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A PROCEDURE FOR EVALUATING VEHICLE BRAKING PERFORMANCE

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16. Abstract

This report describes a method for determining vehicle braking efficiency whereby actual stopping distances achieved by the vehicle are compared to ideal stopping distances theoretically achievable if the vehicle brake system were able to modulate the brakes such that the tires produced peak braking forces throughout the stop. Test procedures are described for both vehicle and tire tests, along with the method employed for calculating ideal stopping distance. Results from tests and calculations are presented for two vehicles equipped with four-wheel antilock systems, which demonstrate that the method is viable and realistic. A means of comparing tire peak capability on a given surface to that produced by a standard tire is also discussed.

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TABLE OF CONTENTS

FIGURES

TABLES

1. INTRODUCTION

This report presents the findings, conclusions, and recommendations derived by the Highway Safety Research Institute (HSRI) in a research study for the National Highway Traffic Safety Administration (NHTSA) entitled, "Braking Efficiency Evaluation Procedure - Vehicles Equipped with Four-Wheel Antilock Systems." The objectives of this study were two-fold: to develop viable and realistic procedures for determining the braking efficiency of vehicles equipped with four-wheel antilock systems, and to determine by means of these procedures braking efficiencies currently attainable by vehicles equipped with production and/or developmental four-wheel antilock systems.

Braking efficiency is a measure of the ability of the vehicle-brake system combination to utilize the braking forces available in the tire-road interface. Heretofore, braking efficiency has been defined (1)* by:

$$E = \frac{100\ (\bar{A}_x/g)}{\mu} \qquad (1)$$

where: $\bar{A}_x$ is the average maximum sustained deceleration, in ft/sec^2, a vehicle can achieve without wheel lock up on a given surface

g is the gravity constant, 32.2 ft/sec^2

μ is the coefficient of friction of the given tire-road interface

Using this definition, braking efficiencies can be calculated from vehicle and brake system design data for a wide range of surfaces and for any vehicle load condition (2). This definition, however, presumes a constant coefficient of friction at the tire-road interface for all tire loads and vehicle speeds. While this assumption may be reasonably correct for braking on surfaces with a high coefficient of friction, test data indicate that for wet or slippery surfaces, the coefficient of friction is highly speed and load dependent. Thus a methodology was proposed and implemented in this program, wherein:

1. Vehicle tests were conducted on dry and slippery surfaces under various load conditions to determine stopping distance for a programmed brake pedal application from a given velocity.
2. Tire-road interface tests were conducted using a mobile tire tester to determine peak tire-road coefficients for the same range of speed and load conditions experienced in the vehicle tests. From these peak coefficients ideal stopping distances were calculated for each load and surface condition.
3. Brake system efficiency was then determined from the following equation:

$$E = 100\ \frac{D_i}{D_a} \qquad (2)$$

*Numbers in parentheses indicate references listed in Section 5 where:

D_i is the ideal stopping distance

D_a is the actual stopping distance

Results from this study indicate that the test procedures developed in this program are viable and realistic, and that vehicles equipped with current production or developmental four-wheel antilock systems can achieve brake system efficiencies in the range of 60 to 70% on a dry surface and 75 to 98% on a wet slippery surface.

In the following section, procedures are described for both the vehicle and the tire tests, along with a presentation of test results. Information on test vehicles, test equipment, and instrumentation is also provided. An analysis of the test results is presented in Section 3.0, showing tire peak force versus velocity characteristics and calculated braking efficiencies for various load and surface conditions. The data from tests of standard tires and locked wheel stops using the test vehicles are also discussed. Conclusions and recommendations are given in Section 4. The μ-slip characteristics generated from each series of tire tests are given in the Appendix. References are listed at the end of the report.

2. TEST PROGRAM

2.1 VEHICLE TESTS

Two vehicles were tested: a 1971 Chrysler Imperial equipped with "Sure-Brake," a four-wheel antilock system developed jointly by Chrysler and Bendix (3); and a 1971 Buick Riviera equipped with an experimental four-wheel antilock system developed by General Motors. Both vehicles were equipped with disc brakes on the front wheels and drum brakes on the rear wheels. Both vehicles were equipped with Goodyear Polyglas tires: the Chrysler with L84-15, load range B; the Buick with H78-15, load range B. Test vehicles are pictured in Fig. 1.

Prior to testing, the brakes in both vehicles were burnished according to the procedure specified in SAE Recommended Practice J873b. Apparatus and instruments installed in the vehicles for the tests (see Fig. 2) included:

1. longitudinal accelerometer for measuring vehicle deceleration
2. fifth wheel for measuring vehicle velocity and stopping distance
3. pressure transducer to measure brake line pressure
4. wheel rotation indicators which produce one pulse per revolution of the wheel
5. thermocouple to measure brake temperatures
6. brake pedal application device which provided ramp fronted pedal displacements for making uniform brake applications
7. light beam oscillograph for recording continuous signals

FIGURE 1-A. TEST VEHICLE: 1971 CHRYSLER IMPERIAL

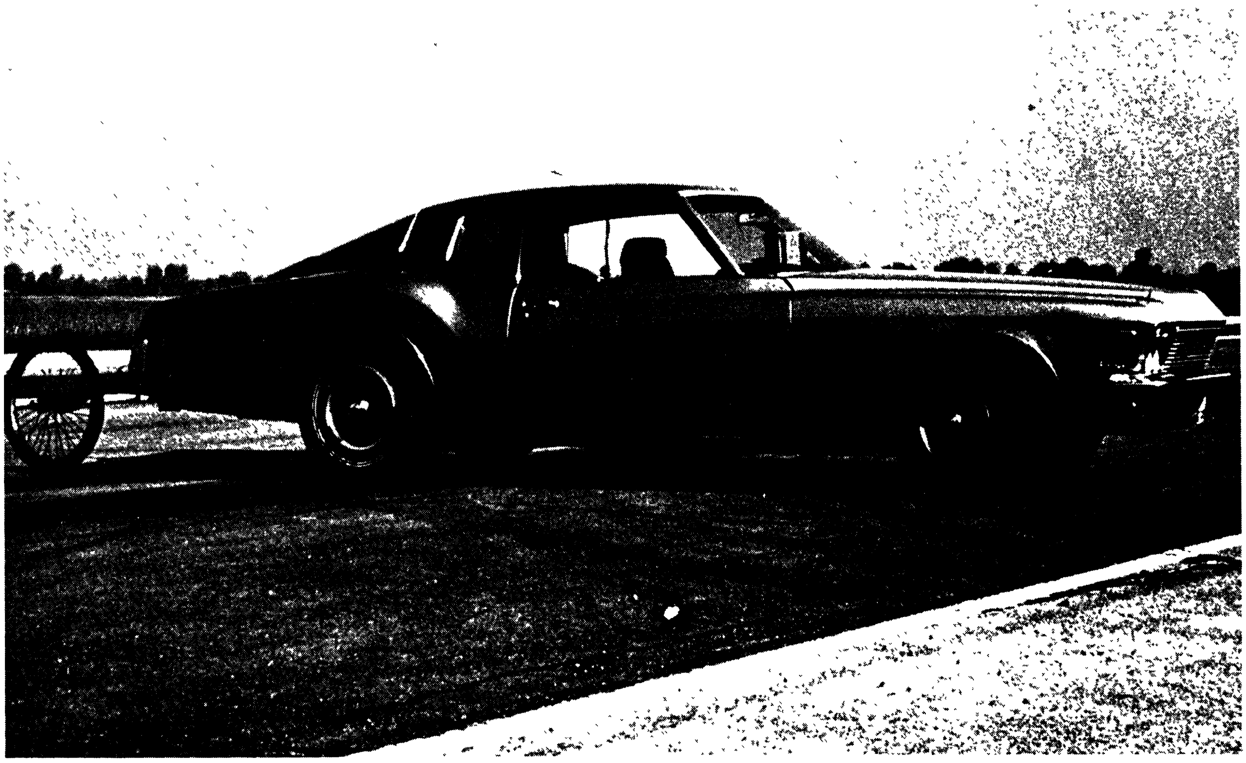

FIGURE 1-B. TEST VEHICLE: 1971 BUICK RIVIERA

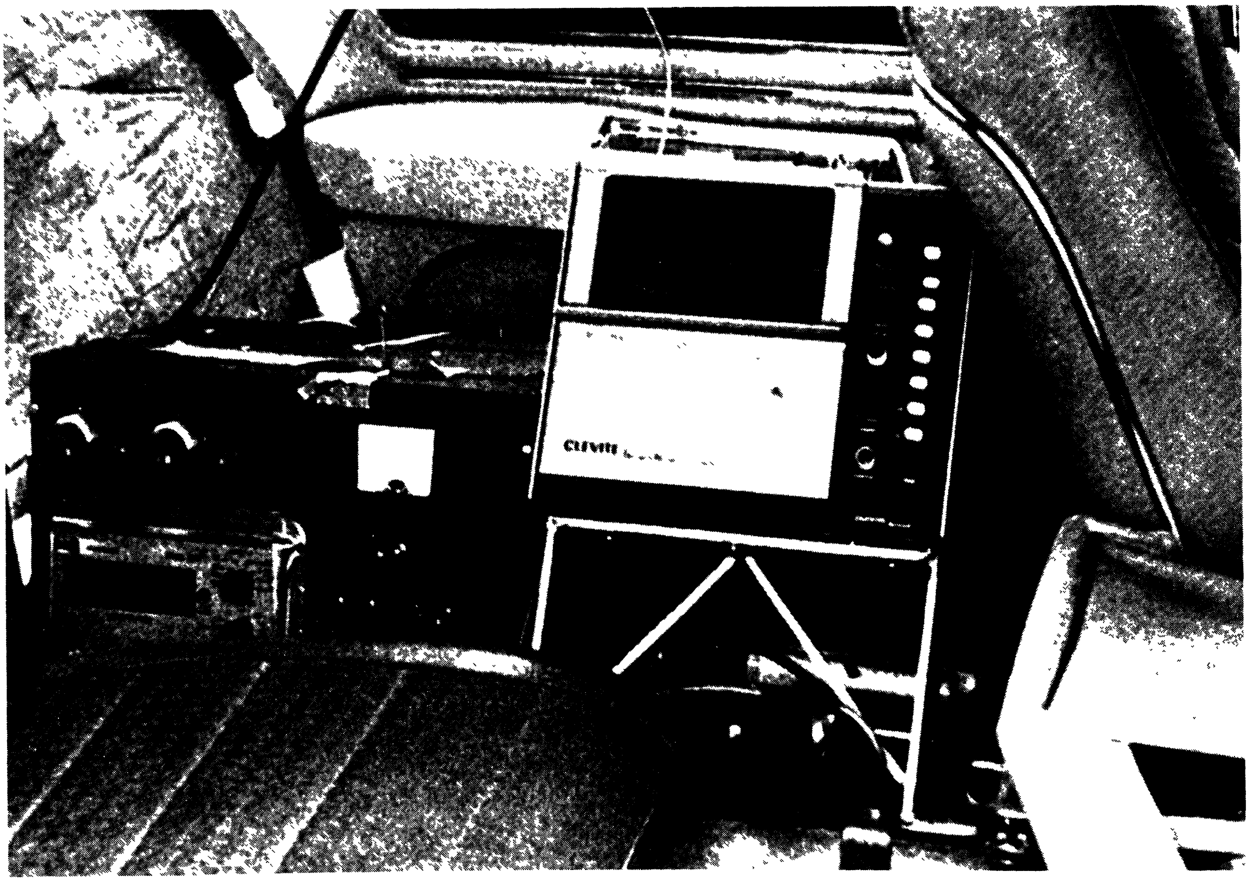

FIGURE 2. INSTRUMENTS AND RECORDER INSTALLED IN THE BUICK

The brake pedal application device (see Fig. 3) consisted of a hydraulic actuator, fixed by a floor bracket, which was powered by a small hydraulic power supply and controlled by means of a servo-valve. Pedal position could be controlled to any value up to four inches, and pedal application rate could be varied from 2 in/sec to 130 in/sec. After a series of preliminary runs on each vehicle, it was determined that a brake pedal application consisting of approximately 3 inches displacement in 0.1 second (30 in/sec application rate) would be appropriate for the tests. With the 3 inch displacement, the brake line pressures developed were high enough (1200 to 1600 psi) to take advantage of the maximum torque capability of the brakes. The 30 in/sec application rate was deemed the maximum allowable for use during the tests, despite the fact that informal tests conducted at HSRI have shown that two male subjects could apply the brake pedal in spike stops at rates of 150 to 180 in/sec. It was feared that rates higher than 30 in/sec, used on a repeated basis, would produce impact stresses severe enough to damage the brake system.

Stopping distance was measured by an electronic counter fed by pulses (1 pulse per foot) from the fifth wheel. The counter started counting pulses at the same instant that the pedal application devise was activated. When the vehicle stopped, the count was held until the digital display was read and reset.

All tests were conducted on the HSRI skid pad, a 100 ft by 700 ft resurfaced asphalt area of the East Ramp of Willow Run Airport. Tests requiring a high coefficient of friction surface were conducted on dry asphalt and those requiring a low coefficient surface were conducted on a portion of the skid pad which had been treated with an asphalt sealant (Jennite) and wetted for the tests.

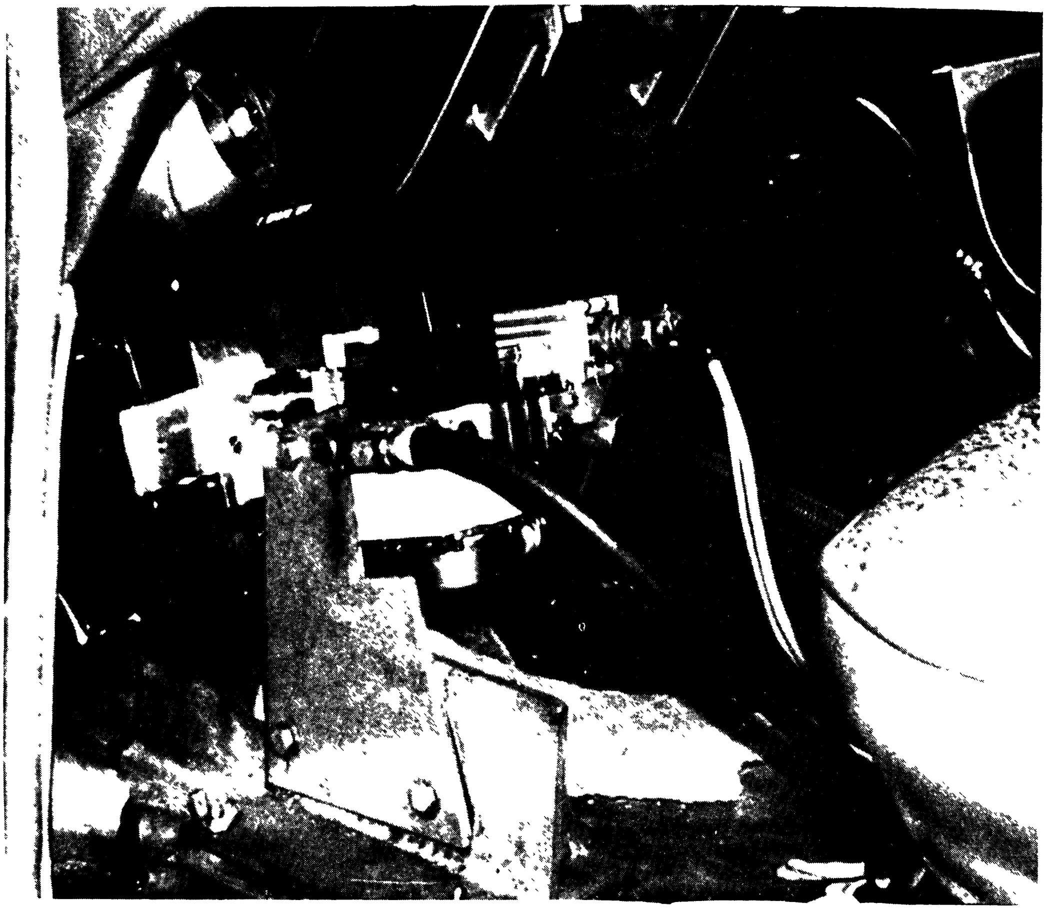

FIGURE 3. BRAKE PEDAL APPLICATION DEVICE INSTALLED IN THE CHRYSLER

Sprinklers were turned on for a set time period before each run. The time period was adjusted each day to meet weather conditions, such that the amount of water on the track was sufficient to cover the track, but not so great as to cause hydroplaning. ASTM skid numbers for the high and low coefficient surfaces are 85 and 23, respectively.

Vehicles were tested under 3 loading conditions (see weights "as tested" Table 1), on the high and low coefficient surfaces. These loading conditions corresponded to curb weight plus weight of optional equipment installed plus a percentage of the vehicle capacity weight. Vehicle load capacity for both the Chrysler and the Buick (6 passengers plus 200 pounds luggage) was 1100 pounds.

With the vehicle traveling initially in a straight line at 60 mph with cold brakes*, the brake application device was activated to bring the vehicle to a stop, while the driver steered to maintain the vehicle in the 12 ft lane. Vehicles were tested at the three specified weights on the high and low coefficient surfaces, with the antilock system functioning and with the antilock system disabled. Each test condition was repeated 10 times. For the stops in which wheel lockup was allowed, pedal force was maintained only if the vehicle remained directionally stable throughout the stop. It became obvious early in the program that vehicle stability could not be maintained in locked wheel stops from 60 mph on the wet surface, and therefore the test speed under these conditions was reduced to 30-34 mph.

Typical traces from the oscillograph recorder for a locked wheel stop are given in Fig. 4 and for a stop with the antilock system operational in Fig. 5. Note that in both cases the pedal was displaced approximately 3 inches in 0.1 seconds. This displacement

TABLE 1. SUMMARY OF WEIGHT DATA FOR TEST VEHICLES

	CHRYSLER IMPERIAL				BUICK RIVIERA			
	Percent Capacity Weight	Static Weight, Pounds Front Wheels	Rear Wheels	Total	Percent Capacity Weight	Static Weight, Pounds Front Wheels	Rear Wheels	Total
Curb Weight[a]	0	2939	2387	5326	---	2570	2134	4704
G.V.W.[a]	100			6426	100	2871	3038	5909
As Received	3	2970	2390	5360	17	2710	2180	4890
As Tested	40	3110	2720	5830	60	2850	2510	5360
As Tested	60	3230	2040	6070	80	2730	2890	5620
As Tested	100	3245	3245	6490	100	2870	3030	5900

[a]Manufacturers Specifications

*Temperature as measured on any brake did not exceed 200°F

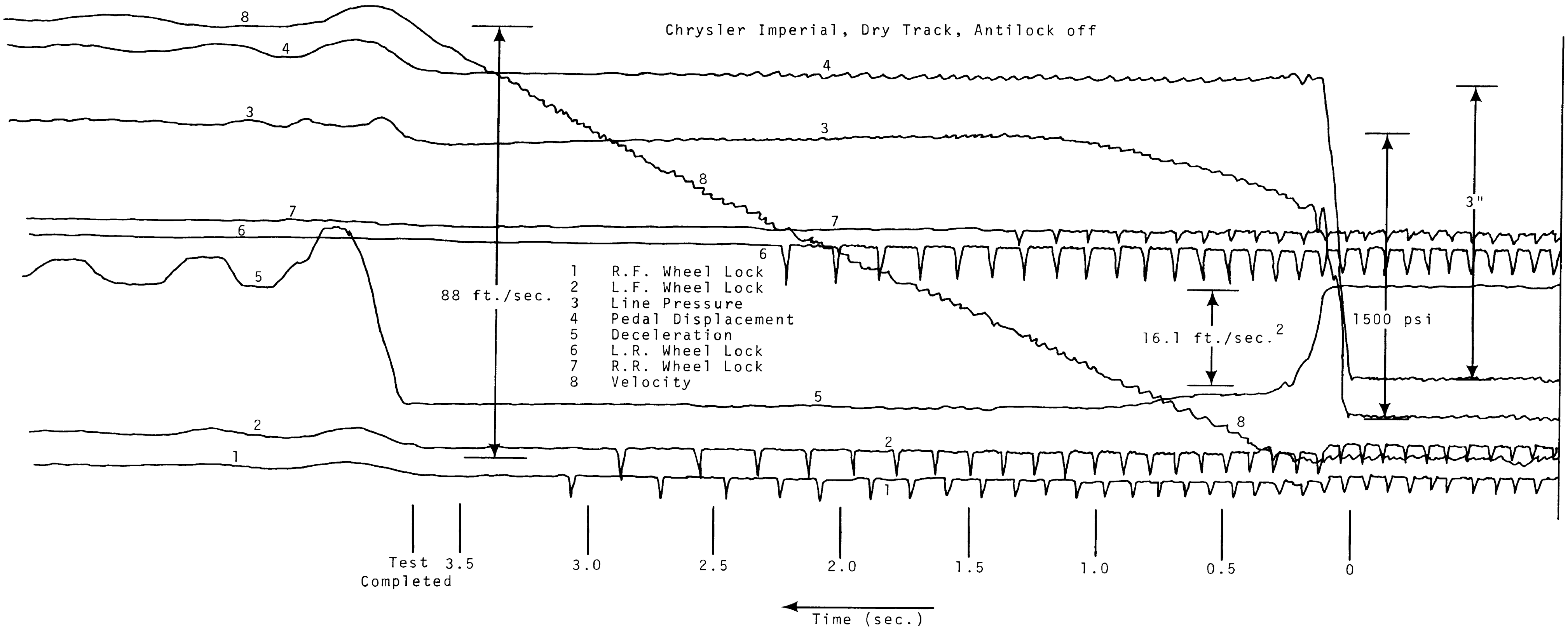

FIGURE 4. OSCILLOGRAPH RECORD FOR STOP WITH ANTILOCK SYSTEM DISABLED, HIGH COEFFICIENT SURFACE, CHRYSLER IMPERIAL

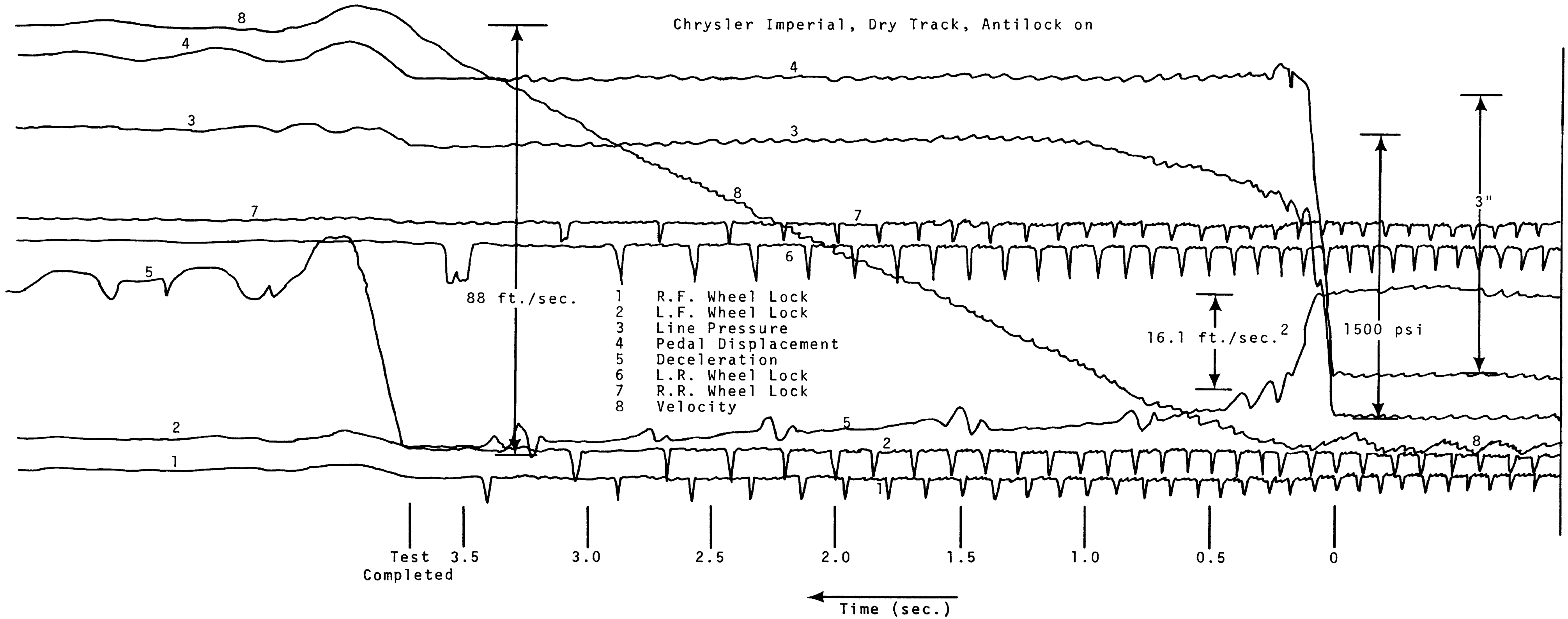

FIGURE 5. OSCILLOGRAPH RECORD FOR STOP WITH ANTILOCK SYSTEM OPERATIONAL, HIGH COEFFICIENT SURFACE, CHRYSLER IMPERIAL

resulted in a brake line pressure slightly higher than 1100 psi occurring in about 0.32 sec., but climbing to steady state value of 1500 psi. The deceleration trace follows the line pressure trace quite closely, except for a slight lag. In Fig. 4, it can be seen that even though maximum brake line pressure was applied, the rear wheels of the Chrysler did not lock until the vehicle velocity had decreased to 30 mph, and the front wheels did not lock until the vehicle was nearly stopped. The wheel rotation traces also show (each "blip" indicates 1 revolution of the wheel) that the right rear wheel locked before the left rear. In Fig. 5, of course, no wheel lock occurs, but it is interesting to note that the acceleration trace indicates intermittent oscillations due to the operation of the antilock system.

In the so-called "locked wheel stops," on the dry pavement, not all wheels locked immediately upon brake activation. Table 2 summarizes the data for the locked wheel stops. Note than even without antilock system operational, wheel lock up on the Chrysler Imperial at 100% load on the high coefficient surface did not occur until the vehicle was nearly stopped.

Stopping distance data* from all the vehicle tests are given in Tables 3, 4, 5, and 6, in which for each set of 10 stops the mean, standard deviation, and the $\pm 1\sigma$ and the $\pm 3\sigma$ values are given.

2.2 TIRE-ROAD INTERFACE TESTS

The purpose of the tire-road interface tests was to establish the μ-slip characteristics of the tires used on the test vehicles and a standard tire** on the high and low coefficient surfaces at various speeds. Of particular interest in these tests was the peak coefficient resulting at a given velocity, because it is upon these peak values that the efficiency calculations are based.

The HSRI Mobile Tire Tester was used to determine the μ-slip characteristics of the tires for this program. It is a high speed, over-the-road device, consisting of a retracting tire-wheel dynamometer mounted on a modified tandem-axle commercial tractor (see Fig. 6). The tire mounted on the test wheel can be steered, and driven or braked. The data recording system, with the associated signal conditioning and control instrumentation, is mounted in the cab. The pneumatic system for raising and lowering of the wheel, the hydraulic system for controlling wheel speed, and the test wheel assembly are all mounted on a specially constructed platform on the vehicle frame.***

The test procedure was as follows: with the tester traveling at a given speed, and the test tire freely rolling on the test surface, the automatic slip control was activated, causing the test tire to proceed from freely rolling (0% slip) to fully locked (100% slip) in 3.5 seconds. When the fully locked condition was achieved, the tire was lifted from the surface and allowed to cool before initiating the next test sequence. During the test sequence, vehicle speed, test tire speed, normal load, and longtudinal force are measured and continuously recorded.

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*All stopping distances have been corrected to 60 mph using the equation: $S_{60} = S_t \frac{3600}{V_t^2}$ where S_{60} is the corrected stopping distance, and S_t is the actual stopping distance measured from test speed in mph, V_t.

**The standard tire used for these tests was an 8.45-15 Traction Test Standard manufactured by the General Tire Company.

***A complete description of the Mobile Tire Tester is given in Reference 4.

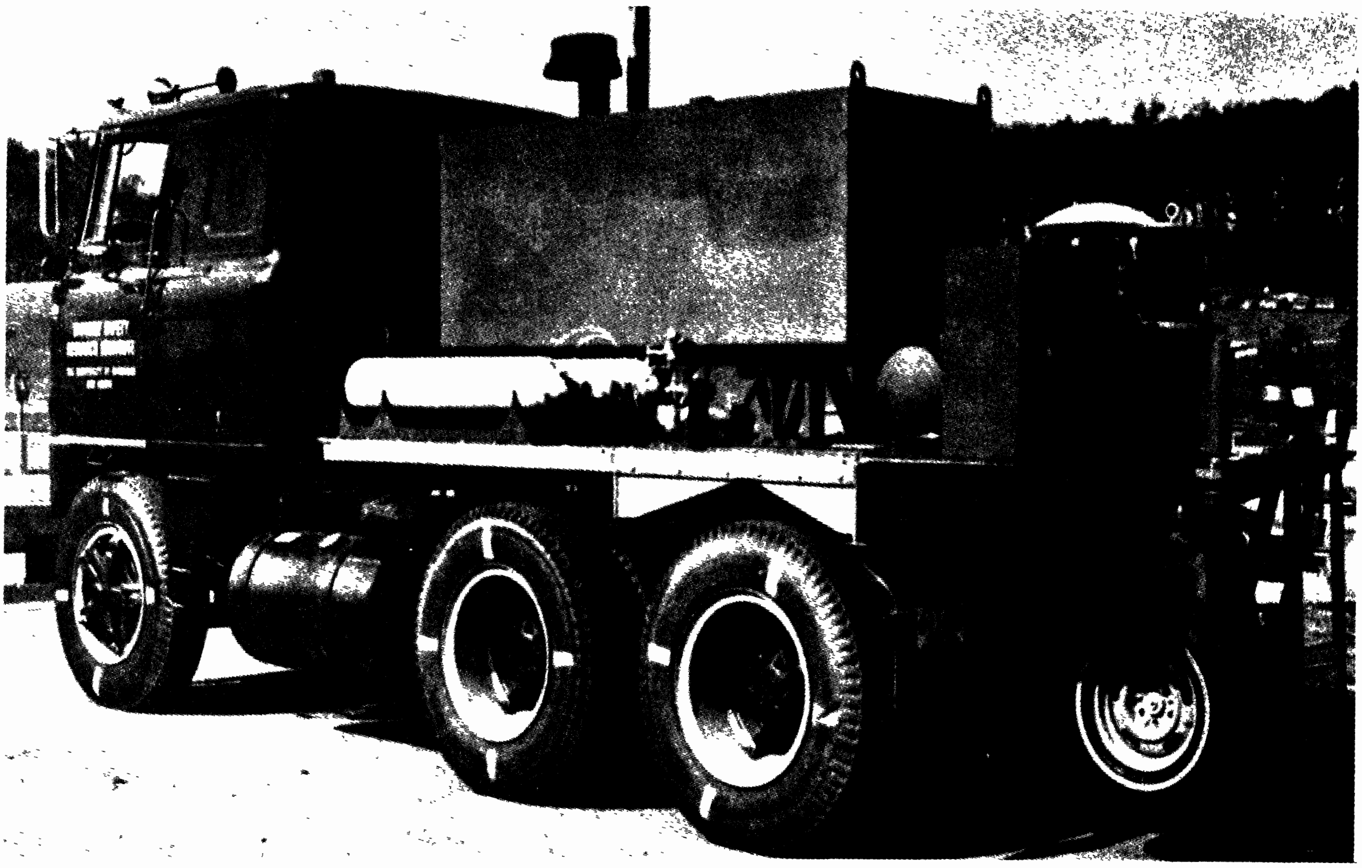


FIGURE 6. HSRI MOBILE TIRE ROAD INTERFACE TESTER

When possible, tire tests were interspersed with vehicle tests to ensure that variations in test results due to differences in temperature, humidity, wind velocity, surface characteristics, and water film depth on the low coefficient surface would be minimized.

On the low coefficient surface, tests were conducted at 5, 15, 30 and 45 mph, with tire loads equivalent to the three average static loads on the vehicle tires. It would have been desirable to test the tires at 60 mph, but space constraints at the test site prevented doing so. It was impossible to test at the same loads and speeds on the dry surface, because the tire-road interface forces generated at such loads and speeds exceeded the capacity of the Mobile Tire Tester. However, an attempt was made to assess the load and speed sensitivity (if any) of the tires on the dry surface, by testing each tire at several loads at a given speed and several speeds at a given load.

Each tire test under each condition was repeated 10 times. A summary of the mean peak force values from each test is given in Table 7. Tables 8, 9, and 10 present the mean, one sigma and 3 sigma values for each tire tested. It is immediately evident that on the wet surface, the peak tire-road interface coefficients were highly velocity sensitive, and somewhat load sensitive. No real correlation either to load or to velocity was found for the dry surface tests, except for the standard tire, which showed the peak force to be quite sensitive to velocity.

The μ-slip characteristics for each tire averaged for the ten runs made under each test condition, are given in the appendix, in Fig. A-1 through A-6.

TABLE 2. DATA SUMMARY - LOCKED WHEEL STOPS

Vehicle	Track Condition	Load %	Speed (MPH)	Average Deceleration (g-units)	Stop Distance (feet)*	Remarks on Wheel Lock-Up
CHRYSLER	Dry	40	60	0.68	178	Front wheels did not lock until vehicle was nearly stopped. Rear wheels locked at speeds from 42 to 50 mph.
	Dry	60	60	0.66	183	Front wheels did not lock until vehicle was nearly stopped. Rear wheels locked at speeds from 20 to 50 mph.
	Dry	100	60	0.61	198	Front wheels did not lock until vehicle was nearly stopped. Rear wheels locked at speeds from 14 to 18 mph.
	Wet	40	30	0.16	770	All wheels locked shortly after brake application
	Wet	60	30	0.16	744	All wheels locked shortly after brake application
	Wet	100	34	0.14	888	All wheels locked shortly after brake application
BUICK	Dry	60	54	0.74	163	Front wheels locked at speeds from 40 to 20 mph. Rear wheels locked at speeds from 45 to 12 mph.
	Dry	80	60	0.69	175	Front wheels locked shortly after brake application. Rear wheels locked at low speed.
	Dry	100	60	0.65	187	Left front wheel locked shortly after brake application. Rear wheels locked at low speed.
	Wet	60	30	0.13	803	All wheels locked shortly after brake application
	Wet	80	30	0.18	656	All wheels locked shortly after brake application
	Wet	100	32	0.18	682	All wheels locked shortly after brake application

*Where initial speed is other than 60 mph, stopping distance is corrected to 60 mph initial speed.

TABLE 3.
STOPPING DISTANCE DATA, CHRYSLER IMPERIAL, ANTILOCK SYSTEM OPERATIONAL

Track Condition	Loading Condition	Corrected Mean Stopping Distance*	One Standard Deviation σ	Three Standard Deviations 3σ	Mean $+1\sigma$	Mean -1σ	Mean $+3\sigma$	Mean -3σ
Dry	40%	183	3.8	11.4	187	179	194	172
Dry	60%	184	3.6	10.8	188	180	195	173
Dry	100%	194	6.7	20.1	201	188	224	174
Wet	40%	419	20.6	61.8	440	398	481	357
Wet	60%	421	12.5	37.5	434	409	458	384
Wet	100%	385	17.4	52.2	402	368	437	333

*All distances are in feet

TABLE 4.
STOPPING DISTANCE DATA, CHRYSLER IMPERIAL, ANTILOCK SYSTEM DISABLED

Track Condition	Loading Condition	Corrected Mean Stopping Distance*	One Standard Deviation σ	Three Standard Deviations 3σ	Mean $+1\sigma$	Mean -1σ	Mean $+3\sigma$	Mean -3σ
Dry	40%	178	3.9	11.8	182	174	190	176
Dry	60%	183	3.0	9.0	186	180	192	174
Dry	100%	198	2.1	6.3	200	196	204	192
Wet	40%	770	50.6	151.8	821	719	922	618
Wet	60%	744	47.3	141.9	791	697	886	602
Wet	100%	889	40.6	121.8	930	848	1011	767

*All distances are in feet

TABLE 5.
STOPPING DISTANCE DATA, BUICK RIVIERA, ANTILOCK SYSTEM OPERATIONAL

Track Condition	Loading Condition	Corrected Mean Stopping Distance*	One Standard Deviation σ	Three Standard Deviations 3σ	Mean $+1\sigma$	Mean -1σ	Mean $+3\sigma$	Mean -3σ
Dry	60%	180	6.4	19.2	186	174	199	161
Dry	80%	181	5.7	17.1	187	175	198	144
Dry	100%	193	6.6	19.8	200	186	213	173
Wet	60%	535	32.7	98.1	568	502	633	437
Wet	80%	498	18.8	56.3	517	479	554	442
Wet	100%	528	20.6	61.8	549	507	590	466

*All distances are in feet

TABLE 6.
STOPPING DISTANCE DATA, BUICK RIVIERA, ANTILOCK SYSTEM DISABLED

Track Condition	Loading Condition	Corrected Mean Stopping Distance*	One Standard Deviation σ	Three Standard Deviations 3σ	Mean $+1\sigma$	Mean -1σ	Mean $+3\sigma$	Mean -3σ
Dry	60%	163	8.4	25.2	171	155	188	138
Dry	80%	175	5.4	16.2	180	170	191	159
Dry	100%	187	4.4	13.2	191	183	200	174
Wet	60%	803	60.5	181.5	864	743	985	622
Wet	80%	657	54.3	162.9	711	603	820	494
Wet	100%	682	65.5	196.5	748	617	878	486

*All distances are in feet

TABLE 7.
SUMMARY OF MEAN PEAK VALUES FROM TIRE-ROAD INTERFACE TESTS

Tire	Speed (MPH)	Track Condition	Load, in Pounds									
			800	850	1000	1050	1200	1250	1350	1450	1500	1600
L84-15	10	dry		1.04								
	15	dry	0.91		0.92					0.92		
	30	dry		1.00								
H78-15	10	dry		1.00								
	15	dry		1.07		1.00		1.03		0.98		
	30	dry		1.04								
Traction Test Standard 8.45-15	5	dry				1.03						
	15	dry				0.90						
	30	dry				0.81						
L84-15	5	wet				0.55				0.49	0.51	0.51
	15	wet				0.47				0.38	0.41	0.43
	30	wet				0.42				0.33	0.34	0.40
	45	wet				0.34				0.28	0.32	0.31
H78-15	5	wet				0.56			0.54	0.55		
	15	wet				0.43			0.46	0.40		
	30	wet				0.38			0.38	0.33		
	45	wet				0.34			0.27	0.26		
Traction Test Standard 8.45-15	5	wet				0.44						
	15	wet				0.38						
	30	wet				0.34						
	45	wet				0.25						

TABLE 8.
SUMMARY OF TIRE-ROAD PEAK FORCE DATA, GOODYEAR POLYGLAS L84-15 TIRE

Surface	Load (lbs)	Speed (MPH)	Tire-Road Friction Coefficient						
			Mean	1σ	3σ	Mean +1σ	Mean -1σ	Mean +3σ	Mean -3σ
dry asphalt	800	15	0.91	0.04	0.12	0.95	0.87	1.03	0.79
dry asphalt	850	10	1.04	0.05	0.15	1.09	0.99	1.19	0.91
"	"	30	1.00	0.04	0.12	1.04	0.96	1.12	0.88
dry asphalt	1000	15	0.92	0.06	0.18	0.98	0.86	1.10	0.76
dry asphalt	1450	15	.0.92	0.02	0.06	0.94	0.90	0.98	0.86
wet jennite	1050	5	0.55	0.04	0.12	0.59	0.51	0.67	0.43
"	"	15	0.47	0.03	0.09	0.50	0.44	0.55	0.38
"	"	30	0.42	0.03	0.09	0.45	0.39	0.51	0.33
"	"	45	0.34	0.04	0.12	0.38	0.30	0.46	0.22
wet jennite	1450	5	0.49	0.02	0.06	0.51	0.47	0.55	0.43
"	"	15	0.38	0.02	0.06	0.40	0.36	0.54	0.32
"	"	30	0.33	0.04	0.12	0.37	0.29	0.45	0.21
"	"	45	0.28	0.04	0.12	0.32	0.24	0.40	0.16
wet jennite	1500	5	0.51	0.03	0.09	0.54	0.48	0.60	0.42
"	"	15	0.40	0.02	0.06	0.42	0.38	0.46	0.34
"	"	30	0.34	0.05	0.15	0.39	0.29	0.49	0.19
"	"	45	0.32	0.04	0.12	0.36	0.28	0.44	0.20
wet jennite	1600	5	0.51	0.03	0.09	0.54	0.48	0.60	0.42
"	"	15	0.43	0.03	0.09	0.46	0.40	0.52	0.34
"	"	30	0.40	0.03	0.09	0.43	0.37	0.49	0.31
"	"	45	0.31	0.05	0.15	0.36	0.26	0.46	0.16

TABLE 9.
SUMMARY OF TIRE-ROAD PEAK FORCE DATA, GOODYEAR POLYGLAS H78-15 TIRE

Surface	Load (lbs)	Speed (MPH)	Tire-Road Interface Coefficient						
			Mean	1σ	3σ	Mean +1σ	Mean -1σ	Mean +3σ	Mean -3σ
dry asphalt	850	10	1.00	0.07	0.21	1.07	0.93	1.21	0.79
"	"	15	1.07	0.04	0.12	1.11	1.03	1.19	0.95
"	"	30	1.04	0.04	0.12	1.08	1.00	1.16	0.92
dry asphalt	1050	15	1.00	0.02	0.06	1.02	0.98	1.06	0.94
dry asphalt	1250	15	1.03	0.04	0.12	1.07	0.99	1.15	0.91
dry asphalt	1450	15	0.98	0.01	0.03	0.99	0.97	1.02	0.95
wet jennite	1050	5	0.56	0.04	0.11	0.60	0.52	0.68	0.44
"	"	15	0.43	0.03	0.09	0.46	0.40	0.52	0.34
"	"	30	0.38	0.04	0.12	0.42	0.34	0.50	0.26
"	"	45	0.34	0.02	0.06	0.36	0.32	0.40	0.28
wet jennite	1350	5	0.54	0.02	0.06	0.56	0.54	0.60	0.48
"	"	15	0.46	0.03	0.09	0.49	0.43	0.55	0.37
"	"	30	0.38	0.03	0.09	0.41	0.35	0.47	0.29
"	"	45	0.27	0.03	0.09	0.30	0.24	0.36	0.18
wet jennite	1450	5	0.55	0.04	0.12	0.59	0.51	0.67	0.43
"	"	15	0.40	0.04	0.12	0.44	0.36	0.52	0.28
"	"	30	0.33	0.04	0.12	0.37	0.29	0.45	0.21
"	"	45	0.26	0.06	0.18	0.32	0.20	0.44	0.08

TABLE 10.
SUMMARY OF TIRE-ROAD PEAK FORCE DATA, GENERAL 8.45-15 STANDARD TIRE

Surface	Load (lbs)	Speed (MPH)	Tire-Road Interface Coefficient						
			Mean	1σ	3σ	Mean +1σ	Mean -1σ	Mean +3σ	Mean -3σ
dry asphalt	1050	5	1.03	0.07	0.21	1.10	0.96	1.24	0.82
"	"	15	0.90	0.04	0.12	0.94	0.86	1.02	0.78
"	"	30	0.81	0.05	0.15	0.86	0.76	0.96	0.66
wet jennite	1050	5	0.44	0.07	0.21	0.51	0.37	0.65	0.23
"	"	15	0.38	0.05	0.15	0.43	0.33	0.53	0.23
"	"	30	0.34	0.04	0.12	0.38	0.30	0.46	0.22
"	"	45	0.25	0.03	0.09	0.28	0.22	0.34	0.16

3. ANALYSIS OF RESULTS

The determination of brake system efficiency as defined by Equation 2 (see Section 1) requires the calculation of ideal stopping distance, which is defined as the distance to stop the car from 60 mph if the brake torque on each wheel were modulated such that peak tire-road forces were produced throughout the stop. The results of the tests of the L84-15 and the H78-15 tires on the dry asphalt surface, cited in the last section and plotted in Fig. 7, show little, if any, sensitivity of tire peak force to load or speed. However, on the low coefficient surface, the test results show considerable speed and load sensitivity, as demonstrated in Figs. 8 and 9.

3.1 CALCULATION OF IDEAL STOPPING DISTANCE

Since no real dependency of tire-road peak coefficient values on speed or load was noted for the dry surface tests, a constant value equal to the average of the peak coefficients for each tire was used to calculate ideal stopping distance on dry asphalt. Thus:

$$\mu = \mu_{av} \tag{3}$$

which gives $\mu = 0.95$ for the L84-15 tire and $\mu = 1.02$ for the H78-15 tire. To describe the variation of μ with velocity on the low coefficient surface, a second degree polynomial was fitted to the data points generated at various speeds for the given tire at a given load using the least squares technique (5).

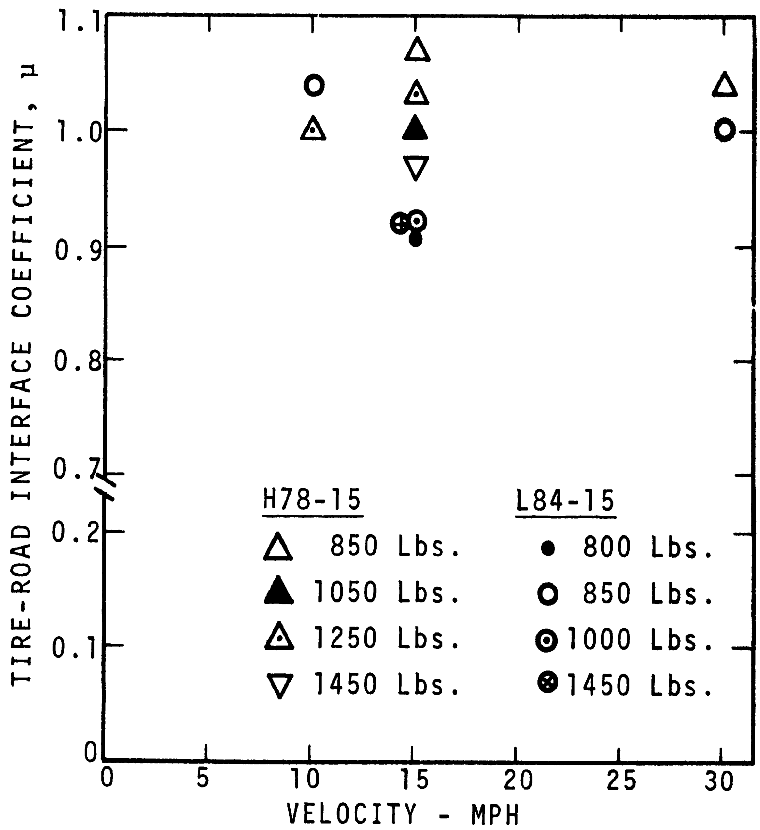


FIGURE 7. PEAK TIRE-ROAD INTERFACE FRICTION COEFFICIENTS ON DRY ASPHALT SURFACE, GOODYEAR POLYGLAS L84-15 and H78-15 TIRES

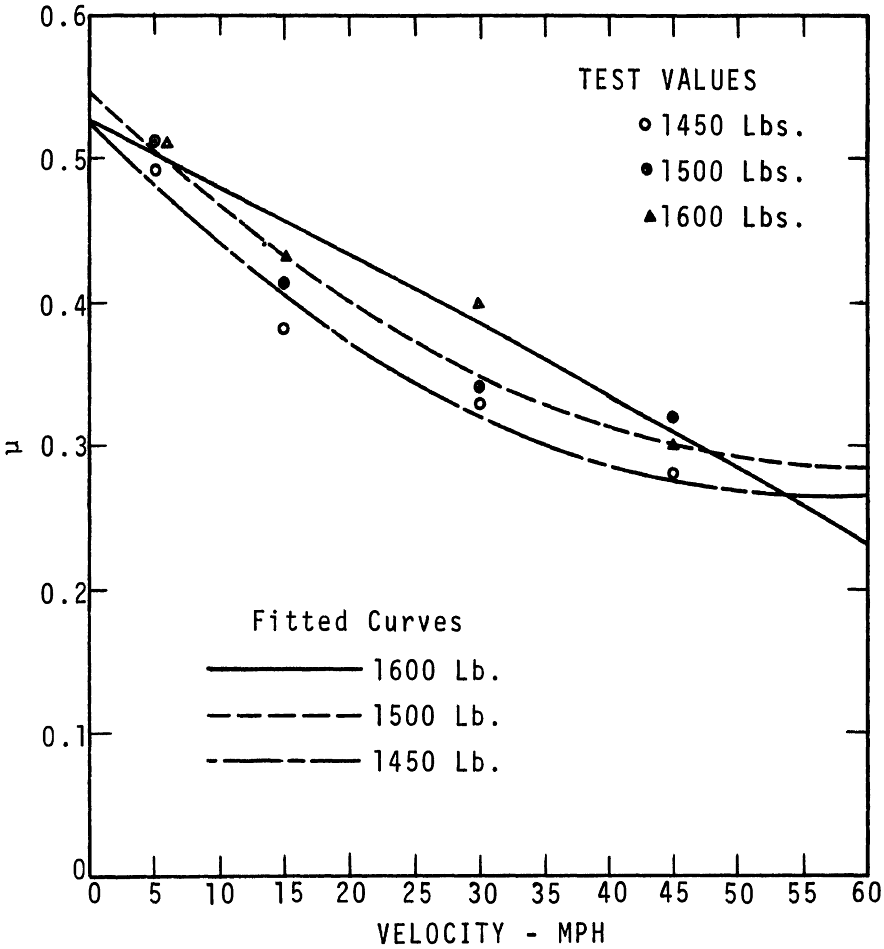


FIGURE 8. PEAK TIRE-ROAD INTERFACE FRICTION COEFFICIENTS ON LOW-COEFFICIENT SURFACE, GOODYEAR L84-15 POLYGLAS TIRE

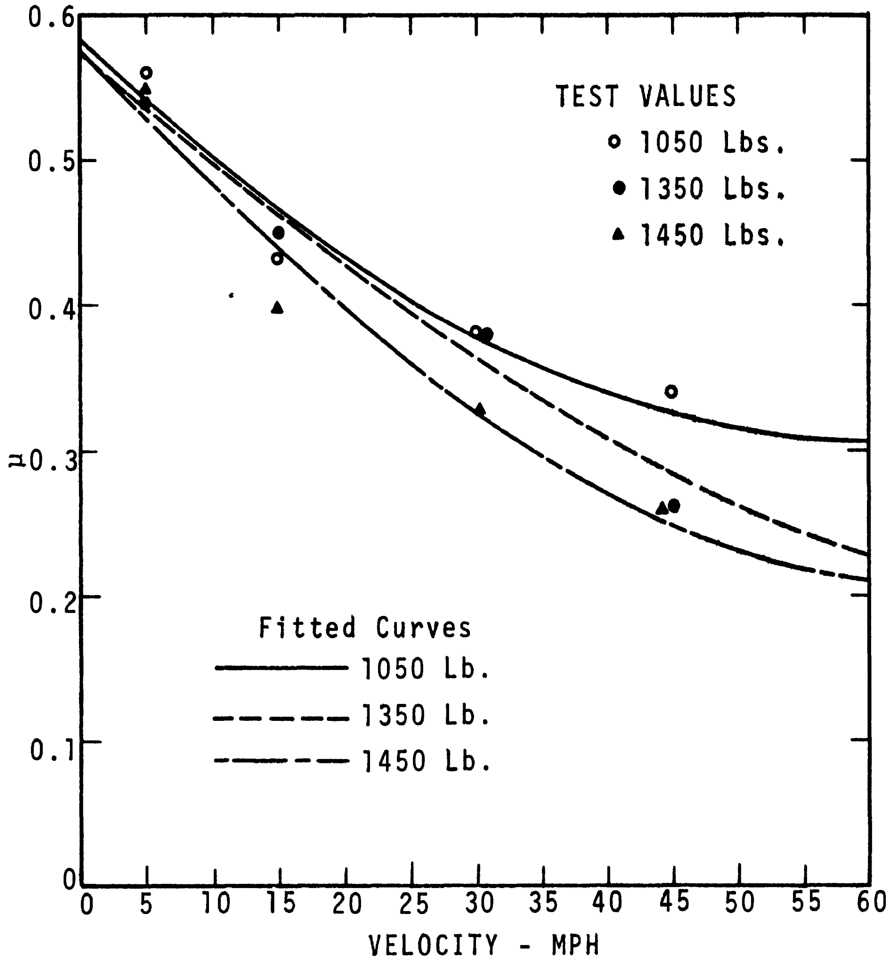


FIGURE 9. PEAK TIRE-ROAD INTERFACE FRICTION COEFFICIENTS ON LOW-COEFFICIENT SURFACE, GOODYEAR H78-15 POLYGLAS TIRE

The resulting expression for the peak tire-road coefficient as a function of velocity for a given load is:

$$\mu = AV^2 + BV + C \tag{4}$$

where V is the velocity in ft/sec and A, B, and C are the constants derived using the least squares curve fitting technique. In Table 11 are listed the values of A, B, and C for each tire in each of three loading conditions. The resulting curves for the polynomials are plotted on the same graph with the experimental data points in Figs. 8 and 9.

If the peak tire-road coefficient is expressed as in Equation 4, the ideal stopping distance can be calculated by

$$D_i = \frac{1}{g} \int_0^{V_1} \frac{V}{AV^2 + BV + C}\, dV \tag{5}$$

where g = the gravity constant, 32.2 ft/sec^2

V = vehicle velocity, ft/sec

V_1 = initial test speed, ft/sec

A, B, and C are as defined for Equation 4.

Note that the effect of the variation in vertical load on the tires due to load transfer during testing is not included in this equation. The net effect on the final calculation would indeed be small since the gain in the peak force capability of the front tires would be offset by a loss of such capability on the rear tires.

TABLE 11.
POLYNOMIALS DESCRIBING PEAK TIRE-ROAD INTERFACE COEFFICIENTS AS A FUNCTION OF VELOCITY ON WET SEALED ASPHALT

$\mu = AV^2 + BV + C$ where V is in ft/sec

Tire	Load, Pounds	A $\times 10^6$	B $\times 10^4$	C $\times 10^2$
L84-15	1450	35.6	-60.1	51.6
"	1500	31.1	-56.5	53.8
"	1600	-3.6	-30.1	52.3
H78-15	1050	36.5	-63.8	58.8
"	1350	19.5	-57.4	57.8
"	1450	37.1	-75.3	58.3

The integral in Equation 5 has a solution in closed form which may be found in a table of integrals (6). However, if the peak tire-road interface coefficient is constant as in Equation 3, the expression for the ideal stopping distance becomes

$$D_i = \frac{V_1^2}{2g\,\mu_{AV}} \qquad (6)$$

Although it is necessary to calculate ideal stopping distance in order to assess how well the vehicle is taking advantage of the peak forces available in the tire-road interface, it is also necessary to assess how the frictional characteristics of a given tire compare to those of a recognized standard tire when tested on the same surface. For this reason, the standard 8.45-15 tires were tested at various speeds on the high- and low-coefficient surfaces under a vertical load of 1050 lbs. The test data given on Table 10 indicate that the peak tire-road interface coefficient is speed sensitive on both surfaces. Again, the least squares technique was used to fit second degree polynomials to the test data, resulting in the following expressions:

$$\mu = 49.0 \cdot 10^{-6}V^2 - 83.2 \cdot 10^{-4}V + 1.07 \qquad (7)$$

for the dry asphalt surface and

$$\mu = 8.5 \cdot 10^{-6}V^2 - 37.3 \cdot 10^{-4}V + 0.469 \qquad (8)$$

for the wet sealed asphalt surface. The curves resulting from Equations 7 and 8 are plotted on the same graph with the experimental data points in Fig. 10. From these expressions, ideal stopping distances for the standard tire can be calculated from Equation 5 for comparison to the other tires.

3.2 EVALUATION OF VEHICLE-BRAKE SYSTEM AND TIRE-ROAD PERFORMANCE

Brake system efficiency was calculated for each vehicle under each test condition using Equation 2:

$$E = 100\,\frac{D_i}{D_a} \qquad (2)$$

It should be emphasized that this efficiency does not evaluate tire-road performance but is a measure of how well the vehicle-brake system has utilized the frictional forces available in the tire-road interface. To fix the tire-road performance to a common, readily acceptable reference level, ideal stopping distances for the standard tire were determined and the test tires were evaluated against this standard by the following expression:

$$F = \frac{D_{i\,(STD)}}{D_i}, \qquad (9)$$

where F is defined as the Tire Factor

D_i (STD) is the ideal stopping distance for the standard tire

D_i is the ideal stopping distance for the test tire

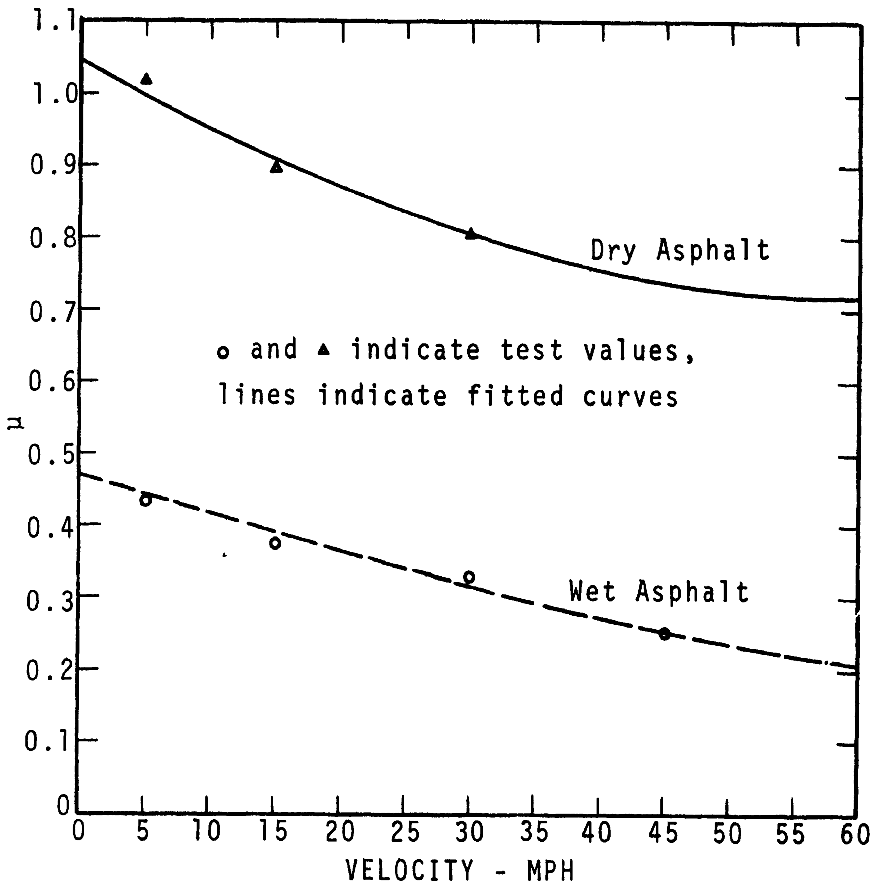


FIGURE 10. PEAK TIRE-ROAD INTERFACE COEFFICIENTS ON HIGH- AND LOW-COEFFICIENT SURFACES FOR GENERAL 8.45-15 STANDARD TIRE

The product of the Braking System Efficiency and the Tire Factor then yeilds a number defined as the Brake System Rating:

$$R = EF \tag{10}$$

This rating compares the actual brake system performance to that which could be achieved with the standard tire.

An evaluation of wet to dry performance can be made for the test vehicle by dividing the ratio of ideal stopping distances, wet to dry, for the standard tire by the ratio of actual vehicle stopping distances, wet to dry.

This may be expressed as:

$$M = \frac{\left.\dfrac{D_i\ (\text{wet})}{D_i\ (\text{dry})}\right|_{\text{STD}}}{\left.\dfrac{D_A\ (\text{wet})}{D_A\ (\text{dry})}\right|_{\text{Vehicle}}} \tag{11}$$

where M is defined as the Wet to Dry Performance Rating.

A summary of the results of this study is given in Table 12, in which are listed the actual stopping distances from the vehicle tests, ideal stopping distances calculated from the tire test data, and the brake system efficiency, tire factor, brake system rating, and the wet to dry performance rating.

TABLE 12. SUMMARY OF TEST RESULTS AND CALCULATIONS

Vehicle	Test Surface	Percent Capacity Weight	Actual Stopping Distance, Ft. Antilock On	Actual Stopping Distance, Ft. Antilock Off	Ideal Stopping Distance Ft.	Brake System Eff.	Tire Factor	Brake System Rating	Wet to Dry Perf. Rating
Chrylser	Dry Asphalt	40	183	178	126	68.8	1.25	86.0	
"	"	60	184	183	126	68.5	1.25	85.6	
"	"	100	194	198	126	65.0	1.25	81.3	
"	Wet Jennite	40	419	770	408	97.2	1.09	106.0	1.23
"	"	60	421	744	377	89.6	1.18	106.0	1.24
"	"	100	385	889	377	97.8	1.18	115.0	1.42
Buick	Dry Asphalt	60	180	163	118	65.5	1.33	87.2	
"	"	80	181	175	118	65.2	1.33	86.7	
"	"	100	193	187	118	61.1	1.33	81.3	
"	Wet Jennite	60	535	803	400	74.8	1.11	83.0	0.95
"	"	80	498	657	425	85.4	1.04	88.8	1.02
"	"	100	528	682	455	86.1	0.97	83.5	1.03
Standard Tire	Dry Asphalt				157				
Standard Tire	Wet Jennite				443				

The results shown in Table 12 indicate that the vehicles tested were able to achieve brake system efficiencies ranging from 60 to 70% on the dry surface and 75 to 98% on the low coefficient surface. These efficiencies may at first glance seem to be disappointingly low. However, when these results are compared to those achieved in driver controlled tests with a vehicle not equipped with an antilock system, the advantages of an antilock system are clearly made manifest. In tests conducted at HSRI in 1969, it was shown that although the vehicle had a braking efficiency of better than 95% for the range of surfaces tested, drivers in general were not able to modulate the brakes to achieve better than 65% of the vehicles' braking capability on dry asphalt, and no better than 45% on wet painted asphalt (7).

The data indicate that the Buick was able to achieve slightly shorter stopping distances on the dry asphalt surface than the Chrysler, while on the wet sealed asphalt surface the reverse was true. However, the ideal stopping distance calculations show that the Buick tires should produce higher peak forces on the dry surface, so that the braking efficiencies calculated for the Chrysler are actually higher. For the low coefficient surface, the ideal stopping distances for the Buick tires were considerably larger than those for the Chrysler tires, yet, because of the larger actual stopping distances, achieved a lower ideal brake system efficiency.

The tire factor gives a reasonable assessment of how well each tire compares to a standard tire on a given surface. In all cases except one, the vehicle tires showed superior peak force characteristics when compared to the standard tire. The brake system rating, on the other hand, compares the actual stopping distance achieved by the vehicle to the ideal stopping distance calculated for the standard tire. The ratings in all cases are higher than 80%, and for the Chrysler on the low coefficient surface, were in excess of 100%.

The wet to dry performance rating compares the wet to dry performance of the standard tire to the wet to dry performance of the vehicle. If the ratio is 1.0, the changes were equal. If greater than 1.0 then the vehicle performance changed less than the standard tire which is desirable. Conversely, at a ratio less than 1.0, the vehicle experiences more change than the standard tire and this is undesirable. With the exception of one point both vehicles exhibited less change than the standard tire.

3.3 STATISTICAL TOLERANCE ON BRAKING EFFICIENCY CALCULATIONS

The determination of braking efficiency is based upon estimates of mean values of D_a and μ which were derived from relatively small samples (10 test replications) of a total population. The statistical tolerance on the values for braking efficiency so derived is estimated from the variations found in each test sample. For the dry surface tests, ideal stopping distance was calculated from Equation 6, and braking efficiency was calculated from Equation 2. Combining Equations 2 and 6 results in

$$E = \frac{V_1^2}{2g\, D_a\, \mu_{AV}} \qquad (12)$$

Since estimates of the standard deviation of D_a and μ_{AV} have been made (see Tables 3, 5, 8, and 9), an estimate of the standard deviation of E can be calculated from the following equation (8):

$$s_E = \sqrt{\left(\frac{\partial E}{\partial D_a}\right)^2 s_{D_a}^2 + \left(\frac{\partial E}{\partial \mu_{AV}}\right)^2 s_{\mu_{AV}}^2} \tag{13}$$

where s_E is the estimated standard deviation of E.

s_{D_a} and $s_{\mu_{AV}}$ are estimates of the standard deviations of D_a and μ_{AV}, respectively.

Performing the indicated differentiations on E as expressed in Equation 12, Equation 13 becomes:

$$s_E = \sqrt{\left(\frac{E}{D_a}\right)^2 s_{D_a}^2 + \left(\frac{E}{\mu}\right)^2 s_{\mu_{AV}}^2} \tag{14}$$

Using this equation and the appropriate data from the vehicle and tire tests, s_E was calculated for each corresponding E. Results are summarized in Table 13. Despite the fact that there is considerable variation in the estimates of the standard deviation for the actual stopping distance, the estimate on the corresponding tolerance on the efficiency calculation varies only from 4.0 to 4.6%.

For the tests on the low coefficient surface, the determination of tolerance on the efficiency calculation was greatly complicated by the introduction of the least squares technique to determine the functional relationship between μ_{PEAK} and velocity. An approach similar to that used in the dry surface tests was not possible since for a given set of tests of a tire at a given load and several speeds, the estimate of the standard deviation of μ_{PEAK} was different for each speed. Thus, some estimate of the standard deviation in D_i had to be made before estimating the tolerance on the braking efficiency calculation. Two procedures were used to get an estimate of σ_{D_i} for the wet tests.

The first consisted of determining the variation in ideal stopping distance which would result from using the mean plus one sigma values of μ_{PEAK} at each velocity to calculate

$$\mu_{+1\sigma} = A_+V^2 + B_+V + C_+ \tag{15}$$

and the mean minus one sigma values of μ_{PEAK} at each velocity to calculate

$$\mu_{-1\sigma} = A_-V^2 + B_-V + C_- \tag{16}$$

The values of A_+, B_+, and C_+ were substituted for A, B, and C in Equation 5 to calculate an ideal stopping distance, D_+, and the values of A_-, B_-, and C_- were likewise used to calculate another ideal stopping distance D_-. Results from these calculations are given in Table 14. The variation listed in the last column in the table is the average of (D_i-D_+) and (D_--D_i).

TABLE 13. TOLERANCE ON BRAKING SYSTEM EFFICIENCY CALCULATIONS FOR TESTS ON DRY ASPHALT SURFACE

Vehicle	Tire	Percent Capacity Load	D_a ft	σ_{D_a} ft	μ_{AV}	$\sigma_{\mu_{AV}}$	E %	s_E %
Chrysler	L84-15	40	183	3.8	0.95	0.06	68.8	4.6
"	"	60	184	3.6	0.95	0.06	68.5	4.2
"	"	100	194	6.7	0.95	0.06	65.0	4.6
Buick	H78-15	60	180	6.4	1.02	0.05	65.5	4.5
"	"	80	181	5.7	1.02	0.05	65.2	4.4
"	"	100	193	6.7	1.02	0.05	61.1	4.0

TABLE 14. VARIANCE IN IDEAL STOPPING DISTANCE CALCULATIONS FROM TESTS ON LOW COEFFICIENT OF FRICTION SURFACE USING USING MEAN ±1σ VALUES FOR μ

Vehicle	Tires	Percent Capacity Weight	Ideal Stopping Distance, ft			Average Variation, ft
			D_i	D_+	D_-	
Chrysler	L84-15	40	408	362	473	55
"	"	60	377	340	440	50
"	"	100	377	326	423	48
Buick	H78-15	60	400	356	449	46
"	"	80	425	363	492	64
"	"	100	455	371	535	82

The second procedure used to assess the variation in D_i consisted of the following. Test data from the H78-15 tire at 1450 lbs load and various speeds were listed in an array as follows:

5 MPH	15 MPH	30 MPH	45 MPH
$\mu_{1,1}$	$\mu_{2,1}$	$\mu_{3,1}$	$\mu_{4,1}$
$\mu_{1,2}$	$\mu_{2,2}$	$\mu_{3,2}$	$\mu_{4,2}$
.	.	.	.
.	.	.	.
.	.	.	.
$\mu_{1,10}$	$\mu_{2,10}$	$\mu_{3,10}$	$\mu_{4,10}$

The order of occurrence of the values in the columns is the same order as the test results were obtained. Then values from the first row were used to derive a functional relationship $\mu = f(V)$ using the least squares curve fitting technique, which when repeated for each succeeding row produced the following array:

$$\mu_1 = A_1V^2 + B_1V + C_1$$

$$\mu_2 = A_2V^2 + B_2V + C_2$$

$$\vdots$$

$$\mu_{10} = A_{10}V^2 + B_{10}V + C_{10}$$

Using Equation 5, each of the ten sets of constants A, B, and C yielded an ideal stopping distance, for a total of 10 values. The mean value was 387 ft. Using small sampling theory, the following equation can be written (9):

$$P\left[\left(\bar{D} - \frac{2.262s}{\sqrt{N}}\right) < D < \left(\bar{D} + \frac{2.262s}{\sqrt{N}}\right)\right] = 0.95 \qquad (15)$$

where
P = probability of occurrence
$\bar{D}$ = mean value
D = any element of the sample
N = number of elements in the sample

$$S = \sqrt{\frac{\Sigma(D-\bar{D})^2}{N-1}}$$

This equation states that 95% of all the values of D_i so calculated will lie between the values of 387 ± 85 ft.

The tolerance on the values of braking efficiency calculated for the wet tests can be estimated in a similar manner, resulting in the following expression:

$$s_E = \sqrt{\left(\frac{E}{D_a}\right)^2 s_{D_a}^2 + \left(\frac{1}{D_a}\right)^2 s_{D_i}^2} \tag{16}$$

Tolerance values on braking efficiency for the tests on the wet sealed asphalt are listed in Table 15. It should be noted that the second term under the radical sign in Equation 16, namely, $\left(\frac{1}{D_a}\right)^2 s_{D_i}^2$ is negligible in comparison to the first term, which indicates that the ratio of $\frac{s_{D_i}}{D_a}$ would have to be greater than 1 before it would significantly affect the value of s_E.

3.4 INFLUENCE OF MEASUREMENT AND COMPUTATIONAL ERRORS ON BRAKING EFFICIENCY CALCULATIONS

Average errors in measurement of input data are estimated to be the following:

Stopping distance on dry asphalt surface, 0.60%

Stopping distance on wet sealed asphalt surface, 0.24%

Velocity measurement, fifth wheel calibration, 0.2%

TABLE 15. TOLERANCE ON BRAKING SYSTEM EFFICIENCY CALCULATIONS FOR TESTS ON THE LOW COEFFICIENT OF FRICTION SURFACE

Vehicle	Tire	Percent Capacity Load	D_a ft	σ_{D_a} ft	D_i ft	$\sigma^*_{D_i}$ ft	E %	s_E %
Chrysler	L84-15	40	419	20.6	408	50	97.2	5.0
"	"	60	421	12.5	377	50	89.6	2.7
"	"	100	385	17.4	377	48	97.8	4.4
Buick	H78-15	60	535	32.7	400	46	74.8	4.6
"	"	80	498	18.8	425	60	85.4	3.3
"	"	100	528	20.6	455	82	86.1	3.4

*Estimated from Table 14.

Error in reading oscillograph traces for velocity is estimated to be 0.85%.

Combined errors in the determination of μ_{PEAK} are estimated to be:

Dry asphalt surface, 2.0%

Wet sealed asphalt surface, 4.0%

Errors in least squares curve fit, averaged for the given data points, was 6.4%.

Assuming the sources of error to be independent of each other, the combined errors in the determination of braking efficiency may be estimated using the sqaure root of the sum of the squares of each of the individual errors (10). For the tests on the dry asphalt surface the combined error is 2.3% and for the wet sealed asphalt surface 6.2%.

4. CONCLUSIONS AND RECOMMENDATIONS

Results from this study indicate that the test procedures developed in this program are viable and realistic. Brake system efficiency, as defined in Equation 2, and calculated from data generated by both vehicle and tire tests, is a reasonable measure of how well the vehicle and brake system take advantage of the peak frictional forces available in the tire-road interface to retard vehicle motion. Use of the programmed pedal application device allowed open loop inputs which were free of driver error and run to run variations. The tire factor, defined in Equation 9, measures the capability of the vehicle tires to produce peak braking forces as compared to the standard tire. By specifying limits on the brake system efficiency, the tire factor, and the wet to dry performance rating, the desired braking performance of the vehicle can be fairly well defined.

Although the test procedures and computational techniques utilized in this study were very adequate to produce the desired results, the following recommendations are made to improve these procedures and techniques:

1. Locked wheel stops cannot be made on the low coefficient of friction surface from 60 mph without compromising vehicle stability. Since the information provided by these stops, either on the high or low coefficient surface, is not necessary for calculation of brake system efficiency, it is recommended that locked wheel stops not be required as part of the test procedure.

2. If the necessary testing equipment is available, tire tests on the dry surface should be made at the average static loads experienced by the vehicle tires at test speeds of 5, 15, 30, 45, and 60 mph to accurately characterize the peak tire-road interface coefficient.

3. Adequate run-out area should be provided for the tire tester such that tests of tires on the low coefficient surface can be made at the same speeds as those recommended for the dry surface tests. The data points at 5 and 60 mph are necessary to define the functional relationship of μ_{PEAK} and velocity.

4. A weighted least squares curve fitting technique should be used to define the $\mu_{PEAK} = f(V)$ relationship which will take into account differences in the estimated values of σ_μ at various velocities. Such a technique would permit a more simplified approach to the determination of the statistical tolerance in the final result.

APPENDIX

This appendix contains all the curves defining the averaged μ-slip characteristics from the tire tests conducted for this program.

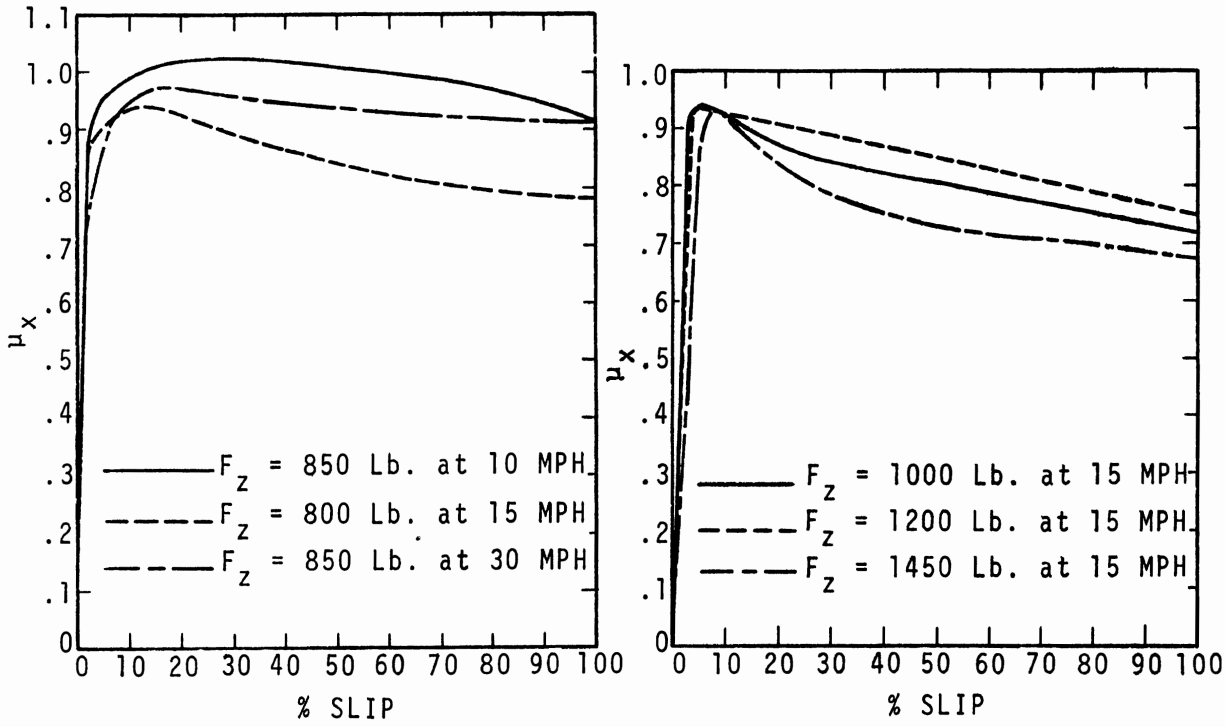


FIGURE A-1. μ - SLIP CHARACTERISTICS FOR GOODYEAR POLYGLAS L84-15 TIRE ON DRY ASPHALT FOR VARIOUS SPEEDS AND LOADS

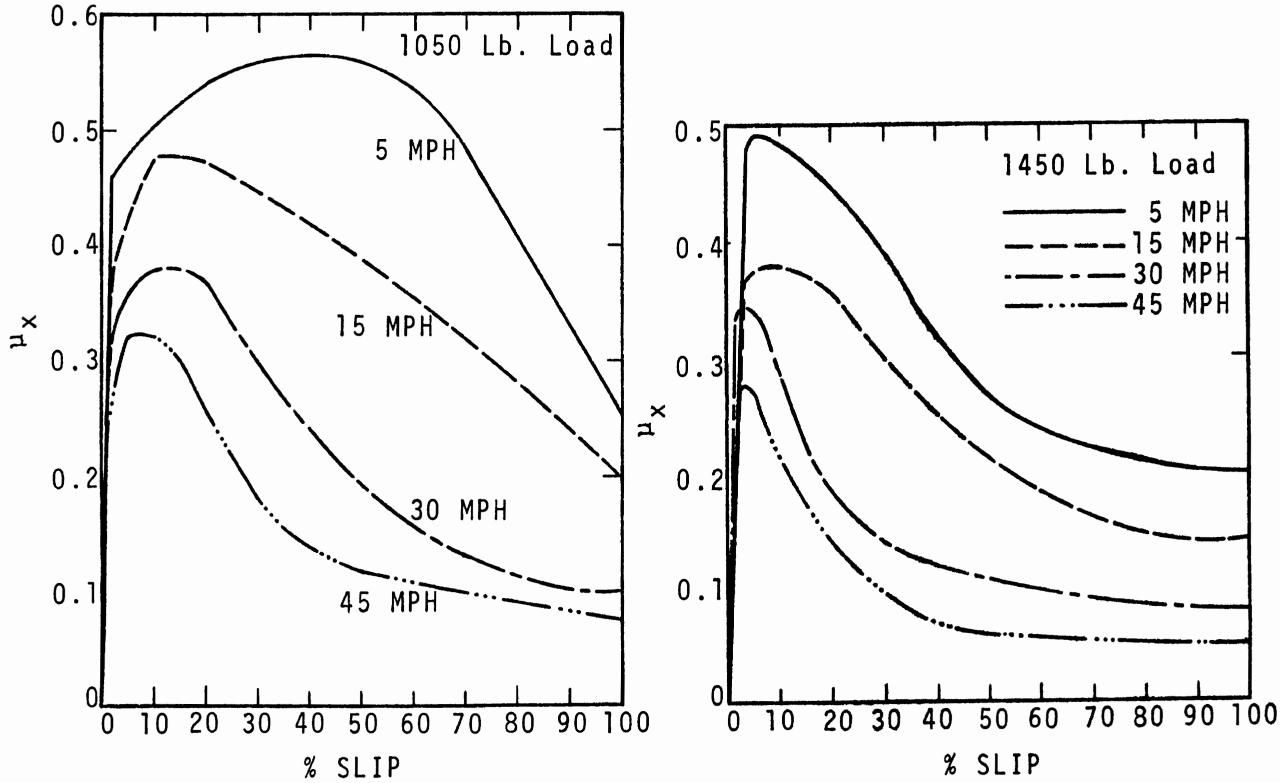


FIGURE A-2. μ - SLIP CHARACTERISTICS FOR GOODYEAR POLYGLAS L84-15 TIRE ON WET JENNITE FOR VARIOUS SPEEDS AT 1050 AND 1450 LB. LOAD

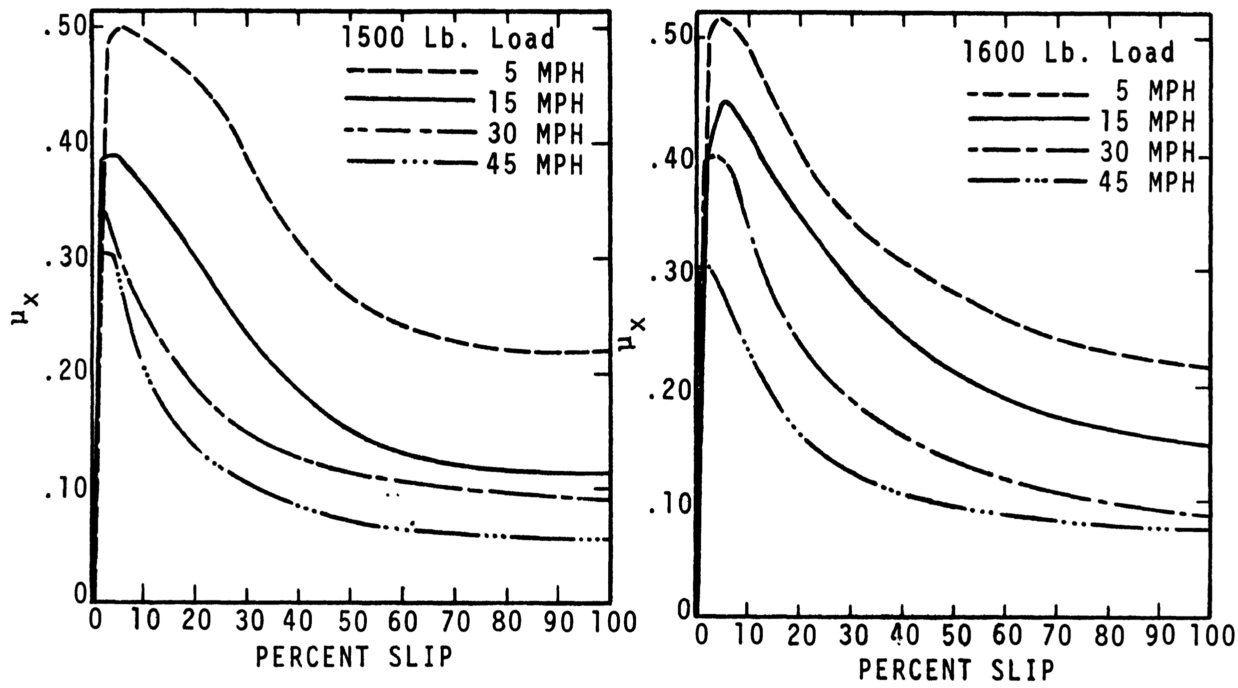


FIGURE A-3. μ - SLIP CHARACTERISTICS FOR GOODYEAR POLYGLAS L84-15 TIRE ON WET JENNITE FOR VARIOUS SPEEDS AT 1500 and 1600 LB. LOAD

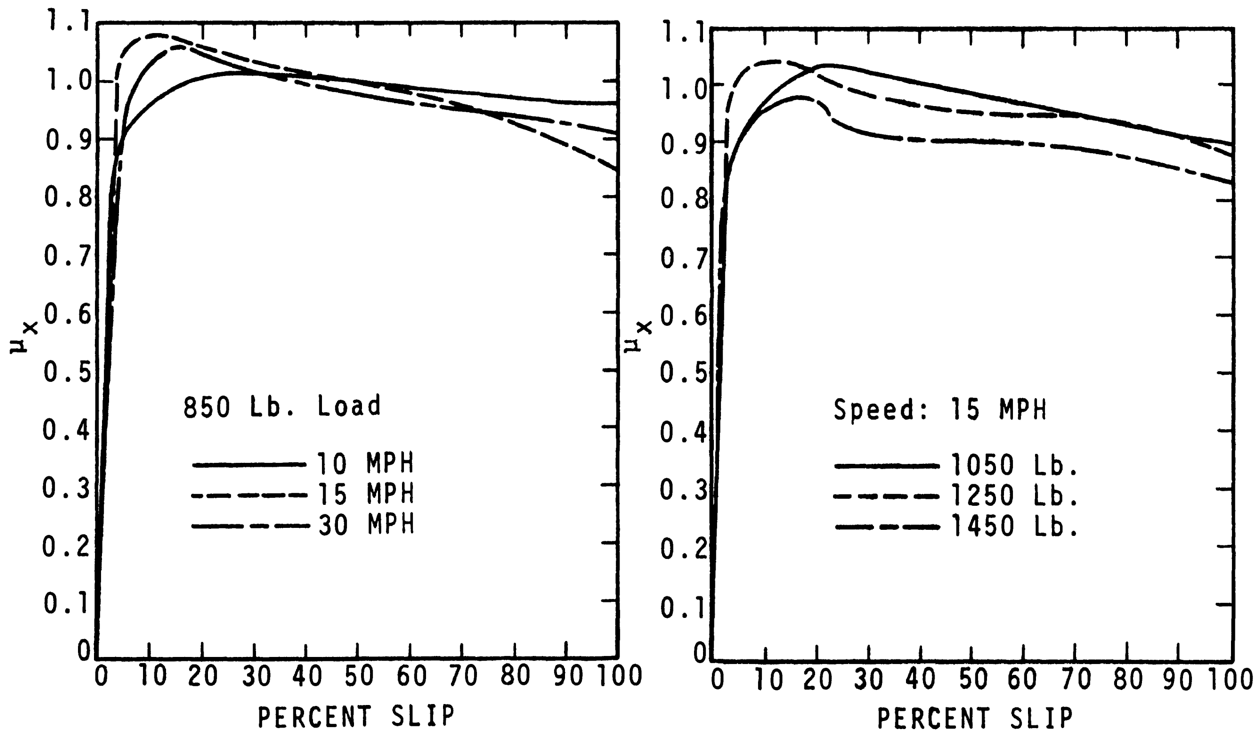


FIGURE A-4. μ - SLIP CHARACTERISTICS FOR GOODYEAR POLYGLAS H78-15 TIRE ON DRY ASPHALT FOR VARIOUS SPEEDS AND LOADS

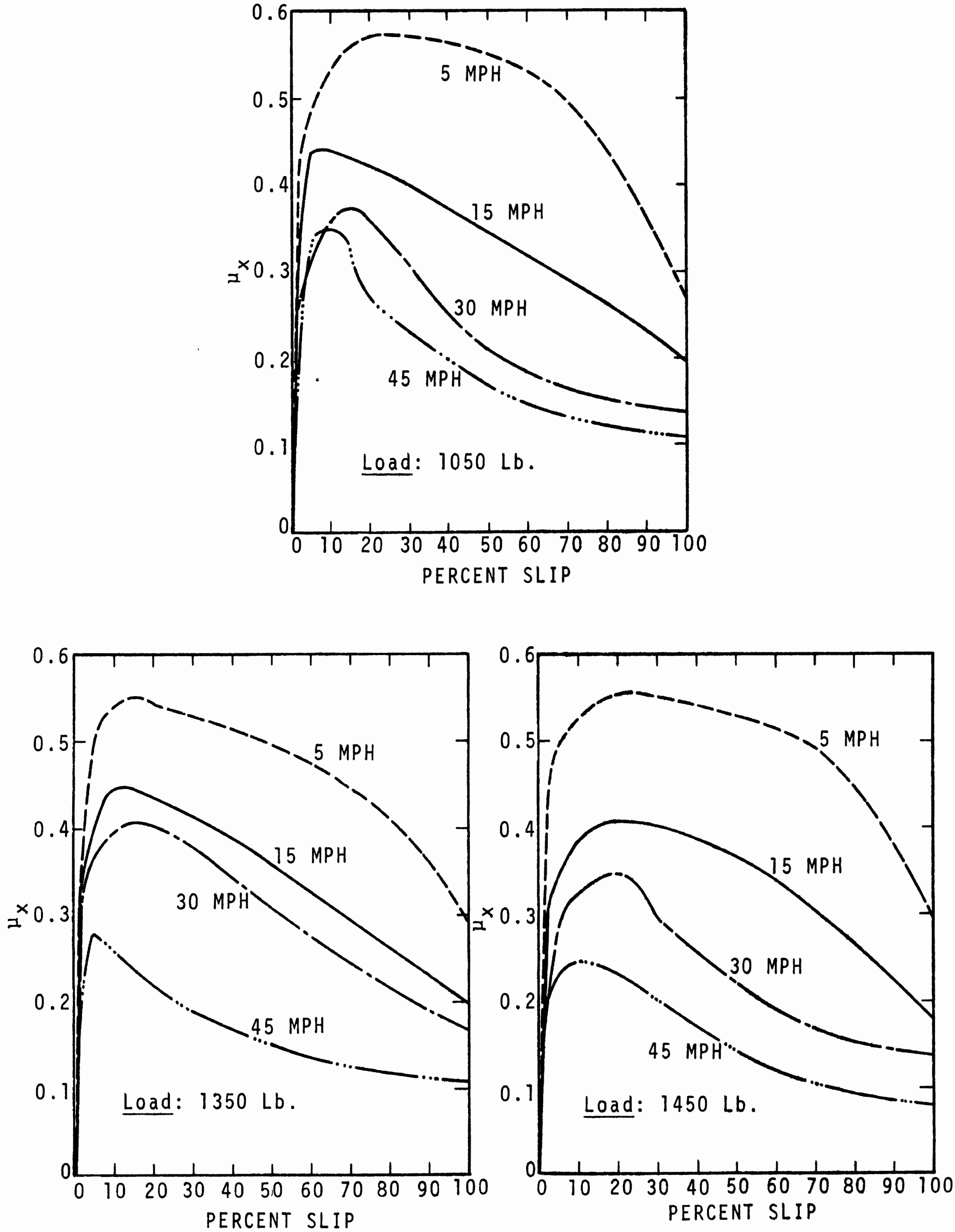
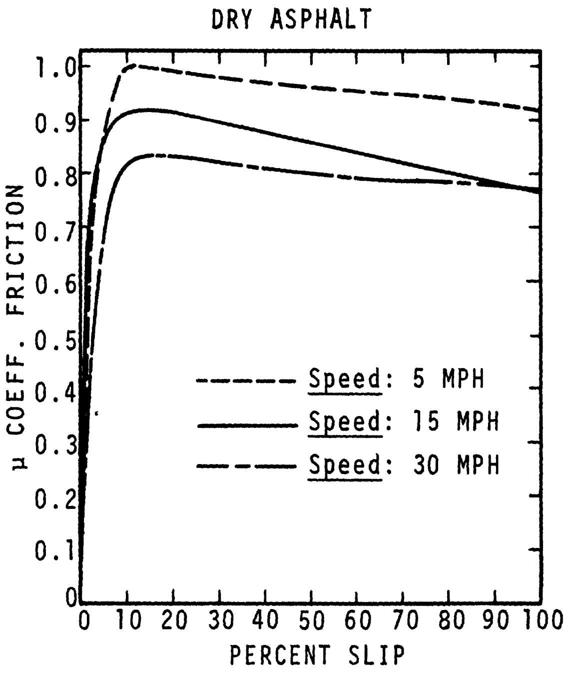


FIGURE A-5. μ - SLIP CHARACTERISTICS FOR GOODYEAR POLYGLAS H78-15 TIRE ON WET JENNITE FOR VARIOUS SPEEDS AND LOADS



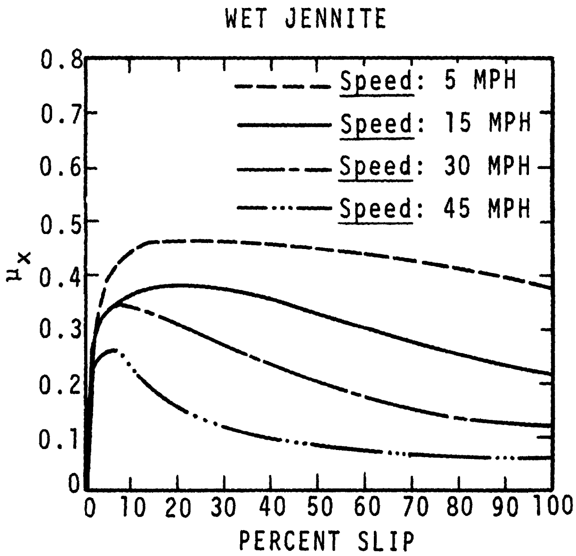


FIGURE A-6. μ - SLIP CHARACTERISTICS FOR GENERAL 8.45-15 STANDARD TIRE ON DRY ASPHALT AND WET JENNITE FOR 1050 LB. LOAD AT VARIOUS SPEEDS

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