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Diffraction of a Dipole Field by a Perfectly Conducting Half Plane[1]

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The various forms of solution for the problem of an arbitrarily oriented electric or magnetic dipole in the presence of a half plane are reviewed and presented from a unified point of view. Some new results are given and errors in the literature are corrected.

1. Introduction

The problem of the diffraction of a dipole field by a perfectly conducting half plane or wedge has attracted considerable attention and the literature dates back over half a century. One of the earliest investigations was that of Bromwich (1915) who sought to deduce the solution from a Hertz vector made up of the known solution for the corresponding scalar problem, but it was later realized that the resulting vector solution violates the edge conditions for almost all orientations of the dipole.

The correct solution for the particular case of an electric dipole with axis normal to the half plane was obtained in 1953. The procedure was to represent the source field as an angular spectrum of plane waves, and thereby synthesize the solution from the known (Sommerfeld) solution for the diffraction of a plane wave. It was found that the expressions for the field components are composed of terms which are derivatives of the scalar solutions for the diffraction of a point-source field by an acoustically hard or soft half plane, plus terms corresponding to a source-free solution of Maxwell's equations. Many of the later investigations resulted in this same type of representation of the solution, and the additive or source-free contribution essential for the correct edge behavior is now known for all orientations of dipoles, both electric and magnetic (Vandakurov, 1954; Woods, 1957; Williams, 1957; Jones, 1964).

In contrast with this mode of solution, most of the Russian literature has been directed at a representation of the Hertz vector in the form of a Sommerfeld contour integral. Several quite elegant results for the exact solutions appropriate to electric and magnetic dipoles have been obtained, as well as various uniform asymptotic developments of the solutions (Malyuzhinets and Tuzhilin, 1963; Tuzhilin, 1964). Nevertheless, some of the elementary expressions that are deducible from these do not seem to have appeared in the literature, and, in addition, the relationships between the results of the two contrasting types of representation have not been fully explored.

It is the purpose of this paper to unify these forms of representation of the solution, and, at the same time, to give some additional results which are either new or do not appear in the open literature. An attempt has been made throughout to correct the numerous, and sometimes elusive, errors that have crept into the various treatments of the problem. For brevity, much of the analysis will be omitted.

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2. Representations for the Hertz Potential

The half plane is assumed infinitely thin and perfectly conducting, and is defined in terms of the rectangular Cartesian coordinates (x, y, z) by the equation $y=0$, $x \geqslant 0$. It is also convenient to introduce the circular cylindrical coordinates (ρ, ϕ, z) in terms of which $x=\rho \cos \phi$, $y=\rho \sin \phi$, as well as the spherical polar coordinates (r, θ, ϕ) where $\rho=r \sin \theta$, $z=r \cos \theta$. In the first two systems the edge of the half plane is coincident with the z axis, and in the latter two, the upper and lower surfaces of the half plane are given by the equations $\phi=0$ and $\phi=2\pi$, respectively.

Given an arbitrarily oriented dipole situated at (ρ_0, ϕ_0, z_0) with free-space electric or magnetic Hertz potential[2]

$$\underline{\Pi}_0=\hat{c}\,\frac{e^{ikR}}{kR} \tag{1}$$

where

$$\hat{c}=\hat{x}\sin\Theta\cos\Phi+\hat{y}\sin\Theta\sin\Phi+\hat{z}\cos\Theta, \tag{2}$$

a contour integral representation of the total electric Hertz potential $\underline{\Pi}^e$ and of the total magnetic Hertz potential $\underline{\Pi}^m$ is (Malyuzhinets and Tuzhilin, 1963),

$$\underline{\Pi}^{e,\,m}=\frac{1}{8\pi i}\int_C \frac{e^{ikR(\alpha)}}{kR(\alpha)}\underline{\pi}^{e,\,m}(\alpha)\,d\alpha, \tag{3}$$

in which

$$\underline{\pi}^{e,\,m}(\alpha)=\hat{e}(\pi-\alpha-\phi+\phi_0-\Phi)\cot\frac{\pi-\alpha-\phi+\phi_0}{4}$$

$$\mp\hat{e}(\pi-\alpha-\phi-\phi_0+\Phi)\cot\frac{\pi-\alpha-\phi-\phi_0}{4},$$

$$\hat{e}(\alpha)=\hat{x}\sin\Theta\cos\alpha-\hat{y}\sin\Theta\sin\alpha+\hat{z}\cos\Theta,$$

and

$$R(\alpha)=\{\rho^2+\rho_0^2+2\rho\rho_0\cos\alpha+(z-z_0)^2\}^{1/2}.$$

The contour C is the Sommerfeld integral contour shown in figure 1, where the shading indicates those regions in which the kernel in (3) vanishes exponentially as $|\text{Im.}\ \alpha|\to\infty$ on the upper Riemann sheet. The kernel has branch points at $\alpha=(2n+1)\pi\pm i\lambda, n=0, \pm1, \pm2, \ldots$ with

$$\lambda=2\cosh^{-1}\frac{R_1}{2\sqrt{\rho\rho_0}},$$

where

$$R_1=R(0)$$

is a parameter associated with the edge diffraction. We also introduce the parameter

$$R'=R(\pi-\phi-\phi_0)$$

representing the distance from the image source, and note that in (1)

$$R=R(\pi-\phi+\phi_0),$$

$$\hat{c}=\hat{e}(-\Phi).$$

In the case of an electric dipole, this expression for $\underline{\Pi}^e$ can be reduced to

[2] M.K.S. units are employed and a time-factor $e^{-i\omega t}$ suppressed.

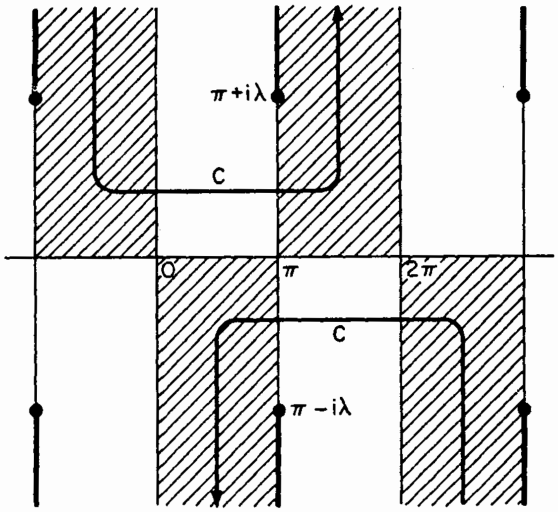

FIGURE 1. *The Sommerfeld contour.*

$$\underline{\Pi}^e=\hat{x}\left[\ell V^s+\frac{i}{k\sqrt{\rho\rho_0}}H_0^{(1)}(kR_1)\left(\ell\sin\frac{\phi_0}{2}-m\cos\frac{\phi_0}{2}\right)\sin\frac{\phi}{2}\right]$$
$$+\hat{y}\left[mV^h-\frac{i}{k\sqrt{\rho\rho_0}}H_0^{(1)}(kR_1)\left(\ell\sin\frac{\phi_0}{2}-m\cos\frac{\phi_0}{2}\right)\cos\frac{\phi}{2}\right]$$
$$+\hat{z}nV^s, \tag{4}$$

where $\ell=\sin\Theta\cos\Phi$, $m=\sin\Theta\sin\Phi$, and $n=\cos\Theta$ are the directional cosines of $\hat{c}$. The values V^s and V^h are, respectively, the acoustically soft and acoustically hard (scalar) point-source solutions whose contour integral representations are (Carslaw, 1899).

$$V^{s,h}=\frac{1}{8\pi i}\int_C\frac{e^{ikR(\alpha)}}{kR(\alpha)}v^{s,h}(\alpha)d\alpha \tag{5}$$

where
$$v^{s,h}(\alpha)=\cot\frac{\pi-\alpha-\phi+\phi_0}{4}\mp\cot\frac{\pi-\alpha-\phi-\phi_0}{4}$$

and the upper and lower signs refer to the soft (Dirchlet boundary condition at the surface of the half plane) and hard (Neumann boundary condition) solutions, respectively.

To derive the result given in (4), $\pi^e(\alpha)$ is expanded using the half-angle trigonometric relations, retaining only the terms of period 4π in α that are odd functions about $\alpha=\pi$. It can be shown that the neglected terms yield a vanishing contribution to the contour integral. Aside from such terms, therefore, $\underline{\pi}^e(\alpha)$ may be written as

$$\underline{\pi}^e(\alpha)=\hat{e}(-\Phi)\cot\frac{\pi-\alpha-\phi+\phi_0}{4}-\hat{e}(\Phi)\cot\frac{\pi-\alpha-\phi-\phi_0}{4}$$

$$-4\cos\frac{\alpha}{2}\left(\ell\sin\frac{\phi_0}{2}-m\cos\frac{\phi_0}{2}\right)\left(\hat{x}\sin\frac{\phi}{2}-\hat{y}\cos\frac{\phi}{2}\right),$$

and the desired result now follows from the identity (Senior, 1953)

$$\frac{i}{k\sqrt{\rho\rho_0}} H_0^{(1)}(kR_1) = -\frac{1}{2\pi i}\int_C \frac{e^{ikR(\alpha)}}{kR(\alpha)} \cos\frac{\alpha}{2}\, d\alpha$$

and from the definitions of V^s and V^h.

The solution (4) is remarkable in that the scalar solutions V^s and V^h are explicitly involved along with certain additive correction terms which obey the source-free wave equation. This is advantageous for numerical purposes since V^s and V^h can be expressed in the real integral form (Macdonald, 1915)

$$V^{s,h} = i\int_{-M}^{\infty} \frac{H_1^{(1)}(\mu^2+kR)}{\sqrt{\mu^2+2kR}}\, d\mu \mp i\int_{-M'}^{\infty} \frac{H_1^{(1)}(\mu^2+kR')}{\sqrt{\mu^2+2kR'}}\, d\mu \tag{6}$$

$$M = 2\sqrt{\frac{k\rho\rho_0}{R_1+R}}\cos\frac{1}{2}(\phi-\phi_0),\; M' = 2\sqrt{\frac{k\rho\rho_0}{R_1+R'}}\cos\frac{1}{2}(\phi+\phi_0).$$

The additive terms in (4) vanish if $\ell = m\cot\frac{\phi_0}{2}$, and the electromagnetic field is then determined by V^s and V^h alone. If, furthermore, $\ell = m = 0$, the field is determined by V^s only. In the case of other dipole orientations, however, the additive terms are necessary to provide the correct edge behavior. In general, the Hertz potential $\underline{\Pi}^e$ and the electromagnetic field derived from it are $O(\rho^{-1/2})$ as $\rho \to 0$. The additive terms for the Hertz potential are equivalent to those derived by Vandakurov (1954) and are analogous, but not equivalent, to those presented for the electromagnetic field quantities by Senior (1953) in the case of the vertical (y-oriented) dipole and by Woods (1957) and Jones (1964) in the case of the arbitrarily oriented dipole. In these last references, the electromagnetic field quantities are expressed as derivatives with respect to both source and observer coordinates, and the consequent additive correction terms are not immediately derivable from a Hertz potential.

A precisely similar analysis is applicable to the magnetic dipole, and leads to

$$\underline{\Pi}^m = \hat{x}\left[\ell V^h + \frac{i}{k\sqrt{\rho\rho_0}} H_0^{(1)}(kR_1)\left(\ell\cos\frac{\phi_0}{2} + m\sin\frac{\phi_0}{2}\right)\cos\frac{\phi}{2}\right]$$
$$+\hat{y}\left[mV^s + \frac{i}{k\sqrt{\rho\rho_0}} H_0^{(1)}(kR_1)\left(\ell\cos\frac{\phi_0}{2} + m\sin\frac{\phi_0}{2}\right)\sin\frac{\phi}{2}\right] + \hat{z}nV^h. \tag{7}$$

In this case the additive terms vanish if $\ell = -m\tan\frac{\phi_0}{2}$. The electromagnetic field is then determined by V^s and V^h alone, and if, furthermore, $\ell = m = 0$, only V^h appears. For other dipole orientations, however, the additive terms are necessary to provide the correct edge behavior.

For $k\rho\rho_0/R_1 \gg 1$ (source and observation point far from the edge), a convenient decomposition of the total Hertz potential is

$$\underline{\Pi}^{e,m} = \underline{\Pi}_g^{e,m} + \underline{\Pi}_d^{e,m} \tag{8}$$

where $\underline{\Pi}_g^{e,m}$ is the geometrical optics contribution given by

$$\underline{\Pi}_g^{e,m} = \eta(\pi+\phi_0-\phi)\,\hat{e}(-\Phi)\,\frac{e^{ikR}}{kR} \mp \eta(\pi-\phi_0-\phi)\,\hat{e}(\Phi)\,\frac{e^{ikR'}}{kR'} \tag{9}$$

with
$$\eta(\psi) = \begin{cases} 1 \text{ for } \psi > 0 \\ 0 \text{ for } \psi < 0 \end{cases},$$

and $\underline{\Pi}_d^{e,m}$ is the diffracted contribution. This last is discontinuous at $\phi=\pi\pm\phi_0$ in order to compensate for the discontinuities in $\underline{\Pi}_g^{e,m}$. Complete and uniform asymptotic expansions of $\underline{\Pi}_d^{e,m}$ have been provided by Tuzhilin (1964). If $kR_1 \gg 1$, a first-order approximation to $\underline{\Pi}_d^{e,m}$ is obtained by combining the results of Tuzhilin (1964) and Macdonald (1915), and is

$$\begin{aligned}\underline{\Pi}_d^{e,m} \sim \frac{e^{i(kR_1+\pi/4)}}{\sqrt{2\pi kR_1}}\,\frac{1}{2k\sqrt{\rho\rho_0}}&\left\{\frac{\hat{e}(-\Phi)-\hat{e}(\pi+\phi_0-\phi-\Phi)}{\cos\frac{\phi-\phi_0}{2}} \mp \frac{\hat{e}(\Phi)-\hat{e}(\pi-\phi_0-\phi+\Phi)}{\cos\frac{\phi+\phi_0}{2}}\right\}\\
&-\sqrt{\frac{2}{\pi kR_1}}e^{-i\pi/4}\left\{\operatorname{sgn}(\pi+\phi_0-\phi)\,\frac{\hat{e}(-\Phi)e^{ikR}}{\sqrt{k(R_1+R)}}F\left[\sqrt{k(R_1-R)}\right]\right.\\
&\left.\mp\operatorname{sgn}(\pi-\phi_0-\phi)\,\frac{\hat{e}(\Phi)e^{ikR'}}{\sqrt{k(R_1+R')}}\,F\left[\sqrt{k(R_1-R')}\right]\right\}\end{aligned} \tag{10}$$

where $\operatorname{sgn}(\psi)$ is the signum function, $\operatorname{sgn}(\psi)=\pm 1$ for $\psi \gtrless 0$, and $F(\tau)$ is the Fresnel integral

$$F(\tau) = \int_\tau^\infty e^{i\mu^2}d\mu.$$

If, in addition, $k(R_1-R)$, $k(R_1-R') \gg 1$, asymptotic expansion of the Fresnel integrals in (10) leads to the rather elegant expression

$$\underline{\Pi}_d^{e,m} \sim -\frac{e^{i\left(kR_1+\frac{\pi}{4}\right)}}{\sqrt{2\pi kR_1}}\,\frac{1}{2k\sqrt{\rho\rho_0}}\left\{\frac{\hat{e}(\pi+\phi_0-\phi-\Phi)}{\cos\frac{\phi-\phi_0}{2}} \mp \frac{\hat{e}(\pi-\phi_0-\phi+\Phi)}{\cos\frac{\phi+\phi_0}{2}}\right\} \tag{11}$$

Formulae for the total electric and magnetic fields can be obtained from (11) by application of the usual differential operators in Cartesian coördinates to the Hertz potential.

3. Eigenfunction Expansions

Solutions of the half-plane problem in cylindrical and spherical coordinates lead naturally to eigenfunction expansions. In the case of an electric dipole, the total electric field is represented as

$$\underline{E}^e = 4\pi k\underline{\underline{G}}^e(\underline{r}|\underline{r}_0)\cdot\hat{c}, \tag{12}$$

and in the case of a magnetic dipole, we write the total magnetic field as

$$\underline{H}^m = 4\pi k\underline{\underline{G}}^m(\underline{r}|\underline{r}_0)\cdot\hat{c}, \tag{13}$$

where $\underline{\underline{G}}^{e,m}(\underline{r}|\underline{r}_0)$ are the electric and magnetic dyadic Green's functions for the half plane. In cylindrical coordinates (Tai, 1954)

$$\frac{4\pi}{k}\underline{\underline{G}}^{e,m}(\underline{r}|\underline{r}_0)=\left\{\frac{\hat{\rho}}{\rho}\frac{\partial}{\partial\phi}-\hat{\phi}\frac{\partial}{\partial\rho}\right\}\left\{\frac{\hat{\rho}_0}{\rho_0}\frac{\partial}{\partial\phi_0}-\hat{\phi}_0\frac{\partial}{\partial\rho_0}\right\}U^{h,s}+\left\{\hat{\rho}\frac{\partial^2}{\partial\rho\partial z}+\frac{\hat{\phi}}{\rho}\frac{\partial^2}{\partial\phi\partial z}\right.$$

$$\left.+\hat{z}\left(\frac{\partial^2}{\partial z^2}+k^2\right)\right\}\left\{\hat{\rho}_0\frac{\partial^2}{\partial\rho_0\partial z_0}+\frac{\hat{\phi}_0}{\rho_0}\frac{\partial^2}{\partial\phi_0\partial z_0}+\hat{z}_0\left(\frac{\partial^2}{\partial z_0^2}+k^2\right)\right\}\frac{U^{s,h}}{k^2}, \tag{14}$$

where
$$U^{s,h}=\frac{1}{2}\sum_{n=0}^{\infty}\epsilon_n\left\{\cos\frac{n}{2}(\phi-\phi_0)\mp\cos\frac{n}{2}(\phi+\phi_0)\right\}T_{n/2}, \tag{15}$$

with
$$T_\tau=\frac{i}{2k}\int_{-\infty}^{\infty}\frac{dt}{k^2-t^2}e^{it(z-z_0)}J_\tau\left(\rho_<\sqrt{k^2-t^2}\right)H_\tau^{(1)}\left(\rho_>\sqrt{k^2-t^2}\right). \tag{16}$$

Since
$$\left(\frac{\partial^2}{\partial z_0^2}+k^2\right)U^{s,h}=V^{s,h},$$

the solution for a z-oriented dipole again follows from a scalar point-source solution.

In spherical coordinates, on the other hand (Tilston, 1952),

$$\frac{4\pi}{k}\underline{\underline{G}}^{e,m}(\underline{r}|\underline{r}_0)=\left\{\frac{\hat{\theta}}{\sin\theta}\frac{\partial}{\partial\phi}-\hat{\phi}\frac{\partial}{\partial\theta}\right\}\left\{\frac{\hat{\theta}_0}{\sin\theta_0}\frac{\partial}{\partial\phi_0}-\hat{\phi}_0\frac{\partial}{\partial\theta_0}\right\}U^{h,s}+\left\{\hat{r}\left(\frac{\partial^2}{\partial r^2}+k^2\right)\right.$$

$$\left.+\frac{\hat{\theta}}{r}\frac{\partial^2}{\partial r\partial\theta}+\frac{\hat{\phi}}{r\sin\theta}\frac{\partial^2}{\partial r\partial\phi}\right\}\left\{\hat{r}_0\left(\frac{\partial^2}{\partial r_0^2}+k^2\right)+\frac{\hat{\theta}_0}{r_0}\frac{\partial^2}{\partial r_0\partial\theta_0}+\frac{\hat{\phi}_0}{r_0\sin\theta_0}\frac{\partial^2}{\partial r_0\partial\phi_0}\right\}\frac{rr_0U^{s,h}}{k^2}, \tag{17}$$

where $U^{s,h}$ is as given in (15), but with T_τ (see (16)) replaced by

$$T_\tau=ie^{-2i\tau\pi}\sum_{s=0}^{\infty}\frac{\Gamma(s+2\tau+1)}{s!}\frac{2s+2\tau+1}{(s+\tau)(s+\tau+1)}j_{s+\tau}(kr_<)\,h_{s+\tau}^{(1)}(kr_>)P_{s+\tau}^{-\tau}(\cos\theta)P_{s+\tau}^{-\tau}(\cos\theta_0). \tag{18}$$

The spherical Bessel and associated Legendre functions are as defined in Stratton (1941).

Since
$$r_0\left(\frac{\partial^2}{\partial r_0^2}+k^2\right)(r_0U^{s,h})=V^{s,h},$$

the solution for a radial dipole (that is, $\hat{c}=\hat{r}_0$) now follows immediately from the point-source solution.

If $kr\ll 1$ and $kr_0\gg 1$, the representation in (17) is rapidly convergent, and for the electric dipole, the dominant term leads to (Felsen, 1957)

$$\underline{E}^e\sim\frac{e^{ikr_0+i\frac{\pi}{4}}}{r_0}\sqrt{\frac{2k}{\pi\rho\sin\theta_0}}\left\{\hat{\rho}\sin\frac{\phi}{2}+\hat{\phi}\cos\frac{\phi}{2}\right\}\left\{(\hat{\theta}_0\cdot\hat{c})\cos\theta_0\sin\frac{\phi_0}{2}+(\hat{\phi}_0\cdot\hat{c})\cos\frac{\phi_0}{2}\right\}$$

$$\underline{H}^e\sim\sqrt{\frac{\epsilon_0}{\mu_0}}\frac{e^{ikr_0+i\frac{\pi}{4}}}{r_0}\sqrt{\frac{2k}{\pi\rho\sin\theta_0}}\left\{\hat{\rho}\cos\frac{\phi}{2}-\hat{\phi}\sin\frac{\phi}{2}\right\}\left\{(\hat{\theta}_0\cdot\hat{c})\sin\frac{\phi_0}{2}+(\hat{\phi}_0\cdot\hat{c})\cos\theta_0\cos\frac{\phi_0}{2}\right\}, \tag{19}$$

whereas for the magnetic dipole,

$$H^m \sim \frac{e^{ikr_0+i\frac{\pi}{4}}}{r_0}\sqrt{\frac{2k}{\pi\rho \sin\theta_0}}\left\{\hat{\rho}\cos\frac{\phi}{2}-\hat{\phi}\sin\frac{\phi}{2}\right\}\left\{(\hat{\theta}_0\cdot\hat{c})\cos\theta_0\cos\frac{\phi_0}{2}-(\hat{\phi}_0\cdot\hat{c})\sin\frac{\phi_0}{2}\right\}$$

$$E^m \sim \sqrt{\frac{\mu_0}{\epsilon_0}}\frac{e^{ikr_0+i\frac{\pi}{4}}}{r_0}\sqrt{\frac{2k}{\pi\rho\sin\theta_0}}\left\{\hat{\rho}\sin\frac{\phi}{2}+\phi\cos\frac{\phi}{2}\right\}\left\{(\hat{\theta}_0\cdot\hat{c})\cos\frac{\phi_0}{2}-(\hat{\phi}_0\cdot\hat{c})\cos\theta_0\sin\frac{\phi_0}{2}\right\}, \tag{20}$$

with

$$\hat{\theta}_0\cdot\hat{c}=\cos\theta_0\sin\Theta\cos(\phi_0-\Phi)-\sin\theta_0\cos\Theta$$

$$\hat{\phi}_0\cdot\hat{c}=-\sin\Theta\sin(\phi_0-\Phi).$$

These equations make explicit the behavior of the electromagnetic field near to edge.

4. References

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The scalar point-source solutions on pp. 592 and 593 are in error by a factor k. This error carries over to the dipole solutions on p. 594. The "additional" terms in the expression for B are also in error by a factor 4π.

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In eq (8), replace -2Ψ by 2Ψ.

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Due to normalization errors in both the θ and ϕ integrations, a factor $2\pi/\phi_0$ is omitted and $\Gamma\left(\frac{2m\pi}{\phi_0}+2n+1\right)$ should be replaced by $\left(\frac{2m\pi}{\phi_0}+2n+1\right)$ throughout. Further, $mB\sin\frac{m\pi\phi}{\phi_0}$ should read $\frac{m\pi B}{\phi_0}\sin\frac{m\pi\phi}{\phi_0}$ whenever it appears (pp. 32, 33, 37). On the same pages replace C by $-C$ and on pp. 36, 37 replace A, B, C, by $-A, -B, -C$, respectively. In eq (3.48), multiply the right-hand side by -1, and in eq (3.59) divide the summand by $n!\left(\frac{m\pi}{\phi_0}+n\right)\left(\frac{m\pi}{\phi_0}+n+1\right)$.

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In the line following eq (8), replace ϕ by Φ. The integral in eq (10) should be multiplied by sgn $(\beta-4n\Phi)$. In eq (14) replace ϕ_n by $4n$, and in the line following, the first Π^1_m should read Π^2_m.

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The solutions given are correct if the scalar point sources are defined in terms of a Hertz potential $(kR)^{-1}e^{-ikR}$, not $R^{-1}e^{-ikR}$ as indicated.

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