A SIMPLIFICATION OF THE CUTTING PLAN FOR THE OUSTER WORKING CIRCLE, HARNEY NATIONAL FOREST, BLACK HILLS.

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A SIMPLIFICATION OF THE CUTTING PLAN FOR THE CUSTER WORKING

CIRCLE: HARDY NATIONAL FOREST, BLACK HILLS.

BY

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PREPARED FOR FORESTRY COURSE #284

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SCHOOL OF FORESTRY AND CONSERVATION
UNIVERSITY OF MICHIGAN
A SIMPLIFICATION OF THE CUTTING PLAN FOR THE CUSTER WORKING CIRCLE, HARNEY NATIONAL FOREST, BLACK HILLS.

OBJECT: To arrive at a basis for annual cut by direct, systematic, simple, and sound reasoning with the data at hand, which is given in the Management Plan for the Custer Working Circle, Harney National Forest, which was approved by the Regional Forester in 1924.

DATA USED: Natural Rotation - 160 years

Technical " 140 " 4 cutting cycles of 35 years each.

Survey Data (forest is uneven aged):

0-20 year stuff 29,411.6 acres
21-80 " 28,008.1 "
80 + " 109,838.4 " with 468,969.5 M or 4.27M per acre (trees over 10" D.B.H.)

Total producing area 167,258.1 "

Only 75% of the 109,838.4 acres is covered with reproduction and poles. The other areas are assumed to be stocked.

Character of Stand: 10" D.B.H. and up - 80 years
6-10" " 20-80 years
under 6" " 1-20 "

Future Growth: 150 bd. ft. per acre per year, for merchantable timber, after the first cut for a 35 year period.

ALLOWABLE ANNUAL CUT FOR THE FIRST CUTTING CYCLE:

Other than the following method for arriving at an allowable annual cut for the first cutting cycle no way was found that was entirely satisfactory. It would have been possible to have used a form of the "step-up" method if cubic foot volume tables had been available for the Black Hills and which would give volumes for trees having small diameters.
Check Calculation on Yield

Harney National Forest

On page 8 of the Working Plan, the acreage of the various size classes are stated to be as follows:

Class I (Seedling & Sapling) 26,205 acres
Class II (Poll) 99,041 acres
Class III (Merchantable) 119,030 acres

Seventy five percent of the area of Class III has been estimated to carry a good distribution of classes I and II in mixture with the merchantable timber, the remaining 25% of the area of Class III being fully stocked with merchantable timber only. If the assumption that 75% of 119,030 acres or 89,272 acres of the Class III area is equally stocked to all three size classes is correct then the further assumption that effectively only 1/3 of this area is fully occupied by any one class, ...
may be allowed. If such is the case then the effective area of the various size classes may be tabulated as under:

Class I by survey 26,205 acres
\[ \frac{1}{3} \times 75\% \text{ of Class III} \times \frac{89,272}{3} = 29,757 \text{ acres} \]
Total 55,962 - 32.1%

Class II by survey 29,041 acres
\[ \frac{1}{3} \times 75\% \text{ of Class III} \times 89,272 = 29,757 \text{ acres} \]
Total 58,798 - 33.7%

Class III - 25% survey data 29,758 acres
\[ \frac{1}{3} \times 75\% \text{ of } \frac{89,272}{3} = 29,757 \text{ acres} \]
Total 59,515 - 34.2%

Total forested area 174,275 - 100%

On page 10 of the plan the total estimated volume of merchantable timber 9.6 inches and up is placed at 5,378,817 MBF. This represents an average stand of \[ \frac{5,378,817 \text{ MBF}}{1190.30 \text{ acres}} \] or 4,518 board feet per acre for the total
1190.90 acres of second III stands. Effectively above this stand occurs on 595.15 acres being the reduced area of second III as above calculated. Calculating the stand per acre on the basis of this reduced area we get \[
\frac{537.817 \text{ M}^{3}}{595.15 \text{ acre}} = 0.9036 \text{ board feet per effective acre of mature timber carrying trees 9.6" in diameter and up.}
\]

In calculating the annual allowable cut it is next essential to figure the actual area cut over on the ground annually under a plan which carries four 25-year cycles in a rotation of 140 years, but only that portion of the effective producing area which will be cut annually. As the rotation is 25 at 140 years this area will be \[
\frac{174.975 \text{ acres}}{140 \text{ years}} = 1237.6 \text{ acres. If we allow for no growth on the stand for the first cycle then the annual allowable cut would be 1237.6 acres.}
\]
9036 board feet or 11,172,953 board feet.

The assumed condition of the stand is indicated by the attached diagram. On this diagram the base line represents the rotation of 140 years and the total area of the forest. The vertical scale represents volume per acre and the stand of 9036 feet has been plotted in on the 110 year ordinate on the assumption that this is the mean age of the Class III timber \( \frac{80-140}{2} = 110 \). The potential production line has been plotted from 0 to 9036 feet on the 110 year ordinate and projected to the 140 year ordinate. If it can be safely assumed that the average age of the present merchantable timber is 110 years.
these timbers at 140 years show an average of 5,000 feet per acre approximately. As the present one, mature timber is removed, the cut should rise toward this yield in later cycles or toward a maximum of 12,376 acres x 11.5-50 feet per 14,232.4 M.B.M., as the allowable annual cut when the present pole and sapling stumps are ready for cutting.

The above is not an unreasonable yield to expect as it amounts to only \( \frac{11.5\text{-}50}{140} \) or 82.1 feet per acre per year as a mean annual increment.
The following method is very theoretical but is the best of any found. It is a well established fact that a forest having all age classes present can be represented very closely by a triangle, in which the area is equal to the timber volume on the forest and the base is equal to the years in the rotation. It is a geometrical fact that areas of similar triangles are directly proportional to the squares of their bases. With these conditions existing and with the data given on page one, the total growing stock can be calculated for a forest, providing a known volume exists for some particular age class.

The 109,838.4 acres is the only area that will produce merchantable timber for the first cutting cycle. Hence an average acre will be used and compared with a triangle. Such an acre will have all the age classes present and the cruised data establishes 4.27 M per acre as the volume for the 80 plus age class. The following diagram shows the condition of such an average acre before a cut takes place.
The percentage volumes that should be present for the three age classes will now be calculated. Then by proportion and with the calculated percentage for the 80-160 year age class and with the known volume of 4.27 M for the same age class, it will be possible to calculate the theoretical volumes that should be present for the two younger age classes.

<table>
<thead>
<tr>
<th>AGE CLASS</th>
<th>VOLUME PER CENT</th>
<th>VOLUME M B M</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-20</td>
<td>$\frac{20^2 \times 100}{160^2}$ or 1.56%</td>
<td>0.09</td>
</tr>
<tr>
<td>21-80</td>
<td>$\frac{80^2 \times 100 - 1.56}{160^2}$ or 23.44%</td>
<td>1.35</td>
</tr>
<tr>
<td>81-160</td>
<td>100 - 25 or 75%</td>
<td>4.27</td>
</tr>
</tbody>
</table>

Only 75% of the area (109,838.4 x .75 or 82,379 acres) is covered by the two younger age classes.

Growing Stock: The total growing stock for the 109,838.4 acres will now be calculated (there is no need for calculating the growing stock for the entire 167,258.1 acres because only the 109,838.4 acres will produce the cut for the first cutting cycle).

<table>
<thead>
<tr>
<th>AGE CLASS</th>
<th>VOLUME M B M</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-20</td>
<td>.09 x 82,379.4 or 7,414</td>
</tr>
<tr>
<td>21-80</td>
<td>1.35 x 82,379.4 or 111,212</td>
</tr>
<tr>
<td>81-160</td>
<td>Cruise Data or 468,969</td>
</tr>
</tbody>
</table>

Total growing stock on which to calculate annual cut for the first cutting cycle 587,595

Annual Cut: By using Von Mantel's formula the annual cut for the first cutting cycle is equal to the total growing stock divided by one-half the rotation (technical). The natural rotation will be shortened to 140 years because growing conditions will be so improved by a cut that a rotation of 140 years will produce timber fully as well as will the
The natural rotation of 160 years.

Annual cut for the first cutting cycle equals 587,595 or 8,394 M 70

This cut will come from 109,838.4 or 3,138 acres. The cut per acre
will be $8,394 \times 2.68$ M. This will leave $4.27 - 2.68$ or $1.59$ M of mer-
chantable timber on each acre after the cut.

The total cut for the first cycle will be $35 \times 8,394$ or 293,790 M.

The per cent of the total growing stock to be removed will be

\[
\frac{293,790 \times 100}{587,595} \approx 50\%.
\]

The per cent of the total merchantable volume to be removed will be

\[
\frac{293,790 \times 100}{468,969} \approx 62.7\%.
\]

COMPARISON OF RESULTS WITH THE FOREST SERVICE PLAN:

<table>
<thead>
<tr>
<th>METHOD</th>
<th>TOTAL CUT FIRST<em>ANNUAL CUT <em>CUT PER ACRE</em>% OF TOTAL</em>% OF TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CUTTING CYCLE *</td>
</tr>
<tr>
<td></td>
<td>*</td>
</tr>
<tr>
<td>*</td>
<td>#</td>
</tr>
<tr>
<td>*</td>
<td>*</td>
</tr>
</tbody>
</table>

| For. Ser. | 245,000 | 7,000 | 2.24 M | total Ga was 70\%(mar-
not calculated king rule
52.2\% if the
annual cut
is 7,000 M

| Triangle | 293,790 | 8,394 | 2.68 M | 50 | 62.7 |

The results obtained by the Forest Service plan are incorrect
because when the cut was calculated by Von Mantel's formula, the wrong
growing stock figure was used. Merchantable growing stock was used
as though it were total growing stock. This meant that the growing
stock for the two younger age classes was left entirely out of con-
sideration. The cut was therefore calculated incorrectly and the
resulting figure is too low. The figure obtained by the triangle
method is very theoretical but gives a fairer result.
The marking rule figure, 70%, which is the amount that shall be removed on a basis of merchantable volume is too large. A fairer figure is 62.7% or 63%.

**ALLOWABLE ANNUAL CUT FOR THE SECOND CUTTING CYCLE:**

A rough approximation for allowable cut for the second cutting cycle is desired only in order to illustrate that the forest is capable of producing a satisfactory return, without possibility of gaps between cuts. The approximation need be only rough because the plan will be revised at least every five years. Better data are needed and will be obtained in time. Future plans will thus have better data to work with and they will give better results.

The same fundamental method will be used in calculating the cut for the second cutting cycle as was used for the first cutting cycle. The total merchantable growing stock present for the 80-140 year age class (the rotation has been reduced from 160 years to 140 years by the first cut) is not given for the 109,838.4 acres (the other two areas are immature and will not be cut over during the second cutting cycle) and will need to be calculated for the second cutting cycle. The merchantable volume left per acre after the first cut is 1.59 M. This volume will be increased by 150 bd.ft. per acre per year for 35 years and is 5.25 M for such a period. The total merchantable volume for the second cutting cycle just before a cut is 1.59 M plus 5.25 M or 6.84 M. This volume is present for the 80-140 age class at the time of the second cut. With this as a basis and by using the same theory as was used in calculating the growing stock per acre for each age class and for the first cutting cycle, it will now be possible to calculate the volumes corresponding to the volume percentage for each age class and for the second cutting cycle.
GROWING STOCK: The total growing stock will now be calculated. This is to be done on 109,838.1 acres only because these are the only acres producing mature timber for the second cutting cycle.

0-35 yr. age class (100% stocking) 109,838 x 0.65M or 71,400 M

36-115 " " (75% " ) 82,379 x (0.93+5.3) or 512,500 M

116 plus " " (100% " ) 109,838 x 3.30M or 362,100 M

Total growing stock 946,000 M

ANNUAL CUT: The annual cut for the second cutting cycle is calculated by use of Von Mantel's formula and will be:

\[
\text{TOTAL GROWING STOCK OR ONE-HALF THE ROTATION}
\]

\[
946,000 \div 70 = 13,514 M
\]

This cut will also come from 3,138 acres annually.

The total expected yield for the second cutting cycle is

13,514 M x 35 yrs. or 473,000 M.

The total expected yield as predicted by the Forest Service plan is 432,138 M. This yield is inaccurate because the marking rule figure, which says that 70% of the merchantable volume should be cut, was used
in the calculation and the results obtained by the triangle method for the first cutting cycle gave no basis for its use during that cutting cycle. Therefore the 70% marking rule figure should not be used blindly in obtaining the cut for the second cutting cycle. The annual cut of 13,514 M seems to be the fairer figure of the two. This is even more apparent when the fact is known that an arithmetical error was made, in addition to the error in theory. The Forest Service plan should have obtained the total expected yield by its incorrect theory as follows, for the second cutting cycle:

\[ 140,691 \, \text{M} \]

the volume remaining after 70\% of the merchantable volume was cut for the first cutting cycle. Increment for the stand was at the average rate of 150 bd. ft. per acre per year and is \( 150 \times 109,838 \, \text{acres} \times 35\,\text{yrs.} \) or 576,000 M. The Forest Service figure here was 476,649 M and no satisfactory reason can be given for such a result except the presence of an arithmetical error. The increment 576,000 M is added to the 140,691 M in order to obtain the total merchantable growing stock of 716,691 M. The expected yield will then be 70\% of 716,691 M or 501,684 M. The annual yield is \( \frac{501,684}{35} \) or 14,350 M. This yield is calculated in order to obtain a corrected Forest Service theory result, so that a comparison can be made with the triangle theory result.

**FUTURE YIELDS:** Yields will not be calculated for the third cycle or thereafter because conditions will almost assuredly be much different by that time and new plans will need to be drafted.

The entire area of 167,258.1 acres will be a producing unit by the end of the first rotation. The 28,008.1 acres which now have 20-80 yr. timber for the oldest age class present, will come into production the third cutting cycle. The 29,411.6 acres, which now have 0-20 yr. timber, will come into production during the fourth cutting cycle.
ADDITIONAL CHECK CALCULATIONS.
CHECK CALCULATION NUMBER ONE

The total number of acres in the forest will be distributed in such a manner between the various age classes that each age class will be represented by an area which is stocked 100% by only a single age class.

The 80 plus year age class is present on every acre of 109,838.4 acres. This class only occupies 50% of the available growing space on each acre because (page 17 of the F.S. plan) if a 140 year rotation is used this class need not be carried for more than 70 years (one-half of the rotation) in order to prevent a hiatus in the supply of timber ready for cutting. The natural rotation is 160 years, and 80 years is equal to half of the natural rotation. Hence \( \frac{80}{160} = \frac{1}{2} \) of each acre is taken up by this class. 25% of the 109,838.4 acres is not covered by stuff 0-80 years of age, but is covered 100% by the 80 plus age class. Thus 109,838.4 x 0.25 or 27,459 acres is covered entirely by this age class. 75% of the 109,838.4 acres has the younger ages below 80 years thereon and they cover 50% of each acre. The area covered by the 80 plus timber is 109,838.4 x 0.75 x 0.50 or 41,189 acres. The total area covered 100% by the 80 plus age class is 68,649 acres.

The 20-80 year age class is present on 75% of the 109,838.4 acres or 82,379 acres. 50% of this 82,379 acres or 41,189 acres is taken up by the 80 plus age class. 60 of 41,189 acres (age allotment ) or 30,891 acres is covered 100% by the 20-80 year age class (41,189 minus 30,891 or 10,298 acres is covered by 0-20 year stuff). The 20-80 year age class is present 100% on 60 of 28,008.1 acres or 21,006 acres (28,008 - 21,006 or 7,002 acres is covered 100% by the 0-20 year stuff). The total area covered 100% by 20-80 year timber is the sum of 30,891 and 21,006 or 51,897 acres.

The total area covered by 0-20 year stuff is the sum of 29,411.6,
10,298, and 7,002 or 46,712 acres.

There are 468,969.5 M of 80 plus timber and this is present on 68,649 acres which are 100% stocked with this single age class. The volume per acre is 468,969.5 or 6.83 M.

With this knowledge at hand it will be possible to construct a diagram which will illustrate the condition of the forest and will determine a cut. The same method of constructing the diagram is used as was used by D.M. Matthews in his article "Use Of The Schematic Diagram To Assess The Cutting Budget," which appears in the Journal of Forestry, year 1929, page 229.

The following data is needed before such a diagram can be constructed (For diagram see Plate I):

<table>
<thead>
<tr>
<th>AGE CLASS</th>
<th>AREA</th>
<th>%</th>
<th>NO. OF ACRES IN A SAMPLE AREA OF 140 ACRE</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-20</td>
<td>46,712 acres</td>
<td>28</td>
<td>39</td>
</tr>
<tr>
<td>21-80</td>
<td>51,897</td>
<td>31</td>
<td>44</td>
</tr>
<tr>
<td>80 plus</td>
<td>68,649</td>
<td>41</td>
<td>57</td>
</tr>
<tr>
<td>Total</td>
<td>167,258</td>
<td>100%</td>
<td>140 acres</td>
</tr>
</tbody>
</table>

For 57 years there will be an annual cut of 6.83 M per acre on 167,258 or 1,195 acres. The total annual cut for this period will be 1,195 x 6.83 or 8,169 M.

After the 57th year the potential production line shows a possible annual cut of 8.7 M per acre (from a fully stocked single age class acre containing the oldest age class). The annual cut will then be 1,195 x 8.7 M or 10,400 M. The forest will at that time be practically in the condition shown by the potential production line and irregularities will be inconsequential.
Each acre is treated as being 100% stocked by one age class
Present Condition of the Forest
The following method is very similar to that used in the previous calculation. In this case however only the 109,838.4 acres will produce the total growing stock because this is the only area which is furnishing mature timber during the first and second cutting cycles. The areas designated for each single age class are 100% stocked and are taken from the preceding calculation.

<table>
<thead>
<tr>
<th>AGE CLASS</th>
<th>AREA IN ACRES</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-20</td>
<td>10,298</td>
</tr>
<tr>
<td>21-80</td>
<td>30,891</td>
</tr>
<tr>
<td>81 plus</td>
<td>68,649 6.83 M per acre</td>
</tr>
<tr>
<td>Total</td>
<td>109,838 acres</td>
</tr>
</tbody>
</table>

A similar diagram was constructed to the one in the preceding calculation. The following data is necessary for such construction:

<table>
<thead>
<tr>
<th>AGE CLASS</th>
<th>AREA</th>
<th>%</th>
<th>NO. OF ACRES IN A SAMPLE AREA OF 140 ACRES</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-20</td>
<td>10,298</td>
<td>9.4</td>
<td>13.2</td>
</tr>
<tr>
<td>21-80</td>
<td>30,891</td>
<td>28.1</td>
<td>39.3</td>
</tr>
<tr>
<td>81 plus</td>
<td>68,649</td>
<td>62.5</td>
<td>87.5</td>
</tr>
<tr>
<td>Total</td>
<td>109,838</td>
<td>100.0</td>
<td>140.0</td>
</tr>
</tbody>
</table>

The distribution which should exist if the forest were regulated is as follows:

<table>
<thead>
<tr>
<th>AGE CLASS</th>
<th>AREA</th>
<th>%</th>
<th>NO. OF ACRES</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-20 years of age</td>
<td>14.3%</td>
<td>20 acres in sample 140, 15,700 acres all</td>
<td></td>
</tr>
<tr>
<td>21-80</td>
<td>42.85%</td>
<td>60 &quot; &quot; &quot; &quot; 47,069 &quot; &quot;</td>
<td></td>
</tr>
<tr>
<td>81-140</td>
<td>42.85%</td>
<td>60 &quot; &quot; &quot; &quot; 47,069 &quot; &quot;</td>
<td></td>
</tr>
</tbody>
</table>
What the actual volume percentage would be for each age class were present in the correct amounts is as follows:

0-20  \( \frac{20^2}{140^2} \times 100 \) or 2.04%

21-80  \( \frac{80^2}{140^2} \times 100 \) - 2.04 or 32.7 - 2.04 or 30.66%

81-140  100 - 32.7 or 67.3%

The actual volume which would be present for each age class if the forest were regulated can now be calculated with the present actual stand of merchantable timber as a basis. The merchantable volume per acre is 6.83 M on 47,069 acres. The total merchantable volume will be 321,800 M and this amounts to 67.3% of the stand. By proportion and with the volume percentages known for the other age classes it will be possible to calculate their volumes as follows:

- 67.3% merchantable timber - 321,800 M on 47,069 acres or 6.83 M per acre
- 30.66% poles 146,500 M 47,069 3.11 M
- 2.04% reproduction 9,940 M 15,700 0.63 M

The actual present approximate amount of growing stock can now be figured as under:

68,649 acres at 6.83 M per acre or 468,969 M 81-140 year timber
30,891 " 3.11 M " " 96,000 M 21-80 " 
10,298 " 0.63 M " " 6,470 M 0-20 " 
109,838 " total 571,439 M

The annual cut by the use of Von Mamtel's formula is equal to TOTAL GROWING STOCK OR 571,439 or 8.163 M. According to the triangle (Plate II) this would be attainable forever.
Present condition of 109.824 acres. Which bear mature timber (each acre is treated as though it were 100 acres, or 0.494 acres).
COMPARISON OF ANNUAL CUTS OBTAINED BY ALL METHODS.

FIRST CUTTING CYCLE

FOREST SERVICE PLAN:

Marking Rule Figure- 9,414 M
Von Mantel's formula-7,000 M

TRIANGLE METHOD: 8,394 M
CHECK CALCULATION # I 8,169 M
" " # II 8,163 M

A cut of about 8,000 M annually is recommended for the first cutting cycle. The Forest Service is undercutting at the present time if it cuts 7,000 M annually and is overcutting if they rigidly adhere to their marking rule figure of 70% of merchantable.

SECOND CUTTING CYCLE

FOREST SERVICE PLAN:

AS IS: 12,346 M
CORRECTED: 14,350 M
TRIANGLE METHOD: 13,514 M
CHECK CALCULATION # I 10,400 M
" " # II 8,163 M

The results are varied for the second cutting cycle but it is safe to say that the annual cut will increase over what it was for the first cycle. A cut of over 10,000 M annually will very likely be gotten in the second cutting cycle. This is a sufficiently rough approximation for the present.