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Asymmetric Price Movements and Borrowing Constraints: A Rational Expectations Equilibrium Model of Crises, Contagion, and Confusion

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ABSTRACT

This study proposes a rational expectations equilibrium model of crises and contagion in an economy with information asymmetry and borrowing constraints. Consistent with empirical observations, the model finds: (1) Crises can be caused by small shocks to fundamentals; (2) market return distributions are asymmetric; and (3) correlations among asset returns tend to increase during crashes. The model also predicts: (1) Crises and contagion are likely to occur after small shocks in the intermediate price region; (2) the skewness of asset price distributions increases with information asymmetry and borrowing constraints; and (3) crises can spread through investor borrowing constraints.

IN THE LAST TWO DECADES, asset markets have shown numerous large price movements. This phenomenon can be seen both historically and globally: the U.S. stock market crash of 1987, the burst of Japan's stock market bubble in 1989, the European exchange-rate debacle in 1992 to 1993, the Mexican crisis of 1995, the Asian crisis of 1997 to 1998, the default of the Russian government in 1998, the sharp depreciation of the real in Brazil in 1999, and more recently, the default of the Argentinian government and the collapse of its currency board regime in 2001.

Despite much research, several empirical features of such large price movements remain puzzling. First, asset price collapses occur without any preceding public news. For example, Culter, Poterba, and Summers (1989) document that many of the biggest movements in the S&P500 index have occurred without any particularly dramatic news event. Second, such movements tend to be downward rather than upward.[1] Finally, large downward price movements (crises)

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[1] See Pindyck (1984), French, Schwert, and Stambaugh (1987), and Bekaert and Wu (2000).

are contagious. In other words, idiosyncratic shocks unique to one market affect asset prices in the other markets. Two interesting findings regarding this correlation emerge. One finding is that contagion cannot be explained by fundamentals. For example, both Karolyi and Stultz (1996) and Connolly and Wang (1998) find that macroeconomic announcements and other public news do not affect the comovements of the Japanese and American stock markets. King and Wadhwani (2000) also find that observable economic variables explain only a small fraction of international stock market comovements. The second finding is that contagion is asymmetrical. Ang and Chen (2002); Connolly and Wang (2003); Longin and Solnik (2001); and Boyer, Kumagai, and Yuan (2002) observe that the correlations among markets are larger in market downturns than in market upturns.

To study the phenomenon of downward price movements, this paper develops a rational expectations equilibrium (REE) model of financial crises and contagion in which asymmetric and correlated price movements arise endogenously from the interaction of information asymmetry and borrowing constraints.

In reality, market-making and arbitrage are carried out by a relatively small number of highly informed professional investors who have limited capital and are borrowing constrained. Moreover, borrowing constraints are more likely to be binding when informed investors most need outside financing. For instance, when the asset price is extremely low, but the informed investor's signal is high, it can be difficult for informed investors to raise arbitrage capital. For example, between August 1998 and October 1999, Tiger Securities, the second largest hedge fund group in the United States, fell from \$20 billion in assets under management to about \$8 billion. This is due to the massive redemption by private investors who were dissatisfied with Tiger's "temporary" dismal investment results. The hedge fund manager, Julian Robertson, blamed the withdrawal of European investors who "are obsessed with short-term profits" for the plight of his funds. To deter more investors from selling, he forced the investors to withdraw only twice-yearly rather than on a quarterly basis. Effectively, Mr. Robertson was trying to loosen the borrowing constraint faced by his funds (*Financial Times*, October 13, 1999).

Motivated by these empirical observations, our model employs a standard information asymmetry framework in which informed investors receive a noisy signal about the asset payoff and uninformed investors observe only the price from which they extract the signal. In our model, the asset price is not fully revealing because the asset supply is random. In addition, some informed investors are borrowing constrained. The borrowing constraint is a function of the price. The lower the asset price, the harder it is for informed investors to raise outside financing to invest in the risky asset.

When informed investors are not constrained, the asset price is informative because the unconstrained trading of informed investors transmits their signal to the asset price. Conversely, when a small adverse shock to the fundamentals lowers the price, informed investors may become borrowing constrained and thus unable to trade on the signal. In this case, their demand for the risky asset is no longer informative. As a result, the price becomes increasingly noisy as it drops. Thus, a high asset price is informative and leads uninformed investors to accommodate a large amount of the random asset supply. In contrast, a low, uninformative asset price makes uninformed investors unwilling to absorb any selling because they cannot separate liquidity selling from informed investor information-based selling. Uninformed investors may find it increasingly difficult to extract the informed signal from the falling price, and thus choose to bail out when the price falls. This behavior exacerbates the downward price movement and creates a backward-bending demand for the risky asset. This backward-bending demand induces several feedback effects. For example, the falling asset price tightens informed investor borrowing constraints. Hence, an asset crisis may occur because of panic selling by uninformed, but rational, investors. Note that crises are equilibrium outcomes even when all investors are rational.

In our model, the backward-bending demand for the risky asset arises endogenously from the optimal behavior of uninformed, but rational, investors. These uninformed investors are rational since they recognize that a low price may indicate distressed, rather than speculative, selling from informed investors. However, even with this realization, uninformed investors may become "confused" due to an increasing lack of informativeness of prices as price levels decline.[2] When an asset is priced in a region where uninformed investors may become confused about price signal precision, a crisis may occur. Since the degree of uncertainty regarding informed investor borrowing constraint status is high in the intermediate price region, crises in this model are not tail-end events, but likely to occur after small shocks in the intermediate price region.

Note that our model of investor behavior during large price movements requires both information asymmetry and borrowing constraints. Without borrowing constraints, price informativeness remains the same following any large or small price movement. That is, uninformed investors demand the same price premium in a bear or bull market. Without information asymmetry, a price drop could be explained by borrowing constraints, which are common knowledge and thus no surprise.

In short, our model outlines how small trigger shocks can create asset price meltdowns. It also explains why large upward price reactions to small favorable trigger shocks are unlikely. In this case, any misalignments between the price and the fundamental will be arbitraged away by borrowing-unconstrained informed investors. Thus, the model predicts an asymmetric price distribution.[3]

[2] Later in this paper, we formally define "confusion" in the context of substitution and informative effects.

[3] In another study, we propose a generalized non-linear REE model to examine the interaction between shortsale constraints and information asymmetry (Yuan (2003)). Interestingly, shortsale constraints accentuate the asymmetry of large price movements. That is, as a positive shock moves the price above the fundamentals, uninformed investors rationally infer that informed investors are more likely to be shortsale-constrained. Hence, uninformed investors are less willing to accommodate the noise selling activity in a "heated" market, and thus dampen the upward price movements.

Furthermore, in our model, contagion occurs endogenously in a two-asset setting. Specifically, a price drop in one market makes asset prices noisier signals for the underlying fundamentals in all markets where borrowing-constrained informed investors trade actively. This effect generates a universal downward price pressure. Nevertheless, contagion occurs less often than single market crises as uninformed investors have more price signals.

The present study relates to a growing literature studying the effects of wealth constraints on crises and contagion. For example, Xiong (2001) studies wealth constraint as an amplification mechanism, while Kyle and Xiong (2001) study it as a spillover mechanism. In another study, Gromb and Vayanous (2002) develop an equilibrium model of arbitrage trading with margin constraints to explain contagion. Our approach is complementary to these studies, as we show that information asymmetry amplifies the wealth effect on price movement.

Note that this paper is not the first attempt to use a constrained information asymmetry framework to explain crises and contagion. Related studies include Genotte and Leland (1990), Romer (1993), Kodres and Pritsker (2002), Hong and Stein (2003), and Barlevy and Veronesi (2003). In Genotte and Leland (1990), crises occur because the amount of portfolio insurance is imperfectly revealed, causing "an erroneous assumption on the functional form of the equilibrium price." In Kodres and Pritsker (2002), contagion due to portfolio rebalancing occurs in the absence of news when shocks in one market are misinterpreted in other markets due to information asymmetry. Markets in their study are linked through the existence of common, random factors affecting the fundamental values, as well as through misinterpretations by asymmetrically informed agents. In our model, markets are linked through the potential borrowing constraint on informed investors that alters their demand (and thus that of uninformed investors) across markets. Our model also relates to a study by Barlevy and Veronesi (2003) that shows high prices are more informative than low prices. As a result, uninformed investors demonstrate backward-bending demand for the risky asset. Our model differs from theirs in the model setup. They focus on wealth and shortsale-constrained risk-neutral agents, with asset supply exponentially distributed. By contrast, we study borrowing-constrained, constant-absolute-risk-averse agents, with asset fundamentals and supplies normal distributed. Our setup allows us to study the effects of price-dependent borrowing constraints.

Finally, Romer (1993) and Hong and Stein (2003) provide alternative explanations of crises and contagion. They argue that small events can reveal substantial information to partially informed agents, thus causing traders to significantly reallocate their portfolios in response to small changes in the underlying environment. This reallocation eventually leads to large changes in stock prices. In contrast, our model predicts that crises are driven by uninformed investors uncertain about fundamentals, rather than by uncertain agents who become informed.

The remainder of this paper is organized as follows. In Section I, the model setup for an economy with one risky asset is first developed. Specifically, the concept of constrained rational expectation equilibrium for an economy with asymmetric information is defined and the solution method is explained. Section I also presents the solution for the problem of uninformed investors' rational learning and optimization and defines "confusion." Finally, the properties of crises are explored. Section II extends the results to an economy with two risky assets and explores the properties of contagion. Section III concludes.

I. The Model of Crises and Investor Confusion

A. The Model

A.1. An Economy with Information Asymmetry

The following model is a generalization of Grossman and Stiglitz (1980) to incorporate a borrowing constraint. The model includes two dates, time 0 and time 1. At time 0, investors trade competitively in the market based on their private information. At time 1, payoffs from the assets are realized and consumption occurs.

The model also includes an underlying probability space, $(\Omega, \mathcal{F}, \mathbf{Q})$, on which all random variables are defined. A state of nature is denoted by $\omega \in \Omega$. It is also assumed that all random variables belong to a linear space, $\mathcal{N}$, of joint normal distributed random variables on Ω.

There are one risk-free and one risky asset. The risk-free asset pays R units, while the risky asset pays $\widetilde{v}$ units of the single consumption good. Taking the risk-free asset to be the numeraire, we let $\widetilde{P}$ be the price for the risky asset. Investors divide their initial wealth, W_0, between the risk-free and the risky asset. We let D_k be the risky asset's holding by agent k.[4] Thus, agent k's final wealth is given by[5]

$$\widetilde{W}_{1,k} = W_{0,k}R + D_k(\widetilde{v} - R\widetilde{P}). \tag{1}$$

In the model, each agent maximizes the expected utility of consumption based on his or her own information set. For informed agents, this information set consists of the equilibrium price and the realization of a private information signal, $\widetilde{s}$, which is correlated with $\widetilde{v}$. By contrast, the uninformed agent's information set consists of only the equilibrium price.

The model also incorporates "noise" in the form of a random supply of the risky asset, so that the no-trade theorem will not apply (Milgrom and Stokey (1982)). Specifically, the net supply of assets is assumed to be the realization of a random vector, $\widetilde{m}$.

With the above general structure, the following assumptions are made. For agent k, the utility function exhibits constant absolute risk aversion, i.e., $E_0[-e^{-\widetilde{w}_{1,k}/\rho}]$, where E_0 is the expectation operator, conditional on investor information at time 0. Again, to simplify notation, we assume that all investors have the same risk aversion parameter, ρ. Generalization to the heterogenous risk aversion parameter is straightforward and is shown in Admati (1985).

[4] We denote informed agents by i, uninformed agents by ui, and generic agents by k.

[5] Assume that the risk-free asset is the numeraire asset and $R = 1$.

In our model, investors are competitive and form a continuum with measure 1. These investors are either informed or uninformed.[6] Prior to trading, informed investors receive private information related to the payoff of the risky asset. The signal, $\widetilde{s}$, is a noisy signal of the asset final payoff, $\widetilde{v}$, given as follows: $\widetilde{s} = \widetilde{v} + \widetilde{\epsilon}_s$, where $\widetilde{\epsilon}_s$ represents the noise of the signal and is independent of $\widetilde{v}$.

In addition, $\widetilde{v}, \widetilde{m}$, and $\widetilde{\epsilon}_s$ are mutually independent and jointly normal distributed with mean $(0, \overline{m}, 0)$ and variances $(\sigma_v^2, \sigma_m^2, \sigma_s^2)$, respectively.[7]

A.2. The Borrowing Constraint

Empirically, investors face borrowing constraints. Typically, borrowing constraints arise when the stock price is low relative to the fundamental. Wealth-constrained informed investors cannot justify a holding position on a beaten-down stock to outside lenders. To reflect this situation, we model borrowing constraints as a restriction on informed investor demand that depends on asset prices.[8]

More specifically, we assume that only a fraction (w_i^c) of informed investors face constraints. Their constrained demand $(\widetilde{D}_i^c)$ is expressed in the following definition and is illustrated in Figure 1.[9]

DEFINITION 1: *Borrowing-constrained informed investor demand is constrained to a set, $\mathcal{A}$, characterized as follows*

$$\mathcal{A} = \{y \in \mathcal{R} : y \leq n(\widetilde{P})\}, n(\widetilde{P}) = a\widetilde{P} + b, \tag{2}$$

where $a > 0$ and $a < w_i^{uc}\rho(\tau_v + \tau_s)/w_i^c$.[10]

Informed investor demand is price-dependent, since their borrowing capacity is tied to asset values. Borrowing capacity determines their ability to arbitrage

[6] We denote the measure of informed investors as w_i and the measure of uninformed investors as w_{ui}, where $w_i + w_{ui} = 1$. We denote the measure of unconstrained informed investors as w_i^{uc} and the measure of constrained informed investors as w_i^c, where $w_i^c + w_i^{uc} = w_i$.

[7] The value $\widetilde{v}$ is assumed to have a mean of 0 to save on notation in the derivations. For a general case where $\widetilde{v}$ has a mean of $\overline{v}$, the derivation remains the same for $\widetilde{v} - \overline{v}$.

[8] Asset pricing and macroeconomics literature models credit constraints as a function of asset prices (Kiyotaki and Moore (1997); Aghion, Bacchetta, and Banerjee (1998); Stein (1995)). In this study, we model endogenous constraints on demand for two reasons. First, this approach simplifies the uninformed investor inference problem. Second, endogenous constraints on demand carry the same intuition as endogenous constraints on wealth and can be derived from endogenous constraints on wealth. Footnote 11 provides a simple example that links a constraint on wealth to a constraint on demand.

[9] The model assumes that only a fraction of informed investors are constrained, for the sake of generality. Constraints on uninformed investors in this type of problem are normally immaterial, since such constraints will not affect the inference problem of uninformed investors.

[10] The first restriction on a is to ensure that it is a borrowing constraint. The second is to ensure that the demand curve of constrained and unconstrained informed investors combined remains downward-sloping with respect to $\widetilde{P}$ so that the result of possible multiple equilibria is not trivial. We use τ to denote the precision of a random variable, that is, the inverse of the variance; later on, we use 1 to denote indicator functions.

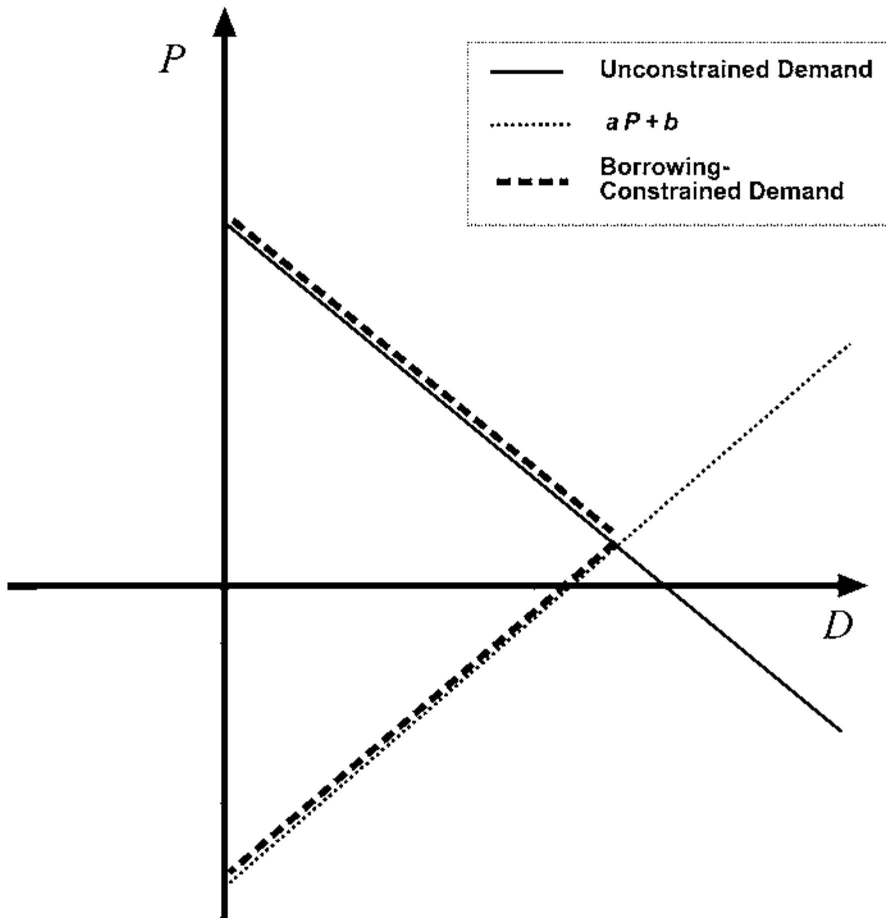

Figure 1. An example of the borrowing constraint. For a given signal, the solid line in the figure represents the demand schedule for the risky asset submitted by an unconstrained informed investor. The dotted line represents the upper bound of demand for the risky asset by a borrowing-constrained informed investor. The dashed line represents the demand schedule for the risky asset submitted by a borrowing-constrained informed investor.

away mispricing in asset markets and make asset prices more information-efficient.[11]

[11] The financial constraint on informed investor demand is stylized but realistic. For example, investors often establish margin accounts with dealers. Let us assume the investor has a margin account for the risky asset and the margin requirement is 30%. At the trading date, an investor's wealth consists of a position (long or short) in the risky asset (Q shares) and a position (long or short with a value of A) in the riskfree asset ($\widetilde{W} = Q\widetilde{P} + A$). He can leverage up using the margin account (70%W). The upper bound of his position in the risky asset is $(1 + 70\%)Q + 70\%A/\widetilde{P}$, which is endogenous in price. Thus, our definition can be considered as a linearized version of this constraint.

A.3. Equilibrium Concept

This section defines the equilibrium concept for the above-specified constrained economy. It is based on the rational expectations developed by Grossman (1976) and Hellwig (1980). The following is a standard equilibrium definition.

DEFINITION 2: *A constrained REE in a constrained economy is a price, $\widetilde{P}$, and allocation function, $\widetilde{D}$, such that*

- *$\widetilde{P}$ is $(\widetilde{s}, \widetilde{m})$ measurable.*
- *For an unconstrained agent k, $\widetilde{D}_k \in \arg\max_{\widetilde{D}_k \in \mathcal{R}^n} E(U(\widetilde{W}_k) \mid \mathcal{F}_k)$, where $\mathcal{F}_k$ is the agent k's information set.*
- *For a constrained agent i, $\widetilde{D}_i^c \in \arg\max_{\widetilde{D}_i \in \mathcal{A}} E(U(\widetilde{W}_i) \mid \mathcal{F}_i)$, where $\mathcal{F}_i$ is the constrained agent i's information set and $\mathcal{A}$ is the constrained demand set.*
- *The market clearing condition is satisfied by: $w_i^{uc}\widetilde{D}_i + w_i^c \widetilde{D}_i^c + w_{ui}\widetilde{D}_{ui} = \widetilde{m}$, where $\widetilde{D}_i$ is unconstrained informed investor demand, $\widetilde{D}_i^c$ is constrained informed investor demand, and $\widetilde{D}_{ui}$ is uninformed investor demand.*

All REE models have a peculiar property; that is, equilibrium prices have dual effects: a substitution effect and an information effect (Admati (1985)). Besides clearing the market, as in Walrasian models, equilibrium prices also affect agent (here, uninformed investors) information sets by revealing information about the underlying, $\widetilde{v}$. In short, uninformed investors infer $\widetilde{v}$ from equilibrium prices. In addition, rational expectations equilibria require that prices are self-fulfilling. That is, equilibrium prices are such that, given shocks, the rational expectations embedded in the allocation functions (demand) are, in fact, realized. Hence, market clearing requires that equilibrium prices are established such that the following is true:

$$w_i^{uc}\widetilde{D}_i(\widetilde{s}, \widetilde{P}) + w_i^c \widetilde{D}_i^c(\widetilde{s}, \widetilde{P}) + w_{ui}\widetilde{D}_{ui}(\widetilde{P}) = \widetilde{m}. \tag{3}$$

The above equation can be rearranged into the following form:

$$w_i^{uc}\widetilde{D}_i(\widetilde{s}, \widetilde{P}) + w_i^c \widetilde{D}_i^c(\widetilde{s}, \widetilde{P}) = \widetilde{m} - w_{ui}\widetilde{D}_{ui}(\widetilde{P}). \tag{4}$$

We now consider a fictitious economy where asset supply is given by $\widetilde{m}^{fic} = \widetilde{m} - w_{ui}\widetilde{D}_{ui}(\widetilde{P})$. Suppose that the equilibrium price in the fictitious economy is given by $\widetilde{P}^{fic}$, the market clearing condition is then expressed as

$$w_i^{uc}\widetilde{D}_i(\widetilde{s}, \widetilde{P}^{fic}) + w_i^c \widetilde{D}_i^c(\widetilde{s}, \widetilde{P}^{fic}) = \widetilde{m}^{fic}. \tag{5}$$

Clearly, $\widetilde{P}^{fic} = \widetilde{P}$ is the equilibrium price of this fictitious economy. The values $\widetilde{P}$ and $\widetilde{P}^{fic}$ are equivalent sufficient statistics for $\widetilde{v}$ in the Blackwell sense (Blackwell Theorem, page 373, DeGroot (1986)).

Hence, prices for a fictitious economy with only informed agents fully characterize the information content of equilibrium prices for an economy with both informed and uninformed agents. In other words, using the prices for a fictitious economy with only informed agents to infer $\widetilde{v}$ is equivalent to using the prices for the economy with both informed and uninformed agents. This equivalence allows us to study uninformed investor inference and optimization behavior as well as how shocks are magnified and transmitted to asset prices through borrowing constraints and information asymmetry.

In the rest of this section, we first solve the uninformed investor inference and optimization problem and then obtain the equilibrium prices to study the properties of crises.

B. The Uninformed Investor Inference and Optimization Problems: The Source of Confusion

B.1. Sufficient Statistics and the Information about $\widetilde{v}$ Revealed by $\widetilde{P}^{fic}$

A sufficient statistic for uninformed investors to infer $\widetilde{v}$ is the equilibrium price for a fictitious economy with only informed investors (P^{fic}). To solve for P^{fic}, we first need to solve for informed investor demand. An informed investor maximizes expected payoff based on the signal ($\widetilde{s}$) received. However, some informed investors might be borrowing constrained. Lemma 1 outlines informed investor total demand.

LEMMA 1: *Informed investor demand is*

$$\widetilde{D}_i = w_i^c\left(1_{\{uc\}}\widetilde{D}_i^{uc} + 1_{\{bc\}}\widetilde{D}_i^{bc}\right) + w_i^{uc}\widetilde{D}_i^{uc}, \tag{6}$$

where[12]

$$1_{\{uc\}} = 1\left\{\widetilde{s} \le \kappa_1^{bc}\widetilde{P} + \kappa_0^{bc}\right\}, \quad \widetilde{D}_i^{uc} = d_s^{uc}\widetilde{s} - d_p^{uc}\widetilde{P}^{uc}, \tag{7}$$

$$1_{\{bc\}} = 1\left\{\widetilde{s} > \kappa_1^{bc}\widetilde{P} + \kappa_0^{bc}\right\}, \quad \widetilde{D}_i^{bc} = d_s^{bc}\widetilde{s} - d_p^{bc}\widetilde{P}^{bc} + d_0^{bc}. \tag{8}$$

All constants ($d_s^{uc}, d_p^{uc}, d_s^{bc}, d_p^{bc}, d_0^{bc}, \kappa_1^{bc}, \kappa_0^{bc}$) are defined in Appendix A.

Supply in this fictitious economy is represented by $\widetilde{m} - (1 - w_i)D_{ui}$. The equilibrium price is the one that clears the market, as expressed in Lemma 2.

LEMMA 2: *If the asset supply of this fictitious economy with only informed investors is $\widetilde{m} - (1 - w_i)D_{ui}$, denoted as $\widetilde{m}^{fic}$, there exists a unique equilibrium price:*

$$\widetilde{P}^{fic} = 1_{\{uc\}}\widetilde{P}^{uc} + 1_{\{bc\}}\widetilde{P}^{bc}, \tag{9}$$

where

$$1_{\{uc\}} = 1\left\{\widetilde{s} \le \kappa_1^{bc}\widetilde{P}^{fic} + \kappa_0^{bc}\right\}, \quad \widetilde{P}^{uc} = p_s^{uc}\widetilde{s} - p_m^{uc}\widetilde{m}^{fic}, \tag{10}$$

$$1_{\{bc\}} = 1\left\{\widetilde{s} > \kappa_1^{bc}\widetilde{P}^{fic} + \kappa_0^{bc}\right\}, \quad \widetilde{P}^{bc} = p_s^{bc}\widetilde{s} - p_m^{bc}\left(\widetilde{m}^{fic} - w_c b\right). \tag{11}$$

All constants ($p_s^{uc}, p_m^{uc}, p_s^{bc}, p_m^{bc}, \kappa_1^{bc}, \kappa_0^{bc}$) are defined in Appendix A.

[12] We use *bc* to denote being borrowing constrained and *uc* to denote being unconstrained.

To illustrate the intuition, we conduct a numerical analysis for an example chosen to reflect "reasonable" parameters, where risky securities are interpreted as the stock market portfolio. Boyer and Zheng (1998) find that, since the late 1980s, institutional investors other than pension funds and insurance companies hold about 15% of total market capitalization. Accordingly, we assume that 15% of investors are informed. We further assume that, among informed investors, a majority (in this case 14%) are borrowing-constrained. A close examination of equilibrium prices shows that the fraction of investors and their risk tolerances are not separately identified. For simplicity, we normalize investor risk tolerance to one. A key parameter of the model is the quality of the information signal received by informed investors. We assume that informed investors receive a high quality signal: the signal-to-noise ratio is 30. We also specify the parameters that describe noise trading (σ_m and $\overline{m}$) and the underlying fundamental asset (σ_v and $\overline{v}$). These parameters are chosen to equate the expected return on the risky security to 6% and the standard deviation to 20%.[13]

A second key parameter of the model is the borrowing constraint. We consider a case where an investor puts zero money down. He borrows money to invest in the stock market and faces a 30% margin constraint. A linearized version of the borrowing constraint yields an a of 3.825 and a b of -2.55.[14]

In this fictitious equilibrium, price is a piece-wise linear function of the state variables. More specifically, equilibrium prices are represented by kinked contour lines in the $(\widetilde{s}, \widetilde{m}^{fic})$ state space. The state space consists of two subspaces: borrowing-constrained and unconstrained (Figure 2).

From the graph in Figure 2, we see that contour lines have a steeper slope and are more densely distributed in constrained than unconstrained states. This means that, in a constrained region, a small positive (negative) supply shock can have a large downward (upward) price impact. Prices reveal information regarding $\widetilde{s}$ at different rates in these two regions.

B.2. The Uninformed Investor Inference and Optimization Problems

As stated earlier, uninformed investors are rational. They learn about primary state variables $(\widetilde{s}, \widetilde{m})$ from $\widetilde{P}^{fic}$. The previous section shows that $\widetilde{P}^{fic}$ reveals information about $\widetilde{s}$ differently depending on whether informed investors are constrained. Therefore, uninformed investors must infer both $\widetilde{s}$ and the informed investor constraint status. We denote the conjecture of the uninformed

[13] The interest rate is normalized to 0 in the model. Thus, the assumed return of 6% represents a 6% premium over the riskfree interest rate.

[14] Suppose that an investor borrows \$1 to invest in the risky asset. Since the riskfree asset is the numeraire asset, a CARA investor would price an asset with an expected 6% return and 20% standard deviation as 0.67, where 0.67 is the solution of $(1.06P - P)/0.2^2 = 1$. With \$1, he can purchase 1.5 shares. With a 30% margin, the maximum the investor can borrow is $0.7(1.5P - 1)$ and the maximum share of the risky asset he can hold is capped under $(1 + 0.7)(1.5 - 1/P)$. When we linearize this constraint around the mean of P (0.67), we get $a = 1.7/0.67^2 = 3.825$ and $b = -1.7/0.67 = -2.55$.

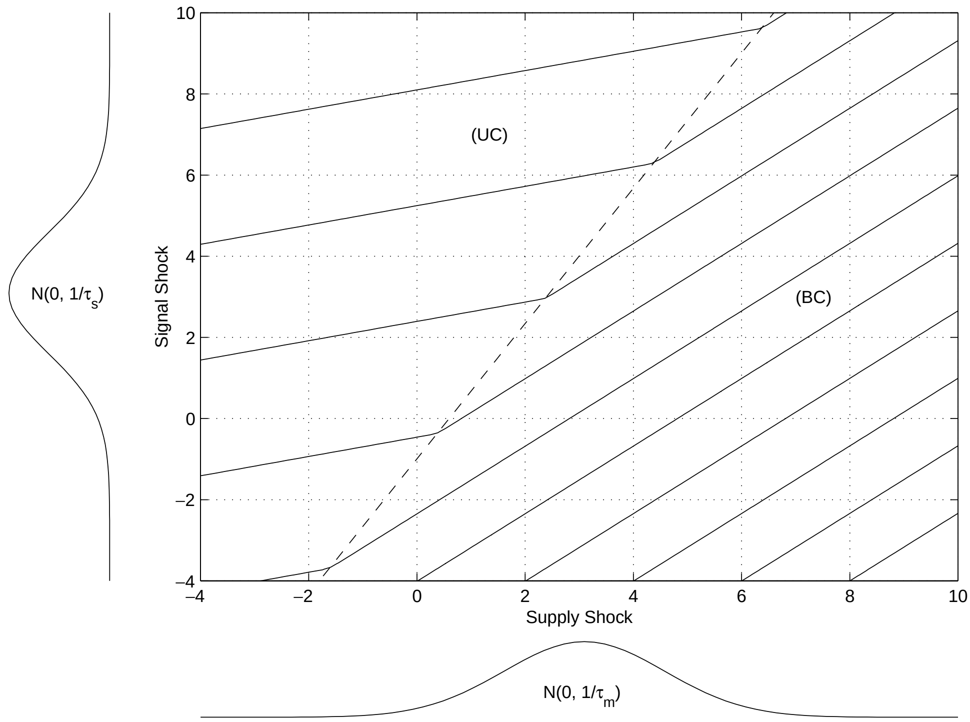

Figure 2. State space and price contours. The solid lines in the figure represent iso-price lines in the supply shock and signal shock $(\tilde{m}, \tilde{s})$ state space. The dashed line represents the boundary between the region where informed investors are not borrowing constrained and the region where some are borrowing constrained.

investor's optimal demand as $\widetilde{D}^*_{ui}$. The uninformed investor inference problem can be expressed as in Lemma 3.

LEMMA 3: *Given the equilibrium price function for the fictitious economy, uninformed investors estimate the conditional moments of the random payoff as*[15]

$$E[\tilde{v} \mid \widetilde{P}, \widetilde{D}_{ui}] = Pr^{uc}\widetilde{E}^{uc} + (1 - Pr^{uc})\widetilde{E}^{bc}, \tag{12}$$

$$\mathrm{Var}[\tilde{v} \mid \widetilde{P}, \widetilde{D}_{ui}] = Pr^{uc}\widetilde{V}^{uc} + (1 - Pr^{uc})\widetilde{V}^{bc}. \tag{13}$$

If informed investors are borrowing constrained, the conditional moments are[16]

$$\widetilde{E}^{bc} = e^{bc}_P\widetilde{P} - e^{bc}_{D_{ui}}\widetilde{D}^*_{ui} + e^{bc}_\lambda\tilde{\lambda}^{bc+} + e^{bc}_0, \; \widetilde{V}^{bc} = v_0 + v^{bc}_\delta(1 - \delta^{bc+}). \tag{14}$$

[15] We let Pr^{bc} and Pr^{uc} denote the conditional probabilities of informed investors being borrowing constrained and unconstrained, respectively.

[16] Unlike the general information asymmetry problem, the inferences that uninformed investors have to make, given the informed investor borrowing constraint, are on the truncated normal distributed variables. The closed-form expressions for conditional moments of truncated normal variables are known.

If informed investors are not borrowing constrained, the conditional moments are

$$\widetilde{E}^{uc} = e_P^{uc}\widetilde{P} - e_{D_{ui}}^{uc}\widetilde{D}_{ui}^{*} + e_{\lambda^{bc}}^{uc}\widetilde{\lambda}_{uc}^{bc-} + e_0^{uc},\ \widetilde{V}^{uc} = v_0 + v_\delta^{uc}\left(1 - \delta_{uc}^{bc-}\right). \qquad (15)$$

All constants ($e_P^{bc}, e_{D_{ui}}^{bc}, e_\lambda^{bc}, e_0^{bc}, v_0, v_\delta^{bc}, \delta^{bc+}, e_P^{uc}, e_{D_{ui}}^{uc}, e_{\lambda^{bc}}^{uc}, e_0^{uc}, v_\delta^{uc}, \delta_{uc}^{bc-}$) *are defined in Appendix B.*

For the simplicity of illustration, we use the mean-variance preference to specify the uninformed investor optimization problem[17]

$$\max_{\widetilde{D}_{ui}} (Pr^{bc}\widetilde{E}^{bc} + Pr^{uc}\widetilde{E}^{uc})\widetilde{D}_{ui} - \widetilde{D}_{ui}^2(Pr^{bc}\widetilde{V}^{bc} + Pr^{uc}\widetilde{V}^{uc})/(2\rho).$$

Hence, uninformed investor demand can be expressed as an implicit function. Uninformed investor optimal demand becomes a fixed point problem, as expressed in Result 1.

RESULT 1: *Uninformed investor optimal demand is a unique fixed point of the following:*

$$\widetilde{D}_{ui}^{*} = \frac{\rho(Pr^{bc}\widetilde{E}^{bc} + Pr^{uc}\widetilde{E}^{uc} - \widetilde{P})}{(Pr^{bc}\widetilde{V}^{bc} + Pr^{uc}\widetilde{V}^{uc})}. \qquad (16)$$

Proof: Given $g(\widetilde{D}_{ui}) = \frac{\rho(Pr^{bc}\widetilde{E}^{bc} + Pr^{uc}\widetilde{E}^{uc} - \widetilde{P})}{(Pr^{bc}\widetilde{V}^{bc} + Pr^{uc}\widetilde{V}^{uc})} - \widetilde{D}_{ui}$, it is immediate that $\lim_{\widetilde{D}_{ui}\to\infty} g(\widetilde{D}_{ui}) < 0$ and $\lim_{\widetilde{D}_{ui}\to-\infty} g(\widetilde{D}_{ui}) > 0$. By the Intermediate Value Theorem, at least one solution to $g(\widetilde{D}_{ui}) = 0$ exists. As $g(\widetilde{D}_{ui})$ is a decreasing function, there is only one solution to $g(\widetilde{D}_{ui}) = 0$. Hence, $\widetilde{D}_{ui}$ exists and is unique. Q.E.D.

The solution to the above fixed point problem is a unique real root of a simple cubic equation. The closed-form solution exists, but its expression is quite

[17] Maximizing a CARA utility function with truncated normal distributions results in the standard mean-variance result plus a correction term for the truncation. The correction term is a non-linear function of the truncation point, mean, variance, and risk tolerance. As demonstrated later in this section, the standard Grossman–Stiglitz setup leads to the linearity results (where the equilibrium price is a linear function of state variables and the demand of uninformed investors is a linear function of prices), and, hence, it is impossible to obtain a large equilibrium price decline in response to a small negative shock in a standard Grossman–Stiglitz setup. In contrast, the main results of this paper, such as the backward-bending demand of uninformed investors, arise from non-linearity. The mean-variance preference specification makes the non-linearity result harder to obtain as the additional non-linear term of mean, variance, and risk tolerance is dropped from the optimization problem on the CARA utility with a truncated normal distribution. In fact, the non-linearity of uninformed investor demand (i.e., the backward-bending demand) and equilibrium price function are more pronounced in the case of the CARA utility function with truncated normal distributions, except that the uninformed investor optimal demand is specified by a more complicated implicit function (Yuan (2003)). Therefore, to save on notation and for the clarity of illustration, we use mean-variance investors in our model.

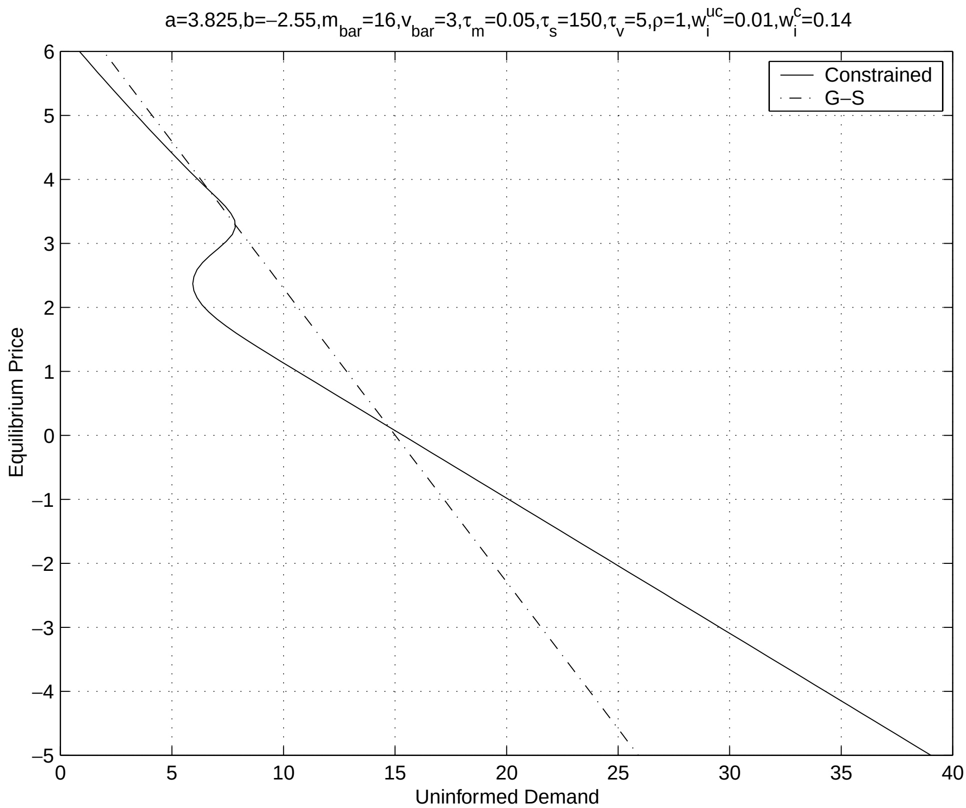

Figure 3. Uninformed investor optimal demand. The solid line in the figure represents the uninformed investor optimal demand schedule when some informed investors face borrowing constraints. The dotted line represents uninformed investor optimal demand in a Grossman–Stiglitz setup where no agents face borrowing constraints.

involved (Korn and Korn (1968)). Consequently, we provide a numeric example to clarify the intuition for the solution.

It is important to emphasize that uninformed investor demand is a nonlinear function of the equilibrium price. Figure 3 illustrates an example of uninformed investor demand. In the two extreme price regions (when prices are extremely high or extremely low), there is little uncertainty regarding informed investor constraint status and thus uninformed investor demand is linear. However, note that the slope in the intermediate region is not a simple average of the slopes in the two extreme regions. Instead, the demand curve can be backward-bending or steeper than either extreme region. Uninformed investors may appear "confused" when they demand more when the price rises and less when the price falls. In this environment, uninformed investors may submit a backward-bending demand curve as higher prices provide more information about the asset's fundamental value.

This variation of uninformed investor optimal investment behavior with respect to price results from the unique interaction between the substitution and the information effects in our model. In REE models, in addition to the Walrasian substitution effect, a price change may affect demand by affecting uninformed investor inferences in an information effect (Admati (1985)). This information effect captures changes in equilibrium price signal precision. If the asset price increases, uninformed investors may decrease demand for the asset due to the substitution effect. However, a higher asset price may also signal to uninformed investors that the underlying ($\widetilde{v}$) is more valuable. This information effect prompts uninformed investors to increase their demand upon a price increase.

In a linear Grossman–Stiglitz (1980) setup (See Gennotte and Leland (1990) and Wang (1994)), uninformed investors always exhibit a downward-sloping demand for the risky asset, indicating that the substitution effect always dominates the information effect. In addition, the information effect is fixed as prices reveal the same amount of information regardless of level. In fact, the downward-sloping demand curve indicates that uninformed investors never increase their estimate of the underlying value by more than the price increase.

In this setting, due to the borrowing constraint imposed on informed investors, the information effect varies with price. The signal precision of the equilibrium price may decrease as prices drop, because some informed investors may be constrained out of the market and thus unable to transmit their private signals to prices. To see this result clearly, we distinguish the uninformed investor inference problem from the standard G-S setting (Grossman and Stiglitz (1980)):

$$\frac{\partial E[\widetilde{v} \mid P, D_{ui}]}{\partial P} = \underbrace{Pr^{uc}\frac{\partial E^{uc}}{\partial P}}_{\text{G-S}} + \underbrace{(1 - Pr^{uc})\frac{\partial E^{bc}}{\partial P}}_{\text{G-S}} + \underbrace{\frac{\partial Pr^{uc}}{\partial P}(E^{uc} - E^{bc})}_{\text{Extra Information Effect}}, \tag{17}$$

$$\frac{\partial V[\widetilde{v} \mid P, D_{ui}]}{\partial P} = \underbrace{Pr^{uc}\frac{\partial V^{uc}}{\partial P}}_{\text{G-S}} + \underbrace{(1 - Pr^{uc})\frac{\partial V^{bc}}{\partial P}}_{\text{G-S}} + \underbrace{\frac{\partial Pr^{uc}}{\partial P}(V^{uc} - V^{bc})}_{\text{Extra Information Effect}}. \tag{18}$$

In addition to the standard G-S terms, uninformed investor inferences of conditional mean and conditional variance both have an extra term. Since $\widetilde{E}^{uc}$ differs from $\widetilde{E}^{bc}$ and $\widetilde{V}^{uc}$ differs from $\widetilde{V}^{bc}$, a price change may lead to a large change in the conditional mean estimation and a large change in conditional variance estimation, causing the information effect to vary with prices. More specifically, two factors determine the variation of price informativeness: the degree of uncertainty regarding informed investor constraint status (Pr^{uc}) and the information structure (including the sharpness of the informed investor's private signal, τ_s, and the fraction of informed investors, w_i^c and w_i^{uc}). The uncertainty regarding the informed investor's constraint status is a unique and additional uncertainty in this constrained economy (as compared with the standard Grossman–Stigliz setup). In this economy, uninformed investors demand different risk premiums as this uncertainty varies with prices. Information structure variables, such as

τ_s, may exacerbate the effect of this additional uncertainty on the uninformed investor's estimation problem regarding the underlying asset, because these variables influence the magnitude of information loss in the case of constrained informed investors.[18]

The degree of uncertainty regarding informed investor constraint status varies with price. A high price indicates a smaller probability that informed investors are borrowing constrained. In this case, the information effect may overwhelm the substitution effect, causing uninformed investors to increase the asset value expectation by more than the price increase because of greater certainty regarding informed investor constraint status. Conversely, as prices fall, signal precision deteriorates. In this case, uninformed investors may decrease the asset value expectation by more than the price drop because of the greater uncertainty regarding informed investor constraint status. Uninformed investors may become rationally confused about the quality of the price as a signal for the underlying asset.[19] Therefore, confusion in this study is defined as the occasion when the information effect dominates the substitution effect.

C. Borrowing Constraints and Information Asymmetry as Shock Multipliers: Crises

C.1. Asset Market Crises: Definition and Source

To define the asset market crisis, we first need to solve for the equilibrium price. The equilibrium price can be obtained from the market clearing condition, where the excess demand is $\widetilde{m} - w_{ui}\widetilde{D}_{ui} - w_i\widetilde{D}_i$.

Figure 4 graphs the total demand for the risky asset for the parameters in our earlier example. It shows that the market demand becomes extremely inelastic when the price is between (2, 3). Therefore, a small supply shock could have a destabilizing effect.

Figure 4 also illustrates a case of multiple equilibria. Many studies have used multiple equilibria as an explanation for large asset price movements (Krugman (1998), Calvo (1999), Drazen (1999), Gennotte and Leland (1990)), although multiplicity arises from entirely different reasons in these studies. The source of multiple equilibrium prices in our model is due to increased confusion among uninformed investors rather than the informed investor's borrowing constraint. The graph in Figure 4 shows that the risky asset is not a Giffen good for informed investors; rather, these investors exhibit less elastic demand

[18] Analytically, the elements of the information structure (the sharpness of the informed investor's signal and the fraction of informed investors) are encompassed in the conditional means and variances.

[19] In the intermediate price region, when informed investors are probably borrowing-constrained, the extra information effect term $\frac{\partial Pr^{uc}}{\partial \widetilde{P}}(E^{uc} - E^{bc})$ dominates. As $E^{uc} > E^{bc}$, an increase in price may result in a larger increase in expectation, that is, $\frac{\partial E[\widetilde{v} \mid \mathcal{F}_{ui}]}{\partial \widetilde{P}} > 1$.

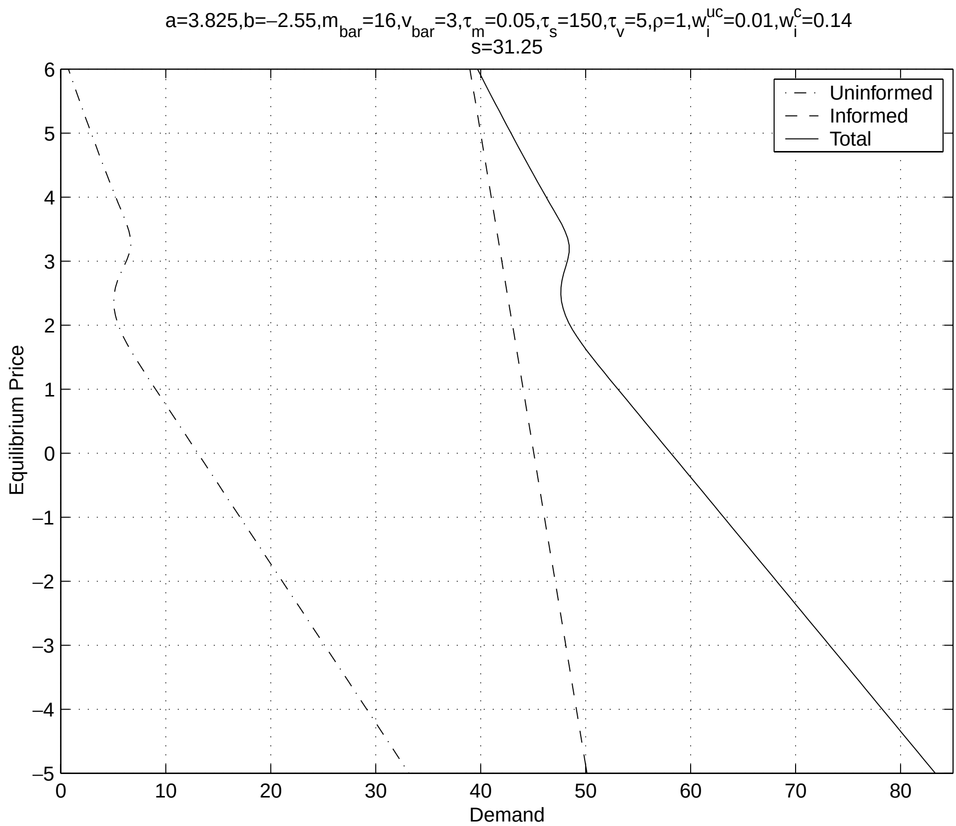

Figure 4. Demands for the risky asset. The dash line, the dash-dotted line, and the solid line represent informed, uninformed, and combined demand for the risky asset, respectively.

when prices are too low, due to their borrowing constraints. On the other hand, the risky asset is a Giffen good for uninformed investors, whose backward-bending demand (graphed in Figure 3) drives the shape of the excess demand function in Figure 4.

However, multiple equilibria are not the only explanation for market crises in our model. Even if multiple equilibria do not occur, market demand elasticity could drop precipitously due to uninformed investors' reluctance to support asset prices. As a result, equilibrium price sensitivity to supply and signal shocks can change dramatically with prices (Figure 5). Asset market prices are more sensitive to shocks when prices are low because of both the confusion of uninformed investors and the inability of informed investors to take advantage of arbitrage opportunities. Therefore, we define an asset market crisis as a large price drop in response to a small shock to the economic environment. Specifically, we interpret shocks as shocks to either the asset's noise supply ($\widetilde{m}$) or the asset's fundamental payoff ($\widetilde{v}$). The crisis is caused by a

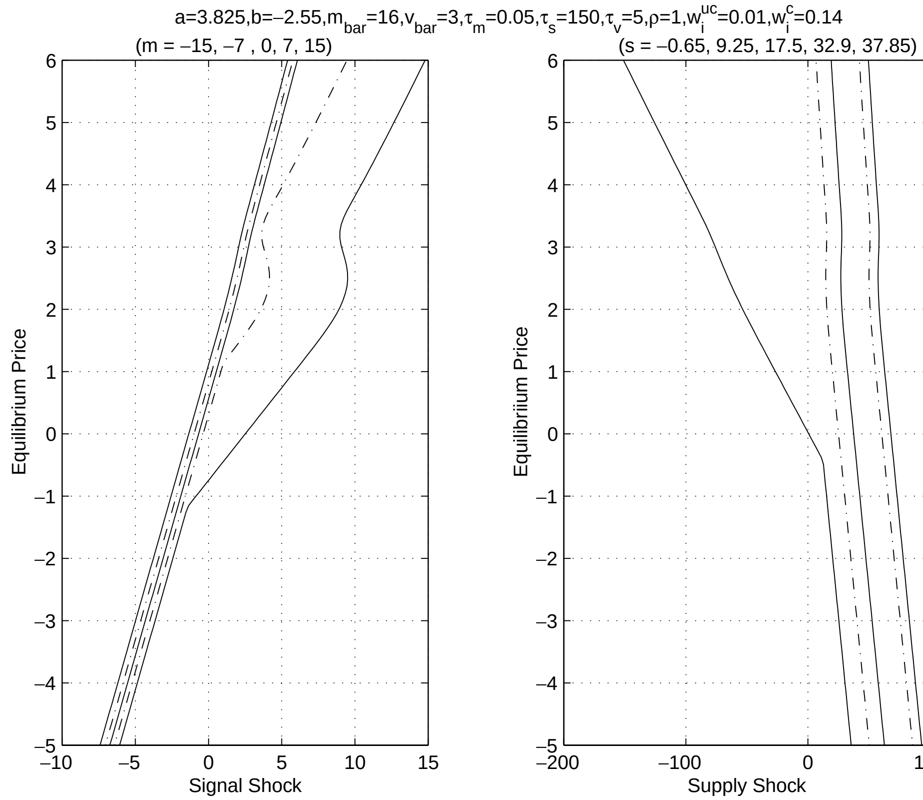

Figure 5. Price sensitivity to signal and supply shocks. The lines in the left graph represent equilibrium price as a function of the signal shock when the supply shock is -15, -7, 0, 7, and 15, respectively. The lines in the right graph represent equilibrium price as a function of the supply shock when the signal shock is -0.65, 9.25, 17.5, 32.9, and 37.85, respectively.

backward-bending or very steep excess demand curve. The following result characterizes the reaction of the equilibrium price to shocks.

RESULT 2: *The rational expectations equilibrium price function, P, for the borrowing-constrained economy is given by the following implicit function:*

$$\widetilde{P} = 1_{uc}\left(p_s^{uc}\widetilde{s} - p_m^{uc}\widetilde{m} + p_{D_{ui}}^{uc}\frac{\rho(E(\widetilde{v}\mid\widetilde{P}) - \widetilde{P})}{Var(\widetilde{v}\mid\widetilde{P})}\right) + 1_{bc}\left(p_0^{bc} + p_s^{bc}\widetilde{s} - p_m^{bc}\widetilde{m} + p_{D_{ui}}^{bc}\frac{\rho(E(\widetilde{v}\mid\widetilde{P}) - \widetilde{P})}{Var(\widetilde{v}\mid\widetilde{P})}\right), \tag{19}$$

where

$$1_{uc} = 1(\widetilde{s} \le \kappa_1^{bc}\widetilde{P} + \kappa_0^{bc}),\ 1_{bc} = 1(\widetilde{s} > \kappa_1^{bc}\widetilde{P} + \kappa_0^{bc}). \tag{20}$$

All constants ($p_s^{uc}, p_m^{uc}, p_{D_{ui}}^{uc}, p_0^{bc}, p_s^{bc}, p_m^{bc}, p_{D_{ui}}^{bc}, \kappa_1^{bc}, \kappa_0^{bc}$) *are defined in Appendix C.*

C.2. Properties of Asset Market Crises

To better understand what determines the likelihood, magnitude, and nature of asset market crises in a constrained information asymmetry environment, we conduct comparative static analyses on the equilibrium price derived in Result 2. Several observations emerge from the model.

Observation 1: The crisis result is the unique interaction of two market imperfections: borrowing constraints and information asymmetry. To show the interaction of these two market imperfections, in Figure 6 we graph the sensitivity of equilibrium prices to signal and supply shocks for a fictitious informed-investors-only economy, for a standard Grossman–Stiglitz setup (1980) without borrowing constraints, and for the borrowing-constrained economy studied in this paper.[20] The multiplier effect of borrowing constraints and information asymmetry can be seen clearly in graphs in Figure 6. Although each imperfection generates some market demand inelasticity on its own, the equilibrium asset price becomes sensitive to shocks in the intermediate price region when both imperfections are present. The magnitude of such sensitivity increases with the degree of information asymmetry and the level of binding borrowing constraints.

Observation 2: Crises may occur independent of the asset's value and are more likely to occur in the intermediate region of price changes. As long as an asset is priced in a region where uninformed investors may become confused about price signal precision, a crisis may occur regardless of the value of the underlying asset. In the intermediate price region, the degree of uncertainty about informed investor constraint status is highest, and hence crises are more likely to occur. This observation is important, since it distinguishes our model from alternative explanations such as Romer (1993) and Hong and Stein (2003), where crises result in the release of information.

Observation 3: The equilibrium price exhibits asymmetry. Equilibrium prices are more sensitive to shocks when prices are lower. In Figure 7, a kernel estimation of the price distribution for the borrowing-constrained economy, based on 200 random realizations of equilibrium price, has a fatter tail than the standard Grossman–Stigliz model (1980) predicts and is negatively skewed.[21] This observation provides a contrast to models where market crises are as likely as bubbles.

[20] By construction, the direct effect of the borrowing constraint on the asset price is quantifiable by examining asset price determination in our fictitious informed-investors-only economy. Information asymmetry compounds this borrowing constraint by affecting uninformed investor investment behavior. This indirect effect has no explicit form, but we can gauge its effect by examining the standard Grossman–Stiglitz economy (1980).

[21] It is interesting to note that the shape of the price distribution for the borrowing-constrained economy is two-humped. The dip between the two humps is the intermediate price region where uncertainty about the constraint status of informed investors is greatest and where the risk premium demanded by uninformed investors is largest to cover for this additional risk.

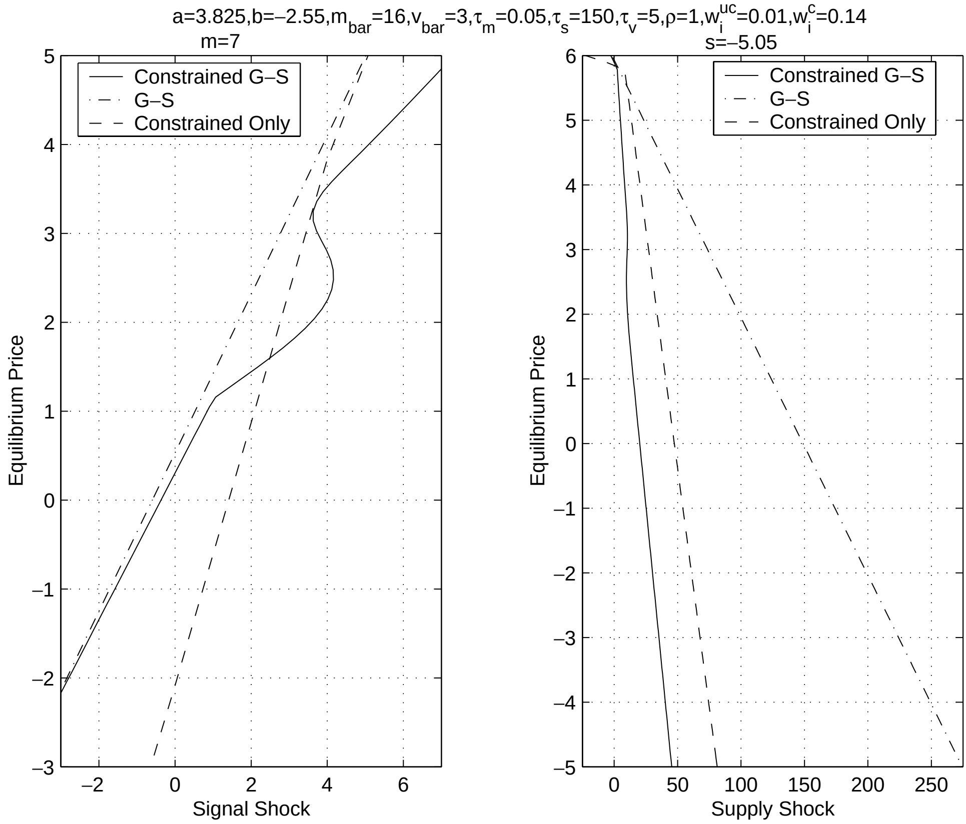

Figure 6. Price sensitivity to shocks: Decomposition. The solid line in the left (right) graph represents equilibrium price as a function of the signal (supply) shock when the realized supply (signal) shock is 7 (−5.05) in an economy with borrowing constraints and asymmetric information. The dashed line in the left (right) graph represents equilibrium price as a function of the signal (supply) shock in an economy with borrowing constraints. The dash-dotted line in the left (right) graph represents equilibrium price as a function of the signal (supply) shock in an economy with asymmetric information.

II. Borrowing Constraints and Information Asymmetry as Spill-Over Mechanisms: Contagion

In a one-risky-asset non-linear REE economy, a large price movement can occur with only a small shock to the asset's fundamental value. In this section, we demonstrate that, in a two-risky-asset non-linear REE economy, a large price movement can occur even without such a shock. More specifically, while there are no shocks to an asset, its price can be affected by a shock to another asset even though these two assets have independent values. This contagion effect is defined as a phenomenon where idiosyncratic shocks unique to one asset market affect asset prices in unrelated markets.

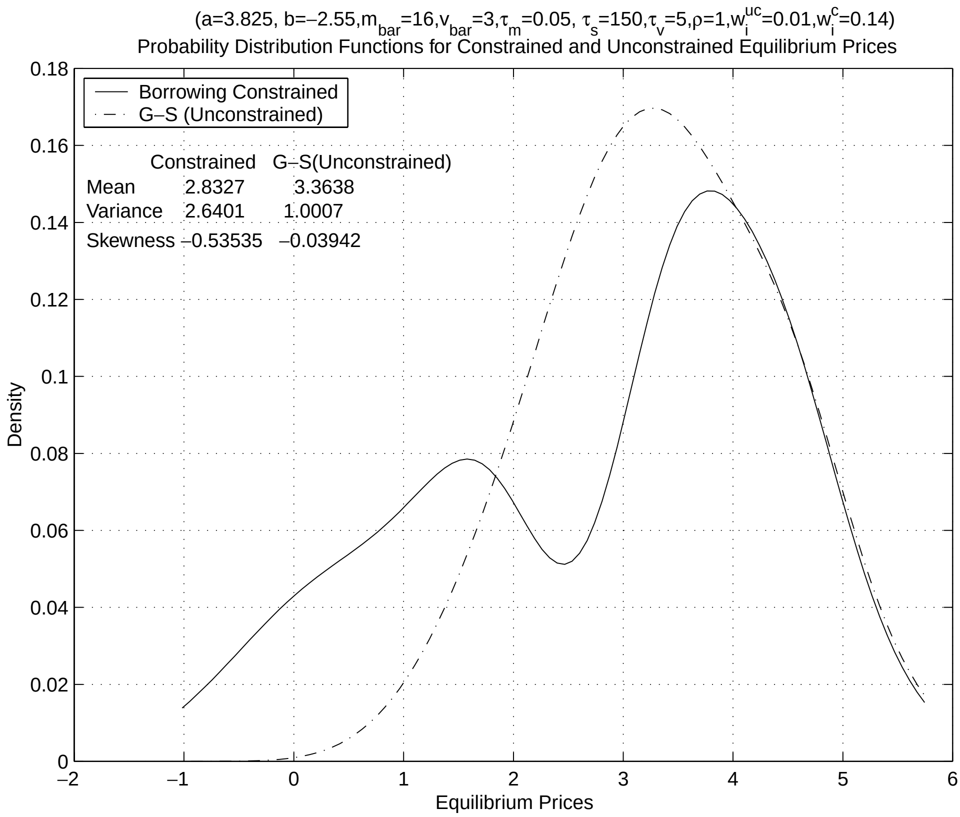

Figure 7. Kernel estimation of the equilibrium price distribution. The solid line in the figure represents the kernel estimation of the equilibrium price distribution based on 200 draws of $(\widetilde{s}, \widetilde{m})$ in an economy with both borrowing constraints and asymmetric information. The dash-dotted line represents the kernel estimation of the equilibrium price distribution for an economy with only asymmetric information.

To illustrate this contagion result, we investigate the same economy as defined in Section II, but now with one risk-free asset and *two identical and independent* risky assets. For this economy, we demonstrate that contagion occurs even when the fundamental values of the risky assets are uncorrelated ($\text{cov}(\widetilde{v}_1, \widetilde{v}_2) = 0$, $\text{cov}(\widetilde{m}_1, \widetilde{m}_2) = 0$), and informed investors receive independent signals regarding $\widetilde{v}_1$ and $\widetilde{v}_2$. The borrowing constraint for this economy is the same as that defined in Section I. This leads to Definition 3.

DEFINITION 3: *Let j index assets* 1 *and* 2; *the borrowing-constrained informed investor demand for asset j, $D^c_{j_i}$, is constrained to a set, $\mathcal{A}_j$, characterized as follows:*[22]

[22] Again we assume that only a fraction of informed investors are constrained. The rationales for restrictions on a_1 and a_2 are stated in Footnotes 9 and 10, respectively.

$$\mathcal{A}_j = \{y \in \mathcal{R} : y \le n(\widetilde{P}_1, \widetilde{P}_2)\}, n(\widetilde{P}_1, \widetilde{P}_2) = a_1\widetilde{P}_1 + a_2\widetilde{P}_2 + b, \tag{21}$$

where $0 < a_1 < w_i^{uc}\rho(\tau_{v_1} + \tau_{s_1})/w_i^c$ *and* $0 < a_2 < w_i^{uc}\rho(\tau_{v_2} + \tau_{s_2})/w_i^c$.

The intuition behind this constraint on informed investor demand is straightforward. The upper bound of informed investor demand is the borrowing constraint. However, unlike the single-risky-asset case, the upper demand bound is a function of both asset prices. Again this definition is not surprising: Speculators in each respective asset market can use one risky asset as collateral to borrow and invest in the other market. Hence, borrowing capacity is determined by the asset prices of both markets.

In our two-risky-asset case, informed investors act as market speculators by arbitraging the mispricing between the two risky assets. However, due to borrowing constraints, their arbitrage ability is limited and consequently their investment decisions transmit little information. Such uninformative asset prices may confuse uninformed investors who have prices as signals for asset values. This confusion may cause contagion in each of the two asset markets. To illustrate this point, we consider the case when asset 1 is underpriced due to a negative idiosyncratic shock and informed investors must hold asset 2 to obtain the speculative funds needed to arbitrage asset 1's underpricing. Thus, borrowing constraints link the prices of these two independent assets. In this case, uninformed investors, who correctly infer that the information content of each asset price is lower, demand an information-disadvantaged premium to hold both assets. This demand places a downward pressure on the prices of both assets, thus creating contagion. Moreover, contagion across markets is more likely when prices decline. When prices are high, informed investors are not borrowing-constrained and can arbitrage against over-priced assets. In this section, we first discuss how the borrowing constraint in the two-risky-asset setting creates price linkages as well as confusion among uninformed investors. We then illustrate how contagion might arise in this setting.

A. Linkages of Equilibrium Prices Due to Borrowing Constraints

In a two-risky-asset economy, unconstrained informed investors submit a demand schedule for asset j conditional on both the received signal and the observed price:

$$\widetilde{D}_{j_i}^{uc} = \rho\tau_{s_j}\widetilde{s}_j - \rho(\tau_{v_j} + \tau_{s_j})\widetilde{P}_j. \tag{22}$$

However, the demand schedules for borrowing-constrained informed investors are more complicated. Four possible scenarios exist for these constrained informed investors: (1) not constrained; (2) borrowing constrained in Market 1 and unconstrained in Market 2; (3) borrowing constrained in Market 2 and unconstrained in Market 1; and (4) borrowing constrained in both markets. These scenarios are outlined in Lemma 4.

LEMMA 4: *Informed investor total demand for asset j ($j = 1, 2$) is:*

$$\widetilde{D}_{j_i} = w_i^{uc}\widetilde{D}_{j_i}^{uc} + w_i^{c}\left(1_1\widetilde{D}_{j_i}^{1} + 1_2\widetilde{D}_{j_i}^{2} + 1_3\widetilde{D}_{j_i}^{3} + 1_4\widetilde{D}_{j_i}^{4}\right), \tag{23}$$

where $1_l(l = 1, 2, 3, 4)$ and l index four possible scenarios:

$$\widetilde{D}_{2_i}^{3} = \widetilde{D}_{2_i}^{4} = \widetilde{D}_{1_i}^{2} = \widetilde{D}_{1_i}^{4} = a_1\widetilde{P}_1 + a_2\widetilde{P}_2 + b, \tag{24}$$

$$\widetilde{D}_{1_i}^{1} = \widetilde{D}_{1_i}^{3} = D_{1_i}^{uc} = \rho\tau_{s_1}\widetilde{s}_1 - \rho(\tau_{v_1} + \tau_{s_1})\widetilde{P}_1, \quad and \tag{25}$$

$$\widetilde{D}_{2_i}^{1} = \widetilde{D}_{2_i}^{2} = D_{2_i}^{uc} = \rho\tau_{s_2}\widetilde{s}_2 - \rho(\tau_{v_2} + \tau_{s_2})\widetilde{P}_2. \tag{26}$$

All constants and indicator functions are defined in Appendix D.

Given these scenarios, we can now specify the equilibrium prices for an informed-investor-only economy in Lemma 5.

LEMMA 5: *Suppose the supply for asset 1 is $\widetilde{m}_1^{fic} (= \widetilde{m}_1 - (1 - w_i)D_{1_{ui}})$ and for asset 2 is $\widetilde{m}_2^{fic} (= \widetilde{m}_2 - (1 - w_i)D_{2_{ui}})$; there exist unique equilibrium prices for an economy with only informed investors:*

$$\widetilde{P}_j = 1_1\widetilde{P}_j^{1} + 1_2\widetilde{P}_j^{2} + 1_3\widetilde{P}_j^{3} + 1_4\widetilde{P}_j^{4}, \tag{27}$$

where asset index $j = 1, 2$,

$$\widetilde{P}_2^{1} = \widetilde{P}_2^{2} = p_{s_2}^{uc}\widetilde{s}_2 - p_{m_2}^{uc}\widetilde{m}_2^{fic}, \tag{28}$$

$$\widetilde{P}_2^{3} = w_i^{uc}\rho\tau_{s_2}\widetilde{s}_2/d_{bc_2} + a_1 w_i^{c}\widetilde{P}_1^{3}/d_{bc_2} + b w_i^{c}/d_{bc_2} - \widetilde{m}_2^{fic}/d_{bc_2}, \tag{29}$$

$$\widetilde{P}_2^{4} = w_i^{uc}\widetilde{s}_2/d_{bc_2} - \widetilde{m}_2^{fic}/d_{bc_2} + w_i^{c}\big(a_1\widetilde{P}_1^{4} + b\big)/d_{bc_2}, \tag{30}$$

$$\widetilde{P}_1^{1} = \widetilde{P}_1^{3} = p_{s_1}^{uc}\widetilde{s}_1 - p_{m_1}^{uc}\widetilde{m}_1^{fic}, \tag{31}$$

$$\widetilde{P}_1^{2} = w_i^{uc}\rho\tau_{s_1}\widetilde{s}_1/d_{bc_1} + a_2 w_i^{c}\widetilde{P}_2^{2}/d_{bc_1} + b w_i^{c}/d_{bc_1} - \widetilde{m}_1^{fic}/d_{bc_1}, \quad and \tag{32}$$

$$\widetilde{P}_1^{4} = w_i^{uc}\widetilde{s}_1/d_{bc_1} - \widetilde{m}_1^{fic}/d_{bc_1} + w_i^{c}\big(a_2\widetilde{P}_2^{4} + b\big)/d_{bc_1}. \tag{33}$$

All constants ($p_{s_2}^{uc}, p_{m_2}^{uc}, d_{bc_2}, p_{s_1}^{uc}, p_{m_1}^{uc}, d_{bc_1}$) are defined in Appendix E.

Note that, in this fictitious economy, the prices of assets 1 and 2 are independent when potentially constrained informed investors are not constrained. On the other hand, if borrowing-constrained informed investors are constrained in Market 1 (2), the price of asset 1 (2) will be affected by an idiosyncratic supply or signal shock in Market 2 (1), because the price of asset 2 (1) determines informed investor borrowing capacity. If borrowing-constrained informed investors are constrained in both markets, either market's supply or signal shock will affect prices in the other market.

B. Contagion as an Equilibrium Outcome in an Economy with Borrowing Constraints and Information Asymmetry

In Section II.A, we established the link between the equilibrium prices of two independent asset markets due to informed investor borrowing constraints. Here, we study the magnitude of this co-movement under borrowing constraints and information asymmetry. Specifically, we solve for the uninformed investor's optimization problem and then obtain the equilibrium price for each asset market.

Similar to the one-risky-asset economy, uninformed investors estimate the state variables $(\widetilde{s}, \widetilde{m})$ from their own information set. To do so, they first estimate the probability of each of the four possible scenarios of informed investor constraint given their information set. They then submit a demand that maximizes their utility, conditional on the information content of the market price. To illustrate, we use mean-variance preference to derive uninformed investor demand for each risky asset. Again, all conditional moments are functions of $\widetilde{P}_1$, $\widetilde{P}_2$, $\widetilde{D}^*_{1_{ui}}$, and $\widetilde{D}^*_{2_{ui}}$. $\widetilde{D}^*_{1_{ui}}$ and $\widetilde{D}^*_{2_{ui}}$ are conjectures of uninformed investor optimal demands and are included in their information sets. Hence, uninformed investor demand can be expressed in an implicit function, and solving for uninformed investor optimal demand becomes a fixed-point problem, as in the one-risky-asset case. The following result is similar to that for the one-risky-asset case.

LEMMA 6: *Uninformed investor optimal demand for asset $j (j = 1, 2)$ is a unique fixed point of the following:*

$$\widetilde{D}^*_{j_{ui}} = \frac{\rho\left(E\left[\widetilde{v}_j \mid \widetilde{P}_1, \widetilde{P}_2, \widetilde{D}^*_{1_{ui}}, \widetilde{D}^*_{2_{ui}}\right] - \widetilde{P}\right)}{\mathrm{Var}\left[\widetilde{v}_j \mid \widetilde{P}_1, \widetilde{P}_2, \widetilde{D}^*_{1_{ui}}, \widetilde{D}^*_{2_{ui}}\right]}. \tag{34}$$

The equilibrium price is obtained from the market clearing condition for each asset market. This leads to Result 3.[23]

RESULT 3: *The rational expectations equilibrium price function, P, for the borrowing-constrained economy is given by the following implicit function:*

$$\widetilde{P}_1 = 1_{uc_1}\left(p^{uc}_{s_1}\widetilde{s}_1 - p^{uc}_{m_1}\widetilde{m}_1 + p^{uc}_{D_{1ui}}\frac{\rho(E(\widetilde{v}_1 \mid \widetilde{P}_1, \widetilde{P}_2) - \widetilde{P}_1)}{\mathrm{Var}\,(\widetilde{v}_1 \mid \widetilde{P}_1, \widetilde{P}_2)}\right)$$

$$+\, 1_{bc_1}\left(p^{bc}_{0_1} + p^{bc}_{s_1}\widetilde{s}_1 - p^{bc}_{m_1}\widetilde{m}_1 + p^{bc}_{D_{1ui}}\frac{\rho(E(\widetilde{v}_1 \mid \widetilde{P}_1, \widetilde{P}_2) - \widetilde{P}_1)}{\mathrm{Var}\,(\widetilde{v}_1 \mid \widetilde{P}_1, \widetilde{P}_2)}\right), \tag{35}$$

[23] The proofs for this section are omitted as the results are straightforward extensions of the results in Section II.

$$\widetilde{P}_2 = 1_{uc_2}\left(p^{uc}_{s_2}\widetilde{s}_2 + p^{uc}_{m_2}\widetilde{m}_2 - p^{uc}_{D_{2ui}}\frac{\rho(E(\widetilde{v}_2 \mid \widetilde{P}_1, \widetilde{P}_2) - \widetilde{P}_2)}{\mathrm{Var}(\widetilde{v}_2 \mid \widetilde{P}_1, \widetilde{P}_2)}\right)$$
$$+ 1_{bc_2}\left(p^{bc}_{0_2} + p^{bc}_{s_2}\widetilde{s}_2 + p^{bc}_{m_2}\widetilde{m}_2 - p^{bc}_{D_{2ui}}\frac{\rho(E(\widetilde{v}_2 \mid \widetilde{P}_1, \widetilde{P}_2) - \widetilde{P}_2)}{\mathrm{Var}(\widetilde{v}_2 \mid \widetilde{P}_1, \widetilde{P}_2)}\right), \quad (36)$$

where

$$1_{uc_1} = 1\{\kappa^{bc}_{1\widetilde{P}_1}\widetilde{P}_1 + \kappa^{bc}_{1\widetilde{P}_2}\widetilde{P}_2 + \kappa^{bc}_{0^1} \geq \widetilde{s}_1\}, 1_{bc_1} = 1\{\widetilde{s}_1 > \kappa^{bc}_{1\widetilde{P}_1}\widetilde{P}_1 + \kappa^{bc}_{1\widetilde{P}_2}\widetilde{P}_2 + \kappa^{bc}_{0^1}\},$$
$$1_{uc_2} = 1\{\kappa^{bc}_{2\widetilde{P}_2}\widetilde{P}_2 + \kappa^{bc}_{2\widetilde{P}_1}\widetilde{P}_1 + \kappa^{bc}_{0^2} \geq \widetilde{s}_2\}, 1_{bc_2} = 1\{\widetilde{s}_2 > \kappa^{bc}_{2\widetilde{P}_2}\widetilde{P}_2 + \kappa^{bc}_{2\widetilde{P}_1}\widetilde{P}_1 + \kappa^{bc}_{0^2}\}.$$

All constants $(p^{uc}_{s_j}, p^{uc}_{m_j}, p^{uc}_{D_{jui}}, p^{bc}_{0_j}, p^{bc}_{s_j}, p^{bc}_{m_j}, p^{bc}_{D_{jui}}, \kappa^{bc}_{1\widetilde{P}_1}, \kappa^{bc}_{1\widetilde{P}_2}, \kappa^{bc}_{0^1}, \kappa^{bc}_{2\widetilde{P}_2}, \kappa^{bc}_{2\widetilde{P}_1}, \kappa^{bc}_{0^2},$ $j = 1, 2)$ *are defined as in Result* 2.

It is evident from Result 3 that the equilibrium price of one asset market is affected by idiosyncratic shocks in the other market. Again, uninformed investor inference and demand behavior creates contagion across the two independent markets. Even when informed investors are not constrained, uninformed investors rationally incorporate the probability of informed investors being constrained into their inferences about the value of the underlying assets. Thus, their demand schedule can be backward-bending in each asset price, because higher prices are more informative. For example, informed investors are not borrowing constrained in the case shown in Figure 8. Given both signals, their demand for the risky asset 1 is a function of only the asset 1 price, and is independent of the asset 2 price. In contrast, in this example, uninformed investor demand for the risky asset 1 is backward-bending, and the size of the backward bending region is smaller when the asset 2 price is higher (Figure 8). This pattern is due to the fact that the uninformed investor's inferences about Market 1 could be influenced by price movements in Market 2. Uninformed investors are rational. They incorporate asset 2 prices into their information set to infer the asset 1 payoff. Higher asset 2 prices indicate a lower likelihood that informed investors are constrained in Market 1. Hence, asset 1 prices are more informative. Conversely, a lower asset 2 price indicates a less informative asset 1 price. This inference problem causes the contagion phenomenon, creating a positive correlation between the prices of two assets with independent fundamental values.

Our contagion result is an extension of the crisis result in the one-risky-asset case. In addition to crisis properties, contagion exhibits the following characteristics:

Observation 1: Contagion can occur in asset markets with independent values. We have shown that an idiosyncratic shock specific to one asset can affect the price of another asset even when their values are *independent*. There is no other connection between markets except the potential of borrowing constraint

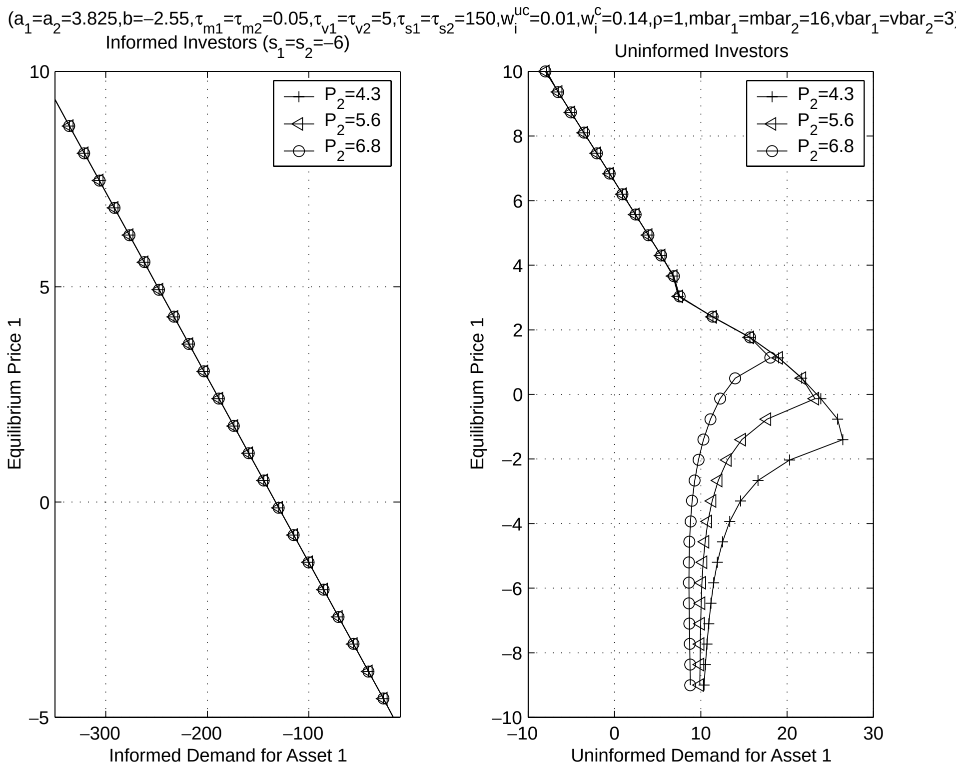

Figure 8. Price and investor demand for asset 1. The lines with $+$, with $\triangleleft$, with $\circ$ in the left (right) graph represent informed (uninformed) investor demand for asset 1 as a function of asset 1 price when asset 2 price is 4.3, 5.6, and 6.8, respectively. The informed investor demand schedules in the left graph are drawn when the realized signal shocks in Markets 1 and 2 are both -6.

on informed investors that alters their demands (and thus those of uninformed investors) in the other market. Kodres and Pritsker (2002) also find contagion occurs without the realization of fundamentals. However, their mechanism is through the existence of common, random factors affecting the fundamental value along with the misinterpretations of asymmetrically informed agents.

Observation 2: Contagion is more likely during market downturns than upturns. This distinction reflects the greater likelihood of constraints in a down market. Hong and Stein (2003) also find that contagion occurs asymmetrically. However, they attribute this result to a short-sales constraint.

Observation 3: Contagion occurs less frequently than crises. In the case of multiple risky assets, uninformed investors observe multiple prices and hence receive more signals than they do in the single-risky-asset case. Thus, their inference about the underlying asset value is more precise. The corresponding empirical prediction is that not all crises are followed by contagion.

III. Concluding Remarks

In this paper, we present a non-linear REE model of asset prices and use this model to study the determinants of asset market crises and contagion in an economy with borrowing constraints and information asymmetry.

In this borrowing-constrained economy, prices shape uninformed investor expectations. However, due to borrowing constraints, the informed investor's ability to transmit information to prices varies with the asset prices. When a small negative shock occurs, uninformed investors are uncertain whether informed investors are restricted in transmitting information to prices, and hence demand a large information-disadvantaged premium to hold risky assets. This demand exacerbates the price decline. Therefore, even though the underlying asset fundamental remains the same, a crisis may occur. This model of constrained information asymmetry explains most empirical irregularities associated with crises: large price movements following small fundamental shocks as well as asymmetric price distribution.

A two-risky-asset extension of the model explains how a negative idiosyncratic shock to one risky asset market may lead uninformed investors to rationally demand information-disadvantaged premiums across risky asset markets. This demand is due to the uncertainty about informed investor borrowing constraint status. Hence, markets where asset values are independent and share no risk factors may be vulnerable to contagion, with assets highly correlated during market downturns. It also shows that contagion occurs less frequently than single-market crises.

In addition, our model leads to several empirical predictions. First, crises and contagion are not tail-end events, but are likely to occur after small shocks. Second, extreme price movements are likely to occur in markets where information asymmetry is most severe and borrowing constraints are most binding, especially during market downturns. That is, asset returns exhibit more skewness in markets where investments are highly specialized and agency problems are more acute. Thirdly, crises propagate not through common shocks or hedging demands, but through investor borrowing constraints. For example, asset markets in countries with the same group of investors should be linked in times of financial upheaval.

Finally, the model also has implications for investment and macroeconomic policies. If falling prices inhibit borrowing and the transmission of information through trading, government may intervene in the stock market by providing liquidity and preventing borrowing constraints that could aggravate a stock market crisis or contagion.[24]

[24] In August 1998, HKMA injected HK$118 billion ($15.1 billion) into the Hong Kong stock market to fend off speculative attacks on Hong Kong dollars (*The Economist*, October 9, 1999). This move aims to keep stock market prices stable, since falling stock prices could inhibit borrowing based on asset collateral.

Appendix A

Proofs of Lemma 1 and Lemma 2: The constrained informed investor optimization problem is

$$\max_{\widetilde{D}_i^c} E_0\big[U\big(\widetilde{v}, \widetilde{D}_i^c \,\big|\, \widetilde{s}, \widetilde{m}^{fic}\big] - \lambda\big(\widetilde{D}_i^c - a\widetilde{P} - b\big).$$

The unconstrained informed investor optimization problem is

$$\max_{\widetilde{D}_i^{uc}} E_0\big[U\big(\widetilde{v}, \widetilde{D}_i^{uc}\big) \,\big|\, \widetilde{s}, \widetilde{m}^{fic}\big].$$

The equilibrium market prices for the informed-only economy are determined by the market clearing conditions:

$$w_i^{uc}\widetilde{D}_i^{uc} + w_i^c\widetilde{D}_i^c = \widetilde{m}^{fic}. \tag{A1}$$

To prove Lemma 1, we solve the above optimization and find that informed investor demand is

$$\widetilde{D}_i = w_i^c\big(1_{\{uc\}}\widetilde{D}_i^{uc} + 1_{\{bc\}}\widetilde{D}_i^{bc}\big) + w_i^{uc}\widetilde{D}_i^{uc}, \tag{A2}$$

where

$$1_{\{uc\}} = 1\left\{\widetilde{s} \le \kappa_1^{bc}\widetilde{P} + \kappa_0^{bc}\right\}, \quad \widetilde{D}_i^{uc} = d_s^{uc}\widetilde{s} - d_p^{uc}\widetilde{P}^{uc}, \tag{A3}$$

$$1_{\{bc\}} = 1\left\{\widetilde{s} > \kappa_1^{bc}\widetilde{P} + \kappa_0^{bc}\right\}, \quad \widetilde{D}_i^{bc} = d_s^{bc}\widetilde{s} - d_p^{bc}\widetilde{P}^{bc} + d_0^{bc}. \tag{A4}$$

We denote the inference constants as follows:

$$d_s^{uc} = \rho\tau_s, d_p^{uc} = \rho(\tau_s + \tau_v), d_s^{bc} = 0, d_p^{bc} = -a,$$
$$\kappa_1^{bc} = (\tau_s + \tau_v + a/\rho)/\tau_s, d_0^{bc} = b, \kappa_0^{bc} = b/(\rho\tau_s).$$

To prove Lemma 2, we now suppose the asset supply of this economy with only informed investors is $\widetilde{m}^{fic}(= \widetilde{m} - (1 - w_i)D_{ui})$. We then solve for the uniquely determined equilibrium price that clears the market, by setting $\widetilde{D}_i = \widetilde{m}^{fic}$:

$$\widetilde{P} = 1_{\{uc\}}\widetilde{P}^{uc} + 1_{\{bc\}}\widetilde{P}^{bc}, \tag{A5}$$

where

$$1_{\{uc\}} = 1\left\{\widetilde{s} \le \kappa_1^{bc}\widetilde{P} + \kappa_0^{bc}\right\}, \widetilde{P}^{uc} = p_s^{uc}\widetilde{s} - p_m^{uc}\widetilde{m}^{fic}, \tag{A6}$$

$$1_{\{bc\}} = 1\left\{\widetilde{s} > \kappa_1^{bc}\widetilde{P} + \kappa_0^{bc}\right\}, \widetilde{P}^{bc} = p_s^{bc}\widetilde{s} - p_m^{bc}\big(\widetilde{m}^{fic} - w_i^c b\big). \tag{A7}$$

We denote the following constants:

$$p_s^{uc} = \tau_s/(\tau_s + \tau_v), \quad p_m^{uc} = 1\big/\big(\big(w_i^{uc} + w_i^c\big)\rho(\tau_s + \tau_v)\big),$$
$$p_s^{bc} = \tau_s\big/\big(\tau_s + \tau_v - w_i^c a\big/\big(w_i^{uc}\rho\big)\big), \quad p_m^{bc} = 1\big/\big(w_i^{uc}\rho(\tau_s + \tau_v) - w_i^c a\big).$$

Q.E.D.

Appendix B

Proof of Lemma 3: We denote the following:

$$\theta_{uc}^{bc} = \kappa^{bc} \Big/ \sqrt{1/\tau_v + 1/\tau_{p^{uc}}}, \ \ \theta^{bc} = \kappa^{bc} \Big/ \sqrt{1/\tau_v + 1/\tau_{p^c}}, \ \ \theta = \kappa^{bc} \Big/ \sqrt{1/\tau_v + 1/\tau_s},$$

$$\frac{1}{\tau_{P^{uc}}} = \left(\frac{1}{\rho\left(w_i^{uc} + w_i^c\right)\tau_s}\right)^2 \frac{1}{\tau_m}, \ \ \frac{1}{\tau_{P^c}} = \left(\frac{1}{\rho w_i^{uc}\tau_s}\right)^2 \frac{1}{\tau_m}, \ \ \epsilon_{p^{uc}} = \left(\frac{1}{\rho\left(w_i^{uc} + w_i^c\right)\tau_s}\right)\epsilon_m,$$

$$\eta^{bc} = (aP + b)\left(w_i^{uc} + w_i^c\right), \ \ \kappa^{bc} = \kappa_1^{bc}P + \kappa_0^{bc}, \ \ \epsilon_{p^c} = \left(\frac{1}{\rho w_i^{uc}\tau_s}\right)\epsilon_m,$$

$$\sigma_{ss}^2 = \frac{1}{\tau_{ss}} = \frac{1}{\tau_s} + \frac{1}{\tau_v}, \ \ Pr^{uc} = Pr\left(\widetilde{s} < \kappa_1^{bc}P + \kappa_0^{bc}\right) = \Phi(\theta), \ \ Pr^{bc} = 1 - Pr^{uc}.$$

We can now express the conditional moments of the truncated normal variables in closed-form (Greene (1990) (pp. 707–708), Johnson and Kotz (1974), and Maddala (1986)):

$$\begin{aligned}
\widetilde{E}^{bc} &= E\left[\widetilde{v} \,\middle|\, \left\{\widetilde{P} = \widetilde{P}^{bc}, \widetilde{D}_{ui} = \widetilde{D}_{ui}^*, (\widetilde{s}, \widetilde{m}) \in \{BC\}\right\}\right] \\
&= E\left[\widetilde{v} \,\middle|\, \left\{\widetilde{P} = p_s^{bc}\widetilde{s} - p_m^{bc}\left(\widetilde{m}^{fic} - w_i^c b\right), \widetilde{D}_{ui} = \widetilde{D}_{ui}^*, \widetilde{s} \geq \kappa^{bc}, \widetilde{m}^{fic} \geq \eta^{bc}\right\}\right] \\
&= e_P^{bc}\widetilde{P} - e_{D_{ui}}^{bc}\widetilde{D}_{ui}^* + e_\lambda^{bc}\widetilde{\lambda}^{bc+} + e_0^{bc},
\end{aligned} \tag{B1}$$

where

$$e_P^{bc} = \frac{\tau_s}{\tau_s + \tau_v}\frac{\tau_{p^c}}{\tau_{p^c} + \tau_{ss}}\left(\frac{\tau_s + \tau_v}{\tau_s} - \frac{w_i^c a}{w_i^{uc}\tau_s\rho}\right), \quad e_{D_{ui}}^{bc} = \frac{\tau_s}{\tau_s + \tau_v}\frac{\tau_{p^c}}{\tau_{p^c} + \tau_{ss}}\left(\frac{1 - w_i}{w_i^{uc}\tau_s\rho}\right),$$

$$e_\lambda^{bc} = \frac{\tau_s}{\tau_s + \tau_v}\frac{1}{\sqrt{\tau_{p^c} + \tau_{ss}}}, \quad e_0^{bc} = \frac{\tau_s}{\tau_s + \tau_v}\frac{\tau_{p^c}}{\tau_{p^c} + \tau_{ss}}\left(\frac{\overline{m} - w_i^c b}{w_i^{uc}\tau_s\rho}\right),$$

and

$$\begin{aligned}
\widetilde{V}^{bc} &= \mathrm{Var}\left[\widetilde{v} \,\middle|\, \left\{\widetilde{P} = \widetilde{P}^{bc}, \widetilde{D}_{ui} = \widetilde{D}_{ui}^*, (\widetilde{s}, \widetilde{m}) \in \{BC\}\right\}\right] \\
&= \mathrm{Var}\left[\widetilde{v} \,\middle|\, \left\{\widetilde{P} = p_s^{bc}\widetilde{s} - p_m^{bc}\left(\widetilde{m}^{fic} - w_i^c b\right), \widetilde{D}_{ui} = \widetilde{D}_{ui}^*, \widetilde{s} \geq \kappa^{bc}, \widetilde{m}^{fic} \geq \eta^{bc}\right\}\right] \\
&= v_0 + v_\delta^{bc}\left(1 - \delta^{bc+}\right),
\end{aligned} \tag{B2}$$

where

$$v_0 = \frac{1}{\tau_v + \tau_s}, \quad v_\delta^{bc} = \left(\frac{\tau_s}{\tau_v + \tau_s}\right)^2\left(\frac{1}{\tau_{p^c} + \tau_{ss}}\right).$$

$$\begin{aligned}
\widetilde{E}^{uc} &= E\big[\widetilde{v}\,\big|\,\big\{\widetilde{P}=\widetilde{P}^{uc}, \widetilde{D}_{ui}=\widetilde{D}^{*}_{ui}, (\widetilde{s},\widetilde{m})\in\{UC\}\big\}\big],\\
&= E\left[\widetilde{v}\,\middle|\,\left\{\widetilde{P}=p^{uc}_{s}\widetilde{s}-p^{uc}_{m}\widetilde{m}^{fic}, \widetilde{D}_{ui}=\widetilde{D}^{*}_{ui}, \widetilde{s}\le\kappa^{bc}, \widetilde{m}^{fic}\le\eta^{bc}\right\}\right],\\
&= e^{uc}_{P}\widetilde{P}-e^{uc}_{D_{ui}}\widetilde{D}^{*}_{ui}+e^{uc}_{\lambda^{bc}}\widetilde{\lambda}^{bc-}_{uc}+e^{uc}_{0},
\end{aligned} \tag{B3}$$

$$e^{uc}_{P}=\frac{\tau_s}{\tau_s+\tau_v}\frac{\tau_{p^{uc}}}{\tau_{p^{uc}}+\tau_{ss}}\frac{\tau_s+\tau_v}{\tau_s},\quad e^{uc}_{D_{ui}}=\frac{\tau_s}{\tau_s+\tau_v}\frac{\tau_{p^{uc}}}{\tau_{p^{uc}}+\tau_{ss}}\frac{(1-w_i)}{w_i\rho\tau_s},$$

$$e^{uc}_{\lambda^{bc}}=\frac{\tau_s}{\tau_s+\tau_v}\frac{\tau_{p^{uc}}}{\tau_{p^{uc}}+\tau_{ss}}\frac{1}{\sqrt{\tau_{p^{uc}}+\tau_{ss}}},\quad e^{uc}_{0}=\frac{\tau_s}{\tau_s+\tau_v}\frac{\tau_{p^{uc}}}{\tau_{p^{uc}}+\tau_{ss}}\frac{\overline{m}}{w_i\rho\tau_s},$$

and

$$\begin{aligned}
\widetilde{V}^{uc} &= \mathrm{Var}\big[\widetilde{v}\,\big|\,\big\{\widetilde{P}=\widetilde{P}^{uc}, \widetilde{D}_{ui}=\widetilde{D}^{*}_{ui}, (\widetilde{s},\widetilde{m})\in\{UC\}\big\}\big],\\
&= \mathrm{Var}\left[\widetilde{v}\,\middle|\,\left\{\widetilde{P}=p^{uc}_{s}\widetilde{s}-p^{uc}_{m}\widetilde{m}^{fic}, \widetilde{D}_{ui}=\widetilde{D}^{*}_{ui}, \widetilde{s}\le\kappa^{bc}, \widetilde{m}^{fic}\le\eta^{bc}\right\}\right],\\
&= v_0+v^{uc}_{\delta}\left(1-\delta^{bc-}_{uc}\right),
\end{aligned} \tag{B4}$$

where

$$v_0=\frac{1}{\tau_v+\tau_s}, v^{uc}_{\delta}=\left(\frac{\tau_s}{\tau_v+\tau_s}\right)^2\left(\frac{1}{\tau_{p^{uc}}+\tau_{ss}}\right). \qquad \text{Q.E.D.}$$

Appendix C

Proof of Result 2: Case 1: Borrowing-constrained informed investors are not constrained. The market clearing condition can be expressed as

$$w_i\rho\tau_s\widetilde{s}-w_i\rho(\tau_s+\tau_v)\widetilde{P}=\widetilde{m}-(1-w_i)\widetilde{D}_{ui}. \tag{C1}$$

Hence,

$$\widetilde{m}-\widetilde{D}_{ui}=w_i\rho\tau_s\widetilde{s}-w_i\rho(\tau_s+\tau_v)\widetilde{P} \tag{C2}$$

$$\Rightarrow \widetilde{P}=p^{uc}_{s}\widetilde{s}-p^{uc}_{m}\widetilde{m}+p^{uc}_{D_{ui}}\frac{\rho(E(\widetilde{v}\mid\widetilde{P})-\widetilde{P})}{\mathrm{Var}\,(\widetilde{v}\mid\widetilde{P})}, \tag{C3}$$

where

$$p^{uc}_{s}=\frac{\tau_s}{\tau_s+\tau_v},\quad p^{uc}_{m}=\frac{1}{w_i\rho(\tau_s+\tau_v)},\quad p^{uc}_{D_{ui}}=\frac{1-w_i}{w_i\rho(\tau_s+\tau_v)}.$$

Case 2: Borrowing-constrained informed investors are constrained. The market clearing condition can be expressed as

$$\widetilde{m}-(1-w_i)\widetilde{D}_{ui}=w_i^c(a\widetilde{P}+b)+w_i^{uc}\rho\tau_s\widetilde{s}-w_i^{uc}\rho(\tau_s+\tau_v)\widetilde{P} \tag{C4}$$

$$\Rightarrow \widetilde{P}=p_0^{bc}+p_s^{bc}\widetilde{s}-p_m^{bc}\widetilde{m}+p_{D_{ui}}^{bc}\frac{\rho(E(\widetilde{v}\mid\widetilde{P})-\widetilde{P})}{\operatorname{Var}(\widetilde{v}\mid\widetilde{P})}, \tag{C5}$$

where

$$p_s^{bc}=\frac{w_i^{uc}\rho\tau_s}{w_i^{uc}(\tau_s+\tau_v)+w_i^c a},\quad p_m^{bc}=\frac{1}{w_i^{uc}(\tau_s+\tau_v)+w_i^c a},$$

$$p_0^{bc}=\frac{1}{w_i^{uc}(\tau_s+\tau_v)+w_i^c a}w_i^c b,\quad p_{D_{ui}}^{bc}=\frac{1-w_i}{w_i^{uc}(\tau_s+\tau_v)+w_i^c a}w_i^c b.$$

Q.E.D.

Appendix D

Proof of Lemma 4: The constrained informed investor optimization problem is

$$\max_{\widetilde{D}_{1_i}^c,\widetilde{D}_{2_i}^c} E_0\left[U\left(\widetilde{v}_1,\widetilde{v}_2,\widetilde{D}_{1_i}^c,\widetilde{D}_{2_i}^c\right)\mid\widetilde{s}_1,\widetilde{s}_2,\widetilde{m}_1^{fic},\widetilde{m}_2^{fic}\right]$$
$$-\lambda_1\left(\widetilde{D}_{1_i}^c-a_1\widetilde{P}_1-a_2\widetilde{P}_2-b\right)-\lambda_2\left(\widetilde{D}_{2_i}^c-a_1\widetilde{P}_1-a_2\widetilde{P}_2-b\right).$$

The unconstrained informed investor optimization problem is

$$\max_{\widetilde{D}_{1_i}^{uc},\widetilde{D}_{2_i}^{uc}} E_0\left[U\left(\widetilde{v}_1,\widetilde{v}_2,\widetilde{D}_{1_i}^{uc},\widetilde{D}_{2_i}^{uc}\right)\mid\widetilde{s}_1,\widetilde{s}_2,\widetilde{m}_1^{fic},\widetilde{m}_2^{fic}\right].$$

The equilibrium market prices for the informed-only economy are determined by the market clearing conditions:

$$w_i^{uc}\widetilde{D}_1^{uc}+w_i^c\widetilde{D}_1^c=\widetilde{m}_1^{fic}\quad\text{and}\quad w_i^{uc}\widetilde{D}_2^{uc}+w_i^c\widetilde{D}_2^c=\widetilde{m}_2^{fic}. \tag{D1}$$

We denote the constants as follows:

$$d_{e_i}=\tau_{s_i}/(\tau_{v_i}+\tau_{s_i}),\quad d_{v_i}=1/(\rho(\tau_{v_i}+\tau_{s_i})),\quad \text{where } i=1,2,$$

$$d_{bc_1}=w_{uc}\rho(\tau_{v_2}+\tau_{s_2})-w_i^c a_2,\quad d_{bc_2}=w_i^{uc}\rho(\tau_{v_1}+\tau_{s_1})-w_i^c a_1,$$

$$\kappa_{1\widetilde{P}_1}^{bc}=(\tau_{s_1}+\tau_{v_1}+a_1/\rho)/\tau_{s_1},\quad \kappa_{2\widetilde{P}_2}^{bc}=(\tau_{s_2}+\tau_{v_2}+a_2/\rho)/\tau_{s_2},$$

$$\kappa_{1\widetilde{P}_2}^{bc}=a_2/(\rho\tau_{s_1}),\quad \kappa_{0^1}^{bc}=b/(\rho\tau_{s_1}),\quad \kappa_{2\widetilde{P}_1}^{bc}=a_1/(\rho\tau_{s_2}),\quad \kappa_{0^2}^{bc}=b/(\rho\tau_{s_2}).$$

The first order condition for the unconstrained informed investor optimization problem leads to

$$\widetilde{D}_{1_i}^{uc}=\rho\tau_{s_1}\widetilde{s}_1-\rho(\tau_{v_1}+\tau_{s_1})\widetilde{P}_1,\quad D_{2_i}^{uc}=\rho\tau_{s_2}\widetilde{s}_2-\rho(\tau_{v_2}+\tau_{s_2})\widetilde{P}_2. \tag{D2}$$

The first order condition for the constrained informed investor optimization problem leads to

$$\widetilde{D}^3_{2_i} = \widetilde{D}^4_{2_i} = \widetilde{D}^2_{1_i} = \widetilde{D}^4_{1_i} = a_1\widetilde{P}_1 + a_2\widetilde{P}_2 + b, \tag{D3}$$

$$\widetilde{D}^1_{1_i} = \widetilde{D}^3_{1_i} = \rho\tau_{s_1}\widetilde{s}_1 - \rho(\tau_{v_1} + \tau_{s_1})\widetilde{P}_1, \tag{D4}$$

$$\widetilde{D}^1_{2_i} = \widetilde{D}^2_{2_i} = \rho\tau_{s_2}\widetilde{s}_2 - \rho(\tau_{v_2} + \tau_{s_2})\widetilde{P}_2, \tag{D5}$$

where

$$\text{case 1: } 1\left\{\kappa^{bc}_{1\widetilde{P}_1}\widetilde{P}_1 + \kappa^{bc}_{1\widetilde{P}_2}\widetilde{P}_2 + \kappa^{bc}_{0^1} \geq \widetilde{s}_1;\ \kappa^{bc}_{2\widetilde{P}_2}\widetilde{P}_2 + \kappa^{bc}_{2\widetilde{P}_1}\widetilde{P}_1 + \kappa^{bc}_{0^2} \geq \widetilde{s}_2\right\}, \tag{D6}$$

$$\text{case 2: } 1\left\{\widetilde{s}_1 > \kappa^{bc}_{1\widetilde{P}_1}\widetilde{P}_1 + \kappa^{bc}_{1\widetilde{P}_2}\widetilde{P}_2 + \kappa^{bc}_{0^1};\ \kappa^{bc}_{2\widetilde{P}_2}\widetilde{P}_2 + \kappa^{bc}_{2\widetilde{P}_1}\widetilde{P}_1 + \kappa^{bc}_{0^2} \geq \widetilde{s}_2\right\}, \tag{D7}$$

$$\text{case 3: } 1\left\{\kappa^{bc}_{1\widetilde{P}_1}\widetilde{P}_1 + \kappa^{bc}_{1\widetilde{P}_2}\widetilde{P}_2 + \kappa^{bc}_{0^1} \geq \widetilde{s}_1;\ \widetilde{s}_2 > \kappa^{bc}_{2\widetilde{P}_2}\widetilde{P}_2 + \kappa^{bc}_{2\widetilde{P}_1}\widetilde{P}_1 + \kappa^{bc}_{0^2}\right\}, \tag{D8}$$

$$\text{case 4: } 1\left\{\widetilde{s}_1 > \kappa^{bc}_{1\widetilde{P}_1}\widetilde{P}_1 + \kappa^{bc}_{1\widetilde{P}_2}\widetilde{P}_2 + \kappa^{bc}_{0^1};\ \widetilde{s}_2 > \kappa^{bc}_{2\widetilde{P}_2}\widetilde{P}_2 + \kappa^{bc}_{2\widetilde{P}_1}\widetilde{P}_1 + \kappa^{bc}_{0^2}\right\}. \tag{D9}$$

Q.E.D.

Appendix E

Proof of Lemma 5:

Case 1: Borrowing-constrained informed investors are not constrained in either market. The market clearing condition leads to

$$\widetilde{P}^1_2 = p^{uc}_{s_2}\widetilde{s}_2 - p^{uc}_{m_2}\widetilde{m}^{fic}_2,\ \widetilde{P}^1_1 = p^{uc}_{s_1}\widetilde{s}_1 - p^{uc}_{m_1}\widetilde{m}^{fic}_1. \tag{E1}$$

Case 2: Borrowing-constrained informed investors are constrained in Market 1 and unconstrained in Market 2. The market clearing condition leads to

$$\widetilde{P}^2_1 = w^{uc}_i\rho\tau_{s_1}\widetilde{s}_1/d_{bc_1} + a_2 w^c_i\widetilde{P}^2_2/d_{bc_1} + \left(bw^c_i - \widetilde{m}^{fic}_1\right)/d_{bc_1}, \tag{E2}$$

$$\widetilde{P}^2_2 = p^{uc}_{s_2}\widetilde{s}_2 - p^{uc}_{m_2}\widetilde{m}^{fic}_2, \tag{E3}$$

where

$$d_{bc_1} = w^{uc}_i\rho(\tau_{s_1} + \tau_{v_1}) - a_1 w^c_i.$$

Case 3: Borrowing-constrained informed investors are constrained in Market 2 and unconstrained in Market 1. The market clearing condition leads to

$$\widetilde{P}_1^3 = p_{s_1}^{uc}\widetilde{s}_1 - p_{m_1}^{uc}\widetilde{m}_1^{fic}, \tag{E4}$$

$$\widetilde{P}_2^3 = w_i^{uc}\rho\tau_{s_2}\widetilde{s}_2/d_{bc_2} + a_1 w_i^c \widetilde{P}_1^3/d_{bc_2} + \big(bw_i^c - \widetilde{m}_2^{fic}\big)/d_{bc_2}, \tag{E5}$$

where

$$d_{bc_2} = w_i^{uc}\rho(\tau_{s_2} + \tau_{v_2}) - a_2 w_i^c.$$

Case 4: Borrowing-constrained informed investors are constrained in both markets. The market clearing condition leads to

$$\begin{aligned}\widetilde{P}_1^4 &= w_i^{uc}\rho\tau_{s_1}\widetilde{s}_1/d_{bc_1} - \widetilde{m}_1^{fic}/d_{bc_1} + w_i^c\big(a_2\widetilde{P}_2^4 + b\big)/d_{bc_1}\\ &= \big(d_{bc_1}^4 + d_{bc_1}^4\big(\tau_{s_1}\widetilde{s}_1 - \widetilde{m}_1^{fic}/(\rho w_i^{uc})\big)\\ &\quad + \big(w_i^{uc}\rho - d_{bc_1}^4\big)\tau_{s_2}\widetilde{s}_2 - \big(1 - d_{bc_1}^4\big)\widetilde{m}_2^{fic}\big)/d_{bc_1}^{bc}, \end{aligned} \tag{E6}$$

$$\begin{aligned}\widetilde{P}_2^4 &= w_i^{uc}\rho\tau_{s_2}\widetilde{s}_2/d_{bc_2} - \widetilde{m}_2^{fic}/d_{bc_2} + w_i^c\big(a_1\widetilde{P}_1^4 + b\big)/d_{bc_2}\\ &= \big(d_{bc_2}^4 + d_{bc_2}^4\big(\tau_{s_2}\widetilde{s}_2 - \widetilde{m}_2^{fic}/(\rho w_i^{uc})\big)\\ &\quad + \big(w_i^{uc}\rho - d_{bc_2}^4\big)\tau_{s_1}\widetilde{s}_1 - \big(1 - d_{bc_2}^4\big)\widetilde{m}_1^{fic}\big)/d_{bc_2}^{bc}, \end{aligned} \tag{E7}$$

where

$$\begin{aligned} d_{bc_1}^4 &= w_i^c ba/(\tau_{s_1} + \tau_{v_1}), \quad d_{bc_2}^4 = w_i^c ba/(\tau_{s_2} + \tau_{v_2}),\\ d_{bc_1}^{bc} &= w_i^{uc}\rho(\tau_{s_2} + \tau_{v_2}) - aw_i^c(\tau_{s_2} + \tau_{v_2})/(\tau_{s_1} + \tau_{v_1}) - w_i^c a_2,\\ d_{bc_2}^{bc} &= w_i^{uc}\rho(\tau_{s_1} + \tau_{v_1}) - aw_i^c(\tau_{s_1} + \tau_{v_1})/(\tau_{s_2} + \tau_{v_2}) - w_i^c a_1. \end{aligned}$$

Q.E.D.

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