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HYSTERESIS MODELS FOR STEEL MEMBERS SUBJECTED TO CYCLIC BUCKLING OR CYCLIC END MOMENTS AND BUCKLING

(USER'S GUIDE FOR DRAIN-2D:EL9 AND EL10)

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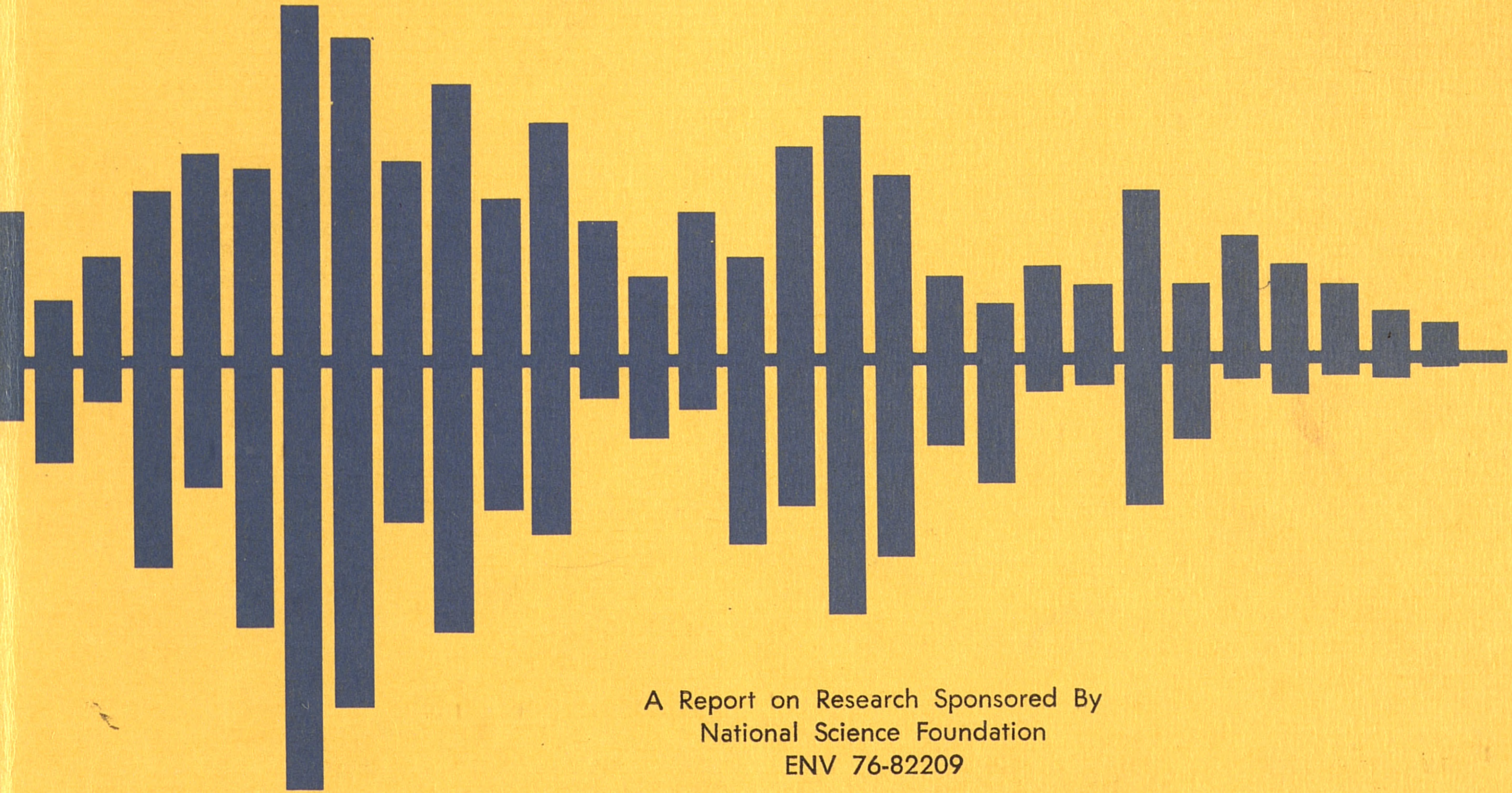

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HYSTERESIS MODELS FOR STEEL MEMBERS SUBJECTED TO CYCLIC BUCKLING OR CYCLIC END MOMENTS AND BUCKLING

(USER'S GUIDE FOR DRAIN-2D:EL9 AND EL10)

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ABSTRACT

Elements EL9 and EL10 are general purpose programs for steel members subjected to cyclic buckling, or cyclic end moment-buckling, respectively. These elements are developed for use with DRAIN-2D computer program. This manual describes the essential features of these two new elements along with their FORTRAN listing. The development of axial load-axial displacement hysteresis model as used for these elements has been described in a previous report.

ACKNOWLEDGEMENTS

The computer programs for the Buckling Element (EL9) and End Moment-Buckling Element (EL10) have been written on the basis of analytical and experimental research conducted at the University of Michigan over the period of past ten years. The financial assistance for the current effort was provided by the National Science Foundation, Grant ENV 76-82209 and the American Iron and Steel Institute, Project 301. Constructive suggestions from American Iron and Steel Institute Task Force on Project 301, Earthquake Resistant Design of Braced Steel Frame Structures, consisting of A. L. Collin, L. W. Lu, M. H. Mark, W. A. Milek, Jr., D. R. Sherman, E. P. Popov and L. A. Wyllie, Jr., were very helpful. Special thanks are due to Professor R. D. Hanson for introducing the idea of writing subroutine VRTX9. The authors are also thankful to Dr. J. B. Scalzi of NSF and Mr. A. C. Kuentz of AISI for the encouragement they provided in carrying out this research.

For compatability and consistency, the format of presentation of this manual has been styled after the other DRAIN-2D elements developed by A. E. Kanaan, G. H. Powell and Pritam Singh.

TABLE OF CONTENTS

CHAPTER

CHAPTER 1

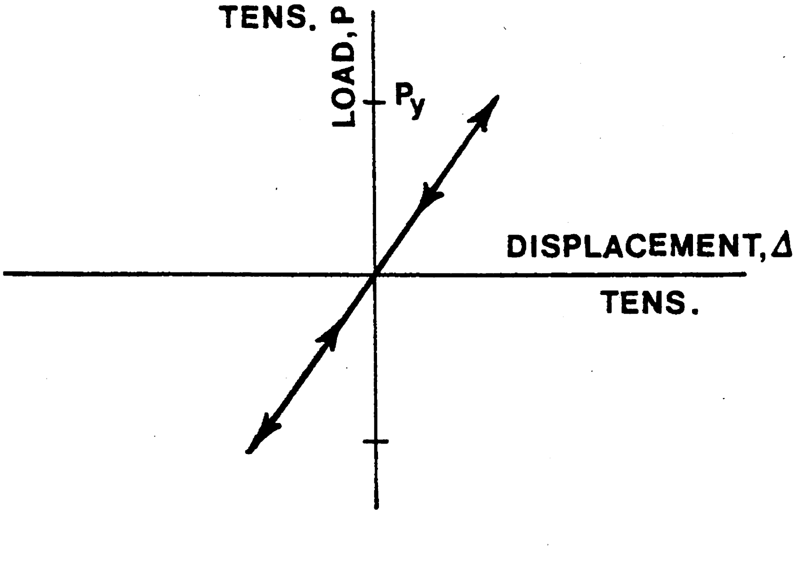
INTRODUCTION

Tall braced frame structures are constructed in seismically active regions throughout the world. Such frames are generally more efficient in terms of lateral stiffness per unit volume of material than open moment resisting steel frames. American Petroleum Institute Code (API-RP2A, Ref. 1) now contains strength and ductility requirements for offshore braced steel platforms. The strength requirements insure that the structure is adequately sized for strength and stiffness to maintain all nominal stresses within yield or buckling for the level of earthquake activity which is normally expected during the life of the structure. The ductility requirements are intended to insure that the structure has sufficient energy absorption capacity to prevent its collapse during rare intense earthquake motions. Bracing members are considered effective earthquake resistant elements as they help satisfy the above two requirements when used in a frame.

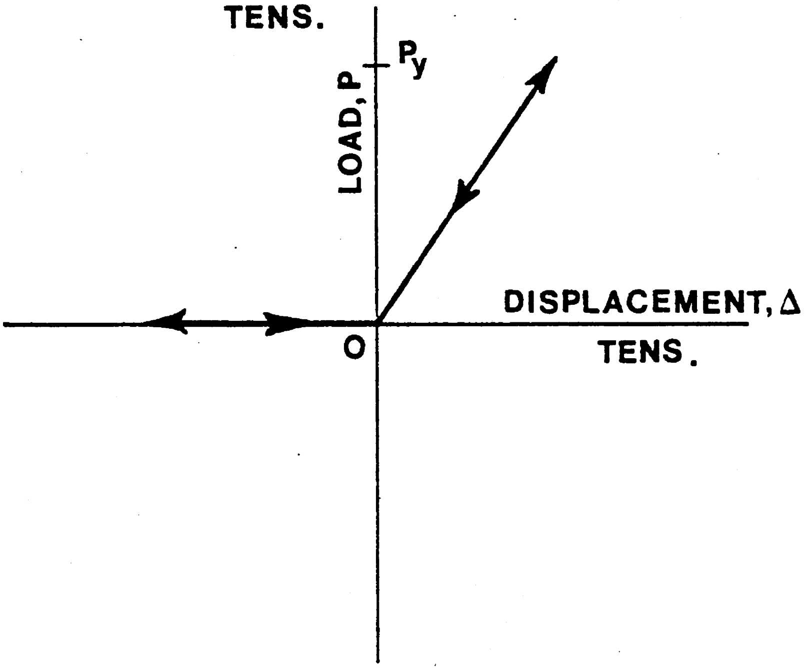
Different member arrangements and proportions are used in braced frames (3,4,5). Bracing arrangements may be either concentric or eccentric type. The connections of bracing members may be designed as simple or moment resistant. The members in the former situation are generally treated as primarily axially loaded, whereas, the latter type may develop significant end moments.

In past studies of braced frames, the hysteresis behavior of primarily axially loaded bracing members has been modeled in one of several ways, such as: elastic in tension and compression (Figure 1a), tension-only elastic model (Figure 1b), tension-yield and compression-yield (Figure 1c), or tension-yield and compression-buckling (Figure 1d). These models neglected the energy dissipation characteristics of bracing members in the post-buckling range. Later, Higginbotham and Hanson (Ref. 2, Figure 1e), Nilforoushan (Ref. 12, Figure 1f), Prathuangsit (Ref. 14), Singh (Ref. 15, Figure 1g), Wakabayashi (Ref. 17), and Marshall (Ref. 10, Figure 1h) presented hysteresis models which represented the post-buckling behavior of bracing members in a more realistic manner. Experimental studies (3,8) on small specimens have pointed out two significant characteristics in the hysteresis behavior which were not included in these analytical models. These characteristics are: increase in member length and reduction in compressive strength with number of cycles. Jain (Ref. 3, Figure 2) presented a hysteresis model which accounts for these two parameters. Minor changes have been made in this model and the latest version is described in Chapter 2 of this manual. This model is called as Buckling Element (EL9) for use in DRAIN-2D Computer program (9).

There is one model available for primarily end

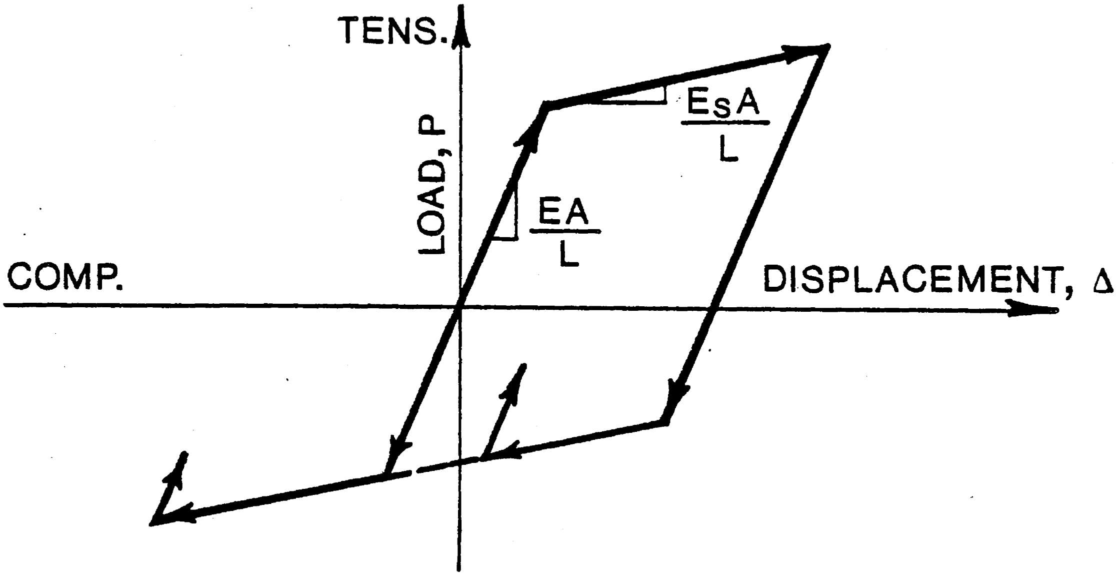


(a) Elastic Tension — Compression Model

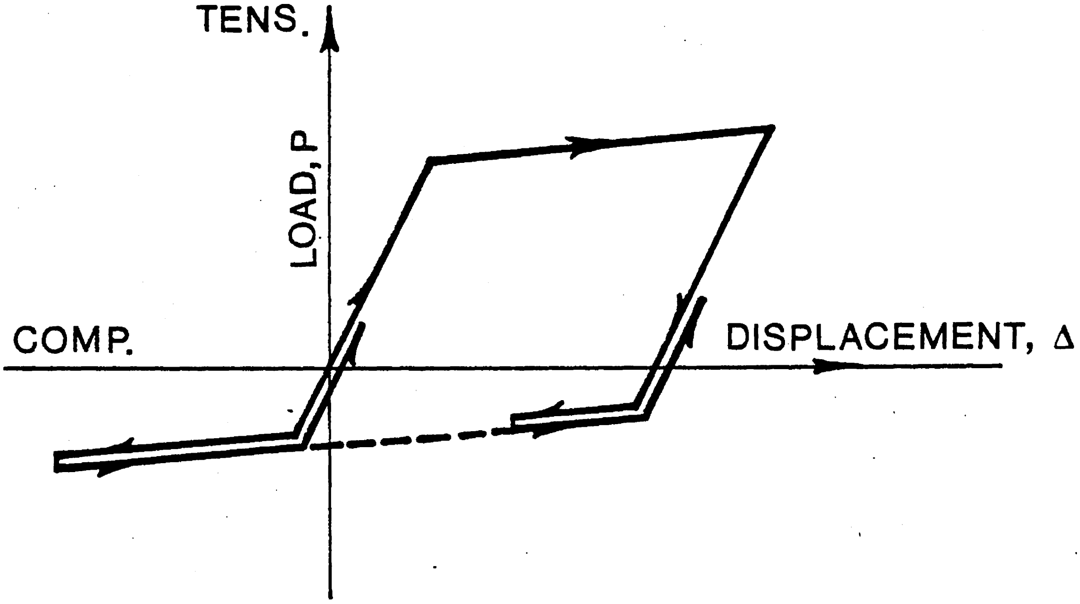


(b) Elastic Tension — Only Model

Figure 1 - Axial Load - Displacement Behavior

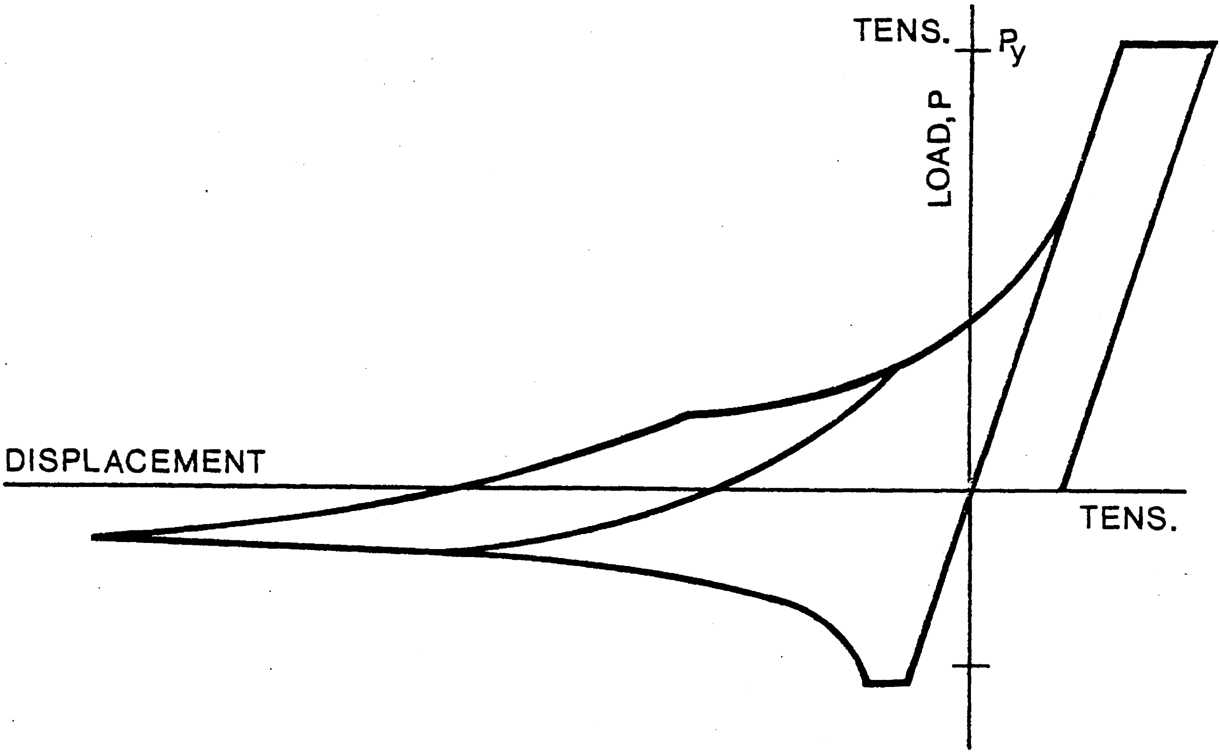


(c) Yield in Tension and Compression

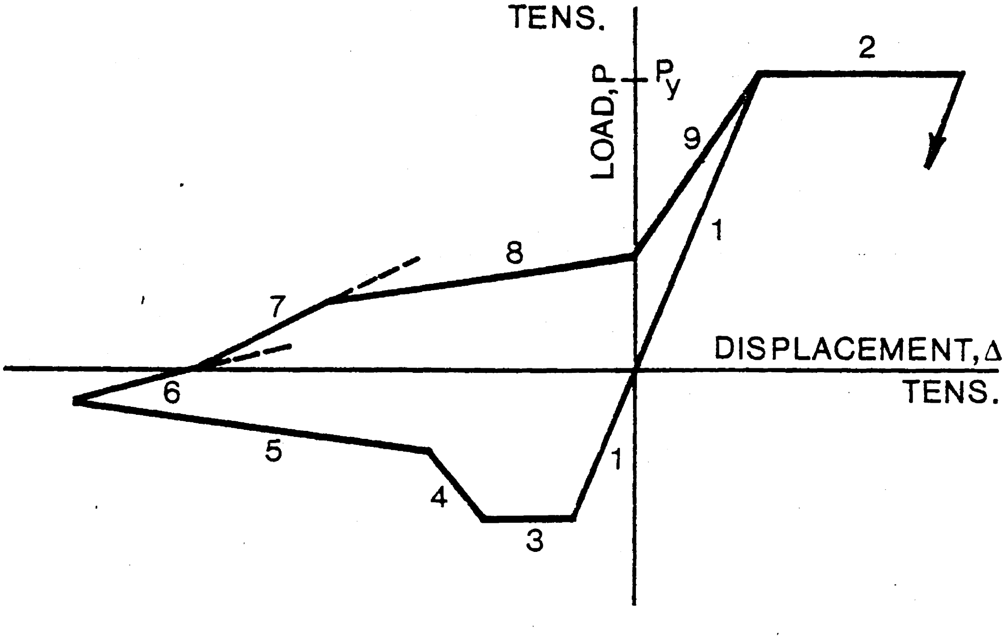


(d) Yield in Tension, Buckling in Compression

Figure 1 - Axial Load - Displacement Behavior (Cont.)

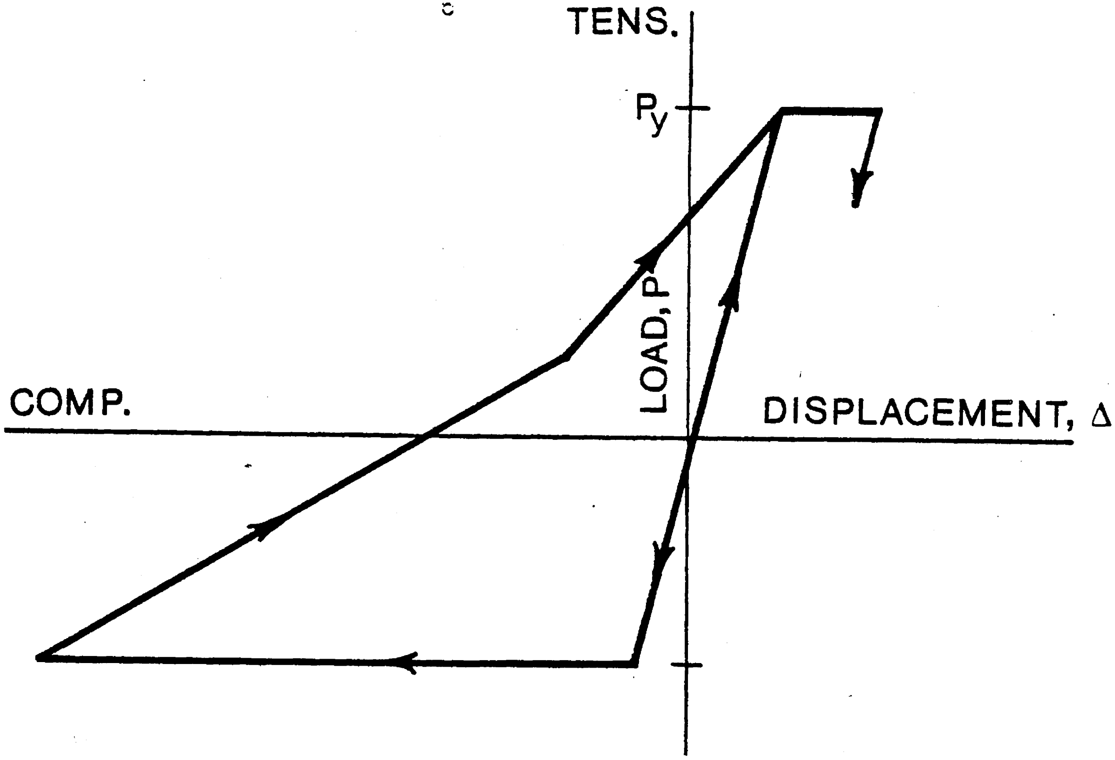


(e) **HIGGINBOTHAM AND HANSON**, Ref. 2

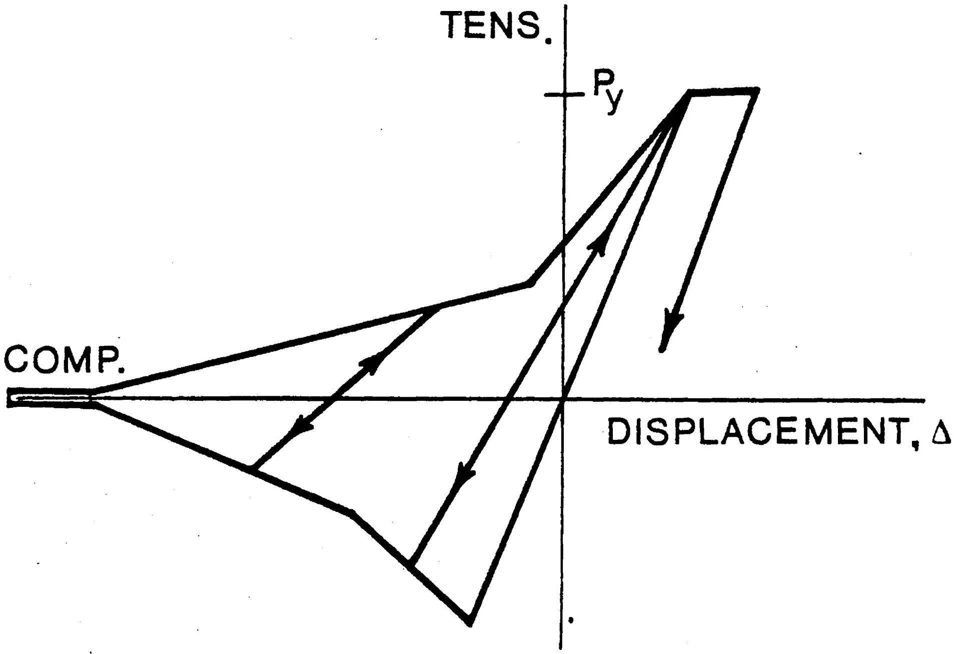


(f) **NILFOROUSHAN**, Ref. 12

Figure 1 - Axial Load-Displacement Behavior (cont.)



(g) **Singh,** Ref. 15



(h) **Marshall,** Ref. 10

Figure 1 - Axial Load-Displacement Behavior (cont.)

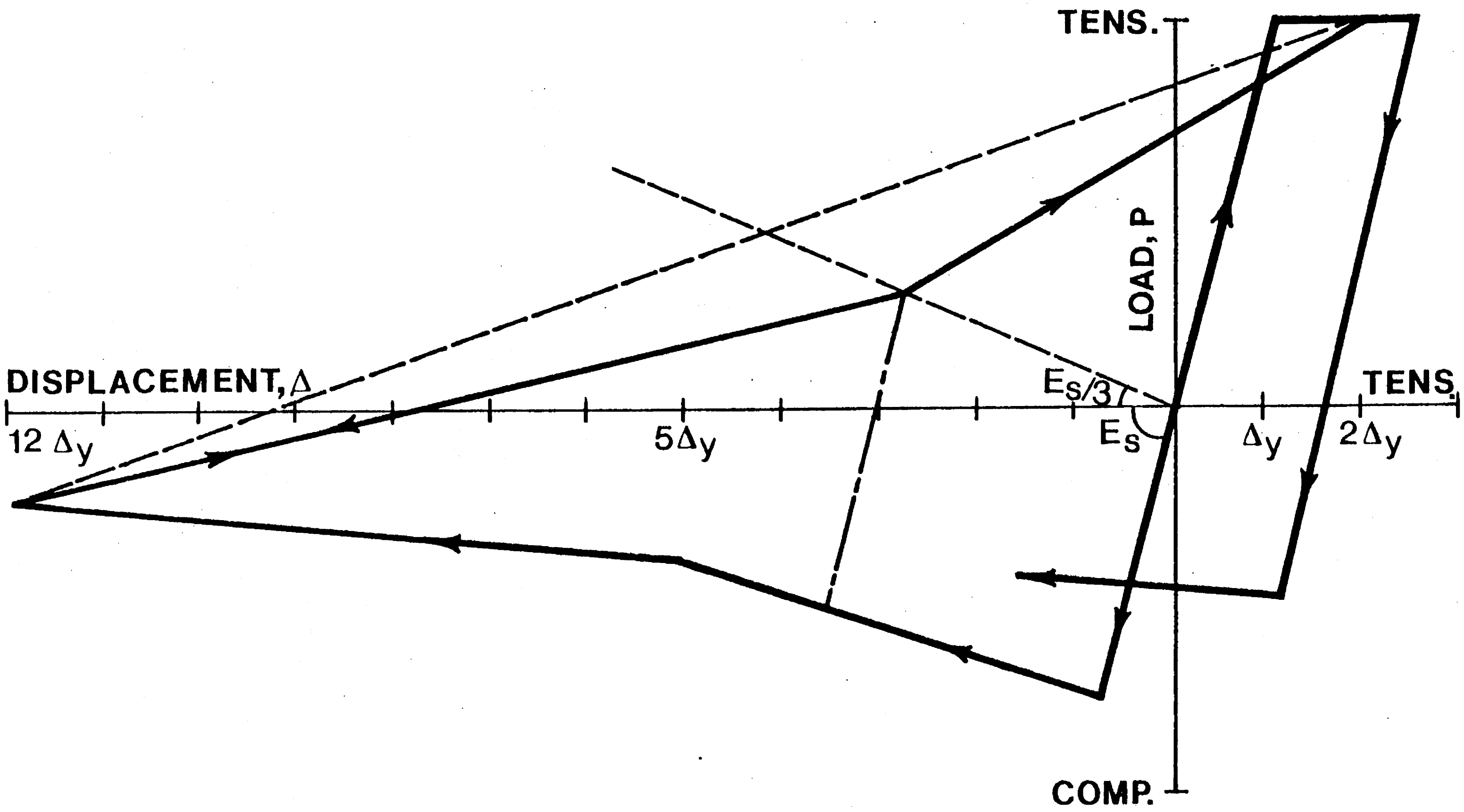


Figure 2 - Axial Load-Displacement Behavior, Ref. 3

moment resisting members and is known as beam-column element (Ref. 9, Figure 3). This model does not consider buckling and, therefore, should be used for full moment connected (or rigid-connected) non-buckling type bracing or column members.

Jain (3) analyzed 18 concentrically braced (X and K) and eccentrically braced (open or split K) frames under monotonic elastic, monotonic inelastic and dynamic loading conditions. The purpose of this analysis was (i) to determine the situations in which end moments dominate over axial forces in bracing members and vice versa, and (ii) to develop an understanding of the inelastic dynamic response of these frames with different member proportions. It was concluded that there is a need to develop a hysteresis model for rigid-connected buckling type steel members. Such a model has been developed by combining buckling element (EL9) and beam-column element (EL2) and is described in Chapter 3 of this manual. This model is called as End Moment-Buckling Element (EL10) for use in DRAIN-2D computer program (9).

DRAIN-2D COMPUTER PROGRAM

DRAIN-2D is a general purpose computer program for the inelastic response of plane structures subjected to earthquake forces, and was developed by Kannan and Powell (9). The program concepts and features are described in Reference 9. User's Guide (13) describes the extensions made to the program and presents input data procedures.

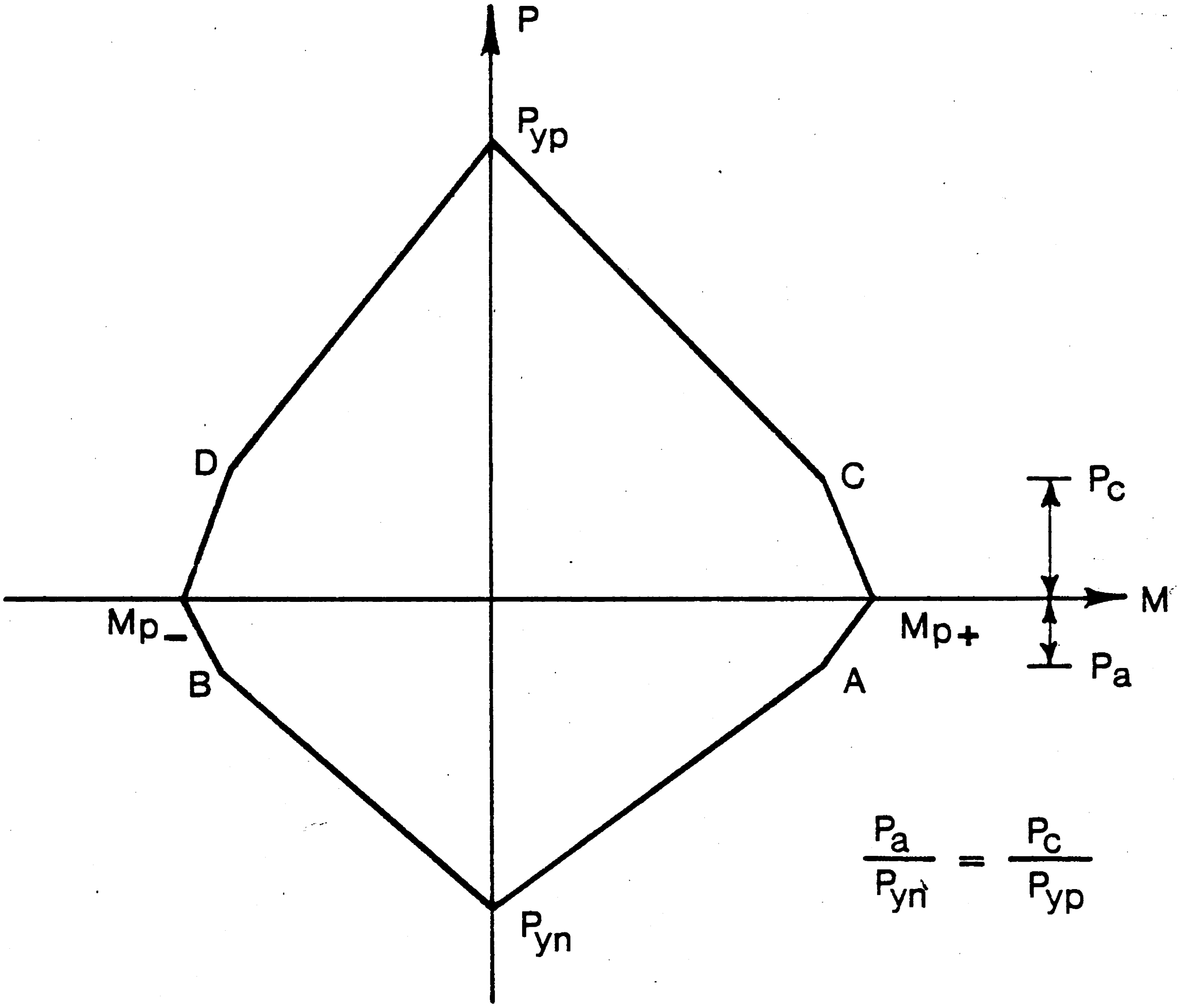


Figure 3 - Beam-Column Model, Ref. 9

This manual supplements References 3, 9 and 13, and should be used in conjunction with them. For compatibility the format of presentation in this manual has also been styled after these references. The procedure followed in adding the new elements EL9 and EL10 to the DRAIN-2D conforms to Chapter 4 of Reference 9. The four main subroutines developed for each element are as follows. The number at the end of the subroutine name corresponds to the element type.

1. INEL9, INEL10: Input and initialization of element data.
2. STIF9, STIF10: Calculation of element tangent stiffness at different time steps.
3. RESP9, RESP10: Determination of increments of element deformations (strains) and forces (stresses), determination of yield status, and output of time history results. This may be called as "state determination phase".
4. OUT9, OUT10: Output of final envelope values for element deformations and forces.

This arrangement is used in DRAIN-2D program and is taken directly from it. The variable names have been kept the same as for other elements (9,15). FORTRAN listing for elements EL9 and EL10 are given in Appendices A-1 and A-2, respectively. Several COMMENT statements are given for understanding the underlying logic. These programs have been used on AMDAHL 470V/6 computer at the University of Michigan using MTS. It is believed that they can be easily used on other systems.

If the user has other element subroutines which are also called either EL9 or EL10, then the suffix 9 or 10 from all the subroutines of these elements including CALL statements should be changed. The new suffix should be less than 10, otherwise, significant additions and changes would have to be made in the main DRAIN-2D program (Cards B to AB) in order to accomodate more than ten elements.

CHAPTER 2

BUCKLING ELEMENT (EL9)

Singh (15) presented a multilinear hysteresis model (EL7) for axially loaded pin-ended bracing members and used in the seismic analysis of multistory braced frames. Jain, Goel and Hanson (7) compared their experimental hysteresis curves with analytical curves obtained by using Singh's model and suggested that this model could be improved if modifications were made in compression envelope and tension envelope regions to include the change in compression loads and residual elongation. The new buckling model accounts for these two parameters, yet, retains the simplicity of Singh's model.

The buckling model is described in the following section. Tension load and displacement are taken as positive, and compression load and displacement are taken as negative.

GENERAL CHARACTERISTICS

Assuming that an initially straight member is loaded first in tension, the member follows segment AE elastically as shown in Figure 4a (computer print out code for this segment = 0). The member yields at E and follows segment EE' (Code = 9). If the direction of displacement is reversed at E', the member unloads elastically, parallel to the initial elastic slope AE (code = 0). Continued compression will result in the first buckling of the member

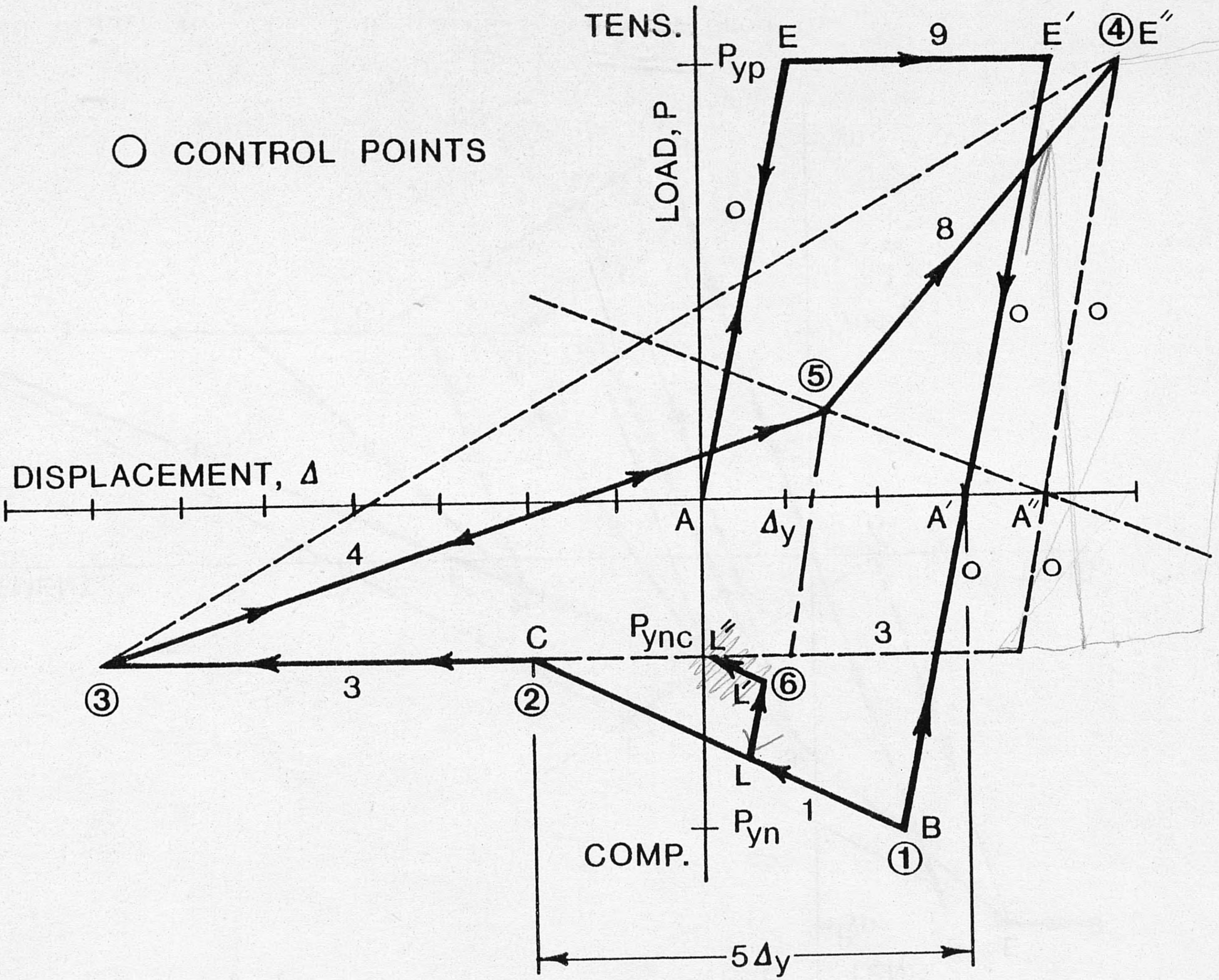


Figure 4a - Axial Hysteresis Behavior Used in Buckling Model

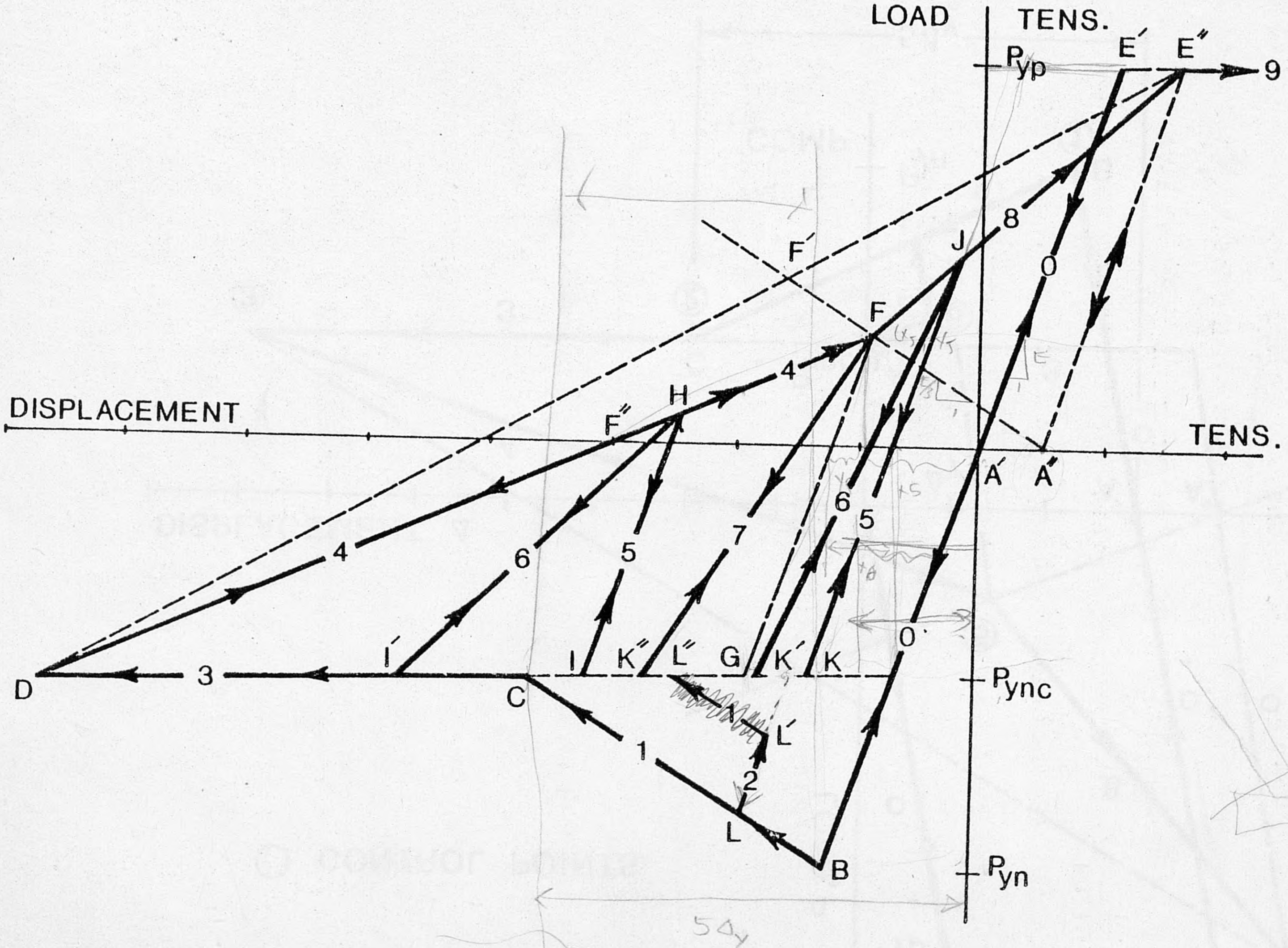


Figure 4b - Axial Hysteresis Behavior Used in Buckling Model (cont.)

at point B. The load at point B corresponds to the first cycle buckling load P_{yn} for the member which is significantly higher than the buckling load P_{ync} used for subsequent cycles. After buckling at point B the member follows segment BC (code = 1). The point C corresponds to a compression displacement equal to five times the tension yield displacement Δy of the member (Ref. 3). If the direction of axial displacement is reversed at L, the member follows segment LL' (code = 2), parallel to the initial elastic slope AB until it hits the post-buckling load level, P_{ync}. However, if the direction of axial displacement is reversed at L', the member follows segment L'L" (code = 1) which is parallel to segment BC. L" lies at the P_{ync} load level.

Once the member hits the post-buckling load level P_{ync}, it comes out of Subroutine VRTX9 and reenters into the main Subroutine RESP9 for further state determination (Figure 4b).

From point C or L", continued compression results in segment CD or L"D, respectively (code = 3). If the direction of axial displacement is reversed at D, it results in compression load decreasing to zero and followed by an increasing tensile load along the path DFE" (code = 4 for segment DF and code = 8 for segment FE").

To locate the point F, a line A"F' is drawn from the new origin A" (A'A" = E'E") at a slope of 1/3 times the initial elastic slope A'E' or A'B, which intersects the line DE" at F'. Intercept A"F is taken as 60/(KL/r) times the distance A"F'. E'E" is the residual elongation term and is calculated from equation 2.1. This equation was developed from the data obtained on small square steel tubes in Reference 3. The coefficients used in this equation can be changed very easily, if program user so desires. One card for RE (residual elongation expressed in terms of residual strain) should be changed in each of the Subroutines RESP9 and LAW9.

RE = 0.0175 * (0.55*X3RE/SLEND+0.0002*X3RE**2) 2.1

The line FG drawn parallel to the initial elastic slope is used to distinguish the loading history in the region DFG from that in the region A"E"FG. If the direction of axial displacement is changed along DF" the member follows the same path (code = 4). If the member reverses at point H, it follows segment HI (code = 5) which is parallel to segment A'B. Continued compression results in segment II' (code = 3). If the direction of displacement is again reversed at I', the member follows segment I'H (code = 6). If the direction of axial displacement is reversed at J, the member follows segment JK (code = 5). Continued compression results in segment KK' (code = 3) or in segment KK" (code = 3). If reversal occurs at K', the member follows segment K'J (code = 6). If member surpasses point G and returns from K", it follows K"F

$$RE = \left[\frac{0.55\Delta}{Kl/r} + 0.0002\,\Delta^2\right]$$

(code = 7) and then FE" (code = 8). New control points are determined if the member surpasses points D or E" during a cycle. In subsequent cycles the maximum compression load is P_{ync}.

The load P_{ync} is input as a fraction of maximum compressive load P_{yn} in the first cycle. The load P_{yn} can be calculated by using AISC equations (3,16). Data obtained in Reference 3 suggests that the strength reduction factor PHI ($\phi = P_{ync}/P_{yn}$) can be approximated as 0.3-0.6 depending upon effective slenderness ratio of the member.

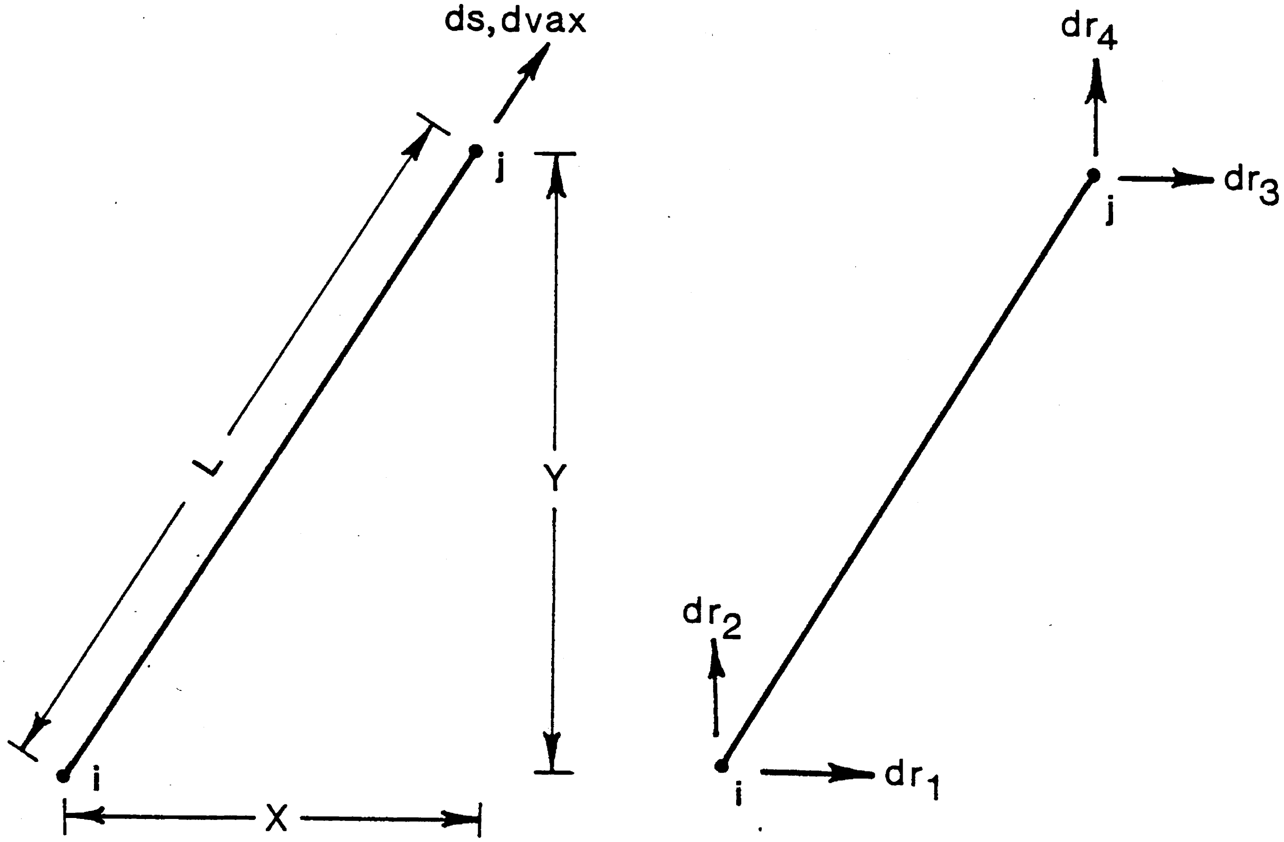
ELEMENT DEFORMATIONS

The buckling element has four degrees of freedom. The only deformation to be considered is its axial extension. The displacement transformation relating increments of deformation and displacement (Figure 5) is:

$$dv = \begin{bmatrix} -\frac{X}{L} & -\frac{Y}{L} & \frac{X}{L} & \frac{Y}{L} \end{bmatrix} \begin{Bmatrix} dr_1 \\ dr_2 \\ dr_3 \\ dr_4 \end{Bmatrix} \qquad 2.2$$

$$\text{or,} \quad \{dv\} = [a]\ \{dr\} \qquad 2.3$$

Large displacement effects are not taken into account. X, Y and L are assumed to remain constant and the displacement transformation matrix also remains constant.



(a) (b)

Figure 5 - Deformations and Displacements

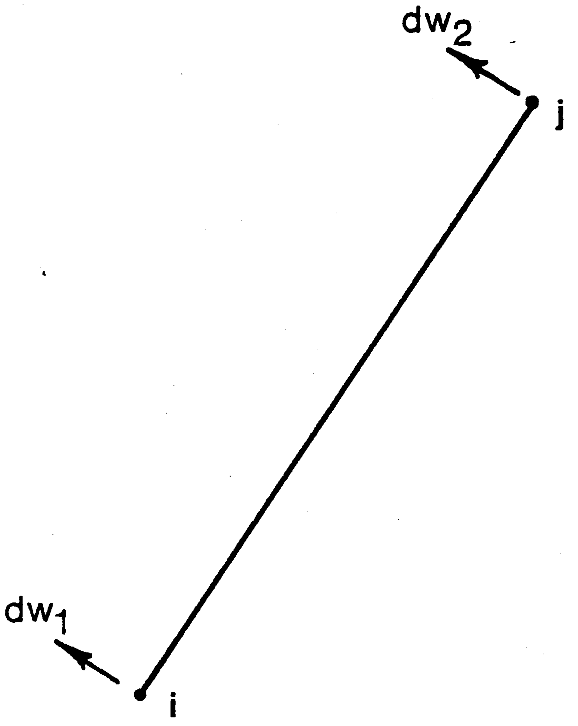


Figure 6 - Displacement for Geometric Stiffness

ELEMENT STIFFNESS

The tangent stiffness in term of deformations is

$$dS = \frac{E_T A}{L} \; dv \qquad 2.4$$

or, $$\{dS\} = [k_T] \; \{dv\} \qquad 2.5$$

where, E_T = tangent modulus in current state, and A = element cross sectional area.

The tangent stiffness in terms of nodal displacements is

$$[K_T] = [a]^T \; [k_T] \; [a] \qquad 2.6$$

where, [a] is given by equations 2.2 and 2.3.

The geometric stiffness in the element coordinates dw_1 and dw_2 is (Figure 6):

$$[k_G] = \frac{S}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \qquad 2.7$$

or, in terms of nodal displacements

$$[K_G] = [a_1]^T \; [k_G] \; [a_1] \qquad 2.8$$

where, $[a_1]$ is given by

$$\begin{Bmatrix} dw_1 \\ dw_2 \end{Bmatrix} = \begin{bmatrix} -\frac{Y}{L} & \frac{X}{L} & 0 & 0 \\ 0 & 0 & -\frac{Y}{L} & \frac{X}{L} \end{bmatrix} \begin{Bmatrix} dr_1 \\ dr_2 \\ dr_3 \\ dr_4 \end{Bmatrix}$$

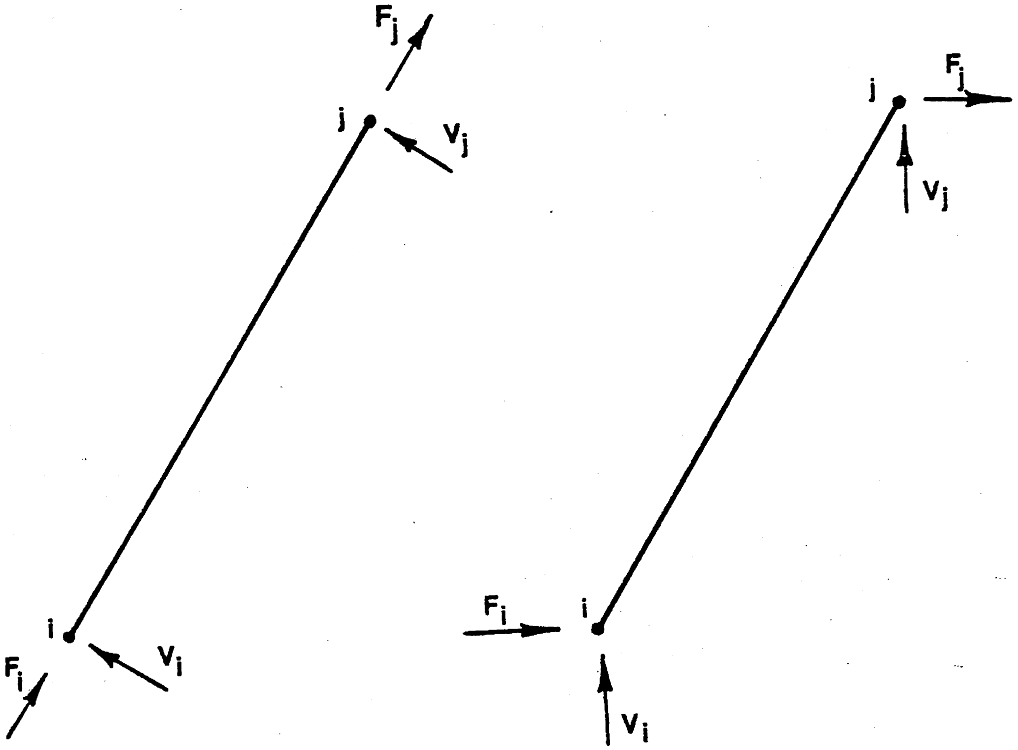
$$= [a_1] \; \{dr\} \qquad 2.9$$

FIXED END AND INITIAL FORCES

The effects of static loads applied along the element length rather than at the nodes can be taken into account by specifying fixed end force patterns. Static thermal effects can also be considered in the same way. The forces to be specified are the forces on the element ends required to prevent them from displacing, with the sign convention shown in Figure 7. If axial forces having different magnitudes at ends i and j are specified, the average value is assumed for determining the yield status of the element and for computing the geometric stiffness.

Elements may be stressed under static load but it may be incorrect or inconvenient to determine the element forces by applying static loads to the structure. To allow for such cases, provision is made for initial forces to be specified in the elements. These forces will typically be the forces in the elements under static loading as calculated by a separate analysis. For consistency, they should be in equilibrium with the static load producing them, but this is not essential. The computer program does not make corrections for any equilibrium unbalance resulting from the specification of initial forces.

To satisfy the requirement that the structure remain elastic under static loading, the initial element forces should be less than the yield strengths of the element. If desired, static loads as well as initial forces may be specified. The element forces will then be the sum of the



(a) CODE = 0

(b) CODE = 1

Figure 7 - End Clamping and Initial Forces

initial forces and those due to the static loads. The geometric stiffness effect is not included in the static analysis.

OUTPUT RESULTS

The following results are printed for the static loading condition (time = 0) and at each output time if a time history is requested. The static results are output for all elements, and the time history results for only those elements for which time histories are requested.

1. Yield code: 0 to 9 as explained earlier in Figures 4a and 4b.
2. Axial force, tension positive.
3. Net axial extension.
4. Accumulated positive and negative plastic extensions up to the current time.

These accumulated deformations are computed by accumulating the plastic extensions during all positive and negative plastic excursions. These accumulated deformations, together with the maximum positive and negative total extensions, provide information on the amount of plastic deformation imposed on the element. The maximum positive and negative values of axial force, maximum positive and negative extension and accumulated plastic extension are printed at the time intervals requested for results envelopes. The times at which the maximum forces and extensions were produced are also printed.

INPUT DATA PREPARATION

E9. BUCKLING ELEMENTS - EL9

Number of words of information per element = 53.

E9(a). CONTROL INFORMATION FOR GROUP (4I5) - ONE CARD

Columns 5: Punch 9 (to indicate that group consists of buckling elements).

6 - 10: Number of elements in group.

11 - 15: Number of different element stiffness types (max. 40). See Section E9(b).

16 - 20: Number of different fixed end force patterns (max. 40). See Section E9(c).

E9(b). STIFFNESS TYPES (I5, 7F10.0) - ONE CARD FOR EACH STIFFNESS TYPE

Columns 1 - 5: Stiffness type number, in sequence beginning with 1.

6 - 15: Young's modulus of elasticity.

16 - 25: Average cross sectional area.

26 - 35: Tension yield force, P_{yp}

36 - 45: Compression yield force, P_{yn} (first cycle)

46 - 55: Radius of gyration

56 - 65: Effective length factor

66 - 75: Strength reduction factor, PHI

E9(c). FIXED END FORCE PATTERNS (2I5, 4F10.0) - ONE CARD FOR EACH FIXED END FORCE PATTERN

Omit if there are no fixed end forces. See Figure 7.

Columns 1 - 5: Pattern number, in sequence beginning with 1.

10: Axis code, as follows:

Code = 0: Forces are in the element coordinate system, as in Figure 7a.

Code = 1: Forces are in the global coordinate system, as in Figure 7b.

Columns 11 - 20: Clamping force F_i.

21 - 30: Clamping force V_i.

31 - 40: Clamping force F_j.

41 - 50: Clamping force V_j.

E9(d). ELEMENT GENERATION COMMANDS (9I5, 2F5.0, F10.0) - ONE CARD FOR EACH GENERATION COMMAND

Elements must be specified in increasing numerical order. Cards for the first and last elements must be included. See Note 7 of User's Guide (13) for explanation of generation procedure.

Columns 1 - 5: Element number, or number of first element in a sequentially numbered series of elements to be generated by this command.

6 - 10: Node number of element end i.

11 - 15: Node number of element end j.

16 - 20: Node number increment for element generation. If zero or blank, assumed to be equal to 1.

21 - 25: Stiffness type number.

30: Code for including geometric stiffness. Punch 1 if geometric stiffness is to be included. Leave blank or punch zero if geometric stiffness is to be ignored.

35: Time history output code. If a time history of element results is not required for the elements covered by this command, punch zero or leave blank. If a time history printout, at the intervals specified on card D1, is required, punch 1.

Colums 36 - 40: Fixed end force pattern number for static dead loads on element. Leave blank if there are no dead loads. See note below.

41 - 45: Fixed end force pattern number for static live load on element. Leave blank if there are no live loads.

46 - 50: Scale factor to be applied to fixed end forces due to static dead loads. Leave blank if there are no dead loads.

51 - 55: Scale factor to be applied to fixed end forces due to static live loads. Leave blank if there are no live loads.

56 - 65: Initial axial force on element, tension positive.

NOTE: If static load code, Card C1, is zero but fixed end forces are still specified for some elements, an inconsistency results. In effect any such fixed end forces will be treated as initial element forces.

CHAPTER 3

END MOMENT-BUCKLING ELEMENT (EL10)

The end moment-buckling element is a combination of beam-column element (EL2) and buckling element (EL9). This element considers the interaction between the end moments and axial force in the beam-column component EL2, the axial force being determined by the buckling element EL9. In this formulation, the flexural stiffness is assumed to be independent of the axial force. Workman (18) studied the influence of axial force-flexural stiffness interaction in the elastic state on the seismic response of braced steel frames. He concluded that the effect of this interaction was not significant for the structural response. Nigam (11) proposed a more consistent procedure for considering the interaction between forces existing at sections where yielding occurs, but it is very complex and, therefore, not considered for this interactive element EL10. It is believed that the axial force-end moment interaction as modeled herein should be adequate for practical applications.

GENERAL CHARACTERISTICS

End moment-buckling element has six degrees of freedom and may be arbitrarily oriented in the X-Y plane. The element possess axial and flexural stiffnesses. Variable cross-sections can be considered by specifying average area and appropriate flexural stiffness

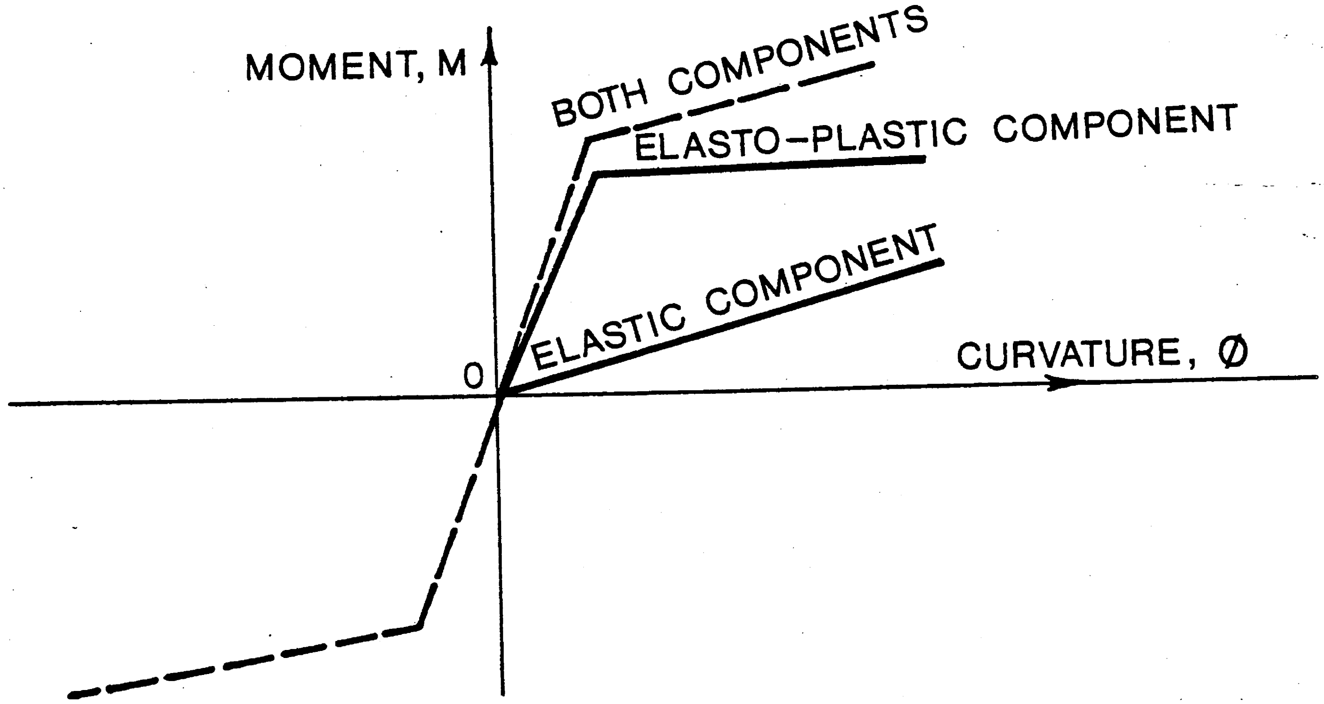


Figure 8 - Moment-Curvature Relation

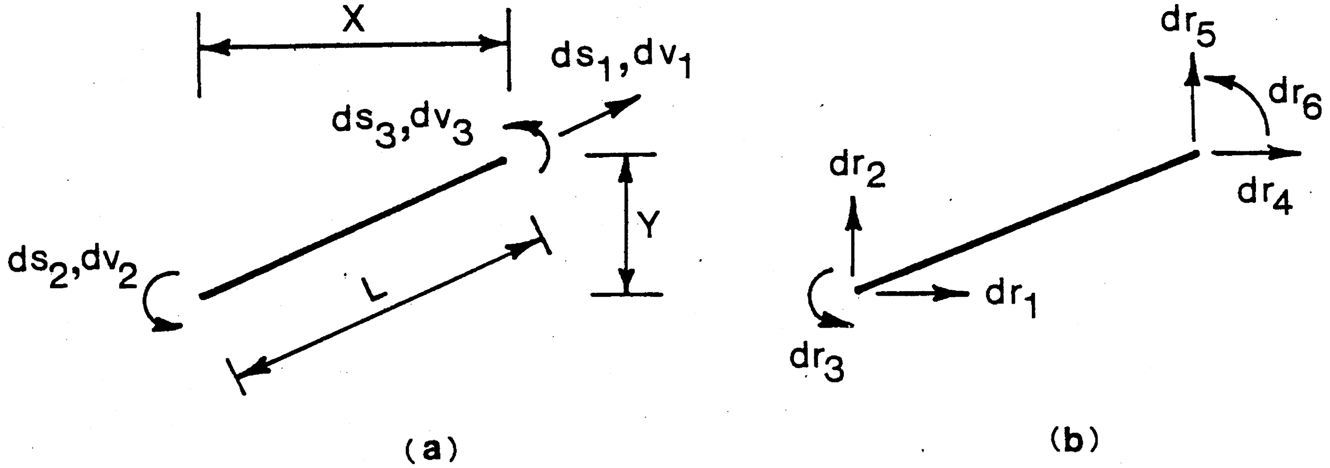


Figure 9 - Deformations and Displacements

coefficients. Flexural shear deformations can also be taken into account.

Strain hardening is considered in the moment-rotation relationship but not in the axial force-axial displacement relationship. Strain hardening is approximated by assuming that the element consists of elastic and elasto-plastic components in parallel as shown in Figure 8. Flexural yielding may take place only in concentrated plastic hinges at the ends of the element. The plastic hinges in the elasto-plastic component rotate under constant moment, but the moment in the elastic component may continue to increase.

The plastic moment capacities may be specified to be different at the two ends of an element and also for positive and negative bending at each end. If tension yield and compression strengths are different at the two ends of an element, minimum values of tension yield and compression strengths are used.

Static loads applied along any element length may be taken into account by specifying fixed end force values. The results of separate static load analyses can be imposed by specifying initial force values.

Large displacement effects may be approximated in the dynamic analysis by including simple geometric stiffnesses based on the element axial forces under static load.

ELEMENT DEFORMATIONS

An end moment-buckling element has three modes of deformation, namely axial extension, flexural rotation at end i, and flexural rotation at end j. The displacement transformation relating increments of deformation and displacement (Figure 9) is:

$$\begin{Bmatrix} dv_1 \\ dv_2 \\ dv_3 \end{Bmatrix} = \begin{bmatrix} -X/L & -Y/L & 0 & X/L & Y/L & 0 \\ -Y/L^2 & X/L^2 & 1 & Y/L^2 & -X/L^2 & 0 \\ -Y/L^2 & X/L^2 & 0 & Y/L^2 & -X/L^2 & 1 \end{bmatrix} \begin{Bmatrix} dr_1 \\ dr_2 \\ dr_3 \\ dr_4 \\ dr_5 \\ dr_6 \end{Bmatrix} \qquad 3.1$$

or, $\{dv\} = [a]\ \{dr\}$ 3.2

As for the buckling element, X, Y and L are assumed to remain constant.

A plastic hinge forms when the moment in the elasto-plastic component of the element reaches its plastic moment. A hinge is then introduced into this component, the elastic component remaining unchanged. The measure of flexural plastic deformation is the plastic hinge rotation.

For any increments of total flexural rotation, dv_2 and dv_3, the corresponding increments of plastic hinge rotation, dv_{p2} and dv_{p3}, are given by

$$\begin{Bmatrix} dv_{p2} \\ dv_{p3} \end{Bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{Bmatrix} dv_2 \\ dv_3 \end{Bmatrix} \qquad 3.3$$

where, A, B, C and D are given in Table 1.
Unloading occurs at a hinge when the increment in hinge rotation is opposite in sign to that of the bending moment.

Inelastic axial deformations obey the same hysteresis law as the Buckling Element EL9 does (Figures 4a and 4b).

INTERACTION SURFACES

The End Moment-Buckling Element uses two types of interaction surfaces. For axial force-axial displacement interaction, it uses the same as used by Buckling Element EL9. For axial force-end moment interaction it uses the envelope as shown in Figure 10.

Knowing the axial deformations, the program first determines the axial state of the element as for EL9. It calculates the axial force and the unbalanced axial force, if any. Then, it calculates the yield moment by using the axial force-moment interaction curve as for beam-column element EL2. If the moment lies on or outside the surface, a plastic hinge is introduced at that end. Combinations outside the yield surface are permitted only temporarily, being compensated for by applying corrective loads in the succeeding time step (Figures 11a and 11b).

Once the axial load in the post-buckling range becomes equal to P_{ync}, the program redefines the four branches of M-P interaction curve in the compression region as shown in Figure 10. Maximum compressive strength of the member for all subsequent cycles remains

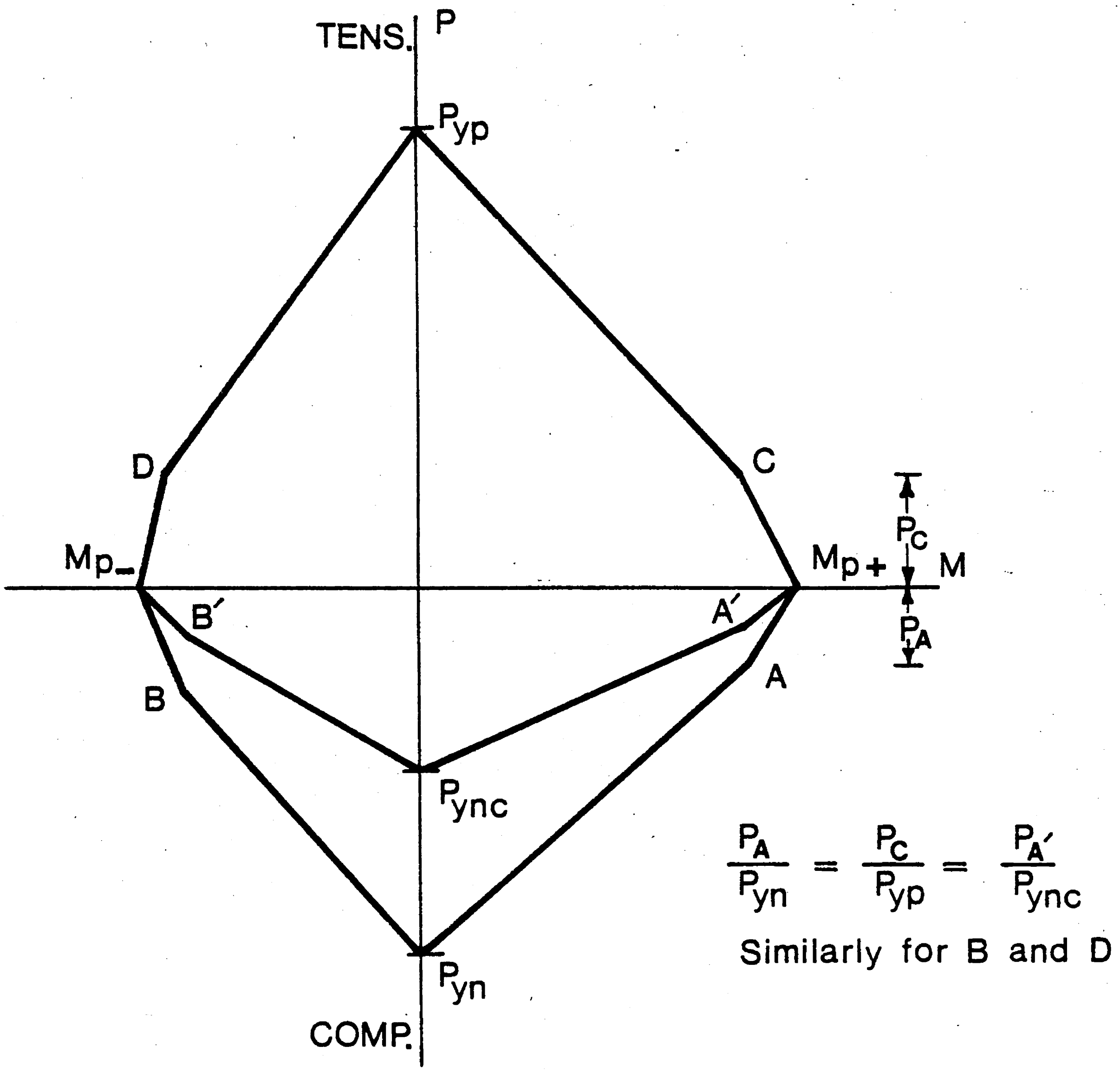


Figure 10 - M-p Interaction Curve Used in EL10

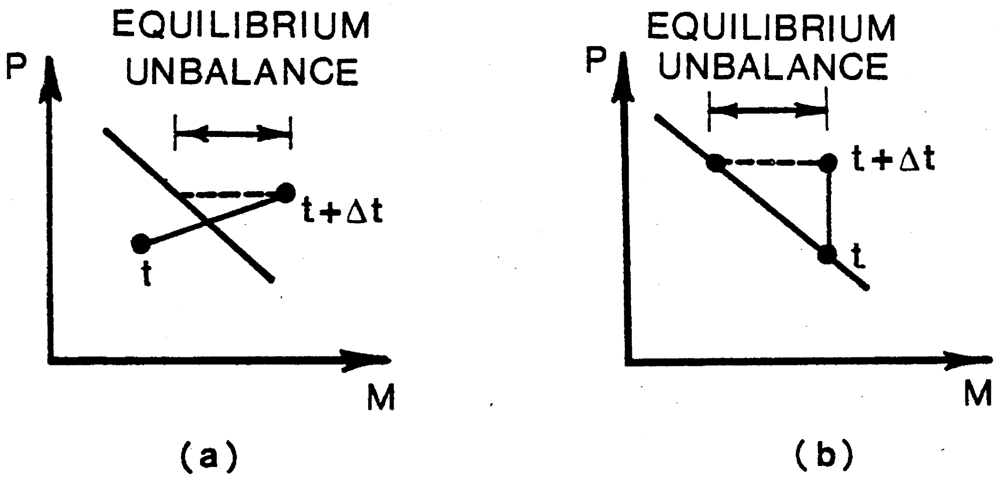


Figure 11 - Equilibrium Correction for Yield Surface Overshoot

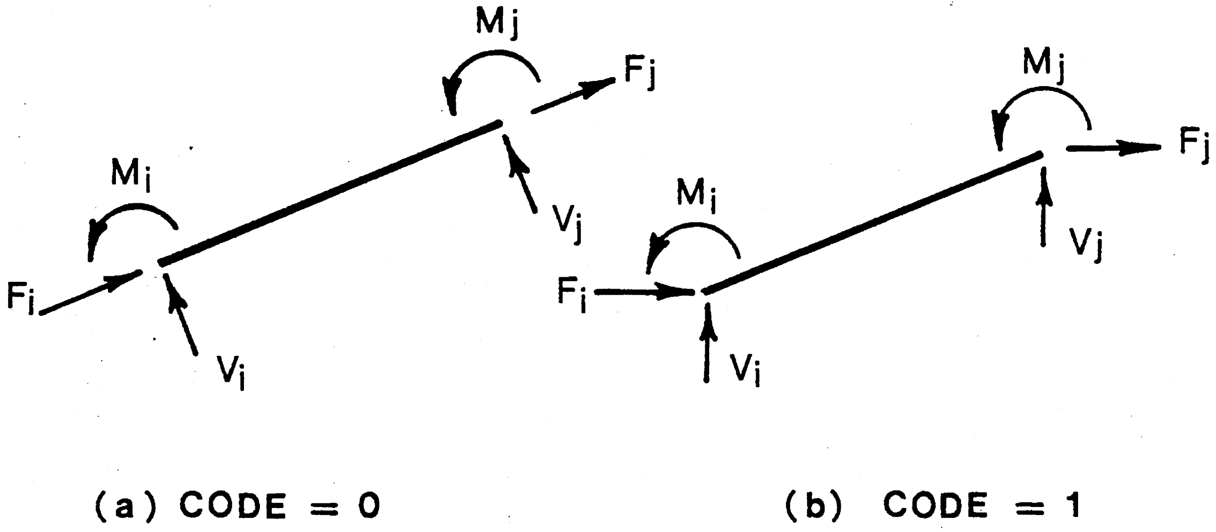


Figure 12 - End Clamping and Initial Forces

at P_{ync}. When the axial load is either P_{yp} (axial yield code = 9), P_{yn} or P_{ync} (axial yield code = 3), the member behaves as a pin-ended member in bending.

ELEMENT STIFFNESS

The element deformations and displacements are shown in Figures 9a and 9b. The axial stiffness is given by

$$dS_1 = \frac{E_T A}{L} \; dv_1 \qquad 3.4$$

where, E_T = tangent modulus in current state, and A = average cross sectional area.

The elastic flexural stiffness is given by

$$\begin{Bmatrix} dS_2 \\ dS_3 \end{Bmatrix} = \frac{EI}{L} \begin{bmatrix} k_{ii} & k_{ij} \\ k_{ij} & k_{jj} \end{bmatrix} \begin{Bmatrix} dv_2 \\ dv_3 \end{Bmatrix} \qquad 3.5$$

where, I = reference moment of inertia; and k_{ii}, k_{ij}, k_{jj} are coefficients which depend on the cross section variation. For a uniform element, I = actual moment of inertia, $k_{ii} = k_{jj} = 4$, and $k_{ij} = 2$. The coefficients must be specified by the program user, and may, if desired, account for such effects as shear deformations and non-rigid end connections as well as cross section variations.

After one or more hinges form, the coefficients for the elasto-plastic component change to k'_{ii}, k'_{ij} and k'_{jj}, as follows:

$$k'_{ii} = k_{ii}(1-A) - k_{ij}C \qquad 3.6$$

$$k'_{ij} = k_{ij}(1-D) - k_{ii}B \qquad 3.7$$

$$k'_{jj} = k_{jj}(1-D) - k_{ij}B \qquad 3.8$$

TABLE 1

COEFFICIENTS FOR PLASTIC ROTATIONS

Yield Condition	COEFFICIENT			
	A	B	C	D
Both Ends Elastic	0	0	0	0
Plastic hinge at end i only	1	k_{ij}/k_{ii}	0	0
Plastic hinge at end j only	0	0	k_{ij}/k_{jj}	1
Plastic hinges at both ends i and j	1	0	0	1

Note:
Stiffness Coefficients k_{ii}, k_{ij}, and k_{jj} are defined by Equation 3.5

where, A, B, C and D are defined in Table 1.

Stiffness in term of nodal displacements is obtained as

$$[K_T] = [a]^T [k_T] [a] \qquad 3.9$$

where, [a] is given by equations 3.1 and 3.2.

The geometric stiffness used is exactly the same as for the buckling element. This is not the exact geometric stiffness for an end moment-buckling element, but is sufficiently accurate for taking into account the P-Δ effect in building frames.

FIXED END AND INITIAL FORCES

Static loads applied along the lengths of end moment-buckling elements may be taken into account by specifying end clamping forces as shown in Figure 12. These forces are those which must act on the element ends to prevent end displacement.

Initial member forces may be specified for structures in which static analyses are carried out separately. The sign convention for these forces is as shown in Figure 12a. These forces are not converted to loads on the nodes of the structure but simply used to initialize the element end actions. Any end forces due to other loadings are then added to the initial forces.

Initial element forces may be specified in addition to static nodal loads and element end clamping forces in which case the element forces due to the static loading are added to the initial forces. The geometric stiffness, if used, is based on the initial axial force plus any axial force due to static loading, and is included only for the dynamic loading, not for the static loading.

Fixed end and initial forces are defined as standard patterns, and each element can be identified with a standard pattern for dead load fixed end force, live load fixed end force and initial force. In addition multiplication factors for scaling the standard patterns can be specified.

LIVE LOAD REDUCTION

Live load reductions based on area supported may have important effects in buildings and, therefore, should be taken into account. The fixed end forces specified for any element, after scaling by the factors specified for the element, should account for any live load reductions permitted for that element.

The fixed end forces for any element will, when changed in sign, constitute static loadings on the nodes to which the element connects, and these loadings are taken into account by the program. Frequently, however, the live load reduction factor permitted for a column in a building will exceed that for the beams it supports, because columns support tributary loads from several floors. Therefore, if the full live load fixed end shears for each beam are applied at the structure nodes the accumulated loads on the columns may be unnecessarily large. This could be compensated for by reducing the fixed end shears to provide the correct column loads but the shear forces computed for the beams would then be too low. A preferable approach is to take advantage of the live load reduction factors which may be specified with the fixed end force patterns and are used as follows.

For initialization of the element shear and axial forces the full specified fixed end forces are used. However, for computation of the static loads on the nodes connected to the element, the fixed end shear and axial forces due to live load (but not the moments) are first multiplied by the specified reduction factor. The forces producing axial loads in the columns may, therefore, be reduced to account for difference in permissible live load reductions between the beams and columns, yet the shear forces computed for the beams will still be correct. The reduction factor is ignored for dead loads.

SHEAR DEFORMATIONS

If desired, effective flexural shear areas may be specified. The program then modifies the flexural stiffness to account for the additional shear deformations. However, the fixed end forces are not changed, so that if shear deformations may be important the specified fixed end force patterns should take these deformations into account.

OUTPUT RESULTS

The following results are printed for the static loading condition (all elements, time = 0) and at each output time if a time history is requested. The time-history results are output only for those elements for which time histories are requested.

1. Yield Code:
    (a) Flexural yield code (at each end of an element). Zero indicates the element end is elastic, and 1 indicates that a plastic hinge has formed.

(b) Axial yield code (for the whole element).
0 to 9 as shown in Figures 4a and 4b.

2. Bending moment, shear force and axial force acting at each end of an element, with the sign convention as shown in Figure 12a.
3. Current plastic hinge rotations at each end.
4. Accumulated positive and negative plastic hinge rotations up to the current time.
5. Net axial extension, positive means extension, negative means shortening.

The maximum positive and negative values of bending moment, shear force, axial force, plastic hinge rotations and axial extension, with their time of occurrence, are printed at the time intervals requested for envelopes.

The envelope values of accumulated positive and negative plastic hinge rotations (PRACP(2), PRACN(2)) as well as of accumulated positive and negative axial elongations (VPACP, VPACN) are not printed, although they are computed within the program. Program users interested in these values can easily insert appropriate print statements in Subroutine OUT10.

E10. END MOMENT-BUCKLING ELEMENTS - EL10

Number of words of information per element = 170.

E10(a) CONTROL INFORMATION FOR GROUP (6I5) - ONE CARD.

Columns 1 - 5: Punch 10 (to indicate that group consists of end moment-buckling elements).

6 - 10: Number of elements in group.

11 - 15: Number of different element stiffness types (max. 40). See Sections E10(b) and E10(c).

16 - 20: Number of different yield interaction surfaces for cross sections (max. 40). See Section E10(d).

21 - 25: Number of different fixed end force patterns (max. 35). See Section E10(e).

26 - 30: Number of different initial element force patterns (max. 30). See Section E10(f).

E10(b). STIFFNESS TYPES (I5, 4F10.0, 3F5.0, 2F10.0) - ONE CARD FOR EACH STIFFNESS TYPE.

Columns 1 - 5: Stiffness type number, in sequence beginning with 1.

6 - 15: Young's modulus of elasticity.

16 - 25: Strain hardening modulus, as a proportion of Young's modulus.

26 - 35: Average cross sectional area.

36 - 45: Reference moment of inertia.

46 - 50: Flexural stiffness factor k_{ii}.

Columns 51 - 55: Flexural stiffness factor k_{jj}.

56 - 60: Flexural stiffness factor k_{ij}.

61 - 70: Effective shear area. Leave blank or punch zero if shear deformations are to be ignored, or if shear deformations have already been taken into account in computing the flexural stiffness factors.

71 - 80: Poisson's ratio (used for computing shear modulus, and required only if shear deformations are to be considered).

E10(c). INPUT RADIUS OF GYRATION, K AND PHI FACTORS (I5, 3F10.0) - ONE CARD FOR EACH STIFFNESS TYPE

Columns 1 - 5: Stiffness type number, in sequence beginning with 1.

6 - 15: Radius of gyration.

16 - 25: Effective length factor.

26 - 35: Strength reduction factor, PHI.

E10(d). CROSS SECTION M-P YIELD INTERACTION SURFACES (I5, 4F10.0, 4F5.0) - ONE CARD FOR EACH YIELD SURFACE.

See Figure 10 for explanation.

Columns 1 - 5: Yield surface number, in sequence beginning with 1.

6 - 15: Positive plastic moment, M_{p+}

16 - 25: Negative plastic moment, M_{p-}

26 - 35: Compression yield force in first cycle, P_{yn}.

36 - 45: Tension yield force, P_{yp}.

46 - 50: M - coordinate of balance point A, as a

proportion of M_{p+}.

Columns 51 - 55: P - coordinate of balance point A, as a proportion of P_{yn}.

56 - 60: M - coordinate of balance point B, as a proportion of M_{p-}.

61 - 65: P - coordinate of balance point B, as a proportion of P_{yn}.

E10(e). FIXED END FORCE PATTERNS (2I5, 7F10.0) - ONE CARD FOR EACH FIXED END FORCE PATTERN.

Omit if there are not fixed end forces. See Figure 12.

Columns 1 - 5: Pattern number, in sequence beginning with 1.

10: Axis code, as follows:

Code = 0: Forces are in the element coordinate system, as in Figure 12a.

Code = 1: Forces are in the global coordinate system, as in Figure 12b.

11 - 20: Clamping force, F_i.

21 - 30: Clamping force, V_i.

31 - 40: Clamping moment, M_i.

41 - 50: Clamping force, F_j.

51 - 60: Clamping force, V_j.

61 - 70: Clamping moment, M_j.

71 - 80: Live load reduction factor, for computation of live load forces to be applied to nodes.

E10(f). INITIAL ELEMENT FORCE PATTERNS (I5, 6F10.0) - ONE CARD FOR EACH INITIAL FORCE PATTERN.

Omit if there are no initial forces. See Figure 12a.

Columns 1 - 5: Pattern number, in sequence beginning with 1.
6 - 15: Initial axial force, F_i.
16 - 25: Initial shear force, V_i.
26 - 35: Initial moment, M_i.
36 - 45: Initial axial force, F_j.
46 - 55: Initial shear force, V_j.
56 - 65: Initial moment, M_j.

E10(g). ELEMENT GENERATION COMMANDS (11I5, 2F5.0, I5, F5.0) - ONE CARD FOR EACH GENERATION COMMAND.

Elements must be specified in increasing numerical order. Cards for the first and last elements must be included. See Note 7 of User's Guide (13) for explanation of generation procedure.

Columns 1 - 5: Element number, or number of first element in a sequentially numbered series of elements to be generated by this command.
6 - 10: Node number at element end i.
11 - 15: Node number at element end j.
16 - 20: Node number increment for element generation. If zero or blank, assumed to be equal to 1.
21 - 25: Stiffness type number.
26 - 30: Yield surface number for element end i.
31 - 35: Yield surface number for element end j.
40: Code for including geometric stiffness. Punch 1 if geometric stiffness is to be included. Leave blank or punch zero if geometric stiffness is to be ignored.
45: Time history output code. If a time history

of element results is not required for the element covered by this command, punch zero or leave blank. If a time history printout, at the intervals specified on card D1, is required, punch 1.

Columns	46 - 50:	Fixed end force pattern number for static dead loads on element. Leave blank or punch zero if there are no dead loads. See Note below.
	51 - 55:	Fixed end forces pattern number for static live loads on element. Leave blank or punch zero if there are no live loads.
	56 - 60:	Scale factor to be applied to fixed end forces due to static dead loads.
	61 - 65:	Scale factor to be applied to fixed end forces due to static live loads.
	66 - 70:	Initial force pattern number. Leave blank or punch zero if there are not initial forces.
	71 - 75:	Scale factor to be applied to initial element forces.

Note: If the static load code, Card C1, is zero but fixed end forces are still specified for some elements, an inconsistency results. In effect, any such fixed end forces will be treated as initial element forces.

CHAPTER 4

EXAMPLE

The braced frame example shown in Figure 13 may be used to check the execution of the DRAIN-2D program with elements EL9 and EL10 when it is implemented on a computer installation. Program decks received through the Department of Civil Engineering, University of Michigan or the National Information Service for Earthquake Engineering will include a data deck and computer output for this example.

The input cards for the structure are listed in Table 2 and identified by the corresponding sections in the User's Guide (13) and in this report. The user should be able to obtain guidance in data preparation procedures by studying this sample data.

The columns in the example structure are represented by element EL10 in group 1, the beams are represented by element EL5 in group 2 and the bracing members (assumed as pin-connected at the ends) are represented by element EL9 in group 3. Node numbers are shown at the ends of the members and element numbers are shown near the middle.

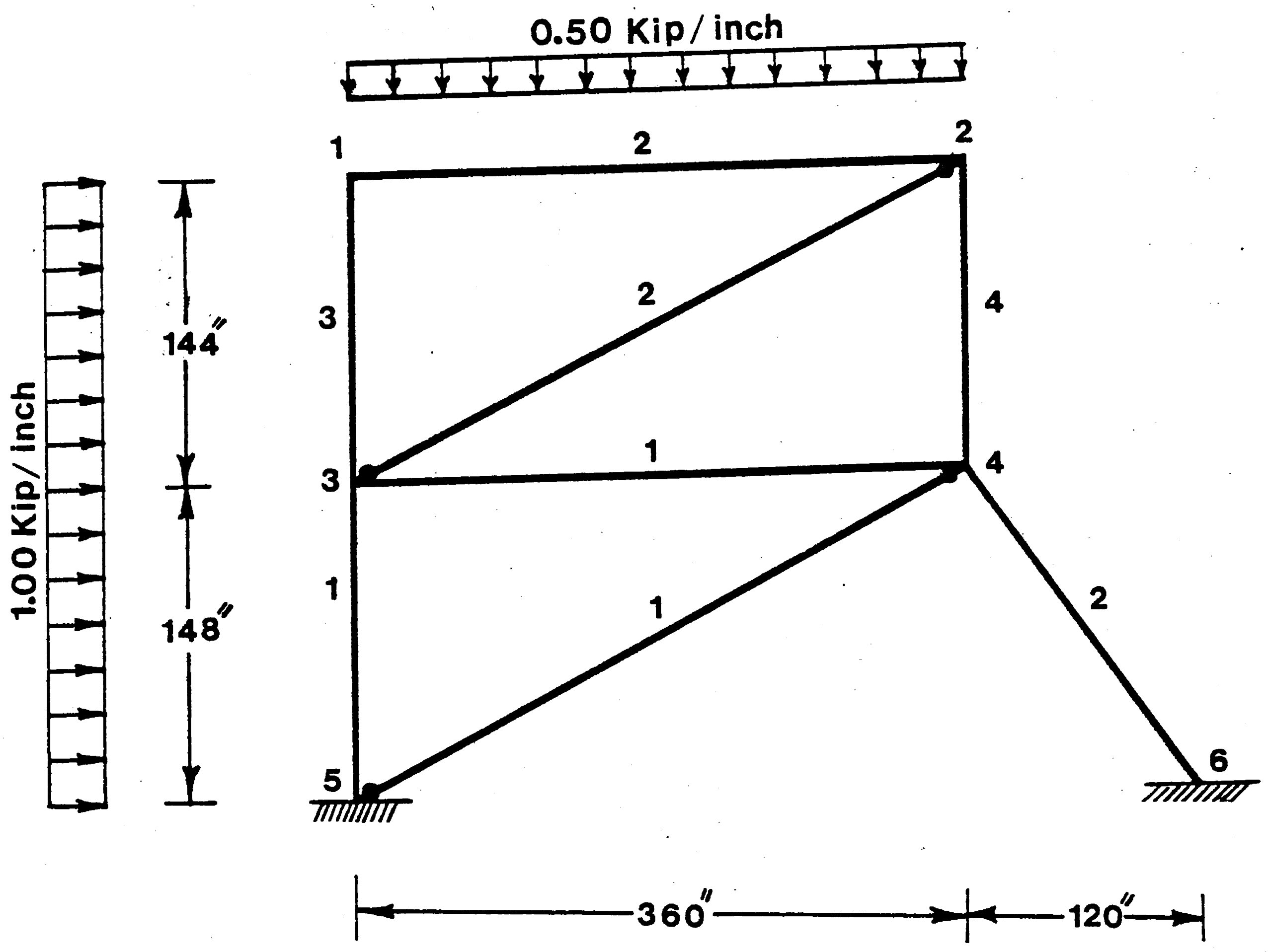


Figure 13 - Test Example

TABLE 2 - SAMPLE INPUT DATA

```
START     TEST EXAMPLE FOR MICHIGAN EL9 AND EL10 ELEMENTS                                     A
     6     6           1     2     1     3                                                    B1
     1            292.
     2 360.       292.
     3            148.                                                                        B2
     4 360.       148.
     5
     6 480.
     5     1     1     1     6                                                                B4
     1     2     1     2
     1     2     3     4                                                                      B5
     1 1.5                                     3    2 1.                                      B6
     1        100  0.01     579.6         1.          1.         1.          20.              C1
     4           1      IMPULSE LOADING
              0.08   1.0    0.10           2.0                                                C3
0                                                                                             C4
    10    10           2     4     4
     1     3
     1     2     3     4                                                                      D
     1     2     3     4
    10     4     2     2     2
     1 29000.     0.01       49.1         2020.      4.    4.    2.
     2 29000.     0.01       35.0         1370.      4.    4.    2.
     1 4.01       0.7        0.5
     2 3.75       0.7        0.4                                                              E10
     1 10900.     -10900.     -1700.      1770.      1.    0.15  1.    0.15
     2 7600.      -7600.      -1200.      1275.      1.    0.15  1.    0.15
     1                  74.         1825.                  74.         -1825.
     2     1 -72.                  1728.      -72.                    -1728.
     1     5     3           1     1     1     1     1           1     1.5
     2     4     6           1     1     1     1     1
     3     3     1           2     2     2     1     1           2     1.25
     4     2     4           2     2     2     1     1
     5     2     1           2     1
     1 29000.     0.05       41.8         3410.      4.    4.    2.
     1      12850.      -12850.
     2      40000.      -40000.                                                               E5
     1                  90.         5400.                  90.         -5400.
     1     3     4           1           1     2          1
     2     1     2           1           1     1          1     1      1.5
     9     2     2
     1 29000.     17.9        350.        -250.      2.45        0.90        0.55
     2 30000.     15.6        300.        -200.      1.90        1.00        0.35             E9
     1     5     4           1     1     1                       50.
     2     3     2           2     1     1                       40.
STOP
```

REFERENCES

1. American Petroleum Institute, "Recommended Practice for Planning, Designing and Constructing Fixed Offshore Platforms," No. API-RP2A, Washington, D.C., November, 1977.

2. Higginbotham, A.B. and Hanson, R.D., "Axial Hysteretic Behavior of Steel Members," Journal of the Structural Division, ASCE, Vol. 102, No. ST7, July, 1976, pp. 1365-1381.

3. Jain, A.K., "Hysteresis Behavior of Bracing Members and Seismic Response of Braced Frames With Different Proportions," Ph.D. Thesis, University of Michigan, Ann Arbor, Michigan, July, 1978. [University Microfilms Access No. 78-22923; Also published as Civil Engineering Report No. UMEE 78R3.]

4. Jain, A.K. and Goel, S.C., "Seismic Response of Braced Steel Structures," in preparation.

5. Jain, A.K., Goel, S.C., and Hanson, R.D., discussion of "Eccentrically Braced Steel Frames for Earthquakes," by C.W. Roeder and E.P. Popov, Journal of the Structural Division, ASCE, Vol. 105, No. ST3, March 1979.

6. Jain, A.K., Goel, S.C., and Hanson, R.D., "Inelastic Response of Restrained Steel Tubes," Journal of the Structural Division, ASCE, Vol. 104, No. ST6, June, 1978, pp. 897-910.

7. Jain, A.K., Goel, S.C., and Hanson, R.D., "An Experimental Verification of Hysteresis Behavior of Axially Loaded Steel Members," Proceedings, Central American Conference on Earthquake Engineering, San Salvador, El Salvador, C.A., Vol. 2, January, 1978.

8. Jain, A.K., Goel, S.C., and Hanson, R.D., "Static and Dynamic Hysteresis Behavior of Steel Tubular Members With Welded Gusset Plates," Report No. UMEE 77R3, Civil Engineering Department, The University of Michigan, Ann Arbor, Michigan, June, 1977.

9. Kanaan, A.E. and Powell, G.H., "General Purpose Computer Program for Inelastic Dynamic Response of Plane Structures," Report No. EERC 73-6, University of California, Berkeley, April, 1973.

10. Marshall, P.W., "Design Consideration for Offshore Structures Having Nonlinear Response to Earthquakes," ASCE Annual Convention and Exposition, Chicago, October, 1978, Preprint No. 3302, pp. 148-172.

11. Nigam, N.C., "Yielding in Framed Structures Under Dynamic Loads," Journal of the Engineering Mechanics Division, ASCE, Vol. 96, No. EM5, May, 1970, pp. 687-709.

12. Nilforoushan, R., "Seismic Behavior of Multistory K-Braced Frame Structures," Ph.D. Thesis, University of Michigan, Ann Arbor, Michigan, November, 1973. [University Microfilms Access No. 74-15813.]

13. Powell, G.H., "DRAIN-2D User's Guide," Report No. EERC 73-22, University of California, Berkeley, October, 1973.

14. Prathuangsit, D., Goel, S.C., and Hanson, R.D., "Axial Hysteresis Behavior With End Restraints," Journal of the Structural Division, ASCE, Vol. 104, No. ST6, June, 1978, pp. 883-896.

15. Singh, P., "Seismic Behavior of Braces and Braced Steel Frames," Ph.D. Thesis, University of Michigan, Ann Arbor, Michigan, July, 1977.

16. Specification for the Design, Fabrication and Erection of Structural Steel for Buildings, American Institute of Steel Construction, New York, November, 1978.

17. Wakabayashi, M., Mastui, C., and Mitani, I., "Cyclic Behavior of a Restrained Steel Brace Under Axial Loading," Preprints VI World Conference on Earthquake Engineering, Vol. 11, New Delhi, India, January, 1977, pp. 189-194.

18. Workman, G.H., "The Inelastic Behavior of Multistory Braced Frame Structures Subjected to Earthquake Excitation," Ph.D. Thesis, The University of Michigan, Ann Arbor, Michigan, September, 1969.

APPENDIX A-1

FORTRAN LISTING OF BUCKLING ELEMENT EL9

```
      SUBROUTINE INEL9 (/KCONT/,/FCONT/,/NDOF/,/NINFC/,/ID/,/X/,/Y/,/NN/
     1)
C
      COMMON /INPEL/ IMEM,KST,LM(4),KGEOM,EAL,FL,COSA,SINA,PFAC,RATIO,
     1               DELTY,KODYX,KODY,XPP,X3RE,EAL1,EAL4,EAL6,EAL7,EAL8,
     2               X5,Y5,P5,X6,X5D,Y5D,P5D,INDRE,IVRTX,
     3               IELOG,SEP,VTOT,XTOT,VPACP,VPACN,VENP,TVENP,VENN,
     4               TVENN,SENP,TSENP,SENN,TSENN,SDFO,NODI,NODJ,KOUTDT,
     5               PYP,PYN,PYNC,REST(147)
      COMMON /WORK/ FTYP(40,7),FEF(40,4),KDFEF(40),DD(4),GA(4,4),
     1              FFEF(4),SFF(4),SSFF(4),NMEM,NMBT,NFEF,SLOP1,INEL,
     2              INODI,INODJ,INC,IINC,IMBT,IIMBT,IKGM,IKDT,KFDL,
     3              IKFDL,KFLL,IKFLL,FDL,FFDL,FLL,FFLL,FINIT,FFINIT,
     4              XL,YL,AREA,RAD,SLEND,W(1460)
C
      DIMENSION KCONT(1),ID(NN,1),X(1),Y(1),COM(1)
      DIMENSION AST(2),YESNO(2)
      DATA AST/2H  ,2H */
      DATA YESNO/4H YES,4H NO /
      EQUIVALENCE (IMEM,COM(1))
C
C     DATA INPUT, BUCKLING ELEMENTS
C
      NDOF=4
      NINFC=53
      NMEM=KCONT(2)
      NMBT=KCONT(3)
      NFEF=KCONT(4)
      PRINT 10, (KCONT(I),I=2,4)
   10 FORMAT (27H BUCKLING ELEMENTS (TYPE 9)////
     1        25H NO. OF ELEMENTS          =I4/
     2        25H NO. OF STIFFNESS TYPES   =I4/
     3        25H NO. OF F.E.F. PATTERNS   =I4)
C
C     INPUT STIFFNESS PROPERTIES
C
      PRINT 20
   20 FORMAT (////16H STIFFNESS TYPES//
     1        5H TYPE,6X,7H YOUNGS,6X,8H SECTION,3X,
     2        16H    YIELD FORCES,10X,10H RADIUS OF,5X,
     3        12H EQU. LENGTH,5X,19H STRENGTH REDUCTION/
     4        5H  NO.,5X,8H MODULUS,6X,7H  AREA ,5X,
     5        8H TENSION,5X,8H  COMPN ,6X,9H GYRATION,4X,
     6        12H COEFFICIENT,9X,7H FACTOR/)
C
      DO 50 IT=1,NMBT
      READ 30, I,(FTYP(IT,J),J=1,7)
   50 PRINT 40, IT,(FTYP(IT,J),J=1,7)
   30 FORMAT (I5,7F10.0)
   40 FORMAT (I4,E14.4,6F13.2)
C
C     FIXED END FORCE PATTERNS
C
      IF (NFEF.EQ.0) GO TO 100
      PRINT 60
```

```
   60 FORMAT (////25H FIXED END FORCE PATTERNS//
     1         8H PATTERN,3X,4HAXIS,2(7X,5HAXIAL,7X,5HSHEAR)/
     2         8H    NO. ,3X,4HCODE,2(7X,5HAT  I),2(7X,5HAT  J)/)
C
      DO 90 NF=1,NFEF
      READ 70, L,KDFEF(NF),(FEF(NF,J),J=1,4)
   70 FORMAT (2I5,4F10.0)
   80 FORMAT (I6,I8,1X,4F12.2)
   90 PRINT 80, NF,KDFEF(NF),(FEF(NF,J),J=1,4)
C
C     ELEMENT DATA
C
  100 PRINT 110
  110 FORMAT(////22H ELEMENT SPECIFICATION//
     1        3X,4HELEM,3X,4HNODE,2X,4HNODE,2X,4HNODE,2X,4HSTIF,2X,
     2        4HGEOM,2X,4HTIME,3X,12HFEF PATTERNS,3X,17HFEF SCALE FACTORS,
     3        5X,7HINITIAL/
     4        3X,4H NO.,3X,4H  I ,2X,4H  J ,2X,4HDIFF,2X,4HTYPE,2X,
     5        4HSTIF,2X,4HHIST,3X,12H  DL    LL  ,3X,17H   DL        LL   ,
     6        5X,7H FORCE /)
C
      KODYX=0
      KODY=0
      KST=0
      XPP=0.
      DO 120 J=19,47
  120 COM(J)=0.
C
      IMEM=1
  130 READ 140, INEL,INODI,INODJ,IINC,IIMBT,IKGM,IKDT,IKFDL,IKFLL,FFDL,F
     1FLL,FFINIT
  140 FORMAT(9I5,2F5.0,F10.0)
      IF (INEL.GT.IMEM) GO TO 170
  150 NODI=INODI
      NODJ=INODJ
      INC=IINC
      IF (INC.EQ.0) INC=1
      IMBT=IIMBT
      KGEOM=IKGM
      KOUTDT=IKDT
      YNG=YESNO(2)
      IF (KGEOM.NE.0) YNG=YESNO(1)
      YNT=YESNO(2)
      IF (KOUTDT.NE.0) YNT=YESNO(1)
      KFDL=IKFDL
      KFLL=IKFLL
      FDL=FFDL
      FLL=FFLL
      FINIT=FFINIT
      ASTT=AST(1)
      IF (INEL-NMEM) 130,170,130
C
  160 NODI=NODI+INC
      NODJ=NODJ+INC
      ASTT=AST(2)
```

```
C
  170 PRINT 180, ASTT,IMEM,NODI,NODJ,INC,IMBT,YNG,YNT,KFDL,KFLL,FDL,FLL,
     1FINIT
  180 FORMAT(A2,I4,I7,3I6,3X,A4,2X,A4,I7,I6,F11.2,F10.2,F11.2)
C
C     LOCATION MATRIX
C
      DO 190 L=1,2
      LM(L)=ID(NODI,L)
  190 LM(L+2)=ID(NODJ,L)
      CALL BAND
C
C     ELEMENT PROPERTIES
C
      XL=X(NODJ)-X(NODI)
      YL=Y(NODJ)-Y(NODI)
      FL=SQRT(XL**2+YL**2)
      COSA=XL/FL
      SINA=YL/FL
      AREA=FTYP(IMBT,2)
      EAL=FTYP(IMBT,1)*AREA/FL
      PYP=FTYP(IMBT,3)
      PYN=-ABS(FTYP(IMBT,4))
      PHI=FTYP(IMBT,7)
      PYNC=PHI*PYN
      RAD=FTYP(IMBT,5)
      AK=FTYP(IMBT,6)
      SLEND=AK*FL/RAD
      PFAC=ABS(PYNC/PYP)
      SLOP1=(PFAC*(1.-PHI))/(PFAC-5.*PHI)
      EAL1=SLOP1*EAL
      XBRE=-PFAC
      RATIO=60.0/SLEND
      DELTY=PYP/EAL
C
C     LOADS DUE TO FIXED END FORCES
C
      SFEF=0.
      IF (KFDL+KFLL.EQ.0) GO TO 310
      DO 200 I=1,NDOF
      DO 200 J=1,NDOF
  200 BA(I,J)=0.
      BA(1,1)=COSA
      BA(1,2)=SINA
      BA(2,1)=-SINA
      BA(2,2)=COSA
      BA(3,3)=COSA
      BA(3,4)=SINA
      BA(4,3)=-SINA
      BA(4,4)=COSA
      DO 210 I=1,4
      SFF(I)=0.
  210 SSFF(I)=0.
      IF (KFDL.EQ.0) GO TO 250
      DO 220 I=1,4
```

```
  220 FFEF(I)=FEF(KFDL,I)*FDL
      IF (KDFEF(KFDL).EQ.0) GO TO 230
      CALL MULT (GA,FFEF,SFF,4,4,1)
      GO TO 250
  230 DO 240 I=1,4
  240 SFF(I)=FFEF(I)
C
  250 IF (KFLL.EQ.0) GO TO 290
      DO 260 I=1,4
  260 FFEF(I)=FEF(KFLL,I)*FLL
      IF (KDFEF(KFLL).EQ.0) GO TO 270
      CALL MULT(GA,FFEF,SSFF,4,4,1)
      GO TO 290
  270 DO 280 I=1,4
  280 SSFF(I)=FFEF(I)
C
  290 DO 300 I=1,4
  300 SSFF(I)=SSFF(I)+SFF(I)
C
      CALL MULTT (GA,SSFF,DD,4,4,1)
      CALL SFORCE (DD)
C
C     INITIALIZE ELEMENT FORCE
C
      SFEF=(SSFF(3)-SSFF(1))*0.5
  310 FF=FINIT+SFEF
      SEP=FF
      IF (FINIT.LT.0.) GO TO 320
      SENP=FINIT
      SENN=0.
      GO TO 330
  320 SENN=FINIT
      SENP=0.
C
  330 CALL FINISH
C
C     GENERATE MISSING ELEMENTS
C
      IF (IMEM.EQ.NMEM) RETURN
      IMEM=IMEM+1
      IF (IMEM.EQ.INEL) GO TO 150
      GO TO 160
C
      END
```

```
      SUBROUTINE STIF9 (/MSTEP/,/NDOF/,/NINFC/,/COMS/,/FK/,/DFAC/)
C
      COMMON /INFEL/ IMEM,KST,LM(4),KGEOM,EAL,FL,COSA,SINA,PFAC,RATIO,
     1               DELTY,KODYX,KODY,XPP,X3RE,EAL1,EAL4,EAL6,EAL7,EAL8,
     2               X5,Y5,P5,X6,X5D,Y5D,P5D,INDRE,IVRTX,
     3               IELOG,SEP,VTOT,XTOT,VPACP,VPACN,VENP,TVENP,VENN,
     4               TVENN,SENP,TSENP,SENN,TSENN,SDFO,NODI,NODJ,KOUTDT,
     5               PYP,PYN,PYNC,REST(147)
      COMMON /WORK/ STIF,STIFF,SST(2,2),AA(2,4),AATK(4,2),FFK(4,4),
     1              W(1962)
C
      DIMENSION COM(1),COMS(1),FK(4,4)
      EQUIVALENCE (IMEM,COM(1))
C
C     STIFFNESS FORMULATION, BUCKLING ELEMENTS
C
      DO 10 J=3,23
   10 COM(J)=COMS(J)
C
C     CURRENT STIFFNESS
C
      CALL FST9 (STIF,KODY)
C
C     PREVIOUS STIFFNESS
C
      IF (MSTEP.LT.2) GO TO 20
      CALL FST9 (STIFF,KODYX)
C
C     STIFFNESS DIFFERENCE
C
      STIF=STIF-STIFF
   20 FK(1,1)=STIF*COSA**2
      FK(1,2)=STIF*SINA*COSA
      FK(1,3)=-FK(1,1)
      FK(1,4)=-FK(1,2)
      FK(2,2)=STIF*SINA**2
      FK(2,3)=FK(1,4)
      FK(2,4)=-FK(2,2)
      FK(3,3)=FK(1,1)
      FK(3,4)=FK(1,2)
      FK(4,4)=FK(2,2)
      DO 30 I=2,4
      JJ=I-1
      DO 30 J=1,JJ
   30 FK(I,J)=FK(J,I)
      IF (MSTEP.GT.1) GO TO 80
C
C     INITIAL STIFFNESS FOR STEP 0, BETA-0 ALLOWANCE FOR STEP 1
C
      CC=1.
      IF (MSTEP.EQ.1) CC=DFAC
      DO 40 I=1,16
   40 FK(I,1)=FK(I,1)*CC
C
C     ADD GEOMETRIC STIFFNESS
```

```
C
      IF (MSTEP.EQ.0.OR.KGECM.EQ.0) GO TO 80
      PFL=COMS(34)/FL
      DO 50 I=1,4
   50 SST(I,1)=PFL
      DO 60 I=1,8
   60 AA(I,1)=0.
      AA(1,1)=-SINA
      AA(1,2)=COSA
      AA(2,3)=SINA
      AA(2,4)=-COSA
      CALL MULTST (AA,SST,AATK,FFK,4,2)
      DO 70 I=1,16
   70 FK(I,1)=FK(I,1)+FFK(I,1)
C
   80 RETURN
      END
```

```
      SUBROUTINE RESP9 (/NDOF/,/NINFC/,/KBAL/,/KPR/,/COMS/,/DDISM/,/DD/
     1,/TIME/,/VELM/,/DFAC/,/DELTA/)
C
      COMMON /INFEL/ IMEM,KST,LM(4),KGEOM,EAL,FL,COSA,SINA,PFAC,RATIO,
     1               DELTY,KODYX,KODY,XPP,X3RE,EAL1,EAL4,EAL6,EAL7,EAL8,
     2               X5,Y5,P5,X6,X5D,Y5D,P5D,INDRE,IVRTX,
     3               IELOG,SEP,VTOT,XTOT,VPACP,VPACN,VENP,TVENP,VENN,
     4               TVENN,SENP,TSENP,SENN,TSENN,SDFO,NODI,NODJ,KOUTDT,
     5               PYP,PYN,PYNC,REST(147)
      COMMON /WORK/ EALE,DSL,DSEP,SLIN,FAC,FACTOR,FACAC,DSUB,FVRTX,
     1              SLOP6,SLOP7,DVP,DS4,DS5,DS6,DS7,DS8,POUT(1983)
C
      DIMENSION COM(1),COMS(1),DDISM(1),DD(1),VELM(1)
      EQUIVALENCE (IMEM,COM(1))
C
C     STATE DETERMINATION, BUCKLING ELEMENTS
C
      DO 10 I=1,NINFC
   10 COM(I)=COMS(I)
      KODYX=KODY
      IF (IMEM.EQ.1) THED=0
C
C     EXTENSION INCREMENT
C
      DVAX=COSA*(DDISM(3)-DDISM(1))+SINA*(DDISM(4)-DDISM(2))
      VTOT=VTOT+DVAX
C
C     LINEAR FORCE INCREMENT
C
      CALL FSTS (EALE,KODY)
      SLIN=SEP+EALE*DVAX
C
C     INITIALIZE
C
      FVRTX=0.
C
C     CHECK VERTEX STATE
C
      IF (IVRTX.EQ.0) CALL VRTX9 (FVRTX,DVAX,TIME)
C
      IF (IVRTX.EQ.0) GO TO 120
C
      FACAC=FVRTX
C
   20 FACTOR=1.-FACAC
      KODYI=KODY+1
      GO TO (30,120,120,40,50,60,70,80,90,100),KODYI
C
C     ON SLOPE 0, ELASTIC, GET FACTOR FOR STATUS CHANGE
C
   30 DSEP=EAL*DVAX
      IF (DSEP) 31,110,32
   31 FAC=(PYNC-SEP)/DSEP
      IF (FAC.GE.FACTOR) GO TO 33
      FACTOR=FAC
```

```
      SEP=PYNC
      KODY=3
      GO TO 110
C
   32 FAC=(PYP-SEP)/DSEP
      IF (FAC.GE.FACTOR) GO TO 33
      FACTOR=FAC
      SEP=PYP
      KODY=9
      GO TO 110
C
   33 SEP=SEP+FACTOR*DSEP
      GO TO 110
C
C     ON SLOPE 3, BUCKLING AND CONTINUING
C
   40 IF (DVAX.GT.0.) GO TO 41
C
C     UPDATE PLASTIC DEFORMATIONS
C
      DVP=FACTOR*DVAX
      VPACN=VPACN+DVP
      GO TO 120
C
C     BUCKLING AND UNLOADING
C
   41 XTOT=(VTOT-DVAX)/DELTY
      IF (XTOT.GE.X3RE) GO TO 42
C
C     ESTABLISH NEW STIFFNESS FOR REVERSE
C
      X3RE=XTOT
      IFLOG=1
      CALL LAW9
      P5=Y5*PYP
      KODY=4
      GO TO 50
C
C     USE OLD STIFFNESS FOR REVERSE
C
   42 IF (INDRE.EQ.2.AND.XTOT.LE.X6) GO TO 43
      IF((X5D-XTOT).EQ.0.) GO TO 44
      SLOP6=(Y5D+PFAC)/(X5D-XTOT)
      EAL6=EAL*SLOP6
      KODY=6
      GO TO 70
C
   43 SLOP7=(Y5+PFAC)/(X5-XTOT)
      EAL7=EAL*SLOP7
      KODY=7
      GO TO 80
   44 KODY=5
      GO TO 60
C
C     ON SLOPE 4, GET FACTOR FOR STATUS CHANGE
```

```
C
   50 DS4=EAL4*DVAX
      IF (DS4) 51,110,54
   51 IF (SEP) 52,52,53
   52 FAC=(PYNC-SEP)/DS4
      IF (FAC.GE.FACTOR) GO TO 55
      FACTOR=FAC
      SEP=PYNC
      KODY=3
      GO TO 110
   53 X5D=VTOT/DELTY
      Y5D=SEP/PYP
      P5D=SEP
      INDRE=1
      KODY=5
      GO TO 60
   54 FAC=(P5-SEP)/DS4
      IF (FAC.GE.FACTOR) GO TO 55
      FACTOR=FAC
      SEP=P5
      KODY=8
      GO TO 110
   55 SEP=SEP+FACTOR*DS4
      GO TO 110
C
C     ON SLOPE 5, GET FACTOR FOR STATUS CHANGE
C
   60 DS5=EAL*DVAX
      IF (DS5) 61,110,62
   61 FAC=(PYNC-SEP)/DS5
      IF (FAC.GE.FACTOR) GO TO 65
      FACTOR=FAC
      SEP=PYNC
      KODY=3
      GO TO 110
   62 FAC=(P5D-SEP)/DS5
      IF (FAC.GE.FACTOR) GO TO 65
      FACTOR=FAC
      SEP=P5D
      GO TO (63,64),INDRE
   63 KODY=4
      GO TO 110
   64 KODY=8
      GO TO 110
   65 SEP=SEP+FACTOR*DS5
      GO TO 110
C
C     ON SLOPE 6, GET FACTOR FOR STATUS CHANGE
C
   70 DS6=EAL6*DVAX
      IF (DS6) 71,110,72
   71 FAC=(PYNC-SEP)/DS6
      IF (FAC.GE.FACTOR) GO TO 75
      FACTOR=FAC
      SEP=PYNC
```

```
      KODY=3
      GO TO 110
   72 FAC=(P5D-SEP)/DS6
      IF (FAC.GE.FACTOR) GO TO 75
      FACTOR=FAC
      SEP=P5D
      GO TO (73,74),INDRE
   73 KODY=4
      GO TO 110
   74 KODY=8
      GO TO 110
   75 SEP=SEP+FACTOR*DS6
      GO TO 110
C
C     ON SLOPE 7, GET FACTOR FOR STATUS CHANGE
C
   80 DS7=EAL7*DVAX
      IF (DS7) 81,110,82
   81 FAC=(PYNC-SEP)/DS7
      IF (FAC.GE.FACTOR) GO TO 83
      FACTOR=FAC
      SEP=PYNC
      KODY=3
      GO TO 110
   82 FAC=(P5-SEP)/DS7
      IF (FAC.GE.FACTOR) GO TO 83
      FACTOR=FAC
      SEP=P5
      KODY=8
      GO TO 110
   83 SEP=SEP+FACTOR*DS7
      GO TO 110
C
C     ON SLOPE 8, GET FACTOR FOR STATUS CHANGE
C
   90 DS8=EAL8*DVAX
      IF (DS8) 91,110,92
   91 X5D=VTOT/DELTY
      Y5D=SEP/PYP
      P5D=SEP
      INDRE=2
      KODY=5
      GO TO 60
C
   92 FAC=(PYP-SEP)/DS8
      IF (FAC.GE.FACTOR) GO TO 93
      FACTOR=FAC
      SEP=PYP
      KODY=9
      GO TO 110
   93 SEP=SEP+FACTOR*DS8
      GO TO 110
C
C     ON SLOPE 9, YIELDED BUT CONTINUING
C
```

```
  100 IF (DVAX.LT.0.) GO TO 101
C
C     UPDATE PLASTIC DEFORMATIONS
C
      DVP=FACTOR*DVAX
      XPP=XPP+DVP/DELTY
      VPACP=VPACP+DVP
      GO TO 120
C
C     YIELDED BUT UNLOADING
C
  101 KODY=0
C
C     RESIDUAL ELONGATION, RE
C
      IF (IELOG.NE.1) GO TO 105
      SLEND=60./RATIO
      RE=0.0175*(0.55*X3RE/SLEND+0.0002*X3RE**2)
      RE=RE*FL/DELTY
      XPP=XPP+RE
      IELOG=0
C
  105 X3RE=XPP-PFAC
      GO TO 30
C
C     CHECK FOR COMPLETION OF CYCLE
C
  110 FACAC=FACAC+FACTOR
      IF (FACAC.LT.0.9999999) GO TO 20
C
C     NEW FORCE, UNBALANCED FORCE DUE TO YIELD
C
  120 ST=SEP
      DSUB=SLIN-SEP
      IF (ABS(DSUB).GT.1.E-8) KBAL=1
C
C     DEFORMATION RATE FOR DAMPING
C
      IF (DFAC.EQ.0.0.AND.DELTA.EQ.0.0) GO TO 140
      IF (TIME.EQ.0.) GO TO 150
      KBAL=1
      DVAX=COSA*(VELM(3)-VELM(1))+SINA*(VELM(4)-VELM(2))
C
C     BETA-0 DAMPING FORCE
C
      IF (DFAC.EQ.0.) GO TO 130
      DSUB=DSUB+DFAC*FAL*DVAX
C
C     STRUCTURAL DAMPING FORCE
C
  130 IF (DELTA.EQ.0.) GO TO 140
      DSL=DELTA*SIGN(ABS(ST),DVAX)
      DSUB=DSUB-DSL+SDFO
      SDFO=DSL
C
```

```
C     UNBALANCED LOAD VECTOR
C
  140 IF (KBAL.EQ.0) GO TO 150
      DD(3)=DSUB*COSA
      DD(4)=DSUB*SINA
      DD(1)=-DD(3)
      DD(2)=-DD(4)
C
C     EXTRACT ENVELOPES
C
  150 IF (SENP.GE.ST) GO TO 160
      SENP=ST
      ISENP=TIME
      GO TO 170
  160 IF (SENN.LE.ST) GO TO 170
      SENN=ST
      ISENN=TIME
  170 IF (VENP.GE.VTOT) GO TO 180
      VENP=VTOT
      IVENP=TIME
      GO TO 190
  180 IF (VENN.LE.VTOT) GO TO 190
      VENN=VTOT
      IVENN=TIME
  190 CONTINUE
C
C     PRINT TIME HISTORY
C
      IF (KPR.LT.0) GO TO 200
      IF (KPR.EQ.0.OR.KOUTDT.EQ.0) GO TO 240
  200 IF (IHED.NE.0) GO TO 220
      KKPR=IABS(KPR)
      PRINT 210,KKPR,TIME
  210 FORMAT(///18H RESULTS FOR GROUP,I3,
     1       27H, BUCKLING ELEMENTS, TIME =,F8.3
     2       //5X,5H ELEM,3X,4HNODE,3X,4HNODE,3X,5HYIELD,8X,5HAXIAL,4X,
     3       9H   NET   ,3X,25HACCUM. PLASTIC EXTENSIONS/5X,
     4       5H  NO.,3X,4H  I ,3X,4H  J ,3X,5H CODE,8X,5HFORCE,4X,
     5       9HEXTENSION,5X,8HPOSITIVE,5X,8HNEGATIVE/)
      IHED=1
  220 PRINT 230, IMEM,NODI,NODJ,KODY,ST,VTOT,VPACP,VPACN
  230 FORMAT(I9,2I7,I8,F14.2,3F13.5)
C
C     SET INDICATOR FOR STIFFNESS CHANGE
C
  240 KST=0
      IF (KODYX.NE.KODY) KST=1
C
C     UPDATE INFORMATION IN COMS
C
      DO 250 J=15,47
  250 COMS(J)=COM(J)
      COMS(2)=COM(2)
C
      RETURN
      END
```

```
      SUBROUTINE OUT9 (/COMS/,/NINFC/)
C
      COMMON /INFEL/ IMEM,KST,LM(4),KGEOM,EAL,FL,COSA,SINA,PFAC,RATIO,
     1               DELTY,KODYX,KODY,XPP,X3RE,EAL1,EAL4,EAL6,EAL7,EAL8,
     2               X5,Y5,P5,X6,X5D,Y5D,P5D,INDRE,IVRTX,
     3               TELOG,SEP,VTOT,XTOT,VPACP,VPACN,VENP,TVENP,VENN,
     4               TVENN,SENP,TSENP,SENN,TSENN,SDFO,NODI,NODJ,KOUTDT,
     5               PYP,PYN,PYNC,REST(147)
C
      DIMENSION COM(1),COMS(1)
      EQUIVALENCE (IMEM,COM(1))
C
C     ENVELOPE OUTPUT, BUCKLING ELEMENTS
C
      DO 10 J=1,NINFC
   10 COM(J)=COMS(J)
C
      IF (IMEM.EQ.1) PRINT 20
   20 FORMAT (27H BUCKLING ELEMENTS (TYPE 9)////
     1        5H ELEM,3X,4HNODE,3X,4HNODE,11X,20HMAXIMUM AXIAL FORCES,
     2        19X,18HMAXIMUM EXTENSIONS,12X,25HACCUM. PLASTIC EXTENSIONS/
     3        5H  NO.,3X,4H   I ,3X,4H   J ,5X,7HTENSION,3X,4HTIME,
     4        6X,5HCOMPN,3X,4HTIME,5X,8HPOSITIVE,3X,4HTIME,
     5        3X,8HNEGATIVE,3X,4HTIME,7X,8HPOSITIVE,5X,8HNEGATIVE/)
C
      PRINT 30,IMEM,NODI,NODJ,SENP,TSENP,SENN,TSENN,VENP,TVENP,VENN,TVEN
     1N,VPACP,VPACN
   30 FORMAT(I4,I7,I7,2X,2(F11.2,F7.2),2X,2(F11.5,F7.2),2X,2F13.5)
C
      RETURN
      END
```

```
      SUBROUTINE FST9 (/STIF/,/KOD/)
C
C
C     FORM AXIAL STIFFNESS
C
      COMMON /INFEL/ IMEM,KST,LM(4),KGEOM,EAL,PL,COSA,SINA,PFAC,RATIO,
     1               DELTY,KODYX,KODY,XPP,X3RE,EAL1,EAL4,EAL6,EAL7,EAL8
     2               REST(177)
      KYY=KOD+1
      GO TO (10,20,30,40,50,60,70,80,90,100),KYY
   10 STIF=EAL
      GO TO 110
   20 STIF=EAL1
      GO TO 110
   30 STIF=EAL
      GO TO 110
   40 STIF=0.001*EAL
      GO TO 110
   50 STIF=EAL4
      GO TO 110
   60 STIF=EAL
      GO TO 110
   70 STIF=EAL6
      GO TO 110
   80 STIF=EAL7
      GO TO 110
   90 STIF=EAL8
      GO TO 110
  100 STIF=0.001*EAL
C
  110 RETURN
      END
```

```
      SUBROUTINE LAW9
C
C     GENERATE P-DELTA HYSTERESIS CURVE
C
      COMMON /INFEL/ IMEM,KST,LM(4),KGEOM,EAL,FL,COSA,SINA,PFAC,RATIO,
     1               DELTY,KODYX,KODY,XPP,X3RE,EAL1,EAL4,EAL6,EAL7,EAL8,
     2               X5,Y5,P5,X6,X5D,Y5D,P5D,INDRE,IVRTX,REST(168)
C
C     RESIDUAL ELONGATION, RE
C
      SLEND=60./RATIO
      RE=0.0175*(0.55*X3RE/SLEND+0.0002*X3RE**2)
      RE=RE*FL/DELTY
C
      XPP=XPP+RE
      XPP1=1.+XPP
      BETA=1./3.
      DENOM=(XPP1-X3RE)+(1.+PFAC)/BETA
      XNUMR=XPP-X3RE-PFAC
      Y5=RATIO*XNUMR/DENOM
      X5=XPP-Y5/BETA
      SLOP4=(Y5+PFAC)/(X5-X3RE)
      EAL4=EAL*SLOP4
      SLOP8=(1.-Y5)/(XPP1-X5)
      EAL8=EAL*SLOP8
      X6=X5-Y5-PFAC
      XPP=XPP-RE
C
      RETURN
      END
```

```
      SUBROUTINE VRTX9 (/FACAC/,/DVAX/,/TIME/)
C
C     AXIAL STATE DETERMINATION IN TENSION AND VERTEX REGIONS
C
      COMMON /INFEL/ IMEM,KST,LM(4),KGEOM,EAL,PL,COSA,SINA,PFAC,RATIO,
     1               DELTY,KODYX,KODY,XPP,X3RE,EAL1,EAL4,EAL6,EAL7,EAL8,
     2               X5,Y5,P5,X6,X5D,Y5D,P5D,INDRE,IVRTX,
     3               TELOG,SEP,VTOT,XTOT,VPACP,VPACN,VENP,TVENP,VENN,
     4               TVENN,SENP,TSENP,SENN,TSENN,SDFO,NODI,NODJ,KOUTDT,
     5               PYP,PYN,PYNC,REST(147)
C
C     INITIALIZE
C
      FACAC=0.
   10 FACTOR=1.-FACAC
      KODYI=KODY+1
      IF (KODYI.EQ.10) KODYI=4
      GO TO (20,30,40,50),KODYI
C
C     ON SLOPE 0, ELASTIC, GET FACTOR FOR STATUS CHANGE
C
   20 DSEP=EAL*DVAX
      IF (DSEP) 21,60,22
   21 FAC=(PYN-SEP)/DSEP
      IF (FAC.GE.FACTOR) GO TO 23
      FACTOR=FAC
      SEP=PYN
      KODY=1
C
      SENN=PYN
      TSENN=TIME
      GO TO 60
C
   22 FAC=(PYP-SEP)/DSEP
      IF (FAC.GE.FACTOR) GO TO 23
      FACTOR=FAC
      SEP=PYP
      KODY=9
      GO TO 60
C
   23 SEP=SEP+FACTOR*DSEP
      GO TO 60
C
C     ON SLOPE 1, GET FACTOR FOR STATUS CHANGE
C
   30 DS1=EAL1*DVAX
      IF (EAL1.NE.0) GO TO 34
      KODY=3
      IVRTX=1
      GO TO 80
C
   34 IF (DS1) 31,60,32
C
C     BUCKLED BUT LOADING
C
```

```
   31 KODY=2
      GO TO 40
C
C     BUCKLING AND CONTINUING
C
   32 FAC=(PYNC-SEP)/DS1
      IF (FAC.GE.FACTOR) GO TO 33
      FACTOR=FAC
      SEP=PYNC
      KODY=3
C
C     UPDATE PLASTIC DEFORMATION
C
      VPACN=VPACN+FACTOR*DVAX
      GO TO 70
C
   33 SEP=SEP+FACTOR*DS1
C
C     UPDATE PLASTIC DEFORMATION
C
      VPACN=VPACN+FACTOR*DVAX
      GO TO 60
C
C     ON SLOPE 2, GET FACTOR FOR STATUS CHANGE
C
   40 DS2=EAL*DVAX
      IF (DS2) 41,60,42
C
C     BUCKLING AND CONTINUING
C
   41 KODY=1
      GO TO 30
C
C     BUCKLED BUT LOADING
C
   42 FAC=(PYNC-SEP)/DS2
      IF (FAC.GE.FACTOR) GO TO 43
      FACTOR=FAC
      SEP=PYNC
      KODY=3
      GO TO 70
C
   43 SEP=SEP+FACTOR*DS2
      GO TO 60
C
C     ON SLOPE 9, TENSION YIELDING
C
   50 IF (DVAX.LT.0) GO TO 51
      DVP=FACTOR*DVAX
      XPP=XPP+DVP/DELTY
      VPACP=VPACP+DVP
      GO TO 80
C
C     YIELDED BUT UNLOADING
C
```

```
   51 KODY=0
      X3RE=XPP-PFAC
      GO TO 20
C
   60 FACAC=FACAC+FACTOR
      IF (FACAC.LT.0.9999999) GO TO 10
      RETURN
C
   70 FACAC=FACAC+FACTOR
      IVRTX=1
C
   80 RETURN
      END
```

APPENDIX A-2

FORTRAN LISTING OF END MOMENT-BUCKLING ELEMENT EL10

```
      SUBROUTINE INEL10(/KCONT/,/FCONT/,/NDOF/,/NINFC/,/ID/,/X/,/Y/,/NN/
     1)
C
      COMMON /INFEL/ IMEM,KST,IM(6),KGEOM,FL,COSA,SINA,A(2,6),EK11,
     1               EK22,EK12,ESH,EAL,EK11H,EK22H,      KODYX(2),
     2               KODY(2),BMTOT(2),SFTOT(2),FTOT(2),PRTOT(2),SENP(8),
     3               SENN(8),TENP(8),TENN(8),PRACP(2),PRACN(2),BMEP(2),
     4               SDACT(3),EMY(2,2),NODI,NODJ,KOUTDT,PR12,PR21,
     5               PMX(3,2,2),A1(4,2,2),A2(4,2,2),
     6               PFAC,RATIO,DELTY,KODYX1,KODY1,XPP,X3RE,EAL1,EAL4,
     7               EAL6,EAL7,EAL8,X5,Y5,P5,X6,X5D,Y5D,P5D,INDRE,
     8               IVRTX,IELCG,VTOT,XTOT,VPACP,VPACN,VENP,TVENP,VENN,
     9               TVENN,PYP,PYN,PYNC,REST(30)
      COMMON /WORK/ SFF(8),SSFF(8),DD(6),GA(6,6),FFEF(6),FF(6),
     1               PMAX(5,2,40),AA1(4,2,40),AA2(4,2,40),XM(5,2),
     2              FTYP(40,9),FEF(35,7),KDFEF(35),FINIT(30,6),
     3              NMEM,NMBT,NSURF,NFEF,NINT,INODI,INODJ,INC,IINC,
     4              IMBT,IIMBT,IKSFI,IKSFJ,IKGM,IKDT,KFDL,IKFDL,KFLL,
     5              IKFLL,FDL,FFDL,FLLF,AK,SLEND,PYP1,PYP2,PYN1,PYN2,
     6              KS1,KS2,XL,YL,DET,PLL,SS,W(25)
C
      DIMENSION KCONT(1),ID(NN,1),X(1),Y(1),COM(1),AST(2),YESNO(2),
     1          KSF(2),FTYP1(40,3)
      EQUIVALENCE (IMEM,COM(1))
      DATA AST/2H  ,2H */
      DATA YESNO/4H YES,4H NO /
C
C     DATA INPUT, END MOMENT-BUCKLING ELEMENTS
C
      NDOF=6
      NINFC=170
      KCOM=KCONT(1)
      NMEM=KCONT(2)
      NMBT=KCONT(3)
      NSURF=KCONT(4)
      NFEF=KCONT(5)
      NINT=KCONT(6)
      PRINT 10, (KCONT(I),I=2,6)
   10 FORMAT(39H END MOMENT-BUCKLING ELEMENTS (TYPE 10)////
     1       34H NO. OF ELEMENTS                    =I4/
     2       34H NO. OF STIFFNESS TYPES             =I4/
     3       34H NO. OF YIELD SURFACES              =I4/
     4       34H NO. OF FIXED END FORCE PATTERNS =I4/
     5       34H NO. OF INITIAL FORCE PATTERNS      =I4)
C
C     INPUT STIFFNESS PROPERTIES
C
      PRINT 20
   20 FORMAT(////16H STIFFNESS TYPES//
     1       5H TYPE,6X,7H YOUNGS,4X,9HHARDENING,6X,7HSECTION,
     2       3X,9HREFERENCE,6X,26HFLEXURAL STIFFNESS FACTORS,
     3       8X,5HSHEAR,5X,7HPOISSON/
     4       5H  NO.,6X,7HMODULUS,4X,9H  RATIO  ,6X,7H  AREA ,
     5       3X,9H INERTIA ,6X,26H  II          JJ          IJ    ,
     6       8X,5H AREA,5X,7H RATIO /)
```

```
      DO 30 N=1,NMBT
      READ 40, I,(FTYP(N,J),J=1,9)
   30 PRINT 50, N,(FTYP(N,J),J=1,9)
   40 FORMAT(I5,4F10.0,3F5.0,2F10.0)
   50 FORMAT(I4,E14.4,E13.4,2F12.2,3X,3F10.3,F13.2,F11.3)
C
C     INPUT RADIUS OF GYRATION , K AND PHI FACTORS
C
       PRINT 60
   60 FORMAT(////16H STIFFNESS TYPES//
     1         5H TYPE,6X,10H RADIUS OF,6X,18H EQUIVELENT LENGTH,
     2         6X,19H STRENGTH REDUCTION/
     3         5H  NO.,6X,10H GYRATION ,12X,12H COEFFICIENT,
     4         12X,7H FACTOR)
      DO 70 N=1,NMBT
      READ 80, I,(FTYP1(N,J),J=1,3)
   70 PRINT 90, N,(FTYP1(N,J),J=1,3)
   80 FORMAT(I5,3F10.0)
   90 FORMAT(I4,4X,F10.3,2(10X,F10.3))
C
C     INPUT M-P YIELD SURFACE PROPERTIES
C
      PRINT 110
  110 FORMAT(////25H YIELD SURFACE PROPERTIES//
     1         8H SURFACE,             9X,13HYIELD MOMENTS,15X,
     2         12HYIELD FORCES, 9X,16HCOORDINATES OF A,6X,
     3         16HCOORDINATES OF B/
     4         8H   NO.   ,            5X,8HPOSITIVE,5X,8HNEGATIVE,8X,
     5         5HCOMPN,6X,7HTENSION,6X,16HMOMENT      FORCE,6X,
     6         16HMOMENT      FORCE/)
      DO 200 IYT=1,NSURF
      READ 120, I,       (SFF(J),J=1,8)
      PRINT 130, IYT,       (SFF(J),J=1,8)
  120 FORMAT(I5,4F10.0,4F5.0)
  130 FORMAT(I5,F15.2,3F13.2,2(2X,2F10.3))
      SFF(2)=-ABS(SFF(2))
      SFF(3)=-ABS(SFF(3))
      SFF(4)=ABS(SFF(4))
      IF (SFF(6).EQ.0.) SFF(6)=1.E-6
      IF (SFF(8).EQ.0.) SFF(8)=1.E-6
C
C     STEEL TYPE
C
      PMAX(1,1,IYT)=SFF(3)
      PMAX(1,2,IYT)=SFF(3)
      PMAX(2,1,IYT)=SFF(3)*SFF(6)
      PMAX(2,2,IYT)=SFF(3)*SFF(8)
      PMAX(3,1,IYT)=0.
      PMAX(3,2,IYT)=0.
      PMAX(4,1,IYT)=SFF(4)*SFF(6)
      PMAX(4,2,IYT)=SFF(4)*SFF(8)
      PMAX(5,1,IYT)=SFF(4)
      PMAX(5,2,IYT)=SFF(4)
      XM(1,1)=0.
      XM(1,2)=0.
```

```
      XM(2,1)=SFF(1)*SFF(5)
      XM(2,2)=SFF(2)*SFF(7)
      XM(3,1)=SFF(1)
      XM(3,2)=SFF(2)
      XM(4,1)=XM(2,1)
      XM(4,2)=XM(2,2)
      XM(5,1)=0.
      XM(5,2)=0.
      DO 190 J=1,2
      PP2=PMAX(1,J,IYT)
      XM2=XM(1,J)
      DO 190 I=1,4
      PP1=PP2
      XM1=XM2
      PP2=PMAX(I+1,J,IYT)
      XM2=XM(I+1,J)
      DENOM=XM1*PP2-XM2*PP1
      AA2(I,J,IYT)=(PP2-PP1)/DENOM
  190 AA1(I,J,IYT)=(XM1-XM2)/DENOM
  200 CONTINUE
C
C     FIXED END FORCE PATTERNS
C
      IF (NFEF.EQ.0) GO TO 250
      PRINT 210
  210 FORMAT(////25H FIXED END FORCE PATTERNS//
     1       8H PATTERN,3X,4HAXIS,7X,5HAXIAL,7X,5HSHEAR,6X,6HMOMENT,
     2       7X,5HAXIAL,7X,5HSHEAR,6X,6HMOMENT,5X,8HLL. RED./
     3       8H  NO.   ,3X,4HCODE,7X,5HAT  I,7X,5HAT  I,6X,6H AT I ,
     4       7X,5HAT  J,7X,5HAT  J,6X,6H AT J ,5X,8H FACTOR /)
      DO 220 N=1,NFEF
      READ 230, I,KDFEF(N),(FEF(N,J),J=1,7)
  220 PRINT 240, N,KDFEF(N),(FEF(N,J),J=1,7)
  230 FORMAT(2I5,7F10.0)
  240 FORMAT(I5,I9,F13.2,5F12.2,F12.3)
C
C     INITIAL FORCE PATTERNS
C
  250 IF (NINT.EQ.0) GO TO 300
      PRINT 260
  260 FORMAT(////28H INITIAL END FORCE PATTERNS //
     1       8H PATTERN,7X,5HAXIAL,7X,5HSHEAR,6X,6HMOMENT,7X,5HAXIAL,
     2       7X,5HSHEAR,6X,6HMOMENT/
     3       8H  NO.   ,7X,5HAT  I,7X,5HAT  I,6X,6H AT I ,7X,5HAT  J,
     4       7X,5HAT  J,6X,6H AT J /)
      DO 270 N=1,NINT
      READ 280, I,(FINIT(N,J),J=1,6)
  270 PRINT 290, N,(FINIT(N,J),J=1,6)
  280 FORMAT(I5,6F10.0)
  290 FORMAT(I5,3X,6F12.2)
C
C     ELEMENT SPECIFICATION
C
  300 PRINT 310
  310 FORMAT(////22H ELEMENT SPECIFICATION//
```

```
     1        3X,4HELEM,3X,4HNODE,2X,4HNODE,2X,4HNODE,2X,4HSTIF,
     2        2X,14HYIELD SURFACES,2X,4HGEOM,2X,4HTIME,3X,
     3        12HFEF PATTERNS,3X,17HFEF SCALE FACTORS,3X,
     4        16H INITIAL FORCES /
     5        3X,4H NO.,3X,4H  I ,2X,4H  J ,2X,4HDIFF,2X,4HTYPE,
     6        2X,14H END I  END J ,2X,4HSTIF,2X,4HHIST,3X,
     7        12H  DL     LL  ,3X,17H   DL          LL  ,3X,
     8        17H NO.   SCALE FAC./)
C
      DO 320 J=36,84
  320 COM(J)=0.
      KODYX(1)=0
      KODYX(2)=0
      KODY(1)=0
      KODY(2)=0
      KST=0
      DO 325 J=145,167
  325 COM(J)=0.
      KODYX1=0
      KODY1=0
      XPP=0.
C
      IMEM=1
  330 READ 340, INEL,INODI,INODJ,IINC,IIMBT,          IKSFI,IKSFJ,IKGM,IKDT,
     1IKFDL,IKFLL,FFDL,FFLL,IINIT,FFINIT
  340 FORMAT(11I5,2F5.0,I5,F5.0)
C
      IF (INEL.GT.IMEM) GO TO 380
  350 NODI=INODI
      NODJ=INODJ
      INC=IINC
      IF (INC.EQ.0) INC=1
      IMBT=IIMBT
      KSF(1)=IKSFI
      KSF(2)=IKSFJ
      KGEOM=IKGM
      KOUTDT=IKDT
      YNG=YESNO(2)
      IF (KGEOM.NE.0) YNG=YESNO(1)
      YNT=YESNO(2)
      IF (KOUTDT.NE.0) YNT=YESNO(1)
      KFDL=IKFDL
      KFLL=IKFLL
      FDL=FFDL
      FLLM=FFLL
      FLLF=1.
      IF (KFLL.EQ.0) GO TO 360
      FLLF=FEF(IKFLL,7)
      IF (FLLF.EQ.0.) FLLF=1.E-6
  360 INIT=IINIT
      FINT=FFINIT
      ASTT=AST(1)
      IF (INEL-NMEM) 330,380,330
C
  370 NODI=NODI+INC
```

```
      NODJ=NODJ+INC
      ASTT=AST(2)
C
  380 PRINT 390, ASTT,IMEM,NODI,NODJ,INC,IMBT,      KSF(1),KSF(2),YNG,YNT
     1,KFDL,KFLL,FDL,FLLM,INIT,FINT
  390 FORMAT(A2,I4,I7,3I6,2I7,5X,A4,2X,A4,I7,I6,F11.2,F10.2,I7,F11.2)
C
C     LOCATION MATRIX
C
      DO 400 I=1,3
      LM(I)=ID(NODI,I)
  400 LM(I+3)=ID(NODJ,I)
      CALL BAND
C
C     ELEMENT PROPERTIES
C
      XL=X(NODJ)-X(NODI)
      YL=Y(NODJ)-Y(NODI)
      FL=SQRT(XL**2+YL**2)
      COSA=XL/FL
      SINA=YL/FL
      YMOD=FTYP(IMBT,1)
      PSH=FTYP(IMBT,2)
      PPSH=1.-PSH
      PSH=PSH/PPSH
      AREA=FTYP(IMBT,3)
      EAL=YMOD*AREA/FL
      RAD=FTYP1(IMBT,1)
      AK=FTYP1(IMBT,2)
      SLEND=AK*FL/RAD
      RATIO=60./SLEND
      KS1=KSF(1)
      KS2=KSF(2)
      PYP1=PMAX(5,1,KS1)
      PYP2=PMAX(5,1,KS2)
      IF(PYP1.LE.PYP2) GO TO 420
      PYP=PYP2
      GO TO 421
  420 PYP=PYP1
  421 CONTINUE
      PYN1=PMAX(1,1,KS1)
      PYN2=PMAX(1,1,KS2)
      IF(PYN1.LE.PYN2) GO TO 425
      PYN=PYN2
      GO TO 426
  425 PYN=PYN1
  426 CONTINUE
      PHI=FTYP1(IMBT,3)
      PYNC=PHI*PYN
      PFAC=ABS(PYNC/PYP)
      SLOP1=(PFAC*(1.-PHI))/(PFAC-5.*PHI)
      EAL1=SLOP1*EAL
      X3RE=-PFAC
      DELTY=PYP/EAL
      EIL=YMOD*FTYP(IMBT,4)*PPSH/FL
```

```
      FACL=FTYP(IMBT,5)
      FACR=FTYP(IMBT,6)
      FACLR=FTYP(IMBT,7)
      IF (FACL.EQ.0.) FACL=1.E-6
      IF (FACR.EQ.0.) FACR=1.E-6
      IF (FTYP(IMBT,8).EQ.0.) GC TO 430
      SHFAC=EIL/(FTYP(IMBT,1)/(2.*(1.+FTYP(IMBT,9)))*FTYP(IMBT,8)*FL*PPS
     1H)
      DET=FACL*FACR-FACLR**2
      FII=FACR/DET+SHFAC
      FJJ=FACL/DET+SHFAC
      FIJ=-FACLR/DET+SHFAC
      DET=FII*FJJ-FIJ**2
      FACR=FII/DET
      FACL=FJJ/DET
      FACLR=-FIJ/DET
  430 EK11=EIL*FACL
      EK22=EIL*FACR
      EK12=EIL*FACLR
      EK11H=EK11-EK12**2/EK22
      EK22H=EK22-EK12**2/EK11
      PR12=EK12/EK22
      PR21=EK12/EK11
C
C     M-P YIELD SURFACE EQUATICN DATA FOR EACH END OF AN ELEMENT
C
      DO 450 K=1,2
      KK=KSF(K)
      DO 450 J=1,2
      DO 440 I=1,3
  440 PMX(I,J,K)=PMAX(I+1,J,KK)
      DO 450 I=1,4
      A2(I,J,K)=PPSH/AA2(I,J,KK)
  450 A1(I,J,K)=AA1(I,J,KK)*A2(I,J,K)
C
C     DISPLACEMENT TRANSFORMATICN
C
      A(1,1)=-SINA/FL
      A(1,2)=COSA/FL
      A(1,3)=1.
      A(1,4)=-A(1,1)
      A(1,5)=-A(1,2)
      A(1,6)=0.
      A(2,1)=A(1,1)
      A(2,2)=A(1,2)
      A(2,3)=0.
      A(2,4)=A(1,4)
      A(2,5)=A(1,5)
      A(2,6)=1.
C
C     LOADS DUE TO FIXED END FCRCES
C
      DO 480 I=1,6
      SFF(I)=0.
  480 SSFF(I)=0.
```

```
      IF (KFDL+KFLL.EQ.0) GO TO 610
      DO 490 I=1,6
      DO 490 J=1,6
  490 GA(I,J)=0.
      GA(1,1)=COSA
      GA(1,2)=SINA
      GA(2,1)=-SINA
      GA(2,2)=COSA
      GA(3,3)=1.
      GA(4,4)=COSA
      GA(4,5)=SINA
      GA(5,4)=-SINA
      GA(5,5)=COSA
      GA(6,6)=1.
C
      IF (KFDL.EQ.0) GO TO 530
      DO 500 I=1,6
  500 FFEF(I)=FEF(KFDL,I)*FDL
      IF (KDFEF(KFDL).EQ.0) GO TO 510
      CALL MULT (GA,FFEF,SFF,6,6,1)
      GO TO 530
  510 DO 520 I=1,6
  520 SFF(I)=FFEF(I)
C
  530 IF (KFLL.EQ.0) GO TO 570
      DO 540 I=1,6
      FLL=FLLF*FLLM
      IF (I.EQ.3.OR.I.EQ.6) FII=FLLM
  540 FFEF(I)=FEF(KFLL,I)*FLL
      IF (KDFEF(KFLL).EQ.0) GO TO 550
      CALL MULT (GA,FFEF,SSFF,6,6,1)
      GO TO 570
  550 DO 560 I=1,6
  560 SSFF(I)=FFEF(I)
C
  570 DO 580 I=1,6
  580 FF(I)=SFF(I)+SSFF(I)
C
      CALL MULT (GA,FF,DD,6,6,1)
      CALL SFORCE (DD)
C
C     MODIFY TO GET INITIAL ELEMENT FORCES
C
      DO 600 I=1,6
      FLL=1./FLLF
      IF (I.EQ.3.OR.I.EQ.6) FIL=1.
  600 SFF(I)=SFF(I)+SSFF(I)*FII
C
C     INITIAL FORCES
C
  610 IF (INIT.EQ.0) GO TO 630
      DO 620 I=1,6
  620 SFF(I)=SFF(I)+FINIT(INIT,I)*FINT
C
C     INITIALIZE ARRAYS
```

```
C
  630 BMEP(1)=SFF(3)*PPSH
      BMEP(2)=SFF(6)*PPSH
      FTOT(1)=SFF(1)
      FTOT(2)=SFF(4)
      SFTOT(1)=SFF(2)
      SFTOT(2)=SFF(5)
      BMTOT(1)=SFF(3)
      BMTOT(2)=SFF(6)
      DO 650 I=1,6
      SS=BMTOT(I)
      IF (SS.LT.0.) GO TO 640
      SENP(I)=SS
      GO TO 650
  640 SENN(I)=SS
  650 CONTINUE
C
C     YIELD MOMENTS FOR INITIAL FORCE STATE
C
      CALL YMOM10
C
      CALL FINISH
C
C     GENERATE MISSING ELEMENTS
C
      IF (IMEM.EQ.NMEM) RETURN
      IMEM=IMEM+1
      IF (IMEM.EQ.INEL) GO TO 350
      GO TO 370
C
      END
```

```
      SUBROUTINE STIF10 (/MSTEP/,/NDOF/,/NINFC/,/COMS/,/FK/,/DFAC/)
C
      COMMON /INFEL/ IMEM,KST,LM(6),KGEOM,FL,COSA,SINA,A(2,6),EK11,
     1               EK22,EK12,PSH,EAL,EK11H,EK22H,       KODYX(2),
     2               KODY(2),EMICT(2),SFTOT(2),FTOT(2),PRTOT(2),SENP(8),
     3               SENN(8),TENP(8),TENN(8),PRACP(2),PRACN(2),BMEP(2),
     4               SDACT(3),BMY(2,2),NODI,NODJ,KOUTDT,PR12,PR21,
     5               PMX(3,2,2),A1(4,2,2),A2(4,2,2),
     6               PFAC,RATIO,DELTY,KODYX1,KODY1,XPP,X3RE,EAL1,EAL4,
     7               EAL6,EAL7,EAL8,X5,Y5,P5,X6,X5D,Y5D,P5D,INDRE,
     8               IVRTX,IFLOG,VTOT,XTOT,VPACP,VPACN,VENP,TVENP,VENN,
     9               TVENN,PYP,PYN,PYNC,REST(30)
      COMMON /WORK/ ST(2,2),STT(2,2),ATK(6,2),AA(2,6),PFL,AXK,FAC,
     1              FFK(6,6),FSK(6,6),W(1893)
C
      DIMENSION COM(1),COMS(1),FK(6,6)
      EQUIVALENCE (IMEM,COM(1))
C
C     STIFFNESS FORMULATION, END MOMENT-BUCKLING ELEMENTS
C
      DO 10 J=3,35
   10 COM(J)=COMS(J)
      DO 15 J=141,149
   15 COM(J)=COMS(J)
C
C     CURRENT AXIAL STIFFNESS
C
      CALL FST10A (STIF,KODY1)
C
C     PREVIOUS STIFFNESS
C
      IF(MSTEP.LT.2) GO TO 20
      CALL FST10A (STIFF,KODYX1)
C
C     STIFFNESS DIFFERENCE
C
      STIF=STIF-STIFF
   20 CONTINUE
      DO 30 I=1,36
   30 FSK(I,1)=0.
      AXK=STIF*COSA**2
      FSK(1,1)=AXK
      FSK(1,4)=-AXK
      FSK(4,4)=AXK
      AXK=STIF*SINA**2
      FSK(2,2)=AXK
      FSK(2,5)=-AXK
      FSK(5,5)=AXK
      AXK=STIF*SINA*COSA
      FSK(1,2)=AXK
      FSK(1,5)=-AXK
      FSK(2,4)=-AXK
      FSK(4,5)=AXK
      DO 40 I=1,6
      DO 40 J=I,6
```

```
   40 FSK(J,I)=FSK(I,J)
C
C     CURRENT FLEXURAL STIFFNESS, ELASTO-PLASTIC PART
C
      CALL FST10B (ST,KODY)
C
C     PREVIOUS STIFFNESS
C
      IF(MSTEP.LT.2) GO TO 50
      CALL FST10B (STT,KODYX)
C
C     STIFFNESS DIFFERENCE
C
      DO 60 I=1,4
   60 ST(I,1)=ST(I,1)-STT(I,1)
      CALL MULTST (A,ST,ATK,FK,6,2)
C
C     GET TOTAL STIFFNESS
C
      DO 70 I=1,6
      DO 70 J=1,6
   70 FK(I,J)=FK(I,J)+FSK(I,J)
      RETURN
C
C     ORIGINAL STIFFNESS AT STEP 0, BETA-0, CORRN AT STEP 1
C
   50 FAC=1.
      IF (MSTEP.NE.1) GO TO 85
      FAC=DFAC
      DO 80 I=1,36
   80 FSK(I,1)=FSK(I,1)*FAC
   85 CC=(1.+PSH)*FAC
      DO 90 I=1,4
   90 ST(I,1)=ST(I,1)*CC
      CALL MULTST (A,ST,ATK,FK,6,2)
C
C     GET TOTAL INITIAL STIFFNESS
C
      DO 100 I=1,6
      DO 100 J=1,6
  100 FK(I,J)=FK(I,J)+FSK(I,J)
C
C     ADD GEOMETRIC STIFFNESS
C
      IF (MSTEP.EQ.0.OR.KGEOM.EQ.0) GO TO 120
      PFL=(COMS(41)-COMS(40))/(2.*FL)
      DO 110 I=1,4
  110 ST(I,1)=PFL
      DO 130 I=1,12
  130 AA(I,1)=0.
      AA(1,1)=-SINA
      AA(1,2)=COSA
      AA(2,4)=SINA
      AA(2,5)=-COSA
      CALL MULTST (AA,ST,ATK,FFK,6,2)
```

```
      DO 140 I=1,36
  140 FK(I,1)=FK(I,1)+FFK(I,1)
C
  120 RETURN
C
      END
```

```
      SUBROUTINE RESP10(/NDOF/,/NINFC/,/KBAL/,/KPR/,/COMS/,/DDISM/,/DD/,
     1/TIME/,/VELM/,/DFAC/,/DELTA/)
C
C     STATE DETERMINATION, END MOMENT-BUCKLING ELEMENTS
C
      COMMON /INFEL/ IMEM,KST,IM(6),KGEOM,FL,COSA,SINA,A(2,5),EK11,
     1               EK22,EK12,PSH,EAL,EK11H,EK22H,         KODYX(2),
     2               KODY(2),BMTOT(2),SFTOT(2),FTOT(2),PRTOT(2),SENP(8),
     3               SENN(8),TENP(8),TENN(8),PRACP(2),PRACN(2),BMEP(2),
     4               SDACT(3),BMY(2,2),NODI,NODJ,KOUTDT,PR12,PR21,
     5               PMX(3,2,2),A1(4,2,2),A2(4,2,2),
     6               PFAC,RATIO,DELTY,KODYX1,KODY1,XPP,X3RE,EAL1,EAL4,
     7               EAL6,EAL7,EAL8,X5,Y5,P5,X6,X5D,Y5D,P5D,INDRE,
     8               IVRTX,IETOG,VTOT,XTOT,VPACP,VPACN,VENP,TVENP,VENN,
     9               TVENN,PYP,PYN,PYNC,REST(30)
      COMMON /WORK/ DVR(2),DPP(2),DBM(2),BBMTOT(2),BML(2),BMEL(2),
     1              DVAX,SLIN,FACAC,FACTOR,FAC,DSF,BMIUB,BMJUB,SFUB,
     2              DSEP,DVP,SLOP6,SLOP7,DS4,DS5,DS6,DS7,DS8,EALE,FOUB,
     3              KBAL1,KST1,W(1966)
C
      DIMENSION COM(1),COMS(1),DDISM(1),DD(1),VELM(1),NOD(2)
      EQUIVALENCE (IMEM,COM(1)),(NODI,NOD(1))
C
      DO 10 J=1,NINFC
   10 COM(J)=COMS(J)
      KODYX(1)=KODY(1)
      KODYX(2)=KODY(2)
      KODYX1=KODY1
      IF (IMEM.EQ.1) IPED=0
C
C     DEFORMATION INCREMENTS
C
      DVAX=COSA*(DDISM(4)-DDISM(1))+SINA*(DDISM(5)-DDISM(2))
      ROT=(SINA*(DDISM(4)-DDISM(1))+COSA*(DDISM(2)-DDISM(5)))/FL
      DVR(1)=DDISM(3)+ROT
      DVR(2)=DDISM(6)+ROT
      VTOT=VTOT+DVAX
      SEP=(FTOT(2)-FTOT(1))*0.5
C
C     AXIAL FORCE INCREMENT
C
      CALL FST10A (EALE,KODY1)
      SLIN=SEP+EALE*DVAX
C
C     AXIAL STATE DETERMINATION
C
      FVRTX=0.
C
C     CHECK VERTEX STATE
C
      IF (IVRTX.EQ.0) CALL VRTX10 (FVRTX,DVAX,SEP,TIME)
C
      IF (IVRTX.EQ.0) GO TO 120
C
      FACAC=FVRTX
```

```
C
   20 FACTOR=1.-FACAC
      KODYI=KODY1+1
      GO TO (30,120,120,40,50,60,70,80,90,100),KODYI
C
C     ON SLOPE 0, ELASTIC, GET FACTOR FOR STATUS CHANGE
C
   30 DSEP=EAL*DVAX
      IF (DSEP) 31,110,32
   31 FAC=(PYNC-SEP)/DSEP
      IF (FAC.GE.FACTOR) GO TO 33
      FACTOR=FAC
      SEP=PYNC
      KODY1=3
      GO TO 110
C
   32 FAC=(PYP-SEP)/DSEP
      IF (FAC.GE.FACTOR) GO TO 33
      FACTOR=FAC
      SEP=PYP
      KODY1=9
      GO TO 110
C
   33 SEP=SEP+FACTOR*DSEP
      GO TO 110
C
C     ON SLOPE 3, BUCKLING AND CONTINUING
C
   40 IF (DVAX.GT.0.) GO TO 41
C
C     UPDATE PLASTIC DEFORMATIONS
C
      DVP=FACTOR*DVAX
      VPACN=VPACN+DVP
      GO TO 120
C
C     BUCKLING AND UNLOADING
C
   41 XTOT=(VTOT-DVAX)/DELTY
      IF (XTOT.GE.X3RE) GO TO 42
C
C     ESTABLISH NEW STIFFNESS FOR REVERSE
C
      X3RE=XTOT
      IELOG=1
      CALL LAW10
      P5=Y5*PYP
      KODY1=4
      GO TO 50
C
C     USE OLD STIFFNESS FOR REVERSE
C
   42 IF (INDRE.EQ.2.AND.XTOT.LE.X6) GO TO 43
      IF((X5D-XTOT).EQ.0.) GO TO 44
      SLOP6=(Y5D+PFAC)/(X5D-XTOT)
```

```
      EAL6=EAL*SLOP6
      KODY1=6
      GO TO 70
C
   43 SLOP7=(Y5+PFAC)/(X5-XTOT)
      EAL7=EAL*SLOP7
      KODY1=7
      GO TO 80
   44 KODY1=5
      GO TO 60
C
C     ON SLOPE 4, GET FACTOR FOR STATUS CHANGE
C
   50 DS4=EAL4*DVAX
      IF (DS4) 51,110,54
   51 IF (SEP) 52,52,53
   52 FAC=(PYNC-SEP)/DS4
      IF (FAC.GE.FACTOR) GO TO 55
      FACTOR=FAC
      SEP=PYNC
      KODY1=3
      GO TO 110
   53 X5D=VTOT/DELTY
      Y5D=SEP/PYP
      P5D=SEP
      INDRE=1
      KODY1=5
      GO TO 60
   54 FAC=(P5-SEP)/DS4
      IF (FAC.GE.FACTOR) GO TO 55
      FACTOR=FAC
      SEP=P5
      KODY1=8
      GO TO 110
   55 SEP=SEP+FACTOR*DS4
      GO TO 110
C
C     ON SLOPE 5, GET FACTOR FOR STATUS CHANGE
C
   60 DS5=EAL*DVAX
      IF (DS5) 61,110,62
   61 FAC=(PYNC-SEP)/DS5
      IF (FAC.GE.FACTOP) GO TO 65
      FACTOR=FAC
      SEP=PYNC
      KODY1=3
      GO TO 110
   62 FAC=(P5D-SEP)/DS5
      IF (FAC.GE.FACTOR) GO TO 65
      FACTOR=FAC
      SEP=P5D
      GO TO (63,64),INDRE
   63 KODY1=4
      GO TO 110
   64 KODY1=8
```

```
      GO TO 110
   65 SEP=SEP+FACTOR*DS5
      GO TO 110
C
C     ON SLOPE 6, GET FACTOR FOR STATUS CHANGE
C
   70 DS6=EAL6*DVAX
      IF (DS6) 71,110,72
   71 FAC=(PYNC-SEP)/DS6
      IF (FAC.GE.FACTOR) GO TO 75
      FACTOR=FAC
      SEP=PYNC
      KODY1=3
      GO TO 110
   72 FAC=(P5D-SEP)/DS6
      IF (FAC.GE.FACTOR) GO TO 75
      FACTOR=FAC
      SEP=P5D
      GO TO (73,74),INDRE
   73 KODY1=4
      GO TO 110
   74 KODY1=8
      GO TO 110
   75 SEP=SEP+FACTOR*DS6
      GO TO 110
C
C     ON SLOPE 7, GET FACTOR FOR STATUS CHANGE
C
   80 DS7=EAL7*DVAX
      IF (DS7) 81,110,82
   81 FAC=(PYNC-SEP)/DS7
      IF (FAC.GE.FACTOR) GO TO 83
      FACTOR=FAC
      SEP=PYNC
      KODY1=3
      GO TO 110
   82 FAC=(P5-SEP)/DS7
      IF (FAC.GE.FACTOR) GO TO 83
      FACTOR=FAC
      SEP=P5
      KODY1=8
      GO TO 110
   83 SEP=SEP+FACTOR*DS7
      GO TO 110
C
C     ON SLOPE 8, GET FACTOR FOR STATUS CHANGE
C
   90 DS8=EAL8*DVAX
      IF (DS8) 91,110,92
   91 X5D=VTOT/DELTY
      Y5D=SEP/PYP
      P5D=SEP
      INDRE=2
      KODY1=5
      GO TO 60
```

```
C
   92 FAC=(PYP-SEP)/DS8
      IF (FAC.GE.FACTOR) GO TO 93
      FACTOR=FAC
      SEP=PYP
      KODY1=9
      GO TO 110
   93 SEP=SEP+FACTOR*DS8
      GO TO 110
C
C     ON SLOPE 9, YIELDED BUT CONTINUING
C
  100 IF (DVAX.LT.0.) GO TO 101
C
C     UPDATE PLASTIC DEFORMATIONS
C
      DVP=FACTOR*DVAX
      XPP=XPP+DVP/DELTY
      VPACP=VPACP+DVP
      GO TO 120
C
C     YIELDED BUT UNLOADING
C
  101 KODY1=0
C
C     RESIDUAL ELONGATION, RE
C
      IF (IELOG.NE.1) GO TO 105
      SLEND=60./RATIO
      RE=0.0175*(0.55*X3RE/SLEND+0.0002*X3RE**2)
      RE=RE*FL/DELTY
      XPP=XPP+RE
      IELOG=0
C
  105 X3RE=XPP-PFAC
      GO TO 30
C
C     CHECK FOR COMPLETION OF CYCLE
C
  110 FACAC=FACAC+FACTOR
      IF (FACAC.LT.0.9999999) GO TO 20
C
C     NEW FORCE, UNBALANCED FORCE DUE TO YIELD
C
  120 FTOT(2)=SEP
      FTOT(1)=-SEP
      FOUB=SLIN-SEP
      IF (ABS(FOUB).GT.1.E-8) KBAL1=1
C
C     ACCUMULATE EXTENSION ENVELOPES
C
      IF (VENP.GE.VTOT) GO TO 180
      VENP=VTOT
      TVENP=TIME
      GO TO 190
```

```
  180 IF (VENN.LE.VTOT) GO TO 190
      VENN=VTOT
      TVENN=TIME
  190 CONTINUE
C
C     SET INDICATOR FOR AXIAL STIFFNESS CHANGE
C
      KST1=0
      IF (KODYX1.NE.KODY1) KST1=1
C
C     FLEXURAL STATE DETERMINATION
C
C     LINEAR MOMENT INCREMENTS
C
      CALL BM10
      BML(1)=BMEP(1)+DBM(1)
      BML(2)=BMEP(2)+DBM(2)
      BMEL(1)=BMTOT(1)-BMEP(1)
      BMEL(2)=BMTOT(2)-BMEP(2)
C
C     TRACE OUT NONLINEAR PATH ON M-P SURFACE
C
      FACAC=0.
      KBAL=0
  250 FACTOR=1.-FACAC
C
C     PLASTIC HINGE ROTATIONS
C
       IF (KODY(1)+KODY(2)-1) 280,260,270
  260 DPR(1)=DVR(1)+PR21*DVR(2)
      DPR(2)=DVR(2)+PR12*DVR(1)
      GO TO 280
  270 DPR(1)=DVR(1)
      DPR(2)=DVR(2)
C
  280 KFAC=0
      DO 320 IEND=1,2
C
C     ELASTIC, GET FACTOR FOR STATUS CHANGE
C
      IF (KODY(IEND).NE.0) GO TO 310
      IF (DBM(IEND))290,320,300
  290 FAC=(BMY(IEND,2)-BMEP(IEND))/DBM(IEND)
      IF (FAC.GE.FACTOR) GO TO 320
      FACTOR=FAC
      BBMY=BMY(IEND,2)
      KFAC=IEND
      GO TO 320
  300 FAC=(BMY(IEND,1)-BMEP(IEND))/DBM(IEND)
      IF (FAC.GE.FACTOR) GO TO 320
      FACTOR=FAC
      BBMY=BMY(IEND,1)
      KFAC=IEND
      GO TO 320
C
```

```
C     CONTINUING TO YIELD - POSITIVE PLASTIC WORK
C
  310 IF  (BMEP(IEND)*DPP(IEND).GE.0.)  GO  TO  320
C
C     UNLOADING - PLASTIC WORK NEGATIVE
C
      FACTOR=0.
      KFAC=0
      KODY(IEND)=0
C
  320 CONTINUE
C
C     UPDATE MOMENTS AND HINGE ROTATIONS
C
      DO  360  IEND=1,2
      IF  (IEND.EQ.KFAC)  GO  TO  350
      IF  (KODY(IEND).NE.0)  GO  TO  330
      BMEP(IEND)=BMEP(IEND)+FACTOR*DBM(IEND)
      GO  TO  360
  330 DPPR=FACTOR*DPR(IEND)
      PRTOT(IEND)=PRTOT(IEND)+DPPR
      IF  (DPPR.LT.0.)  GO  TO  340
      PRACP(IEND)=PRACP(IEND)+DPPR
      GO  TO  360
  340 PRACN(IEND)=PRACN(IEND)+DPPR
      GO  TO  360
  350 BMEP(IEND)=BBMY
      KODY(IEND)=1
  360 CONTINUE
C
C     CHECK COMPLETION OF CYCLE
C
      FACAC=FACAC+FACTOR
      IF  (FACAC.GT.0.99999)  GO  TO  370
      CALL  BM10
      KBAL=1
      GO  TO  250
C
C     YIELD MOMENTS FOR NEXT STEP
C
  370 CALL  YMOM10
C
C     CHECK FOR OVERSHOOT OF M-P SURFACE
C
      DO  420  IEND=1,2
      IF  (KODY(IEND).EQ.0)  GO  TO  400
      IF  (BMEP(IEND).LE.BMY(IEND,1))  GO  TO  380
      BMEP(IEND)=BMY(IEND,1)
      KBAL=1
      GO  TO  420
  380 IF  (BMEP(IEND).GE.BMY(IEND,2))  GO  TO  390
      BMEP(IEND)=BMY(IEND,2)
      KBAL=1
      GO  TO  420
  390 IF  (BMEP(IEND).LT.BMY(IEND,1)*0.98.AND.BMEP(IEND).GT.BMY(IEND,2)*0
```

```
     1.98) KODY(IEND)=0
      GO TO 420
  400 IF (BMEP(IEND).LE.BMY(IEND,1)) GO TO 410
      BMEP(IEND)=BMY(IEND,1)
      KBAL=1
      KODY(IEND)=1
      GO TO 420
  410 IF (BMEP(IEND).GE.BMY(IEND,2)) GO TO 420
      BMEP(IEND)=BMY(IEND,2)
      KBAL=1
      KODY(IEND)=1
  420 CONTINUE
C
C     ELASTIC AND TOTAL FORCES
C
      BBMTOT(1)=BMTOT(1)
      BBMTOT(2)=BMTOT(2)
      BMTOT(1)=BMEP(1)+BMEL(1)+(EK11*DVR(1)+EK12*DVR(2))*PSH
      BMTOT(2)=BMEP(2)+BMEL(2)+(EK12*DVR(1)+EK22*DVR(2))*PSH
      DSF=(BMTOT(1)-BBMTOT(1)+BMTOT(2)-BBMTOT(2))/FL
      SFTOT(1)=SFTOT(1)+DSF
      SFTOT(2)=SFTOT(2)-DSF
C
C     UNBALANCED LOADS DUE TO YIELD
C
      BMIUB=0.
      BMJUB=0.
      IF (KBAL.EQ.0) GO TO 430
      BMIUB=BML(1)-BMEP(1)
      BMJUB=BML(2)-BMEP(2)
C
C     DEFORMATION RATES FOR DAMPING
C
  430 IF (DFAC.EQ.0.0.AND.DELTA.EQ.0.0) GO TO 460
      IF (TIME.EQ.0.) GO TO 470
      KBAL=1
      DVAX=COSA*(VELM(4)-VELM(1))+SINA*(VELM(5)-VELM(2))
      ROT=(SINA*(VELM(4)-VELM(1))+COSA*(VELM(2)-VELM(5)))/FL
      DVR(1)=VELM(3)+ROT
      DVR(2)=VELM(6)+ROT
C
C     BETA-O DAMPING
C
      IF (DFAC.EQ.0.) GO TO 450
      FAC=DFAC*(1.+PSH)
      BMIUB=BMIUB+(EK11*DVR(1)+EK12*DVR(2))*FAC
      BMJUB=BMJUB+(EK12*DVR(1)+EK22*DVR(2))*FAC
      FOUB=EAL*DVAX*DFAC+FOUB
C
C     STRUCTURAL DAMPING LOAD
C
  450 IF (DELTA.EQ.0.) GO TO 460
      SDMI=DELTA*ABS(BMTOT(1))*SIGN(1.,DVR(1))
      SDMJ=DELTA*ABS(BMTOT(2))*SIGN(1.,DVR(2))
      SDFO=DELTA*ABS((FTOT(2)-FTOT(1))/2.)*SIGN(1.,DVAX)
```

```
      BMIUB=BMIUB-SDMI+SDACT(1)
      BMJUB=BMJUB-SDMJ+SDACT(2)
      FOUB=FOUB-SDFO+SDACT(3)
      SDACT(1)=SDMI
      SDACT(2)=SDMJ
      SDACT(3)=SDFO
C
C     SET UP UNBALANCED LOAD VECTOR
C
  460 IF (KBAL+KBAL1.EQ.0) GO TO 470
      KBAL=1
      SFUB=(BMIUB+BMJUB)/FL
      DD(1)=-SFUB*SINA-FOUB*COSA
      DD(2)=SFUB*COSA-FOUB*SINA
      DD(3)=BMIUB
      DD(4)=-DD(1)
      DD(5)=-DD(2)
      DD(6)=BMJUB
C
C     EXTRACT ENVELOPES
C
  470 DO 490 I=1,8
      S=BMTOT(I)
      IF (S.LE.SENP(I)) GO TO 480
      SENP(I)=S
      TENP(I)=TIME
  480 IF (S.GE.SENN(I)) GO TO 490
      SENN(I)=S
      TENN(I)=TIME
  490 CONTINUE
C
C     PRINT TIME HISTORY
C
      IF (KPR.LT.0) GO TO 500
      IF (KPR.EQ.0.OR.KOUTDT.EQ.0) GO TO 540
  500 IF (IHED.NE.0) GO TO 520
      KKPR=IABS(KPR)
      PRINT 510, KKPR,TIME
  510 FORMAT(///18H RESULTS FOR GROUP,I3,
     1       38H, END MOMENT-BUCKLING ELEMENTS, TIME =,F8.3///5X,
     2       5H ELEM,4X,4HNODE,2X,10HFLX. YIELD,2X,7HBENDING,7X,5HSHEAR,
     3       7X,5HAXIAL,12X,23HPLASTIC HINGE ROTATIONS,
     4       15X,5HAXIAL/5X,
     5       5H  NO.,4X,4H NO.,3X,5H CODE,6X,7H MOMENT,7X,5HFORCE,
     6       7X,5HFORCE,8X,7HCURRENT,4X,9HACC. POS.,3X,9HACC. NEG.,
     7       3X,10HYIELD CODE,3X,9HEXTENSION/)
      IHED=1
  520 PRINT 530, IMEM,(NOD(I),KODY(I),BMTOT(I),SFTOT(I),FTOT(I),PRTOT(I)
     1,PRACP(I),PRACN(I),I=1,2),KODY1,VTOT
  530 FORMAT(I9,I8,I7,3X,3F12.2,3X,3F12.5/
     1       9X,I8,I7,3X,3F12.2,3X,3F12.5,I10,F11.5)
C
C     SET INDICATOR FOR STIFFNESS CHANGE
C
  540 KST=0
```

```
      IF (KODYX(1).NE.KODY(1).OR.KODYX(2).NE.KODY(2))  KST=1
      IF (KST.EQ.0.AND.KST1.NE.0) KST=1
C
C     UPDATE INFORMATION IN COMS
C
      DO 550 J=32,88
  550 COMS(J)=COM(J)
      COMS(2)=COM(2)
      DO 560 J=141,167
  560 COMS(J)=COM(J)
C
      RETURN
      END
```

```
      SUBROUTINE OUT10 (/COMS/,/NINFC/)
C
      COMMON /INFEL/ IMEM,KST,LM(6),KGEOM,FL,COSA,SINA,A(2,5),EK11,
     1               EK22,EK12,ESH,EAL,EK11H,EK22H,       KODYX(2),
     2               KODY(2),BMTCT(2),SFTOT(2),FTOT(2),PRTOT(2),SENP(8),
     3               SENN(8),TENP(8),TENN(8),PRACP(2),PRACN(2),BMEP(2),
     4               SDACT(3),BMY(2,2),NODI,NODJ,KOUTDT,PR12,PR21,
     5               PMX(3,2,2),A1(4,2,2),A2(4,2,2),
     6               PFAC,RATIO,DELTY,KODYX1,KODY1,XPP,X3RE,EAL1,EAL4,
     7               EAL6,EAL7,EAL8,X5,Y5,P5,X6,X5D,Y5D,P5D,INDRE,
     8               IVRTX,IEICG,VTOT,XTOT,VPACP,VPACN,VENP,TVENP,VENN,
     9               TVENN,PYP,PYN,PYNC,REST(30)
C
      DIMENSION COM(1),COMS(1)
      EQUIVALENCE (IMEM,COM(1))
C
C      ENVELOPE OUTPUT, END MOMENT-BUCKLING ELEMENTS
C
      DO 10 J=1,NINFC
   10 COM(J)=COMS(J)
C
      IF (IMEM.EQ.1) PRINT 20
   20 FORMAT(39H END MOMENT-BUCKLING ELEMENTS (TYPE 10)///
     1         5H ELEM,3X,4HNODE,17X,7HBENDING,14X,5HSHEAR,14X,5HAXIAL,
     2         13X,8HPL HINGE,7X,14H      MAX. AXIAL/
     3         5H  NO.,3X,4H NO.,17X,7H MOMENT,3X,4HTIME,7X,5HFORCE,3X,
     4         4HTIME,7X,5HFORCE,3X,4HTIME,6X,8HROTATION,3X,4HTIME,
     5         5X,9HEXTENSION,3X,4HTIME/)
C
      PRINT 30, IMEM,NODI,(SENP(I),TENP(I),I=1,7,2),VENP,TVENP,(SENN(I),
     1TENN(I),I=1,7,2),VENN,TVENN,NODJ,(SENP(I),TENP(I),I=2,8,2),(SENN(I
     2),TENN(I),I=2,8,2)
   30 FORMAT(I4,I7,5X,8HPOSITIVE,3(F12.2,F7.3),2(F14.5,F7.3)/
     1         16X,8HNEGATIVE,3(F12.2,F7.3),2(F14.5,F7.3)/
     2         7X,I4,5X,8HPOSITIVE,3(F12.2,F7.3),F14.5,F7.3/
     3         15X,8HNEGATIVE,3(F12.2,F7.3),F14.5,F7.3/)
C
      RETURN
      END
```

```
      SUBROUTINE FST10A (/STIF/,/KOD/)
C
      COMMON /INFEL/ IMEM,KST,LM(6),KGEOM,FL,COSA,SINA,A(2,6),EK11,
     1               EK22,EK12,PSH,EAL,EK11H,EK22H,      KODYX(2),
     2               KODY(2),BMTOT(2),SFTOT(2),FTOT(2),PRTOT(2),SENP(8),
     3               SENN(8),TENP(8),TENN(8),PRACP(2),PRACN(2),BMEP(2),
     4               SDACT(3),BMY(2,2),NODI,NODJ,KOUTDT,PR12,PR21,
     5               PMX(3,2,2),A1(4,2,2),A2(4,2,2),
     6               PFAC,RATIO,DELTY,KODYX1,KODY1,XPP,X3RE,EAL1,EAL4,
     7               EAL6,EAL7,EAL8,POUT(51)
C
C     FORM AXIAL STIFFNESS
C
      KYY=KOD+1
      GO TO (10,20,30,40,50,60,70,80,90,100),KYY
   10 STIF=EAL
      GO TO 110
   20 STIF=EAL1
      GO TO 110
   30 STIF=EAL
      GO TO 110
   40 STIF=0.001*EAL
      GO TO 110
   50 STIF=EAL4
      GO TO 110
   60 STIF=EAL
      GO TO 110
   70 STIF=EAL6
      GO TO 110
   80 STIF=EAL7
      GO TO 110
   90 STIF=EAL8
      GO TO 110
  100 STIF=0.001*EAL
C
  110 RETURN
      END
```

```
      SUBROUTINE LAW10
C
      COMMON /INFEL/ IMEM,KST,LM(6),KGEOM,FL,COSA,SINA,A(2,6),EK11,
     1               EK22,EK12,PSH,EAL,EK11H,EK22H,      KODYX(2),
     2               KODY(2),BMTOT(2),SFTOT(2),FTOT(2),PRTOT(2),SENP(8),
     3               SENN(8),TENP(8),TENN(8),PRACP(2),PRACN(2),BMEP(2),
     4               SDACT(3),EMY(2,2),NODI,NODJ,KOUTDT,PR12,PR21,
     5               PMX(3,2,2),A1(4,2,2),A2(4,2,2),
     6               PFAC,RATIO,DELTY,KODYX1,KODY1,XPP,X3RE,EAL1,EAL4,
     7               EAL6,EAL7,EAL8,X5,Y5,P5,X6,X5D,Y5D,P5D,INDRE,
     8               REST(44)
C
C     GENERATE P-DELTA HYSTERESIS CURVE
C
C     RESIDUAL ELONGATION, RE
C
      SLEND=60./RATIO
      RE=0.0175*(0.55*X3RE/SLEND+0.0002*X3RE**2)
      RE=RE*FL/DELTY
C
      XPP=XPP+RE
      XPP1=1.+XPP
      BETA=1./3.
      DENOM=(XPP1-X3RE)+(1.+PFAC)/BETA
      XNUMR=XPP-X3RE-PFAC
      Y5=RATIO*XNUMR/DENOM
      X5=XPP-Y5/BETA
      SLOP4=(Y5+PFAC)/(X5-X3RE)
      EAL4=EAL*SLOP4
      SLOP8=(1.-Y5)/(XPP1-X5)
      EAL8=EAL*SLOP8
      X6=X5-Y5-PFAC
      XPP=XPP-RE
C
      RETURN
      END
```

```
      SUBROUTINE VRTX10 (/FACAC/,/DVAX/,/SEP/,/TIME/)
C
C     AXIAL STATE DETERMINATION IN TENSION AND VERTEX REGIONS
C
      COMMON /INFEL/ IMEM,KST,LM(6),KGEOM,FL,COSA,SINA,A(2,6),EK11,
     1               EK22,EK12,PSH,EAL,EK11H,EK22H,       KODYX(2),
     2               KODY(2),BMTOT(2),SFTOT(2),FTOT(2),PRTOT(2),SENP(8),
     3               SENN(8),TENP(8),TENN(8),PRACP(2),PRACN(2),BMEP(2),
     4               SDACT(3),BMY(2,2),NODI,NODJ,KOUTDT,PR12,PR21,
     5               PMX(3,2,2),A1(4,2,2),A2(4,2,2),
     6               PFAC,RATIO,DELTY,KODYX1,KODY1,XPP,X3RE,EAL1,EAL4,
     7               EAL6,EAL7,EAL8,X5,Y5,P5,X6,X5D,Y5D,P5D,INDRE,
     8               IVRTX,IELOG,VTOT,XTOT,VPACP,VPACN,VENP,TVENP,VENN,
     9               TVENN,PYP,PYN,PYNC,REST(30)
C
C     INITIALIZE
C
      FACAC=0.
   10 FACTOR=1.-FACAC
      KODYI=KODY1+1
      IF (KODYI.EQ.10) KODYI=4
      GO TO (20,30,40,50),KODYI
C
C     ON SLOPE 0, ELASTIC, GET FACTOR FOR STATUS CHANGE
C
   20 DSEP=EAL*DVAX
      IF (DSEP) 21,60,22
   21 FAC=(PYN-SEP)/DSEP
      IF (FAC.GE.FACTOR) GO TO 23
      FACTOR=FAC
      SEP=PYN
      KODY1=1
C
      SENP(5)=-PYN
      SENN(6)=PYN
      TENP(5)=TIME
      TENN(6)=TIME
      GO TO 60
C
   22 FAC=(PYP-SEP)/DSEP
      IF (FAC.GE.FACTOR) GO TO 23
      FACTOR=FAC
      SEP=PYP
      KODY1=9
      GO TO 60
C
   23 SEP=SEP+FACTOR*DSEP
      GO TO 60
C
C     ON SLOPE 1, GET FACTOR FOR STATUS CHANGE
C
   30 DS1=EAL1*DVAX
      IF (EAL1.NE.0) GO TO 34
      KODY1=3
      IVRTX=1
```

```
      GO TO 80
C
   34 IF (DS1) 31,60,32
C
C     BUCKLED BUT LOADING
C
   31 KODY1=2
      GO TO 40
C
C     BUCKLING AND CONTINUING
C
   32 FAC=(PYNC-SEP)/DS1
      IF (FAC.GE.FACTOR) GO TO 33
      FACTOR=FAC
      SEP=PYNC
      KODY1=3
C
C     UPDATE PLASTIC DEFORMATION
C
      VPACN=VPACN+FACTOR*DVAX
      GO TO 70
C
   33 SEP=SEP+FACTOR*DS1
C
C     UPDATE PLASTIC DEFORMATION
C
      VPACN=VPACN+FACTOR*DVAX
      GO TO 60
C
C     ON SLOPE 2, GET FACTOR FOR STATUS CHANGE
C
   40 DS2=EAL*DVAX
      IF (DS2) 41,60,42
C
C     BUCKLING AND CONTINUING
C
   41 KODY1=1
      GO TO 30
C
C     BUCKLED BUT LOADING
C
   42 FAC=(PYNC-SEP)/DS2
      IF (FAC.GE.FACTOR) GO TO 43
      FACTOR=FAC
      SEP=PYNC
      KODY1=3
      GO TO 70
C
   43 SEP=SEP+FACTOR*DS2
      GO TO 60
C
C     ON SLOPE 9, TENSION YIELDING
C
   50 IF (DVAX.LT.0) GO TO 51
      DVP=FACTOR*DVAX
```

```
      XPP=XPP+DVP/DELTY
      VPACP=VPACP+DVP
      GO TO 110
C
C     YIELDED BUT UNLOADING
C
   51 KODY1=0
      X3RE=XPP-PFAC
      GO TO 20
C
   60 FACAC=FACAC+FACTOR
      IF (FACAC.LT.0.9999999) GO TO 10
      RETURN
C
   70 FACAC=FACAC+FACTOR
      IVRTX=1
C
   80 PHI=PYNC/PYN
C
C     MODIFY THE SLOPES OF 4 SEGMENTS ON M-P CURVE IN COMPRESSION
C
      DO 90 K=1,2
      DO 90 J=1,2
      DO 90 I=1,2
   90 A1(I,J,K)=A1(I,J,K)/PHI
C
      DO 100 K=1,2
      DO 100 J=1,2
  100 PMX(1,J,K)=PHI*PMX(1,J,K)
C
  110 RETURN
      END
```

```
      SUBROUTINE FST10B (/ST/,/KOD/)
C
C     FORM 2*2 FLEXURAL STIFFNESS
C
      COMMON /INFEL/ IMEM,KST,LM(6),KGEOM,FL,COSA,SINA,A(2,6),EK11,
     1               EK22,EK12,PSH,EAL,EK11H,EK22H,REST(169)
      DIMENSION ST(2,2),KOD(2)
C
      KYY=KOD(1)+2*KOD(2)+1
      GO TO (10,20,30,40), KYY
   10 ST(1,1)=EK11
      ST(2,2)=EK22
      ST(1,2)=EK12
      GO TO 60
   20 ST(1,1)=0.
      ST(2,2)=EK22H
      GO TO 50
   30 ST(1,1)=EK11H
      ST(2,2)=0.
      GO TO 50
   40 ST(1,1)=0.
      ST(2,2)=0.
   50 ST(1,2)=0.
   60 ST(2,1)=ST(1,2)
C
C
      RETURN
      END
```

```
      SUBROUTINE BM10
C
C     END MOMENT INCREMENT
C
      COMMON /INFEL/ IMEM,KST,LM(6),KGEOM,FL,COSA,SINA,A(2,5),EK11,
     1               EK22,EK12,ESH,EAL,EK11H,EK22H,      KODYX(2),
     2               KODY(2),REST(165)
      COMMON /WORK/ DVR(2),DPR(2),DBM(2),W(1994)
C
      KYY=KODY(1)+2*KODY(2)+1
      GO TO (10,20,30,40), KYY
   10 DBM(1)=EK11*DVR(1)+EK12*DVR(2)
      DBM(2)=EK12*DVR(1)+EK22*DVR(2)
      GO TO 50
   20 DBM(1)=0.
      DBM(2)=EK22H*DVR(2)
      GO TO 50
   30 DBM(1)=EK11H*DVR(1)
      DBM(2)=0.
      GO TO 50
   40 DBM(1)=0.
      DBM(2)=0.
C
C
   50 RETURN
      END
```

```
      SUBROUTINE YMOM10
C
C     YIELD MOMENTS FOR CURRENT AXIAL FORCE
C
      COMMON /INFEL/ IMEM,KST,LM(6),KGEOM,FL,COSA,SINA,A(2,6),EK11,
     1               EK22,EK12,PSH,EAL,EK11H,EK22H,      KODYX(2),
     2               KODY(2),EMTCT(2),SFTOT(2),FTOT(2),PRTOT(2),SENP(8),
     3               SENN(8),TENP(8),TENN(8),PRACP(2),PRACN(2),BMEP(2),
     4               SDACT(3),BMY(2,2),NODI,NODJ,KOUTDT,PR12,PR21,
     5               PMX(3,2,2),A1(4,2,2),A2(4,2,2),POUT(63)
C
      FACC=-1.
      DO 30 IEND=1,2
      FFT=FTOT(IEND)*FACC
      FACC=1.
      FAC=1.
      DO 30 J=1,2
      FAC=-FAC
      DO 10 I=1,3
      IF (FFT.LT.PMX(I,J,IEND)) GO TO 20
   10 CONTINUE
      I=4
   20 BBMY=A2(I,J,IEND)-A1(I,J,IEND)*FFT
      IF (FAC*BBMY.GT.0.) BBMY=0.
   30 BMY(IEND,J)=BBMY
      FAC=-BMY(1,1)
      BMY(1,1)=-BMY(1,2)
      BMY(1,2)=FAC
C
      RETURN
      END
```