10 mHz signal corresponding to a linearly polarized traveling wave superposed with a small amount of random background noise is shown. Panel 1 shows the Poynting vector in the direction of net energy transfer in the time domain, and the same result is shown for the frequency domain in Panel 2. The next two panels show the same data, but for the second test. In this case, the synthetic signal is composed of Gaussian wave packets with random center times and random durations (ranging from approximately 2-12 wave periods); all wave packets propagate in the same direction. Panels 5 and 6 are for the third test; this is the same as the second test, except each wave packet may propagate in the positive or negative direction with different probability (2/3 for positive, 1/3 for negative).

To model the error, we varied the number of wave packets, the wave frequency, and the number of FFT windows (length of data interval) in these synthetic signals. For each FFT window, we compare the time domain averaged Poynting vector (the mean of the Poynting vector for the signal segment that is Fourier transformed, St_avg) with the frequency domain averaged Poynting vector (the sum of the frequency domain Poynting vector over all frequencies for that FFT window, Sf_avg). The second quantity should match the first quantity if errors due to the Fourier analysis of non-stationary signals are small. The metric for comparison between different tests is then Sf_err=(Sf_avg-St_avg)/St_avg. Over many tests with different synthetic signals, we found Sf_err to be

very small, provided we exclude FFT windows where St_avg is very close to 0 (because this is the denominator of Sf_err). Sf_err is usually less than 0.01 for any given FFT window. The effects of outlier FFT windows with large Sf_err (due to rapid changes in the Poynting vector) are minimal if median or quartiles rather than mean values of S are considered, as is done in the present study.