To explain a procedure, strategy, or skill		Such as the procedure for factoring a difference of two squares $x^2 - a = (x + \sqrt{a}) \cdot (x - \sqrt{a})$
A teacher would	Which means	For example
Focus the procedure • **What for?** • **When?** • **As opposed to…?**	• Establishes clearly <u>what kind of problem</u> the procedure targets. • Demonstrates circumstances <u>when</u> students may encounter such problem (and thus need to use the procedure). • Promotes flexibility by comparing pros and cons of the target procedure with those of alternative ones that might be used in similar situations.	• This procedure, like all cases of factoring aims at expressing a quadratic polynomial as the product of linear binomials, which are simpler to handle. • If you had to graph a function like $f(x) = \frac{x^2 - 4}{x - 2}$ you could use factoring the numerator into (x + 4) · (x – 2), since that would tell you that the function has exactly the same graph as f(x) = x+2 except for x=2 where it is undefined. Graphing y = x+2 is easy. • Now, if you did not know how to factor the numerator you might have to do a table for that rational function and it could get complicated.
Establishing the procedure • **Describing** • **Representing** • **Justifying**	• Identify the concepts that make the procedure possible and prescribe the steps that the student needs to take to execute the procedure • Justify why steps in the application of a procedure are mathematically valid as well as strategically appropriate moves toward the goal of the procedure	• Show that the first step is to find what are the terms that one will think of as "squares," (x^2 and a) then find the square root of those, x and $\sqrt{a}$, then fit them in the pattern (□+○)(□–○). • Show that when one multiplies through the indicated product, one gets back the original polynomial • $(x - \sqrt{a})(x + \sqrt{a}) = x^2 - x\sqrt{a} + x\sqrt{a} - \sqrt{a} \cdot \sqrt{a} = x^2 - a^2$
Demonstrating the procedure • **How to…?** • **Extreme cases**	• Choose a few generic cases where to demonstrate how the procedure is done. • Use the generic cases to help students envision possible complications or simplifications that a student might run into and how the procedure might then change.	• Factor $x^2 - a^2$, $x^2 - b$, $b - 2x^2$, and $x^{10} - y^6$ (possibly also $x^2 + a^2$). Some done with numerical examples. • Help students notice that properties like $a = (\sqrt{a})^2$, $a = -(-a)$, and $a^{2k} = (a^k)^2$ help them think about some polynomial differences as differences of squares. • Note that sometimes the squares in the difference are made of several terms themselves, such as $x^2 - y^2 - 2y - 1 = x^2 - (y + 1)^2$
Connecting the procedure to relevant prior knowledge	Make explicit all the component elements of the procedure and recall what they mean, stressing vocabulary inasmuch as it helps understanding, but concentrating on how and why to use them	Connect this procedure to other factorizations of polynomials and to factoring numbers (e.g., $45 = 3^2 \bullet 5$), possibly also 144–25 = (12–5)(12+5) Connect to properties of exponents and to the problem of finding zeroes of a polynomial.

Identifying conditions of use and means of control	Gives students the opportunity to • Justify why the procedure may or may not be applied • Justify why the result is or is not correct Provide metacognitive strategies to monitor performance	Give a rational function and ask students: Would it make sense to factor the denominator as a difference of two squares? What could you do with that? Give them factorizations done by others, correct and incorrect, and ask them to explain why they are correct or not. Provide self-monitoring questions for students to use, like • what are the two squares whose difference I want to factor? • how can I make sure these are the right factors?
Identifying tools, notations, and requirements	Establishes some reasonable rules but leaves other matters to individual choice in regard to tool use, symbol systems, and other requirements.	Say to students—it does not really matter which base is first and which one is second in the factoring. You could factor $x^2 – 9$ as $(x – 3)(x + 3)$ or as $(3 + x) (-3 + x)$ but note how the signs need to change to preserve the fact that the 9 is negative. In many procedures it is important for students to realize what requirements are important to mind because of the mathematics and which ones are just convenience related.
Holding students accountable	Holds students accountable for performing the procedure, as well as for using it in context of more complicated problems, for describing and justifying the steps of the procedure	Give students problems that span the range of difficulty for the procedure and that illustrate how the procedure is useful in other problems. For example, in the test that contains factoring of a difference of two squares, it would be very appropriate to ask students to describe what the graph of $y = \frac{x^2 - 4}{x+2}$ would look like.