Online appendix for "Investing in a shared supplier in a competitive market: Stochastic capacity case"

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A.1. Exclusive capacity contract

Proof of Lemma 1 Without loss of generality, we assume $k_1 \geq k_2$. Using concavity of the objective function, we obtain the best response function of firm 1 and firm 2 as:

$$q_1^*(q_2) = \begin{cases} \frac{a-c-bq_2}{2b} & \text{if } \frac{a-c-bq_2}{2b} < (\frac{k_0}{2}+k_1)\xi, \\ (\frac{k_0}{2}+k_1)\xi & \text{if } \frac{a-c-bq_2}{2b} \geq (\frac{k_0}{2}+k_1)\xi. \end{cases} \quad q_2^*(q_1) = \begin{cases} \frac{a-c-bq_1}{2b} & \text{if } \frac{a-c-bq_1}{2b} < (\frac{k_0}{2}+k_2)\xi, \\ (\frac{k_0}{2}+k_2)\xi & \text{if } \frac{a-c-bq_1}{2b} \geq (\frac{k_0}{2}+k_2)\xi. \end{cases}$$

Solving for the intersection of the best response functions $q_1^*(q_2)$ and $q_2^*(q_1)$, we can obtain the equilibrium order quantities and hence the equilibrium profits shown in Lemma 1. □

Discussion about when $\frac{a-c}{3b(\frac{k_0}{2}+k_2)} > 1$. We observe that when $\frac{a-c}{3b(\frac{k_0}{2}+k_2)} > 1$, there are two cases.

Case 1. When $\frac{a-c}{b(\frac{3k_0}{2}+2k_1+k_2)} \leq 1$, we have the equilibrium order quantity and ex post profit as follows:

realized yield ξ	order quantity (q_1^*, q_2^*)	ex post profit $(\pi_1^e,\ \pi_2^e)$
$0 \le \xi \le \frac{a-c}{b(\frac{3k_0}{2}+2k_1+k_2)}$	$\left((\frac{k_0}{2}+k_1)\xi, (\frac{k_0}{2}+k_2)\xi\right)$	$\left(m_2(\frac{k_0}{2}+k_1)\xi, m_2(\frac{k_0}{2}+k_2)\xi\right)$
$\frac{a-c}{b(\frac{3k_0}{2}+2k_1+k_2)} < \xi \le 1$	$\left(\frac{a-c-b(\frac{k_0}{2}+k_2)\xi}{2b}, (\frac{k_0}{2}+k_2)\xi\right)$	$\left(\frac{m_1[a-c-b(\frac{k_0}{2}+k_2)\xi]}{2b}, m_1(\frac{k_0}{2}+k_2)\xi\right)$

Case 2. When $\frac{a-c}{b(\frac{3k_0}{2}+2k_1+k_2)} > 1$, we have the equilibrium order quantity and ex post profit as follows:

realized yield ξ	order quantity (q_1^*, q_2^*)	ex post profit $(\pi_1^e,\ \pi_2^e)$
$0 \le \xi \le 1$	$\left((\frac{k_0}{2}+k_1)\xi, (\frac{k_0}{2}+k_2)\xi\right)$	$\left(m_2(\frac{k_0}{2}+k_1)\xi, m_2(\frac{k_0}{2}+k_2)\xi\right)$

With the ex post profit derived, the firm's expected profit can be obtained similarly as in equation (1) and (2), and the analysis follows. The discussions for the first-priority case (Lemma 2) are similar and omitted for space.

Proof of Proposition 1 We prove the proposition in two steps. We first illustrate that given the number of investing firms, the equilibrium capacity investment satisfies equation (3) or (4) respectively, and show the monotonicity of the equilibrium capacity with respect to the variable capacity cost w. Then we show the monotonicity of the number of investing firms with respect to the fixed capacity cost w_0 by constructing the equilibrium switching curves, and finally show the monotonicity of the equilibrium switching curves.

Both firms investing: If both firms decide to invest in the supplier, we first show that the equilibrium capacity satisfies $k_1 = k_2$, and then the equilibrium capacity is the k^e as shown in condition (3).

We first show that if $k_1 > k_2$, then (k_1, k_2) cannot be the equilibrium capacity. Following equation (1) and (2), we have

$$\frac{\partial V_1^e}{\partial k_1} = \int_0^{\frac{a-c}{b(\frac{3k_0}{2}+2k_1+k_2)}} \left[a-c-b\left(\frac{3k_0}{2}+2k_1+k_2\right)\xi\right]\xi f(\xi)d\xi - w \qquad \text{(A1)}$$

$$\frac{\partial V_2^e}{\partial k_2} = \int_0^{\frac{a-c}{b(\frac{3k_0}{2}+2k_1+k_2)}} \left[a-c-b\left(\frac{3k_0}{2}+k_1+2k_2\right)\xi\right]\xi f(\xi)d\xi$$

$$+\int_{\frac{a-c}{b(\frac{3k_0}{2}+2k_1+k_2)}}^{\frac{a-c}{3b(\frac{k_0}{2}+k_2)}} \frac{a-c-b(k_0+2k_2)\xi}{2}\xi f(\xi)d\xi - w \tag{A2}$$

Let $k_1 = k+\epsilon$ and $k_2 = k$ where $\epsilon > 0$, then we have

$$\left.\frac{\partial V_1^e}{\partial k_1}\right|_{(k+\epsilon,k)} - \left.\frac{\partial V_2^e}{\partial k_2}\right|_{(k+\epsilon,k)}$$
$$= \int_0^{\frac{a-c}{b(\frac{3k_0}{2}+3k+2\epsilon)}} -b\epsilon\xi^2 f(\xi)d\xi - \int_{\frac{a-c}{b(\frac{3k_0}{2}+3k+2\epsilon)}}^{\frac{a-c}{3b(\frac{k_0}{2}+k)}} \frac{a-c-b(k_0+2k)\xi}{2}\xi f(\xi)d\xi \le 0 \tag{A3}$$

Therefore, at least one of the two firms will have incentive to deviate from the current capacity investment level, and we have (k_1, k_2) where $k_1 > k_2$ cannot be an equilibrium. Similarly, we have (k_1, k_2) where $k_1 < k_2$ cannot be an equilibrium.

We next show that $k_1 = k_2 = k^e$ is indeed an equilibrium by showing that neither firm has incentive to deviate in this case. We focus on the analysis for firm 1 as the analysis for firm 2 is similar. If firm 1 deviates from k^e to $k^e+\epsilon$, then we have

$$\left.\frac{\partial V_1^e}{\partial k_1}\right|_{(k^e+\epsilon,k^e)} - \left.\frac{\partial V_1^e}{\partial k_1}\right|_{(k^e,k^e)} = \int_0^{\frac{a-c}{b(\frac{3k_0}{2}+3k^e+2\epsilon)}} -2b\epsilon\xi^2 f(\xi)d\xi$$
$$-\int_{\frac{a-c}{b(\frac{3k_0}{2}+3k^e+2\epsilon)}}^{\frac{a-c}{b(\frac{3k_0}{2}+3k^e)}} \left[a-c-b\left(\frac{3k_0}{2}+3k^e\right)\xi\right]\xi f(\xi)d\xi \le 0 \tag{A4}$$

That is, firm 1 will have incentive to decrease its capacity investment from $k^e+\epsilon$. On the other hand, if firm 1 deviates from k^e to $k^e-\epsilon$. Then we have

$$\left.\frac{\partial V_1^e}{\partial k_1}\right|_{(k^e-\epsilon,k^e)} - \left.\frac{\partial V_1^e}{\partial k_1}\right|_{(k^e,k^e)}$$
$$= \int_0^{\frac{a-c}{b(\frac{3k_0}{2}+3k^e)}} 2b\epsilon\xi^2 f(\xi)d\xi + \int_{\frac{a-c}{b(\frac{3k_0}{2}+3k^e)}}^{\frac{a-c}{b(\frac{3k_0}{2}+3k^e-\epsilon)}} \left[a-c-b\left(\frac{3k_0}{2}+3k^e-2\epsilon\right)\xi\right]\xi f(\xi)d\xi$$
$$+\int_{\frac{a-c}{b(\frac{3k_0}{2}+3k^e-\epsilon)}}^{\frac{a-c}{3b(\frac{k_0}{2}+k^e-\epsilon)}} \frac{a-c-b(k_0+2k^e-2\epsilon)\xi}{2}\xi f(\xi)d\xi \ge 0 \tag{A5}$$

Therefore, firm 1 will have incentive to increase its capacity investment from $k^e-\epsilon$. To conclude, we have shown that if both firms invest in the supplier, the equilibrium capacity investment k^e is the same for both firms and is characterized by condition (3).

Then we implicitly differentiate k^e with respect to w in equation (3), and obtain that

$$\frac{\partial k^e}{\partial w} = -\frac{1}{3b\int_0^{\frac{a-c}{3b(\frac{k_0}{2}+k^e)}} \xi^2 f(\xi)d\xi} \leq 0 \tag{A6}$$

Therefore, it follows that k^e decreases in w.

One firm investing: If only one firm invests, we analyze the case where the investing firm is firm 1 below. When the investing firm is firm 2, the analysis is the same and therefore omitted for space. First, we have the first order derivative of firm 1's expected profit as

$$\left.\frac{\partial V_1^e}{\partial k_1}\right|_{(k_1,0)} = \int_0^{\frac{a-c}{b(\frac{3k_0}{2}+2k_1)}} \left[a-c-b\left(\frac{3k_0}{2}+2k_1\right)\xi\right]\xi f(\xi)d\xi - w \tag{A7}$$

It follows that the second order derivative

$$\left.\frac{\partial^2 V_1^e}{\partial k_1^2}\right|_{(k_1,0)} = \int_0^{\frac{a-c}{b(\frac{3k_0}{2}+2k_1)}} -2b\xi^2 f(\xi)d\xi \leq 0 \tag{A8}$$

Therefore, firm 1 will choose k_1^e which satisfies the condition that $\frac{\partial V_1^e}{\partial k_1} = 0$.

Similarly, we implicitly differentiate k_1^e with respect to w in equation (4), and obtain that

$$\frac{\partial k_1^e}{\partial w} = -\frac{1}{2b\int_0^{\frac{a-c}{b(\frac{3k_0}{2}+2k_1^e)}} \xi^2 f(\xi)d\xi} \leq 0 \tag{A9}$$

Therefore, it follows that k_1^e decreases in w.

Monotonicity of number of investing firms: To simplify the notations, we define $L_i^e(k_1,k_2;w) \triangleq E\left[\pi_i^e(k_1,k_2,\boldsymbol{\xi})\right] - wk_i$. For given w, we write $L_i^e(k_1,k_2;w)$ as $L_i^e(k_1,k_2)$ when there is no confusion. Then we have $V_i^e(k_1,k_2) = L_i^e(k_1,k_2) - w_0\mathbf{1}_{\{k_i>0\}}$. When only one firm invests, we still label the investing firm as firm 1. We prove the results in three steps. First, we show in a technical lemma that $k^e \leq k_1^e \leq 2k^e$. Then we show that $L_1^e(k_1^e,0) \geq L_1^e(0,0)$ and $L_2^e(k^e,k^e) \geq L_2^e(k_1^e,0)$. Finally, we prove the monotonicity in the number of investing firms by deriving the switching curves, and then show the monotonicity of the equilibrium switching curves.

Lemma A1 **(Monotonicity in the capacity investment)**. *$k^e \le k_1^e \le 2k^e$.*

Proof: By comparing equation (3) and (4), we have $3k^e = 2k_1^e$. Therefore, the results follow.□

We next show that $L_1^e(k_1^e,0) \ge L_1^e(0,0)$ and $L_2^e(k^e,k^e) \ge L_2^e(k_1^e,0)$. First, as $L_1^e(k_1^e,0) \triangleq \max_{k\ge 0} L_1^e(k,0)$, it follows that $L_1^e(k_1^e,0) \ge L_1^e(0,0)$. Second, using equation (3) and (4) we have

$$\begin{aligned}
L_2^e(k^e,k^e) &= \int_0^{\frac{a-c}{b(\frac{3k_0}{2}+3k^e)}} m_2(\frac{k_0}{2}+k^e)\xi f(\xi)d\xi + \int_{\frac{a-c}{3b(\frac{k_0}{2}+k^e)}}^1 \frac{m_0(a-c)}{3b} f(\xi)d\xi - wk^e \\
&= \int_0^{\frac{a-c}{b(\frac{3k_0}{2}+3k^e)}} \left[(a-c-b(k_0+k^e)\xi)\frac{k_0}{2} + b(k^e)^2\xi\right]\xi f(\xi)d\xi \\
&\quad + \int_{\frac{a-c}{3b(\frac{k_0}{2}+k^e)}}^1 \frac{m_0(a-c)}{3b} f(\xi)d\xi \\
&\triangleq E[f_2^e(k^e,k^e,\boldsymbol{\xi})] \qquad \text{(A10)}\\
L_2^e(k_1^e,0) &= \int_0^{\frac{a-c}{b(\frac{3k_0}{2}+2k_1^e)}} m_2\frac{k_0}{2}\xi f(\xi)d\xi + \int_{\frac{a-c}{b(\frac{3k_0}{2}+2k_1^e)}}^{\frac{2(a-c)}{3bk_0}} m_1\frac{k_0}{2}\xi f(\xi)d\xi + \int_{\frac{2(a-c)}{3bk_0}}^1 \frac{m_0(a-c)}{3b} f(\xi)d\xi \\
&\triangleq E[f_2^e(k_1^e,0,\boldsymbol{\xi})] \qquad \text{(A11)}
\end{aligned}$$

For any realization of ξ, the integrand $f_2^e(k^e,k^e,\xi) \ge f_2^e(k_1^e,0,\xi)$ following Lemma A1. Therefore, we have $L_2^e(k^e,k^e) \ge L_2^e(k_1^e,0)$.

Finally, we define $\overline{w}_0^e(w) \triangleq L_1^e(k_1^e,0;w) - L_1^e(0,0;w)$, $\underline{w}_0^e(w) \triangleq \min\{L_2^e(k^e,k^e;w) - L_2^e(k_1^e,0;w), \overline{w}_0^e(w)\}$, and $w_0^e(w) \triangleq L_i^e(k^e,k^e) - L_i^e(0,0)$. It follows that when $w_0 \ge \overline{w}_0^e(w)$, neither firm has incentive to deviate from the status quo (neither firm invests in the supplier); when $\underline{w}_0^e(w) \le w_0 < \overline{w}_0^e(w)$, only one firm invests in the supplier; when $w_0 < \underline{w}_0^e(w)$, both firms invest in the supplier. When $w_0^e(w) \le w_0 < \underline{w}_0^e(w)$, both firms invest in the supplier but both firms earn a lower profit than they do when neither firm invests in the supplier. Therefore, both firms are trapped in a prisoner's dilemma.

For the monotonicity of the equilibrium switching curves, by envelope theorem, we have

$$\frac{\partial \overline{w}_0^e(w)}{\partial w} = -k_1^e \leq 0. \tag{A12}$$

That is $\overline{w}_0^e(w)$ decreases in w. We also define $\hat{w}_0^e(w) \triangleq L_2^e(k^e, k^e; w) - L_2^e(k_1^e, 0; w)$. Then we have

$$\begin{aligned}\frac{\partial \hat{w}_0^e(w)}{\partial w} &= \int_0^{\frac{a-c}{3b(\frac{k_0}{2}+k^e)}} -\frac{b(k_0+2k^e)}{2}\xi^2 f(\xi)d\xi\frac{\partial k^e}{\partial w} - k^e + \int_0^{\frac{a-c}{b(\frac{3k_0}{2}+2k_1^e)}} \frac{bk_0}{2}\xi^2 f(\xi)d\xi\frac{\partial k_1^e}{\partial w} \\ &= -\frac{8k^e+k_0}{12} \leq 0 \end{aligned} \tag{A13}$$

The second equality follows equation (A6) and (A9). As $\underline{w}_0^e(w) \triangleq \min\{\hat{w}_0^e(w), \overline{w}_0^e(w)\}$, we have $\underline{w}_0^e(w)$ decreases in w. $\square$

Remark: One may observe that when $\overline{w}_0^e(w) < L_2^e(k^e, k^e) - L_2^e(k_1^e, 0)$ and $w_0 \in [\overline{w}_0^e(w), L_2^e(k^e, k^e) - L_2^e(k_1^e, 0)]$, in theory there exist two equilibria: both firms investing in the supplier, and neither firm investing in the supplier. However, as the status quo of this game is that neither firm invests in the supplier at the first place (and both firms are deciding simultaneously about whether they should invest in the supplier) and the continuity of decisions is preserved as the region where both equilibria exist is adjacent to the region where neither firm investing is the only equilibrium, the final equilibrium outcome of this game is still that neither firm invests in the supplier. This is why we define $\underline{w}_0^e(w) \triangleq \min\{L_2^e(k^e, k^e) - L_2^e(k_1^e, 0), \overline{w}_0^e(w)\}$. That is, we always have $\underline{w}_0^e(w) \leq \overline{w}_0^e(w)$.

A.2. First-priority capacity contract

The proof in this section is similar to the proofs in Section 3. Therefore, we will sketch the proof and illustrate details of the important steps for the interest of space.

Proof of Lemma 2 The proof is similar to the proof of Lemma 1. Without loss of generality, we assume $k_1 \geq k_2$. By concavity of the objective function, we obtain the best response functions as:

$$q_1^*(q_2) = \begin{cases} \frac{a-c-bq_2}{2b} & \text{if } \frac{a-c-bq_2}{2b} < (\frac{k_0}{2}+k_1)\xi, \\ (\frac{k_0}{2}+k_1)\xi & \text{if } \frac{a-c-bq_2}{2b} \geq (\frac{k_0}{2}+k_1)\xi. \end{cases} \qquad q_2^*(q_1) = \begin{cases} \frac{a-c-bq_1}{2b} & \text{if } \frac{a-c-bq_1}{2b} < k_s - q_1, \\ k_s - q_1 & \text{if } \frac{a-c-bq_1}{2b} \geq k_s - q_1. \end{cases}$$

Similarly, we solve for the intersection of the best response functions $q_1^*(q_2)$ and $q_2^*(q_1)$, and obtain the equilibrium order quantities and profits in Lemma 2. □

Before we prove Proposition 2, we first derive the expressions for firms' profit based on Lemma 2, assuming $k_1 \geq k_2$.

$$\begin{aligned} V_1^f(k_1,k_2) = &\int_0^{\frac{a-c}{b(\frac{3k_0}{2}+2k_1+k_2)}} m_2\left(\frac{k_0}{2}+k_1\right)\xi f(\xi)d\xi + \int_{\frac{2(a-c)}{3b(k_0+k_1+k_2)}}^1 \frac{m_0(a-c)}{3b} f(\xi)d\xi \\ &+ \int_{\frac{a-c}{b(\frac{3k_0}{2}+2k_1+k_2)}}^{\frac{2(a-c)}{3b(k_0+k_1+k_2)}} m_2\left(\frac{a-c}{b}-k_s\right) f(\xi)d\xi - w_0\mathbf{1}_{\{k_1>0\}} - wk_1 \end{aligned} \tag{A14}$$

$$\begin{aligned} V_2^f(k_1,k_2) = &\int_0^{\frac{a-c}{b(\frac{3k_0}{2}+2k_1+k_2)}} m_2\left(\frac{k_0}{2}+k_2\right)\xi f(\xi)d\xi + \int_{\frac{2(a-c)}{3b(k_0+k_1+k_2)}}^1 \frac{m_0(a-c)}{3b} f(\xi)d\xi \\ &+ \int_{\frac{a-c}{b(\frac{3k_0}{2}+2k_1+k_2)}}^{\frac{2(a-c)}{3b(k_0+k_1+k_2)}} m_2\left(2k_s-\frac{a-c}{b}\right) f(\xi)d\xi - w_0\mathbf{1}_{\{k_2>0\}} - wk_2 \end{aligned} \tag{A15}$$

We also prove the following technical lemma that will be used in the proof of Proposition 2.

LEMMA A2. *For a finite differentiable function $h(x)$ defined on $x \in \mathbb{R}^+$, if $h'(0) \geq 0$ and $\lim_{x\to\infty} h'(x) = 0$, then $x^*(w) = \arg\max_x\{h(x) - wx\}$ decreases in w.*

Proof of Lemma A2 If $h(x)$ is monotone, the proof is trivial. We prove the lemma for the case where $g(x;w) \triangleq h(x) - wx$ may have two local maxima, $\hat{x}_1(w) \leq \hat{x}_2(w)$. For the cases where $g(x;w)$ has more than two local maxima, the lemma can be proved similarly. We denote the local minimum between the two local maxima as $\underline{x}(w)$.

To prove the lemma, it is sufficient to show that if $g(\hat{x}_1(w);w) \geq g(\hat{x}_2(w);w)$, then for $\hat{w} > w$, we have $g(\hat{x}_1(\hat{w});\hat{w}) \geq g(\hat{x}_2(\hat{w});\hat{w})$. First, as the function $g(x;w)$ has at most two local maxima, it follows that $\hat{x}_1(w)$ and $\hat{x}_2(w)$ decrease in w, and $\underline{x}(w)$ increases in w. Second, we have

$$\begin{aligned}
g(\hat{x}_2(\hat{w})) - g(\hat{x}_1(\hat{w})) &= \int_{\hat{x}_1(\hat{w})}^{\underline{x}(\hat{w})} g'(x;\hat{w})dx + \int_{\underline{x}(\hat{w})}^{\hat{x}_2(\hat{w})} g'(x;\hat{w})dx \\
&= \int_{\hat{x}_1(\hat{w})}^{\underline{x}(\hat{w})} (h'(x) - \hat{w})dx + \int_{\underline{x}(\hat{w})}^{\hat{x}_2(\hat{w})} (h'(x) - \hat{w})dx \\
&\leq \int_{\hat{x}_1(w)}^{\underline{x}(w)} (h'(x) - w)dx + \int_{\underline{x}(w)}^{\hat{x}_2(w)} (h'(x) - w)dx \\
&= g(\hat{x}_2(w)) - g(\hat{x}_1(w)) \leq 0 \qquad \text{(A16)}
\end{aligned}$$

Therefore, we have proved that $g(\hat{x}_1(\hat{w})) \geq g(\hat{x}_2(\hat{w}))$. □

Proof of Proposition 2 The proof is similar to the proof of Proposition 1 with two key steps. We first derive the subgame perfect equilibrium capacity investment given the number of investing firms. Then we characterize the monotonicity of number of investing firms.

Both firms investing: If both firms investing in the supplier, we first obtain the first order derivative of firms' profit with respect to its capacity investment as follows, assuming $k_1 > k_2$.

$$\begin{aligned}
\frac{\partial V_1^f}{\partial k_1} &= \int_0^{\frac{a-c}{b(\frac{3k_0}{2}+2k_1+k_2)}} \left[a - c - b\left(\frac{3k_0}{2} + 2k_1 + k_2\right)\xi\right] \xi f(\xi)d\xi \\
&\quad + \int_{\frac{a-c}{b(\frac{3k_0}{2}+2k_1+k_2)}}^{\frac{2(a-c)}{3b(k_0+k_1+k_2)}} -2[a - c - b(k_0 + k_1 + k_2)\xi]\xi f(\xi)d\xi - w \qquad \text{(A17)}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial V_2^f}{\partial k_2} &= \int_0^{\frac{a-c}{b(\frac{3k_0}{2}+2k_1+k_2)}} \left[a - c - b\left(\frac{3k_0}{2} + k_1 + 2k_2\right)\xi\right] \xi f(\xi)d\xi \\
&\quad + \int_{\frac{a-c}{b(\frac{3k_0}{2}+2k_1+k_2)}}^{\frac{2(a-c)}{3b(k_0+k_1+k_2)}} [3(a - c) - 4b(k_0 + k_1 + k_2)\xi]\xi f(\xi)d\xi - w \qquad \text{(A18)}
\end{aligned}$$

Then following similar steps as in proof of Proposition 1, we have (k_1, k_2) where $k_1 \neq k_2$ cannot be an equilibrium, and $k_1 = k_2 = k^f$ where k^f is defined in Proposition 2 is indeed an equilibrium. The details are omitted for space. Then we implicitly differentiate k^f with respect to w in equation (5), and obtain that

$$\frac{\partial k^f}{\partial w} = -\frac{1}{3b\int_0^{\frac{a-c}{3b(\frac{k_0}{2}+k^f)}} \xi^2 f(\xi)d\xi} \leq 0 \tag{A19}$$

Therefore, it follows that k^f decreases in w.

Only one firm investing: If only one firm invests, we analyze the case where the investing firm is firm 1. The other case where the investing firm is firm 2 is similar and omitted for space. We obtain the following first order derivative of firm 1's expected profit with respect to its capacity investment.

$$\begin{aligned}\left.\frac{\partial V_1^f}{\partial k_1}\right|_{(k_1,0)} &= \int_0^{\frac{a-c}{b(\frac{3k_0}{2}+2k_1)}} \left[a-c-b\left(\frac{3k_0}{2}+2k_1\right)\xi\right]\xi f(\xi)d\xi \\ &+ \int_{\frac{a-c}{b(\frac{3k_0}{2}+2k_1)}}^{\frac{2(a-c)}{3b(k_0+k_1)}} -2[a-c-b(k_0+k_1)\xi]\xi f(\xi)d\xi - w \end{aligned} \tag{A20}$$

We observe that $\lim_{k_1\to\infty} \left.\frac{\partial V_1^f}{\partial k_1}\right|_{(k_1,0)} = -w$. In addition, $V_1^f(k_1,0)$ is a continuous and differentiable function in k_1. Therefore, there exists a finite $k_1 = k_1^f$ where $\max_{k_1\geq 0} V_1^f(k_1,0)$ is attained, and the k_1^f satisfies the condition specified by the first order condition as shown in equation (6). The decrease of k_1^f with respect to w follows Lemma A2.

Monotonicity of number of investing firms: To simplify the notations, we similarly define $L_i^f(k_1,k_2) \triangleq E\left[\pi_i^f(k_1,k_2,\boldsymbol{\xi})\right] - wk_i$. Then we have $V_i^f(k_1,k_2) = L_i^f(k_1,k_2) - w_0\mathbf{1}_{\{k_i>0\}}$. When only one firm invests, we still label the investing firm as firm 1. We prove the results in two steps. First, we show that $L_1^f(k_1^f,0) \geq L_1^f(0,0)$. Then we define $\overline{w}_0^f(w) \triangleq L_1^f(k_1^f,0) - L_1^f(0,0)$, $\underline{w}_0^f(w) \triangleq \min\{(L_2^f(k^f,k^f) - L_2^f(k_1^f,0))^+, \overline{w}_0^f(w)\}$, and $w_0^f(w) \triangleq L_i^f(k^f,k^f) - L_i^f(0,0)$. It follows that when $w_0 \geq \overline{w}_0^f(w)$, neither firm has incentive to invest in the supplier; when $\underline{w}_0^f(w) \leq w_0 < \overline{w}_0^f(w)$, only one firm invests in the supplier; when $w_0 < \underline{w}_0^f(w)$, both firms invest in the supplier. When $w_0^f(w) \leq w_0 < \underline{w}_0^f(w)$, both firms invest in the supplier but both firms earn a lower profit than they do when neither firms in the supplier. Therefore, both firms are trapped in a prisoner's dilemma.

For the monotonicity of the equilibrium switching curve $\overline{w}_0^f(w)$, by envelope theorem, we have

$$\frac{\partial \overline{w}_0^f(w)}{\partial w} = -k_1^f \leq 0. \tag{A21}$$

That is $\overline{w}_0^f(w)$ decreases in w. □

A.3. Spillover effect: comparing exclusive and first-priority capacity

When only one firm invests, we label the investing firm as firm 1. Before we proceed to prove propositions in this section, we first prove two technical lemmas.

LEMMA A3 **(Over-investment with exclusive capacity)**. $k_1^f \leq k_1^e$.

Proof: Assume $k_1^f > k_1^e$, then following equation (6) and (4), we have

$$\begin{aligned}
&\int_0^{\frac{a-c}{b(\frac{3k_0}{2}+2k_1^f)}} \left[a-c-b\left(\frac{3k_0}{2}+2k_1^f\right)\xi\right]\xi f(\xi)d\xi \\
&+\int_{\frac{a-c}{b(\frac{3k_0}{2}+2k_1^f)}}^{\frac{2(a-c)}{3b(k_0+k_1^f)}} -2\left[a-c-b\left(k_0+k_1^f\right)\xi\right]\xi f(\xi)d\xi - w \\
\leq &\int_0^{\frac{a-c}{b(\frac{3k_0}{2}+2k_1^f)}} \left[a-c-b\left(\frac{3k_0}{2}+2k_1^f\right)\xi\right]\xi f(\xi)d\xi - w \\
< &\int_0^{\frac{a-c}{b(\frac{3k_0}{2}+2k_1^e)}} \left[a-c-b\left(\frac{3k_0}{2}+2k_1^e\right)\xi\right]\xi f(\xi)d\xi - w = 0
\end{aligned} \tag{A22}$$

This contradicts the condition specified in equation (6). Therefore, we conclude that $k_1^f \leq k_1^e$. □

LEMMA A4. $V_1^e(k_1^e,0) \geq V_1^f(k_1^f,0)$; $V_2^f(k_1^f,0) \geq V_2^e(k_1^e,0)$.

Proof: We first prove that $V_1^e(k_1^e,0) \geq V_1^f(k_1^f,0)$ by showing that $V_1^e(k_1^e,0) \geq V_1^e(k_1^f,0) \geq V_1^f(k_1^f,0)$. We observe that $V_1^e(k_1^e,0) \triangleq \max_{k\geq 0} V_1^e(k,0)$, so we have $V_1^e(k_1^e,0) \geq V_1^e(k_1^f,0)$. Next, by equation (1) and (A14), we have

$$V_1^e(k_1^f,0)-V_1^f(k_1^f,0)=\int_{\frac{a-c}{b(\frac{3k_0}{2}+2k_1^f)}}^{\frac{2(a-c)}{3b(k_0+k_1^f)}}\left\{\frac{[a-c-b(\frac{k_0}{2})\xi]^2}{4b}-\frac{(a-c-bk_s)^2}{b}\right\}f(\xi)d\xi$$
$$+\int_{\frac{2(a-c)}{3b(k_0+k_1^f)}}^{\frac{2(a-c)}{3bk_0}}\left\{\frac{[a-c-b(\frac{k_0}{2})\xi]^2}{4b}-\frac{(a-c)^2}{9b}\right\}f(\xi)d\xi \quad \text{(A23)}$$

We note that $\frac{[a-c-b(\frac{k_0}{2})\xi]^2}{4b}-\frac{(a-c-bk_s)^2}{b}\geq 0$ when $\frac{a-c}{b(\frac{3k_0}{2}+2k_1^f)}\leq\xi\leq\frac{2(a-c)}{3b(k_0+k_1^f)}$, and $\frac{[a-c-b(\frac{k_0}{2})\xi]^2}{4b}-\frac{(a-c)^2}{9b}\geq 0$ when $\frac{2(a-c)}{3b(k_0+k_1^f)}\leq\xi\leq\frac{2(a-c)}{3bk_0}$. Therefore, it follows that $V_1^e(k_1^f,0)\geq V_1^f(k_1^f,0)$. Then we have proved that $V_1^e(k_1^e,0)\geq V_1^f(k_1^f,0)$.

We next prove that $V_2^f(k_1^f,0)\geq V_2^e(k_1^e,0)$ by showing that $V_2^f(k_1^f,0)\geq V_2^e(k_1^f,0)\geq V_2^e(k_1^e,0)$. Similarly, by equation (2) and (A15), we have

$$V_2^f(k_1^f,0)-V_2^e(k_1^f,0)=\int_{\frac{a-c}{b(\frac{3k_0}{2}+2k_1^f)}}^{\frac{2(a-c)}{3b(k_0+k_1^f)}}\left[(a-c-bk_s)\left(2k_s-\frac{a-c}{b}\right)-\frac{(a-c-b\frac{k_0}{2}\xi)k_0}{4}\xi\right]f(\xi)d\xi$$
$$+\int_{\frac{2(a-c)}{3b(k_0+k_1^f)}}^{\frac{2(a-c)}{3bk_0}}\left[\frac{(a-c)^2}{9b}-\frac{(a-c-b\frac{k_0}{2}\xi)k_0}{4}\xi\right]f(\xi)d\xi \quad \text{(A24)}$$

We note that $(a-c-bk_s)\left(2k_s-\frac{a-c}{b}\right)-\frac{(a-c-b\frac{k_0}{2}\xi)k_0}{4}\xi\geq 0$ when $\frac{a-c}{b(\frac{3k_0}{2}+2k_1^f)}\leq\xi\leq\frac{2(a-c)}{3b(k_0+k_1^f)}$, and $\frac{(a-c)^2}{9b}-\frac{(a-c-b\frac{k_0}{2}\xi)k_0}{4}\xi\geq 0$ when $\frac{2(a-c)}{3b(k_0+k_1^f)}\leq\xi\leq\frac{2(a-c)}{3bk_0}$. Therefore, we obtain that $V_2^f(k_1^f,0)\geq V_2^e(k_1^f,0)$. Then from equation (2), we have

$$\left.\frac{\partial V_2^e}{\partial k_1}\right|_{(k_1,0)}=\int_0^{\frac{a-c}{b(\frac{3k_0}{2}+2k_1+k_2)}}-\frac{bk_0\xi^2}{2}f(\xi)d\xi\leq 0 \quad \text{(A25)}$$

In addition, by Lemma A3, we have $k_1^f\leq k_1^e$ and hence $V_2^e(k_1^f,0)\geq V_2^e(k_1^e,0)$. Then we have proved $V_2^f(k_1^f,0)\geq V_2^e(k_1^e,0)$. □

Proof of Proposition 3 i and iii) We prove this part by showing that $w_0^e(w)=w_0^f(w)$ and $\underline{w}_0^e(w)\geq\underline{w}_0^f(w)$. First, if $k_1=k_2$, we have $V_i^e(k_1,k_2)=V_i^f(k_1,k_2)$. In addition, by definition we have $w_0^e(w)=V_i^e(k^e,k^e)-V_i^e(0,0)$ and $w_0^f(w)=V_i^f(k^f,k^f)-V_i^f(0,0)$. Therefore, we have $w_0^e(w)=w_0^f(w)$. Second, we have $\underline{w}_0^e(w)=\min\{L_2^e(k^e,k^e)-L_2^e(k_1^e,0),\overline{w}_0^e(w)\}\geq\underline{w}_0^f(w)=\min\{(L_2^f(k^f,k^f)-L_2^f(k_1^f,0))^+,\overline{w}_0^f(w)\}$ because (1) we have

$[L_2^e(k^e,k^e) - L_2^e(k_1^e,0)] - [L_2^f(k^f,k^f) - L_2^f(k_1^f,0)]^+ \geq 0$ because $L_2^e(k^e,k^e) \geq L_2^e(k_1^e,0)$ (see the proof of Proposition 1), $L_2^e(k^e,k^e) = L_2^f(k^f,k^f)$, and $L_2^e(k_1^e,0) - L_2^f(k_1^f,0) = V_2^e(k_1^e,0) - V_2^f(k_1^f,0) \leq 0$ by Lemma A4; (2) we have $\overline{w}_0^e(w) = L_1^e(k_1^e,0) - L_1^e(0,0)$ and $\overline{w}_0^f(w) = L_1^f(k_1^f,0) - L_1^f(0,0)$, so $\overline{w}_0^e(w) - \overline{w}_0^f(w) = L_1^e(k_1^e,0) - L_1^f(k_1^f,0) = V_1^e(k_1^e,0) + wk_1^e - V_1^f(k_1^f,0) - wk_1^f \geq 0$ by Lemma A3 and A4.

ii) We first observe by equation (3) and (5) that $k^e = k^f$. Then by Lemma A1, we have $k^e \leq k_1^e \leq 2k^e$. By Lemma A3, we have $k_1^f \leq k_1^e$. Therefore, when $0 \leq w_0 < \underline{w}_0^f(w)$, both firms invest with either capacity type so the total capacity investment $2k^e = 2k^f$; when $\underline{w}_0^f(w) \leq w_0 < \underline{w}_0^e(w)$, both firms invest with exclusive capacity but only one firm invests with first-priority capacity, so the total capacity investment $2k^e \geq k_1^f$; when $\underline{w}_0^e(w) \leq w_0 < \overline{w}_0^f(w)$, only one firm invests with either capacity type, so the total capacity investment $k_1^e \geq k_1^f$; when $\overline{w}_0^f(w) \leq w_0 < \overline{w}_0^e(w)$, one firm invests with exclusive capacity while neither firm invests with first-priority capacity, so the total capacity investment $k_1^e \geq 0$; when $w_0 \geq \overline{w}_0^e(w)$, neither firm invests with either type of capacity. Therefore, the total capacity investment is higher with exclusive capacity. Similar analysis hold for the non-overlapped one-firm-investing region case. □

Proof of Proposition 4 In the proof, when only one firm invests, we analyze the case where the investing firm is firm 1. This is because in these cases, when the investing firm is firm 2, the analysis is symmetric.

i) When $0 \leq w_0 < \underline{w}_0^f(w)$, both firms invest in the supplier with either capacity type, so we have $V_i^e(k^e,k^e) = V_i^f(k^f,k^f)$ and firms are indifferent between exclusive and first-priority capacity.

ii) When $\underline{w}_0^f(w) \leq w_0 < \overline{w}_0^f(w)$, only one firm invests with first-priority capacity and both firms invest with exclusive capacity. We have $V_1^f(k_1^f,0) \geq V_1^f(k^f,0) \geq V_1^f(k^f,k^f) =$

$V_1^e(k^e,k^e)$ and $V_2^f(k_1^f,0) \geq V_2^f(k^f,k^f) = V_2^e(k^e,k^e)$. Therefore, both firms prefer the first-priority capacity.

iii) When $\overline{w}_0^f(w) \leq w_0 < \underline{w}_0^e(w)$, both firms invest with exclusive capacity and neither firm invests with first-priority capacity. In this case, we show that $\overline{w}_0^f(w) \geq w_0^e(w)$, i.e., the lower end of the region is greater than the prisoner's dilemma threshold. Therefore, both firms are better off under the first-priority capacity where neither firm invests in the supplier. Note that following a similar argument in part (ii), we have $\overline{w}_0^f(w) = L_1^f(k_1^f,0) - L_1^f(0,0) \geq L_1^f(k^f,k^f) - L_1^f(0,0) = L_1^e(k^e,k^e) - L_1^e(0,0) = w_0^e(w)$. Therefore, both firms are better off with the first-priority capacity.

iv) When $\underline{w}_0^e(w) \leq w_0 < \overline{w}_0^e(w)$, one firm invests with exclusive capacity while neither firm invests with first-priority capacity, $V_1^e(k_1^e,0) \geq V_1^e(0,0) = V_1^f(0,0)$ and $V_2^e(k_1^e,0) \leq V_2^e(0,0) = V_2^f(0,0)$ by equation (A25).

v) When $w_0 \geq \overline{w}_0^e(w)$, neither firm invests with either type capacity. Therefore, the firms are indifferent between the exclusive and first-priority capacity, i.e., $V_i^e(0,0) = V_i^f(0,0)$. □

Proof of Corollary 1 When $\underline{w}_0^e(w) \leq w_0 < \overline{w}_0^f(w)$, only one firm invests with either type of capacity. If the investing firm is the same one under both capacity types (say firm 1), then we have $V_1^e(k_1^e,0) \geq V_1^f(k_1^f,0)$ and $V_2^e(k_1^e,0) \leq V_2^f(k_1^f,0)$ by Lemma A4.

If the investing firm is different under both capacity types, then let firm 1 be the investing firm under the exclusive capacity and firm 2 be the investing firm under the first-priority capacity. Then firm 2 prefers the first-priority capacity because $V_2^e(k_1^e,0) \leq V_2^e(0,0) = V_2^f(0,0) \leq V_2^f(0,k_1^f)$. For firm 1, we show that it prefers the exclusive capacity because $V_1^e(k_1^e,0) \geq V_1^f(0,k_1^f)$ as follows. First, we have

$$V_1^f(0,k_1^f) - V_1^f(0,0)$$

$$= \int_0^{\frac{a-c}{b(\frac{3k_0}{2}+2k_1^f)}} -\frac{bk_1^f k_0 \xi^2}{2} f(\xi)d\xi + \int_{\frac{2(a-c)}{3b(k_0+k_1^f)}}^{\frac{2(a-c)}{3bk_0}} \left[\frac{(a-c)^2}{9b} - (a-c-bk_0\xi)\frac{k_0}{2}\xi\right] f(\xi)d\xi$$
$$+ \int_{\frac{a-c}{b(\frac{3k_0}{2}+2k_1^f)}}^{\frac{2(a-c)}{3b(k_0+k_1^f)}} \left\{\left[a-c-b(k_0+k_1^f)\xi\right]\left[2(k_0+k_1^f)\xi - \frac{a-c}{b}\right] - (a-c-bk_0\xi)\frac{k_0}{2}\xi\right\} f(\xi)d\xi$$

Then, by equation (A23), we have

$$\left[V_1^e(k_1^e,0) - V_1^f(k_1^f,0)\right] - \left[V_1^f(0,k_1^f) - V_1^f(0,0)\right]$$
$$\geq \left[V_1^e(k_1^f,0) - V_1^f(k_1^f,0)\right] - \left[V_1^f(0,k_1^f) - V_1^f(0,0)\right]$$
$$\geq \int_{\frac{a-c}{b(\frac{3k_0}{2}+2k_1^f)}}^{\frac{2(a-c)}{3b(k_0+k_1^f)}} \left\{b\left[\frac{a-c}{2b} - \left(\frac{3}{4}k_0 + k_1^f\right)\xi\right]^2 + \frac{bk_0k_1^f\xi^2}{2}\right\} f(\xi)d\xi$$
$$+ \int_{\frac{2(a-c)}{3b(k_0+k_1^f)}}^{\frac{2(a-c)}{3bk_0}} \left[\frac{(a-c)^2}{36b} + \frac{(a-c)k_0}{4}\xi - \frac{7bk_0^2}{16}\xi^2\right] f(\xi)d\xi \geq 0$$

Therefore, we have

$$V_1^e(k_1^e,0) - V_1^f(0,k_1^f) \geq V_1^f(k_1^f,0) - V_1^f(0,0) \geq 0$$

The second inequality follows that only one firm invests in equilibrium under the first-priority capacity. □

Proof of Proposition 5 When only one firm invests, the supplier's expected profit remains the same no matter which buying firm invests. Therefore, we show the case where firm 1 is the investing firm. The analysis for firm 2 being the only investing firm is similar.

i) By equation (7) and (8), if $k_1 = k_2$, we have $V_s^e(k_1,k_2) = V_s^f(k_1,k_2)$. Therefore, when $0 \leq w_0 < \underline{w}_0^f(w)$ or $w_0 \geq \overline{w}_0^e(w)$, we have $V_s^e(k^e,k^e) = V_s^f(k^f,k^f)$ and $V_s^e(0,0) = V_s^f(0,0)$ respectively.

ii) When $\underline{w}_0^f(w) \leq w_0 < \overline{w}_0^f(w)$, both firms invest with exclusive capacity and only one firm invests with first-priority capacity, so by equation (7) and (8), we have

$$V_s^e(k^e,k^e) = \int_0^{\frac{a-c}{b(\frac{3k_0}{2}+3k^e)}} c(k_0+2k^e)\xi f(\xi)d\xi + \int_{\frac{a-c}{b(\frac{3k_0}{2}+3k^e)}}^1 \frac{2c(a-c)}{3b} f(\xi)d\xi \qquad \text{(A26)}$$

$$V_s^f(k_1^f,0) = \int_0^{\frac{2(a-c)}{3b(k_0+k_1^f)}} c(k_0+k_1^f)\xi f(\xi)d\xi + \int_{\frac{2(a-c)}{3b(k_0+k_1^f)}}^1 \frac{2c(a-c)}{3b} f(\xi)d\xi \tag{A27}$$

We note that $2k^e = 2k^f \geq k_1^e \geq k_1^f$ (see Lemma A1 and A3.) Therefore, it follows that $V_s^e(k^e,k^e) \geq V_s^f(k_1^f,0)$.

Similarly, when $\overline{w}_0^f(w) \leq w_0 < \underline{w}_0^e(w)$, both firms invest with exclusive capacity and neither firm invests with first-priority capacity, so we have $V_s^e(k^e,k^e) \geq V_s^f(0,0)$. When $\underline{w}_0^e(w) \leq w_0 < \overline{w}_0^e(w)$, one firm invests with exclusive capacity and neither firm invests with first-priority capacity, so we have $V_s^e(k_1^e,0) \geq V_s^f(0,0)$. The details are omitted for space. □

A.4. Discussions on sequence of events

In Section 2, we have described the sequence of events as follows. First, firms invest to improve the supplier's capacity, and then place an order *after* the capacity is realized. Finally, firms use all the ordered components to produce the final product and serve the end market. In this case, the wholesale price c in our model may include two parts: the ordering cost c_o and the production cost c_p, where $c = c_o + c_p$.

However, one may also think about an alternative sequence of events where the order might be placed *before* the capacity uncertainty resolves. Same as in our base model, firm i first invests k_i in the supplier's capacity incurring the investment cost $w_0 \mathbf{1}_{\{k_i>0\}} + wk_i$. Then firm i orders $\hat{q}_i$ from the supplier at the cost of c_o per unit. After the capacity uncertainty is resolved, firm i produces q_i to serve the market. There are two scenarios in this phase: on the one hand, if the realized capacity is less than $\hat{q}_i$, firms may only utilize a portion of the ordered components to produce the final products at the cost of c_p. That is, the number of the produced product $q_i \leq \hat{q}_i$. On the other hand, if there are still leftover in the invested capacity, firms may produce with all the ordered components at the cost of c_p per unit, and then order and produce from the extra capacity at the cost of $c_o + c_p$. In this case, the number of the produced product $q_i > \hat{q}_i$.

In this section, we use the exclusive capacity as an example to show that these two sequences of events are equivalent under mild assumptions. That is, under the second sequence of events, firms are indifferent between ordering a positive quantity before the capacity uncertainty resolves, or do not order anything before the capacity uncertainty resolves in an equilibrium. In other words, how many products should be ordered from the supplier are essentially decided after the capacity uncertainty resolves, even if firms have the opportunity to order before that. We first consider the case where firms do not incur an ordering cost before the capacity uncertainty resolves, i.e., $c_o = 0$ and $c_p = c$. Doing so allows us to isolate the impact of the timing to incur the ordering costs. We summarize the results in the following proposition.

PROPOSITION A1. *When $c_o = 0$ and $c_p = c$, for any k_i, k_j and $\hat{q}_j$, firm i is indifferent between ordering $\hat{q}_i = 0$ and $\hat{q}_i > 0$ before the capacity uncertainty resolves.*

The proposition implies that when it is free to preorder the components before the capacity uncertainty resolves, firms are indifferent between not ordering anything and ordering a positive quantity. The result follows the fact that the capacity investment has dual functions. On the one hand, it increases the supplier's capacity. On the other hand, it secures the firms' priority in accessing its invested capacity. The second function of the investment coincides with the purpose of placing an order before the capacity uncertainty resolves. Therefore, it is redundant to place an order in addition to the capacity investment before the capacity uncertainty resolves, and firms are indifferent between ordering a positive quantity and not ordering anything before the capacity uncertainty resolves because it is costless to preorder. We next examine the case where a part of the ordering cost is incurred before the capacity uncertainty resolves, i.e., $c_o > 0$ and $c_o + c_p = c$, with the assumption that the firm's order quantity before the capacity uncertainty resolves cannot exceed the unconstrained Cournot quantity $\frac{a-c_o-c_p}{3b}$.

PROPOSITION A2. *Assume that $\hat{q}_i \leq \frac{a-c_o-c_p}{3b}$, when $c_o > 0$ and $c_o + c_p = c$, for any k_i and k_j, there exists an equilibrium where $\hat{q}_i = 0$ for $i = 1, 2$.*

The proposition shows that it is indeed an equilibrium where neither firm will order anything before the capacity uncertainty resolves. To understand this, consider the following. If firm i deviates the equilibrium and places an order before the uncertainty resolves ($\hat{q}_i > 0$), there are two possible scenarios after the uncertainty resolves: the realized capacity might be smaller than or equal to the order quantity, $(\frac{k_0}{2} + k_i)\xi \leq \hat{q}_i$, or the realized capacity might be greater than the order quantity, $(\frac{k_0}{2} + k_i)\xi > \hat{q}_i$. In the first case, firm i could have saved the ordering cost if the order quantity is smaller, as the extra ordering cost is incurred for the parts beyond the realized capacity. In the second case, if firm i needs more components beyond the order quantity, it should be able to access the extra invested capacity by paying the ordering and production costs at once. Therefore, the firm is indifferent between ordering the same quantity or ordering less. To summarize both scenarios, firm i will find it an equilibrium when $\hat{q}_i = 0$.

Under the mild assumption that under the first-priority capacity, the supplier is able to leverage the leftover capacity after the capacity uncertainty resolves, similar equivalence between the sequence of events in this setting and the one in the basic model can be established. Therefore, the two sequences of events are equivalent as the actual ordering decisions are made after the capacity uncertainty resolves.

Proof of Proposition A1 We prove the proposition by showing that given k_i, k_j and $\hat{q}_j$, for any realization of ξ, firm i's best response function is independent of its choice of

$\hat{a}_i$ when $c_o = 0$ and $c_p = c$. In the following analysis, we highlight the dependency of the formulas on $\hat{q}_i$.

Let firm i's order quantity be $\hat{q}_i$ in the first stage (before the uncertainty resolves.) In the second stage, the equilibrium production quantities are determined by solving the following:

$$\begin{aligned}\hat{\pi}_i^e(k_1,k_2,\hat{q}_1,\hat{q}_2,\xi) &= \max_{q_i \le (\frac{k_0}{2}+k_i)\xi} q_i P(q_1,q_2) - c_p \min\{q_i,\hat{q}_i\} - (c_p+c_o)(q_i-\hat{q}_i)^+ \\ &= \max_{q_i \le (\frac{k_0}{2}+k_i)\xi} q_i P(q_1,q_2) - c_p q_i - c_o(q_i-\hat{q}_i)^+ \\ &= \max_{q_i \le (\frac{k_0}{2}+k_i)\xi} q_i P(q_1,q_2) - c q_i, \;\; i=1,2 \end{aligned} \quad \text{(A1)}$$

Following the concavity of firm i's profit in q_i, we obtain the best response function of firm i as follows.

$$q_i^*(q_j;\hat{q}_i) = \begin{cases} \frac{a-c-bq_j}{2b}, & \text{if } \frac{a-c-bq_j}{2b} < (\frac{k_0}{2}+k_i)\xi \\ (\frac{k_0}{2}+k_i)\xi. & \text{if } (\frac{k_0}{2}+k_i)\xi < \frac{a-c-bq_j}{2b} \end{cases}$$

We first observe that $q_i^*(q_j;\hat{q}_i)$ does not change with respect to $\hat{q}_i$. Therefore, the second stage equilibrium quantity does not change with $\hat{q}_i$. Let q_i^* denote the equilibrium order quantity and $\hat{\pi}_i^e(k_1,k_2,\hat{q}_1,\hat{q}_2,\xi)$ denote the second stage subgame perfect equilibrium profit. As $\hat{\pi}_i^e(k_1,k_2,\hat{q}_1,\hat{q}_2,\xi)$ does not change with $\hat{q}_i$ we suppress the dependence of $\hat{\pi}_i^e$ on $\hat{q}_i$ and write it as $\hat{\pi}_i^e(k_1,k_2,\xi)$. Then we have the first stage expected profit given k_i, k_j, $\hat{q}_i$, and $\hat{q}_j$ as

$$\begin{aligned}\hat{V}_i^e(k_1,k_2,\hat{q}_1,\hat{q}_2) &= E\left[\hat{\pi}_i^e(k_1,k_2,\hat{q}_1,\hat{q}_2,\boldsymbol{\xi})\right] - c_o\hat{q}_i - w_0\mathbf{1}_{\{k_i>0\}} - wk_i \\ &= E\left[\hat{\pi}_i^e(k_1,k_2,\boldsymbol{\xi})\right] - w_0\mathbf{1}_{\{k_i>0\}} - wk_i, \;\; i=1,2 \end{aligned} \quad \text{(A2)}$$

As equation (A2) does not change with respect to $\hat{q}_i$, firm i is indifferent between ordering any quantity before the uncertainty resolves. $\square$

Proof of Proposition A2 We prove the proposition by showing that given k_i, k_j and $\hat{q}_j = 0$, firm i does not have incentive to increase $\hat{q}_i$ from $\hat{q}_i = 0$ when $c_o > 0$ and $c_o + c_p = c$, assuming $\hat{q}_i \leq \frac{a-c_0-c_q}{3b}$. In the following analysis, we also highlight the dependency of the formulas on $\hat{q}_i$.

Let firm i's order quantity be $\hat{q}_i$ before the uncertainty resolves. In the second stage, the equilibrium production quantities are determined by solving the following:

$$\hat{\pi}_i^e(k_1, k_2, \hat{q}_1, \hat{q}_2, \xi) = \max_{q_i \leq (\frac{k_0}{2}+k_i)\xi} q_i P(q_1, q_2) - c_p q_i - c_o(q_i - \hat{q}_i)^+, \; i = 1, 2 \tag{A3}$$

Following the concavity of the profit in q_i, we obtain two best response functions $q_i^*(q_j; \hat{q}_i)$ as follows, depending on the relationship between $\hat{q}_i$ and $(\frac{k_0}{2} + k_i)\xi$. When $\hat{q}_i \leq (\frac{k_0}{2} + k_i)\xi$, we have

$$q_i^*(q_j; \hat{q}_i) = \begin{cases} \frac{a-c_p-bq_j}{2b}, & \text{if } \frac{a-c_p-bq_j}{2b} < \hat{q}_i \\ \hat{q}_i, & \text{if } \frac{a-c_p-bq_j}{2b} \geq \hat{q}_i > \frac{a-c_p-c_o-bq_j}{2b} \\ \frac{a-c_p-c_o-bq_j}{2b}, & \text{if } (\frac{k_0}{2} + k_i)\xi \geq \frac{a-c_p-c_o-bq_j}{2b} > \hat{q}_i \\ (\frac{k_0}{2} + k_i)\xi. & \text{if } (\frac{k_0}{2} + k_i)\xi < \frac{a-c_p-c_o-bq_j}{2b} \end{cases}$$

When $\hat{q}_i > (\frac{k_0}{2} + k_i)\xi$, we have

$$q_i^*(q_j; \hat{q}_i) = \begin{cases} \frac{a-c_p-bq_j}{2b}, & \text{if } \frac{a-c_p-bq_j}{2b} < (\frac{k_0}{2} + k_i)\xi \\ (\frac{k_0}{2} + k_i)\xi. & \text{if } (\frac{k_0}{2} + k_i)\xi < \frac{a-c_p-bq_j}{2b} \end{cases}$$

We show the analysis for the case where $k_i \geq k_j$. The analysis for the other case is similar. Under the assumption that $\hat{q}_i \leq \frac{a-c_o-c_p}{3b}$, we have the subgame perfect equilibrium order quantities given $(k_i, k_j, \hat{q}_i, \hat{q}_j)$ (note that $\hat{q}_j = 0$) the same as the one characterized in Lemma 1. Therefore, let $\hat{\pi}_i^e(k_1, k_2, \hat{q}_1, \hat{q}_2, \xi)$ denote the second stage subgame perfect equilibrium profit, and then we have the first stage expected profit $\hat{V}_i^e(k_1,k_2,\hat{q}_1,\hat{q}_2)$ given k_i, k_j, $\hat{q}_i>0$, and $\hat{q}_j=0$ as

$$
\begin{aligned}
&\hat{V}_i^e(k_1,k_2,\hat{q}_1,0)\\
=&\int_0^{\frac{a-c_o-c_p}{b(\frac{3k_0}{2}+2k_i+k_j)}}\left\{(a-b(k_0+k_i+k_j)\xi-c_p)\left(\frac{k_0}{2}+k_i\right)\xi-c_o\max\left[\left(\frac{k_0}{2}+k_i\right)\xi,\hat{q}_i\right]\right\}f(\xi)d\xi\\
&+\int_{\frac{a-c_o-c_p}{b(\frac{3k_0}{2}+2k_i+k_j)}}^{\frac{a-c_o-c_p}{3b(\frac{k_0}{2}+k_j)}}\left\{\frac{[a-b(\frac{k_0}{2}+k_j)\xi-c_p]^2-c_o^2}{4b}-c_o\max\left[\frac{a-c_o-c_p-b(\frac{k_0}{2}+k_j)\xi}{2b},\hat{q}_i\right]\right\}f(\xi)d\xi\\
&+\int_{\frac{a-c_o-c_p}{3b(\frac{k_0}{2}+k_j)}}^{1}\frac{(a-c_o-c_p)^2}{9b}f(\xi)d\xi-w_0\mathbf{1}_{\{k_1>0\}}-wk_1\\
\leq&\int_0^{\frac{a-c_o-c_p}{b(\frac{3k_0}{2}+2k_i+k_j)}}\left\{(a-b(k_0+k_i+k_j)\xi-c_p-c_o)\left(\frac{k_0}{2}+k_i\right)\xi\right\}f(\xi)d\xi\\
&+\int_{\frac{a-c_o-c_p}{b(\frac{3k_0}{2}+2k_i+k_j)}}^{\frac{a-c_o-c_p}{3b(\frac{k_0}{2}+k_j)}}\left\{\frac{[a-b(\frac{k_0}{2}+k_j)\xi-c_p-c_o]^2}{4b}\right\}f(\xi)d\xi\\
&+\int_{\frac{a-c_o-c_p}{3b(\frac{k_0}{2}+k_j)}}^{1}\frac{(a-c_o-c_p)^2}{9b}f(\xi)d\xi-w_0\mathbf{1}_{\{k_1>0\}}-wk_1\\
=&\hat{V}_i^e(k_1,k_2,0,0)
\end{aligned}
$$

Therefore, firm i does not have incentive to increase its order quantity from 0 before the capacity uncertainty resolves. $\square$