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# Economic Spacing Of Roads

# When Skidding To A Landing With Tractors And Arches

# On Slopes Ranging Up To 60%.

May 22, 1939.

William E. Scott.

## Foreword.

It is recognized that natural conditions may vary widely on two logging operations located within a few miles of each other. To present a rigid plan for determining the economic spacing of roads is therefore largely wasted effort. No practical use could be made of such a plan.

The first part of this thesis appears to be in direct opposition to the above statement. The excuse for this is that it is necessary to demonstrate general ideas with specific cases. On page 34, the theoretical road spacings are listed for slopes ranging up to 60 per cent. The times will be rare when it will be possible to locate a transportation system which will meet these theoretical conditions.

The second part of this work deals with the practical application of the principles. An economic transportation system is planned for a portion of a tract known as the Strawberry Logging Unit located on the Stanislaus National Forest in California, and here the ideas presented with such mathematical precision at first are molded to fit the actual conditions on the ground.

This study requires cost data in a form which is not readily obtainable at this time. This has made it necessary to assume certain facts, but as more accurate information becomes available, such material can be applied to any specific problem without any change in the fundamental principles involved.

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# Part I.

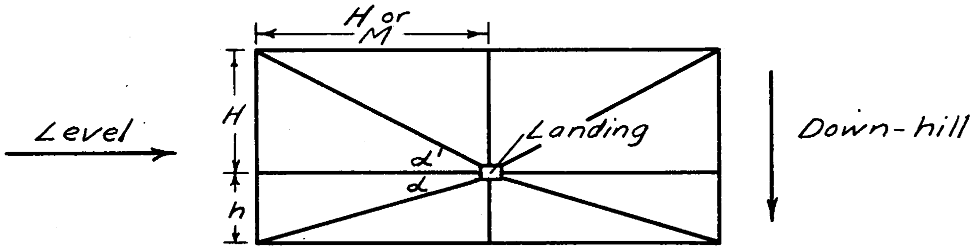
## Theoretical Aspects of Planning An Economic Logging Transportation System.

## Summary.

1. The cost of skidding increases with distance, and the cost of skidding up-hill is greater than the cost of skidding down-hill. It follows that for a given amount of money, logs can be skidded a greater distance down-hill than up-hill.

2. Two things must be known to compute the cost of skidding. These are slope and the average distance skidded.

3. The average skidding distance is determined by sub-dividing the area served by a landing into component triangles.




4. Where the slope is less than the maximum which a loaded tractor can negotiate directly, the relative lengths of h and M can be determined by means of a break even calculation.

$$\text{Tan } \alpha = \frac{h}{M}$$

5. Where the slope exceeds the maximum which a loaded tractor can negotiate directly,

$$\text{Sin } \alpha = \frac{\text{Allowable grade } \%}{\text{Actual grade } \%}$$

6. With angles $\alpha$ and $\alpha'$ known, H, h, and M can be expressed as percentages of the road spacing.

7. On slopes which are gradual enough to permit hauling directly to the landing, the average skidding distance will closely approximate the distance to the landing from the center of gravity of the triangle involved. If greater accuracy is desired,

the exact average skidding distance equals .67 HB.[1] These values have been curved and can be read directly for any slope (Figure 16) in terms of M. The average skidding distance is then expressed as a percentage of the road spacing.

8. Where the slope is so steep as to necessitate a switchback movement to reach the landing, the average skidding distance equals two-thirds the length of the hypotenuse of the triangle concerned.

9. Steep slopes will require a switchback movement up-hill to the landing, but the return trip can be made directly down the slope.

10. The average skidding distance for the entire area serving a landing is determined by prorating the skidding distances of the component areas.

11. The basic cost of skidding per MBM per 100' of distance on level ground can be adjusted for any slope by deducting 4% of the available tractor drawbar pull for each 1% of adverse grade.

12. The average cost of all skidding is determined by weighting the charges for up-hill and down-hill skidding.

13. The cost of a road per MBM varies inversely with the area (volume) tapped. Wider road spacing gives lower total road construction costs.

14. The cost of skidding per MBM on a given slope varies directly with the distance hauled. Wider road spacing causes higher skidding costs.

15. The average cost of skidding equated against the cost of road construction gives the economic spacing of roads.

(1) H = Economic skidding distance on level ground.

$$B = \left[ \frac{\left(1+\frac{b^2}{h^2}\right)^{\frac{1}{2}}}{2} + \frac{h}{2b} \log_e \left( \frac{b}{h} + \left(1+\frac{b^2}{h^2}\right)^{\frac{1}{2}} \right) \right]$$

Skidding On Slopes

With Tractors And Arches.

# Skidding On Slopes With Tractors and Arches.

Logging transportation systems have always been dependent on topography. In the days of the bull team, yarding was a downhill operation, and loads were limited by the strength of the teams. Conversely, yarding with donkey engines is an up hill process. An abundance of power is available and even the largest logs can be skidded up the steepest slopes with ease. High lead donkey yarding is not adapted to down-hill skidding because the logs overrun the tackle and there is no control over the load.

The most recent machine to be used for skidding logs is the tractor. The early models were not nearly as powerful as are those of today, and operators considered them as mechanized teams to be used for skidding down-hill. Some up-hill skidding on unfavorable slopes was attempted, but for the most part, skidding was confined to down-hill operations. This practice has prevailed, and it is the purpose of this discussion to investigate the desirability of using tractors for up-hill skidding on slopes ranging as high as 60%.

Hourly costs of operating a tractor remain about the same regardless of the amount of timber skidded per hour. Therefore, low skidding costs are obtained by operating tractors with full loads.

With every 1% increase in adverse grade, 4% must be deducted from the available drawbar pull of a tractor. This reduction in pulling power represents the energy required to lift the tractor up the hill. On a 10% adverse slope, a tractor can skid only 60% as much timber per turn as it can on level ground. This is a serious reduction in the output of the machine and represents the reason why skidding is not practiced on adverse slopes which are steeper than 10%.

When skidding on slopes of 0%-10% to a landing, hauling is done on grades which range from a maximum pitch at right angles to the contours to level hauls parallel to the contours. Skidding is done directly to the landing for both up-hill and down-hill hauls on these slopes.

On steeper slopes, direct skidding down-hill is practiced, but up-hill skidding must be done at an angle to the contours so that the maximum adverse slope traversed is 10%.

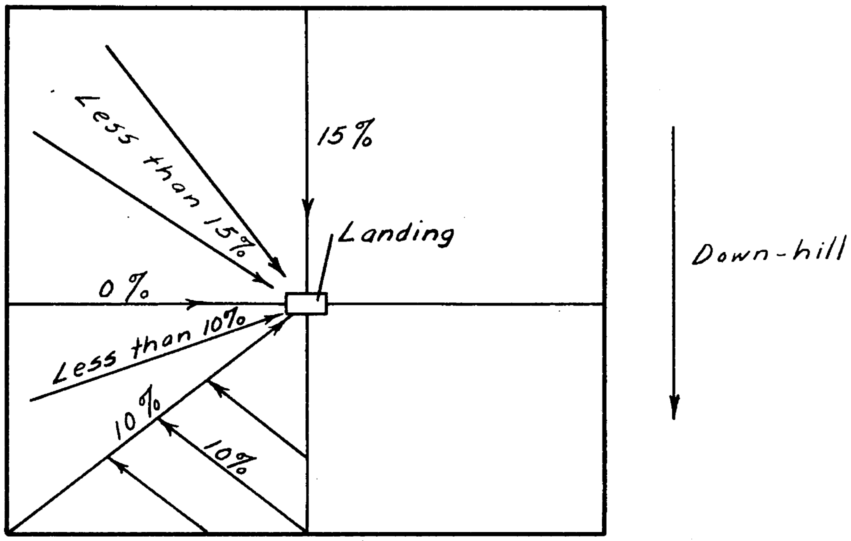



Figure 1. Skidding on 15% slope.

On slopes of 15%-25%, skidding is done directly down-hill to the landing. 25% represents a safe maximum down which a tractor equipped with an arch can bring a full load. It also represents the steepest slope which an unloaded tractor can climb.

Where slopes are 30% or greater, indirect skidding is required for both up-hill and down-hill hauls. A D-8 Caterpillar tractor has a 78" gauge and can safely skid a load along a 60% side hill without danger of capsizing. Where slopes exceed 60%, special skid roads will usually be required.

It will be convenient to divide slopes into the following three groups:

1. Slopes 0% - 10%. Direct skidding up-hill and down-hill.
2. Slopes 15% - 25%. Indirect skidding up-hill, direct skidding down-hill.
3. Slopes 30% - 60%. Indirect skidding up-hill and down-hill.

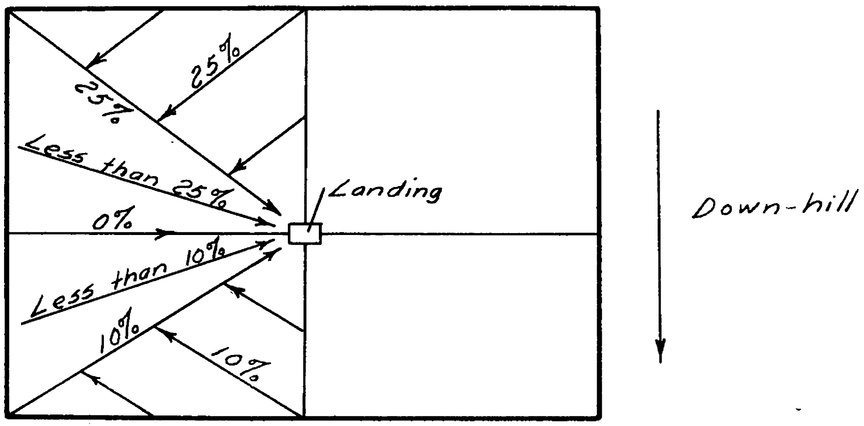



Figure 2. Skidding on 40% slope.

# Skidding Up-hill And Down-hill

# With Tractors And Arches On Slopes

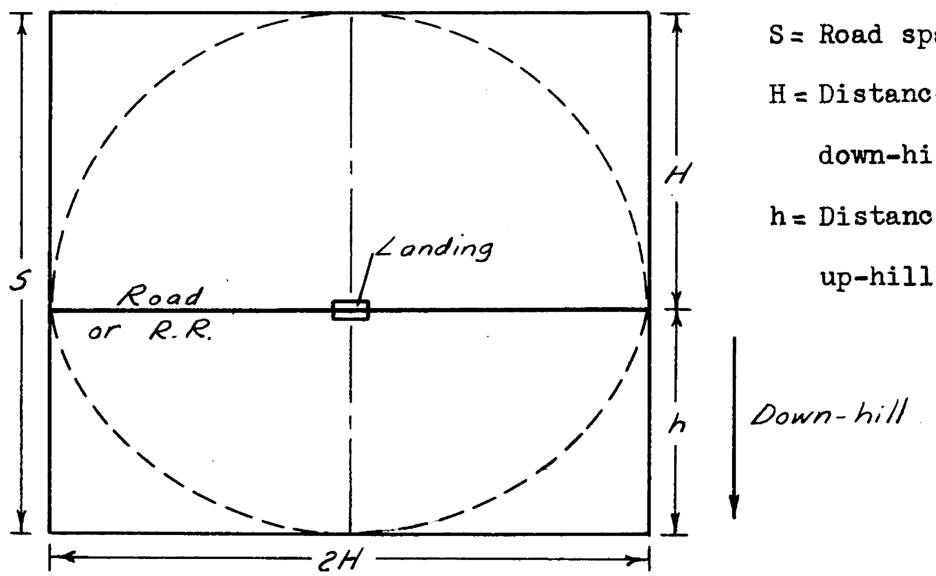
# 0% - 10%

## Slopes 0% - 10%.

Skidding up-hill and down-hill with tractors and arches on slopes which do not exceed the maximum adverse grade the machine is capable of negotiating.

The tractor will be able to skid directly to the landing from any point in the area being served. The cost of skidding down-hill will not vary materially from the cost of skidding on the level, and these two costs will here be considered to be equal. When this is the case, the economic skidding distance will be the same for skidding along the contour as it is for skidding down-hill, and the area served by down-hill skidding will be a rectangle which is twice as wide as it is deep.

The cost of skidding up-hill is greater than the cost of skidding on the level. It follows that the economic distance for up-hill skidding is less than the economic distance for skidding on the level. Up-hill skidding will be done from a rectangle which is more than twice as wide as it is deep.




S = Road spacing.

H = Distance skidded down-hill.

h = Distance skidded up-hill

Figure 3. Economic skidding distance.

The dotted line in Fig. 3 represents the economic skidding distance, or the distance at which the cost of skidding just equals the cost of road construction. It will later be shown that there is a tolerance in the economic skidding distance, and that it is possible to bring in all the logs within the rectangle at only a slightly increased skidding cost.

The relationship between the distance skidded down-hill and the distance skidded up-hill can be determined by a break-even calculation.

$H =$ Distance skidded down-hill.

$h =$ Distance skidded up-hill.

$C =$ Cost of skidding down-hill.

$C' =$ Cost of skidding up-hill.

$$H\,C = h\,C'$$

Example:

10% slope. Tractor is 60% efficient on up-hill skidding.

If $C = 3.4¢/M/100'$

$$C' = \frac{3.4}{.60} = 5.7¢/M/100'$$

Let $H = 10$

$H\,C = h\,C'$ or $10(3.4) = h(5.7)$ or $h = 6.0$

$S =$ Road spacing.

$S = H + h$ or $S = 10 + 6$ or $S = 16$

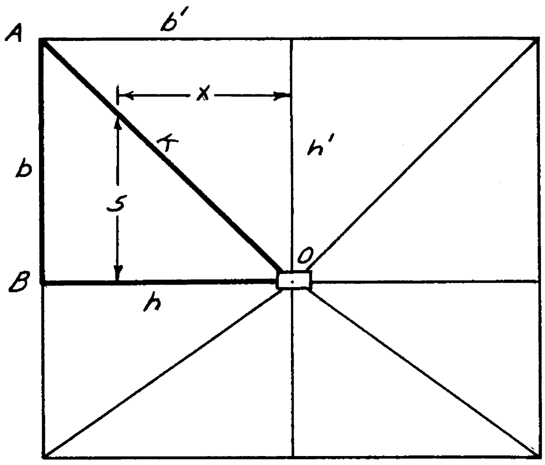
$H = \frac{10}{16} = 62.5\%\ S$ $\qquad h = \frac{6}{16} = 37.5\%\ S$

After the shape of the area served by a landing has been defined, the next step is to determine the average distance the logs must be skidded to the landing in terms of the road spacing.

The average skidding distance to a landing can be computed accurately for any slope, but this requires a rather lengthy analysis involving calculus. For practical application, such precision is not necessary, and two approximate methods of determining the average skidding distance are also presented. These methods are easy to apply and are sufficiently accurate.

The area being served by a landing is divided into a number of triangles (Fig. 4.), and for each of these, the average hauling distance is calculated. The average skidding distance for the whole area is obtained by weighting the average hauls of the component triangles.

To determine accurately the average external yarding distance:




O the point (Landing)

AB the line.

Figure 4.

$$\text{Average haul} = \frac{\int_0^b dx\ (x^2 + h^2)^{\frac{1}{2}}}{\int_0^b dx = b}$$

$$= \frac{\frac{1}{2}\left[x(x^2 + h^2)^{\frac{1}{2}} + h^2 \log_e(x + (x^2 + h^2)^{\frac{1}{2}})\right]_0^b}{b}$$

$$= \frac{1}{2b}\left[\left(bk + h^2 \log_e (b + k)\right) - 0 - h^2 \log_e h\right]$$

$$\text{Average haul} = \frac{k}{2} + \frac{h^2}{2b} \log_e \frac{b + k}{h}$$

$$\frac{s}{x} = \frac{b}{h} \qquad \text{or} \qquad s = \frac{xb}{h}$$

Substitute s for b where $s = \frac{xb}{h}$

Substitute for k $\quad (x^2 + s^2)^{\frac{1}{2}} = (x^2 + x^2 \frac{b^2}{h^2})^{\frac{1}{2}} = x(1 + \frac{b^2}{h^2})^{\frac{1}{2}}$

Substitute for h $\longrightarrow$ x

$$\text{Av. haul to element strip} = \frac{x(1 + \frac{b^2}{h^2})^{\frac{1}{2}}}{2} + \frac{x^2}{\frac{2xb}{h}} \log_e \left[\frac{(\frac{xb}{h}) + x(1 + \frac{b^2}{h^2})^{\frac{1}{2}}}{x}\right]$$

$$\text{Average external haul} = x\left[\frac{(1 + \frac{b^2}{h^2})^{\frac{1}{2}}}{2} + \frac{h}{2b} \log_e \left(\frac{b}{h} + (1 + \frac{b^2}{h^2})^{\frac{1}{2}}\right)\right]$$

$$\text{Average external haul} = \underline{\underline{x\ B}} \qquad \text{where } B = \left[\frac{(1 + \frac{b^2}{h^2})^{\frac{1}{2}}}{2} + \frac{h}{2b} \log_e\left(\frac{b}{h} + (1 + \frac{b^2}{h^2})^{\frac{1}{2}}\right)\right]$$

Average actual skidding distance for any right triangle:

$$\text{Average actual haul} = \frac{\int_0^b s \cdot dx \cdot x \cdot B}{\int_0^b s \cdot dx} = \frac{\int_0^b x \frac{b}{h} \cdot x \cdot dx \cdot B}{\int_0^b x \frac{b}{h} \, . \, dx}$$

$$= \frac{\left. \frac{x^3}{3} \, B \right]_0^b}{\left. \frac{x^2}{2} \right]_0^b} = \frac{2}{3} \, h \, B$$

The average actual skidding distance for a rectangle:

$$\text{Average actual haul} = \frac{\frac{2}{3} h_1 B_1 + \frac{2}{3} h_2 B_2}{2}$$

Av. actual haul $(.33 \, h_1 B_1) + (.33 \, h_2 B_2)$

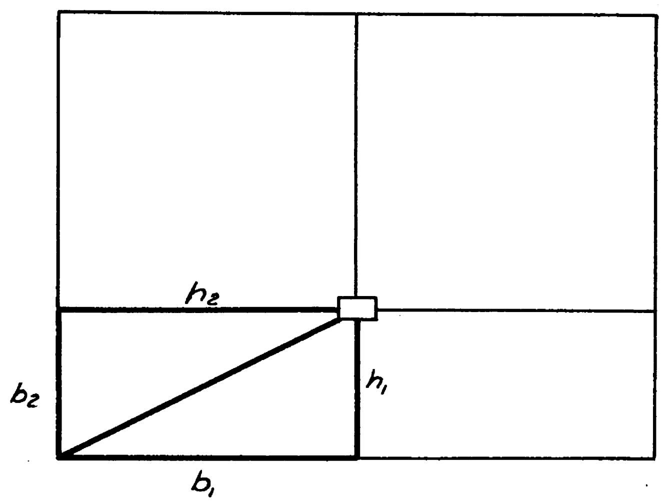



Figure 5.

Down-hill Skidding.

Down-hill skidding done from two square areas. $h_1 = h_2 = b_1 = b_2$

Average skidding distance $= \frac{2}{3} h B$

$$B = \left[ \frac{\left(1 + \frac{b^2}{b^2}\right)^{\frac{1}{2}}}{2} + \frac{b}{2b} \log_e\left(\frac{b}{b} + \left(1 + \frac{b^2}{b^2}\right)^{\frac{1}{2}}\right) \right] = \frac{1.414}{2} + \frac{1}{2} \log_e(2.41)$$

$$B = .707 + .5(.8796) = 1.147$$

Average skidding distance to landing from a square $= .67(1.147)\ h = .768\ h$

Down-hill skidding on all slopes of 25% or less is done from two square areas. For all such areas, the average actual skidding distance $= .768\ H$ where H is a side of the square.

With a 10% slope:

$H = .625\ S$

Average down-hill haul $= .625\ S \times .768 = .48\ S$

Up-hill skidding on 0% - 10% slopes is done directly to the landing, and the average skidding distance can be obtained by application of the same formula that was used for down-hill skidding.

Up-hill Skidding.

Average skidding distance $= (.33\ h_1 B_1) + (.33\ h_2 B_2)$

With a 10% slope:

$h_1 = .375\ S = b_2$

$b_1 = .625\ S = h_2$

$$B_1 = \left[ \frac{\left(1 + \frac{.625^2}{.375^2}\right)^{\frac{1}{2}}}{2} + \frac{.375}{2(.625)} \log_e\left(\left(\frac{.625}{.375}\right) + \left(1 + \frac{.625^2}{.375^2}\right)^{\frac{1}{2}}\right) \right]$$

$$B_1 = \left[\frac{(1+2.78)^{\frac{1}{2}}}{2} + .30 \log_e(1.67 + 3.78^{\frac{1}{2}})\right] = \frac{1.94}{2} + .30 \log_e(1.67 + 1.94)$$

$$B_1 = .97 + .30(1.2837) = .97 + .39 = 1.36$$

$$B_2 = \left[\frac{\left(1 + \frac{.36^2}{.62^2}\right)^{\frac{1}{2}}}{2} + \frac{.625}{2(.375)} \log_e\left(\frac{.375}{.625} + \left(1 + \frac{.375^2}{.625^2}\right)^{\frac{1}{2}}\right)\right]$$

$$B_2 = \left[\frac{(1+.36)^{\frac{1}{2}}}{2} + .83 \log_e(.60 + 1.36^{\frac{1}{2}})\right] = .58 + .83 \log_e(1.77)$$

$$B_2 = .58 + .83(.5710) = .58 + .47 = 1.06$$

Average actual skidding distance $= (.33)(.375\ S)(1.36) + (.33)(.625\ S)(1.06)$

Average up-hill haul on 10% slope $= .17\ S + .22\ S = .39\ S$

An approximate average skidding distance can be obtained without the use of calculus.

If skidding was limited to the economic yarding distance, down-hill skidding would be done from sectors of a circle, and up-hill skidding would be done from areas approaching the shape of sectors of a circle. The area of a circle varies with the square of the radius. The average skidding distance $= \sqrt{.5}$ or .707 x radius. For up-hill hauls, the av. skidding distance .707 x av. external haul.

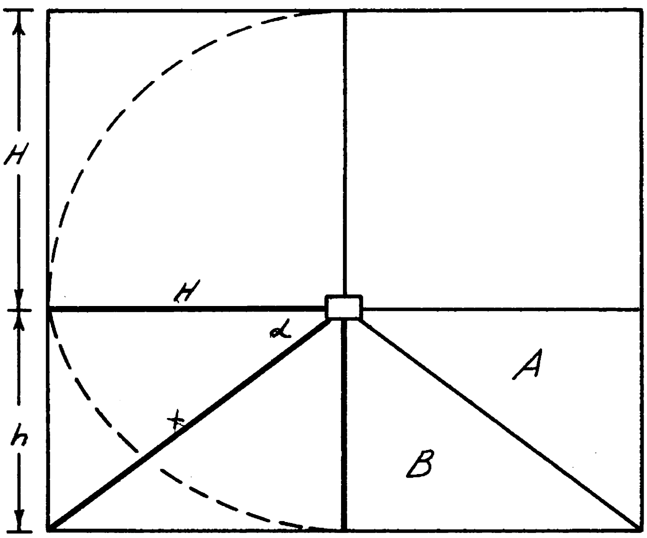



Figure 6.

Down-hill Skidding:

Down-hill skidding will be done from two squares. Average skidding distance has been determined to be .768 H for any square.

With a 10% slope:

H = .625 S    Average haul = .768(.625 S) = .480 S

Up-hill Skidding:

Up-hill skidding must be divided into two sections of equal area;

| A. | B. |
|---|---|
| Maximum haul = x | Maximum haul = x |
| Minimum haul = H | Minimum haul = h |
| Av. haul = $.707\frac{(x+H)}{2}$ | Av. haul $.707\frac{(x+h)}{2}$ |

With a 10% slope:

$$\text{Tan}\,\alpha = \frac{h}{H} = \frac{.375\ S}{.625\ S} = .600$$

$$\alpha = 30^\circ\ 58'$$

$$\text{Cos}\,\alpha = .85747 \qquad x = \frac{.625\ S}{.85747}$$

| | |
|---|---|
| H = .625 S | h = .375 S |
| x = .729 S | x = .729 S |

| Average skidding distance up-hill (Approximate) | Corresponding accurate figures. |
|---|---|
| $.707\ \frac{(.625\ S + .729\ S)}{2} = .707 \times .677\ S \times .50 = .239\ S$ | .22 S |
| $.707\ \frac{(.375\ S + .729\ S)}{2} = .707 \times .552\ S \times .50 = .195\ S$ | .17 S |
| Average haul up-hill = .434 S | .39 S |

This approximate method gives too large a figure for the average skidding distance. This will raise the cost of skidding, and the result will be a somewhat closer spacing of roads than the true figure will give. It is more costly to space roads too closely together than too far apart as is demonstrated in Figure 9, and consequently it is desirable to consider another approximate method of determining the average skidding distance which does not have this disadvantage.

The average skidding distance will closely approximate the distance to the landing from the center of gravity of any triangle being served. This is proven graphically in Figure 16 where the dotted line gives values which agree closely with the exact values of the solid line. The dotted line gives the distance to the landing from the centers of gravity of the triangles which are served on different slopes. (For further proof, see Appendix A).

The center of gravity of a right triangle is located at the point of intersection of lines which are perpendicular to the sides of a triangle at a distance from the right angle equal to one third of the lengths of the sides. The average skidding distances will be equal to the lengths of the heavy lines in Figure 7.

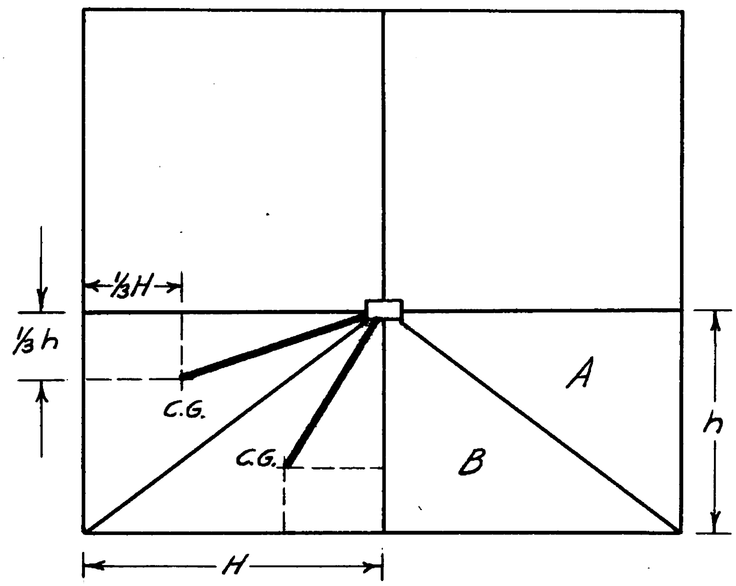



Figure 7.

Up-hill skidding must be divided into two sections of equal area. (Fig. 7.)

A

$$\text{Average haul} = \sqrt{(.67\ H)^2 + (.33\ h)^2}$$

B

$$\text{Average haul} = \sqrt{(.67\ h)^2 + (.33\ H)^2}$$

With a 10% slope:

$H = .625\ S$

$h = .375\ S$

A. $$\text{Average haul} = \sqrt{(.67 \times .625\ S)^2 + (.33 \times .375\ S)^2}$$
$$= \sqrt{(.42\ S)^2 + (.12\ S)^2} = \sqrt{.18\ S^2 + .01\ S^2} = \sqrt{.19\ S^2}$$

Average haul = $.44\ S$

B. $$\text{Average haul} = \sqrt{(.67 \times .375\ S)^2 + (.33 \times .625\ S)^2}$$
$$= \sqrt{(.25\ S)^2 + (.21\ S)^2} = \sqrt{.06\ S^2 + .04\ S^2} = \sqrt{.10\ S^2}$$

Average haul = $.32\ S$

Average up-hill skidding distance $= .5(.44\ S + .32\ S) = .38\ S$

This checks well with the accurate average up-hill skidding distance for a 10% slope of .39 S.

After the average down-hill and up-hill skidding distances have been computed, it is possible to determine the average skidding distance to a landing for the area as a whole.

With a 10% slope:

Let A = average skidding distance.

$A = .48\ S(.625) + .39\ S(.375) = .30\ S + .15\ S = .45\ S$

The cost of skidding.

Each operator must determine what his skidding costs are per MBM per 100' of distance. This information can be made to fit any particular show by determining the average slope on which up-hill skidding will be done, and converting these costs to fit the conditions.

With a 10% slope:

Maximum slope = 10%

Minimum slope = 0%

Average slope = 5%

Deduct 4% drawbar pull from tractor for every 1% of adverse slope.

Let

Cost of skidding on level = Cl = 3.4¢/M/100'

Cost of skidding on 5% slope = Cu = $\frac{3.4}{.80}$ = 4.3¢/M/100'

Where Cd = cost of skidding down-hill.

Cu = cost of skidding up-hill.

P = percent of timber skidded up-hill.

1-P = percent of timber skidded down-hill.

Average cost of skidding up-hill and down-hill = C = Cu(P) + Cd(1-P)

With a 10% slope:

Cd = 3.4¢

Cu = 4.3¢

P = .375% of area served.

1-P = .625% of area served.

C = 4.3(.375) + 3.4(.625)

C = 1.61 + 2.12

C = 3.73¢

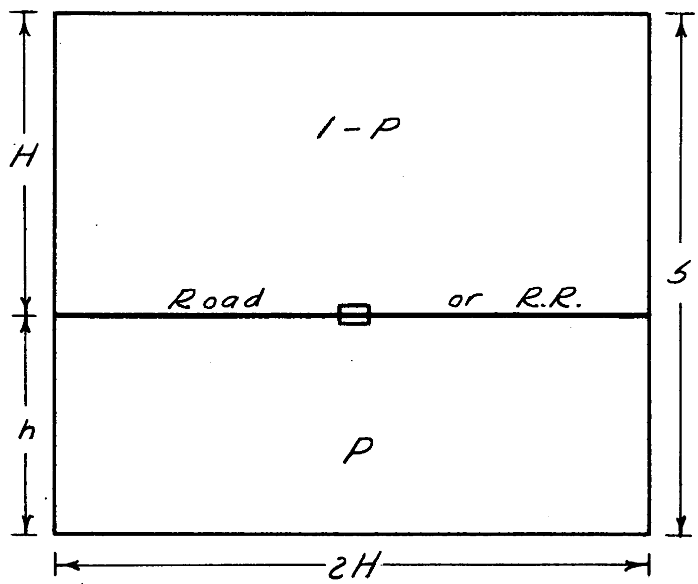



Figure 8.

Cost of road varies with 2Hr where r = cost of road per 100' station.

Area served by landing = 2HS or Area = $\frac{2HS}{4.356}$ acres.

Volume per acre = V

$$\text{Cost of road per MBM} = \frac{2Hr}{V\,\frac{2HS}{4.356}} = \frac{4.356\ r}{VS}$$

As timber is skidded greater distances, roads must be spaced farther apart. This means that as the cost of skidding rises due to increased hauling distance, the cost of road construction falls.

Figure 9 demonstrates that the most economic spacing of roads is obtained when the cost of skidding just equals the cost of road construction. The reason for this is that as road spacing is increased from a very narrow figure, road costs fall faster than skidding costs increase. Conversely, a similar increase in an already wide spacing of roads will reduce the cost of road construction only slightly whereas the cost of skidding will mount at a constant rate. Where these two values cross, the tangent of the road cost curve is equal and opposite in slope to the skidding cost value.

Example:

What is the most economical road spacing to use when

| | |
|---|---|
| Slope = 10% | A = .45 S |
| r = $150 | V = 30 M |
| C = 3.7¢ | S = ? |

| Road Spacing | Cost of Skidding A x C | Cost of Roads $\frac{4.356\ r}{VS}$ | Total Cost per MBM |
|---|---|---|---|
| 500' | $ .08 | $4.35 | $4.43 |
| 1000 | .17 | 2.18 | 2.35 |
| 1500 | .25 | 1.45 | 1.70 |
| 2500 | .42 | .87 | 1.29 |
| 3500 | .58 | .62 | 1.20 |
| 4500 | .75 | .48 | 1.23 |
| 5500 | .92 | .40 | 1.32 |
| 6500 | 1.08 | .34 | 1.42 |

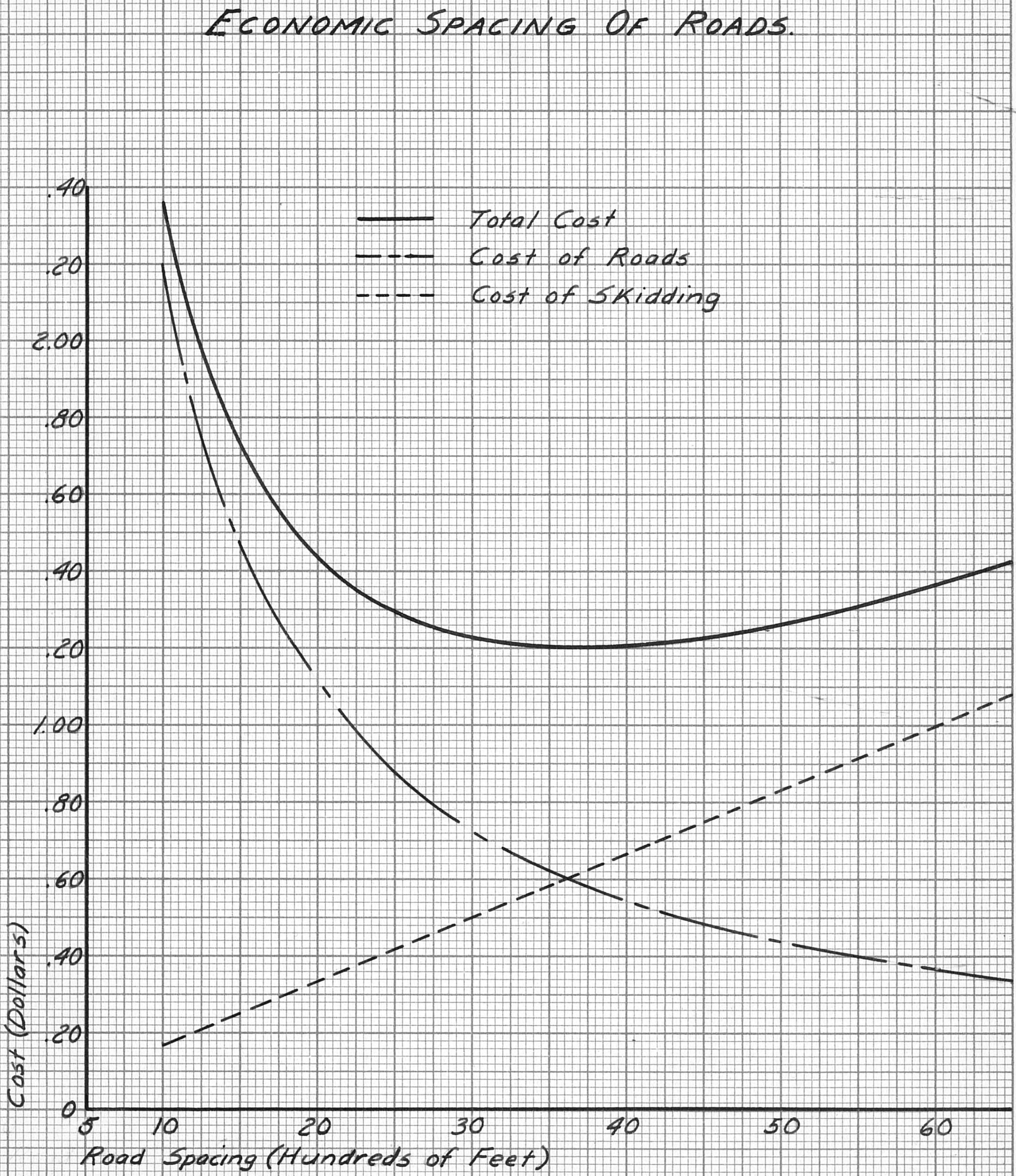

ECONOMIC SPACING OF ROADS.
Total Cost
Cost of Roads
Cost of Skidding
.40
.20
2.00
.80
.60
.40
.20
1.00
.80
.60
.40
.20
0
Cost (Dollars)
5
10
20
30
40
50
60
Road Spacing (Hundreds of Feet)

Figure 9.

Figure 9 shows that roads can be spaced 3000'- 4500' apart without much variation in total road and skidding costs.

The very gradual increase in total cost when timber is skidded beyond the economic skidding distance shows why it is desirable to skid timber to a landing from a rectangular area even though part of the area lies beyond the economic skidding distance (Figure 3). If hauling is to be restricted to the economic skidding distance, one of two choices must be made. Either the road spacing can be kept constant and the islands of timber beyond the economic reach of the tractors left standing, or the road spacing can be reduced so that all the area can be covered without exceeding the economic skidding distance. Either of these plans is considerably more costly than is the case where tractors are operated to their full economic skidding distance both on the level hauls and on the hauls directly up and down the slopes involved.

It has been shown that the economic spacing of roads is obtained when the cost of skidding equals the cost of road construction.

A = Average skidding distance.

C = Average skidding cost/M/100'

Cost of road construction per M $= \frac{4.356\ r}{VS}$

$$AC = \frac{4.356\ r}{VS} \qquad \text{or} \qquad AS = \frac{4.356\ r}{VC}$$

With a 10% slope:

$A = .45\ S$ $\qquad AS = \frac{4.356\ r}{VC}$

$V = 30\ M$

$r = \$150$ or $15{,}000¢$ $\qquad .45\ S^2 = \frac{4.356\ r}{VC}$

$C = 3.7¢$

For a 10% slope, $\quad S = \sqrt{\frac{9.68\ r}{VC}}$

$$S = \sqrt{\frac{9.68 \times 15{,}000}{30 \times 3.7}} = \sqrt{1310} \quad \text{or} \quad S = 36.2 \text{ hundred ft.}$$

Economic road spacing = 3,620 feet.

Skid down-hill 3,620 x .625 = 2,260 feet.

Skid up-hill 3,620 x .375 = 1,360 feet.

Space landings 2 x 2,260 = 4,520 feet apart.

Cost of skidding = (.45 x 36.2) x 3.7 = 60¢ per M

Cost of roads $= \frac{4.356 \times 15{,}000}{30 \times 36.2} =$ 60¢ per M

Total = 120¢ per M

This checks with the graphical solution in Figure 9.

This can be compared with the cost of skidding down-hill only. When all the timber is skidded down-hill, the cost of skidding is reduced, but the area served by a landing is also decreased.

D. M. Matthews has shown that for one way skidding

$$S = \sqrt{\frac{5.11\ r}{VC}} = \sqrt{\frac{5.11 \times 15{,}000}{30 \times 3.4}} = \sqrt{752} = 27.5 \text{ hundred feet.}$$

$$\text{Cost of skidding} = \frac{.707C(S + \sqrt{2\ S^2})}{2} = \frac{.707 \times 3.4(27.5 + \sqrt{2 \times 27.5^2})}{2} = 79¢/M$$

$$\text{Cost of roads} = \frac{4.356\ r}{VS} = \frac{4.356 \times 15{,}000}{30 \times 27.5} = 79¢/M$$

Total = 158¢/M

A savings in skidding and road costs of 24% can be realized on a 10% slope by skidding to a landing from both down-hill and up-hill directions.

# Skidding Up-hill and Down-hill

## With Tractors And Arches On Slopes

## 15% - 25%

Skidding On Slopes 15% - 25%

Skidding up-hill and down-hill with tractors and arches on slopes which exceed the maximum adverse grade the machine is capable of negotiating directly.

Tractors with arches can skid directly downhill on 25% slopes. On slopes of 15% - 25%, up-hill hauling must be done at an angle to the contours, so that skidding will be confined to slopes of 10% or less.

Demonstrating with a 25% slope;

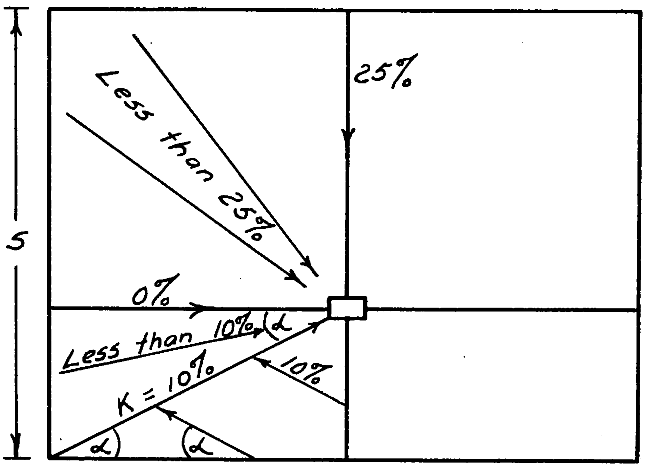



Figure 10.

Direct skidding used on all down-hill hauls. Direct skidding used for up-hill skidding above diagonal k. Indirect skidding used for up-hill skidding below diagonal k.

With a 25% slope;

Allowable grade = 10%. Must travel 250' to rise 25'.

Actual grade = 25%. Must travel 100' to rise 25'.

$$\sin \alpha = \frac{\text{allowable grade \%}}{\text{actual grade \%}}$$

$$\sin \alpha = \frac{10}{25} = .4000 \qquad \alpha = 23^\circ\ 35'$$

$\alpha$ = angle to contours at which up-hill skidding must be done for area below diagonal k.

Down-hill skidding will be done from a rectangular area which is twice as wide as it is deep, and with $\alpha$ known, it is possible to ascertain the shape of an area being served by a landing. With this information, the percentages of timber skidded up-hill and down-hill can be determined.

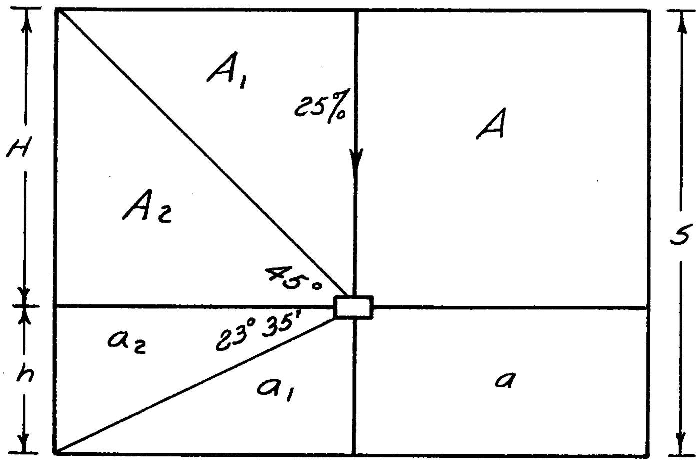



Figure 11. Skidding on 25% slope.

With a 25% slope:

Let H = 10

$h = \tan 23^\circ 35' \times 10 = .4365 \times 10 = 4.3654$

Area A = 10 x 10 = 100.0 $\quad \frac{100}{143.7} = 70\%$ of timber

Area a = 10 x 4.3654 = 43.7 $\quad = 30\%$ of timber

Total: 143.7

$S = H + h \qquad H = .70\ S$

$\qquad\qquad\quad h = .30\ S$

Down-hill skidding.

On slopes of 15% - 25%, down-hill skidding will be done directly to the landing from a rectangular area composed of two squates. It has been shown that the average skidding distance from a square = .768 H.

On 25% slope, H = .70 S Av. down-hill haul = .768(.70 S) = .54 S

Up-hill Skidding:

For the half of the up-hill skidding where the slope is 10% or less, the average skidding distance $= \sqrt{(.67\ H)^2 + (.33\ h)^2}$

With a 25% slope:

$H = .70\ S$ Average haul $= \sqrt{(.67 \times .70\ S)^2 + (.33 \times .30\ S)^2}$

$h = .30\ S$ $= \sqrt{(.47\ S)^2 + (.10\ S)^2}$

Average haul $= .48\ S$

For the half of the up-hill skidding where the slope exceeds 10%, the average skidding distance can be determined as follows;

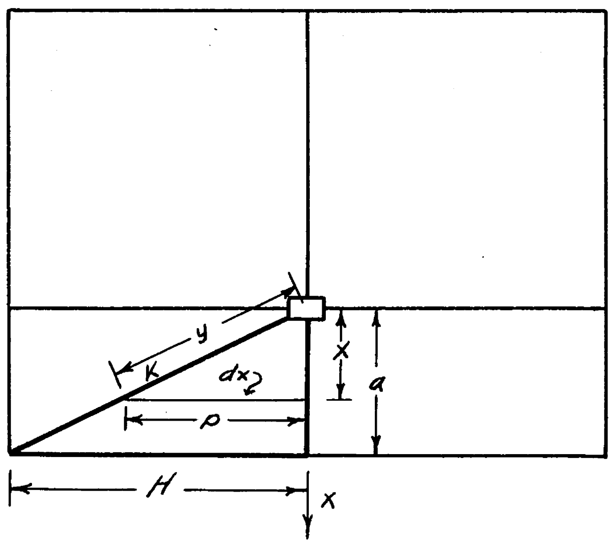



Figure 12.

The hauling distance to the landing from any point on p dx = y. This may be observed in fig. 12. The hauling distance from any point on p dx to the diagonal k makes an isosceles triangle with the diagonal.

$$\frac{a}{H} = \frac{x}{p} \qquad \text{or} \qquad p = \frac{H}{a}\,x$$

$$y = \sqrt{p^2 + x^2}$$

$$\text{Average skidding distance} = \frac{\int p \cdot dx \cdot y}{\int p \cdot dx}$$

$$= \frac{\int_0^a \frac{H}{a} \cdot x \cdot \left(\frac{H^2}{a^2}\, x^2 + x^2\right)^{\frac{1}{2}}\, dx}{\int_0^a \frac{H}{a}\, x \cdot dx} = \frac{\int_0^a \frac{H}{a}\, x^2 \left(\frac{H^2}{a^2}+1\right)^{\frac{1}{2}}\, dx}{\int_0^a \frac{H}{a}\, x \cdot dx}$$

$$= \frac{\frac{H}{a}\, \frac{x^3}{3} \left(\frac{H^2}{a^2}+1\right)^{\frac{1}{2}}\Big]_0^a}{\frac{H}{a} \cdot \frac{x^2}{2}\Big]_0^a} = \frac{\frac{H}{a} \cdot \frac{a^3}{3} \left(\frac{H^2}{a^2}+1\right)^{\frac{1}{2}} - 0}{\frac{H}{a} \cdot \frac{a^2}{2} - 0}$$

$$= \frac{2}{3}\, a \left(\frac{H^2}{a^2}+1\right)^{\frac{1}{2}}\Big] = \frac{2}{3}\, a \frac{(H^2+a^2)^{\frac{1}{2}}}{a^2} = \frac{2}{3}\,(H^2+a^2)^{\frac{1}{2}}$$

Average skidding distance = $\frac{2}{3}$ times the length of the diagonal k.

(Geometric solution Appendix A).
With a 25% slope:

$H = .70\ S$

$h = .30\ S$

$$\text{Average haul} = \frac{2}{3} \sqrt{(.70\ S)^2 + (.30\ S)^2}$$

$$\text{Average haul} = \frac{2}{3} \sqrt{.49\ S^2 + .09\ S^2} = .51\ S$$

While it is necessary for loaded tractors to skid uphill indirectly where slopes exceed 10%, the unloaded tractors can return to the woods by the most direct route.

$$\text{Average distance} = \sqrt{\left(\frac{2}{3}\, h\right)^2 + \left(\frac{1}{3}\, H\right)^2}$$

With a 25% slope:

$H = .70\ S$

$h = .30\ S$

$$\text{Average distance} = \sqrt{(.67 \times .30\ S)^2 + (.33 \times .70\ S)^2}$$
$$= \sqrt{(.20\ S)^2 + (.23\ S)^2} = .30\ S$$

Average skidding distance up-hill where the slope exceeds 10% $= \dfrac{.51\ S + .30\ S}{2} = .41\ S$

Both halves of up-hill skidding have equal weights.

Average hauling distance for all up-hill skidding $= \frac{.48\ S + .41\ S}{2}$

Average up-hill skidding distance = .44 S on a 25% slope.

A = Average skidding distance for up and down-hill skidding.

With a 25% slope:

$A = .54\ S(.70) + .44\ S(.30) = .38\ S + .13\ S \qquad A = .51\ S$

Cost of skidding.

Down-hill skidding costs are here considered to equal costs of skidding on the level.

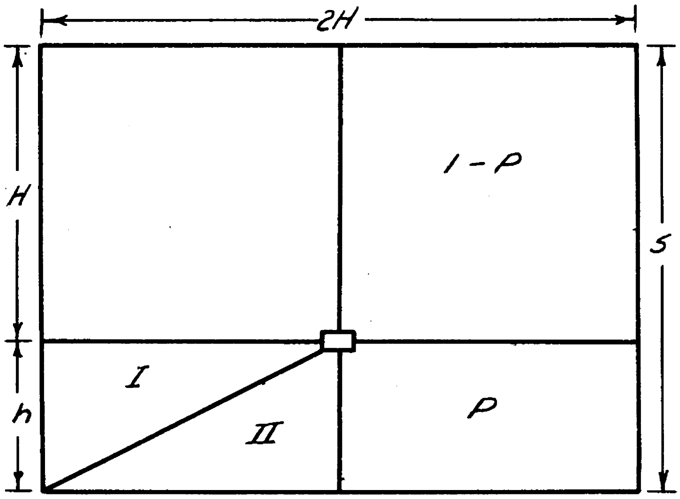



Figure 13.

Skidding in area I varies from 0% - 10% and averages 5%.

All skidding in area II is done on a 10% slope.

When the cost of skidding on the level is 3.4¢/M/100',

| | | |
|---|---|---|
| 10% adverse slope | 3.4¢/.60 = 5.67¢ x .50 = | 2.84¢ |
| 5% adverse slope | 3.4¢/.80 = 4.25¢ x .50 = | 2.12¢ |
| | | 4.96¢ |

Average cost of up-hill skidding = 5.0¢/M/100'

Cost of down-hill skidding = 3.4¢/M/100'

Where $Cd =$ Cost of skidding downhill.

$Cu =$ Cost of skidding up-hill.

$P =$ Percent of timber skidded up-hill.

$1 - P =$ Percent of timber skidded down-hill.

Average cost of all skidding = $Cu\,(P) + Cd\,(1-P)$

With a 25% slope:

$Cd = 3.4¢$

$Cu = 5.0¢$

$P = 30\%$

$1-P = 70\%$

Average cost of all skidding = $5.0(.30) + 3.4(.70)$

Average skidding cost = 3.9¢/M/100'.

Cost of road = 2Hr when r = cost of road per 100' station.

Area served by landing = $\frac{2HS}{4.356}$ acres.

V = volume per acre.

$$\text{Cost of road per MBM} = \frac{2\,Hr}{V\,\frac{2\,HS}{4.356}} = \frac{4.356\,r}{V\,S}$$

A = Average skidding distance

C = Average skidding cost per MBM per 100'.

For economic spacing of roads, road costs should equal skidding costs.

$$A\,C = \frac{4.356\,r}{V\,S} \qquad \text{or} \qquad A\,S = \frac{4.356\,r}{V\,C}$$

With a 25% slope:

$A = .51\,S$

$V = 30\ M$

$r = \$150$ or $15000¢$

$C = 3.9¢$

$$.51\,S^2 = \frac{4.356\,r}{V\,C} \qquad S = \sqrt{\frac{8.54\,r}{VC}}$$

$$S = \sqrt{\frac{8.54 \times 15,000}{30 \times 3.9}} \qquad S = \sqrt{1095}$$

S = 33 hundred feet.

Space roads = 3300 feet apart.

Space landings $2(.70 \times 33) \approx 4600$ feet apart.

Economic Spacing of Roads gives the following costs;

With a 25% slope:

$r = 15{,}000¢$

$V = 30\ M$ Skidding cost $= AC = (.51 \times 33)3.9 = 66¢/M$

$C = 3.9¢$ Road cost $= \frac{2Hr\ 4.356}{V\ 2H\ S} = \frac{4.356 \times 15{,}000}{30 \times 33} = 66¢/M$

$H = .70\ S$

Total $= 132¢/M$

$A = .51\ S$

$S = 33$ hundred ft.

This can be compared to corresponding costs involved when all of the timber is skidded down-hill to the landing. The lower skidding costs are more than offset by the smaller area served by a landing.

$r = 15{,}000¢$

$V = 30\ M$ $S = \sqrt{\frac{5.11\ r}{V\ C}}$ Proven by D. M. Matthews.

$C = 3.4¢$

$$S = \sqrt{\frac{5.11 \times 15{,}000}{30 \times 3.4}} = \sqrt{752} = 27.5 \text{ hundred feet.}$$

$$\text{Cost of skidding} = \frac{.707\ C(S+\sqrt{2S^2})}{2} = \frac{.707 \times 3.4(27.5+\sqrt{2 \times 27.5^2}}{2} = 79¢/M$$

$$\text{Cost of roads} = \frac{4.356\ r}{VS} = \frac{4.356 \times 15{,}000}{30 \times 27.5} = 79¢/M$$

Total $= 158¢/M$

By skidding both up and down-hill to a landing, a saving of 16% can be realized on skidding and road costs on a 25% slope.

Skidding Up-hill and Down-hill

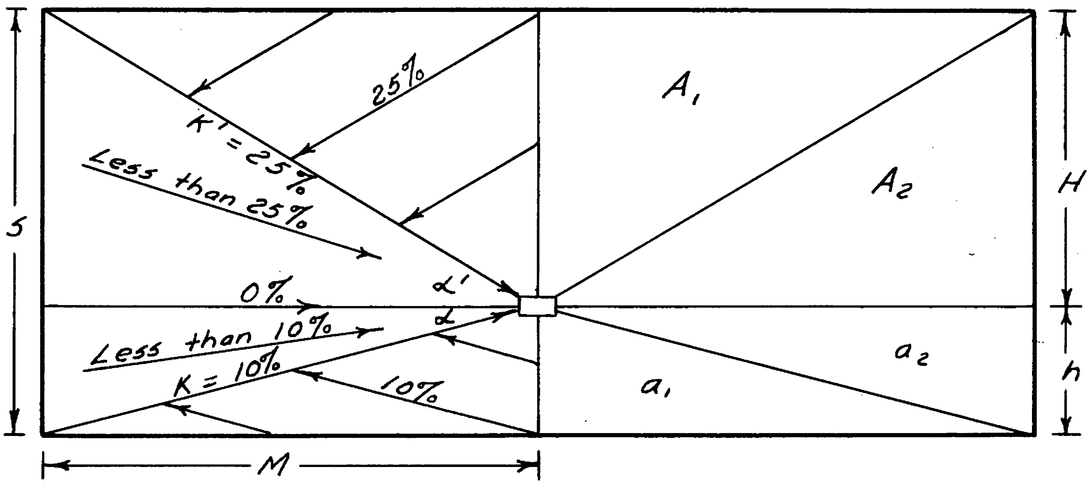
With Tractors And Arches On Slopes

30% - 60%

## Slopes 30% - 60%.

Skidding up-hill and down-hill with tractors and arches on slopes which exceed the maximum grade a loaded tractor can descend directly and up which an unloaded tractor must climb at an angle.

Tractors equipped with arches can skid a full load directly down a maximum slope of 25%. This also represents the maximum slope an unloaded tractor can climb directly. It is necessary to employ indirect skidding for both down-hill and up-hill hauls where slopes exceed 25%.




Skidding on 60% slope.

Figure 14.

$\sin\alpha = \frac{10}{60} = .167$ $\alpha = 9^\circ\ 36'$

$\sin\alpha' = \frac{25}{60} = .417$ $\alpha' = 24^\circ\ 39'$

With a 60% slope:

Let $M = 10.00$

Area $A = A_1 + A_2 = HM$

$\text{Tan}\,\alpha' = \frac{H}{M}$ or $H = \text{Tan}\,\alpha' \times M$

$H = .4589 \times 10.00 = 4.59$

Area $A = 45.9$

Area $a = a_1 + a_2 = hM$

$\text{Tan}\,\alpha = \frac{h}{M}$

$h = .1691 \times 10.00 = 1.69$

Area $a = 16.9$

Area $A = 45.9$

Total $= 62.8$

$A$ = Down-hill skidding $= \frac{45.9}{62.8} = 73\%$

$a$ = Up-hill skidding $= 27\%$

$S = H + h$

$\text{Tan}\,\alpha = \frac{h}{M}$ or $M = \frac{h}{\text{Tan}\,\alpha}$

$M = \frac{.27\ S}{.169} = 1.60\ S$

$H = .73\ S$

$h = .27\ S$

$M = 1.60\ S$

Down-hill skidding:

Tractors must climb and descent as indicated in Fig. 14.

$A_1$ Indirect skidding. Average haul $= .67\sqrt{H^2 + M^2}$

$A_2$ Direct skidding. Average haul $= \sqrt{(.33H)^2 + (.67\ M)^2}$

Average down-hill skidding distance $= \frac{1}{2}(A_1 + A_2)$

Up-hill skidding:

All hauls to the landing will be done as indicated in Fig. 14. However, return trips to the woods can be made directly to any point as an unloaded tractor can safely descend steep slopes.

$a_1$ { Indirect skidding to landing. Av. haul $= .67\sqrt{h^2 + M^2}$
Direct skidding to woods. Av. haul $= \sqrt{(.67h)^2 + (.33\ M)^2}$ }

$a_1$ Indirect skidding. Av. Haul $= \frac{.67\sqrt{h^2+M^2} + \sqrt{(.67\ h)^2 + (.33\ M)^2}}{2}$

$a_2$ Direct skidding. Av. Haul $= \sqrt{(.33\ h)^2 + (.67\ M)^2}$

Average up-hill skidding distance $\frac{1}{2}(a_1 + a_2)$

With a 60% slope:

Down-hill skidding.

$$\text{Average haul} = \frac{.67\sqrt{(.73\ S)^2 + (1.60\ S)^2} + \sqrt{(.33 \times .73\ S)^2 + (.67 \times 1.60\ S)^2}}{2}$$

$$\text{Average haul} = \frac{.67\sqrt{3.09\ S^2} + \sqrt{.06\ S^2 + 1.14\ S^2}}{2} = \frac{1.18\ S + 1.10\ S}{2}$$

Average down-hill skidding distance = 1.14 S

Up-hill skidding.

$$\text{Average haul} = \frac{.5\left(.67\sqrt{h^2 + M^2} + \sqrt{(.67\ h)^2 + (.33\ M)^2}\right) + \sqrt{(.33\ h)^2 + (.67\ M)^2}}{2}$$

$$= \frac{.5(.67\sqrt{(.27\ S)^2 + (1.60\ S)^2} + \sqrt{(.67 \times .27\ S)^2 + (.33 \times 1.60\ S)^2}}{2}$$

$$+ \frac{\sqrt{(.33 \times .27\ S)^2 + (.67 \times 1.60\ S)^2}}{2}$$

$$= \frac{.5(.67\sqrt{2.63\ S^2} + \sqrt{.03\ S^2 + .28\ S^2}) + \sqrt{.01\ S^2 + 1.14\ S^2}}{2}$$

$$= \frac{.5(1.09\ S + .56\ S) + 1.07\ S}{2} = \frac{.82\ S + 1.07\ S}{2} = .95\ S$$

Average up-hill skidding distance = .95 S

A = Average skidding distance up-hill and down-hill.

A = .73(1.14 S) + .27(.95 S) = .83 S + .26 S

A = 1.09 S on a 60 % slope.

Cost of skidding:

It has previously been determined that when the cost of skidding on the level is 3.4¢/M/100',

Average cost of up-hill skidding = 5.0¢/M/100'

Cost of down-hill skidding = 3.4¢/M/100'

Average cost of all skidding = Cu(P) + Cd(1-P)

60% slope P = .27

1-P = .73 Average cost of skidding = 5.0(.27) + 3.4(.73)

Cu = 5.0¢ Average skidding cost 3.8¢/M/100'.

Cd = 3.4¢

Economic Spacing of Roads.

For economic spacing of roads, road costs should equal skidding costs.

Cost of road = 2 Mr where r cost of road per 100' station.

Area served by landing = $\frac{2\ MS}{4.356}$ acres.

$$\text{Cost of road per MBM} = \frac{2\ Mr}{V\ \frac{2\ MS}{4.356}} = \frac{4.356\ r}{VS}$$

Skidding costs = AC

$$AC = \frac{4.356\ r}{VS} \quad \text{or} \quad AS = \frac{4.356\ r}{VC}$$

With a 60% slope:

$A = 1.09\ S$ $\qquad S = \sqrt{\frac{4.356\ r}{1.09\ VC}}$ $\qquad S = \sqrt{\frac{4.00\ r}{VC}}$

$V = 30\ M$

$r = 15{,}000¢$ $\qquad S = \sqrt{\frac{4.00 \times 15{,}000}{30 \times 3.8}}$ $\qquad S = \sqrt{527}$

$C = 3.8¢$ $\qquad S =$ 23 hundred feet.

Space roads 23 hundred feet apart.

Space landings $2 \times 1.60 \times 23 =$ 74 hundred feet apart.

Skidding costs = $AC = (1.09 \times 23)3.8 = 95¢/M$

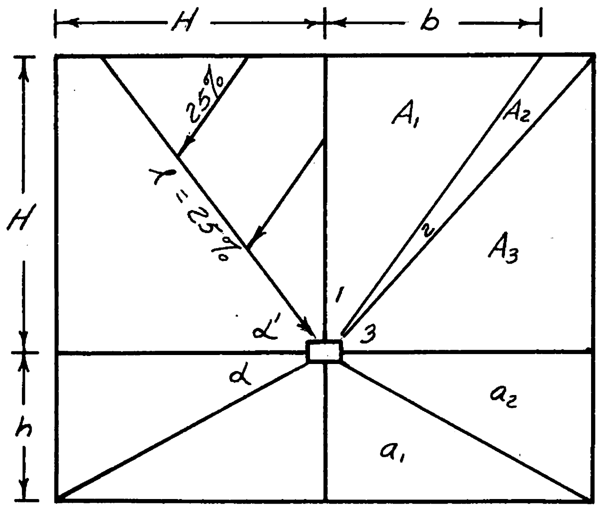
Costs of roads $= \frac{4.356\ r}{VS} = \frac{4.356 \times 15{,}000}{30 \times 23} = 95¢/M$

The formula $S = \sqrt{\frac{5.11\ r}{VC}}$ is true for direct skidding only. On slopes of 30%-60%, indirect skidding is necessary and therefore no comparisons can be made of relative costs.

Special Case - 30% Slope.

The solution used to find the economic road spacing on a 60% slope is applicable to all slopes of 35% - 60%. To determine the economic skidding distance on a 30% slope, one further calculation is required.

A 30% slope represents a special case, because on this grade, $\alpha'$ is greater than 45°.




$$\sin \alpha' = \frac{25}{30} = .834$$

$$\alpha' = 56°\ 25'$$

$$\beta' = \angle 1 = 33°\ 35'$$

$$\alpha' = \begin{cases} \angle 2 = 11°\ 25' \\ \angle 3 = 45°\ 00' \end{cases}$$

Figure 15.

Skidding on a 30% slope.

When $\alpha'$ is greater than 45°, 1 does not divide the down-hill skidding area into component triangles. This must be done by adding the diagonal k' and computing the average skidding distance for each of the triangles and giving them their proper weights.

$$\sin \alpha = \frac{10}{30} = .333$$

$$\alpha = 19°\ 27'$$

$$\tan \alpha = .3531$$

Let H = 10.00

h = 3.53

Area a = 10 x 3.531 = 35.31

Area A = 10 x 10 = 100.00

A + a = 135.31

S = h + H

$h = \frac{3.531}{13.531}$ .26S  a = 26% of timber.

H = .74S  A = 74% of timber.

Down-hill skidding:

Area $A = A_1 + A_2 + A_3 = 100$

Angle 1 (Figure 15.) = 33° 35'

$A_1$ $\quad \sin 33° 35' = \frac{b}{H}$ $\quad$ Let $H = 10$

$b = .553 \times 10 = 5.531$ $\quad \frac{1}{2}(10 \times 5.531) =$ Area $A_1 = 27.7$

$A_2$ $\quad$ Area $A_2 = \frac{10 \times 10}{2} = 50$ $\quad$ Area $A_2 = 50.0$

$A_3$ $\quad$ Area $A_3 = 100 - 77.7$ $\quad$ Area $A_3 = 22.3$

$A_1$ $\quad$ Average haul $= .67 \sqrt{H^2 + b^2}$ $\quad H = .74\ S$ $\quad b = .55\ H = .41\ S$

Average haul $= .67 \sqrt{(.74\ S)^2 + (.41\ S)^2} = .67 \sqrt{.72\ S^2} = .57\ S$

$A_2$ $\quad .768\ H = .768\ (.74\ S) = .57\ S =$ Average haul

$A_3$ $\quad$ Maximum haul $= 1.41\ H = 1.41\ (.74\ S) = 1.04\ S$

Minimum haul $= l =$

$\cos 33° 35' = \frac{H}{l}$ $\quad l = 1.20\ H$

$.8331 = \frac{10}{l}$ $\quad l = 1.20 \times .74\ S = .89\ S =$ Min. haul.

$$\text{Average haul} = \frac{.707(.89\ S + 1.04\ S)}{2} = \frac{.707(1.93\ S)}{2} = .68\ S$$

Average down-hill haul $\quad .57S(.28) + .57S(.50) + .68S(.22) = .60S$

Up-hill skidding:

Average actual skidding distance $\frac{1}{2}(a_1 + a_2)$

$$a_1 = \frac{.67\sqrt{H^2 + h^2} + \sqrt{(.67\ h)^2 + (.33\ H)^2}}{2}$$

$$a_2 = \sqrt{(.67\ H)^2 + (.33\ h)^2}$$

$$\text{Average haul} = \frac{.5\left(.67\sqrt{(.74S)^2 + (.26S)^2} + \sqrt{(.67 \times .26S)^2 + (.33 \times .74S)^2}\right)}{2} + \frac{\sqrt{(.67 \times .74S)^2 + (.33 \times .26S)^2}}{2}$$

Average up-hill skidding distance $= .47\ S$

Average skidding distance for whole area A.

$A = .26(.47S) + .74(.60S)$ $\quad$ or $\quad A = .56\ S$

Cost of road per MBM $= \frac{4.356\ r}{VS}$

Cost of skidding $=$ AC

Economic spacing of roads on 30% slope:

$$AC = \frac{4.356\ r}{VS} \qquad \text{or} \qquad AS = \frac{4.356\ r}{VC}$$

$$A = .56\ S \qquad\qquad .56\ S^2 = \frac{4.456\ r}{VC}$$

$$S = \sqrt{\frac{4.356\ r}{.56\ VC}} \qquad\qquad S = \sqrt{\frac{7.78\ r}{VC}}$$

Summary Of Data For Specific Slopes.

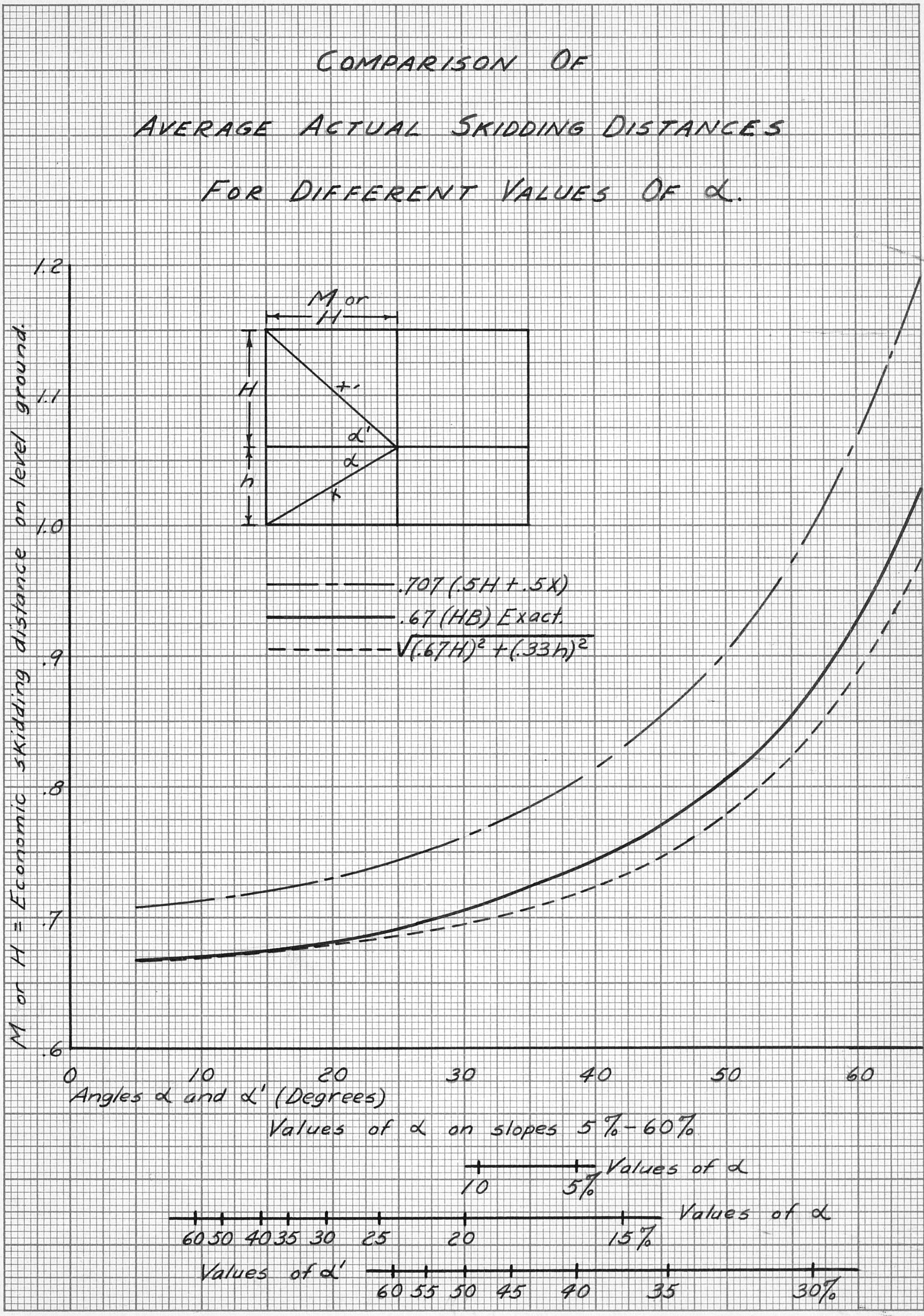

COMPARISON OF
AVERAGE ACTUAL SKIDDING DISTANCES
FOR DIFFERENT VALUES OF α.
M or H
H
h
α'
α
.707 (.5H + .5X)
.67 (HB) Exact.
√((.67H)² + (.33h)²)
M or H = Economic skidding distance on level ground.
1.2
1.1
1.0
.9
.8
.7
.6
0
10
20
30
40
50
60
Angles α and α' (Degrees)
Values of α on slopes 5% - 60%
Values of α
10
5%
Values of α
60 50 40 35 30 25 20 15%
Values of α'
60 55 50 45 40 35 30%

Figure 16.

TABLE I.

Data for Economic Spacing of Roads on Specific Slopes.

| % Slope | $\alpha$ ° ' | $\beta$ ° ' | $\alpha'$ ° ' | $\beta'$ ° ' | h | H | M | Av. Haul Up-hill. | Av. Haul Down-hill. | Av. Skidding. Dist. | Timber skidded Down-hill. | Economic road spacing. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 45°00' | 45°00' | 45°00' | 45°00' | .50S | .50S | - | - | - | .38S | - | $\sqrt{\frac{11.46\ r}{VC}}$ |
| 5 | 38 40 | 51 20 | 45 00 | 45 00 | .44S | .56S | - | .38S | .43S | .41S | 56% | $\sqrt{\frac{10.62\ r}{VC}}$ |
| 10 | 30 58 | 59 02 | 45 00 | 45 00 | .38S | .62S | - | .39S | .48S | .45S | 62% | $\sqrt{\frac{9.68\ r}{VC}}$ |
| 15 | 41 50 | 48 10 | 45 00 | 45 00 | .47S | .53S | - | .40S | .41S | .41S | 53% | $\sqrt{\frac{10.62\ r}{VC}}$ |
| 20 | 30 00 | 60 00 | 45 00 | 45 00 | .37S | .63S | - | .43S | .48S | .46S | 63% | $\sqrt{\frac{9.47\ r}{VC}}$ |
| 25 | 23 35 | 66 25 | 45 00 | 45 00 | .30S | .70S | - | .44S | .54S | .51S | 70% | $\sqrt{\frac{8.54\ r}{VC}}$ |
| 30 | 19 27 | 70 33 | 56 25 | 33 35 | .26S | .74S | - | .47S | .60S | .56S | 74% | $\sqrt{\frac{7.78\ r}{VC}}$ |
| 35 | 16 37 | 73 23 | 45 38 | 44 22 | .23S | .77S | .77S | .47S | .66S | .62S | 77% | $\sqrt{\frac{7.03\ r}{VC}}$ |
| 40 | 14 29 | 75 31 | 38 41 | 51 19 | .24S | .76S | .94S | .57S | .75S | .70S | 76% | $\sqrt{\frac{6.22\ r}{VC}}$ |
| 45 | 12 50 | 77 10 | 33 47 | 56 13 | .25S | .75S | 1.11S | .67S | .85S | .81S | 75% | $\sqrt{\frac{5.38\ r}{VC}}$ |
| 50 | 11 32 | 78 28 | 30 00 | 60 00 | .26S | .74S | 1.28S | .77S | .95S | .90S | 74% | $\sqrt{\frac{4.84\ r}{VC}}$ |
| 55 | 10 29 | 79 31 | 27 04 | 62 56 | .27S | .73S | 1.43S | .85S | 1.04S | .99S | 73% | $\sqrt{\frac{4.40\ r}{VC}}$ |
| 60 | 9 36 | 80 24 | 24 39 | 65 21 | .27S | .73S | 1.60S | .95S | 1.14S | 1.09S | 73% | $\sqrt{\frac{4.00\ r}{VC}}$ |

See next page.

In many cases, the spacing figure obtained for a given set of costs will not change much with different slopes. However, the percentage of timber skidded down-hill will vary greatly, and the spacing formula, along with the information given in Table I, is needed in determining the best location of a spur within a skidding area.

# Part II.

## Practical Aspects of Planning An Economic Logging Transportation System.

## Practical Aspects of Planning
## An Economic Logging Transportation System.

In planning a transportation system for a logging operation, a compromise must be made between what is theoretically desirable and what can actually be accomplished. Limitations in equipment and variations in topography are apt to require many modifications of the material presented in Part I, but the underlying principles need not be abandoned.

Flexibility and low construction costs are both of great value in planning an economic transportation system. A railroad, with its relatively flat grade must be fitted closely to the topography in mountainous country, and the opportunity for locating the grade in the most desirable place is greatly reduced.

The high construction costs call for long skidding distances and wide spacings of landings for which the topography, if irregular, may not be suited. These difficulties are considerably lessened with truck transportation because of the feasibility of steeper grades and the lower costs of road construction.

In so far as topography will permit, the road system will follow the plan shown in Figure 17.

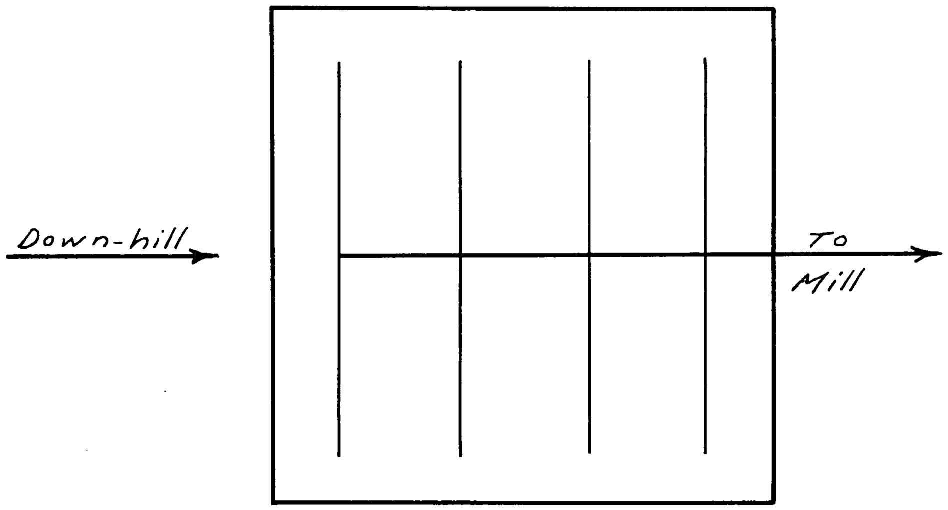



Figure 17. Schematic Plan of Road System.

For purposes of demonstration, a road system is planned for a section of mountainous topography. While certain advantages could be realized by using a larger sample, an area of this size is large enough to demonstrate the technique of laying out a transportation system. Every logging operation will present individual problems, and these must be met as they arise.

The demonstration area being used is Section 28, Township 4 N., Range 18 E., Mount Diablo Meridian. A topographic map along with pertinent data are presented on page 44. It is suggested that this map be left unfolded for reference to the material presented in the following pages.

Skidding is to be done by tractors and arches to landings. Logs are to be transported by truck to the mill which is located several miles west of Section 28.

## Location of Main Woods Road.

The Tuolumne River divides the section into two portions. No roads will be proposed for the area north of the river as this tract is an integral part of a separate transportation system.

The remainder of the section has a northwest aspect and it is apparent that the main woods road should follow the creek canyon which runs diagonally across the section. The maximum allowable favorable slope of 15% is to be avoided as much as possible. The proposed road location lies on a slope of 10%. This is entirely satisfactory and the main woods road is plotted, as proposed, on the transparent sheet on page 43 which can now be superimposed on the map.

It is obvious that the main woods road should enter the section at a distance from the river which can be determined by use of the proper spacing formula.

The slope for some distance south of the river averages 25%.

From Table I, $S = \sqrt{\frac{8.54\ r}{VC}}$ 70% of timber skidded down-hill.

When the cost of skidding on level ground is 3.5¢/M/100',

Up-hill skidding.

10% adverse grade 3.5¢/.60 = 5.83¢ x .50 = 2.92¢/M/100'

5% adverse grade 3.5¢/.60 = 4.37¢ x .50 = 2.18¢

Average cost of up-hill skidding = 5.10¢/M/100'.

Down-hill skidding. Average cost of skidding = 3.5 ¢/M/100'.

$C = Cu(P) + Cd(1-P)$ or $C = 5.1(.30) + 3.5(.70) = .153 + .245$

$C = 4.0¢/M/100'$ $S = \sqrt{\frac{8.45 \times 4500}{40 \times 4.0}} = \sqrt{238} = 15.5$ hundred ft.

$r = 4500¢$

$V = 40$ MBM $S = 23$ chains.

The road should be located above the river a distance of .30 x 23 = 7 chains.

In Figure 18-A, the main woods road enters the section at a point 7 chains south of the river.

After the main woods road is plotted, the next step is to locate the spurs and landings. Landings should be located to take advantage of any favorable topographic features lying within reasonable distance of the site indicated by the spacing formula.

## Location of Spurs and Landings.

Spur S-1.

Measuring parallel to and back from the river a distance of 23 chains, the average slope is found to range from 25% near the main woods road to 10% near the north boundary of the section.

The previous computations for a 25% slope indicate that the spur should start about 7 chains from the river. As the slope becomes more gradual, the distance from the river and the north boundary of the section should increase.

10% Slope: $S = \sqrt{\dfrac{9.68\ r}{VC}}$ 62% of timber skidded down-hill.

$C = 4.4¢(.38) + 3.5¢(.62) = 1.67 + 2.17$ or $C = 3.8¢/M/100'$.

$S = \sqrt{\dfrac{9.68 \times 4500}{40 \times 3.8}} = \sqrt{286} = 1700'$ or 26 chains.

With this information, the spur and the boundary of the area which it serves can be plotted (Figure 18-A).

The landings for S-1 are the next consideration. The theoretical distance between landings is obtained from the road spacing formulae.

The 25% slope calls for a road spacing of 23 chains of which 16 chains are skidded along the contour. The indicated distance between landings is 2 x 16 = 32 chains. Farther out on the spur, the 10% slope also calls for a distance between landings of 32 chains.

No skidding is permitted across the main woods road and a landing is necessary at the junction of this road and S-1. The loading rig can handle logs from both sides of the road.

The next landing is to be located some 28 chains out along the spur. This point is a suitable location for a landing. The boundary between the areas served by the two landings is shown in Figure 18-A.

Spur S-2.

The average slope for S-2 is 25%. Previous calculations show that for this slope, the spur should be 7 chains above the lower boundary of an area 23 chains wide.

The indicated distance between landings is again 32 chains. The area being served by S-2 is more than large enough for one landing, but it is not large enough for two. A study of the topography shows a suitable landing site to be in the ravine where the slope is only 5%. The intermittent stream would not be running during the logging season. Only one landing need be established for this area.

Spur S-3.

The average slope for S-3 varies from less than 15% near the main woods road to 30% on a point some 30 chains out. Beyond this point, it again flattens out to less than 15%.

15% Slope: $S = \sqrt{\frac{10.62\ r}{VC}}$ 53% of timber skidded down-hill.

$C = 5.1¢(.47) + 3.5¢(.53) = 2.40 + 1.85 = 4.3¢/M/100'.$

$S = \sqrt{\frac{10.62 \times 4500}{40 \times 4.3}} = \sqrt{278} = 1670' = 25$ chains.

$.47 \times 25 = 12$ chains.

The spur should be 12 chains above its lower boundary. The area served by the spur should be 25 chains wide.

Landings should be spaced 2 x 13 = 26 chains apart. The area is large enough to justify two landings.

Spur S-4.

The average slope is 35%. $S = \sqrt{\frac{7.03\ r}{VC}}$

77% of timber skidded down-hill.

$$C = 5.1(.23) + 3.5(.77) = 1.17 + 2.70 = 3.9¢$$

$$S = \sqrt{\frac{7.03 \times 4500}{40 \times 3.9}} = \sqrt{202} = 1420' = 22 \text{ chains.}$$

The spur should be located .23 x 22 = 5 chains above the lower boundary of the area served. It is not feasible to give the area a constant width of 22 chains due to the irregularity of the topography. This boundary is represented by a light dotted line for reasons which will later be explained.

The quantity M for a 35% slope is .77 S (Table I). The indicated distance between landings is 2 x .77 x 22 = 34 chains. The area is large enough for the establishment of two landings.

Spur S-5.

The area served by S-5 is a small plateau-like projection. An inspection of the topography indicates the most logical location of a landing to be somewhere on top of this plateau. The percentage of slope varies widely and it is difficult to determine an average figure. By comparison with the other areas in the section, it is evident that one landing is adequate. The site of this landing is arbitrarily selected.

Spur S-5 services all the remaining area north of the creek.

Spur S-6.

The area above S-4 could be served by another spur, but this area is quite small and the question arises as to whether it is cheaper to skid all the timber above S-4 down to a landing on that road, or whether it is cheaper to construct another spur to handle the timber on this area. A decision can be reached by comparing the costs involved under the two plans.

Plan I. Skid all of the timber in the upper area to S-4.

The area beyond the economic skidding distance covers 45 acres. Volume on the area = 45 x 40 M = 1800 M. This must all be skidded an average distance of 18 chains or 1200' which would be avoided by constructing another spur.

The added cost of skidding = 3.5¢ x 1200' x 1800M = $760.

Plan II. Construct a spur to service the upper area.

In order to reach the upper area with a spur, an additional 25 chains or 1650' of road must be built @ $45 per 100' = $735.

The cost of an extra landing would also have to be included. This would more than even up the costs under the two plans. In addition, there is no really suitable location for a landing in the upper area. The slope is rather steep and a landing would be both expensive and inconvenient. Therefore, all the timber will be skidded to the landing on S-4.

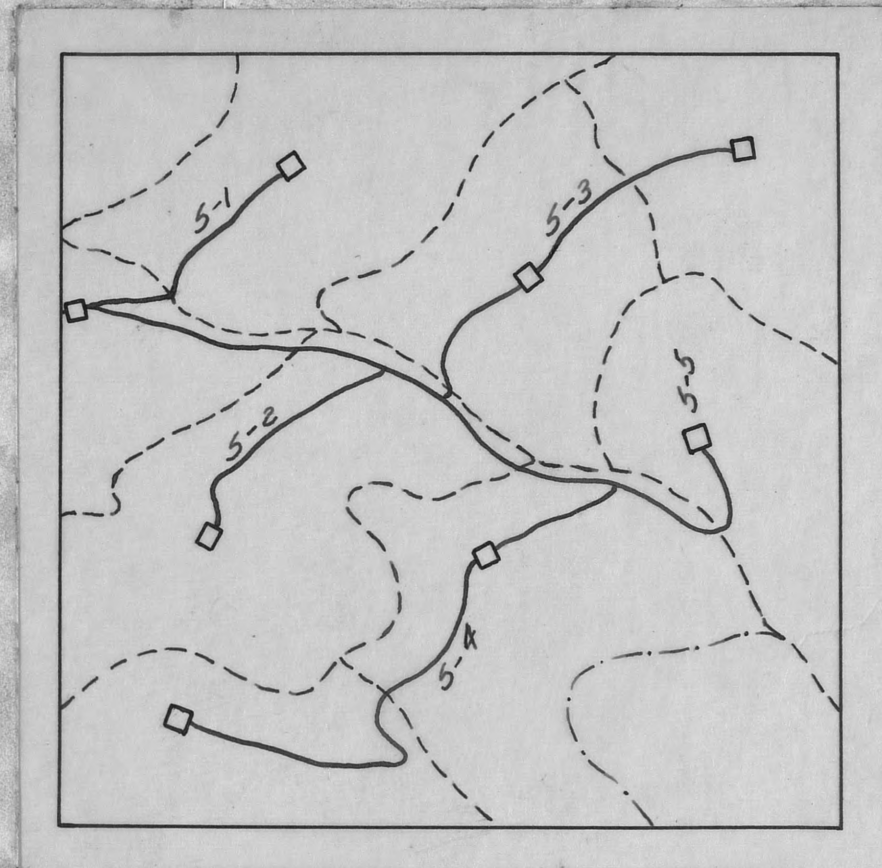



Figure 18-A.

——— Road. □ Landing.

- - - - Skidding Area Boundary.

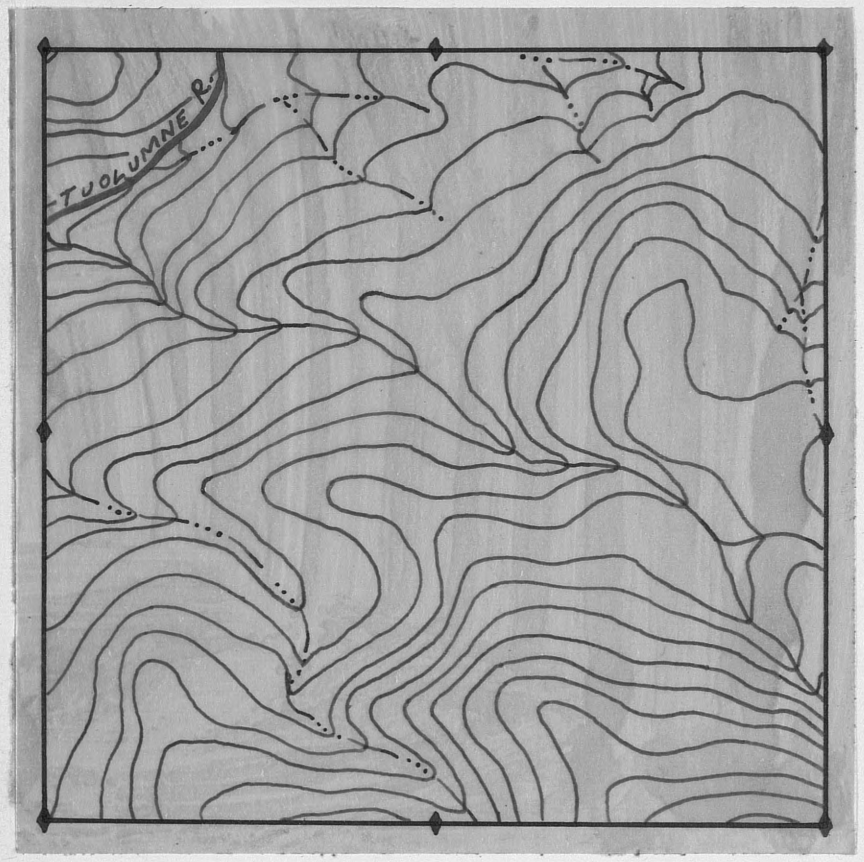



Figure 18.

Sec. 28, T. 4 N., R. 18 E., Mount Diablo Meridian.

Scale 4" 1 mile. Contour interval 50'.

——...—— Intermittent stream.

Merchantable volume per acre - - - - - - - - 40 MBM.

Maximum allowable favorable road slope - - - - 15%.

Maximum allowable unfavorable road slope - - - 10%.

APPENDICES.

# Appendix A.

## Graphical Demonstrations of Average Skidding Distances.

I. Proof that the average skidding distance of a triangular area closely approximates the distance to the landing from the center of gravity of the triangle.

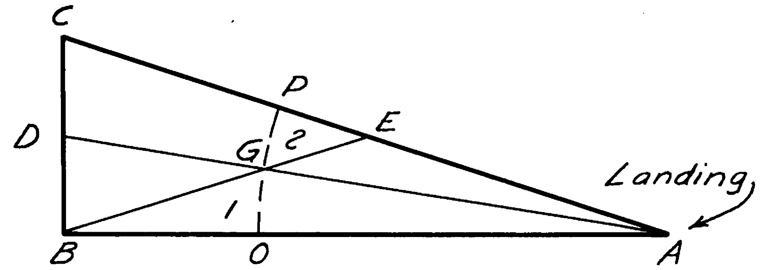



Figure a.

1. Draw AD to the mid point of BC.
2. Draw BE to mid point of AC.
3. Point of intersection G is the center of gravity of the triangle.
4. Area ACD = Area ABD.
5. Area ABE = Area BCE.
6. Average skidding distance lies on arc passing through G.
7. Area 1 tends to equalize Area 2. The discrepancy is a small part of the total area.
8. Therefore the average skidding distance lies practically along arc OP.

By inspection, Area 1 is larger than Area 2. This indicates that the arc OP is somewhat short of the true average skidding distance. Verification of this is obtained with the curves which are presented in Figure 16.

II. Proof that the average distance from any point in a right angle triangle to one apex is two-thirds of the hypotenuse when the distance is measured along lines which make an isosceles triangle with the hypotenuse.*

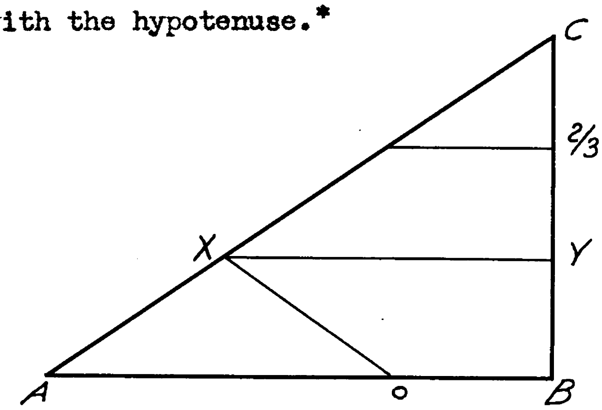



Figure b.

The maximum distance will be AC for any point as o on AB. The maximum distance will be o at point C. The average distance will be XC when XY is the average distance from CB to AC. This average distance with right angle triangles is a perpendicular to the altitude measured at a point one-third from the base. As distances measured along the altitude lay off proportional distances on the hypotenuse, the average distance is two-thirds the hypotenuse.

* This proof was developed by Professor D. M. Matthews.

## Appendix B.

## Derivation of Formulae for Economic Spacing of Roads As Listed in Table I.

For reference in the following derivations, turn to the diagrams on page 56.

**Level Ground.**

Average skidding distance = .768 H $\qquad$ H = .50 S

Average actual skidding distance = .768(.50 S) = .38 S = A.

$$AC = \frac{4.356\ r}{V\ S} \quad \text{or} \quad AS = \frac{4.356\ r}{V\ C} \quad \text{or} \quad .38\ S^2 = \frac{4.356\ r}{V\ C}$$

$$S = \sqrt{\frac{4.356\ r}{.38\ VC}} \qquad S = \sqrt{\frac{11.46\ r}{V\ C}}$$

<u>5% Slope.</u>

Let $H = 10$ Maximum slope for up- hill skidding = 5%.

$h = 8$ 80% of tractor drawbar pull is available.

$S = H + h$

$H = .56\ S$ 56% of timber skidded down-hill.

$h = .44\ S$ 44% of timber skidded up-hill.

Down-hill skidding. Average haul $= .768\ H = .768 \times .56 = .43\ S$

Up-hill skidding. Average hauling distance,

$$\text{Average haul} = \frac{\sqrt{(.67x.56S)^2 + (.33x.44S)^2} + \sqrt{(.67x.44S)^2 + (.33x.56S)^2}}{2}$$

$$= \frac{\sqrt{.14\ S^2 + .02\ S^2} + \sqrt{.09\ S^2 + .03\ S^2}}{2} = \frac{.40\ S + .35\ S}{2}$$

Average haul $= .38\ S$

All skidding:

$$A = .43\ S(.56) + .38\ S(.44) = .24\ S + .17\ S = .41\ S$$

$$AS = \frac{4.356\ r}{V\ C} \qquad .41\ S^2 = \frac{4.356\ r}{V\ C}$$

$$S = \sqrt{\frac{4.356\ r}{.41\ VC}} \qquad S = \sqrt{\frac{10.62\ r}{V\ C}}$$

$\text{Tan}\ \alpha = \frac{h}{H} = .8000$ $\alpha = 38^\circ\ 40'$ $\beta = 51^\circ\ 20'$

<u>10% Slope.</u>

See page 4.

15% Slope.

$\sin\alpha = \frac{10}{15} = .667 \qquad \alpha = 41^\circ\ 50' \qquad \beta = 48^\circ\ 10'$

Let $H = 10$

$h = \tan\ 41^\circ\ 50' \times 10 = 8.9515$

$$\begin{array}{llr} \text{Area} & A = 10 \times 10 & = 100.0 \\ \text{Area} & a = 10 \times 8.95 & = \underline{\ \ 89.5} \\ & & 189.5 \end{array} \qquad \frac{100.0}{189.5} = 53\% \text{ of the timber is skidded down-hill.}$$

$S = H + h \qquad H = .53\ S \qquad h = .47\ S$

Down-hill skidding.

Average haul = $.768 \times .53\ S = .41S$

Up-hill skidding.

$a_1$ Av. haul $= \dfrac{.67\sqrt{(.47S)^2 + (.53S)^2} + \sqrt{(.67x.47S)^2 + (.33x.53S)^2}}{2}$

$$\frac{.67\sqrt{.22\ S^2 + .28\ S^2} + \sqrt{.10\ S^2\ .03\ S^2}}{2} = \frac{.47S + .36S}{2}$$

$a_1$ Av. haul $= .42\ S$

$a_2$ Av. haul $= .753\ H = .753(.53\ S) = .40\ S$ (See Figure 16).

Average haul up-hill $= \dfrac{.42\ S + .40\ S}{2} = .41\ S$

All Skidding:

$A = .41\ S(.56) + .41\ S(.44) = .41\ S$

$$S = \sqrt{\frac{4.356\ r}{.41\ VC}} \qquad S = \sqrt{\frac{10.62\ r}{VC}}$$

20% Slope.

$\sin \alpha = \frac{10}{20} = .500$ $\alpha = 30°00'$ $\beta = 60°00'$

Let $H = 10$

$h = \tan 30° \times 10 = 5.773$ $S = h + H$

Area $A = 10 \times 10 = 100.0$ $H = .63\ S$

Area $a = 10 \times 5.77\ \underline{57.7}$ $h = .37\ S$

157.7

63% of timber is skidded down-hill.

Down-hill skidding: Av. haul = $.768 \times .63\ S = .48\ S$

Up-hill skidding:

$a_1$ Av. haul $= \dfrac{.67\sqrt{(.63S)^2 + (.37S)^2} + \sqrt{(.67x.37S)^2 + (.33x.63S)^2}}{2}$

$= \dfrac{.67\sqrt{.41\ S^2 + .14\ S^2} + \sqrt{.06\ S^2 + .04\ S^2}}{2}$

$= \dfrac{.67(.74\ S) + .32\ S}{2} = \dfrac{.49\ S + .32\ S}{2} = .41\ S$

$a_2$ Av. haul $= .705(.63\ S) = .44\ S$

Average haul up-hill $= \dfrac{.41\ S + .44\ S}{2} = .43\ S$

All skidding:

$A = .48\ S(.63) + .43\ S(.37) = .30\ S + .16\ S = .46\ S$

$S = \sqrt{\dfrac{4.356\ r}{.46\ VC}}$ $S = \sqrt{\dfrac{9.47\ r}{VC}}$

25% Slope.

See page 19.

30% Slope.

See page 30.

35% Slope.

$\sin \alpha = \frac{10}{35} = .286$  $\alpha = 16^\circ\ 37'$  $\beta = 73^\circ\ 23'$

$\sin \alpha' = \frac{25}{35} = .714$  $\alpha' = 45^\circ\ 38'$  $\beta' = 44^\circ\ 22'$

Let $H = 10$

$h = \tan 16^\circ\ 37' \times 10 = 2.984$  $S = H + h$

Area $A = 10 \times 10 = 100.0$  $H = .77\ S$

Area $a = 10 \times 2.98 = \underline{29.8}$  $h = .23\ S$

$129.8$

77% of timber is skidded down-hill.

Down-hill skidding:

$A_1$ Av. haul $= .67\sqrt{(.77S)^2 + (.77S)^2} = .67\sqrt{.59S^2 + .59S^2}$

$= .67\sqrt{(1.18\ S^2)} = .67\ (1.09\ S) = .73\ S$

$A_2$ Av. haul $= .768\ (.77\ S) = .59\ S$

Average haul down-hill $= \frac{.73\ S + .59\ S}{2} = .66\ S$

Up-hill skidding:

$a_1$ Av. haul $= \frac{.67\sqrt{(.23S)^2 + (.77)^2} + \sqrt{(.67x.23S)^2 + (.33x.77S)^2}}{2}$

$= \frac{.67\sqrt{.05S^2 + .59S^2} + \sqrt{.02S^2 + .06S^2}}{2} = \frac{.54\ S + .28\ S}{2}$

$= .41\ S$

$a_2$ Av. haul $= .677(.77\ S) = .52\ S$

Average haul up-hill $= \frac{.41\ S + .52\ S}{2} = .47\ S$

All skidding:

$A = .66\ S(.77) + .47\ S(.23) = .51\ S + .11\ S = .62\ S$

$S = \sqrt{\frac{4.356\ r}{.62\ VC}}$  $S = \sqrt{\frac{7.03\ r}{VC}}$

<u>40% Slope.</u>

$\sin \alpha = \frac{10}{40} = .250$ $\alpha = 14^\circ\ 29'$ $\beta = 75^\circ\ 31'$

$\sin \alpha' = \frac{25}{40} = .625$ $\alpha' = 38^\circ\ 41'$ $\beta' = 51^\circ\ 19'$

Let $M = 10$

$\tan \alpha = \frac{h}{M}$ $\tan \alpha' = \frac{H}{M}$ Area $a = 2.583 \times 10 = 25.83$

Area $A = 8.007 \times 10 = 80.07$

$105.90$

$.2583 \times 10 = h$ $.8007 \times 10\ H$

$h = 2.583$ $H = 8.007$ $M = .94\ S$

$h = .24\ S$ $H = .76\ S$

Down-hill skidding:

$A_1$ Av. haul $= .67\sqrt{(.76\ S)^2 + (.94\ S)^2} = .67\sqrt{.58S^2 + .88S^2} = .81\ S$

$A_2$ Av. haul $= \sqrt{(.67 x .94S)^2 + (.33x.76S)^2} = \sqrt{.40S^2 + .06S^2} = .68\ S$

Average haul down-hill $= \frac{.81\ S + .68\ S}{2} = .75\ S$

Up-hill skidding:

$a_1$ Av. haul $= \frac{.67\sqrt{(.24S)^2 + (.94S)^2} + \sqrt{(.67x.24S)^2 + (.33\ x\ .94S)^2}}{2}$

$= \frac{.67\sqrt{.06S^2 + .88S^2} + \sqrt{.03S^2 + .10S^2}}{2} = \frac{.65S + .36S}{2}$

$a_1$ Av. haul $= .51\ S$

$a_2$ Av. haul $= .675(.94\ S) = .63\ S$

Average haul up-hill $= \frac{.51\ S + .63\ S}{2} = .57\ S$

All skidding:

$A = .75\ S(.75) + .57\ S(.25) = .56\ S + .14\ S = .70\ S$

$S = \sqrt{\frac{4.356\ r}{.70\ VC}}$ $S = \sqrt{\frac{6.22\ r}{VC}}$

<u>45% Slope.</u>

$\sin \alpha = \frac{10}{45} = .222$ $\alpha = 12^\circ\ 50'$ $\beta = 77^\circ\ 10'$

$\sin \alpha' = \frac{25}{45} = .556$ $\alpha' = 33^\circ\ 47'$ $\beta' = 56^\circ\ 13'$

Let $M = 10$

$\tan \alpha = \frac{h}{M}$ $\tan \alpha' = \frac{H}{M}$ Area $a = 2.278 \times 10 = 22.78$

$.2278 \times 10 = h$ $.6690 \times 10 = H$ Area $A = 6.690 \times 10 = \underline{66.90}$

$89.68$

$h = 2.278$ $H = 6.690$

$h = .25\ S$ $H = .75\ S$ $M = 1.11\ S$

Down-hill skidding:

$$A_1 = .67\sqrt{(1.11S)^2 + (.75S)^2} = .67\sqrt{1.23S^2 + .56S^2} = .67\sqrt{1.79S^2} = .90\ S$$

$$A_2 = \sqrt{(.67x1.11S)^2 + (.33x.75S)^2} = \sqrt{.55S^2 + .06S^2} = \sqrt{.61\ S^2} = .80\ S$$

$$\text{Average down-hill haul} = \frac{.90\ S + .78\ S}{2} = .85\ S$$

Up-hill skidding:

$$a_1 = \frac{.67\sqrt{(1.11S)^2 + (.25S)^2} + \sqrt{(.67x.25S)^2 + (.33x1.11S)^2}}{2}$$

$$= \frac{.67\sqrt{1.23S^2 + .06S^2} + \sqrt{.03S^2 + .13S^2}}{2} = \frac{.76S + .40S}{2} = .58\ S$$

$$a_2 = .672(1.11\ S) = .75\ S$$

$$\text{Average haul up-hill} = \frac{.58\ S + .75\ S}{2} = .67\ S$$

All skidding:

$$A = .67\ S(.25) + .85\ S(.75) = .17\ S + .64\ S = .81\ S$$

$$S = \sqrt{\frac{4.356\ r}{.81\ VC}} \qquad S = \sqrt{\frac{5.38\ r}{VC}}$$

50% Slope.

$\sin \alpha = \frac{10}{50} = .200 \qquad \alpha = 11^\circ 32' \qquad \beta = 78^\circ 28'$

$\sin \alpha' = \frac{25}{50} = .500 \qquad \alpha' = 30^\circ 00' \qquad \beta' = 60^\circ 00'$

Let $M = 10$

$\tan \alpha = \frac{h}{M} \qquad \tan \alpha' = \frac{H}{M} \qquad \text{Area } a = 2.041 \times 10 = 20.41$

$.2041 \times 10 = h \qquad .5774 \times 10 = H \qquad \text{Area } A = 5.774 \times 10 = \underline{57.74}$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad 78.15$

$h = 2.041 \qquad H = 5.774$

$h = .26\ S \qquad H = .74\ S \qquad M = 1.28\ S$

Down-hill Skidding:

$$A_1 = .67\sqrt{(.74S)^2 + (1.28S)^2} = .67\sqrt{.55S^2 + 1.64S^2} = .99\ S$$

$$A_2 = \sqrt{(.67 \times 1.28S)^2 + (.33 \times .74S)^2} = \sqrt{.74S^2 + .06S^2} = .90\ S$$

$$\text{Average haul down-hill} = \frac{.99\ S + .90\ S}{2} = .95\ S$$

Up-hill Skidding:

$$a_1 = \frac{.67\sqrt{(1.28S)^2 + (.26S)^2} + \sqrt{(.67 \times .26S)^2 + (.33 \times 1.28S)^2}}{2}$$

$$= \frac{.67\sqrt{1.64S^2 + .07S^2} + \sqrt{.03S^2 + .18S^2}}{2} = \frac{.88S + .46S}{2} = .67\ S$$

$$a_2 = .671(1.28S) = .86\ S$$

$$\text{Average haul up-hill} = \frac{.67\ S + .86\ S}{2} = .77\ S$$

All skidding:

$$A = .77\ S(.26) + .95\ S(.74) = .20\ S + .70\ S = .90\ S$$

$$S = \sqrt{\frac{4.356\ r}{.90\ VC}} \qquad S = \sqrt{\frac{4.84\ r}{VC}}$$

## 55% Slope.

$\sin \alpha = \frac{10}{55} = .182 \qquad \alpha = 10^\circ\ 29' \qquad \beta = 79^\circ\ 31'$

$\sin \alpha' = \frac{25}{55} = .455 \qquad \alpha' = 27^\circ\ 04' \qquad \beta' = 62^\circ\ 56'$

Let $M = 10$

$\tan \alpha = \frac{h}{M} \qquad \tan \alpha' = \frac{H}{M} \qquad$ Area $a = 1.850 \times 10 = 18.50$

$.1850 \times 10 = h \qquad .5110 \times 10 = H \qquad$ Area $A = 5.110 \times 10 = \underline{51.10}$

$69.60$

$h = 1.850 \qquad H = 5.110$

$h = .27\ S \qquad H = .73\ S \qquad M = 1.43\ S$

Down-hill Skidding:

$$A_1 = .67\sqrt{(.73S)^2 + (1.43S)^2} = .67\sqrt{.53S^2 + 2.05S^2} = 1.08\ S$$

$$A_2 = \sqrt{(.67x1.43S)^2 + (.33x.73S)^2} = \sqrt{.92\ S^2 + .06\ S^2} = .99\ S$$

Average haul down-hill $= \frac{1.08\ S + .99\ S}{2} = 1.04\ S$

Up-hill skidding:

$$a_1 = \frac{.67\sqrt{(.27S)^2 + (1.43\ S)^2} + \sqrt{(.67x.27S)^2 + (.33x1.43S)^2}}{2}$$

$$= \frac{.67\sqrt{.07S^2 + 2.05S^2} + \sqrt{.03S^2 + .22S^2}}{2} = \frac{.98\ S + .50\ S}{2} = .74\ S$$

$$a_2 = .670(1.43\ S) = .96\ S$$

Average haul up-hill $= \frac{.74\ S + .96\ S}{2} = .85\ S$

All Skidding:

$$A = .85\ S(.27) + 1.04\ S(.73) = .2\ 3\ S + .76\ S = .99\ S$$

$$S = \sqrt{\frac{4.356\ r}{.99\ VC}} \qquad S = \sqrt{\frac{4.40\ r}{VC}}$$

## 60% Slope.

See page 26.

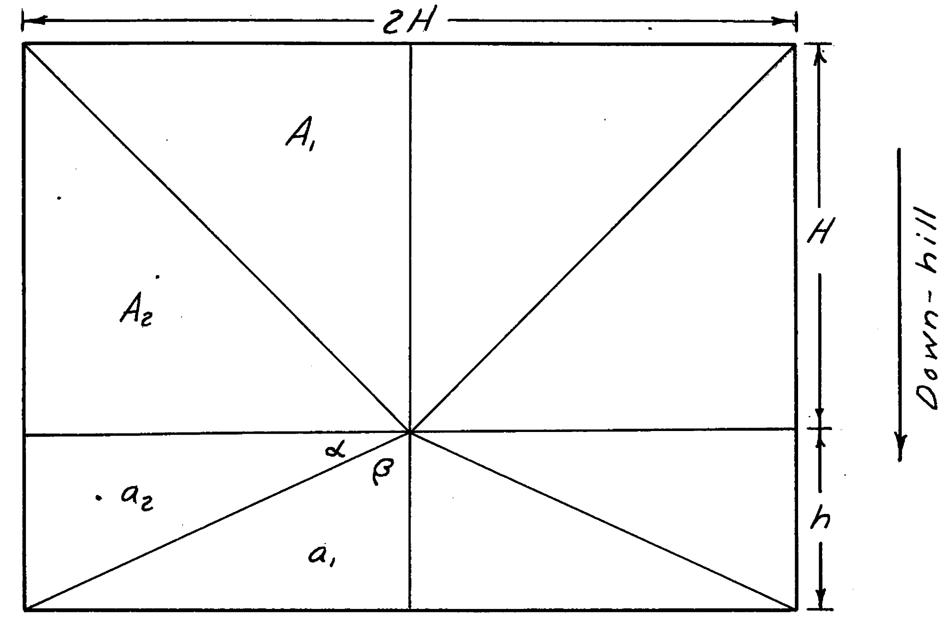



Figure c. Skidding on Slopes 0% - 25%.

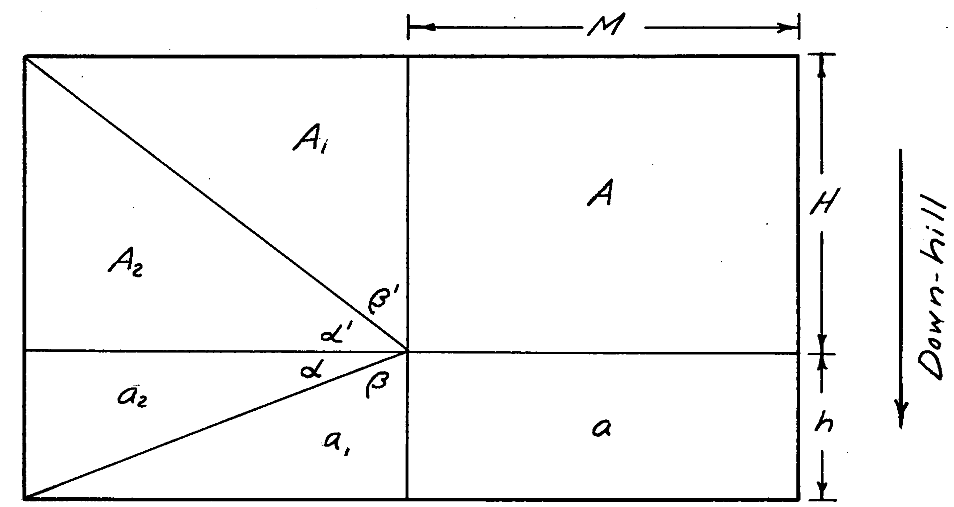



Figure d. Skidding on slopes 30% - 60%.

# Appendix C.

## Theoretical Shape of Area Served by Landing on Slopes Greater Than 10%.

When planning an economic transportation system, tractors should be operated as closely as possible to their economic skidding distance. It has been demonstrated that this distance must be exceeded somewhat in order to serve a landing from a rectangular area (Page 17.).

The closest practical application of the principle of limiting tractor hauls to the economic skidding distance is demonstrated in Figure e. When the maximum adverse slope to be negotiated by a loaded tractor is 10%, the length of line k is limited to .60 M instead of being extended to form the diagonal of its component rectangle as was done in Part I. Similarly, when the cost of skidding down-hill equals the cost of skidding on level ground, the line k' is restricted to equal the length of line M.

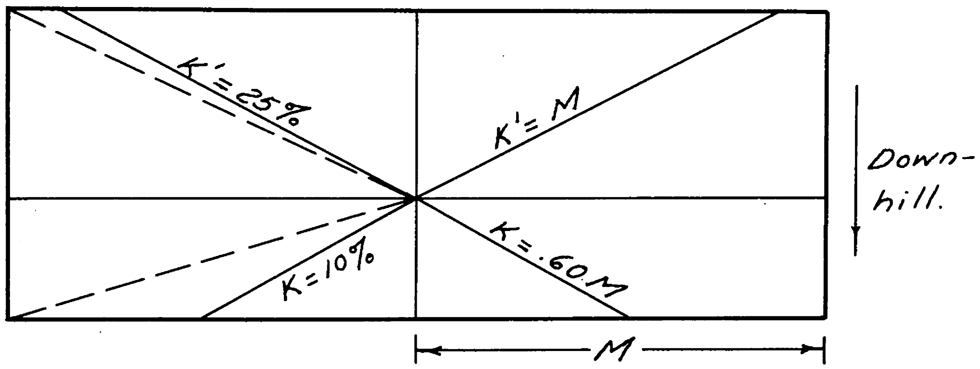



Figure e.

There are disadvantages to limiting k and k' in this manner. In the first place, the determination of the average actual skidding distance is unduly complicated by the increased number of component triangles in the area served by a landing. In the second place, when k and k' are restricted to their economic lengths, the spacing of roads is reduced, and finally, this method is somewhat more expensive than the method presented in Part I due to the reduced amount of down-hill skidding.

Demonstrating with a 25% Slope.

Turn to page 66 for diagram.

Let $H = 10.$

| | | |
|---|---|---|
| $h = \cos\beta \times k$ | $S = H + h$ | $H = \frac{10.0}{12.4} = .81\ S$ |
| $k = .60\ H$ | $H = 10.0$ | $h = .19\ S$ |
| $h = \cos 66^\circ\ 25' \times 6$ | $h = \underline{2.4}$ | $k = .60H = .49\ S$ |
| $h = .4000 \times 6 = 2.4$ | $S = 12.4$ | |

$\text{Tan} \angle 1 = \frac{h}{H} = \frac{2.4}{10.0} = .240$

$\angle 1 = 13^\circ\ 29'$

$\angle \alpha = 23^\circ\ 35'$

$\angle 2 = \angle\alpha - \angle 1 = 10^\circ\ 06'$

$\angle 3 = \angle\beta = 66^\circ\ 25'$

$\text{Sin}\beta = \frac{D}{k}$

$\text{Sin } 66^\circ\ 25' = \frac{D}{.60\ H}$

$.9165 = \frac{D}{.60\ H}$

$D = .55\ H$

$D = .45\ S$

$$n = \sqrt{h^2 + H^2} = \sqrt{(.19S)^2 + (.81S)^2} = \sqrt{.04S^2 + .66S^2} = \sqrt{.70\ S} = .84\ S$$

To sum up the properties of the area:

| | |
|---|---|
| $H = .81\ S$ | $\left.\begin{array}{l}\angle 1 = 13^\circ\ 29' \\ \angle 2 = 10^\circ\ 06'\end{array}\right\} = \angle\alpha$ |
| $h = .19\ S$ | |
| $k = .49\ S$ | $\angle 3 = 66^\circ\ 25' = \beta$ |
| $n = .84\ S$ | |
| $D = .45\ S = .55\ H$ | |

Since up-hill and down-hill skidding areas have a common side, 19% of timber will be skidded up-hill and 81% of timber will be skidded down-hill.

Down-hill skidding:

On slopes of 15% - 25%, down-hill skidding is done from a rectangle twice as wide as it is deep. Average actual skidding distance = .768 H for any square.

H = .81 S    Av. skidding distance down-hill = .768(.81 S) = .62 S

Up-hill skidding:

The average skidding distance for the component triangles must be determined and weighted.

Using half of the up-hill skidding area as a representative sample:

Area $a_1 = \frac{H \times h}{2} = \frac{10 \times 2.4}{2} = 12$    $\frac{12}{24} = 50\%$

Area $a_2 = \frac{(H-D)h}{2} = \frac{4.5 \times 2.4}{2} = 5.4$    $\frac{5.4}{24} = 22\%$

Area $a_3 = \frac{D \times h}{2} = \frac{5.5 \times 2.4}{2} = 6.6$    $\frac{6.6}{24} = 28\%$

Average actual skidding distances:

$a_1$  $\angle 1 = 13^\circ\ 29'$    From Figure 16, average haul = .674 H

Average haul = .674(.81 S) = .546 S

$a_2$  Maximum external haul = n = .84 S

Minimum external haul = k = .49 S

Average haul = $.707\frac{(.84S + .49S)}{2} = .470$ S

$a_3$  Average skidding distance to landing = .67 k. Average distance back to woods = $\sqrt{(.67h)^2 + (.33D)^2}$

Average haul = $\frac{.67(.49S) + \sqrt{.02S^2 + .02S^2}}{2} = \frac{.328S + .200S}{2} = .264$ S

Average actual skidding distance up-hill:

Av. haul = .546S(.50) + .470S(.22) + .264S(.28)

= .273 S + .104 S + .074 S = .451 S = .45 S

Average skidding distance for whole area served by landing:

A = .62 S(.81) + .45 S(.19) = .50 S + .09 S = .59 S

Cost of skidding:

Down-hill skidding costs are here considered to equal costs of skidding on level ground.

Up-hill skidding costs will vary with slope. Skidding in areas $a_1$ and $a_2$ varies from 0% to 10% and averages somewhat less than 5%. This area represents less than 15% of the total area served by a landing, and it is not worth while to compute this average slope exactly, as an exact calculation is quite lengthy. A close approximate figure is 4%. In area $a_3$, all skidding is done on a 10% slope.

When the cost of skidding on level ground is 3.4¢/M/100',

4% adverse slope $3.4¢/.84 = 4.05 \times .72 = 2.92¢$

10% adverse slope $3.4¢/.60 = 5.67 \times .28 = \underline{1.59}$

$4.51¢$

Average cost of up-hill skidding $= 4.5¢/M/100'$

Average cost of down-hill skidding $= 3.4¢/M/100'$

Average cost of all skidding $= C = Cu(P) + Cd(1-P)$

With a 25% slope:

$Cd = 3.4¢$

$Cu = 4.5¢$ $\quad C = 4.5(.19) + 3.4(.81) = .86 + 2.75$

$P = 19\%$ $\quad C = 3.61 = 3.6¢/M/100'$

$1 - P = 81\%$

Cost of Road Construction:

Cost of road $= 2Hr$ when $r$ = cost of road per 100' station.

Area served by landing $= \frac{2HS}{4.356}$ acres. $V$ = volume per acre.

$$\text{Cost of road per MBM} = \frac{2\,Hr}{V\,\frac{2HS}{4.356}} = \frac{4.356\,r}{VS}$$

For an economic spacing of roads, road costs should equal skidding costs.

$$AC = \frac{4.356\ r}{VS} \qquad \text{or} \qquad AS = \frac{4.356\ r}{VC}$$

With a 25% slope:

$A = .59\ S$

$V = 30$ MBM

$r = 15{,}000¢$

$C = 3.6¢$

$$.59\ S^2 = \frac{4.356\ r}{VC}$$

$$S = \sqrt{\frac{7.38\ r}{VC}}$$

$$S = \sqrt{\frac{7.38 \times 15{,}000}{30 \times 3.6}} = \sqrt{1030} = 32.2 \text{ hundred feet.}$$

Space roads 3200' apart.

Space landings 2 (.81 x 32) = 5200 feet apart.

Economic spacing of roads gives the following costs:

| | |
|---|---|
| $r = 15{,}000¢$ | Skidding costs |
| $V = 30$ MBM | $AC = .59(32.2)3.6 = 68¢$ |
| $C = 3.6¢$ | Roads $= \frac{4.356 \times 15{,}000}{30 \times 32.2} = \underline{68}$ |
| $H = .81\ S$ | 136¢/MBM |
| $A = .59\ S$ | This can be compared with the costs |
| $S = 3220'$ | listed on page 25. |

Demonstrating with a 60% slope.

See Figure g on page 66.

Indirect skidding used for down-hill hauls above line k' and for up-hill hauls below line k.

Direct skidding used for down-hill hauls below line k' and for up-hill hauls above line k.

$\alpha = 9^\circ\ 36'$ $\alpha' = 24^\circ\ 39'$

$\beta = 80^\circ\ 24'$ $\beta' = 65^\circ\ 21'$

Let $M = 10$

$h = \cos\beta \times k$ $H = \cos\beta' \times k'$ $S = H + h$

$h = .16677 \times 6$ $H = .4171 \times 10$ $S = 4.17 + 1.00$

$h = 1.001$ $H = 4.171$ $S = 5.17$

$h = \frac{1.00}{5.17} = .19\ S$ $H = \frac{4.17}{5.17} = .81\ S$ $M = \frac{10.0}{5.17} = 1.93\ S$

$k = .60\ M = 1.16S$ $n = \sqrt{h^2 + M^2} = \sqrt{.04\ S^2 + 3.72\ S^2} = 1.94\ S$

$k' = M = 1.93S$ $n' = \sqrt{H^2 + M^2} = \sqrt{.65\ S^2 + 3.72\ S^2} = 2.09\ S$

Down-hill skidding:

$\angle 1' = 22^\circ\ 46'$ $\tan \angle 1' = \frac{H}{M} = \frac{.81S}{1.93S} = .4197$

$\angle 2' = 1^\circ\ 53'$

$\angle 3' = 65^\circ\ 21'$ $\angle 1' = 22^\circ\ 46'$

Using half of the down-hill skidding area as a representative sample, area served = $M \times H = 10 \times 4.171 = 41.71$

Area $A_1 = \frac{M \times H}{2} = \frac{10 \times 4.17}{2} = 20.85$

Area $A_2 = \sin \angle 3' = \frac{D'}{k'}$ or $.90887 = \frac{D'}{M}$ or $D' = .909\ M$

Area $A_2 = \frac{H(M-D')}{2} = \frac{4.17(10-9.09)}{2} = \frac{3.79}{2} = 1.90$

Area $A_3 = \frac{D' \times H}{2} = \frac{9.09 \times 4.17}{2} = 18.95$

Area $A_1 = 20.85 = 50\%$

Area $A_2 = 1.90 = 5\%$

Area $A_3 = \frac{18.95}{41.70} = \frac{45\%}{100\%}$

Average actual skidding distances:

$A_1$ $\angle 1' = 22°46'$

Average haul $= .686\ M = .686(1.93\ S) = 1.32\ S$

$A_2$ Maximum external haul $= n' = 2.09\ S$

Minimum external haul $= k' = 1.93\ S$

Average haul $= .707\left(\frac{2.09\ S + 1.93\ S}{2}\right) = 1.42\ S$

$A_3$ Average haul $= .67\ k' = .67(1.93\ S) = 1.29\ S$

Average down-hill skidding distance:

Average haul $= 1.32\ S(.50) + 1.42\ S(.05) + 1.29\ S(.45)$

$= .66\ S + .07\ S + .58\ S = 1.31\ S$

Up-hill skidding:

$\angle 1 = 5°\ 37'$ Tan $\angle 1 = \frac{h}{M} = \frac{.19\ S}{1.93\ S} = .0984$

$\angle 2 = 3°\ 59'$

$\angle 3 = 80°\ 24'$ $\angle 1 = 5°\ 37'$

Using half of up-hill skidding area as representative sample,

Area served $= M \times h = 10 \times 1.001 = 10.01$

Area $a_3 = \frac{h(D)}{2} = \frac{1.00(5.92)}{2} = 2.96$

Area $a_2 =$ Sin $\angle 3 = \frac{D}{K}$ or $.9860 = \frac{D}{.60M}$

$D = .592\ M$ or $D = 1.14\ S$

Area $a_2 = \frac{(M-D)h}{2} = \frac{(10-5.92)1.00}{2} = \frac{4.08}{2} = 2.04$

Area $a_1 = \frac{M \times h}{2} = \frac{10 \times 1.00}{2} = 5.00$

Area $a_1 = 5.00 = 50\%$

Area $a_2 = 2.04 = 20\%$

Area $a_3 = \frac{2.96}{10.00} = \frac{30\%}{100\%}$

Average actual skidding distances:

$a_1$ Average haul = .667 M = .667(1.93 S) = 1.29 S

$a_2$ Maximum external haul = n = 1.94 S

Minimum external haul = k = 1.16 S

$$\text{Average haul} = .707\frac{(1.94\ S + 1.16\ S)}{2} = 1.10\ S$$

$a_3$ Average skidding distance to landing = .67 k. Average distance back to woods = $\sqrt{(.67h)^2 + (.33S)^2}$

$$\text{Average haul} = \frac{.67(1.16\ S) + \sqrt{.02\ S^2 + .14\ S^2}}{2} = \frac{.78\ S + .40\ S}{2} = .59\ S$$

Average actual skidding distance up-hill:

Average haul = 1.29 S(.50) + 1.10 S(.20) + .59 S(.30)

= .645 S + .220 S + .177 S = 1.04 S

Average skidding distance for whole area:

A = 1.31 S(.81) + 1.04 S(.19) = 1.06 S + .20 S = 1.26 S

Cost of skidding:

When down-hill skidding costs 3.4¢/M/100'

Up-hill skidding costs

| | | | | |
|---|---|---|---|---|
| 4% | adverse slope | 3.4/.84 = 4.05. | 4.05 x .70 = | 2.835¢/M/100' |
| 10% | adverse slope | 3.4/.60 = 5.67 | 5.67 x .30 = | 1.701 |
| Average cost of up-hill skidding | | | | 4.536¢/M/100' |

Average cost of all skidding:

C = 3.4(.81) + 4.5(.19) = 2.75 + .86 = 3.61¢/M/100'

$$AS = \frac{4.356\ r}{VC} \qquad 1.26\ S^2 = \frac{4.356\ r}{VC}$$

$$S = \sqrt{\frac{4.356\ r}{1.26\ VC}} \qquad S = \sqrt{\frac{3.46\ r}{VC}}$$

With a 60% slope.

$A = 1.26\ S$ $\qquad S = \sqrt{\dfrac{3.46 \times 15{,}000}{30 \times 3.6}} = \sqrt{480}$

$V = 30\ M$

$r = 15{,}000¢$ $\qquad S = 21.9$ hundred feet.

$C = 3.6¢$

Space landings $2M = 2(1.93 \times 21.9) = 8450$ feet apart

Skidding costs $= AC = 1.26(21.9)\ 3.6 = 99¢$

Costs of roads $= \dfrac{4.356\ r}{VS} = \dfrac{4.356 \times 15000}{30 \times 21.9} = \underline{99}$

198¢/MBM

This can be compared with costs obtained on page 29.

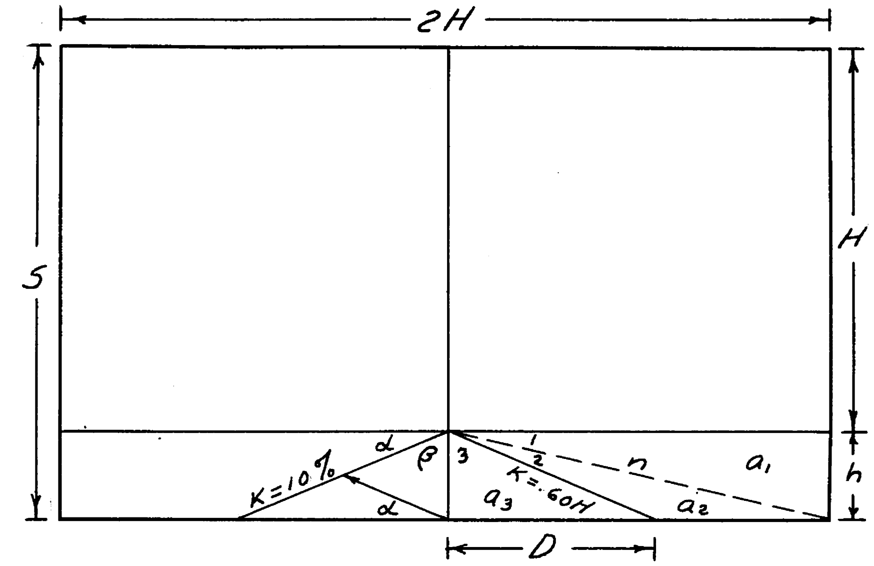



Figure f. Skidding on a 25% slope.

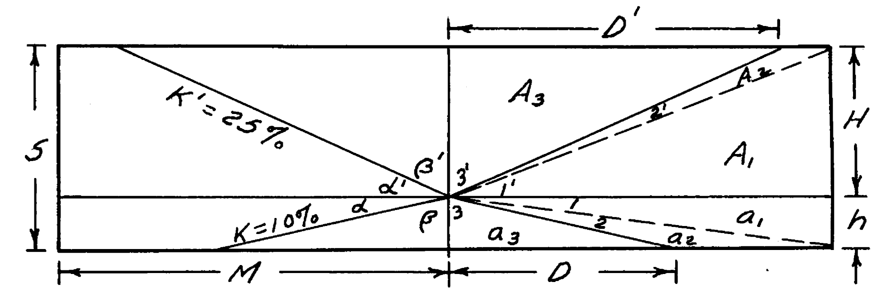



Figure g. Skidding on a 60% slope.

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