PICTURING NUMBER IN THE CENTRAL MIDDLE AGES

by

Megan C. McNamee

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Doctoral Committee:

Professor Elizabeth L. Sears, Chair
Associate Professor Catherine Brown
Professor Megan L. Holmes
Assistant Director Elizabeth C. Teviotdale, Western Michigan University
Associate Professor Achim Timmermann

DEDICATION

for my parents

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LIST OF ABBREVIATIONS

LIBRARIES AND ARCHIVES

BAV	Vatican City, Bibliotheca Apostolica
BB	Bern, Burgerbibliothek
BL	London, British Library
BLB	Karlsruhe, Badische landesbibliothek
BM	France (general), Bibliothèque municipale
BN	Naples, Biblioteca Nazionale Vittorio Emanuele III
BNE	Madrid, Biblioteca nacional de Espãna
BnF	Paris, Bibliothèque nationale de France
BR	Leiden, Bibliotheek der Rijksuniversiteit
BRB	Brussels, Bibliothèque royale de Belgique
BSB	Munich, Bayerische Staatsbibliothek
CCCC	Cambridge, Corpus Christi College
DB	Cologne, Dombibliothek
FB	Gotha, Forschungsbibliothek
HA	Cologne, Historisches Archiv
HAB	Wolfenbüttel, Herzog August Bibliothek
HB	Bernkastel-Kues, Hospitalbibliothek
NkČ	Prague, Národní knihovna České republiky
NL	Chicago, Newberry Library
ÖNB	Vienna, Österreichische Nationalbibliothek
SB	Bamberg, Staatsbibliothek
StiftsB	Einsiedeln, Stiftsbibliothek
UB	Utrecht, Universiteitsbibliotheek
UBJ	Jena, Universitätsbibliothek
UL	Leiden, Universiteit Leiden
ULC	Cambridge, University Library
WAB	Erfurt, Wissenschaftliche Allgemeinbibliothek

SERIES

ABMA	Auctores Britannici Medii Aevii
CSEL	Corpus Scriptorum Ecclesiasticorum Latinorum
CCSL	Corpus Christianorum Series Latina
DOML	Dumbarton Oaks Medieval Library
EETS	Early English Text Society
MGH SS	Monumenta Germaniae Historica Scriptores

ABSTRACT

Numeracy was as highly valued as literacy in the schools of Latin-speaking Europe around the year 1000, and the skills inculcated by masters, engendering specific modes of seeing and imagining, had demonstrable impact on contemporary visual culture. The *trivium*—grammar, rhetoric, and dialectic—continued to be taught as the foundation of learning, but the *quadrivium,* the four disciplines of number—arithmetic, geometry, astronomy, and music—received new emphasis. Two of the era's greatest intellects, Gerbert of Aurillac (Pope Sylvester II; c.940–1003) and Abbo of Fleury (c.944–1004), gained renown for their mathematical prowess and charismatic teaching. They educated a generation of Europe's powerful elites—including Emperor Otto III—and a host of anonymous clerics, monks, and priests. In the closed economy of the central middle ages, these men were also the primary patrons, makers, and viewers of objects. Works of the time, like the Pericope Book of Henry II, reveal new qualities when examined through the lens of number. This project is located at the cathedral school of Reims and the monastery school of Saint-Benoît-sur-Loire (Fleury)—where Gerbert and Abbo were masters, epicenters of a pan-European network of exchange linking monastic, episcopal, and lay institutions. Numeric knowledge was drawn from late antique and early medieval tracts by such figures as Boethius, Calcidius, Macrobius, Martianus Capella, Cassiodorus, Isidore of Seville, and Bede. Manuscript copies of these works produced and used at Reims and Fleury c.1000 give evidence of active engagement with their content, visual as well as verbal. Diagrammatic images earlier devised to explicate numeric concepts were now adapted and artfully elaborated for classroom use. This is evident in important introductions to the quadrivial disciplines prepared by Abbo (*Explanatio in Calculo Victorii*), Abbo's student Byrhtferth of Ramsey (*Enchiridion*), and Gerbert (*Isagoge geometriae*). Accompanying images to these tracts are witness to contemporary notions of materiality, sight, and the limits of representation. Students of arithmetic became freshly attuned to placement and order. Computistic study developed an active, agile, and

"curious" eye, while the practice of geometry exercised the intellectual eye, sharpening it, according to Gerbert, "for contemplating spiritual things and truths."

CHAPTER 1
Introduction

In a letter to Otto III written around 997, Gerbert of Aurillac expressed his pleasure at the emperor's enthusiasm for numeric study:

> Unless you were not firmly convinced that the power of numbers (*vis numerorum*) contained both the origins of all things in itself and explained all from itself, you would not be hastening to a full and perfect knowledge of them [numbers] with such zeal.[1]

Gerbert (c.945–1003), future Pope Sylvester II, then Archbishop of Reims, wrote the above-quoted lines in response to an invitation from Otto III (980–1002) to join the imperial court and "explain to us [Otto] this book on arithmetic."[2] Fanning the young ruler's ardor for arithmetic was a recent gift from Gerbert—the book in need of explanation—a manuscript copy of Boethius's *De institutione arithmetica* (*On the Fundamentals of Arithmetic*). The *De arithmetica*, a late Latin translation and commentary on a Greek treatise written by Nicomachus of Gerasa (c.50–c.150), was the most comprehensive account of arithmetic then available.[3] Gerbert is thought to have presented Otto with a deluxe and storied copy of the treatise: Bamberg, SB MS Msc.Class.5, which had been made for Charles the Bald (823–77) at Tours around 845.

The magnificent manuscript evinces the respect, even reverence, accorded to Boethius's text throughout the medieval period.[4] It opens with a full-page, heavily pigmented

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[1] Gerbert, *Epistulae* 187; ed. Pierre Riché and Jean-Pierre Callu (Paris: Les Belles Lettres, 1993), 484; trans. Harriet Pratt Lattin in Gerbert, *The Letters of Gerbert with His Papal Privileges as Sylvester II* (New York: Columbia University Press, 1961), 296: "Nisi enim firmum teneretis ac fixum vim numerorum vel in se omnium rerum continere primordia vel ex sese profundere, non ad eorum plenam perfectamque noticiam tanto festinaretis studio." In a subsequent letter, Gerbert writes that he will send more mathematical problems, suggesting that their discussion of number was ongoing.

[2] Ibid. 186; ed. Riché and Callu, 482; trans. Lattin, 295: "nos arithmeticae librum edoceatis."

[3] On the *De arithmetica* see below.

[4] E. K. Rand dated the manuscript to c.832, basing his assessment on ruling technique and script. The association with Charles the Bald is inferred from the date of the manuscript's making and the dedicatory verses

illumination of the tract's author and original patron: Boethius and Symmachus in Roman military garb sit beside one another, their names written above them in rustic capitals with silver ink (fig. 1.1).[5] The composition pivots on the book at its center, which is touched with shimmering gold. This is a kind of image that was, in the ninth century, rarely (if ever) seen in secular works.[6] In the folios that follow, the chapter lists are framed by delicately elaborated arcades of a type generally associated with Eusebian canon tables in gospel books (fig. 1.2–3).[7] Such lavish treatment was not limited to the opening pages, but is seen throughout the manuscript in eighty-five mathematical *descriptiones* and *formulae*—what today we might call diagrams—that were an essential part of the tract (figs. 1.4–6).[8] With this splendid object before him, it is little wonder that the emperor was so zealously "hastening" to a better understanding of number.

The exchange between Gerbert and Otto III is somewhat at odds with standard conceptions of the state and status of numeric learning in Latin-speaking Europe during the central middle ages, roughly the mid-ninth through the eleventh centuries. Historians of all stripes have tended to treat this period as one that saw the eclipse of the hard sciences—a preconception that will be addressed further below. Yet here we have a monarch clamoring to be taught the intricacies of arithmetic.[9] Otto was not exceptional in this respect. Numeracy

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of a poem found on fols. 1v–2r, 62v–63r, 139r–139v. The poem, written in alternating gold, silver, and red script on a field of purple, refers to a monarch with the same name as his grandsire ("Invicto pollens nomine, Caesar, avi"). Charles the Bald, the grandson of Charlemagne, fits the criterion, but so does Otto III. Indeed, it has been suggested that Gerbert was the author of the poem, which then would have been added (on blank pages) on the occasion of the manuscript being presented to Otto III. Hartmut Hoffmann notes that there is no hard evidence that this manuscript was owned or known to the emperor. Edward Kennard Rand, *A Survey of the Manuscripts of Tours*, Vol. 1, Text (Cambridge: The Mediaeval Academy of America, 1929), 131–32; Hartmut Hoffmann, *Bamberger Handschriften des 10. und 11. Jahrhunderts* (Hannover: Hahn, 1995), 33; a thorough description of the manuscript can be found in Gude Suckale-Redlefsen, *Die Handschriften des 8. bis 11. Jahrhunderts der Staatsbibliothek Bamberg* (Wiesbaden: Harrassowitz, 2004), 30–39 and 198–212.

5 The two seem to be mislabeled. Boethius was significantly younger than Symmachus. The senator was both his adopted father and father-in-law. Hence, it is likely that the grey-bearded man on the right was intended to represent the elder statesman. The vector of the gift, moreover, is from left to right, as indicated by the pointing finger of the more virile man on the left, who appears to push the book into the hand of his patron.

6 A point made by Margaret Gibson, "Illustrating Boethius: Carolingian and Romanesque Manuscripts," in *Medieval Manuscripts of the Latin Classics: Production and Use*, ed. Claudine A. Chavannes-Mazel and Margaret M. Smith (London: The Red Gull Press, 1996), 119.

7 See, for example, London, BL Add. MS 10546 (the "Moutier-Grandval Bible"), fols. 408v–409r.

8 This is the terminology used by Boethius in the dedicatory letter to the *De arithmetica*, discussed below, 18.

9 On the applicability of Boethian arithmetic to the business of kings see Alison Peden, "Unity, Order, and Ottonian Kingship in the Thought of Abbo of Fleury," in *Belief and Culture in the Middle Ages: Studies Presented to Henry Mayr-Harting*, ed. Richard Gameson and Henrietta Leyser (Oxford: Oxford University Press, 2001), 158–68.

was on the rise. Some of the greatest minds of the era, Gerbert and Abbo of Fleury (c.944–1004) chief among them, gained renown for their numeric knowledge and authored new works on calculation, time-reckoning, geometry, music, and the astrolabe. Perhaps more important: these men were teachers, who made sustained, even passionate, efforts to spread numeric knowledge and secure its perpetuation. They were far more successful than is generally recognized. Journeying as masters throughout present-day France, England, Germany, and Italy, Gerbert and Abbo developed interpersonal networks that were extensive and enduring.[10] Together they educated a generation of Europe's rulers and abbots, bishops and administrators, as well as a host of anonymous clerics, monks, and priests, some of whom themselves became schoolmasters, extolling number's "power" to yet another generation. In the relatively closed economy of the central middle ages, these same men were also the primary patrons, makers, and viewers of objects.[11] That they were numerate is, I hold, vital to understanding those objects.

This dissertation investigates the nature of numeracy around the turn of the first millennium, the character of numeric training, and the quality of its impact on visual culture. I suggest that the study of number strongly shaped what Michael Baxandall called the "cognitive style" of viewers, "the interpreting skills ..., the categories, the model patterns and the habits of inference and analogy," and deeply informed contemporary modes of representation.[12] In *Painting and Experience in Fifteenth-Century Italy* Baxandall wrote, "The beholder must use on the painting such visual skills as he has ... and he is likely to use those skills that his society esteems highly. The painter responds to this; his public's visual capacity must be his medium."[13] It was Baxandall's belief that taught skills, rather than inborn abilities, were most valued by Renaissance society and readily applied to pictures.[14] His assertion holds, I suggest, for the central middle ages. At this time education changed. While the *trivium*, the arts of grammar, rhetoric, and dialectic, continued to be taught as the foundation of all learning, the *quadrivium,* the four disciplines of number—arithmetic, geometry, astronomy, and music—received new emphasis. In what follows I assume a deep-

[10] See chapter 2.
[11] See chapter 6.
[12] Michael Baxandall, *Painting & Experience in Fifteenth-Century Italy*: *A Primer in the Social History of Pictorial Style*, 2nd ed. (Oxford: Oxford University Press, 1988), 29–30.
[13] Ibid., 40.
[14] Ibid. 37–38.

seated relationship between education and aesthetics and try to show how numeracy fostered particular approaches to and expectations of images on the part of makers and viewers. That, theoretically, such a relationship ought to exist was stated explicitly by Augustine of Hippo (354–430) in his *De libero arbitrio* (*On Free Choice of the Will*).[15] Augustine argued that the same patterns of number found in nature might also be encountered in the works of men:

> Craftsmen, who fashion all bodily forms, have numbers in their craft which they apply to their works. They use their hands and tools in designing, until what is formed externally achieves its consummation when it conforms as much as possible to the inward light of numbers ... ask what moves the craftsman's hands. It will be number, for their movements are also full of number. Then inspect the beauty of a sculpted body. Its numbers are held in place.[16]

For Augustine the artisan's application of numeric rationale is both intentional and inevitable. Order and beauty are deeply intertwined; maker and object alike are numerate. Number is also the source of enjoyment for the viewer—a point made in passing in the passage above, but driven home elsewhere in the *De libero*: "whatever delights you in a body and entices you through your bodily senses is full of number."[17] Here, the frisson of aesthetic pleasure experienced in front of an object, be it man-made or of nature, is caused by recognition of its underlying order, its geometry or number. One learned to recognize or, perhaps better because of its scientific overtones, observe number through study of the quadrivium.

Nowhere was the quadrivium more vigorously pursued than at the cathedral school of Reims and the monastic school of Saint-Benoît-sur-Loire (more commonly known as

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[15] Not well known today, the *De libero* was owned by numerous institutions and actively copied in artistic centers including Reichenau and Regensburg in the central middle ages. About a dozen surviving copies have been dated to this early period. The work is listed in numerous 9th-, 10th-, and 11th-century catalogues. Reichenau had at least two copies. One of the oldest surviving copies was owned by the abbey of St. Emmeram, Regensburg, and it was this manuscript, or one very like it, that gave rise to several new copies around the year 1000. On the transmission of the *De libero* see Karla Pollmann and Willemien Otten, ed., *The Oxford Guide to the Historical Reception of Augustine* (Oxford: Oxford University Press, 2013), 1:329; William McAllen Green, in *Sancti Aurelii Augustini*: *Contra academicos, De beata vita, De ordine, De libero arbitrio*, CCSL 74 (Turnhout: Brepols, 1970), xiv; and Gustav Becker, *Catalogi Bibliothecarum Antiqui* (Bonn: Max Cohen and Sons, 1885), 5 (no. 6), 16 (no. 8).

[16] Augustine, *De libero arbitrio* 2.16.42; ed. Green, 165–67: "Et omnium quidem formarum corporearum artifices homines in arte habent numeros quibus coaptant opera sua, et tamdiu manus atque instrumenta in fabricando mouent, donec illud quod formatur foris ad eam quae intus est lucem numerorum relatum, quantum potest ... quaere deinde artificis ipsius membra quis moueat: numerus erit, nam mouentur etiam illa numerose ... inspice iam pulchritudinem formati corporis: numeri tenentur in loco."

[17] Ibid. 2.16.15; ed. Green, 265: "Quidquid te delectat in corpore et per corporeos inlicit sensus, videas esse numerosum."

Fleury), where Gerbert and Abbo were masters. I locate the present project at these centers of numeric study, which acted as epicenters of a pan-European network of exchange linking monastic, episcopal, and lay institutions; and ground it in the quadrivial manuscripts that were made, copied, and used by the members of these communities around the year 1000. At least 600 extant manuscripts have been attributed to the Fleury scriptorium; more than 60 of these contain quadrivial material.[18] The library remained largely intact until the sixteenth century, after which the collection of books was gradually dispersed, with large portions going to Bern, Rome, Paris, and Orléans.[19] A select group of Fleury manuscripts, which had been taken from the monastery by Pierre Daniel, an amateur philologist, was given to the governing body of Bern where they have remained in the Burgerbibliothek. Queen Christina of Sweden (1626–89), who had in her personal collection a significant number of items from Fleury, left these to the Vatican where they are still housed. Numerous Fleury manuscripts made their way to the Bibliothèque nationale de France. After the Revolution, those manuscripts still remaining at Fleury were transferred to Orléans, where they can be found in the Bibliothèque municipale. A small but significant group of manuscripts, associated with Abbo, who taught for a time at Ramsey in Cambridgeshire, are now in British collections.[20] Relatively few manuscripts, on the other hand, can be traced with certainty to Reims. Of these, many were produced in the ninth century under Archbishop Hincmar (845–882), twenty-two of which bear his *ex dono*.[21] Today, all but two of these manuscripts are housed in the Bibliothèque municipale de Reims.[22] Others can be tied to Gerbert, who became tutor to Emperor Otto III in 996 and is thought to have dispatched a number of manuscripts—the deluxe *De arithmetica* among them—for the edification of his royal charge. These are held as a group in the Staatsbibliothek Bamberg.[23] Gerbert's epistles are filled with frequent

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[18] For the number of extant Fleury manuscripts I take a more conservative estimate than that offered by Marco Mostert, *The Library of Fleury: A Provisional List of Manuscripts* (Hilversum: Verloren Publishers, 1989), 16.
[19] Mostert, *The Library of Fleury*, 29–33.
[20] On Abbo's stint at Ramsey see chapters 2 and 4.
[21] The best resource for the pre-1100 Reims manuscripts remains Frederick Carey, "The Scriptorium of Reims during the Archbishopric of Hincmar (845–882 A.D.)," in *Classical and Mediaeval Studies in Honor of Edward Kennard Rand Presented upon the Completion of his Fortieth Year of Teaching*, ed. Leslie Webber Jones (Freeport, NY: Books for Libraries Press, 1968), 41–60.
[22] Ibid., 49–56. The two manuscripts not in Reims can be found at the library of Pembroke College, Cambridge and in Paris at the Bibliothèque nationale de France.
[23] The theory that Otto III's library was either taken by or legally passed to Henry II, who then gifted a substantial part of it to Bamberg Cathedral, was first proposed by Ludwig Traube and Hans Fischer. A number of these manuscripts—all didactic works, many showing signs of having at least passed through the Reims

demands and requests for manuscripts on quadrivial topics. Scholars have identified many of the exemplars, most now scattered across Europe in municipal collections, from which Gerbert commissioned copies.

I have, in the course of research, examined in-person most of those manuscripts from Fleury and Reims containing quadrivial material. I have also surveyed the larger family of quadrivial manuscripts to which these belong in order to map shifts in the appearance of quadrivial images as they were copied throughout Europe from the eighth to the thirteenth centuries. A diachronic approach has allowed me to isolate alterations specific to the tenth and eleventh centuries and, in a few cases, to trace these to the circles of Abbo and Gerbert. Looking beyond these circles made me realize that numeracy was pervasive; quadrivial tracts, ubiquitous. The *De arithmetica*, for example, is preserved in eighty-eight copies dated to the ninth through the eleventh centuries.[24] The *Ars Donati*, the premier grammatical primer, in comparison, survives in fifty-nine manuscripts from the same period;[25] and Augustine's *De civitate Dei*–arguably his most popular work—in around a hundred complete and fragmentary copies.[26] Number may have been most intensely explored at Fleury and

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scriptorium—are thought to have been given by Gerbert to the Emperor. Hartmut Hoffmann has suggested that these manuscripts were, instead, part of Gerbert's personal library, and were eagerly seized after *his* death in 1003, possibly by Henry II, but more likely by an intermediary like Leo of Vercelli (c.965–1026). Hoffmann stresses that both hypotheses are merely conjectures and that little is known for certain of the trajectories taken by many of the books in the Bamberg collection. Traube, *Paläographische Forschungen* (Munich: Verlag der K. Akademie, 1904), 4: 6–14; Fischer, "Die kgl. Bibliothek in Bamberg und ihre Handschriften," *Zentralblatt für Bibliothekswesen* 24 (1907): 364–93**.** Numerous other scholars have supported and expanded on the Otto III hypothesis. See, especially, Florentine Mütherich, "The Library of Otto III"; and Rosamond McKitterick, "Ottonian Intellectual Culture in the Tenth Century and the Role of Theophanu," *Early Medieval Europe* 2, no. 1 (1993): 59–62. Hoffmann, *Bamberger Handschriften*, 5–34, esp. 26–29. Hoffmann's hypothesis about Gerbert's ownership is indebted in part to two articles by Marina Passalacqua: "Un Papa e tre codici (Silvestro II ed Erlangen, Universitätbibl., 380; Bamberg, Staatsbibl., Misc. Class. 25; Bamberg, Staatsbibl., Hist. 5)," *Scriptorium* 48 (1994): 147–51; and eadem, "Gerbert di Reims e il codice Erlangen, Universitätsbibliothek 380," in *Dicti studiosus: Scritti di filologia offerti a Scevola Mariotti dai suoi allievi* (Urbino: QuattroVenti, 1990), 322–27.

[24] Guillaumin, in Boèce, *Institution arithmétique*, lxv–lxxxiv. Study of the Boethian *corpus* is increasingly facilitated by the Warburg Institute's *Codices Boethiani* series, a catalogue of all the Latin works of Boethius. Unfortunately the volumes covering France and Germany are not yet complete.

[25] This number includes complete and partial copies of the *Ars minor* and *maior* comes from Louis Holtz's *Donat et la tradition de l'enseignement grammatical: Etude sur l'*Ars Donati *et sa diffusion (IV*[e]*–IX*[e] *siècle) et édition critique* (Paris: Centre national de la recherche scientifique, 1981), 352–423.

[26] This number includes excerpts. It is a tally of pre-10th c, items (58 in total) listed by Michael Gorman's 1982 article on the oldest manuscripts of the *De civitate Dei* and the 10th and 11th c. copies (42) included by A. Wilmart in his earlier list. When comparing numbers it should be remembered the tremendous effort made by scholars to recover and record all traces of Augustine's writing. It is difficult to believe that the same diligence has not been applied to the works of Boethius. Gorman, "A Survey of the Oldest Manuscripts of St. Augustine's

Reims, but numeracy was clearly cultivated elsewhere.[27] It can be said with some confidence that the entirety of the educated classes, lay and ecclesiastical alike, were numerate to a greater or lesser degree.

1. *VIS NUMERORUM*: THE POWER OF NUMBER IN THE CENTRAL MIDDLE AGES

When Gerbert wrote to Otto III that number "contained both the origins of all things in itself and explained all from itself," he was paraphrasing the *De arithmetica*:

> From the first birth of things, all things whatever (that) have been constructed seem to be formed according to numerical rationale. Number was the principal exemplar in the mind of the creator. From it was derived the multiplicity of the four elements, the turning of the seasons, the movement of the stars, and the cycle of the heavens.[28]

Boethius derived this number-based model of Creation from Plato's *Timaeus*, a brief dialogue on the genesis and structure of the cosmos. According to Plato all matter was made up of four elements: fire, air, water, and earth. All else, the whole of the material world, was deemed to be generated from various combinations of these elements. Numeric proportions governed their bonding: fire to air, air to water, water to earth. Each element was related to a number and a geometrical solid. Shape dictated matter's sensible qualities—color, temperature, texture, weight, smell, and sound. Hence fire, a pyramid, was hot because of the "fineness of its edges" and the "sharpness of its angles."[29] In this Platonic scheme, points, lines, planes, and solids were, so to speak, nature's joints, its sinew, skin, and body. Their articulation had been divinely wrought, "formed according to numeric rationale."

De civitate Dei," *Journal of Theological Studies* 33 (1982): 398–410; and Wilmart, "La tradition des grands ouvrages de Saint Augustin," *Miscellaenea Agostiniana* 2 (1931): 279–92.

[27] The cathedrals of Canterbury and Cologne, St. Michael's church in Hildesheim, and the monasteries of St. Colombanus, St. Gall, Saint-Germain—to name a few—had impressive collections of quadrivial manuscripts.

[28] Boethius, *De arithmetica* 1.2.1; ed. Jean-Yves Guillaumin (Paris: Les Belles Lettres, 1995), 11; my translation: "Omnia quaecumque a primaeua rerum natura constructa sunt, numerorum uidentur ratione formata. Hoc enim fuit principale in animo conditoris exemplar. Hinc enim quattuor elementorum multitudo mutuata est, hinc temporum uices, hinc motus astrorum caeli que conuersio."

[29] On the *subtilitas* of fire see Calcidius, *Commentary* 2.7–9; ed. Jan Hendrick Waszink (London: Warburg Institute, 1975), 72.

The Platonic, ultimately Pythagorean, notion of number as a divine idiom and the basis of materiality found its echo in the Old Testament.[30] God was said to have "ordered all things in measure, number, and weight" (Wisdom 11:21).[31] This passage from the book of Wisdom had been married to Platonic theory since at least the time of Philo of Alexandria (c.20 BCE–40 CE) and was firmly embedded in Christian exegesis by the Church Fathers.[32] Indeed, the chain of influence from pagan to Christian was extended and reversed in the central middle ages. The inversion is captured in a simple distich written in a ninth-century mathematical miscellany from Cologne:

> Adam was the first to discover number when he said to Eve his wife, "Behold bone of my bones, flesh of my flesh, and they will be two in one flesh." And after him Moses in the Hebrew language and Pythagoras in the Greek.[33]

Number, as discovered by Adam and described in the *De arithmetica*, was perceived to be constitutive, and had an atom-like quality. *Numerus* is among the thousands of words analyzed in Isidore of Seville's encyclopedic *Etymologiae sive origines* (*Etymologies or Origins*), composed in the seventh century. As the work's name suggests, etymology was Isidore's chief method of interpretation. He explained that the term "coin" (*nummus*) gave its name to "number" (*numerus*) because of the "pervasiveness" of both.[34] Number, like money, was utterly ubiquitous, the koine of the universe. It was commonly defined as a collection of unities—a definition that cast unity as something essential to but distinct from number.[35]

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[30] The *Timaeus* was the primary conduit of Pythagorean notions of number. Christiane Joost-Gaugier, *Measuring Heaven: Pythagoras and His Influence on Thought and Art in Antiquity and the Middle Ages* (Ithaca: Cornell University Press, 2006), 118.

[31] "Sed omnia in mensura et numero et pondere disposuisti."

[32] Russell Peck observed that Augustine alone quoted this passage more than twenty times. Peck, "Number as Cosmic Language," 60.

[33] Cologne, DB MS 83(II), fol. 16r: "Adam primus invenit numerum ubi dixit de Eva coniuge sua, ecce os de ossibus meis et caro de carne mea, et erunt duo in carne una. Et post eum moyses in ebrea lingua, et phitagoras in graeca." An early 11th-century *accessus* on Plato discussed by Anna Somfai wove together accounts of the philosopher's knowledge of the Old Testament found in Ambrose and Jerome. The Church Fathers agreed that Plato had journeyed to Egypt to study the laws of Moses and the words of the prophets, thus establishing a direct link between Plato's conception of the cosmos and the Old Testament. Augustine reiterated Ambrose's account in *De civitate dei* (8.11) and *De doctrina christiana (2.107–108).* Henry Mayr-Harting, *Church and Cosmos in Early Ottonian Germany: The View from Cologne* (Oxford: Oxford University Press, 2007), 178; Anna Somfai, "The Eleventh-Century Shift in the Reception of Plato's 'Timaeus' and Calcidius's 'Commentary'," *Journal of the Warburg and Courtauld Institutes* 65 (2002): 11–12.

[34] Isidore, *Etymologiae* 3.3.1; ed. Gasparotto and Guillaumin (Paris: Les Belles Lettres, 2009), 8; my translation unless otherwise noted.

[35] Ibid.: "Numerus autem est multitudo ex unitatibus constituta." Boethius, *De arithmetica* 1.3.2; ed. Guillaumin, 12: "Numerus est unitatum collectio, uel quantitatis aceruus ex unitatibus profusus."

Boethius called unity the "primary unit of all numbers" and the "generator" of multitude.[36] The fifth-century writer and synthesizer Macrobius described unity, the monad, in no uncertain terms in his commentary on the sixth book of Cicero's *De re publica* (*On the Commonwealth*) known as the *Somnium Scipionis* (*Dream of Scipio*)—a staple of quadrivial study in the central middle ages. Macrobius declared the monad to be: "the supreme god ... itself not numbered, it nevertheless produces from itself, and contains within itself innumerable patterns of created things."[37] God was not only numerate, he was "one."

The elision of unity and deity was the ultimate justification for study of mathematics in a Christian culture and it largely determined the nature and aims of numeric study. Scholars explored and marveled at the properties of numbers, and if these have been more thoroughly studied and expanded today than they had been by the year 1000, the phenomenon under investigation, of course, remains unchanged.[38] Two odd numbers when added yield an even sum, three points make a plane, etc. Much different, however, was the significance then assigned to numeric order and, with this, the motivations that fueled its pursuit. The power of numbers to reveal the ways and workings of the universe, what has been called their "unreasonable effectiveness" in the natural sciences, continues to amaze and to inspire zeal, but not (for most) because they are a divine idiom.[39] Whereas modern mathematicians might offer the tongue-in-cheek explanation "because God is a mathematician," for Gerbert and the young Otto III there was nothing coy about such a sentiment. The whole of the universe, from the four elements to the farthest stars, was thought to be devised and ordered by numeric rationale. Mathematical understanding illuminated mysteries of the Christian religion. The apparent paradoxes of a triune God and the dual nature of Christ were resolved through arithmetic example and geometric argument. Hence, quadrivial practice was both a prayerful and practical activity; quadrivial

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[36] Boethius, *De arithmetica* 1.7.2; ed. Guillaumin, 15; trans. Michael Masi, *Boethian Number Theory: A Translation of the De Institutione Arithmetica* (New York: Radopi, 1983), 79: "Hoc idem in omnibus numeris evenit, usquedum ad unitatis terminum perveniri queat."

[37] Macrobius, *Commentarii in Somnium Scipionis* 1.6.7; ed. James Willis (Leipzig: B. G. Teubner, 1963), 19; trans. William H. Stahl in *Commentary on the* Dream of Scipio *by Macrobius* (New York: Columbia University Press, 1990), 100: "ipse non numerus sed fons et origo numerorum"; and Ibid. 1.6.8, ed. Willis, 19; trans. Stahl, 101: "cum que utpote una non sit ipsa numerabilis, innumeras tamen generum species et de se creat et intra se continet."

[38] That said, entire categories of numbers (e.g., imaginary, irrational) did not exist in the central middle ages.

[39] See Eugene Wigner's influential article, "The Unreasonable Effectiveness of Mathematics in the Natural Sciences," *Communications in Pure and Applied Mathematics*, 13, no. 1 (1960): 1–14.

manuscripts, and the graphic elements therein, served as sites of contemplation as well as teaching tools; and mastery of the quadrivium was a sign not only of intelligence, but of grace.

2. *Ad plenam perfectamque noticiam*: Numeracy and the quadrivium c.1000

A "full and perfect knowledge of number" was achieved through study of the quadrivium. Translated literally, *quadrivium* means "crossroads" or "the place where four ways meet." Boethius in the *De arithmetica* was the first to apply the term to the four numeric disciplines:[40]

> It is the *quadrivium* by which we bring a superior mind from knowledge offered by the senses to the more certain things of the intellect. There are certain steps and fixed measures of development by which the mind is able to ascend and make progress so that by means of the eye of the mind ... truth can be investigated and beheld.[41]

Boethius's "certain steps" and "fixed measures of development" suggest a particular order, a curriculum, by means of which the mind was developed. During the central middle ages that curriculum comprised the seven liberal arts: grammar, rhetoric, dialectic, arithmetic, geometry, music, and astronomy.[42] The tenth-century scholar, Remigius of Auxerre, who briefly taught at Reims, explained that knowledge of the liberal arts had been erased by original sin.[43] Hence, learning the arts was more a process of recovery and remembrance than of acquisition. What was recovered were habits of mind specific to each subject. As will be

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[40] Only later, by Alcuin (735–804), would the term *trivium* be applied to the arts of the letter. Jean-Yves Guillamin, "Boethius's *De institutione arithmetica* and its Influence on Posterity," in *A Companion to Boethius in the Middle Ages*, ed. Noel Kaylor and Philip Edward Phillips (Leiden: Brill, 2012), 136–37.
[41] Boethius, *De arithmetica* 1.1.7, ed. Guillaumin, 8; trans. Masi, 73: "Hoc igitur illud quadriuium est, quo his uiandum sit, quibus excellentior animus a nobis cum procreatis sensibus ad intellegentiae certiora perducitur. Sunt enim quidam gradus certae que progressionum dimensiones, quibus ascendi progredi que possit, ut animi illum oculum ... quod eo solo lumine uestigari uel inspici ueritas queat, hunc inquam oculum demersum orbatum que corporeis sensibus hae disciplinae rursus illuminent."
[42] There was no doubt of the number and names of the liberal arts in the central middle ages. Quadrivial authorities, Boethius as well as Martianus, Cassiodorus, and Isidore, all listed them unequivocally as if they were an ancient construct (which they were not). For a recent rethinking of the historical emergence of the canonical seven disciplines, see Danuta Shanzer, "Augustine's Disciplines: *Silent diutius Musae Varronis*?" in *Augustine and the Disciplines: From Cassiciacum to Confessions*, ed. Karla Pollmann and Mark Vessey (Oxford: Oxford University Press, 2005), 69–112.
[43] The idea that the liberal arts were inborn and lost in The Fall was derived from Eriugena. Cora E. Lutz, "Remigius' Ideas on the Origin of the Seven Liberal Arts," *Medievalia et Humanistica* 10 (1956): 32–49; and Mayr-Harting, *Church and Cosmos*, 167.

suggested here and become clearer in the following chapters, each of number's disciplines (the arts of word as well) demanded as well as engendered its own rationale, what historian Gillian Evans called "thinking schemes."[44] Evans's phrase carries a sense of the topographic, of an articulated cognition with a particular form or shape evinced and recoverable—to a certain extent—in the verbal and visual mechanisms by means of which the disciplines were taught.[45]

The quadrivial disciplines were simultaneously distinct and deeply integrated. Boethius defined each against the others: arithmetic was the study of pure number, music that of interrelationships. Geometry was the investigation of static forms, astronomy of forms in motion.[46] These complementary relationships were given visual expression in Bamberg, Msc.Class.5, the copy of the *De arithmetica* discussed above, where they were pictured together as four women (fig. 1.7). The illuminator was clearly familiar with Martianus Capella's allegory of the liberal arts, the *De nuptiis Philologiae et Mercurii* (*On the Marriage of Philology and Mercury*). Written in the fifth century, the treatise enjoyed enormous popularity throughout the middle ages.[47] Martianus personified the arts, endowing each with a comely form and unforgettable attributes. In Msc.Class.5, the four quadrivial arts, carefully labeled, are distributed on a ground line. *Geometria*, leaning over her gold-framed abacus, stands facing *Arithmetica*, who is stationed a little forward of the rest. Here, as in the Boethian definitions, arithmetic and geometry are given precedence.[48] Likewise, in the *De nuptiis*, Martianus introduced them first and as a pair. The artist has depicted the dependence of music on arithmetic and astronomy on geometry through placement and posture. *Musica* stands beside *Arithmetica*, and, though each engages in her own discipline-specific activity, the position of their hands is identical. Likewise, *Astronomia* appears next to *Geometria*, their bodies so close that they appear to be joined. In this case their bond is expressed through chiastic opposition: *Astronomia* casts her eyes upward to the heavens which she

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[44] Gillian Evans, "The Influence of Quadrivium Studies in the Eleventh- and Twelfth-Century Schools," *Journal of Medieval History* 1, no. 2 (1975): 151–64.

[45] Roughly equivalent to Erwin Panofsky's "mental habits" and Michael Baxandall's "conceptual structures." Panofsky, *Gothic Architecture and Scholasticism* (New York: Meridian Books, 1957); Baxandall, *Patterns of Intention: On the Historical Explanation of Pictures* (New Haven: Yale University Press, 1985), 106.

[46] Boethius, *De arithmetica* 1.1.4.

[47] For the medieval reception and circulation of the *De nuptiis*, see below.

[48] Boethius called arithmetic the "mother" (*mater*) of the rest of the numeric disciplines and argued at length for its being "prior" (*prior*) to the rest of the quadrivium (*De arithmetica* 1.1.8.)

studies while *Geometria* gazes down at the abacus and, beyond this, toward the earth which she measures.

Rarely, in current historical scholarship, do we find the disciplines gathered in this way. Study of the quadrivium has not so much been neglected as allowed to become deeply fragmented. Classicists, medievalists, and historians of science and mathematics have tended to focus their efforts on individual texts and/or disciplines.[49] The rift between studies of the trivium and quadrivium is even more pronounced—a schism that no doubt reflects the present divide between the sciences and the humanities in the academy today. Yet, in the cathedrals and monasteries of the tenth and eleventh centuries interdisciplinarity was the norm, not the exception. Otto III called Gerbert a *philosopher*, "lover of wisdom" (*philosophus*), an accolade reserved for those who mastered all seven of the liberal arts.[50] Wisdom was not achieved through specialization, but proven by one's facility in every discipline. Comprehension of the medieval course of study, the mental practices it engendered, and its textual and pictorial tools requires a similar outlook. The fruitfulness of such an approach has been demonstrated by, among others, Gillian Evans, Alison Peden, Anna Somfai, and Anna Grotans, who, following the precepts of those whom they study, have looked across disciplinary bounds.[51] Though the two "prior" disciplines of arithmetic and geometry will be the focus here, I necessarily begin with a survey of the works forming the quadrivial canon—the core set of texts used to study the four disciplines of number.[52]

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[49] Numerous examples of each of these approaches are provided in the survey of the quadrivial canon that follows.

[50] Otto addressed Gerbert as, "Girberto dominorum peritissimo atque tribus philosophiae partibus laureato." Gerbert, *Epistulae* 186; ed. Riché and Callu, 480.

[51] Especially, Gillian R. Evans, "Introductions to Boethius's 'Arithmetica' of the Tenth to the Fourteenth Century," *History of Science* 16 (1978): 22–41; Evans and Alison Peden. "Natural Science and the Liberal Arts in Abbo of Fleury's *Commentary* on the Calculus *of Victorius of Aquitaine*," *Viator* 16 (1985): 109–27; Anna Somfai, "Calcidius' *Commentary* on Plato's *Timaeus* and Its Place in the Commentary Tradition: The Concept of *Analogia* in Text and Diagrams," in *Philosophy, Science and Exegesis in Greek, Arabic and Latin Commentaries*, ed. Peter Adamson, Hans Baltussen, and M. W. F. Stone (London: Institute of Classical Studies, School of Advanced Study, University of London, 2004), 203–20; and Anna Grotans, *Reading in Medieval St. Gall* (Cambridge: Cambridge University Press, 2006).

[52] A truly comprehensive investigation of the ways that number was pictured across the quadrivium in the central middle ages is beyond the scope of this dissertation. Such a study would best be accomplished by a team of scholars. The effectiveness of such an approach is seen in the online and print publications overseen by Mariken Teeuwen on the commentaries on Martianus's *De nuptiis*: Mariken Teeuwen and Sinéad O'Sullivan, ed., *Carolingian Scholarship and Martianus Capella: Ninth-Century Commentary Traditions on* De nuptiis *in Context* (Turnhout: Brepols, 2011); and Mariken Teeuwen et al., ed., *Carolingian Scholarship and Martianus Capella: The Oldest Commentary Tradition*. Digital edition, 1st ed. November 2008, online at http://martianus.huygens.knaw.nl/, accessed May 6, 2015.

The quadrivium crowned learning. Boethius wrote: "It is clearly obvious that hardly anyone has been able to reach the highest perfection of the disciplines of philosophy unless the nobility of such wisdom was investigated by him in a certain four-part study, the quadrivium."[53] This does not, however, mean that it was the exclusive remit of the elite. Though Gerbert and Abbo are the protagonists of this story I tell, what is at issue is less their unusually high level of numeric knowledge than how they chose to transmit this knowledge in the schoolroom and on the page. Nor, despite my focus on two men and the male communities they guided, was the study of number a gendered enterprise. Women also strove for mastery of all seven of the liberal arts, trivium *and* quadrivium. The prolific Hrotsvit (c.935–c.1002) of Gandersheim in lower Saxony, for example, rehearsed the basic properties of number with fluency and verve in a monologue spoken by personified wisdom, *Sapientia*, to the attentive emperor Hadrian.[54] The content and structure of this and other dramatic works reveal her knowledge of arithmetic and music as well as an urge to trasmit such learning to a larger audience, theatrically.[55] There were in the central middle ages as many levels and kinds of numeracy as there were numerate individuals. Used here, the term "numeracy," the "ability with or knowledge of numbers," encompasses a broad spectrum of skills and a capacious body of knowledge. I use the term expansively to refer to any and all activities associated with the quadrivium from the mechanics of finger calculation to esoteric harmonics. To be sure, certain subjects, especially music, were held to be extraordinarily difficult, and their mastery inspired considerable respect. But *some* knowledge of each field was achieved by many, and even rudimentary understanding, I hold, affected habits of thought, the "cognitive style" of the era and, with this, ways of representing the world, divine and human.

Art historians in other subfields have looked to scientific paradigms to help explain representational trends, notably the use of perspective in the Renaissance, the desire for verisimilitude in seventeenth-century Dutch painting, or the rise of abstraction in the modern era. Such studies are grounded on scientific *terra firma*: undisputed periods of scientific

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[53] Boethius, *De arithmetica* 1.1.1; ed. Guillaumin, 6; trans. Masi, 71: "Inter omnes priscae auctoritatis uiros, qui Pythagora duce puriore mentis ratione uiguerunt, constare manifestum est haud quemquam in philosophiae disciplinis ad cumulum perfectionis euadere, nisi cui talis prudentiae nobilitas quodam quasi quadruuio uestigatur."
[54] Hrosvit, *Sapientia* 3.9–22, in *Opera omnia*, ed. Walter Bershin (Leipzig: K. G. Saur, 2001), 249–52. See also Katharina M. Wilson, "Hrotsvit of Gandersheim, *Rara avis in Saxonia*?: A Collection of Essays," ed. eadem (Ann Arbor: MARC Publishing Co., 1987), 99–112.
[55] Wilson, "Mathematical Learning," 99–112.

activity (e.g., the Scientific Revolution, the Enlightenment, the Industrial Revolution).[56] Historian of science Sven Dupré has critiqued this tendency observing that:

> Implicit in all efforts to show intersections between art and science in a particular period is the (anachronistic) assumption of a rift. That is to say, if an intimate relationship between these two modes of human exploration was assumed, such evidence would not be necessary.[57]

With Dupré, I hold that overlap and interpenetration of these pursuits existed in all eras, but his statement is premised on a universal and comprehensive history of science that does not yet exist. Certain places and periods have been and continue to be overlooked, among them Latin-speaking Europe in the central middle ages.

In his seminal work, *The Renaissance of the Twelfth Century*, medievalist Charles Homer Haskins wrote off the tenth and eleventh centuries as an era without science, pointing to the period between 1100 and 1200 as the *first* moment of "intellectual revival" since the Carolingian *renovatio* of the ninth century.[58] Haskins saw the history of European science as "twofold," concerned on the one hand with "the recovery and assimilation of the science of antiquity" and, on the other, with the "advance of knowledge by the processes of observation and experiment in western Europe."[59] The central middle ages met, in his eyes, neither condition. At this time the Latin-speaking world did not have access to the full legacy of Greek philosophical and mathematical texts. The story of Euclid's *Elements* is characteristic of the fate of many ancient works in the medieval period as well as the way that this fate has been interpreted. From the time of its writing in the third century BCE, the *Elements* was,

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[56] Examples include: Martin Kemp, *Seen | Unseen: Art, Science, and Intuition from Leonardo to the Hubble Telescope* (Oxford: Oxford University Press, 2006); idem, *The Science of Art: Optical Themes in Western Art from Brunelleschi to Seurat* (New Haven: Yale University Press, 1990); Pamela Smith, *The Body of the Artisan: Art and Experience in the Scientific Revolution* (Chicago: University of Chicago Press, 2006); Brian S. Baigrie, *Picturing Knowledge: Historical and Philosophical Problems Concerning the Use of Art in Science* (Toronto: University of Toronto Press, 1996); Svetlana Alpers, *The Art of Describing: Dutch Art in the Seventeenth Century* (London: Penguin Books, 1989); and Peter Galison, "Aufbau/Bauhaus: Logical Positivism and Architectural Modernism," *Critical Inquiry* 16, no. 4 (1990): 709–52.

[57] Sven Dupré, "The Historiography of Perspective and *Reflexy-Const* in Netherlandish Art," *Nederlands Kunsthistorisch Jaarboek* 61 (2011): 35.

[58] Haskins lifted up the twelfth century by putting down the central middle ages. Such tactics are, perhaps, inevitable in the construction of Renaissances. His chapter on science is filled with statements along the lines of "That the science of Vincent's [of Beauvais] time was superior in quality as well as quantity to that current before 1100 is clearly evident ..." He is kinder in his first chapter, "The Historical Background," where he notes the dearth of sources from the tenth and eleventh century, which impedes understanding of the period. Haskins, *The Renaissance of the Twelfth Century* (Cambridge, MA: Harvard University Press, 1927), 309.

[59] Haskins, *Studies in the History of Mediaeval Science* (Cambridge: Harvard University Press, 1924), vii.

more often than not, *the* textbook on geometry. Euclid synthesized all prior discussions of geometry's elements or principles; even more enduring was his axiomatic method, which became the paradigm for virtually all scientific and philosophical argument. As a result, the title of the work and name of its author were synonymous with geometry practically from the time of its writing almost to the present day.[60] The collapsing of author, text, and discipline has meant that the history of geometry has been told largely *vis-à-vis* the transmission and fate of the *Elements.* Hence, at times and in places where this work was lost to view or existed only in garbled, fragmentary form, so too—according to the standard narrative—was geometry extinguished. Such was the case during the central middle ages in Latin-speaking Europe. Only fragments of the *Elements* were available. It was not until the twelfth century with the translation of Byzantine and Arabic copies that Euclid's text resurfaced in its entirety.

The translation movement of the twelfth century, which brought the complete *Elements* to light as well as nearly all the works of Aristotle, Plato, and Archimedes, has generally defined the limits of medieval science and mathematics.[61] That this boundary largely remains intact is clear from what is currently the standard textbook on the history of science, David Lindberg's *Beginnings of Western Science: The European Scientific Tradition in Philosophical, Religious, and Institutional Context, Prehistory to A.D. 1450*. Lindberg devotes the greater part of his study to science of the Greeks (four chapters) and the middle ages of the twelfth century and after (four chapters). The Carolingian era and central middle ages are covered in a scant eight pages. Even as historians have moved away from a text-based approach to history (of science or otherwise) and have called into question the idea that European intellectual culture was principally bound up with the recovery and transformation of the Classical past, the translation movement has remained the dominant discourse in the history of science in the middle ages; this is probably because it is at this time that medieval science and mathematics become more recognizable—with respect to their sources and spirit

[60] Describing a spider's precision and artistry, Aelian, in *On the characteristics of animals*, written in the 2nd c. BCE, wrote that the creature constructed shapes with its threads that "lacked nothing of Euclid." In this statement the author of the *Elements* stands as synonym for geometry, and has remained so almost to the present day. A summary of the key role of the *Elements* in the history of geometry see J. L. Heilbron, *Geometry Civilized: History, Culture, and Technique* (Oxford: Clarendon Press, 1998), 4–22.
[61] Lindberg, *Beginnings of Western Science*, 2nd edn. (Chicago: University of Chicago Press, 2007).

of inquiry—and thus somewhat more relevant to modern audiences.[62] Yet, as we shall see, geometry was actively practiced in the central middle ages; it just was not especially Euclidian.[63] Likewise arithmetic was undertaken without Arabic numerals or algorithms, and astronomy pursued without recourse to Ptolemy's *Almagest.*

Haskins's model has proven remarkably durable, helped in no small part by a perceived lack of evidence—textual and visual—to demonstrate the contrary. C. Stephen Jaeger, in *The Envy of Angels*, compellingly accounted for the dearth of textual evidence of intellectual activity, arguing that the arts of letter were largely conveyed "charismatically" through the body of the schoolmaster (e.g., gestures, expressions, movement, voice) and that the instruction was thus ephemeral.[64] Jaeger limited his investigation to the tenth- and eleventh-century cathedral schools and focused on the arts of the trivium.[65] I suggest that the charismatic model of instruction extended to all seven of the liberal arts and to monastery as well as cathedral schools.[66]

Jaeger also described the preference among tenth- and eleventh-century authors for Latin poetry that was, in his words, highly "complex" and "intentionally obscure," qualities that have largely deterred its investigation by today's scholars.[67] Indeed, the perception that documentary sources for this period are scarce is due, I suggest, to the kind of composition that was favored in the era. Didactic writing generally took the form of introductions to and commentaries on canonical works, rather than new independent treatises. Little of this

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[62] To be sure, our understanding of the complex mechanisms as well as the significant and widespread effects of this fascinating exchange is far from complete; shifts toward interdisciplinary and transcultural approaches along with recent interest in the movement of people, objects, and ideas across the Mediterranean have, moreover, reinvigorated its study, offering fresh insights. The idea that contemporary interest in a period is directly proportional to its relevance to the present (with respect to the middle ages) has recently been addressed by Walter Kudrycz, *Historical Present: Medievalism and Modernity* (London: Continuum, 2011), esp. 1–10.
[63] Gillian Evans has called it "sub-Euclidean." Even more surprising and compelling is the ongoing production (and, presumably, use) of works like the *Geometry I* and *II* from the 9th c. *through* the 14th, when printed editions of these compendia were made alongside the complete *Elements*. Gillian Evans, "The 'Sub-Euclidean' Geometry of the Earlier Middle Ages up to the Mid-Twelfth Century," *Archive for the History of Exact Sciences* 16 (1976): 105–18.
[64] C. Stephen Jaeger, *The Envy of Angels: Cathedral Schools and Social Ideas in Medieval Europe, 950–1200* (Philadelphia: University of Pennsylvania Press, 1994), 4.
[65] Jaeger addresses music and *physica* or cosmology briefly in his study, but they figure little in his argument. Ibid., 164–79.
[66] Scholars now largely agree that differences between monastic and cathedral schools have been overstated. See, among others, Grotans, *Reading*, 61; Mayke De Jong, *In Samuel's Image: Child Oblation in the Early Medieval West* (New York: E. J. Brill, 1996), 233 and 237, n. 29; and R. W. Southern in the *Making of the Middle Ages* (New Haven: Yale University Press, 1963, reprinted 1992), 159.
[67] Jaeger, *The Envy of Angels*, 4.

material has made it into edited editions; even less has been translated. Works of this sort written by Abbo; Abbo's student, Byrhtferth; and Gerbert, are the core of this study. In plumbing this material I follow those authors whose efforts have helped us better understand the rigorous intellectual activities of this obscure and fascinating period: Jaeger, Pierre Riché, John Contreni, Gillian Evans, Alison Peden, and Immo Warntjes among others.

Historians of art, no doubt influenced by the dominant narrative of the history of science sketched above which omits the central middle ages, have not looked for scientific explanations or practices in order to explain the preference in the period for abstract representation. Further discouraging this line of questioning are present notions of how scientific thinking might manifest itself visually in the past. The chief characteristics of the Year 1000 styles—the flattening of forms and space, a taste for unmodeled and patterned surfaces, asystematic proportion, and the intermingling of textual and graphic elements—seem inimical to rational apperception. Indeed, for Erwin Panofsky, these qualities epitomized the ascientific. In *Perspective as Symbolic Form*, which has been called the *locus classicus* for study of the intersection of art and science,[68] he cast the middle ages as the antithesis of the Renaissance, contrasting perspective painting, which, he claimed, negated the picture plane, with Romanesque painting, in which surface was "merely" surface, "the unconditionally two-dimensional surface of a material picture support."[69] Whereas, he argued, the arrangement of the former was determined by the rules of optical geometry (Panofsky collapsed optical and mathematical geometry) and evocative of a scientific approach to painting, the latter denoted a strictly non-quantitative and irrational mode of depiction. Whereas Panofsky's claims concerning the "scientific" rigor of perspective have long been disproved, his collateral characterization of the Romanesque and abstraction more generally in medieval art has gone largely unchallenged. Yet, if we turn to tenth- and eleventh-century quadrivial manuscripts, sources of scientific knowledge during this period, we find the same graphic tendencies picked out by Panofsky in pictures constructed and amended to convey quantitative concepts. Such pictures offer evidence that we are, perhaps,

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[68] Dupré,"The Historiography of Perspective": 35. See also Christopher Wood, "Introduction," in Erwin Panofsky, *Perspective as Symbolic Form*, trans. Christopher Wood (New York: Zone Books, 1997), 23.
[69] Panofsky, *Perspective as Symbolic Form*, 51.

too quick to limit our notion of the visual "scientific."[70] A key purpose of the present project is to revisit interconnections between intellectual and visual culture in the central middle ages and consider the period's artistic abstraction in light of the way God and the world were comprehended through numeric study.

3. DIAGRAMS

> The woman about to appear surpasses Apelles and Polyclitus; she is, in fact, so highly reputed to be able to represent any object that we must conclude that she is the offspring of Daedalus, of Labyrinth fame.[71]

The lady in question—deemed greater than the greatest painter, sculptor, and architect of antiquity—is *Geometria* as imagined by Martianus in the *De nuptiis*, a figure we have seen represented in the full-page frontispiece to the Boethian manuscript that Gerbert may have given Otto III (Bamberg, MS Msc.Class.5; fig. 1.7). Geometry's attribute, in the image and *De nuptiis*, is an abacus, which Martianus described as "a beautiful little board covered with a sprinkling of greenish powder," "designed for delineating figures."[72] Geometry demanded visual explication by its very definition. In the discipline's own words, it was the study of corporealities, of *demonstrable* knowledge; as such, it was apprehended primarily by sight. Bending to draw on the powdery surface of the abacus, Martianus's *Geometria* insists that one "could not explain in words" number's recondite qualities, that images were its apposite idiom.[73] It is in just this posture that the personified discipline was pictured by the Carolingian illuminator in Bamberg, Msc.Class.5. Though it was surely by accident that a brown-red stroke of paint was never added for her mouth or the paint flaked off, the absence leaves *Geometria* mute: dependent on demonstration (fig. 1.8). Her abacus is upturned,

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[70] Recent studies following in the footsteps of and thoughtfully reevaluating Svetlana Alpers's *Art of Describing*, have dilated this category. Notably Sachiko Kusukawa, *Picturing the Book of Nature: Image, Text, and Argument in Sixteenth-Century Human Anatomy and Medical Botany* (Chicago: University of Chicago Press, 2012); and, in the domain of medieval art, Jean Givens, *Observation and Image-Making in Gothic Art* (Cambridge: Cambridge University Press, 2004).

[71] Martianus, *De nuptiis* 6.579; ed. Willis, 204; trans. Stahl, 218–19: "ista, quae veniet, Apellen Polyclitum que transcendit; ita quippe memoratur posse omnia effigiare, ut labyrintheus Daedalus eam credendus sit genuisse."

[72] Ibid. 6.575 and 6.579; ed. Willis, 202—204; trans. Stahl, 216–17: "parent denique iam ingressurae Artis obsequio electissimae feminarum, quae decentem quandam atque hyalini pulveris respersione coloratam velut mensulam gestitantes ad medium superi senatus locum fiducia promptiore procedunt" and "res depingendis designandis que opportuna formis."

[73] Ibid. 6.579; ed. Willis, 204; trans. Stahl, 218: "'videbis istic depingi, quicquid verbis [visum] non valeas explicare.'"

permitting a clear view of the little shapes—a circle, a triangle—already traced on its surface. The instrument shares the composition's center with *Arithmetica*'s counting fingers. In the *De nuptiis*, *Arithmetica* enters in silence, signing numbers with such speed that her fingers are said to "vibrate" and their rapid movement "blurs vision."[74] *Arithmetica*'s gesturing hand and *Geometria*'s abacus give visual expression to numeric concepts. Isolated on a blue field and framed by the bodies of the two women, that expression is the focal point of the Bamberg illumination just as—turning the page—the complex, diverse, and artful images are the focus of the *De arithmetica* (figs. 1.4–6).

The visual aspects of quadrivial tracts were not appendages, they were the core, integral to the works in which they appear. As will be made clearer by the survey of the quadrivial canon below, the appearance of these graphic elements varied widely, ranging from austere, monochromatic forms to full-page, vividly pigmented *rotae* and charts exuberantly embellished with figurative and/or abstract designs. Copied and recopied, the texts of these earlier authors changed little, but the pictures underwent alterations that suggest a significant shift in use and an upsurge in interest. Tenth- and eleventh-century makers and users of quadrivial manuscripts—often one and the same in this period—provided pictures to elucidate textual passages that made no mention of them; where pictures were an organic part of the tract, they often added more. They experimented with placement, scale, color, and contours in ways that suggest a keen awareness of contemporary notions of materiality, sight, and the limits of representation that were themselves the product of explorations in the domains of mathematics and science.

Late antique and medieval authors typically named these nontextual elements with a vocabulary as varied as that which they sought to describe: *picturae*, *figurae*, *imagines*, *formulae, schemata*, *descriptiones,* even *paginae.*[75] Modern scholars, on the other hand, overwhelmingly refer to them as "diagrams"—this despite the fact that the Latin cognate *diagramma* was very rarely used by contemporary authors.[76] Although the aforementioned Latin terms were at times used synonymously, their homogenization in modern translations

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[74] Ibid. 7.729; ed. Willis, 261; trans. Stahl, 264: "Digiti vero virginis recursantes et quadam incomprehensae mobilitatis scaturrigine vermiculati."
[75] For the instability of terminology in antique mathematical texts see Reviel Netz, *The Shaping of Deduction in Greek Mathematics: A Study in Cognitive History* (Cambridge: Cambridge University Press, 1999), 36.
[76] Vitruvius used *diagramma* and *diagrammate* in *De architectura* (5.4.1, 5.5.6, and 6.1.7). It became more common in the 13th c. when Thomas Aquinas, for example, employed the term with some frequency.

and secondary literature obscures what were, in many cases, meaningful distinctions.[77] Attention to the varied language by which these images were designated yields insight into their diverse uses and perceived status. Shifts in terminology, especially within a single tract or an author's *oeuvre*, generally correspond to differences in image type and/or function. In the course of this study I draw attention to the terminology preferred by each author and, when possible, adopt the vocabulary of the tract under discussion.

Though not inaccurate, "diagram" is a relatively lackluster rubric, not clearly defined by art historians.[78] Often set in contradistinction to "picture," "diagram" suggests an absence of artistic interest and aesthetic value.[79] It is even more frequently opposed to "narrative image," a pairing that invests diagrams with a kind of static, timeless quality.[80] The term's ambiguity and elasticity has meant that it has been widely applied to a diverse array of images and served as a sort of umbrella category.[81] It is—at least in part—for this reason that much of the literature on medieval diagrams is given over to classification. Michael Evans's invaluable "Geometry of the Mind" and the indispensable *Album of Science: Antiquity and*

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[77] An idea that I explored in an invited paper titled "*Formula*, *Figura*, *Pictura*: *A Study in Terms*" presented at the Index of Christian Art's syposium, "Maps and Diagrams," at Princeton University, Princeton, NJ on March 16, 2013.

[78] Harry Bober, several of whose articles have become touchstones in the study of medieval diagrams, saw a sharp divide between "diagrams" and "*schemata*" which has since been blurred by most authors. He defined "*schemata*" as "geometric figures with inscribed text." In contrast, he cited the rainfall figure from Macrobius (an example of which is fig. 1.89) as "typical" of the "unschematic form of those [Macrobian] diagrams"; Bober, "In Principio. Creation before Time," in *De artibus opuscula XL: Essays in Honor of Erwin Panofsky*, ed. Millard Meiss (New York: New York University Press, 1961), 1: 19 and 23, The difficulty of defining the term is witnessed in John Bender and Michael Merrinan's 2010 book, *The Culture of Diagram*, which focuses on the images in Diderot and d'Alembert's *Encyclopédie*. The authors define a diagram as, "a proliferation of manifestly selective packets of dissimilar data correlated in an explicitly process-oriented array that has some of the attributes of a representation but is situated in the world like an object." Bender and Merrinan, *The Culture of Diagram* (Stanford: Stanford University Press, 2010), 7

[79] "Pictorial" and "diagrammatic" are, likewise, also frequently contrasted.

[80] Oppositions among these terms has, of course, yielded significant and productive insights into the mechanics of medieval objects in which the so-called narrative and diagrammatic modes often meet. Wolfgang Kemp in his investigation of the "the composite physical and compositional layering so characteristic of medieval art," explored the rhetorical effect of combining multiple discursive modes: spatial/cosmological, narrative/historic, and the "figure" (which is, according to Kemp, "purely geometrical" "form of a purely graphic nature"). More recently, Danielle Joyner thoughtfully explored the impact of interweaving narrative and diagrammatic/schematic images in the *Hortus deliciarum*. Kemp, "Medieval pictorial systems," in *Iconography at the Crossroads*, ed. Brendan Cassidy (Princeton: Index of Christian Art, Dept. of Art and Archaeology, Princeton University, 1993), 121–37, esp. 129–30; Joyner, "A Timely History: Images and Texts in the *Hortus Deliciarum*," (PhD diss., Harvard University, 2007), esp. 43–97.

[81] Steffen Bogen and Felix Thürlmann historicize and revel in the rich ambiguity of *schema* and *diagramma*. "Jenseits der Opposition von Text und Bild: Überlegungen zu einer Thoerie des Diagramms und des Diagrammatischen," in *Die Bildwelt der Diagramme Joachims von Fiore: Zur Medialität religiös-politischer Programme im Mittelalter*, ed. Alexander Patschovsky (Ostfildern: J. Thorbecke, 2003), 3.

the Middle Ages by John Murdoch sort diagrams according to shape, use, and content.[82] Murdoch drew his examples from manuscripts dating from the eighth through the fifteenth centuries. In so doing, he showed the prevalence and persistence of diagrams and the diagrammatic mode of representation throughout the medieval period.[83] The *Album*, is, however, set up in such a way that diagrams from the same tract or manuscript are, for the most part, separated and out of context. The arrangement points to another common tendency: diagrams have often been treated more as a kind of image or stylistic phenomenon rather than as distinct artifacts—individual instantiations with a particular history and context.

Harry Bober, in his 1956 article on what he called an "illustrated medieval school-book" (Walters MS 73), wrote, "as a class, manuscripts with predominantly schematic drawings ... are practically an unexplored world."[84] A fresh upswing in interest among art historians in medieval "schematic drawings" or diagrams means that book by book, this world is slowly being discovered. Recent studies on, for example, the *Liber floridus* by Lambert of Saint-Omer (1090–1120), Herrad of Hohenbourg's *Hortus deliciarum* (d.1195), the works of Joachim of Fiore (c.1132–1202), and Lothar of Segni's (c.1160–1216) *De missarum mysteriis* subject the manifold diagrams within these works to the same kind of scrutiny and attention accorded more traditional objects of art historical inquiry.[85] It will be

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[82] Michael Evans, "The Geometry of the Mind." *Architectural Association Quarterly* 12, (1980): 32–53; John Murdoch, *Album of Science: Antiquity and the Middle Ages* (New York: Scribner, 1984); others who have taken a classificatory approach or focused on a particular formal typology (e.g., "quincunx," tree) include Karl August Wirth, "Von mittelalterlichen Bildern und Lehrfiguren im Dienste der Schule," in *Studien zum städtischen Bildungswesen des späten Mittelalters und der frühen Neuzeit*, ed. Bernd Moeller, et al. (Göttingen: Vandenhoeck & Ruprecht, 1983), 256–297; Bianca Kühnel, *The End of Time in the Order of Things*; Steffen Bogen, "Träumt Jesse? Eine ikonographische Erfindung im Kontext diagrammatischer Bildformen des 12. Jahrhunderts," in *Ästhetik des Unsichtbaren: Bildtheorie und Bildgebrauch in der Vormoderne*, ed. David Ganz, Thomas Lentes and Georg Henkel (Berlin: Reimer, 2004), 219–40. Elizabeth Sears discussed the diagrammatic within a study of a medieval topos: *The Ages of Man: Medieval Interpretations of the Life Cycle* (Princeton: Princeton University Press, 1986).

[83] A point made by Jeffery Hamburger, "*Haec figura demonstrat*: Diagrams in an Early Thirteenth Century Parisian Copy of Lothar de Segni's *De missarium mysteriis*," *Wiener Jahrbuch für Kunstgeschichte* 58, no. 1 (2009): 9n11.

[84] Bober, "An Illustrated Medieval School-Book of Bede's 'De natura rerum,'" *Journal of the Walters Art Gallery* 19/20, (1956/57): 65.

[85] On the *Liber floridus* see Albert Derolez, *The Making and Meaning of the* Liber Floridus: *A Study of the Original Manuscript, Ghent, University Library, MS 92* (Tunhout: Brepols, Harvey Miller, 2015); and Hannah Vorholt's forthcoming, as yet untitled monograph on the transmission of the *Liber Floridus*. On the *Hortus* see Danielle Joyner, "A Timely History"; on the output of Joachim see Alexnder Patschovsky, ed. *Die Bildwelt der Diagramme Joachims von Fiore: Zur Medialität religiös-politischer Programme im Mittelalter* (Ostfildern: J. Thorbecke, 2003). On the *De missarum mysteriis* see Jeffery Hamburger, "*Haec figura demonstrat*."

noted that these works are, largely, products of the twelfth century, an epoch long identified as the heyday of schematic modes of representation.[86] They are also exceptionally beautiful examples of this kind of graphic device.[87] Bober's statement still holds, I posit, for the far more common and frequently commonplace quadrivial manuscripts. The texts and figures in these books remain, for the most part, *terra incognita* for historians of art. In undertaking this study I join historians of science, musicologists, and a small, but growing cadre of art historians, especially Eva-Maria Engelen, Barbara Obrist, and Katrin Müller, who have shown the potential of this material for shedding light on contemporary modes of representation, aesthetic attitudes, and cognition.[88]

That diagrams demanded as well as engendered particular mental and visual attitudes on the part of medieval viewers is now largely taken for granted. Michael Camille suggested that the diagrams in medieval schoolbooks "represent ways of thinking," and as such might

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[86] On the identification of the 12th c. with schematic expression see, especially, Henri Focillon, *Art d'Occident: Le moyen âge, roman et gothique*, 2nd ed. (Paris: A. Colin, 1947), 55; C. M. Kauffmann, *Romanesque Manuscripts, 1066-1190* (London: H. Miller, 1975), 45–46; and Jean-Claude Schmitt, "Les images classificatrices," *Bibliothèque de l'École des Chartes* 147, no. 1 (1989): 311–41, esp. 312, Romanesque, the prevalent style of the 12th century, is also associated with the schematism, so much so that Madeline Caviness made a point of distinguishing between diagrammatic modes of representation and the Romanesque in her "Images of Divine Order and the Third Mode of Seeing," *Gesta* 22 (1983): 110 and 116. See also Meyer Schapiro, "On Geometrical Schematism in Romanesque Art," in *Romanesque Art: Selected Papers* (New York: George Braziller, 1977, rpt. 1993), 265–85.

[87] It should be noted that the *De missarum mysteriis*, extant in some 194 manuscripts (not all with diagrams), has many more modest manifestations than that discussed by Hamburger, who focused his article on the earliest (and previously unidentified) Gotha manuscript (Gotha, FB Cod. Memb. I 123), which he described as, "a schoolbook of the very highest quality." Hamburger, "*Haec figura demonstrat*," 23.

[88] My choice to focus primarily on arithmetical and geometrical tracts was determined in part by the excellent work already done by these scholars and scholars in other fields on astronomical/cosmological and musical imagery. Barbara Obrist's ongoing investigation and explanation of astronomical figures and computus material (especially Abbo's *oeuvre*) has been vital to my own work. Likewise, Katrin Müller's analysis of the figures in Calcidius's *Commentary* on the *Timaeus* and the *De arithmetica* has shaped my thinking on the function of figures in these works and more generally. See in particular Engelen, *Zeit, Zahl und Bild: Studien zur Verbindung von Philosophie und Wissenschaft bei Abbo von Fleury* (Berlin: Walter de Gruyter, 1993); Obrist, *La cosmologie médiévale: Textes et images*, Vol. 1, Les fondements antiques (Florence: SISMEL edizioni del Galluzzo, 2004); eadem, "Les tables et figures Abboniennes dans l'histoire de l'iconographie des recueils de comput," in *Abbon de Fleury: Philosophie, sciences et comput autour de l'an mil*, ed. Barbara Obrist (Paris: C.N.R.S. and E.P.H.E., 2006), 141–86; Katrin Müller, "*Admirabilis forma numeri.* Diagramm und Ornament in mittelalterlichen Abschiften von Boethius' *De arithmetica*," in *Ornament. Motiv–Modus–Bild*, ed. Vera Beyer and Christian Sies (Paderborn: Wilhelm Fink, 2012), 180–205; eadem, "Theorie und Materialisierung der Zahl in Boethius' *De institutione arithmetica*," in *Was Zälht: Ordnungsangebote, Gebrauchsformen und Erfahrungsmodalitäten des "numerus" im Mittelalter*, ed. Moritz Wedell (Vienna: Böhlau, 2012), 81–102; and eadem, "Visuelle Weltaneignung: Astronomische und kosmologische Diagramme in lateinischen Handschriften des 11. bis frühen 14. Jahrhunderts" (PhD diss., University of Hamburg, 2005).

offer "ways of understanding the dynamics of physical as well as conceptual construction."[89] Following Camille as well as Erwin Panofsky, Ernst Gombrich, Michael Baxandall, and others who have connected patterns of cognition with modes of representation, it seems worthwhile to probe and interrogate quadrivial pictures for what they can tell us of the ideas and abilities that informed their appearance.[90] Anna Esmeijer and Mary Carruthers were among the first to successfully demonstrate how such images stimulated specific heuristic practices and/or mnemonic and devotional techniques.[91] The efficacy of any syntax relies, of course, on recognition and this is acquired through training. Esmeijer tied the rise of visual exegesis to education.[92] Likewise, I believe that the heavy use of diagrammatic conventions, their potent transferability and widespread legibility, was possible because they were absorbed in the schools in conjunction with habits of eye and mind.[93] Madeline Caviness in her oft-cited, "Images of Order and the Third Mode of Seeing," connected certain graphic aspects derived from diagrams with a "higher," "spiritual" sight.[94] Caviness premises her discussion of diagrams on the assumption—prevalent among medieval art historians—that "spiritual seeing" was an inborn ability. Quadrivial, especially geometrical, tracts make it

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[89] The statement appears in Camille's analysis of the so-called square of opposition in Aristotle's *Analytica Posteriora* in London, BL Burney MS 275, a deluxe compendium of the liberal arts. In this article, Camille pushes hard against what he called the "vague analogies between artistic styles and medieval conceptual thinking" of Panofsky's *Gothic Architecture and Scholasticism*, seeing in the figures that populate logical texts objects that give form to rational dynamics. Michael Camille, "Illuminating Thought: The Trivial Arts in British Library, Burney Ms. 275," in *New Offerings, Ancient Treasures: Studies in Medieval Art for George Henderson*, ed. Paul Binski and William Noel (Thrupp, Gloucestershire: Sutton, 2001), 343–66, esp. 358–59. Camille made similar claims in "'Seeing and Lecturing': Disputation in a Twelfth-Century Tympanum from Reims," in *Reading Medieval Images: The Art Historian and the Object*, ed. Elizabeth Sears and Thelma K. Thomas (Ann Arbor: University of Michigan Press, 2002), 75–87.

[90] Panofsky, *Gothic Architecture and Scholasticism* (New York: Meridian Books, 1957); Gombrich, *Art and Illusion; A Study in the Psychology of Pictorial Representation*. (New York: Pantheon Books, 1960); Baxandall, *Giotto and the Orators: Humanist Observers of Painting in Italy and the Discovery of Pictorial Composition, 1350-1450*, (Oxford: Clarendon Press, 1971); idem, *Painting and Experience in Fifteenth Century Italy: A Primer in the Social History of Pictorial Style* (Oxford: Oxford University Press, 1988); and Carruthers, *The Craft of Thought: Meditation, Rhetoric, and the Making of Images, 400–1200* (New York: Cambridge University Press, 1998).

[91] Anna Esmeijer, *Divina Quaternitas: A Preliminary Study in the Method and Application of Visual Exegesis* (Assen: Van Gorcum, 1978); Mary Carruthers, *The Book of Memory: A Study of Medieval Memory and Culture* (Cambridge: Cambridge University Press, 1990), esp. 221–57.

[92] Esmeijer, *Divina Quaternitas*, 30–31. See also Michael Curschmann, "Imagined Exegesis: Text and Picture in the Exegetical Works of Rupert of Deutz, Honorius Augustodunensis, and Gerhoch of Reichersberg." *Traditio* 44 (1988, publ. 1990): 145–69.

[93] Historians of art have long recognized the didactic roots of diagrams and the diagrammatic mode of representation. Esmeijer points to Bober as the first to make this connection.

[94] Akin to Richard of St. Victor's "third mode" referred to in the article's title, from the commentary on St. John. Caviness, "Images of Divine Order," esp. 114–16.

clear that this was not deemed to be the case and that it was, moreover, precisely this faculty that medieval masters sought to inculcate through the teaching of number.

4. The Quadrivial canon

The deluxe Tours copy of the *De arithmetica* with which this chapter began is but one of several manuscripts—many on quadrivial topics—that Gerbert may have dispatched to Otto III, whom he tutored. Neither Gerbert nor Abbo left behind a reading list of the texts that they found essential to quadrivial study, but the names of authors and titles of works they relied on and considered authoritative surface in their writing, both epistolary and didactic.[95] Surviving manuscripts associated with the two men and their students corroborate and extend this list. From this evidence we see that a core group of tracts were deemed more or less indispensable for numeric study, what might be called the quadrivial canon. This "canon" included a range of texts of different kinds: specialist treatises, like the *De arithmetica*, as well as compendia devoted to a particular discipline; tracts that might be classed as metaphysical or philosophical, grounded in the Pythagorean/Platonic tradition; so-called encyclopedic works containing summaries of all the liberal arts; and discursive theological texts, many by the Church Fathers. Despite the fact—touched on above—that they survive in astounding numbers, these works are often now considered obscure, their images virtually unknown to any but a small number of specialists. What follows, therefore, is a survey of the quadrivial canon grounded in manuscripts associated with Gerbert and Abbo, and focused on their visual aspects.

* * *

> Now Otto comes, the power of Rome to raise.
> And to Boethius's name renews the praise.
> Your picture in the Emperor's hall is hung.
> For ever now be your high merit sung.
>
> *Nunc decus imperii, summas qui praegravat artes*
> *Tertius Otto sua dignum te iudicat aula*
> *Aeternumque tui statuit monimenta laboris*

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[95] See chapter 2.

Et bene promeritum meritis exornat honestis. [96]

These laudatory lines, attributed to Gerbert, were part of a longer poem that may have adorned a lost "monument" (*monimenta*) to Boethius (c.480–c.524) commissioned by Otto III.[97] Poem and monument capture just how highly esteemed Boethius was c.1000. Abbo praised him for translating Greek treasures;[98] he was given the etymologically based sobriquet "helper" in the brief biographies that were then in circulation.[99] The moniker was well deserved. A witness to the waning of the Roman Empire, Boethius had set himself the task of translating and commenting on certain texts in Greek that had long been the backbone of Roman education, including the complete works of Aristotle (in his own words: "every work that he could get his hands on"), Plato's dialogues, Euclid's *Elements*, Ptolemy's *Almagest*, Nicomachus of Gerasa's *De arithmetica*, and texts on music by these last two authors.[100] Copies of the *De arithmetica*, preserved in its entirety, and the *De musica*, preserved in part, proliferated throughout the middle ages (and after).[101] They were the premier textbooks on these subjects around the year 1000. Fragments of Boethius's translation of the *Elements* also survived and became the nexus of geometric miscellanies.

A dedicatory poem in Bamberg, Msc.Class.5 written in gold and silver on a deep purple background cues the reader to the precious and comprehensive nature of the *De arithmetica* (fig. 1.9).

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[96] MGH *Poetae* 5, num. 13, 475 (www.mgh.de/dmgh/resolving/MGH_Poetae_5,1.2_S._475 accessed Sept. 22, 2014); trans. adapted from R. W. Southern in *Making*, 178.
[97] On the attribution to Gerbert see Southern, *Making*, 178; and Mütherich, "Library of Otto III," 20.
[98] Abbo, *Explanatio* 3.24, ed. Alison Peden in *Commentary on the* Calculus *of Victorius of Aquitaine* (Oxford University Press, 2003), 87: "Unde Anicius viciorum et maxime invidiae iaculis invictus, calamitatibus seu errore ignorantiae deiectis, auxiliando in suis commentariis, quos de Grecis deliciis in thesaurum Latinae orationis summa karitate transtulit."
[99] Noel Harold Kaylor, Jr., "Introduction: The Times, Life and Work of Boethius," *A Companion to Boethius in the Middle Ages* (Leiden: Brill, 2012), 7. In Greek, Boethius (Βοηθεια) resembles the Greek term for "helper" (βοηθός).
[100] For an in-depth discussion of this great salvage project and its later impact, see Andreas Kirchner, Thomas Jürgasch, and Thomas Böhm, ed. *Boethius as a Paradigm of Late Ancient Thought* (Boston: De Gruyter, 2014); Noel Kaylor and Philip Phillips, ed., *A Companion to Boethius in the Middle Ages*, which includes an extensive annotated bibliography on Boethius and his works; Henry Chadwick, *Boethius: The Consolations of Music, Logic, Theology, and Philosophy* (Oxford: Clarendon Press, 1981); Margaret Gibson, ed., *Boethius: His Life, Thought, and Influence* (Oxford: Basil Blackwell, 1981); Michael Masi, ed., *Boethius and the Liberal Arts* (Bern: Peter Lang, 1981).
[101] In the most recent edition and French translation of the *De arithmetica*, Jean-Yves Guillaumin notes the survival of 186 complete and partial copies of that work. Calvin Bower assembled a handlist of the 137 surviving partial and complete copies of the *De musica*, 65 of which have been dated to the 9th through the 11th c. Guillaumin, in Boèce, *Institution arithmétique*, lxv; Calvin M. Bower, "Boethius *De Institutione Musica*: A Handlist of Manuscripts," *Scriptorium* 42 (1988): 205–51.

Anything [that] exists by means of number,
is learned by means of [that] same number
the order of which you hold, O reader, in this book…

Quae numero constant, numero discuntur eodem,
Cuius in hoc seriem codice, lector, habes…[102]

The *De arithmetica* was deemed indispensable for numeric study.[103] The prologue served as an introduction to the quadrivium as a whole and offered a framework for numeric pursuit. According to Boethius, the ultimate end of quadrivial study was, as already stated, philosophy, which could not be obtained without mastery of mathematics, which began with arithmetic. The *De arithmetica* swings from technical explanation to metaphysical reverie and back again. Boethius encourages a sense of awe at number's power and remarkable behaviors. Book one is given over to definitions and properties. Among the topics addressed are the sequence of natural numbers, evens and odds, equalities and inequalities. Book two looks forward to the study of geometry and music, introducing plane and solid numbers, and identifying numeric ratios. An annotator working around the year 1000 neatly summarized the *De arithmetica* in this way: "In the previous book [Boethius] demonstrated how inequality proceeded from equality. Here [in book two] it will be demonstrated how all inequality reverts to that equality from which it proceeded."[104] Hence, to study arithmetic was to observe the tide of numbers flowing from and returning to unity—a model of the creation of the cosmos and its end.

Boethius gave these numeric currents form in a panoply of explanatory figures. He called the graphic elements "formulas" (*formulae*) and "descriptions" (*descriptiones*). We tend to think of calculation in its notational form. It should be remembered that mathematical notation is a relatively late invention and that the signs used today to denote addition, multiplication, etc., replaced words (verbal equations) and the arrangement of numbers on a page.[105] In the *De arithmetica* numbers are laid out, ordered, and patterned in ways that make

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[102] Bamberg, SB MS Msc.Class.5, fol. 62v. For the full poem see, MGH *Poetae* 4*, Carmina varia*, 3.
[103] On the number of surviving copies see above.
[104] Henry Mayr-Harting, *Church and Cosmos*, 188: "In superiori libro demonstravit quomodo inaequalitas ab aequalitate processerit. Hic vero demonstraturus est quemadmodum omnis inaequalitas in ipsam unde processit aequalitatem revertatur."
[105] Harriet Lattin, "The Origin of Our Present System of Notation according to the Theories of Nicholas Bubnov," *Isis* 19, no. 1 (1933): 181–94.

their interrelationships apparent, and are set off from the text by frames. The value that tenth- and eleventh-century audiences placed on the *descriptiones* and *formulae* is clear from signs of their often meticulous copying and correction. This was not only true in manuscripts of the *De arithmetica*, but—we shall see—of virtually all the quadrivial tracts.

Few early copies of the *De arithmetica* were without the full complement of over one hundred pictures.[106] Bamberg, Msc.Class.5, despite (probably, in part, because of) its beauty—is among the rare exceptions to this rule and the omissions proved untenable to later users.[107] In this manuscript the numerical sequences were integrated into the text rather than being distinguished in some manner.[108] The majority of the missing *descriptiones* were of the most basic and common type encountered in the *De arithmetica*: a series of numbers enclosed in a frame or grid (figs. 1.10–12). Perhaps surprisingly, even these were considered essential, as evidenced by the actions of a tenth-century annotator, who drew slashes—breaking up the line of numbers—and added brief explanatory notes in the margins (figs. 1.13–15). These say things like "the numbers must be drawn out in three lines" and "these [numbers] must also be figured in three [lines]."[109] Gerbert may have found the undifferentiated *descriptiones* unacceptable as well. It was, perhaps, for this reason that we find a second, corrected copy of the *De arithmetica* (Bamberg, Staatsbibliothek, MS Msc.Class.8) made in northern France with annotations added in Reims among the manuscripts generally associated with Otto III.[110] The same *descriptiones* omitted in Msc.Class.5 were also missing from Msc.Class.8, but these were later added in the margins of the latter manuscript—in some instances quite artfully (figs. 1.16–18).[111] Indeed, two more copies of the *De arithmetica*, both products of the tenth century, were also part of the

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[106] About 123 images total. An exact tally is difficult. I count certain clusters of related figures as a single entity.
[107] The failure to distinguish the *descriptiones* was likely due to a deficient exemplar.
[108] Save through standard punctuation: each number is followed by a point.
[109] Bamberg, SB MS Msc.Class.5, fol. 45v: "numeri in tres lineas [per?]ducendi" (the beginning of the last word is cut off); and on fol. 46: "hii quoque tri[?] figurandi." Similar action was taken by the annotator on fols. 26v, 52r, and 55r–55v.
[110] For a description of Bamberg, SB MS Msc.Class.8, see Suckale-Redlefsen, *Die Handschriften*, 41–44 (no. 24); the manuscript was included by Carey in his handlist of manuscripts from Reims, "The Scriptorium of Reims," 58.
[111] The text of Bamberg, Msc.Class.5 and Msc.Class.8 is not close enough to posit a common exemplar, but the two manuscripts certainly belong to the same family. The missing *descriptiones* were later added at Reims in the 10th c. to Msc.Class.8. It seems likely that they were not added to Msc.Class.5 because they would have spoiled the look of the manuscript. For the dating and localization of these interventions see Hoffmann, *Bamberger Handschriften*, 126.

Bamberg cathedral library, though how they came to be part of the collection is unknown.[112] A side-by-side comparison of the four manuscripts shows the amount of creative effort expended on making the *descriptiones* legible, accurate, and even beautiful in the central middle ages (figs. 1.19–22).

Certain problems in the *De arithmetica* became points of focus, analyzed and resolved by the great mathematical minds of the day. Abbo, Gerbert, and Notker of Liège (940–1008) all commented on the chapter titled "How every inequality is reduced to equality."[113] It was in this chapter that Boethius explained the process of Creation arithmetically, by working backward and showing how a series of unequal terms can be reduced to equal terms, and, finally, to unity. Constantine, a monk of Fleury and student of Gerbert,[114] wrote to the schoolmaster of Reims asking him to clarify how this feat was achieved. Gerbert responded, taking his friend step-by-step through the problem to its solution.[115] Of particular interest is the emphasis he placed on spatial relations, echoing admonitions made by Boethius.[116] Clearly he had assimilated the attention to layout, which permeates the *De arithmetica*, and was emulating it in his teaching. Gerbert closed his letter:

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[112] Bamberg, SB MS Msc.Class.6 and Msc.Class.7. These are described in Suckale-Redlefsen, *Die Handschriften*, 71–73 (no. 55) and 132–60 (no. 80); and Hoffmann, *Bamberger Handschriften*, 125–26.

[113] Boethius, *De arithmetica* 2.1; ed. Guillaumin, 78–81; my translation: "Quemadmodum ad aequalitatem omnis inaequalitas reducatur." Abbo and Notker's comments are preserved in two manuscript copies of the *De arithmetica*: Munich, SBS Clm 3517, fol. 15v; and Cambrai, BM MS 928 (827), fol. 55. Nikolaus Bubnov, ed., *Gerberti, postea Silvestri II papae, Opera mathematica, 972–1003* (Berlin: R. Friedländer und Sohn, 1899), 32, n. 3.

[114] Later, abbot of Micy, see chapter 2.

[115] Gerbert, *Epistulae*; ed. Riché and Callu, appendix 5, no. 6, p. 693–99. Boethius and Gerbert each begin with three proportional terms. Boethius uses 8, 32, 128, and Gerbert 16, 20, 25. Gerbert states that these terms should be written in a line, one beside the other, smallest to largest. Below the first line of numbers a second line (the first reduction) will be written. Gerbert instructs Constantine to subtract the first term (16) from the second (20) and write the first (16) in the first position of the second line (see below). The difference, 4, should be written in the second place. From the third term (25), subtract the first term (16) and twice the second term (2x4 or 8), this leaves 1. So the second line is 16, 4, 1. These numbers then need to be put in reverse order (1, 4, 16) and the same operations will be done again, leaving 1, 3, 9, and again, leaving 1, 2, 4, and again leaving 1, 1, 1. The whole problem, written out would like something like this:

xvi	xx	xv
i	iiii	xvi
i	iii	viiii
i	ii	iiii
i	i	i

[116] The *descriptiones* and *formulae* make the same point visually.

> See, therefore, how all uneven quantity is reduced to three equal terms, that is unities: i i i not confusedly, but in an orderly manner, just as it was created in the beginning (*a principio*). This, then, is the true nature of number.[117]

Here, the numbers written on the page are reminiscent of divine order. What is at stake in their arrangement is not merely mathematical misunderstanding, but misrepresentation of Creation—a kind of visual heresy. Whether Gerbert enclosed the numerical series in frames or grids, as in the *De arithmetica*, is not known (fig. 1.23). His letter survives only in later copies, which do not preserve this feature (fig. 1.24). Copies of some of his other letters do contain figures.[118] When, for example, Gerbert was asked by Constantine to explain a knotty problem in the *De musica*, he adopted the graphic idiom characteristic of that work, providing at least one *descriptio* to explain a section of the tract that had no figures (fig. 1.25).[119]

That even a star pupil like Constantine would struggle with the *De musica* is not surprising.[120] The work was notoriously difficult. Boethius claimed that he wrote the treatise in order to train "musicians" (*musici*). The true musician, according to Boethius, was not one who played or composed music, but one who formed "judgments according to speculation or reason" about rhythms, consonances, etc.[121] A reader of the challenging text learned the names and qualities of the various ratios, proportions, consonances, and tones as well as how these might be demonstrated on a monochord, which—according to contemporary sources—Gerbert used to teach music.[122] He was, no doubt, inspired by Boethius, who said that the

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[117] Gerbert, *Epistulae*; ed. Riché and Callu, appendix 5, no. 6, p. 698, "Vides igitur, quemadmodum tota quantitas sesquarti redacta est ad tres aequales terminos, id est unitate: i, i, i, non confuse, sed ordinatim, sicut fuerat a principio procreata. Haec est igitur vera natura numerorum."

[118] The letter has been identified in at least six manuscripts listed by Bubnov, *Opera Mathematica*, 31.

[119] Boethius, *De institutione musica* 2.21; ed. Godfried Friedlein, in *Anicii Manlii Torquati Severini Boetii De institutione arithmetica libri duo; De institutione musica libri quinque* (Leipzig, 1867), 253–54. Gerbert introduced the figure in the text of the letter: "quod monstrat praesens descriptio." Gerbert, *Epistulae*; ed. Riché and Callue, appendix 5, no. 5, p. 689. The letter has been identified in at least four manuscripts listed by Bubnov, *Opera Mathematica*, 30. In Brussels, BRB MS 5444-46, fol. 63, the letter includes two *descriptiones*.

[120] Gerbert's letters to Constantine on the *De musica* are dated to before 982. Constantine eventually became quite adept in the discipline. See chapter 2.

[121] Boethius, *De musica* 1.34; ed. Friedlein, 224; trans. Calvin Bower in Boethius, *Fundamentals of Music*, ed. Claude V. Palisca (New Haven: Yale University Press, 1989), 51: "isque est musicus, cui adest facultas secundum speculationem rationemve propositam"

[122] Unlike Boethius's other didactic works, the *De musica* was not a translation of and commentary on a single Greek text but was based on a number of works on the subject, possibly a 6th-c. compendium of some sort. The first four books seem to be a translation of a now lost *Isagoge musica* by Nicomachus. The fourth book, on the monochord, may have been a work redacted by Nicomachus and appended to his *Isagoge musica*. The fifth

theory of consonances could be "made clear" and "learned beyond all doubt" with the single-stringed, instrument invented purely for didactic purposes.[123]

Visually, the *De musica* resembled the *De arithmetica*. In clarifying musical concepts, Boethius relied as heavily on graphic demonstration as he had to clarify arithmetic concepts, and many of the same conventions introduced in the *De arithmetica* appear also in the *De musica*: rectangular grids denote numeric progression and semicircular arcs represent intervals (fig. 1.26). Unsurprisingly, this last convention proliferated in a work devoted to related quantities. Indeed, in a copy from Fleury semicircular arcs were added to *descriptiones* of which they were not an organic part (figs. 1.27–28).[124] The complexity of the discipline's precepts is apparent in the often complicated and intricate figures. Scribes struggled to fit them on the page, negotiating between legibility on the one hand and continuity and integrity on the other. In an eleventh-century manuscript some of the *descriptiones* were accommodated on special long pages that unfolded to three times the width of the book (figs. 1.29–32).[125] In several witnesses, figures (many supplementary) drawn on parchment fragments were tipped-in (figs. 1.33–37).[126] Two copies of the tract, probably from Fleury, include a set of duplicate figures grouped together at the end of the *De musica*, an arrangement that underscores the independence of the images and suggests their use apart from the written text (fig. 1.38).[127] Indeed a fascicle of thirty-six figures from the

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book is based on the first book of Ptolemy's *Harmonica*. This last breaks off abruptly in the middle of chapter nineteen. Eleven more chapters were included in the list of chapters provided, which were either never finished or lost. For an in-depth summary of the contents see Bower, in Boethius, *Fundamentals of Music*, ed. Claude V. Palisca (New Haven: Yale University Press, 1989), xxix–xxxviii; on Gerbert's use of the monochord see chapter 2.

[123] Boethius, *De musica* 4.18; ed. Friedlein, 348; trans. Bower, 160: "Ut vero indubitanter consonantiarum ratio colligatur, tali brevissimo ac simplici effici poterit instrumento."

[124] Paris, BnF lat. MS 7200.

[125] Brussels, BRB MS 5444–46.

[126] See, for example, a copy now in Einsiedeln, StiftsB Cod. 358(610), 145–269, which was made in Reims, Trier, or Fleury at the end of the 10th century; Munich, BSB Clm. 14272, fols. 1–62, produced in Chartres and brought to St. Emmeram by Hartwic (discussed briefly in chapter 6); Chicago, NL MS F. 9, fols. 1–64; Cologne, HA MS W 331, fols. 94v–111r; and Madrid, BNE MS 9088, fols. 41r–94v. Bowers, "Boethius *De Insitutione Musica*," 217 and 224.

[127] Paris, lat. 7200, fols. 86r–93r. Calvin Bower wrote that though the origin and provenance of the manuscript remains in doubt, "the independent quire of diagrams definitely relates the MS to Fleury." The manuscript is dated to the period of Abbo: 945–1004, as is another 10th-c. copy: Paris, BnF MS lat. 7297, fols. 92v–102, also from Fleury. Fleury owned at least four complete copies of the *De musica* in the central middle ages. Ibid., 231 and 233.

De musica was folded into another Fleury manuscript, a miscellany of quadrivial tracts (fig. 1.39).[128]

The *De musica* traveled independently, but was also encountered bound with other tracts, especially the *De arithmetica*.[129] The combination of these subjects with their complementary definitions was natural. Both took number, that is multitude or quantity, as their object of study, but whereas arithmetic was the study of number *per se*, music, as we have seen, was the study of numeric relationships.[130] That the tracts were both the work of Boethius strengthened the bond between them, and they were often written and bound as a pair. Omnibus manuscripts of Boethius's quadrivial works were also produced.[131] It was in this context that the *De arithmetica* and the *De musica* came into the orbit of the Boethian translation of Euclid's *Elements*.

Boethius had translated all thirteen books of *Elements*, but only the first four survived much beyond his own time.[132] These fragments were added to collections of geometric tracts culled from an array of late antique and medieval sources. Gerbert sent such a compilation to Otto III: Bamberg, Staatsbibliothek, MS Msc.Class.55. The manuscript is a miscellany of short excerpted tracts on geometry and astronomy. Like arithmetic and music, these disciplines had complementary definitions and were thus ligated. The geometrical tracts found in Bamberg, Msc.Class.55 were excerpted from the so-called *Geometria I*. Several geometric miscellanies, fairly stable in content, circulated around the year 1000, among these the *Corpus agrimensorum Romanorum* and a compendium dubbed the *Geometria I*. The former, as its name suggests, is a collection of texts written by Roman "land surveyors"

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128 Paris, BnF lat. 8663, fols. 51v–57v. The manuscript includes Hyginus's *Astronomica* and Macrobius's *Commentary* on the *Somnium Scipionis*. Bower dated the *De musica* fascicle to the 10th c. Ibid., 246.

129 The *De arithmetica* and *De musica* were combined in twelve manuscripts dating from the 9th through the 11th c. Boethius referred to the *De arithmetica* by its title in the *De musica* once, and also as the "arithmetic books." Ibid., 211–46.

130 Boethius, *De arithmetica* 1.1.4; he repeats these definitions in the *De musica* 2.3.

131 Surviving omnibus manuscripts from the 10th and early 11th c. include Einsiedeln, StiftsB Cod. 358; London, BL Harley MS 3595; Prague, NkČ MS IX. C. 6. (Y. II. 3. n. 29.); and Vienna, ÖNB Cod. 55; Bower, "Boethius *De Insitutione Musica*," 217, 221, 236, and 242–43.

132 The *Geometria I* and *II* are generally called the work of the "Pseudo Boethius." Only recently have scholars recognized the fragments of the genuine Boethian translation within its layers of textual accretions. For the history of Euclid's *Elements* in Latin see Menso Folkerts, *The Development of Mathematics in Medieval Europe: the Arabs, Euclid, Regiomontanus* (Aldershot: Ashgate Variorum, 2006), 39–64; on the Boethian translation see idem, "The Importance of the Pseudo-Boethian *Geometria* During the Middle Ages," in *Boethius and the Liberal Arts*, ed. Michael Masi (Bern: Peter Lang, 1981), 187–209; and Wesley Stevens, "Euclidean Geometry in the Early Middle Ages," in *Villard's Legacy: Studies in Medieval Technology, Science, and Art in Memory of Jean Gimpel*, ed. Marie-Therese Zenner (Aldershot: Ashgate, 2004), 229–64.

(*agrimensores*):[133] Sextus Julius Frontinus, Siculus Flaccus, Hyginus, Marcus Iunius Nipsus, and Balbus, about whom little is known beyond their names.[134] They wrote technical treatises on such topics as methods of land measure and division, surveyors' instruments, types of boundary markers, and laws regarding field limits. These specialized tracts were prefaced by lessons on geometry's fundamentals—the definitions of a point, line, plane, and solid; kinds of planes, solids, and angles, etc.—as well as esoteric musings on the nature and aims of geometric thought and practice.[135] In the ninth century at the monastery of Corbie, the *Corpus agrimensorum* was adapted for use in the schools.[136] The *Geometria I* was the result of this effort. It comprises extracts from the *Corpus agrimensorum*, Boethius's translation of the *Elements*, and his *De arithmetica*; passages from Isidore's *Etymologiae* and the *Institutiones* of Cassiodorus (discussed below); and the *Altercatio duorum geometricorum de figuris, numeris et mensuris* (*Debate of Two Geometers on Figures, Numbers, and Measures*)—a dialogue derived from Augustine's *De quantitate animae* (*On the Measure of the Soul*, also discussed below) and the *Soliloquies*.[137] Gerbert based his *Isagoge geometriae* (*Introduction to Geometry*) on this compilation, synthesizing the various tracts. His *Isagoge* was, in turn, folded into an updated geometric miscellany known as the *Geometria II*.[138]

Gerbert wrote the *Isagoge* while serving as abbot at the monastery of St. Columbanus in Bobbio,[139] where he may have come across the earliest witness to the *Corpus agrimensores,* a manuscript known as the *Arcerianus*.[140] A product of the sixth or seventh

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[133] Also called "gromatics" after the *groma*, a surveyor's measuring rod.
[134] For a summary of what is known of these authors see Brian Campbell, *The Writings of the Roman Land Surveyors: Introduction, Text, Translation, and Commentary* (London: The Society for the Promotion of Roman Studies, 2000), xxxix–xl.
[135] The basics of geometry were essential knowledge for the accurate measurement and division of fields. Indeed, the discipline was said to have originated in Egypt where, because of the annual flooding of the Nile, they had need of an orderly method of dividing the land and settling disputes over borders. The word "geometry," is derived from the Greek terms for "earth" and "measure." See chapter 5.
[136] B. L. Ullmann, "Geometry in the Medieval Quadrivium," in *Studi di bibliografia e di storia in onore di Tammaro de Marinis* (Verona: Stamperia Valdonega, 1964), 263–85.
[137] Folkerts, "The Importance of the Pseudo-Boethian *Geometria*," 189–90. On the *Altercatio* see idem, "Die *Altercatio* in der *Geometrie I* des Pseudo-Boethius. Ein Beitrag zur Geometrie im mittelalterlichen Quadrivium," in *Fachprosa-Studien: Beitrage zur mittelalterlichen Wissenschafts- und Geistesgeschichte*, ed. A. Van de Vyver (Berlin: Erich Schmidt, 1982), 84–114.
[138] The *Geometria II* was edited by Menso Folkerts, "*Boethius*" Geometrie II: *Ein mathematisches Lehrbuch des Mittelalters* (Wiesbaden: F. Steiner, 1970).
[139] On Gerbert's time in Bobbio see chapters 2 and 5.
[140] Wolfenbüttel, HAB Cod. Guelf. 36.23 Aug. 2°. The manuscript was designated "Arcerianus" after Joannes Arcerius of Utrecht, a former owner. There is some dispute about whether Gerbert encountered the Arcerianus

century, the Arcerianus is a deluxe manuscript replete with colorful and detailed illuminations (figs. 1.40–42).[141] The largest and most elaborate images picture diverse scenarios that an *agrimensor* might encounter in the field and strategies for coping with a range of physical obstacles such as valleys, broken ground, trees, buildings, walls, rocks, mountains, and the like.[142] The various elements that compose each scene are rendered in vivid washes of red, blue, brown, orange, green, yellow, and purple pigments; forms are modeled and shaded to give a sense of plasticity (fig. 1.43). Later copies of the *Corpus agrimensorum* dispensed with the kind of surface detail seen in the *Arcerianus* in favor of monochrome representations (figs. 1.44). Gerbert included only unadorned geometric shapes and linear constructions in the *Isagoge* (figs. 1.45–46). Likewise, the pages of Bamberg, Msc.Class.55, contain exclusively line drawings executed in plain brown ink (figs. 1.47–48). The diminutive figures that accompany the first book of Euclid's *Elements* in this manuscript are of a type that multiplied in this period.[143] Adjacent to the verbal definition of a right angle, two lines labeled *recti lineus angulus* were drawn intersecting at ninety degrees (figs. 1.49). Similar figures, most simple polygons, fill the margins of this page and its reverse (figs. 50–51). The same scribe added a near identical set of figures to the margins in another book that may have passed from Gerbert to Otto III: Bamberg, Staatsbibliothek, MS Msc.Class.39, a copy of Martianus Capella's *De nuptiis* (figs. 1.52–55).[144]

Reading the *De nuptiis*, the young emperor would have encountered the most thorough-going account of *all* seven disciplines contained in a single, relatively compact volume. Martianus or Felix Capella's fifth-century tract was widely read in the central middle ages. Ninth- and tenth-century copies are disproportionately represented among the 241 extant manuscripts, fragments, and excerpts; and the title often appears more than once in

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at Bobbio and if the manuscript should be identified as the one described in a letter to Archbishop Adalbero. For a full discussion of these matters see chapter 5.

[141] The antiquity of the illuminations of the *agrimensores* is uncertain. It is possible that they are only as old as the Arcerianus. Campbell, *Roman Land Surveyors*, xxv–xxvi. The most thorough examination of the pictures remains James Nelson Carder's published dissertation: *Art Historical Problems of a Roman Land Surveying Manuscript: The Codex Arcerianus A Wolfenbüttel* (New York: Garland Publishing, 1978).

[142] This is the list given by Julius Frontinus (*De arte mensoria*; ed. and trans. Campbell in *The Writings of the Roman Land Surveyors*, 14–15.

[143] Bamberg, Msc.Class.55, fols. 7–7v. On the proliferation of this kind of figure see chapter 5.

[144] Bamberg, Msc.Class.39, fols. 156v–158.

contemporary library lists.[145] Both Abbo and Gerbert quoted the work. The annotated copy presented to Otto III was, in fact, written at Fleury in the ninth century and annotated at Reims at the end of the tenth. The appeal of the treatise lay in Martianus's chosen genre. The *De nuptiis* is a complex allegory, a didactic love story. Martianus describes the nuptials of Philology, the love of learning, and Mercury, god of communication and eloquence. The liberal arts are presented as a gift from Mercury to his new bride. Like Philology herself, the arts are personified. Each holds forth on her discipline in turn, before a bawdy and distractible audience of Greek and Roman gods and intellectuals.

The personified arts' self-contained speeches covered much of what was known about each subject, though they sometimes skipped over basic terminology, assuming a certain level of knowledge on the part of readers. By the tenth century, any such lacunae had been filled by Carolingian annotators, notably John the Scot (Eriugena; died c.877) and Remigius of Auxerre (c.841–c.908).[146] Gerbert, Abbo, and their contemporaries generally read Martianus's work through the lens of this ninth-century material and continued the tradition of annotation. Images were not an organic part of the *De nuptiis*, yet many copies of the treatise contain graphic elements sketched in the margins or gathered together at the end. A set of ten astronomical figures created to enhance an anonymous ninth-century gloss became quite standard, having been assimilated into Remigius's much-circulated commentary.[147] The group included simple figures—a spoked wheel demonstrating the relationship between the

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[145] Over eighty manuscripts from the 9th through the 11th c. contain some part of the *De nuptiis*. Claudio Leonardi, who assembled a handlist of complete copies and excerpts, noted that of the fifty or so complete copies of the *De nuptiis*, over half could be dated to the 9th and 10th c. Bruce Eastwood stressed that most of the 9th-c. copies are from the latter half of the century. Leonardi, "I Codici de Marziano Capella," *Aevum* 33, no. 5/6 (1959): 462; Bruce Eastwood, *Ordering the Heavens: Roman Astronomy and Cosmology in the Carolingian Renaissance* (Leiden: Brill, 2007), 179–303.

[146] On the commentaries see Ilaria Ramelli, ed., *Tutti i commenti a Marziano Capella: Scoto Eriugena, Remigio di Auxerre, Bernardo Silvestre e Anonimi* (Milano: Bompiani, 2006); On the oldest (c.830) anonymous annotations see the collaborative work overseen by Mariken Teeuwen (http://martianus.huygens.knaw.nl/path) and her summary article of the project: eadem, "The Pursuit of Secular Learning: The Oldest Commentary Tradition on Martianus Capella," *Journal of Medieval Latin* 18 (2008): 36–51. Cora Lutz edited the commentary of John the Scot, *Johannis Scotti Annotationes in Marcianum* (Cambridge: Medieval Academy of America, 1939), and also wrote on Remigius of Auxerre's commentary: eadem, "The Commentary of Remigius of Auxerre on Martianus Capella," *Medieval Studies* 19 (1957): 137–56.

[147] Bruce Eastwood and Gerd Graßhoff, *Planetary Diagrams for Roman Astronomy in Medieval Europe, ca. 800–1500* (Philadelphia: American Philosophical Society, 2004), 8. The so-called Anonymous Commentary, preserved in around twenty manuscripts, has been attributed to Dunchad and Martin of Laon, but is currently considered to have been a co-operative endeavor. The full text of the Anon. Commentary is edited online by Mariken Teeuwen, et al. http://martianus.huygens.knaw.nl/path/the_manuscripts/descriptions/leiden_vossianus_48.

size of an arc and the length of a radius—as well as more complex clusters of figures picturing different models of planetary motion (figs. 1.56–57).[148] In the case of Bamberg, Msc.Class.39, the Reims scribe filled the blank margins flanking *Geometria*'s definitions of basic geometric forms (e.g., a plane angle, square, circle, parallel lines) with figures adapted from contemporary geometric compilations.[149]

If Otto III, having absorbed the lessons of the Boethian corpus and the *De nuptiis*, sought a more in-depth treatment of the quadrivium, he could go to the very source of numeric theory: Plato's *Timaeus*. He may have had at hand a handsome copy of this work, Bamberg, MS Msc.Class.18, which included some sophisticated graphic glosses associated with Gerbert and his intellectual circle (figs. 1.58–60).[150] The *Timaeus* was the only work of Plato continuously available to the Latin West.[151] The dialogue took its title from its primary speaker, Timaeus, a fictive character well-versed in astronomy and devoted to the study of the nature of the universe. Timaeus's wide-ranging description of the cosmos embraced

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[148] For a more detailed explanation of this figure see Bruce Eastwood, "The Power of Diagrams: The Place of the Anonymous Commentary in the Development of Carolingian Astronomy and Cosmology," in *Carolingian Scholarship and Martianus Capella: Ninth-Century Commentary Traditions on* De nuptiis *in Context*, ed. Mariken Teeuwen and Sinéad O'Sullivan (Turnhout: Brepols, 2011), 200–201, especially n. 23. The full set of figures was included at the end of Paris, BnF MS lat. 8669, fols. 121v–122v. The ninth-century manuscript, possibly from Soissons, is described by Claudio Leonardi, "I Codici di Marziano Capella," 436–37 (no. 160).

[149] The verbal definitions are derived from Euclid's *Elements.*

[150] Bamberg, SB MS Msc.Class.18 was probably produced in Würzburg. Michel Huglo believed that Gerbert used this copy in writing his treatise on organ pipes. A copy of the treatise almost certainly existed at Reims or in the vicinity. Valenciennes, BM MS 293, was written at Reims at the end of the 9th c. (during the abbacy of Fulk, when Hucbald was teaching at Reims). The manuscript was in the possession (perhaps copied for the esteemed master) of Hucbald at the time of his death in 930. It seems unlikely that he would have taken Reims's only copy with him. The graphic annotations added to Bamberg, Msc.Class.18 derive from a much more extensive 10th-c. commentary best preserved in Brussels, BRB MS 9625–26. This manuscript and its unusual gloss is discussed in depth by Anna Somfai, who associated it with Gerbert and his intellectual circle. Mütherich, "Library of Otto III," 21; Huglo, "Gerberto, teorico musicale, visto dall'anno 2000," in *Gerberto d'Aurillac da Abate di Bobbio a Papa dell'Anno 1000*, ed. Flavio Nuvolone (Bobbio: Associazione culturale Amici di *Archivum Bobiense*, 2001), 229; McKitterick, "Knowledge of Plato's *Timaeus* in the Ninth Century: The Implications of Valenciennes, Bibliothèque Municipale MS 293," in *From Athens to Chartres: Neoplatonism and Medieval Thought: Studies in Honour of Edouard Jeauneau*, ed. Jan Westra Haijo (Leiden: Brill, 1992), 85–96, esp. 90–91; Anna Somfai, "The Brussels Gloss: a Tenth-century Reading of the Geometrical and Arithmetical Passages of Calcidius's *Commentary* (ca. 400 AD) to Plato's *Timaeus,*" in *Scientia in margine: Études sur les marginalia dans les manuscrits scientifiques du moyen âge à la renaissance*, ed. D. Jacquart and Charles Burnett (Geneva: Droz, 2005), 139–69.

[151] The whole of the *Timaeus* was not available during the middle ages, but two partial translations of the work did circulate: that of Cicero (27D–47B with some passages omitted; the numbering of the *Timaeus* refers to pages of an early printed edition: Stephanus, 1578) and Calcidius (17A–53C). Cicero's translation is preserved in only seven copies (from the early and central middle ages); Calcidius's survives in twenty-seven copies suggesting that it was the more popular and influential of the two. See Anna Somfai, "The Eleventh-Century Shift in the Reception," 1–21, esp. 8.

topics as diverse as the four elements, the relationship between the immortal soul and the mortal body, the mechanics of vision, and the nature of space. Creation, according to Plato, was the work of a supreme divinity, which ordered disorder, fashioning the universe in its own image according to numeric rationale. The Platonic notion of number as divine idiom and the basis of materiality became, as we have seen, the dominant theory of Creation, mapped by early Christian thinkers onto the biblical account of the Old Testament.[152]

During the central middle ages the *Timaeus* was known primarily through the translation and commentary of Calcidius, an obscure fourth-century author.[153] The dialogue had had other commentaries before it, but Calcidius's proved most enduring, possibly because he bound his explanation to the disciplines of number.[154] At the outset of the *Commentary*, Calcidius observed that even the ancients found the *Timaeus* difficult. He explained that the difficulty arose not from Plato's words but from the mistaken rationale with which the work had been approached.[155] As a treatise about number, the *Timaeus*, Calcidius insisted, could only be comprehended through the quadrivium.[156] Much of the *Commentary* is given over to shoring up Plato's often cursory statements on the mechanics of the universe with systematic arguments that were grounded in the numeric disciplines, many of which pivoted on graphic demonstrations.[157]

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[152] The handful of changes made by Hadoard, *custos* of Corbie (c. 850), to excerpts from the Ciceronian translation of the *Timaeus* (e.g., he changed *dei* to *deus*, *gignere* to *creare*) preserved in the so-called *Collectaneum Hadoardi* (Vatican City, BAV Reg. lat. MS 1762) show how easily the work could be redacted for a Christian audience; Somfai, "Shift in Reception," 11–12.

[153] From the 11th c. Calcidius was identified as the deacon or archdeacon of Ossius (c.257–c.357), Bishop of Cordoba; ibid., 12. Twenty-four manuscripts (with the *Commentary* and *Timaeus*) survive from the period between 975 and 1125; fifteen from the 11th c. alone. Many of these are glossed, showing lively contemporary interest. On the early impact of Calcidius see Paul Dutton, "Medieval Approaches to Calcidius," in *Plato's* Timaeus *as a Cultural Icon*, ed. Gretchen J. Reydams-Schils (Notre Dame: University of Notre Dame Press, 2003), 183–84; and Anna Somfai, "The Transmission and Reception of Plato's *Timaeus* and Calcidius's *Commentary* during the Carolingian Renaissance" (PhD Diss., University of Cambridge, 1998).

[154] The Calcidian commentary is based on the lost work of Adrasto Afrodisia (2nd c. CE); Claudio Moreschini, in Calcidius, *Commentario al Timeo di Platone* (Milano: Bompiani, 2003), ix.

[155] Calcidius, *Commentary* 1.1. Calcidius gives two clear examples: (to paraphrase) if one has some confusion regarding the movements of the stars, one would look to the discipline of astronomy; if about tones, to music; if illness, to medicine.

[156] Calcidius, *Commentary* 1.1–2. He did not refer to the numeric disciplines by this term.

[157] Calcidius did not translate the whole of the *Timaeus* (though he may have intended to). He omitted later sections on the geometric structure of the elements, the material basis of sensations, and the workings of the human body (some of these topics, however, were woven into his *Commentary*). He also did not address all aspects of what he did translate evenly: certain sections are passed over entirely, others expounded in great depth.

The *Commentary* included twenty-seven figures, the majority of which can be classed as cosmological. Calcidius pictured the divisions of the heavens and the order and orbits of the planets (figs. 1.61–63). He crafted visual explanations for the varying length of the seasons and eclipses of the sun and moon (figs. 1.64–69). He explained the disappearance of objects over the horizon geometrically, due to the sphericity of water and the earth (fig. 1.70), and demonstrated the mechanics of epicycles and how this kind of motion caused the appearance of uneven or erratic motion of the planets when seen from the earth (figs. 1.71–75).[158] Several figures were devoted to the four elements. Calcidius sought to demonstrate mathematically the process of Creation, especially how fire and earth were joined by means of two intermediate elements, air and water (figs. 1.58–60 and 1.76–78).[159] These figures inspired the most detailed annotations in Msc.Class.18. Indeed, the commentator reworked the Calcidian figures, adding a number of new figures of his own (figs. 1.58–60).[160]

The three *psicogoniae* figures were perhaps the most successful images in the *Commentary*, at least in terms of their active reuse by medieval scribes and authors (figs. 1.79–81).[161] Scholars now typically refer to these figures as *lambda* diagrams on account of their resemblance to the Greek letter. The *psicogoniae* figure was, in its simplest manifestations, an inverted **v** with the Roman numeral for one at its apex and various, interrelated numeric sequences running down its sides.[162] Abbo wrote admiringly that the *psicogoniae* made "manifest" (*manifestat*) the whole discourse on the nature of unity.[163] At Fleury and elsewhere such figures were systematically drawn into the margins of another essential quadrivial text: Macrobius's *Commentary on the Dream of Scipio* (figs. 1.82–85).[164]

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[158] The cosmological figures are discussed in depth by Eastwood, *Ordering the Heavens*, 313–72; and Eastwood and Graßhoff. *Planetary Diagrams*, 73–116.
[159] Barbara Obrist, *La cosmologie médiévale: Textes et images*, Vol. 1, Les fondements antiques (Florence: SISMEL edizioni del Galluzzo, 2004), 263–73; and Anna Somfai, "The Concept of *Analogia*," 203–20.
[160] These are discussed in chapter 5.
[161] Descriptions of all three figures can be found in Michel Huglo, "La réception de Calcidius," 4.
[162] In its most basic form the progression of even numbers—2, 4, 8—runs down the left side of the figure and odd—3, 9, 27—down the right side. Abbo credited Plato with the figure's invention, though it was not part of the *Timaeus*, but of Calcidius's *Commentary*. This kind of figure (there are many variants) was probably first devised by the Greek philosopher Crantor (*fl.*3rd and 4th c. BCE). Somfai, "The Concept of *Analogia*," 213; described also by Harry Bober, "*In principio*. Creation before Time," 16.
[163] Abbo, *Explanatio* 3.2; ed. Peden, 73–74; all translations of the *Explanatio* are my own unless otherwise cited: "Idque manifestat praedicta *psicogoniae* figura, arithmeticae, geometricae, musicae et astronomiae subtilitatibus contenta ac hoc modo mirabiliter expressa."
[164] Adjacent to a passage citing the *Timaeus* and describing the simple figure: Macrobius, *Commentary* 6.46.

Macrobius's *Commentary* was a staple of scientific study.[165] Abbo carried a copy with him when he went to teach at Ramsey in England.[166] At Fleury the tract was required reading and mass produced for use in the school. Six copies survive from the monastery and scholars speculate that there were more.[167] A Roman senator active in the early fifth century, Macrobius had based his wide-ranging commentary on the sixth and only surviving book of Cicero's *De re publica* known as the *Somnium Scipionis*.[168] The *Somnium* described the dream-ascent of the Roman statesman Scipio Africanus Minor from earth to the empyrean. Cicero's words served as stepping-off points to address an array of diverse topics, from types of dreams and their meanings to the structure and stuff of the cosmos. Macrobius embroidered Scipio's journey with technical information: the nature of unity and numbers; the spherical shape of the Earth, its measure and divisions; the distances of the planets, their orbits and harmony; the relationship between body and soul, and the notion of man as a microcosm—to name just a few of the many topics touched on in the work.[169] Macrobius quoted and paraphrased the *Timaeus* at length and covered much of the same ground as Calcidius. Abbo considered the two authors equal authorities on number. Both commentaries could be made to serve any of the quadrivial disciplines. The *Commentary on the Somnium* was, however, shorter and better suited to beginners.

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[165] Forty-five manuscripts and fragments of the *Commentary* were produced between the 9th and 11th c. and survived to the 19th c. Thirty-one of these were dated to the 11th c. Not all are extant. For a detailed account of the medieval manuscript tradition see Bruce Barker-Benfield, "Macrobius," in *Texts and Transmission: A Survey of the Latin Classics*, ed. L. D. Reynolds (Oxford: Oxford University Press, 1983), 222–32; and idem, "The Manuscripts of Macrobius' *Commentary* on the *Somnium Scipionis*" (PhD diss., Oxford University, 1975).
[166] Perhaps Paris, BnF lat. MS 7299. On this and other manuscripts that Abbo is thought to have brought with him to Ramsey, see chapter 4.
[167] These include Paris, BnF lat. MS 16677; Paris, BnF lat. MS 6365; Paris, BnF lat. MS 8663; Paris, BnF lat. MS 16678, fols. 1–8 + lat. MS 6620 (missing figures); and Leiden, BR Voss. lat. fol. MS 12β + Voss. lat. fol. MS 122 + London, BL Royal MS 15 B xii, fols. 1–2. Bruce Barker-Benfield, who undertook the most thorough investigation of medieval copies of Macrobius's *Commentarii* to date, has shown that "more extant copies were owned by Fleury than any other medieval center" and links the "reduplication of complete texts" to their use as teaching tools. Bruce Barker-Benfield, "A Ninth-Century Manuscript from Fleury: *Cato de senectute cum Macrobio*," in *Medieval Learning and Literature*, ed. J. Alexander and M. Gibson (Oxford: Oxford University Press, 1976), 145–65, esp. 156.
[168] On the identity of Macrobius, see Alan Cameron, "The Date and Identity of Macrobius," *The Journal of Roman Studies* 56 (1966): 25–38.
[169] For a comprehensive list of the extraordinary number of topics covered by Macrobius, see William Harris Stahl, in *Commentary on the Dream of Scipio by Macrobius*, 39–40.

The tract's pedagogical potential was enhanced through annotation, verbal and visual.[170] As already noted, the Calcidian *lambda* figure was frequently added in the margin beside its verbal description. Likewise, the verbal definitions of points, lines, planes, solids were pictured in the margins (figs. 1.86–87), just as polygons were drawn beside their descriptions in the Bamberg copy of the *De nuptiis* (MS Msc.Class.39) discussed above. Macrobius had furnished his treatise with five figures showing the order of the planets, the movement of bodies toward the Earth, the division of the Earth into five climatic zones and the relationship of these zones to those of the heavens, and the distribution of land and water over the surface of the Earth (figs. 1.88–92).[171] These images were deeply integrated into the text of the *Commentary* and essential to its arguments.[172] They also became fixtures in miscellanies made for the science of time-reckoning or computus.[173] Abbo included all five figures in his *Computus*, best preserved in Berlin, Staatsbibliothek, MS 138 (figs. 1.93–94).[174]

Much of what we might now designate as astronomical, cosmological, geographical, and geometrical, was subsumed, in late antiquity and the early middle ages, under the heading of computus.[175] Synchronizing the calendar was a secular and theological imperative for Charlemagne in governing his vast empire. His *Admonitio generalis* (*General Warning*) of 789 made the study of computus a requirement for all those educated in monastery and

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[170] Alison Peden identified a set of related glosses that she dubbed the "Theoprotus" set. Even within the set, the commentary remained fluid and there are variations among manuscripts. Alison Peden (née White), "Glosses Composed before the Twelfth Century in Manuscripts of Macrobius' *Commentary* on Cicero's *Somnium Scipionis*" (PhD diss., Oxford University, 1981), 16–33.
[171] The Macrobian figures are described in Eastwood, *Ordering the Heavens*, 31–94; Eastwood and Grasshoff, *Planetary Diagrams*, 49–72; and Obrist, *La cosmologie médiévale*, 180–194 and 307–10.
[172] Macrobius does not merely state that a figure will follow, but provides instruction for its construction. Most of the figures include letters which were used in the text to indicate points. The special relationship between this kind of "lettered diagram," a fixture of ancient geometry, and its accompanying text is touched on in chapter 4.
[173] Steven C. McCluskey, *Astronomies and Cultures in Early Medieval Europe* (New York: Cambridge University Press, 1998), 117.
[174] On Abbo and computus see chapters 2 and 4. For a full description of this manuscript and its contents see Valentine Rose, *Die Handschriften-Verzeichnisse der königlichen Bibliothek zu Berlin*, *Verzeichnis der lateinischen Handschriften* (Berlin: A. Asher, 1893), 308–15.
[175] Discussed in chapter 2. The most complete discussion on the place and nature of computus in Latin-speaking Europe remains Arno Borst, *Computus: Zeit und Zahl in der Geschichte Europas* (Berlin: Verlag, 1999); for an up-to-date bibliography on the subject see Immo Warntjes, *The Munich Computus: Text and Translation: Irish Computistics between Isidore of Seville and the Venerable Bede and Its Reception in Carolingian Times* (Stuttgart: Steiner, 2010), xxxiii–xxxiv.

cathedral schools.[176] Spurred by official requirement, computus was vigorously explored and expanded from the eighth through the eleventh centuries. Interest is evidenced in the vast number of manuscripts or parts of manuscripts devoted to the subject that were produced during this period.[177] Key texts for the study of computus were written by the great English intellectual Bede (c.673–735), a monk of the Northumbrian monastery of Wearmouth-Jarrow. Bede's *De temporibus* (*On Times*) and the more thoroughgoing *De temporum ratione* (*On the Reckoning of Time*) largely determined the content of computus for subsequent centuries.[178] The *De temporum ratione* defined and explored time on all scales from the briefest *momentum* to the Six Ages of the world.[179] Bede explained in detail the mechanics of calendar reckoning and introduced readers to the tools and formulas requisite for accurately determining the date of Easter and setting the liturgical calendar. Most essential to this endeavor was the Julian solar calendar with its associated tables and formulae, and the Paschal table. These items were the defining core of a computus manuscript, but they were accompanied by an ever-evolving corpus of auxiliary material on time, which included a significant number of figures drawn from a variety of sources.[180]

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[176] This mandate was followed in 805 by an official notice—a list of fifteen items—entitled, "What Priests Should Learn" (*Quae a presbyteris discenda sint*). Study of computus was number eight on the list. John Contreni and Faith Wallis provide an overview and commentary on the effect of the Carolingian reforms on computus in these articles and elsewhere: Contreni, "Counting, Calendars, and Cosmology: Numeracy in the Early Middle Ages," in *Word, Image, Number: Communication in the Middle Ages*, ed. John J. Contreni and Santa Casciani (Turnhout: Sismel, 2002), 47–48; Wallis, "'Number Mystique' in Early Medieval Computus Texts," in *Mathematics and the Divine: A Historical Study*, ed. T. Koetsier and L. Bergmans (Amsterdam: Elsevier, 2005), 184.

[177] Historian of science Wesley M. Stevens estimated that around 1,500 surviving manuscripts dating to the 8th through the 10th c. contain "numeric" material, much of it tied to the calendar. Faith Wallis has noted the high level of production of Bede's *De temporum ratione* in the 8th and 9th c. and ties this to teaching. That the work was actively used is clear from regular modifications that kept it up to date. Stevens, *Cycles of Time and Scientific Learning in Medieval Europe* (Aldershot: Variorum, 2005), vii; Wallis in Bede, *The Reckoning of Time* (Liverpool: Liverpool University Press, 2004), lxxxix.

[178] One, the other or even both of these works or excerpts from them were included in most computus manuscripts. Indeed, where they are absent, as in two Fleury computus manuscripts, Berlin 138 and Bern BB Cod. 250, one can assume that a separate copy was owned by the institution.

[179] Bede, *De temporum ratione* 3 and 66.

[180] Computus compilations varied in their contents. Two highly influential compendia known as the three-book and seven-book computus were the product of a conference on calendar reform called by Charlemagne in 809 and served as models. A once-complete copy of the seven-book computus,Vatican City, BAV lat. 645, was made at or near Reims (830–860). See Arno Borst, "Alcuin und die Enzyklopädie von 809," in *Science in Western and Eastern Civilization in Carolingian Times*, ed. Paul Leo Butzer and Dietrich Lohrmann (Basel: Birkhäuser Verlag,1993), 53–78.

In Berlin 138, the calendar and its associated tables are followed by five folios replete with figures, many of them cosmological (fols. 1.93–100).[181] Timekeeping was, of course, tied to the movements of the sun, moon, and planets. For this reason computus manuscripts contained a great deal of astronomical material including extracts from Pliny the Elder's *Naturalis Historia* (*Natural History*).[182] The first-century encyclopedist had quantified celestial order. Thus, his descriptions of the physical arrangement of the planets and the paths and periods of their "orbits" (*apsides*) were deemed indispensable for time-reckoning as were the figures that had been contrived by ninth-century glossators to accompany these extracts (figs. 1.101–104). [183] In Berlin 138 some of these share space with the five Macrobian figures and thirteen cosmological images from Calcidius's *Commentarius* (figs. 1.93, 1.97–100). Like the Plinian figures, the images from Macrobius's *Commentarii* were standard computus fare. Given this and the heavy use of Macrobius's work at Fleury, their inclusion by Abbo is unsurprising. The Calcidian figures, on the other hand, were an unusual addition and reflect Abbo's personal interests as well as a growing engagement with Calcidius, which reached its peak in the eleventh century.[184]

Following the figures from Pliny, Macrobius, and Calcidius in Berlin 138, are two circular figures from Isidore of Seville's *De natura rerum* (*On the Nature of Things*; fig. 1.99).[185] Unlike the Calcidian figures, these *rotae* would have been instantly recognizable to

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[181] Berlin 138, fols. 35r–39v.
[182] These extracts were drawn from Book Two, *On the Heavens*. Major studies of the Plinian excerpts were undertaken by Karl Rück, *Auszüge aus der Naturgeschichte des C. Plinius Secundus in einem astronomisch-komputistischen Sammelwerke des achten Jahrhunderts* (Munich: Straub, 1888); and Vernon H. King, "An Investigation of Some Astronomical Excerpts from Pliny's *Natural History* Found in Manuscripts of the Earlier Middle Ages" (Bachelor's thesis, Oxford University, 1969).
[183] On these figures see Bruce Eastwood, "Plinian Astronomical Diagrams in the Early Middle Ages," in *Mathematics and Its Applications to Science and Natural Philosophy in the Middle Ages: Essays in Honor of Marshall Clagett*, ed. E. Grant and J. E. Murdoch (New York: Cambridge University Press, 1987), 141–72, esp. 144; idem, *Ordering the Heavens*, 5 and 95–178; and Eastwood and Graßhoff, *Planetary Diagrams*, 23–48.
[184] Bruce Eastwood, assessing the inclusion and its significance wrote, "The unusual choice, for a computus, of such elements [the Calcidian figures] means an increased interest in reasoned, or geometrical, explanation of certain entities such as the differing lengths of the seasons and the retrograde motions of the planets." Eastwood, "Calcidius's *Commentary* on Plato's *Timaeus* in Latin Astronomy of the Ninth to Eleventh Centuries," in *Between Demonstration and Imagination: Essays in the History of Science and Philosophy Presented to John D. North*, ed. Lodi Nauta and Arjo Vanderjagt (Leiden: Brill, 1999), 171–209, esp. 178–86.
[185] Above the two *rotae* is an *horologium* explained by Barbara Obrist, "Les tables et figures Abboniennes dans l'histoire de l'iconographie des recueils de comput," in *Abbon de Fleury: Philosophie, sciences et comput autour de l'an mil*, ed. B. Obrist (Paris: C.N.R.S. and E.P.H.E., 2006), 178–82.

a contemporary audience even without their corresponding verbal explanations.[186] The *De natura rerum* was in regular and widespread circulation from the time of its writing in the early seventh century through the central middle ages.[187] Much essential information could be gained from the slim work, concerning, in the words of its author, "The nature or causes of things ... the disposition of the heavens, the virtues of the elements, the divisions of time, the cycle of the year, and the articulation of the stars."[188] The tract was pedagogical in intent. Isidore, who served as unofficial tutor to the Visigothic kings, dedicated the treatise to his, "king and son," the young Sisebut, who ruled from 612–21.[189] Echoing the gesture of Isidore, Gerbert presented a ninth-century copy to Otto III (Bamberg, Staatsbibliothek, MS Msc.Nat.1), inscribed with a poem praising the emperor (figs. 1.105–06).[190] Otto may have owned a second, older copy of the work, Bamberg, Staatsbibliothek, MS Msc.Patr.61, made in Montecassino at the end of the eighth century.[191]

Six books of the *De natura rerum* include graphic elements announced in the text.[192] The figures represent the uneven length of the months, the characteristics of the four seasons,

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[186] It will be noted that Abbo included the verbal descriptions from Macrobius (in and around the figures). Redacted excerpts from Calcidius (fols. 37r–37v) were included with the Calcidian figures. The *rotae* from the *De natura rerum*, on the other hand, are entirely unmoored from the accompanying passages.
[187] Jacques Fontaine, the modern editor of Isidore's *oeuvre* including the *De natura rerum*, dated the tract to between 612 and 615. 7th-c. copies (from Fleury and Cambrai) show that the work was copied almost immediately after its writing. Thirty-three copies date from the 7th through the 9th c., seventeen of which are pre-Carolingian. The diverse origins of early copies, written in what is now southern Germany, Austria, and northern Italy—show broad dissemination of and demand for the work. Fontaine, in *Isidore de Séville: Traité de la nature*, 69–85. Fontaine's *stemma* and list of early manuscripts was updated by Michael Gorman, "The Diagrams in the Oldest Manuscripts of Isidore's *De Natura Rerum*," *Studi medievali* 42 (2001): 534–36 and 540–41.
[188] Isidore, *De natura rerum* Praef. 1.1–2, ed. and trans. Jacques Fontaine (Bordeaux: Féret, 1960), 167–69; my translation: "Ex rerum natura vel causis ... Ipse mihi dedit horum quae sunt scientiam veram, ut sciam dispositionem coeli, et virtutes elementorum, conversionum mutationes, et divisiones temporum, annorum cursus, et stellarum dispositiones."
[189] Jacques Fontaine, *Isidore de Séville: Genèsee et originalité de la culture Hispanique au temps des Wisigoths* (Turnhout: Brepols, 2000), 129–43.
[190] Bamberg, Msc.Nat.1, fol. 1r: *Otto vir invictus laetetur caesar amatus* (*Otto, unconquered hero, beloved emperor, rejoices*). Mütherich wrote that the "florid style of the dedication verses" suggests Gerbert's authorship. "Library of Otto III," 19; the line is reproduced in MGH *Poetae* 5, 1.2, 476 (www.mgh.de/dmgh/resolving/MGH_Poetae_5,1.2_S._476, accessed Sept. 22, 2014). The manuscript is described in Suckale-Redlefsen, *Die Handschriften*, 24–26 (no. 18).
[191] On Otto III's ownership of Bamberg, Msc.Patr.61 see Mütherich, "Library of Otto III," 21. Descriptions of the manuscript can be found in Suckale-Redlefsen, *Die Handschriften*, 3–10 (no.1); and E. A. Lowe, *Codices latini antiquiores: A Palaeographical Guide to Latin Manuscripts Prior to the Ninth Century*, vol. 8 (Oxford: Clarendon Press, 1959), 143.
[192] These are seven in number: book 4: *De mensibus*; book 7, *De temporibus*; book 10, *De quinque circulis mundi*; book 11, *De partibus mundi* (two figures); book 23, *De positione septem stellarum errantium*; book 37,

the Earth's climatic zones, qualities of the four elements, and the position of the planets and the duration of their orbits (figs. 1.107–13).[193] In MS Msc.Nat.1 they dominate the page, filling large breaks in the textblock. In many copies of the treatise, among them MS Msc.Patr.61, the *rotae* were also colored with eye-catching hues (figs. 1.114–15.). So prominent were the figures that King Sisebut called the treatise the "book of wheels" (*liber rotarum)* on account of their circular shape.[194] The moniker stuck and was used to refer to the *De natura rerum* throughout the middle ages.[195] Indeed, the relationship between the *De natura rerum* genre and Isidore's *rotae* was so strong that the circular figures were inserted into copies of Bede's treatise of the same name.[196]

In the deluxe copy in Bamberg, MS Msc.Patr.61, the *De natura rerum* is preceded by Flavius Magnus Aurelius Cassiodorus Senator's (c.484–c.584) *Institutiones divinarum et saecularum litterarum* (*Institutions of Divine and Secular Learning*).[197] Unlike the works discussed above, the *Institutiones* was rarely referenced by schoolmasters in the central middle ages, nor was it copied in numbers that suggest regular use in the schools. Still, it had a significant impact since it largely determined the quadrivial canon. Cassiodorus wrote the

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De nominibus ventorum. This last *rota* is not named in the text and may not have been part of the original treatise.

193 In contrast to many quadrivial images, the figures in the *De natura rerum* have attracted the attention of modern scholars and historians of art. See, especially, Bianca Kühnel, "Carolingian Diagrams, Images of the Invisible," in *Seeing the Invisible in Late Antiquity and the Early Middle Ages*, ed. Giselle de Nie, Karl Frederick Morrison, and Marco Mostert (Turnhout: Brepols, 2005), 359–89; eadem, *The End of Time in the Order of Things: Science and Eschatology in Early Medieval Art* (Regensburg: Schnell & Steiner, 2003), 123–36; Bruce Eastwood, "The Diagram of the Four Elements in the Oldest Manuscripts of Isidore's *De Natura Rerum*," *Studi Medievali* 42, no. 2 (2001): 547–64; Michael Gorman, "The Diagrams in the Oldest Manuscripts of Isidore's *De Natura Rerum*"; Barbara Obrist, *La cosmologie médiévale*, 273–83; eadem, "Wind Diagrams and Medieval Cosmology," *Speculum* 72, no. 1 (1997): 33–84; eadem, "Le diagramme isidorien des saisons, son contenu physique et les représentations figuratives," *Mélanges de l'Ecole française de Rome. Moyen-Age* 108, no. 1 (1996): 95–164; Fontaine in Isidore, *Traité de la nature*, 15–18; Bernard Teyssèdre, "Un Example de survie de la figure humaine dans les manuscrits précarolingiens: Les Illustrations du *De natura rerum* d'Isidore," *Gazette des beaux-arts* 56 (1960): 19–34.

194 Sisebut wrote a poem *De libro rotarum* that often circulated with the *De natura rerum.* Fontaine included the poem in his edition (*Traité de la nature*, 151–61). It should be noted that one of the Isidorian figures is not a *rota*.

195 Thus, the library lists of St. Riquier (831) and St. Gall (9th c.) include the "wheels (*rotarum*) of Isidore." Becker, *Catalogi*, 26 (no. 11), 34 (no. 15), and 47 (no. 22).

196 Kühnel, *The End of Time*, 99, n. 159; and Bober, "An Illustrated Medieval School-Book."

197 On the transmission and influence of the *Institutiones* see Louis Holtz, "Quelques aspects de la tradition et de la diffusion des *Institutions*," in *Atti della Settimana di studi su Flavio Magno Aurelio Cassiodoro: Cosenza-Squillace, 19–24 settembre 1983*, ed. Sandro Leanza (Soveria Mannelli: Rubbettino, 1983), 281–312; Leslie W. Jones, "The Influence of Cassiodorus on Mediaeval Culture," *Speculum* 20 (1945): 433–42; idem, "Further Notes concerning Cassiodorus' Influence on Mediaeval Culture," *Speculum* 22 (1947): 254–56; and Mark Vessey, "Introduction," in *Cassiodorus:* Institutions of Divine and Secular Learning *and* On the Soul, trans. James W. Halporn (Liverpool: Liverpool University Press, 2004), 1–97.

Institutiones primarily for the monastic community of Vivarium, which he had established on a family estate in the southwest corner of Italy.[198] The tract was intended as a guide to the library, listing works in the collection that he considered authoritative on particular topics. As its title makes clear, the work was divided into two distinct parts: the first focusing on scripture, the second on the liberal arts.[199] Rudimentary knowledge of the quadrivial disciplines could be found in the second book, which circulated independently.[200]

Though Cassiodorus emphasized his role as redactor and conduit, he did more than copy the works of his predecessors: he epitomized, edited, and arranged—ordering knowledge into complex hierarchies of ideas and things.[201] He gave these hierarchies spatial expression in thirty-seven *schemata* or *stemma*.[202] The *schemata* resemble complex root systems, expanding and dividing, their ramifications sometimes even criss-crossing as they run down the page. Bamberg, Msc.Patr.61 is the oldest and most elaborately illuminated witness of the *Institutiones*.[203] In it*, schemata* emerge from the mouths of ornamental amphorae and bellies and beaks of a menagerie of vividly pigmented birds and beasts (figs. 1.116–17).[204] Here, Otto III would have encountered the four disciplines of number tied to the paws of a fierce looking leopard and the divisions of number legated to the beak and

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[198] Near Squillace, in modern-day Calabria, Italy.

[199] For a detailed summary of the contents and rhetoric of both books see Vessey, "Introduction," 42–63 and 75.

[200] The *Institutiones saecularum* often appeared on its own in medieval manuscripts and in catalogues where it was titled *De septem artibus* or *De VII liberalibus artibus*, as, for example, in the 9th-c. booklist of St. Gall and the 12th-c. booklist of St. Bertin (Becker, *Catalogi*, 35 [no. 15], 182 [no. 77]). The theory that *De septem artibus* was conceived of as a complete and separate entity (rather than an early draft that somehow made its way into circulation apart from the final, two-book work) was first suggested by Fabio Troncarelli, *Vivarium: I libri i destini* (Turnhout: Brepols, 1989), 12–21.

[201] Cassiodorus, *Institutiones divinarum et saecularum litterarum* Praef. 1, ed. R. A. B. Mynors (Oxford: Clarendon Press, 1937), 3–4.

[202] On the graphic aspects of the *Institutiones* see Giulia Orofino, "Da Montecassino a Nonantola: La tradizione illustrativa delle Institutiones di Cassiodoro," in *Il monachesimo italiano dall'età longobarda all'età ottoniana, sec. VIII-X: Atti del VII Convegno di studi storici sull'Italia benedettina, Nonantola (Modena), 10-13 settembre 2003*, ed. Giovanni Spinelli (Cesena: Badia di Santa Maria del Monte, 2006), 553–67; Michael Gorman, "The Diagrams in the Oldest Manuscripts of Cassiodorus' *Institutes*," *Revue Bénédictine* 110 (2000): 27–41; and Troncarelli, *Vivarium*, 67–78; idem, "Una pietà più profonda. Scienza e Medicina nella cultura monastica medievale Italiana," in *Dall'eremo al cenobio: La civilita monastica in Italia dalle origini all'età di Dante*, ed. G. P. Carratelli (Milan: Libri Scheiwiller, 1987), 703–27.

[203] The ultimate source of these *schemata* is unknown. Various linear classificatory schemes appear as *scholia* in early copies of Greek logical and mathematical tracts. See Robert S. Brumbaugh, "Logical and Mathematical Symbolism in the Platonic *Scholia*." *Journal of the Warburg and Courtauld Institutes* 24, no. 1/2 (1961): 45–58; and William Chase Greene, *Scholia Platonica* (Haverford, PA: Philological Society of America, 1938).

[204] Bamberg, Msc.Patr.61 is one of a handful of early copies of the *Institutiones*, which include these playful, varied, and elaborate figures. These include Paris, Bibliothèque Mazarine MS 660, fols. 75–142 (10th c.); San Gall, StiftsB. Cod. 855, fols. 180–351 (9th c.); London, BL Harley MS 2637, fols. 1–52 (9th c.); Karlsruhe, BLB Augiensis MS 214 (late 9th c.). Mynors in Cassiodorus, *Institutiones* (Oxford: Clarendon, 1937), xxii–xxiii.

claws of a multicolored bird. More austere, nonzoomorphic versions of the *schemata* are encountered in slightly later versions of the work (fig. 1.118–19).[205] Copies of this kind were owned by Fleury and Reims and known to Abbo and Gerbert (fig. 120).[206]

Without the whimsical animals and objects, the individual *schemata* would have been less memorable, but, it seems, no less useful.[207] From at least the mid-ninth century, simple *schemata* showing the division and stratification of knowledge became a common addition in the margins of manuscripts produced for the schools (figs. 1.121–22). Set beside passages on diverse subjects, they served—as Cassodorus's *schemata* did—as locators, placing information immediately at hand into a larger structure. That superstructure was also imaged in more elaborate and ambitious *schemata* that encompassed *all* knowledge. These covered whole pages and even openings in manuscripts and existed outside of books on large, unbound parchment leaves (fig. 1.123). The existence of such objects is attested in an episode from Richer of Saint-Rémi's *Historia francorum* (*History of the Franks*) in which Gerbert's right to teach hinged on the proper articulation of such an image.[208]

The above-discussed works were the core of the quadrivial canon, but in places where these more specialized treatises were not available some knowledge of number could still be obtained. No doubt many were first exposed to the liberal arts through Isidore of Seville's encyclopedic *Etymologiae*, which was one of the most widely disseminated tracts in the

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[205] As, for example, Paris, BnF lat. MS 2200 (9th c.); St. Gall, StiftsB. Cod. 199 (from the third quarter of the 9th c.); and Paris, BnF lat. MS 12963 (10th-c. copy; Corbie).

[206] Berlin, SB Cod. 176 (*olim* Phillipps MS 1780) is a 10th-c. manuscript from Fleury. It contains the first two chapters (on grammar and rhetoric) of the second book of the *Institutiones* and may have once included the full tract. A copy of book two of the *Institutiones* is in Reims, BM MS 975, fols. 34–75v; Carey included the first part of this manuscript (fols. 1–33, the *De arithmetica*) in his handlist of Reims manuscripts, which is only concerned with the early library, through the 10th c. ("The Scriptorium of Reims," 59). The *Institutiones* is dated to the beginning of the 12th c. in the *Catalogue général des manuscrits des bibliothèques publiques de France* 39*,* Reims 3 (Paris: Bibliothèque nationale, 1904), 232–34; these dates, however, are not always accurate; the *De arithmetica*, for example, is dated in the *Catalogue* to the 11th c. I believe the *Institutiones* to be about contemporary with the *de arithmetica*. Mynors described Reims MS 975, fols. 34–75v, as written in a "neat, tenth-century hand" (in *Institutiones*, xxxiii). Justin Lake took the manuscript as proof that the *Institutiones* was present in Reims during Gerbert's tenure. See Lake, *Richer of Saint-Rémi: The Methods and Mentality of a Tenth-century historian* (Washington, DC: Catholic University of America Press, 2013), 192.

[207] On the mnemonic value of the images see Fabio Troncelli, "Con la mano del cuore: L'arte della memoria nei codici di Cassiodoro," *Quaderni medievali* 22 (1986), 22–58; and idem, "Alpha e acciuga: Immagini simboliche nei codici di Cassiodoro," *Quaderni medievali* 41 (1996), 6–26.

[208] See chapter 2.

middle ages.[209] Fleury owned several copies, among these a very fine ninth-century manuscript made during Theodulf's abbacy.[210] At least three complete copies were available in the vicinity of Reims, one bearing Archbishop Hincmar's *ex dono*.[211] In writing the *Etymologiae*, Isidore drew heavily on, even paraphrasing at length, works already surveyed, especially Cassiodorus's *Institutiones*.[212] Conceived as a compendium of all worldly knowledge, the ambitious project remained unfinished at Isidore's death in 636; the work was organized and redacted by his friend and colleague, Braulio (c.585–651). Book III, "On Mathematics" (*De mathematica*), provided an outline of the quadrivium. Isidore touched first on the nature of number and the purpose of numeric study and then sketched the fundamentals of each discipline. As might be expected, he focused on terminology, etymologizing the very names of numbers: "'four' (*quattuor*) takes its name from the 'square' (*quadratus*)," "'thirty' (*triginta*) is so called because it 'arises' (*gignere*) from 'three' (*ternarius*) tens."[213] The section on "geometrical figures" included both verbal and visual descriptions of plane and solid shapes (fig. 1.124).[214]

Isidore's pithy explanations proved useful to medieval audiences. The *Etymologiae* was mined in the central middle ages for tidbits of practical information, though it was seldom cited. Abbo relied on it for the names and definitions of fractions, weights, and measures when writing his *Explanatio* on Victorius's *Calculus*.[215] He may not have had to lug out and comb through the whole of the *Etymologiae* to get this information,[216] since the work

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[209] Over 1,100 copies (complete, fragmentary, and excerpts) of the *Etymologiae* are extant. Exact numbers are difficult to come by. Around fifty complete or near-complete copies survive from before 900. A list of these is provided by Gorman in "The Diagrams in the Oldest Manuscripts of Isidore's *De natura rerum*," 542.
[210] Bern, BB Cod. 36; Bern, BB Cod. 101; and Bern, BB Cod. 224 all came from Fleury; the last, Bern 224, was made for Theodulf, who was bishop of Orléans and abbot of Fleury c. 797–817. See chapter 2.
[211] Reims, BM MS 425 bears Hincmar's *ex dono* (Hincmar was archbishop of Reims from 845–882). Carey believes that Reims, BM MS 426 came from St. Thierry; and that Leiden, UL BPL MS 114 was at Saint-Rémi by the 13th c. and probably before. Carey, "Scriptorium of Reims," 45, 55–56.
[212] On the relationship between the two works see Vessey, "Introduction," xxvii; and Jacques Fontaine, "The Editing of Isidore's *Etymologiae*," *The Classical Quarterly* 5, no. 1 (1911): 42.
[213] Isidore, *Etymologiae* 3.3.2 and 3.3.4; ed. Gasparotto and Guillaumin, 89; my translation: "Quattuor uero a figura quadrata nomen sumpserunt" and "Triginta, quod a ternario denario gignantur."
[214] The so-called Hispanic family of eleven (mostly early) manuscripts also included a set of elaborate astronomical and musical figures. On these see Michel M. Huglo, "The Diagrams Interpolated into the *Musica Isidori* and the Scale of Old Hispanic Chant," in *Western Plainchant in the First Millennium: Studies in the Medieval Liturgy and Its Music*, ed. Sean Gallagher (Aldershot: Ashgate, 2003), 243–59.
[215] Peden, in *Abbo of Fleury and Ramsey:* Commentary, xxxv.
[216] When copied in its entirety the *Etymologiae* filled a voluminous, weighty tome, often running to over 170 folios measuring over 250 x 330 mm.

was often excerpted.[217] The first three books, a digest of the liberal arts, were the most frequently copied.[218] Abbo's use of the *Etymologiae* also reveals the tract's utility in places where specialist treatises *were* available. Indeed, in MS SB Msc.Class.6, one of four tenth-century copies of the *De arithmetica* that were part of the Bamberg cathedral library, Isidore's brief summary of arithmetic prefaced Boethius's in-depth treatment of the discipline.[219] Likewise, Isidore's cursory treatment of geometry was a fixture of geometric compilations, and excerpts from the chapters on music were included in music miscellanies like Bamberg, MS Msc.Var.1, which may have been owned by Otto III.[220]

Some notion of number could also be gleaned from theological tracts. In his *Isagoge geometriae* Gerbert wrote:

> The beginning of this art [geometry] and, so to speak, its elements, seems to be a point, a line, plane, and solid. Both Boethius and other writers of both secular and divine works in many places of their writings frequently describe these things, and the most blessed and eloquent doctor of the church, Augustine, most eloquently holds forth in not a few of his books, and especially in that one, which is called *De quantitate animae*.[221]

Here, Boethius and Augustine are presented as equal authorities on geometry, mentioned in the same breath. The *De quantitate animae* (*On the Measure of the Soul*), one of Augustine's early dialogues, contains a surprisingly detailed and lengthy lesson in geometry. It is one of "not a few books" in which Augustine "eloquently held forth" on geometric matters. His expertise was, of course, not limited to the domain of geometry. The young, newly baptized

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217 Abbo could have found this information in one of the many computus miscellanies available at Fleury, including Bern, BB MS 207, fols. 1–24 (779–97); Bern, BB MS 610 (9th c., Fleury?); Paris, BnF lat. MS 3017 (9th c., Fleury?); Paris, BnF lat. MS 5543 (c.847 with 10th and 11th c. additions including an Abbonian calendar); Paris, BnF n. a. l. MS 1615, fols. 1–127 (9th c.); Paris, BnF n. a. l. MS 1616 (mid. 9th c.); Mostert, *Library of Fleury*, 81, 107, 207–208, and 243–44; and Faith Wallis, "Related Manuscripts," McGill University, http://digital.library.mcgill.ca/ms-17/apparatus.php?page=related_manuscripts#sec05 (accessed December 19, 2015).

218 L. D. Reynolds and N. G. Wilson, *Scribes and Scholars: A Guide to the Transmission of Greek & Latin Literature* (Oxford: Oxford University Press, 1968; 4th ed., 2013), 85.

219 On this manuscript, see note 68 above.

220 Bamberg, SB MS Msc.Var.1 also contains Hucbald's *De musica*, a treatise well known at Reims, where Hucbald briefly taught (see chapter 2). On Otto III's ownership see Mütherich, "Library of Otto III," 20; for a description of the manuscript see Suckale-Redlefsen, *Die Hanschriften*, 146–47 (no. 87).

221 Gerbert, *Isagoge* 2.1; ed. Bubnov, 51–52; my translation: "Artis hujus initia et quasi elementa videntur: punctum, linea, superficies, atque soliditas. De quibus cum ipse Boetius aliique tam saecularis, quam divinae litteraturae tractatores in plurimis scriptorum suorum locis satis superque disputent, tum beatissimus et eloquentissimus Ecclesiae doctor, Augustinus, in nonnullis libris suis et praecipue in eo, qui de quantitate animae inscribitur."

Augustine had considered writing a series of books on the disciplines.[222] Though these books were never fully realized, his knowledge of the numeric disciplines is evident throughout his *oeuvre* especially in *De ordine* (*On Order*), *De libero arbitrio*, his commentaries on Genesis, *Confessionum* (*Confessions*), and *De civitate dei* (*On the City of God*). Some of these works (e.g., *De quantitate*, *De libero*), may seem to us obscure, but the numbers of extant copies are not insignificant and they were produced in scriptoria across Latin-speaking Europe.[223] The *Confessionum* and the *De civitate dei* survive in extraordinary numbers.[224] Hence these texts and their quadrivial contents might have been met even in modest collections, at institutions where access to works in the quadrivial canon may have been limited.

In the passage from the *Isagoge* quoted above, Gerbert referred to "other writers of both secular and divine works." The statement reminds us that number—capacious and sacred—was gleaned from tracts of all kinds. Augustine in the *De doctrina Christiana* (*On Christian Teaching*) wrote that "an unfamiliarity with numbers makes unintelligible many things that are said figuratively and mystically in scripture."[225] Hence, numerical knowledge was requisite to comprehending certain books of the Bible, notably Genesis, Wisdom, Job, Psalms, the Gospel of John, and Revelation. Cassiodorus famously keyed his *Expositio psalmorum* (*Explanation of Psalms*) to the seven arts with marginal signs that alerted his reader as to the nature of the adjacent text and the rationale with which it was best approached and understood (fig. 1.125).[226] He marked Psalm 95:13: "He shall judge the

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[222] As stated in the *Retractationes* (1.6). Augustine completed a book on grammar and six books on music. He claims to have started five other books on dialectic, rhetoric, geometry, arithmetic, and philosophy. Only the treatise on music (*De musica*; PL 32) survives.
[223] There are today around 200 copies of the *De quantitate* dating from the 9th through the 16th c. Twelve date to the early and central middle ages (this number does not include excerpts folded into the so-called *Geometry II*). The *De libero* survives in more than 170 manuscripts dating from the 9th to the 14th c.; about a dozen are early (i.e., pre-12th c.). These were made all over Europe: St. Emmeram, Regensburg (Munich, BSB Clm. 14527); Murbach Abbey (Basil, B VIII, 9); Trier (London, BL Add. MS 10940); Mainz (Oxford, Bodl. Laud. Misc. MS 113); Angers (Paris, BnF lat. MS 2700); St. Germain (Paris, BnF lat. MS 1161); St. Augustine's, Canterbury (Oxford, Trinity College, MS 4). For summaries of the pre-modern reception of these tracts see Pollman and Otten, ed. *Historical Reception of Augustine*, 232–34, 329. For a list of manuscripts of the *De libero* see Green in Augustine, *Contra academicos*, 346–50.
[224] There are over 300 extant copies of the *Confessionum* extant and 394 of the *De civitate Dei*. Pollmann, *Historical Reception of Augustine*, 167, 255.
[225] Augustine, *De doctrina christiana* 2.16; ed. and trans. R. P. H. Green (Oxford: Clarendon Press, 1995): "numerorum etiam imperitia multa facit non intellegi translate ac mystice posita in scripturis."
[226] Mark Vessey called this "disciplinary indexing." Early copies of the *Expositio Psalmorum* included headings. The quadrivial headings were: *arithmetica*, *geometrica*, *musica*, and *astronomica*. Words were later

world (*orbem terrae*) with justice, and the people with his truth," with "*Geo*," the rubric for geometry, and in the *Expositio* he reconciled the use of the term *orbis*—"circle" (*circulum*) in Latin—to describe the shape of the earth with another biblical passage where it is called a square.[227] Taken grammatically, scripture would appear to contradict itself; geometrically, however, there was no contradiction, since—Cassiodorus wrote—a circle circumscribes all shapes as "Euclid clearly demonstrated in the fourth book of the *Elements*."[228] At some point around the year 1000, this passage from the *Expositio* was copied into a blank space in a geometric miscellany known to Gerbert (fig. 1.126).[229] The excerpted passage shows the permeability of Gerbert's categories, the symbiotic relationship that existed between the secular and divine works: study of the quadrivium shed light on scripture and scripture confirmed and bolstered scientific and mathematical theories.

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This miscellaneous array of works thus formed the "quadrivial canon." The sheer number of manuscripts copied, corrected, illustrated, supplemented is witness to the importance of these texts in medieval pedagogical programs. Close study of the visual and textual material contained in these manuscripts yields period-specific insights on a range of integrated issues: the relations and tensions between word and image, the nature of cognition, and modes of picturing both the sensible world and that domain which was considered to be beyond the reach of the senses.

5. CHAPTER SUMMARIES

This dissertation seeks to recover, to the extent possible, the notions and practices that shaped the images in quadrivial manuscripts as well as their apperception, through close study of such manuscripts. The heart of the work is three case studies, each centered on a particular discipline, master, school, and tract, and a few related manuscripts. The conclusion and coda mirrors the introduction, taking as its object another deluxe manuscript—a luxury pericope

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replaced by signs. Vessey, "Introduction," 78; and James Halporn, "Methods of Reference in Cassiodorus," *The Journal of Library History* 16, no. 1 (Winter, 1981): 71–91, esp. 82–83.

[227] Psalm 95:13: "Iudicabit orbem terrae in iustitia et populos in veritate sua." The "four angles of the Earth" (quattuor angulos terrae) appear in Revelation 7:1.

[228] Cassiodorus, *Expositio Psalmorum* 95.316: "Sed quomodo quadratus iste demonstrandus intra circulum scribi debeat, euclides in quarto libro elementorum euidenter insinuat."

[229] Naples, BN MS V.A.13, fol. 27v. This manuscript is discussed in chapter 5.

book commissioned by Otto III's successor, Henry II—in order to show how quadrivial learning inflected the making and viewing of objects. I begin by setting given quadrivial manuscripts and their images in precise historical contexts. The biographies of Gerbert and Abbo are sketched in chapter two with an emphasis on their pursuits and achievements in the area of numeric study. The well-stocked libraries of Reims and Fleury provided ample fodder for study and instruction. Gerbert and Abbo actively sought "better" copies of tracts that they already owned as well as titles that were missing from their collections. Considerable effort also went into synthesizing and enriching available works. To make the older tracts useful tools for contemporary study, Gerbert, Abbo, and their contemporaries wrote lengthy introductions and made marginal and interlinear annotations of many kinds. They also prepared original works.

Abbo authored the *Explanatio in calculo Victorii*, a lengthy and discursive commentary on the *Calculus* of Victorius, which served as an introduction to the discipline of arithmetic as well as quadrivial study more generally. As touched on above, he also created his own computus, which was, in turn, copied and augmented by one of his students, Byrhtferth of Ramsey. Byrhtferth penned the *Enchiridion*, a handbook to help his own students grasp the intricacies of computus. Gerbert, as already mentioned, composed the *Isagoge geometriae*, a synthesis of virtually all the material on geometry that was then available. These three original tracts, it is suggested, give a sense of the nature, methods, aims, and application of the quadrivium in the central middle ages and are the focus of chapters three through five.

Chapter three probes the juncture between the trivium and quadrivium. Grammar had primacy in the schools, but calculation and computus were also part of elementary education. Abbo brought students at Fleury and Ramsey from the arts of word to the disciplines of number with his *Explanatio in calculo Victorii*, which he called a "bridge to arithmetic." The metaphor evokes the traffic that existed between these domains. Readers came to the mathematical tables of Victorius's *Calculus* with a grammatical grasp of letters as, in the words of Priscian, "signs of utterances." But in the central middle ages, before the widespread use of Arabic numerals, letters served as signs of numbers as well as words. Abbo worked to reinvent the alphabet as a set of neutral signs especially suited to address the mind. Outside the verbal sphere, letters combined and behaved in new ways and made different demands on viewers. In quadrivial manuscripts tables, calendars, schemata, and figures largely usurp lines of text, radically reforming the page and the habits with which it was approached by one schooled in the trivium. Likely the graphic elements of quadrivial tracts were the first pictures that one encountered in books. Students of calculation and arithmetic became newly attuned to placement and order so as to decipher Victorius's tables and the shaped sequences and series of Boethius's *De arithmetica*. They were also trained to assume that what was pictured was only partial and to imagine infinity. Thus, the study of number opened the possibility of the page and fostered habits of eye and mind as deeply ingrained as reading.

Chapter four considers the numeric imperative to "picture" through Byrhtferth's *Enchiridion*. Computus required an especially active and agile or "curious" eye. Echoing the example of Abbo, Byrhtferth created tables and figures that were as mobile as the heavens that they represented. Ever mindful that the path of the eye was followed also by the mind, the Anglo-Saxon schoolmaster carefully curated the visual trajectories mapped out on the page and crafted figures out of forms—primarily crosses and circles—deemed perfect and holy. The theory of the image underlying the *Enchiridion* is emblematic and old—derived, if only obliquely, from Plato's *Timaeus*. According to the *Timaeus* the eye and, to a lesser extent, the ear were the vehicles of numeric discovery. For Plato the soul was "disciplined" and wisdom achieved through a process of observation and imitation of divine bodies. Calcidius bound this notion to learning (pursuit of the arts and disciplines), as did Abbo and Byrhtferth in his wake. I suggest that efforts to visualize numeric notions, in the *Enchiridion* and elsewhere, were ultimately rooted in this Platonic theory of vision and apprehension.

Chapter five considers period-specific notions of the limits of two-dimensional representation. Gerbert in the *Isagoge geometriae* concerned himself less with the eye of the body, more with the eye of the mind. In tracts and passages devoted to geometry, including Gerbert's, figures of the so-called geometrical elements—points, lines, planes, and solids—were pictured in a manner that invited viewers to take an active role, to see correctly in the mind's eye what could not be properly pictured on the page. The process of visual emendation and manipulation fostered an awareness of the nature of things—their corporeal or incorporeal status, and, with this, their perception through the eye of the body and the intellect. A perspective that was founded scientifically and philosophically, it presumes imperfection or incompleteness on the part of two-dimensional images. I argue that the study of geometry equipped its practitioners with what Michael Baxandall, in his work on the limewood sculptors, referred to as "visual skills and habits" that shaped the expectations one had of pictures and, with this, conditions of representation.[230]

The intersection of the domains of scientific thinking and art-making in the central middle ages had complex ramifications. Whereas in the first five chapters my analysis of the effect of quadrivial pictures and pedagogy remains largely within the confines of the schoolroom, the conclusion does as those whom I study did: puts numeric knowledge to work. In the *De doctrina Christiana* Augustine warned that the liberal arts were not to be studied for themselves, but in order to understand scripture. In this spirit, the inscription on the treasure binding of the Pericope Book of Henry II is reinterpreted in light of the quadrivium, especially geometry, throwing light on the form and content of the precious Ottonian manuscript.

[230] See notably "Chapter IV: The Period Eye" in Michael Baxandall, *The Limewood Sculptors of Renaissance Germany* (New Haven: Yale University Press, 1980), 143–64.

CHAPTER 2

Masters and Manuscripts

> But when God desired that Gaul, which at that time lay mired in darkness, should shine forth once more with a great light, he put it in the mind of Borrell and Hatto to go to Rome to pray. After getting together the necessary provisions, they set out, taking with them the young man who had been entrusted to their care. ... The young man's diligence and love of learning did not escape the notice of the Pope. And because hardly anything was known of music or astronomy in Italy at the time, he sent an envoy to inform King Otto of Germany and Italy that a young man had arrived who possessed a complete (*optime*) understanding of the mathematical arts (*mathesim*) and was capable of providing vigorous instruction for his [Otto's] students.[1]

The passage, describes events around the year 970. It appears in the *Historiae* (*Histories*), a chronicle of West Francia written by the monk Richer of Saint-Rémi between 991 and 998.[2] The boy full of promise is the young Gerbert, to whom Richer eventually dedicated the *Historiae*.[3] Richer painted a dire picture of the state of education in the Latin-speaking world

[1]Richer of Saint-Rémi, *Historiae* 3.44, ed. and trans. Justin Lake, DOML 11 (Cambridge, MA: Harvard University Press, 2011), 2:64–5: "Sed cum divinitas Galliam iam caligantem magno lumine relucere voluit, predictis duci et episcopo mentem dedit ut Romam oraturi peterent. Paratisque necessariis, iter carpunt ac adolescentem commissum secum deducunt. ... Nec latuit papam adolescentis industria simulque et discendi voluntas. Et quia musica et astronomia in Italia tunc penitus ignorabantur, mox papa Ottoni regi Germaniae et Italiae per legatum indicavit illus huiusmodi advenisse iuvenem, qui mathesim optime nosset suosque strenue docere valeret." All translations of the *Historiae* are from Lake unless otherwise noted. The DOML translation is printed side-by-side with the Latin text based on the newest, most complete edition of the *Historiae*: *Richeri historiarum libri IIII*, ed. Hartmut Hoffmann, MGH SS 38 (Hanover: Hahn, 2000).

[2] Richer used the term *Gallia* (*Gaul*) to designate the area bequeathed to Charles the Bald in 843 in the Treaty of Verdun, corresponding roughly with the borders of modern-day France. He goes into some detail describing Gaul and its "parts" (Belgica, Celtic Gaul, and Aquitaine) in *Historiae* (1.2). For a discussion of Richer's geographic terminology see Justin Lake, in Richer, *Historiae*, xxii–xxiii. On the dating of the *Historiae*, which was never completed and survives in a single, autograph copy, Bamberg, Staatsbibliothek MS Msc.Hist.5, see Jason Glenn, "The Composition of Richer's Autograph Manuscript," *Revue d'histoire des textes* 27 (1997): 151–89.

[3] Richer, *Historiae* prologue; ed. and trans. Lake, 1:2–4.

at the close of the tenth century. Gaul "lay mired in darkness," and in Italy there was utter ignorance of music and astronomy, two of the liberal arts. A contemporary reader of the passage would know that wisdom could be achieved only through mastery of *all* the seven arts: grammar, rhetoric, dialectic, arithmetic, geometry, astronomy, and music.[4] The implication is that there were precious few wise men in Italy. It seems they were scarce in Germany as well. Otherwise what can explain Otto I's immediate and eager reply to Pope John XIII (r.965–72), demanding that he, "keep the young man with him and under no circumstances allow him the opportunity to go home."[5] Count Borrell of Barcelona (947–992) and Bishop Hatto of Vic (d.971/972) had, evidently, brought a rare and valuable commodity with them from Barcelona.[6]

The "darkness" of the age might be dismissed as hyperbole,[7] if contemporary sources did not corroborate the paucity of suitable teachers, especially those with "a complete understanding" of *mathesis* (a Greek term that, following Boethius, Richer used to refer to the quadrivium).[8] Abbo of Fleury, in the preface for his *Explanatio* on Victorius of Aquitaine's *Calculus* wrote:

> From the first beginning of early youth, I grieved continuously that the disciplines of the liberal arts had been made unsteady by neglect and carelessness and [that they] had scarcely been restored by a few [men], who set a price for their knowledge.[9]

Abbo wrote the *Explanatio* in the early 980s at the urging of his fellow monks at Saint-Benoît-sur-Loire, who needed help deciphering the mathematical tables compiled by

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[4] Whereas "mired in darkness" might be considered something of a topos, Richer's claim about the lack of teachers skilled in music and astronomy has a particularity to it that rings true. The quadrivium culminated in these two disciplines, which were generally learned last; music was often deemed the most difficult of the disciplines. See chapter 1.

[5] Richer, *Historiae* 4.44; ed. and trans. Lake, 2:64–65: "Mox etaim ab rege papae suggestum est ut iuvenem retineret, nullumque regrediendi aditum ei ullomodo preberet."

[6] The Pope, according to Richer, assured the two men that they would receive remuneration from the emperor if they left the youth. Ibid. 3.44; ed. and trans. Lake, 2:64–7.

[7] Regarding Richer's rhetorical flourishes see Jason Glenn, *Politics and History in the Tenth Century: The Work and World of Richer of Reims* (Cambridge: Cambridge University Press, 2004), 3–4; on Richer as a historian see Hartmut Hoffmann, "Die Historien Richers von Saint-Remi," *Deutsches Archiv* 54 (1998): 445–532.

[8] On this term see chapter 6.

[9] Abbo, *Explanatio* 1.2; ed. Peden, 64: "Nam a primevae aetatis tirocinio iugiter indolui liberalium artium disciplinas quorumdam incuria ac negligentia labefactari et vix ad paucos redigi, qui avare precium suae statuunt arti."

Victorius, a fifth-century mathematician.[10] At least one, possibly several copies of the *Calculus* were housed in Fleury's great library: accessible, but incomprehensible without someone to explain them.[11]

The lack of access to certain fundamental texts was, as has often been pointed out, an impediment to intellectual advance in the domains of science and mathematics during the central middle ages.[12] In the passage cited above, however, Abbo did not bemoan the absence of texts, but of teachers: good, generous men willing to share their knowledge freely. His bitter words arose from personal experience, as attested by Abbo as well as his friend and hagiographer, Aimoin of Fleury (c.965–d.after 1008). In the *Vita et passio sancti Abbonis* (*The Life and Passion of Holy Abbo*), Aimoin recounted how his hero's talents had quickly outstripped the teaching available even at Fleury:

> That very one [Abbo] ... burning to investigate greater scientific secrets, went to wisdom's workshop in diverse places (because he had already fully achieved investigation of grammar, arithmetic, and—not least—dialectic) in order that he might proceed to add the other arts to his own talent.[13]

Abbo was able to acquire knowledge of grammar, arithmetic, and dialectic at home; for the rest of the arts, however, he had to go abroad. He traveled to Paris and Reims, but left disappointed, having mastered only the rudiments of astronomy. In the end, Abbo paid an unnamed cleric in Orléans to teach him music; he taught himself geometry and rhetoric from books.[14] Fleury's exceptional library, with its impressive stock of texts devoted to the subjects of the trivium and quadrivium, sufficed when no teacher could be had.[15]

Books, of course, were not merely the last resort of a desperate scholar, but essential didactic tools. When called across the Channel to teach at the fledgling monastery of Ramsey

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[10] The *Explanatio* is the focus of chapter 3.
[11] As many as five copies may have been available at Fleury. Three are extant: Vatican City, Bibliotheca Apostolica Vaticana, MS Reg. lat. MS 1281; Paris, BnF lat. MS 6620; and Bern, Burgerbibliothek, MS 250. For a complete list of surviving manuscripts and summary of the tradition see Alison Peden, in Abbo, *Commentary*, xxxvi–xlix.
[12] See chapter 1.
[13] Aimoin of Fleury, *Vita et Passio Sancti Abbonis* 3; ed. and trans. Robert-Henri Bautier and Gillette Labory, in *L'Abbaye de Fleury en l'an mil*, Sources d'histoire médiévale 32 (Paris: CNRS Éditions, 2004), 48; all translations of Aimoin's *Vita* are my own unless otherwise noted: "Verum ipse, adhuc maiora gliscens scientiae scrutari arcana, diversorum adiit sapientiae officinas locorum, ut, quia grammaticae, arithmeticae, nec non dialecticae iam ad plenum indaginem attigerat, caeteras ingenio suo pergeret superadjicere artes."
[14] Ibid.
[15] Fleury's library is discussed below.

in Anglo-Saxon England, Abbo carried with him copies of tracts without which he could not teach, among them Macrobius's *Commentary* on Cicero's *Dream of Scipio*.[16] Indeed, he may have overseen the collation of several early copies of the *Commentary* and the mass-production of this new revised version for use in the school. Gerbert rejoiced with his mentor, Adalbero, when, in the course of his travels, he came across a particularly prized manuscript. It is clear from his correspondence that he had been charged with expanding the library of Reims, for he is constantly seen asking his friends for copies of various texts. Some twenty of his 233 extant letters involve procuring books and he himself was constantly on the lookout for new titles wherever he went.

Knowledge in the central middle ages was transmitted through interpersonal exchange, broadly defined to include in-person contact as well as written correspondence. Contemporary histories, treatises, poems, and letters reveal surprisingly extensive and enduring networks of masters and pupils that crisscrossed Latin-speaking Europe, knitting the continent together. This chapter maps these communities. It is divided into two parts: the first focuses on people, the second on books (their avatars in parchment). Part one traces the trajectories of Gerbert and Abbo, who traveled extensively, often expressly for the purpose of expanding or sharing their knowledge of the liberal arts. I close with a consideration of the literary landscape in which they moved. Gerbert and Abbo learned and taught the numeric disciplines through what I have called the quadrivial canon. These tracts had circulated in the ninth century, but their numbers markedly increased in the tenth and eleventh centuries as the quadrivium became a standard part of the curriculum. I consider the character and conent of those libraries with which Gerbert and Abbo were most closely associated: Reims, Fleury and Ramsey.

1. THE MILLENIUM

The *Historiae* begins at the end of the ninth century, after 879. Richer told of the turmoil that followed the death of Louis II, known as "the Stammerer." He wrote:

> Universal harmony lapsed into complete and utter discord, and outbreaks of pillaging, burning, and seizures of property followed. While these acts of

[16] See below.

> savagery were being perpetrated, the pirates who inhabited the territory around Rouen were themselves roused to savage deeds.[17]

Charles III, Louis's youngest son, was born after his father's death. The boy's Carolingian blood, the fact that his family had ruled West Francia for generations—these did not guarantee his succession. The local lords were too powerful (and greedy) to pledge loyalty to the infant and submit to a long regency. They rebelled and vied for power in the vacuum created by Louis's death. Charles was not able to establish his right to rule until 893. In the interim, chaos engulfed the realm.

The central middle ages was an era of upheaval and significant political, cultural, religious, and even chronological fragmentation. The Carolingian Empire, which, at the turn of the ninth century, stretched more than a million square kilometers from the Pyrenees to the Elbe, had slowly dissolved, methodically parsed and passed to several generations of legitimate male heirs.[18] It had been briefly reunited in the 880s when, in the space of two years (882–84), three of the four heirs died suddenly, without sons of an age to inherit. The rule of their kingdoms fell to Charles the Fat, Charlemagne's great-grandson. But Charles was deposed in 887 and died shortly thereafter (888), also heirless. With him, the blood of Charlemagne was not spent, but it ran only in the veins of bastards and women, descendants whose legitimacy would be challenged regularly by powerful local nobles. The stability of their realms would be further disturbed by Vikings, Slavs, and Magyars, who invaded from west and east.[19]

In the West, a few of these outsiders, the Norsemen (or Normans), were eventually granted land in exchange for peace by the weakened monarchy, on the condition that they convert to Christianity. In the East, local nobles gathered to elect a king, first Conrad (911–18), duke of Franconia, and then, on his death, Henry (918–36), duke of Saxony, who successfully fought off and pushed back first the Slavs, then the Magyars. His son, Otto I,

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[17] Richer, *Historiae* 1.4; ed. and trans. Lake, 1:16–17: "Unde et omnium concordia in summam discordiam relapsa est. Hinc direptiones, hinc incendia, hinc rerum pervasiones exarsere. Quae cum immanissime agitarentur, piratae qui Rhodomensem provinciam incolebant ... ad rerum immanitatem incitantur."

[18] On the decline of the Carolingian dynasty see Simon MacLean, *Kingship and Politics in the Late Ninth Century: Charles the Fat and the End of the Carolingian Empire* (Cambridge: Cambridge University Press, 2003), esp. chapters 5 and 6.

[19] Simon Coupland, "The Vikings in Francia and Anglo-Saxon England to 911," in *The New Cambridge Medieval History*, ed. Rosmond McKitterick (Cambridge: Cambridge University Press, 1995), http://dx.doi.org.proxy.lib.umich.edu/10.1017/CHOL9780521362924.010 (accessed July 17, 2014).

crowned emperor in Rome in 962, inherited territories that included much of the central and eastern areas of the old Frankish Kingdom. He gave his name to the Ottonian dynasty.[20] A new and powerful royal line emerged in the west as well. Hugh Capet (941–96) was crowned king of France in 987; his descendants would rule for over 300 years.

The appearance of violent and foreign peoples, and the sudden failure of a powerful, imperial dynasty, are events that might cause unease and social unrest at any time. In Latin Christendom at the close of the first millennium, however, they stirred a particular "fear and hope" of apocalypse, the end of the world and the second coming of Christ vividly described in the book of Revelation.[21] The Vikings and Magyars were seen by some as the armies of Gog and Magog unleashed, men from far-off lands who had been stirred by Satan to do battle against Christ and his followers (Revelation 20:7–10). The fall of the Roman Empire also betokened the nearness of the last days.[22] So successfully had Charlemagne and the Carolingians cast themselves as true heirs of Constantine that the Roman imperium was thought to continue while they ruled. Their end and the crowning of kings of non-Frankish lineage, Henry I in the East and Hugh Capet in the West, marked, for many, the end of Roman rule. These signs were woven into a constellation of portents (e.g., years of famine [1005–06], the appearance of a new star [1006]) that seemed to proliferate around the year 1000, an auspicious date by any reckoning, but one that figured prominently in contemporary annals.

Augustine and others had cautioned that no one but God knew when the end would come; to assume that it might be predicted by means of human calculation was considered hubris, even blasphemy. Despite this prohibition, ominous conjunctions in the liturgical calendar were noted and anxiously anticipated. Especially alarming were instances when Good Friday fell on the twenty-fifth of March, the traditional date of Adam's creation and the Annunciation. This coincidence occurred three times in the second half of the tenth century: in 970, 981, and 992. Equally worrisome was the so-called sabbatical millennium, the notion that the world, which God *made* in six days, resting on the seventh, would *endure* for six "days." A "day" was interpreted as 1000 years as stated in Psalms (90:4): "a thousand years is

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[20] Including East Francia (extended to the east), Lotharingia, the lands of the Danes to the north.
[21] Richard Landes, "Fear of an Apocalyptic Year 1000: Augustinian Historiography, Medieval and Modern," *Speculum* 75, no. 1 (2000): 118–19.
[22] 2 Thessalonians 2:8.

a day in the sight of the Lord." Hence the world would end after 6000 years. Just when this would be depended, of course, on how one calculated the current age of the world. There existed different systems, but according to some the year 1000 or 1033 (if one began counting from Christ's birth or his death) would bring the apocalypse, the "seventh" day or sabbath—a time of rest and rejoicing for all who had labored in the days prior to the coming and the tortures of hell for those who had not.

The tenth and eleventh centuries have often been viewed through the lens of chiliasm.[23] Likewise, the numeric (especially computational) pursuits of Abbo, who wrote a treatise recalculating the age of the universe to show that its 6000 had already passed without event, and of Gerbert have been interpreted in this light.[24] Recent discussions of eschatological effects present a tempered and more nuanced picture. Scholars continue to take chiliastic concerns and motivations into consideration but allow for more varied reactions to the inauspicious date, including nonreaction and suppression.[25] The impending end of time was one among several factors that spurred reform of monastic and apostolic communities and sparked renewed interest in the project of education, the liberal arts, and number.

2. Masters and Pedagogy

Richer, Gerbert, Abbo and their contemporaries were well aware of the turbulence of their times and sought refuge in the life of the mind. On the eve of setting out to fight on the eastern border of the empire, Otto III sat conversing with Gerbert and other intellectuals at court. Together they marvelled at man's ability to have "depths of thoughts" amidst the strife of war.[26] Gerbert and Abbo both reported finding comfort in the liberal arts. Abbo referred to philosophy as a tranquil retreat.[27] Gerbert frequently protested that he far preferred the

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[23] Landes provides an exhaustive bibliography "Fear of an Apocalyptic Year 1000," 98, n. 6.
[24] Ibid., 123–30.
[25] See the collection of essays edited by Richard Landes, Andrew Colin Gow, and David C. Van Meter: *The Apocalyptic Year 1000: Religious Expectation and Social Change, 950-1050* (Oxford: Oxford University Press, 2003).
[26] The expression appears in a letter Gerbert wrote in 997 to Otto III recounting this meeting; the letter accompanied a treatise written as a gift for Otto: the tract *De rationali et ratione uti*. Treatise and letter are reproduced by Olleris under the title of *Libellus de rationale et ratione uti* (*Oeuvres de Gerbert*, 297–310).
[27] The statement appears in the so-called *Apologeticus* (PL 139: 461) written by Abbo to Hugh Capet. Extracts of the *Apologeticus* were included by Aimoin in the *Vita* (chapters 8 and 9). The work is translated and discussed in depth by Mostert, *Political Theology*, 48–51.

"certain leisure of studies, to the uncertain business of wars."[28] Abductions, sieges, wars, riots, raids, even apocalypse may have slowed or diverted the flow of knowledge during the central middle ages, but these events certainly did not arrest it. The promise of philosophy was too great: to achieve wisdom was to see God.[29]

Just as the apostles, witnesses of Christ, were touched by the Holy Spirit and so tasked with spreading what they had experienced and beheld firsthand, so pedagogy was cast as part of the apostolic mission. Byrhtferth, a schoolmaster of Ramsey abbey in England, was instructed in his youth by Abbo.[30] Like his master, he wrote didactic tracts on computus. In one such work, he traced his teaching lineage from Abbo and Bede all the way back to the Church Fathers and the apostles.[31] According to Byrhtferth, the Holy Spirit granted, "wisdom of the great chronographers and grammarians"—knowledge of number and letter—to these "distinguished successors of heavenly bliss."[32] He himself called on the Holy Spirit to give him words to convey the complexities of computus, quoting Arator's *Historia apostolica* (1:226–7): "Come, Holy Spirit: without your support your name cannot even be pronounced; Grant the gifts of speech, you who give the gift of tongues."[33] The apostolic metaphor was pervasive. Aimoin described Abbo as having, "a dove-like simplicity coupled with serpentine cunning," thus fulfilling Christ's command to his apostles to be "wise as serpents and simple as doves" (Matt. 10:16).[34] Thierry of Fleury called the *armarius* one "equal to an apostle."[35]

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[28] Gerbert, *Epistulae* 45; ed. Riché and Callu,110–12; trans. Lattin, 91: "delegimus certum otium studiorum, quam incertum negotium bellorum."
[29] On the aims of philosophy see chapter 1.
[30] Byrhtferth is explicit about his debt to Abbo in his *Epilogus* to Bede's *De temporibus* (extant in a single manuscript: Oxford, St. John's College MS 17), printed and translated in Byrhtferth, *Enchiridion*, Appendix A, 4.17–53; ed. and trans. Baker and Lapidge, 375–77.
[31] Ibid. To summarize, Byrhtferth starts with the apostles, "bounteously inspired by the inexpressible breath of the Holy Spirit," then to the Church Fathers: Jerome, Augustine, and Gregory—"endowed with the eloquence of ecclesiastical doctrine" and "abundantly taught in the holy wisdom of the great chronographers," then to Bede. Paraphrasing and elaborating on the book of Wisdom, Byrhtferth wrote that Bede had been given "the true knowledge of things that are: to know the disposition of the whole world, and the virtues of the elements, the beginning, and ending, and midst of the times...the revolutions of the year, and the dispositions of the stars" (Wisdom 7:17–19). After Bede comes Abbo, "alumnus of St. Benedict, through whose kindness I [Byrhtferth] received my understanding of this subject [computus] as well as my knowledge of other things."
[32] Ibid., Appendix A, 4.5–6; ed. and trans. Baker and Lapidge, 375: "facundia ... aecclesiastici dogmatis; sacra sophia inlustrium chronoraforum ortographorumque superni gaudii clarissimi dehinc extitere secutores."
[33] Byrhtferth, *Enchiridion* 2.3.264–66; ed. and trans. Baker and Lapidge, 120–21: "Spiritus alme, veni, sine quo non diceris umquam; Munera da lingue, qui das in munere linguas."
[34] Aimoin, *Vita* 2; ed. and trans. Bautier and Labory, 46. The statement appears as Abbo begins his schooling. Aimoin finishes the phrase: "to both beguile good men through gentleness and avoid sly men through the sharpness of mental forethought." (Inerat ei columbina simplicitas serpentinae copulata astutiae, ut et per mansuetudinem leniret benevolos, et per providae mentis acumen devitaret subdolos.)

The lives of Gerbert and Abbo give us some idea of how a complete education was achieved in the tenth century and how they, like apostles, spread their knowledge through their person and with their pens.

GERBERT OF AURILLAC

> An Aquitanian by birth raised from boyhood and instructed in grammar at the monastery of the holy confessor Gerald.[36]

This line from Richer's *Historiae* sets the tone for the rest of the twenty-two chapters devoted to Gerbert, which are concerned primarily with his intellectual pursuits.[37] Scholars assume that Gerbert was born around 945. Richer specifies that he was from Aquitaine, but exactly where in the vast area stretching from the Loire to the Pyrenees is unknown. Gerbert was, in his own words, "neither noble nor wealthy."[38] His low birth was at once an impediment to advancement and that which made him legendary. An eleventh-century chronicler wrote of him: "Gerbert, a lover of wisdom (*philosophus*) ... made pope on account of his incomparable knowledge."[39] At a time when ecclesiastical office was often reserved for the nobility and simony was openly practiced, that a boy without family connections or financial advantage could become a powerful political player—one "who un-makes and makes kings"—archbishop (twice), and pope, all owing to his astonishing mental acuity, was extraordinary.[40]

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[35] Thierry of Fleury, *Consuetudines floriacenses antiquiores* 9; ed. Anselme Davril and Lin Donnat, in *L'Abbaye de Fleury en l'an mil*, Sources d'histoire médiévale 32 (Paris: CNRS Éditions, 2004), 182: "... pro apostolo habeatur."

[36] Richer, *Historiae* 3.43; ed. and trans. Lake, 2:62–3: "Qui Aquitanus genere, in coenobio sancti confessoris Geroldi a puero altus et grammatica edoctus est."

[37] Book 3, chapters 43–65 are devoted to Gerbert. The long section on Adalbero and Gerbert sits uneasily within Richer's historical narrative and in the manuscript in which the unfinished *Historiae* is preserved. Jason Glenn has convincingly argued, based on the style and content of these chapters as well as paleographical and codicological evidence, that these chapters were written separately, probably prior to the *Historiae,* and then added to the work; "The Lost Works of Richer: The *Gesta Adalberonis* and *Vita Gerberti*," *Filologia mediolatina: Rivista della Fondazione Ezio Franceschini* 4 (1997): 153–90.

[38] Gerbert, *Epistulae* 217; ed. Riché and Callu, 648; trans. Lattin, 258. Regarding his extraordinary fortune, Gerbert quotes the Psalms (112:7–8): "lifting up the poor out of the dunghill; that he may place him with princes."

[39] J. B. Haudiquier and C. Haudiquier, ed., *Recueil des historiens des Gaules et de la France* (Poitiers: Henri Oudin, 1874), 10:324: "Girberti Philosophi ... merito incomparabilis scientiae."

[40] Gerbert, *Epistulae* 163; ed. Riché and Callu, 404; trans. Lattin, 200: "qui reges deponerem regesque ordinarem."

Most of what is known of Gerbert's early life and education comes from the *Historiae* written by Richer, a monk of the monastery of Saint-Rémi, located on the outskirts of Reims.[41] Richer was a colleague, friend, and admirer of Gerbert. As already mentioned, he dedicated the *Historiae* to him. Couched in an account of reforms enacted by Adalbero, then Archbishop of Reims, the chapters on Gerbert recount the reinvigoration of the liberal arts in Gaul. Adalbero, Richer wrote, was seeking someone to instruct the "sons of his church" when "God himself sent Gerbert." Of particular interest is the stress laid on Gerbert's knowledge of number. As we saw in the passage from the *Historiae* with which this chapter began, it was Gerbert's unparalleled mathematical (*mathesis*) ability that the pope and then emperor found so desirable. Indeed, for Richer, the arrival of quadrivial knowledge—first in Rome, then in Reims—was cast as an event of historical import, a divine act. The *Historiae* tells us little of Gerbert's episcopal career and his life outside of Reims, but where Richer's account leaves off, Gerbert's letters take over. Alarmed at the loss of a letter "not received" early in his career, Gerbert began to copy his correspondence. Some 230 letters survive, providing a wealth of details about their author, his contemporaries, and his times.[42] Many can be classed as pedagogical. Just as in Richer's account of Gerbert's teaching, which focuses overwhelmingly on the quadrivial disciplines, in the letters mathematical topics (especially arithmetic, geometry, and music) are disproportionately represented. In contemporary sources Gerbert's greatness cannot be separated from his knowledge of number.

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Gerbert began his studies at the prestigious monastery of Saint-Géraud, Aurillac, under the tutelage of a monk named Raymond. He affectionately referred to Raymond as his "master" and "father" throughout his life, long after he had surpassed his first teacher in learning.[43] Raymond was clearly a competent grammarian—more than once Gerbert invited him to Reims to teach—but he knew little, perhaps nothing, of the arts of number. For this reason Gerald, the abbot, asked Count Borrell II of Barcelona, who had stopped at Aurillac to

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[41] The most recent and thorough-going account of Richer is Jason Glenn's *Politics and History in the Tenth Century.*

[42] The collection, though extensive, is not exhaustive. On the selectivity, ordering, and possible redaction of the letters see Riché and Callu, in Gerbert, *Correspondence*, xxii–xxxii.

[43] See, for example, Gerbert, *Epistulae* 45; ed. Riché and Callu, 112.

pray, if there were men in Spain with "perfect" or complete knowledge of the liberal arts.[44] Borrell responded unhesitatingly that there were.[45] Hearing this, Gerald asked if the count would take one of the boys of the monastery with him to be educated. Borrell agreed and Gerbert was unanimously chosen by the community. Borrell delivered the youth into the hands of Hatto, Bishop of Vic (957–71), a cathedral city in Catalonia located less than fifty miles due north of Barcelona and close to the monastery of Ripoll.[46]

Gerbert remained in Vic from 967 to 970, during which period, according to Richer, he "made great strides in the study of *mathesis*."[47] The statement is tantalizing and vague.[48] Few particulars are known about Gerbert's course of study in Catalonia. Though there was certainly a cathedral school at Vic, the two tenth-century book lists that survive name largely ecclesiastic works, nothing of use in teaching the quadrivium.[49] A later book list from the nearby monastery of Ripoll shows that by 1047 that library contained a number of quadrivial works, some of which can be securely dated to the tenth century and may have been present at the time of Gerbert's sojourn.[50] Scholars in our own time have pointed out that these are standard texts, available elsewhere in Europe, and contain nothing that would have dazzled the intellectual elites of the papal and imperial courts. But it should be remembered that Gerbert went to Catalonia seeking masters, not manuscripts. The same texts may have existed in other libraries, but without masters skilled in deciphering them, they lay dormant.

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[44] Richer, *Historiae* 3.43; ed. and trans. Lake, 2:62–63: "post sermones quotlibet an in artibus perfecti in Hiapaniis habeantur sciscitatur." In 967, Borrell II, count of Barcelona, Gerona, Urgell, and Osona, made a journey to Aquitania to ask (and obtain) in marriage Ledgarda, daughter of Ramon I, count of Rouergue. It was on the return journey that he stopped at Aurillac. Marco Zuccato, "Gerbert of Aurillac and a Tenth-Century Jewish Channel for the Transmission of Arabic Science to the West," *Speculum* 80 (2005): 747–48.
[45] Richer, *Historiae* 3.43; ed. and trans. Lake, 2:62–63: "Quod cum promptissime assereret."
[46] Richer refers to Gerbert as an *adolescens* at this point in the *Historiae*. Ibid.
[47] Ibid.; ed. and trans. Lake, 2:62–4: "Apud quem etiam in mathesi plurimum et efficaciter studuit."
[48] Scholars have sought to flesh out the contours of Gerbert's course of study in Catalonia, working backward from Richer's description of his revolutionary teaching techniques (described below) and Gerbert's writing, particularly his treatises on the astrolabe and the abacus. The problem with the latter is that the corpus of works attributed with certainty to Gerbert is unstable. See Guy Beaujouan, "Les Apocryphes mathématiques de Gerbert," in *Gerberto: Scienza, storia e mito* (Bobbio: Archivi storici bobiensi, 1985), 645–55, esp. 646.
[49] There are two extant "catalogues" for Vic, one from 957 and the other from 971, both of which contain only ecclesiastical texts. This may mean that the cathedral did not possess any so-called secular texts for teaching the liberal arts, or it may mean that school texts were housed separately and not recorded together with the ecclesiastical holdings. Eduard Junyent i Subirà, ed., *Diplomatari de la Catedral de Vic (segles IX-X)*, vol. 1 (Vic: Publicacions del Patronat d'Estudis Osonencs and Publicacions de l'Arxiu, Biblioteca i Museu Episcopals de Vic, 1980–96), nos. 303 and 413.
[50] For the manuscripts of Ripoll, see Rudolf Beer, "Die Handschriften des Klosters Santa Maria de Ripoll, I and II," *Sitzungsberichte der philosophisch-historischen Klasse der Kaiserlichen Akademie der Wissenschaften, Wien* 155, no. 3 (1908): 1–112 and 158, no. 2 (1908): 1–117.

It was, moreover, Gerbert's unusual methods of instruction that would later set him apart from his peers. He would employ tactics and tools unprecedented in the Latin-speaking world, but *not* in Arabic-speaking Spain. This fact has led scholars persuasively to assume that Gerbert was exposed to Arabic sources in Catalonia.[51]

Hatto is known to have had access to such sources. He and Gerbert's other guardians in Spain—Borrell and Miró Bonfill—were part of the intellectual circle of Gotmar, Bishop of Girona. Gotmar had spent time in Cordoba as ambassador to the caliph on behalf of Borrell's father, Count Sunyer (911–47). He had travelled to Cordoba with his Andalusian counterpart, a Jew, Abū Yusuf Hasdāy ben Ishāq ben Shaprūt, the caliph's chief advisor and one of the brilliant *literati* of the Cordoban court. Hasdāy passed at least three months in Catalonia, where he may have been known to the Latin-speaking community by the moniker "Joseph," in reference to the Joseph of the Old Testament: a Jewish savant, who advised the Pharaoh. Years after his sojourn in Spain, Gerbert wrote to Miró Bonfill, a cousin of Borrell, who by then had been made Bishop of Girona (971–84), to request a copy of "Joseph the Wise's *De multiplicatione et divisione numerorum*" for the library of Reims, very possibly a work by Hasdāy.[52] Historian Marco Zuccato has convincingly argued that it was via diplomatic channels, through men like Gotmar and Hasdāy, that Arabic science first began to flow into the Latin West.[53] Put in Hatto's charge, Gerbert was well positioned to receive, absorb, and eventually transmit this new knowledge.

In 970/71 Gerbert travelled to Rome and joined the court of Otto I. He stayed with the emperor only briefly (possibly serving as a tutor to Otto II). Richer writes how Gerbert expressed his desire to flesh out his understanding of the trivium. Otto, sympathetic, granted

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[51] Just decades after his death it was said that Gerbert had gone to the heart of Arab Spain, to Cordoba, the seat of the Caliphate and center of learning in the region: "Gerbertus vero natione Aquitanus, monacus Aureliacensis sancti Geraldi ecclesiae, causa sophiae primo Franciam, deinde Cordobam lustrans, cognitus ab imperatore, archiepiscopatu Ravennae donatus est." This testimony appears in the third book of the *Historia Francorum* written by Adhemar of Chabannes (d. c.1030); *Ademari Historiarum libri III*, ed. G. Waitz, MGH SS 4 (Hanover: Hahn, 1841), 130. At stake is the role of Gerbert (and Catalonia) in the early dissemination of Arabic science. Marco Zuccato has offered the most recent reassessment (reaffirming Gerbert's role) in two articles: "Gerbert of Aurillac and a Tenth-Century Jewish Channel for the Transmission of Arabic Science to the West," *Speculum* 80 (2005): 742–63; and "Arabic Singing Girls, the Pope, and the Astrolabe: Arabic Science in Tenth-Century Latin Europe," *Viator* 45, no. 1 (2014): 99–120. The following is deeply indebted to Zuccato's work.

[52] Gerbert, *Epistulae* 25; ed. Riché and Callu, 50. In another letter of the same year he requested a copy of the same work, this time naming the author as "Joseph the Spaniard." Ibid, 17, 36. For the identification of "Joseph" with Hasdāy see Zuccato, "Gerbert of Aurillac," 754–55.

[53] Zuccato, "Gerbert of Aurillac," 742–63, esp. 746 and 763.

Gerbert permission to study with Gerannus of Reims, a master of the verbal arts, who happened to be sent to Otto I's court on a diplomatic mission (c.972).[54] When Gerannus returned to Reims, Gerbert went with him. His arrival in Reims marked the beginning of his career as a teacher (972–982, 984–991). Gerannus became his first pupil, as eager to master the quadrivium as Gerbert was the trivium. Both men applied themselves with zeal, but music proved too difficult for Gerannus, who abandoned the endeavor. Gerbert, on the other hand, made "great progress in a short amount of time."[55] His mental talents recommended him to Archbishop Adalbero of Reims, who took the young man on as secretary, a position he held intermittently until 991, and asked him to teach at the cathedral school.

Several extended passages in the *Historiae* vividly capture Gerbert's unconventional approach to teaching. Dialectic, Richer reports, came first.[56] Gerbert taught the art of logic through a "fixed sequence of texts."[57] These are minutely enumerated. Richer provides titles in both Greek and Latin and gives the names of authors, commentators, and translators. Thus, for example, we are told that Gerbert began with "Porphyry's *Isagoge* (that is, *Introduction*) according to the translation of the rhetorician Victorinus and the commentary by Boethius."[58] The bookishness of the section on dialectic is striking,[59] especially as it is juxtaposed with his treatment of rhetoric, taught, we are told, by means of active argument with a trained sophist, and the quadrivium, for which no tracts are mentioned. Though Richer's account is truly remarkable for its detail, it is not exhaustive. Richer says nothing, for example, about a teaching device that Gerbert enthusiastically described in a letter to Bernard, a monk at

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[54] The name of the logician sent by King Lothar to Otto in Italy is given in MS Misc.Hist.5. only as "G Remensium archidiaconus." Richer, *Historiae* 3.45; ed. and trans. Lake, 2:66. For the identification of "G" as Gerannus, see Hoffmann, *Richeri historiarum* 193, n. 2.

[55] Richer, *Historiae* 3.45; ed. and trans. Lake, 2:68–69: "in brevi admodum profecit."

[56] Richer does not mention grammar, probably because most students would have come to Reims with a firm grasp of the art (see chapter 3). Apart from this omission, the order in which the disciplines are introduced by Richer is the same as that given by Augustine and Martianus Capella. Cassiodorus, however, in the *Institutiones*, addressed rhetoric first and Isidore in the *Etymologiae*, followed suit. Mark Vessey notes that Varro had earlier put dialectic before rhetoric and suggests that this was the "standard" order in the Latin-speaking world; Cassiodorus, in his view, may have followed a Greek model. I would add that in the context of the *Historiae*, the teaching of rhetoric, which culminated in sparring with a sophist—shifting away from book-centered study, provides a seamless transition to Gerbert's more active and hands-on manner of teaching the quadrivium described by Richer. Mark Vessey, "Introduction," 72.

[57] Richer, *Historiae* 3.46; ed. and trans. Lake, 2:68–69: "Dialecticam ergo ordine librorum percurrens."

[58] Ibid.: "Imprimus enim Porphirii ysagogas, id est introductiones, secundum Victorini rhethoris translationem, inde etiam easdem secundum Manilium explanavit."

[59] The marginal heading for this section is "The sequence of books that he [Gerbert] maintained in teaching"

Aurillac, namely a large rectangular table constructed of twenty-six sheets of parchment (2 sheets by 13 sheets) made for "fixing the materials of the rhetoricians in the mind."[60] Hence, Richer's silence on the subject of tracts for teaching rhetoric and the quadrivial disciplines does not mean that Gerbert did not rely on books to teach these subjects. Ample evidence suggests that he did.[61] Teaching with books, however, was not *new*, as such it perhaps had no place in an account of "reformed education." That Richer chose to highlight the texts used in the teaching of dialectic, namely the cluster of Aristotle's logical works, known as the *logica vetus*, may have merited mention because in 970 they were a novelty, still largely unknown in the Latin-speaking world.[62]

"I do not think it would be out of place to mention how much effort (*sudor*) he [Gerbert] expended on the mathematical disciplines."[63] In this way Richer turned from the trivium to the quadrivium. A marginal heading beside the above-quoted sentence in the Bamberg manuscript reads: "The labor he expended on the mathematical arts."[64] Both statements emphasize the physicality of the endeavor: when it came to mathematics Gerbert toiled, he sweated—a lot (*quantus*). Indeed, he seems to have put on a sort of one-man pedagogical pageant involving fancy props that—we know from his letters—were expensive and time-consuming to create.[65] He plucked out music's tones and semi-tones on the monochord,[66] and he taught the art of geometry with the help of an abacus. A board divided

("Quem ordinem librorum in docendo servaverit"). These headings are part of the original work, found in Bamberg, Misc.Hist.5.

[60] Gerbert, *Epistulae* 92; ed. Riché and Callu, 220, trans. Lattin, 140–41: "ad res rethorum fugaces et caliginosissimas comprehendendas atque in animo colocandas."

[61] Discussed below.

[62] The *logica vetus* included Porphery's *Isagoge*, Aristotle's *Categories*, and the *Peri hermeneias*; Richer also lists the *Topica*, translated by Cicero. This cluster of texts quickly became a mainstay of dialectical study, but Gerbert was at the vanguard. As was Abbo. On Gerbert's teaching of the *logica vetus* see Harriet Lattin, "The Eleventh Century MS Munich 14436: Its Contribution to the History of Co-ordinates, of Logic, of German Studies in France," *Isis* 38, no. 3/4 (1948): 223. On the emergence and influence of Aristotle's logical works more generally see John Casey, "Boethius's Works on Logic in the Middle Ages," in *A Companion to Boethius in the Middle Ages*, ed. Noel Kaylor, Jr. and Philip Phillips (Leiden: Brill, 2012), 193–220, esp. 212; and Osmund Lewry, "Boethian Logic in the Medieval West," in *Boethius: His Life, Thought and Influence*, ed. Margaret Gibson (Oxford: Blackwell, 1981), 90–134.

[63] Richer, *Historiae* 3.49; ed. and trans. Lake, 2:72.

[64] Bamberg, Msc.Hist.5, fol. 35: "Qui labor ei in mathematicis impensus sit." The heading, at the bottom right of the page, is illegible in the digital images available online.

[65] In two letters to Remi (a monk in Trier, later Abbot of Mettlach) Gerbert describes the process of the making of a sphere as an onerous activity; Remi requested one in exchange for copying the *Achilleidos*. Gerbert, *Epistulae* 134 and 148; ed. Riché and Callu, 328–30 and 362; trans. Lattin, 172–73 and 184–85.

[66] Richer, *Historiae* 3.49; ed. and trans. Lake, 2:72–74.

into twenty-seven columns, each of which corresponded to a power of ten,[67] the abacus made the abstract processes of arithmetic visible and tangible and, as such, geometrical; with it, numbers were not only seen, but grasped, and manipulated.[68] Richer tells of the swiftness with which Gerbert's hands moved small counters, performing difficult calculations (multiplication and division) with awe-inspiring speed and ease.[69] The rapidity of his manual movements made manifest his mental agility. The account is rich in material specifics—the abacus board is made of wood covered in parchment, the counters of horn—that appeal to reader's senses, bringing these scenes to life.

Gerbert taught astronomy, the science of solid bodies in motion, with three-dimensional, mobile models. These were described by Richer in hyper-detail so that, in his words, "the ingenuity of such a great man may be appreciated and the reader may be duly convinced of the effectiveness of his methods."[70] Gerbert used a wooden sphere to help students recognize the shapes of the constellations and envision the rising and setting of the stars in the night sky (fig. 2.1). He encouraged his students to turn their eyes from the sphere to the heavens and observe firsthand the position and movement of the constellations.[71] He made the "invisible circles" known as "parallels" or "equidistants" visible through another instrument: the compound sighting tube. The object, which could be rotated, comprised several tubes (one for each celestial circle in the northern hemisphere: the arctic circle, tropic of cancer, and equator) set at fixed angles (fig. 2.2).[72] Anyone peering through the tubes

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[67] Ibid. 3.54; ed. and trans. Lake, 2:82. The term *abacus* has been applied to diverse tools. Derived from the Greek αβαξ, the word in antiquity designated a board or table covered in colored dust or sand, which was wetted and tamped down in preparation for use. Though such a surface could serve any discourse, the habit of drawing in dust or sand was most closely associated with the arts of geometry and astronomy. By the 10th c. an *abacus* was an oblong object—a board or a large piece of parchment—divided into columns each of which corresponded to a power of ten (the first column is for ones (100), the second for tens (101), etc.). Numbers were represented and manipulated by means of counters, which were made of some easily shaped material and marked with characters 1 through 9 (Arabic ciphers were used to represent all but the number one). On the term *abacus* and the changing shape of the instrument it described, see Mariken Teeuwen, *The Vocabulary of Intellectual Life in the Middle Ages* (Turnhout: Brepols, 2003), 353–54.

[68] On the didactic tactic of making numbers tangible for students, see Gillian Evans, "The 'Sub-Euclidean' Geometry," 107.

[69] Performing large calculations in the mind is always difficult, Roman numerals—cumbersome and unwieldy—made even basic calculations a challenge.

[70] Richer, *Historiae* 3.50; ed. and trans. Lake, 2:74–75: "Ratio vero astronomiae quanto sudore collecta sit dicere inutile non est ut et tanti viri sagacitas advertaturet artis efficacia lector commodissime capiatur."

[71] Ibid.; ed. and trans. Lake, 2:76.

[72] Ibid. 3.51; ed. and trans. Lake, 2:76–78.

could trace with their eye and "see" the paths of the equidistants, which notionally divided the heavens.

Gerbert demonstrated the planetary orbits by means of an armillary sphere, described by Richer as "a sphere made up entirely of circles." Around this Gerbert wound more circles delineating the "incidents" (two longitudinal celestial circles: one passing through the solstices and the other through the equinoxes), the equidistants, and the zodiac. He made a fourth sphere, its surface enlivened with fine iron and brass wires outlining the constellations. A tube passed through the center of the solid sphere between the north and south poles. The constellations on the sphere's surface could be aligned with the night sky by sighting the North Star through the tube. Richer, clearly awed by this object, exclaimed:

> There was also something miraculous about this instrument, for if someone who knew nothing of astronomy were shown one constellation with this sphere, he could then use it to identify all the other constellations without the need for a teacher.[73]

Likewise, he wrote with wonder of the power of the compound sighting tube:

> This instrument was so efficacious in design that if the diameter were pointed toward the North Pole and the curved part of the circle were rotated, it would reveal those circles that are invisible to the eye and store them deep within the memory.[74]

Significantly, Richer in these passages explains not only how these objects were constructed, but also how they worked. That is, how they affected both body and mind, instilling new knowledge: by directing the eye, making phenomena visible, aiding the beholder's apprehension, impressing information on memory.[75]

The use of spheres to teach astronomy was unheard of in the Latin-speaking world before Gerbert.[76] Texts and passages in authoritative works devoted to the discipline—Plato, Pliny, Calcidius, Macrobius, Martianus—include a range of pictures, some quite colorful, detailed, and of considerable measure, but all two-dimensional, flat (figs. 1.93–94 and

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[73] Ibid. 3.53; ed. and trans. Lake, 2:80–83: "Illud quoque in hac divinum fuit, quod cum aliquis artem ignoraret, si unum ei signum demonstatum foret, absque magistro cetera per speram cognosceret."
[74] Ibid. 3.52; ed. and trans. Lake, 2:78–79: "Cuius instrumenti ratio in tantum valuit, ut ad polum sua diametro directa, ac semicirculi productione superius versa, circulos visibus inexpertos scientiae daret atque alta memoria reconderet."
[75] See chapter 4.
[76] Zuccato, "Gerbert of Aurillac," esp. 755–61.

1.103).[77] Neither were such spheres a standard feature of the classroom in the Arabic-speaking context, where the astrolabe was the astronomer's instrument of choice. At least one contemporary thinker, however, was championing the pedagogical power and efficacy of spheres. Dunāsh ibn Tamīm ibn Ya' qūb al-Isrā'īlī al-Qarawī, philosopher, physician, and astronomer at the Fatimid court in Qayrawan (modern-day Tunisia), wrote an astronomical treatise, the first part of which was given over to the topic of celestial spheres. He dedicated the work to a Jew named Hasdāy, almost certainly the same Hasdāy whom Gotmar encountered in Cordoba and whose work on arithmetic Gerbert so eagerly sought for the library at Reims. The treatise does not survive, but another work by Dunāsh on the armillary sphere offers some insight into the lost work's contents. Of particular interest is the introduction, in which Dunāsh enumerates the advantages of a spherical model over a superficial one like the astrolabe. First and foremost is the fact that the shape of the former echoes the actual shape of the heavens, if in miniature. Once the geometry of the heavens has been grasped by means of such models, he believed, then the astrolabe, a transposition of the three-dimensional heavens onto a two-dimensional surface, could be better understood. Gerbert's use of spheres and Richer's descriptions, which chime with Dunāsh's words, suggest that the master of Reims had encountered Dunāsh's treatise while in Catalonia.[78]

As a master capable of instructing students in *all* the liberal arts, Gerbert made Reims, an ecclesiastical and political center, into a center of learning as well.[79] Richer speaks of the

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[77] For examples of these images see chapter 1; on the use of a circle to represent a sphere see chapter 4. Lattin has argued that Macrobius's description of an armillary sphere was complete and that one of his proofs (at 1.18.12) referred to a sphere (Lattin, "Use of a Sphere by Macrobius," *Isis* 39, no. 3 [1948]: 168–69). For a detailed study of the use of spheres, see Emilie Savage-Smith "Islamicate Celestial Globes: Their History, Construction and Use," *Smithsonian Studies in History and Technology* 46 (1985): 3–60. On the astrolabe as a teaching tool, see Arianna Borelli (*Aspects of the Astrolabe:* Architectonica ratio *in Tenth- and Eleventh-Century Europe*, Sudhoffs Archiv. 57 (Stuttgart: Franz Steiner Verlag, 2008) and Marco Zuccato (2014). For the introduction of these tools into Latin-speaking Europe, see Richard Lorch ("The *Sphera solida* and Related Instruments," *Centaurus* 24 [1980]: 153–61) and Charles Burnett,"King Ptolemy and Alchandreus the Philosopher: The Earliest Texts on the Astrolabe and Arabic Astrology at Fleury, Micy and Chartres," *Annals of Science* 55, no. 4 (1998): 329–68.

[78] Richer describes the *armilla*, the horizon line, for the first time in the West as pointed out by Marco Zuccato ("Gerbert of Aurillac," 757). I strongly disagree with his statement that "the orthodox method of absorbing classical sources *was replaced by the practical use of actual spherical instruments*" [emphasis mine]. As stated above, Richer's account of Gerbert's teaching should not be considered exhaustive. I suggest that models and manuscripts complemented each other, were used in tandem, and that the models should be seen as a physical extension of the text and its images.

[79] The primates of Reims called their city a "second Rome," repeating the legendary story of its founding by Romulus's brother, Remus, since Rome was too small to contain the twins' ambitions. The story is told in Flodoard's *Historia Remensis ecclesiae* (MGH SS 36 i.i, 61–3). Reims was known as the "crowning city." The

"crowds of students" who, eager for instruction, flocked to Reims from "diverse regions," their number "increasing every day."[80] Gerbert's fame, he tells us, carried as far as the Tyrrhenian and Adriatic seas.[81] Around 979/80, a young man travelled from Saxony to study at Reims.[82] Otric, master at the cathedral school of Magdeburg, had tasked him with recording the system of knowledge taught by Gerbert. The young man copied Gerbert's schema of the division of philosophy and returned with it to Germany. When shown the *figura Gerberti philosophica* (*philosophical figure of Gerbert*) Otric was horrified and triumphant. For, Richer wrote:

> Although Gerbert had arranged physics as a part of the same genus as mathematics, in the version that this man [from Saxony] brought back physics was subordinated to mathematics as species to genus. It is not known whether he did this deliberately or if it was a mistake.[83]

Otric publicly ridiculed and denounced Gerbert for subordinating physics to mathematics, deeming him unfit to teach philosophy. He brought the offending *figura* to the palace to show the emperor and the intellectuals at his court. Otto II was skeptical. He doubted Gerbert would make such a blunder, having "seen and heard him [Gerbert] engaging in disputations on more that one occasion."[84]

A year later, when the emperor and Gerbert crossed paths in Pavia, he invited the schoolmaster of Reims to Ravenna. There he arranged for a debate with Otric in order to test Gerbert's knowledge of philosophy. The emperor gathered the learned men in his entourage to witness the contest, which was staged as entertainment for the Christmas season. Archbishop Adalbero of Reims and Adso, the abbot of Montier-en-Der, joined their ranks.

Holy Ampulla, housed in the Abbey of Saint-Rémi, was thought to contain oil used by St. Remigius, the founder of Reims, to baptize Clovis and was said to be divinely replenished. Louis the Pious was anointed in Reims, as were other kings. Even when coronations were performed elsewhere, it was, by tradition, the Bishop of Reims, who did the anointing. Michael Moore, "Prologue: Teaching and Learning History in the School of Reims, c.800–950," in *Teaching and Learning in Northern Europe, 1000–1200*, ed. Sally N. Vaughn and Jay Rubenstein (Turnhout: Brepols, 2006), 19–49, esp. 22.

[80] Richer, *Historiae* 3.55; ed. and trans. Lake, 2:84–85: "numerusque discipulorum in dies accrescebat. Nomen etiam tanti doctoris fetebatur non solum per Gallias, sed etiam per Germaniae populos dilatabatur. Transiitque per Alpes ac diffunditur in Italiam usque Thirrenum et Adriaticum." Gerbert's students are discussed below.

[81] Ibid.

[82] Ibid.

[83] Ibid. 3.56; ed. and trans. Lake, 2:86–87: "Etenim cum mathematicae phisica per atque coaeva a Gerberto posita fuisset, ab hoc mathematicae eadem phisica ut generi species subdita est. Incertumque utrum industria an errore id factum sit."

Richer minutely described the disputation in the *Historiae*. The offending *figura* was brought before Gerbert, who quickly condemned it as a false representation. Otric then challenged Gerbert to "make a complete division [of philosophy]."[85] Gerbert complied and then successfully rebuffed several more challenges from the Magdeburg schoolmaster, who, according to Richer, had been ordered by Otto II to be relentless in his questioning. The discussion ranged from the cause of Creation to the nature of shadows. In the end, Gerbert proved himself more than worthy of the title of philosopher and "returned to Gaul covered in glory."[86] The episode highlights an enthusiasm for esoteric topics beyond the bounds of the schoolroom as well as the diverse settings in which such ideas were broadcast. Otto II's comment that he had "seen and heard Gerbert engaging in disputations more than once" suggests that the debate in Ravenna was not a unique event, though few records of such proceedings survive.[87] Though we do not know exactly how Otric first came to hear of Gerbert's methods and ideas nearly 500 miles away from Reims, it is clear that there were conduits—letters, word-of-mouth—for transmitting such information.

The debate pushed Gerbert into the imperial spotlight once again. Too valuable a vassal to be wasted, Otto II named him abbot (981/82) of the monastery of St. Columbanus in Bobbio.[88] The once-great monastery was in near ruin, stripped bare of land, resources, and even the relics of its patron.[89] Gerbert struggled in his new post. His letters read as a litany of rebukes and accusations leveled at the former abbot, nearby farmers, and even Empress Adelaide, concerning the monastery's extensive land holdings, which had been parceled away as gifts or bound up in leases that yielded the abbey no profit.[90] Without direct intervention on the part of Otto II, busy campaigning in the south, Gerbert was helpless to

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[84] Ibid. 3.60; ed. and trans. Lake, 2:94: "Augustus vero, cum et ipse talium studiosissimus haberetur, an Gerbertus erraverit admirabatur. Viderat etenim illum et non semel disputantem audierat."
[85] Ibid. 3.60; ed. and trans. Lake, 2:94–95: "Ad haec Otricus, innuente augusto, sic ait: 'Quoniam philosophiae partes aliquot breviter attigisti, ad plenum oportet ut divisionem enodes'."
[86] Ibid. 3.65; ed. and trans. Lake, 106–107: "in Gallias clarus remeavit."
[87] Ibid. 3.56; ed. and trans. Lake, 86–88: "Viderat etenim illum et non semel disputantem audierat."
[88] On the issues of dating this appointment, Michele Tosi, "Il Governo abbaziale di Gerberto a Bobbio," in *Gerberto: Scienza, storia e mito* (Bobbio: Archivi storici bobiensi, 1985), 97–107.
[89] The monastery was founded in the 7th c. by the Irish monk and missionary St. Columbanus (c.540–640). Gerbert wrote in a letter of the desperate conditions: "I see my monks wasting away from hunger and suffering from nakedness." In 929 the relics of Columban were moved temporarily to Pavia. Tosi, "Il Governo Abbaziale di Gerberto a Bobbio," 87; Gerbert, *Epistulae* 2; ed. Riché and Callu, 4–6; trans. Lattin, 49.
[90] Adelaide was the widow of King Lothar of Italy (d.950) and Otto I (d.973). She controlled more land in Italy than Otto II. Gerbert, *Epistulae* 1–15; ed. Riché and Callu, 2–30.

bring stability in an increasingly acrimonious situation and was reduced to haggling over hay.[91]

The vast trove of books at St. Columbanus seems to have been Gerbert's chief consolation.[92] A tenth-century booklist from the abbey includes over 660 entries. It describes a comprehensive collection of Christian and classical works alike—everything needed to offer instruction in the quadrivium. Whether Gerbert continued to teach while at Bobbio is unclear. A letter suggests that he may have or that he was at least willing to. Egbert (d.933), Archbishop of Trier, asked Gerbert if he might send pupils to him in Italy for instruction.[93] It was around this time that Gerbert wrote the *Isagoge geometriae* (*Introduction to Geometry*), a synthesis of geometric tracts then in circulation. He may have been inspired by a geometric miscellany discovered at Bobbio, about which he wrote to Adalbero, describing the "beautiful figures of geometry" that filled its pages.[94]

When Otto II died on December 7, 983, Gerbert fled St. Columbanus, the situation having deteriorated to the point that he feared for his life.[95] He returned to Reims and resumed his position as secretary to the archbishop and master at the school. Gerbert's letters, some written in his own name, some written on behalf of Adalbero, Duke Charles, Queen Emma, and Hugh Capet, show that he was deeply involved in the political maneuverings of the period.[96] Adalbero died in 989. Gerbert, an intimate of Hugh Capet, who had been elected king after the death of Louis V (d.987), and well versed in the duties of the archbishop, was certain the see of Reims would be his, but Hugh appointed Arnulf, son of King Lothar, instead. Disappointed, Gerbert continued to serve as secretary until the new archbishop showed his disloyalty to Hugh, by opening the gates of Reims to his rival for the throne,

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[91] A letter written to Boso on July 15, 982 requests the return of a quantity of hay wrongly taken from lands of St. Columban (Gerbert *Epistulae* 4; ed. Riché and Callu, 8–9). Gerbert, like many friends of the Ottonian crown, who were placed by Otto in isolation at the head of institutions scattered across Italy to secure his conquests, lacked the support of nearby monarchs. His letters seeking advice of the local bishop were ignored and his relationship with the former abbot was contentious at best (this last had, under the auspices of an obscure Roman law, bequeathed to himself some of the lands of St. Columban); Tosi, "Il Governo Abbaziale di Gerberto a Bobbio," 93.

[92] The library may have been the largest in the Latin-speaking world at that time. See Becker, *Catalogi*, 64–73.

[93] The letter from Egbert is lost. Gerbert's response leaves the decision up to the Archbishop of Trier. Gerbert, *Epistulae* 13; ed. Riché and Callu, 26–27.

[94] Ibid. 8; ed. Riché and Callu, 16–18. On the manuscript(s) described, see chapter 5.

[95] Ibid. 14; ed. Riché and Callu, 28–30.

[96] See, for example, Gerbert, *Epistulae* 26 (Adalbero), 32 (Charles), 97 (Emma), and 107 (Hugh).

Duke Charles.[97] In 991 Gerbert was made archbishop of Reims in place of the treacherous Arnulf, but the verdict and appointment, made by a council of bishops without papal approval, was disputed by the pope and others: those loyal to Arnulf and those who feared the growing power of the bishops.[98] Gerbert clung to the title and acted as archbishop for six years, but he was ever uneasy in his post, always watchful. He suffered excommunication by the pope and was forced to run from Reims more than once. At one point he took refuge at the court of Otto III, where he tutored and advised the young emperor, assuming the position of organ master. It was during this period that the two became close.[99]

Otto III resolved the dispute over the see of Reims by making Gerbert archbishop of Ravenna (April 28, 998). Soon after the appointment the pope died (February 999) and Otto III offered his former tutor the papal crown. Gerbert took the name of Sylvester II in honor of Sylvester I (d.335), bishop of Rome during the reign of Constantine, and, by legend, the Emperor's spiritual instructor and guide. According to the spurious *Donatio Constantini* (*Donation of Constantine*) said to have been written by Constantine and offered to Sylvester, the emperor pledged to share the imperial crown with the Bishop of Rome, successor of St. Peter.[100] Thus, taking the name of Sylvester broadcast the intimate, fruitful, and mutually beneficial relationship between pope and emperor. At the time of his election Gerbert wrote: "This event will accomplish what I have always wished, have always hoped for, by making me his [Otto III's] inseparable companion so that we can guide the exalted empire for him. What, therefore, could be sweeter?" The sweetness, however, did not last. Otto died in 1002, and Gerbert, a year later.

Late medieval legend transformed Gerbert's brilliance with numbers and penchant for scientific demonstration into a kind of sorcery.[101] By the eleventh century it was rumored that his meteoric rise to power was the product of a pact with the devil. The fact that he had spent

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[97] Charles, son of Louis IV, saw himself as the true heir of Louis V. He had been appointed the duke of Lower Lotheringia by Otto II in 977.
[98] The actions of the bishops were never sanctioned by the pope and were thus deemed by him and all succeeding popes to be illegitimate and void.
[99] Gerbert, *Epistulae* 186–87; ed. Riché and Callu, 480–86. On Gerbert as tutor to Otto III see chapter 1.
[100] The *Donatio* was probably written c.750. The oldest known witness is a 9th-c. manuscript, Paris, BnF lat. MS 2777.
[101] This may be in part due to Richer's unusual use of the term *mathesis* to describe Gerbert's quadrivial knowledge in the *Historiae*. In the high and late middle ages the term came to be associated with astrology, alchemy, and the occult. On the "Dark Legend" see Massimo Oldoni, "Gerberto e la sua storia," *Studi medievali* 3rd ser., 18 (1977): 629–704.

time in Arabic-influenced Spain only deepened suspicions. William of Malmesbury wove fact and fiction into a compelling and enduring narrative in the *Gesta regum anglorum* (1125). As in Richer's *Historiae,* the Gerbert of William's *Gesta* revived arithmetic, geometry, astronomy, and music in Gaul "where they had for a long time been wholly obsolete," but he also practiced necromancy, invoked the devil in times of need, had the gift of prophecy, and created impossible and uncanny wonders such as an automaton in the form of a head that spoke the truth.[102] According to William, Gerbert died a cruel and painful death, confessing his sins to the college of cardinals in Rome and crying for them to maim his body and deliver it to the devil.

ABBO OF FLEURY

Abbo, instead of being demonized after death, was canonized: in 1004 he was martyred in Gascony in the course of his duties as a reform abbot. Soon after his death, sometime between 1005 and 1008, Aimoin, a monk of Fleury and intimate of Abbo, wrote the *Vita et passio sancti Abbonis*. It is from this hagiographical account that the details of Abbo's life and death are known.[103] Aimoin was part of the small party that accompanied Abbo on his final journey to La Rèole, and hence his account of the events leading up to the abbot's death—his passion—are recorded in some detail. Aimoin included a few letters written to and by Abbo in the *Vita*. Besides these, only a handful of Abbo's letters survive.[104] He also listed works that Abbo authored—all didactic treatises, several devoted to the quadrivial topics. These give us some idea of Abbo's methods of teaching, though the report is not nearly as vivid as Richer's account of Gerbert in the classroom.

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[102] William of Malmesbury, *Gesta regum anglorum* 2.167–72, ed. G. Waitz, MGH SS 10 (1852), 461–64.

[103] Two other documents of the period help us to build a picture of Abbo's environs. Aimoin scrupulously recorded the miracles performed by Saint Benedict's remains along with information on fires, floods, and the built environment of Fleury in the *Miracula Sancti Benedicti* (*Miracles of Saint Benedict*). Further insight into Abbo's experience at Fleury can be gleaned from Thierry of Fleury's (later Amorbach) detailed description of the customs of Saint-Benoit-sur-Loire written for monks at an unnamed monastery in Germany (possibly dedicated to Berward of Hildesheim). Aimoin of Fleury, *Miracula Sancti Benedicti*, ed. G. Waitz, MGH SS 9 (1851), 347–76; Thierry of Fleury, *Consuetudines.*

[104] Most of Abbo's surviving letters are preserved in a single manuscript, London, BL Add. MS 10972, written at Fleury shortly after Abbo's death. The collection is clearly not comprehensive and it is so eclectic that any unifying factor beyond authorship is difficult to discern. Unlike Gerbert, Abbo was not in the habit of copying letters before sending them. Thus the hodgepodge of letters in BL Add. MS 10972 probably reflects the gathering of surviving copies of letters that remained at Fleury after Abbo's death.

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Abbo, Aimoin tells us, was born in the countryside near Orléans.[105] The year of his birth is not known for certain, but it was probably between 940 and 950, making him a contemporary of Gerbert.[106] His parents were of "proud blood, not swollen of empty nobility," but free men.[107] Despite this modest background, Abbo was not the first in his family brought to be educated by the good clerics of St. Peter's of Fleury,[108] one of the three churches within the monastic compound, or to become a monk of Fleury.[109] His entry into the community, however, at least according to his hagiographer, was particularly auspicious. On the occasion when the boy was pledged an oblate, Abbot Wulford asked his name. Hearing that it was Abbo, the prescient Abbot remarked that if the last letter was changed from "o" to "a" it would be *abba*, "father" in Aramaic—what Christ called his father in heaven—thus foretelling the boy's destiny as abbot (*abbas*), a spiritual "father" of men.[110]

By the time Abbo took the monastic habit, he had learned the art of letters, that is grammar. Even as a little boy, he is said to have pursued his studies with such urgency and concentration that he had only to hear a thing once and it was "firmly fixed in the secret spaces of his heart."[111] Aimoin recounts how the adolescent Abbo fought off the vices of youth and subdued his body through "frequent meditation" and with "continual exercise of

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105 Aimoin, *Vita* 1; ed. Bautier and Labory, 42: "Venerabilis igitur Abbo Aurelianensi ortus est in pago."
106 For the dating of Abbo's birth see, among others, Pierre Riché, *Abbon de Fleury: Un Moine savant et combatif (vers 950–1004)* (Turnhout: Brepols, 2004), 16.
107 Aimoin, *Vita* 1; ed. and trans. Bautier and Labory, 42: "quos quidem non vana tumens de nobilitate superbus alebat sanguis, sed tamen avis attavisque derivata eos honestabat libertas."
108 Ibid.: "in Floriacensi monasterio scholae clericorum ecclesiae sancti Petri obsequentium traditur litteris imbuendus."
It is not clear whether there was a dual school system—an internal and external school—at Fleury. See Guerreau-Jalabert, in Abbo, *Questions*, 18–19. There were three churches at Fleury: Saint-André, Saint-Pierre, and the main church dedicated to the Virgin, Notre Dame.
109 Two men on his mother's side, Gombaudus and Christianus, entered before him. Gombaudus became a monk, Christianus a cleric, both were ordained. Giselbert, another relative, held the abbacy of Saint-Cyprien, Poitiers. Aimoin, *Vita* 1; ed. and trans. Bautier and Labory, 44.
110 Ibid. 2; ed. and trans. Bautier and Labory, 44–45: "Ante quem cum in conventu fratrum fuisset adductus, ab eo quo vocaretur nomine est inquisitus. Ille continuo interroganti responsum dedit Abbonem se dici asseruit. Tum sanctus vir Wulfaldus, prophetico mox spiritu afflatus, alludens ad nomen: 'Abbo, ait, si ultima immutetur littera, patrem sonat in Achivorum lingua.' Et conversus ad puerum: 'Hincjam, inquit, puerorum dulcissime, talem te Christo exhibe ut dignus inveniaris tui nominis appellatione. Annuat hoc omnipotens Pater, a quo omnis paternitas in coelo nominatur et terra [Ephes. 3:25], teque sua faciat servare praecepta, ut ad gaudia possis pervenire aeterna.'"
111 Ibid.; ed. and trans. Bautier and Labory, 46: "Jam vero litterariae artis profunda tanta adhuc puerulus rimabatur instantia, ut a didascalis semel audita firmiter intra cordis conderet arcana."

letters."[112] Study, we are told, was a solace to him; as soon as he was finished with his prayers he took up "practice of the liberal arts."[113] Precocious, he "stuck to the sides of his elders" even when it was inappropriate to do so.[114] In very little time he was drawn into their ranks and enlisted to teach new boys *lectio*, the psalms, and chant, a role he held for a few years.[115] During this time he mastered grammar, arithmetic, and dialectic.

Wanting to complete his education, Abbo set out for Paris and Reims to study with "men, who professed a love of wisdom."[116] Aimoin does not specify which philosophers or say where, exactly, they taught. Pierre Riché speculates that Abbo may have been drawn to Saint-Germain-des-Prés with its well-stocked library, where Remigius of Auxerre had taught at the turn of the tenth century. At Reims, Abbo would have encountered Adalbero (who, according to Richer, instructed youths in the "liberal sciences" before becoming archbishop) and Gerannus. It is uncertain if he crossed paths with Gerbert, who may still have been in Spain.[117] As was touched on in the introduction to this chapter, Abbo left these places having learned less than he had hoped.[118]

Disappointed, but undaunted, Abbo paid a cleric in Orléans "not a little money" to teach him "the sweetness of the art of music."[119] Aimoin here adds a curious aside: that Abbo attended these lessons in secret, on account of "jealous men."[120] The comment suggests that many would have liked to attain such knowledge, but few were able, perhaps due to lack of

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[112] Ibid.: "Egressus pueritiae tempora. frequenti meditatione adolescentiae satagebat reprimere vitia, sciens scriptum quia 'frequens meditatio camis est afflictio' (Eccle. 12:12). Quam idcirco jugi litterarum exercitio domare curabat ut spiritui eam servire cogeret."
[113] Ibid.: "quasi proquodam levamine, post precum ad Dominum missa libamina, liberalium artium sumebantur exercitia."
[114] Strict rules separated the oblates from the monks and governed their interactions. See Benedict of Nursia, *The Rule of Saint Benedict* 48.21 and 63.18–19, ed. and trans. Bruce L. Venarde, in DOML 6 (Cambridge: Harvard University Press, 2011), 162 and 204.
[115] *Lectio* was the first stage of reading. On the teaching of grammar at Fleury see chapter 3.
[116] Aimoin, *Vita* 3; ed. and trans. Bautier and Labory, 48: "Verum ipse, adhuc majora gliscens scientiae scrutari arcana, diversorum adiit sapientiae officinas locorum, ut, quia grammaticae, arithmeticae, nec non dialecticae jam ad plenum indaginem attigerat, caeteras ingenio suo pergeret superadjicere artes. Quapropter Parisius atque Remis ad eos qui philosophiam profitebantur profectus."
[117] Scholars disagree about whether Abbo and Gerbert overlapped at Reims. There is no evidence to suggest they did.
[118] Ibid.: "sed non quantum cupierat, apud eos profecit."
[119] Ibid.: "Inde Aurelianis regressus, musicae artis dulcedinem, quamvis occulte, propter invidos, a quodam clerico non paucis redemit nummis."
[120] Ibid.: "quamvis occulte, propter invidos."

funds or talents.[121] Indeed, Aimoin stresses just how rare, unique even, Abbo's intellectual ability was. He writes that having mastered five of the liberal arts, Abbo stood above "all his peers with respect to wisdom."[122] Still, two of the arts, rhetoric and geometry, remained largely unlearned, even if Aimoin is quick to point out that Abbo was not entirely ignorant of these subjects. On rhetoric he had read Marius Victorinus, "whom Jerome, translator of divine law, boasted as having had as a tutor."[123] No sources are cited for geometry, but Aimoin states that Abbo was sufficiently "skilled in the manipulation of geometrical numbers," possibly a reference to work with the abacus.[124]

Aimoin is vague about Abbo's activities after his return to Fleury. Likely, he resumed teaching, now armed with a thorough knowledge of five of the liberal arts and partial knowledge of all seven.[125] He may have served as *armarius*.[126] The title appears for the first time in Thierry of Fleury's *Consuetudines Floriacenses antiquiores* (*Customary of Fleury*), a detailed description of the traditions and organization of Saint-Benoît. Though written between 1010 and 1022, it describes the workings of the monastery during Thierry's time there, before 1002.[127] According to Thierry, the *armarius* served as keeper of manuscripts, overseer of the scriptorium, and master of the school—a position of considerable responsibility and power. As *armarius*, Abbo would have been well placed to write. Aimoin

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[121] It will be remembered that music was the subject that stymied Gerannus at Reims. Pierre Riché speculated that men may have been jealous of the cleric, who was gaining a profit for his knowledge, and perhaps did not want others to know about it. Riché, *Abbon de Fleury*, 28.
[122] Aimoin, *Vita* 3; ed. and trans. Bautier and Labory, 48: "Itaque quinque ex his quas liberales vocant, plenissime imbutus artibus, sapientiae magnitudine cunctos praeibat coaetaneos."
[123] Ibid.: "Nam et de rhetoricae ubertate facundiae Victorinum, quem divinae interpres legis Hieronymus praeceptorem se habuisse gloriatur, legit." The work refered to is probably the 4th-century Marius Victorinus's *Explanationes* on Cicero's *De inventione*. Works by Victorinus are preserved in some surviving manuscripts from Fleury: Paris, BnF n. a. l. MS 1611 and Bern, Burgerbibliotek Cod. 338 and 207. It should be remembered that the claim of being self-taught was a *topos* known at least from Augustine, *Conf.* 4.16.28: "et quid mihi proderat, quod omnes libros artium, quas liberales uocant, tunc nequissimus malarum cupiditatum seruus per me ipsum legi et intellexi, quoscumque legere potui? ... quidquid de arte loquendi et disserendi, quidquid de dimensionibus figurarum et de musicis et de numeris sine magna difficultate nullo hominum tradente intellexi, scis tu, domine deus meus, quia et celeritas intellegendi et dispiciendi acumen donum tuum est."
[124] Ibid.:"et geometricorum multiplicitatem numerorum non mediocriter agnovit."
[125] By the time he went to England, he had been appointed to the office of deacon (*levita*), but this is the only title given by Aimoin. *Vita* 4; ed. and trans. Bautier and Labory, 54.
[126] Aimoin uses the term *levita*, a title that is not defined in the *Consuetudines*. The terms *Decano* and *Subdecano* are defined. Both were roles of power in the monastery. The *Decano* was second only to the Abbot and was the liaison between abbot and community. Thierry, *Consuetudines* 4.
[127] Thierry, *Consuetudines* 9; ed. Davril and Donnat, 182–84. The term was also used by Byrhtferth in the *Vita Sancti Oswaldi* 3.7; ed. and trans. Michael Lapidge, in *The Lives of St. Oswald and St. Ecgwine* (Oxford: Clarendon Press, 2009), 66.

lists four works that Abbo authored at this time: a dialectical treatise, in which he unraveled "certain knots of logical syllogisms"; a computus, in which he constructed "colorful and delightful" tables with elaborate care;[128] and two short astronomical tracts on the orbits of the sun, the moon, and the planets.[129] To these can be added the *Explanatio* on the *Calculus*. Abbo called the *Explanatio* a "little book on number, measure, and weight," and said that it had been written at the request of his brothers.[130] In it, he claimed that nothing pleased him more than "industrious learning and teaching."[131] These statements, along with his writings of this period, which are all didactic and show easy access to a well-stocked library, make it clear that Abbo was looked to for instruction and explanation, whether or not he held the title of *armarius*.

In 985, when a delegation came from England asking that "one of the wise men" of Fleury be given to them to teach at the fledgling Benedictine monastery of Ramsey, Abbo, "in whom divine and human knowledge shone singularly," was nominated.[132] After a long and harrowing journey, he arrived at Ramsey, located in the Cambridgeshire fens.[133] The monastic complex had been built on an island, a rare spit of solid land in the peat marsh,

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[128] Aimoin *Vita* 7 and 13; ed. and trans. Bautier and Labory, 60 and 100.
[129] Ibid. 7; ed. and trans. Bautier and Labory, 60: "Sic demum, vivaci mentis ingenio haec universa strenuae assecuto, facilis erat eorum operatio. Denique quosdam dialecticorum nodos syllogismorum enucleatissime enodavit compotique varias ac delectabiles secularium in morem tabularum texuit calculationes. De solis quoque ac lunae seu planetarum cursu a se editas disputationes scripto posterorum mandavit notitiae." Aimoin refers to these works a second time in the *Vita* and adds other works to the list, including a florilegium of extracts from the Church Fathers. (Aimoin explains that the latter had been lost, but that it served as the basis for Abbo's defense of the rights of Fleury against the bishop of Orléans.) On Abbo's *oeuvre* see Pierre Riché, *Abbon de Fleury: Un moine savant et combatif (vers 950–1004)*. (Turnhout: Brepols, 2004) 280–83; and A. Van de Vyver, "Les oeuvres inédites d'Abbon de Fleury," *Revue bénédictine* 47 (1935): 125–69.
[130] Abbo, *Quaestiones grammaticales* 50, ed. Anita Guerreau-Jalabert (Paris: Belles lettres, 1982), 275; all translations of the *Quaestiones* are my own unless otherwise cited: "disserui in libellulo quem precibus fratrum coactus de numero, mensura et pondere." It is curious that Aimoin omits this work. Possibly he considered it part of Abbo's computus. The two works are paired in at least one extant manuscript from Fleury: Berlin, Staatsbibliotek Cod. 138.
[131] Abbo, *Explanatio* 1.2; ed. Peden, 64: "nihil unquam fuerit in vita iocundius quam discere aut docere attentius."
[132] Aimoin, *Vita* 4; ed. and trans. Bautier and Labory, 50. Aimoin states that the selection was approved by Oylbold, who had been elected abbot (possibly over Abbo) at the death of Amalbert in 985. Some of the monks in the community, in Aimoin's view, were convinced that Oylbold's intentions toward Abbo were ill, that he was sending his bright, charismatic competitor on the dangerous trip across the Channel in the hopes that he would not return. Abbo allegedly dismissed these speculations and accepted his abbot's order, quoting Paul: "to them that love God, all things work together unto good." (Romans 8:28).
[133] The journey from Fleury was about 250 miles to a port near Calais; the company crossed the Channel by boat; and then it was another 125 miles overland to Ramsey. Six boats were lost in the crossing owing to high seas. Aimoin reckoned that Abbo's safe passage in these conditions was a miracle due to the power of the "man

accessible only by boat.[134] It was a far cry from Fleury, and Abbo and others characterized his time in England as "exile."[135] But the fenland fog concealed a young and vigorous monastic institution. Ramsey had been established in the mid-960s with the support of King Edgar (959–75) by the powerful and strong-willed Bishop Oswald of Worcester (961–92) and later York, and Æthelwine (d.992), a wealthy local landowner and nobleman.[136] The site was chosen for its isolation; the abundance of fish, fowl, wood, and eels; and its ready access to the Barnack quarries. By the mid-eleventh century, Ramsey was the richest monastery in England.[137] At the time of Abbo's sojourn, however, it was a modest institution, comprising a number of wooden outbuildings and a cruciform church made of stone with a tower at its crossing.[138] The forty or so monks it housed adhered wholeheartedly to their vows of poverty, chastity, and obedience, reveling in their "other Elysium."[139] They welcomed the

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of God's" prayers. Aimoin, *Vita* 4; ed. and trans. Bautier and Labory, 52–54; Dachowski, *First Among Abbots*, 65–66.

[134] Technically Ramsey was built on a peninsula, but the ground connecting it to the mainland was unstable and waterlogged. Abbo described it as an island in a poem he wrote celebrating the monastery: "There gleams an exquisite island with its woodland waters" ("insula silvoso gurgite pulchra nitet"). He also references the isolation of the place: "secluded by spreading waters" ("amplis que claudere stagnis"). Dachowski points out that Ramsey remained isolated despite its growing prestige. At the writing of the Domesday Book a century later, no towns were listed in the fenlands. The exact layout of the monastery is not clear. It is possible that boats or a bridge were needed to cross from the monastic buildings to the church. Dachowski, *First Among Abbots*, 67.

[135] Such terminology is used by Abbo in the *Quaestiones* and the *Summa sacer* (an acrostic celebrating Dunstan) and by Oylbold in a letter calling Abbo back to Fleury (included by Aimoin in the *Vita*). Abbo, *Quaestiones* 3; ed. Guerreau-Jalabert, 211; Gwara, "Three Acrostic Poems," *Journal of Medieval Latin* 2 (1992): 216; Aimoin, *Vita* 6; ed. and trans. Bautier and Labory, 54–56.

[136] Byrhtferth, *Vita S. Oswaldi* 3.14; ed. and trans. Lapidge, 82 and 84. Æthelwine was the ealdorman of East Anglia, and Ramsey was located on his lands. Ramsey was the first successful reformed Benedictine monastery in England. Other like-minded communities soon followed, a number of which (Ely, Thorney, Peterborough, Crowland, and St. Neots) were located nearby in the fens, where the ideal of isolation could be easily met. What is known about Oswald and the founding of Ramsey comes from Byrhtferth of Ramsey's *Vita S. Oswaldi*, written between 997 and 1002, and an anonymous chronicle (*Chronicon abbatiae Ramesiensis*) written c.1170. Cyril Hart, "The Foundation of Ramsey Abbey," *Revue bénédictine* 104 (1994): 295–327.

[137] According to the Domesday surveys of 1068, Ramsey had an annual income of £358 5s. Lapidge, in Byrhtferth, *Vita S. Oswaldi*, xv.

[138] Likely Abbo, on arrival, found himself on a construction site. Sometime in the 980s a crack was discovered in the western tower of the church. The tower was deconstructed, rebuilt with stronger cement, and reconsecrated in 991. Ibid., xxv–xxvi and Byrhtferth, *Vita S. Oswaldi* 4.2; ed. and trans. Michael Lapidge, 98.

[139] The term is used in the Ramsey Chronicle. *Chronicon,* 38.

new master from Fleury and showed themselves eager for instruction.[140] Abbo, according to Aimoin, taught "some of them" the "science of letters."[141]

Though only at Ramsey for two years,[142] Abbo's specter lingered long, not least in the figure and works of one of his students, Byrhtferth. Little is known about Byrhtferth of Ramsey (fl. c.986–c.1016) beyond what can be gleaned from his writings.[143] He was, no doubt, among those to whom Abbo taught grammar. Byrhtferth claimed that it was through Abbo's "kindness" that he received an "understanding of *computus*" and "knowledge of other things."[144] It seems likely that Abbo, though enlisted to teach the trivium, instructed some of

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[140] In the *Quaestiones*, which was written for the monks of Ramsey, Abbo writes of his pupils with great fondness, stating that it was their benevolence that brightened his time in exile. Abbo, *Quaestiones* 3, 211. Byrhtferth spoke of Abbo with respect and reverence in his writing. See chapter 4.

[141] Aimoin, *Vita* 5; ed. and trans. Bautier and Labory, 54: "aliquos monachorum scientia imbuit litterarum." Byrhtferth in the *Vita S. Oswaldi* is a little more specific, stating that Gerbert taught grammar (3.18).

[142] The *Vita* (5; ed. and trans. Bautier and Labory, 54) states that Abbo was in England for *duorum annorum* ("two years"); some scholars have been more specific and calculate that he spent eighteen months there. Dachowski believes the journey to Ramsey would have taken about two months each way, possibly longer, so perhaps Abbo was away from Fleury for about two years. *First among Abbots*, 64–67.

[143] Byrhtferth authored a range of didactic and historic/hagiographic figures and texts including a (now lost) *computus* manuscript for Ramsey with an extended *Epilogus* (Byrhtferth thought this Greek term meant "prologue") and an elaborate figure (written between 988 and 996), the *Passio SS. Æthelberhti* and *Æthelredi* (part of the *Historia regum*; c.991), the aforementioned *Vita S.* Oswaldi, the *Enchiridion* (c.1011), the *Vita S. Ecqwini*, some verse, and a chronicle of English history. The *computus* and *Enchiridion* both name Byrhtferth as their author. Other works have been attributed to him based on his—in one scholar's words—"highly distinctive," "idiosyncratic," "unmistakable," style as well as historical context. Byrhtferth has also been credited with writing the "glossae Bridferti," extensive and very learned glosses, which take the form of comparative excerpts from a host of other authorities, on Bede's *De natura rerum* and *De tempore ratione*. The attribution was made by Johannes Herwagen the Younger (1534–60), who printed Bede's *opera* and included the glosses in that edition. There is no known surviving manuscript witness to the glosses. For the most recent work in support of Byrhtferth's authorship of these glosses see Michael Lapidge, "Byrhtferth of Ramsey and the *Glossae Bridferti in Bedam*," *Journal of Medieval Latin* 17 (2007): 384–400. A counter argument (which summarizes the historiography of the question) was made by John J. Contreni in "'Old Orthodoxies Die Hard': Herwagen's *Bridferti Ramesiensis Glossae*." *Peritia* 22–23 (2011–12): 15–52. Contreni pinpoints what is at stake in the debate: "How much depends on linking the glosses to an unknown continental teacher c.900 or to Byrhtferth of Ramsey c.1000? In my view, Herwagen's *Bridferti Ramesiensis glossae* represents a *summa* of a century's worth of glossing the DNR and DTR in Carolingian schools. In Lapidge's view, the glosses provide valuable insight into Byrhtferth's culture, late Anglo-Saxon scholarship, and the library at Ramsey. They are "one of the most remarkable monuments of late Anglo-Saxon learning" ("Old Orthodoxies," 18). The debate is of interest here since the attribution affects our understanding of Ramsey's library at the time of Abbo's visit and during Byrhtferth's lifetime. When Lapidge describes Ramsey's holdings (and, with this, lists texts that were present and known in Anglo-Saxon England in his *Anglo Saxon Library*), he includes works cited in the glosses, but which do not appear in his other works (e.g., Ambrose's *Hexaemeron*, the poems of Eugenius of Toledo, Martianus's *De nuptiis*, the comedies of Terence). For a summary of Byrhtferth's life and works see Lapidge, "Byrhtferth of Ramsey (*fl. c.*986–*c.*1016)," *Oxford Dictionary of National Biography* (Oxford: Oxford University Press, 2004). [http://www.oxforddnb.com.proxy.lib.umich.edu/view/article/4268, accessed 22 Aug 2014].

[144] Byrhtferth, *Computus* 4.85–88; ed. and trans. Baker and Lapidge, 379: "Abbonis sophitae dicta, alumni Benedicti patris, per cuius benivolentiam percepimus huius rei intelligentiam necnon aliarum rerum peritiam."

his more gifted pupils, like Byrhtferth, in the other arts, including arithmetic and astronomy. Byrhtferth went on to become schoolmaster at Ramsey. He authored a number of works including a computus manuscript, copied in part from Abbo's own computus, and a bilingual manual that explained its use.[145] In this way, Abbo remained a model for teaching at Ramsey even after his departure, as did the manuscripts he carried with him that generated local copies.

Abbo made as great an impression on local churchmen and aristocracy as he did on his students. Aimoin wrote of the powerful and distinguished company Abbo kept while abroad.

> [He] visited the king of the English [Ethelred II 978–1016] and his ealdorman Æthelwine. From the king he received only words, but from the ealdorman he had gifts worthy of his sanctity and was treated with much reverence by him as long as he was in his company. He also became the close friend of those venerable men, ... Oswald and Dunstan ... the first was archbishop of York the second archbishop of Canterbury.[146]

Abbo's friendship and esteem for Dunstan are made apparent in two acrostic poems he addressed to the saintly bishop, celebrating his virtues.[147] Oswald and Dunstan's affection for Abbo took the form of rich gifts—golden bracelets, collars, a chalice; fine priestly vestments—given to him when he left England and returned home.[148]

In 987 Abbo had received a letter from Abbot Oylbold urging him to return to Fleury as soon as possible.[149] Heeding the call, Abbo left England. Soon after, between the summer of 987 and early in 988, Oylbold died, and Abbo was elected abbot—a post he filled for seventeen years.[150] After briefly describing the election, which was contested, Aimoin adds that "Abbo, being a man who understood the importance of what he had been given, tried

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[145] The *Enchiridion* and Byrhtferth's other computus figures are the topic of chapter 4.

[146] Aimoin, *Vita* 5; ed. and trans. Bautier and Labory, 54–56; trans. Grandsen, in "Abbo of Fleury's *Passio Sancti Eadmundi*," *Revue bénédictine* 150 (1995): 21: "Intra eorumdem sane spatium annorum regem adiit Anglorum ac eius ducem Hehelguinum. Sed a rege verba tantum, a duce vero condigna sui sanctitati suscepit munera atque in magna apud eum. quamdiu cum ipso fuit, est habitus reverentia. Pontificum quoque venerabilium sepe nominati Oswaldi et beatae memoriae Donstani amica sibi familiaritate usus est. Horum prior Heboracensium sequens Cantuariorum archiepiscopus fuit."

[147] The *Summa sacer* and *O presul Dunstane*, which Byrhtferth included in the *Vita S Oswaldi.* He also included the *Summa sacer* in his *computus*. Scott Gwara's "Three Acrostic Poems by Abbo of Fleury," 203–35.

[148] Aimoin describes how these gifts symbolized and made manifest Abbo's purity, innocence, and eloquence. Aimoin, *Vita* 6; ed. and trans. Bautier and Labory, 58.

[149] Ibid. 6; ed. and trans. Bautier and Labory, 56–58.

hard to act in a manner that was beyond reproach."[151] The comment communicates something of the pressure that must have come with the prestigious appointment and the earnestness with which Abbo met the challenge. As *abba* to the community, Abbo encouraged his monks to root out their vices and tame their bodily urges through a regimen of prayer, fasting, and intellectual pursuits, namely "the study of letters" and "written exercises."[152] With Abbo in charge, the liberal arts became fodder for contemplation. Abbo led by example. Aimoin describes him constantly engrossed in study of "letters," so that hardly a moment passed when he was not "reading, writing, or composing."[153] But his days as schoolmaster were over. The abbot of Fleury was, in the words of Pope Gregory IV (d.844), the first of the abbots of Gaul," answerable only to the pope.[154] Placed in this powerful position, Abbo bent his efforts and applied his learning to defending the interests of Fleury and fostering the project of reformed monasticism more broadly, protecting monastic autonomy, and keeping the power of nobles and bishops at bay.

King Louis V died in the spring of 987—an event that effectively marked the end of Carolingian rule and the rise of Capetians. Hugh Capet was crowned in December of that same year. His right to rule was quickly contested by Duke Charles of Lorraine, the Carolingian claimant to the throne, who captured Laon and—with the help of his nephew, the traitorous Archbishop Arnulf—Reims. Arnulf, as noted above, had been given the see of Reims by Hugh (over Gerbert), after swearing loyalty to the new king. But shortly thereafter he conspired with his uncle. He opened the gates of the city to Charles and his army, thus betraying Hugh and the people of Reims, who, as their bishop, Arnulf was charged to protect. He was tried at the council of Saint-Basle in 991, a gathering of the bishops of Gaul. Abbo was one of three abbots who came forward to defend Arnulf. Though not a staunch supporter of the man, Abbo was concerned about the larger implications of the council: the growing power of the monarchy and episcopacy. He and his fellow abbots believed that in calling the

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[150] Ibid. 7 and 16; ed. and trans. Bautier and Labory, 58–60 and 106.
[151] Ibid. 7; ed. and trans. Bautier and Labory, 60.
[152] Ibid.: "ad quod explendum, post orationum vota, post jejuniorum virilia certamina, multum prodesse censebat Iitterarum studia maximeque dictandi exertitia."
[153] Ibid.: "Quarum ipse perstudiosus existens, nullum pene intermitebat tempus quin legeret, scriberet dictaretve."
[154] M. Cuissard-Gaucheron, *L'école de Fleury-sur-Loire a la fin du dixième siècle et son influence* (Orléans: H. Herluison, 1875), 69: "Addimus etiam ut quia venerabilis pater Benedictus monachorum legislator et dominus, dux est religionis monasticae, sit etiam qui eidem coenobio praefuerit primus inter abbates Galliae."

council, king and bishops were overreaching their traditional authority and setting a dangerous precedent. They argued that only the pope had the right to deprive a bishop of his see. In so doing, they advocated for limiting royal and episcopal power more generally and—ultimately—the sway they had over monasteries. The council at Saint-Basle was but one episode in a protracted dispute over the archbishopric of Reims—one, as discussed above, that closely involved Gerbert, whom Hugh and the bishops—deaf to Abbo's arguments—appointed archbishop of Reims in Arnulf's stead.

Abbo achieved assurance of Fleury's independence and freedom from episcopal control in 997, when Pope Gregory V (r.996–99) issued a bull reaffirming the monastery's privileges and primacy.[155] The papal privilege is the first of several documents described by Aimoin in the *Vita.* He also noted the contents of letters written to important personages: the pope, Otto III, and Abbot Odilo of Cluny. To Otto, Abbo sent an acrostic poem (c.998)—*Otto, valens Caesar, nostro tu cede coturno* ("Otto, powerful emperor, yield to our plaintive song!")—inspired, according to Aimoin, by the fourth-century poet Porfyrius, who was known for the complicated and colorful acrostics that he constructed for Constantine.[156] The poem was contrived in such a way that the first line (*Otto valens...*), embedded in the verses and repeated six times, was laid out like a cross and outlined four squares framing the terms: *Otto, caesar, Abbo, abbas* (fig. 2.3). Despite Aimoin's claim that the poem "could not be compared with the poetry of contemporary savants," Abbo's acrostic may have resembled a *carmen figuratum* by Gerbert also written for Otto (fig. 2.4).[157] Abbo imitated Porfyrius's acrostics in function as well as form. The late-antique poet used his poems to ask the emperor for pardon and permission to return from exile.[158] Likewise, Abbo sought the support of Otto

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[155] As had been done by Pope John VIII in 878 and Pope Leo VII in 938. The bull was dated November 13, 997. According to Aimoin the privilege prohibited the bishop of Orléans from interfering with the monastery, among other things. The contents of the privilege are described in the *Vita* (12; ed. and trans. Bautier and Labory, 92–96). A full bibliography on the papal document can be found in Bautier and Labory's edition of the *Vita* (93, n. 91).

[156] Scott Gwara edited, translated, and reconstructed the poem and commented on its influences and structure in "Three Acrostic Poems by Abbo of Fleury," 203–35, esp. 227–31. Elizabeth Dachowski offers a subtle assessment of the poem in light of the historical moment in which it was written, *First among Abbots*, 198–201.

[157] Aimoin, *Vita* 13; ed. and trans. Bautier and Labory, 98: "nullis sapientium istius temporis comparandum carminibus." Gerbert's poem is best preserved in Paris, BnF lat. MS 776, fol. 1v. See Clyde Brockett, "The Frontispiece of Paris, Bibliothèque nationale, ms. lat. 776: Gerbert's Acrostic Pattern Poems," *Manuscripta* 39, no. 1 (1995): 3–25.

[158] On Porfyrius and his poems to Constantine see T. D. Barnes, "Publilius Optatianus Porfyrius," *The American Journal of Philology* 96, no. 2 (1975): 173–86.

III for Pope Gregory V, who had been driven from Rome by powerful local factions.[159] The letter to Odilo answered a question on the Eusebian canon tables that had been put to Abbo by the Cluny community on one of his visits to the abbey.[160] Thus the letters included by Aimoin in the *Vita* serve as written testaments to Abbo's political clout and intellectual stature.

Abbo's abbacy came to an abrupt end on November 13, 1004 at the remote abbey of La Rèole, 330 miles southwest of Fleury in Gascony. In 977 the local lords had handed over the monastery to Fleury for reform. Neither the residents of the small, isolated town nor the local monks took kindly to the newcomers with their foreign ways and strict interpretation of the Rule. Abbo had travelled to La Rèole in 1003. He left behind a delegation, who soon returned to Francia in fear with stories of the unruly Gascons. He sent other, more trusted monks, but it quickly became clear that he himself would again have to intervene. Aimoin describes the trip and events leading up to Abbo's death in excruciating detail. There was a cry from women in the town, which, Aimoin wrote, "by custom marked the beginnings of a riot or a man's death." In this case it heralded both. An angry mob had formed and quickly became violent: Frankish staffs were swung, the Gascons retaliated by throwing rocks and dirt. Abbo, still filling every empty moment with study, was in the cloister dictating computus. Hearing the clamor, he left off the little problem he had been working out.[161] Seeking to calm the crowd, he entered the melee and was fatally wounded: pierced by a soldier's lance through his side. Abbo became a martyr and was made a saint. Having died, in Aimoin's words, "because he had been killed for truth, which is Christ," he was buried in the church at La Rèole, beside the altar dedicated to St. Benedict.[162]

Aimoin carefully shaped the *Vita* in such a way that Abbo's last days and final hours were (not-so-subtly) patterned after Christ's passion.[163] Given the liberties taken with details, it is striking that he describes Abbo in his last moments alone, "repeating aloud a little

[159] The Crescentii, who elected John Philagathus as antipope.
[160] Discussed below.
[161] Aimoin, *Vita* 20; ed. and trans. Bautier and Labory, 122: "Interea, vir Domini Abbo intra c1austrum monasterii residens et quasdam compoti ratiunculas dictitans, tumultuantium clamore exaudito."
[162] Ibid. ; ed. and trans. Bautier and Labory, 126: "quia innocenter ac pro veritate quae Christus est interemptus erat, in ecclesia poneretur."
[163] Dachowski goes into considerable detail about these parallels and discusses the *Vita* in light of contemporary hagiographies. Dachowski, *First among Abbots*, 254–59.

reckoning of computus."[164] Its placement in the narrative parallels Christ's agony in the garden. At Gethsemane, Christ, knowing and accepting the cruel fate that awaited him, prayed for mercy and deliverance. In the *Vita*, Aimoin records a number of prescient statements made by Abbo on the road to La Rèole suggesting that he too knew his end was near. That his last minutes were spent not in prayer but in mathematical calculation, may represent historic fact. Abbo excelled at computus; and he wrote on several occasions of the comfort he took in philosophy. But perhaps in this context there is another nuance—the activity also suits Aimoin's narrative program. Computus, the art of time reckoning, of mapping in advance the movements of the heavens, is after all a science of prediction.

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Though close contemporaries with overlapping interests, Abbo and Gerbert were not friends.[165] Their offices and convictions set them in direct conflict more than once.[166] As abbot of Fleury, Abbo worked tirelessly to free his and other monasteries from episcopal and lay control. Gerbert, on the other hand, as archbishop and then pope, envisioned a close, symbiotic relationship between church and state. But their opposing ideologies should not obscure the remarkable similarities between the two men, especially in the shape of their early lives. Both came from humble backgrounds, were raised in reformed monastic communities, and rose to positions of influence and power almost solely on account of their

[164] In the *Explanatio*, Abbo tells students learning the multiplication tables to "chant" or "recite" ("decantare") them aloud. Speaking out or verbalizing mathematical problems seems—given the prevalence of verbs like *decantare* and *dictitare*—to have been standard practice. Abbo, *Explanatio* 3.90; ed. Peden, 125; Aimoin, *Vita* 20; ed. and trans. Bautier and Labory, 122.

[165] On the relationship between Abbo and Gerbert see Pierre Riché, "Nouvelles vies paralleles: Gerbert d'Aurillac et Abbon de Fleury," *Media in Francia: Recueil de mélanges offerts à Karl Ferdinand Werner à l'occasion de son 65e anniversaire par ses amis et collègues français*, ed. Georges Duby (Maulévrier: Hérault-Editions, 1989), 419–27; on their relative influence see Marco Mostert, "Gerbert d'Aurillac, Abbon de Fleury et la culture de l'an mil: Étude comparative de leurs oeuvres et de leur influence," in *Gerberto d'Aurillac da Abbate di Bobbio a Papa dell'anno 1000*, ed. Flavio G. Nuvolone (Bobbio: Associazione culturale Amici di Archivum Bobiense, 2001), 397–431.

[166] Opposing ideologies pitted them against each other in the dispute over the see of Reims. Abbo advocated for papal preeminence and sided with Arnulf. In his many letters, Gerbert mentioned Abbo by name only once (and only his first initial): in concern over the zealous and influential abbot's trip to Rome to seek papal intervention on behalf of Arnulf. When violent riots brought the episcopal council at St. Denis to an abrupt end, Gerbert was among those who accused Abbo of fomenting opposition on the part of the peasantry and local monks against the bishops. Abbo was temporarily excommunicated for the crime.

mental gifts.[167] Their biographies—contemporary and modern—read largely as a sequence of intellectual feats. Among these feats was their mastery (or, in the case of Abbo, near mastery) of learning that had been largely lost: the quadrivium.[168] At Fleury and Reims Abbo and Gerbert taught the disciplines of arithmetic, geometry, astronomy, and music alongside the arts of the trivium. The impact of this comprehensive curriculum was widely felt, throughout Europe and at the highest social and political levels. To borrow an expression from Richer: all Gaul shone with their splendor. So did England, Italy, and Germany.

3. Networks

> Gerbert pursued his studies with tremendous energy, and the number of his students increased every day. His fame as a great teacher was not only carried throughout Gaul, but also circulated among the peoples of Germany, and it crossed the Alps and spread into Italy all the way to the Tyrrhenian and Adriatic seas.[169]

In this passage from the *Historiae*, Richer exaggerates only slightly. Among those who sought Gerbert out were Otto I, "who was in search of someone with complete understanding of the mathematical arts and was capable of providing vigorous instruction for his students"—who may have included his son, Otto II; his grandson, Otto III; Heribert, future chancellor to Otto III; Robert II and Ingomen, sons of Hugh Capet; and three sons of King Lothar, among them Arnulf, the future archbishop of Reims. To this distinguished company can be added at least six future abbots and five future bishops and archbishops.[170] Abbo's

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[167] Odo of Cluny reformed Aurillac c.925. John of Salerno described the process of reform at Fleury in detail in the *Vita Odonis* 3.8–9; ed. and trans. George Sitwell, in *The Life of St. Odo of Cluny by John of Salerno and the Life of St. Gerald of Aurillac by St. Odo* (New York: Sheed and Ward, 1958), 79–82.

[168] In a letter to Constantine of Fleury, who studied with both men, Gerbert dismisses the opinion of "an incomplete philosopher," who may have been Abbo. Whereas Gerbert had mastered all seven of the liberal arts, making his claim to the title of "philosopher," indisputable, Abbo's pedigree was not so perfect. As touched on above, Abbo, though, according to Aimoin, was "not wholly ignorant" of rhetoric and geometry, a master of these subjects he was not—a fact that Aimoin openly admitted in the *Vita*, suggesting it was common knowledge. Gerbert, *Epistulae* Annexe 5, 1; ed. Riché and Callu, 664.

[169] Richer, *Historiae* 3.55; ed. and trans. Lake, 84–85: "Fervebat studiis, numerusque discipulorum in dies accrescebat. Nomen etiam per Germaniae populos dilatabatur. Transiitque per Alpes ac diffunditur in Italiam usque Thirrenum et Adriaticum."

[170] Heriger, Abbot of Lobbes (990–1007); Constantine (d.1014), abbot Saint-Mesmin, Micy; Ingon (d.1025), abbot of Saint-Martin, Massay and St. Germain des Prés; Richard (d.1048), abbot of Saint-Vannes; Ayard, abbot of St. Thierry; Herbert, abbot of Chagny; Girard, bishop of Cambrai (1012–51); Leutheric, archbishop of Sens; Bruno, bishop of Langres (980–1015); John, bishop of Auxerre (996–99); and Fulbert, bishop of Chartres (1007–28).

pupils, too, assumed positions of power and influence. Bernard, who shared Abbo's zeal for liberal studies, was made Abbot at Solignac (c.997) and Beaulieu, and was later given the see of Cahors (1005).[171] Hervé (d.1022) became a canon and the treasurer at Tours, where he oversaw the rebuilding of the cathedral of Saint Martin.[172] Gauzlin, a bastard son of Hugh Capet, was named Abbot of Fleury after Abbo, from whom he had learned the "sacred sciences."[173] He was Bishop of Bourges from 1013 to 1030. Berno (Bern) of Prüm, later Abbot of Reichenau (c.978–1048), was educated at Fleury while Abbo was abbot (c.994), as was Constantine of Fleury (d.1020/21), who would become abbot at the nearby monastery of Micy on the Loire.[174]

It was in the figure of Constantine and at the abbey of Micy that the legacy of Gerbert and Abbo, Reims and Fleury met. Constantine had entered Fleury at a young age and was almost certainly educated by Abbo.[175] He may have taught at Fleury during Abbo's absence in England.[176] It has been conjectured that he and Abbo were both considered for the office of abbot after the death of Oylbold.[177] When Abbo was elected, Constantine may have stayed on and taught. Gerbert wrote many letters to Constantine. The tone of these letters is tender and intimate.[178] Constantine was clearly a friend and a confidant; it is assumed that the two spent some time together and that Constantine went to Reims to study, as had Abbo before him. His own letters show that he continued to consult with his former master after returning to Fleury, especially on mathematical matters. In one letter Constantine asked Gerbert to explain the principles of calculation with an abacus. Gerbert responded, "Only the

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[171] Bernard, son of Hugh, an Aquitainian noble, was sent at an early age to study at Fleury. Abbot Richard (successor of Wulfaldus, 962–c.978) put Abbo in charge of Bernard's education. According to Aimoin, Bernard took after his master in his zeal for liberal studies, which Abbo taught him whenever he had time. Aimoin, *Vita* 10; ed. and trans. Bautier and Labory, 76.
[172] Dachowski, *First among Abbots*, 260; Riché, *Abbon de Fleury*, 73; Guy Oury, "L'Idéal monastique dans la vie canonicale: Le bienheureux Hervé de Tours," *Revue Mabillon* 52, no. ser. 3, no. 207 (1962): 16–31.
[173] Helgaud, *Vie de Robert le Pieux* 25; ed. and trans. Robert Henri Bautier and Gillette Labory (Paris Éditions du Centre national de la recherche scientifique, 1965), 120.
[174] He was appointed Abbot by Bishop Arnulf of Orlèans (970–1003), exactly when is not clear. Gerbert first addresses him as "Abbot of Micy" in a letter written in 996 (*Epistulae* 191; ed. Riché and Callu, 498). Dachowski, *First among Abbots*, 45; Riché, *Abbon de Fleury*, 74.
[175] Very little is known of Constantine. See F. M. Warren, "Constantine of Fleury, 985–1014," *Transactions of the Connecticut Academy of Arts and Sciences* 15 (1909): 287.
[176] Deduced from Gerbert's letters to Constantine, dated by Havet to 986 (Abbo was in England from 985–87), in which he addresses Constantine as *scolasticus*. See, for example, Gerbert, *Epistulae* 86; ed. Riché and Callu, 202.
[177] A hypothesis first put forward Warren, "Constantine of Fleury," 285.
[178] See, for example, Gerbert, *Epistulae* Annexe 5 (nos. 1–6); ed. Riché and Callu, 662–99.

compulsion of friendship reduces the nearly impossible to the possible. Otherwise how could we strive to explain the rules of the abacus unless urged by you, O Constantine, sweet solace of my labors?"[179] He protests that it had been at least a *lustra* (a period of four or five years) since he had "either a book or any practice in this sort of thing," and then recounts the rules from memory.[180] Constantine was interested not only in the abacus, but also in problems he encountered in Boethius's *De arithmetica* and *De musica*. Gerbert responded to these questions in lengthy letters with verbal and visual explanations.[181]

Alongside known personages, Gerbert and Abbo taught ranks of clerics, monks, and priests. A few of these men would themselves become teachers, extending the reach of their masters farther afield and to future generations. Scattered across the map of Europe, these one-time students fostered bustling networks conveying teaching strategies and tools—celestial spheres, schemata, tables, abaci, astrolabes (or at least drawings and descriptions of them), and manuscripts. Even while "pressed by secular duties" when filling the office of archbishop in Ravenna, Gerbert found time to explain (at some length) to his friend, Adalbold of Liège, how to calculate the area of a triangle and volume of a sphere arithmetically and geometrically (fig. 2.5).[182] For Adalbar, a teacher at St. Vincent in Metz, Gerbert wrote an epitaph praising his learning.[183] To a man named Adam, who was, perhaps, a priest or canon somewhere north of Reims, Gerbert sent information for the construction of *horologia* charts for uneven length of days and advice on how to calibrate the *horologia* with a waterclock to fit local needs.[184] To Gerald of Aurillac, he sent organs from Italy and to Bernard of Aurillac, he promised guides for playing the organs as well as materials on rhetoric (possibly a copy of the aforementioned chart) and music.[185] To Remi, a monk and schoolmaster at Trier (appointed abbot of Mettlach before 993), he explained the intricacies

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[179] Ibid., Annexe 5, no. 1: "Vis amicitiae impossibilia redigit ad possibilia. Nam quomodo rationes numerorum abaci explicare contenderemus, nisi te adhortante, o mi dulce solamen laborum, Constantine?"
[180] Ibid.: "Itaque cum aliquot lustra iam transierint, ex quo nec librum, nec exercitium harum rerum habuerimus."
[181] See chapter 1.
[182] Bubnov, *Opera*, 41–45 and 302–309. Adalbold was a student of Notker of Liège and Heriger of Lobbes. He served as master at Lobbes and then bishop of Utrecht (d.1026). Lattin, in Gerbert, *Letters*, 301, n. 1.
[183] Gerbert, *Epistulae* 77; ed. Riché and Callu, 184.
[184] Ibid. 153; ed. Riché and Callu, 374–79. On the identity of Adam, see Lattin, in Gerbert, *Letters*, 190, n. 1.
[185] Ibid. 70 and 92; ed. Riché and Callu, 172–74 and 218–20.

of arithmetic with the abacus.[186] Remi eventually wrote a treatise on the abacus, suggesting that he used it as a teaching tool. That he was eager to adopt Gerbert's unconventional methods of teaching is clear from a series of letters pestering Gerbert to make a celestial sphere for him.[187]

In the absence of a like number of letters, Abbo's scholarly network is more difficult to reconstruct. He forged strong ties with Rome and the local monarchs. To Pope Gregory V he sent a copy of the history of the translation of Benedict's relics to Fleury and, upon request, a "beautiful" missal.[188] He addressed the erudite *Apologeticus* and *Collectio canonum* to the Capetian kings Hugh and Robert II.[189] He carefully constructed acrostic poems as gifts for Archbishop Dunstan of Canterbury and Otto III, praising the recipients (fig. 2.3 and 2.6).[190] Fulbert, Bishop of Chartres, a great friend and admirer, addressed Abbo: "O great philosopher ... your friendship is the favor of a master; my fidelity is that of a student."[191] For the monks of Cluny, Abbo wrote a lengthy explanation of the concordance of the gospels and a how-to guide to the Eusebian canon tables, which he addressed to their abbot, Odilo (d.1049). Aimoin, as already mentioned, included the letter in the *Vita* prefaced by a few lines on its genesis.[192] The question of the concordance of scripture arose while Abbo was visiting Cluny and conversing with some members of the community. He did not have the time (nor, perhaps, the resources) to respond in full on the spot, so he treated the topic in a letter. Such informal and ephemeral "teaching moments" must have occurred with some frequency as learned men like Abbo and Gerbert made their way to and from their home institutions.

Abbo traveled extensively during his abbacy. As abbot, he would have been obliged to visit Fleury's many priories, scattered throughout the diocese of Orléans, and to keep an

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[186] Ibid. 134; ed. Riché and Callu, 328–30. Remi went on to write a treatise on the abacus, the *Regulae de divisionibus abaci*.
[187] Ibid. 148 and 152; ed. Riché and Callu, 362 and 372–74.
[188] Cuissard-Gaucheron, "L'Ècole de Fleury à la fin du X[e] siècle et son influence," 95.
[189] Mostert, *Political Theology*, 48–54.
[190] The poem to Otto III is discussed above. Gwara analyzed the poems in his edition and translation: "Three Acrostic Poems by Abbo of Fleury," 203–35.
[191] Fulbert, *Epistulae* 1, ed. and trans. Frederick Behrends, *The Letters and Poems of Fulbert of Chartres* (Oxford: Clarendon Press, 1976), 2: "O magne phylosophe ... ac perennem fidelitatis habitum amicitiae tuae rependo, hac scilicet differentia tuam benivolentiam meamque distinguens, ut illa pro maiestate persone gratia vocetur ut domini, ista fidelitas ut alumni."
[192] Aimoin, *Vita* 13; ed. and trans. Bautier and Labory, 100.

eye on its extensive land holdings, which reached to the very edges of Gaul.[193] Sources record trips—sometimes several—that he made to Ramsey, Cluny, La Réole, Paris, Poitiers, Reims, Rome, and Tours.[194] Gerbert led an even more peripatetic life than Abbo. Whereas Abbo called Fleury home for most of his days, Gerbert was ever on the move. He lived in Aurillac, Barcelona, Rome, Reims, Bobbio, Sasbach,[195] Pavia, Ravenna, and Rome. The *Vita* and his letters show him crisscrossing West Francia, Germany, and northern and central Italy. While abbot of St. Columbanus, Gerbert had responsibility for the most extensive monastic property in Italy; as archbishop of Reims, he oversaw the ten surrounding sees: Senlis, Soissons, Beauvais, Amiens, Thérouanne, Tournai, Noyon, Laon, Châlons-sur-Marne, and Cambrai. The scale of these duties increased with his tenure as archbishop of Ravenna and pope. The point to make is that Abbo and Gerbert were prominent members of a community that extended well beyond the bounds of Fleury and Reims. This community shared ideas and objects amongst its members, and with others. Few mastered the quadrivium, but through these networks many would come to know its fundamental premises.

4. Manuscripts

In the *Historiae*, at the very moment in the narrative when the fate of the usurper and would-be king Charles of Lorraine, who had been taken hostage by the Bishop of Laon, was being decided, Richer interrupted his narrative to tell of a trip undertaken for the purpose of reading a book.[196] Richer declared that he had been thinking "often and at length" about the liberal arts.[197] Hence, when he received an invitation from his friend, Heribrand, a cleric of Chartres,

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[193] Dachowski, *First among Abbots*, 84–86; Riché, *Abbon de Fleury,* 56–58; *Lumières de l'an mil en Orléanais: Autour du millénaire d'Abbon de Fleury*, (Turnhout: Brepols, 2004), 12–18. Gauzlin increased Fleury's land holdings and developed especially close ties with Oliva (d.1046), Abbot of Ripoll and St. Michel-de-Cuxa (e.1008), and later Bishop of Vic (e.1017).

[194] Dachowski, *First among Abbots* 85, esp. n. 6.

[195] In southeastern Germany (either in Baden, between Achern and Biihl, or north of Breisach), where he was granted an estate by Otto III.

[196] The tale is the only autobiographical aspect of the *Historiae*. It is written on fols. 49–51 of Bamberg, Msc.Hist.5. Folios 49 and 50 are an added bifolium and folio 51 is a tipped-in scrap. The added text is keyed to the narrative with a large asterisk. The material evidence shows that Richer went to great effort to insert the story of his trip to Chartres where he did. Jason Glenn suggests that Richer, who shows loyalty to the Carolingian cause elsewhere, was perhaps being careful to explain his flight from Reims during a period of political turmoil in order to protect himself. The reason for the insertion does not affect (or may even strengthen) the point made here: that Richer's contemporaries would not have found it strange that a monk might travel great distances to read a book. Glenn, *Politics and History*, 252–66, esp. 261–63.

[197] Richer, *Historiae* 4.50; ed. and trans. Lake, 2:304: "de studiis liberalibus saepe et multum cogitarem."

to examine a copy of the *Aphorisms* of Hippocrates, he jumped at the chance.[198] He then provided a surprisingly lengthy and vivid account of his harrowing journey: through dark woods, in torrential rain, and across a dilapidated bridge riddled with so many holes of such great size that the knight with whom he travelled had to lay his shield over the openings for them to walk on.[199] There was even the strange, abrupt death of his servant's horse, which suddenly collapsed "as if struck by lightning!"[200]

The inclusion of what we might deem a minor tale of little or no bearing on the larger events recorded in the *Historiae*—a chronicle largely devoted to the affairs of kings—and its insertion *in medias res*, at a point of tension and suspense in the historical narrative, may seem eccentric and bizarre.[201] Yet such juxtapositions appear with some frequency in literature of the period. Abbo, as we have seen, dictated calendric calculations until the shrieks of the murderous mob forced him to stop. These examples give us some idea of just how vigorously learning was pursued despite, or because of, the violence and upheavals of the era. They also shed light on the crucial role that books and their dissemination played in that pursuit. Richer was willing to risk his life as well as the lives of those who accompanied him, to read a work that was, evidently, not to be found at Saint-Rémi or any libraries nearby—Reims, Saint-Nicaise, Saint-Denis, Saint-Thierry, and Hautvilliers.[202] The book in question proved a disappointment; Richer wrote that he learned from it nothing he did not

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[198] The *Aphorisms* is a medical text.
[199] Richer, *Historiae* 4.50; ed. and trans. Lake, 2:304–308.
[200] Ibid.; ed. and trans. Lake, 2:306: "velut fulgure traiectus."
[201] In the prologue, Richer states his aim in writing the *Historiae* (prologue; ed. and trans. Lake, 1:4–5) clearly: "My particular goal is to recall to memory in writing the frequent wars waged by the Gauls during the reigns of these kings [of Gaul], their various struggles, and the different reasons for their undertakings. If the affairs of others are mentioned, let it be assumed that this is due to incidental reasons that could not be avoided." ("Quorum temporibus bella a Gallis saepenumero patrata variosque eorum tumultus ac diversas negotiorum rationes ad memoriam reducere scripto specialiter propositum est. Si qua vero aliorum efferantur, ob incidentesrationes quae vitari non potuerunt id evenisse putetur.")
[202] That Hippocrates' *Aphorisms*, a fairly common medical treatise, was not among the works to be found in any of the collections in the vicinity of Saint-Rémi is surprising. The contents of Saint-Rémi's 10th-c. library are not well known, but they are thought to have been fairly extensive. On January 15, 1774 the library was almost completely destroyed by fire. According to an eyewitness, 50 or 60 volumes of a collection that numbered more than 1,000 were thrown from a window and saved. Frederick Carey has compiled a list of around seventy manuscripts that retain numbers in red added by a 13th-c. scribe. Glenn, *The Work and World of Richer*, 20; Moore, "Teaching and Learning History in the School of Reims," 23–24; Carey, "The Scriptorium of Reims" 45–47.

already know.[203] But the perilous journey was redeemed when he stumbled across another work in Chartres library: *On the Concordance of Hippocrates, Galen, and Soranus*.

Books attracted eager scholars in much the same way a famous schoolmaster might. More often than not, famed libraries and renowned masters went hand in hand. At the end of the tenth century the schools of Fleury and Reims each boasted both. Fleury's library, a mature collection with depth and breadth in all areas, flourished first under Theodulf, a bright star of the Carolingian court, who was abbot from 798 to 818.[204] Charlemagne, with help from scholars like Theodulf, fostered the assemblage of an extraordinary collection of classical texts at court.[205] Emulating the emperor, Theodulf built up the libraries of Orléans, where he was bishop, and Fleury. The library at Reims, on the other hand, was slightly younger and more modest, expanded largely in the late ninth and tenth centuries, through the heroic efforts of a few of its archbishops: Hincmar, Fulk, and later Adelbero and Gerbert.[206]

Reims

Archbishop Fulk (883–900) took charge of Reims after the schools (one for canons, another for priests and clerics) had been disbanded in the wake of invasions by Norse raiders in 882. Wanting to establish the cathedral as an intellectual center, he lured the famed Remigius (841–908) from Auxerre, and Hucbald (840–930), renowned for his knowledge of music, from Saint-Amand in Flanders.[207] Surviving manuscripts suggest that Fulk simultaneously built up the library, furnishing these men with works they needed to teach, among these,

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203 The statement is remarkably similar to that voiced by Abbo in the *Vita* about his experience in Paris and Reims.
204 A summary of the library's history is given by Mostert in *Library of Fleury*, 19–28. On the early library see Gremont and J. Hourlier, "La plus ancienne bibliotheque de Fleury," *Studia Monastica* 21 (1979), 253–64.
205 Bernhard Bischoff, "The Court Library of Charlemagne," *Manuscripts and Libraries in the Age of Charlemagne*, trans. Michael M. Gorman (Cambridge: Cambridge University Press, 1994), 56–75.
206 On the library of Reims in the early and central middle ages see Carey, "Scriptorium of Reims," 41–60; Pierre Riché, "La bibliothèque de Gerbert d'Aurillac," *Bulletin de la Société nationale des antiquaires de France* (1987): 110–13; and Michel Sot, *Un historien et son église au X^e^ siècle: Flodoard de Reims* (Paris: Fayard, 1993), 68–77; and Jean Vezin, "La repartition du travail dans les 'scriptoria' carolingiens," *Journal des savants* 3 (1973): 212–27; idem, "Hincmar de Reims et de Saint-Denis, à propos de deux manuscrits du *De Trinitate* de Saint Hilaire," *Revue d'histoire des textes* 9 (1979): 289–98; and idem, "Reims et Saint Denis au IX^e^ siècle, l'ancêtre du manuscrit 118 de la Bibliothèque Municipale de Reims," *Revue Benedictine* 94 (1984): 315–25.
207 It may be that these masters also had manuscripts made at Reims for their own personal use and collections. Hucbald, for example, gifted a number of manuscripts to St. Amand, as recorded in the *index maior* of the library. Among those listed in the *index* are three products of the Reims scriptorium now at Valenciennes,

tracts they themselves had penned.[208] Fulk, however, was not starting from scratch. There was a modest clutch of manuscripts already at Reims at the time of Archbishop Tilpin (c. 735–c. 800).[209] Almost nothing is known of book production in Reims during the bishopric of Ebo (816–35), save that there were scribes and artists in the area capable of making such masterpieces as the Ebo Gospels and Utrecht Psalter (figs. 2.7–8).[210] During his long episcopacy, Hincmar (845–882), Fulk's immediate predecessor, expanded the collection, quadrupling its size.[211] But Hincmar's library comprised mostly patristic texts and works of canon law; it included few tracts by classical authors useful for teaching the liberal arts.[212] While Hincmar was abbot, the brightest students went elsewhere—Laon, Auxerre—to complete their education. Fulk changed that by bringing great masters to Reims, but his experiment was cut short by his murder following the elevation of Charles III (r. 893–923, d. 929) to the throne of West Francia.[213] In the wake of his death the schools soon disbanded. Remigius and Hucbald abandoned Reims for Paris, but the books amassed by Fulk remained.

When Gerbert arrived in Reims sometime after 972 he encountered a library with strengths in history, patristic texts, law, and probably also rhetoric, for which Gerannus was well known.[214] He set about equipping the cathedral (and himself, as one of the masters at the

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Bibliothèque municipale: the *Timaeus* with the Calcidian *Commentary* (MS 293), and Seneca's *Apocolocyntosis* and Publilius Maximus (MS 411). McKitterick, "Knowledge of the *Timaeus*," 93–94.

[208] Frederick Carey compiled a list of the extant manuscripts produced during Fulk's tenure. The list provides shelf numbers only and includes MSS made at the cathedral as well as the monasteries of St. Basol, Hautvillers, St. Nicaise, Saint-Rémi, and St. Thierry. Michel Sot redacted Carey's list to include just the manuscripts from the cathedral and both titles and shelfmarks. On this list are two copies of John Scottus's *De divisione naturae* (Paris, BnF lat. MS 12964 and Reims, Bibliothèque municipale, MS 875) and Haimo of Auxerre's *Expositio super Apocalypsim* (Reims, BM MS 126). Carey, "The Scriptorium of Reims," 58–59; Sot, *Un historien et son église*, 72–75.

[209] Little is known of the early collection. Carey, "The Scriptorium of Reims," 42–43, 57.

[210] Épernay, BM MS 1; and Utrecht, UB MS 32. It has been argued that the Utrecht Psalter was produced later, under Hincmar. Celia Chazelle, "Archbishops Ebo and Hincmar of Reims and the Utrecht Psalter," *Speculum* 72 (1997): 1055–77.

[211] A great deal is known about the library under Hincmar; more was written down during his episcopacy than in the periods before and after. Flodoard describes the books Hincmar gifted to the cathedral. Around 100 manuscripts survive from this period, most are housed in the municipal library of Reims. Many of the manuscripts given by Hincmar bear his *ex dono*. Carey, "The Scriptorium of Reims," 43–5 and 49–56.

[212] Carey, "Scriptorium of Reims," 44–5.

[213] Charles III, the son of Louis the Stammerer (d.879), was elevated to the throne at the age of fifteen. His rule was contested by Robert (d.866) the brother of King Odo (r.888–898), who was allied with Count Baldwin II of Flanders (d.918). Baldwin arranged Fulk's murder, thinking that without the archbishop—one of Charles's staunchest and most powerful allies—the crown could be transferred to Robert. The events are described by Richer (*Historiae* 1.12–17).

[214] Richer implies that before Gerbert's arrival the curriculum at Reims was limited (*Historiae* 3.42), but it should be remembered that its reputation and resources were enough to hold Gerannus and to attract Abbo (who

school) with a more comprehensive library that would enable teaching *all* the arts. He approached this task with vigor, sparing no expense or effort. As already noted, twenty of his 233 extant letters include negotiations over manuscripts. A letter to Ebrard, the Abbot of St. Julien (976–91) in Tours, gives an idea of Gerbert's process:

> I am diligently forming a library. Just a short time ago in Rome, and in other parts of Italy, in Germany also, and in Lorraine, I used large sums of money to pay copyists and to acquire copies of authors, permit me to beg that this be done likewise in your locality and through your efforts so that I may be aided by the kindness and zeal of friends who are my compatriots. The writers whom we wish to have copied we shall indicate at the end of this letter. Not unmindful, moreover, of your kindnesses, we shall send parchment for the copyists and necessary funds.[215]

He appended to the letter a list of desired titles, now, unfortunately lost. It is unclear whether the list was a wish list, to be fulfilled by the recipient to the extent possible, or if it reflected certain knowledge—works that Gerbert knew were held at St. Julien or the great library of St. Martin, Tours.[216] If the latter, it is not clear just how Gerbert came to know the specific location of particular texts. In other instances, of course, Gerbert had firsthand knowledge. As discussed above, he dispatched a letter to Miro Bonfill, the Bishop of Girona, requesting that he send a copy of the arithmetical tract titled *On the Multiplication and Division of Numbers* which he had encountered in Spain.[217]

No doubt Gerbert depended as heavily on word of mouth and hearsay as on the "kindness and zeal of friends." Just as Heribrand, aware of his comrade's interest in medicine, alerted Richer to the copy of the *Aphorisms* at Chartres, so Gerbert's many connections must have acted as manuscript scouts. In a letter to a monk at Bobbio, Gerbert requested that three

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was in Reims sometime between 958 and 965) and Gerbert. From 900 to 950 manuscripts continued to be produced. A few schoolbooks date from this time, including a copy of Boethius's *De arithmetica* (Bamberg, Msc.Class. 8), the complete works of Horace (Paris, BnF lat. MS 7974), Cassiodorus's *Institutiones* (Book II only; Reims, BM MS 975, fols. 34r–75r).

215 Gerbert, *Epistulae* 44; ed. Riché and Callu, 106–08; trans. Lattin, 90: "bibliothecam assidue comparo. Et sicut Romae dudum ac in aliis partibus Italiae, in Germania quoque et belgica, scriptores auctorumque exemplaria multitudine numerorum redemi, adjutus benivolentia, ac studio amicorum comprovincialium, sic identidem, apud vos fieri ac per vos, sinite ut exorem. Quos scribi velimus, in fine epistolae designabimus. Scribentibus membranas suptusque necessarios ... dirigemus."

216 On libraries of Tours, see Lesne, *Histoire de la Propriété*, 4:560–65.

217 On this tract and its author see above, 62.

works be copied.[218] The titles he gave correspond in order and in idiosyncrasies in spelling and attribution to an extant booklist from the library.[219] The implication is that Gerbert had access to a copy of the list and referred to it when he made his requests.[220] Some of his requests seem to have been based more on the general reputation of a collection. In a letter to Constantine of Fleury, Gerbert refers to the monastery's impressive stash of Ciceronian texts. He asks Constantine to "let the Tullian works accompany your journey" and then names a few titles of particular interest including the *De re publica*, a complete copy of which Fleury almost certainly did not own.[221]

FLEURY

Fleury's reputation as a place where rare texts like the *De re publica* and quality copies might be preserved was not unfounded.[222] The collection was old even in the tenth century. The monks who brought Saint Benedict's remains from Monte Cassino (July 11, 703) in a successful *furta sacra* made off with some of the southern Italian monastery's manuscripts as well.[223] These imports formed the foundation of a library that was then vastly expanded by Theodulf, Bishop of Orléans and Abbot of Fleury from 798 to 818 (d.821). A Visigoth from Spain who served Charlemagne, Theodulf is best known today for his poetry and the

[218] Gerbert, *Epistulae* 130; ed. Riché and Callu, 318–20.
[219] Becker, *Catalogi*, 64–73 (no. 32), entries 384–86, 387, and 399; Lattin, in Gerbert, *Letters*, 169; Tosi "Il Governo abbaziale di Gerberto," 130–39, 197–223.
[220] There is some evidence that a sort of informal system of listing books comparable to that developed in the 9th c. continued to be used or was revived in the 10th. On the circulation of library lists among institutions and individuals in the Carolingian era see Rosamond McKitterick, *Carolingians and the Written Word* (Cambridge: Cambridge University Press, 1989), 192–210.
[221] Fleury was known as a repository of the works of Cicero, the earliest manuscripts with Ciceronian content are 10th-c. copies: Paris, BnF lat. MS 7696; Leiden, Voss. lat. MS F 70 + Orléans, BM MS 277, and Paris, BnF n.a.l. MS 1611 + Orléans, BM MS 267. As already discussed in chapter 1, the *Somnium Scipionis* is the sixth book of Cicero's *De re publica* and the only part of this work to survive. In the central middle ages it travelled together with Macrobius's *Commentary*. It is difficult to believe that a copy of this work was not available at Reims. It seems likely, therefore, that Gerbert was on the hunt for Cicero's full treatise.
[222] The avid collector and corrector of classical texts, Lupus of Ferrières, depended on the library for copies of a number of secular works. His hand has been recognized in fifteen items from Fleury. Mostert, "The Tradition of Classical Texts in the Manuscripts of Fleury," in *Medieval Manuscripts of the Latin Classics: Production and Use: Proceedings of the Seminar in the History of the Book to 1500, Leiden, 1993*, ed. Claudine Chavannes-Mazel and Margaret M. Smith (Los Altos Hills, CA: Anderson-Lovelace, 1996), 21.
[223] Marco Mostert counted thirty-nine manuscripts written before the 8th c. E. A. Lowe noted that, "the oldest Fleury manuscripts are importations" with the majority likely coming from Italy, with Merovingian additions that suggest they were in France by the 7th c. This evidence may give ground to the speculation that books as well as bones were swiped from Monte Cassino. Mostert, *The Political Theology*, 33; Lowe, *CLA*, vol. 6, xviii–

anonymous tract on use of images, the *Opus Caroli Regis contra synodum*.[224] His extant writings evince a taste for and thorough knowledge of classical literature.[225] Likewise, the capitularies he issued show considerable effort to enact Charlemagne's edicts on education.[226] He sought to make instruction widely available, by establishing schools *per villas et vicos* throughout his see.[227] Theodulf's interests and ambitions left their mark on the monastery's library. No book list survives from the ninth century, but books do, many of which were clearly intended for teaching, especially the trivium.[228]

The survival of so much of the early library is remarkable considering that Fleury was raided, burned, and rebuilt at least three times in the ninth century.[229] Significant gaps in the collection began to be filled when the monastery was reformed by Odo of Cluny (879–942). Production of books went hand in glove with the renewed emphasis on education brought by

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xxi; a list of some of the pre-800 manuscripts is given in Guerreau-Jalabert, in Abbo, *Questions grammaticales*, 14, n. 15.

[224] On the *Opus Caroli* see Thomas Noble, *Images, Iconoclasm, and the Carolingians* (Philadelphia: University of Pennsylvania Press, 2009), esp. 159–206.

[225] On references to Classical poetry in Theodulf's verses, see Ann Freeman, "Theodulf of Orlèans: A Visigoth at Charlemagne's Court," in *L'Europe héritière de l'Espagne wisigothique*, ed. Jacques Fontaine and Christine Pellistrandi (Madrid: Casa de Velázquez, 1992), 188, esp. n. 32. He wrote a poem describing an image (imagined or real) of the personified liberal arts in the branches of a tree. On the image and poem see Anna Esmeijer "De VII liberalibus artibus in quadam pictura depictis: Een reconstructie van de arbor philosophiae van Theodulf van Orléans," in *Album amicorum J. G. Van Gelder*, ed. J. Bruyn et al. (The Hague: Nijhoff, 1973), 102–15.

[226] Anita Guerreau-Jalabert suggests that Theodulf's policies and efforts were akin to those of Alcuin at Tours, in Abbo, *Questions grammaticales*, 16, esp. n. 22.

[227] Theodulf, *Capitula episcoporum* 1 n. 20, ed. P. Brommer, MGH Capit. episc. 1 (Hannover: Hahnsche Buchhandlung, 1984), 116: "Presbyteri per villas et vicos scolas habeant." Priests were encouraged to send "nephews and other [male] blood relatives" to schools (*scolae*) at the cathedral in Orlèans, or the monasteries of Saint-Aignan, Saint-Lifard, Saint-Benoît-sur-Loire, and other monasteries.

[228] Bern, BB Cod. 207 + Paris, BnF lat. MS 7520, dated to the end of the 8th c., contains Donat's *Ars minor* and *Ars maior*; three 9th c. manuscripts (Orléans, BM MS 259 + Paris, BnF n. a. l. MS 1621; Orléans, BM MS 294; and Orléans, BM MS 295 also contain Donat's texts; part of Quintilian's *Ars oratoria* is preserved in Bern, Burgerbibliothek, MS 351, copied in the 9th century (probably) at Fleury. Orléans, Bibliothèque municipale, MS 270, written at the beginning of the 9th c. contains Boethius's *Consolation of Philosophy*; the 9th-c. manuscript Paris, BnF lat. MS 16677 contains Macrobius's *Commentary* and Cicero's *Somnium Scipionis*; Caesar's *Gallic Wars* is preserved in Paris, BnF lat. MS 5763 (written in the 9th c.); there are pre-9th c. fragments of Sallust (and later copies of his works). Marco Mostert, "The Tradition of Classical Texts in the Manuscripts of Fleury," 19–39; Elisabeth Pellegrin, "La tradition des textes classiques latins dans l'abbaye de Fleury-sur-Loire," *Revue d'histoire des textes* 14–15 (1984–85): 155–67.

[229] In 854, 865, and again before December of 883. Perhaps more remarkable is the persistent productivity of the scriptorium throughout this rocky period. Mostert, *Library of Fleury*, 24, n. 39; Guerreau-Jalabert, in Abbo, *Questions grammaticales*, 149–50.

Cluniac reform.[230] From Aimoin's *Vita*, we know that by the mid-tenth century boys at Fleury could learn grammar, dialectic, and arithmetic; Abbo, as we saw above, was even able to teach himself rhetoric and some geometry drawing on books in the library.[231] Under Abbo and then Gauzlin manuscript making at Fleury boomed and the library reached its zenith.[232] While Gerbert looked outward, expanding the more limited holdings at Reims by scouring institutions near and far, often out-sourcing production, Abbo looked inward, plumbing the collection he had at hand. There were, in Fleury's library, multiple copies of texts.[233] Some of these give evidence of thoughtful and selective collation: desirable elements of one copy were married to those of another to produce a new master exemplar.[234] For this work, Abbo tapped the talents of scribes at Fleury's own scriptorium.

The scriptorium, at least according to Thierry of Fleury's *Consuetudines* (c.1022),[235] was headed by the *armarius*, an impressive individual, who wore "the robes of a philosopher" and was considered "equal to an apostle."[236] To him fell the care of books as

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[230] See Jean Vezin, "La production et la circulation des livres dans l'Europe du X[e] siecle," in *Gerbert L'Européen: Actes du colloque d'Aurillac, 4–7 juin 1996*, ed. Nicole Charbonnel and Jean-Eric Iung (Cantal: Société des lettres, sciences et arts "La Haute Auverge," 1997), 214–15.
[231] See above, 75.
[232] There may have been a scriptorium as early as the end of the 7[th] or the beginning of the 8[th] c. The only extant "quadrivial" manuscript produced at Fleury in this early period is a copy of Isidore's *De natura rerum* (Paris, BnF lat. MS 6400G, fols. 112–45).
[233] Large, well-known collections had many books as opposed to many titles. Lesne, *Histoire de la propriété*, 4:762–9.
[234] This process is best observed in a group of related copies of Macrobius's *Commentary* on Cicero's *Dream of Scipio.* Bruce Barker-Benfield has identified seven extant copies of this text that once belonged to Fleury. (He also notes, "There are still more which possibly belonged to Fleury, but for which there is no firm evidence of ownership. Other manuscripts, which contain only the text of the *Somnium Scipionis* or excerpts from Macrobius's *Commentary*, can also be shown definitely to have belonged to Fleury.") Paris, BnF lat. MS 6365 is one of three manuscripts made at the Fleury scriptorium in the first half of the 11th century. Its contents were drawn from three older manuscripts: the text of the *Commentary* comes from Paris, BnF lat. MS 16677; the figures from Paris, BnF lat. MS 7299; and the text of the *Somnium* from a third, unidentified source. Both Paris, lat.16677 and 7299 contain text *and* figures for the *Commentary* and the *Somnium*, hence either might have served as an exemplar for the contents of Paris, lat. 6365. That said, the figures in Paris, lat. 16677 show divergence from what is considered the standard set. Those of Paris, lat. 7299, on the other hand, conform to the "standard" and show signs of correction. Presumably, Paris, lat. 6365 unites the "best" elements of the three manuscripts from which it descends. It gives us some idea of what was considered desirable: smaller, "correct" figures (rather than the full-page figures of Paris, lat. 16677) and a two-column format. Antia Guerreau-Jalabert observes the same multiplication of trivial texts. There were, for example five copies of Sedulius's *Carmen paschale* and hymns. Bruce Barker-Benfield, "The Manuscripts of Macrobius' *Commentary*," 84; Ibid., "A ninth-century manuscript from Fleury," 145–65.
[235] Though Thierry wrote the *Consuetudines* after arriving in Germany, likely between 1018 and 1022, his account records the workings of Saint-Benoit prior to his departure (for Italy) in 1002 and corresponds with Abbo's abbacy.
[236] Thierry, *Consuetudines* 9; ed. Davril and Donnat, 182: "philosophica redimitus toga ... pro apostolo habeatur."

well as all the materials and tools of the scriptorium. He was also in charge of the school. The dual role of the *armarius* underscores the intimate connection between the making of books and the molding of minds.[237] Indeed, in his description of the tasks of *armarius*, Thierry ranges from reprimanding young students to emending texts.[238] More mature students probably served as scribes. As the fame of the school increased, students and scholars came from diverse locales, bringing with them different styles of writing, as evidenced in the manuscripts.[239] During Abbo's tenure, there was in their ranks a Beneventan scribe and another, Leofnoth, who came from England.[240] Both made an effort to imitate the characteristically pointy Carolingian minuscule of the Loire and employ the ligatures and abbreviations that were standard in that region,[241] but neither was able to entirely erase the habits of their homeland. Those who came from nearby, especially Micy, Auxerre, and Ferrières, would have written almost as Fleury natives. Book production in these Loire-based communities was so integrated that manuscripts made at one or the other institution were and are virtually indistinguishable.[242]

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237 See chapter 3.

238 Thierry, *Consuetudines* 9; ed. Davril and Donnat, 184: "Non tepidus. non timidus, sed salva caritate animo cordatissimus, cujus vocem increpationis discipuli ac si tonitrum perhorrescant, ad cujus constantiam vigoris infantum vultus pallescant. Emendatio librorum et termini lectionum et responsio fidei catholice et hereticorum confutatio et, si quid *sane doctrine* obstiterit, ilium attinet."

239 A steady stream of young men were sent from England to be trained at Fleury. While Gauzlin was abbot a similar relationship developed with Ripoll in Spain.

240 On the Beneventan scribe see E. A. Lowe, *Palaeographical Papers, 1907–1965* (Oxford: Clarendon Press, 1972), 2:479; on Leofnoth see JeanVezin, "Leofnoth: Un scribe anglais à Saint-Benoît-sur-Loire," *Codices manuscripti* 4, no. 3 (1977): 109–19.

241 The Fleury script is exaggeratedly spikey and angular; half-uncial **a**; the common ligatures are: **ct**, **ra**, **re**, **rt**, **st**; ligatures are prominent, especially **ct;** capital **N** is often used mid-word; and *est* is typically abbreviated with a dash. There is no comprehensive work on the Fleury scriptorium. Frederick M. Carey wrote his dissertation on the subject, but his findings have been criticized, and he never published the work. Elisabeth Pellegrin's work on the *membra disiecta* remains the best guide. Barker-Benfield, "A Ninth-Century Manuscript at Fleury," 155; Frederick M. Carey, "*De scriptura Floriacensi*" (PhD diss., Harvard University, 1923); Marco Mostert, *Library of Fleury*, 23; Elisabeth Pellegrin, "Membra disecta Floriacensia I," *Bibliothèque de l'École des Chartres* 117 (1959): 5–57; eadem, "Membra disiecta Floriacensia II," in *Miscellanea codicologica F. Masai*, ed. P. Cockshaw, Monique-Cécile Garand, and Pierre Jodogne (Ghent: E. Story-Scientia S.P.R.L., 1979), 83–103.

242 From the 9th c. these institutions, geographically close, were linked and their *scriptoria* interdependent. Bruce Barker-Benfield has observed that the ties that bound these institutions in the early middle ages remained strong through the 11th c. See Rosamond McKitterick, "Carolingian Book Production: Some Problems," *The Library* 12 (1990): 9 (who cites John Marenbon, *From the Circle of Alcuin to the School of Auxerre: Logic, Theology and Philosophy in the Early Middle Ages* [Cambridge: Cambridge University Press, 1981], 88–115); and Mostert, *Library of Fleury*, 23.

RAMSEY

Aimoin tells us that Abbo was "magnificently outfitted" for his journey across the Channel to Ramsey Abbey.[243] With him he carried copies of key quadrivial texts, including Helperic of Auxerre's *De computo ecclesiastico,* Hyginus's *De astronomia*, and Macrobius's *Commentary on the Dream of Scipio*—unknown in England prior to his visit.[244] Some of these manuscripts found their way back to Fleury, either with Abbo or with one of the many individuals passing between the two institutions in the tenth and eleventh centuries. Abbo's visit was part of a longer tradition of exchange. To spend time in the reformed community, near the blessed Benedict's remains, seems to have been regarded in England as something of a rite of passage. Oda, Archbishop of Canterbury (941–58), may have resided there for a time (in the 930s), and he is documented as having sent his nephew, Oswald, future bishop of Worcester and archbishop of York to Fleury, where he took the habit. It was sometime during his eight years there that Oswald met Germanus, who helped him found Ramsey and who served as abbot of Winchcombe for a time.[245] The extent of the library that Abbo encountered at Ramsey cannot be known. The monastery was dissolved in 1536; no pre-Conquest manuscripts have, with certainty, been assigned to the abbey.[246] Anglo-Saxon libraries were,

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[243] Aimoin, *Vita* 4; ed. and trans. Bautier and Labory, 50–52: "Verum aliter se rem habere ostendit apparatus itineris qui magnitieus a domno Oylboldoa proficiscenti collatus est."

[244] These works do not pre-date Abbo's visit, nor were they cited (before the Norman Conquest) by any Anglo-Saxon authors other than Byrhtferth. A copy of Macrobius's *Commentary* written at Fleury, but with a quire added containing a liturgical calendar suitable for Ramsey—Paris, BnF lat. 7299—was probably the manuscript that Abbo brought with him to Ramsey and brought back on his return to Fleury. London, BL Harley MS 2506, was written by a Fleury scribe, illuminated by an English artist, and later glossed and corrected in Anglo-Norman minuscule (c.1000), suggesting that it too may have gone to England. Michael Lapidge, *The Anglo-Saxon Library* (Oxford: Oxford University Press, 2006), 51; Barker-Benfield, "A Ninth-Century Manuscript from Fleury," 152; and Patrick McGurk, "*Computus Helperici*: Its Transmission in England in the Eleventh and Twelfth Centuries," *Medium Aevum* 43 (1974): 1–2.

[245] Vezin, "Leofnoth," 109. Vezin also writes that Aethelwold, Abbot of Abingdon (c.955) and Bishop of Winchester (963), dispatched the monk Osgar to learn *mores* at Fleury. Osgar replaced Aethelwold as abbot at Abingdon, when Aethelwold was appointed bishop. Adhelgar, one of Dunstan's followers, also ended up at Fleury after Dunstan's death in 988.

[246] Without a contemporary inventory or a strong physical link—a manuscript that would give us some idea of what a book from Ramsey looked like—there is no way to assign any of the 800+ extant pre-conquest manuscripts to the monastery's collection. Lapidge estimates that there may have been as many as 100 manuscripts at Ramsey c.1000, a number based almost exclusively on references to various works encountered in Byrhtferth's writings. (The number includes references from the *Glossae Bridferti*, a work of contested authorship [see above note 144]. If the possibly spurious *Glossae Bridferti* are taken out of the equation, then the holdings would still be impressive, though less cosmopolitan.) Michael Lapidge, in Byrhtferth, *Vita S. Oswaldi*, xxiii–xxiv; idem, *Anglo-Saxon Library*, 266–74; Helmut Gneuss, *Handlist of Anglo-Saxon Manuscripts: A List of Manuscripts and Manuscript Fragments Written or Owned in England up to 1100*

on the whole, far more modest than their continental cousins.[247] But Oswald had great ambitions for Ramsey, which he modeled on Fleury.[248] This ideal, together with the great wealth of the monastery and its patrons, may mean that Ramsey had a significant library even in its early years.

Abbo's and Byrhtferth's writings provide some insight into the breadth of the collection around the year 1000. Abbo probably worked on two tracts while at Ramsey: a hagiographical account of the East Anglian martyr-king, Edmund, the *Passio Sancti Eadmundi* (*Passion of Saint Edmund*), and the *Quaestiones grammaticales* (*Grammatical Questions*), though the latter is likely to have been completed at Fleury. To write the *Passio*, Abbo drew heavily on Bede's *Ecclesiastical History*, Gregory the Great's *Letters* and *Dialogues*, and a number of early hagiographies—possibly a collection of such texts—all standard fare in England as on the continent.[249] For the *Quaestiones* he drew on other sources. This text takes the form of a letter comprising responses to a series of difficult questions on the finer points of grammar—questions that were, according to the prologue, posed by Abbo's Anglo-Saxon pupils.[250] Abbo found answers to the questions—which have been characterized as "esoteric, detailed, and intelligent"— primarily in Priscian's *Institutes* and Isidore's *Etymologies*. Ramsey almost certainly had a copy of the latter,[251] but it seems unlikely that a fledgling monastery in Anglo-Saxon England owned a complete copy of Priscian's highly technical, multivolume work.[252] Fleury's library, on the other hand, included

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(Tempe: Arizona Center for Medieval and Renaissance Studies, 2001), 3; Cyril Hart, "The Foundation of Ramsey Abbey," 319.

[247] Lapidge, *Anglo-Saxon Libraries*, 127–32, esp. 127.

[248] Byrhtferth recounts Oswald's time at Fleury (c.950–958) in the *Vita S. Oswaldi* (2.9; ed. Lapidge, 50). Dachowski, *First among Abbots*, 63.

[249] In the dedicatory letter that prefaces the *Passio*, Abbo states that he wrote the work at the urging of his brothers at Ramsey. He dedicated the work to Dunstan. Antonia Grandsen argues that Abbo wrote the work at Ramsey. The dedicatory letter was written while Dunstan was alive. Abbo returned to Fleury at some point in 987 and Dunstan died in May 988. Abbo borrowed phrases from a number of hagiographies, leading scholars to believe that it must have been written in proximity to Fleury's library, but Grandsen has shown that Abbo may have been working from a collection of saints' lives assembled on the continent that was in circulation in England c.1000. Grandsen, "Abbo of Fleury's *Passio,*" 22–23.

[250] Roger Wright, "Abbo of Fleury in Ramsey (985–987)," in *Conceptualizing Multilingualism in England, c. 800–1250*, ed. Elizabeth M. Tyler (Turnhout: Brepols, 2011), 108.

[251] Nineteen copies of the *Etymologiae* survive from pre-Conquest England. If Ramsey did not have a copy of the *Etymologiae*, it soon got one. Byrhtferth quotes Isidore regularly in his *Computus*, the *Enchiridion*, and the *Vita S Ecgwini*. See Lapidge, *Anglo-Saxon Libraries*, 125, 271, and 326.

[252] Seven copies of Priscian's *Institutes* survive from pre-Conquest England. According to Louis Holtz, Donatus and the many commentaries on his *Ars minor* and *Ars maior* reigned supreme in the early and central middle ages, it was only with the rise of the university that Priscian came to the fore. In the 10th century, the

at least one copy of Priscian's masterwork as well as virtually every other treatise, commentary, and gloss on grammar that was then in circulation.[253] The situation was likely similar in the domain of science and mathematics. Ramsey, thanks in no small part to Abbo, had the basics but little more: Isidore's *Etymologies*, Helperic's *De computo*, the Macrobian *Commentary on the Dream of Scipio*, and Boethius's *De arithmetica*. It is perhaps for this reason that the Anglo-Saxon monks continued to make their way to Fleury to complete their education: in search of masters and manuscripts.

Conclusion

In histories if the central middle ages, Gerbert and Abbo regularly recieve starring roles, especially in discussions of the intellectual culture of the era. An example is R. W. Southern's seminal *Making of the Middle Ages*. 972, the year that Gerbert travelled from Barcelona to Rome, served Southern as the starting point for his story. He described Gerbert's eventual accomplishments and their impact with an enthusiasm that rivaled Richer's:

> The works which he [Gerbert] wrote, the methods of teaching he devised and the pupils he taught at Rheims became the most important factor in the advancement of learning in nothern Europe during the next two generations.[254]

Gerbert was an exceptional thinker and teacher, but he was not the isolated genius-hero that Southern made him out to be. As the foregoing discussion makes clear, he and Abbo rose to positions of power on account of their relationships as well as their wits. Their success as educators lay in the strength of their scholarly networks, which proved durable over time and across great distances. Access to manuscripts—good ones—was crucial to their pedagogic programs. The following chapters consider the nature of those programs, their didactic strategies and aims, and their effect on visual culture.

Institutiones was used primarily for reference and by more advanced students and schoolmasters, not for teaching grammar's fundamentals. Anita Guerreau-Jalabert has emphasized how unwieldy (linguistically and physically) manuscript copies of the *Institutes* were/are. Abbo relied on both works to write the *Quaestiones*. Lapidge, *Anglo-Saxon Library*, 311; Louis Holtz, "Les nouvelles tendances de la pédagogie," 163–73; Guerreau-Jalabert, "Caractères de la culture Latine," in *Questions Grammaticales*, 123.

[253] For a list of the grammatical manuscripts as well as copies of the classical and Christian poets see Guerreau-Jalabert, in *Questions Grammaticales*, 180–9.

[254] Southern, *The Making of the Middle Ages*, 11.

CHAPTER 3
Abbo: Bridging to Arithmetic

In the early 980s, the monks of Fleury asked Abbo, to explain the *Calculus*, a work written by the fifth-century mathematician, Victorius of Aquitaine.[1] Even a glance at Victorius's work is sufficient to show why it would have baffled Abbo's brothers. In Bern, Burgerbibliothek Cod. 250, a ninth-century copy that may well have been at Fleury in Abbo's day and was certainly there by the twelfth century, the *Calculus* occupies its own extended gathering of twelve leaves (figs. 3.1–6). It begins with a brief preface—a text the length of a page transcribed in two columns (fig. 3.1). Victorius wrote in a clear, straightforward prose, but his technical language—his talk of *unciae,* composite numbers, remainders, duplication, etc.—takes for granted a rudimentary knowledge of number and would have left anyone unfamiliar with such specialized vocabulary entirely at sea. The cursory preface would certainly not have helped a novice navigate the pages of letter- and symbol-filled folios that followed. The greater part of the *Calculus* comprises a set of multiplication tables (figs. 3.2–4). In Bern 250 the tables are divided vertically by vivid orangey-red lines into five narrow columns each populated with Roman numerals and glyphs—shorthand notations for fractions (fig. 3.5). For those newly trained (or being trained) in the arts of the *trivium*, who might have had only a tenuous grasp of grammar and virtually no understanding of arithmetic, the *Calculus*, with its unusual content and layout, would have seemed utterly inscrutable.

Unable to refuse his beloved brothers, Abbo complied with their request. He wrote a commentary titled—in two other Fleury manuscripts—"an explanation of the *Calculus* of Victorius, which it is pleasing to call an introduction to arithmetic" (figs. 3.7–8).[2]

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[1]Abbo recounts his brothers' request in the first few lines of the *Explanatio* (1.1).

[2] Berlin, Staatsbibliothek Cod. 138, fol. 7v: Incipit explanatio in calculo victorii quam isagogen arithmetica placeat dicere. All translations of the *Explanatio* are mine unless otherwise noted.

Significantly, Abbo characterized the *Explanatio* as a "bridge to arithmetic."[3] The metaphor is worth investigation for it suggests the larger, far more ambitious goal that Abbo had set for himself. He intended the *Explanatio* to close the divide between the *trivium* and *quadrivium*, and, more specifically, between grammar and arithmetic. This chapter considers through close study of the *Explanatio* the nature of such a feat and just how it was accomplished. Arithmetic was, in theory and practice, the first of the quadrivial disciplines to be learned. Indeed, the rudiments of arithmetic in the guise of computus had been part of elementary education in monastic and cathedral schools since at least the time of Charlemagne. As such, they were absorbed along with basic grammar, often before rhetoric and dialectic. Grammar, the ability to read and speak Latin—the tongue of church, state, and the liberal arts—was, however, emphasized above all else in the schools.[4]

Grammar in this period was, moreover, a remarkably visual activity. Literary historian Martin Irvine emphasized the deeply "textual" (as opposed to oral) bent of medieval grammarians in *The Making of Textual Culture: 'Grammatica' and Literary Theory, 350–1100*.[5] He argued that in this early period grammar "placed writing at the center of its discourse," and theory of speech "was merged into the theory of letters and writing," written *and* read.[6] Hence, all writing—from entire libraries down to the very last letter—was imbued with a certain grammaticity. Grammar, in short, had dibs on books.[7] Irvine marshaled statements by a host of early medieval authors to this effect, among them Rabanus Maurus

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[3] Abbo, *Explanatio* 1.2; ed. Peden, 65: "sub expositionis tenore ad arithmeticam introductionis pontem construo." The title "explanatio in calculo Victorii quam ysagogen arithmeticae placeat dicere" was given to the work in the two copies most closely associated with Abbo and Fleury (Vatican City, BAV Reg. lat. MS 1281 and Berlin, SB Cod. 138 [*olim* Phillipps 1833]), but the title is unstable in the manuscript tradition, and sometimes omitted altogether. In one copy the term *commentum* was used. I will refer to the tract as the *Explanatio* in order to avoid confusion with other commentaries under discussion.

[4] The early and central middle ages might be called the era of Big Grammar. The number of extant grammatical texts from the medieval era is staggering: around 700 copies of Priscian's *Institutiones* and close to 300 of Donatus's *Ars.* These numbers come from Margaret Gibson, "Priscian *Institutiones grammaticae*, a Handlist of Manuscripts," *Scriptorium* 26 (1972): 102–23; and Louis Holtz, "La Typologie des manuscrits grammaticaux latins," *Revue d'histoire des textes* 7 (1977): 248. Martin Irvine traced grammar's rise and described how it absorbed the other verbal arts in his *The Making of Textual Culture: 'Grammatica' and Literary Theory, 350–1100* (Cambridge: Cambridge University Press, 1994).

[5] Both Martin Irvine and Vivian Law (see below, note 9) were indebted to Michael Clanchy's seminal *From Memory to Written Record*, which first appeared in 1979 and described the extraordinary "growth in the uses of literacy" in England between the 12th and 13th c. According to Clanchy, the revolution began (if "haltingly") with the monarchy and in monasteries, where writing had long been taught. Clanchy, *From Memory to Written Record, England, 1066-1307* (Cambridge: Harvard University Press, 1979), 1.

[6] Irvine, *Making*, 95–96.

[7] Ibid., 2.

(d.856), monk of Fulda, the so-called *praeceptor Germaniae*, who, when offering advice on the instruction of clerics, called grammar "the judge of all books."[8]

Grammar, as arbiter, also took on the role of artisan, a shaper of format and as well as the content of books. [9] Vivian Law, who traced changes in grammar tracts from antiquity through the Renaissance, described how grammatical terminology that drew attention to the written form of words emerged in the seventh century; she wrote of: "a shift from a predominantly aural way of conceptualizing language to one which was predominantly visual," and observed in manuscripts the increased practice of setting out paradigms in columns and tables, rather than in the more traditional form of a running text, as in a late-eighth century copy of Donatus's *Ars minor* from Fleury (fig. 3.9).[10] Historian Paul Saenger connected consistent use of word separation along with punctuation, notation, and abbreviation in the late tenth and eleventh centuries as evidence that literacy was increasingly inculcated by visual means.[11] Both tendencies gained ground in the eighth and ninth centuries.[12] In the latter half of the tenth century, several centers of study including Fleury

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[8] Rabanus Maurus, *De institutione clericorum* 2.18; PL 107.395: "omnium iudex librariorum" (cited by Irvine, *Making*, 7 and 335). Rabanus was playing with the last of the four categories of grammatical interpretation: *lectio, enarratio, emendatio, and iudicium*.

[9] Irvine uses the term "objectify": "grammatical discourse ... objectified a library of texts which simultaneously authorized its [grammar's] rules." *Making*, 108.

[10] Law charted the rise of two new terms for "word" in a general sense (that is, not a particular part of speech, but the whole generic category of words) *litteratura* and *superficies*, which "connote the written rather than the spoken aspects of the word." These terms appear in grammars of the 7th c. and gained popularity and acceptance in the 8th and 9th c. Law, "The Transmission of Early Medieval Elementary Grammars: A Case Study in Explanation," in *Formative Stages of Classical Traditions: Latin Texts from Antiquity to the Renaissance*, ed. Oronzo Pecere and Michael D. Reeve (Spoleto: Centro italiano di studi sull'Alto medioevo, 1995), 256–57, 259; and eadem, "From Aural to Visual: Medieval Representations of the Word," in *Grammar and Grammarians in the Early Middle Ages* (London: Longman, 1997), 255.

[11] This was Paul Saenger's basic thesis in his article "Separated Script at Reims and Fleury at the Time of Gerbert and Abbo" (expanded in "The Early Protoscholastics" chapter in *Space between Words*). He included a handlist of manuscripts containing works by Abbo, Gerbert, and their circles, which were written with separated script and other characteristics of written literacy. Irvine made similar claims for a much longer period (from the 9th through the 11th c.) focusing more on the overall layout of the page: space between lines, size of margins, and script hierarchies. He included a handlist of "compilations that served grammatical culture in early and medieval textual communities." On the list are several manuscripts from Fleury and Reims. The features and layout Irvine described are common features of literary and mathematical texts alike, though the latter contain far less marginal commentary—which Irvine links to the task of *enarratio* or interpretation—probably due to the terse "quadrivial" style of writing (discussed below). Saenger, "Separated Script at Reims and Fleury at the Time of Gerbert and Abbo," *Le Livre et l'historien: Études offertes en l'honneur du Professeur Henri-Jean Martin*, ed. Frédéric Barbier and Henri Jean Martin (Geneva: Droz, 1997), 3–23; idem, *Space between Words: The Origins of Silent Reading* (Stanford: Stanford University Press, 1997), 143–64 and 279–90; Irvine, *Making*, 384–405, esp. 384.

[12] Possibly even earlier. M. B. Parkes made a similar point discussing what he called a "fundamental shift of attitude" in the status of letters. He cited Isidore and an interpolation introduced into the family of Visigothic

and Reims began producing manuscripts that physically facilitated grammatical interpretation and engagement, especially the processes of *lectio* (reading) and *enarratio* (interpretation) (figs. 3.10–11).[13] Lines of text were widely spaced and surrounded by generous margins to accommodate and invite interlinear gloss and marginal commentary.[14] This was true of tracts devoted to any and all subjects—quadrivial material included—especially those to be used in the schools. All this is to say that when students, especially beginners, opened a book, they were trained to utilize skills of *lectio* and *enarratio* that they had acquired in the study of grammar, and, increasingly, texts were laid out and annotated in a way that further fostered this impulse.

This chapter considers how, in the context of this visually muscular grammar, Abbo introduced arithmetic. Grammar may have been presented using devices that were ever more visual, but arithmetic, I suggest, expanded the expectations and abilities of manuscript users.[15] Its pursuit truly opened the possibilities and potential of the page and engaged the

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manuscripts of the *Etymologiae* (1.3.1) which reads: "Letters introduce words through the eyes not through the ears." Commenting on this statement Parkes said: "writing was no longer merely the record of the spoken word but could signal directly to the mind through the eye."*Pause and Effect: An Introduction to the History of Punctuation in the West* (Aldershot: Ashgate Publishing Ltd, 1992), 21, esp. n. 14.

[13] Milan, Biblioteca Ambrosiana MS C. 128 is the earliest copy of the *Saltus Gerberti*, Gerbert's scholium on Boethius's *De arithmetica* (2.1). The manuscript contains both the *De arithmetica* and the *De musica*. Saenger notes that words were very generously separated, terminal forms used, along with emblematic punctuation, and capitals. Orléans, Bibliothèque Municipale MS 277 (233) is a collection of logical texts written at Fleury (fig. 3.11 is the incipit page of Boethius's translation of Aristotle's *Categories*). In this manuscript words are generously spaces (at "two times the unity of space" according to Saenger).

[14] Holtz, "Typologie," 249–50. Irvine built on Holtz's argument, claiming, "the appearance of glosses and commentary in a manuscript is the primary sign that a text was part of a grammatical culture, either as part of the canon of *auctores* or as a text which is first constituted as such by grammatical methodology." *Making*, 372.

[15] The disciplines of number had always had a strong visual aspect and graphic elements had long been lodged within text, in both rolls and codices. Historian Reviel Netz has described arithmetic equations and geometric figures as "leaping" off the page and considers this a Greek invention. According to Netz and others, the most striking characteristic of Greek (and Latin) papyri is their uniformity on the macro (from roll to roll and text to text) and micro level (between letters, lines, passages, etc.). Textual interpretation, the parsing of words, phrases, and ideas, happened not on the page, but in the imagination or in vocalization and performance. Netz has stressed how the transmission of mathematics in written tracts, which we might take for granted, was by no means given. He has convincingly argued that it was, rather, the inevitable side-effect of very specific cultural and geographic conditions. Greek "mathematicians" were part of the social elite and they pursued mathematics according to the rules of their class, which largely prohibited practical application. Hence, in order not to transgress socioeconomic boundaries, advanced mathematics was pursued intellectually, as an esoteric theoretical thought exercise, which, if applied at all, served only as an aid to philosophical argument. They married mathematics to the elite literary culture. Advanced mathematics was, in addition, not a particularly popular activity or common hobby; enthusiasts were few and far between. Netz calculates that there were actually very few mathematicians, even in the period now referred to as "golden" or the heyday of Greek mathematics. Mathematical discourse thus happened across distances, geographical and temporal, and mathematical ideas were, of necessity, communicated in letters and tracts. Writing, in short, was requisite. Netz, *Shaping*, 271–312.

mind in new ways. In the *Calculus* long lines of text give way to numbers arrayed in columns, along the sides of triangles, in grids, etc. Separate rules and assumptions governed these spaces; successfully coping with them required different habits of thought: Abbo had to make readers into viewers. There is a tendency among art historians to assume that pictures in books were "read" and deciphered in the manner of text, and that reading was the default or dominant hermeneutic applied to all aspects of a book. Reading, however, even broadly defined to include rhetorical and logical modes of interpretation, was only what one did with words. The letter-filled tables and figures that comprised the medieval mathematical corpus remind us of the work that letters did outside the verbal sphere, as Roman numerals and *notae* ("signs"). In this period before Arabic numerals were adopted for writing numbers, the alphabet did double duty. Abbo could be certain that his readers came to the *Calculus* with a grasp of letters as the "primary elements of grammar." He both built on and troubled this knowledge dwelling on the mercurial nature of signs; mapping new, strange signs (for fractions, measures, etc.) onto the familiar system; and rearticulating the page.

1. Early education at Fleury

> That very one [Abbo] burning to investigate still greater scientific secrets, went to wisdom's workshop in diverse places (because he had already fully achieved investigation of grammar, arithmetic, and—not least—dialectic), in order that he might proceed to add the other arts to his own talent.[16]

We return to this passage from Abbo's *Vita* discussed in chapter two to examine more closely Aimoin's aside: Abbo left Fleury having "already fully achieved investigation of grammar, arithmetic, and—not least—dialectic."[17] These three of the seven arts were, evidently, taught by Fleury's mid-tenth-century masters.[18] With a *nec non* ("not least!") Aimoin draws attention to dialectic, for it was not everywhere that one could learn the intricacies of logic

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[16] Aimoin of Fleury, *Vita* 3; ed. Bautier and Labory, 48: "Verum ipse, adhuc maiora gliscens scientiae scrutari arcana, diversorum adiit sapientes officinas locorum ut, quia grammaticae, arithmeticae, nec non dialecticae iam ad plene indaginem attigerat, ceteras ingenio suo pergeret superadjicere artes."

[17] Aimoin wrote that Abbo was "imbued with letters" (*litteris imbuendus*) at the school (*scola*) of the church of St. Peter's at Fleury. Ibid. 1; ed. Bautier and Labory, 42.

[18] Anita Guerreau-Jalabert has confirmed the historic accuracy of Aimoin's brief description of the curriculum at Fleury, noting that in the library (*vis-à-vis* the extant manuscripts) the disciplines of grammar and dialectic are far better represented than rhetoric. Guerreau-Jalabert, in *Abbon de Fleury Questions grammaticales* (Paris: Belles Lettres, 1982), 169.

"fully" (*ad plenum*).[19] Fleury, with its comprehensive collection of dialectical tracts, was special in that regard.[20] His stress also implies that the former two subjects, grammar and arithmetic, were quotidian, standard fare.

The pairing of grammar and arithmetic, which Aimoin set before dialectic and the rest of the arts, reflects both an ideal and historic truth. In discussions of the liberal arts, these subjects held special positions, both were seen as "prior," or constituent: grammar to all learning, arithmetic to the rest of the quadrivium.[21] As such, they were deemed requisite knowledge and their precepts had, from at least the time of Charlemagne, been widely taught *first*. Writing (*notas*), which included transcription and hand signing, came before all else, preceding *both* grammar and arithmetic, a point to which I shall return. "Basic" grammar comprised the study of syllables, parts of speech, meter, morphology, and orthography, and this was followed by a focus on building vocabulary and deciphering simple sentences.[22] Elementary arithmetic involved writing and signing numbers, recitation and memorization of simple equations (i.e., addition, subtraction, multiplication, and division), basic finger calculation as well as learning to manipulate fractions and to use tables. Though these fields of study could be and were pursued at a high level by quite advanced scholars, their

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[19] Similarly, Gerbert, according to Richer, taught dialectic before rhetoric at Reims. Introductory dialectic texts—Aristotles's *Categories* and *Peri hermeneias* were read, according to Anna Grotans, at the "early intermediate level" at St. Gall. Focused on words and statements, the two works were easily assimilated into grammatical studies. Grotans, *Reading*, 73 and 84.

[20] Porphyry's *Isagoge*, Aristotle's *Categories*, *Peri hermeneias*, and *Topica* (with commentaries by Boethius and Cicero) were all available at Fleury. All the above-listed works are contained in the late 10th/early 11th c. manuscript (now dispersed): Leiden, UL MS V. F. 70, fols. 1–50 + Orléans, BM MS 277 p. 1–55 + Paris, BnF NAL MS 1630, fols. 14–16 + Leiden, UL MS V. F. 70, fols. 51–66 + lacuna + Orléans, BM MS 277, fols 62–77 + lacuna. For a list of extant manuscripts containing dialectical tracts see Guerreau-Jalabert, in *Abbon de Fleury: Questions*, 189–93. On the teaching of dialectic under Abbo see F. Schupp, "Abbon de Fleury et la logique: Quelques questions historiques et systématiques," in *Abbon de Fleury: Philosophie, sciences et comput autour de l'an mil*, ed. Barbara Obrist, Oriens–Occidens 6 (Paris: C.N.R.S. and E.P.H.E., 2006), 43–60.

[21] Boethius stated in no uncertain terms that arithmetic was the first of the disciplines to be learned: "Which of these disciplines, then, is the first to be learned but that one which holds the principal place and position of a mother to the rest? This is arithmetic. It is prior to all not only because God the creator of the massive structure of the world considered this first discipline as the exemplar of his own thought and established all things in accord with it" (*De arithmetica* 1.1.8; ed. Guillaumin, 8–9; trans. Masi, 73–74). Abbo in the *Explanatio* (3.83) is less definitive. None of the arts is naturally "first," he says, but (to paraphrase) in the mathematical disciplines prior knowledge of arithmetic was useful in the study of, for example, music.

[22] An overview of early medieval primary education is provided in Anna Grotans, *Reading*, 71–76. Alcuin had divided grammatical study into elementary (*litteratio*) and advanced (*litteratura*) levels. The model seems to have persisted in the 10th c. See Irvine, *Making*, 316–17.

fundamentals were learned by "tender little boys" (*pueruli tenelli*).[23] This was certainly the case at Fleury, where Abbo had been charged with teaching grammar long before his own education was complete, possibly when he was still in his teens.[24]

What we know about the order of education at Fleury reveals surprisingly strong links between grammar and arithmetic. Grammar was, to be sure, pre-eminent in the elementary curriculum, but it was not exclusive. Numbers and calculation—the rudiments of arithmetic—had been part of early education at Fleury since Theodulf. The interpenetration of these subjects is evidenced in the oldest extant collection of Latin grammatical tracts, Paris, Bibliothèque nationale de France, MS lat. 7530, copied at Monte Cassino between 779 and 796, a compendium of grammatical, rhetorical, and computus material.[25] Similar combinations are encountered in manuscripts made at Fleury, as, for example, the tenth-century Paris, Bibliothèque nationale de France, MS lat. 7518, in which Helperic's *De computo*—an introduction to computus—sits between the fifth-century grammarian Phocas's elementary treatise on grammar (known as the *Ars de nomine et verbo*) and Cicero's *Synonyma*.[26]

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[23] The turn of phrase comes from Ælfric, who in the preface to his *Grammar* is apologetic about the elementary nature of the tract and references *pueruli tenelli* (tender little boys) already familiar with the eight parts of speech in the *Ars minor* of Donatus. Notker, schoolmaster of St. Gall at the end of the 9th c. and beginning of the 10th, defined grammar as the art that, "teaches *rectiloquium,* that is to speak correctly and something that even little children can learn, as we hear every day." This passage from Notker's translation of Boethius's *Consolation of Philosophy* was cited and translated by Aelfric, Abbot of Eynsham, *Aelfric's prefaces*, ed. and trans. Jonathan Wilcox, Durham Medieval Texts 9 (Durham: Durham Medieval Texts, 1994), 114–15; Grotans, in *Reading*, 24; and Joyce Hill, "Learning Latin in Anglo-Saxon England: Traditions, Texts and Techniques," in *Learning and Literacy in Medieval England and Abroad*, ed. Sarah Rees Jones (Turnhout: Brepols, 2003), 12.
[24] Pierre Riché specified that a master was typically twenty-five years of age (the age when one became a deacon), but this was not a firm rule and there are examples of younger masters. Ekkehard IV (late 10th c.—after 1056), a monk of St. Gall, who left a detailed history of the abbey rich with details about the community, wrote that both Gerald and Ratpert had been "magistri" from adolescence. Anna Grotans suggests that they may have served as teaching assistants, called *semi-magistri*, *submagistri*, or *adiutores scolarum*; or exclusively as a *precantor*. The same may have been true of Abbo. Riché, *Les écoles et l'enseignement dans l'Occident chrétien de la fin du Ve siècle-milieu du XIe siècle* (Paris: Aubier Montaigne, 1979),195–97; and Grotans, *Reading*, 65.
[25] Faith Wallis has observed a certain selectivity with regard to the kind of grammatical material that was added to computus manuscripts. She wrote, "The nature of the grammar materials included in computus manuscripts suggests that their compilers also perceived some congruence between the study of words and the study of numbers and time. ... When compilers of computus manuscripts elect to devote more space to grammar, they show a distinctive preference for the elements of prosody: pronunciation, orthography, quantity, accent, and meter." Luis Holtz, "La Typologie," 248–49, esp. n. 1; idem, "Le Parisinus latinus 7530: Synthèse cassinienne des arts libéraux." *Studi Medievali,* 3rd series, 16, no. 1 (1975): 97–152; and Wallis, "The Calendar & The Cloister: Oxford, St. John's College MS 17," McGill University, http://digital.library.mcgill.ca/ms-17/folio.php?p=159v&showitem=159v-175r_13GrammarOverview (accessed October 23, 2014).
[26] Van de Vyver wrote that Paris 7518, "vraisemblablement originaire de Fleury"; "Les Œuvres," 149.

The *Quaestiones grammaticales* (*Grammatical Questions*), a lengthy letter written by Abbo to his "most-beloved, English Brothers in Christ, particularly those in the monastery of the Holy Father Benedict [i.e., Ramsey]," exhibits the same easy admixture of grammar and arithmetic.[27] The letter is a set of responses to twenty-three questions on grammatical matters written near the end or just after Abbo's sojourn in England.[28] In writing the epistle Abbo drew on a number of late-antique and medieval sources including the *Ars minor* and *maior* (*Minor* and *Major Arts*) of Donatus; Priscian's *Institutiones grammaticae* (*Grammatical Instruction*); Isidore's works on synonyms, spelling, and the first book, *De grammatica* (*On Grammar*), of the *Etymologiae*; the grammatical works of Bede, and Alcuin's *Ars grammatica* (*Art of Grammar*). These works were part of the standard grammatical corpus and were present at Fleury; they may also have been available at Ramsey.[29]

The *Quaestiones* provides insight into what Abbo taught his Anglo-Saxon charges, what they knew already (from him or other sources), and what he did not have to explain. It also shows how he wove together arguments and proofs from diverse fields. Grammar and arithmetic existed cheek-by-jowl and were mutually reinforced. Absorbed locally and in tandem or quick succession, grammatic and arithmetic rationales, so distinct, were thus intimately bound and proximate: in the space of the schoolroom, manuscripts, and the minds of students. Given this, the ability to switch from one mode of thinking to the other would have been second nature. Consideration of such a skill helps us begin to understand how quadrivial manuscripts with their unusual admixture of verbal passages and visual elements were navigated. Perhaps more importantly, we are reminded that there were other habits of thought as deeply ingrained as reading.

Abbo began the *Quaestiones* well within the bounds of grammar with discussions of accent, pronunciation, verb tense and mood, etc. The final two questions, however, veer into the domain of number. The penultimate question posed by the Ramsey monks concerned the

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[27] Abbo, *Quaestiones* 1; ed. Guerreau-Jalabert, 209: "Dilectissimis in Christo Angligenis fratribus, maximeque in monasterio sancti patris Benedicti" The *Quaestiones* is preserved in two manuscripts, both written at Fleury during Abbo's lifetime: Vatican City, BAV Reg. lat. MS 596, fols. 10–23 (beginning of the 11th c.); and London, BL Add. MS 10972, fols. 39v–48 (first quarter of the 11th c.).

[28] In the manuscripts, the treatise is labeled only "Epistula Abbonis." It was christened the "Quaestiones" by Migne (PL 139.522). Roger Wright, "Abbo of Fleury in Ramsey (985–987)," in *Conceptualizing Multilingualism in England, c. 800–1250*, ed. Elizabeth M. Tyler, (Turnhout: Brepols, 2011), 105–20, esp. 111–12 and 119.

[29] On the Ramsey library see chapter 2.

meaning of the term *olympiad*. Abbo, as he provided an explanation, first likened the Greek *olympiad*, a period of four years, to the Latin *lustrum*, a period of four or five years. He then gave authoritative examples of the term's use: Christ was born in the 194th *olympiad* and Diocletian renewed the persecution of the Christians in the 267th *olympiad*.[30] The subject matter is computistic, but the method of explanation grammatic. Abbo then added an arithmetic coda: a formula. He explained that multiplying the number of *olympiads* by four gives the periods "stated in the chronicles."[31] The observation assumes competence in calculation (or access to a set of multiplication tables like the *Calculus*) and shows an interdisciplinary approach. Vocabulary was often built through the use of synonyms, by pairing an unfamiliar word with one that was more common. Here, Abbo supplies a mathematical synonym. Abbo's readers were unused to seeing the date of Christ's birth in *olympiads*; more familiar were the numbers in the chronicles. Hence, the formula is a sort of *id est* statement arrived at by means of multiplication: 194 *olympiads*, that is 776 years.

It is, however, in his response to the final question that Abbo moves fully into the realm of arithmetic. The question concerns the the controversial line from the Athanasian Creed: "The Holy Spirit not made by, nor created by, nor born of the Father and the Son, but proceeding from [them]."[32] Abbo argues for the truth of this verse, grammatical and otherwise, despite seeming contradictions. Particularly problematic is the nature of the Holy Spirit, if it is neither begotten nor begets. Abbo first provides examples of entities that are neither one thing, nor the opposite, but something in-between for which no suitable verbal expression exists, for example, a person who is neither in good health nor sick; someone who does not see, but is not blind.[33] He then takes a different tack, proposing a numerical proof:

> It is, without doubt, understood that God—who has proclaimed his Trinity and ordered all ... according to number, measure, and weight—rejoices in odd

[30] Abbo, *Quaestiones* 43; ed. Guerreau-Jalabert, 265: "Unde et dominus fertur centesima nonagesima tertia olympiade natus; et in libro *De laude uirginum* ducentesima sexagesima septima olympiade fuisse Diocletianum nouimus persecutorem Christianorum."

[31] Ibid.: "A prima enim olympiade quicquid factum dicitur, per quaternarium multiplicata numeri summa manifestatur, sicut in cronicis dicitur."

[32] Ibid. 44; ed. Guerreau-Jalabert, 265–66: "Spiritus Sanctus a Patre et Filio non factus, nec creatus, nec genitus, sed procedens."

[33] The examples are lifted directly from Boethius's translation and commentary on Aristotle's *Peri hermeneias* and fall into an unusual category of words that Boethius translated as "infinite names" (or "nouns"). Daniel Heller-Roazen, "Four Remarks on Infinite Names," (lecture, Harvard University, Cambridge, MA, November 14, 2014).

> [numbers]. And for this reason one can explain, to a certain extent, the difference between the persons of the Trinity by the nature of three odd numbers ... : the first, five; the second, seven; and the third, nine; ... the first [five] begets, but is not begotten; the second [seven] neither begets nor is begotten; the third [nine], however, is begotten, but does not beget.[34]

Abbo assigns meaning to the arithmetic properties of five, seven, and nine.[35] He goes on to explain: five is generative or "begets" because it combines with two to produce ten.[36] A number is "begotten" if it is a product of numbers other than one (which was not considered a number) and itself, that is, if it is not prime. Five and seven, both prime numbers, are thus "not begotten."[37] Nine, on the other hand, is the square of three and, thus, begotten. Seven and nine could, of course, also be multiplied, but only to produce numbers outside the decad (i.e., the first ten numbers, which Abbo is concerned with here), hence they "do not beget." Abbo remarks that he could continue in this vein, but he stops himself, saying:

> But, it seems to me, that I said enough about such things in the modest little work that I was compelled to write by the prayers of my brothers *On Number*, *Measure*, *and Weight* on the *Calculus* of Victorius.[38]

It is in the *Explanatio* that we find the far end of Abbo's trivial-quadrivial bridge.

2. THE *CALCULUS* OF VICTORIUS AND ABBO'S *EXPLANATIO*

Victorius was known in his own time as the "calculator of infinitesimals"—an apt moniker.[39] For the *Calculus* he worked out the products of 1000 to $^{1}/_{144}$ multiplied by numbers from 2 to

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[34] Abbo, *Quaestiones* 48–49; ed. Guerreau-Jalabert, 271–75: "Constat nimirum quia numero Deus inpare gaudet, qui Trinitatis sue assertor extitit, dum omnia in numero, mensura et pondere constituit. Et ideo Trinitatis quodam modo differentiam per trium inparium se ... sequentium censeo naturam discutiendam, quorum primus sit quinarius, secundus septenarius, tertius nouenarius. ... horum prescriptorum inparium primus gignit, sed non gignitur; secundus nec gignit nec gignitur; tertius autem gignitur, sed non gignit."
[35] This kind of arithmological exegesis and reasoning was not unusual. It was practiced and championed by Augustine, Martianus, Macrobius, and Isidore among others.
[36] Pythagoras had assigned special powers to the decad; Plato followed suit as did Neo-Platonists, including Augustine and Boethius.
[37] Nine, on the other hand, is the square of three and, thus, begotten.
[38] Abbo, *Quaestiones* 50; ed. Guerreau-Jalabert 275: "Sed quia de his, ut mihi uisum est, satis disserui in libellulo quem precibus fratrum coactus De numero, mensura et pondere olim edidi super Calculum Victorii."
[39] *scripulorum calculator* from Gennadius, *De viris illustribus* 88; ed. W. Herdin (Leipzig: Teubner, 1879): 108.9–25 (cited by Wallis, in *Bede: The Reckoning of Time*, li, n. 93). Victorius's fame and facility with numbers led Pope Leo the Great (440–61) to ask him to create a set of Easter Tables that might settle debates on the timing of the moveable feasts, an issue that was causing great friction within the church, especially between Alexandria and Rome. An excellent discussion of Victorius and his 5th-c. milieu can be found in Warntjes, *The Munich Computus*, xxxvi–xli, esp. xxxviii.

50. In a copy of the work that was produced and owned by Fleury around the year 1000, Berlin, Staatsbibliothek Cod. 138, the large pages are divided into five narrow columns drawn with a rule in the same bright orange pigment used for rubrication (fig. 3.12).[40] Each opening contains ten columns. The columns are capped with peaked "roofs" and labeled with the names of the proportions below: "doubles" (*dupli*), "triples" (*tripli*), "quadruples" (*quadrupli*), etc., to fifty.[41] Beneath the names of the multipliers, each column splits into two vertical lines of numbers: on the right are the multiplicands (in descending order from 1000 to $^{1}/_{144}$), on the left are the products.[42] Thus, to find the product of 2 and 900, for example, one would go to the *duplis* column and scan down the line of numbers on the right to 900 (***dcccc***). To the left of this number, in the same row, is the answer, 1800 (***īdccc***). To divide, one would do the reverse.

Over five hundred years after their making, Victorius's multiplication tables were still in use. They had over time, attracted into their orbit a collection of anonymous tables, lists, and compact tracts devoted to related topics including tables of square numbers, a list of signs used for fractions and another of signs for weights, and notes on measures of oil, linear measures, medicinal measures (figs. 3.4 and 3.6). The chronology of these accretions is difficult to ascertain.[43] Abbo referred to Victorius as the author of all the elements, meaning that the compilation was travelling as a unit by the third quarter of the tenth century at the latest. Indeed, in the ninth-century copy of the *Calculus* already cited, Bern 250, there is little visual difference between Victorius's tables and the additional material.[44] In this and other

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[40] This manuscript—like many copies of the *Explanatio*—does not contain all of the *Calculus*, only Victorius's preface and the multiplication tables to sixteen (and even these are incomplete).

[41] The columns are labeled differently from copy to copy. In Bern 250 the labels appear only on the first page of tables (the first six columns). The sixth column is labeled *sescupli* where we would expect *sextupli*.

[42] Not every number is addressed. From 1000 to 100 the decrease is in increments of 100 (e.g., 900, 800, 700), from 100 to 10 the numbers decrease by increments of 10 (e.g., 100, 90, 80), then by ones, then by $^{1}/_{12}$ (an *uncia*), then by multiples of an *uncia*.

[43] Peden dates the two prose elements (short commentaries on two of the additional tables) to the 9th c. or earlier. For a summary of contents of the *Calculus* see Peden, in *Abbo of Fleury and Ramsey*: *Commentary,* xv–xvii and xliv–xlv.

[44] That is, they are enclosed within frames formed of a single line. The arrangement of numbers, however, differs from table to table. As will be discussed below, the additional tables include text and tend to be "read" from left to right.

copies, tables and lists run into each other, divided only by horizontal lines, and are differentiated one from the next by content alone (fig. 3.13).[45]

The *Explanatio*, Abbo's explication of the *Calculus* written for the bemused monks of Fleury, survives in eight witnesses, all contemporary or near-contemporary with its writing.[46] Several copies were made at and/or owned by Fleury, which naturally served as a center for the work's dissemination. The fairly broad circulation of the work in France and Germany suggests that it was eagerly sought and deemed useful well beyond the bounds of Saint–Benoît. One of the earliest copies written at the beginning of the eleventh century was at Bamberg cathedral library by mid-century.[47] It may have been among the manuscripts gifted by Gerbert to Otto III, or, perhaps more likely, it was brought to Germany by Berno of Reichenau, who studied at Fleury and—as Abbot of the imperial abbey of Reichenau—had strong ties to Henry II.[48]

Abbo observed in his preface to the *Explanatio*, that the *Calculus* was an especially handy tool for computus.[49] It was in this context that Victorius's tables were encountered at Fleury.[50] Likewise, the *Explanatio* was placed in a similar setting.[51] Bibliotheca Apostolica Vaticana, MS Reg. lat. 1281, the single extant manuscript combining a full copy of

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[45] The only exception is a table of fractions of measures multiplied by two, which is labeled "The Door of Calculation" (*Ianua calculi*) in large, orange majuscules. Discussed below and in chapter 4.
[46] Vatican City, BAV Reg. lat. MS 1281, fols. 37v–52r (Fleury?, 11th c.); Bamberg, SB MS Msc.Class.53 (*olim* H.I.IV.24), fols. 5–46 (German, first quarter of the 11th c.); Bernkastel-Kues, HB MS 206, fols. 1r–41r (German, 11th c.); Berlin 138, fols. 7v–21v (Fleury, end of the 10th/beginning of the 11th c.); Brussels, BRB MS 10078, fols. 97r–114r (St. Peter's, Gembloux; 11th c.); Karlsruhe, BLB MS 504, fols. 90r–99r (incomplete, *Explanatio* ends at 3.36; St. Michael's, Bamberg; end of the 11th/beginning of the 12th c.; copied by Prior Thiemo [d. 1119]); Darmstadt, Hessisches Landes- und Hochschulbibliothek, MS 756, fols. 92–116v (St. James, Liège; 11th c.); Vienna, ÖNB Cod. 2269, fols. 134r–40r (German?, 11th c.).
[47] Hoffmann, *Bamberger Hanschriften*, 136.
[48] Mütherich, "The Library of Otto III," 19–21; Peden, "Unity, Order, and Ottonian Kingship," 166–67. On Berno see chapters 2 and 6.
[49] Abbo, *Explanatio* 1.3.
[50] The multiplication tables were part of the seven-book computus compiled in the 9th century. Peden, in *Abbo of Fleury and Ramsey: Commentary*, xvi.
[51] In Vatican, Reg. lat. 1281 the *Calculus* and *Explanatio* are preceded by several of Abbo's letters on chronology, the table of Dionysius Exiguus (a table of annual solar concurrents for the Paschal cycle of 532 years); an excerpt from Chapter 47 of Bede's *De temporum ratione* explaining the rationale behind the duration of the so-called Great Paschal Cycle (532 years; the 19 year lunar cycle multiplied by the 28 year solar cycle, beginning with Christ's incarnation); the Paschal tables; and, interestingly, an excerpt from Porphyry's *Isagoge* to Aristotle's *Categories*. In Berlin 138 the *Calculus* (incomplete) is preceded by excerpts from Macrobius. It is followed by the Abbo's *Explanatio*, *Computus*, and letters.

Victorius's *Calculus* (always difficult to reproduce),[52] and the *Explanatio* was made in the eleventh century, probably at Fleury.[53] In this omnibus computus compendium, Abbo's text is joined by his letters on chronology, the table of Dionysius Exiguus (a table of annual solar concurrents for the Paschal cycle of 532 years); an excerpt from Bede's *De temporum ratione* explaining the rationale behind the duration of the so-called Great Pascal Cycle (532 years; the 19 year lunar cycle multiplied by the 28 year solar cycle, beginning with Christ's incarnation); the Pascal tables; and, interestingly, an excerpt from the third-century polymath Porphyry's *Introduction* (*Isagoge*) to Aristotle's *Categories*—a tract that was among the set of texts used to introduce students to dialectic. The manuscript thus embodies Aimoin's description of early education at Fleury discussed above.[54]

The *Explanatio*, addressed all parts of the *Calculus*: Victorius's preface, the multiplication tables, and the hodgepodge of anonymous lists and tables that travelled with it. The appearance of the verb *comminiscor* ("to make up" or "to devise") in the preface gave Abbo the opportunity to reflect on the very role of a commentator. He wrote, "We call those people commentators, who, with the light of explanation, illuminate truth, which was wrapped up in thoughts obscurely expressed, by fashioning many things having the appearance of truth, which things they call 'commentaries' (*commentarios*)."[55] A commentary on the nature of commentating, Abbo's statement offers insight into his own aims and methods. Earlier in this same passage, he equates *comminiscor* with *fingere* (to

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[52] None of the extant copies are free of error. Copying the tables of the *Calculus* can be compared to copying the Eusebian Canon Tables. Patrick McGurk has shown how mistakes of alignment and incorrect numbers were made in even the most meticulously executed manuscripts as, for example, the Lindesfarne Gospels. "The Disposition of Numbers in Latin Eusebian Canon Tables," in *Philologia Sacra: Biblische und patristische Studien für Hermann J. Frede und Walter Thele zu ihrem siebzigsten Geburtstag*, ed. Roger Gryson (Freiburg: Herder, 1993), 242–58, esp. 247.
[53] The majority of extant copies of the *Explanatio* are either joined to an abbreviated version of the *Calculus* or without it. It is possible that the *Explanatio* circulated and was used independent of the *Calculus*. Indeed, the whole of Victorius's preface is reiterated, if piecemeal, in the *Explanatio*, which breaks the text into lemmata. It is also likely that institutions that came to own the *Explanatio* would have already had a copy of the *Calculus*—as Fleury did—and would not have needed a second copy of the work, which, as discussed above, was difficult to reproduce accurately. Peden, in *Abbo of Fleury and Ramsey: Commentary*, xliv.
[54] That the *Explanatio* served—beyond the walls of Fleury—as a teaching text is even more evident in Vienna, ÖNB Cod. 2269: an 11th-c. manuscript made in Germany, described by Alison Peden as a "compendium of the liberal arts." Peden, in *Abbo of Fleury and Ramsey: Commentary*, xli.
[55] Abbo, *Explanatio* 3.41; ed. Peden, 98: "Unde commentatores eos vocamus, qui veritatem aliquo modo obscuris sententiis involutam multa verisimilia fingendo expositionis luce inluminant, quae inventa 'commentarios' vocant." He is commenting on the line in the prologue of the *Calculus*: "Ad huius divisionis compendium tale calculandi argumentum antiqui commenti sunt."

create/produce/transform), *invenire* (to invent), and *excogitare* (to devise).[56] For Abbo, commentating was a creative and generative enterprise, as the *Explanatio* bears out.

His exposition of the first line of Victorius's preface gives some idea of the rich didactic potential that he saw in the *Calculus*.

> Unity, from which all multitude of numbers proceeds, which belongs properly to the discipline of arithmetic, can in no way be divided, because it is truly simple and is not composed of any parts.[57]

Victorius wrote for readers already familiar with the basic terminology and properties of numbers. He therefore passed over and compressed elementary concepts carried by terms such as "unity," "multitude," "number," "discipline," "arithmetic," and "division." Abbo, working as a grammarian and for beginners, held forth on each key term in turn.[58] He devoted roughly a tenth of the *Explanatio* to unpacking this one sentence—which in fact encapsulates much of the content of the first book of Boethius's *De arithmetica*. The result is a comprehensive introduction to number that ranges from the esoteric (e.g., the nature of the Trinity) to the mundane (e.g., divisions of liquid measure). Between these poles Abbo uncovered arithmetic's fundamentals: the definition and properties of unity, even and odd numbers, corporeality and incorporeality, fractions and their manipulation (division, multiplication, etc.), types of inequalities, tools used for calculation (tables, abacus, and fingers), numeric symbols, and units of measure.

In his preface, Abbo justified his endeavor in part, as we have seen, because he had been invited by the Fleury community to write a commentary, but also, as he stated, because he was concerned for the present state of education, particularly the scarcity of men intellectually and morally qualified to teach.[59] He laid out the moral stakes of mathematics in the next section of the *Explanatio*, which he subtitled "On Number, Measure, and Weight." In this mini-treatise, he contextualized the art of calculation, asserting that the purpose of

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56 Ibid.:"Sed quod ait commenti sunt, id est finxerunt, videtur sonare eos non ipsam veritatis naturam tenuisse; quia 'fingere' non solum conponere verum etiam quandam vultus simulationem solet praetendere. Hoc tamen loco *commenti sunt* significat 'invenerunt' vel 'excogitaverunt,' ut Terentius: 'Facite, fingite, invenite,' licet fictio, id est simulatio, veritatis sit imaginatio."

57 Victorius, *Calculus* Preface; ed. Peden, 3: "Unitas illa, unde omnis numerorum multitudo procedit, quae proprie ad arithmeticam disciplinam pertinet, quia vere simplex est et nulla partium congregatione subsistit, nullam utique recipit sectionem."

58 Abbo, *Explanatio* 3.1–4.

59 Ibid. 1.2

numeric study was to achieve wisdom, "the contemplation of unchangeable truths."[60] Paraphrasing Proverbs (9:1), Abbo explained that Wisdom's temple rested on the seven columns of the liberal arts and that their pursuit takes man from understanding the visible and corporeal to perceiving that which is invisible and incorporeal, that is, the divine. Indeed, philosophy, the love of wisdom, is tantamount to loving God, who *is* Wisdom. Clearly, the study of number was for Abbo a wondrous and transformative activity that engaged both body and soul. Of note is the care with which he distinguished between the sensible and intelligible. God, he tells us, did not create the universe from numerable, measureable, and weighable *things*; these were part of created order, changeable, and understood through the senses.[61] Number, measure, and weight, on the other hand, were utterly ineffable qualities perceived by the soul. The lessons of the *Explanatio* vacillated between these states.

After meticulously dissecting Victorius's short preface, Abbo unveiled the significance of the tables and lists that comprised, by far, the bulk of the *Calculus*. [62] Victorius, of course, addressed only the multiplication tables, which he himself had authored. The so-called additional tables were transmitted without introduction or explanation by their anonymous creators.[63] Abbo provided both, along with a great deal of supplemental information, much of it practical advice intended to facilitate use of the *Calculus* and served students in their larger arithemetic endeavors: these included an excursus on finger calculation and ruminations on the benefits of reciting and memorizing equations. Throughout the *Explanatio* he added graphic elements of his own—visual commentaries that show his awareness of the power of learning through the eyes. It is in these that we can observe Abbo's approach to visual pedagogy: the way that students were trained to cope with nonnarrative material.

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[60] Ibid. 2.1–15.

[61] Ibid. 2.14; ed. Peden, 71: "Anima videt mensuram, pondus et numerum per se, videt mensurabilia, penderabilia et numerabilia per corpus. Ergo per corpus videt quae sunt similia corpori, ide est corpora; incorporea vero per se, quibus est ipsa similis."

[62] Ibid. 3.1–100.

[63] That said, whereas Victorius's tables comprise only numbers, the additional tables include words and phrases that clarify the relationship between columns, such as *et* in the addition tables, *tolle*, in the subtraction table, and *in se* in the table of square numbers. Hence, though referred to as "tables" by contemporary scholars they are

3. LETTERS AS NUMBERS

Abbo began teaching arithmetic at the level of the letter. The challenge he faced and the strategy he adopted is clear in his analysis of Victorius's use of the terms *littera* and *nota*. After defining unity—that which is "truly simple" has "no parts" and "accepts no division"—and distinguishing it from things that are merely *called* "one" (e.g., *a* horse, *a* man, *a* day), Victorius moved from notion to notation, that is, from the concept of unity to its physical expression. He wrote:

> In this argument *unity* is called *assis*, parts of which [i.e., fractions] are themselves distinguished with their own names according to their proportionality. Also *notae* were invented to this end, through which these same names are expressed, so that by discrete names and the *notis* appointed to names, the knowledge of each fraction could be more easily obtained. And indeed *assis* [unity] (which is expressed through the letter i[64] just as it is customary to write "one" in the case of numbers) has xii parts.[65]

The *notae* that Victorius refers to—letter-like symbols for the duodecimal fractions—appear alongside Roman numerals in every column of his multiplication tables (figs. 3.2 and 3.5). Hence, their recognition and comprehension was essential to deciphering the *Calculus*.

In the *Explanatio*, Abbo first described and then pictured the *notae* matching the signs with their corresponding number. Hence the sign for $^1/_2$ was paired with **ii**, $^1/_3$ with **iii** and so on. The *notae* were generally integrated into the text. Thus in the Fleury manuscript Berlin 138, for example, each *nota* is flanked by *punctūs* and the corresponding Roman numerals, each also separated by *punctūs*, hover above them like an interlinear gloss (fig. 3.14).[66] Hence, the scribe treated *notae* for fractions as if they were numbers. By the central middle ages, Roman numerals were habitually set off from words in some way. A *punctus*—a sign of punctuation which generally indicated a pause and introduced a new thought in a verbal

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perhaps better understood as framed lists of equations in which each line is an equation the parts of which are aligned with the line above and below so they roughly form columns.

64 When letters are used in quotations to represent Roman numerals they will not be set off from the text in any way (unless I am quoting from a MS that includes punctuation). Individual letters in my narrative will be bold.

65 Victorius, *Calculus,* Preface; ed. Peden, 3: "In hoc argumento unitas assis vocatur, cuius partes iuxta proportionalitatem suam propriis sunt insignitae vocabulis. notis etiam ad hoc excogitatis, per quas eadem vocabula exprimantur, ut per discretionem nominum et notas nominibus affixas uniuscuiusque particulae notio facilius advertatur. Et assis quidem, qui per i litteram, sicut in numeris unum scribi solet, exprimitur, xii partes habet."

66 Berlin 138, fol. 15v; and Bamberg, Msc.Class.53, fol. 30.

narrative—came before and after a Roman numeral when one was used in a sentence.[67] The accompanying verbal description in the *Explanatio* would have made it clear just what kind of number—a fraction—was being represented. One would think that here Abbo's task of explanation was complete, it had, however, only just begun. The above-quoted statement served Abbo as *entrée* to a surprisingly lengthy and wide-ranging discourse on signs. A closer look at the quote as it appears in Bern 250 reveals the knotty problem Abbo faced and why he deemed a fairly thoroughgoing lesson in sign theory necessary (fig. 3.15). The passage can be transcribed thus: *Et assis quidem qui p(er) .i. litteram . sicut in numeris unum scribi solet exprimitur .xii partes habet.* The physical and grammatical juxtaposition of *litteram* and ***i*** makes the issue evident: in the era before the widespread use of Arabic numerals in Latin-speaking Europe, letters served as signs of numbers as well as words.

One of the most oft-quoted passages (then and now) on the power and function of letters comes from the *Etymologiae*. Isidore wrote:

> The common letters [of the alphabet] are the primary elements of the art of grammar, which [letters] scribes (*librarii*) and calculators (*calculatores)* follow. Indeed, letters are tokens of things, the signs of words, and they have so much force that the utterances of those who are absent speak to us without a voice ... The use of letters was invented for the sake of remembering things, which are bound by letters lest they slip away into oblivion.[68]

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[67] On the use of points to set off roman numerals Malcolm Parkes wrote, "As handwriting became more compressed in the twelfth century, and the space between words decreased in size, the *punctus* became the most common mark of punctuation. It was used to indicate all kinds of pauses, to introduce quotations, and to separate. In this last function it was used to prevent the false association of roman numerals with the letters which preceded and followed them." This "pointed" separation, standardized in the 12th c., was very old, seen, for example, in Roman inscriptions. By the end of the 10th c., techniques for marking numerals, which set them off from the text and made them more legible, became common on the continent. Long numbers were parsed by either spaces or points into smaller, more legible units, and the terminal **i** was elongated to a **j** in order to make clear where a number ended. These conventions were adopted, respectively from the British Isles and Spain. Parkes, *Pause and Effect*, 42; Karl Menninger, *Number Words and Number Symbols: A Cultural History of Numbers*, trans. by Paul Broneer (Cambridge, MA: MIT Press, 1969), 260; and Saenger, *Space between Words*, 135–36.

[68] Isidore, *Etymologiae* 3.3.1–3; ed. Gasparotto and Guillaumin; trans. Barney, 39: "Primordia grammaticae artis litterae communes existunt, quas librarii et calculatores sequuntur. Quarum disciplina uelut quaedam grammaticae artis infantia est; unde et eam Varro litterationem uocat. Litterae autem sunt indices rerum, signa uerborum, quibus tanta uis est, ut nobis dicta absentium sine uoce loquantur. [Verba enim per oculos non per aures introducunt]. Vsus litterarum repertus propter memoriam rerum. Nam ne obliuione fugiant, litteris alligantur. In tanta enim rerum uarietate nec disci audiendo poterant omnia, nec memoria contineri. Litterae autem dictae quasi legiterae, quod iter legentibus praestent, uel quod in legendo iterentur."

Isidore's statement has made it easy for modern scholars to focus on the function of letters as grammatical objects.[69] Likewise, those involved in the study of memory have welcomed his vivid account of writing as a mnemonic.[70] Such interpretations largely overlook the opening sentence: "letters," Isidore wrote, are followed by "scribes *and* calculators." The latter, *calculator*, derived from the verb, *calculare*, was not a common term in Isidore's time or after.[71] Victorius used it in the *Calculus* and Abbo followed suit, explaining that *calculus* was the diminutive form of *calco* meaning, "a tiny stone, with which the ancients, who did not yet know the science of numbers, calculated."[72] They were, in other words, the proto numeral. Elsewhere in the *Explanatio*, he wrote that *calculatori* were those "concerned" with "weight and measure," and glossed Victorius's *calculandi* with *numerandi*.[73] The point to make is that, though Isidore identified letters as grammatical elements, he also acknowledged their utility for those who work with number.[74]

In order to explain the *Calculus*, Abbo had to reestablish the disciplinary neutrality of letters, to reambiguate the alphabet. I say "reestablish" and "reambiguate" because letters—spoken and signed with the hand, as will be shown—were learned before words and thus before grammar. Letters as written objects in books and on wax tablets, were, in contrast, learned as parts of words, *with* grammar. The *Explanatio* must be understood as an introduction to *written* arithmetic. Thus one of Abbo's chief objectives was matching aural/oral and embodied number with what was written on the page—a point to which I shall return.

Abbo began the process of ambiguation by establishing letters as belonging to a much broader category of signs. The section of the *Explanatio* in which this is accomplished is worth quoting at length:

> *Notae* are signs of utterances, because, just as we use movements of the eyes and various motions of the hands in lieu of the voice in order to make our

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[69] See, for example, Irvine, *Making*, 29, 211–43, esp. 213–214.
[70] See, for example, Carruthers, *Book of Memory*, 107–21, esp. 111.
[71] Derived from the verb *calculo*, *calculare.* For use of the term see TLL 3.0.140.30–45.
[72] Abbo, *Explanatio* 3.41; ed. Peden, 98: "'Calculus' autem diminuitur a calco qui est lapis parvissimus quo antiqui necdum scientes numerare supputabant."
[73] Ibid. 1.3 and 3.42; ed. Peden, 65 and 98: "quaestio inesset de mensura et pondere, quae omnia calculatori sunt curae."
[74] Indeed, it should be remembered that the earliest known examples of writing—tally sticks and the clay impressions used for credit—were numerical. Denise Schmandt-Besserat, *Before Writing: From Counting to Cuneiform* (Austin: University of Texas Press, 1992), 1:184–94

desires manifest to someone, so letters and *notae* speak to the person who understands in some way. Each [*notae* and letters], as a matter of fact, are the sort of signs (*signa*) by means of which we often direct thoughts both to persons absent and to persons present, so that they carry the same affections to the minds of them exactly as if they had been utterances. Whence the most-wise orator, after he said, "he winketh with the eyes, presseth with the foot," added, "he speaketh with the finger" [Proverbs 6:13], since we express words or numbers by the articulated movements of the hand. We see some followers of the religious life, in the silence of their seriousness, refrain from this speech of fingers at the appropriate time, as if from the superfluous speaking of words. That point which is the beginning and end of all lines is also called a *nota* or sign (*signum*). We are accustomed to use such outward signing when we designate a particular man without saying his name by means of a spot or scar, which, for him, became thick from a wound. Rightly, therefore, he [Victorius] says that the *notas* of duodecimal fractions are expressed with names (*vocabula*) because while whatever *nota* is subject to intellectual vision it comes as a sound and already sounds in the heart's ears, for which it [the heart] is one and the same power/function to sift through the differences in a thing seen or heard.[75]

The debt to Isidore's passage on letters quoted above is evident. Abbo married this well-worn statement to the definition of *nota* by Boethius, for whom the term did heavy lifting. In his translation of Aristotle's *Peri herimeneias*, Boethius replaced the Greek "sign" (*semeion*) and "symbol" (*symbolon*) with *nota.* Boethius wrote, "spoken sounds are *notae* of *passiones* in the soul, and written marks *notae* of spoken sounds."[76] Thus *notae* and letters are closely related to speech; vehicles that make intangible notions manifest to the intellect.

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[75] Abbo, *Explanatio* 3.50; ed. Peden, 104: "Notae autem sunt signa vocum, quoniam ut nutu oculorum varioque motu manuum ad manifestandas alicui voluntates nostras pro voce utimur, sic litterae et notae intelligenti aliquo modo loquuntur. Utraque etenim talia signa sunt, quibus tam ad praesentes quam ad absentes persaepe cogitata dirigimus, ut eorum animis easdem passiones ingerant ac si voces fuissent. Unde spientissimus orator, postquam dixit: "Annuit oculo, terit pede," adiecit "digito loquitur," quoniam articulari motu manus verba seu numeros exprimimus. A qua loquela digitorum quosdam suae [seu] gravitatis taciturnitate religiosae professionis sectatores videmus cavere opportuno tempore, ac si superflua verborum locutione. Dicitur tamen nota vel signum etiam punctus, qui est principium finisve omnis lineae. Qua significatione solemus uti cum tacito nomine aliquem designamus macula aut cicatrice, quae illi occalluit ex vulnere. Bene ergo dixit notas unciarum vocabula exprimi, quoniam dum se quaelibet nota visui intelligentis subicit, vocalis venit ac iam in aure cordis sonuit, cui una eademque potestas est colligere differentias visae vel auditae rei."

[76] Boethius, *Commentarii in librum Aristotelis Peri hermēneias* 1.1; ed. Karl Meiser (Leipzig: Teubner, 1877), 2:25: "Sunt ergo ea quae sunt in voce earum quae sunt in anima passionum notae, et ea quae scribuntur eorum quae sunt in voce." For more on Boethius's use of the term see John Magee, *Boethius on Signification and Mind*, Philosophia antiqua 52 (Leiden: Brill, 1989), 56–63; and Taki Suto, *Boethius on Mind, Grammar, and*

Notae as described by Abbo have an active almost animate quality. Indeed, the relationship between the nominal form of the word (*nota*) and its verbal cognates (*notare*, *novi*sse, and *noscere*) would have been readily apparent to readers and was often highlighted by authors. Boethius, in his commentary on the *Topica* of Cicero, a difficult dialectical text that was available at Fleury and which Abbo studied, defined *nota* as "that which designates any thing. Hence, every name (*nomen*) is a *nota* because it makes known the thing **(***notam facit rem*) of which it is predicated."[77] Similarly, Isidore claimed that signs used for shorthand "are called *nota*e because they designate (*notare*) words and syllables by predetermined characters and recall them to the knowledge (*notitia*) of readers."[78] Thus *notae* as defined in the *Explanatio* are sensible expressions that promote knowing by working directly on the mind and memory, making them an ideal medium for teaching. Likewise letters. Both were, in Abbo's words, "great signs" (*utraque etenim talia signa sunt*). It is of note that Abbo did not elide *litterae* and *notae*. Rather, he aligned them (*litterae et notae*). The reason for maintaining a distinction can only be guessed at, but it probably reflects Victorius's use of the terms; in the *Calculus* signs for fractions are *notae*, while the characters that comprise Roman numerals are referred to as *litterae*.

Abbo included both letters and *notae* in a capacious category of meaningful marks the bounds of which were broad enough to encompass the telltale scar or birthmark of a man as well as the points that designate a line. All these, we are told, should be classed as "conventional" signs:

> None of these [signs] not an utterance (*vox*), nor letter (*littera*), nor point (*signum*)*,* nor *nota* is according to nature, but rather all [are signs] according to convention by the customs of whatever peoples or inhabitants of diverse regions.[79]

Logic: A Study of Boethius' Commentaries on Peri hermeneias, Philosophia antiqua 128 (Leiden: Brill, 2012), esp. chapter two.

[77] Boethius, *In Ciceronis topica* 8.35–37; ed. J. C. Orelli and G. Baiterus, in *Ciceronis opera* (Zurich: Fuesslini, 1833), 5:336: "Nota vero est, quae rem quamque designat. Quo fit, ut omne nomen nota sit, idcirco, quod notam facit rem, de qua praedicatur."

[78] Isidore, *Etymologiae* 1.22.2, ed. W. M. Lindsay (Oxford: Clarendon Press, 1911); trans. Barney, 51:"Notae autem dictae eo, quod uerba uel syllabas praefixis characteribus notent et ad notitiam legentium reuocent." The pages of Lindsay's edition are not numbered. The new Belles Lettres edition and translation of Book One, is not yet available.

[79] Abbo, *Explinatio* 3.50; ed. Peden, 105: "Ceterum nec vox, nec littera, nec signum, nec nota est secundum naturam, sed magis omnia secundum quod convenit moribus quarumlibet gentium aut habitatoribus diversorum locorum."

Abbo had certainly read Augustine's *De doctrina christiana*, the classic work on what a Christian might draw on from liberal arts to inform reading of scripture.[80] He thus had access to the most thoroughgoing explanation of "natural" (*natura*) and "given" (*data*) signs available to the middle ages.[81] Augustine had written:

> Some signs are natural, others given. Natural signs are those which without a wish or any urge to signify cause something else besides themselves to be known from them ... Given signs are those which living things give to each other, in order to show, to the best of their ability, the emotions (*motus*) of their minds, or anything that they have sensed or perceived.[82]

No doubt Abbo had this passage in the back of his mind as he commented on the *Calculus.* But if Augustine's definitions were in the background, a passage by Boethius, found in the *De arithmetica*, was at the forefront, and takes us to Abbo's ruminations on specifically numeric signs.

Midway through the *De arithmetica*, Boethius reminds his reader of the artificiality of Roman numerals: "*nature* does not arrange the figure of five, v, or of ten, that is x, and other figures of this type which we write; rather *custom* fixes them."[83] He announces his intention to avoid using Roman numerals in the subsequent section of the *De arithmetica*. When he "wishes to demonstrate five" he will instead "make five strokes ... drawn in this way: iiiii." Boethius contends that this method was "more natural than little notes (*notulae*)" because "the sign is comprised of as many elements as the quantity expressed." In the *Explanatio*, we find Boethius's statement turned on its ear:

> When the ancients wished to mark a number, however small, [they] often wrote a letter for however many unities in that number. [A system] which later

[80] Abbo quoted the *De doctrina christiana* in the *Explanatio* (2.17).
[81] Augustine, *De doctrina christiana* 2.1–11 (esp. 2–3), ed. and trans. R. P. H. Green (Oxford: Clarendon Press, 1995), 58–63.
[82] Augustine, *DDC* 2.2–3, ed. Green, 57 (with slight adjustments to Green's translation): "Signorum igitur alia sunt naturalia, alia data. Naturalia sunt, quae sine voluntate atque ullo appetitu significandi, praeter se aliquid aliud ex se cognosci faciunt... Data vero signa sunt, quae sibi quaeque viventia invicem dant ad demonstrandos, quantum possunt, motus animi sui, vel sensa, aut intellecta quaelibet." Augustine preferred the term *signa* to *nota* for sign. He only uses *nota* once in the *DDC* and that is to reference shorthand (2.26).
[83] Boethius, *De arithmetica* 2.4.3, ed. Guillaumin, 89; trans. Masi, 128: "Ut enim quinarii subiectam notulam fingant de v, uel denarii quam descripsimus, id est de x, et alias huiusmodi, non natura posuit, sed usus affinxit."

> men, wearied by the tedious length, improved, and for five or ten unities [they] began to write v or x, and l or c for fifty or one hundred.[84]

In this compact history of Roman numerals, Abbo describes a primitive mode of numerical notation, closely tied to counting, that was used by "the ancients," who represented one by a single mark, two by two marks, and so on. In his narrative, the system proved so tiresome that it was abandoned in favor of a more streamlined approach that replaced groups of ones with single symbols: letters of the alphabet. The echoes of Boethius are evident. Both authors acknowledge the tedium of making a mark for every unity in a number, but they then part ways. Boethius shrugs off the extra labor, deeming the ancients' system more "natural" and, for this reason, better. Abbo, on the other hand, stresses the utility of Roman numerals, which, of course, populate Victorius's tables. It is, to be sure, difficult (if not impossible) to imagine the tables being of any use without a shorthand method for writing numbers.[85] Thus in taking a pro-Roman numeral stance, he does the practical work of preparing his audience to navigate the tables.

Abbo goes further, not only favoring the use of Roman numerals, but also stressing that they are *litterae*. He introduces the symbols for five, ten, fifty and one hundred in the Latin system—**i**, **v**, **l**, and **c**—calling them, as Victorius had, "letters" (*litterae*).[86] He even applies the term to the simple, vertical marks made by "the ancients."[87] This is in contrast to Boethius, who referred to Roman numerals as "little signs" (*notulae*). It also departs strongly from Isidore's account of Roman numerals. In the first book, *De grammatica*, of the *Etymologiae*, which Abbo referenced with some frequency, Isidore specified that "All the letters in Greek compose words and also make numbers ... Latin speakers, however, do not assign numbers to the letters, but only use them to form words, with the exception of the letters i and x."[88] Here, Isidore contrasts the Greek and Latin use of letters as numeric signs,

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[84] Ibid.: "antiqui quemcumque numerum subnotare voluissent, totiens litteram scribebant, quot unitates in numero illo fuissent. Quod posteriores longitudinis taedio affecti correcerunt et pro quinque vel decem unitatibus v vel x scribere coeperunt et pro quinquaginta seu centum l vel c."

[85] Indeed, for us even Roman numerals seem cumbersome.

[86] He mentions **d**, for 500, a little further on in the passage, but omits **m**, 1000, probably because it had fallen out of common use since the Roman period, and had been replaced by an **x** or an **i** with a line over it. The latter convention is used in Berlin 138.

[87] Boethius, in contrast, called these marks *virgulae* (*De arithmetica* 2.4.3).

[88] Isidore, *Etymologiae* 1.3.10–11; ed. Lindsay; trans. Barney, 40: "Omnes autem litterae apud Graecos et uerba conponunt et numeros faciunt... Latini autem numeros ad litteras non conputant, sed sola uerba componunt, excepto I et x littera, quae et figura crucem significat et in numero decem demonstrat."

opposing Athens with Rome. Abbo, on the other hand, though he acknowledged a difference, chose instead to emphasize the similarities between the two systems. He praised the Greeks "before whom nothing was done with numbers," for sensibly matching each number with a letter and then stated, "the Latins use letters for numbers, that is two vowels (*vocales*) i and v, one double (*duplicem*) x, a mute (*mutam*) d, and a liquid (*liquidam*) l."[89] "Vowel," "double," "mute," and "liquid"—these terms would have evoked the standard classification for letters according to their three essential qualities: name, shape, and function. To us this scheme, relating to the aural/oral quality of letters, may seem unequivocally grammatical. Yet one gets the sense from Abbo's application of the terms to Roman numerals (in, moreover, a treatise on arithmetic) that this was neutral information, almost extra-disciplinary.

In the encyclopedic setting of the *Etymologiae*, letters inhabit a sort of disciplinary no-man's land between the more general discussions of learning and those sections focused exclusively on grammar.[90] The chapters devoted to the Hebrew, Greek, and Latin alphabets immediately follow an overview of wisdom and liberal arts. (The terms used by Abbo are encountered in Isidore's description of the Latin alphabet.[91]) Only *after* these chapters does Isidore turn his attention to grammar *per se*. Given this, we might see Abbo's classification of Roman numerals according to the sound of the letter as less grammatical, more technical. Indeed, the most common so-called grammatical element that was added to computus manuscripts was the passage from Martianus's *De nuptiis* in which lady *Grammatica* instructs her audience on how each letter of the alphabet should be pronounced. "A," she declares, is uttered "with the mouth open, with a single suitable breath."[92] Thus, by including

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[89] Abbo, *Explanatio* 3.51; ed. Peden, 105: "Sed et Graeci suis litteris vel caracteribus id maxime faciunt, apud quos nulla numero vacat, cum Latini tantum quinque habeant, quas pro numeris simplices aut crebro repetitas sriptitant, id est duas vocales i et v, unam duplicem x, unam mutam d, et unam liquidam l."

[90] The order of the *Etymologiae* may be the work of Braulio rather than Isidore. On the state of the *Etymologiae* at Isidore's death and Braulio's role as redactor see chapter 1.

[91] Isidore's descriptions shed light on Abbo's terminology: "Vowels are so called because they make a complete 'vocal sound' (*vox,* gen. *vocis*) on their own"; "**x** is called a double letter, because it is used for **cs**"; "the mutes (*mutus*, i.e. the voiced stops) are so called because, without vowels joined to them, they are never released"; and "in Latin there are two sounds, **l** and **r**, which melt (*liquescere*)." Isidore, *Etymologiae* 1.4.3–8, 28–29; ed. Lindsay; trans. Barney, 40–41.

[92] Martianus, *De nuptiis* 3.261; ed. Willis, 68; trans. Stahl, 75: "Namque a sub hiatu oris congruo solo spiritu memoramus." The passage follows an extended discussion of letters and their combination. Its placement is somewhat similar to Isidore's discussion of letters, i.e., before *Grammatica* speaks of literacy and reading, that is grammar in earnest. Faith Wallis named a few examples of computus manuscripts that include the passage, among these: Munich, BSB Clm 14456, fol. 63v (9th c.); Paris, BnF lat. MS 12117, fol. 146v (11th c.); and London, BL Cotton Vitellius MS A.XII, fol. 44v. The first manuscript is from St. Emmeram, Regensburg, the

the voiced names of letters used for numbers, Abbo returned them to a pre-grammatical, extra-disciplinary state.

Letters, oral or written, were building blocks, for scribes and calculators, the trivium and the quadrivium, and this accounts in part for the initially curious fact that alphabets were a standard feature of computus manuscripts. In Bern 250, for example, a full Latin alphabet written in elegant, classicizing majuscule letters three-lines high fills the space between Victorius's *Calculus* and Abbo's own *Computus* (fig. 3.16).[93] One bifolium in Bern, Burgerbibliothek, Cod. 207, a compendium of grammatical and arithmetic material from Fleury, contains a host of exotic alphabets followed by a brief tract by Bede known as the *De computo vel loquela per gestum digitorum* (*On Calculating or Speaking with the Fingers*), which circulated as the first chapter of the *De temporum ratione* and—just as often—on its own (figs. 3.17–19).[94] The title is somewhat misleading since Bede gave instructions for counting, not calculating, though for reckoning involving large numbers an intermediate term might be "held" on the hand while further calculation was done in the head.[95] The habit of counting and calculating with the fingers is first documented in Roman sources, but the practice may be much older.[96] It was from Bede's work that Abbo and his contemporaries learned how to sign numbers from 1 to1,000,000. Bede assigned each number a unique finger

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second from Saint-Germain-des-Prés, and the third from Salisbury. The latter two include works by Abbo, suggesting some association with Fleury. Wallis, "Calendar and Cloister," McGill University, http://digital.library.mcgill.ca/ms-17/folio.php?p=159v&showitem=159v-175r_13GrammarOverview (accessed October 23, 2014).

[93] See also Bern, BB Cod. 417, fols. 94r–99r (France?, 9th c.); and Paris, BnF lat. MS 5239, fols. 235r–36r (St. Martial, Limoges; mid-10th c.).

[94] Charles Jones wrote that "manuscripts containing a part or whole of chapter one, divorced from a complete text of *DTR* [*De temporum ratione*], are too numerous to list." The isolated reproduction of this chapter occurred quite early on; it stands alone, for example, in a late 8th-c. manuscript in Beneventan script (Paris, BnF lat. MS 7530). Jones, *Bedae pseudepigrapha: Scientific Writings Falsely Attributed to Bede* (Ithaca: Cornell University Press, 1939), 22–3.

[95] For a discussion of this omission see Faith Wallis in Bede, *The Reckoning of Time*, trans. Faith Wallis, Translated Texts for Historians 29 (Liverpool: Liverpool University Press, 2004), 254–61.

[96] Augustine referred to finger calculation with some frequency, especially in his sermons, assuming knowledge of it on the part of his audience. Indeed, some mention of the practice can be found in the works of most of the Church Fathers. Bede shows himself aware of this fact. In defense of the practice he wrote: "For one ought not to despise or treat lightly that rule with which almost all the exegetes of Holy Scripture have shown themselves well acquainted no less than they are with verbal expressions." (*De temporum ratione* 1). A statement by Augustine is particularly interesting in the context of the *computus*: "Truly he [God] relaxes the fingers of all who calculate concerning this subject [the end of time] and orders them to be quiet, who says: it is not for you to know the time which the father has placed in his own power." Augustine, *Civitas Dei* 18.53; as cited in Burma P. Williams and Richard S. Williams, "Finger Numbers in the Greco-Roman World and the Early Middle Ages," *Isis* 86, no. 4 (1995): 596.

position. "When you say 'one,'" he instructed, "bend the little finger of the left hand and fix it on the middle of the palm."[97] Two was formed in the same manner with the ring finger; ten by touching the nail of the index finger to the middle joint of the thumb. The left hand represented units and tens, the right hundreds and thousands. Larger numbers involved rather gymnastic gestures with the arms; there is little evidence for their use. Bede's verbal descriptions were sometimes supplemented with pictures of hands demonstrating each figure (fig. 3.20).[98] It was the visible aspect of the practice that appealed to Abbo. He urged readers of the *Explanatio* to calculate with the fingers because it put numbers on the hand in front of the eyes—literally making its logic clearer.[99]

In the passage from the *Explanatio* quoted earlier, speech includes certain physical gestures: "movements of the eyes" and "motions of the hands." The inclusion reflects, I think, dominant modes of communication at Fleury, where sign language was often preferred in certain parts of the abbey complex and during periods of strict liturgical silence.[100] The Cluniac reforms that shaped Fleury in the tenth century included strict enforcement of monastic silence. Communal living made some communication necessary, but words, especially spoken words, were avoided. At Cluny, hand signs, memorized by novices before they were ordained as monks, made such silence possible.[101] The earliest documentary evidence that hand signs were used at Fleury is a lexicon of 154 signs dated to 1078.[102] The list of signs is so much like that of Cluny that it is assumed that the system of silent communication had been introduced by Odo when he reformed Fleury in 938.[103] Abbo's statement that, "we express words or numbers by the articulated movements of the hand" suggests that hand signs were used at Fleury in the 980s, when he wrote the *Explanatio*.[104]

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[97] Bede, *De temporum ratione* 1, ed. Charles W. Jones, in *Opera de temporibus*, Mediaeval Academy of America 41 (Cambridge: MAA, 1943), 179; trans. Faith Wallis, in Bede, *The Reckoning of Time*, 10: "Cum ergo dicis unum, minumum in laeva digitum inflectens, in medium palmae artum infiges."
[98] Jones lists a number of manuscripts that included pictures. Jones, *Bedae Pseudepigrapha*, 54–55.
[99] Abbo, *Explanatio* 3.67; ed. Peden, 114: "Quorum omnium planior erit expositio, si multiplicatorum repetatur a minimis progressio, perspecto quoniam digitis praefixa multiplicationis ratio convenit."
[100] Scott G. Bruce, *Silence and Sign Language in Medieval Monasticism: The Cluniac Tradition c. 900–1200* (Cambridge: Cambridge University Press, 2007), 61–62.
[101] Evidence for when the sign language was learned is largely anecdotal. In the *Vita Odonis*, for example, a child signs permission to go to relieve himself in the middle of the night. John of Salerno, *Life of St. Odo* 1.33; ed. and trans. Gerard Sitwell (New York: Sheed and Ward, 1958), 34.
[102] Bruce, *Silence*, 105.
[103] Ibid., 103.
[104] Abbo, *Explanatio* 3.50; ed. Peden, 104: "quoniam articulari motu manus verba seu numeros exprimimus."

The Fleury lexicon overwhelmingly comprises nouns: food, apparel, liturgical objects, and the various titles held by monks in the monastery. There are a few very common verbs: read, sing, talk, hear, etc. Finger numbers are not specified, likely because they had already been well described by Bede.[105] It is assumed that they were interwoven into the Cluniac lexical system and used to express anything quantitative and, possibly, to spell out unusual words or proper nouns for which no sign existed.

Bede described how the finger numbers could be used for silent speech in the *De loquela*. A letter was matched to each of the first twenty-three numbers, according to the order of the alphabet (e.g., a=1, b=2). Like any alphabet, these alphanumeric hand signs could be strung together into words and phrases. By way of example, Bede suggested that one might "warn a friend who is among traitors" by signing the numbers 3-1-20-19-5 and 1-7-5, thus silently telling them to *caute age* ("act cautiously").[106] Likely it was for this reason that the *De loquela* was added to the pages of alphabets in Bern 207. Hence, though the primary method of *recording* numbers at Fleury and in Latin-speaking Europe was *not* an alphanumeric system (as in Greek or Hebrew), such a system *did exist*, was well known and probably learned at a young age, alongside or even prior to Roman numerals. Thus it was in an embodied form rather than written one that numbers were first encountered: shaped with the mouth and the hand. Transferring or, perhaps better, extending this knowledge to written signs was among the tasks of teaching arithmetic. How this was done is best demonstrated by one of Abbo's students, Byrhtferth of Ramsey.

We know from Byrhtferth that Abbo taught grammar and computus in England during his brief tenure at Ramsey.[107] Hence, his *Enchiridion* or "handbook" to computus—an

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105 On the "thin" documentary evidence for monastic sign language see Bruce, *Silence*, 100–101.
106 Bede, *De tempore ratione* 1; ed. Jones, 181; trans. Wallis, 11: "Verbi gratia, si amicum inter insidiatores positum ut caute se agat admonere desideras, iii, et i, et xx, et xix, et v, et i, et vii, et v, digitis ostende; huius namque ordinis literae, 'caute age' significant." Abbo, *Explanatio* 3.50. Interestingly, Thierry in the *Consuetudines* mentions a gesture of reproof enacted by the *circators* when they caught someone speaking. Bruce, *Silence and Sign Language*, 104; Thierry of Fleury, *Consuetudines* 10, ed. Anselme Davril and Lin Donnat, in *L'Abbaye de Fleury en l'an mil*, 186.
107 Byrhtferth, *Vita Oswaldi* 3.17; ed. Lapidge, 90–91: "Philosophus Abbo veniens ex Floriaco glorioso arcisterio ... atque doctrinam grammaticae artis affluenter suos erudiens discipulos." Byrhtferth stated that he learned computus from Abbo in his *Proemium* to Bede's *De temporibus*, which is thought to have been a part of his *Computus.* He wrote, "It is fitting to direct my pen to a summary of the entire book, since after its conclusion you have some writings of the scholar Abbo, an alumnus of St Benedict [i.e., of Fleury], through whose kindness I received my understanding of this subject [computus] as well as my knowledge of other things." Byrhtferth, *Computus*; ed. and trans. Baker and Lapidge, Appendix A, 379.

aid to teaching time reckoning and number—can serve as an indirect witness to pedagogy at Fleury. In a substantial section devoted to signs of all sorts including letters, Byrhtferth introduced the alphabets with characteristic flourish:

> Now we will reveal to rustic priests the mysteries of the letters according to the learning of the Romans. First we will write them down together, and afterwards we will show their divisions in the manner that writers have and hold, and we will also wrap up separately the letters that the gracious priests have in their counting, and then we will expound the Hebrew and Greek alphabets. And we intend to make known the numerical value of the letters, for we know that it may be of use.[108]

In Oxford, Bodleian Library Ashmole MS 328, the most complete extant copy of the *Enchiridion*, three tables follow.[109] The first contains a number of narrow columns with the Latin and Anglo-Saxon alphabets (fig. 3.21). Beside these is a much broader and more prominent column labeled in Old English: "These are the letters in Latin counting."[110] Under the title are the letters **i**, **v**, **x**, **l**, **c**, **d**, and **m**; next to each numeral the name of the number was written in Old English. Hence the first line reads: "i. means 'one'," the second, "v. means 'five'," and so on.[111] The list continues with short lines (equations really) that begin to show how the "counting" letters were paired and grouped to form numbers not designated by a single letter:

> i. and v. six[112]
> x. and i. eleven
> x. and l. forty
> l. and [x.] sixty
> x. and c. [ninety]
> d. and c. six hundred
> duo .cc. two hundred
> .ccc. th[ree] h[undred]

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[108] Byrhtferth, *Enchiridion* 3.3.330–37; ed. and trans. Baker and Lapidge, 184–87: "Heræfter we wyllað geopenian uplendiscum preostum þæra <stafena> gerena æfter Lydenwara gesceade. Ærest we willað hig amearkian togædere, and syððan heora todælednyssa we willað gekyðan on þa wisan þe þa boceras habbað and healdað, and eac we willað þa stafas onsundron gewriðan þe þa esfullan preostas on heora getæle habbað, and þæræfter Ebreiscra abecede we willað geswutelian and Grecisra. And þæt getæl þæra stafena we þencað to cyðanne, forþon we witon þæt hyt mæg fremian."
[109] On Ashmole 328 see chapter 4.
[110] Byrhtferth, *Enchiridion* 3.3; ed. and trans. Baker and Lapidge, 186, fig. 26.
[111] Ibid.
[112] It should be remembered that, at this time, four was formed with four strokes, thus: iiii.

.cccc. f[our] h[undred]
d. and cccc. nine hundred
m thousand[113]

It is a remarkably compact lesson in the Latin number system devised for someone—a "gracious priest"—able to read Old English and who might need to identify Roman numerals. The more important lesson—essential to computus and which would carry over to the next two tables—is the shifting identity of letters from sound to number, and number to name.

The second, most lavish of the tables holds the sacred alphabets: Latin, Greek, and Hebrew (fig. 3.22). In Ashmole 328, three semicircular arches surmount the table and contain headings that read from left to right: "the alphabet of the Latins," "[the alphabet] of the Greeks with numbers," and "[the alphabet] of the Hebrews with meanings."[114] Framed together and set in parallel columns, the table shows the difference between the diverse alphabets as well as their equivalence. The round arches would have brought to mind the Eusebian canon tables, which demonstrated the concordance of the four gospels. Byrhtferth, as promised, aligned the Greek alphabet with the corresponding Roman numerals in the adjacent column. Numbers were thus treated as a kind of alphabet and given pride of place at the table's center. The majority of the Greek letters were spelled out in Latin as they would be pronounced (e.g., *alpha*, *beta*, *gamma*); a handful were represented by approximations of the Greek letter-forms followed by their names (again, spelled out in Latin). Likewise the Hebrew letters were written as they would be pronounced (transliterated into Latin) and were presented *cum significationibus*: one (*aleph*) means "doctrine" (*doctrina*), two (*beth*) "home" (*domus*), three (*gimel*) means "many" (*plenitudo*). Thus stress is laid on sound and sense—numeric and symbolic.

The final and most extensive table contains Roman numerals from 1 to 100,000 (figs. 3.23–24). Here Byrhtferth named numbers in two languages. He paired the Roman numerals from one to twenty-two with the Latin ordinal and Old English cardinal, the rest were written only in Old English. The table would allow students to recognize numbers as either words or

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[113] I have adapted Baker and Lapidge's English translation to show the missing elements (in parenthesis). Byrhtferth, *Enchiridion* 3.3; ed. and trans. Baker and Lapidge, 186–87, fig. 26: "i. and v. syx x and i. endlufon x and l. feowertig l and <x.> syxtig x and c. hund <nigontig> d and c. syx hundred duo .cc. twa hundred .ccc. þ<reo> h<undred> .cccc. f<eower> h<undred> d and cccc. nigon hundred m̄. þusend."
[114] Ibid.; ed. and trans. Baker and Lapidge, 188, fig. 27: "Abecedarium Latinorum Grecorum cum numeris Hebreorum cum significationibus."

numerals and to speak them: to learn to pronounce **x** as *decimus* ("tenth") or as *tyn* ("ten"). At the bottom of the table four equations are distributed across the four columns:

> Ten ten times multiplied, one hundred makes
> One hundred ten times multiplied, one thousand makes
> One thousand ten times multiplied, a myriad completes
> Twenty-two thousands, two myriads make.[115]

The lines, like the tables, capture the ephemeral practice of speaking number. In promoting this practice, Byrhtferth was standing solidly in the tradition of Abbo, who recommended internalizing numbers and equations by giving them voice. In his explanation of one of the more elementary additional tables of Victorius's *Calculus*, he stated that the table was best memorized through "spoken recitation" (*decantare*).[116] It is, in fact, the only table in Bern 250 made up entirely of words rather than numbers—all the numbers are spelled out—thus facilitating its speaking (compare figs. 3.6 and 3.25). Abbo's choice of words—*decantare*—emphasizes the singsong and repetitive nature of the task, seen also in Byrhtferth's verses.

For Abbo and Byrhtferth the alphabet proved to be an incredibly flexible sign system. Learning to cope with this flexibility was not easy. Both men fostered an awareness of the arbitrary and protean nature of a particular sign at the elementary level. Abbo's computus—an assemblage of calendars and tables for calculating the date of Easter and the moveable feasts—includes particularly striking examples of the kinds of tools devised for this purpose.[117] The knowledge that letters could serve multiple functions was particularly essential for navigating computus material. In the hands of Abbo, the calendar and its associated tables became ever more complex and sophisticated; he often employed several notational systems—Roman numerals, the Greek and Latin alphabets, and adaptations of these alphabets (e.g., various colors, scripts)—at once.[118] He enjoyed playing with the

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[115] Ibid. 3.3.422–25, ed. Baker and Lapidge, 194. In this instance the translation is mine. I have tried to translate these equations in a manner that maintains the terminal rhyme. The following transcription is taken directly from Ashmole 328: "Decem decies multiplicati centum faciunt / Centum decies multiplicati mille faciunt / Mille decies multiplicati miriandam perficiunt / Viginti duo milia duas miriadas faciunt."
[116] Abbo, *Explanatio* 3.90; ed. Peden, 125–26: "Quibus instructi absque ulla difficultate valent memoriter decantare
quid unicuique summae subtractum, quid additum, quid relictum sit, et omnino omnium numerorum causas augmenti aut detrimenti, seu per se <seu> ad se invicem teneanta relationem proportionalitatis."
[117] Abbo's computus is discussed and described in greater detail in chapter 4.
[118] Faith Wallis wrote, "Until the early twelfth century, Greek letter-numbers were not uncommonly used in *computus* tables, in preference to, or in parallel to Roman numerals. Why the designers of these tables preferred Greek numerals is not altogether clear. A plausible explanation is that they are more compact, and therefore

different systems, creating redundant twin tables that existed side by side, in which the only difference was that one alphabet had been substituted for another (fig. 3.26). The effect is a powerful visual lesson in semiotics and the slippery nature of signs.

Abbo's *Ephemerida*, an acrostic poem that takes the form of a computus table, is perhaps the most dramatic example of this kind of object. In Bern 250 the poem opened Abbo's computus. Situated on the threshold to the study of number it served as a point of contact between reading and reckoning.[119] The acrostic was set out in a square grid of thirty by thirty compartments, each holding a character (fig. 3.27).[120] Hence, each of the thirty lines of verse was thirty letters long, dimensions that reflect the approximate length of the lunar cycle.[121] Reading left to right, top to bottom yielded a poem describing the movements of the heavens. A characteristic feature of acrostics, the *versus intexti*, embedded poems or phrases, take a most unusual form in the *Ephemerida.* In Bern 250 the first, sixth, twelfth, eighteenth, and twenty-fourth columns are physically distinguished from the rest—bounded with red minum lines—and certain letters within these columns are larger and highlighted in colored ink (fig. 3.28). Rather than spelling out a second poem or message, the contents of these columns could be used in conjunction with the calendar and tables to which they were keyed to determine a wealth of computistic information.[122]

Instructions that typically accompanied the poem explained how it worked.[123] The sixth column, for example, could be used to find the present year in what was known as the 19-year cycle. This cycle, spanning, as its name suggests, nineteen years was a tool for

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easier to inscribe into the restricted and inelastic spaces of a grid than are Roman numerals." To this I would add that it may also be residual and have to do with the fact that the Latin computus and calendar tradition is rooted in the Greek. Wallis, in Bede, *The Reckoning of Time*, 263; on the Greek origins of computus tables see Jones in Bede, *Opera de temporibus*, 34–77.

119 Not unlike the *Explanatio*. Indeed the two works were sometimes paired. For a list of manuscripts containing the poem see Michael Lapidge and Peter S. Baker, "More Acrostic Verse by Abbo of Fleury," *The Journal of Medieval Latin* 7 (1997): 3.

120 Lapidge and Baker note that it is not an acrostic in "the strictest sense of the word." Ibid., 2.

121 The cycle would be 29.5 days in a month, but—for the sake of simplicity—alternated from month to month between 29 and 30 days.

122 See Ibid., 4–6.

123 Two versions of these instructions circulated with the *Ephemerida*: a brief rather cryptic explanation known as the *Quadratus hic equilaterus*; and a longer and more detailed explanation, the *Quoniam brevitatem*. The first was more common in copies of Abbo's *Computus* produced on the continent. The second, thought to have been written after the *Quadratus hic equilaterus* in order to make the acrostic more accessible to a broader, less accomplished audience, is found in most copies of the *Computus* that originated in England as well as computus manuscripts associated with Byrhtferth. Lapidge and Baker edited both versions. Ibid., 3, 16–21.

predicting the age of the moon on any given day in a year—essential information for determining the date of Easter.[124] Abbo's directions for the column's use were explicit:

> If you wish to find out the year of the 19-year cycle, with this same letter "A," [written in green] count on your fingers up to the letter that stands for the epact of that year [the age of the moon on 22 March], and however many spaces from "A" you find it, that will be the year of the 19-year cycle.[125]

The actions here described by Abbo are strikingly different from those typically involved in reading. The eye moves up and down the column counting the spaces, and even the fingers get involved. Letters do double duty. The green "A" in the sixth column begins the word *aequali* in the line *Hic semper serus, haec est velocior, etsi ambos aequali cursu ciet unius ordo* ("He is always late, she is swifter, even though the order of the one propels them both on an equal course") and, at the same time, marks the starting point from which one would count to find the number of the year in the nineteen-year cycle. The letter's meaning and function depended not on placement, but how it was viewed: whether it was read horizontally, as word, or vertically, as number. Here, grammar and arithmetic were seamlessly interwoven. Diverse ways of looking and, with this, thinking together yielded a more comprehensive body of knowledge about the moon.

4. Making space for arithmetic

In the instructions to the *Ephemerida* Abbo told viewers to "cast the eyes around" (*circumfer oculos*) to find the age of the moon.[126] This kind of directive, promoting new ways of looking at and investigating the page, was, as we shall see, regularly encountered in quadrivial works. Understanding numeric elements in manuscripts—Victorius's multiplication tables, calendars and computus tables, and the so-called *demonstrationes* and *formulae* of the *De*

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[124] Bede describes the 19-year cycle in the *De tempore ratione*: "Eusebius ... first devised the sequence of the 19-year cycle in order to find the fourteenth Moon of the Paschal feast and the day of Easter itself [typically, the Sunday following the fourteenth moon] because after this period of time a Moon of a given age will recur on the same day of the solar year." The use of such a cycle, which helped reconcile the solar calendar with the lunar cycle, allowed the date of Easter and the other moveable feasts to be set in advance. Bede, *The Reckoning of Time*, 121.

[125] Abbo, *Quoniam brevitatem*; ed. and trans. Lapidge and Baker, in "More Acrostic Verse," 18: "Si uero annum cycli decemnouenalis quaeris cum ipsa littera .A., in digitis supputa usque ad litteram quae illo anno pro epacta ponitur, et quoto loco ab .A. remotam reppereris, talis erit annus cicli decemnouenalis."

[126] Ibid. The directive was not out of the ordinary when dealing with computus tables. Bede used the same expression. See, for example, Bede, *De tempore ratione* 19; ed. Jones, 220.

arithmetica—demanded habits of viewing and thinking markedly different from those cultivated through the study of grammar. As remarked at the beginning of this chapter, students would have approached the page ready to read; and, in the central middle ages, the pages of books, regardless of subject, were increasingly shaped in a manner that encouraged this response. Masters at Fleury and elsewhere developed and inculcated strategies—some visual—to help students make sense of sentences. A brief excursus on grammatical tactics that were promoted by Abbo and his contemporaries will show some of the assumptions that were brought to bear on books—the kind of rationale that had to be either co-opted or disrupted for numerical study.

One skilled in the art of grammar approached lines of text on the manuscript page, any manuscript, even those devoted to quadrivial topics, with techniques of *lectio* ("reading") and *ennaratio* ("understanding" and "interpretation") at the ready. This was true of the original scribes and later readers as well, who, in the closed system of the monastery or cathedral, were often one and the same. *Lectio* began with the identification of syllables and their accenting, which was especially important for poetic texts.[127] Syllables were assembled into words, which in turn needed to be comprehended. Glosses helped with this task. Synonyms, often more than one per term, were the most common interlinear addition. They simplified difficult texts and served to build vocabulary. Once each word was understood, lines of text could be parsed and punctuated into syntactic, semantic, and metrical units, a process known as *discretio* ("separation"). *Discretio* involved *continuatio*, understanding the syntax of words in combination, and *separatio*, isolating and understanding the syntax of individual words that might be ambiguous.

The application of the techniques of *lectio* and *ennaratio* are evident, for instance, in a tenth-century copy of the *De arithmetica* produced in and owned by Fleury, Paris, Bibliothèque nationale de France lat. MS 6401 (fig. 3.29). Boethius's prologue and the prefaces to books one and two, rhetorically rich and stylistically complex, were favorite sites

[127] Irvine, *The Making*, 70. *Lectio* was divided into four parts: *accentus*, identifying and accenting syllables and words; *discretio*, parsing and punctuation of texts into syntactic, semantic, and metrical units; *pronuntiatio*, correct intonation of sentences; and *modulatio*, regulating the voice. I focus on the first two, more elementary activities.

of grammatical study, and the glosses were often dense.[128] In the Paris manuscript, the interlinear and marginal gloss, written by a contemporary hand, simplify Boethius's Latin for an amateur reader, bringing the syntax and vocabulary to a more manageable level. The first line of the prologue, a letter to Symmachus, the dedicatee, can serve as example (fig. 3.30). Boethius wrote:

> In giving and receiving gifts, such courtesies are rightly esteemed, especially if it will be clearly understood among those who consider themselves important, that nothing is given with more generosity nor anything is received with greater benevolence.
>
> *In dandis accipiendis que muneribus ita recte officia inter eos praecipue qui sese magni faciunt aestimantur, si liquido constabit nec ab hoc aliud quod liberalius afferret inuentum nec ab illo umquam quod iucundius beneuolentia complecteretur acceptum.*[129]

The prepositional phrase that opens the text: "In giving and receiving gifts" was rewritten in the interlinear gloss as a simple subordinate clause: "In gifts, which are given and received."[130] Many terms were glossed with synonyms, sometimes several. Thus, above *aestimantur* ("understood") is written *ponderatur* and, next to it, a tie mark—in this instance a squiggle with a point—directed the reader to the margin where, beside the same sign, were two more synonyms: *putant*(*ur*) and *credunt*(*ur*).

Sentences were not only rephrased and reworded, but their components mentally and sometimes physically reordered so as to make content more legible. The tenth- or eleventh-century collection of poems by Sedulius, Orléans, Bibliothèque municipale MS 303—probably a Fleury manuscript—contains a treatise on word order called the *De ordinatione constructionis* ("On the Order of Verbal Construction").[131] The anonymous and undated tract

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[128] Marco Masi, who translated the *De arithmetica* into English, noted that, "Boethius uses the 'precious style' in his preface. ... a highly ornate and artificial mode of writing with involved syntax and sentence structure. It contrasts sharply with the simple and more direct style found in the body of the treatise." Such shifts in style, according to Reviel Netz, were standard in mathematical and scientific treatises, see below, note 144. Masi in Boethius, *Boethian Number Theory: A Translation of the* De Institutione Arithmetica (Amsterdam: Rodopi, 1983), 66, n. 1.

[129] Boethius, *De arithmetica ,* Praef. 1, ed. Guillaumin, 1; trans. Masi, 66. I have replicated the punctuation in Paris, lat. 6401, fol. 87.

[130] The simple, subordinate clause *quae danturq*(*ue*) *accipiuntur* was written above the participles *dandis* and *accipiendis*.

[131] Preserved in Orlèans, BM MS 303, p. 145–48. A list of the manuscript's contents can be found in Guerreau-Jalabert, 188 (no. B22); Grotans, *Reading*, 168; Saenger, *Space between Words*, 161–62.

advocated for strict subject-verb-object order. Such straightforward sentence construction was also promoted by Priscian, who called it "natural." But Priscian also observed that authors often departed from so-called natural order for reasons of clarity, rhythm, and euphony.[132] Where this was the case, students were encouraged to rearrange sentences in their heads. Active use of this technique is attested in manuscripts in which sentences are entirely rewritten in the margin.[133] Construe marks, numbers or letters written above words or clauses of a difficult sentence, were added as an aid for or by readers. When these were read in numerical or alphabetical order the sentence was recast in subject-verb-object order. Such marks are encountered in an eleventh-century Fleury copy of the *Mathesis*, a tract on astrology written by the fourth-century Sicilian, Firmicus Maternus.[134]

Grammatical techniques and strategies were regularly applied to quadrivial manuscripts to unlock meaning of narrative passages, as is clear from the above-mentioned examples. A marginal note in a heavily annotated tenth-century copy of the *De arithmetica* produced for St. Augustine's monastery, Canterbury (now Cambridge, Corpus Christi College MS 352) offers a glimpse into contemporary pedagogical theory: "In the seven arts .iii. teach, that is grammar, rhetoric, dialectic, and .iv. are taught, that is arithmetic, geometry, music, and astronomy" (figs. 3.31–32).[135] Medieval historian Gillian Evans highlighted this particular note and interpreted it in this way: "the three arts that 'teach' do so in the sense that they provide methods of interpretation, techniques of argumentation, criteria of assessment, which may be applied to the learning of the four mathematical arts."[136] Ancient and medieval grammarians offer corroboration. The philosopher Sextus Empiricus (*fl.* 180–200 CE)

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[132] Priscian, *Institutiones* 18.105–06; Grotans, *Reading*, 165.

[133] Anna Grotans described the situation in this way, "the *ordo naturalis* was applied already early in the Middle Ages to pedagogic ends in order to rearrange the words of a sentence in such a way that the grammatical relationships between the elements and their meaning would be clear. In effect, early medieval classroom practice brought what before had been and was soon again to become linguistic speculation back to the domain of empirical observation." Grotans, *Reading*, 167.

[134] Saenger, *Space between Words*, 162. Saenger cited Mostert, *Library of Fleury*, no. BF804, which is Orlèans, BM MS 303, p. 145–48. These pages contain the *De ordinatione constructionis*, not the *Mathesis*. Indeed, the *Mathesis* is not included in this compendium, which is devoted to the poems of Sedulius. I know of two 11th-c. copies of the *Mathesis* produced and owned by Fleury: Paris, BnF lat. MS 7311, fols. 4r–19r and Vatican City, BAV Reg. MS 1244, fols. 2r–50r. I have not yet been able to verify the presence of sequential construction notes mentioned by Saenger in either MS. Mostert, *Library of Fleury*, 215 and 279; and Pellegrin and Bouhot, *Catalogue des manuscrits*, 409.

[135] CCCC 352, 3v: "In vii artibus tres docent i(d est) gramatica rethorica dialectica iiii docentur i(d est) arithmetica geometria musica astronomia."

[136] Evans, "Introductions to Boethius's 'Arithmetica'," 38.

considered Grammar an essential tool: "almost everything depends on it [grammar], and without it, it is impossible to teach anything necessary to others, and it will be impossible to learn anything profitable from another."[137] Likewise Isidore called grammar the "origin and foundation of liberal letters."[138] Hence, teaching, whatever the topic, depended on the verbal arts. Still the lessons yielded by grammatical analysis had their limits. Evans observed that Boethius's introduction and prefatory material (the letter to Symmachus discussed above and the opening chapters of book one and book two already mentioned) "lent themselves far more readily to discussion in dialectical, grammatical and other non-mathematical terms, than did the solid core of mathematical material which makes up the bulk of the *De arithmetica*."[139] What is not clear in Evans's assessment is the largely graphic nature of what she refers to as the "mathematical core"—the more than 120 *descriptiones* and *formulae* that dominate the pages of manuscripts like CCCC 352 and Paris, lat. 6401 (figs. 1.4–6, 1.18–23, and 3.33).

Boethius pointed out to his patron, Symmachus, and subsequent readers how the graphic elements of the *De arithmetica* made numeric notions "evident" or "vivid" (*evidentia*).[140] In the body of the work, when describing the mechanics and habits of numbers, Boethius dispensed with the flowery language and rhetorical flourishes that characterized the sections of the tract that attracted so much "trivial" annotation. His language becomes, for the most part, exceptionally plain and clear.[141] Hence as the density of

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[137] Sextus Empiricus, *Pro Mathematikos (Adversus Mathematicos)* 1.52. Quoted in Irvine, *Making*, 29. Irvine characterized Sextus as a transmitter of "the presuppositions of the textual culture of Hellenistic, Roman, and medieval eras."

[138] Isidore, *Etymologiae* 1.5.1; ed. Lindsay; trans. Barney, 42: "Grammatica est scientia recte loquendi, et origo et fundamentum liberalium litterarum."

[139] Evans, "Introductions to Boethius's 'Arithmetica'," 38. A similar opinion was voiced by Henry Mayr-Harting, who studied a corpus of glosses to the *De arithmetica* preserved (with variations) in at least seven manuscripts. He characterized the majority of additions as "gritty efforts to clarify, or to state in slightly different words, what the text says; that is, their interest at first sight appears to be purely arithmetical." The glosses attached to the beginning of Book 1 are, on the other hand, described by Mayr-Harting as "remarkable" and show the "cosmic ethical awareness of medieval minds." Mayr-Harting, *Church and Cosmos*, 149–50.

[140] A sentiment shared and amplified by medieval annotators. In Cologne 183, fol. 1v the statement is glossed: "Ieiuna oratio est sua imperfectione ieiunos reddit. ut illa nichomachi quae ad evidentiam rerum istius formulis ac descriptionibus eguit." The gloss is preserved in six other MSS, the earliest extant copy of which is dated to the mid-10th c. Mayr-Harting, *Church and Cosmos*, 248.

[141] The terse style of mathematical explanation has a long history. Its origins were investigated by Reviel Netz, who found the Greek mathematical lexicon (that of Euclid, Archimedes, etc.) to be "dramatically small—not only in specifically mathematical words, but in any words including the most common Greek grammatical words. It is strongly repetitive ... And it follows, on the whole, a principle of one-concept-one-word." It was

descriptiones increased, Boethius relied less on vivid verbal description. In medieval copies, the annotation added to this part of the *De arithmetica* is sparse and can be characterized as arithmetical: Roman numerals are written above numbers spelled out in the text and relationships between numbers are verbalized. The change in quantity and kind of commentary clearly corresponds to a marked shift in the form—visual and verbal—of the work. Abbo worked in and went beyond this tradition when he turned his attention to Victorius's *Calculus* and prepared the *Explanatio* for the monks of Fleury.

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Arguably Abbo's chief duty in the *Explanatio* was to teach the concept of multiplication, the nature of *multiplex* numbers ("multiples," numbers containing another number more than once) and, with this, the logic that generated and underlay Victorius's multiplication tables. He did so in a lengthy passage culminating with a table of his own design. He introduced the table in this way:

> Let us now teach how the whole multitude of numbers is multiplied by means of a certain compendium (*conpendio*) of proof (*argumenti*), in which it will be possible to observe (*animadvertere*) what is the nature first of unities in the "singles," what the second [nature is] in the "tens," what the third [is] in the "hundreds," and what the fourth is in the "thousands," [these kinds of numbers] are written below in a single, orderly arrangement.[142]

In the passage, Abbo calls the table variously a *conpendium argumenti*, *unus ordo*, and *figura*. All three terms shed light on the purpose and mechanics of the image, and the visual elements of the *Explanatio* more generally. Each will be considered here, but first it is necessary to take a closer look at the object to which they refer. In Berlin 138 the table, which I will call the *multiplex* figure, is integrated into the column of text and drawn in the same ink as the words around it (figs. 3.34–35). With it students could find the product of numbers 1 through 9 and factors of 10 up to 100,000. Rounded arches cap six columns with pointed bases. The term "latitude" (*latitudo*), broken into its four syllables, inhabits the

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also deeply formulaic. Netz, *Shaping*, 120 (Netz devotes chapters three and four to the language of Greek mathematics).

[142] Abbo, *Explanatio* 3.59; ed. Peden, 10: "Doceamus qualiter omnis numerorum multitudo multiplicetur quodam argumenti conpendio, ubi animadverti poterit, quae sit in sungularibus prima unitatis natura, quae in decenis secunda, quae in centenis tertia, quae in millenis quarta, quorum subscriptio talis sit uno ordine disposita."

spaces below the curves, and its antonym, "longitude" (*longitudo*), clings to the left side of the table. By "singles," "tens" etc. Abbo referred to the columns, each so-labeled at the bottom in the bright orange pigment used for rubrication. The blind ruling for the text divides the table horizontally into nine rows. In the "singles" column Roman numerals one through nine are written in alternating orange pigment and brown ink. Likewise multiples of ten from ten to ninety were written in the "tens" column, and so on. The product of two factors was found at the intersection of their respective rows and columns.

The table is set off by leafy flourishes that curl from its corners, resembling the *fleur-de-lis* tails of the dog-headed birds that compose the zoomorphic initial at the top of the page. Even such modest ornamentation is rare in Berlin 138. Victorius's tables and the graphic aspects of the *Explanatio* are, in this copy and others, austere, monotone entities, executed in ink and separated from the verbal narrative by swaths of blank space and lines that form simple, spindly frames (figs. 3.1–6 and 3.36–40). Yet demarcation, even by the most minimal means, was crucial to teaching mathematics with manuscripts for it created a field for quadrivial study in which numbers were, in the words of Abbo, "arranged" (*disponere*, *ordinare*) and "placed" (*ponere*) ready not to be read, but to be seen and inspected. Abbo's *multiplex* table and the adjacent text help us understand the kind of logic that governed these spaces and made them so essential.

One might expect Victorius's tables to have served as Abbo's primary inspiration for the *multiplex* figure. They did not. Instead Abbo created a kind of graphic epitome of the many images of and tools for multiplication available to him: Victorius's tables, the abacus, and *descriptiones* from Boethius's *De arithmetica*. Side by side comparison of the *mutiplex* figure with the tables of the *Calculus* reveals significant differences in content and layout (figs. 3.12 and 3.35). Abbo's figure is both more expansive and less comprehensive than the tables. He begins with unity rather than two as Victorius did. He also included multiplicands up to 100,000, but only as multiples of 10. In so doing, he created a figure reminiscent of an abacus, a device for calculation with which he was familiar. In one copy of the *Explanatio*, a scribe copied out a brief poem in which Abbo was referred to as the "teacher of the abacus" (*abaci doctor*).[143] Abbo may have used the abacus in the classroom for demonstration as, we are told, Gerbert did.[144]

Some time at the end of the tenth century a drawing of an abacus was inserted at the front of Bern 250, just before the *Calculus* (fig. 3.41).[145] The image spans the whole of the page and is oriented differently from the rest of the manuscript, so that the book would have to be turned clockwise 90° for it to be read or used. It consists of four long rows each divided into thirty narrow compartments or columns. Above the top row, a series of ten large semicircular arches gather the columns into clusters of three. Columns were designated with the letters **m**, **d**, and **c**, standing for units, tens, and hundreds. Each compartment represented a multiple of ten and was so labeled. Thus the fourth compartment frrom the left in the uppermost row contains an **i**, the next, an **x**, and so on.[146] To use the drawing as an abacus, one would need tokens of some sort marked with signs for numbers (sometimes Arabic numerals) one through nine. These would be placed in compartments and could represent any number from 1 to 999,999,999,999,999,999,999,999,999.[147] It should be remembered that the abacus was, above all, a didactic tool for demonstration.[148] As such its form was—even without tokens—an evocative symbol of multiplication.

There are obvious visual parallels between the drawing and Abbo's *multiplex* figure, primarily the columns surmounted by semicircular arches containing powers of ten.[149] But

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[143] Brussels, BRB MS 10078, fol. 114r. The epithet is part of a short poem: "Nunc manibus fer aquas quia sat iam sumpsimus escas / Qui erit in lacrimis, recipit hic gaudia messis / His abbas abaci doctor dat se Abbo quieti."
Peden assumed the author was the scribe since the lines do not appear in any other copies of the *Explanatio*. Charles Burnett thought it likely that the lines were written by Abbo and that the phrase *abaci doctor* should be understood to mean "master of teaching calculation." Peden, in *Abbo of Fleury and Ramsey: Commentary*, xxxviii; Burnett, "Abbon de Fleury *Abaci doctor*," in *Abbon de Fleury: Philosophie, sciences et comput autour de l'an mil*, ed. Barbara Obrist (Paris: C.N.R.S. and E.P.H.E., 2006), 139.
[144] See chapter 2.
[145] On other drawings of the abacus in circulation at this time see Charles Burnett, "The Abacus at Echternach in ca. 1000 A.D." *Sciamus* 3 (2002): 91–108, esp. 92, n. 5.
[146] The first three compartments contain signs for fractions.
[147] Practical reckoning was more easily accomplished with tables such as those in the *Calculus* or with finger calculation. A detailed description of calculation with the abacus is found in K. Vogel, "Gerbert von Aurillac als Mathematiker, *Acta historica Leopoldina* 16 (1985): 9–23. On early use of the abacus see Gillian Evans, "*Difficillima et Ardua*: Theory and Practice in Treatises on the Abacus, 950–1150," *Journal of Medieval History* 3 (1977): 21–38; and eadem, "From Abacus to Algorism: Theory and Practice in Medieval Arithmetic," *The British Journal for the History of Science* 10, no. 2 (1977): 114–31.
[148] Burnett, "The Abacus," 92.
[149] Charles Burnett, who considered Abbo's figure in light of various contemporary treatments of the abacus, observed, "He [Abbo] used elements arising from treatises on the abacus ... in order to reveal the marvelous

though he incorporated certain aspects of the abacus into his figure, he also departed from the tool in important ways, especially with respect to orientation. In the abacus, smaller quantities appear to the right with numbers increasing to the left, so that a number mapped out on the board would resemble its verbal and written forms (e.g., *viginti unus* ["twenty-one"] and **xxi**). The reverse is true in Abbo's figure. This might seem a small difference, but it points to an accommodation to reading practice as well as an adaptation of Boethian logic. Unlike Victorius's tables and the abacus, the graphic aspects of the *De arithmetica* were embedded in an extended verbal narrative. Hence the logical "flow" of the work as a whole is that of a viewer reading from left to right, top to bottom. It is perhaps for this reason that Boethius chose the orientation that he did, aligning numeric increase from unity, the overarching, cosmic theme of the *De arithmetica*, with the textual vectors in order to highlight its importance.[150] Abbo did the same in his *multiplex* figure. In Berlin 138, the left-right, top-bottom order is further underscored by the terms *latitudo* and *longitudo* written across the top and along the side of the figure.[151] In many copies of the *De arithmetica*, these same terms were written along the sides of the *tetragona* or "squares" figure (which Boethius referred to as both a *descriptio* and a *formula*) to which Abbo's figure is closely related (fig. 3.42).[152]

Boethius conceived the *tetragona* as a summing up of the numeric relationships discussed in the pages that precede it:[153] *multiplex*,[154] *tetragona*, and all kinds of *super- and sub-particulars*.[155] As such, it served as a kind of overview of multiplication. Abbo's figure

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power of numbers." [translated from the French]. A statement that holds for the visual as well as the written elements. Burnett, "Abbon de Fleury *Abaci doctor*," 138.

150 There are numerous points in the *De arithmetica* where Boethius goes against the normative reading pattern, but to do so required applying extra verbal and visual effort, i.e., explicit instructions to look this way or that, or to re-arrange an image in the mind's eye.

151 The same arrangement is found in other copies of the *Explanatio* such as Bamberg, SB MS Misc.Class.23, fol. 33v.

152 Indeed Abbo's *multiplex* figure follows a discussion of *tetragoni* (square numbers) drawn largely from the *De arithmetica*.

153 It may be for this reason that in certain copies of the *De arithmetica* (Bamberg, SB MS Msc.Class.8 and MS Msc.Class.5) the preceding set of figures, made redundant by the *tetragona*, were omitted. These omissions are discussed in chapter 1.

154 Multiples of 1, 2, 3, 4, ... including 1, 2, 4, 6, 8, ... the "double" (*duplex*); 1, 3, 6, 9, 12, ... the "triple" (*triplex*); etc.

155 In Boethius's words, "a number compared to another in such a way that it has in itself the entire smaller number and a fractional part of it." For example, 3 is a superparticular with respect to 2, which it contains along with 1. This kind of relationship, in which the larger (3) number contains the smaller (2) and its *half* (1) is called a *sesqualiter*. If the larger number contains the smaller number and a third of that number it would be a

was crafted in a similar spirit. Indeed, he might have imported the Boethian *tetragona* wholesale. Instead, he adjusted the image in ways that, arguably, clarified the act of multiplication for a novice audience and emphasized the infinite nature of *multiplex* numbers. The *tetragona* is a square grid, ten compartments wide and long. Its layout and mechanics are near-identical to the *multiplex* figure. The series of natural numbers from one to ten occupy the uppermost row and the left-most column; their products extend along the rest of the rows and columns so that the second row from the top and the second column from the left contain the multiples of two, likewise the third row and third column hold multiples of three, and so on to ten. Boethius drew attention in the verbal narrative to the series of "squares" that form a diagonal stretching from unity at the upper-left corner of the square to 100 in the lower-right corner. The figure is symmetrical along this diagonal, which was often picked out with graphic embellishments (figs. 3.42–45). Abbo adopted the form of the Boethian *tetragona*, but not its content. His adaptation was geared to making arithmetic processes sensible and thus knowable.

Abbo called the rows and columns of the *multiplex* figure "verses" (*versi*) a term that evoked simultaneously lines of poetry and geometry. Both qualities—the poetic (sonic, grammatical) and geometric (visual, spatial, directional, quadrivial) surfaced in his description of the figure. He drew the reader/viewer's attention to the pattern of each row as the numbers in the "singles" column increased by multiples of ten:

> Truly, each latitude line is marked with numbers, which draw [their] names from the singles [column] placed ahead [of them] ... *binarius* and, for instance, *viginti*—which is almost called ***bi**-ginti* (that is ten twice born, ***bis** geniti*)—and two-hundred (***du**centorum*), and two-thousand (***duum** milium*), and twenty thousand (***vi**ginti milium*), and two-hundred thousand (***du**centorus milium*).[156]

Whereas elsewhere in the *Explanatio* numbers in the verbal narrative were written as Roman numerals, here they were spelled out so that the presence of two (*duo, bis*) literally *in* each

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sesquitertius, a fourth would be *sesquisuartus*, etc. Boethius, *De arithmetica* 1.24.1; ed. Guilluamin, 50; trans. Masi, 103.

[156] Emphasis added. Abbo, *Explanatio* 3.61; ed. Peden, 111: "Singulae nimirum latitudinis lineae insignitae sunt numeris, qui sortiuntur vocabula ex singularibus antepositis ... binarius nanque viginti, qui dicti sunt quasi biginti, id est denario bis geniti, et ducentorum et duum milium et viginti milium et ducentorus milium." My translation and understanding of this passage is indebted to Charles Burnett's translation (in French) and discussion of the larger passage to which these lines belong. "Abbon de Fleury *Abaci doctor*," 134–35.

number became etymologically emphatic.[157] Abbo even cheated a little, likening *viginti* to *bi-ginti*, and *bis geniti*. The relationships to two are just as apparent in the figure, where each character is physically doubled: twin **ii**'s are followed by two **x**'s and two **c**'s, etc.[158] In contrast, recitation of the multiples of two in Boethius's *tetragona* would have yielded the series *duo*, *quattuor*, *sex*, *octo*, ..., *duodeviginti, viginti*. The relationship between these numbers and to two would have been instantly recognizable to anyone familiar with *duplex* ("double") relationship, but their origin, the twoness of them, cannot be heard or seen. The sequence of Roman numerals in the figure—**ii**, **iiii**, **vi**, **viii**, ..., **xviii**, **xx**—only sporadically (e.g., **iiii** and **xx**) looks to be derived from two. Seeking to make underlying principle of multiplication evident, Abbo selected a set of products which clearly "show" their factors to the ear and eye.

The senses, however, had their limits. Explaining *multiplex* numbers, all of them, Abbo brushed up against the problem of conveying the notion of infinite progression to his charges. The range of the numbers represented in the *multiplex* figure differs markedly from its models. Abbo restricted the latitude—the length of the figure—to the first nine numbers below ten. These were said to generate all other numbers, hence the choice was obvious. The limit of the figure's width or longitude to 900,000 seems, on the other hand, somewhat arbitrary. Victorius's tables extended to 50,000; the abacus went as high as 999,999,999,999,999,999,999,999,999. Perhaps 900,000 was simply a happy medium.[159]

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[157] Abbo's use of Roman numerals (versus spelling numbers out) in the verbal narrative is difficult to determine from extant manuscripts. Ron Thomson, who examined 18 manuscripts for his edition of two short computus tracts: the *Sententia Abbonis de ratione spere* (also known as the *De differentia circule et spere*) and the untitled, "Denique luna totius zodiaci signa..." summed the problem up in this way: "It is often difficult to be certain, even from the earliest manuscripts, of what Abbo wrote, i.e., whether he used roman numerals or wrote the figures out in full. This is particularly so where case endings are added ("ii^{bus}" for *duobus*)—are these truly roman numerals or simply scribal abbreviations of the words? Sometimes one will find a variety of forms throughout the manuscripts ("xii, "xii^{cim}", *duodecim*) and sometimes the same manuscript will present a number in one way and in a second way just a few lines later." I have noticed similar inconsistency in copies of the *Computus*. Whether intentional or not, the phenomenon demonstrates the fluidity of numeric form and significant flexibility on the part of readers. Thomson, "Two Astronomical Tractates," 118–19.

[158] Patterns are less evident for those numbers greater than four since the number of characters does not correspond with the multiplier. Still, the general shape or pattern of a numeral (i.e., **vii**, **lxx**, **dcc**, ...) is repeated and visible.

[159] It should be noted that the figure consisted of six columns and nine rows: both numbers were numerologically significant. See Heinz Meyer and Rudolf Suntrup, *Lexikon der mittelalterlichen Zahlenbedeutungen*, Münstersche Mittelalter-Schriften 56 (Munich: Wilhelm Fink Verlag, 1987), cols. 442–79 and 581–90.

Boethius had not pushed so far; the largest number represented in the *De arithmetica* was 20,736.[160] He addressed the problem of understanding infinity as postulate in the *proemium*:

> Nothing, indeed, that is infinite can be encompassed by science or comprehended by the mind (*mente*), but from there [i.e., from the infinite] reason itself chooses that in which it [reason] is able to exercise its truth-seeking skill. For it selects from the plurality of infinite multitude a limit of finite quantity and, having rejected a section of unending magnitude, it demands for itself a defined space for cognition/investigation.[161]

The mind's inability to grasp the infinite does not, Boethius assures his reader, bar the study of number, for reason "demarcates a manageable field of study." In other words, one understands arithmetic by means of numeric synecdoche. Boethius assumes the role of reason in the *De arithmetica*, delegating boundaries "of finite quantity to the plurality of infinite multitude." Nearly every definition and discussion in the work pivots on finite sets of numbers that serve as examples of patterns that could continue without limit. He reminded his reader of the infinite nature of number and numeric progression with dogged persistence. *Multiplex* numbers, for example, are said to "move to infinite possibility," and "proceed to infinity without obstacle."[162] What is explicit in the *De arithmetica* is implicit in the *Explanatio*.

Seen in light of Boethius's ruminations, Abbo's use of the term *conpendium* to describe the *multiplex* figure takes on greater significance:

> Let us now teach how *the whole multitude of numbers* is multiplied by means of a certain compendium of proof (*conpendio argumenti*).[163]

Conpendium argumenti, might be translated as "abridgement," "abbreviation," or "summary" of an *argumentum*, "proof" or even "the matter which lies at the basis of any written or

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[160] 20,736 is the product of 72 and 288. It is among the multiples in the "description pertaining to the nature of the oddly even [i.e., odd times even]" ("Descriptionis ad impariter paris naturam pertinentis expositio"). Boethius, *De arithmetica* 1.12; ed. Guillaumin, 29–30.
[161] Ibid. 1.1.6; ed. Guillaumin, 8; my translation: "Nihil enim quod infinitum est uel scientia potest colligi uel mente comprehendi, sed hinc sumpsit sibi ipsa ratio [corrupt?], in quibus possit indagatricem ueritatis exercere sollertiam. Delegit enim de infinitae multitudinis pluralitate finitae terminum quantitatis et, interminabilis magnitudinis sectione reiecta, definita sibi ad cognitionem spatia depoposcit."
[162] Ibid. 1.23.4; ed. Guillaumin, 48; trans. Masi, 102: "Cum autem naturaliter multiplicitas infinita sit, eorum quoque species per proprias generationes in infinita consideratione versantur." And Ibid.; ed. Guillaumin, 49; trans. Masi, 102: "idemque in ceteris in infinitum sumentibus sine aliquo impedimento procedit."
[163] Abbo, *Explanatio* 3.59; ed. Peden, 110: "Doceamus qualiter omnis numerorum multitudo multiplicetur quodam argumenti conpendio."

artistic representation."[164] The *multiplex* figure is an "abridgement" or "abbreviation" of the infinite series of *multiplex* numbers. Indeed, the figure is characterized as such in the twelfth-century computus Oxford, St. John's College MS 17, associated with both Byrhtferth and Abbo (figs. 3.46–47). Above the *multiplex* figure, a *titulus* declares: "In this figure, infinite number is described (*descriptus est*); [It, i.e. infinite number] begins from one and comes forth all the way to 900,000."[165] The statement might seem a contradiction, but for one familiar with the conceit of arithmetic demonstration there is no conflict: *every* numeric figure pictured infinity through the finite. Perhaps, by including a quantity so great as 900,000, Abbo gave a nod toward the unfathomable, unimaginable infinite, while still acting the part of reason.

The term *figura*, encountered in the *titulus* in St. John's 17, was used by Abbo as well.[166] The word points to the visual and graphic nature of arithmetic explanation. At its most basic, *figura* meant outward appearance, the shape of a thing. Stemming from *fingere* meaning "to mold or shape," and related to terms like *figulus* and *fictor* "potters, sculptors, makers," it carried connotations of the man-made and plasticity. Abbo used it in the *Explanatio* to describe a range of things from rhetorical figures and dialectical arguments[167] to geometric planes,[168] letters, and other written signs.[169] A comparison with the *Etymologiae* shows Abbo's usage to be standard. *Figura* appears more than one hundred times in the *Etymologiae*, in nearly every book (there are twenty). The remarkable spread is due to the concept of *figura* as a quality of a thing, anything—letters of the alphabet, animals, constellations. It is used with some regularity in the first book *De grammatica* in the chapters

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[164] *Conpendium* is a common variant of *compendium*.
[165] Oxford, St. John's 17, fol. 35r: "In hac figura descriptus est numerus infinitus: incipit enim ab uno p(er)venitq(ue) usq(ue) ad nongentesimum millesimum."
[166] St. John's 17 contains a significant amount of Abbonian material. The manuscript was probably produced at Thorney Abbey, a few miles from Ramsey. Some of the contents are clearly indebted to a Ramsey exemplar(s). It has been argued that the manuscript relied heavily on Byrhtferth's *Computus*, no longer extant, and it has been used to help reconstruct that work. Byrhtferth included a version of Abbo's *multiplex* figure in the *Enchiridion*. He introduced the figure: "The number 1000 is perfect, as the number itself, and the following figure, make utterly clear." (The figure was amended to include 1,000,000—the square of 1,000.) Indeed, the figure seems to have served as a sort of memorial to Abbo. Byrhtferth wrote below it: "It is fitting for us to savor the significance which Abbo of precious memory attributed to this number [1000]. The miracles following his death reveal how greatly his distinction shown out in this life. For he was accomplished in scientific learning and perfect in his knowledge." Byrhtferth, *Enchiridion* 4.1.402–403 and 404–497; ed. and trans. Baker and Lapidge, 228–29.
[167] E.g., Abbo, *Explanatio* 3.15, 3.29, 3.33, 3.35, 3.48.
[168] E.g., Ibid. 3.58 and 3.100.

on alphabets and letterforms, but never as the letter itself, always as "the figure, by which the letter is designated." Indeed the term describes marks and notations of all sorts from words and accents to geometric shapes. It appears most often in book three devoted to *Mathematica*, in the sections on geometry and astronomy. Within the space of just a few lines Isidore refers to both incorporeal geometric entities—points, lines, and planes; as well as the little line drawings of planes and solids all as "figures." Hence it is not surprising that Abbo applied the term to the *multiplex* figure as well as another graphic element in the *Explanatio*, an image of the *psicogonia* or the soul's creation and harmony (fig. 3.36 and 3.48).[170] His word choice suggests that shape was a key quality of both images.

Typically referred to as a *lambda* on account of its resemblance with the Greek letter, the *psicogonia* figure is, in its simplest manifestations, an inverted **v** with the Roman numeral for one at its apex; two, four, and eight appear at equal intervals on its left side; and three, nine, and twenty-seven on the right side.[171] In Berlin 138, the legs of the figure are bands with the numerals written in their narrow interiors. Arcs of various diameters connect numeric pairs and their ratios are spelled out: *duplus* between one and two, *triplus* between one and three, etc. These ratios are translated into musical relationships in the text that surrounds the upper half of the figure (e.g., *duplus est dyapason* "the double relationship is an octave"). An abstract form, probably a fountain, inhabits the space between the legs of the *lambda.* Arrow-like elements issue from it, pointing upwards to unity, while two frond-like forms first rise and then curl downward descending to the figure's base. *Fleurs-de-lis* spring from the point where these "fronds" meet the *lambda* legs. Notably, it is the only other graphic element besides the *multiplex* figure in Berlin 138 to receive even modest decoration—an indication, perhaps, of their metaphysical significance.

Abbo marveled over the way the *psicogonia* made "manifest" (*manifestat*) the whole discourse on the nature of unity and that it "marvelously expressed (*mirabiliter expressa*) the

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[169] E.g., Ibid. 3.51–54.
[170] Abbo consistently called the *psicogonia* (more commonly spelled *psychogonia*) image a "figure": *Explanatio* 3.1–2 and 3.30.
[171] Abbo claimed that the *psicogonia* was "invented" by Plato. The figure was not part of the *Timaeus*, but of Calcidius's *Commentary*. It was also habitually copied into Macrobius's *Commentary* on the *Somnium*, as already discussed in chapter 1. Just below the figure, Abbo cited Calcidius and Macrobius as authorities on the nature of unity (*Explanatio* 3.3) suggesting that he associated the shape with both authors. This kind of figure (there are many variants) was probably first devised by the Greek philosopher Crantor (*fl.*3rd and 4th c. BCE) as touched on by Anna Somfai, "Calcidius' Commentary on Plato's *Timaeus*," 213.

subtleties of arithemetic, geometry, music, and astronomy."[172] Such a feat was considerable, for in the same way that the infinite was incomprehensible, so too unity—indivisible and unchanging—was, in the words of Abbo, "discerned by the reason of the soul (*ratione animi*)."[173] Numbers were incorporeal, as such they could not be seen, at least not with the eyes of the body. Early in the *Explanatio* Abbo explained, "the soul sees measure, weight, and number through itself; it sees measurable, weighable, and numerable things through the body."[174] The assumed status and nature—whether incorporeal or corporeal—of the graphic elements of the *Explanatio* and other mathematical treatises will be addressed elsewhere.[175] Here, suffice it to say that to be seen by body *or* soul, number needed an appearance, a *figura*. Abbo's language draws attention to the visual aspect of all the graphic elements in the *Explanatio.* They were things to be "seen" and "beheld." [176] Abbo calls them "descriptions" (*descriptiones*)—as Boethius did—"orderly arrangements" (*dispositiones*), and, as already noted, *unus ordo*, which might variously be translated as a "a series," "a regular succession," or "a methodical arrangement." Numbers are "placed" and "set out" (*ponere, disponere*), "ordered" (*ordinare*) and "articulated" within their bounds—the terminology is topological.

In many of the additional tables that had accrued to Victorius's *Calculus*, layout and language together serve to clarify relationships between numbers (figs. 3.6 and 3.13). "And" (*et* or *&*) was used in tables for addition to join one column of numbers with another (fig. 3.49). In a table for subtraction, each row was patterned after the simple phrase "from 1000 take 100 900 remains" (*De .ī. tolle c remanent dcccc*). Victorius's tables, on the other hand, were wordless. Their meaning depended entirely on arrangement. Thus numbers within a single column share a multiplicand; two numbers in the same column that are horizontally aligned are factor and product. Failure in alignment yielded a loss of meaning. A missed

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172 Abbo, *Explanatio* 3.2; ed. Peden, 73–74: "Idque manifestat praedicta *psicogoniae* figura, arithmeticae, geometricae, musicae et astronomiae subtilitatibus contenta ac hoc modo mirabiliter expressa."

173 Ibid., The statement is part of a longer passage likening unity to a point in geometry, ending: "proptereaque sub nollos sensus venit; est tamen et cernitur ratione animi." ("and for this reason [because it is indivisible] it [unity/a point] comes to none of the senses; nevertheless it exists and is discerned by reason of the soul.")

174 Abbo continued, driving the point home, "Thus, it sees through the body those things that are similar to the body, that is bodies; incorporeal things truly [it sees] through itself, to which [incorporeal things] it is similar." ("Anima videt mensuram, pondus et numerum per se, videt mensurabilia, penderabilia et numerabilia per corpus. Ergo per corpus videt quae sunt similia corpori, ide est corpora; incorporea vero per se, quibus est ipsa similis.") Ibid. 2.14; ed. Peden. 71.

175 See chapter 5.

176 "Ut vides in dispositione superiori" (3.22); "Ecce dum multiplices ubique fundamenti loco disponimus" (3.27).

number in a table in Bern 250 put an entire row out of synch, a grave mistake (fig. 3.4 and 3.50–51).[177] To correct the error, the scribe drew lines connecting each factor with the product to which it belongs. The purely visual emendation highlights the fact that in the domain of pure number, place *was* meaning.

Abbo wrote admiringly of Victorius's "so great arrangement" (*talis dispositio*) when describing how different kinds of inequalities emerge from three equal terms (fig. 3.52).[178] In this case the numbers are laid out in a simple grid of three columns and six rows. The first row shows the three equal terms, in this case the number two. The following rows contain numbers in different kinds of multiplex relationships: doubles in the second row; *sesqualteri* numbers, which contain the smaller number and their half,[179] in the third row; *superbiterii* numbers, which comprise the smaller number that precedes them and $^{2}/_{3}{}^{\text{rds}}$ of that number,[180] in the fourth; etc, all of which are multiples of two. The arrangement allows Abbo to speak of "first," "second," and "third," terms; and their "right" (*rectus*; left to right) and "reversed" (*conversus*; right to left) order.[181] He notes how orderly placement helps create meaning, "And what an extraordinary [thing] if species of different kinds mingled together with each other in a sequence of procreation, *not crowded or confused.*"[182]

Abbo voiced a similar sentiment in relation to a list of fractions and the number of *scrupuli* ($^{1}/_{288}$) (fig. 3.53). In Berlin 138 the list, written in the same orangey-brown ink of the text, appears within the left column of text and makes use of the blind ruling to define each row. It is enclosed in a rectangular frame. Within the frame is a column of symbols for fractions from one, the *as*, to the *dimidia sextula* ($^{1}/_{144}$). Beside this is a column containing the symbol for the *scripulus* followed by a Roman numeral (the number of *scripuli* in a particular fraction, e.g., beside the *as* is the number 288). The framing of a simple list might be dismissed as necessary only because of its intrusion into the textblock. But Abbo writes how the "setting out" (*exponere*) of the fractions in this way makes them "more easily

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[177] Bern 250, fol. 6v. The error occurred in the column for multiples of forty-nine. The product of 49 and 70 should be 3,430. This number was skipped and then squeezed in.
[178] Abbo, *Explanatio* 3.24; ed. Peden, 87: "Tali descriptione erit expeditior."
[179] 8, 12, 18. 12=8+4, 18=12+6. The next number in the series would be 27, which equals 18+9.
[180] 18, 30, 50. 30=18+(2x6), 50=30+(2x10). The next number in the series would be $83^{1}/_{3}$ (i.e., 50+(2x$16^{2}/_{3}$)).
[181] Abbo, *Explanatio* 3.63–4; ed. Peden, 112–13.
[182] Ibid. 3.22; ed. Peden, 86: "Et quid mirum si diversorum generum species non vulgo neque confuse permiscentur sibi in procreationis serie."

understood" (*facilius intelligantur*).[183] Framing or bounding information emphasized the unified and singular nature of what was within while marking its difference from whatever was without.

Within frames or the outlines of figures, Abbo urges his readers to direct their gaze or cast their eyes in particular ways. In the text just below the *multiplex* figure (fig. 3.34), he points out how the nine rows and six columns yield meaning to a particular kind of inspection, permitting each to be examined on its own and, as part of a collectivity, in relation to the others.[184] Further on he asks:

> Do you not see how each line of verse consecutively follows in the figure, with their little sums examined from here and from there, while 400 and 40,000 are drawn from four, of which one proceeds from twenty the other from 200?[185]

The figure's rows and columns were to be inspected from different angles and directions. Abbo encouraged his audience to follow the contours of the figure, to trace the horizontal and vertical lines of numbers, but also to observe connections between the even numbers, in orange, and the odd numbers, in brown. All this looking would reveal number's nature and its powers—a claim predicated on the belief that articulation of information affects cognition and that conveying numeric notions required a departure from the standard, "grammatical" *mise-en-page*. The following chapter will consider figures as dynamic objects, contrived to move the eyes and with this, the mind.

CONCLUSION: RETHINKING "READING" IMAGES

Historians have demonstrated how medieval images evince cognitive patterns promoted by the three disciplines of the *trivium*. Such figures as Michael Camille and Mary Carruthers have used analytical tools provided by the verbal arts to interpret nonverbal elements in books.[186] These approaches are premised in part on the privileged place of word (*logos*) in

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[183] Ibid. 3.54; ed. Peden, 107.
[184] Ibid. 3.61; ed. Peden, 110–11: "Cum itaque novem versus sint longitudinis, sex vero latitudinis, singulos considerare oportet et per se et ad invicem collatos, ut omnem multiplicandi formulam alterutrum doceant."
[185] Ibid.: "Videsne singulas versuum lineas quomodo se sequantur hinc inde perspectis summulis in figura, dum a quaternario deriventur quadrigenti et xl milia, quorum alter ex xx, alter processit ex cc?"
[186] See, especially, Camille, "Seeing and Reading: Some Visual Implications of Medieval Literacy and Illiteracy," *Art History* 8 (1985): 26–49; idem, "Oxford University Textbook"; Mary Carruthers, *The Book of*

medieval epistemology as well as the primacy of the verbal disciplines over those of number in the medieval curriculum. Word *was* privileged. John the Evangelist gave word an ontological status and redemptive powers; his God spoke the world and his son into existence. As discussed in chapter 1, number also had metaphysical and ontological aspect. The God of the Old Testament was a mathematician, who, according to Genesis, crafted the universe through processes proper to number, dividing the firmament, multiplying living creatures. Hence, there was biblical authority for regarding both word and number as divine idioms in the middle ages. As we have seen in this chapter, numeracy was regarded as being fully as essential as literacy in the schools. The basics of grammar and arithmetic were taught at a young age, and this suggests that numeric rationales, as well as verbal skills, were deeply ingrained in the minds of those educated in monastic and cathedral schools.

The present bias toward what one might call verbal modes of viewing, reception, and signification has deep historiographical roots. Recent art history has been profoundly influenced by semiotics, linguistics, and the study of literature; there is no lack of metaphors in the field likening the visual to the verbal.[187] The analogies drawn between reading and seeing, that align verbal with pictorial modes of representation are not, of course, without historic antecedents: Horace's "as painting, so is poetry" or Gregory the Great's famed statement that "in picture, those who do not know letters, read."[188] Both *dicta* were known and repeated throughout the middle ages; the latter, from the mouth of a Church Father and bearing directly on the role of images in worship and devotion, has exercised particular sway among medievalists as it did their medieval predecessors.[189]

Memory; and eadem, *The Craft of Thought*. Their work can be seen as continuing in the tradition of (among others) Emile Mâle, Ernst Kitzinger, Otto Pächt, and Erwin Panofsky.

[187] The influence of linguistics on the study of images is summarized by W. T. J. Mitchell, in *Picture Theory: Essays on Verbal and Visual Representation* (Chicago: University of Chicago Press, 1994), 11–15.

[188] Horace, *Ars Poetica* 361, ed. D. R. Shackleton Bailey, *Q Horatii Flacci Opera*, 3rd ed. (Stuttgart: Teubner, 1995), 325: "Ut pictura poesis." Gregory the Great, *S. Gregorii Magni Registrum epistularum* 11.10; ed. Dag Norberg, CCSL 140A (Turnhout: Brepols, 1982), 874: "Nam quod legentibus scriptura, hoc idiotis praestat pictura cernentibus, quia in ipsa etiam ignorantes vident quid sequi debeant, in ipsa legunt qui litteras nesciunt." Abbo quotes the *Ars poetica* (325–30) in the *Explanatio* (3.48). Fleury had at least one copy of the work (Paris, BnF lat. MS 7971), but Abbo could have lifted these lines from another source (a grammar or a *florilegium*). He did, however, cite the work in full: "Horatius Flaccus in sua *Poetria* exsequens ait ..." (*Explanatio* 3.48).

[189] Discussed by many art historians, see Celia Chazelle, "Pictures, Books and the Illiterate: Pope Gregory I's Letter to Serenus of Marseilles," *Word & Image* 6 (1990): 138–53, esp. 150, n. 1; eadem, "Memory, Instruction, Worship: 'Gregory's' Influence on Early Medieval Doctrines of the Artistic Image," in *Gregory the Great: A Symposium*, ed. John C. Cavadini (Notre Dame: Notre Dame University Press, 1996), 181–215.

Camille and Carruthers both cited the twelfth-century restatement of Gregory's *dictum* by Gilbert Crispin, Abbot of Westminster (1085–1117): "Just as letters are the shapes and signs of spoken words, pictures exist as the representations and signs of writing."[190] For Carruthers, the statement, made in defense of the narrative historical images that adorned the walls of churches, served as a good general definition of *pictura.*[191] She wrote:

> Letters and other images are signs not primarily by virtue of imitating an object but by virtue of recalling something that is past to memory. This understanding requires that pictures themselves function 'textually,' as a type of writing and not something different from it in kind.[192]

Her position about the semiotic role of images—that pictures function as a "type of writing"—is persuasive, but it does not necessarily follow that pictures function "textually." Carruthers's analysis serves as a well-known and respected example of a larger tendency in medieval scholarship to collapse writing and text. But, whereas text is inherently verbal, writing, as we have seen, is not. I would build on Carruthers's statement to say that certain pictures, especially historical and narrative images, yield to analysis that sees them functioning as text, but that virtually all images gain in nuance if we approach them as writing. I would claim that for a fuller and more nuanced understanding of books, and the pictures therein, much depends on our distancing them from verbal modes of cognition. This was, of course, among the tasks that Abbo set himself in the *Explanatio*.

The study of Abbonian manuscripts and quadrivial classics kept and produced at Fleury makes it clear that teachers and the makers of books (often one and the same person in this period) worked hard to disrupt grammatical training and assumptions. Even very young readers brought strategies of *lectio* and *enarratio* to the page, and these, as we have seen, were indeed essential to deciphering quadrivial texts. Quadrivial learning did not erase or usurp modes of cognition cultivated through the *trivium*, nor was this its aim. Learning was and remains an additive process. Teachers then and now sought to supplement, not supplant;

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[190] Gilbert Crispin, *Disputatio Judex et Christiani*; ed. B. Blumenkranz (Utrecht: Spectrum, 1956), 67. Quoted in Camille, "The book of Signs: Writing and Visual Difference in Gothic Manuscript Illumination," *Word & Image* 1 (1985): 135; and Carruthers, *The Book of Memory*, 222.
[191] Carruthers, *The Book of Memory*, 222. In a Fleury manuscript of Gregory's letters (Paris, BnF lat. MS 2278, fol. 45v), a late 10th c. annotator summarized the passage in the margin, specified that Gregory was concerned with "any picture in a church" (*quid pictura in aecclesia*). For an in-depth discussion on the original context of the letters see Celia Chazelle, "Pictures, Books and the Illiterate," 138–53.
[192] Carruthers, *The Book of Memory*, 222.

to build bridges between diverse domains of knowledge. However the "mathematical core" of quadrivial manuscripts the *descriptiones* and *figurae*, required different modes of looking and thinking. This was a process that began with writing. Abbo reminded readers of the alphabet's semiotic flexibility and helped them to match the numbers they knew so well from their fingers with written forms, and methods of manual calculation with mathematical tables and the abacus. As numbers, letters were ordered and arranged in new ways. The traditional text block was interrupted to make—paraphrasing Boethius—a field for reason to investigate finite series and, in so doing, contemplate the infinite. Within these spaces number, incorporeal, was given a concrete, visible form.

CHAPTER 4

Byrhtferth and the "Curious Eye" of Computus

> The number five is perfect, and is divisible into its parts, for it rejoices to be adorned with three and two. Three pertains to the mystery of the holy Trinity; two pertains to love of God and of one's neighbor … It is unequal; it is also equal. It is odd in respect of three, but even in respect of two.[1]

In this way, the Benedictine monk Byrhtferth of Ramsey (c.970–c.1016) introduced the number five in his *Enchiridion*, a handbook of computus, thought to have been composed around 1010.[2] At this time, the middle-aged monk was the schoolmaster of the fledgling monastery of Ramsey located in the fens north of Cambridge.[3] The passage on five is part of a numerological tract near the end of the work, following the tables of alphabets and Roman numerals with their names that were briefly touched on in the last chapter. For Byrhtferth, computus, the study of time, was, in essence, about number—a conviction he shared with Abbo, who had taught him as a boy.

With expressive enthusiasm, in this numerological subsection of the *Enchiridion*, Byrhtferth guides his readers through the arithmological properties and sacred associations of numbers from one to one thousand. He does not treat all numbers in depth; after twenty he skips whole decades and hundreds entirely.[4] Those numbers he does address he lauds. To five's aforementioned qualities, Byrhtferth adds its relation—when doubled—to the Decalogue.[5] He explains that five's perfection is evident in the preponderance of noteworthy

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[1] Byrhtferth, *Enchiridion* 4.1.87–93; eds. and trans. Baker and Lapidge, 202–205: "Quinarius numerus perfectus est et in suis partibus constat divisus nam gloriatur se ternario atque binario esse comptum. Ternarius ad sanctae trinitatis pertinet mysterium; binarius vero ad dilectionem Dei et proximi ... Est inaequalis; est et aequalis. Inparilis est in ternario, parilis in binario."
[2] Baker and Lapidge (in Byrhtferth, *Enchiridion*, xxv–xxxiv) deduced Byrhtferth's date of writing from internal evidence within the *Enchiridion*.
[3] On the early history of Ramsey see chapter 2.
[4] Along with the descriptions of numbers, this section, which is discussed in greater detail below, includes instructions for multiplying large numbers by factors of ten (to 10,000) by means of finger calculation and a brief account of the six world ages. This is followed by a "friendly admonition" in Old English.
[5] Byrhtferth, *Enchiridion* 4.1.99–100; eds. and trans. Baker and Lapidge, 202.

fives encountered in both scripture and nature: the five senses, the five wise virgins, and the first five books of the Old Testament written by Moses.[6] But words are not adequate to convey the wonders of five; he concludes his treatment: "It is appropriate to introduce a figure (*figura*) into our discussion, so that what we have said in speech (*ore*, "with the mouth") will stand out more clearly than light to the eyes."[7] In Oxford, Bodleian Library Ashmole MS 328 a transcription of Byrhtferth's tract made not long after he wrote it, possibly in Canterbury—this conviction was realized on the page opposite, in a half-page figure drawn in vivid red and green pigments (fig. 4.1).[8]

In anticipation of rendering the figure, the eleventh-century scribe of Ashmole 328 left eleven of the twenty lines of the ruled textblock blank. The image of the properties of five fills the space entirely and extends slightly into the margins (figs. 4.2–3). Drawn with a straightedge and compass, it takes the shape of a Latin cross, outlined in double red lines. *Quinarius*, "fivefold" or "containing five," is written across the cross's patible.[9] Its beams terminate in rectangular green plaques. Each plaque is labeled with a Roman numeral, I through V, as well as its corresponding Latin ordinal, "first" through "fifth." The plaques are thus revealed as "parts" of five. Byrhtferth placed the *primus* plaque at the top of the stipes of the cross. The *secundus* and *tertius* plaques occupy, respectively, the left and the right ends of the transverse axis; *quartus* is near the bottom of the stipes, and *quintus* at its base. Red circles—some partial, some complete—embrace the plaques like halos. A horizontal red line below the figure serves to ground the cross and separate the drawing from the text below.

Byrhtferth's emphatic words, the manuscript's *mise-en-page*, and the figure's careful execution leave little doubt that the cruciform device was considered essential to demonstrating the arithmetical significance of five by its author as well as the makers of Ashmole 328. Throughout the *Enchiridion* verbal narrative cedes to visual explanation,

[6] Ibid.
[7] Ibid. 4.1.103–104; eds. and trans. Baker and Lapidge, 203–04: "Placet huic operi nostro figuram adicere, quae quod ore dicimus oculis luce clarius illustret."
[8] The provenance of Ashmole 328 is unknown. Baker and Lapidge suggest Glastonbury or Christ Church, Canterbury (leaning toward the latter). They date the manuscript to the mid-11th c., based on its distinctive script. The text of Ashmole 328 was written first and the figures added soon after (they clearly belong to the same campaign). Significant gaps were left between passages to accommodate some of the larger figures that—in the exemplar—must have extended into the adjacent text. The script is otherwise remarkably even. Ashmole 328 is the only copy of the *Enchiridion* to survive except for two short excerpts in Cambridge, UL MS Kk.5.32, fols. 49–60, and Cambridge, CCC MS 421. Baker and Lapidge, in Byrhtferth, *Enchiridion*, cxv–cxxiv.
[9] All but the lowermost semicircle were drawn with a compass. A few of the horizontal lines may have been drawn without a rule, but use the text ruling as a guide.

mouth to eyes. Nearly three dozen figures, schemata, tables, and frames disrupt the manuscript's tidy lines of text. In almost every instance, their appearance is announced. Byrhtferth writes: "the following figure sets out in clear view, for all those wishing to learn, the things which we have established verbally";[10] "We set down here the epacts and the lunar regulars so that they will more plainly and boldly stand before the priest's sight";[11] "Take heed when there is an embolism, for three months will then have a thirty-day-old moon, as I will joyfully illustrate here" (figs. 4.4–6).[12] In these and similar passages, Byrhtferth joins the verbal to the visual, stating how these objects represent, reveal, clarify, and demonstrate for the eye those qualities of number and numerical relations that could not best or most fully be conveyed with words.

This chapter considers what those qualities were, how they were conveyed on the page, and—insofar as is possible—their perceived efficacy by Byrhtferth and the Ramsey community for whom he wrote. Following in the footsteps of his master, Byrhtferth created spaces for quadrivial contemplation. The *Enchiridion*, treated briefly in the last chapter as part of a consideration of the instruction Abbo offered to Byrhtferth and his fellow monks at Ramsey, will be the focus here. Byrhtferth was the first to apply the Greek term to his tract; "enchiridion," he explained, meant "*manualis* in Latin and *handboc* in [Old] English."[13] The pocket-sized work was, above all, a teaching document, intended to help readers learn *computus*. In the tenth century and through the eleventh, *computus* might still refer to computation in a general way but the term was gradually becoming synonymous with time reckoning, and more specifically, with calculating the date of Easter.[14] As mentioned in chapter one, the Carolingian court had been keenly interested in consistency and orthodoxy of practice within the church and did much to set precedents. Charlemagne called for a

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[10] Byrhtferth, *Enchridion* 1.1.134–35; eds. and trans. Baker and Lapidge, 12–13: "Sequens figura hec que ore sanximus cunctis puro lumine prodit scire volentibus."
[11] Ibid. 1.2.351–55; eds. and trans. Baker and Lapidge, 44–45: "Her we hig wyllað amearkian, þa epactas and eac þa regulares lunares, þæt hig openlicre and orpedlice standun beforan þæs preostes gesyhðe."
[12] Ibid. 3.2.152–53; eds. and trans. Baker and Lapidge, 144–45: "Warna þe þonne se embolimus beo, forþon þænne þry monðas sceolon habban þrittig nihta ealdne monan, swa ic þe wille nu þis don to wynsumere bysne."
[13] Ibid. 2.3.248; eds. and trans. Baker and Lapidge, 120: "We gestton on þissum enchiridion þæt ys manualis on lyden and handboc on Englisc." Both terms signal the small size of the work (Ashmole 328 measures only 198 x 127mm), which suggests that it was intended for use by an individual. The *Enchiridion* shrinks even further when set beside Oxford, St. John's College MS 17 (340 x 250 mm) a copy of the computus it is thought to have served.
[14] On the history of the term see Wallis, in *Bede: The Reckoning of Time*, 425–26.

uniform liturgy; all masses were to follow the Roman rite.[15] *When* feasts were celebrated was as important to establish as *how* they were to be celebrated. Proper setting of the liturgical calendar and synchronized celebration of the moveable feasts was central to liturgical reform. The only way to ensure uniformity of practice across the empire was to train local priests and clerics in the art of computus. This charge was conveyed to regional church, monastery, and lay officials by means of widely dispersed documents like the *Admonitio generalis* (789). These were adhered to and guided practice long after their issue, well into the central middle ages. Bishops were tasked with the implementation of imperial directives, but the ultimate success of these statutes relied as much if not more on local polities and monasteries, as they did on episcopal enforcement.[16] It also depended on the abilities of schoolmasters like Byrhtferth to make computus comprehensible to a diverse audience of varying abilities and backgrounds. The *Enchiridion* was a product of that effort at Ramsey.

Those schooled in computus required the tools necessary to determine the date of Easter correctly and independently, namely, a Julian Calendar, the Paschal Tables (tables projecting the date of Easter into the future), and *argumenta* (computus formulas).[17] A manuscript containing these elements was called a computus. Just as Carolingian insistence on literacy in Latin led to a boom in production of grammatical texts, so too the requirement to learn and practice computus sparked an upswing in the making of computus manuscripts.[18] Examples survive in great numbers, suggesting that even the most remote chapel was supplied with one.[19] Yet the precise form that these books took varied significantly. The

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[15] Giles Brown, "Introduction: The Carolingian Renaissance," in *Carolingian Culture: Emulation and Innovation*, ed. Rosamond McKitterick (Cambridge: Cambridge University Press, 1994), 20–21.
[16] Marios Costambeys, Matthew Innes, and Simon MacLean, *The Carolingian World* (Cambridge: Cambridge University Press, 2011), 115; and Brown, "Carolingian Renaissance," 44–45. Byrhtferth mentions in passing in (*Enchiridion* 1.2.323–25) how the priests will be tested by their bishop on the subject of epacts.
[17] For an explanation of the mechanics of setting the date of Easter see Faith Wallis, "Computus: The Problem of Determining Easter," in *Medieval Science, Technology, and Medicine: An Encyclopedia*, eds. Thomas Glick, Steven J. Livesey, and Faith Wallis (New York: Routledge, 2005), 139–40.
[18] Ælfric, Abbot of Eynsham (c.955–c.1010) in a letter to Archbishop Wulfstan of York stated that priests must have "spiritual arms," that is "divine books" and then lists numerous liturgical books (e.g., a missal, lectionary, gradual). A *compotum* or "computus manuscript" is included among these essential tracts. Ælfric states that these books should be "well corrected" and "known" by the priest. ("Presbyter debet habere etiam spiritalia arma, id sunt divinos libros, scilicet missalem, lectionarium, quod quidam vocant epistolarium, psalterium, nocturnalem, gradalem, manualem, passionalem, paenitentialem, compotum, et librum cum lectionibus ad nocturnas ... Hos libros debet ipse [presbyter] habere bene correctos et hos scire.") Ælfric, *The Letters, Second Latin Letter to Wulfstan* XIV, ed. M. D. Elliot and T. J. Major, University of Toronto, http://groups.chass.utoronto.ca/aelfric/AEL2.html (accessed December 16, 2014).
[19] For approximate numbers of MSS containing computus material see chapter 1, note 133.

calendar and tables at the core of every computus were joined by a farrago of material on time, broadly defined. The composition and character of this halo of associated texts changed over time.[20] During the central middle ages, computus underwent what might be called a visual turn. Faith Wallis has described the "extraordinary outburst of creativity in the contrivance of *computus* tables" during this period.[21] The chief catalyst of that change was Abbo. He created a computus outstanding in its plentitude of profoundly visual devices, which would be heavily copied on the continent and in England by, among others, Byrhtferth (figs. 1.93–100 and 4.7–11).[22] Byrhtferth adopted the core of Abbo's computus and added elaborate, original, full-page figures (figs. 4.12–16).

The exact contents of Byrhtferth's computus are disputed by scholars.[23] No contemporary copy survives, and extant witnesses were made at least a century later.[24] The lost work is thought to be best preserved in a twelfth-century manuscript, Oxford, St. John's College MS 17, produced at nearby Thorney Abbey (about eleven miles from Ramsey as the

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[20] Computus manuscripts are stubbornly "uncanonical"—a phrase coined by Faith Wallis to describe medical manuscripts of the early and central middle ages. Wallis defined an "uncanonized" text as, "a text which is deliberately reorganized, interpolated, abbreviated or otherwise altered." She used the term in relation to pre-Salernitan (6th to the 11th c.) medical manuscripts, but it applies as well to computus manuscripts, though these sometimes *contained* canonical texts like the works of Bede and excerpts from named authorities including Macrobius and Isidore (see chapter 1); Faith Wallis, "The Experience of the Book: Manuscripts, Texts, and the Role of Epistemology in Early Medieval Medicine," in *Knowledge and the Scholarly Medical Traditions*, ed. Don Bates (Cambridge: Cambridge University Press, 1995), 103–04.

[21] Wallis notes that computus was inherently visual, since it had at its core not texts, but tables and arithmetical *argumenta*. Wallis, "Images of Order in the Medieval Computus," in *Ideas of Order in the Middle Ages,* ed. Warren Ginsberg (Binghamton, NY: The Center for Medieval and Early Renaissance Studies, 1990), 45–68, esp. 54; and subsequent publications and talks.

[22] Abbo's computus is known to be preserved (at least in part) in thirty-three extant manuscripts (thirty-four if the destroyed Chartres, BM MS 55 [*olim* 75] is counted). Twenty-five of these were produced on the continent; eight in England. Of the eight manuscripts of English origin three are copies of Byrhtferth's computus. The most complete list of manuscripts can be found in Barbara Obrist, ed., *Abbon de Fleury: Philosophie, sciences et comput autour de l'an mil*, Oriens–Occidens (Paris: C.N.R.S. and E.P.H.E., 2006), 239–40.

[23] On Byrhtferth's authorship see Baker and Lapidge, in Byrhtferth, *Enchiridion*, xxv–xxvi; and Peter S. Baker, "More Diagrams by Byrhtferth of Ramsey," in *Latin Learning and English Lore: Studies in Anglo-Saxon Literature for Michael Lapidge*, ed. Katherine O'Brien O'Keeffe and Andy Orchard (Toronto: University of Toronto Press, 2005), 53–73; and Wallis, "2. Computus Related Materials: 20. Byrhtferth's Diagram," McGill University, http://digital.library.mcgill.ca/ms-17/folio.php?p=7v&showitem=7r_2ComputusRelated_20ByrhtferthsDiagram (accessed January 28, 2015); and Wallis, "3. *Computus* Tables and Texts I: 20. Byrhtferth's *Proemium*," McGill University, http://digital.library.mcgill.ca/ms-17/folio.php?p=12v&showitem=12v-13r_3ComputusTablesTextsI_20ByrhtferthProemium# (accessed January 28, 2015).

[24] As already observed, computus manuscripts fall into a class of "uncanonical" works, notoriously elastic, constantly subjected to alterations and redactions.

crow flies), sometime between 1102 and 1113.[25] Byrhtferth's computus and probably his *Enchiridion* were among the chief sources that the Thorney monks relied on when making St. John's 17, which contains Abbo's revised calendar, tables, and his brief astronomical tracts as well as items attributed to Byrhtferth.[26] Byrhtferth's name appears twice in the manuscript. He is credited as author of an extended preface[27] and as designer of a complex figure showing temporal and spatial concordances (fig. 4.15).[28] This last, usually referred to as Byrhtferth's diagram, resembles in form and content the many figures that populate the *Enchiridion*.[29] Modern scholars have tended to read and understand the *Enchiridion* as a companion or commentary to a particular computus manuscript. The *Enchiridion* does not itself contain a Julian calendar; to put its lessons into practice one would certainly have to

[25] On the dating and place of origin of St. John's 17 see Wallis, "Background Essay: St. John's 17: Location and Dating," McGill University, http://digital.library.mcgill.ca/ms-17/apparatus.php?page=MS_17_Location_and_Dating (accessed December 18, 2014).

[26] Wallis, who has studied St. John's 17 in depth, describes the relationship between it, Byrhtferth's *Computus*, and his *Enchiridion* in this way: "[St. John's 17] incorporates the computus manuscript of Byrhtferth of Ramsey, and in so doing, the computus of Abbo of Fleury that lay behind it; but it also includes Abbonian material not found in Byrhtferth's writings or the other manuscripts connected to him. As well, it picked up items from two non-Abbonian English lineages, as well as older Carolingian anthologies." Ibid.

[27] St. John's 17, fols. 12r–13v. Byrhtferth mistakenly called his prologue an *epilogus*. The misuse of this Greek term helps confirm his authorship, since he used the same term—again incorrectly—to refer to prefatory material in the *Vita S. Ecgwini*, which he also wrote. Heindrich Henel seized on the term to argue that the prologue belonged to the *Enchiridion* and would have bridged Byrhtferth's handbook with "authoritative" tracts on computus by Bede, Abbo, and Helperic, which are mentioned in the prologue. Part of the strength of Henel's argument lay in the fact that a formal prologue is a computus oddity. If, however, we think of computus as a quadrivial discipline and computus manuscripts as a kind quadrivial manuscript the addition seems less strange—part of the genre of introductory commentaries like Abbo's *Explanatio* and Gerbert's *Isagoge* that proliferated in the period. Henel, "Byrhtferth's Preface: The Epilogue of His Manual?," *Speculum* 18, no. 3 (1943): 288–302; and Michael Lapidge, "Byrhtferth of Ramsey and the Vita S. Ecgwini," *Mediaeval Studies* 41 (1978): 337, n. 32.

[28] St. John's 17, fol. 7v. Text in the upper left corner of the page reads, "Bryhtferð, monk of Ramsey monastery, set forth this figure concerning the concordance of the months and elements" (*Hanc figuram edidit bryhtferð monachus ramesiensis coenobii de concordia mensium atque elementorum*).

[29] Charles and Dorthea Singer identified the elaborate concordance figure in St. John's 17 as a copy of a full-page figure that had been cut from Ashmole 328. More recently, scholars have questioned the idea of a one-to-one correlation between the two images. It seems likely the St. John's 17 figure was created by or at least based on a figure by Byrhtferth, given the contemporary attribution to "Bryhtferð" as well as the striking visual and thematic similarities between it and the images of the *Enchiridion*. Charles Singer and Dorothy Singer, "A Restoration: Byrhtferth of Ramsey's Diagram of the Physical and Physiological Fours" *Bodleian Library Record* 2 (1917–19): 47–51; Baker and Lapidge, in Byrhtferth, *Enchiridion*, 256, n. on fig. 3; and Wallis, "2. Computus Related Materials: 20. Byrhtferth's Diagram," *The Calendar and the Cloister: Oxford, St John's College MS17*, 2007, McGill University Library, Digital Collections Program, http://digital.library.mcgill.ca/ms-17/folio.php?p=7v&showitem=7r_2ComputusRelated_20ByrhtferthsDiagram#sec04 (accessed January 19, 2015). Wallis's treatment of the diagram remains the most thorough-going to date. The figure has been discussed by historians of art, noteably Kuhnel, *End of Time*, 181–83; Esmeijer, *Divina Quaternitas*, 38 and 60; and Sears, *Ages of Man*, 34–35.

turn to a computus like St. John's 17. Though perhaps written with this purpose in mind, the work need not be considered only as an ancillary document, but also as a textbook in its own right.

The *Enchiridion* offers unusual insight into the pedagogic role that one contemporary teacher assigned to the visual aspects of computus, and, to a certain extent, to quadrivial images more generally, since computus encompassed arithmetic, geometry, and astronomy. Byrhtferth reworked the opening lines of Genesis so as to bring their numeric significance to the fore:

> As Genesis (that is, the first book of the Bible) says, all-creating God fashioned two mighty luminaries and the planets and all the stars. He established two solstices, one of which will always fall on 21 December, the other always on 20 June. He adorned and arranged the twelve months in two equinoxes, he, who through his might alone furnished the year with four seasons and twice two elements, just as the following formula (*formula*) will demonstrate (*demonstrabit*) to those who examine (*cernentibus*) it.[30]

Here, Creation is tantamount to crafting the calendar. Time is reduced to number: two solstices, twelve months, two equinoxes, four seasons, four (twice two) elements. Numeric congruence—all the twos, fours, and twelves—is divine. Byrhtferth believed that the recognition and discovery of number in the cosmos, computus, could, in turn, lead one back to the Creator. The above-quoted passage was followed by a figure (Byrhtferth used the term *formula*) that revealed the numeric order of Creation to those "who probed it with their senses" (*cernentibus*), especially the sense of sight (fig. 4.17). According to Byrhtferth, pictures did not merely facilitate numeric rationale, but were essential to engendering such thinking. Indeed, the figures in the *Enchiridion* yield meaning to a certain kind of scrutiny very different from that demanded by words. I suggest that this way of seeing was taught along with such essential computus matters such as the calculation of concurrents, and that graphic aspects of the *Enchiridion* served as sites for ocular exercise and revelation.

[30] Byrhtferth, *Enchiridion* 1.1.79–88; eds. and trans. Baker and Lapidge, 6–9: "Cum omnipotentia magnitudinis Domini cuncta mirabiliter creasset, "omnia," ut divina ait scriptura, "in mensura et in numero et in pondere" constituit. Fecit Deus omniparens, ut Genesis testatur, ... duo magna luminaria et stellas et omnia sidera. Constituit duo solstitia, unum quod erit .xii. kalendas Ianuarii, alterum quod erit .xii. kalendas Iulii. Exornavit atque ordinavit .xii. menses in binis equinoctiis qui sola potentia annum compsit quattuor temporibus et bis binis elementis, sicuti formula sequens demonstrabit cernentibus." Byrhtferth restated the passage word-for-word in Old English—this was, evidently, a lesson to be understood by all his readers. The Old English even concludes with the same statement as the Latin: that a figure will show what has been written to "all who look at it" ("swa þis gefeig ætywð eallum þe hyt sceawiað").

Byrhtferth had himself been taught these habits of eye and mind, this kind of computus, by Abbo.

1. Abbo and computus at Ramsey

> O noble throng of Ramsey, secluded by spreading waters,
> You strive to be purer than gold for God's sake.
> The vast fen, abounding in fish, yields its secrets
> So that new confinements of the wilderness may be found for you.
> For where the destroyer of the raven-bearing Hydra arises,
> There gleams an exquisite island with its woodland waters;
> And where the reins of gleaming Bootes sink,
> There is a land-bridge accessible to all the English;
> Where the Lesser Bear rules its fixed orbit of the sky,
> There the eel-filled waters know no bounds.
> From there the sun's unsteady light draws back its sinister shadows;
> The earth lies open, linked up by no shallows.
> In this place I, a stranger, luckily gave myself over to unknown students:
> May Christ always save, honor and love them![31]

Byrhtferth folded these twelve verses, written by his teacher, Abbo of Fleury, into the *Life of St. Oswald* (*Vita Sancti Oswaldi*). In the *Vita*, the poem serves as a physical description of Ramsey abbey, which Oswald founded c.966. It conveys the seclusion of the place—which Abbo called an "island" (*insula*), despite the land bridge—as well as the bounty of the environment, and the virtue of its inhabitants. As described in chapter two, we know from Abbo's own *Vita* that Archbishops Dunstan of Canterbury and Oswald of York had invited

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[31] Byrhtferth, *Vita S. Oswaldi* 3.18; ed. and trans. Lapidge, 92–93:
"O Ramesiga cahors, amplis que claudere stagnis,
purior obrizo niteris esse Dea.
Vasta palus, piscosa nimis, sua dindima pandit,
ut noua sint heremi claustra reperta tibi.
Nam qua coruifere consurgit proditor Hidre
insula silvoso gurgite pulchra nitet;
et qua splendentis se mergunt Iora Bootis,
pons est inde suis peruius Angligenis;
qua Cynosura poli fixum regit undique girum,
anguillosa palus nescit habere rnodum.
Inde refert umbras vaga lux Phebea sinistras;
terra patet nullo continuata vado.
Qua me sorte dedi ignotis, ignotus, alumnis:
quos Christus semper saluet, honaret, amet!"

Fleury's brilliant *armarius* across the Channel to England to teach.[32] Byrhtferth was among the "unknown" students mentioned in the poem, who welcomed Abbo as their schoolmaster between 985 and 987 and cherished the memory of the "lover" of God and wisdom long after his visit.[33]

Abbo's poem, written for the Ramsey community, assumes knowledge of computus,[34] specifically, the positions and movements of the constellations in the night sky above Ramsey: the "hydra destroying" Hercules, which rose in the east; "Bootes" (also known as the Wagoner or *Arcas*), which set to the west; and the Lesser Bear, which—never rising or setting—pivoted overhead to the north.[35] Information on the constellations was found in Cicero's *Aratea*, a Latin translation of the Greek *Phaenomena* (*Celestial Phenomena*) by Aratus (c.315–before 240 BCE), and in the *De astronomia*, an explanation of the *Aratea* by the first-century librarian Hyginus.[36] Abbo knew both works intimately—he had penned an epitome on the third book of the *De astronomia*, which was, essentially, a star catalog.[37] He had good reason to assume familiarity with the work on the part of his English audience, for he probably had introduced them to it. British Library Harley MS 2506, a cannily crafted and deftly illuminated cosmological collection, is thought to have been produced at Fleury but owned by Ramsey, where it may have been brought by Abbo (fig. 4.18).[38] The manuscript

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[32] The invitation should be considered part of widespread reform efforts that reinvigorated monasteries in this era, especially in England and Gaul.

[33] Byrhtferth, ever eager to show off his knowledge of Greek, calls Abbo a *phylosophus* and *theophilo*. Abbo probably taught the boy when he was in his late teens. Baker and Lapidge, in Byrhtferth, *Enchiridion*, xxxiii.

[34] The poem was paired with the *Quaestiones grammaticales* in Erfurt, WAB MS Amplon. 4^{o}.53, f. 73v; and London, BL Add. MS 10972, 47v. It will be remembered that the *Quaestiones*, discussed in chapter 3, was also written for Abbo's Anglo-Saxon charges. Byrhtferth was not the only one to quote the poem; Andrew of Fleury included it in the *Vita Gauzlini*. See Lapidge, in *Byrhtferth of Ramsey: The lives of St Oswald and St Ecgwine*, 91–92, n. 176.

[35] Ibid., 92, nn. 177–80.

[36] The identity of Hyginus is not conclusive, but he likely was C. Julius Hyginus, keeper of the Palatine Library in Rome.

[37] Van de Vyver, "Les œuvres," 141; and idem, "Les plus anciennes traductions latines médiévales Xe-XIe de traités d' astronomie et d' astrologie," *Osiris* 1 (1936): 678. The second book of the *De astronomia* lists the constellations, and the third book describes them, including information about the arrangement of stars, their mythic names, and their relative positions in the sky. It is unclear exactly when Abbo wrote his summary of book three. It is almost certain that he authored two minor astronomical tracts while at Ramsey: *De duplici signorum ortu vel occasu* (*The Double Rising and Setting of Signs*) and *De quinque circulis* (*Concerning the Five Circles*). Both (very brief) tracts explain basic concepts and were probably written especially for the Ramsey community, who seem to have had little exposure to astronomical material prior to his visit. All but one of the five extant copies of these works were written in England. For editions see Ron B. Thomson, "Further Astronomical Material of Abbo of Fleury," *Mediaeval Studies* 50 (1988): 671–73.

[38] Richard Gameson and Jean Vezin agree that this manuscript was written at Fleury, illuminated by an English artist, and glossed by an English hand. Van de Vyver states that either this manuscript or its exemplar "almost

opens with the *De astronomia*; the *Aratea* is at its heart.[39] Byrhtferth quoted Hyginus in the prologue of Oswald's *Vita*. Indeed, he is the only Anglo-Saxon author who shows knowledge of the *De astronomia*, suggesting that he had special access to it.[40]

Ramsey, when Abbo arrived, well equipped with texts for teaching the trivium (at least grammar), was less well stocked with quadrivial works.[41] There is little doubt that the fenland monastery owned copies of the English Bede's scientific oeuvre, including the *De temporum ratione*, a staple of computistic study, and the *De natura rerum*, his treatise on cosmology, as well as Isidore's encyclopedic *Etymologiae*.[42] But Abbo introduced new works. The cosmological collection Harley 2506 is one of a number of manuscripts that passed between Ramsey and Fleury during this period. Significantly, many of these books can be classed as quadrivial, including Helperic's *De computo*, Macrobius's *Commentary on the Dream of Scipio*,[43] and the *De arithmetica*. Abbo also brought with him a copy of his computus. We have encountered Abbo's computus already in chapters one and three. The work is thought to be best preserved in Berlin 138, which was produced at Fleury while

certainly" (*vraisemblablement*) went with Abbo to Ramsey ("Les œuvres," 149–50). Likewise, Lapidge wrote that Harley 2506 was illuminated "either at Fleury or in England." He would like to attribute a number of glosses to Byrhtferth himself and states that the manuscript was "probably a Ramsey book" (Lapidge, *The Anglo-Saxon Library*, 51–52). If Harley 2506 was, in fact, brought to Ramsey by Abbo, or—more likely—sent later, perhaps as a gift, then it would have served as the vehicle of transmission of Hyginus's *De astronomia* to England. The manuscript—carefully arranged and finely executed—also includes Abbo's astronomical tracts and two of his acrostic poems, which frame the *De astronomia*, along with excerpts from Pliny, Macrobius, and Martianus. Richard Gameson, "An Itinerant English Master around the Millennium," in *England and the Continent in the Tenth Century: Studies in Honour of Wilhelm Levison (1876–1947)*, eds. D. W. Rollason, Conrad Leyser, and Hannah Williams (Turnhout: Brepols, 2010), 87–134, esp. 100, n. 22, 100–105, and 124–25; Jean Vezin, "Leofnoth: Un Scribe anglais à Saint-Benoît-sur-Loire," *Codices manuscripti* 3 (1977): 109–20.

[39] Harley 2506, fols. 1r–30r and 36r–48v. Between the two works are Priscian's *De duodecim signis* (*On the Twelve Signs*), fol. 30r–30v; two short tracts by Abbo, *De differentia circuli et sphere* (*On the Difference between a Circle and a Sphere*) and *De cursu septem planetarum per zodiacum circulum* (*On the Orbits of the Seven Planets through the Zodiac*), fols. 30v–32r, and an anonymous tract on the names of the stars, *De nominibus stellarum*, fols. 33r–35r.

[40] Byrhtferth, *Vita S. Oswaldi* Prologus; ed. and trans. Lapidge, 4.

[41] For a more thorough discussion of the make-up of Ramsey's library at the time of Abbo's visit see chapter 2. On the sources of the *Enchiridion* see Baker and Lapidge, in Byrhtferth, *Enchiridion*, lxxiv–xciv.

[42] In their introduction to the *Enchiridion* Baker and Lapidge write, "In a word B[yrhtferth]'s debt to Isidore's encyclopedia is pervasive." Nineteen copies of the *Etymologiae* can be connected to Anglo-Saxon libraries. Baker and Lapidge, in Byrhtferth, *Enchiridion*, lxxxii; and Lapidge, *Anglo Saxon Libraries*, 311.

[43] It has been suggested that Abbo brought Paris, BnF lat. MS 7299, which contains Macrobius's *Commentary* and Helperic's *De computo* (as redacted by Abbo), with him to Ramsey and, perhaps, back to France when he returned to Fleury. The manuscript once also contained the *Calculus* of Victorius and, possibly, Abbo's *Explanatio*. Its connection to Ramsey is manifest in a calendar (between the now-lost *Calculus* and the *De computo*) of English (Ramsey, or the vicinity) origin and written by an English hand. See also chapter 2.

Abbo was abbot, presumably under his supervision, and completed just after his death.[44] The manuscript begins with excerpts from Macrobius, an abridged version of Victorius's *Calculus*, and a complete text of the *Explanatio*.[45] These are followed by a modified Julian calendar linked to a set of elaborate tables (figs. 4.7–11).[46]

The standard calendar, of the sort described and promoted by Bede, generally comprised four columns: one for numbers representing the date; another, broader column, to the right of the date column, for the martyrology and notes; and two narrow columns to the left, filled with what are called key letters (fig. 4.19).[47] These key letters were to be used in conjunction with tables for determining essential information such as the day of the week, the age of the moon on a particular day, or its position in the zodiac (fig. 4.20). Though Bede, the premier authority on computus, included such tables and gave instructions for their use in *De temporum ratione*, he was wary of them, stating that that they promoted ignorance and laziness, since they replaced the need for reckoning.[48] It was precisely for this reason that they had been developed: in order to avoid arithmetical errors that might arise through careless or unskilled calculation. Rather than using a mathematical *argumentum* or formula to determine, for example, the age of the moon, one only had to note the key-letter for a given day in the calendar, flip to the corresponding table, locate that letter in the appropriate column, and scan across to a line of numbers representing the moon's age. Unlike Bede, Abbo approved of and encouraged this kind of mechanical, book-based computus.

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[44] Another contemporary copy of Abbo's computus from Fleury is Bern 250, discussed in the last chapter. The contents of this manuscript are enumerated by Hermann Hagen, *Catalogus codicum Bernensium* (Bern: Haller, 1875), 286.
[45] Macrobian excerpts, fol. 1r; *Calculus*, fols. 1v–3r; excerpts from Rabanus's *De computo*, Isidore's *Etymologiae*, Bede's *De temporum ratione*, Alcuin's *De rhetorica*, and Ambrose's *Hexameron*, fols. 3v–7; Abbo's *Eplanatio,* fols. 7v–21v. A detailed description of the contents of Berlin 138 is provided by Valentine Rose in *Die Handschriften*, 308–15.
[46] Spanning fols. 23r–34v.
[47] Bede discusses the Julian calendar in the *De temporum ratione*, chapters 5–41.
[48] Chapter 19 of the *De temporum ratione* describes the *pagina regularis*, a table for determining the position of the moon in the zodiac. Chapter 23 describes the *litterae punctatae* table for finding the age of the moon on a particular day (so-called because some of the letters are preceded and/or followed by points). Faith Wallis suggests that Bede's distain for such tables stemmed from a concern about their accuracy, since they were notoriously difficult to copy correctly. She also points out that Abbo's tables, because they are patterned (see, for example, fig. 4.10), are particularly easy to copy, since an error is immediately apparent. Wallis, "4. Calendar fols. 16r–21v Overview," *The Calendar and the Cloister: Oxford, St John's College MS17*, 2007, McGill University Library, Digital Collections Program, http://digital.library.mcgill.ca/ms-17/folio.php?p=16r&showitem=16r-21v_4Calendar_Overview#sec03 (accessed December 22, 2014).

Abbonian calendars, like that in Berlin 138, contain as many as eight key-letter columns corresponding to at least as many tables (figs. 4.7 and 4.21). The tables travelled as a unit immediately following the calendar. Five of the eight columns of Abbo's calendar contained key-letter sequences for determining the age of the moon. Such redundancy was intended and even emphasized. In Berlin 138, tables are paired and arranged in a way that highlights similarities. The two so-called A-E-I-O-U tables on the recto of folio 34, named for the series of vowels that serve as their key letters, mirror each other (fig. 4.10 and 4.22–23).[49] Both tables are cross-shaped and drawn in a vivid red pigment. They are joined by a row of numbers that spans the space between them.[50] "The following figure," Abbo explains in the adjacent text, referring to the lower table, "is similar in all respects to that one placed above with the exception that the lunations and the years of the nineteen-year cycle are noted with Greek letters" (figs. 4.22–23).[51] Inspection of the lower table reveals that, in fact, all numbers, written in Roman numerals in the upper table, have been replaced by Greek alpha-numeric signs in the table below. The two tables are, in all other respects, identical.[52]

The A-E-I-O-U tables are one of several table pairs in Abbo's computus. One might only *need* a single table to determine the moon's age, but the schoolmaster of Fleury believed that there was something to be gained by having many tables, yielding the same information differently. The result is a computus that—counter to Bede's concerns—requires an unusually high degree of visual attention, a vigorous gaze or, in Abbo's words, a "curious eye" (*oculus curiosus*). The phrase appears in Abbo's short tract on the nature of number and the motion of the moon.[53] Abbo instructs his reader to look at (*intuens*) the moon in relation

[49] See Wallis, "5. Computus Tables and Texts II: 8. Lunar Letters AEIOV: Table and Text," *The Calendar and the Cloister: Oxford, St John's College MS17*, 2007, McGill University Library, Digital Collections Program, http://digital.library.mcgill.ca/ms-17/folio.php?p=24v (accessed December 20, 2014).
[50] This row represents years sixteen through nineteen of the nineteen year cycle. The last six rows that extend below the lower table are also shared.
[51] Berlin 138, fol. 34r: "Sequens figura per omnia precedenti similis est. exepto quod lunationes et anni cicli .xviiii.lis grecis notantur litteris." Swapping sign systems is extremely common in Berlin 138, though it is not always so explicitly referenced in the accompanying verbal narrative, as with the A-E-I-O-U tables.
[52] The Greeks used several different counting systems. Here and elsewhere in Berlin 138 a place-value method is employed, which uses only the first nine alpha-numeric symbols that are combined with new symbols for different tens and hundreds. For a description of the three systems see Karl Menninger, *Number Words and Number Symbols: A Cultural History of Numbers*, trans. Paul Broneer (Cambridge: MIT Press, 1969), 262–63.
[53] The tract often travelled with Abbo's computus and with Helperic's *De computo.* Regarding the context of this and Abbo's other so-called astronomical tracts, their making and use, Thomson followed Van de Vyver, who saw them as "an appendix to Abbo's version of Helperic's *De computo* or ... a companion piece to the star catalogues." Thomson, "Two Astronomical Tractates of Abbo of Fleury," in *The Light of Nature: Essays in the*

to the stars with an *oculo curioso* and observe its position against the backdrop of the zodiac.[54] Similar prompts, connecting scrutiny to discovery and comprehension, are common in the explanatory passages that Abbo wrote to accompany each of the tables in his computus. Hence, the same ocular interrogation of the heavens was to be applied to the page. Indeed, Abbo's computus is a profoundly "codified" work that both relies on and exploits the codex structure.[55] Abbo conceived the calendar and tables as a suite to be experienced in sequence. In many copies, the order of the key-letter columns in the calendar corresponds to the order of the tables.[56] Placement of paired tables back-to-back or side-by-side drew attention to their likeness and redundancy. Their use required flipping back and forth. In the explanatory passages that accompany each table Abbo instructed the reader/viewer to fetch a datum from one page and apply it to another. Thinking spatially, he routinely referred to the placement of figures on the page in relation to each other and to the text "above," "below," "beside," even "right" and "left"—a tendency we also saw in the verbal narrative associated with the images in the *Explanatio*. The mapping of computus calendars and tables to the page is captured in the use of the term *paginula* by Bede and Abbo to refer to these elements.[57]

In the *Vita Abbonis*, Aimoin wrote of Abbo's computus: "He [Abbo] composed (*texuit*) certain computistical calculations both various (*varius*) and delightful (*delectabilis*) after the fashion of tables of years [i.e., calendar tables]."[58] The choice of words captures the particular nature of Abbo's contribution to computus and the features of his tables that made them so distinct, and, perhaps, so popular and long-lived. Abbo, Aimoin writes, painstakingly "wove" (*texuit*) his calculations into tables as one would weave textiles. The verb aptly

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History and Philosophy of Science Presented to A.C. Crombie, eds. John David North and J. J. Roche (Dordrecht: Martinus Nijhoff Publishers, 1985), 115–18.

[54] Abbo, *De cursu vii planetarum per zodiacum circulum* 98–101; ed. Ron Thomson, in "Two Astronomical Tractates," 127: "Cognitis quoque punctis partibus et quo signa movetur sicut argumenta produnt, per omnium lunationum singulos dies curioso oculo per singulas noctes intuens eadem luna quo loco sil et quas stellas in circuitu sui habeat."

[55] This was, to a certain extent, also true of other computus texts that preceded Abbo's computus. Bede's direction for how to use the key letters of the calendar began: "with the book having been opened..." (*aperto codice*; Bede, *De temporum ratione* 29.20; ed. Jones, 220). References to the book and to individual pages was, however, more pronounced in Abbo's computus.

[56] In some copies of Abbo's computus the order of the key-letter columns determined the order of the tables, as in St. John's 17. In Berlin 138 the tables were not so-ordered, but a contemporary hand added notes at the top of each page containing a table directing the user to the relevant column in the calendar (the term "line" *linea* was used for "column"). This information was also embedded in Abbo's verbal explanations.

[57] See, for example, Berlin 138, fol. 34r: "Paginula ista."

[58] Aimoin, *Vita* 3; eds. Bautier and Labory, 48: "compotique varias et delectabiles, saecularium in morem tabularum, texuit calculationes."

evokes the crisscrossing of the vertical and horizontal lines that compose each table as well as the careful amalgamation of calendar and tables into an integrated whole. Aimoin goes on to describe the tables as *varius* and *delectabilis*. The first term, which might be translated as "variegated," "manifold," or "various," may refer to Abbo's careful use of color to enhance legibility, how he reformatted traditional tables so that their contents would fall into repeat patterns, or the way that he reshaped standard, rectangular tables into meaningful forms—mainly circles and crosses, as he did the A-E-I-O-U tables. In instances where more than one version of a traditional table was available, as, for example, the table of epacts, which had been passed down in both a rectangular and a circular form, Abbo consistently chose the more dynamic, circular option. Hence, through processes of invention and selection, Abbo created a strikingly visual form of computus. *Delectabilis*, the second term Aimoin applied to the tables, tells us of the pleasurable and stimulating effect that such innovations had on contemporary viewers, Byrhtferth included.

Byrhtferth wrote that it was through Abbo that he first encountered *computus*. His extant works on the subject, his computus (988–96) and *Enchiridion*, bear out this claim. We can see what might be called Abbonian tactics at play in his use of repetition and redundancies, his structural use of color, and a strong preference for meaningful forms discussed below. Abbo had streamlined computus, stripping away the ungainly accretions of loosely related material, cutting out redundant texts and inserting a handful of brief explanatory tracts.[59] At the same time, however, he *added* complexity and new redundancies to the computus corpus.[60] He reconfigured and reshaped the standard calendar and tables into what historian of medicine and science Faith Wallis has called "evocative icons of order"

[59] Abbo's contribution to computus is described by Van de Vyver, "Les œuvres inédites," 140–58. Baker and Lapidge describe early computi as "unwieldy, redundant, and inconsistent ... Many of their tables were all but useless, being unaccompanied by explanations." Abbo, they say, "pruned these old books, supplied explanations for both the tables he retained and those he composed, and imposed order on the resulting, more tractable collection of materials ... That it [Abbo's computus] achieved great popularity, then is of no surprise; with it a reasonably well-educated priest could calculate the dates of the moveable feasts for himself. He no longer had to rely on possibly inaccurate Easter tables or wade through long textbooks and hundreds of *argumenta*" Baker and Lapidge, in Byrhtferth, *Enchiridion*, xlv. I am skeptical about the earlier tables being "useless," without explanations. The language of the *Enchiridion* suggests that oral explanation was extremely important to transmitting computus. Handbooks like the *Enchiridion* may have been typical, but travelled in more ephemeral forms, possibly as limp bound pamphlets. Baker and Lapidge asserted that it was the "utility" of the Abbonian computus (referring to the explanatory passages added by Abbo) that made it so popular (repeated in "More Acrostic Verse by Abbo of Fleury," 2). I have been unable to find evidence that would support this claim.
[60] Discussed in Wallis, "Images of Order," 54; and Obrist, "Les tables et figures Abboniennes," 141–86.

(figs. 4.7–12).[61] In some instances, these changes arguably would have impeded the legibility and utility of traditional tables. Indeed their addition may, at first glance, seem at odds with Abbo's more pragmatic interventions, which clearly facilitated teaching and practice of computus. Squaring these impulses requires a reevaluation of what we imagine was being taught, the ultimate aim of computistic study at Fleury and Ramsey.

2. BYRHTFERTH'S *ENCHIRIDION*

Byrhtferth was the schoolmaster of Ramsey when he wrote the *Enchiridion*. He was a thoughtful and utterly pragmatic teacher who deeply considered how best to convey the complexities of computus. Throughout the work, he berates and cajoles, scolding even whilst pondering how best to educate charges he deemed "ignorant," "lazy," and "rustic."[62] Byrhtferth reserved his strongest epithets for clerics and priests, who seem to have lacked the motivation and discipline of monks. Such language leaves little doubt that Byrhtferth found the task of teaching difficult and daunting. Nevertheless, it was one that promised significant satisfaction, even eternal reward. He found solace in the Old Testament, where it was written that "the wise man there will be a very honorable reward in the sight of God, if he imparts the knowledge of wisdom to a faithful young man."[63] Indeed, failure to do so might be deemed sinful: "in the sight of the just judge both will be guilty—those who do not wish to learn, and those who do not wish to teach."[64]

Fueled by such thoughts, Byrhtferth bent all his efforts to inculcating computus. Rarely in the *Enchiridion* does he miss a teaching moment, be it in the main text, the interlinear gloss, or the marginal commentary, all of which he is thought to have authored.[65] Even as he announces the start, the *incipit*, of the work, he regales his reader with synonyms

[61] Wallis "Images of Order," 53–54. Similarly, Barbara Obrist observed what she called a curious "surplus of signification" (*surplus de signification*) apparent in many medieval cosmological images. *La cosmologie médiévale*, 25.
[62] Byrhtferth, *Enchiridion* 1.3.1, 1.4.1, 2.3.44; eds. and trans. Baker and Lapidge, 46–47, 52–53, and 106–107: the terms are "imperitus," "deses," and, in Old English, "unplendicea." Such epithets are extremely common in the *Enchiridion*.
[63] Ibid. 3.3.2-4; eds. and trans. Baker and Lapidge, 162–63: "Daet byð snotrum were med swyðe arwurðlic beforan Godes gesihðe, gif he wisdomes lare geleaffullum esne cyð to soðe."
[64] Ibid.1.4.5–6; eds. and trans. Baker and Lapidge, 52–53: "Simul erunt rei in conspectu iusti arbitris: qui nolunt scire et qui nolunt docere."
[65] Peter S. Baker, "The Old English Canon of Byrhtferth of Ramsey," *Speculum* 55, no. 1 (1980): 34–36.

of this most common term—*inchoat, initium sumit,* and *exordium accipit*.[66] He shows himself deeply conscious of didactic method and style, justifying his use of simple prose, saying, "we [Byrhtferth] wish them [readers] to understand this saying, 'Earthen vessels are more useful the cheaper they are.' Although we are weak and we explain these things cursorily, they may do more good than those that are fairly adorned in verse."[67] Byrhtferth likened the *Enchiridion* to an elixir, a remedy to cure ignorance of computus:

> When once I was sitting quietly in an appropriate place and was contemplating the manifold wisdom of the computus, I began inwardly to ruminate on a few things among many, concerning what sort of medicine I might apply to the clerics so they would ease up on the dice and acquire some knowledge of this science.[68]

Continuing in this metaphoric vein, Byrhtferth characterizes the contents of his work as food and drink. The passage reveals a preoccupation with techniques and strategies of teaching, the *sort* of medicine, the *form* of the repast. Pictures and words were each key ingredients, often intermingled. Learning was accomplished through sight and sound. Indeed, the reader cannot but hear the text as spoken. Byrhtferth wrote in the first person and consistently chose verbs that connote speech. He peppered his treatise with breathy interjections like "if I may whisper in your ear!"[69]

Byrhtferth whispered the *Enchiridion* in two languages, Latin and Old English. All through the text he moves from the school tongue to the vernacular. To call this bilingual interplay "translation" does not capture his process. Some of the Latin is rendered word-for-word into Old English, some is paraphrased. Entire sections are written in one language or

[66] Byrhtferth, *Enchiridion* 1.1.5; eds. and trans. Baker and Lapidge, 2: "Incipit, id est inchoat vel initium sumit sive exordium accipit."
[67] Ibid. 1.2.195–200; eds. and trans. Baker and Lapidge, 36–37: "We wyllað þæt hig understandon þisne cwide: 'Vasa fictilia tanto solent esse utiliora quuanto et uiliora. Þeah we wace synand þas þing leohtlice unwreon, hig magon fremian bet þonne þa þe beoð on leoðwisan fægre geglenged."
[68] Ibid. 1.4.9–11, eds. and trans. Baker and Lapidge, 52–53: "Cum quodam tempore silens residerem in loco oportuno et perscrutatus essem multiplicem computandi prudentiam, cepi cordentus ruminare pauca ex plurimis, quali medicamine possem clericis proficere ut alee ludo relaxarent et huius artis notitiam haberent."
[69] Ibid. 1.1.195, eds. and trans. Baker and Lapidge, 18–19: "ut tibi in aure dico" The interjection appears again in Byrhtferth's Old English rendition of the passage (1.1.225). Another example: "We could say much more concerning the number six at this point, but so that we do not prolong our discussion, it is fitting to put a finger to our lips" ("Plurima de senario potuissemus calculo hoc in loco affari, sed ne prolongemus orationem nostram placet digitum ori opponere"). Ibid. 4.1.152–54, eds. and trans. Baker and Lapidge, 208–209.

the other.[70] English is the predominant language, presumably because it was spoken and read by clerics, priests, and monks alike. Byrhtferth states that it pleased him to repeat things in his own language, "so that those who cannot take in the sense of Latin may at least understand."[71] Latin, at least according to Byrhtferth, was known only to monks (and perhaps a few priests).[72] Generally, he wanted the matter of the text to be understood by *all* his readers and repeatedly wrote statements like this: "Now let us say these things in a different way so that what is clearly understood by monks may also be known to clerics."[73] Linguistic ability might explain significant redundancies between the *Enchiridion* and the computus that Byrhtferth was likely to have composed or used. The explanatory texts and instructions in St. John's 17 are all in Latin (some were glossed in Old English). *Enchiridion*'s Old English explanations would have permitted someone ignorant of Latin or with an extremely limited grasp of the language (e.g., a priest who knew, for example, the names of the months) to use the calendar and tables.

Pictures were, in a sense, the third idiom by which Byrhtferth sought to convey the nuances of *computus*. Implicit in their inclusion is the understanding that number is ineffable.[74] The short preambles with which Byrhtferth prefaced his pictures (a few of which I quoted in this chapter's introduction) make it clear that he considered them constitutive.

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[70] Following the divisions in the work according to Baker and Lapidge: Part 1 alternates between Latin and Old English, with the latter generally following the former. Part 2 is in Old English (except for some Latin terms in the lists and tables). Part 3 is in Old English (again, except for the odd quote and words in tables, figures, and lists). Part 4 begins in Latin with an interlinear translation (for the first few pages), and finishes in both Latin and Old English. The "friendly warning" at the end is in Old English. On "translation" Byrhtferth wrote: "We have written many things about computus in our own tongue because we wished that young men would therefore understand the Latin more easily and speak with old priests about these things more fully, and we wished these things to be known to them because they have been revealed to mankind with great labor." *Enchiridion* 2.3.248–53; eds. and trans. Baker and Lapidge, 120–21.

[71] Byrhtferth, *Enchiridion* 1.4.13–15; eds. and trans. Baker and Lapidge, 51–52: "Plurima affati sumus que iterum placet renovari nostro eloquio, ut qui Latinitatis elogium non potuerint sumere accipiant saltim vulgarem notsrum sermonem." He did not always relish the task of translation. See, for example, ibid. 1.1.214–16; eds. and trans. Baker and Lapidge, 20–21: "It seems to us too tedious to explain this whole passage to clerks in English."

[72] Byrhtferth wrote (in Old English): "Now we will set these things down for priests in Latin" (*Enchiridion* 1.3.87; eds. and trans. Baker and Lapidge, 50–51). The names of the months are written in Latin in the list of the lunar regulars that follows the statement. This is the only instance where Latin is directed at a priest. It should be remembered that Byrhtferth was himself a priest. When, in a passage written in Old English, Byrhtferth quotes the monks, their words are in Latin (e.g., *Enchiridion*, 2.2.20–22). Rather cruelly, Byrhtferth saves his harshest critiques of clerics for his Latin passages. See, for example, *Enchiridion* 1.4.1.

[73] Ibid. 1.1.115–16; eds. and trans. Baker and Lapidge, 12–13: "Iam alio modo dicamus qualiter sint clericis nota que monachis sint perspicue cognita"

[74] On the inherent materiality (and thus visibility) of numbers and objects for counting and calculation more generally see Brian Rotman, *Mathematics as Sign: Writing, Imaging Counting* (Stanford: Stanford University Press, 2000), especially 44–70.

Such statements are common in quadrivial manuscripts; we have encountered them already in the *De arithmetica* and Abbo's *Explanatio*. Macrobius's *Commentary on the Dream of Scipio*, a work Byrhtferth knew through Abbo, furnishes some of the most telling examples. The treatise, as we have seen, included five figures (figs. 1.88–92). Macrobius called these graphic elements "pictures" (*picturae*) or "descriptions" (*descriptiones*) and claimed that they "minimized the labor" of proving a point.[75] Frequently he voices a preference for visual explanation. In his discussion of the five climatic zones of the earth he stated that, "an idea (*concepta ratio*) slips (*inlabor*) more easily into the intellect by means of an image (*discriptione*) than by words."[76]

That images—material and mental—were understood by classical and medieval thinkers to play a crucial role in cognition, that they were, in fact, indispensable, is a notion the ramifications of which have been fruitfully plumbed by scholars of the middle ages, not least by Mary Carruthers. In *The Craft of Thought* (and subsequent publications), Carruthers expanded the definition of "image" to draw out distinctive features of medieval theories of cognition.

> Medieval and ancient writers do not distinguish between what we call "verbal" and "visual" memory; that the letters used for writing were considered to be as visual as what we call "images" today; and that as a result the page as a whole, the complete parchment with its lettering and all its decoration, was considered a cognitively valuable "picture."[77]

Yet medieval authors did not allow distinctions between the written and pictorial elements on a page to be entirely blurred. Both Byrhtferth and Macrobius state plainly that words did not work on the mind in the same way as "what we call 'images' today." The intellect was understood to be more easily accessed via vision. A point spelled out by Macrobius when introducing an image of the zodiac: "and because the easier way to the intellect is through the eyes, let that which speech has described be assigned to vision"[78] Pictures served the eye and sight, the implication being that the inscribed text reached a different faculty. Abbo argued in

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[75] Macrobius's preferred term was, by far, *descriptio,* He used *pictura* only once, to describe an image that shows the relationship between the zones of the sky and the zones of the earth: *Commentary* 2.7.31; ed. Willis, 117; trans. Stahl, 208: "sed hic quoque adserendi quod dicitur minuemus laborem oculis subiciendo picturam."
[76] Ibid. 2.5.13; ed. Willis, 112; my translation: "et quia animo facilius inlabitur concepta ratio descriptione quam sermone."
[77] Mary Carruthers, *Craft of Thought*, 122; also eadem, *The Book of Memory*, esp. 221–57.
[78] Macrobius, *Commentary* 1.21.3, ed. Willis, 85; trans. Stahl, 175: "et quia facilior ad intellectum per oculos via est, id quod sermo descripsit visus adsignet."

the *Explanatio* that although senses are not transferable (the eye cannot hear, nor the ear see), they can assist each other. He gives the example that the eye perceives numerical ratios which produce audible harmonies.[79] Byrhtferth seems to have internalized this lesson. In the *Enchiridion* eye and ear, approached through picture and word, assist each other, but they are not transferable, especially in the domain of number or "reckoning" (*ratio*). A closer look at the images in the *Enchiridion* reveals assumptions held by Byrhtferth and his circle about what words can do, what pictures can do, and how they might work together to mold minds.

3. Forging visual tools

Ashmole 328 has lost a number of its pictures and not all that remain were finished (figs. 4.4 and 4.17). Among the survivors, a few, like the circular figure of the zodiac, which tells how long the sun stays in each of the twelve signs, were a standard part of the computus corpus (fig. 4.24).[80] Others, like the A-E-I-O-U table of the lunar letters for finding the age of the moon and the circle of the lunar phases were less common, though, because authored by Abbo, they were well-known at Ramsey (figs. 4.6 and 4.25).[81] Most of the figures, however, were unique to the *Enchiridion*—original, if not wholly, of course, without precedent (figs. 4.2, 4.8, 4.26–30). Just as Byrhtferth paraphrased and summarized textual sources readily at hand or in mind, so too he fabricated his figures from preexisting forms and showed himself cognizant of widespread conventions. Drawing on this rich visual repertoire, Byrhtferth crafted figures for his students that were distinctive in form. What is striking is how loose and approximate his borrowings were. Departures from traditional models offer insight into his didactic strategies and aims.

Byrhtferth's process of adaptation is perhaps best captured in a figure showing the fourfold division of the year and a number of congruent quaternities (fig. 4.30).[82] He introduced the figure in this way:

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[79] Abbo, *Explanatio* 3.5; ed. Peden, 76. This passage is briefly analyzed in Evans and Peden,"Natural Science and the Liberal Arts in Abbo of Fleury's Commentary," 124.
[80] Ashmole 328, p. 117. The figure was lifted from Isidore's *De natura rerum*.
[81] Ashmole 328, p. 163 and 168. Both figures appear in St. John's 17 (fols. 24v and 38v). Twinned A-E-I-O-U tables (discussed above), also in a cruciform shape, are encountered in Berlin 138 (fol. 34r) as is a version of the figure of the lunar phases (fol. 39r).
[82] Byrhtferth's affinity for "tetradic thinking" as expressed in his figures and writing was discussed by Elizabeth Sears, who also analyzed this figure in her *Ages of Man*, 33–34.

> There are two solstices, two equinoxes, twelve signs and four seasons in the twelve months, and there are four ages of man, four letters in Adam's name and four elements. We will set down all these things so that the young priest may be the wiser for looking at them. God's name—*Deus*—is also revealed here.[83]

We find this information arrayed on a narrow-beamed cross centered on a set of seven concentric circles drawn in alternating green and red pigment. The cross's intersection appears to be "hidden" behind the three innermost circles while its beams "overlap" the larger, outermost circles dividing them into quarters. The fourfold figure seems to be derived primarily from two sources: the *Divisio anni* ("Division of the year") figure from Calcidius's *Commentary* on Plato's *Timaeus* and two concordance *rotae* from Isidore's *De natura rerum*—one *On the seasons* (*De temporibus*) and the other *On the parts of the world* (*De partibus mundi*). The transformation of these figures by Byrhtferth reveals a distinct interest in ocular motion and meaningful forms.

There is little evidence suggesting that Ramsey owned a copy of Calcidius's *Commentary*. Byrhtferth probably encountered the *Divisio anni* figure in Abbo's computus.[84] It will be remembered from chapter one that Berlin 138 contained a sizeable set of astronomical, cosmological, and computus figures, a number of which came from the Calcidian *Commentary* (figs. 1.97–98). Indeed, Byrhtferth's use of color, if accurately reflected in Ashmole 328, is similar to the use of color in the version of the *Divisio anni* figure in Berlin 138 (fig. 4.31). Rarely was this figure colored in copies of the *Commentary*; Berlin 138 is exceptional in this regard. It may also have been through Abbo's computus that Byrhtferth came into contact with Isidore's *rotae.* Though there is no record of a copy of the *De natura rerum* at Ramsey, several surviving copies were either present or produced in England during Byrhtferth's lifetime, and the title is included in a number of contemporary

[83] Byrhtferth, *Enchiridion* 2.1.321–326; eds. and trans. Baker and Lapidge, 74–75: "Twegen sunstedas synd and twa emniht on þam twelf monðum, and twelf tacna and feower timan, and feower ylda on mankynne and feower stafas on Adames naman and gesceaft. Ealle þas þing we wyllað her amearkian þæt se iunga preost mæge beon þe wisra þe he þas þing gesihð. Eac her ys geswutelod Godes nama, þæt ys Deus."
[84] The figures in this work were typically drawn in the same brown ink as the text, or a combination of brown ink and the red-orange pigment used for rubrication. A standard example is Bamberg, SB MS Msc.Class.18, fol. 34v (fig.1.64).

English inventories.[85] But Byrhtferth need not have ever come into contact with a complete copy in order to have been familiar with Isidore's figures, for excerpts circulated widely.[86] Abbo included both the *De temporibus* and the *De partibus mundi rotae* in his computus. In Berlin 138 they appear side by side (fig. 1.99 and 4.32). Vivid orange *tituli* explicitly identify them as images of concordance. At the center of the *De temporibus* figure is written, "of which [the seasons] communion (*communionis*) this is a figure";[87] the *De partibus mundi* figure is labeled, "On the harmony (*convenientia*) of the cosmos, micro-cosmos, and year."[88] St. John's 17 includes the latter *rota* with the relevant passage and thus it is thought that Byrhtferth included it in his own computus (fig. 4.33).[89]

Searching for a way to express the unity and divisions of the year in a single figure, Byrhtferth seized on the structure of the *Divisio anni* figure, adapting it to the task at hand. The figure comprised three nested circles crossed by a horizontal and a perpendicular line (fig. 4.31). In Berlin 138, the outermost ring contains the names of the signs of the zodiac, written in green pigment, and the names of the seasons in orange. The same vivid orange ink was used to divide this ring radially into twelve segments, and to write four Greek letters—A, B, Γ, Δ—around the figure's periphery. Numbers (the length of each season) fill the middle ring, which was divided radially into fourths by the intersecting lines. These numbers were inscribed in the same dark brown ink used to draw the circles and the intersecting lines. According to the text that surrounds the figure, the circle, ABΓΔ, represents the zodiac with earth, marked by the Greek letter Θ, at its center. A and Γ, we are told, stand for, respectively, the spring and autumn equinoxes; B and Δ the summer and winter solstices.[90] These heavenly events divide the year into its four seasons.

Like Calcidius, Byrhtferth pictured the course of the zodiac and the seasons as a circle cut evenly into quarters by intersecting lines (fig. 4.34). In Byrhtferth's figure,

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[85] Isidore's *De natura rerum* was cited by several Anglo-Saxon authors (Theodore/Hadrian, Aldhelm, Bede, Ælfric, and Byrhtferth) and included in five English library inventories. Seven extant copies of the text are thought to have been owned by Anglo Saxon institutions. Lapidge, *Anglo-Saxon Libraries*, 271 and 310–11.
[86] Faith Wallis, "6. Cosmographical Anthology: 13. Isidore of Seville on the Elements: Text and Diagrams," McGill University, http://digital.library.mcgill.ca/ms-17/folio.php?p=39v#note08 (accessed 29 January, 2015).
[87] Berlin 138, fol. 38v: "cuius communionis hec est figura."
[88] Ibid.: "De convenientia cosmi microcosmi et anni."
[89] St. John's 17, fol. 39v; See Byrhtferth, *Computus* 48; eds. and trans. Baker and Lapidge, Appendix A, 426.
[90] Berlin 138, fol. 36v: "Sit zodiac(us) circ(u)l(i)s Λ.B.Γ.Δ. in cuius medio sit Θ quasi terra p(er) qua(m) du(a)e lineae diriguntur. Q(uam) utraq(ue) diametros intelligat. Ab .Λ In .Γ. et a .B. in Δ. Ergo sol accedens ad .Λ. facit vernu(m) æq(u)inoct(um) et ad .Γ. autu(m)nale. Ad .B. autem c(on)versione(m) æstiuam. Et ad .Δ. hiemale(m). Te(m)pora dividet sequens figura."

however, these lines are broader and within their bounds are written the terms "solstice" and "equinox." The change radically alters the relationship between text and image. The *Divisio anni* figure is a lettered diagram, a figure-type closely associated with geometric argument.[91] Lettered diagrams are, as their name suggests, arrayed with letters—usually from the Greek alphabet—indices that serve as referents in the verbal explanation.[92] Lettering allowed the author of an argument to direct the attention of the reader/viewer to a specific aspect of a figure with a high degree of precision (e.g., line ΑΓ, circle ΑΒΓΔ).[93] Without its text, a lettered diagram remains largely indecipherable; the reverse is also true.[94]

By verbally designating each element, Byrhtferth severed the bond between figure and text so that both might function independently. He did not, however, entirely abandon the conceit of the lettered diagram. His figure is lettered, not with letters of the Greek alphabet, but with letters from the Roman alphabet. These spell the name of Adam, A-D-A-M and of God, D-E-U-S (fig. 4.35).[95] The name of Adam was associated with the cardinal directions. "For the Greeks," Byrhtferth explained, "the east begins with A, the west with D, the north with another A, and the south with M." In the fourfold figure the names of the

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91 Reviel Netz called the lettered diagram the "metonym of mathematics" and discusses this kind of figure in depth. He wrote: "The lettered diagram is the tool which instead of being filtered out by the written mode, was made more central and, with the marginalisation of other tools [e.g. physical models], became the metonym of mathematics." Netz, *Shaping*, 66.

92 For the status of letters as indices see Charles Peirce, "Elements of Logic," in *Collected Papers*, Vol. 2: Principles of Philosophy and Elements of Logic, ed. A. W. Burks (Cambridge: Belknap Press of Harvard University Press, 1932), 171; and Netz, *Shaping*, 50–51.

93 Netz summarized the mechanics and effect of the lettered diagram as follows, "It [the text associated with a lettered diagram] does not identify its objects and leaves the identification to the visual imagination ... it does not *name* its objects—it simply points to them, via indices ... Finally, it does not even hint *what*, ultimately, its objects are; it simply works with an ersatz, as if it were the real thing." Netz, *Shaping*, 56.

94 The manner in which the *Divisio anni* figure is embedded in its explanatory text in Berlin 138 conveys the inseparability of object and verbal description. Eastwood argues that the maker(s) of Berlin 138 initially intended for all the figures and relevant texts from Calcidius to be displayed in this manner, but that the verbal descriptions were too long, so the original layout was abandoned and the text tipped in on a singleton. The figures in Macrobius's *Commentary* were also lettered diagrams. In Berlin 138, they either contain or are surrounded by their explanatory text. See, for example, Berlin 138, fols. 36r (the two circular figures on the left side of the page) and 39v (all the figures are Macrobian). Eastwood, "Calcidius's *Commentary*," 182.

95 The letters of Adam's name are also arrayed in a cross around one of the marginal figures that accompany the numerological description of four (*Enchiridion* 4.1.51; Ashmole 328, p. 210); and "Byrhtferth's Diagram" (fol. 7v) in St. John's 17. In the *Enchiridion* (4.1.67–8), Byrhtferth associates the letters with Greek terms for the cardinal compass points and with Greek numbers: α is one, δ is four, α is one again, and μ is forty, making forty-six (he provides no further interpretation). In their commentary on this part of the *Enchiridion* Baker and Lapidge point to Augustine as the earliest source in the Latin West to identify the name of Adam with the cardinal directions in Greek. Significantly for its spread in England, Augustine's interpretation was taken up by Bede. Hrabanus Maurus dedicated one of the acrostics in the *De laudibus crucis* to the name of ADAM. Baker and Lapidge, in Byrhtferth, *Enchiridion*, 343; and D. Cerbelaud, "Le nom d'Adam et les points cardinaux: Recherches sur un thème patristique," *Vigiliae Christianae* 38 (1984): 285–301.

cardinal directions in Greek—*anatho*(*le*), *misinbrios*, *disis*, *arcton*—are interspersed with the corresponding majuscule letters A, M, D, A. Of interest is how in Byrhtferth's figure the letters, while meaningful in and of themselves as decipherable words, continued functioning in much the same way as Greek letters in a traditional lettered diagram, directing the eye through the image and designating aspects of the figure worthy of being marked, that is, points of interest critical to meaning.

Byrhtferth arrayed the letters of the name of "Adam" (A-D-A-M) and of God (D-E-U-S) in such a way so that, urged by orthography, the reader/viewer's eye would make the sign of the cross (up-down-left-right). So important was this gesture that Byrhtferth sacrificed the standard alignment of the cardinal directions with the seasons. As a rule, spring (*ver*) was paired with the east, summer with the south, autumn with the west, and winter with the north. According to Isidore, this was due to the qualities of a given season and was also etymologically evident.[96] In Byrhtferth's figure, however, this order is reversed so that winter is aligned with south, and summer the north, etc.[97] The form of the cross and the cruciform gesture radiate outward from the name of God. Earlier in this chapter, we saw how Byrhtferth recast Creation as described in the first lines of Genesis as the setting of the celestial calendar. In the passage quoted, Byrhtferth equated the making of the sun and the moon with the setting of the solstices and the division of the year into twelve months and four seasons. The act is visualized in the fourfold figure. Here, the divine act of division—the establishment of solstices and equinoxes—is accomplished through the sign of the cross. At the same time, making the sign of the cross, the eye encounters opposites—fire and water, spring and autumn—and draws them together, connecting that which, in the beginning, was divided.

The text in the fourfold figure served to draw the eye along a different, but also meaningful trajectory: a circle. The months, names of the zodiac, seasons, ages of man, elements, and cardinal directions are spelled out in the rings of the fourfold figure. Reading propels the eye clockwise. Terms used to identify circles—*circulus*, *cyclus*, *rota*—give a sense of both shape and motion. "The circle (*circulus*)," Isidore etymologized, derives its

[96] Isidore, *Etymologiae* 5.35.7–8.
[97] The same anomalous alignment is encountered in another figure thought to have been authored by Byrhtferth in London, BL MS Cotton Tiberius C. i, fol. 5r. The figure is "edited" by Peter Baker, "More Diagrams by Byrhtferth," 66 (no. 1).

name from "going round (*circumducere*)."[98] The use of the circle in the *Enchiridion* and elsewhere suggests that the shape was understood to be inherently dynamic, a quality that made it an especially apt expression of time and explains, at least in part, why the form proliferated in computus manuscripts. In the seminal *The Rise of Pictorial Narrative in Twelfth-Century England*, Otto Pächt wrote:

> The history of narrative art is indeed but a series of repeated attempts to smuggle the time factor into a medium which by definition lacks the dimension of time ... since pictorial form cannot move, ingenious devices have been developed for enlisting the onlooker's help in supplying motion or movement."[99]

Byrhtferth faced essentially the same problem, only in computus time was not merely an aspect of what was being pictured, it was *what was being pictured*. In the fourfold figure, Byrhtferth adopted strategies from the domains of geometry—the lettered diagram—and grammar—reading—to animate the image and show temporal division and cycle. He conveyed the sacred nature of time through shape.

As mentioned above, Byrhtferth organized the fourfold diagram to emphasize the cross. Whereas the thin brown lines of the *divisio anni* figure as it appears in Berlin 138 *might* be interpreted as a cross, such an interpretation is unavoidable in the *Enchiridion*, where the increased length and width of the lines makes the shape emphatic. The lines-*cum*-beams extend beyond the circles. Indeed, the scribe of Ashmole 328 seems to have been working from an exemplar in which the base of the upright beam extended so far that it penetrated the text below (fig. 4.36).[100] Historian of art Bianca Kühnel has drawn attention to the symbolic importance of the cross in medieval cosmological figures more generally and points to Isidore's *De temporibus* and *De partibus mundi rotae* as prime examples of a much larger phenomenon.[101] The articulation of quaternities in the fourfold figure and, perhaps, the prominence of the cross, suggest that these *rotae* served as models for Byrhtferth.

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[98] Isidore, *Etymologiae* 3.12.1; ed. Guillaumin, 45; trans. Barney, 93: "In plano figurarum prima circulus est, figura plana, et circumducta, cujus in medio punctus est, quo cuncta convergunt, quod centrum geometrae vocant, Latini punctum circuli nuncupant." Isidore's definition of a circle is indebted to the Boethian translation of Euclid's *Elements*.

[99] Otto Pächt, *The Rise of Pictorial Narrative in Twelfth-Century England* (Oxford: Clarendon Press, 1962), 1.

[100] Ashmole 328, p. 85. Extra space was left between two words (*iunium* and *cymð*) in the first line of the text below the figure to accommodate the stipes of the cross. Space was also left below another similar figure (Ashmole 328, p. 9).

[101] Kühnel, "Carolingian Diagrams, Images of the Invisible," esp. 363–67.

Book seven of the *De natura rerum*, "On the seasons" (*De temporibus*), and book eleven on the elements, titled "On the parts of the world" (*De partibus mundi*), conclude with figures that, in Isidore's words, "declare commonalities and distinctions."[102] In the latter chapter, Isidore describes the four elements—fire, air, earth, and water—and their qualities—dry, wet, sharp, blunt, mobile, static. Paraphrasing the explanation of creation in Plato's *Timaeus*, Isidore explains that it is on account of common qualities or lack thereof that certain elements may or may not combine, [103] and he describes their combination one to the next in a circuit "comparable to a dance."[104] Isidore depicted both kinship and opposition within the bounds of a circle, its interior crisscrossed by eight overlapping arcs, the resulting interstices filled with words (figs. 4.37–38). In the figure accompanying *De partibus mundi*, the names of the elements, seasons and—less often—the cardinal directions and humors were written in what might be called the primary arcs, which open upwards and downwards, to the left and right. These were often distinguished by eye-catching colors and/or their size. The lesser arcs, denoting the qualities, link the primary arcs, either bridging the space between them or overlapping them.[105]

Kühnel observed that the negative space carved out by the oblique, secondary arcs in these *rotae* takes the form of a Greek cross with flared arms, a cross-type that was not uncommon in the early middle ages.[106] She argues that the cross should not be considered an

[102] Isidore *De natura rerum* 7.4 and 11.3; ed. Fontaine, 203 and 217. On these figures see Gorman, "The Diagrams in the Oldest Manuscripts of Isidore's *De Natura Rerum*," 529–34; Barbara Obrist, "Le diagramme isidorien des saisons," 95–164; and Teyssèdre, "Un example de survie de la figure humaine," 19–34.

[103] This is, of course, the doctorine known as the *syzygia*, though Byrhtferth does not use this term. Plato asserted that numeric proportion was the glue that allowed the elements to combine to make the myriad forms that comprise the universe. Calcidius developed Plato's account verbally and visually. He translated the Greek *analogia* as *ratio*, meaning both "proportion" and "likeness," and made it the focus of the *Commentary*. His mathematical explanations took the form of figures. The logic of continuous proportion, the *ratio* of creation, was pictured "arithmetically" through numerals linked by arcs, a convention seen also in Boethius's *De arithmetica* and, in the context of cosmology, in the *rotae* of Isidore's *De natura rerum*. Isidore represented the elements and their qualities, not as numbers, but by their names, as words. Rather than presenting these terms linearly, as Calcidius did, he turned them in a circle. On the concept of *analogia* as interpreted by Calcidius see Somfai, "Calcidius' *Commentary*," 205.

[104] Isidore, *De natura rerum* 11.3; ed. Fontaine, 215–17: "atque ita sibi per hunc circuitum quasi per quendam chorum concordi societate conueniunt."

[105] The "bridging" type (when the arcs meet end to end) can be seen in Berlin 138, fol. 38v; and the "overlapping" type in St. John's 17, fol. 39v. The two types are paired in Baltimore, WAM MS 73, fol. 8r.

[106] Kühnel gives the example of the Clonmacnoise cemetery cross. Other examples, provided in her book, *The End of Time and the Order of Things*, include the low relief chancel screens from St. Prassede, Rome; a fibula from the Landesmuseum, Trier; and the Soiscél Molaise Reliquary (R 4006) from the National Museum. Kühnel, "Carolingian Diagrams," 382 (fig. 7); and eadem, *The End of Time and the Order of Things*, 136–60, 320 (figs. 58–9), 321 (fig. 63), and 322 (fig. 62).

accidental byproduct of the figure's arrangement, but an essential pictorial element, and one which would have been immediately apparent and meaningful to a medieval viewer. These figures, in her estimation, "show in the most literal sense how the bulk of knowledge acquired from ancient sources was placed under the sign of the cross."[107] In this, her primary thesis, Kühnel is surely correct, and Byrhtferth can be seen in line with tradition. That said, the cross that Kühnel calls attention to is more and less pronounced in particular renditions of Isidore's two diagrams, which vary greatly in individual manuscripts. The cross can be prominent or less so. Isidore's concordance figures were remarkably unstable.[108] Shifts in the color, number, scale, and arrangement of the arcs are common and, in many cases, seem to be intentional, reflecting changes in emphasis and interest on the part of makers.[109] Such alterations may seem minor, but they strongly affect structure and meaning. In St. John's 17, all of the arcs are the same size and two of the "primary" arcs were omitted. Opposing pairs of arcs were colored in vivid shades of red, green, and purple-blue. Outlined in alternating red and green pigments and overlapped by the blue arcs, the cross in this version of the figure recedes (fig. 4.33).[110] In Berlin 138, where the arcs are uniform in size and do not overlap, the cross is all but non-existent (fig. 4.32). Present, but often deeply embedded, the cross in Isidore's *rotae* underlies the order of creation. In comparable figures in the *Enchiridion*, it governs the whole.

It is impossible to know whether Byrhtferth was inspired to bring the cross to the fore by a version of the Isidorian concordance figures in which the shape was particularly prominent. There can be no doubt, however, that he had these figures in mind as he configured the elements within his fourfold figure. Emulating the *rotae* of *De temporibus* and *De partibus mundi*, Byrhtferth centered his figure on a smaller, undivided, interior circle; he aligned related or equivalent information radially; and placed on opposing sides those things in opposition. The flow of time, clockwise, also accords with figures in the *De natura*

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[107] Kühnel, "Carolingian Diagrams," 362.
[108] This is true of a number of the figures in the *De natura rerum* and for mathematical and scientific figures more generally.
[109] Only Isidore's picture of the "elemental solid," has been subjected to any sort of typology/taxonomy and then only the earliest copies of the *De natura rerum.* See Gorman, "The Diagrams in the Oldest Manuscripts of Isidore's *De Natura Rerum*," 531–34; and Eastwood, "The Diagram of the Four Elements in the Oldest Manuscripts of Isidore's *De Natura Rerum,"* 547–64. A more nuanced understanding of this kind of "error" in medieval figures is provided by Kathrin Müller in "Irritierende Variabilität," 415–43.
[110] Kühnel discusses and includes pictures of Dijon, BM MS 448, fol. 73; Laon, BM MS 423, fol. 12; Munich, BSB Clm 16128, fol. 11.

rerum.[111] Importantly, Byrhtferth omitted the crisscrossing, overlapping arcs, arguably the most characteristic feature of the Isidorian *rotae*. The choice, I suggest, reflects the divergent interests of the two authors and, possibly, the capacities of their audiences. Whereas Isidore focused on the "dance"—the way that the four elements were joined by common qualities and how these same qualities also linked the four seasons, four humors, etc. revealing a profound proportional order underlying and binding all of creation, Byrhtferth homed in on the number four itself and its appearances in nature.

Understanding quaternity, the notion of a fourth part, was fundamental to computus. Byrhtferth defined the term in the course of the first lesson in the *Enchiridion* on the solar year. A year, he explains, contains 365 days and 6 hours—a "quadrant" (*quadrans*), "the fourth part of a day, or of other things that can be precisely divided by four."[112] The quadrant had to be constantly kept in mind in order to maintain the calendar accurately. Over the course of four years, quadrants accrued until it was necessary to add a full day, called the *bissextile day*, to the year.[113] The year with the extra day, a leap year, was known as a *bissextile year*; then as now an additional day was added in February. Nearly every computistic calculation had to be adjusted for the leap year, and key-letter tables had to be used with caution—the leap year taken into account. Hence, driving home the importance of four and a fourth, instilling a sense of reverence, respect, and even awe for this number in an audience of future computists, was motivated, at least in part, by practical concerns. These dovetailed with the number's cosmological and spiritual significance, discussed in greater detail below. Hence, just as Byrhtferth rewrote the opening lines of Genesis in a manner that brought numeric significances to the fore, he revealed computus as number in the fourfold concordance figure: the solar months and the signs of the zodiac *are* twelve, the seasons and elements *are* four, the equinoxes and solstices *are* two. At the same time, all these things are circular and cruciform, and as such blessed, divine.

Byrhtferth drew from a limited array of forms when devising the figures of the *Enchiridion*. Cross and circle were his primary building blocks. There was, as we have seen,

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[111] See, for example, the *De mensibus* figure (*De natura rerum* 4.7; ed. Fontaine, 190 [fig. 1]). The *Diviso anni* figure, on the other hand, flows counterclockwise.
[112] Byrhtferth, *Enchiridion* 1.1.37–40; eds. and trans. Baker and Lapidge, 4–5: "Gif hwam gelustfullað to witanne hwæt sy quadrans (þæt ys fyðerlinc) þonne undergite he mid soðre race þæt quadrans byð se feorða dæl þæs dæges oððe oðra þinga þe man mæg rihtlice todælan on feower." The quadrant is defined again and in even greater detail later in the *Enchiridion* (2.1.59–72).
[113] Ibid. 2.1.30–72.

ample precedent for the use of these shapes in figures associated with computus, but Byrhtferth laid particular emphasis on them. This was true of the figures in the first three sections of the *Enchiridion* devoted to computus as well as the final section, on numerology. His choice of these shapes reveals a strong preference for forms that were potent, equivocal, and dynamic—qualities difficult to convey with words. A closer look at of some of the numerological figures gives further indication of the perceived limits of verbal explanation and the work of pictures.

4. Picturing number

Reading sequentially, a reader/viewer was ushered into Byrhtferth's numerological tract by a list of Roman numerals, along with their Latin ordinals and Old English equivalents.[114] These were written in a column eighty-two lines long across two pages (figs. 4.39–40). Byrhtferth pivoted from these artificial signs, written and spoken, to natural signs—number as it was thought to be manifest in scripture and nature. He discussed numbers one through twelve in detail; thirteen through fifteen merited their own brief explanations; sixteen through nineteen he addressed *en masse*, likewise twenty, forty, and fifty.[115] Byrhtferth explicitly "omitted" a discussion of "sixty, eighty, and ninety" lest he "aggravate" his audience, and moved on to 1,000, which was followed by a description of finger calculation useful for multiplying tens up to 1,000,000.[116] Numbers for Byrhtferth were evocative and multivalent. One, he wrote, represented the singularity of divine wisdom *and* God in his guise as the alpha and the omega (Revelation 1:5);[117] two alluded to the Incarnation *and* the apostles, who travelled in pairs; three comprised the world ages *and* the mystery of the Trinity; four stood for the cardinal virtues *and* the elements.[118] These are just a very few of the examples gathered in this section of the *Enchiridion* by Byrhtferth, for whom numeric correspondence was not merely symbolic, but betokened real and significant correlations.

[114] Discussed in chapter 3.
[115] Byrhtferth, *Enchiridion* 4.1.1–390.
[116] Ibid. 4.1.396–98; eds. and trans. Baker and Lapidge, 226–27: "De sexagessimo et septuagessimo et octuagesimo necnon et nonagesimo, supersedimus hoc in loco sermocinari, ne forte perturbemus audientes."
[117] Also Revelation 21:6 and 22:13.
[118] Byrhtferth *Enchiridion* 4.1.1–100.

The dominant language of the numerological tract is Latin, suggesting that it was written primarily for a monastic audience.[119] Elsewhere in the *Enchiridion*, Byrhtferth stated that Latin was included for the benefit of "young monks," who, unlike clerics and priests, "occupied their childhoods with learned books," and "sucked the milk of the catholic church while they were sucking the milk of their human mother."[120] So trained, even the youngest of monks would likely have been acquainted with basic mathematical vocabulary, which appears in this section of the *Enchiridion*. "Even" and "odd," "equal" and "unequal," "major" and "minor," and "perfect"—all terms of particular significance when applied to number—are used in passing. The implication is that Byrhtferth either assumed prior arithmetical knowledge on the part of his reader or that the common or contextual meaning of the term was deemed sufficient.[121] He also assumed a devotional context.

Byrhtferth's numerological tract belongs to a long tradition of Christian numerological exegesis extending from Philo to Hincmar of Reims.[122] Byrhtferth assembled his descriptions of numbers from the Bible, patristic authors, and so-called secular works, sources which did not include images.[123] He pictured numbers four through ten (figs. 4.41–

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[119] It is the lengthiest stretch of Latin in the *Enchiridion*. In Ashmole 328 The first few pages are heavily glossed in Old English. It is impossible to know whether this continued in Byrhtferth's autograph copy. Byrhtferth returned to alternating Latin and Old English in his discussion of the world ages (4.2.1–76). The final sections of the *Enchiridion* (4.2.77–125) are in Old English.

[120] Byrhtferth, *Enchiridion* 2.3.237–47; eds. and trans. Baker and Lapidge, 120–21. The passage is worth quoting in full: "We could add many things from the writings of learned men, but because we know that these things seem complicated enough to clerks and rustic priests, we will now turn our speech to the young monks, who have occupied their childhoods with learned books. I mention a few of many: They have scrutinized Sergius and Priscian, and have read through the sayings of Cato the bald man and the compositions of Bede the venerable writer. Now, as we believe, they have an overpowering thirst for this science. Now that we know their desire, we intend to show them the stream from which they can drink a healthful draught, and also leave to others elegant learning and a legacy." (Manega þing we mihton of þeodwitena gesetnysse herto geicean, ac forþan þe we witon þæt þas þing þincað clericum and uplendiscum preostum genoh mænigfealde, nu wille we ure spræceawendan to þam iungam munecum þe heora cildhad habbað abisgod on cræftigum bocum, swylce ic of manegum feawa hrepige: Hig habbað ascrutnod Serium and Priscianum and þurhsmogun Catus cwydas þæs calwan esnes and Bedan gesetnysse þæs arwurðan boceres. Nu þyrst heom þearle swyðe to þissam cræfte, þæs ðe we wenað. Nu we heora willan cunnon, we þenceað heom þæne burnan to ætywanne þe hig magon halwendne tige of drincan, and eac oðrum fægere lare and lafe forlætan.) Ibid. 4.2.13–15; eds. and trans. Baker and Lapidge, 232–33: "Haec vero monasterialibus viris cognita sunt perfecta ratione, qui ab ipso pubertatis tyrocinio cum lacte carnis genetricis eorum lac sugxerunt aecclesiae catholicae."

[121] "Even" and "odd" (*Enchiridion* 4.1.93 and 4.1.159–60), "equal" and "unequal" (4.1.92), "major" and "minor" (4.1.96–7), and "perfect" (several points including 4.1.112).

[122] The tradition extending as far back as Pythagoras is summarized by Baker and Lapidge, in Byrhtferth, *Enchiridion*, lxi–lxxiv. For the relationship between numerology and computus at the time of Byrhtferth see Wallis, "'Number Mystique,'"181–95.

[123] Byrhtferth's primary sources for this section are the Bible (especially Psalms), Augustine's *De civitate Dei*, Gregory the Great's *Homiliae xl in evangelia* and *Moralia in Job*, Bede's exegetical works, and the homilies of Haymo of Auxerre. Baker and Lapidge, in Byrhtferth, *Enchiridion*, lxxiii–lxxiv. Quadrivial works with

42, 4.2, 4.43–45).[124] These images range in size from tiny, glyph-like objects to framed, half-page figures. Nothing like Byrhtferth's numerological figures have come down to us. Some precedent, however, is found in Boethius's *De arithmetica*.[125] Boethius introduced the concept of plane numbers in a section concerning, "quantity, which consists of itself," that is, pure number.[126] Boethius admitted that these numbers were the "proper consideration of geometry," but, he continued, justifying their inclusion in a treatise on arithmetic: "as the science of geometry is produced from arithmetic as from a root or mother, we find in numbers, which are primary, the seed of those figures."[127] In the *De arithmetica*, plane numbers are pictured as polygons. Paris lat. 6401, a copy of the tract that may have been produced, at least in part, in England, possibly at Ramsey, is a typical example. The figures of plane numbers are drawn in colored pigment, and their interiors bristle with squadrons of **i**'s (fig. 4.46).[128] In order to demonstrate plane numbers, Boethius set aside Roman numerals, using instead tiny upright "strokes" (*virgulae*) their quantity equal to the number being pictured.[129] Byrhtferth seems to have had these figures, called *descriptiones formarum* by Boethius, in mind when he began to add pictures to the margins of his numerological descriptions.

Six diminutive figures—Byrhtferth refers to them as "signs" (*signae*) and "characters" (*caracteres*)—accompany his lengthy discussion of the number four, the first number

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extended "numerological" sections such as Martianus's *De nuptiis* (a description of the powers of each number in the decad; 7.731–42) and Macrobius's *Commentary* (1.5–7; a celebration of seven and all the numbers (1,2, 3, 4, 5, 6) that comprise it) seem also to have influenced his interpretations.

[124] A page is missing from Ashmole 328 between p. 230 and 231, causing a lacuna in Byrhtferth's description of the number twelve. Baker and Lapidge (following Heindrich Henel) suggest in their commentary on this section that the missing text may have resembled Cotton Tiberius C. i, fols. 2–17 + Harley 3667, fol. 5v. The folio contains a brief tract on the number twelve and a large figure representing the number. Hence, Byrhtferth may have also pictured the number twelve in the *Enchiridion*. If this was the case, it would be the only numerological figure to appear within the verbal description rather than to follow it. Baker and Lapidge, in Byrhtferth, *Enchiridion*, 358–59; and Peter Baker, "More Diagrams by Byrhtferth of Ramsey," 53–65 and (no. 2).

[125] Lapidge does not include the *De arithmetica* among those works cited by Byrhtferth, but he does not take pictorial citations into account. Lapidge, *Anglo-Saxon Library*, 266–74, 294.

[126] Boethius, *De arithmetica* 2.4.1; ed. Guillaumin, 88; trans. Masi, 128: "Nunc autem in hac sequentia quaedam de ea quantitate, quae per se ipsam constat."

[127] Ibid. 2.4.2; ed. Guillaumin, 88; trans. Masi, 128: "quae omnia quidem geometricae propriae considerationis sunt, sed sicut ipsa geometriae scientia ab arithmetica uelut quadam radice ac matre producta est, ita etiam eius figurarum semina in primis numeris inuenimus."

[128] Jean-Yves Guillaumin notes that these figures are an adaptation of a system for picturing numbers in the Pythagorean tradition. Where the Pythagoreans used points to represent unity, Boethius used strokes, essentially the Roman numeral for one. Guillaumin, in Boethius, *Institution arithmétique*, 211, n. 33.

[129] Boethius, *De arithmetica* 2.4.3, ed. Guillaumin, 88–9.

pictured in the numerological section. Three of these *signae* are simple shapes outlined in red that enclose the number four written as four vertical strokes (figs. 4.41, 4.47–48). Formally, they strongly resemble Boethius's plane figures; functionally, however, they differ. In the *De arithmetica*, a one-to-one relationship existed between quantity and angle: three formed a triangle; four, a square; five a pentagon, and so on.[130] Hence the number four would always form a square. Byrhtferth, like Boethius, gave number shape, but those shapes derived from what might be deemed symbolic rather than geometric properties, though it is unlikely that he would have made such a distinction. Hence, four *might* manifest as a square, but it could also be pictured in other ways. On page 209 of Ashmole 328, four is pictured as a quadrilateral, a cross, and a circle.

The various forms chosen by Byrhtferth correspond to the manner in which the number was parsed: as the product of two and two, the sum of one and three, and four unities. Each *signa* is juxtaposed with a commensurate passage in the verbal description. Thus, beside the quadrilateral is written: "If you divide this number [four] in equal measure, you get twice two."[131] Parsing four in this way, calling it "twice two," evoked the standard geometrical figure of four, a square (fig. 4.47). The four strokes in the quadrilateral's interior are "equally divided" into two pairs, as they are in Boethius's square in the *De arithmetica* (fig. 4.46). Curiously, the square drawn in the margin of Ashmole 328 has sides of equal length, but their ends do not join. The upper and lower sides extend beyond the intersection with the vertical sides so that the figure resembles an altar or the number two, written in majuscule Roman numerals. It is not quite, or perhaps better, is more than a square.

Above the square a cross with arms of equal length was drawn beside the statement (fig. 4.47):

> Let these words suffice concerning unity and trinity; but we should carefully pray to the loftiness (*celsitudo*) of unity and the magnitude of trinity, that what we confess in our words we may fulfill in our deeds.[132]

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[130] Ibid. 2.6.1–2.17.2, ed. Guillaumin, 93–104.
[131] Byrhtferth, *Enchiridion* 4.1.33–4; eds. and trans. Baker and Lapidge, 198: "Si hunc numerum aequa lance divideris, fit bis binarius."
[132] Ibid. 4.1.28–30; eds. and trans. (the latter adapted) Baker and Lapidge, 198: "Hec de unitate et trinitate dicta sufficiant, sed solle<rter> roganda est unitatis celsitudo trinitatisque magnitudo ut quod ore confitemur op(er)ibus exequamur."

The lines serve as segue from Byrhtferth's brief description of one, two, and three to his extended treatment of four. Hence, the cruciform shape was placed, appropriately, at an intersection. The vertical strokes on the cross's interior were arranged so that one, unity—set off by *punctūs*—was "lofted" above a row of three strokes, trinity, as specified in the text. The articulation of strokes highlights the cross's symbolic flexibility: it represents unity, trinity, and quaternity all alike, simultaneously. The cross was, above all, a symbol of Christ's passion, resurrection, and triumph over death and sin. But in the *Enchiridion* the sign is associated just as closely with the beginning of time and Creation as with the end of time and salvation.[133] In the fourfold figure discussed above, "solstice" and "equinox" were written on the cross's beams. Thus, the cross in this instance represents the ordering of time and matter: the emergence of quaternity from unity. Hence, the shape bridged the temporal and metaphysical gap between the God of the Old Testament, incorporeal unity, who created all things; and Christ, the embodied, earthly God of the Gospels.

Returning to the cruciform figure of four, in the adjacent text quoted above, Byrhtferth enjoins his reader to "pray to the loftiness of unity and the magnitude of trinity." By bounding one and three in the contours of a cross, Byrhtferth made four the object of prayer and confession. It should perhaps be assumed that the ubiquity of the cross in the *Enchiridion* accurately reflects its prevalence in the mental and material landscape of Ramsey.[134] As the central symbol of Christianity, the cross was celebrated daily by the community in the liturgy and constantly invoked by the individual.[135] The shape adorned the abbey's high altar and was embedded in the configuration of the church.[136] In contemporary

[133] In this it resembles the role of the cross in the Veneration ceremony on Good Friday. Sarah Larratt Keefer, "The Performance of the Cross in Anglo-Saxon England," in *Cross and Culture in Anglo-Saxon England: Studies in Honor of George Hardin Brown*, eds. George Hardin Brown, Karen Louise Jolly, Catherine E. Karkov, and Sarah Larratt Keefer (Morgantown: West Virginia University Press, 2008), 203–41.
[134] On the significance and prevalence of the cross in early medieval figures see Kühnel's, "Carolingian Diagrams," 359–89; eadem, *The End of Time and the Order of Things*, 116–159, esp. 139–47; and Cynthia Hahn's "*Visio Dei*: Changes in Medieval Visuality," in *Vision before and beyond the Renaissance*, ed. Robert Nelson (Cambridge: Cambridge University Press, 2000), 176.
[135] The cross was the central object of veneration on the feast of the *Inventio Crucis* (May 3), the *Exaltatio Crucis* (September 14), and Good Friday. Daily, the Passion and cross were contemplated in the so-called little hours and vigils of the Daily Office. Liuzza, "Prayers and/or Charms," 277, esp. nn. 3 and 4.
[136] Byrhtferth wrote of the abbey's construction under Oswald in the *Vita S. Oswaldi*: "And since he [Oswald] built under the venerable sign of the cross, through which we believe ourselves to be saved, so he [Oswald] began to construct the buildings of this place in the form of a cross, with wings toward the east, the south, and the north, and a tower in the middle." Later in the *Vita* he tells of how the monks were standing in the shadow of the cross at the altar when a crack formed in the tower. Byrhtfert, *Vita S. Oswaldi* 4.2; ed. and trans. Lapidge, 98–99.

Anglo-Saxon prayers and charms the cross was the focus of veneration and a powerful emblem of protection, healing, benediction, even a finding tool.[137] Near the end of the *Enchiridion*, Byrhtferth chides his reader to, "Let your food and your sleep always be consecrated to God and blessed with the holy cross."[138] The statement reminds us of the omnipresence of the cross as image and action as well as its infinite potency.[139]

Further down the page, a circle with four strokes at its center was drawn so near the textblock that it seems part of the phrase beside it, a kind of graphic appositive (fig. 4.48). The sentence reads "four is a perfect number" (*quaternarius perfectus est numerus*), with the circular figure between *quaternarius* and *perfectus*. The juxtaposition of word and form implies that four's perfection is like that of a circle or a sphere (spheres were always represented as a circle in pre-modern geometry).[140] In book twelve, "On the heavens" (*De caelo*), of the *De natura rerum*, Isidore located the perfection of spheres and circles in their uniformity—the equidistance of all points from their centers, their total symmetry, and the fact that they were formed of a single plane or line with no beginning and no end.[141] He wrote in the *Etymologiae* of the circle's remarkable ability to contain all forms within its bounds: "Just as every number is within ten, so the perimeter of every figure is enclosed within the circle."[142] Byrhtferth, following the example of Isidore, seems to test this axiom enclosing—in images like the fourfold figure—the whole of Creation and all time within a circle. In the fourfold figure, the letters D-E-U-S embrace the innermost circle, drawn with double red lines and centered on a point, the hole in the parchment left by the compass (fig. 4.35).[143] In traditional lettered diagrams, letters were used in the verbal narrative to "name"

[137] For the cross as a guide or finding agent see Liuzza, "Prayers and/or Charms," 293–94.
[138] Byrhtferth, *Enchiridion* Postscript 91–92; eds. and trans. Baker and Lapidge, 248–49: "Beo æfre þin mete <and> þin rest Gode betæht and mid þære halgan rode gebletsod."
[139] Scholar of Anglo-Saxon literature, Roy Liuzza, has characterized the cross as a kind of portable contact relic "that could be infinitely reproduced in stone, wood, or gesture so it was useful not just for veneration and contemplation but for invocation, benediction, and protection." Liuzza, "Prayers and/or Charms," 292.
[140] Abbo began his tract on planetary motion (*Sententia Abbonis de ratione spere*) by explaining the difference between a circle and a sphere, suggesting that either ambiguous terminology or the habit of using a circle to represent both circles and spheres confused contemporary audiences. He compared the two-dimensional circle to the "circuit of a compass" (*quasi circini circumductio*) and the three-dimensional sphere to the roundness of a "ball" (*sicut pilota*). Like Isidore, he points out that both forms have contours/sides that are equidistant from their centers. The tract was edited by Ron B. Thomson, "Two Astronomical Tractates," 120.
[141] Isidore, *De natura rerum* 12.4–5; ed. Fontaine, 219–21.
[142] Isidore, *Etymologiae* 3.12.7; ed. Guillaumin, 53; trans. Barney, 94: "Sicut autem intra decem omnis est numerus, ita intra circulum omnium figurarum concluditur ambitus."
[143] Attention is drawn to the point left by a compass in a few figures in Ashmole 328 (e.g., the fourfold concordance figure on p. 85, a similar fourfold figure on p. 9). The compass points/centers of a number of the

the element and its parts. Hence, in Calcidius's *divisio anni* figure the circle is ΑΒΓΔ; the line, ΑΓ; and the point, Θ. Thus, Byrhtferth effectively "named" the circle *DEUS*. The designation chimes with the shape's perfection as well as its universal ambit. These three, simple quaternity *signae* epitomize Byrhtferth's use of richly evocative forms and help us understand their significance. They also show how text and figure were deeply integrated, but not equivalent. The verbal narrative is more evocative than descriptive. That is to say: one could not reconstruct the figures, based on the text. This was especially true of the more complex figures like the *quinarius*, with which this chapter began.

Assuming a standard reading pattern, the *quinarius* figure comes "after" the verbal description of five (fig. 4.1).[144] The figure is thus framed and, to a certain extent, determined by the text. The gap between verbal and visual is bridged by the fact that the text adopts a spatial language and refers directly to the figure, although in a manner at once poetical and metaphorical. After announcing his intention to "add a figure," Byrhtferth described it: "Let the primary number be located at the summit of this mountain, the principal unity, also called the number one, until we get to the number six."[145] The *primus* plaque is located at the figure's apex, albeit the top of a cross rather than a mountain *per se*. A reader would presumably "enter" the picture at its uppermost point; hence "first" is encountered first. Propelled by the momentum of reading and guided by the upright beam of the cross, the viewer's eye swiftly segues from *primus* to *senarius.* The number six, though not represented within the figure, is present in the form of its written description, immediately below (fig. 4.2). Indeed, the figure and the passage that follows are juxtaposed so closely in Ashmole 328 that they encroach on each other's space. The upper bowl of the three-line rubricated **S** of *senarii* curls up, almost hooking itself on the base of the *quinarius* figure. Hence, the text provides a way through the picture, but this path was not the only possible route.

Whereas the written description of the properties of five takes the form of an inventory—a list of the number's symbolic meanings one after another in sequence—the picture generates multiple meaningful itineraries. On it, terms whose sum is five are placed

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circular figures in Berlin 138 were also highlighted (see fols. 6r and 35r). This would correspond to the technical definition of a circle, provided in the *Etymologiae* (3.12.1): "a circumscribed point."

[144] That said, text and figure share an opening in Ashmole 328 (p. 214 and 215), which would permit a back-and-forth from the verbal to the visual description; but this is not the case for all the numerological figures and there is no way to know with any certainty if this arrangement was intended by Byrhtferth.

[145] Byrhtferth, *Enchiridion* 4.1.104–6; eds. and trans. Baker and Lapidge, 204–205: "Primas sit in cacumine huius montis, unitas principalis, qui et primus dicitur, quoadusque perveniat ad dignitatem senarii numeri."

opposite one another: two and three on the horizontal beam, one and four on the upright. The cross shape provided Byrhtferth with a symbolic means of picturing the relationship between two and three, which, he wrote, were complementary opposites: two was equal and even, three unequal and odd. Byrhtferth located five's "perfection" in its numeric hermaphrodism.[146] He stated that the opposing qualities of these numbers were *balanced* when joined to make five. Likewise he placed two and three on either end of the horizontal beam, thus transforming the cross into a scale with the two numbers in equilibrium: perfectly balanced *and* opposite. The maneuver had both mathematical and liturgical precedent.[147] Cross and balance, the picture permits simultaneous signification in contrast to the sequential signification of the text.

The reader is told, as quoted above, that five "is divisible into its parts" and, indeed, in the figure we find five particulate. Its first, second, third, fourth, and fifth parts are each enclosed and isolated at or near the extremes of the cross. Elsewhere in the *Enchiridion*, Byrhtferth consistently uses the cross to show opposition and interrelation simultaneously, as in the fourfold concordance figure. The shape plays a similar role here. That said, nowhere in the verbal description does Byrhtferth liken five to a cross.[148] On the contrary, he refers to the fivefold figure as a "mountain." It is, instead, foursomes that he associated with and tends to array on crosses (for obvious structural and symbolic reasons). Indeed, turning back a page, the *quaternarius* description is accompanied by a host of small crosses. With these in mind, we might discern a fourfold figure *within* the figure of five. Fours, as we have seen, are discussed throughout the *Enchiridion.* God imposed a quatrinate pattern on creation: elements, seasons, winds; on man: the name of ADAM, the ages; and salvation: the virtues,

[146] In their commentary to the edition, Baker and Lapidge note that Byrhtferth is not using *perfectus* in its proper technical and mathematical sense (a point Baker returned to in his "More Diagrams by Byrhtferth" article). According to Isidore's *Etymologiae* (3.5.11) a "perfect" number is, "one completely filled up by its own parts." Isidore gives six as an example: it is the sum of its factors: 1, 2, and 3. In discussing the number six, Byrhtferth applied the term and explained it in a manner that shows his awareness of the formal mathematical definition (4.1.112–14). Here and elsewhere in the *Enchiridion*, however, Byrhtferth's use of the term is closer to that of Macrobius. In the *Commentary* on the *Somnium*, Macrobius wrote that the number seven (which, of course, is *not* the sum of its factors): "is truly *perfect* which is begotten from a union of these [an odd and an even] numbers" (1.6.1; trans. Stahl, 99).
[147] Tasking a reader with "imagining a balance" between two elements to demonstrate equality had, interestingly, precedent in ancient geometry, especially in the field of mechanics. Reviel Netz, "Imagined and Layered Ontology in Greek Mathematics," *Configurations* 17 (2009): 22. Venantius Fortunatus (530–609) construed the cross as a balance in the *Vexilla Regis*, a hymn honoring the cross, which was traditionally sung during Holy Week. Cynthia Hahn, *Strange Beauty: Issues in the Making and Meaning of Reliquaries, 400–circa 1204* (University Park: Pennsylvania State University Press, 2012), 93, n. 80.
[148] Though five was certainly associated with the *crucifix*: the five wounds of Christ and the *crux gemmata.*

evangelists, gospels, and the name of Christ envisioned as a cross, rising above five, Byrhtferth's "mount."

A rich and uniquely numeric ambiguity arises, moreover, from the drawing's graphic austerity. It is a quality shared to a greater or lesser extent by all of the numerological figures, but especially those like the *quinarius* figure, the *signae* of four, and the images of six and ten, which do not contain text (figs. 4.2, 4.45, and 4.47–49). The *quinarius* figure's green rectangles serve as spaces for contemplation of the multiple meanings of one, two, three, four, and five provided in the preceding text. In the figure, correspondences are physically congruent, or rather they are potentially congruent since meaning does not abide within the figure's bounds, but had to be brought to bear by a contemplative viewer. Contemplative, but not passive. The cruciform shape compelled the viewer to trace the sign of the cross (top–bottom, left–right), and in so doing, to add one and four, two and three. In the figure, the sacred gesture is transformed into a mathematical one.[149] The reverse is also true. Numbers invite the viewer to tally and combine, to find fives, and, due to their arrangement, the mathematical motion is made sacred. It is the same trick we saw in the fourfold concordance figure, but here it is arithmetic rather than grammar that guides the eye.

Many of Byrhtferth's pictures, even some of the most simple in form, are made more meaningful through ocular motion. The verbal and visual descriptions of the number six share a page with the *quinarius* figure. Byrhtferth described the number thus:

> The number six surpasses the eloquence of our speech by virtue of its magnitude, sublimity and solemnity. Authorities on numbers are accustomed to demonstrate its loftiness and solemnity to the sons of the church as follows. "One plus two plus three makes six, and this number," as St. Augustine says, "is called perfect because it is made up of its constituent parts."[150]

Byrhtferth notes that the world was created in *six* days, that it was on the *sixth* day that God made man in his own image, and that it was the *sixth* age which witnessed the Incarnation.[151] These events do not make six perfect, they are, rather, proof of the number's perfection. According to Byrhtferth (and Augustine) God's preference for six lay in its unusual

[149] One could also move in the opposite direction (adding three to two, four to one), but a reader would, I suggest, naturally follow standard reading patterns and move from the lesser number to the greater.
[150] Byrhtferth, *Enchiridion* 4.1.111–13; eds. and trans. Baker and Lapidge, 204–205: "'Unum enim,' inquit, 'et duo <et> tres sex fiunt. Qui numerus,' ut Aurelius Augustinus ait, 'propterea p<er>fectus dicitur, quia partibus suis completur.'"
[151] Ibid. 4.1.117–21.

mathematical properties. An arithmetically "perfect" number is equal to the sum of its aliquot parts. Such numbers are exceedingly rare and were treated with due reverence.[152]

Byrhtferth pictured six in the margin alongside the opening lines of the verbal description (fig. 4.49). The simple figure comprises an elongated rectangle, resting on a trapezoidal base, that tapers slightly as it rises and is surmounted by a cross. It is internally divided into four compartments, each of which houses a Roman numeral. At the bottom is one, above that is two, then three, and six at the top. Through the arrangement of the numbers (smallest to greatest), the viewer is induced to begin at the bottom and move upward, adding one, two, and three to make six (1+2+3 = 6). Continuing on this trajectory, the eye encountered the cross at the figure's apex. As in the *quinarius* figure, the process of reckoning generated a devotional act, in this case, raising one's eyes to the cross. Such acts remind the reader/viewer that the ultimate purpose of numeric study was spiritual contemplation and salvation. Byrhtferth's figures gave number a shape and vector, cultivating a "curious eye."

CONCLUSION

The notion that number is ineffable and its corollary—that pictures do not merely facilitate numeric rationale, but are essential to engendering such thinking—though ultimately rooted in Pythagorean traditions, was known in the middle ages in the Latin West largely through Calcidius's fourth-century translation of Plato's *Timaeus*.[153] In this dialogue, as evident in the following passage, wisdom and the love of wisdom, philosophy, arise from the ability to see:

> Our ability to see the periods of day and night, of months and of years, of equinoxes and solstices has led to the discovery of number and has given us the idea of time and opened the path to inquiry into the nature of the universe. These pursuits have given us philosophy .... I'm quite prepared to declare this to be the supreme good our eyesight offers us. I pass over other lesser [benefits of sight] lacking which [things] those who are far from philosophy,

[152] Six is the only perfect number in the decad. It is followed by 28, 496, and 8,128. Boethius stated that there was only one perfect number in the first ten, hundred, thousand, and ten thousand numbers (*De arithmetica* 1.20.1–3).

[153] On the influence and transmission of Pythagorean notions about number see Christiane L. Joost-Gaugier, *Measuring Heaven*, esp. 66–76 and 117–20; and David Albertson, *Mathematical Theologies: Nicholas of Cusa and the Legacy of Thierry of Chartres* (Oxford: Oxford University Press, 2014), 1–80.

> debilitated and blind, live sad and lugubrious lives.[154] Let us rather declare that the cause and purpose of the supreme good is this: god invented sight and gave it to use so that we might observe the orbits of intelligence in the heavens and apply them to the revolutions of our own understanding. For there is a similarity between them, even though our revolutions are disturbed, whereas the universal orbits are undisturbed. So once we have come to know them and to share in the ability to make correct calculations according to nature, we should stabilize the straying revolutions within ourselves by imitating the completely unstraying revolutions of god. [155]

Plato, in Calcidian Latin, describes a spiritual and intellectual transformation wrought of investigation, calculation, and imitation. Underlying the account is the assumption that the mind's capacity to comprehend the divine is possible because of an innate "similarity" or kinship—knowing by analogy—and that this inborn likeness can be honed and strengthened.[156] Such change is brought about through "the ability to see periods," to "observe the orbits of intelligence in the heavens" and "the completely unstraying revolutions of god." These orderly, regular, essentially numerical movements serve as models for the reshaping of the wobbly and disturbed revolutions of the human soul. He locates number's quickening in the "enumerated periods" (*curricula dinumerata*) of days, nights, months,

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[154] The Calcidian version of this line (which I have translated and included here) is markedly different from what is taken to be the correct reading by Plato's modern editors. Donald Zeyl's translation reads: "Why then should we exalt all the lesser good things, which a non-philosopher struck blind would lament and bewail in vain?" Plato, *Timaeus*, trans. Donald J. Zeyl (Indianapolis: Hackett Publishing Company, 2000), 35.

[155] Plato, *Timaeus* 47a–c; ed. Waszink, 44; trans. adapted from Plato, *Timaeus*, trans. Donald J. Zeyl (Indianapolis: Hackett Publishing Company, 2000), 35–36: "Visus enim iuxta meam sententiam causa est maximi commodi plerisque non otiose natis atque institutis ob id ipsum quod nunc agimus; neque enim de uniuersa re quisquam quaereret nisi prius stellis sole caeloque uisis. At nunc diei noctisque insinuate nobis alterna uice menses annorumque obitus et anfractus nati sunt eorumque ipsorum dinumeratio et ex di numeratione perfectus et absolutus extitit numerus, tum temporis recordatio, quae naturam uniuersae rei quaeri docuit curamque inuestigationis iniecit mentibus, quasi quoddam seminarium philosophiae pangens, quo bono nihil umquam maius ad hominum genus diuina munificentia commeanit. … Nobis uero causa dicenda demonstrandaque uidetur diuini muneris quod prouidentia commenta est salubriter hactenus: deum oculos hominibus id circo dedisse, ut mentis prouidentiaeque circuitus, qui fiunt in caelo, notantes eorum similes cognatosque in usum redigerent suae mentis, circuitusque animae, qui animaduersiones seu deliberationes uocantur, quam simillimos efficerent diuinae mentis prouidis motibus placidis tranquillisque, perturbatos licet, confirmatoque ingeneratae rationis examine, dum imitantur aplanem mundi intellegibilis circumactionem, suae mentis motus erraticos corrigant."

[156] The relationship between observation of the heavens, cognition, and the refinement of the intellect mapped out by Plato was available through sources other than Calcidius. Macrobius wrote in his *Commentary*: "Man alone was endowed with reason, the power of the mind, the seat of which is in the head ... Accordingly, man possesses reasoning power and perceives and grows (*sentit et crescit*), and solely by his reasoning power has deserved precedence over the other animals, which, because they are always bent forward and have difficulty in looking upwards, have drifted away from the heavens, and which, because they have not received in any part of their bodies a likeness of divine shapes, have been allotted no share of mind and consequently lack reasoning power." *Commentary* 1.14.10–11, ed. Willis, 57; trans. Stahl, 144.

years, and hours, which he calls the "birth of numbers" (*ortus numeri*) and the "genesis of measure" (*genitura dimensionis*).

For Plato, the eye is the nexus of number's discovery and the soul's reshaping: a gift.[157] The benefits of sight depend, however, on its manner of use. The straying motions of the mind are not corrected by passive stargazing; efficacy is bound to activity. The eye of the *Timaeus* is dynamic—seeking, investigating, and examining.[158] Only through this quite vigorous way of seeing are natural phenomena subject to "dinumeration" (*dinumeratio*), a now obscure term, which might be translated in many ways: being numbered, counted, calculated, reckoned. As discussed in chapter one, Plato's cosmological conception of number had been married to the biblical account of creation in the Old Testament since at least the first century and was deeply embedded in *all* available scientific and mathematic discourse available to the central middle ages. So too, I suggest, was his attendant theory of vision. The "supreme good" of the sense of sight could be known, of course, directly from the *Timaeus*, which, at the turn of the first millennium was available throughout the Latin-speaking world. That said, the link between vision, number, and philosophy was, perhaps, most broadly dispersed in the *way* that quantitative knowledge was conveyed: in the many pictures that filled manuscripts devoted to number.

The role of pictures in the *Enchiridion* seems to reflect a familiarity with Platonic ideas about number, vision, and cognition, but Byrhtferth probably never read the *Timaeus*.[159] He absorbed Platonic precepts obliquely, through his master, Abbo, and the books Abbo brought with him to Ramsey.[160] Abbo quotes the *Timaeus* and its Calcidian

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[157] Indeed, the relationship between seeing/vision and knowing (and verbs of cognition and of sight) once assumed to be universal, is now understood to be culturally specific. In many non-Western cultures hearing metaphors, rather than seeing, connote understanding. See Iraide Ibarretxe-Antuñano, "Vision Metaphors for the Intellect: Are they Really Cross-Linguistic?" *Atlantis* 30, no. 1 (2008): 15–33, esp. 25–26; Alfred Gell, "The Language of the Forest: Landscape and Phonological Iconism in Umeda," in *The Anthropology of Landscape: Perspectives on Place and Space*, ed. Eric Hirsche and Michael O'Hanlon (Oxford: Clarendon Press, 1995), 232–54; and Walter J. Ong, "The Shifting Sensorium," in *The Varieties of Sensory Experience: A Sourcebook in the Anthropology of the Senses*, ed. David Howes (Toronto: University of Toronto Press, 1991), 25–30.
[158] These are the English cognates to Plato's terms (as translated into Latin by Calcidius): *quaero*, *investigatio*, *examen*.
[159] Baker and Lapidge do not include it among Byrhtferth's sources, but they do discuss the transmission of Pythagorean/Platonic notions about number and reality through works that were available to Byhrtferth including Macrobius's *Commentary*. A contemporary of Byhrtferth, Lantfred of Winchester (*fl*.975; a monk, originally from Fleury) cited Calcidius. Baker and Lapidge, in Byrhtferth, *Enchiridion*, lxi–lxix and lxxxv–lxxxvi; Lapidge, *The Anglo-Saxon Library* (Oxford: Oxford University Press, 2006), 239–40.
[160] For the strong influence of Plato and Calcidius on Abbo's works and thought see Irene Caiazzo, "Abbon de Fleury et l'héitage Platonicien," in *Abbon de Fleury: Philosophie, sciences et comput autour de l'an mil*, ed.

Commentary with some frequency, often conflating the two works and their authors, as was common practice in this period.[161] He referred to the text as the *locus classicus* for the numerical structure of the World Soul, and he included a host of pictures from the *Commentary* in his computus and his *Explanatio* on the *Calculus* of Victorius.[162] A particularly fine copy of the *Timaeus* (Paris, BnF lat. MS 2164) was owned by Fleury.[163] There is every reason to think that the content and methods of his teaching while at Ramsey were as steeped in Platonic/Calcidian notions as his writing, most of which was didactic in aim. In his *Explanatio in Calculo Victorii* Abbo acknowledges the role that the senses, especially the eye, but also the ear, play in the transmission of numeric knowledge. Perhaps more importantly, he demonstrates this. In his *Explanatio*, Abbo called one of the multiplication tables the "door (*ianua*) of calculation" and claimed that its memorization through spoken recitation (*decantare*) would smooth the "rough" (*rudes*) soul.[164]Abbo referred to his commentary an "Introduction to arithmetic" (*Isagoge arithmeticae*), making clear his intention that the *Explanatio* might prepare the way for further numeric study. Hence, the sight of the *Ianua*'s neat columns and the sound of standard measures multiplied by two (written out, i.e., "twice half a *sescle* is a *sescle*" [*bis media sescle id est sescle*]) and rattled off like a litany in decreasing order, primed the innocent intellect, so that it might absorb the more advanced lessons of the quadrivium.[165] It is Plato's paradigm, but Abbo

Barbara Obrist (Paris: C.N.R.S. and E.P.H.E., 2004), 11–39; Evans and Peden, "Natural Science and the Liberal Arts"; and Eastwood, "Calcidius's *Commentary,"* 178–86.

[161] Dutton, "Medieval Approaches to Calcidius," 193–94.

[162] Abbo, *Explanatio* 3.1–3; ed. Peden, 72–75. Abbo included the so-called *lambda* (*psicogonia*) figure from the *Commentary* (32) in his *Explanatio* (see chapter 3). The same figure was scrupulously added in the margins of Fleury copies of Macrobius's *Commentary* beside passages on the world soul (see chapter 1). The astronomical figures and extracts from Calcidius's *Commentary* included in Berlin 138 are discussed above and in chapter 1.

[163] The juxtaposition of Claudius Mamertus's *De statu animae*—a work that Abbo drew heavily on in writing his *Explanatio*—with the *Timaeus* and the Calcidian *Commentary* suggests Abbo's direct involvement in the production of this manuscript.

[164] Abbo, *Explanatio* 3.90; ed. Peden, 125–26: "Haec est ianua calculi, qua intromittuntur ad hanc disciplinam rudes animi. Quibus instructi absque ulla difficultate valent memoriter decantare quid uniquique summae subtractum quid additum, quid relictum sit, et omnino omnium numerorum causas augmenti aut detrimenti, seu per seu ad se invicem teneanta relationem proportionalitatis."

[165] This sort of meditative or ruminative number crunching was not merely for the *rudes*, nor was it a pedagogical technique unique to Fleury. In "Number Mystique," Faith Wallis discusses the anonymous, 7th-c. *De ratione computandi*, which included problems involving absurdly minute divisions in time. In the same vein, Macrobius's discussion of the sun's diameter in his *Commentary* inspired calculations of infinitesimals. Such exercises can be considered a kind of mathematical ascesis that kept the mind clicking away on divine matters, in this case, pure number. Wallis, "Number Mystique," 190; Peden describes the marginal calculations of the sun's diameter in her dissertation, "Glosses Composed before the Twelfth Century."

performed a sort of sleight-of-hand, replacing the orderly circuits of the heavens with tidy rows of ordinals and measures.

For Plato number was discovered, the soul honed, and wisdom achieved through direct observation of the heavens. By the central middle ages, this was no longer the case.[166] What evidence we have suggests that *seeing* and *the path of sight* was of greater importance than the *thing seen*. Indeed, the skies could deceive and direct observation often had to be aligned with a rationalized representation. At one point in the *Enchiridion* Byrhtferth explained that there were two kinds of days, the *natural* day, of twenty-four hours, and the *vulgar* or *artificial* day, from sunrise to sunset. The former, natural day, was considered truer than the vulgar day, the length of which varied depending on the time of year, weather, and point of observation. Indeed, the "orderly fashion" of the sun's advance was observed not by following the sun through the sky, but by watching the shadow cast by a gnomon pass through points marked on the surface of a sundial.[167]

Another look at the description of Gerbert's teaching provided by Richer in the *Historiae* will give an idea of how didactic tools and representations were thought to function in conjunction with the eye and affect the mind. Richer stressed the vital role of such objects, especially in the domain of astronomy: "Although this subject [astronomy] is virtually inaccessible to the understanding, he [Gerbert] nonetheless made it comprehensible in marvelous fashion by employing certain instruments."[168] Describing these instruments, Richer collapsed the distance between object and intellect. About the compound sighting tube,[169] which was used to "view" artificial divisions known as equidistants in the night sky, he wrote:

[166] A point made by John Murdoch in his *Album of Science*: "The contemporary notion that observation and experiment are necessary ingredients of a discipline called science ... is not a conception properly applicable to antiquity and the Middle Ages. Then, science was *livresque*. It was not just set down in books it was largely carried out in books." Murdoch, *Album of Science: Antiquity and the Middle Ages* (New York: Charles Scribner's Sons, 1984), 3.

[167] Byrhtferth, *Enchiridion* 2.3.1–28; eds. and trans. Baker and Lapidge, 104: "þæt ys se sticca on þam dægmæle, ariht stent, þonne miht ðu asmeagan hu gefædlice seo sunne gesihð on þam dægmæle."

[168] Richer *Historiae* 50; ed. and trans. Lake, 2:74–75: "Ratio vero astronomiae quanto sudore collecta sit dicere inutile non est, ut et tanti viri sagacitas advertatur, et artis efficacia lector commodissime capiatur. Quae cum pene intellectibilis sit, tamen non sine admiratione quibusdam instrumentis ad cognitionem adduxit." Astronomy had, historically, been associated with models and demonstrations. See Netz, *Shaping,* around page 65–66.

[169] Also called a *dioptra*. For a description see chapter 2.

> The instrument was so efficacious in design that if the diameter were pointed toward the North Pole and the curved part of the semicircle were rotated, it would reveal those circles that are invisible to the eye and store them deep within the memory.[170]

Here, the tube works directly on the faculties of cognition and comprehension, transmitting the "invisible circles" into the memory. Gerbert's stellar sphere had a similar effect. Wire images of the constellations were tacked on the sphere's surface; through its center ran a tube for sighting and orientation. One would locate the north star with the tube and then rotate the sphere until the stellar configurations mapped and named on its surface were aligned with the constellations above. A student could then pick out and identify the constellations through a back-and-forth between the sphere and the night sky. Richer declared this device "divine" (*divinus*), for, he tells us, it permitted even one utterly ignorant of astronomy to know the constellations without the aid of a teacher.

Whereas the sighting tube and stellar sphere took the place of Gerbert's pointing hand, the armillary sphere supplanted the sky at which he pointed. It's crisscrossing circles showed the apogees, altitudes, and arrangement of the planetary orbits. With it, one could observe planetary movement "with and against the rotation of the heavens"—a thing impossible to perceive even through careful scrutiny. Gazing at the sky from the Earth, the planets appear to wander back and forth, sometimes disappearing completely. Nothing in Richer's account suggests that the armillary sphere violated or compromised the knowledge it conveyed; like the sighting tube it too "led to cognition." The figures in the *Enchiridion* are perhaps best understood as instruments. Like the sighting tube and stellar sphere they guide the eye and reveal the invisible circles of the heavens, like the sun dial and armillary sphere they served as surrogates, revealing the unobservable order of creation. Rather than looking up to follow the movements of the heavenly bodies, which yield their orderly circuits to the earthbound observer only partially and over time, a student could look down and trace their perfect, rationalized trajectories in three-dimensional instruments and models produced for demonstration or in a figure drawn in a manuscript. As the eye followed the path of an star across the night sky, a line across a page, so did the soul.

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[170] Richer, *Historiae* 3.52; ed. and trans. Lake, 2:78–9: "Cuius instrumenti ratio in tantum valuit, ut ad polum sua diametro directa, ac semicirculi productione superius versa, circulos visibus inexpertos scientiae daret atque alta memoria reconderet."

Historian John Murdoch in the *Album of Science* called the science of the middle ages "*livresque*" meaning that, "it was not just set down in books, it was largely carried out in books."[171] Murdoch wrote in defense of medieval science, explaining that though the science of this period was not "empirical," that is, grounded in observation and experiment carried out in laboratories or other specialized, professional locales, it was still science. Abbo's computus and Byrhtferth's *Enchiridion* show that the inverse of Murdoch's statement was equally true: books became more *scientifique*. That is to say, they became the primary site of observation and experiment; the medieval "laboratory" so-to-speak.

We should, I would argue, think of the pictures in the *Enchiridion* as a field of exercise on which to train the eye and, with it, the mind. In her chapter, "The Concept of *Ductus*, or Journeying through a Work of Art," Mary Carruthers considers the processual nature of "artistic form" (in the medieval sense of *ars*) so often encountered in medieval objects and associates this tendency with *ductus*, a technique of rhetoric. Carruthers defines *ductus* as "the way by which a work leads someone through itself."[172] She identifies the "formal arrangements" of a work as "agents, which cause movements, mental and sensory and—as in the case of architecture—physical."[173] It is an apt description of the workings of the images in the *Enchiridion*. Color and line compel the eye, carefully articulated numbers and letters propel it along certain paths. I would marry the rhetorical/trivial notion of *ductus* to the mathematical/quadrivial concept of orbit (*circuitus*) as described in the *Timaeus.* The paths of rhetoric and mathematics, however, differ. Plato promoted following only the "more perfect" cycles of the heavens. Mindful that each figure would guide and shape the intellect, Byrhtferth drew contours with care, selecting forms that were inherently perfect and divine.

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[171] Murdoch, *Album of Science: Antiquity and the Middle Ages* (New York: Charles Scribner's Sons, 1984), 3. The term is also used by Barbara Obrist to describe the study of cosmology after the 7th c., especially during the Carolingian era. Obrist, *La cosmologie médiévale*, 51–52.

[172] Carruthers, "The concept of *ductus*, or journeying through a work of art," in *Rhetoric beyond Words: Delight and Persuasion in the Arts of the Middle Ages*, ed. Mary J. Carruthers (Cambridge: Cambridge University Press, 2010), 190.

[173] Ibid., 201. I would elide the latter two categories: "sensory" and "physical."

CHAPTER 5

Gerbert, Geometry, and the Limits of Representation

> To be sure, the usefulness of this discipline is extremely great for all lovers of wisdom. ... For it is both the most subtle [means] of exercising the power of spirit and intellect and of sharpening observation, and most delightful for investigating many things (which seem bewildering and inconceivable to many) by means of a certain and true reckoning, as well as most full of subtle speculations for contemplating, admiring, and praising the wondrous force of nature and the power and ineffable wisdom of its creator, who made all things according to number, measure, and weight.[1]

Gerbert of Aurillac (c.940–1003), later Pope Sylvester II (999–1003), wrote the lines above; the discipline he so exuberantly describes is geometry. The passage comes near the beginning of his *Isagoge Geometriae* (*Introduction to Geometry*; hereinafter *Isagoge*), where it served to whet the reader's appetite for a subject of near limitless scope that, he believed, offered quite extraordinary benefits.[2] Geometry, for Gerbert and his contemporaries, was less a body

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[1] Gerbert, *Isagoge* 1.3; ed. Nicolaus Bubnov, in *Opera mathematica*, 50–1; all translations of the *Isagoge* are my own: "Utilitas vero disciplinae hujus omnibus sapientiae amatoribus quam maxima est. ... Nam et ad animi ingeniique vires exercitandas intuitumque exacuendum subtilissima, et ad plurima certa veraque ratione vestiganda, quae multis miranda et inopinabilia videntur jocundissima atque ad miram naturae vim ejusque Creatoris, omnia in numero, mensura et pondere disponentis, potentiam et ineffabilem sapientiam contemplandam, admirandam et laudandam subtilium speculationum plenissima est."

[2] The authorship of the *Isagoge* has been questioned. The work is attributed to Gerbert in a number of manuscripts, but some of the earliest lack this attribution. Bubnov included the text among Gerbert's genuine works in his *Gerberti Opera Mathematica*. Paul Tannery (and later Guy Beaujouan) argued that the treatise shows signs of being redacted over time and is the work of more than one author. Most recently, Anna Somfai has suggested that the first two sections should be attributed to Gerbert. She writes, "This portion of the text displays Gerbert's demonstrative methods and interest, it contains references to the sources that he used elsewhere in his letters and mathematical works." I agree with Somfai's assessment. If Gerbert only wrote the first two sections of the geometry, then his work might be thought of as an introduction in the literal sense: a preface to the more-or-less standard corpus on geometry and it might be likened to the introductions he authored for Boethius's *De arithmetica* and *De musica*. That the *Isagoge* is a genuine product of the central middle ages is more important to the arguments made in this chapter than Gerbert's authorship. Tannery, "La géométrie au XIe siècle," in *Paul Tannery: Mémoires Scientifiques*, Vol. 5: Sciences exactes au Moyen Âge, 1887–1921, ed. J. L. Heiberg (Paris: Gauthier-Villars, 1922), 99–102; Beaujouan, "Les Apocryphes mathématiques de Gerbert," in *Gerberto: Scienza, storia e mito* (Bobbio: Archivi storici bobiensi, 1985), 645–55; Somfai, "Brussels Gloss," 166, n. 63.

of axioms and precepts to be demonstrated and memorized, and more a tool that flexed and sharpened the mind, thus heightening the ability to comprehend things worldly and divine.

Gerbert's language emphasizes the visual nature of the endeavor; a geometer's powers of *observation* are especially well honed; he is one who delights in *searching*; the art is replete with *speculations* that permit him to *gaze* at and *admire* the divine order of creation.[3] Such seeing was ultimately not the work of the corporeal eye, but an exercise of the spirit (*anima*) and the intellect (*ingenia*). Medieval historians, especially historians of art, frequently reference "spiritual seeing" and seeing with the "mind's eye."[4] That medieval pictures were produced in the expectation of complementary acts of intellectual imaging on the part of the viewer has been convincingly argued.[5] Yet, little attention has been given to just how interior imaging was thought to have been accomplished, to the quality of this vision, or to the means by which it was cultivated. Indeed, it has been largely taken for granted that a viewer familiar with theological arguments calling for such an attitude of mind would be readily capable of seeing in this manner. What Gerbert and his sources make clear, however, is that this mode of cognition was regarded as anything but a native talent. It was a skill to be learned and one that was not easily mastered.

Quadrivial study, more generally, furnished a means of fostering such an ability. Boethius, in his preface to the *De arithmetica*, which, as we saw in chapter two, served as an introduction to all four of the numeric arts, wrote:

> It is the *quadrivium* by which we bring a superior mind from knowledge offered by the senses to the more certain things of the intellect. There are certain steps and fixed measures of development by which the mind is able to ascend and make progress so that by means of the eye of the mind ... truth can

[3] The Latin terms used are (respectively): *intuitum*, *vestiganda*, *speculationum*, *contempandam*, *admirandam*.
[4] To list a few: Caviness, "Images of Divine Order"; Herbert L. Kessler, *Spiritual Seeing: Picturing God's Invisibility in Medieval Art* (Philadelphia: University of Pennsylvania Press, 2000); David Ganz, Thomas Lentes, and Georg Henkel, *Ästhetik des Unsichtbaren: Bildtheorie und Bildgebrauch in der Vormoderne, Kultbild: Visualität und Religion in der Vormoderne* (Berlin: Reimer, 2004); Jeffrey F. Hamburger and Anne-Marie Bouché, *The Mind's Eye: Art and Theological Argument in the Middle Ages* (Princeton: Princeton University Press, 2005); and Giselle de Nie, Karl Frederick Morrison, and Marco Mostert, *Seeing the Invisible in Late Antiquity and the Early Middle Ages: Papers From "Verbal and Pictorial Imaging: Representing and Accessing Experience of the Invisible, 400-1000"* (Turnhout: Brepols, 2005).
[5] See, for example, Herbert Kessler, "Real Absence: Early Medieval Art and the Metamorphosis of Vision," in *Spiritual Seeing*, 104–48; idem, "'Hoc Visibile Imaginatum Figurat Illud Invisibile Verum': Imagining God in Pictures of Christ," in *Seeing the Invisible in Late Antiquity and the Early Middle Ages*, 291–325.

> be investigated and beheld. This eye, I say, submerged and surrounded by the corporeal senses, is in turn illuminated by the disciplines of the *quadrivium*.[6]

Boethius's references to "certain steps" and "fixed measures of development" suggest a particular order, a curriculum, by means of which the eye of the mind was developed. Other authorities—Gerbert, but also, Augustine, Macrobius, and Martianus—are more specific. They point to geometry as the proving ground of the intellectual eye. According to Martianus, geometry was "begotten indeed from incorporealities, and fashioned into manifold perceptible shapes from a slight and scarcely comprehensible beginning."[7] Martianus put these words into the mouth of *Geometria*, whom he personified along with the other liberal arts in his *De nuptiis*. *Geometria*'s words evoke a discipline that encompassed the realm of the incorporeal and the corporeal alike and imply that the ability to shift one's perception from bodily to intellectual vision was a requisite of geometric study. Contemporary sources suggest that, in the central middle ages, the capacity to do so was the discipline's primary imperative.[8]

Geometry was part of the standard curriculum taught in tenth- and eleventh-century monastic and cathedral schools by Gerbert, Abbo, and their peers.[9] Like others of the seven liberal arts, it both demanded and engendered its own distinctive rationale. The *agrimensor,* Agennius Urbicus (who wrote in the late fourth or early fifth century), in his preface to a treatise on land disputes that was known to Gerbert, likened the effect on the mind of learning by means of the disciplines to crafting a tool:[10]

> So, just as iron by its nature is not able to cut unless it has received a shape suitable for cutting, similarly the mind, which has the capacity to understand the natural universe, will lack the more subtle reasoning processes unless it is

[6] Boethius, *De arithmetica* 1.1.7, ed. Guillaumin, 8; trans. Masi, 73: "Hoc igitur illud quadriuium est, quo his uiandum sit, quibus excellentior animus a nobis cum procreatis sensibus ad intellegentiae certiora perducitur. Sunt enim quidam gradus certae que progressionum dimensiones, quibus ascendi progredi que possit, ut animi illum oculum, [qui, ut ait Plato, multis oculis corporalibus saluari constitui que sit dignior,] quod eo solo lumine uestigari uel inspici ueritas queat, hunc inquam oculum demersum orbatum que corporeis sensibus hae disciplinae rursus illuminent."
[7] Martianus, *De nuptiis* 6.706, ed. Willis, 251; trans. Stahl, 264: "alia est linearis atque optica huius pulveris erudita cognitio, quae quidem ab incorporeis procreata ac sensim [in] multiplices formas effigiata tenui ac vix intellectuali principio in caelum quoque subvehitur."
[8] I am especially indebted to Gillian Evans's 1976 article "'Sub-Euclidian' Geometry of the Earlier Middle Ages" for first drawing my attention to this fact.
[9] See above, chapter 2 for a more detailed discussion on the schools and curriculum.
[10] Urbicus's treatise was not among the works folded into the *Geometria I* or *II*. It was likely known to Gerbert through the so-called Arcerianus discussed in chapter 1 and below.

> assisted by a fixed, disciplined order. Therefore, above all, we must stimulate our mind with a love of noble arts and strengthen it with the instruments of a sound intellect.[11]

Though it remains common in English to refer to academic subjects as *disciplines*, the nominative form of the word has become somewhat distanced from the verbal notion: to discipline. Not so in Agennius Urbicus's description. For him the disciplines discipline the mind in a manner not unlike that of a parent who disciplines a child, and—given his choice of metaphor—these are not gentle parents. Metalworking is a violent process, one that radically reshapes and transforms material. Seen in this light, the mental imaging of geometry cannot be considered a casual pursuit, but rather a strenuous activity that roused students to move their minds in a manner entirely new to them.

This chapter considers the nature of intellectual sight produced through geometric practice and the implications that this habit of thought had for the making and viewing of pictures. Picturing with the mind's eye can be discussed if not experienced: even today's most advanced neuroimaging techniques do not permit us to enter into another's thoughts, and, as with any intellectual practice, mental imaging would have had as many manifestations as practitioners.[12] What a mind perceives, now and in the past, can be relayed and received only obliquely. This was appreciated in Gerbert's time, and yet, compelled by pedagogical necessity, searching attempts were made to capture and convey, verbally and pictorially, the quality and mechanics of intellectual sight. Manuscript evidence shows how masters charged with teaching the art of geometry and scribes tasked with copying geometrical treatises and works with geometrical content grappled with the problem—a struggle that left its traces in manuscripts.[13]

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[11] Agennius Urbicus, *De controversiis agrorum*, ed. and trans. Brian Campbell, in *The Writings of the Roman Land Surveyors: Introduction, Text, Translation, and Commentary*, Journal of Roman Studies Monograph 9 (London: The Society for the Promotion of Roman Studies, 2000), 18–19: "Ut enim nec ferrum <in> genere secare potest, nisi ad secandum habilem acceperit figuram, sic animus naturalium capax rerum, nisi certo disciplinae ordinem a<di>utus, subtilioribus indiget argumentis. Quam ob rem interpraecipua honestarum amore artium conpun<g>ere animum et bonae mentis instrumentis fundare debemus."

[12] Some scholars are getting quite close to achieving such feats. See notably Olaf Breidbach, *Das Anschauliche oder über die Anschauung von Welt: Ein Beitrag zur neuronalen Ästhetik* (Vienna: Springer, 2000); and Semir Zeki *Inner Vision: An Exploration of Art and the Brain* (Oxford: Oxford University Press, 1999).

[13] It is, of course, possible that these—masters and scribes—were, at times, one and the same.

The chapter will focus on Gerbert and his *Isagoge*, a late tenth-century synthesis of the numerous authoritative tracts on geometry that were then in circulation.[14] Gerbert's work survives in twelve copies, most of which were produced in the eleventh or twelfth century.[15] The structure, language, and content of this treatise, which takes the form of a dialogue, show Gerbert to have been highly selective in his choice of sources and willing to depart from tradition, especially in his use of pictures. For this reason the work serves as a contemporary guide to the discipline as practiced at the turn of the eleventh century by a teacher of significant influence. Gerbert looked to Augustine's *De quantitate animae* as a model for geometry teaching. Hence, this didactic dialogue will be examined in order to better understand the kind of mental activity engendered by geometry and methods of its inculcation. This habit of thought, involving controlled mental imaging, strongly affected methods of picturing in manuscripts and parts of manuscripts devoted to geometric matters. As with Gerbert's *Isagoge*, pictorial annotations added to works like Macrobius's *Commentary* on the *Dream of Scipio* reveal a host of strategies for imaging incorporealities and picturing three dimensions on a flat surface.

In her discussion of Dutch painting, Svetlana Alpers states that "It was a particular assumption of the seventeenth century that finding and making, our discovery of the world and our crafting of it, are presumed to be as one."[16] If "finding" and "discovery" are taken as akin to perception, then a strikingly similar assumption existed in the central middle ages, which gave rise to a mode of picturing radically different from the descriptive model employed by the painters discussed by Alpers. By the early modern era there had been a decisive shift in the primary locus of discovery from the eye of the intellect to that of the body, one that corresponds to a change in geometrical interests from matter and its measure to optical concerns. This chapter begins to show how artistic conventions that emerged in the tenth and eleventh centuries and would develop into what we call the Romanesque—conventions typically construed as unrealistic and a-scientific—may be seen to image reality according to theories of matter and perception of the day.

[14] See chapter 1.
[15] My characterization of the work is based on the three earliest copies, which are generally dated to the 11th c.: Munich, BSB Clm 14836; Paris, BnF lat. MS 7377C; and Brussels, BRB MS 4499–4503.
[16] Alpers, *The Art of Describing*, 27.

1. GERBERT'S *ISAGOGE GEOMETRIAE*

Gerbert in 982 was not enjoying his Italian sojourn.[17] Early that summer, he sent a desperate letter to Otto II:

> What, therefore, am I, a sinner, doing here? If it could be done at the pleasure of my lord, I would prefer to be the only one in need among the French rather than to be begging money with so many in need among the Italians.[18]

Otto had made Gerbert abbot of St. Columbanus of Bobbio (c.981), when the once-great monastery was in near ruins, stripped bare of land, resources, and even the relics of its patron.[19] Without direct intervention on the part of the emperor, Otto, Gerbert knew himself to be helpless.[20] Yet, despite many eloquent entreaties, the unhappy abbot was having difficulty attracting Otto's attention, for the emperor was then focused on gaining and maintaining power over the whole of the Italic peninsula. Gerbert was losing the battle for control of the lands of St. Columban. His letters of the period read as a litany of rebukes and accusations leveled at the former abbot, nearby farmers, and the Empress Adelaide; all concerned the monastery's territorial holdings, which had been parceled away as gifts or bound up in leases that yielded the abbey no profit.[21]

Given the nature of these disputes, it is not surprising that Gerbert turned to study of geometry for comfort and counsel.[22] The word *geometria* was, after all, derived from the Greek terms for "earth" and "measure"; Gerbert was familiar with the story of its

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[17] Opinions vary as to the duration and timing of Gerbert's abbacy of Bobbio in Lombardy. The period is not recorded in Richer's *Historiae*. Michele Tosi assembled a table that includes the dates suggested by all scholarly authorities on Gerbert prior to the time of his writing in 1985. M Tosi, "Il Governo Abbaziale di Gerberto a Bobbio," in *Gerberto: Scienza, Storia e Mito: Atti del Gerberti Symposium (Bobbio 25-27 Luglio 1983)* (Bobbio: Archivi storici bobiensi, 1985), 74.

[18] Gerbert, *Epistulae* 2; eds. Riché and Callu; trans. Lattin, 49: "Quid ergo peccator hic facio? Si cum gratia domini mei fieri posset satius esset me solum apud Gallos egere, quam cum tot egentibus apud Italos mendicare."

[19] In the same letter, Gerbert wrote of the wretched conditions in the monastery: "I see my monks wasting away from hunger and suffering from nakedness." In 929 the relics of Columbanus had been moved to Pavia. Tosi, "Il Governo Abbaziale," 87.

[20] Gerbert, like many friends of the Ottonian crown, who were placed by Otto in isolation at the head of institutions scattered across Italy in order to secure his conquests, lacked the support of nearby monarchs. His letters seeking advice of the local bishop were ignored and his relationship with the former abbot was contentious at best (this last had, under the auspices of an obscure Roman law, bequeathed to himself some of the lands of St. Columban). Ibid., 93.

[21] I am referring to letters 2–31. Adelaide was the widow of King Lothar of Italy (d.950) and Otto I (d.973). She controlled more land in Italy than did Otto II.

[22] The basics of geometry were essential knowledge for the accurate measurement and division of fields.

etymological origins.[23] The art, it was believed, was invented by the Egyptians, who had need of a rational method for dividing and allotting land because of the annual flooding of the Nile, which regularly confused the limits of their fields. The limits of his own fields being confused, Gerbert may have looked to the works of the Roman *agrimensores* ("land surveyors": Sextus Julius Frontinus, Siculus Flaccus, Hyginus, Marcus Iunius Nipsus, Balbus, Agennius Urbicus (whose definition of *disciplina* is quoted in the introduction to this chapter), and a number of anonymous writers.[24] These authors described methods of land measure and division, surveyors' instruments, types of boundary markers, and—perhaps of greatest interest to the besieged abbot—how to settle land disputes.[25]

Gerbert would not have had to go far for copies of the *agrimensores* or for any of the diverse texts through which geometry was studied. During his years at Reims, the contents of some of the greatest libraries in the Latin West were readily available to him and, though wanting in basic necessities, Bobbio remained rich in manuscripts.[26] Hence, as Gerbert put pen to parchment, he had either at hand or in his memory the full canon of texts in Latin that could support the study of geometry. We know from his correspondence with Archbishop Adalbero of Reims that he had come upon a volume at Bobbio with "some most splendid (*praeclarissima*) figures of geometry."[27] Two strikingly different manuscripts have been

[23] The story of geometry's invention is told by a number of authors, among these Isidore and Balbus. Gerbert included it at the beginning of the *Isagoge* (I.2; ed. Bubnov, 49–50): "Hujus inventores primi traduntur Aegyptii, qui propter eluvionem Nili fluminis, agrorum limites inundatione sui saepius confundentis, talis sollertiam artis excogitavere, cujus exercitatione sui quisque quantitatem agelli facilius a continenti posset secernere."
[24] Little is known of these authors. They were likely active in the 1st and 2nd c. CE. Summary biographies can be found in Campbell, in *The Writings of the Roman Land Surveyors*, xxvii–xliv.
[25] It is, perhaps, of note that additions were made in 9th- or 10th c. to a picture in the Arcerianus (discussed below) illustrating Agennius Urbicus's treatise on land disputes, *De controversiis argrorum*. The scene follows a verbal description of how to handle disputes over diverted rain water and laying sewers, when such disputes involve boundaries. Four lines—very difficult to make out—are written in the image: 1) Ego autem dixi amane ..., 2) MISERERE MEI DeuS, 3) Giselbertus, and 4) abis ssturum. It is, of course, the second and third lines that bring Gerbert to mind. The annotations were first pointed out by James Nelson Carder (1978) and more recently addressed by Tosi, "Il governo," 121–22.
[26] For a discussion of Gerbert and the libraries with which he was familiar see chapter 2. On the contents of the library at Bobbio see the extraordinary 10th-c. inventory in Becker, *Catalogi*, 64–73 (no. 32); and Tosi, "Il Governo," 130–39.
[27] Gerbert, *Epistulae* 8; eds. Riché and Callu; trans. Lattin, 15: "octo volumina Boetii de astrologia, praeclarissima quoque figurarum geomeriae." Scholars disagree as to the parsing and translation of this statement. Lattin summed up the issue: "Whether Gerbert means here eight volumes of a Boethius work on astronomy plus the other works mentioned, or eight volumes all together, is unclear, because of the confused Latin text and punctuation." Havet assumed that the eight volumes were of Boethius's works. Lattin, Bubnov, and Tosi understood Gerbert to mean that there were eight volumes all together. Bubnov later reversed his

associated with this reference: the so-called Arcerianus,[28] discussed in chapter one, and Naples, Biblioteca Nazionale Vittorio Emanuele III MS V.A.13.[29]

The Arcerianus, it will be remembered, is the oldest surviving copy of the works of the Roman land surveyors. The deluxe manuscript is best known for its illuminations colored in bright washes (figs. 1.40–42).[30] As discussed in chapter one, the more elaborate half and full-page images show an interest in surface detail and shading (fig. 1.43). Naples V.A.13 is a work-a-day compendium of geometric, astronomical, and computus material. It includes the *Geometria I*, a collection that combines gromatic tracts with passages on geometry from other sources, including Boethius's translation of Euclid's *Elements*, the *Institutiones* of Cassiodorus, and Isidore's *Etymologiae*. The manuscript includes numerous pictures, but these are austere line-drawings, drawn in the same brown ink as the text (fig. 1.44). Though the dominant mode of representation in these two manuscripts is markedly different, each is so densely illuminated that either might have inspired Gerbert's statement to Adalbero. It has been argued that one, the other, or both books served as immediate models for the *Isagoge*.[31]

The *Isagoge* is less a reworking of the texts found in any single manuscript, more a synthesis of material from the full geometrical corpus shaped by eight years of teaching experience.[32] One might say that Gerbert did for geometry what Abbo did for arithmetic and computus: he streamlined an ungainly corpus and readied it for contemporary use. The

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opinion. Lattin, in *The Letters of Gerbert*, 54, n.4; Havet, in *Lettres*, 6, n.8; Bubnov, in *Opera mathematica*, 100, n. 6; and Tosi, "Il Governo," 115.

[28] Wolfenbüttel, HAB Cod. Guelf. 36.23 Aug. 2°.

[29] Nicolaus Bubnov saw a strong resemblance between Naples V.A.13 and Gerbert's *Isagoge*, but wrongly dated the manuscript to the 11th c. Hence, he believed that its *exemplar* was at Bobbio and had been encountered there by Gerbert. Bernhard Bischoff has shown the Naples manuscript to be a product of Corbie, part of it written by Hadoard in the third quarter of the 9th c. Lattin, Ullmann, and others consider the Naples manuscript to be the one mentioned by Gerbert in his letter to Adelbold and used in the writing of the *Isagoge*. The manuscript itself affirms such a connection, for the text was heavily and intelligently redacted around Gerbert's time. In the upper margin of fol. 17r a later (12th?-c.) hand added the statement in Latin: "Here begins the book *Liber geometricae artis* edited by Lord Gerbert, pope and philosopher named also Sylvester the Second." Havet, on the other hand, assumed that Gerbert's letter could only be referring to the Arcerianus, which was "discovered" at Bobbio in 1453. Its whereabouts prior to that date are unknown. Recently, Tosi argued anew for the presence of the Arcerianus at Bobbio and for Gerbert's knowledge of it. Among his most compelling (but still inconclusive) pieces of evidence are 10th-c. additions that include the name "Giselbertus" written into one image (discussed above, note 24). Bubnov, in Gerbert, *Opera mathematica,* 394–98 and 472–76; Bernhard Bischoff, *Mittelalterliche Studien: Ausgewählte Aufsätze zur Schriftkunde und Literaturgeschichte* (Stuttgart: Hiersemann, 1966), 1:56; Lattin, in Gerbert, *The Letters*, 55, n. 5; Ullmann, "Geometry in the Medieval Quadrivium," 278; Havet, in Gerbert, *Lettres*, 6, n. 8; and Tosi, "Il governo," 114–22, esp. 121.

[30] The antiquity of the illuminations in the *agrimensores* manuscripts is uncertain. It is possible that they are only as old as the Arcerianus. Campbell, in *Roman Land Surveyors*, xxv–xxvi.

[31] See above, note 28.

[32] For a discussion of Gerbert's classroom practices see chapter 2.

Isagoge is a compact and concise document.[33] Copies typically fill fewer than twenty folios, often only ten or twelve, depending on format. Owing to its small size, the work rarely (if ever) travelled alone but was bound with other arithmetical and geometrical texts, sometimes with Gerbert's letters and other works attributed to him.[34] The small number of existing copies suggests that the treatise did not enjoy particularly broad circulation, but that does not mean that the brand of geometry expounded therein was not widespread. Gerbert, a renowned teacher, and his students—many of them future schoolmasters—disseminated his theories in classrooms across Europe.[35]

Gerbert began the *Isagoge* by situating geometric study within the quadrivium.[36] He then went on to rehearse the story of the practical origins of the discipline in Egypt and its later abstraction by the Greeks.[37] After enumerating the manifold benefits of geometric practice—the passage with which this chapter began—he got down to the task of teaching in earnest. He writes:

> The beginning of this art [geometry] and, so to speak, its elements seem to be a point, a line, plane and solid. ... A solid body is anything which is extended with three intervals or dimensions ... as is anything which can be comprehended by sight and touch .... We call the limits or surfaces of a solid "planes." Which [a plane] must be understood by means of the intellect (*intellectu*) because it enjoys nothing of height (that is depth) but is content to extend itself by length and width alone. For if depth is added to this, it would no longer be a plane, but ... a solid body. To be sure, the limit or edge of a plane is a line ... which it is necessary [that] you perceive with the mind (*mente*) because it stretches out with an extension of length alone, lacking width. A point, however, determines the beginning and end of a line; it [a point] restricts itself to being intelligible only by reason (*ratione*).[38]

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33 The text was made even more concise in an early redacted version that cut part of the first section and began with the passages devoted to measure (*Isagoge* 2.5). Among the manuscripts that contain this shorter version are Paris lat. 7377C and Brussels 4499–4503.

34 It is possible that the *Isagoge* circulated more broadly as a pamphlet with a limp binding and that over time such examples have been lost or were bound with other works.

35 On Gerbert's scholarly network see chapter 2.

36 Gerbert follows the Boethian order (rather than that of Martianus): arithmetic, music, geometry, and astronomy.

37 Gerbert, *Isagoge* 1.2, ed. Bubnov, 49–50.

38 Ibid. 2.1–2, ed. Bubnov, 51–54; "Artis hujus initia et quasi elementa videntur: punctum, linea, superficies, atque soliditas ... Solidum corpus est quidquid tribus intervallis seu dimensionibus porrigitur, id est quidquid longitudine, latitudine altitudineque distenditur, sicuti est quidquid visu tactuve comprehendi potest, ut haec praesens, in qua scribo, tabella. Hoc autem graece stereon dicitur. Hujus autem termini, seu superobducta planities, superficiei apud nos nomen accepit .... Quae ita intellectu capienda est, ut nihil sibi altitudinis, id est

The passage illustrates the central role that perception, sensible and intellectual, played in defining the elements.[39] We learn, for example, that a line, having only length and no width or depth, cannot exist independent of a body and, for this reason, can only be discerned by the mind. Depth (or height), the third extension in space, conferred a material dimension and, with this, in turn, determined how a thing was comprehended.[40] For Gerbert, who, as we will see, insisted on "accurate depiction," such an alignment of information brought to the fore inherent problems of representation. In the case of points, lines, and planes, all were immaterial; even the finest stroke of ink lent them a sensible reality contrary to their very nature. Picturing a solid's three dimensions on a flat, two-dimensional surface proved equally problematic. Indeed, Gerbert chose not to picture either incorporeal points, lines, and planes; or corporeal solids and he made his choice conspicuous.

There are thirty-seven figures in the *Isagoge*—all rigorously unadorned line drawings demonstrating geometric principles (figs. 1.45–46 and 5.1–2). Referred to in the textual narrative and placed within the lines of text or the textblock, figures are integral to Gerbert's tract. Their general placement within the treatise is, however, strikingly uneven. Pictures only appear in the latter two-thirds of the work, in those sections devoted to measure. Their inclusion accords with measure's innate materiality. Measure is always of a thing, it is the property of a body: the length of a stream, the height of a tree.[41] It is sensible and can (even should), therefore, be pictured. The geometric elements, intelligible, but *not* sensible, were, likewise, *not* pictured. This does not, however, mean that Gerbert intended for them not *to be* pictured. Indeed, he firmly believed that picturing was crucial to their comprehension. His definitions of the geometric elements—quoted above—are replete with admonishments that they be understood "by means of intellect" or seen "with the mind's eye." Exactly what was

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crassitudinis, usurpet, sed tantum longitudine latitudineque contenta se dilatet. Nam, si his altitudinem adjicit iam non superficies, sed corporis pars atque ideo corpus solidum erit. Superficiei vero extremitas sive terminus linea ... est. Quam ita mente percipias oportet, ut latitudinis expers solius longitudinis se rigore producat ne latitudine addita iam non linea, sed superficies sit. Lineae autem principium et extremitatem punctum determinat, quod ita se intelligibili ratione coarctat, ut lineae tantummodo finis existens nullam in eo partis aut alicuius omnino magnitudinis quantitatem obtineat."

[39] The marriage of the Euclidean definitions of the elements with statements on their epistemic nature was not a Gerbertian innovation. Such an approach was essential to arguments made by Calcidius, Macrobius, Augustine, who (among others) were interested in geometry, not for its own sake, but for the what it might reveal about metaphysical matters (e.g., the nature of the soul).

[40] As Gerbert (*Isagoge* 2.2, ed. Bubnov, 53) points out, "height" (*altitudo*) and "depth" (*crassitudo*) were both terms used to describe the third extension.

[41] These are two of the examples given by Gerbert (*Isagoge* 2.5, ed. Bubnov, 55).

meant by these injunctions, the nature of intellectual sight, will be broached in the next section. Of interest here is how these verbal prompts worked. I suggest that their success depended on the expectation of a picture as well as its absence, and that the language and structure of the *Isagoge* were carefully curated to create a sense of want and stimulate mental action in the reader/viewer.

Gerbert followed the definitions of the geometric elements with descriptions of their real-world counterparts: linear measure, area, and volume. Unsurprisingly, these definitions closely resemble those of their less tangible cousins, with one significant difference: length and area are pictured. The two sets of definitions sit in close proximity, one immediately following the other. In some copies of the tract, like Oxford, Bodleian Library, Selden Supra MS 25, they appear on either side of the same folio (figs. 5.3–4). Juxtaposition shines a light on likeness and difference. The language is, moreover, similar, inviting comparison. A line, for example, "stretches out with an extension of length alone, lacking width." Likewise, linear measure, "is any line measured that takes nothing of height or of width."[42] But the definition of linear measure continues: "and is of such a kind (*et est talis*)." *Talis* is followed by a blank space filled by a horizontal line (fig. 5.5). The definition of a line is followed by that of a plane; linear measure by surface area. Again, the two definitions are strikingly similar, but the latter precedes another gap in the text, this one filled by a drawing of a square. The presence of the picture is signaled in the text, this time with the word *ita*: "in this way." In both instances the pictures are referents; without them, the sentences would be incomplete.

With these definitions, Gerbert established a pattern: verbal description, visual description. In so doing, he primed readers to expect that the definition of solid measure would be followed by its picture. He wrote, "A solid ... is equally extended and squared by means of length, width and height, through which solid bodies are measured."[43] Where a reader thought to find a picture of a solid Gerbert instead provided the statement:

> That is to say [solids] hold a form like a cube or die, which cannot clearly be figured on a flat surface, but which can be understood by the intellect, or easily be formed in wax or wood, or another material of this sort. Although

[42] Gerbert, *Isagoge* 2.6, ed. Bubnov 56: "Linearis pes est, per quem lineas ... aliquam metimur nihil interim de altitudine vel latitudine curantes."
[43] Ibid.: "Solidus autem est longitudine, latitudine altitudineque aequaliter distensus et quadratus, per quem solida metiuntur corpora."

> Calcidius—commentating on the *Timaeus* of Plato—depicted, somehow or other (*utcunque*), a solid body on a flat surface.[44]

Although Gerbert omits the figure he shows himself aware of his readers' expectation of one.[45] He includes examples—a cube or die—and justifies the absence of a picture with a simple explanation: three dimensions cannot be depicted in two. As with the incorporeal elements, Gerbert does not discourage picturing a solid, but insists that it be done *accurately* and this meant, not on parchment.

The passage on picturing (or, rather, not picturing) a solid is among the clearest statements on representation and its limits to come down to us from the central middle ages. Significantly, it declares a stance that had little basis in existing tracts and passages on geometry from the late antique or Carolingian era. Had Gerbert wanted to include a drawing of a solid, he would have had a variety of authoritative models to choose from. Solids were pictured with some frequency in Naples V.A.13: in the excerpts from the *agrimensor* Balbus's *Expositio et ratio omnium formarum* (*Description and Analysis of All Forms*), Cassiodorus's *Institutiones*, and Isidore's *Etymologiae* (figs. 5.6–8). Gerbert regarded these authors as authorities, and he mined their work for information included in the *Isagoge*, but he went his own way, evidently, when it came to picturing. Still, I would argue that the images of solids encountered in Naples V.A.13 and elsewhere played a fundamental role in shaping Gerbert's thought on representation, serving not as models, but as proofs: demonstrations that three-dimensions "cannot clearly be figured on a flat surface."

There was, however, one exception. Gerbert wrote that "Calcidius, commenting on Plato's *Timaeus* depicted, somehow or other, a solid body on a flat surface." Plato had cast earth and fire as the primary building blocks of the universe, for, he explained, "nothing could ever become visible without fire, nor tangible without ... earth."[46] Calcidius contrived a series of arithmetic and geometric proofs to demonstrate mathematically how these unlike

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[44] Ibid. 2.6, ed. Bubnov, 56: "Solidus est ... formam videlicet cubi vel tesserae retinens, qui in planitiei quidem aequalitate non potest aperte figurari, sed vel mente intelligi, vel cera, vel ligno, aliave ejusmodi materia facile valet formari, quamvis Calcidius Timaeum Platonis exponens solidum in plano corpus figuratum utcunque descripserit."

[45] Indeed, the expectation was so great that in at least one, later copy of the treatise, Oxford, Selden Supra 25, fol. 119v, the text was erased and rewritten to make space and a figure of a solid was added in the middle of the textblock.

[46] Plato, *Timaeus* 31b, ed. Waszink, 24; my translation: "Et quia corpulentus uisibilisque et contiguus erat merito futurus, sine igni porro nihil uisibile sentitur nec uero tangi quicquam potest sine soliditate, soliditas porro nulla sine terra, ignem terramque corporis mundi fundamenta iecit deus."

elements, fire and earth, might be joined by intermediate elements, air and water.[47] Among these proofs were two Euclidean-type constructions, linear projections. In a copy of the *Timaeus* and its *Commentary* owned by Otto III and probably known to Gerbert, these figures were drawn by the scribe with a rule in the same ink as the text (figs. 5.9–10).[48] The first figure comprises four solid quadrilaterals arranged in two rows of two (fig. 5.11). Roman numerals give the dimensions of each solid. So, for example, the solid in the upper left corner is composed of surfaces with an area of six (two by three), the depth of the solid is four, making its volume twenty-four. As with most geometric constructions, the figure is qualitative, not quantitative. Hence, the sides are pictured as even, despite their unequal lengths. Calcidius explained in the accompanying text that the solids and thus the elements were connected by numbers. In the figure, the numbered cubes are connected with a zigzagging line like a connect-the-dots puzzle—24 to 48 to 96 to 192—in continuous proportion (1:2), which Plato held to be the most perfect bond.[49] The four solids are combined into a single construction in the second figure and labeled with letters from the Greek alphabet (fig. 5.12).

Doubtless Gerbert had these figures in mind when he wrote the above-quoted statement singling out the Calcidian solids. The exact tenor of his comment is difficult to determine. He obviously understood the graphic conceit of projection and acknowledged its success, but he did not then endorse picturing solids in this way. Exactly why is not entirely clear. It is possible that he had observed how easily errors crept into these figures. In many copies of Calcidius's *Commentary* we find minor departures—extra lines or lines that ought to be parallel, but are not—which disrupt the illusion of depth (fig. 5.13). Given the context of Gerbert's statement, however, it seems more likely that his was a broader skepticism: why teach three dimensions through something flat? Better to picture them in the mind's eye or form them out of some malleable substance.

[47] The blind-ruling for the textblock served as a guide for horizontal lines. On the role of these figures in the *Commentary* see Anna Somfai, "Calcidius's *Commentary*," 208–18.
[48] Bamberg, Msc.Class.18, fols. 15r and 16r. The figures on folio 15r were corrected and redrawn in a dark brown ink by a contemporary or near contemporary hand, who also corrected errors in the text. Hoffmann dated the annotations and emendations to the 10th or 11th c. and believed them to be the work of a Würzburg scribe. Hoffmann, *Bamberger Handschriften*, 128–29.
[49] Plato, *Timaeus* 31c–32. On the number and binding of the elements and the mathematical superiority of continuous proportion in Plato, see Francis M. Cornford, in *Plato's Cosmology: The* Timaeus *of Plato* (New York: Humanities Press, 1952), 43–52.

Some verification of this interpretation comes in the form of an extensive and sophisticated tenth-century gloss on the *Commentary* that reflects a strikingly similar attitude toward projection. The gloss, called the "Brussels gloss" by Anna Somfai, who edited and analyzed the annotations, is best preserved in Brussels, Bibliothèque royale de Belgique MS 9625-26.[50] This manuscript was made in northern France and shows close ties to the oldest extant copy of Calcidius's *Commentary*, Valenciennes, Bibliothèque municipale, MS 293, which was written at Reims and later owned by Hucbald of St. Amand.[51] Indeed, Somfai suggests that the gloss may be associated with Gerbert and/or his circle.[52] Whoever the tenth-century annotators may have been, they introduced a number of pictorial additions, graphic explanations of Calcidius's figures. The commentary on the projections is especially dense. In order to explain three-dimensional imaging on a plane, the Calcidian solids were systematically deconstructed and flattened or, in the words of one contemporary reader, "dissolved" (figs. 5.14–15).

The word *dissolutio* was squeezed into the narrow space between the main text and some of the graphic annotations (fig. 5.16). The term captures well the reductive approach of the annotator. Calcidius's four detached solids were each reduced in the gloss to a single plane. Hence the upper-left solid, with a volume of twenty-four, was re-imaged as a rectangle measuring two units by three units (fig. 5.17). Height was added verbally. Below the rectangle was written "on account of solidity let four in height be placed over it [the surface]."[53] The lettered geometric construction in which the four solids were fitted together

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[50] Somfai, "The Brussels Gloss," 139–69; a description of the manuscript is found on page 143.

[51] On Valenciennes 293 see Rosamond McKitterick, "Knowledge of Plato's *Timaeus* in the Ninth Century," 85–96, esp. 90–96. Somfai concluded that Valenciennes 293 was either the exemplar for Brussels 9625-26 or that the two manuscripts shared a parent manuscript at Reims. Somfai, "The Brussels Gloss," 143–44.

[52] Somfai, "The Brussels Gloss," 165–68. On the authorship of the Brussels Gloss, Somfai wrote, "The fact that there was an exemplar of the texts [Plato's *Timaeus* and Calcidius's *Commentary*] at hand, that a need for producing another copy arose, and that two expert scribes could work on the enterprise, suggests an excellent library with a significant scriptorium, and probably a major scholarly center as Brussels's [9625-26] place of origin." Somfai suggests that these conditions only existed at Fleury and Reims, and that Abbo, Gerbert, or someone in their immediate intellectual circles authored the gloss, with Gerbert being the more likely source. Among the evidence that might connect the Brussels manuscript with Fleury is the coincidence of a correct figure (faulty in all other copies of the *Commentary*) in Brussels 9625-26, and Berlin 138, and some shared glosses with Paris, lat. 2164 (ibid., 164–65). It should also be noted that in the copy of the *De arithmetica*—the ultimate source for the oblong bases of the separated solids pictured in the gloss—from Fleury (Paris, lat. 6401) units are "pigeonholed" in the same way that they are in Brussels 9625-26. Somfai does not mention Bamberg, Msc.Class.18 in her discussion, which also includes selections from the Brussels gloss and has been associated with Gerbert.

[53] Brussels 9625-26, fol. 12r: "Haec figura a senaria basi id est(?) de duobus lateribus longitudinis quae est .iii. et latitudinis quae est .ii. quater propter soliditatem in altum sibi superponatur fiunt .xxiiii.

was first exploded and then broken down into planes (fig. 5.18). Again, not all the surfaces were pictured, only the "front" and "back."[54] Text within each plane instructed the reader/viewer to place one surface over another in order to reconstruct the compound solid mentally. On the last plane, ΘΗΤΛ, the annotator wrote: "And then [this plane] is placed over the surface to make a solid. All of which [surfaces-cum-solids] are increased to demonstrate the above-written figure" (fig. 5.19).[55] By giving the component parts and instructions for assembly, the annotator adopts a similar strategy to Gerbert, who defined a solid, gave real-world examples—a die or cube—and then encouraged its mental visualization.

The Brussels gloss shows how three dimensions might be pictured on the page, if need be, though in full awareness that what could be rendered was incomplete without an act of imaginative construction. As an author of an original work, Gerbert could choose not to picture solids in the *Isagoge*. But this option was not available to a scribe tasked with copying the authentic text of the Calcidius's *Commentary* or the many other authoritative tracts that included pictures of solids. The Calcidian projections were deemed essential aspects of the work. Indeed, Calcidius devoted a particularly lengthy passage to explaining the second construction. Without the figure, the passage would have been largely incomprehensible.[56] Hence the projections were reproduced with care in the Brussels manuscript, and then reduced to two dimensions with equal care, making them fit for contemporary use. The verbal prompts in the Brussels gloss speak of extending a plane into three dimensions by adding height or placing one surface above another to make a solid. These actions were not accomplished on the page, but in the mind of the reader/viewer.

2. Picturing with the Intellect

> The beginning of this art [geometry] and, so to speak, its elements, seems to be a point, a line, plane, and solid. Both Boethius and other writers of both

[54] ΔΓΡΣ, as the text on its interior points out, is the surface shared by the two intermediaries. The "rear" surface, ξϒϕο, of the second intermediary was omitted by the annotator of the Brussels manuscript. It was, however, included in Bamberg, Msc.Class.18, fol. 16r (fig. 5.10).
[55] Brussels 9625-26, fol. 16r: "Et sup(er)ponit(ur) tu(m) sup(er)ficiei ad soliditate(m) qu(a)e om(n)ia augm(en)tantur ad figugam ostendendam superscriptam."
[56] On the interdependence of text and image of lettered diagrams see chapter 4. Somfai observed that the glossator adopted Calcidius's terminology and how the gloss was lettered so that it could be used in conjunction with the main text in place of the original figure. Somfai, "Brussels's Gloss," 160.

secular and divine works in many places of their writings frequently describe these things, and the most blessed and eloquent doctor of the church, Augustine, most eloquently holds forth in not a few of his books, and especially in that one which is called *De quantitate animae*.[57]

In this way, citing his sources, Gerbert introduced the definitions of the geometric elements in his *Isagoge*. Foremost among these was St. Augustine's *De quantitate animae* (*On the Measure of the Soul*), which took the shape of a Platonic dialogue between Augustine, the teacher, and Euodius, his curious student.[58] The latter, puzzled by the nature of the soul, asks about its quantity or measure.[59] Through patient inquiry on the part of Augustine, Euodius is made to realize that his question rests on a mistaken premise: being incorporeal, the soul lacks the qualities of a solid body such as length, width, and height—it has no measure (at least not of this sort). This conclusion is arrived at by way of a lesson in geometry that begins with picturing the geometric elements with the mind's eye:

> Augustine: Try your best to imagine length, which has as yet assumed no width.
> Euodius: I can imagine nothing like that. If, for instance, I create in [my] mind a spider's thread ... there comes to mind even in that [fine thread] length, but also width and depth, which—however large or small they are—can surely not be denied to exist.
> A.: Your response is not altogether absurd, but surely, when you recognize these three [dimensions] in the thread of a spider, you separate them [one from the other] and know that they are distinct?
> E.: How could I not know that there is some difference? How else could I see that the thread has three dimensions?!
> A.: You see these things [length, width, and height] by means of a certain understanding, and it is by this [faculty] that you are also able to imagine length by itself, provided you do not set your mind on any bodily thing; for no

[57] Gerbert, *Isagoge* 2.1, ed. Bubnov, 51–52: "Artis hujus initia et quasi elementa videntur: punctum, linea, superficies, atque soliditas. De quibus cum ipse Boetius aliique tam saecularis, quam divinae litteraturae tractatores in plurimis scriptorum suorum locis satis superque disputent, tum beatissimus et eloquentissimus Ecclesiae doctor, Augustinus, in nonnullis libris suis et praecipue in eo, qui de quantitate animae inscribitur."
[58] The *De quantitate* survives in a limited number of witnesses suggesting that it did not enjoy particularly broad circulation in the central middle ages (see chapter 1, note 179). An extended excerpt from the dialogue was, however, included in the *Geometry I*, which survives in more than thirty copies. Hence, the work may have been best know in its excerpted form and in the context of geometry. On the *Geometry I* see Menso Folkerts, "Die Altercatio in der *Geometrie I*," 84–114; and Wesley M. Stevens, "Euclidean Geometry in the Early Middle Ages," 229–64.
[59] Augustine, *De quantitate animae* 1.1, ed. Wolfgang Hörmann, CSEL 89 (Vienna: Hoelder-Pichler-Tempsky, 1986), 131.

matter what it may be, a body will have all of these dimensions. What I want you to think of now is incorporeal. For length alone can be grasped only by the mind; alone it cannot be found in a body.

E.: I understand.

A.: Therefore, should you wish to cut that length—mentally, as it were—lengthwise, you see at all events that it cannot be done, if it were possible, width would also be present.

E.: That is obvious.[60]

"Imagine length." Augustine's command seems simple. Euodius, however, is quick to point out the challenging nature of the task: how can one envision a thing for which no worldly example exists? He tries and fails: even a spider's finest silk—colorless, seemingly without substance—has length, width, and depth! Augustine agrees, but urges his frustrated companion to consider his response more closely: if able to discern these three dimensions in the spider's thread, he should also be able to picture length alone.

Augustine explains that a line is seen with "the soul, by means of a certain interior eye, that is, the intellect." He admits that this mode of seeing is "most subtle," that it demands "other eyes of the mind far different from those used in daily life."[61] For Euodius, seeing with the soul is, at first, an entirely new and difficult endeavor, but as the dialogue progresses, he learns to govern his interior eye. Eventually he "sees" a line, after which he dutifully conjures a circle, triangle, square, and—most challenging of all—a point, which is entirely without measure.[62] Augustine asks his pupil not only to picture but also to manipulate these visions. The images described in the dialogue are remarkably mobile and malleable. Euodius draws

[60] Ibid., 6.10, ed. Hörmann, 142–43; translation heavily adapted from Joseph M. Colleran, in Augustine, *On the Greatness of the Soul*, Ancient Christian Writers 9 (New York: Newman Press, 1978), 27–29: "A: Itaque, enitere cogitare longitudinem, quae adhuc nullam latitudinem assumpserit. E. Nihil possum tale cogitare. Si enim filum araneae in animo constituero, quo nihil exilius solemus videre, occurrit mihi etiam in eo tamen et longitudo per se, et latitudo, et altitudo: quae, qualescumque sint, esse tamen negare non possum. A. Non usquequaque absurda est responsio tua: sed certe cum tria ista in araneae filo intelligis, discernis haec, et quid inter se differant nosti? E. Quidni quid differant nossem? An aliter potui videre nihil horum deesse huic filo? A. Quo igitur intellectu haec discrevisti, hoc potes etiam, seiunctis illis, solam longitudinem cogitare, si nec aliquod corpus vol vas animo: nam, quodeumque fuerit, his omnibus non earebit. In corporeum est enim quod te nunc intetligere cupio. Nam sola longitudo, non nisi, intelligi animo potest, in corpore inveniri non potest. E. Iam intelligo. A. Ergo istam longitudinem, si quasi secare cogitatione per longum vetis, vides profeeto non posse: nam, si potest, inest etiam latitudo. E. Manifestum est."

[61] Ibid., 4.6, ed. Hörmann, 137; my translation: "Verum quia res subtilissima est, et longe alios menti~ oeulos quaerit, quam humana consuetudo in quotidianae vitae actibus habere solita est."

[62] Ibid., 11.18, ed. Hörmann, 154.

and extends, he cuts, revolves, and compares—thus proving geometric notions by means of thought and reason.

Augustine rehearses in words the definitions of the geometric elements, which his student verifies visually. He carefully monitors the progress of his charge. Before beginning a new line of inquiry, Augustine remarks that he *thinks* Euodius has correctly conceived a line. He tests the accuracy of Euodius's mental pictures through probing questions that reveal errors and misunderstandings. In the passage quoted above, Euodius is asked to cut his line lengthwise—an impossible feat, if he has, in fact, pictured a line. Elsewhere, he is told to turn a plane on its edge and slip it through the crack between two doors.[63] The task is a test to prove that he has imagined a plane correctly, as length and width without height. Once assured that Euodius has seen a line, Augustine instructs him to extend the imagined line from either end, but follows this command with a concerned query: "you see this can be done without limit. Or, does this tax the keenness of your mind too much?" Euodius reassures his master that he is able to see infinite length "clearly and with ease."[64] Satisfied, Augustine moves the lesson along.

When Euodius "sees" a line, what does he see? Augustine's words suggest that intellectual sight was not wholly unlike corporeal vision.[65] The two were, at least, comparable. Augustine refers to the mind's eye as "another" or "other eyes," and the language he uses to describe the act of spiritual seeing—"to see" (*videre*), "to reveal" (*manifestare*), "to perceive" (*cernere*)—frequently resembles that used to describe bodily vision. These same terms are, in turn, employed interchangeably with verbs that connote thought, such as "to understand" (*intelligere*), "to think" (*cogitare*). Augustine observed this phenomenon in the *Confessionum*: "for, seeing belongs properly to the eyes; yet we use this

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[63] Ibid., 12.21, ed. Hörmann, 156: "A. Redi ergo ad cogitationem latitudinis, et, si earn quasi iacentem animo figuraveris, erigatur in quodlibet latus, tanquam si eam velles per tenuissimam rimam, ubi se clausae ianuae iungunt, educere."

[64] Ibid., 6.11, ed. Hörmann, 144; my translation: "A: ... Sed linea ista, quam iam, ut opinor, bene intelligis, si porrigatur, sive ex una, sive ex utraque parte, quam in longum porrigi pot est, cernis nullum esse finem. An ad hoc contemplandum minus valet acies mentis tuae? E. Contemplor omnino, ac nihil facilius."

[65] Echoed by Gerbert, who wrote in the *Isagoge* (2.4; ed. Bubnov, 54), "But these three, that is a point, line, and plane cannot exist in nature outside of bodies, although [these] incorporeal things can be discerned by the mind and almost as if understood as if they have their being outside of bodies." ("Sed haec, videlicet, tres: punctum, linea et superficies in rerum natura subsistere nequeunt praeter corpora, mente tamen intelliguntur incorporalia et quasi praeter corpora esse suum habentia.")

word of the other senses also, when we employ them in seeking knowledge."[66] This seamless shift from verbs of seeing to verbs of thinking serves to emphasize the close relationship between intellectual observation and knowing and to define the ontological status of mental pictures: to envision a line is to understand *line*. Indeed, the primary difference between corporeal and intellectual sight is what might be seen and, thereby, known.

Further on in the dialogue Augustine posed the question:

> A.: Now, then, have you ever seen with the eyes of the body such a point, or such a line, or such width [i.e., a plane]?
> E.: No, never. These things are not bodily.
> A.: But if, by some sort of remarkable affinity of realities, bodily things are seen with bodily eyes, it must be that the soul, by means of which we see these incorporeal things is not a body, nor like a body.[67]

The eye of the body sees things with bodies, that is three-dimensional things. The soul, on the other hand, sees those things without bodies. Euodius's ability to see the incorporeal geometrical elements with his intellect's eye serves as proof that the intellect too must be incorporeal. Having determined that the soul is not a body, Augustine and Euodius turn their attention away from geometrical matters.

The *De quantitate* offers a glimpse—albeit idealized—into how geometry was taught in Augustine's own time, but it applies as well to later centuries, not least because the text continued to be read. In a number of manuscripts, extended excerpts from the dialogue prefaced Gerbert's *Isagoge*.[68] The arrangement was natural since Gerbert specifically referred to the work. He summarized it in this way:

> In the *De quantitate* Augustine shows that the eye of the mind, which has been blunted by images of corporeal things, through the practice of geometry,

[66] Augustine, *Confessionum* 10:35.54; ed. James J. O'Donnell, in "The *Confessions* of Augustine: An Electronic Edition," The Stoa Consortium, http://www.stoa.org/hippo/text10.html (accessed July 20, 2015); my translation: "Ad oculos enim proprie uidere pertinet, utimur autem hoc uerbo etiam in ceteris sensibus, cum eos ad cognoscendum intendimus." The statement was repeated by Isidore in the *Etymologiae* (1.1.20).
[67] Ibid., 13.22, ed. Hörmann, 157; my translation: "A. Unquam ne igitur oculis istis corporeis, vel tale punctum, vel talem lineam, vel talem latitudinem vidisti? E. Omnino nunquam. Non enim sunt ista corporea. A. Atqui si corporea corporeis oculis mira quadam rerum cognatione cernuntur; oportet animum quo videmus illa incorporalia, corporeum corpusve non esse."
[68] As, for example, in Brussels 4499–4503 where the excerpt from the *De quantitate* appears on fols. 8v–12r and the *Isagoge* (preceded by a letter of Gerbert on the area of a triangle) begins on fol. 15r.

> is purged and made sharp in no small way for somehow contemplating spiritual things and truths.[69]

Here, the primary purpose of geometry is to strengthen intellectual sight. For Augustine and Gerbert, the definitions of the geometric elements were a means to this end, tools with which the soul was trained to see.

Definitions of the geometric elements were encountered, of course, in specialist treatises on geometry, like the *Isagoge*, but they were also met in works with a broader scope such as Martianus's *De nuptiis*, the *Institutiones* of Cassiodorus, and Isidore's *Etymologiae*; and in discursive theological and philosophical works including commentaries on Genesis, Job, and the psalms; Augustine's *De civitate Dei*; Plato's *Timaeus* with its Calcidian commentary; and Macrobius's *Commentary*. These tracts, as we saw in chapter one, to judge from the number of extant manuscripts were among the most popular of the period, and part of the standard repertory used for teaching.[70] Contemporary and near-contemporary library lists show that even modest monastic collections included at least one and often more of these titles. Larger institutions often boasted a copy of each work and sometimes several. The level of dissemination of the verbal and visual definitions of a point, line, plane, and solid in contexts outside geometrical treatises suggests that at least the rudiments of geometry were widely inculcated.

3. Picturing for the intellect on the page

Euodius used the mental image of a cobweb as a point of departure, an aid for envisioning intangible elements; most medieval readers of the *De quantitate* had recourse to pictures drawn beside the text, in the margins of manuscripts.[71] Hence, on the page, Augustine's verbal definition of a square was, in many manuscripts, accompanied by a drawing of one (figs. 5.20–22). It is, however, unlikely that these images were part of the original fourth-

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[69] Gerbert *Isagoge* 2.1, ed. Bubnov, 52: "copiosissime disserit, ubi etiam mentis oculum, corporearum rerum imaginationibus obtusum, per talium artium exercitia ad spiritalia veraque utcunque contemplanda non modicum purgari et exacui ostendit."

[70] See chapter 1 for descriptions of these tracts, their manuscript traditions, and their place in the quadrivial canon in the central middle ages.

[71] Pictures of geometric figures appear in the margins of at least three 9th-c. copies of the tract that I have seen: London, BL Harley MS 3012, fols. 1v–32v; Paris, BnF lat. MS 13369, fols. 144r–85v; Valenciennes, BM MS 163 (155), fols. 4r–58r.

century treatise.[72] In ancient geometric texts such as Euclid's *Elements*, descriptions of the geometric elements and figures (e.g., circles, triangles, spheres) called "definitions," were considered *a priori* knowledge and required neither proof nor picture.[73] At some point, however, this changed. Exactly when is uncertain. Augustine, Macrobius, Martianus (who were near contemporaries) all provided definitions of the geometric elements and figures in their works, but did not even hint at the presence of actual pictures, suggesting that the conviction these were needed emerged only later.[74]

By the time Isidore of Seville wrote his encyclopedic *Etymologies* in the seventh century, the practice of picturing the definitions had become *de rigueur*.[75] Isidore explicitly refers to pictures:

> The first figure of this art [geometry] is the point, which has no parts. The second figure is the line, a length without breadth. ... A plane has length and breadth only.... The forms of these [a point, line, and plane] are not placed in the preceding figures, because they are found among them.[76]

That Isidore felt it necessary to justify *not* picturing the first three geometric elements implies that he considered graphic representations to be a fundamental component of their definitions. They are only omitted, he tells us, because he had already provided pictures of more complex geometric figures, which comprise points, lines, and planes. Likewise, Gerbert's strenuous objection to picturing solids was a reaction to contemporary expectations that these would be pictured. The geometric figures in the *Etymologiae* give us some idea of the kind of image that was expected. The popularity of the complete work and excerpts of the sections on the liberal arts means that Isidore's geometric figures were far and away the most

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[72] In the earliest surviving copies, the pictures are not integrated into the textblock and most appear to have been written apart from the main text.

[73] Greek geometrical theorems can usually be broken into six distinct parts: the enunciation, setting out, definition of goal, construction, proof, and conclusion. The language of the setting out, construction, and proof often assumes or demands mental and/or physical picturing. Reviel Netz, *Shaping*, 10–11.

[74] Augustine in the passages of the *De quantitate* quoted above; Macrobius, *Commentary* 1.5.11, 2.2.3, and 1.6.35; Martianus, *De nuptiis* 6.708–17.

[75] Michel Huglo claimed that all surviving copies of the *Etymologiae* include the geometric figures. I have yet to see a copy that omits them. The makers of even the earliest manuscripts provided space for the figures within the area generally reserved for text. Huglo and Barbara Haggh-Huglo, review of *Étymologies, Livre III: De mathematica,* by Isidore de Séville, ed. Giovanni Gasparotto, trans. with commentary by Jean-Yves Guillaumin, *Speculum* 88, no. 2 (2013): 527; and Huglo, "The Diagrams Interpolated," 248.

[76] Isidore, *Etymologiae* 3.12.7, ed. Guilaumin, 53; trans. Barney, 94: "Prima autem figura huius artis punctus est, cuius pars nulla est. Secunda linea, praeter latitudinem longitudo .... Superficies uero, quod longitudines et latitudines solas habet. ... quorum formae ideo in superioribus decem figuris positae non sunt, quia inter eas inueniuntur."

widely known in the middle ages. Though references in the text and the *mise-en-page* of early copies strongly suggest that figures were part of the original tract, we do not know for certain what they looked like. The earliest extant copies of the *Etymologiae* belong to three different "families," each preserving a set of figures that differs slightly from the others.[77] The following discussion will, therefore, focus on general tendencies rather than particular forms.

Isidore enumerated the plane and solid figures as a group, and in most manuscripts they appear on a single page. Pictures of a circle and a sphere or square and a cube might be confused if not graphically differentiated in some manner. Placement and verbal labels helped the viewer distinguish between two- and three-dimensional objects. Pictures were carefully aligned to be adjacent to the corresponding definition (fig. 5.23). In some copies they appear within the line of text (fig. 5.24). They were also labeled. Plane figures are marked (*in*) *plano pede*, measured "in flat" or "linear feet"; solids *in solidium*, "in three dimensions" (fig. 5.25) Hence the reader/viewer encountered twice, in the narrative and the label, the name of a figure and its status as a plane or a solid.

In Naples V.A.13, which included excerpts from Book three of the *Etymologiae*, the only visible difference between plane and solid figures is that the latter are filled-in with brown ink (figs. 5.7 and 5.26). A similar strategy was adopted in a late tenth- or eleventh-century copy of the *Etymologiae* from Fleury (fig. 1.124). In this manuscript, the red lead pigment used for rubrication was applied to all the geometrical figures—planes and solids alike; its application was not, however, uniform. The plane figures are inscribed with color; the solids, circumscribed. Color also "fills" the enclosed space of the solids' surfaces. Such selective treatment accords with the scientific account of color, which was conceived as a type of fire, "a flame that streamed off [solid] bodies," the only *visible* quality of a solid.[78] Hence color, even when unmodeled and flatly applied, might signify three-dimensionality and materiality. Subtle graphic cues further disambiguated planes and solids. Isidore described a cube as "a solid figure that consists of length, breadth, and height."[79] In an early tenth-century copy of the *Etymologiae* produced in Fleury this figure was pictured as a

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[77] On these families see. Lindsay, in *Isidori Hispalensis episcopi Etymologiarum,* 1:vi–xi.
[78] Plato *Timaeus* 31b, 45a–d.
[79] Isidore, *Etymologiae* 3.12.3, ed. Guillaumin, 49; my translation: "Cubus est figura propria solida, quae longitudine, latitudine et altitudine continetur."

square with a short diagonal line extending from one of its interior corners to its center (fig. 5.27). The diagonal was, evidently, sufficient to connote "height." Indeed, it expresses visually the only difference between a square and a cube.

The figures in the *Etymologiae* were not designed for a passive viewer; they yielded information to those who approached them actively, mentally dissecting them, and, in the case of the solids, extending them into the third dimension. In the above-quoted passage, Isidore encourages his audience to revisit the drawings and to seek out the incorporeal elements—points, lines, and planes—within them, a directive reminiscent of Augustine's charge to distinguish length, width, and height in a spider's web. Part of the very fabric of Isidore's work, the pictures were treated as authoritative; scribes did not deviate too far from their exemplars. Though they underwent changes in the central middle ages that suggest interest and experiment on the part of copyists, their appearance tells us only so much about pictorial tendencies of that period. The margins of Macrobius's *Commentary*, on the other hand, offered a blank canvas and the pictorial annotations added beside the definitions of the geometric elements provide the clearest example of period preference.

A host of anonymous annotators put pen to parchment to clarify concepts in Macrobius's *Commentary* in the central middle ages. These additions were never standardized. There are nearly as many commentaries on Macrobius's *Commentary* as there are surviving copies of the tract.[80] That said, there was some local homogeneity in content and type of annotation, and, across the tradition,[81] certain sections of the text tended to attract more comment than others. Two extended passages on the geometric elements were among these.[82] Whereas the greater part of the treatise inspired verbal annotations; the definitions of points, lines, planes, and solids gave rise to pictures. As we have seen, pictures had become an essential part of these definitions. Given this, it is unsurprising that coming upon

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[80] Alison Peden studied the commentaries on Macrobius's *Commentary* written prior to the 12th c. She identified a set of related glosses that she dubbed the "Theoprotus" set. Even within the set, the commentary remained fluid and there are variations among manuscripts. Peden (née White), "Glosses Composed before the Twelfth Century in Manuscripts of *Macrobius' Commentary*," esp. 16–33; see also Bruce Barker-Benfield, "Macrobius," in *Texts and Transmission: A Survey of the Latin Classics*, ed. L. D. Reynolds (Oxford: Oxford University Press, 1983), 224.

[81] Manuscripts produced in the same scriptorium or housed in the same library often share a set of annotations or show considerable overlap in their interpolations. For one such example see Bruce Barker-Benfield "A Ninth-Century Manuscript from Fleury," 153–54.

[82] Macrobius *Commentary* 1.5.11 and 2.2.3–7. Macrobius defines the elements again (in passing) at *Commentary* 1.6.35, but, curiously, no pictures were ever added to this section of the text.

Macrobius's verbal definitions, a medieval annotator would have felt compelled to make them visual. Isidore pictured *all* the geometric elements and figures that he named and defined: a circle, a square, a cone, a cube, and so forth. A few of the *Commentary*'s annotators adopted a similar systematic approach (fig. 5.28).[83] Most, however, were more selective, homing in on points in the text that were not easily pictured in the mind or, for that matter, on the page.

A contemporary annotator made several textual corrections on folio 46 of an eleventh-century copy of the *Commentary* now housed in London (British Library Harley MS 2772; fig. 5.29).[84] He then added a visual gloss in the margin. With a straight edge, he drew a small square and, about an inch below this, another small square—a twin of the first—which he enclosed in a larger square (fig. 5.30). To finish, he connected the corners of the larger and the smaller square with four short diagonal lines. Shared characteristics, alignment, and proximity suggest that the upper square and the "embedded squares" below were meant to function as a unit. The adjacent text clarifies their relationship:

> One is represented by a point, two by the drawing of a line ..., and four by points arranged at right angles to each other with lines extending between the points to form a quadrilateral. When these four [lines and points] are duplicated and made eight, forming two equal quadrilaterals, and one is placed above the other, giving the figure height, the result is a cubical figure, which is a solid body.[85]

The passage is ekphrastic. Macrobius took his reader step-by-step from point, to line, to plane, and finally, to solid body. The annotator captured the narrative aspect of the text by making not one, but two figures. Their arrangement implies a sequence that flows down the page with the text: verbal logic transposed into pictorial logic.

Indeed, Macrobius's words might aptly describe the actions of the annotator, who, perhaps, did begin by drawing four points at right angles, connected these with lines to make a quadrilateral, and then repeated the process. But "place one [square] over the other," as the

[83] Oxford, Bodleian MS Auct. T. 2.27, fol. 14v is an example of more "systematic" picturing.
[84] London, BL Harley 2772, is of German provenance and includes only part of the *Commentary* (books 1.2.2–2.15.8).
[85] Macrobius, *Commentary* 1.5.11, ed. Willis, 16; trans. Stahl, 97: "Si quidem unum apud geometras puncti locum obtinet, duo lineae ductum faciunt quae duobus punctis ut supra diximus coercetur, quattuor vero puncta adversum se in duobus ordinibus bina per ordinem posita exprimunt quadri speciem, a singulis punctis in adversum punctum eiecta linea. Haec quattuor ut diximus, duplicata et octo facta, duo quadra similia describunt, quae sibi superposita addita que altitudine formam cybi quod est solidum corpus efficiunt."

text directed, he could not. The ink instantly fixed the figures in place and the surface of the page offered length and width, but no height, no "above." Identical quadrilaterals drawn one-atop-the-other would, of course, look like a single square. Hence, to avoid ambiguity, the annotator made one square larger than the other and used diagonal lines to represent height. The "embedded squares" are a compromise between the demands of the text and the constraints of the page. If imitation is a sign of success, the visual device was deemed satisfactory and useful, for in the margin above the ink drawings someone hastily scratched another embedded-squares figure into the smooth surface of the parchment with a hard, sharp-tipped utensil, probably a *graphium* or *stylus*, an iron-tipped tool typically used to write on a waxed tablet (fig. 5.31).

As accurate representations of a cube, drawings—whether made with pen or stylus—and text were doomed to failure: the text because it is not a picture, the drawing because it is two-dimensional. The authors of each were well aware of the limits of their medium. They worked around the problem by stimulating the reader's visual imagination. Macrobius did so with words. He enjoined his reader to conjure a picture, but not on the page.[86] He did this implicitly, through his use of vivid language, and explicitly. The two descriptions of a cube are part of a longer passage on points, lines, planes and solids—the geometric elements, which Macrobius called "mathematical bodies."[87] He prefaced the definitions of these so-called bodies with a statement on their apperception: "The word body is used here as you conceive it through intelligence and not sense-perception."[88] In the description of the cube beside the stylus sketch, he urged his reader to observe the solid with the mind.[89] It was at

[86] Macrobius shifts in this section to the second person singular form of the verb (e.g., *animadvertis*, *conspicias*), addressing the reader directly.
[87] *corpora mathematica* in Latin. Macrobius, *Commentarii*, 1.5.7, ed. Willis, 16.
[88] Ibid., 1.5.4, ed. Willis, 15; my translation: "sed corpus quod intellegendo, non sentiendo concipias."
[89] *mente conspicias* in Latin. Ibid., 1.5.10, ed. Willis, 16. Active use of the intellect to picture was, in the not-so-distant-past, essential to mathematical thought more generally and especially integral to geometric pursuits. In the very distant past, it was "routine"—a term used by classicist Reviel Netz. Netz has shown how Greek authors of geometrical tracts including Euclid (*fl.* c.295 BCE) and Archimedes (287–12 BCE) used the Greek verb *noein* ("to imagine" or "to conceive") at moments that required "more imagination" or extra effort to picture. *Noein* connotes *more* imagination, because *some* imagination on the part of the reader was always necessary in the realm of mathematics, even where a picture was present. Euclid and his contemporaries consistently demanded "more imagination" at moments when the drawing was particularly inadequate as, for instance, in proofs involving three-dimensional objects (especially conics, spheres, and solids). Significantly, it is at just such a moment that Macrobius added the phrase *mente conspicias* and the medieval annotator a picture, echoing the ancient practice. Reviel Netz, "Imagined and Layered Ontology in Greek Mathematics," *Configurations* 17 (2009): 19–50, especially 23 and 32. On the history of mental imaging in mathematics see Lorraine Daston, "The Physicalist Tradition in Early Nineteenth-Century French Geometry," *Studies in History*

this same point that the medieval annotator drew a figure that simultaneously adheres (physically, in its position on the page, and formally, to the extent possible) to the accompanying text and fails to instantiate it.

In the margin of a tidy, pocket-sized copy of the *Commentary* made at the turn of the first millennium and now housed in Brussels (Bibliothèque royale de Belgique MS 10146), the annotator wrote *Mathematica* in the margin (fig. 5.32).[90] The term is found in the adjacent text, "These [points, lines, and planes] are what are known as *mathematical* bodies, about which geometers dispute with skill and zeal."[91] Lifted from the sentence to which it belonged and isolated in the margin, *mathematica* no longer modifies the word "bodies," but the entire page. Marginal headings were common additions to medieval manuscripts; they acted as finding aids and linked disparate passages in a manuscript (or a series of manuscripts) with related content. They also signaled to a reader the nature of the text at hand and, with this, how it should be read. In the margin, the term *mathematica* alerted the reader to the abstract nature of the material in this section of the *Commentary*, that this was the stuff of geometers to be investigated by means of the intellect. A few pictures of geometric figures appear in the margin below the term. *Mathematica* labels these as well. Yet, even as they are designated by the heading, the pictures serve a similar function, marking the page as mathematical, perhaps with greater immediacy than the verbal label. An integral part of most geometric texts, drawings of geometric figures made manuscripts and parts of manuscripts devoted to the discipline instantly identifiable and signaled to the reader to approach the adjacent material geometrically, that is: primed to picture.

The vast majority of verbal additions to the *Commentary* take the form of glosses. According to the *Etymologiae,* a gloss "defines the utterance in question by means of a single word."[92] Glosses were written between the lines of text, above the terms they qualified. Physically, this created a mirror-like relationship between the authoritative utterance and its

and Philosophy of Science 17, no. 3 (1986): 269–95; and James Franklin, "Diagrammatic Reasoning and Modelling in the Imagination: The Secret Weapons of the Scientific Revolution," in *1543 and All That: Image and Word, Change and Continuity in the Proto-Scientific Revolution*, ed. Guy Freeland and Anthony Corones (London: Kluwer Academic Publishers, 2000), 53–116.

[90] Brussels, BRB 10146, fol. 19v.

[91] Macrobius, *Commentary* 1.5.7; ed. Willis, 15–16; trans. Stahl, 96: "Et haec sunt corpora quae mathematica vocantur, de quibus sollerti industria geometriae disputatur."

[92] Isidore, *Etymologiae* 1.30.1; ed. Lindsay; my translation: "Hanc philosophi adverbium dicunt, quia vocem illam, de cuius requiritur, uno et singulari verbo designat."

rephrasing. Greek words were translated into Latin, and Roman numerals were written over numbers spelled out in the text. Most glosses were synonyms, which helped expand a reader's vocabulary even while explaining difficult terms or words used in an unusual manner. Geometric figures added to the *Commentary* may usefully be regarded as a form of gloss, pictorial reflections of a given term, graphic synonyms or translations. Transmitted verbally, the pictorial nature of geometry was obscured by words. Pictures revealed the visual nature of the discipline, but they could only indirectly convey certain aspects of its objects. The incorporeality of a line, the solidity of a cube—these qualities could not be pictured. Tasked with showing something they could not, annotators relied on the abilities of their viewers.

The annotator of the Brussels copy of the *Commentary* selected key terms—*longitude*, *plane*, and *solid*—from the main text and rewrote them in the margin (fig. 5.33). Beside *longitude* he drew a horizontal line about the width of a fingernail. Next to *plane* he drew a tiny rectangle, thus establishing a one-to-one word-to-picture pattern. A reader would expect the word *solid* to be followed by a single, glyph-like figure. It is not. Rather, the term is followed by a dot, a short vertical line, and another short vertical line joined with a horizontal at a right angle. It is not so much a picture of a solid, but a kind of equation or recipe containing the raw materials necessary to make a solid. In this it resembles the adjacent text of the *Commentary*: "bodies," Macrobius wrote, "acquire solidity by adding height to the other two dimensions."[93] Both author and annotator leave it to the reader to sum up and create a solid from its parts. The annotator does this by juxtaposing the term "solid" with what is clearly not. Proximity serves to emphasize the disjunction between what is written and what is pictured, lending urgency to the task of emendation.

A similar tactic was employed in another copy of Macrobius's *Commentary* now in Paris (Bibliothèque nationale de France, lat. MS 7299). Someone, probably the eleventh-century scribe responsible for the main text, drew a small triangle and square in the margin (fig. 5.34–35). He then added little dots at each of the two shapes' corners and labeled his drawings "triangle" and "square." Above them he wrote: "a body is bounded by a surface,

[93] Macrobius, *Commentary* 1.5.9, ed. Willis, 16; trans. Stahl, 96–97: "soliditas autem corporum constat cum his duabus additur altitudo; fit enim tribus dimensionibus impletis corpus solidum quod stereon vocant, qualis est tessera quae kubos vocatur."

just as surfaces [are bounded] by lines, [and] lines by points."[94] The excerpt is a paraphrase of the main text:

> Indeed, when you designate a whole body, the surface is also included in the name. But even if surfaces are not kept separate from bodies in the material realm, the intellect does distinguish between them. *As the terminus of a body is the surface, so the termini of the surface are lines ... and lines terminate in points*.[95] [emphasis added]

As Macrobius describes it, the relationship between a surface and its elements is so intimate that it cannot be discerned with the eye, let alone drawn with the hand; only the intellect can "distinguish between them." Yet, in the picture, points hover outside the bounds of both triangle and square, well beyond the limits of the lines that they ought to define. These displaced points, the disassembled solid in the Brussels manuscript, and the "embedded squares" in the London manuscript require parsing, assembly, and extension in order to fulfill the demands made by the adjacent text. Macrobius addressed the reader in the imperative, commanding his audience to picture mathematical bodies in the mind. The medieval annotators should be seen as having adopted, I would argue, a similar stance. By mismatching figure and text, they created a visual imperative, a problem or puzzle that catches the eye of the reader and requires mental attention and resolution.

Scholars in our own time have often remarked on the limited nature and repetitive quality of the material available for geometric study in Latin-speaking Europe before the reintroduction of Euclid's *Elements*. The observation is neither unfounded nor original. In the *Isagoge*, Gerbert stated that the rudiments of the discipline—the definitions of points, lines, planes, and solids—had been expounded "enough and more than enough" by innumerable writers, in works both secular and divine.[96] Having said this, he then proceeded with his own version of the definitions. Medieval readers and annotators behaved similarly. Instead of passing over what must have been well-trodden territory as we might expect, they chose to enliven passages on the geometric elements with pictures like those in the London, Brussels,

[94] Paris, lat. 7299, fol. 31v: "Corpus finitur superficiae [more commonly superficie]. Superficies vero lineis. Lineae autem punctis."
[95] Macrobius *Commentary*, 1.5.6, ed. Willis, 15; trans. Stahl, 96: "Cum totum denique corpus nominas, etiam superficies hoc vocabulo continetur: de corporibus eam tamen etsi non res sed intellectus sequestrat. Haec superficies, sicut est corporum terminus, ita lineis terminatur ... punctis lineae finiuntur."
[96] Gerbert, *Isagoge* 2.1, ed. Bubnov, 51-52: "De quibus cum ipse Boetius aliique tam saecularis, quam divinae litteraturae tractatores in plurimis scriptorum suorum locis satis superque disputent."

and Paris manuscripts.[97] Perhaps surprisingly, repetition did not render this information uninteresting or useless to its medieval audiences. Rather, they seem to have intentionally dwelt on the definitions, reveling in their repetition—an attitude that forces a reassessment of their purpose and utility, and the aims of geometry more generally. I suggest that every encounter with the definitions provided an opportunity for geometric practice, that is, for exercising the mind's eye and strengthening intellectual sight. Practice, of course, makes perfect and in the realm of exercise, repeating an action builds and tones muscle, creating habit and memory. From this it follows that scribes might gravitate toward more taxing passages that would challenge the capacities of their reader's intellect.

4. Touch and Thought

The pages of manuscripts might provide a field for mental exercise, but they were not its limit. Gerbert was famed in his day for bringing tangible aids into the classroom. His didactic techniques, which were admired and imitated by his peers, were recorded and praised in Richer's *Historiae*. As will be remembered from chapter two, Richer reports that Gerbert taught astronomy, the science of solid bodies in motion, with mobile models.[98] Gerbert, he says, insisted on teaching music's tones and half-tones through demonstration on the monochord. He taught geometry with an abacus. Richer tells of the swiftness with which Gerbert's hands moved small counters from column to column on the board, performing difficult calculations with awe-inspiring speed and ease. The abacus made the abstract processes of arithmetic visible and tangible and, as such, geometrical; with it, numbers were not only seen, but grasped, and manipulated.[99] The account is rich in material specifics—the abacus board is made of wood covered in parchment, the counters of horn—that appeal to reader's senses, especially sight and touch.

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[97] The passages on the geometric elements in Macrobius's *Commentary* were not the only such descriptions to receive graphic annotations. Boethius described solids in the *De arithmetica*. His description of a cube (*De arithmetica* 2.25) was visualized by an annotator in the Fleury/Ramsey copy Paris, lat. 6401, fol. 133v. As with the pictures of solids discussed in this chapter, there is no attempt at projection. The cube is flattened (interestingly) into the shape of a Greek cross. I have not been able to find either precedent or parallel for this image; it seems to be the original invention of the manuscript's annotator. Geometric figures were also added to a copy of Martianus's *De nuptiis* (6.710–14), Bamberg, Msc.Class.39, fols. 156v–58r, as discussed in chapter 1. Likewise, figures were part of the medieval gloss on Boethius's translation and commentary on Aristotle's *Categories* (the section on quantity) in the Fleury manuscript Orléans, BM MS 269 (225), p. 222–24.

[98] Richer, *Historiae* 3.49–54, ed. and trans. Lake, 2:73–83.

[99] On making numbers tangible for students see Evans, "The 'Sub-Euclidean' Geometry," 107.

Gerbert does not mention the abacus in the *Isagoge*, but he says, as mentioned above, that three-dimensional models of geometric solids might "be formed in wax or wood."[100] Tangible models of this kind would have been relatively easy to produce, though none survives from this period. That said, such manufactured objects may not have been necessary. When defining a solid body, Gerbert provides an example that would have been readily available. He writes: "A solid body is anything that is extended with three dimensions ... as is everything which can be comprehended by sight and touch, like this wax tablet on which I write."[101] Shifting, for a moment, to the first person, Gerbert aligns himself with his reader. He draws attention to his own wax tablet, an object that, if not immediately at hand, would have been within a reader's reach. He coaxes his audience to comprehend—to see and to touch—the properties of a solid body through *anything three-dimensional*. Here, Gerbert echoes Plato, who explained in the *Timaeus* that all that comes to be must have bodily form and be both visible and tangible.[102] Hence, the universe, the whole of creation, comprised myriad solid bodies and provided endless opportunity for geometric practice.

Gerbert followed his statement about a solid by verbally deconstructing the wax tablet, breaking it down into planes, lines, and points, in a manner akin to the verbal/visual dissolution of the Calcidian projections in the Brussels gloss. He invites his reader to "see in" to the object just as Isidore asked his audience to look for the geometric elements in the figures in the *Etymologiae* and Augustine instructed Euodius to pick apart a spider's thread. Gerbert thus avoids misrepresenting either incorporealities (a point, line, and plane) or solids by trying to picture them on the page. A similar sensitivity might also be at play in the graphic glosses of Macrobius's *Commentary*. The square and triangle in the Paris manuscript described above were bounded by exceedingly fine lines; the points at their corners vanishingly small, especially in the case of the triangle. No other additions in the manuscript were drawn or written with such a finely pointed pen. The accompanying text and labels ("*trigonum*" and "*quadratum*") make it clear that the commentator intended his figures to be understood as incorporeal planes. I suggest that their appearance—the fine lines and minute

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[100] Gerbert, *Isagoge* 2.6, ed. Bubnov, 56. Quoted above, 207.
[101] Ibid. 2.2, ed. Bubnov, 52: "Solidum corpus est quidquid tribus intervallis seu dimensionibus porrigitur ... sicuti est quidquid visu tactuve comprehendi potest, ut haec praesens, in qua scribo, tabella."
[102] Plato, *Timaeus* 31b; ed. Waszink, 24.

points—further underscored their immaterial nature. It is possible that the maker of the stylus sketch in the London manuscript was similarly motivated.

The formal relationship between the ink and stylus drawings is clear (figs. 5.30–31). That the same, fairly unusual, form would appear twice on the same page by coincidence is unlikely; [103] one drawing must have inspired the other, but which came first is impossible to know with certainty. The same dark reddish-brown ink of the drawing was used elsewhere on the page and in the manuscript for corrections and annotations, the script of which is very like that of the main text, suggesting that the ink drawings were contemporary additions. The stylus sketch cannot be dated. Sketches in dry-point or lead-point are encountered with some frequency in medieval manuscripts. In the high and late middle ages (from the twelfth through the fifteenth century), as the making of manuscripts was professionalized and they were produced in ever greater numbers, it became common for illuminators to be guided by verbal and pictorial notes that conveyed what should be pictured and how the image was to be composed. Sketches made for this purpose were executed quickly and provided only the most rudimentary details. Dry-point was a favorite medium since it left the page largely unmarred and did not require erasure. In the London manuscript, the dry-point sketch may well have been meant as a cue to the commentator, a reminder to return to the page with a straight-edge and fresh ink to add the figure. As such, it would anticipate the ink drawing and represent an intermediate step between the writing of the text and the writing (and drawing) of the marginal commentary. But sketches for illuminators were rare prior to the twelfth century. In this earlier period, when manuscripts were produced in smaller numbers within the confines of cathedrals and monasteries, there seems to have been less need for this sort of

[103] Editions of the *Agrimensores* include the "embedded squares" figure as the representation of a solid in Balbus's *Description and Analysis of All Forms*. It is recorded as a variant found in, among others, Paris, BnF lat. MS 7377c (11th c., Reims?), Munich, SBS Cod. 23511 (12th or 13th c., German), and Jena, UB Cod. f. 156 (16th c., German [copy of the Arcerianus]). I have encountered the figure only in manuscripts that would either be contemporary with the London manuscript or made after it. These include Brussels, BRB 4499–4503, fol. 1r, and Selden Supra 25, fol. 119v. A similar figure (without the diagonals) was added to a copy of Macrobius's *Commentary* now in Aachen (Ludwig Collection, MS XIII.4, fol. 4r). Friedrich Blume, Karl Lachmann, and Adolf Rudorff, *Die Schriften der römischen Feldmesser* (Berlin: G. Reimer, 1848), figs. 69 and 70; and Menso Folkerts, ed. *"Boethius"* Geometrie II: *Ein mathematisches Lehrbuch des Mittelalters*, (Wiesbaden: F. Steiner, 1970), 147, 241 (fig. 148).

instruction. More likely, the stylus drawing is not a product of the scriptorium, but the work of a canny and impulsive reader.[104]

The placement of the sketch is calculated. It appears beside a description of a cube very like the one adjacent to the ink drawing. Hence, whoever made the sketch imitated the ink drawing in both form and function. He filled a perceived gap, adding an annotation of his own with a tool he likely had in hand. During the middle ages, many more marks were made with a stylus in colored wax than with pen on parchment. Wax tablets were among the few "necessities" that were given to every monk.[105] Isidore called them "the nourishers of children."[106] On them, youths practiced letter forms, authors drafted writings, students made notes. Mistakes could be pressed out with the butt-end of the stylus; when the wax had become black from use, it could be removed and renewed. Parchment, costly and arduous to produce, was for publication.[107] The tattoo bite of the acidic ink was permanent, errors were excised—scratched out with a knife—but roughened patches left on the surface of the page were blemishes that betrayed their making. By their very nature, the inked-in annotations added to the *Commentary* looked forward to future viewers. The faint stylus sketch, on the other hand, difficult to make out even in strong light, may have served only a single viewer, its maker, and then probably only in the moment of its making. Cut in parchment, the sketch in the London manuscript may be a rare example of everyday geometrical note-taking and practice.

The act of drawing put information (quite literally) into the hands of the maker in a way that looking at a picture could not. Indeed, the animate and ephemeral nature of mental pictures was, perhaps paradoxically, best grasped through physical experience, by facture and erasure. The many metaphors for making a mark in wax stress the medium's materiality; it

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[104] On the habitual use of the stylus by readers for mark making in manuscripts during the early and central middle ages see Bernhard Bischoff, "Über Einritzungen in Handschriften des frühen Mittelalters," in *Mittelalterliche Studien: Ausgewählte Aufsätze zur Schrifikunde und Literaturgeschichte* (Stuttgart: Anton Hiersemann, 1966), 88–92.
[105] Benedict, *Regula* LV.19, 143. On wax tablets see Richard H. and Mary A. Rouse, "The Vocabulary of Wax Tablets," *Harvard Library Bulletin*, New Series 1, no. 3 (1990): 12–19 and Mariken Teeuwen, *The Vocabulary of Intellectual Life in the Middle Ages*, CIVICIMA Études sur le vocabulaire intellectuel du moyen âge 10 (Turnhout: Brepols, 2003), 207–208. For their use in the schools see Pierre Riché, *Les écoles*, 461–70.
[106] Isidore, *Etymologiae*, 6.9.1, ed. César Chaparro Gómez (Paris: Les Belles Lettres, 2012), 61; my translation: "parvulorum nutrices."
[107] On the cost of parchment and its availability see Michael T. Clanchy, *From Memory to Written Record*, 92–94. Small, odd-shaped scraps were used in a more casual manner for notes and practice writing. Medieval grammarian Smaragdus of Saint-Mihiel (c.770–826) refers to students studying from pieces of parchment. Smaragdus, *Liber in partibus* prol. 0T, 1.

was imprinted (*imprimere*), plowed (*exarare, sulcare*), and molded (*effigiare*).[108] Wax was written *in* not *on*.[109] Plowing a stylus through wax must have conveyed something of a solid's tangible and haptic qualities as well as the incorporeal nature of the points, lines, and planes that compose it. The stylus cut a narrow channel, a line of shadow and air, less substantial even than a spider's thread and just as fugitive. It reminds us that the study of a text engaged the senses as well as the mind; that scholarly practice often took place beyond the bounds of text: in the margins of a manuscript, on a wax tablet, in the mind of the reader; and that the form of this practice was often not itself textual or even tangible.

CONCLUSION

Martianus's personified character *Geometria* called geometry the study of "demonstrable knowledge, drawn from dust." Seemingly poetic, the description was, in fact, quite literal. *Geometria*'s primary attribute was called an abacus, but it was an instrument very different from the calculation board used by Gerbert. Martianus described *Geometria*'s abacus as "a beautiful little board covered with a sprinkling of greenish powder ... designed for delineating figures."[110] In the ancient world, geometers, whom Plato and Aristotle called "those who make figures," drew pictures and proofs in the earth.[111] The abacus made this mode of demonstration possible even where the ground yielded no dusty or sandy surface suitable for drawing (as, for instance, in the classroom, or the starry realm of the empyrean, the setting for most of the *De nuptiis*). Great claims were made for this simple instrument with its learned dust.[112] *Geometria* boasted that the board could, "represent the entire circumference and the circles of the universe, the shapes of the elements, and the very depths of the earth." Though wax- or clay-covered tablets had largely displaced the dust of antiquity by the early

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[108] The plowing metaphor is particularly apt in the practice of geometry, which had its origins in land surveying. Fields were often delineated and divided by earthen furrows. These are but a few examples of the host of colorful metaphors collected by Rouse and Rouse, in "The Vocabulary of Wax Tablets," 17–18.
[109] Ibid., 15.
[110] On the term *abacus* and the changing shape of the instrument it described see Teeuwen "abacus, abacista," *The Vocabulary of Intellectual Life*, 353–54.
[111] Plato, in the *Euthydemus* and Aristotle in *De caelo* and the *Metaphysics* called geometers "those who make diagrams." A 1st-c. mosaic from Pompeii of a gathering of philosophers may represent the practice. The gaze of the central figure, possibly Plato, is fixed on the ground, where the tip of the long narrow stick that he holds in his right hand, meets the earth. The work is now housed in the Museo archeologico nazionale di Napoli (Inv. No. 124545). Likewise, Euclid draws on the ground while Pythagoras uses a tablet in Raphael Sanzio's *School of Athens* in the Apostolic Palace, Vatican City.
[112] *Pulvis eruditus* as it was called by Cicero in *De natura deorum*, 2.47.

middle ages, the practice of sketching geometric figures persisted.[113] The many pictures of geometric figures preserved in manuscripts should be considered the more durable siblings of those that were drawn in dust and wax; they are, for us, their only legacy. The turn from insubstantial lines of light and shadow, which might be erased with a brush of the hand, rubbed smooth with a stylus or dissolved with heat, to more permanent figures etched in ink on the manuscript page was a significant move, an act of reification, and it was one that was not taken lightly.

Convinced that medium conveyed meaning, Gerbert refused to picture incorporealities and solids in the *Isagoge.* His was, as we have seen, not the only possible stance. Near the end of the tenth century and the beginning of the eleventh, graphic glosses—pictures of solids, planes, lines, even points—were drawn in the margins alongside verbal descriptions, where pictures were not an organic part of the original tract, especially Macrobius's *Commentary* on the *Dream of Scipio*. At first glance, these pictures might seem to undermine the text of the *Commentary*. Macrobius, like Gerbert, stressed the symptomatic and imperceptible nature of the geometrical elements (unlike Gerbert, he counted geometric or "ideal" solids among these); they could only be discerned by the intellect. Close analysis of the pictorial additions to the *Commentary* reveals, however, an aim that, despite their inky materiality, was not entirely at odds with the words of Macrobius, only less stringent, more pragmatic. Taken in the context of contemporary practices of commentary and glossing that endeavored to make texts of earlier eras clearer and more useful to a medieval audience, the pictures can be understood as graphic goads, carefully contrived to provoke geometric practice—mental imaging—which was requisite to understanding the corresponding text. As such, they embody a compromise between accurate representation and the transmission of knowledge and show how one might image for the intellect.

The study of geometry equipped its practitioners with what Michael Baxandall, in his work on the limewood sculptors, referred to as "visual skills and habits" that shaped the

[113] Stevens notes the medieval practice of spreading dust on a table and using it as an informal drawing surface, but I have been unable to verify this reference. Wesley M. Stevens, "*Circulus*, *Triangulus*, *Epidonicus*: Geometrical Difficulties with Latin Lexography," in *Daimonopylai: Essays in Classics and the Classical Tradition Presented to Edmund G. Berry*, eds. R. B. Egan and M. A. Joyal (Winnipeg: University of Manitoba Centre for Hellenistic Studies, 2004), 397–426.

expectations one had of pictures and, with this, conditions of representation.[114] Students of geometry, the makers and viewers of manuscripts, learned to confront geometrical pictures and, by extension, possibly all pictures, with a keen awareness of the limits of representation and corporeal vision. A perspective that was founded scientifically and philosophically, it presumes imperfection or incompleteness on the part of two-dimensional images. Comprehension of the geometric elements fostered, moreover, an awareness of the nature of things—their corporeal or incorporeal status, and, with this, their perception. Images of the geometric elements were contrived in a manner that invited their viewers to take an active role, to see correctly in the mind's eye what could not be properly pictured on the page.

[114] See notably "Chapter IV: The Period Eye" in Michael Baxandall, *The Limewood Sculptors of Renaissance Germany* (New Haven: Yale University Press, 1980), 143–64.

CHAPTER 6
Conclusion and Coda

In a letter to Hervé, the Bishop of Beauvais, written around 996, Gerbert of Aurillac—then Archbishop of Reims—warmly endorsed a talented young artisan known only as "D." He wrote, "Receive, then, this man learned in the liberal disciplines and thoroughly instructed in the craft of artisans (*opificum*) whom many people sought after with much money, but whom we retained."[1] It is a brief statement, but a revealing one. Gerbert places as much emphasis on D's knowledge of the liberal arts as on his ability as a fabricator, probably a metalworker or painter. In so doing, he assumes that the Bishop of Beauvais would similarly value such education in a maker of luxury goods. Indeed, the syntactic parallelism with which he binds disciplined learning and technical training implies that—for Gerbert, at least—knowledge of the liberal arts was a wholly appropriate aspect of an artist's training. The further implication is that such knowledge might be recoverable in his creations.

The letter reminds us that objects produced in monasteries, cathedrals, and courts in the central middle ages were part of a relatively closed economy of persons and things. Those who passed through monastery and cathedral schools—the future patrons, makers, and viewers of objects—were, to a greater or lesser degree, both literate and numerate. Gerbert and Abbo were, of course, themselves patrons. According to his *Vita*, written by Aimoin, Abbo "increased the beauty" of Fleury, adding silver panels to the sides of a golden retable that had been installed by his predecessor, Odylbold, and adorning six other altars with silver.[2] He also had a wooden enclosure made around the tomb of St. Benedict. This too was

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[1] Gerbert, *Epistulae* 200; eds. Riché and Callu, 534: "Suscipite ergo illum in disciplinis liberalibus eruditum, in opificum magisterio edoctum, a multis multa mercede expetitum, sed a nobis obtentum." *In opificum magisterio edoctum* might also be translated as "thoroughly instructed in the teaching of artisans" as pointed out to me by Elizabeth Sears.

[2] Aimoin, *Vita Abbonis* 15; eds. Davril and Donnat, 106: "Narnque anteriorem tabulam altaris sanctae Dei genitricis Mariae ab antecessore ejus domno Oy[l]boldo auro construi coeptam ad perfectum usque perduxit duas circumjacentes argento opertas augmentari fecit et, ut breviter multa perstringamus, sex altaria, tam ejus quam monachorum sub ipsius regimine consistentium devota intentione, argenteis sibi affixis prefulgent laminis."

adorned with precious metal sculpted or engraved with scenes depicting the saint's miracles.[3] Gauzlin, Abbo's student and successor (abbot of Fleury 1004–1030), was even more zealous when it came to the abbey's adornment. André of Fleury, author of the *Vita Gauzlini*, called him "another Solomon" for his rebuilding activities in the wake of a devastating fire in 1026;[4] like Augustus of Rome, he "found a city of brick, and left one of marble."[5] Under Gauzlin's auspices the floor of the choir of the main church, Notre Dame, was made bright with colored marbles, its columns skillfully carved,[6] and numerous altars were covered and clad in the precious stone.[7] He commissioned a new crucifix;[8] had a lectern,[9] throne, and decorative panels cast in bronze;[10] and ordered the western and southern walls of the church of St. Peter to be painted with frescos.[11] To the treasury he added a golden censor, an alb, and a copy of Rabanus Maurus's *De laudibus crucis* "elegantly adorned with gold and silver."[12] This manuscript may still be extant, though stripped of its splendid binding (fig. 6.1). For these projects, Gauzlin enlisted a number of established artists, who are named by André in the *Vita*: the monk Odolric of St. Julian, Tours, "of good talent";[13] the "most skilled,"

[3] Ibid.: "Sed et paries ligneus circa tumulum incliti confessoris Christi Benedi[c]ti locatus. Simili est comptus specie metalli atque in eo quaedam miraculorum eiusdem dilecti domini celatoria arte perspiciuntur expressa." Aimoin explains that these works and more were completed during the abbacy of Abbo by means of the "diligence and direction" (*instantia et procuratione*) of the honorable monk Gauzfredus, who was the custodian of the treasury. The statement gives us the idea of the actors involved in the making of goods for the church, which evidently included Abbot, administrator, and artist.
[4] André of Fleury described the fire in the *Vita Gauzlini* (2.57–58). Two of Fleury's three churches, Saint-André and Saint-Pierre, burned to their foundations. The main church dedicated to the Virgin, Notre Dame, was also severely damaged. Most of the monastic buildings (certainly everything made of wood) were, we are told, entirely consumed.
[5] André, *Vita Gauzlini abbatis floriacensis monasterii* 2.65e; eds. Robert Henri Bautier and Gillette Labory (Paris: Centre national de la recherche scientifique, 1969), 134; my translation unless otherwise cited: "Urbem latericiam repperit, relinquam marmoream." André is paraphrasing Suetonius, *Aug.* 28.3.
[6] Ibid. 2.65b; eds. Bautier and Labory, 134.
[7] Ibid. 1.44b and 2.65d; eds. Bautier and Labory, 80 and 134.
[8] Ibid. 2.65a; eds. Bautier and Labory, 132.
[9] Ibid. 1.44c; eds. Bautier and Labory, 80–82.
[10] Ibid. 2.65b–c; eds. Bautier and Labory, 132–34.
[11] Ibid. 2.62–63; eds. Bautier and Labory, 118–28.
[12] Ibid. 1.44d–g; eds. Bautier and Labory, 82: "Poema Rabani, exaratum in laude sanctae Crucis auro argentoque eleganter adornavit"
[13] Ibid. 2.62; eds. Bautier and Labory, 118: "Quae in melius constructa, aliptico scemate est eleganter defloralata a quodam sancti martiris Juliani Turonicae sedis monacho, nomine Odolrico, bonae indolis viro." Odolric was called to decorate the interior of the newly rebuilt church of St. Peter. It seems that he was responsible for painting, or oversaw the painting (or both) of at least two ambitious fresco cycles. After introducing Odolric, André tells the story of a terrible fall (which the artist miraculously survived, unhurt) and then provides the inscriptions for the images of the Apocalypse that decorated the interior of the west wall and the scenes of the miracles of St. Peter, which adorned the "left" (*sinistro*) wall.

Nivardus of Lombardy;[14] and Radolfus, who is likened to Bezalel of the Old Testament for his ability with metals.[15]

Upon his appointment to the see of Reims, Adalbero undertook a major overhaul of the cathedral. He expanded the nave, shifted the shrine of Pope Callistus, rebuilt the oratory, and dedicated a new altar.[16] This last was fitted with "gleaming chancels" and "golden crosses."[17] As secretary, Gerbert played a significant role in the effort. On behalf of the archbishop, he dispatched a cross to Trier to be finished by the renowned glass workers of that city. While Pope, Gerbert commissioned a golden crown for Stephan, King of Hungary.[18] This may not have been the first crown he had had made. The dazzling *Reichskrone* housed in Vienna,[19] generally thought to have been commissioned by Bruno, archbishop of Cologne (d.965) for his brother, Otto I, ought rather, according to Christina Nielsen, to be associated with Gerbert, who had it made for Otto III (fig. 6.2).[20] Whether the project of Bruno or Gerbert, the crown's creation was overseen by a patron thoroughly (and in both cases famously) imbued with liberal learning.[21] Yet the extent to which such knowledge inflected artistic choices and shaped contemporary experience of such objects is not easily gleaned from written sources. In his letters to Egbert regarding the aforementioned cross for Reims cathedral, Gerbert referred to the *scientia* by which it was elaborated by the Archbishop of Trier.[22] He also describes its "admirable form" (*admirabilem formam*) as "fodder for eye and mind" (*et quae mentem et oculos pascat*).[23] These comments are vague

[14] Ibid. 2.65a; eds. Bautier and Labory,132: "Gauzlinus quodam pictorum peritissimo a Langobardorum regione adscito nomine Nivardo, insignis operis crucifixum componi precepit." Nivardus was also an accomplished sculptor. André tells how the Lombard artist carved the columns in the choir with his "chisel" (*scalpro*).
[15] Ibid. 2.65b; eds. Bautier and Labory, 132: "Rodulfus, in omni arte fusoria peritissimus, velut alter Beseleel"
[16] Richer, *Historiae* 3.22, ed. and trans. Lake, 2:28.
[17] Ibid.; ed. and trans. Lake, 2:28–29: "cancellis radiantibus" and "crucibus aureis." Richer also included fascinating descriptions of a portable altar, candelabrum, an "elegantly worked" reliquery chest, hanging crowns, and windows "depicting diverse stories."
[18] The crown does not survive. See Lattin, "Appendix C: Sylvester II's Privileges, Decrees, Letters, and Acts Whose Texts Are Not Extant," in *The Letters of Gerbert*," 383 (no. 34).
[19] Kunsthistorisches Museum Wien, Weltliche Schatzkammer, Inv.-Nr. SK_WS_XIII_1.
[20] The cross at the front of the crown and the arch inscribed with pearls were later additions. Along with her hypothesis, Nielsen provides an (highly polemical) overview of the various theories posited to date about the crown's genesis. Christina Nielsen, "*Hoc opus eximium*: Artistic Patronage in the Ottonian Empire," (PhD diss., University of Chicago, 2002), 132–93, esp. 151–74.
[21] On Bruno's education, mastery of the quadrivium, and the study of number in 10th-c. Cologne see Mayr-Harting, *Church and Cosmos,* esp. 52–56
[22] Egbert (c.950–93).
[23] These statements come from Gerbert, *Epistulae* 104 and 106; eds. Riché and Callu, 254 and 258: Letter 104: "Destinatio operi designatas mittimus species. *Admirabilem formam, et quae mentem et oculos pascat*, frater

and difficult to decipher. *Scientia* might be translated as "knowledge" or manual "skill" or may have been chosen precisely because it encompassed both terms. Similarly, *forma* could refer to the cross shape in and of itself, its physical beauty, or the object's appearance including its surface decoration, or, again, all aspects.

That a liberal education was part of an artist's qualifications is hinted at in Gerbert's letter to Hervé quoted above. A similar suggestion is encountered in the *Vita* of one of Gerbert and Abbo's near contemporaries: Bernward (d.1023), bishop of Hildesheim. Thangmar, the author of the *Vita Bernwardi*, claimed that "although he [Bernward] was continuously burned by a most fervent fire of the spirit in each liberal science, he imparted no less zeal in the less serious arts (*levioribus artibus*) which are called 'mechanical.'"[24] Bernward excelled, we are told, at writing, painting, casting, carving, laying mosaic floors, building, even brick-making, and the list does not end there.[25] Significantly, Thangmar begins this mind-boggling litany of artistic activity with the "art of writing" and the task of assembling a library of "theological and philosophical codices," again closing the distance between the liberal and the "mechanical" arts.[26]

What then were the implications of widespread numeracy? Students of calculation and arithmetic became newly attuned to placement and order; they were also trained to assume that what was pictured was only partial and to imagine infinity. Computus required an active and agile eye. Geometric practice, according to Augustine, "purged" the eye of the intellect. Picturing geometry's elements—points, lines, planes, and solids—exercised the

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efficiet fratri, soror sorori. Exiguam materiam nostram magnum ac celebre ingenium vestrum nobilitatbit, cum adjectione vitri, tum compositione artificis elegantis." Letter 106: "eo crucem vestra *scientia*, ut speramus, elaboratam, si fieri potest."
The cross is mentioned again in letter 126 (eds. Richer and Callu, 302).

[24] Thangmar, *Vita Bernwardi episcopi Hildesheimensis* 1; ed. G. H. Pertz, MGH SS 4 (Hannover: Hahnsche Buchhandlung, 1841), 758; my translation: "Et quamquam vivacissimo igne animi in omni liberali scientia deflagraret, nichilominus tamen in levioribus artibus quas mechanicas vocant studium impertivit." Authorship and dating of the biography are disputed. It has been argued the the whole work may have been a product of the 12th c. The literature is summed up by Jennifer Kingsley, *The Bernward Gospels: Art, Memory, and the Episcopate in Medieval Germany* (University Park: Pennsylvania State University Press, 2014), 3, n.10. Martina Giese completed the most thorough investigation of the manuscript evidence to date, concluding that significant 12th-c. interpolations were added to an original 11th-c. text. M. Geise, *Die Textfassung der Lebensbeschreibung Bischof Bernwards von Hildesheim* (Hannover: Hahnsche Buchhandlung, 2006).

[25] Ibid.: "In scribendo vero adprime enituit, picturam etiam limate exercuit, fabrili quoque scientia et arte clusoria omnique structura mirifice excelluit, ut in plerisque aedificiis, quae pompatico decore composuit, post quoque claruit." For a recent, in-depth discussion of these lines and the larger passage to which they belong see Jennifer Kingsley, *The Bernward Gospels: Art, Memory, and the Episcopate in Medieval Germany* (University Park: Pennsylvania State University Press, 2014), esp. 3–4.

[26] Thangmar, *Vita Bernwardi* 6; ed. Pertz, 760: "tam divinorum quam philosophicorum codicum"

intellectual eye, making it, according to Gerbert, "sharp in no small way for somehow contemplating spiritual things and truths."[27] What, specifically, might art historians gain by applying medieval varieties of "scientific" thinking to the analysis of objects made and used around the year 1000? A reconsideration of a single work produced in this period, the *Pericope Book of Henry II* (Munich, Bayerische Staatsbibliothek Clm. 4452), will point the way to the kinds of insights that such an investigation might yield (fig. 6.3). This splendid manuscript with its spectacular jeweled cover looks different when the inscription on its treasure binding is read in light of quadrivial, especially geometric, practice. I suggest, to paraphrase the passage from Augustine's *De libero*, quoted in full in the chapter one, that the craftsmen who fashioned the *Pericope Book* had numbers in their craft which they applied to their work; that if we inspect its beauty, we will find numbers held in place; and that what delighted its medieval audience and enticed use through the bodily senses is also full of number. [28]

CODA: THE *PERICOPE BOOK* OF HENRY II: A QUADRIVIAL INTERPRETATION

Henry II tapped two centers of book-production for the making of the *Pericope Book*: the 206 folios were written and painted at the island monastery of Reichenau, while its binding was probably crafted by artisans associated with the abbey of St. Emmeram in Regensburg (figs. 6.3–5).[29] In 1008, Henry II had installed Berno of Prüm as abbot at Reichenau and, in so doing, transplanted the learning of Fleury to the island.[30] Berno had been schooled at

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[27] Gerbert *Isagoge* 2.1, ed. Bubnov, 52: "copiosissime disserit, ubi etiam mentis oculum, corporearum rerum imaginationibus obtusum, per talium artium exercitia ad spiritalia veraque utcunque contemplanda non modicum purgari et exacui ostendit."

[28] Augustine, *De libero arbitrio* 2.16.42. See chapter 1, 14.

[29] The literature on the *Pericope Book* is vast, a few select works are included here: Eliza Garrison, *Ottonian Imperial Art and Portraiture: The Artistic Patronage of Otto III and Henry II* (Burlington, VT: Ashgate, 2012), 124–35; *Pracht auf Pergament. Schätze der Buchmalerei von 780 bis 1180*, eds. Claudia Fabian et al. Bayerische Staatsbibliothek Ausstellungskataloge 86 (Munich: Bayerische Staatsbibliothek and Hirmer Verlag, 2012), 176–81 (no. 36); *Zierde für ewige Zeit: das Perikopenbuch Heinrichs II*, eds. Rainer Kahsnitz, Ulrich Kuder, and Karl Dachs, (Frankfurt am Main: S. Fischer, 1994); Florentine Mütherich, *Das Perikopenbuch Heinrichs II. Clm. 4452 der Bayerischen Staatsbibliothek in München*, 2 vols. (Lachen am Zürichsee: Coron, 1994); Henry Mayr-Harting, *Ottonian Book Illumination: An Historical Study* (London: Harvey Miller, 1991), 1:179–201; Percy Ernst Schramm and Florentine Mütherich, *Denkmale der deutschen Könige und Kaiser*: *Ein Beitrag zur Herrschergeschichte von Karl dem Grossen bis Friedrich II. 768–1250* (Munich: Prestel, 1962), 156–57 (no. 110).

[30] Henry II installed Immo of Gorze at Reichenau prior to the more moderate Berno. Immo's extreme asceticism made him unpopular and drove away some of the abbey's wealthier denizens. Increased attention to learning and the liberal arts, however, would have come with reform.

Fleury during Abbo's abbacy, probably in the last decade of the tenth century.[31] His ready grasp of the quadrivium is evidenced in his writing. He penned tracts on music, including two compendia on the subject that were intended for teaching.[32] At Reichenau, Berno would have found a library equal to that which he left at Fleury and a scriptorium that far surpassed it in the beauty of the books that it produced. The ninth-century booklists include nearly the whole of the quadrivial canon.[33] That Berno encouraged the study of number is clear from the learned output of those who passed through the school during his tenure. Hermann the Lame (c.1013–54), for example, an outstanding scholar of computus and music, was educated entirely at Reichenau.[34] Henry had special ties to Regensburg, where he had been educated by the pious and learned Bishop Wolfgang (c.930–94), who reformed the local religious houses.[35] Wolfgang installed the saintly Ramwold (d.1000) as abbot of St. Emmeram.[36] Together the two revamped the liturgy, reformed the educational curriculum, and added texts to the library—the earlier contents of which are known through two late tenth-century booklists.[37] Classic expositions on number were contained in copies of

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[31] See chapter 2.

[32] The *Prologus in tonarium* and the *De consona tonorum diversitate*. Berno also composed music for the liturgy.

[33] Becker included four 9th-c. booklists dated between c.822 and c.842. The most extensive of these inventories was compiled by a monk named Reginbert and includes 415 "books" (*codices*). Becker, *Catalogi*, 4–13 (no. 6), 16–18 (no. 8), 19 (no. 9), and 19–24 (no. 10). Notably absent from these lists are Macrobius's *Commentary* on the *Dream of Scipio* and Plato's *Timaeus* with its Calcidian *Commentary.* That said, these works were exceedingly rare in the 9th c. Most of the early copies were found in what is now France.

[34] Arno Borst, *Das mittelalterliche Zahlenkampfspiel* (Heidelberg: C. Winter, Universitätsverlag, 1986), 81–97.

[35] By the age of seven Wolfgang had an ecclesiastic tutor at home. Later he was schooled first at Reichenau and then at the cathedral school of Würzburg, where he encountered the noted Italian grammarian, Stephen of Novara. Wolfgang taught at Trier and the monastery of Einsiedeln, where he became a monk. He was made bishop of Regensburg in 972. He tutored Henry II c.985. On Henry's education see Stefan Weinfurter, *Heinrich II (1002–1024): Herrscher am Ende der Zeiten* (Regensburg: Pustet, 1999), 25–27.

[36] Though Ramwold did not have a formal hand in Henry II's education he was deeply respected by the monarch, who, upon Ramwold's death, carried the abbot's body over his shoulder like a cross in the funeral procession. John W. Bernhardt, "King Henry II of Germany: Royal Self-Representation and Historical Memory," in *Medieval Concepts of the Past: Ritual, Memory, Historiography*, eds. Gerd Althoff, Johannes Fried and Patrick J. Geary (New York: Cambridge University Press, 2002), 45.

[37] One list (no. 42) contains the books at St. Emmeram during the time of Ramwold (d.17 May 1000); the other (no. 44) lists the "books of the monk Walther" and is dated to 994. It should be remembered that Theodulf and the Carolingian court were at Regensburg from Christmas of 791 to autumn of 793. The extant manuscripts associated with St. Emmeram have been discussed and catalogued by Bernhard Bischoff. Becker, *Catalogi*, 127–29 (no. 42) and 130–31 (no. 44); and Bischoff, *Die südostdeutschen Schreibschulen und Bibliotheken in der Karolingerzeit* (Wiesbaden: O. Harrassowitz, 1960), 1:171–259.

Boethius's *De arithmetica* and *De musica*, Martianus Capella's *De nuptiis*, Isidore of Seville's *De natura rerum* and *Etymologiae*, as well as numerous computistic miscellanies.[38]

Sometime in the first quarter of the eleventh century a number of tracts were added to the already well-stocked library at St. Emmeram by Hartwic, a monk at the abbey—best known to medieval art historians for his hand in the making of the *Uta Codex*.[39] Hartwic had travelled to Chartres to study the liberal arts. Fulbert, then Bishop of Chartres, had been a pupil of Gerbert and modeled the curriculum of his cathedral school on that of Reims.[40] Hartwic returned to Regensburg with a number of manuscripts, their contents and composition redolent of Gerbert's teaching as described by Richer.[41] Richer's description may have been known at Regensburg through the autograph (and only) copy of the *Historiae*, which the author had dedicated and given to Gerbert. Indeed, the *Historiae* is one of a group of manuscripts connected to Gerbert that were absorbed into the Bamberg cathedral library.[42] Most of these manuscripts are devoted to rhetoric and dialectic, but there are a few quadrivial works as well: Boethius's *De arithmetica*, a geometric and astronomic miscellany, and

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[38] These titles appear in the aforementioned booklists. Among the surviving manuscripts (most now housed in Munich, BSB) are a computus miscellany (Clm. l4456), Boethius's *Consolation of Philosophy* (Clm. 14324); a full copy of Martianus's *De nuptiis* (Clm 14729) and fragments of books six and seven (Clm 14070, fols. 65r–91r), the *Liber de computo* by Hrabanus (Clm 14221), Bede's *De temporibus* (Clm 14725), and the *Etymologiae* of Isidore of Seville (Clm 29051 (b)). Bischoff, *Die südostdeutschen Schreibschulen*, 195–96, 220–21, 223–24, 227, 253, and 257.

[39] We can only loosely pin down the dates of Hartwic's travels to sometime between 1007 and 1029, while Fulbert was bishop. On Hartwic's education and role in the making of the *Uta Codex* see Bernhard Bischoff, "Hartwic von St. Emmeram," *Die deutsche Literatur des Mittelalters. Verfasserlexikon*, eds. Kurt Ruh et al. (New York: Walter de Gruyter, 1981), 3:529–32; Adam Cohen, *The Uta Codex: Art, Philosophy, and Reform in Eleventh-Century Germany* (University Park: Pennsylvania State University Press, 2000), 184–85; and Barker-Benfield, "A Ninth-Century Manuscript from Fleury,"161.

[40] On Fulbert and the school of Chartres see Édouard Jeauneau, *Rethinking the School of Chartres* (Toronto: University of Toronto Press, 2009); Margot Fassler, "Mary's Nativity, Fulbert of Chartres, and the Stirps Jesse: Liturgical Innovation circa 1000 and Its Afterlife," *Speculum* 75, no. 2 (April 2000): 389–434; and Loren MacKinney, *Bishop Fulbert and Education at the School of Chartres*, Texts and Studies in the History of Mediaeval Education 6 (Notre Dame: Mediaeval Institute, University of Notre Dame, 1957).

[41] These are Munich, BSB Clm. 14171, I4272, 14370, I440I, 14436, I4735. Harriet Lattin has added Clm. 14498 to this group, basing the addition largely on content. Macrobius's *Commentary* on Cicero's *Dream of Scipio* and the *Celestial Hierarchy* of the Pseudo-Dionysius were among the texts that he brought with him. Also included in these manuscripts is Gerbert's own *Rationale et ratione uti* and two letters both on mathematical subjects. Bernhard Bischoff, "Literarisches und künstlerisches Leben in St. Emmeram (Regensburg) während des frühen und hohen Mittelalter," *Studien und Mitteilungen zur Geschichte des Benediktiner-Ordens und seiner Zweige* 51 (1933): 107–109; Harriet Lattin, "The Eleventh Century MS Munich 14436: Its Contribution to the History of Co-ordinates, of Logic, of German Studies in France," *Isis* 38, no. 3/4 (1948): 207, n. 16.

[42] Hoffmann, *Bamberger Handschriften*, 26–29; Marina Passalacqua, "Un Papa e tre codici," 147–51; eadem, "Gerbert di Reims e il codice Erlangen, Universitätsbibliothek 380," 322–27.

Martianus's *De nuptiis*.[43] Just how this cache of didactic tracts came to the Bamberg is unclear.[44] Whatever the mechanisms, these manuscripts provided a solid foundation for liberal learning at Bamberg, one that would be built on by later schoolmasters including Meinhard (d.1088), who was educated at Speyer and Reims and whose letters suggest ongoing connections between the scriptoria and schools of Reims and Bamberg.[45] The *Pericope Book* was, in short, produced by numerate individuals, for a numerate audience.

*   *   *

> Whoever seeks to understand the letters of true wisdom
> will rejoice to hold this square full of *mathesis*.[46]
> Behold those who shone (as) the true followers of wisdom.
> King Henry ornaments these perfect teachings with a wreath of honor.
>
> GRAMMATA QUI SOPHI[A]E QU[A]ERIT COGNOSCERE VER[A]E
> HOC MATHESIS PLEN[A]E QUADRATUM PLAUDET HABERE
> EN QUI VERACES SOPHI[A]E FULSERE SEQUACES
> ORNAT PERFECTAM REX HEINRIH STEMMATE SECTAM [47]

These lines were incised in narrow copper-gilt bands set into the cover of the *Pericope Book* (fig. 6.3).[48] Together they celebrate the manuscript's contents, decoration, and illustrious

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[43] Hoffman assigned seventeen manuscripts to this group. These include two manuscripts now in the Universitätsbibliothek in Erlangen: Cicero, *De oratore* (MS 380); Lucan (MS 389); and fifteen now in the Bamberg Staatsbibliothek: Richer's *Historiae* (MS Hist.5); Boethius, *De arithmetica* (MS Msc.Class.8); Boethius, *In Aristolelis peri hermenias* (MS Msc.Class.11); Boethius, *In Aristotelis peri hermenias* (MS Msc.Class.12); Cicero, *Topica* and Boethius *In Ciceronis topica* (MS Msc.Class.14); Grillius, *In Ciceronis librum de inventione* (MS Msc.Class.24); Victorinus, *Commentary* on the *De inventione* (MS Msc.Class.25); Cicero, *De officiis* (MS Msc.Class.26); Livy, *Ab urbe condita* (MS Msc.Class.35); Macrobius, *Saturnalia* (MS Msc.Class.37); Ps. Quintilian, *Declamationes maiores* (MS Msc.Class.44); Quintilian, *Institutio oratoria* (MS Msc.Class.45); a geometrical and astronomical miscellany (Ms Msc.Class.55); Boethius, *Opuscula sacra* (MS Msc.Patr.46); John Scot Eriugena, *Periphyseon* (MS Msc.Ph.2). To these I would add Martianus, *De nuptiis* (MS Msc.Class.39).

[44] See chapter 1, note 49.

[45] On Meinhard see Jaeger, *Envy of Angels*, 60–61. On the connection between the two scriptoria see Hoffman, *Bamberger Handschriften*, 24.

[46] *Mathesis plen(a)e* is, more precisely, "of full knowledge,"—the cases were chosen to maintain meter and rhyme—but this is awkward in English.

[47] Naturally, the richly ambivalent inscription has yielded diverse interpretations and translations. As will be clear from what follows, I understand the words as equivocal, referring simultaneously to the parchment pages and the bejeweled binding. Inscriptions on treasury bindings are rare (but then so are intact treasury bindings). One extant example very close to the *Pericope Book* in execution and date (c.1015) is the so-called Precious Gospels of Bernward (Hildesheim, Dom und Diözesanmuseum, Domschatz 18).

[48] For a detailed technical description of the front and back covers see Hermann Fillitz, "Der Einband," Z*ierde für ewige Zeit: das Perikopenbuch Heinrichs II*, eds. Rainer Kahsnitz, Ulrich Kuder, and Karl Dachs (Frankfurt am Main: S. Fischer, 1994), 103–107. Works that focus on the cover and/or the elements that compose it include Susannah D. Fisher, "Materializing the Word: Ottonian Treasury Bindings and Viewer Reception" (PhD

patron. The "King Henry" named in the fourth line is Henry II, who, as king of East Francia and future emperor, presented the book to the cathedral of Bamberg on the occasion of the building's dedication on 6 May 1012.[49] The "letters of true wisdom," and "perfect teachings," of the first and fourth lines refer to the manuscript's contents: Gospel excerpts selected and ordered for liturgical use.[50] Brightly colored enamels, locally produced, image the authors of the "true letters," the four evangelists, in the form of their symbols. Their round shape sets them off from the twelve Byzantine enamels, containing bust-length images of Christ and the apostles.[51] The apostles, along with the evangelists, are the "true followers of wisdom." All do, quite literally, shine.

At the center of the cover there appears a Carolingian ivory picturing Christ's crucifixion, the three Marys at the tomb, and the *in carne* resurrection of the dead on the last day (fig. 6.6).[52] One translator of the inscription, André Grabar, translated *quadratum* in the second line as *carré* or "square," as I have, and understood it as a reference to the ivory plaque.[53] For Grabar *quadratum* is an instance of synecdoche, shorthand for the complex

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diss. Rutgers, The State University of New Jersey, 2012), esp. 28–35; Rainer Kahsnitz, "Buchdeckel aus Gold, Elfenbein und Edelsteinen—*auro, ebore et gemmis ornatus*," in *Pracht aud Pergament. Schätze der Buchmalerei von 780 bis 1180*, eds. Claudia Fabian et al., Bayerische Staatsbibliothek Ausstellungskataloge 86 (Munich: Bayerische Staatsbibliothek and Hirmer Verlag, 2012), 319–27; Eliza Garrison, "The Art Policy of Emperor Henry II (1002–1024)," (PhD diss. Northwestern University, 2005), 172–4; Nielsen, *"Hoc opus eximium*," 9–52; *Prachteinbände, 870-1685: Schätze aus dem Bestand der Bayerischen Staatsbibliothek München*, ed. Béatrice Hernad and Ulrich Montag (München: Bayerische Staatsbibliothek, 2001), 17–18; Gunter G. Wolf, "Byzantinische Spolien auf dem Buchdeckel des Bamberger Perikopenbuches König Heinrichs II," *Aachener Kunstblätter* 61 (1995/97): 395–98; Peter Lasko, *Ars sacra, 800-1200*. 2nd ed. (New Haven: Yale University Press, 1994), 29 and 32 (on the ivory panel), 55 (on the presentation of the *Codex Aureus* to St. Emmeram, Regensburg by Arnulf and the "restoration" of the MS by Ramwold; its influence on later bookmaking at Regensburg and the possible reuse of ivory in the *Pericope Book*), 123; Frauke Steenbock, *Der kirchliche Prachteinband im frühen Mittelalter, von den Anfängen bis zum Beginn der Gotik* (Berlin: Deutscher Verlag für Kunstwissenschaft, 1965), 131–33 (no. 50).

[49] The date also marked the monarch's thirty-ninth birthday. Henry II's direct influence on the manuscript is emphasized by Mayr-Harting, *Ottonian Book Illumination*, 189–201.

[50] *Perfectam* can also be translated as "complete," underscoring that the compilation is complete despite being made up of excerpts.

[51] On the Byzantine enamels see Rainer Kahsnitz, *Rom und Byzanz: Schatzkammerstücke aus bayerischen Sammlungen*, ed. Reinhold Baumstark (Munich: Hirmer, 1998), 136.

[52] Many have written on the ivory, its origin and iconography. Most notably, Adolph Goldschmidt argued that it may once have been the centerpiece of the (now-lost) back cover of the *Codex Aureus* (removed by Abbot Ramwold when he restored the magnificent manuscript). Goldschmidt, *Die Elfenbeinskulpturen aus der Zeit der karolingischen und sachsischen Kaiser,* 8–11 *Jahrhundert.* Vol. 1. (Berlin: Cassirer, 1914), 25 (no. 41).

[53] André Grabar wrote, "Sur l'ivoire carolingien, de forme rectangulaire, est représentée l'oeuvre du Sauveur: Mort, Résurrection et Ascension du Christ, Résurrection des Morts." *Quadratum* is translated as *Geviert* by Hermann Fillitz, but understood as referring to the four gospels, not as a reference to shape. In this he follows all previous translators, who use *Geviert*. André Grabar, "L'archéologie des insignes médiévaux du pouvoir. Deuxième article," *Journal des savants* (1956): 87; Fillitz, "Der Einband," 103.

iconography contained *within* the square, *on the surface* of the ivory—that which the "seeker of true wisdom rejoices to hold." He assigns no meaning to the shape itself. I would suggest that further nuances emerge if one interprets the entire second line of the inscription, where the term *quadratum* appears—HOC MATHESIS PLEN[A]E QUADRATUM PLAUDET HABERE—in the light of contemporary geometric practice.

The use of the Greek term, *mathesis*, is highly unusual.[54] The word is rarely encountered in Latin texts of this period.[55] Where it does appear, it generally refers to quadrivial knowledge, as, it does, for example, in one of St. Emmeram's library lists, notably that prepared while Ramwold was abbot, sometime between 975 and 1000. Among the works included is an anonymous tract called *De creatione vel Super mathesin*.[56] Here *mathesis* is paralleled and synonymous with "created order." In another work in St. Emmeram's library, Boethius's *Philosophiae consolatio* (*Consolation of Philosophy*), we are reminded why. Boethius's Lady Philosophy, sings of a universe governed "by reason everlasting" its "elements bound by number."[57] This number-based model of Creation, is, by now, familiar to us. Boethius himself used the term *mathesis* in the *De arithmetica* to refer to "the four mathematical disciplines" (*quattuor matheseos disciplinarum de arithmetica*) or *quadrivium*.[58] Very significantly, the late tenth-century chronicler, Richer of Saint-Rémi, would take up the term.[59] *Mathesis* appears no fewer than six times in the *Historiae*, reserved exclusively for descriptions of Gerbert's numeric knowledge and teaching.[60]

[54] Though such Graecisms were not uncommon in Latin poetry of the period. See Elizabeth Teviotdale, "Latin Verse Inscriptions in Anglo-Saxon Art," *Gesta* 35, no. 2 (1996): 100.
[55] An admittedly unscientific search for *mathesis* (in all its forms) yields 19 references in the MGH online, 52 in LLT-A, 22 in LLT-B, for a total of 93 hits. It is often encountered several times in a single work and tends to be used repeatedly by a few authors, especially Claudianus Mamertus, Cassiodorus, Augustine, and Boethius.
[56] Becker, *Catalogi*, 129 (no. 42), entry 510. This may refer to the *Mathesis*, a tract on astrology written by the 4th-c. Sicilian, Firmicus Maternus, though the designation in the catalogue is too vague to make this certain.
[57] Boethius, *Philosophiae consolatio* 3.9.1–10; ed. L. Bieler, in CCSL 94 (Turnhout: Brepols, 1958), 51–52; trans. P. G. Walsh, in Boethius, *The Consolation of Philosophy* (Oxford: Oxford University Press, 1999), 56: "o qui perpetua mundum ratione gubernas" and "tu numeris elementa ligas."
[58] Boethius, *De arithmetica*, Praef.4; ed. Guillaumin, 3: "Cum igitur quattuor matheseos disciplinarum de arithmetica, quae est prima, perscriberem, tu tantum dignus eo munere uidebare, eo que magis inerrato opus esse intellegebam"
[59] Richer accounts for 6.5% of its use. See above, note 56.
[60] Richer, *Historiae* 3.43–45, 3.62–68. It will be remembered that the autograph copy of the *Historiae* (Bamberg, Hist.5.) may have been in Henry II's possession or at Bamberg around the time of the *Pericope Book*'s making as noted above.

Quadratum has been translated as "fourfold" or "four" and understood as a reference to the Gospels.[61] This is surely correct, but I suggest that it was not the sole or, at the time, even the most obvious meaning. For Henry and his contemporaries, *quadratum* primarily described the physical form of a thing. It meant simply "squared," a "four-sided figure," or "quadrate." [62] Applied to a four-sided surface and juxtaposed with *mathesis*, this geometrical sense would almost certainly prevail. Grabar, as noted above, construed the term as a reference to the ivory, but the square-ness of the plaque radiates outward.[63] It is the dominant idiom of the book. On the cover the square is echoed and emphasized in the arrangement of the inscribed bands and the gold, enamel, and gem-studded frame.

It is appropriate to use the term "square," despite the uneven length of the sides, because they have equal "measure" in the geometrical sense.[64] That is, each side is "equal" to the other, with three apostles, or four enamels, or nine enamels and jewels, or six metrical feet of each line of the inscription written in dactylic hexameter.[65] Squaring the rectangle—fitting the same number of elements, arrayed in the same pattern on the short and long sides—required ingenuity and determination on the part of the maker. The effort, and with it the shape, is significant, for square numbers and squares were deemed perfect. Augustine, in the *De quantitate* remarked on "superior uniformity" of a square (compared to a triangle) because of its symmetry.[66] He pointed out how all opposing elements are equal in a square:

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[61] Among others Fisher, "Materializing the Word," 29; Garrison, *Ottonian Imperial Art and Portraiture*, 127; Nielsen, "*Hoc opus eximium*," 13; Schramm and Mütherich, *Denkmale*, 156–7; Olle Källström, "Ein neuentdecktes Majestätsdiadem aus ottonischer Zeit," *Münchner Jahrbuch der bildenden Kunst* 2 (1951): 64. Interestingly, Gunther G. Wolf introduced the idea that *quadratum* referred to the square shape of the cover, but dismissed the interpretation as being too simplistic. Wolf, "Byzantinische Spolien," 395.

[62] Wesley M. Stevens, "*Circulus*, *Triangulus*, *Epidonicus*," 419–20.

[63] And inward. Martin Büchsel understood the personifications of Sol and Luna, who flank the cross on the ivory plaque, and the figures of Oceanus and Gaia in, respectively, the lower left and right corners of the plaque, as analogues to the Evangelists. These figures would thus form another square within the confines of the plaque. Büchsel, "Die Kreuzigung zwischen Antike und Christentum," *Jahrbuch der kunsthistorischen Sammlungen in Wien* 89-90 (1993): 12.

[64] Geometrical figures were qualitative and not quantitative.

[65] My thanks to Jeffrey Hamburger for bringing the even distribution of the lines (and, with this, meter) of the inscription to my attention. It is, in fact, usual for an inscription to be laid out in this manner. Verses are similarly arrayed on an 11th-c. portable altar now at the Musée de Cluny in Paris (cl. 11459). The direction of the letters, however, is different. Whereas the verses appear to wrap around the altar, changing their orientation with the side, the letters in the *Pericope Book* inscription are all upright, suggesting that the book may have been meant to be displayed in this manner. On this kind of "picture inscription" see Teviotdale, "Latin Verse Inscriptions," 99–110, esp. 102–103; Cohen, *Uta Codex*, 63–64; Jeffrey Hamburger, *Script as Image* (Leuven: Peeters, 2014), 45.

[66] Augustine, *De quantitate* 9.14; ed. Hörmann, 147; my translation: "Quid ergo, si et angulus angulo, et linea lineae contraria sit, nonne fateris aequalitatem esse potiorem in ea figura in qua id accidit?"

lines face lines of equal length and angles are opposite angles of equal measure; and likened the force of this geometric equity (*aequitas*) to the virtue justice. Euodius told his teacher that he admired the "truth, equity, equality" of the square "not a little."[67]

The fourfold idiom is, of course, pronounced in the interior of the book, where lections from the gospels according to Matthew, Mark, Luke, and John were united and woven into a single story of Christ's birth, life, death, resurrection, and second coming to be told through the year. Both the fourfold nature of the gospels and the unity of their accounts are captured in the appearance and arrangement of their portraits (figs. 6.7–10). Each evangelist is pictured, according to tradition, in a slightly different pose, captured at a discrete moment in the writing process. They are shown in tandem with their symbols—man/angel, lion, ox, and eagle. The settings, on the other hand, are nearly identical, suggesting that they participate in a single present. Each man sits before a background of shimmering gold, between two fictive marble columns bearing architraves inscribed with golden letters and surmounted by arches decorated with simple geometric and vegetal patterns. The portraits appear together at the beginning of the book, paired in successive openings, joined one to the next by the verses of a single poem. The visual/verbal message is clear: the evangelists number four, but, like a series of bays in an arcade, they are part of a single structure, their writings comprise one book.[68] On the back cover, the busts of the personified cardinal virtues—justice, prudence, fortitude, and temperance—are enclosed in four silver roundels partially gilded and arranged in a square around a large circular medallion with the lamb of god at its center (fig. 6.5). Their appearance mirrors that of symbols of the four evangelists on the corners of the front cover.

"Whoever seeks to understand the letters of true wisdom, will rejoice to hold this square full of *mathesis*." The lines draw attention to the thickness and with this the corporeality of the bound manuscript, suggesting that even these intrinsic qualities were meaningful to a medieval audience (fig. 6.11). Read in tandem, *quadratus* and *mathesis* assume decidedly geometric overtones. *Plenus* meaning "full" furthers the link. It is a word

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[67] Ibid. 9.15; ed. Hörmann, 149; my translation: "A: Hoc ergo, cum me multum moveret, quaerendum abs te visum est, quonam modo ista veritate, aequitate, aequalitate delectarere. E. Iam cerna quid dicas et non mediocriter admiror." Augustine and Euodius then go on to discuss the greater perfection of the circle, in which there is equality of all parts.
[68] On imaging the evangelists see Catherine Karkov, "Evangelist Portraits and Book Production in Anglo-Saxon England," in *The Cambridge Illuminations: The Conference Papers*, ed. Stella Panayotova (London: Harvey Miller, 2007), 55–63.

with ripe, fleshy, physical connotations. The addition transforms a two-dimensional, incorporeal square into a three-dimensional, corporeal, cube.[69] There is reason to believe that the *Pericope Book* would have most often been encountered in its closed, and thus quadrate, state.[70] Henry Mayr-Harting observed that descriptions of deluxe manuscripts in contemporary treasury inventories focused entirely on the covers. From this, he concluded that these books were generally seen closed, an idea bolstered by instructions in one of the earliest liturgical *ordines* specifying that the gospel be kept closed in procession until secured on the lectern.[71] That the processing of deluxe liturgical manuscripts was one of their primary functions and that the binding was tied to this use is clear also from a statement in Thangmar's *Vita Bernwardi* that, "He [Bernward] made a gospel book for the solemn processions on high feasts, which was covered in gold and precious stones."[72] Indeed, the unusually large size and great heft of the *Pericope Book* would have made it particularly unwieldy; to carry it when open would have been a challenging, if not impossible, feat. Translating *habere* as "to hold" rather than (or in addition to) "to have" brings to mind the

[69] *Quadratus*, *mathesis*, and *plenus*: the unusual cluster of terms is found together in Macrobius's *Commentary* on the *Dream of Scipio.* In a passage discussed in chapter 5, Macrobius guides readers step-by-step through the process of mentally making a cube: quod animadvertis si super unum quadratum quale prius diximus alterum tale altius impositum mente conspicias ut altitudo quae illi plano deerat adiciatur fiat que tribus dimensionibus impletis corpus solidum. (Macrobius, *Commentary* 1.5.10). The reader is told to picture twin squares, one above the other, and to fill the space between them to form a cube. The elementary exercise, repeated three times in the *Commentary*, aimed at demonstrating the relationship between two-dimensional incorporealities such as points, lines, and planes and three-dimensional corporealities or solids, like cubes, cones, and spheres. It will be remembered from chapter 5 that Macrobius referred to the geometric elements as "mathematical bodies" ("haec sunt corpora quae mathematica vocantur"; Macrobius, Commentary 1.5.7) and that these passages attracted graphic annotations in the central middle ages. It is unclear whether the anonymous author of the *Pericope Book* inscription was intentionally echoing Macrobius. As touched on above Hartwic brought a complete copy of the *Commentary*, half of which (including the above-quoted passage) he copied himself, back with him to Regensburg sometime in the first quarter of the 11th c. (Clm. 14436, fols. 10–32, 35–58). The inscriptions of the Uta Codex, thought to have been composed by Hartwic, draw heavily on Macrobius's *Commentary*, as demonstrated by Adam Cohen (*Uta Codex*, 35–38). A copy of the *Commentary* made in Italy at the end of the 10th c. or the beginning of the 11th c. was among the manuscripts given to Bamberg cathedral (Bamberg, Msc.Class.38), probably by Henry II.

[70] In the illuminations of the *Pericope Book* Christ is pictured holding a closed book in nearly every scene following the crucifixion. Prior to this point he is pictured with a scroll. When pictured in heaven he either holds an open book (at the assumption, fol. 161v) or does not hold a book (as in the Last Judgement, fol. 202). An open book as also pictured in the image of Pentecost (fol. 135v). A closed book is also present in each of the images of the evnagelists. Hence, it seems that the illuminator(s) associated the body of Christ resurrected with the closed book. It has been suggested that the covers were once part of a *Buchkasten*. See Hermann Fillitz, "Der Einband," 107; and Irmgard Siede, "Buchkasten des Perikopenbuchs Heinrichs II," in *Karolingische und Ottonische Kunst,* ed. Bruno Reudenbach (Munich: Prestel, 2009), 481–82 (no. 247).

[71] Mayr-Harting took this also to be an indication that 1) the value of these books lay largely in their bejeweled bindings, and 2) illuminations in such luxurious objects were largely taken for granted. *Ottonian Book Illumination*, 49.

[72] Thangmar, *Vita Bernwardi* 8; ed.

weight of a three-dimensional object. A colossal book measuring 425 x 320 mm,[73] its weight is considerable.[74] It is indeed a "square full of mathesis."

Yet art historical investigations have tended to treat cover and contents of the *Pericope Book* in isolation, in part owing to standing questions about where these were made. Even scholars interested in the composite quality of Ottonian luxury objects have tended to treat elements of the manuscript in isolation (i.e., the front cover alone).[75] The making of the *Pericope Book* entailed the assembly of a variety of objects new and old, local and exotic. A firm understanding of each of these items is, of course, essential to the comprehension of the object, but this focus on parts has somewhat dissolved its integrity and made it more difficult to appreciate the book's quiddity.

Those who have studied liturgical manuscripts and their luxury bindings more generally have emphasized how scripture could be, quite literally, embodied in bound transcriptions of the gospels. In "The Word Made Visible: The Exterior of the Early Christian Book as Visual Argument," John Lowden described the process in this way:

> When the gospel text was written on the animal-skin sheets of a book the word was made flesh. When the gospel was bound and its exterior decorated with images, the word was made visible. When such a book was displayed in a procession or on an altar, the viewer beheld via its exterior the *doxa* of the word, the glory, that is to say, of God incarnate.[76]

Though Lowden refers here to earlier copies of the gospels, his insights hold for those made in later periods as well. The inscription on the cover of the *Pericope Book* chimes with

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[73] It is telling that, though the manuscript is intact, facsimiles (printed and online) and catalogues—including the most recent *Pracht auf Pergament*—do not generally record or contain images that give an idea of the manuscript's height or breadth. The Bayerische Staatsbibliothek now provides images of the sides of the closed manuscript (spine, top, tail, and front) in their Digitale Bibliothek, but these, in the case of the *Pericope Book*, are not accompanied by indications of scale.

[74] The *Pericope* dwarfs other books of its kind, such as the *Reichenau Evangeliary* (Munich, BSB Clm 4454; 305 x 235 mm) and the *Gospels of Otto III* (Munich, BSB Clm 4453; 335 x 240 mm) with which it was given to the catherdral by Henry II. It is, notably, of roughly the same dimensions as the *Codex Aureus* (Munich, SBS Clm. 14000; 420 x 330 mm). I have been unable to determine the weight of the *Pericope Book*. Catalogue entries (including the description of the manuscript in the facsimile) refer only to a "wooden core" (*Holtzkern*) and are not specific about the kind of wood. The parchment appears to be quite thick, which also would contribute to the overall weight of the object. For a comparative list of large manuscripts and their sizes see George G. Greenia, "The Bigger the Book: On Oversize Medieval Manuscripts," *Revue belge de philologie et d'histoire* 83, no. 3 (2005): 742–45.

[75] This is true even of Ilene Forsyth's masterful "Art with History: The Role of Spolia in the Cumulative Work of Art" *Byzantine East, Latin West: Art Historical Studies in Honor of Kurt Weitzmann*, eds. C. Moss, et al. (Princeton: Princeton University Press, 1995), 153–58.

[76] Lowden, "The Word Made Visible," 47.

Lowden's description, while reminding us that the physical act of making the written word flesh was a mathematical undertaking. We might think of the *Pericope Book* as an example of applied geometry.

A book was a cube, and the cube was emblematic of solidity. In the *Isagoge*, it served Gerbert as his example of three-dimensionality over a pyramid or a sphere. This was, according to Plato, the geometric shape assigned to the element of earth (*terra*), which, along with fire, was not only fundamental to Creation, but also human perception. "Now that which comes to be," stated Plato, "must have bodily form, and be both visible and tangible, but nothing could ever become visible apart from fire, nor tangible without something solid, nor solid without earth."[77] Hence, earth—heavy, immobile, malleable, and stable—was understood to be *essential* to solidity. It was also linked with the earthly sphere as well as its inhabitants. The world had long been pictured as a square, the *mundus tetragonus*, in computus manuscripts.[78] Isidore in the *Etymologiae* explained that though "all the elements were present in all [things]," God had assigned living beings to appropriate elements, "filling heaven with angels, air with birds, water with fish, and earth with humans and other living animals."[79] Thus when God took human form he too would be associated with the earthly element as well as its shape.

The abstract, geometric form of a cube was a potent symbol of Christ incarnate. Paraphrasing Psalm 118, Jesus referred to himself as a "cornerstone" (*lapis in caput anguli*)

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[77] Plato, *Timaeus* 31b, ed. Waszink, 24; my translation: "Et quia corpulentus uisibilisque et contiguus erat merito futurus, sine igni porro nihil uisibile sentitur nec uero tangi quicquam potest sine soliditate, soliditas porro nulla sine terra, ignem terramque corporis mundi fundamenta iecit deus."

[78] References to "the four corners" of the earth are found in the Bible (Isaiah 11:12 and Rev. 7:1). Two well-known Carolingian examples are Vienna, ÖNB Cod. 387, fol. 134r (c.818) and Cologne Diözesanbibliothek Cod. 83 II, fol. 84r (798–805). Adam Cohen has pointed to two examples, one primarily verbal, one visual, that were at Regensburg: a titulus in the *Maiestas* image of the *Codex Aureus* (fol. 6v), which reads: *Christus vita hominum coelorum gloria summa / Librat tetragonum miro discrimine mundum.*(Christ life of man, highest glory of the heavens / Weighs the rectangle-world with exceeding discrimination); and Munich, BSB Clm. 14456, a 9th-c. computus miscellany with an image of "the four rivers of paradise are situated within a square world." Cohen, *Uta Codex*, 71, esp. n. 84; and Herbert Kessler, *The Illustrated Bibles from Tours* (Princeton: Princeton University Press, 1977), 51–52.

[79] Isidore, *Etymologiae* 13.3.3; ed. G. Gasparotto, in *Etymologie: Libro XIII:* De mundo et partibus (Paris: Les Belles Lettres, 2004) 20; trans. Barney, 272: "Quapropter omnia elementa omnibus inesse, sed unumquodque eorum ex eo quod amplius habet accepisse uocabulum. Sunt autem diuina prouidentia propriis animantibus distributa: nam caelum angelis, aerem uolucribus, mare piscibus, terram hominibus ceteris que animantibus Creator ipse inpleuit." The notion is derived from Plato (*Timaeus* 39e–40a). See Fontaine, *Isidore de Séville: Genèse et originalité de la culture hispanique*, 649–51.

in the synoptic gospels.[80] Peter and Paul took up the analogy, speaking of the "stone rejected by the builders, which is become the head of the corner" as an emblem of the Passion and salvation. They tied the statement to the prophecy of Isaiah: "Thus says the Lord God: Behold I will lay a stone in the foundations of Sion, a tried stone, a corner stone, a precious stone, founded in the foundation."[81] The cornerstone was a point of convergence between two walls. These walls were exegetically interpreted in many ways, by the tenth century, however, they had come to represent Christ's dual nature, the union of the terrestrial and celestial, earth and fire.[82] The shape of the "cornerstone" was not established in the Bible, but some in the central middle ages assumed that it was squared.[83] John of Salerno likened Odo of Cluny to a "square cornerstone" (*lapis angularis quadrus*) in which "angel and man" were joined.[84] Significantly, Christ as cornerstone was invoked in the benediction, dedication, and consecration of churches.[85] It will be remembered that Henry II had the *Pericope Book* made for just such an occasion; it contains the readings for the rite of consecration and is thought to have been used for this purpose.[86]

Christ is explicitly likened to a cube in the near-contemporary *Uta Codex*, which was made at Regensburg a little over a decade after the *Pericope Book*.[87] In the so-called Symbolic Crucifixion frontispiece, Christ, crowned and wide-eyed, is nailed to a cross with an exaggeratedly long upright beam (fig. 6.12). Just below the patible is written: "O Christ,

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[80] Psalms 118:22: "Lapidem quem reprobaverunt ædificantes, hic factus est in caput anguli." The statement is recorded in gospels of Matthew (21:42), Mark (12:10), and Luke (20:17) and repeated by Peter (Acts 4:11).

[81] Isaiah 28:16: "hæc dicit Dominus Deus: Ecce ego mittam in fundamentis Sion lapidem, lapidem probatum, angularem, pretiosum, in fundamento fundatum."

[82] The prolific monk of Corbie Paschius Radbertus (c.790–c.865) interpreted the cornerstone as "that which joins heaven and earth" ("Qui coelestia simul et terrena jungeret") in his exegetical writing on Matthew (*In Matth.* 21.42). Gerhart B. Ladner, "The Symbolism of the Biblical Corner Stone in the Medieaeval West," *Mediaeval Studies* 4, no. 1 (1942): 51.

[83] Scholars (including Panofsky) have challenged the meaning of *lapis...anguli* suggesting that it might be better understood as a wedge-shaped coping stone rather than a quadrilateral block or foundation stone. This revised interpretation is based on a handful of images (mostly later) in which the stone is pictured as a pointed, pyramidal stone at the pinnacle of a tower/pyramid and, as has been shown by Ladner, was part of a distinct exegetical thread and *not* the standard interpretation. Ladner, "The Symbolism of the Mediaeval Corner Stone," 43–60.

[84] John of Salerno, *Vita Odonis* 2.5, PL 133, col. 63c: "Erat enim velut lapis angularis quadrus, angelicus videlicet et humanus, largus et gratus."

[85] It appears in the liturgical hymn *Urbs beata Hierusalem dicta pacis visio*, which was sung at the dedication of churches. Ladner, "Symbolism," 53–54.

[86] The theme of dedication and gift giving is embodied in the image on fol. 2v of Henry II and his wife, Cunegunde, with Christ enthroned.

[87] Munich, BSB Clm. 13601. On the date and origin of this manuscript see Cohen, *Uta Codex*, 10–17.

solidifying (*solidans*) faith, conquering well by squaring (*tetrago*[nizans])" (fig. 6.13).[88] Adam Cohen translated *solidans* as "strengthening," a sound interpretation, but, again, not necessarily the most obvious for a contemporary viewer.[89] *Solidans* and *tetragonizans* rhyme in the meter. Joined poetically, I suggest that the reader would assume a mathematical meaning for both terms. The unusual participle *tetragonizans* is derived from the far more common *tetragonus* meaning "square."[90] The term is encountered with some frequency in tracts on arithmetic and geometry. In the *De arithmetica,* Boethius defined a *numerus tetragonus* as the result of multiplying two equal numbers; and he called such numbers "divine."[91] He also used the term geometrically, to describe the square base of a pyramid.[92] Hence, the combination of *solidans tetragonizans*, like *plenus* and *quadratus*, should be understood as referring to a solid square or cube.

Cohen called the complex image, which incorporates numerous other inscriptions as well as musical proportions and terminology, a "harmonization of opposites." He wrote: 'The unification of diametrical opposites is a concept fundamental to the Crucifixion, for Christ subverts death by overcoming the torment on the cross. As both God and man, Christ is the paradigm of unity, and his two-fold nature is dramatically revealed at the Crucifixion."[93] That nature, the power to bind opposites, was, as already noted, also revealed in the image of the cornerstone. Hence the union of cross *and* cube in the Symbolic Crucifixion frontispiece and the *Pericope Book* would have amplified the idea of Christ as a "paradigm of unity."

We see the convergence of these two powerfully evocative symbols in the margins of Paris, lat. 6401, a copy of the *De arithmetica* from Fleury, which was possibly known at

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[88] Munich, Clm. 13601, fol. 3v: "Christe fidem solidans, vincens bene tetrago[nizans]." Cohen followed Bernhard Bischoff in his transcription of this *titulus*, which is difficult to read. Bernhard Bischoff, "Hartwic von St. Emmeram," 531.
[89] Cohen, *Uta Codex*, 60.
[90] Bischoff pointed out that the rare verb *tetragonare* also appeared in Munich, SBS Clm. 14272, fol. 148v in a tract on the abacus written by Hartwic. Bernhard Bischoff, "Literarisches und künstlerisches Leben in St. Emmeram," 80, n. 27; as quoted in Cohen, *Uta Codex*, 60, n. 27.
[91] Boethius, *De arithmetica* 1.27.7; ed. Guillaumin, 56: "Hoc autem in hac est dispositione divinum quod omnes angulares numeri tetragoni sunt. Tetragonus autem dicitur, ut brevissime dicam quod post latius explicabitur, quem duo aequales numeri mutiplicant."
[92] Ibid., 2.22.1; ed. Guillaumin, 109: "Idem si a tetragona basi proficiscatur et ad unum verticem eius lineae dirigantur, erit pyramis quattuor triangulorum per latera, uno tantum tetragono in basi positio super quam ipsa figura fundata est."
[93] Cohen, *Uta Codex*, 71.

Ramsey (figs. 6.14–15).[94] A figure was added beside Boethius's discussion of arithmetic/geometric cubes:

> The tetragon four has two in its side and is born from twice two; twice two makes four. If you multiply these two from the side of the figure equally, the form of the cube would be born. If you make twice two times two, the quantity of eight grows from it, and this is the first cube.[95]

The number eight is the "first cube" born of the first *tetragon*, four. The anonymous annotator pictured the eightfold cube as five conjoined planes. Four of these planes are attenuated and radiate outward from a small, central square forming a Greek cross. Each is marked with a Roman numeral: two on each of the elongated squares, and eight, their total, at the center. The spiritual significance of eight was spelled out by Abbo at the end of the *Quaestiones*, where, as observed in chapter three, he shifted from grammatical to mathematical explanation in order to reconcile verbal paradoxes in the Athanasian Creed. He wrote, "this eight, moreover, born from quadrupling the first even [number], accepts diminution in the guise of a cubic quantity, to indicate that god, perfect, was, briefly, a little reduced from the angels in the guise of a slave (Phil 2:7)."[96]

Like the graphic glosses added to Calcidius's *Commentary* and the *Commentary* of Macrobius, the flattened square in the Paris manuscript was meant to provoke an act of mental imaging on the part of the viewer. It will be remembered that Gerbert, in the *Isagoge*, encouraged readers to apply this kind of thinking to three-dimensional, solid objects as well; to "see" the incorporeal planes, lines, and points that composed and bounded "all things." He used his wax tablet as an example. Similarly, the *Pericope Book* could stand as a demonstration piece, an opportunity to exercise the intellectual eye, and maybe even to seek the incorporeal, the divine, in the corporeal. The structure of the cover can be seen to

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[94] Paris, lat. 6401, fol. 133v.

[95] Boethius, *De arithmetica* 2.25.3; ed. Guillaumin, 114–15; trans. Masi, 147–48: "Nam quattuor tetragonus duos habet in latere et natus est ex bis duobus. Bis enim duo quattuor faciunt. Hos ergo duos ex ipsius latere si multiplices aequaliter, cybi forma nascetur. Nam si bis binos bis facias, octonaria quantitas crescit. Et est primus hic cybus." This figure was described in passing in chapter 5, note 51.

[96] Abbo, *Quaestiones* 48; ed. Guerrau-Jalabert, 273; my translation: "Hic viii uero ex primo pari quadruplicato diminutionem recipit sub ostensione cubice quantitatis, ut perfectum Deum innuant paulo minus minoratum ab angelis sub forma serui."

resemble the graphic goads discussed in chapter five (figs. 5.28–35).[97] Thus the evangelists in their roundels on the four corners of the front cover are points, between them lines are drawn to make a square. Likewise, the four virtues, on the back cover form a second square. The gospels fill the space between these planes, adding flesh, extending the cover into the third dimension.

The inscription and composition of the *Pericope Book* cover were, I suggest, intended to stimulate a kind of cognitive practice made more explicit in other Ottonian contexts. Looking again at the *Uta Codex* we find a set of Ambrosian strophes embedded in the illumination that prefaces excerpts from Matthew's Gospel, which begins with the generations of Christ (fig. 6.16). The poem casts Christ's lineage in expressly geometric terms. The "forefathers" (*archipatribus*) are associated with a point, unnamed Judaic "leaders" (*ductum*) with a line, and King David with a plane (fig. 6.17–19). The verses were written in gold on bands of purple at the top and bottom of the image beside pictures of Christ's predecessors, which occupy squares in corners of the composition. The lines devoted to Christ, adjacent to an image of the Virgin and child, read (fig. 6.20):

> Finally the heavenly offspring
> When through the vitality of the Virgin
> He takes on true flesh
> He erects [makes] as it were the form of a cube.[98]

Here again, Christ incarnate is conceived as a geometrical *corpus*, a cube. His miraculous birth is explained mathematically as a progression from point to line, plane to solid. The last three elements (and possibly a point) were pictured. A vertical line hovers above the word *grammatis*, two lines joined at a right angle follow the term *epiphaniam*, and a projected *cybus* was drawn beside the above-quoted verses. Here a reader/viewer is prompted to picture the geometric elements and Christ's distinguished lineage *with the eye of the intellect*. The tiny drawings in the St. Matthew illumination leave little doubt that the kinds of mental habits cultivated in the school room informed the creation and consumption of luxury liturgical objects like the *Uta Codex*. I believe the same can be said of the *Pericope Book*.

[97] Likewise, the front cover of the Sacramentary of Henry II (Munich, BSB Clm. 4456) made in Regensburg possibly as early as 1002, resembles the "embedded squares" of from geometrical miscellanies of the period, which we saw added to the margins of a copy of Macrobius's *Commentary*: Harley 2772, fol. 46 (fig. 5.30).
[98] Munich, Clm. 13601, fol. 5v; trans. Cohen, in *Uta Codex*, 112: "Tandem p[ro]pago c[a]elica / Per virginis vitalia / Dum carne[m] vera[m] suscipit / Cybi ceu forma[m] subrigit."

Viewing the Pericope Book through the lens of geometry, likening it to a cube, refocuses attention in its appearance. The overall effect of the manuscript, inside and out, is one of muscular materiality or, perhaps better, corporeality. In Calcidius's *Commentary* solids were said to be sensible because of color, surface texture, and weight.[99] We find these qualities exaggerated in the *Pericope Book*. The front cover is studded and made rough with gems and pearls of diverse shapes and sizes. A band of pearl beads once edged the inscription. The deeply cut ivory plaque is framed by undulating acanthus leaves. Gems and enamels give the cover a rainbow-like appearance. Greens and blues dominate, but various shades of red, purple, pink, yellow, and brown are also present. This same supersaturated palette is encountered in the twenty-eight vividly pigmented full-page (sometimes double-page) illuminations and 294 large-scale decorated initials, many of which extend the length of a page, afloat on painted fields of deep, velvety purple (figs. 6.21–24). The manuscript is an example of the Word of God made, not merely manifest, but overtly, even aggressively, *sensible*, based on the most up-to-date understanding of the mechanics of vision.

The smooth surface of the back cover, decorated with pieces of engraved and partially gilded openwork silver depicting the four virtues and the lamb of God,[100] contrasts sharply with the uneven and varied texture of the front (figs. 6.3, 6.5) The choice of form and materials may have been rooted in practical considerations, namely, the need for the manuscript to lie flat. But pragmatic motivation does not, of course, preclude meaning. Indeed, we find colorful, jewel-encrusted surfaces opposed with flat, near-monochromatic ones in reliquaries, such as the *Stephansbursa*, and processional crosses—objects that were not intended to rest on the flat side (figs. 6.25–27).[101] Eliza Garrison has discussed the impact of this kind of pairing in the Lothar Cross. Following observations of O. K. Werckmeister, Garrison suggests that the "decidedly sculptural," jeweled side of the cross "speaks" to the material world of the emperor, whereas the minimal decoration of the crucifixion, which she aptly compares to a "two-dimensional sketch," refers to the spiritual domain.[102] I agree with

[99] Isidore explained that weight was an essential aspect of all things: "Nature has given weight to all corporeal objects"; *Etymologiae* 16.25.1.
[100] The green silk that covers the spine and back of the book is part of the 18th-c. restoration, but it likely replaced material that had deteriorated over time. The same silver gilt openwork over cloth is seen on the back cover of the near-contemporary (c.1015) Bernward Gospels; Fillitz, "Der Einband," 107.
[101] Vienna, Kunsthistorisches Museum, Weltliche Schatzkammer, Inv. Nr. SK_WS_XIII_26.
[102] Garrison, *Ottonian Imperial Art*, 77. Garrison cites an unpublished paper of Werckmeister, "The Donations of Otto III in the Imperial Chapel at Aachen." See also his earlier work on the treasure binding of the Codex

her assessment but would emphasize that the rationale behind these associations was at once biblical and scientific. For engraving itself could be viewed as process with celestial connotations. Isidore in the *Etymologiae* wrote:

> The sky (*caelum*) is so named because, like an engraved (*caelatum*) vessel, it has the lights of the stars pressed into it, just like engraved figures; for a vessel which glitters with figures that stand out is called *caelatus.* God embellished the heaven and filled it with bright light—that is, he adorned it with the sun and the gleaming orb of the moon and the glorious constellations of glittering stars. In a different way, it [sky/heavens] is named from engraving (*caelare*) the superior bodies.[103]

Hence, to engrave was to write with a heavenly light. Like drawings made with a stylus in wax, the engraved image was composed of narrow channels cut into the surface, wholly without substance, wrought of air and shadow.

The exclusive use of gold and silver on the back cover may have been intended to mimic the divine radiance of the heavens.[104] Both metals were thought to shine most brightly when touched by air, the purest of the four elements, the greatest concentration of which was found in the sky.[105] In an inscription on a near-contemporary antependium for Xanten Cathedral, gold was said simply to signify god: *Deus est signatus in auro*.[106] More radiant and sacred than either gold or silver was their combination. The intermingling of the two metals on the back cover may have been intended to evoke the alloy *electrum*,[107] which

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Aureus: Werckmeister, *Der Deckel des Codex aureus von St. Emmeram: ein Goldschmiedewerk des 9. Jahrhunderts* (Baden-Baden: Verlag Heitz, 1963).

[103] Isidore, *Etymologiae* 13.4.1; ed. Gasparotto, 22; trans. Barney, 272: "Caelum vocatum eo quod, tamquam caelatum vas, inpressa lumina habeat stellarum veluti signa. Nam caelatum dicitur vas quod signis eminentioribus refulget. Distinxit enim eum Deus claris luminibus, et inplevit; sole scilicet et lunae orbe fulgenti et astrorum micantium splendentibus signis adornavit. [Alias autem a superiora caelando.]" The last line does not appear in all copies of the *Etymologiae*, but was included by Lindsay in his edition and Barney in his translation. The passage on engraving appears twice in the *Etymologiae*, here in Book 13 on the universe and its parts, and earlier in Book 3 (3.1.1) on Number. Ambrose gave a similar explanation in the *Hexameron* (2.4.15).

[104] Interestingly, in an extant geometric and astronomical miscellany at Bamberg (SB Msc.Class.55) the weights of gold and silver follow the size and measures of the planets. The logic that guided this ordering may have been the divine/celestial associations attached to these metals.

[105] Isidore, *Etymologiae* 16.18.1; ed. José Feáns Landeira, in *Etymologiae XVI: De las piedras y de los metales* (Paris: Les Belles Lettres, 2011), 197.

[106] Kingsley, "UT CERNIS and The Materiality of Bernwardian Art," in *1000 Jahre St. Michael in Hildesheim: Kirche, Kloster, Stitfer*, eds. Gerhard Lutz and Angela Weyer (Petersberg: Michael Imhof Verlag, 2012), 173; on the use of gold to signify divinity in the Christian context more generally see Janes, *God and Gold* (Cambridge: Cambridge University Press, 1989), esp. 61–93.

[107] My attention was drawn to this possibility by the work of Jennifer Kingsley, who, building on the work of Christoph Schulz-Mons and Herbert Kessler, explored the self-conscious use of a silver/gold alloy by Bernward

comprised, according to Isidore, "three parts gold to one part silver."[108] The material figured prominently in the visions of Ezekiel (1:4 and 1:26) in the Old Testament, and its significance was revealed by exegetes, most of whom interpreted it in Christological terms, as a symbol of Christ's dual nature.[109] Gregory the Great synthesized and honed previous interpretations in the *Moralia in Job*, drawing out its connection to the incarnation:

> Now electrum is a metal in which gold and silver arc combined. In this alloy, the silver is made brighter while the brilliance of gold is tempered. What, then is signified by electrum if not the mediator between God and man? For, when he unites a divine and human nature in himself for our sake, his human nature is made brighter by the divinity and his divine nature is tempered by the humanity, adjusted to our capacity to see. In this way, then, thanks to the power of so great a miracle, his divinity brightens his humanity, as gold enhances silver.[110]

Gregory likens Christ, the mediator, to electrum. In Christ, God tempered his divine brilliance so that man might look upon him. The metal's properties physically demonstrate how humanity, like silver, might be enhanced, while the splendor of divinity, like gold, is only a little diminished. The alloy was also associated with the virtues.[111] Both meanings are

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in a pair of candlesticks cast for St. Michael's church, Hildesheim. Kingsley also relates the admixture of gold and silver to the use of these materials in the illuminations of the Bernward Gospels, but not to their application on the back cover, which, like the *Pericope Book*, is decorated with silver-gilt metalwork. I believe that there too the metal served as another way to represent and visually explain the incarnation. Kingsley, "UT CERNIS," 171–84; Christoph Schulz-Mons, *Das Michaeliskloster in Hildesheim: Untersuchungen zur Gründung durch Bischof Bernward (993–1022)* (Hildesheim: Gerstenberg, 2010), 360–70; Herbert Kessler, "Image and Object: Christ's Dual Nature and the Crisis of Early Medieval Art," in *The Long Morning of Medieval Europe: New Directions in Early Medieval Studies*, eds. Jennifer Davis and Michael McCormick (Aldershot: Ashgate, 2008), 291–320; and idem, "The Eloquence of Silver: More on the Allegorization of Matter," in *L'Allégorie dans l'art du Moyen Âge*: *Formes et fonctions*: *Héritages, créations, mutations*, ed. Christian Heck (Turnhout: Brepols, 2011), 49–64.

[108] Isidore, *Etymologiae* 16.24.2; ed. Feáns Landeira, 255; trans. Barney, 332: "... quod fit de tribus partibus auri et argenti una."

[109] As discussed by Nancy van Deusen and Marcia L. Colish, "*Ex utroque et in utroque: Promissa mundo guadia, Electrum*, and the Sequence," in *The Place of the Psalms in the Intellectual Culture in the Middle Ages*, ed. Nancy van Deusen (Albany: State University of New York, 1999), 114–30.

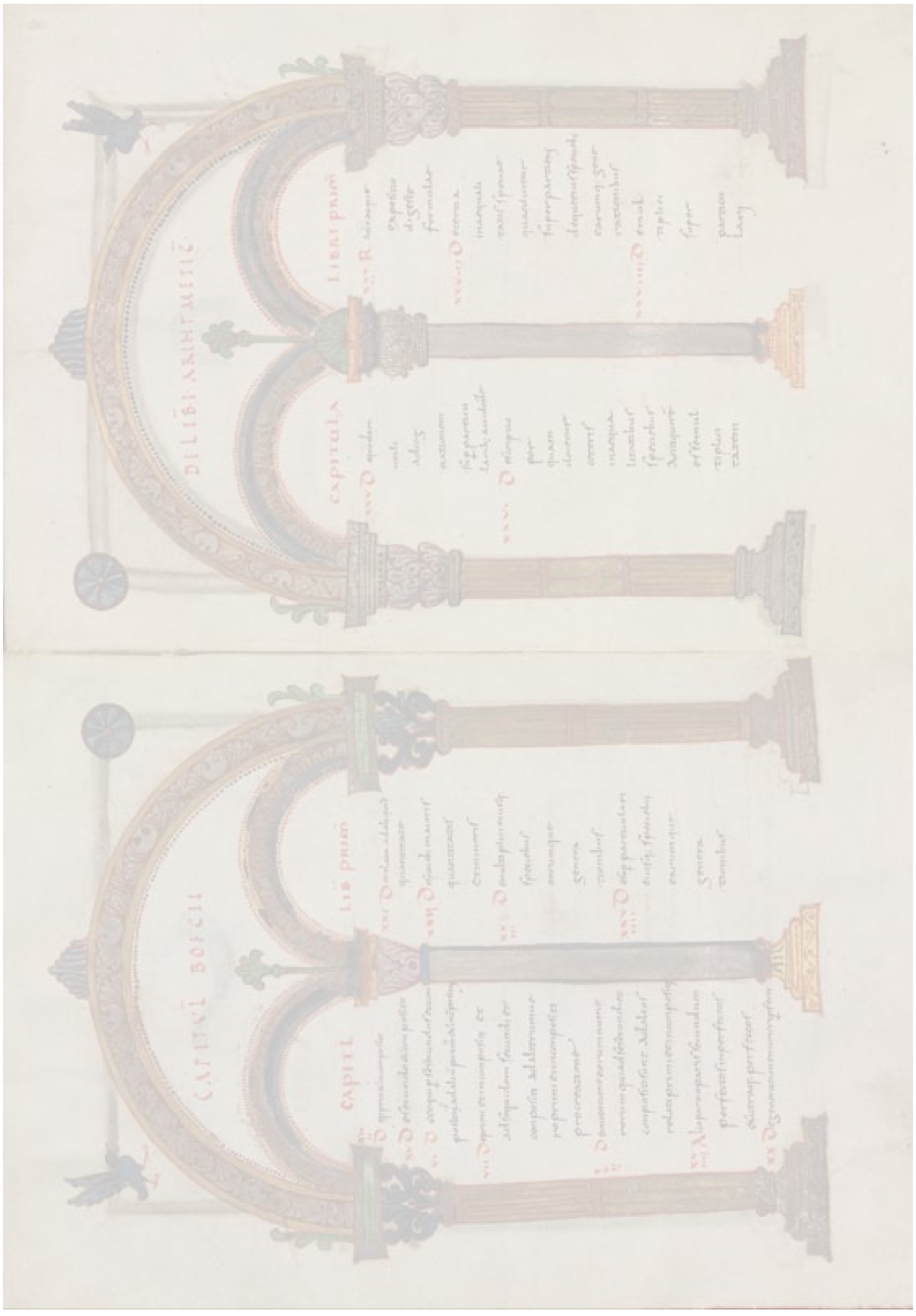
[110] Gregory, *Moralia in Job* 28.1.5; ed. Marcus Andriaen, CCSL 143B (Turnhout: Brepols, 1985), 1397–98; trans. Nancy van Deusen and Marsha Colish, in "Ex utroque et in utroque," 125: "Electrum quippe ex auri argenti que metallo miscetur, in qua permixtione argentum quidem clarius redditur, sed tamen fulgor auri temperatur. Quid ergo in electro nisi mediator dei et hominum demonstratur? Qui dum semetipsum nobis ex diuina atque humana natura composuit, et humanam per deitatem clariorem reddidit, et diuinam per humanitatem nostris aspectibus temperauit. Quia enim uirtute diuinitatis eius tot miraculis humanitas fulsit, ex auro creuit argentum; et quod per carnem deus cognosci potuit quod que per carnem tot aduersa tolerauit, quasi ex argento temperatum est aurum."

[111] van Deusen and Colish, "*Ex utroque et in utroque*," 120.

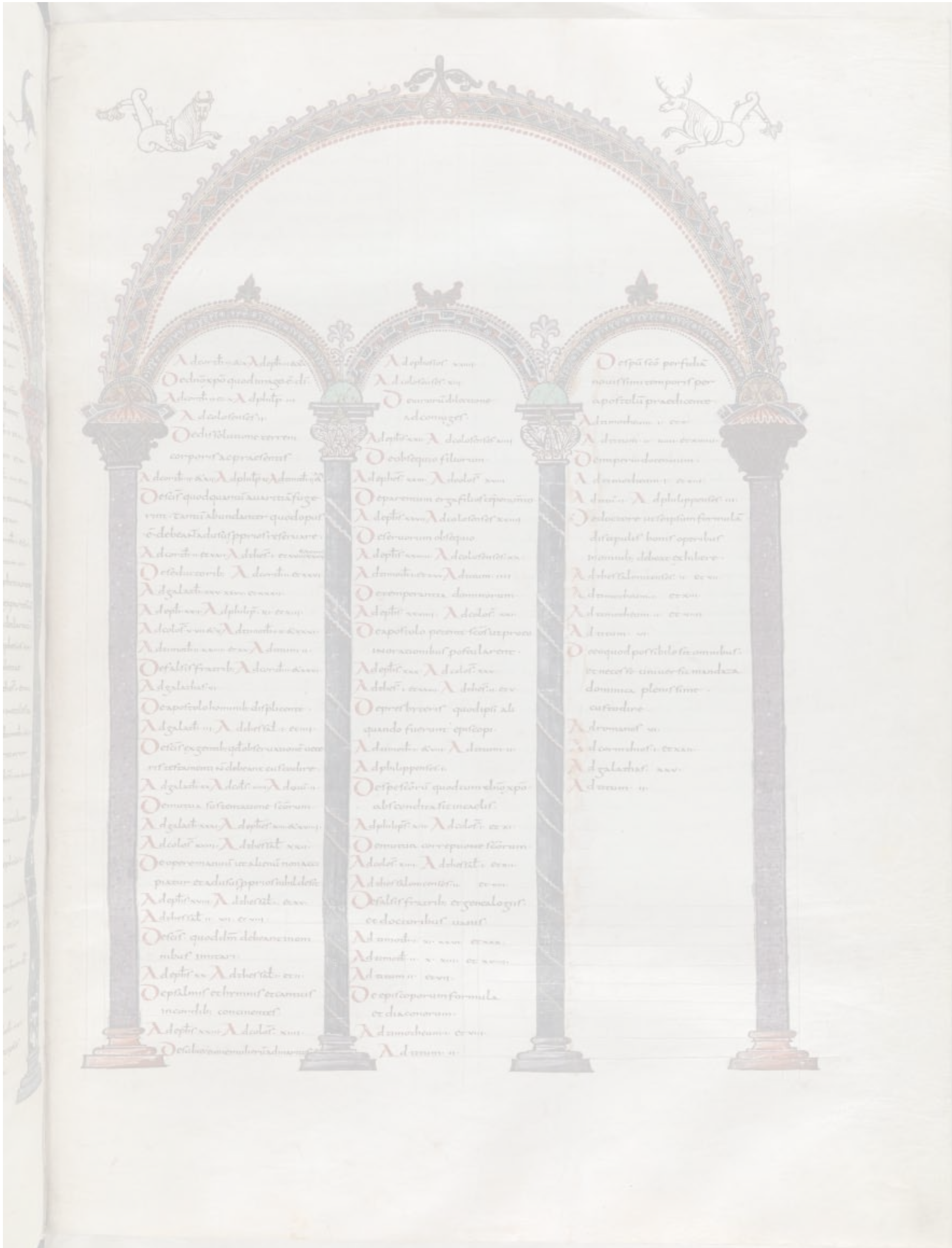
clearly at play on the back cover of the *Pericope Book*, where the personified virtues and the lamb of God were imaged in engraved *electrum* in an instance of iconic synchrony.

The makers of the *Pericope Book* sought to craft theophany. Material, medium, and iconography were all bent to this purpose. Likewise, form, though inherent, was made to signify. "In the guise of the cube," the cornerstone, the codex, God, perfect, incorporeal, became corporeal and thus visible. The front cover of the *Pericope Book* was designed to engage the eyes of the body; the back to serve the intellect or spirit. The book is a demonstration, a "proof" in the mathematical sense, of the incarnation. It was also, of course, a written exposition of that mystery—an observation that returns us the first lines of the inscription: "Whoever seeks to understand the letters of true wisdom will rejoice to hold this square full of *mathesis*." In the *Pericope Book*, Christ as the Word, "the letters of true wisdom," was joined to Christ as number, a "square full of *mathesis*." Hence, to understand the object, trivium and quadrivium, literacy and numeracy must both be brought to bear.

1.1 Bamberg, Staatsbibliothek MS Msc.Class.5, fol. 2v.

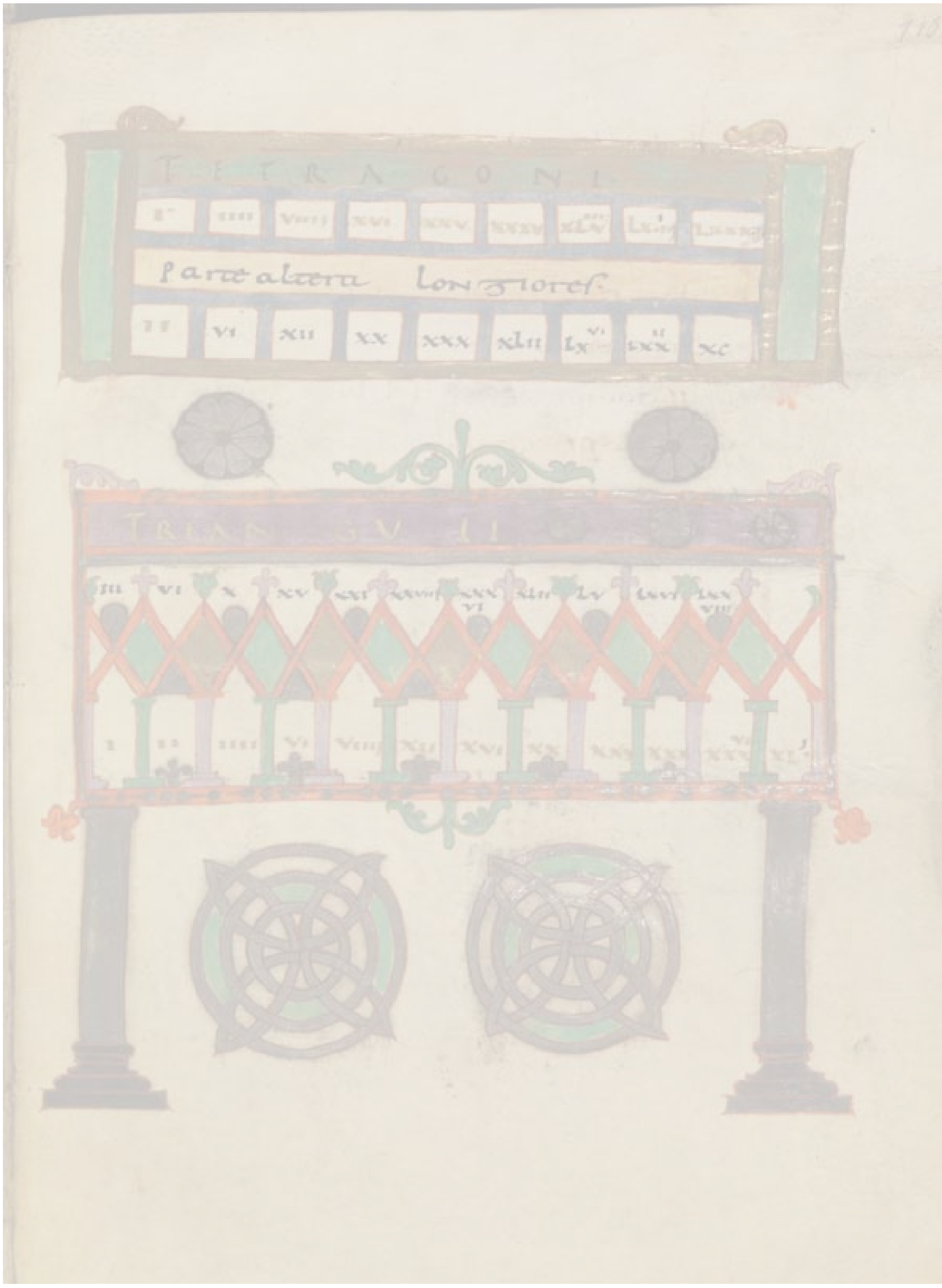


1.2 Bamberg, Staatsbibliothek MS Msc.Class.5, fols. 7v–8r. © Staatsbibliothek Bamberg



1.3 London, British Library Additional MS 10546 (the “Moutier-Grandval” Bible), fol. 409r.

1.4 Bamberg, Staatsbibliothek MS Msc.Class.5, fols. 107v–108r. © Staatsbibliothek Bamberg



1.5 Bamberg, Staatsbibliothek MS Msc.Class.5, fol. 110r.
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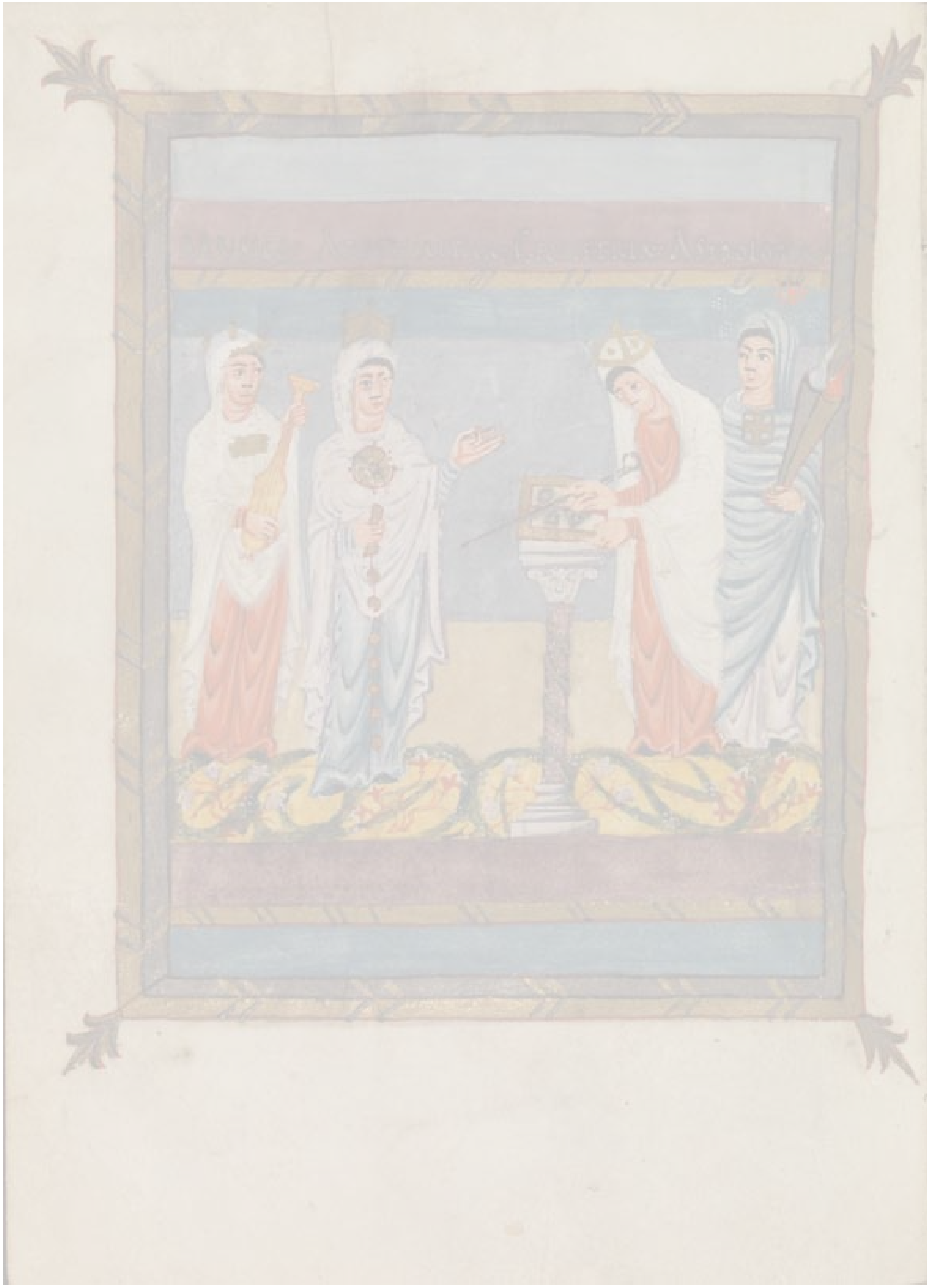
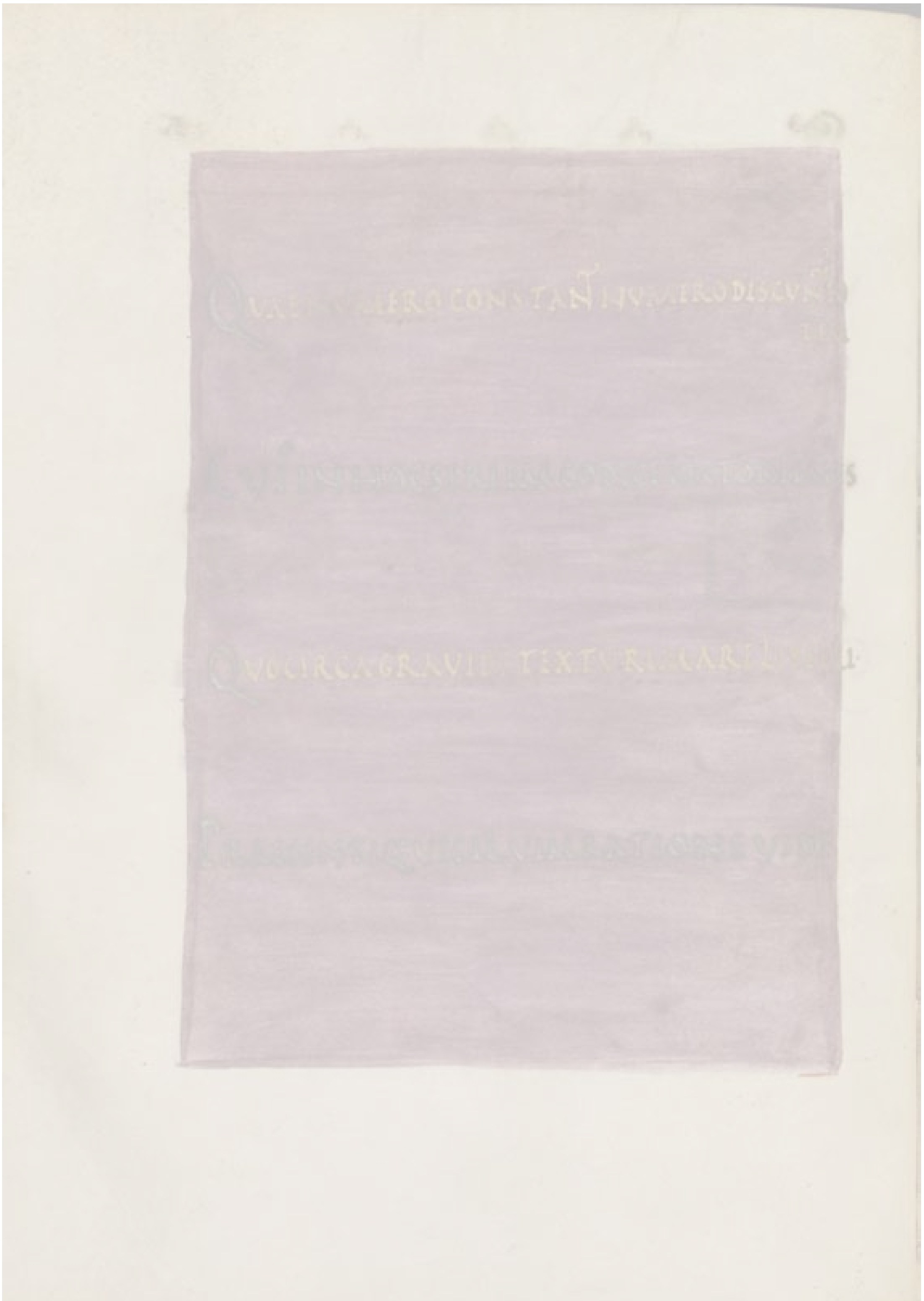
1.6 Bamberg, Staatsbibliothek MS Msc.Class.5, fol. 127r.

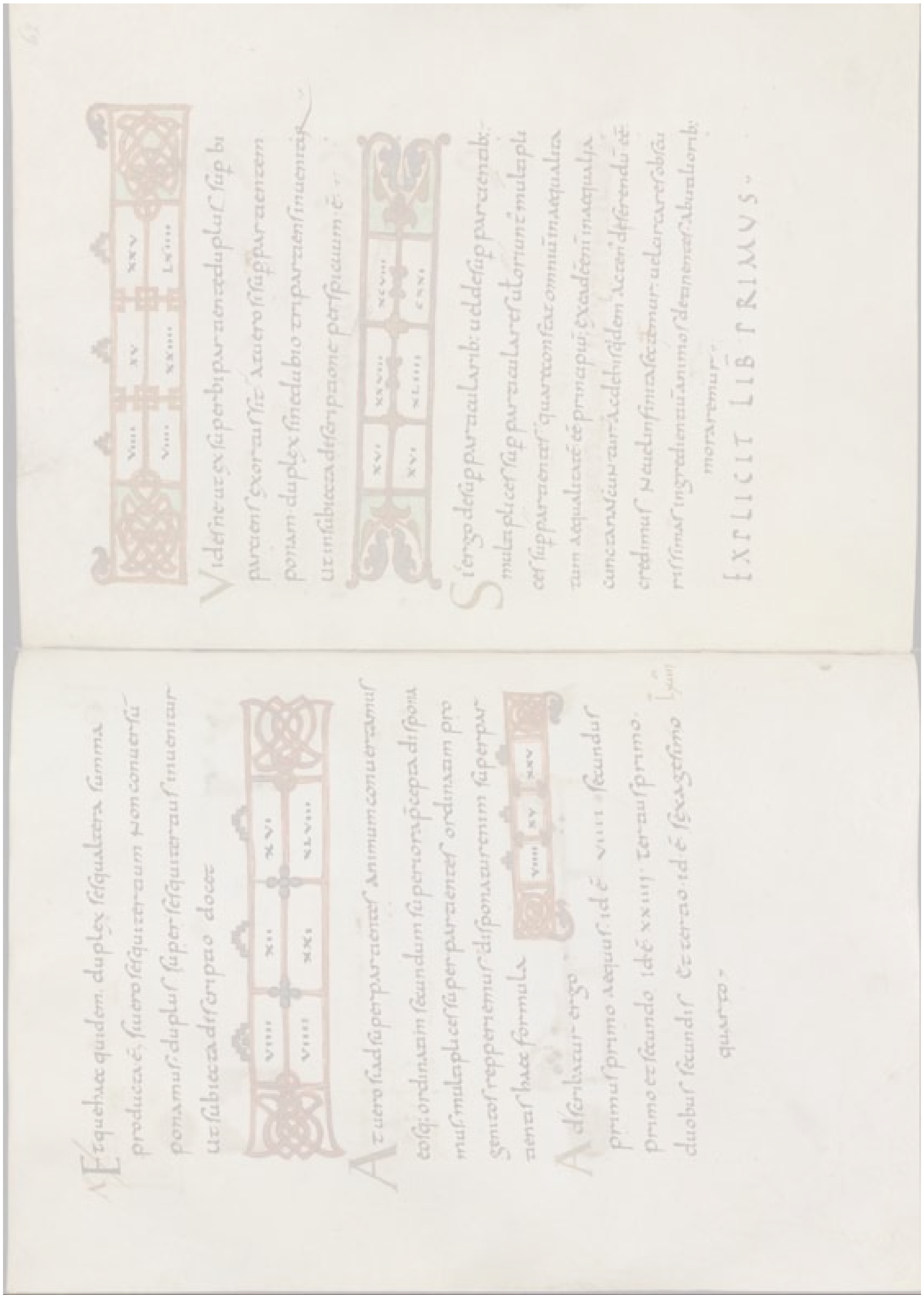


1.7 Bamberg, Staatsbibliothek MS Msc.Class.5, fol. 9v.

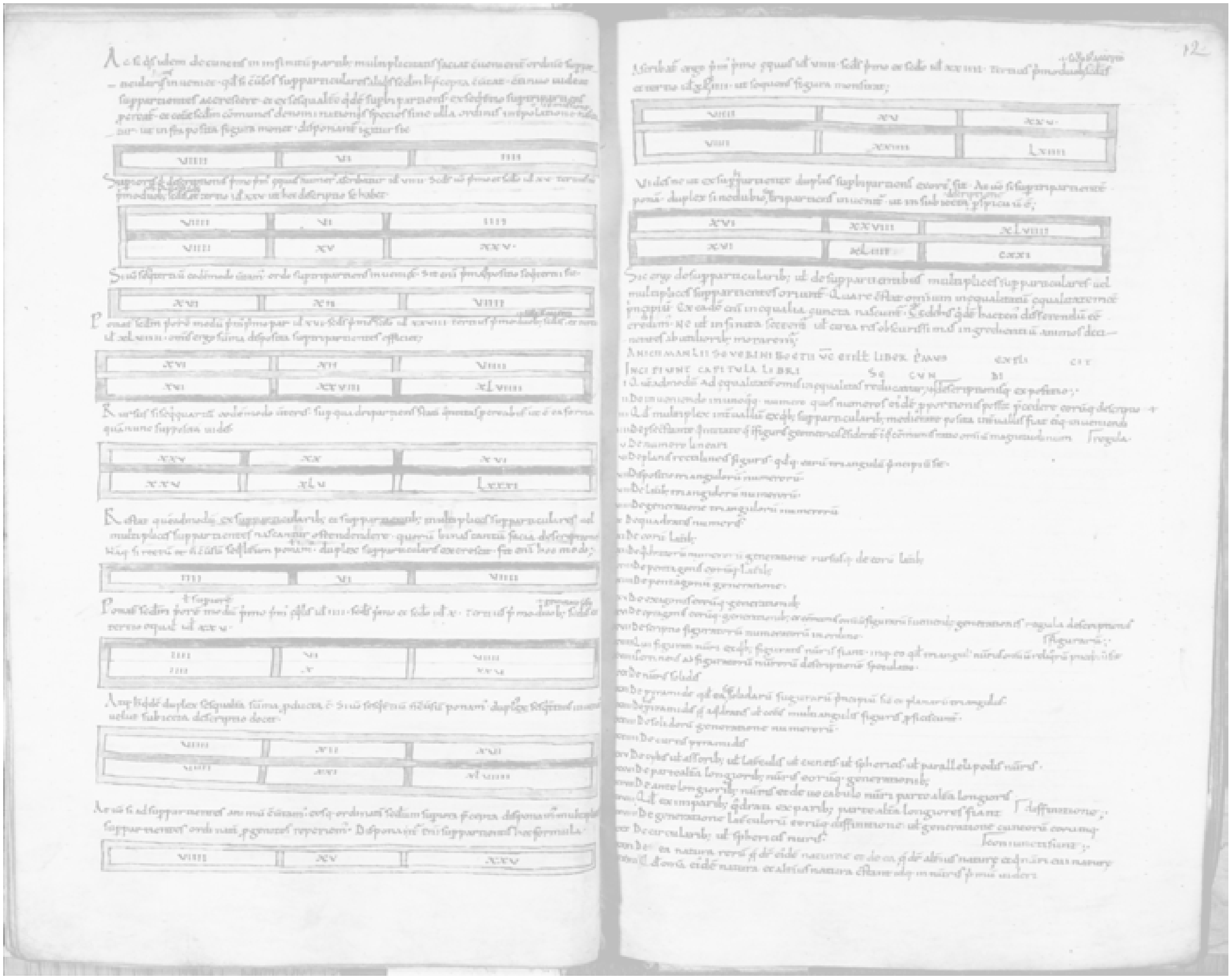
1.8 Bamberg, Staatsbibliothek MS Msc.Class.5, fol. 9v, detail. © Staatsbibliothek Bamberg



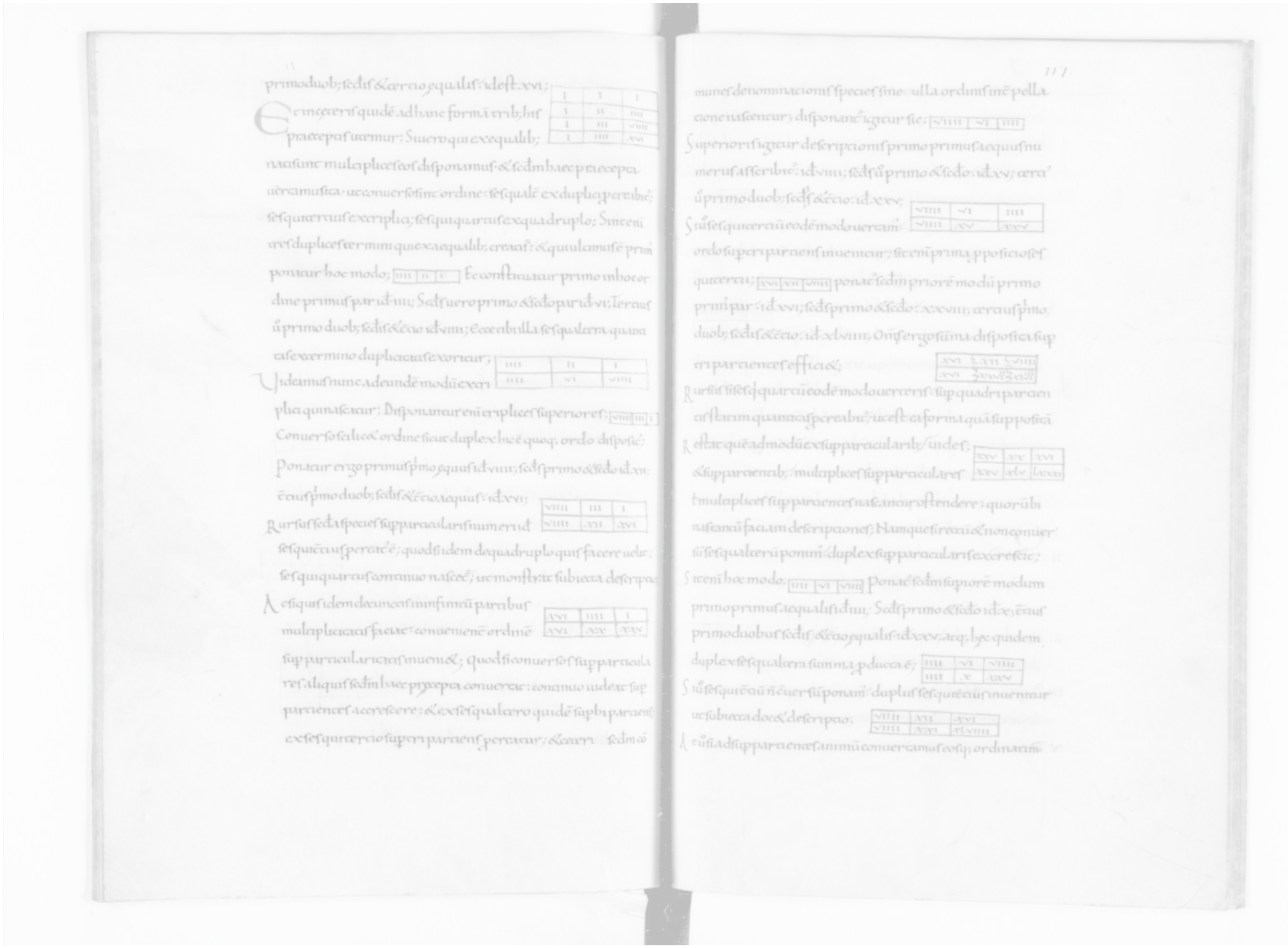
1.9 Bamberg, Staasbibliothek MS Msc.Class.5, fol. 62v.



1.10 Bamberg, Staatsbibliothek MS Msc.Class.5, fol. 61v–62r. © Staatsbibliothek Bamberg



1.11 Reims, Bibliothèque municipale MS 975, fols. 11v–12r. © Bibliothèque municipale Reims



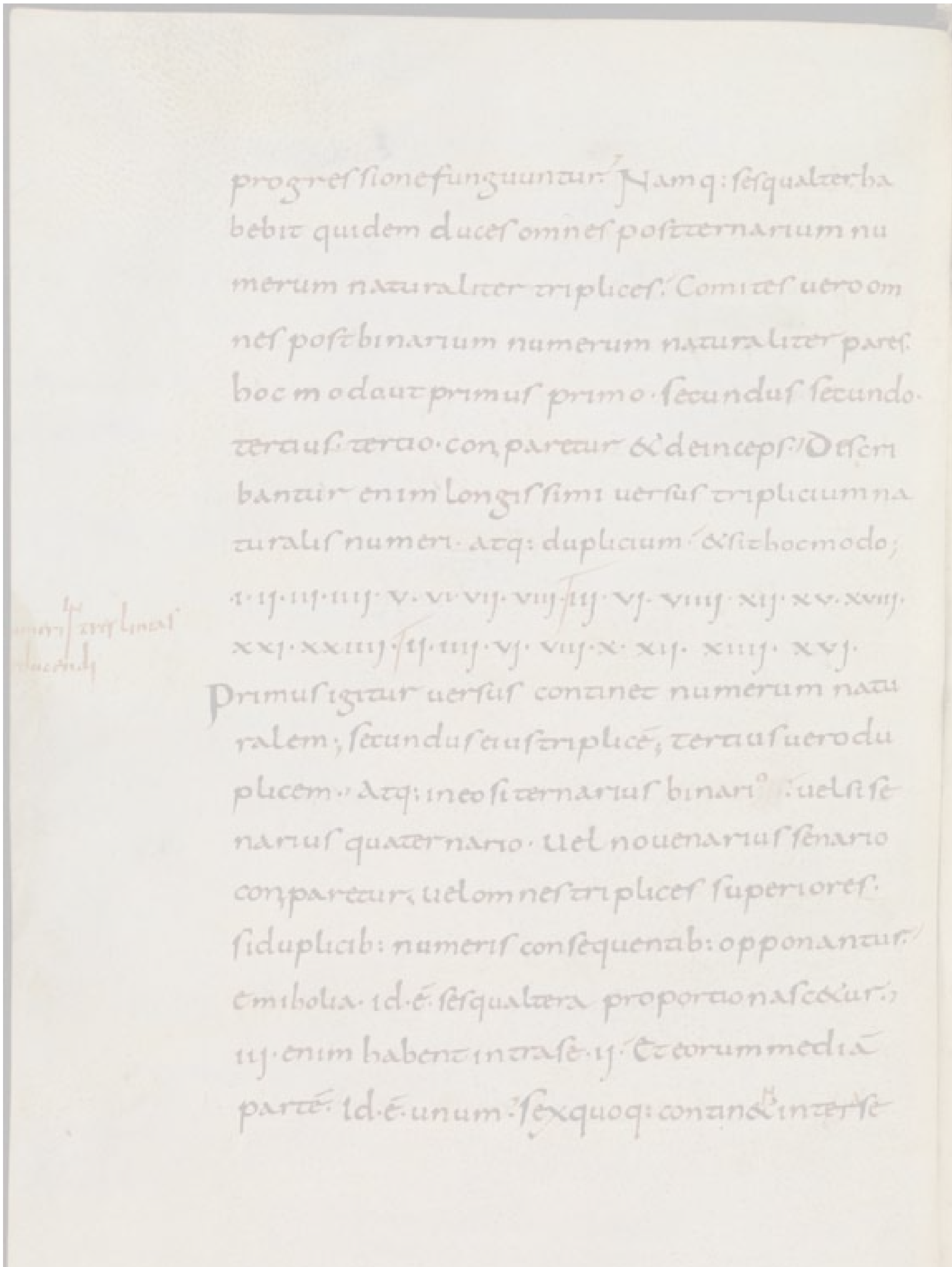
1.12 Paris, Bibliothèque nationale de France lat.MS 6401, fols. 116v–17r. © Bibliothèque nationale de France

&in conpositos ex illa coaceruatione factus sit
totam summam in illum multiplicabis quem
posterius adgregaueras. Si uero coaceruatione
facta primus inuentus non fuerit. sed conpositus
& secundus. hunc transgredere atq; alium qui
adsequitur adgregabis. Si uero necdum fue
rit primus & inconpositus. alium rursus iunge
& uide quid fiat. Quod si primum inconpositumq;
repperies. tunc in ultimam multitudine summe
coaceruationem multiplicabis. Disponantur
enim omnes pariter pares numeri hoc modo.

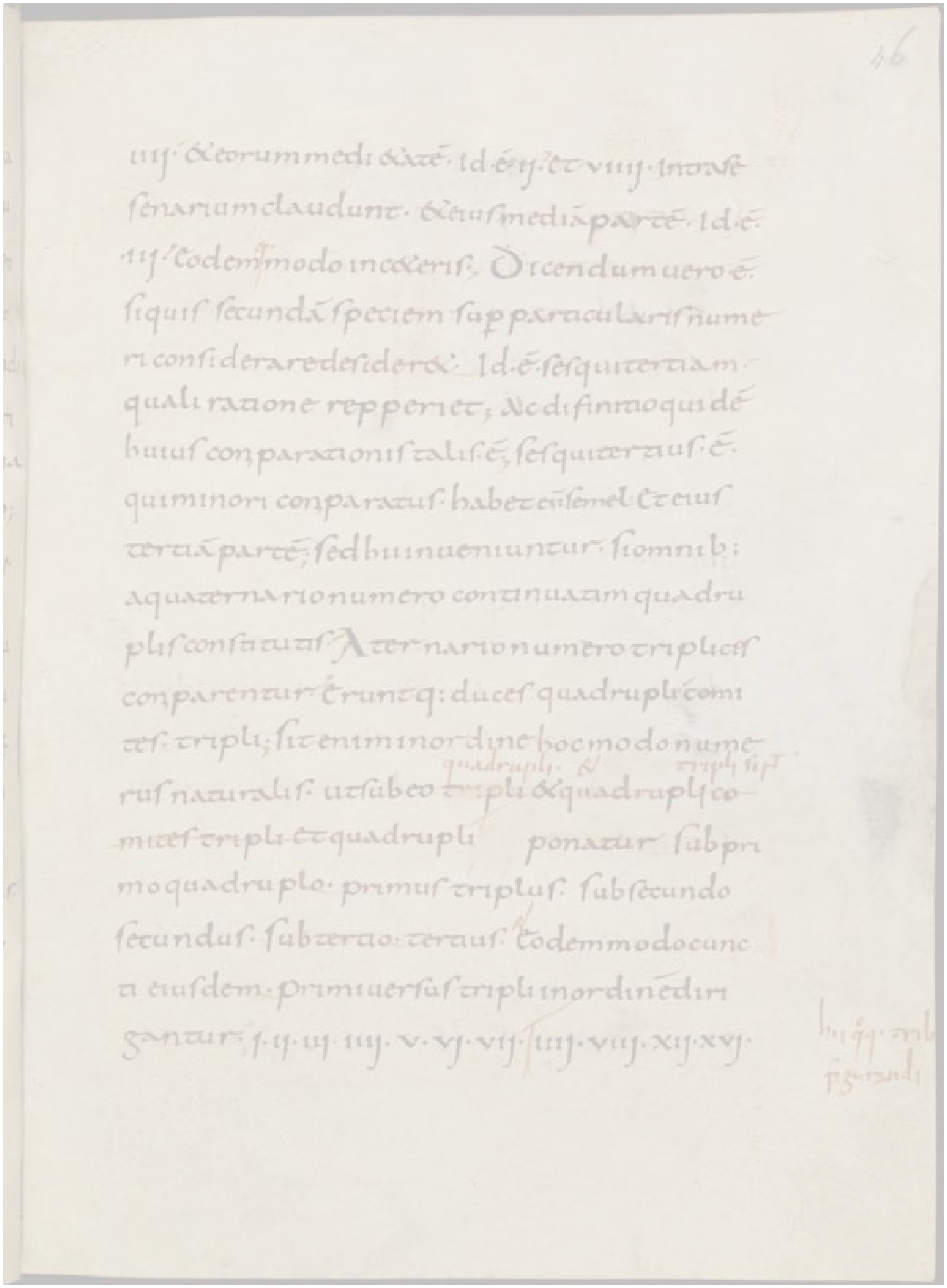
i	ii	iiii	viii	xvi	xxxii	lxiiii	cxxviii

fa
cies ergo ita; pones. i. ei qui sequitur adgregabis. ii.
Tunc respicies ex hac adgregatione qui nume
rus factus sit. inde. iii. qui scilicet primus & in
conpositus est; & pro summitatem ultimum binari
um numeri adgregaueras. fit igitur terna
rium. id est qui ex coaceruatione collectus est. p
binarium multiplices qui est ultimus adgre

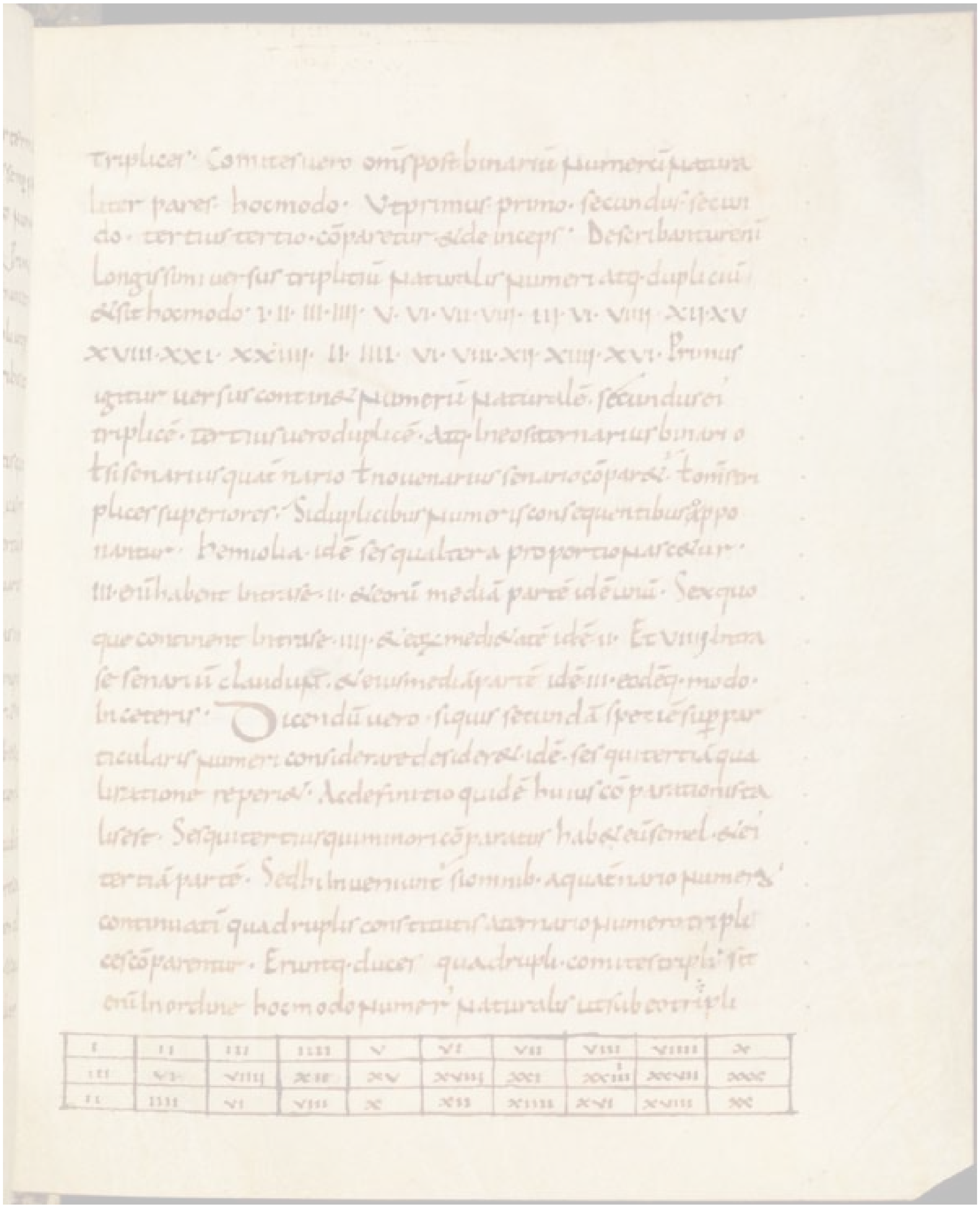
1.13 Bamberg, Staasbibliothek MS Msc.Class.5, fol. 39v.


progressione funguuntur. Namq: sesqualter ha
bebit quidem duces omnes post ternarium nu
merum naturaliter triplices. Comites uero om
nes post binarium numerum naturaliter pares.
hoc modo ut primus primo. secundus secundo.
tertius tertio. con paretur & deinceps. Descri
bantur enim longissimi uersus triplicium na
turalis numeri. atq: duplicium & sit hoc modo;
I. II. III. IIII. V. VI. VII. VIII. III. VI. VIIII. XII. XV. XVIII.
XXI. XXIIII. II. IIII. VI. VIII. X. XII. XIIII. XVI.
Primus igitur uersus continet numerum natu
ralem; secundus eius triplicē; tertius uero du
plicem. Atq: in eo si ternarius binario uel si se
narius quaternario. uel nouenarius senario
con paretur, uel omnes triplices superiores
si duplicib: numeris consequentib: opponantur.
emibolia. id est sesqualtera proportio nascetur.
III. enim habent intra se. II. & eorum mediā
partē. id est unum. sex quoq: continent intra se

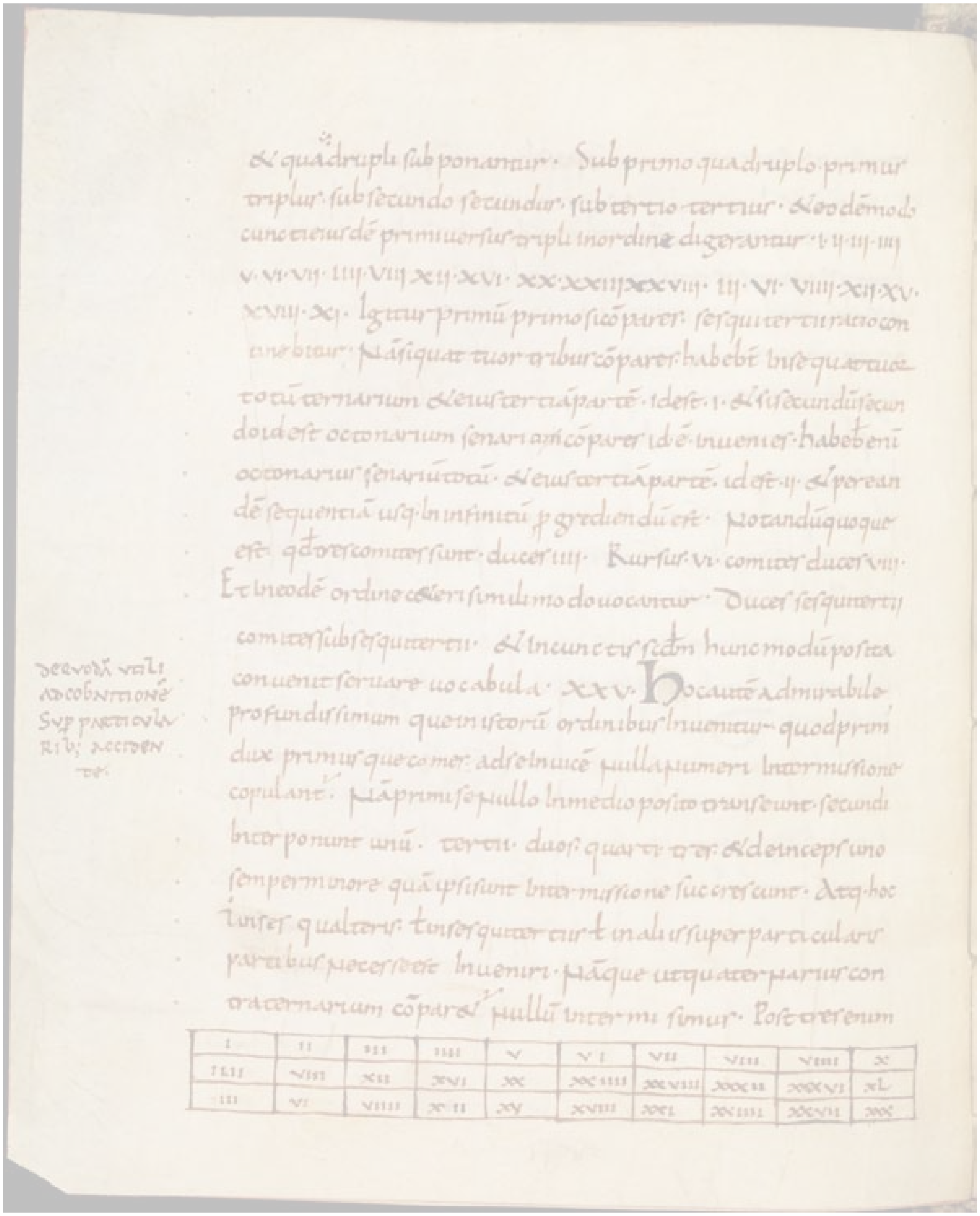
1.14 Bamberg, Staasbibliothek MS Msc.Class.5, fol. 45v.



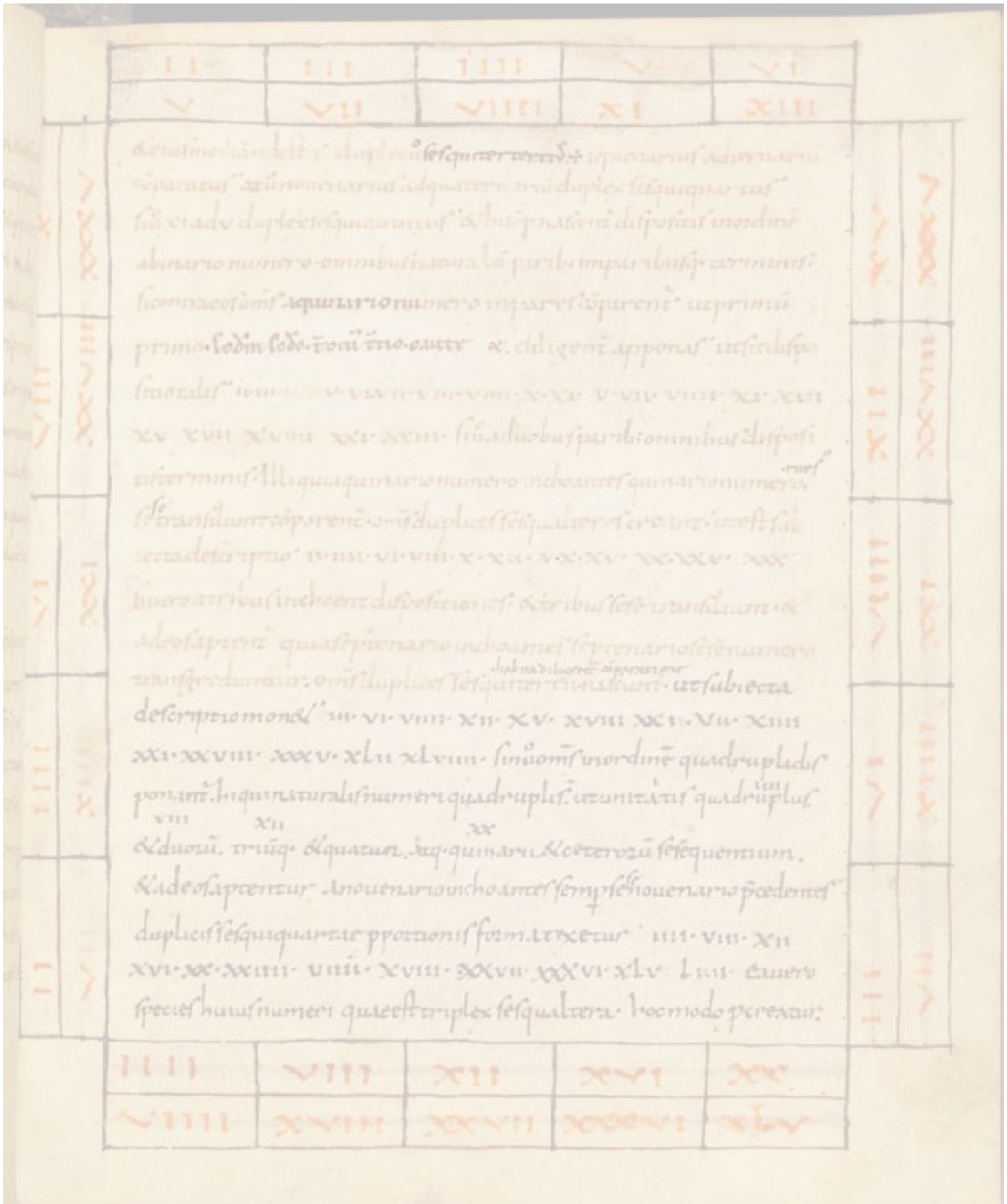
1.15 Bamberg, Staasbibliothek MS Msc.Class.5, fol. 46r.



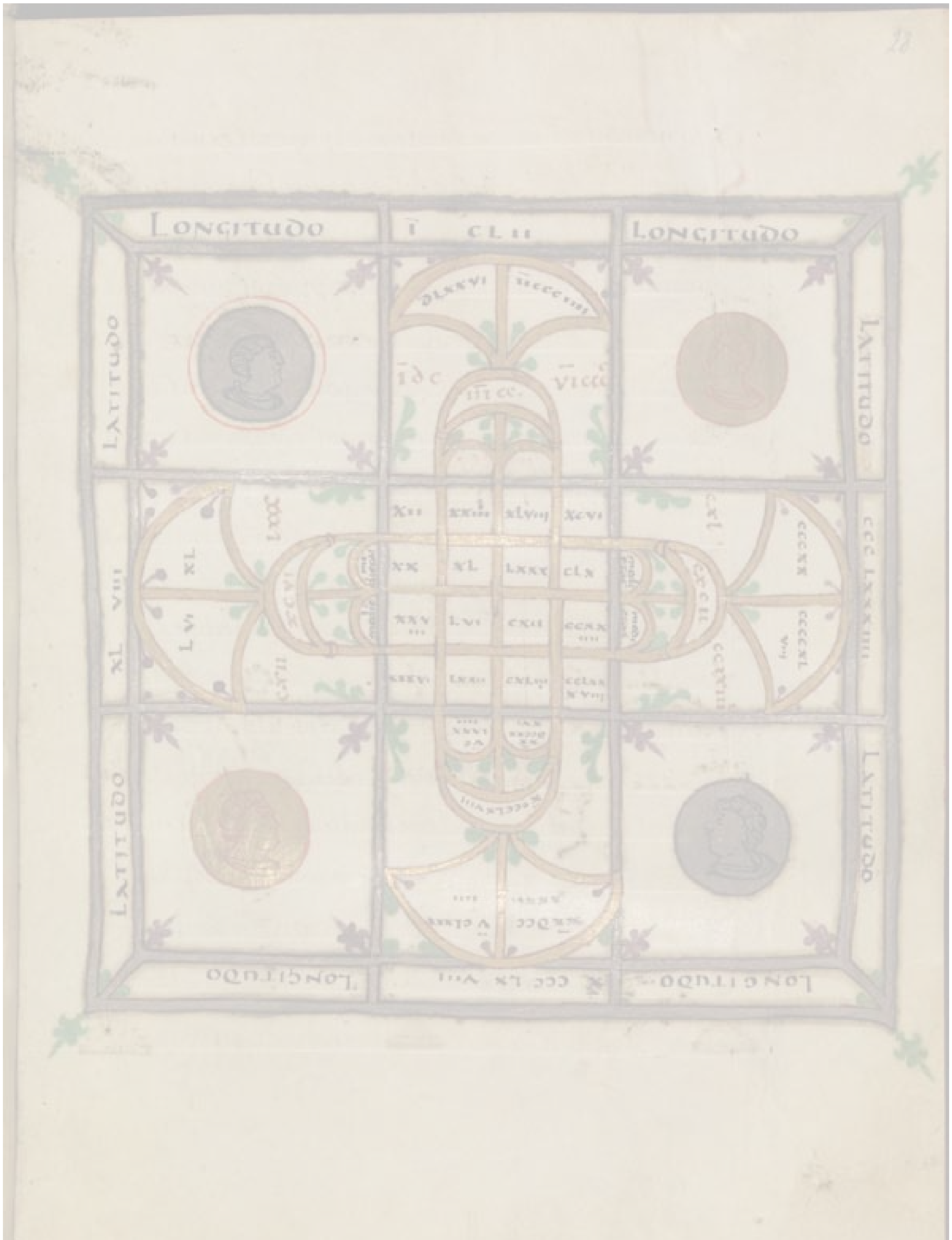
1.16 Bamberg, Staasbibliothek MS Msc.Class.8, fol. 27r.



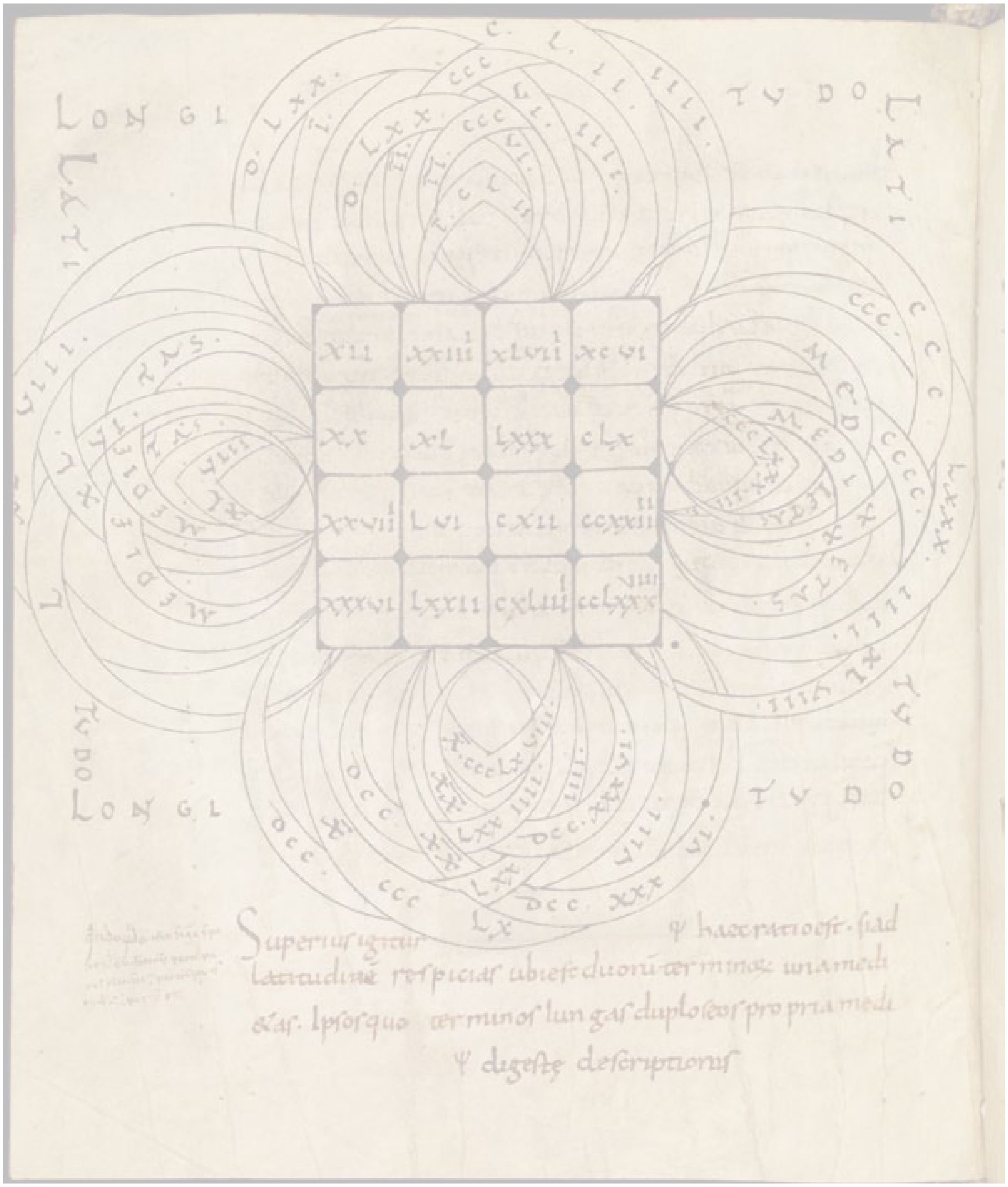
1.17 Bamberg, Staasbibliothek MS Msc.Class.8, fol. 27v.



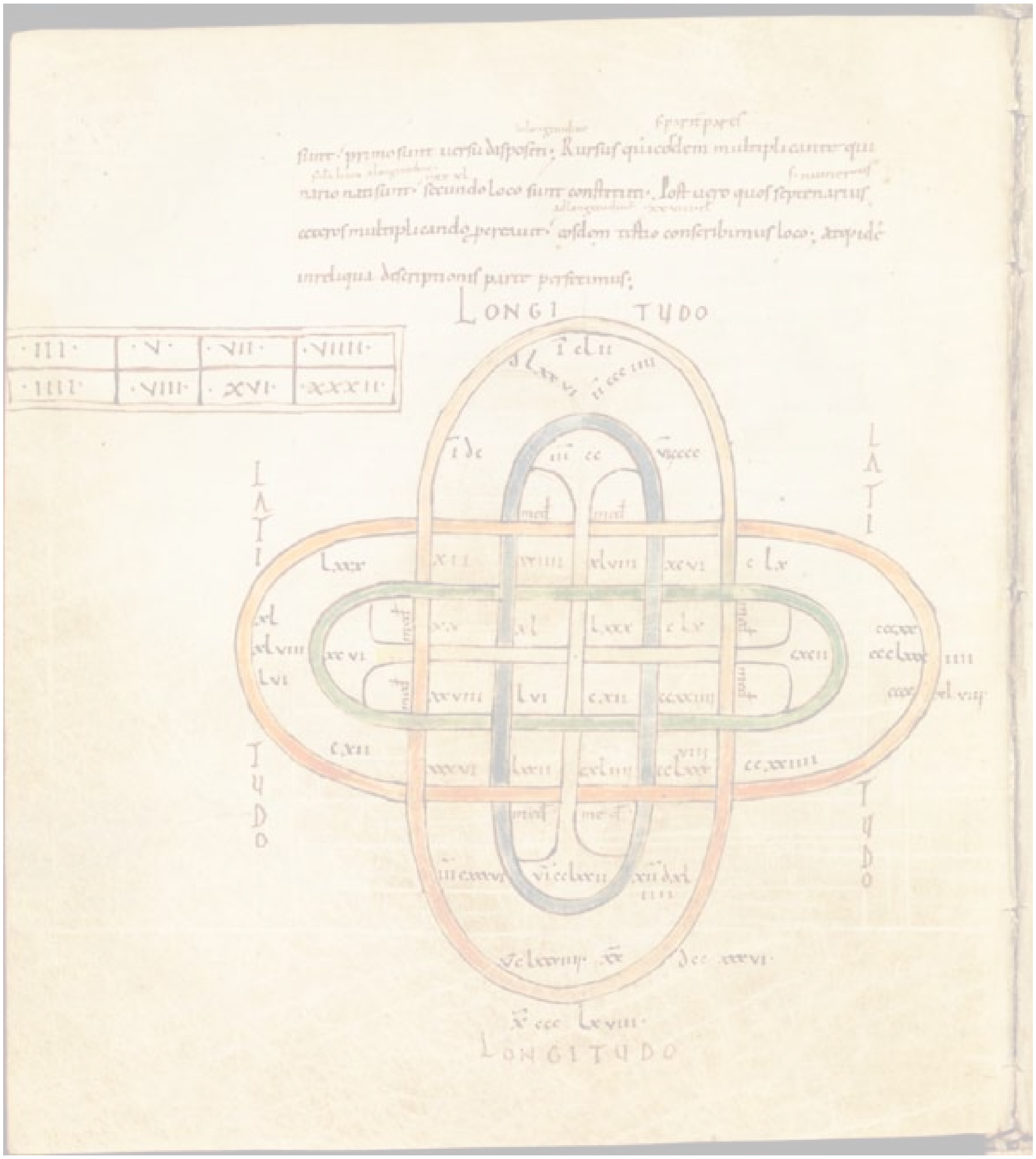
1.18 Bamberg, Staasbibliothek MS Msc.Class.8, fol. 33r.



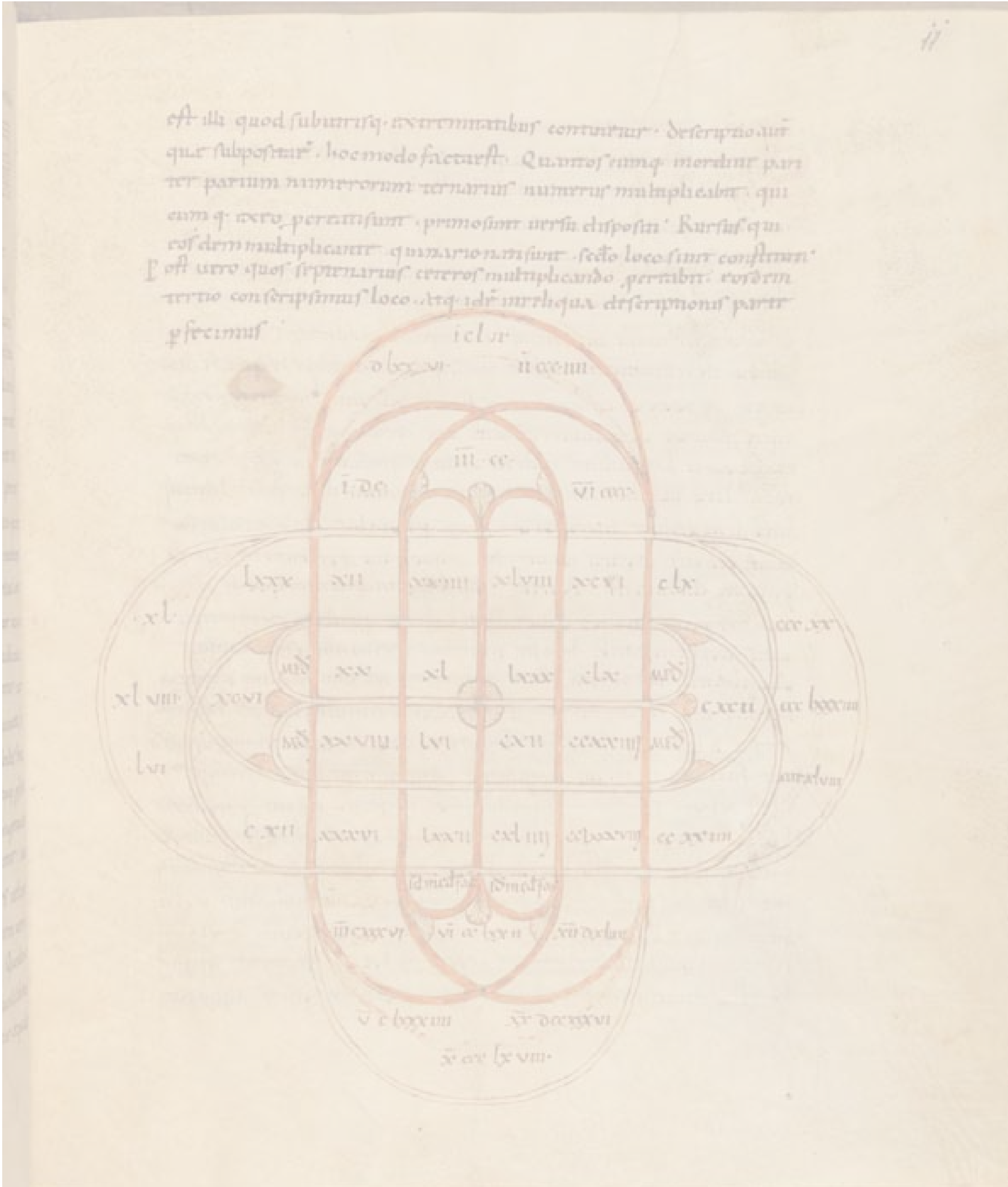
1.19 Bamberg, Staasbibliothek MS Msc.Class.5, fol. 28r.



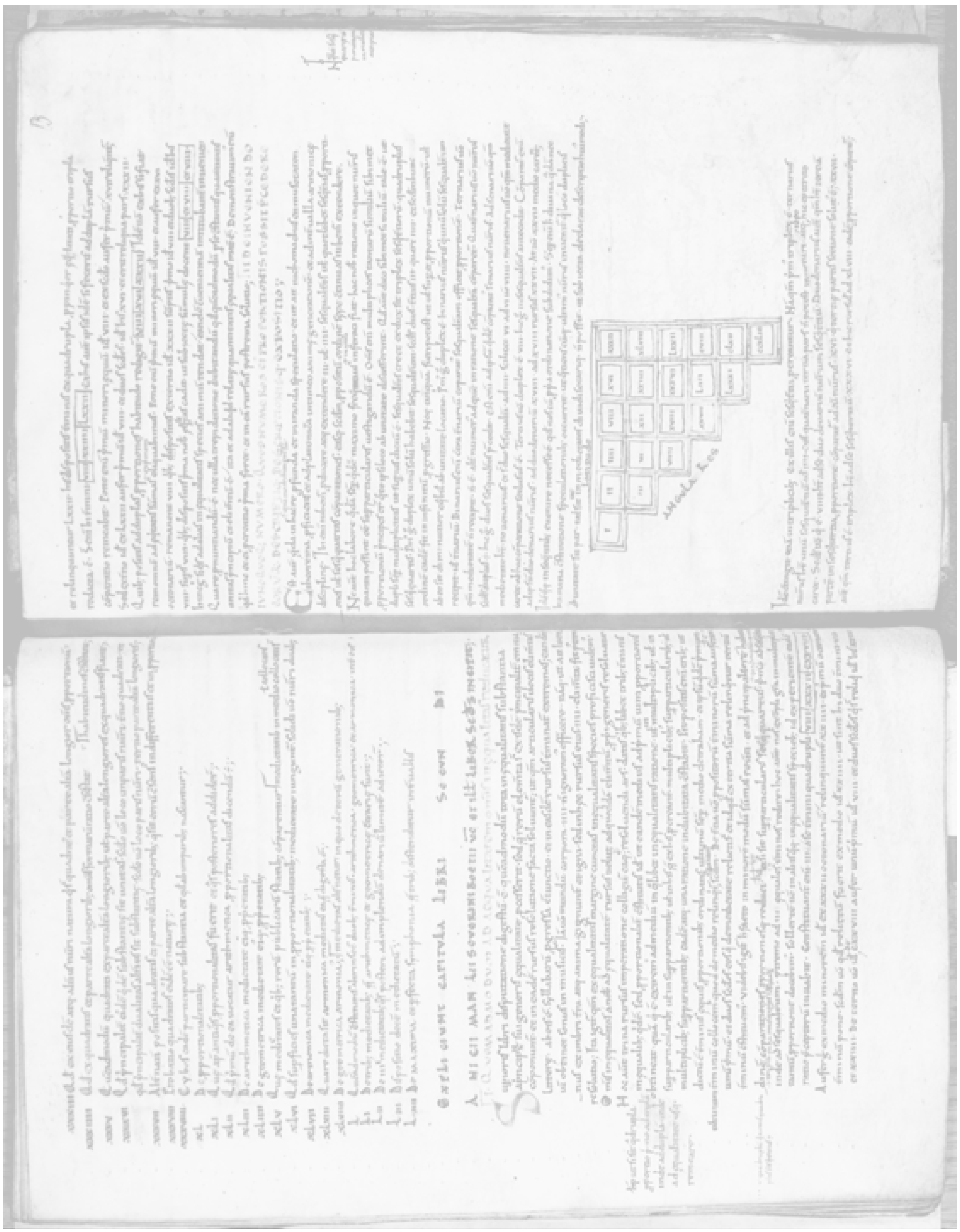
1.20 Bamberg, Staasbibliothek MS Msc.Class.8, fol. 15v.



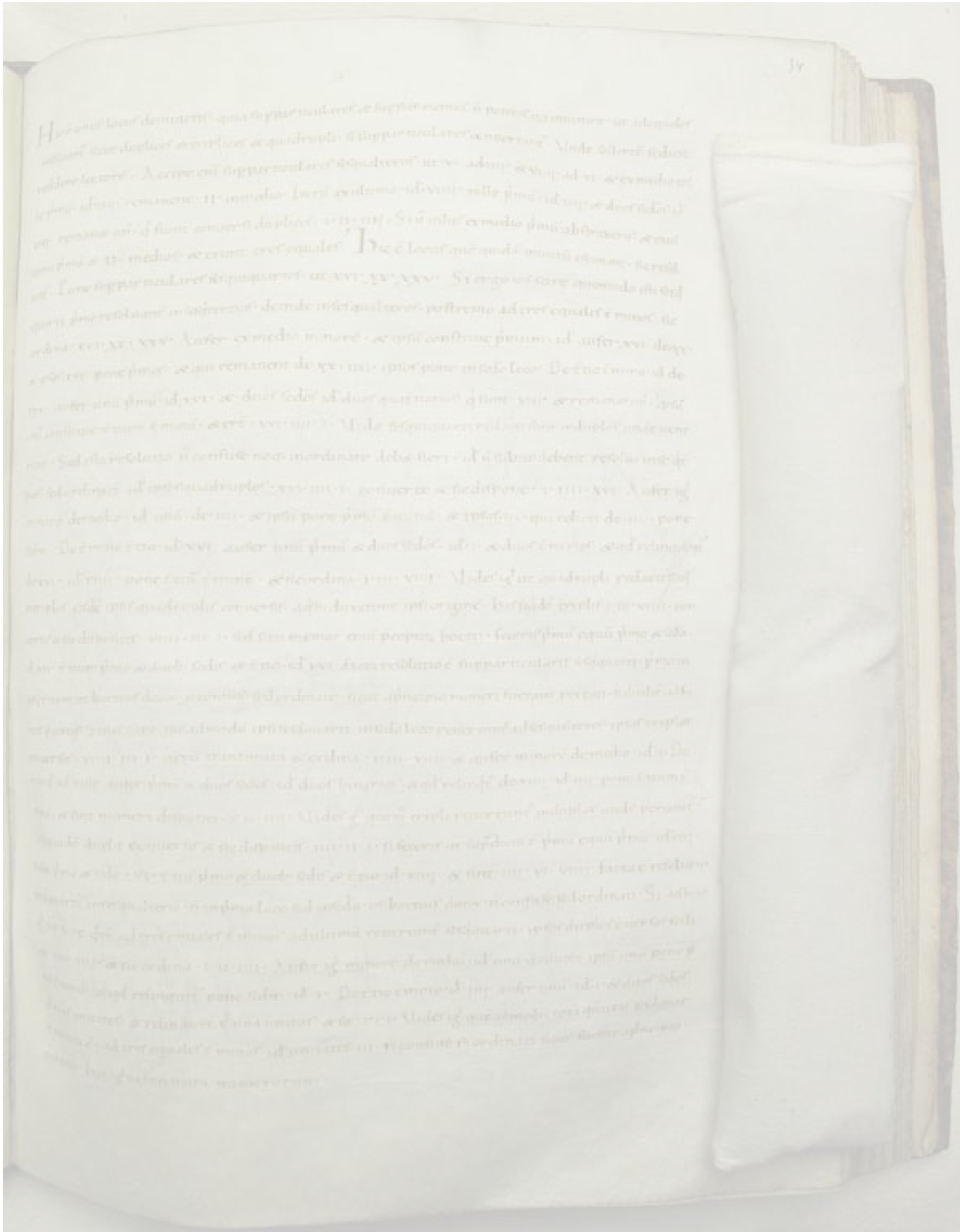
1.21 Bamberg, Staasbibliothek MS Msc.Class.6, fol. 23r.



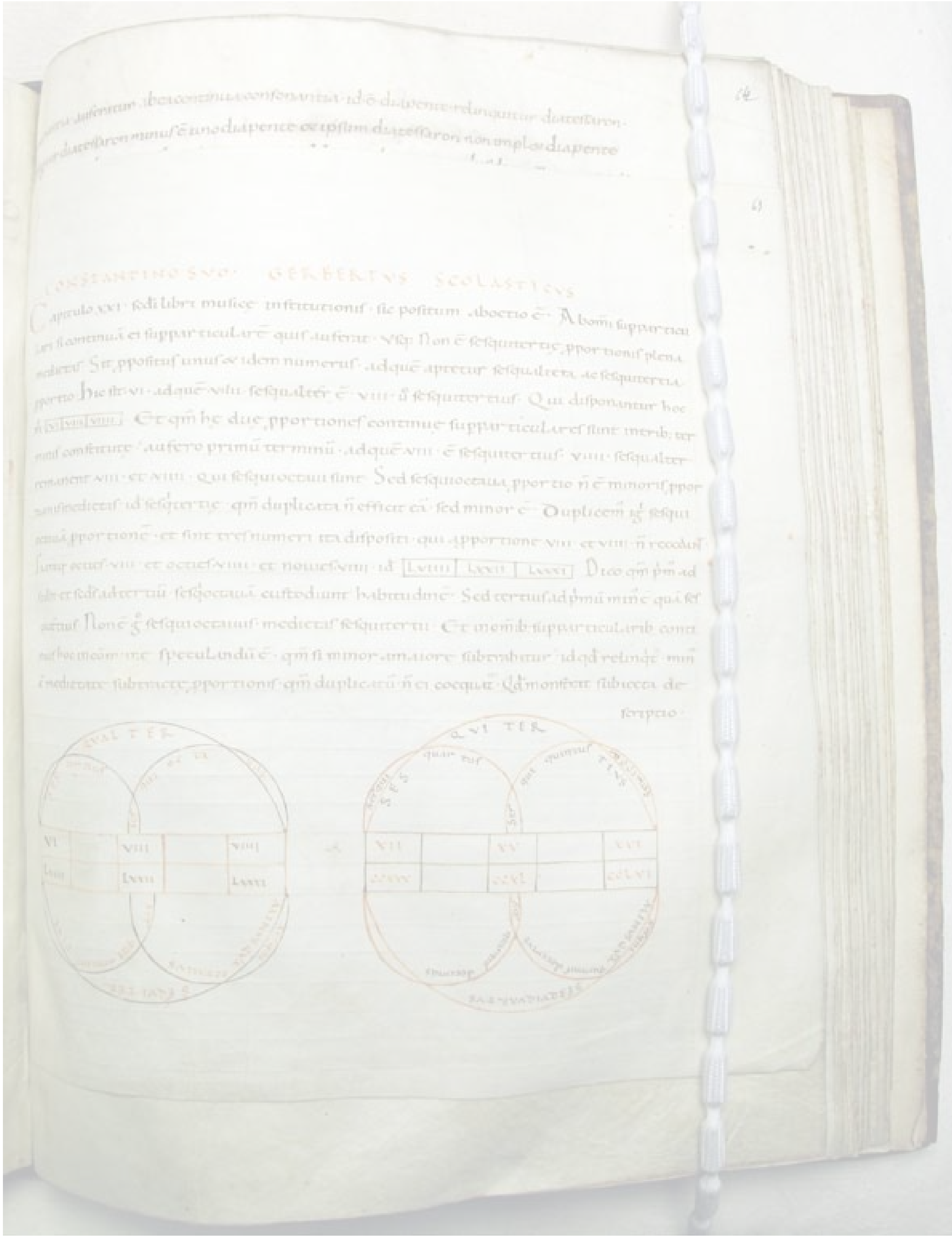
1.22 Bamberg, Staasbibliothek MS Msc.Class.7, fol. 11r.



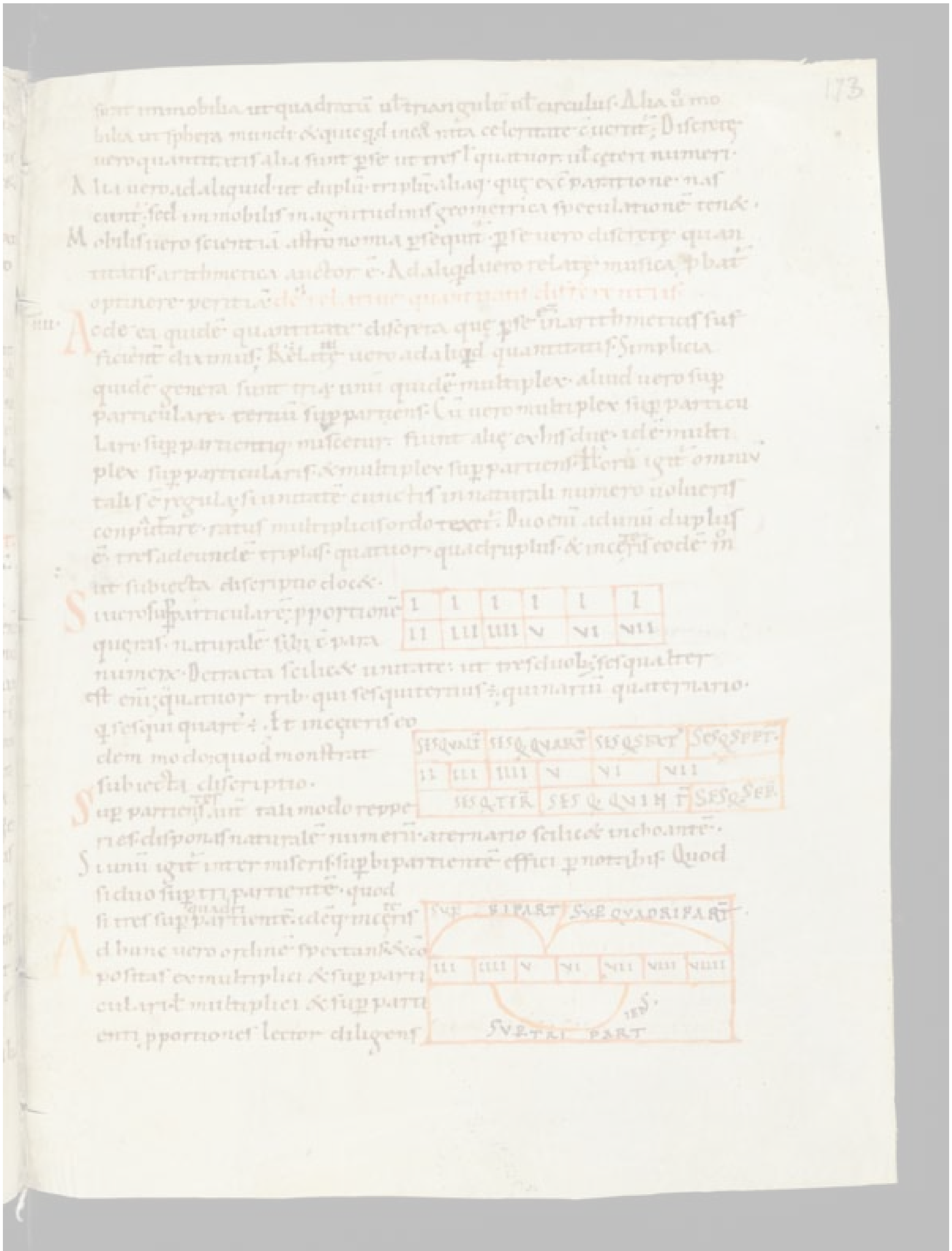
1.23 Reims, Bibliothèque municipale MS 975, fols. 12v–13r. © Bibliothèque municipale Reims



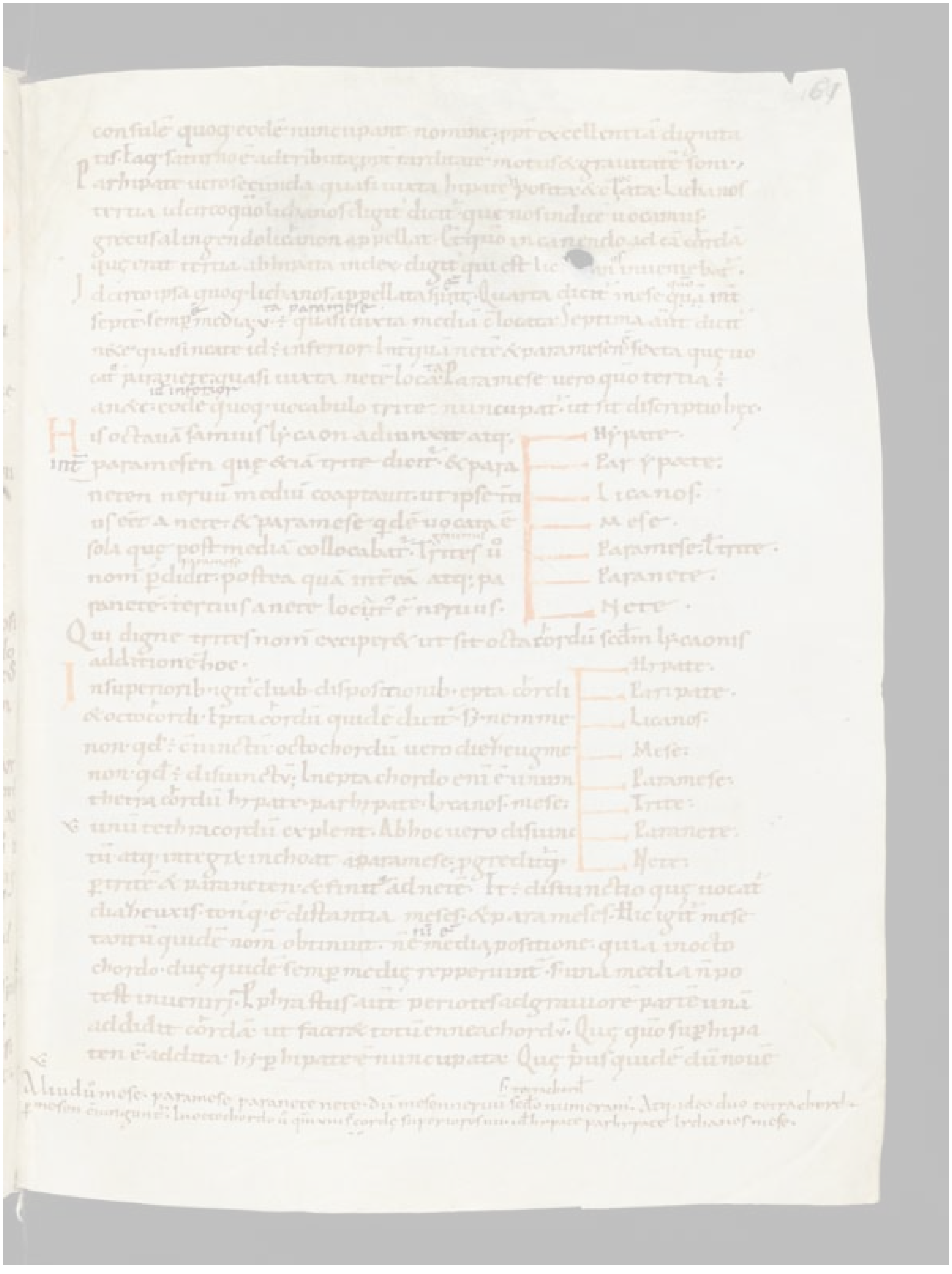
1.24 Brussels, Bibliothèque Royale de Belgique MS 5444-46, fol. 38r.
Photo taken with permission of the Bibliothèque Royale de Belgique



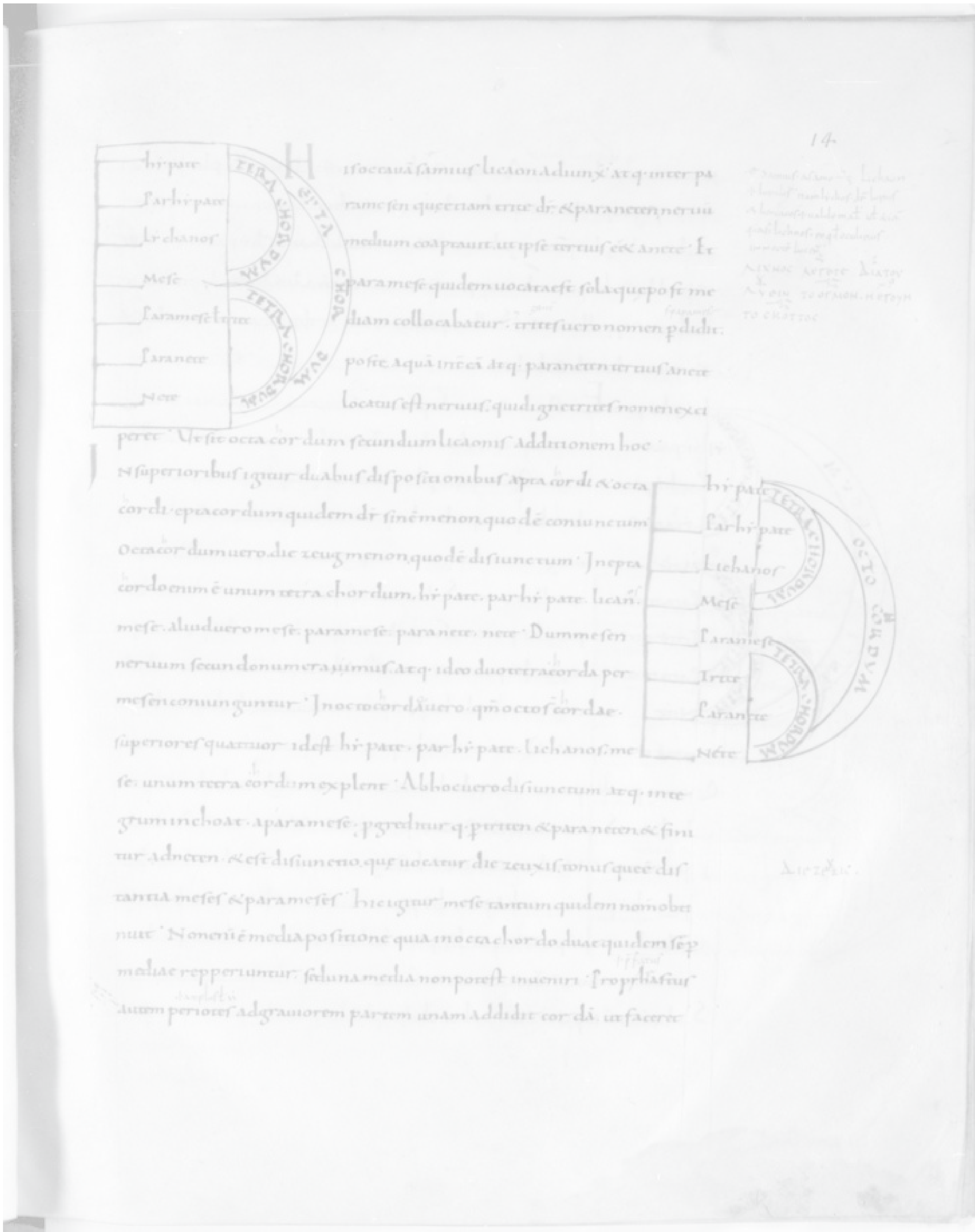
1.25 Brussels, Bibliothèque Royale de Belgique MS 5444-46, fol. 63r.
Photo taken with permission of the Bibliothèque Royale de Belgique



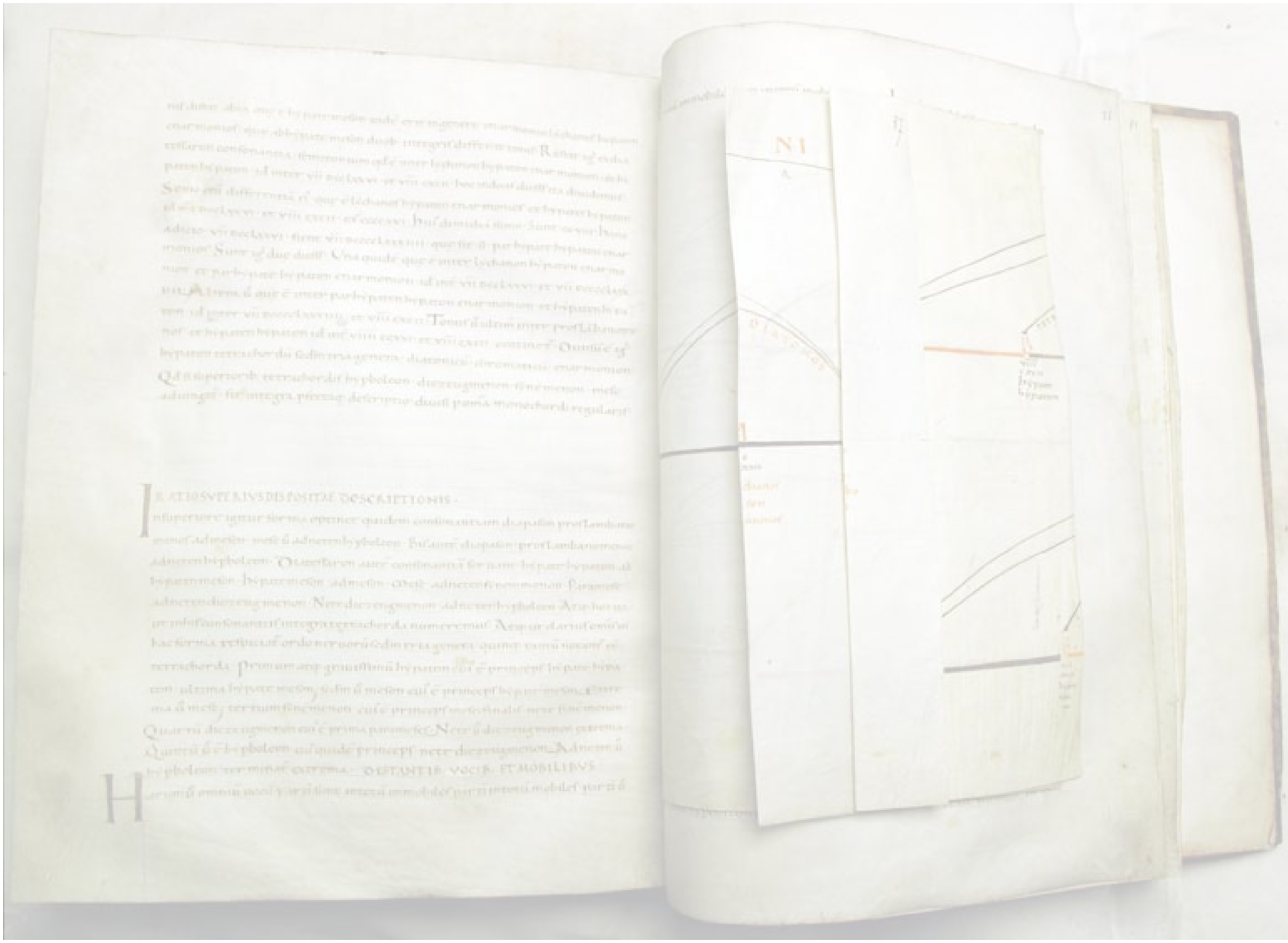
1.26 Einsiedeln, Stiftsbibliothek Codex 358(610), p. 173.



1.27 Einsiedeln, Stiftsbibliothek Codex 358(610), p. 161.

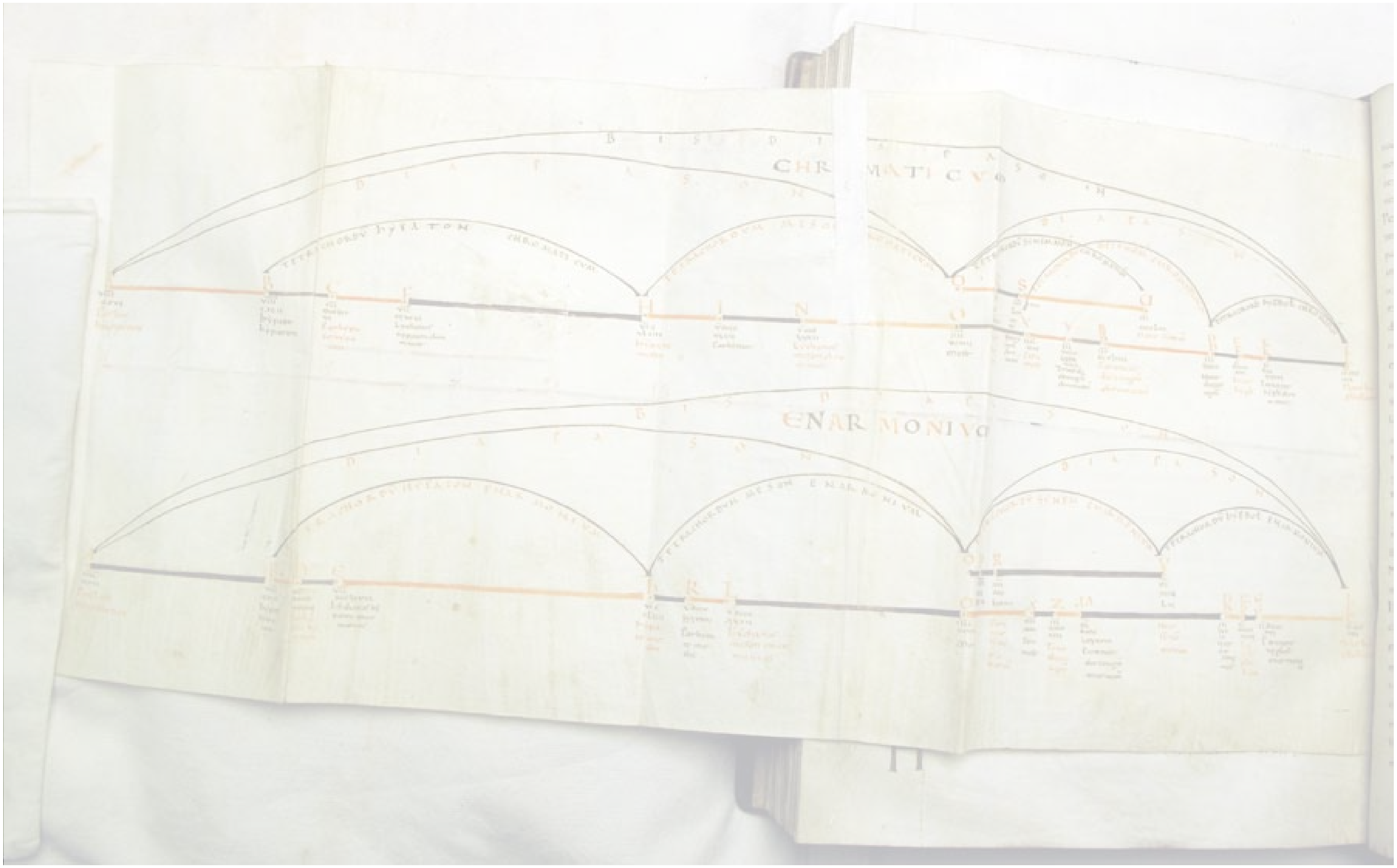


1.28 Paris, Bibliothèque nationale de France lat. MS7200, fol. 14r.

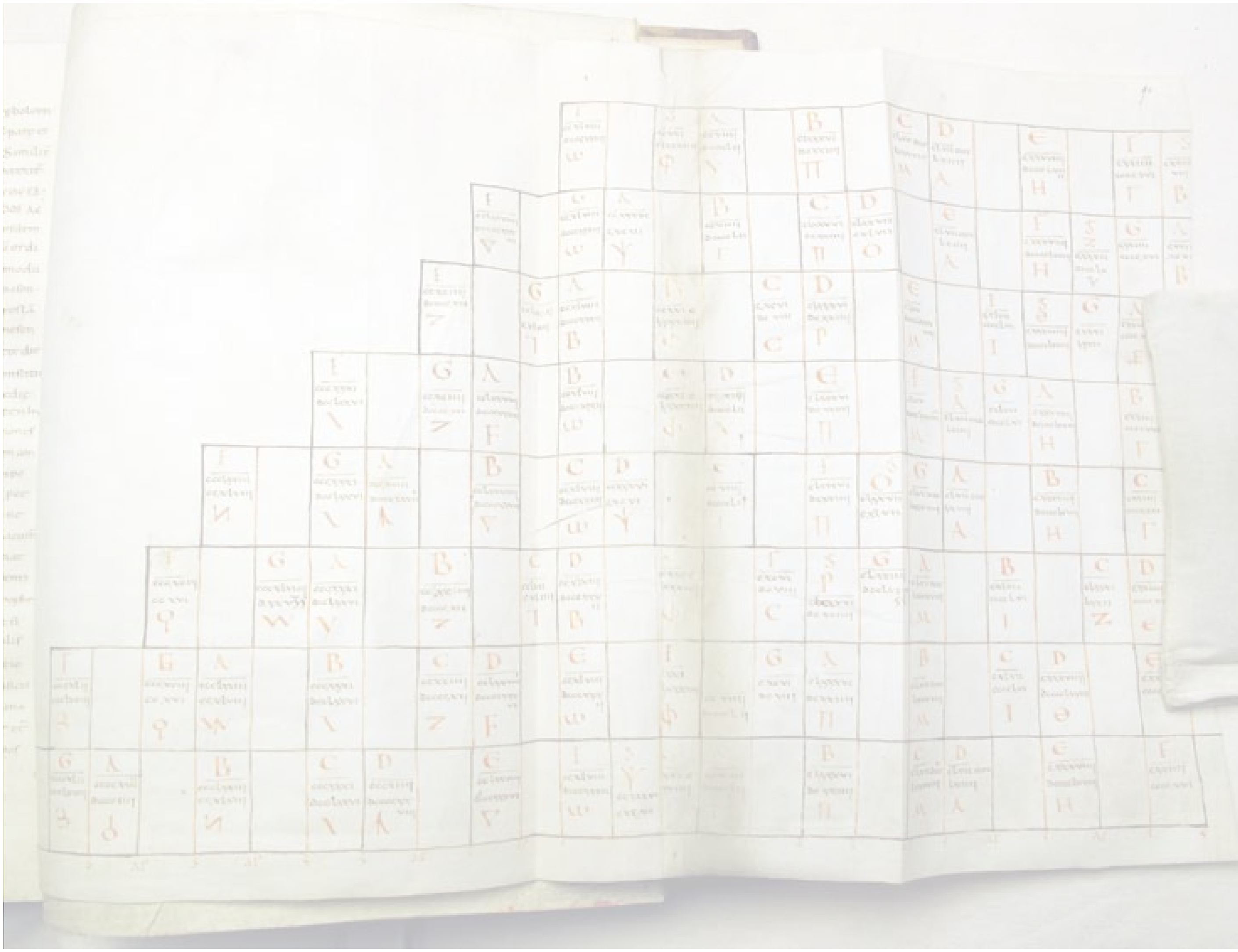


1.29 Brussels, Bibliothèque royale de Belgique MS 5444-46, fol. 87r, folded. Photo taken with permission of the Bibliothèque royale de Belgique

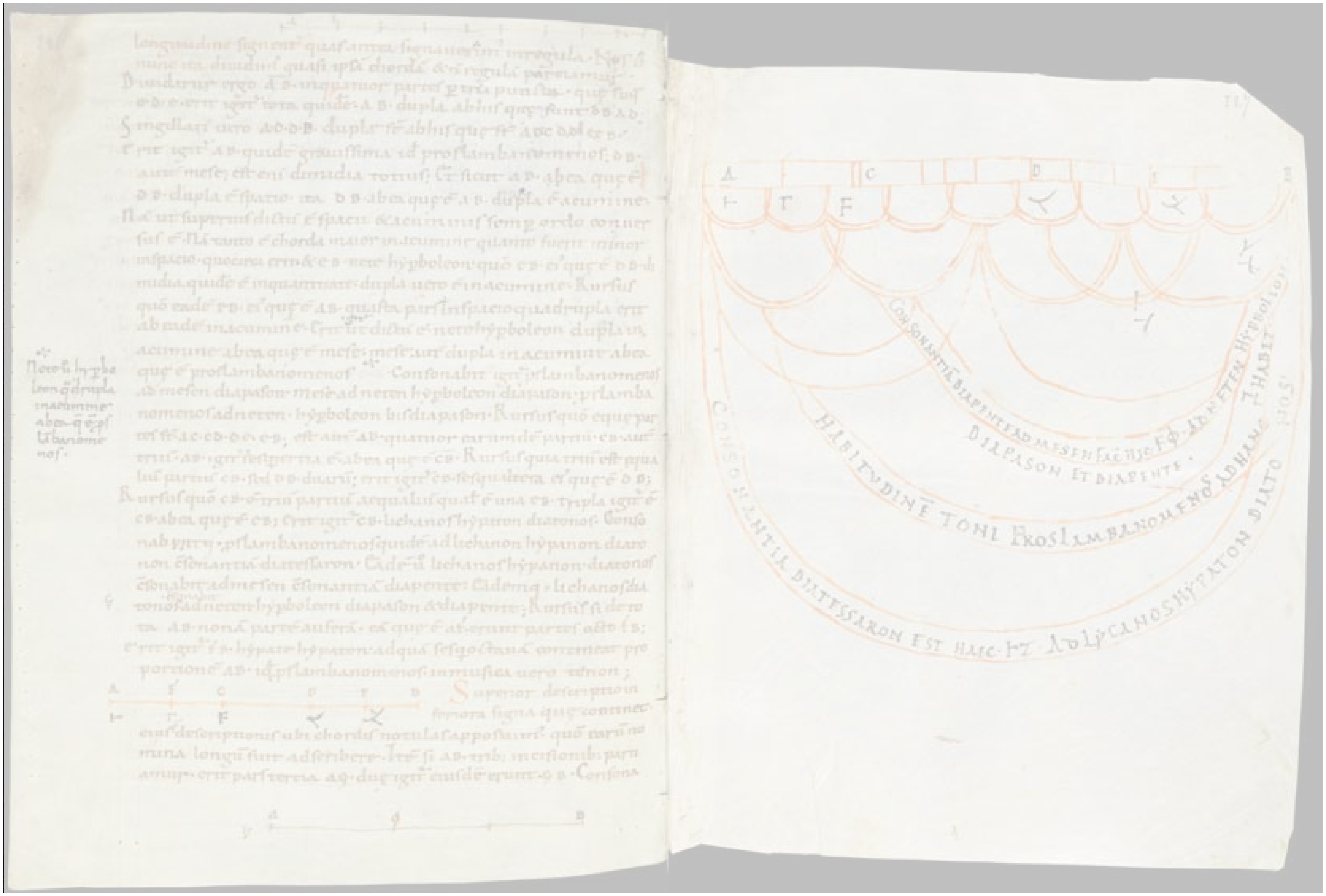
1.30 Brussels, Bibliothèque royale de Belgique MS 5444-46, fol. 87r, unfolded. Photo taken with permission of the Bibliothèque royale de Belgique



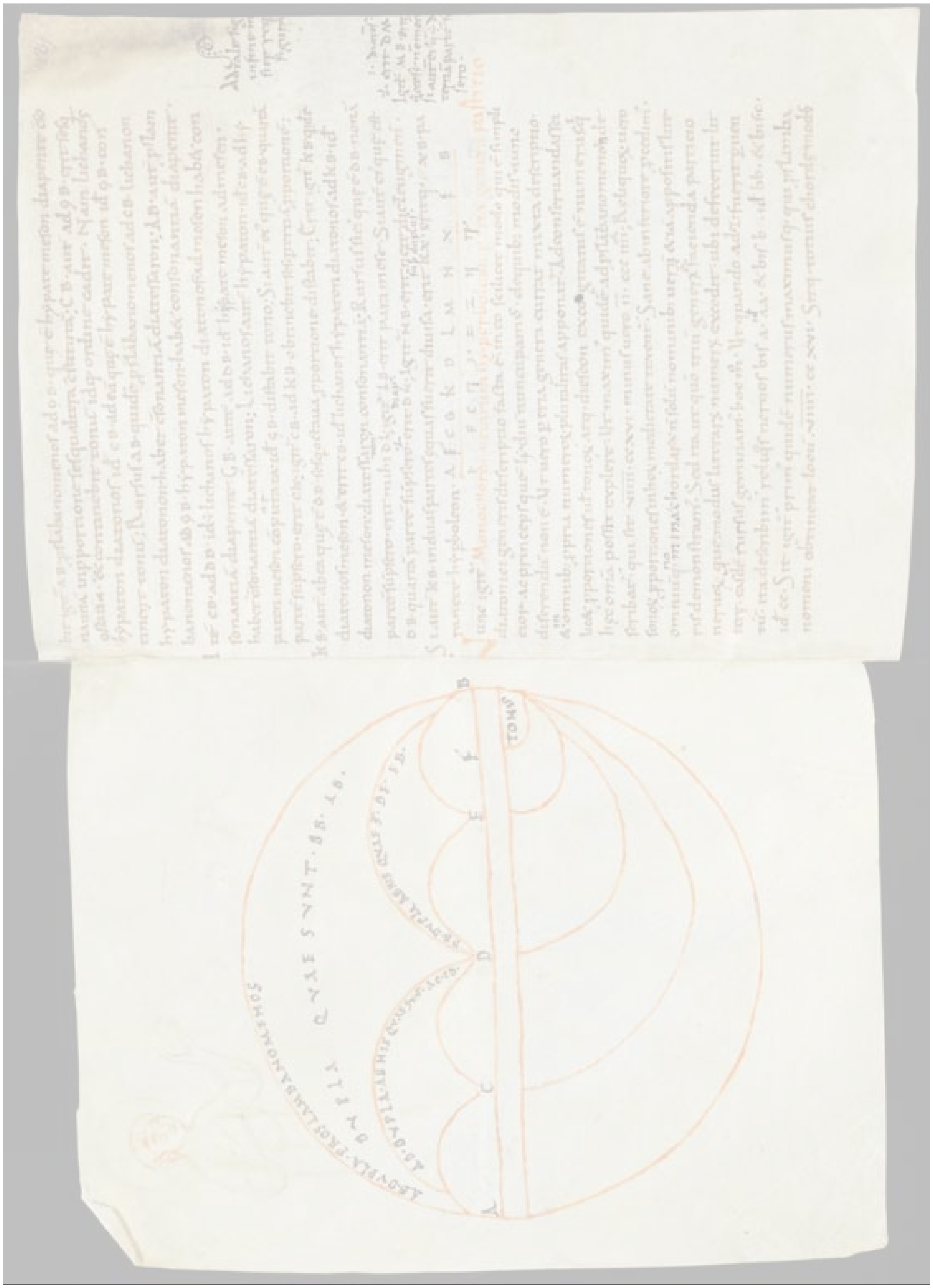
1.31 Brussels, Bibliothèque royale de Belgique MS 5444-46, fol. 87v, unfolded. Photo taken with permission of the Bibliothèque royale de Belgique



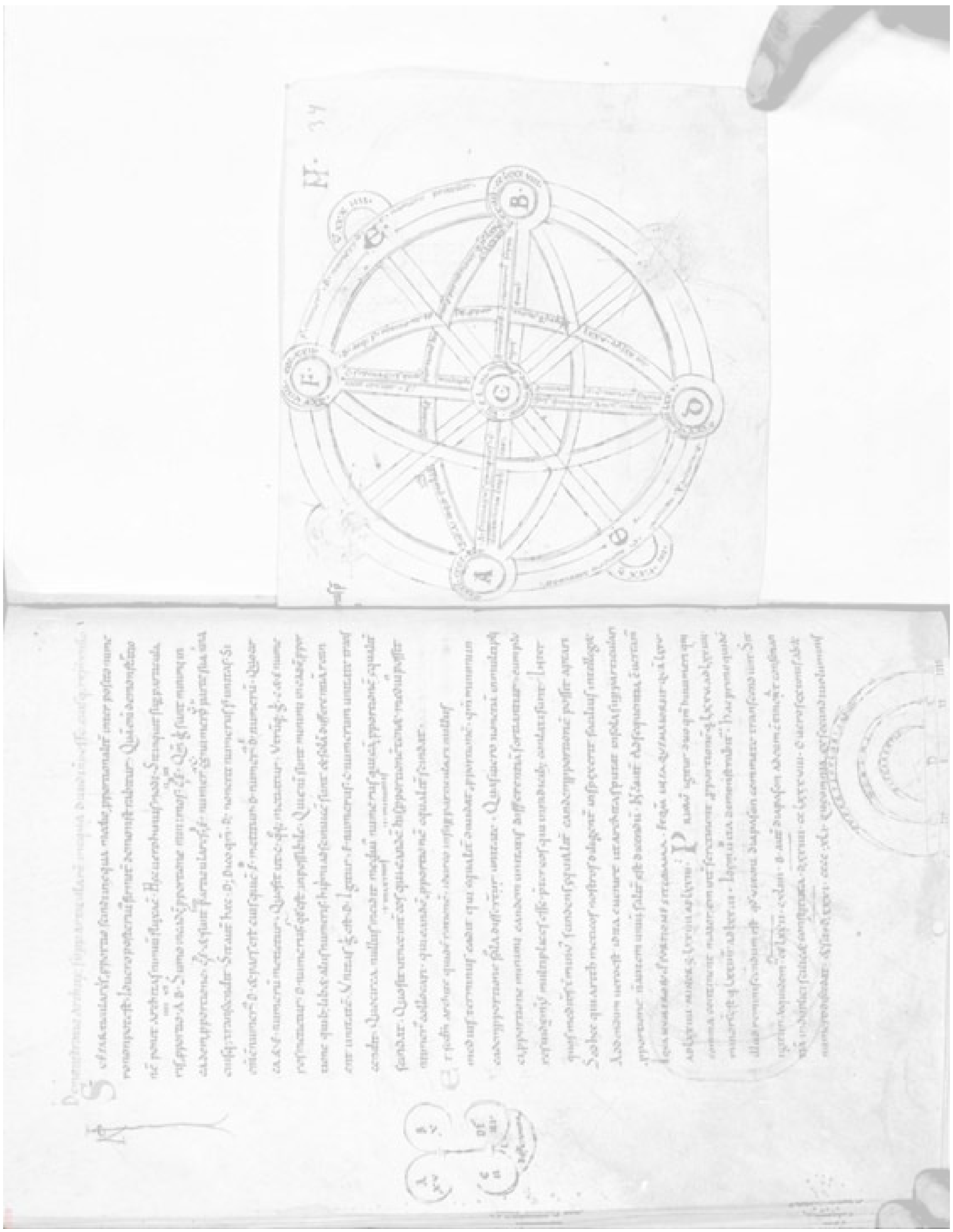
1.32 Brussels, Bibliothèque royale de Belgique MS 5444-46, fol. 90r, unfolded (damaged). Photo taken with permission of the Bibliothèque royale de Belgique



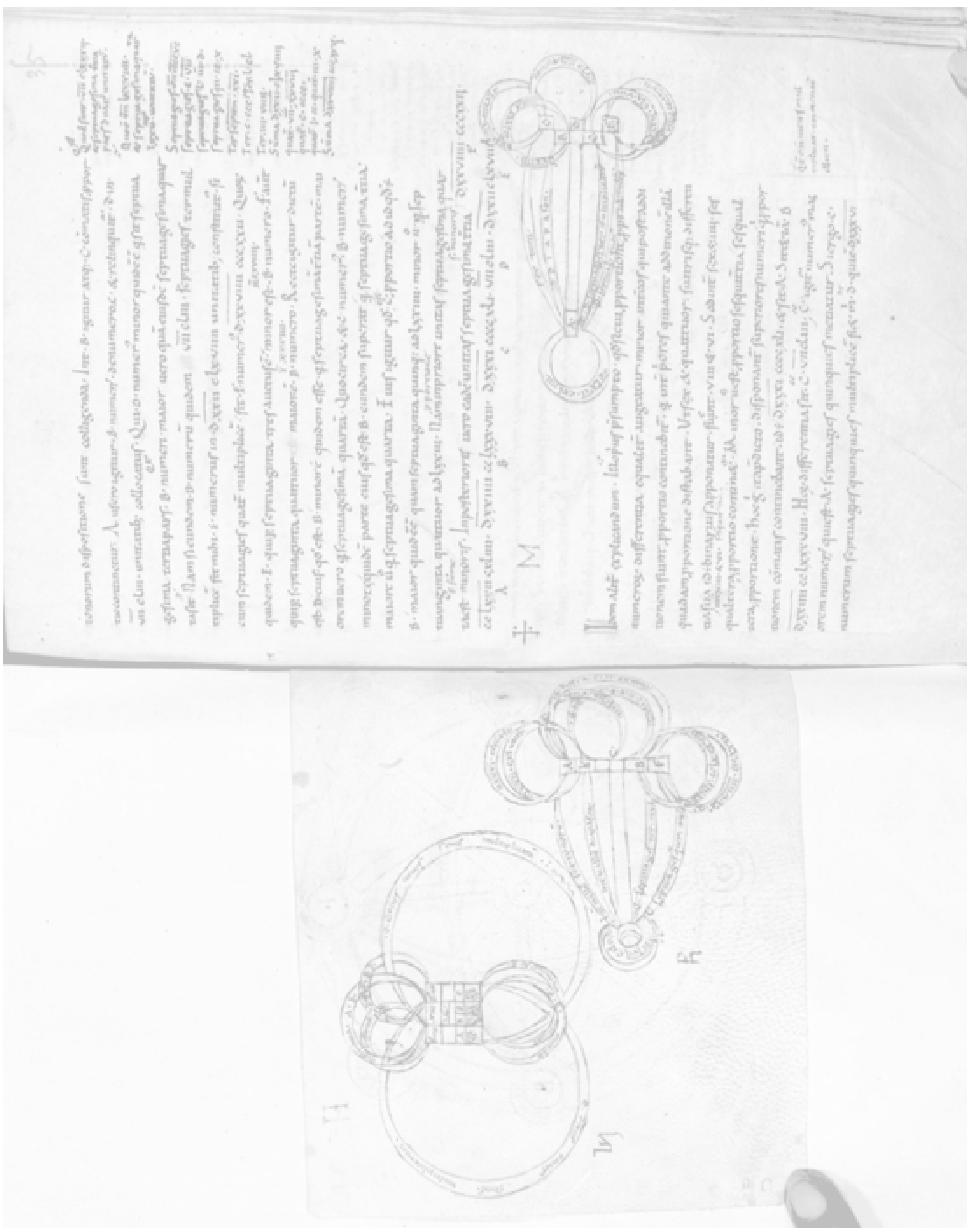
1.33 Einsiedeln, Stiftsbibliothek Codex 358(610), p. 226–27 (additional figure). © Stiftsbibliothek Einsiedeln



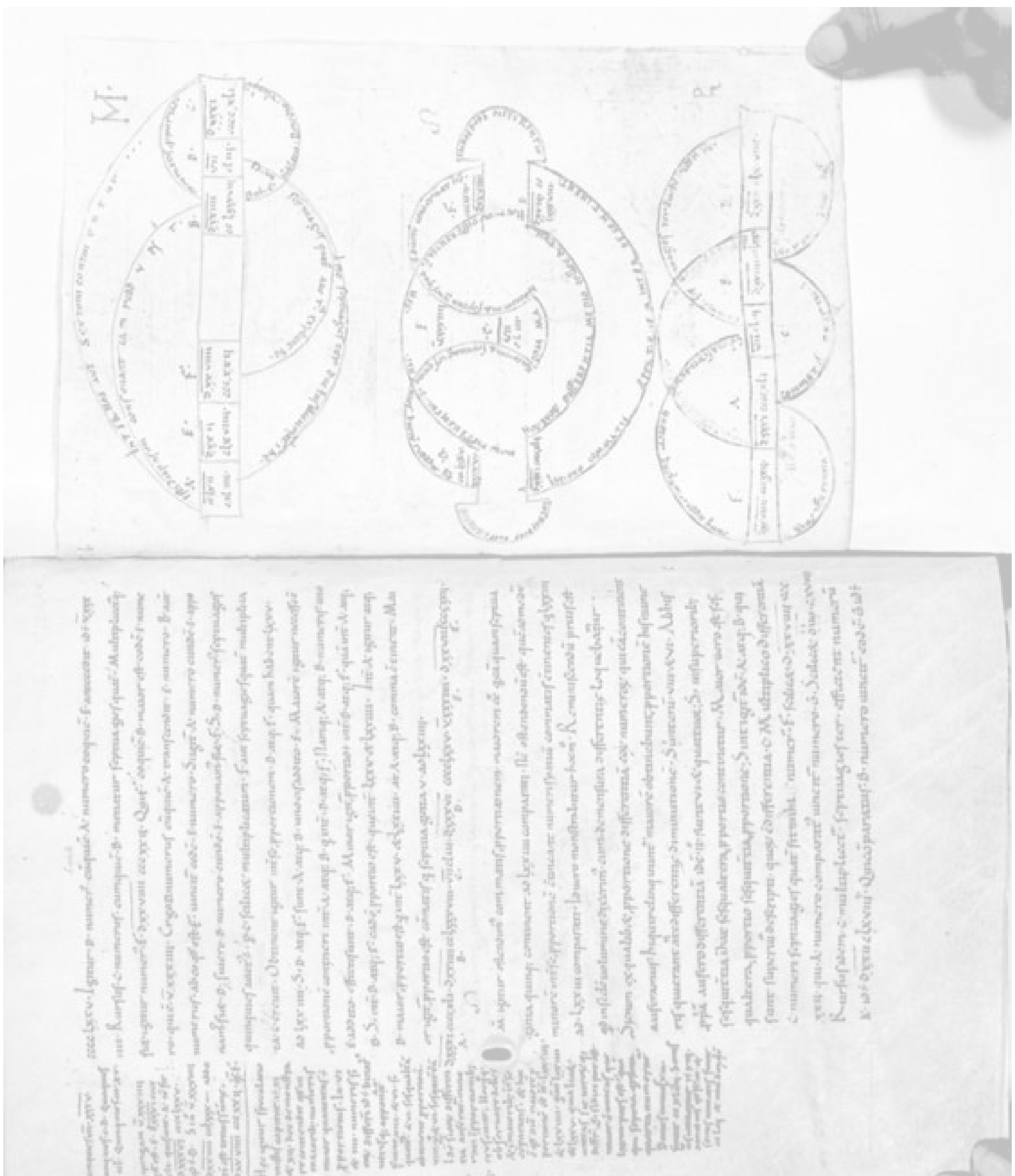
1.34 Einsiedeln, Stiftsbibliothek Codex 358(610), p. 228–29 (additional figure). © Stiftsbibliothek Einsiedeln



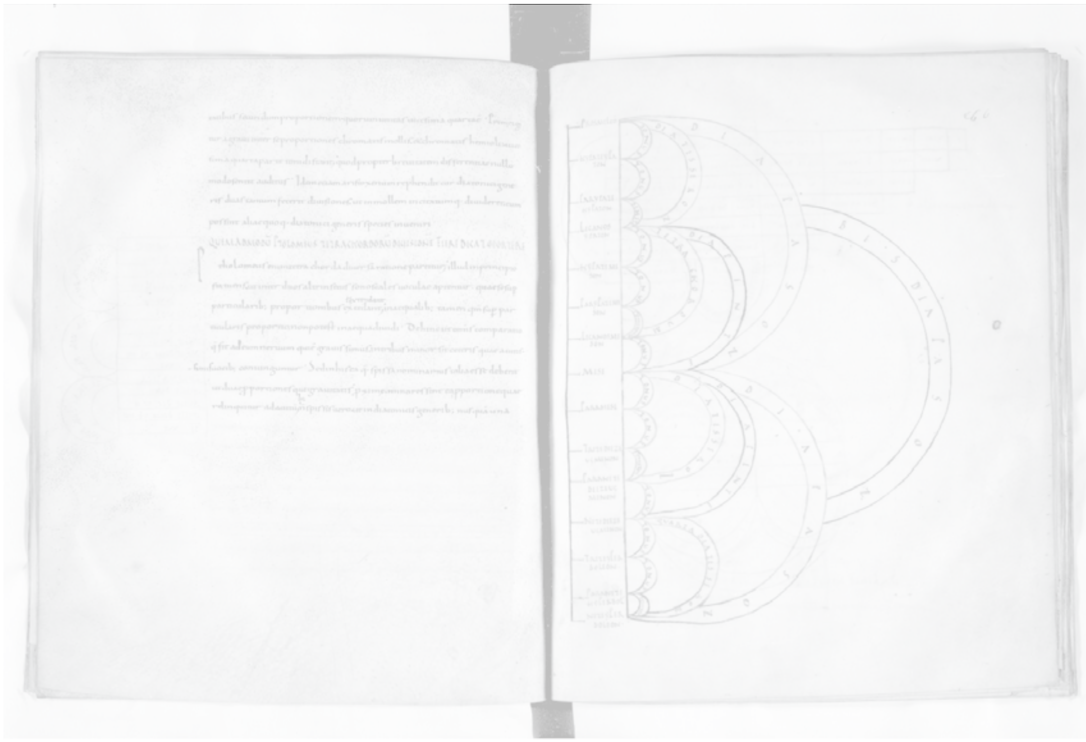
1.35 Munich, Bayerische Staatsbibliothek Clm. 14272, fols. 33v–34. © Bayerische Staatsbibliothek

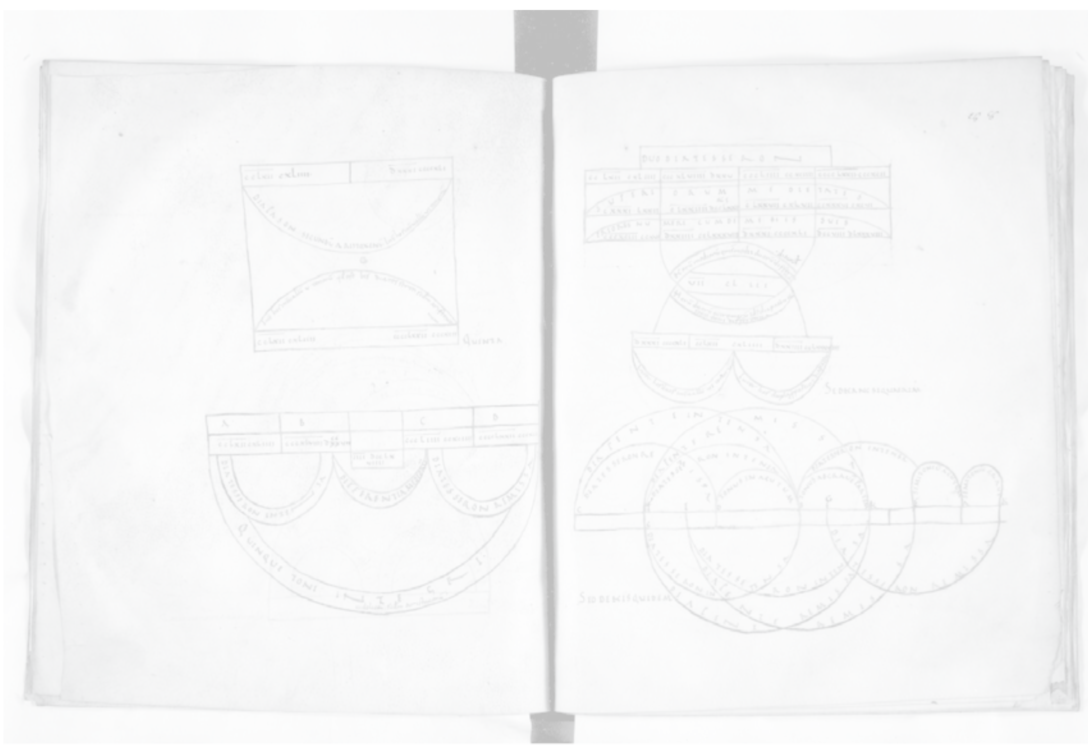


1.36 Munich, Bayerische Staatsbibliothek Clm. 14272, fols. 34v–35r. © Bayerische Staatsbibliothek

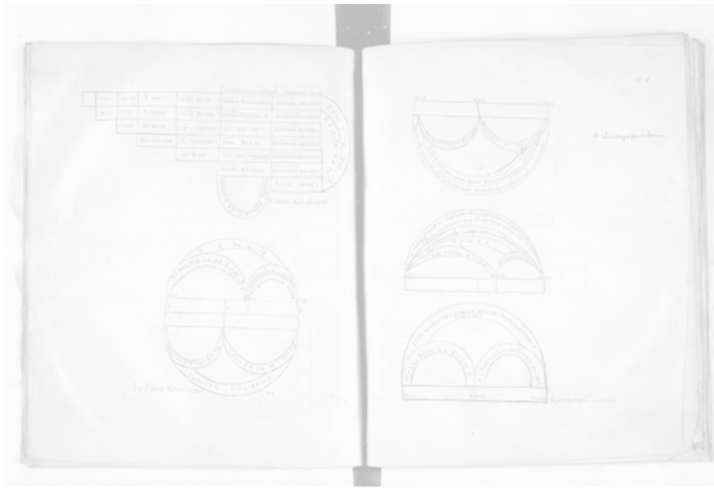


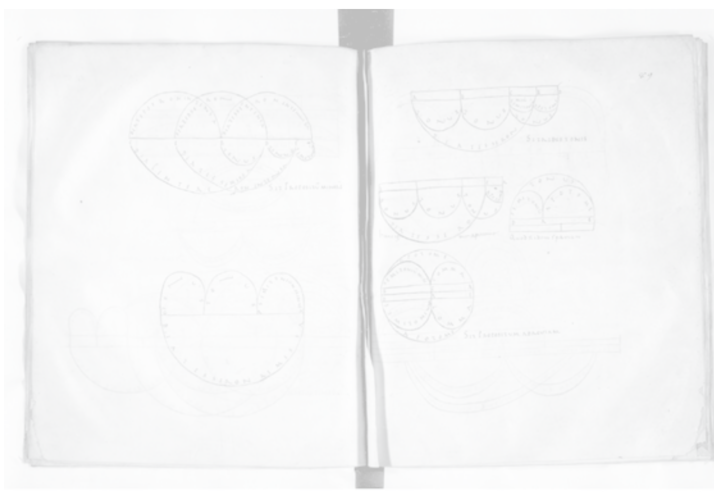
1.37 Munich, Bayerische Staatsbibliothek Clm. 14272, fols. 35v–36r. © Bayerische Staatsbibliothek

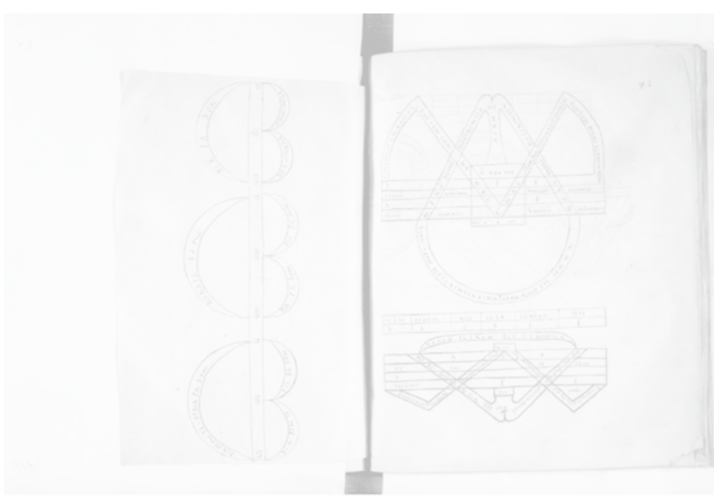


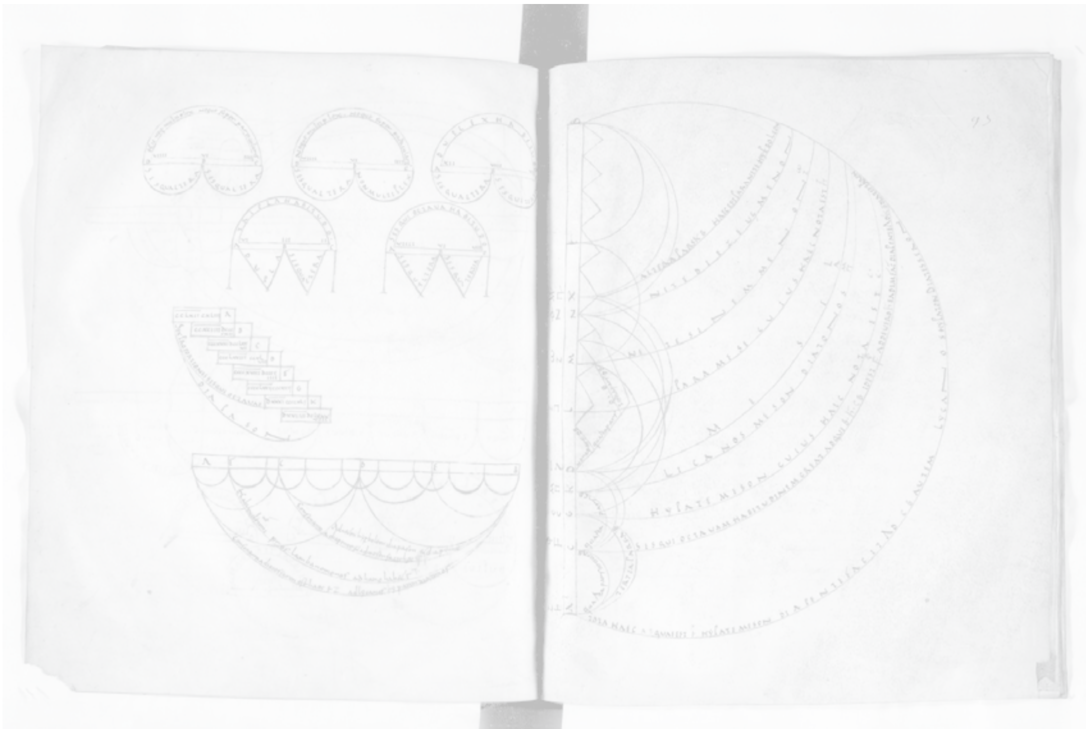


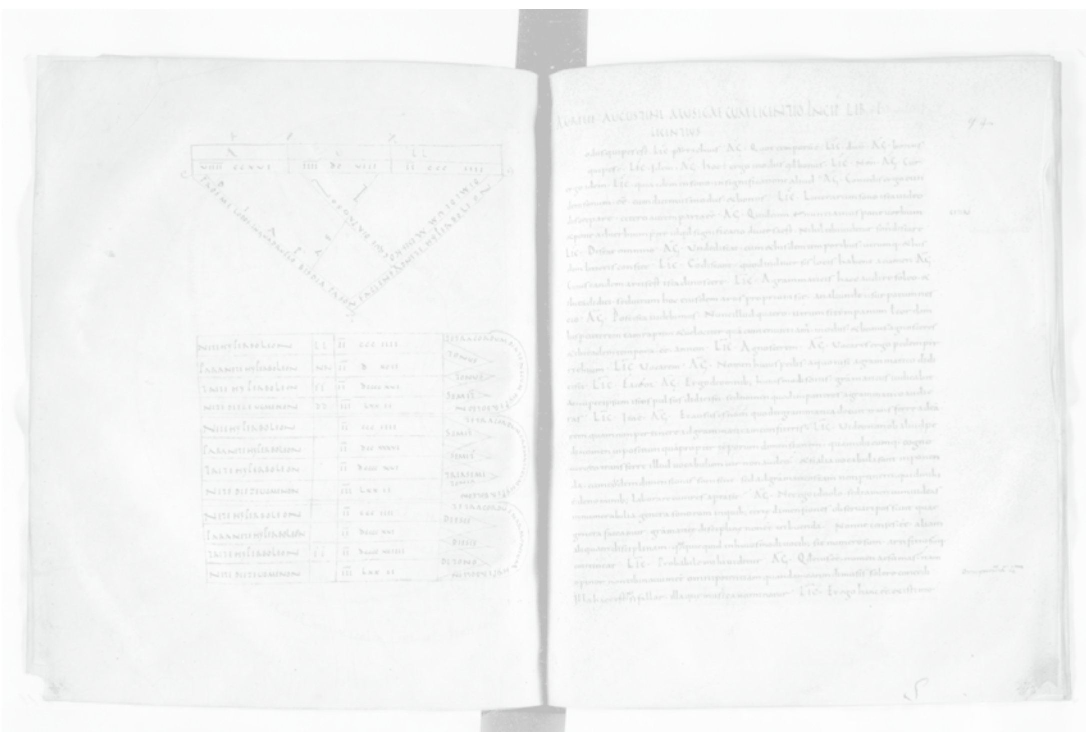
1.38 Paris, Bibliothèque nationale de France lat. MS 7200, fols. 84v–94r (continued on the following pages). © Bibliothèque nationale de France



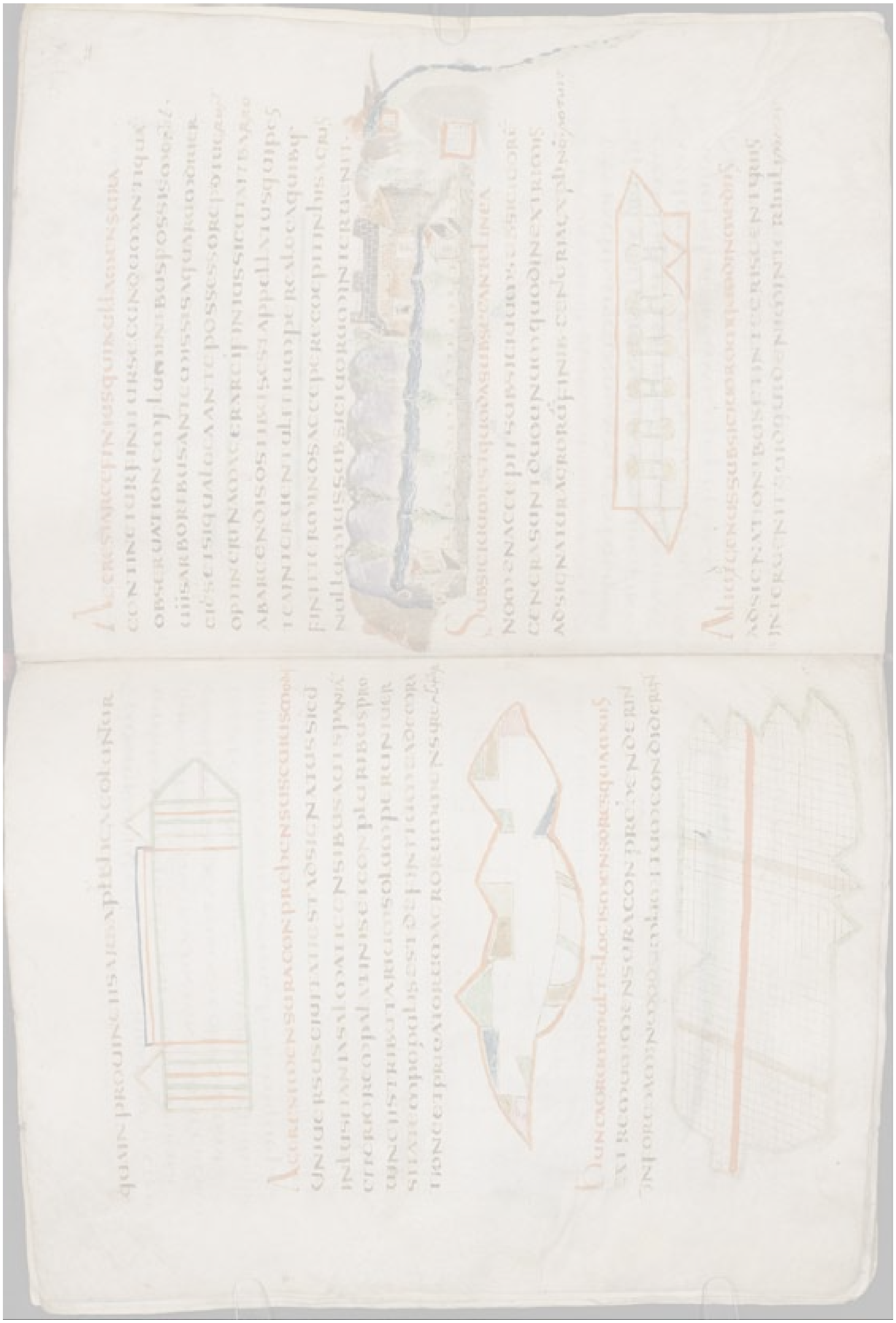




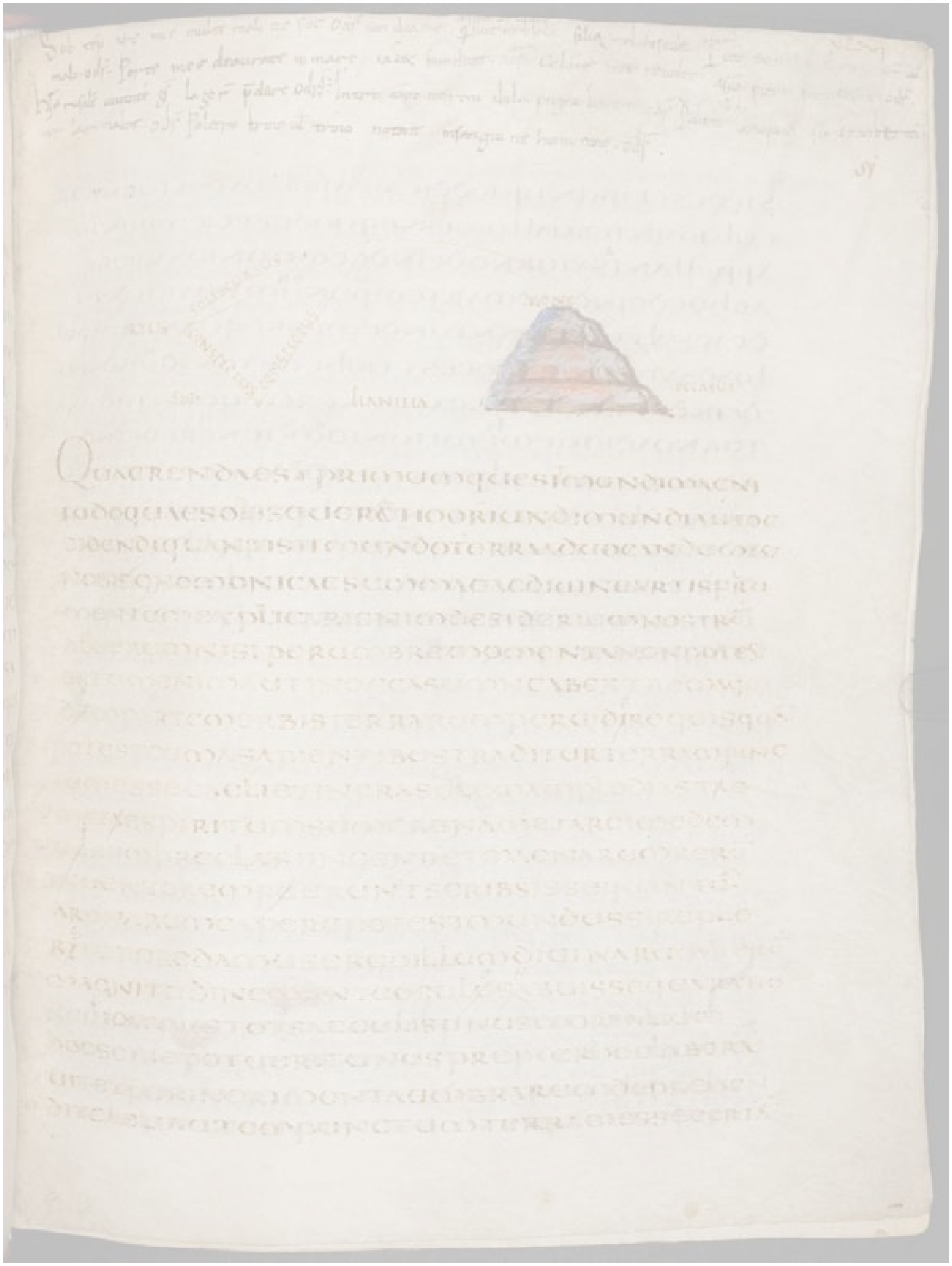




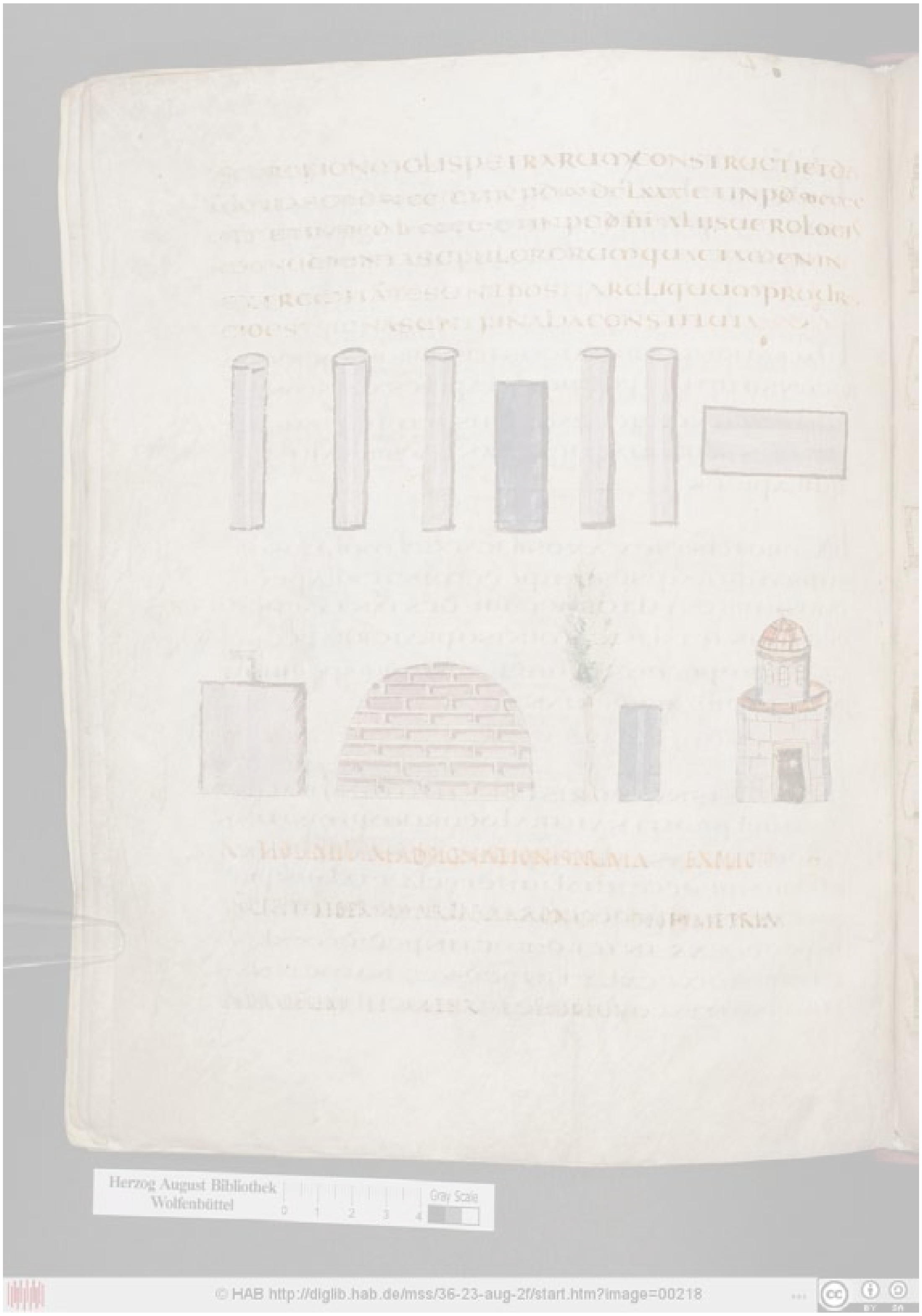
1.39 Paris, Bibliothèque nationale de France lat. MS 8663, fols. 51v–57v (continued on next page). © Bibliothèque nationale de France



1.40 Wolfenbüttel, Herzog August Bibliothek Cod. Guelf. 36.23 Aug. 2o, fols, 16v–17r. © Herzog August Bibliothek



1.41 Wolfenbüttel, Herzog August Bibliothek Cod. Guelf. 36.23 Aug. 2o, fol. 50r.



1.42 Wolfenbüttel, Herzog August Bibliothek Cod. Guelf. 36.23 Aug. 2o, 77v.
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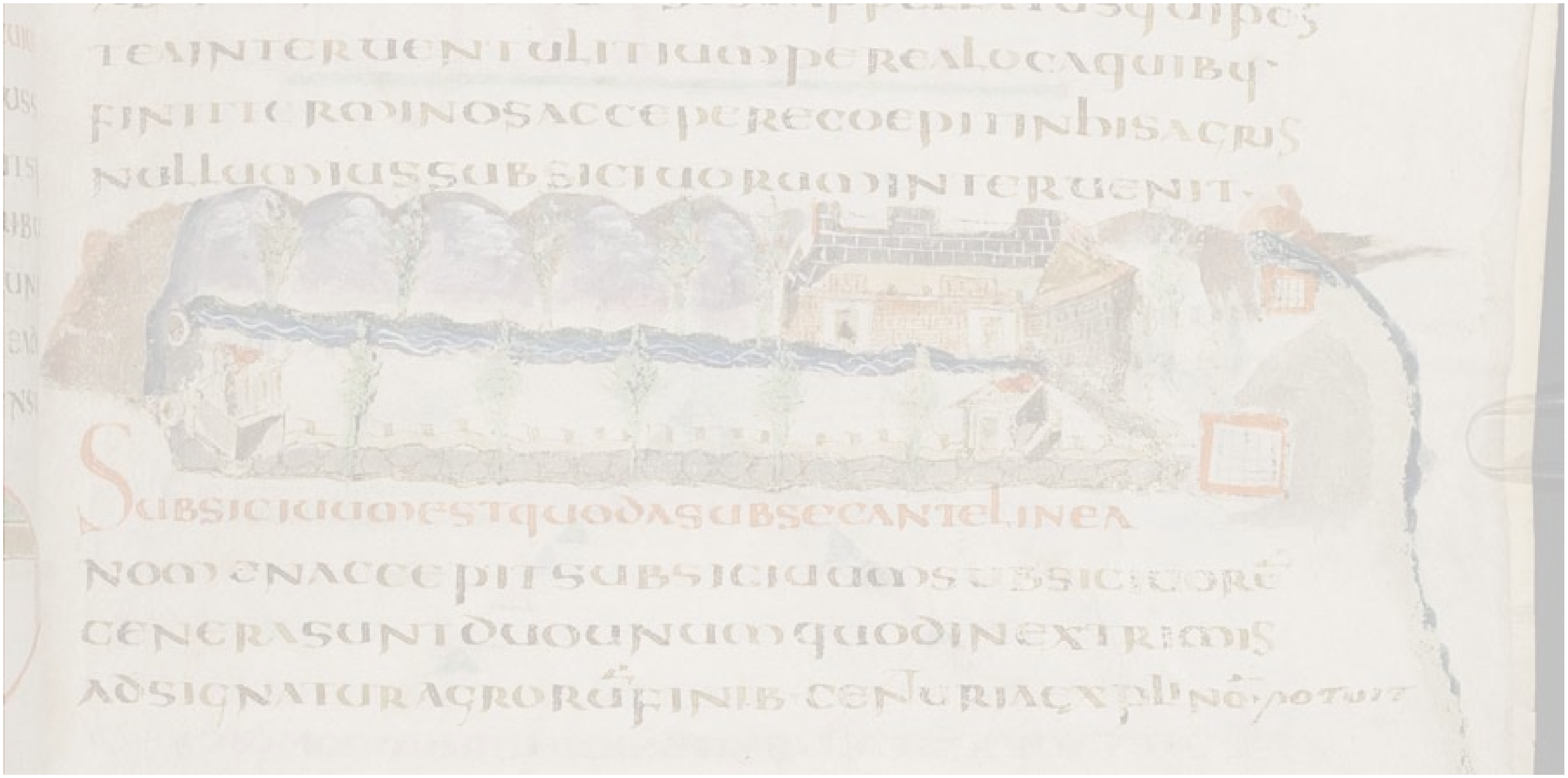
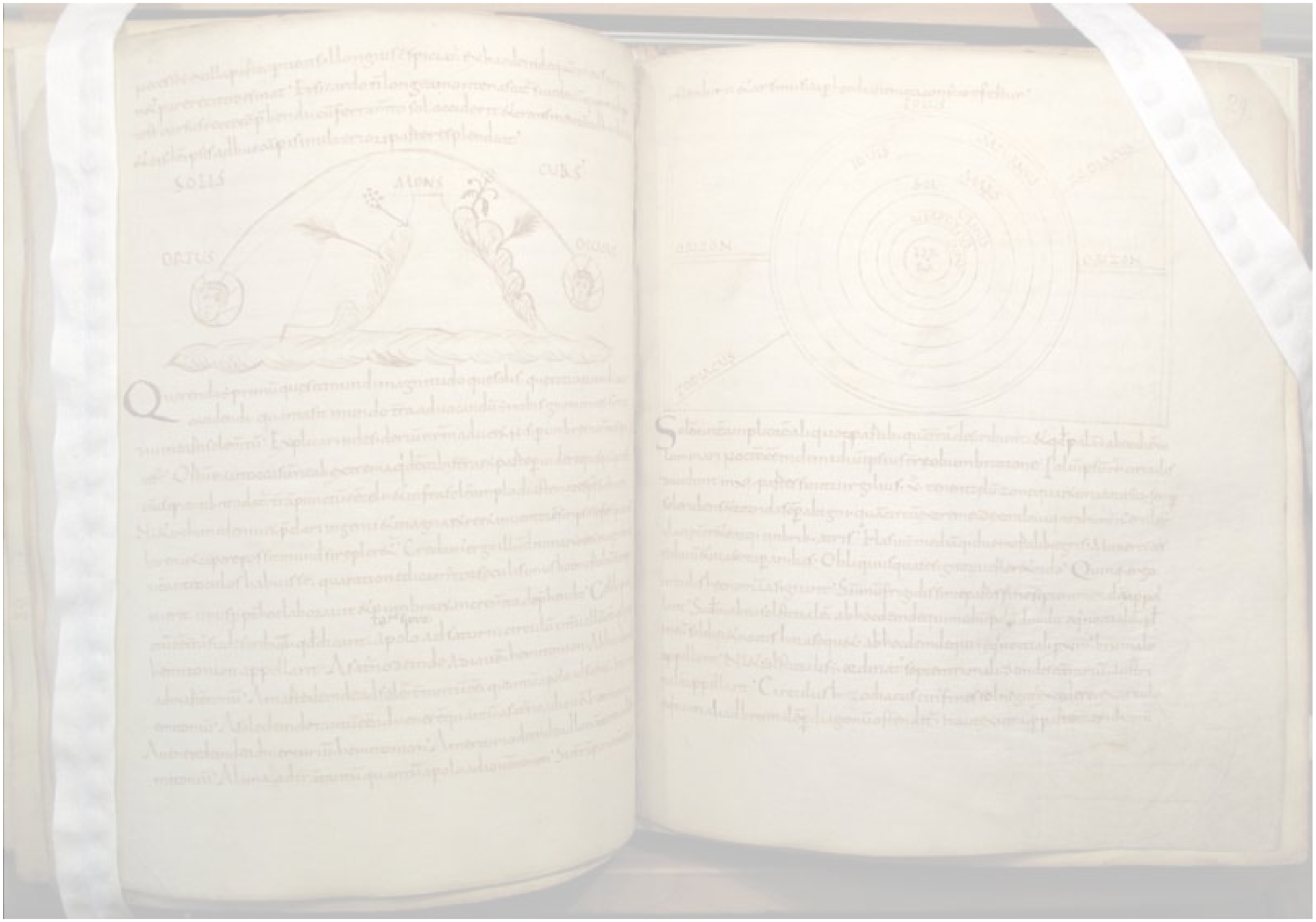
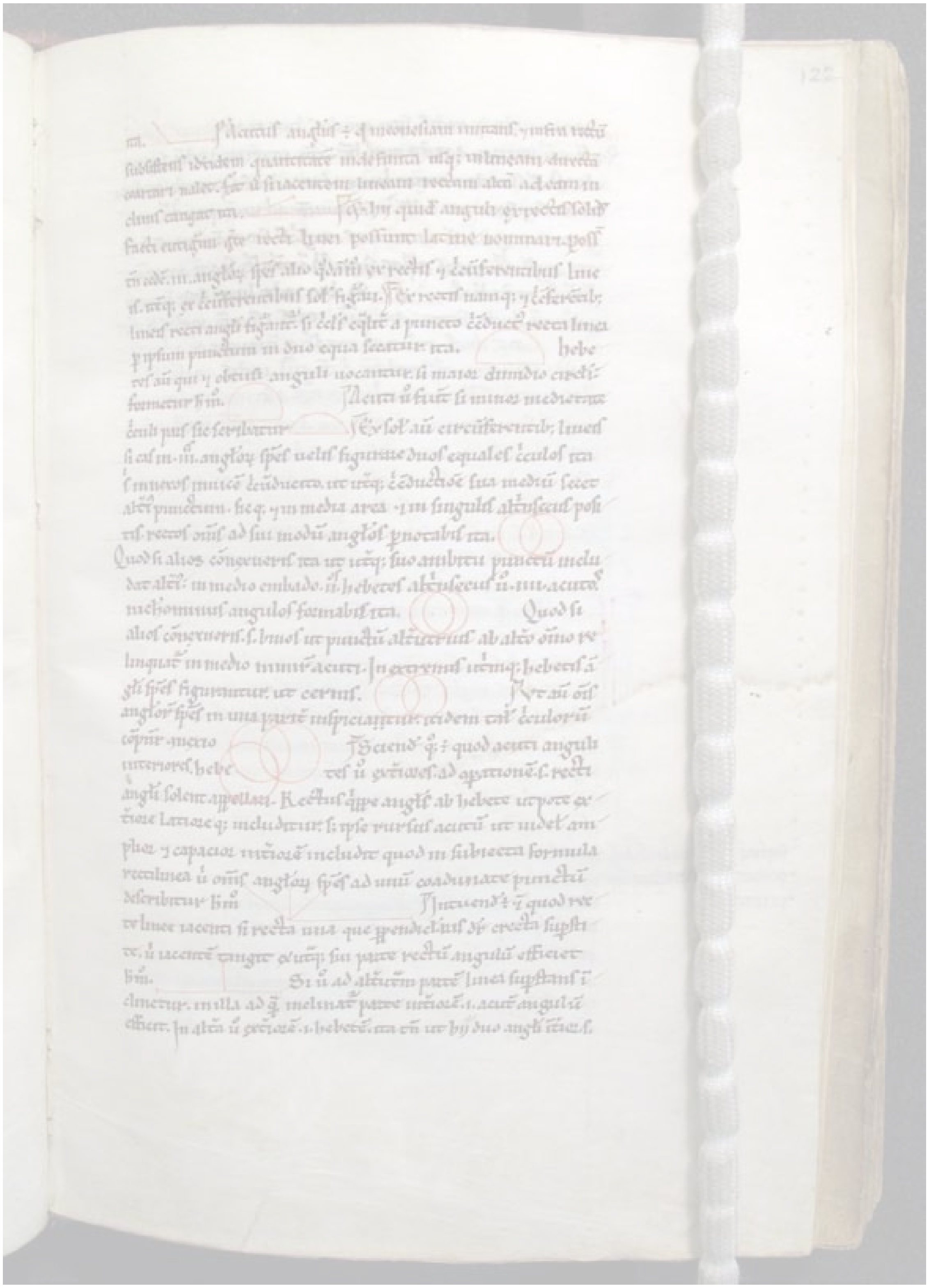


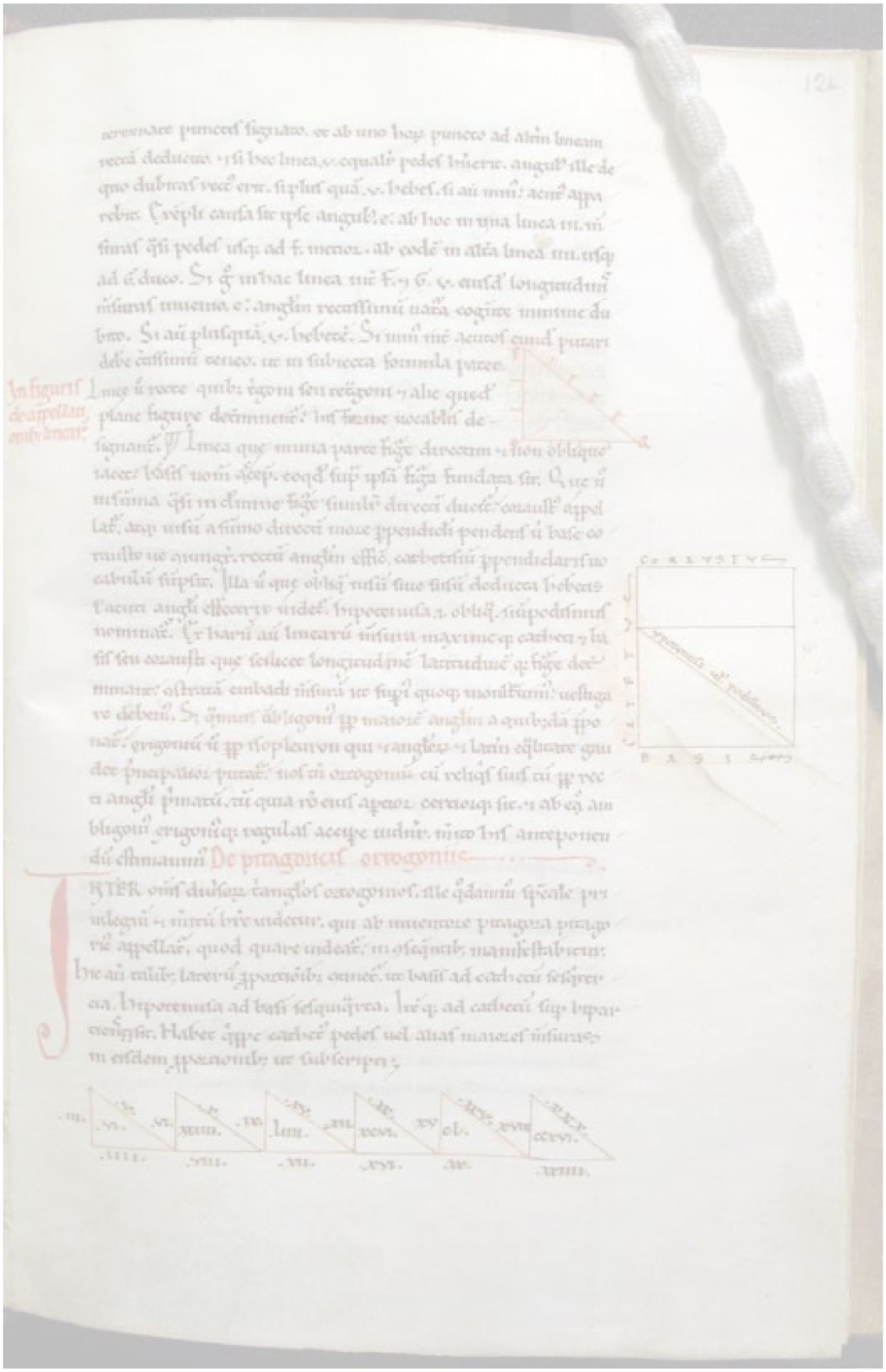
1.43 Wolfenbüttel, Herzog August Bibliothek Cod. Guelf. 36.23 Aug. 2o, fol. 17r, detail. © Herzog August Bibliothek



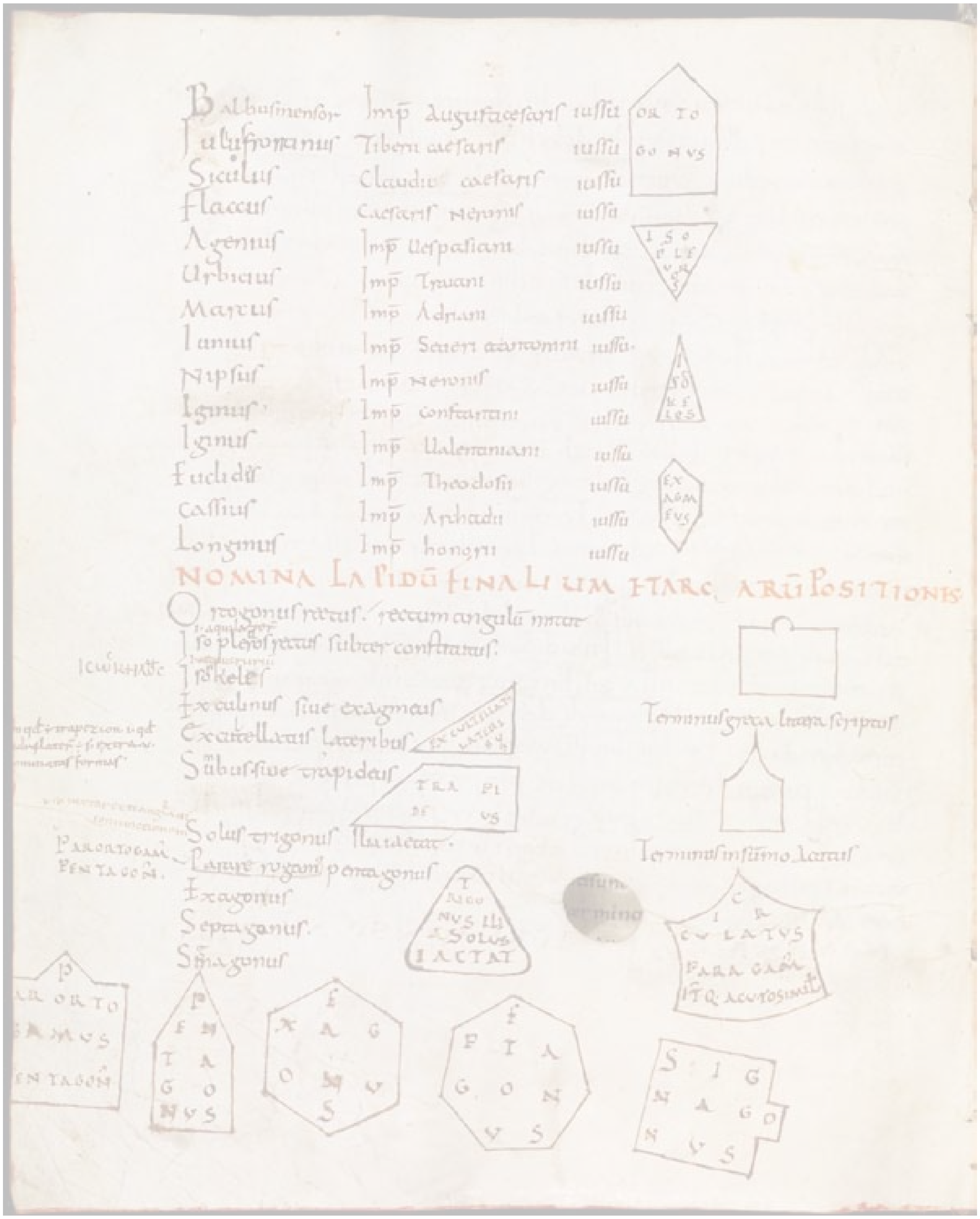
1.44 Naples, Biblioteca Nazionale Vittorio Emanuele III MS V.A.13, fol. 28v–29r. Photo taken with permission of the Biblioteca Nazionale Vittorio Emanuele III



1.45 Oxford, Bodleian Library Selden Supra MS 25, fol. 122r.
Photo taken with permission of the Bodleian Library, Oxford University



1.46 Oxford, Bodleian Library Selden Supra MS 25, fol. 124r.
Photo taken with permission of the Bodleian Library, Oxford University



1.47 Bamberg, Staasbibliothek MS Msc.Class.55, fol. 5v.
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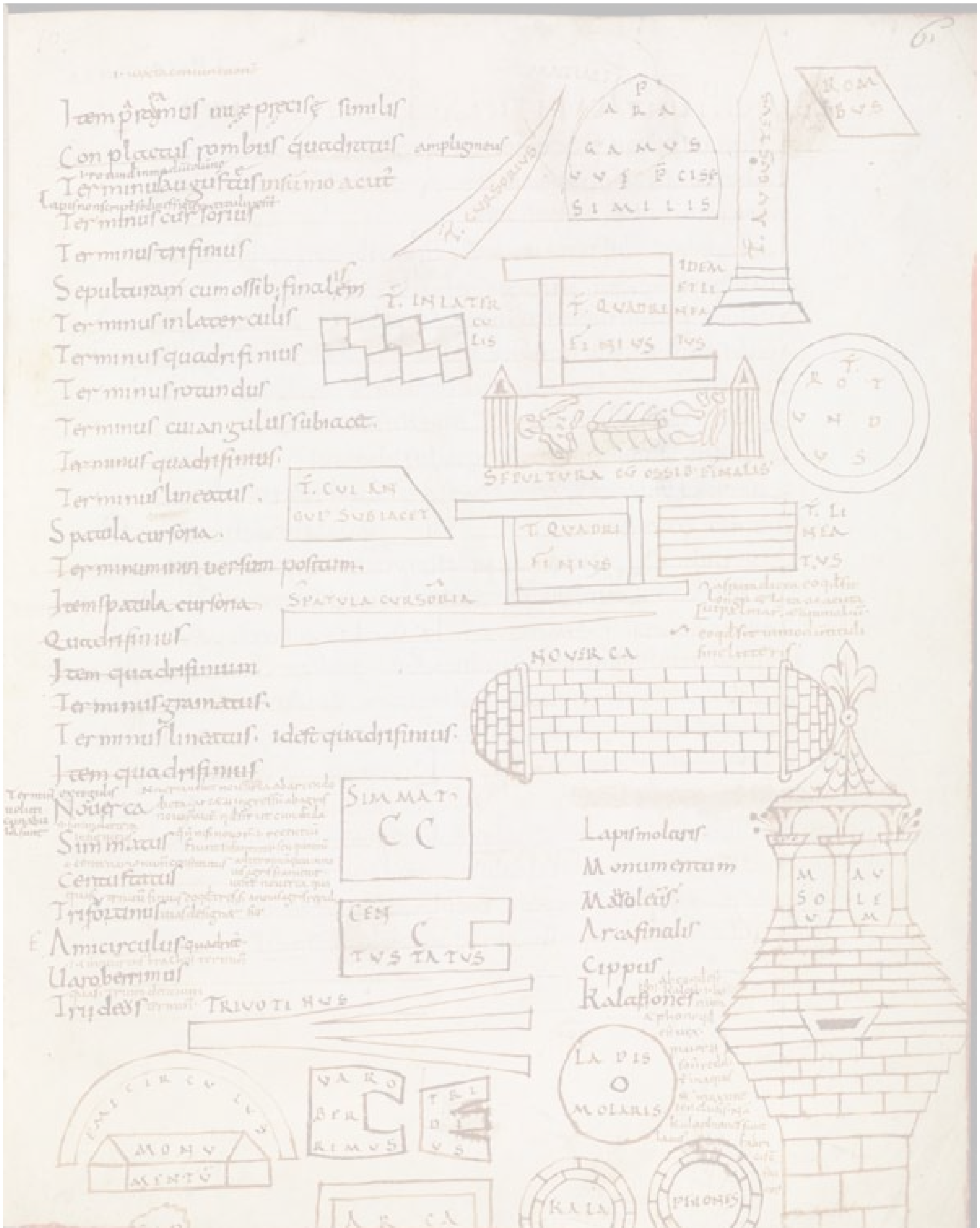
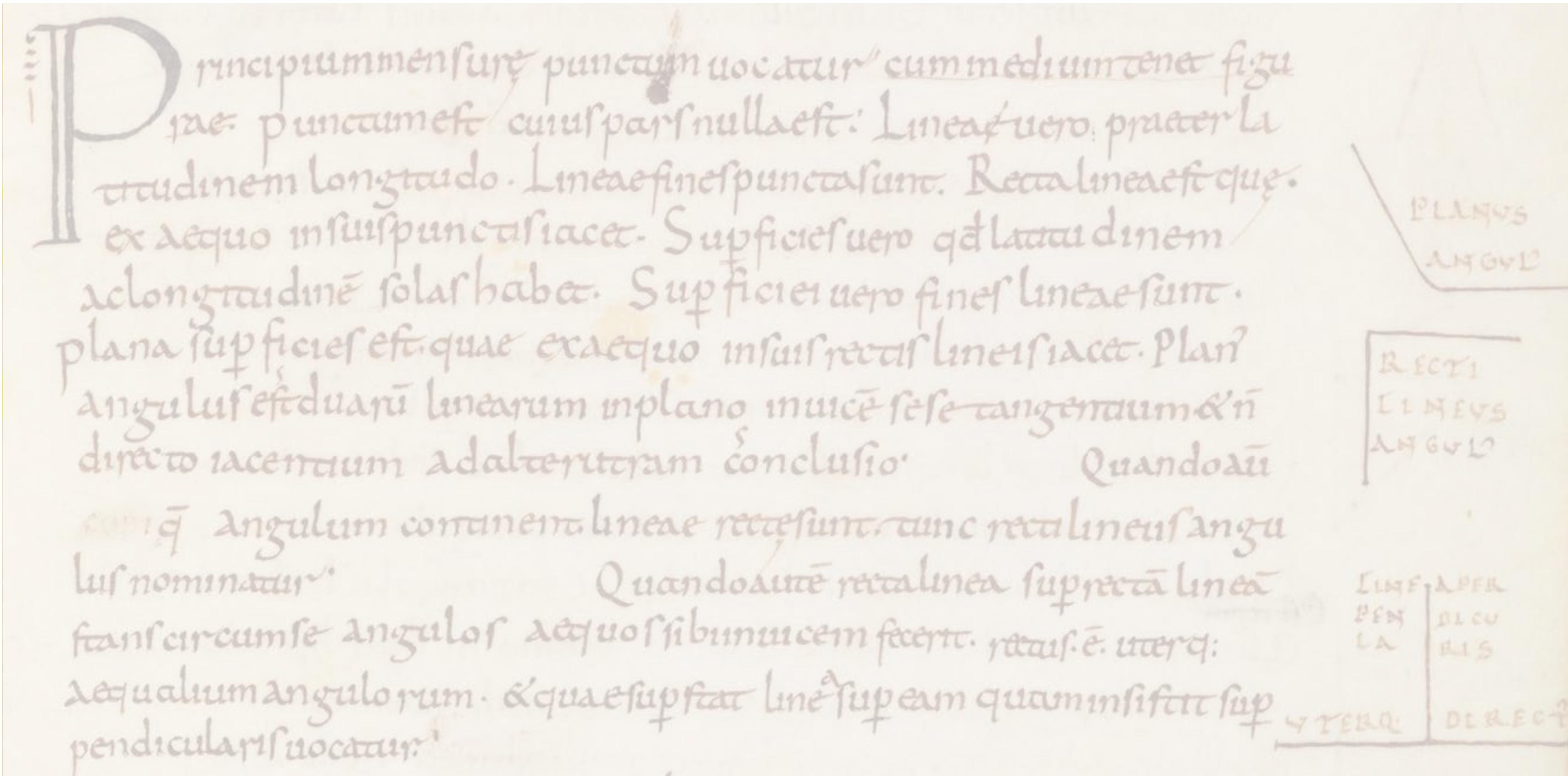
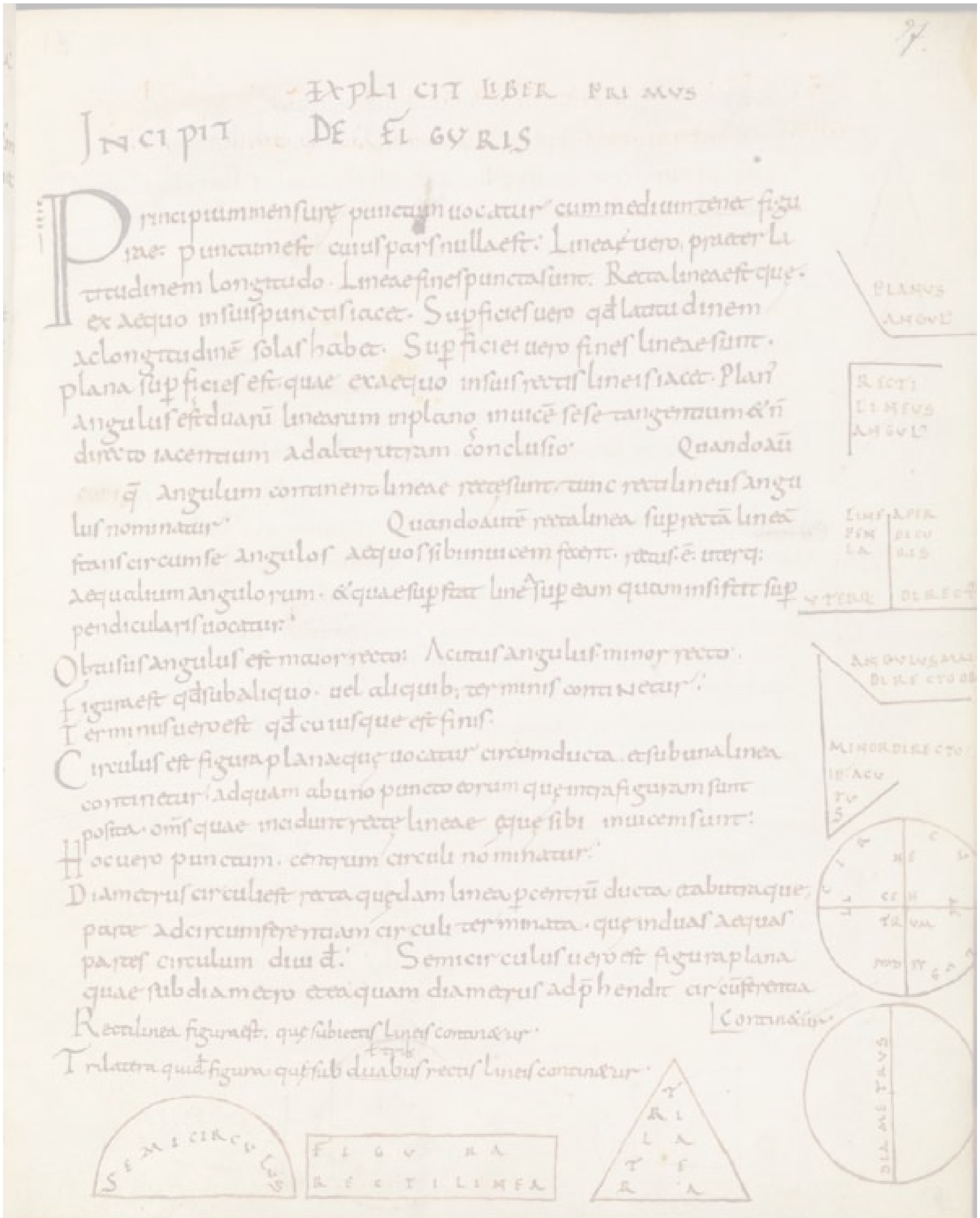


1.48 Bamberg, Staasbibliothek MS Msc.Class.55, fol. 6r.



1.49 Bamberg, Staasbibliothek MS Msc.Class.55, fol. 7r, detail. © Staatsbibliothek Bamberg



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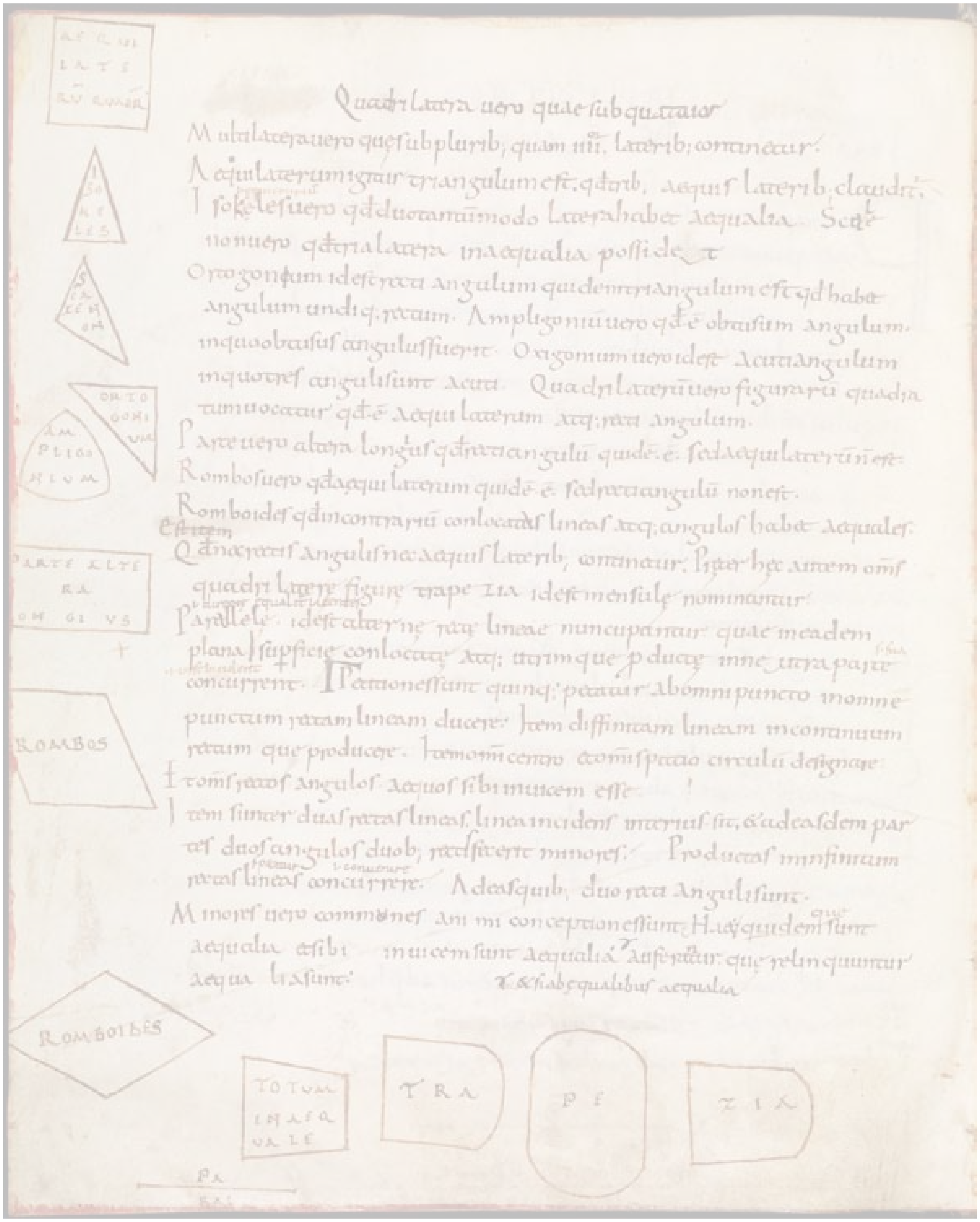
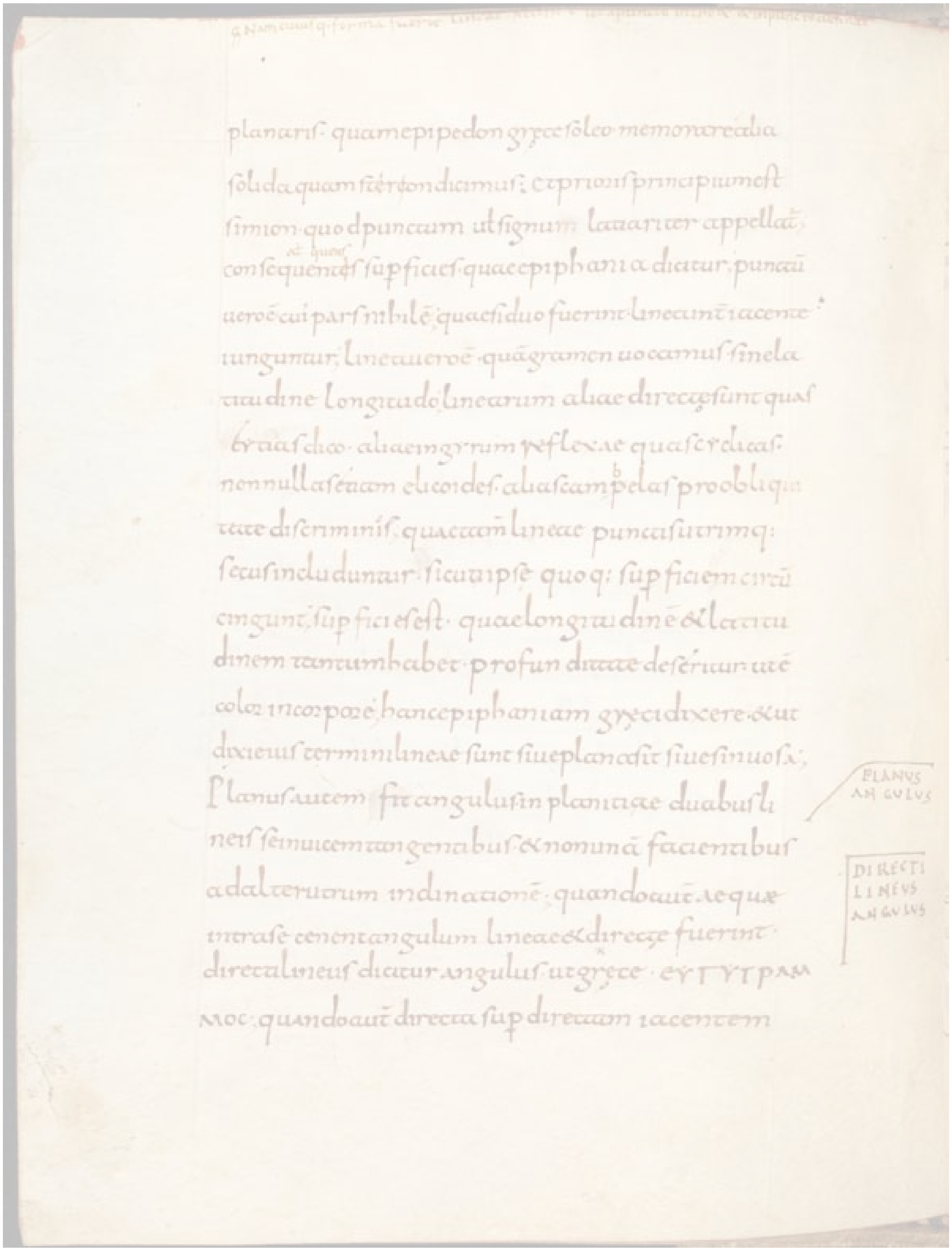
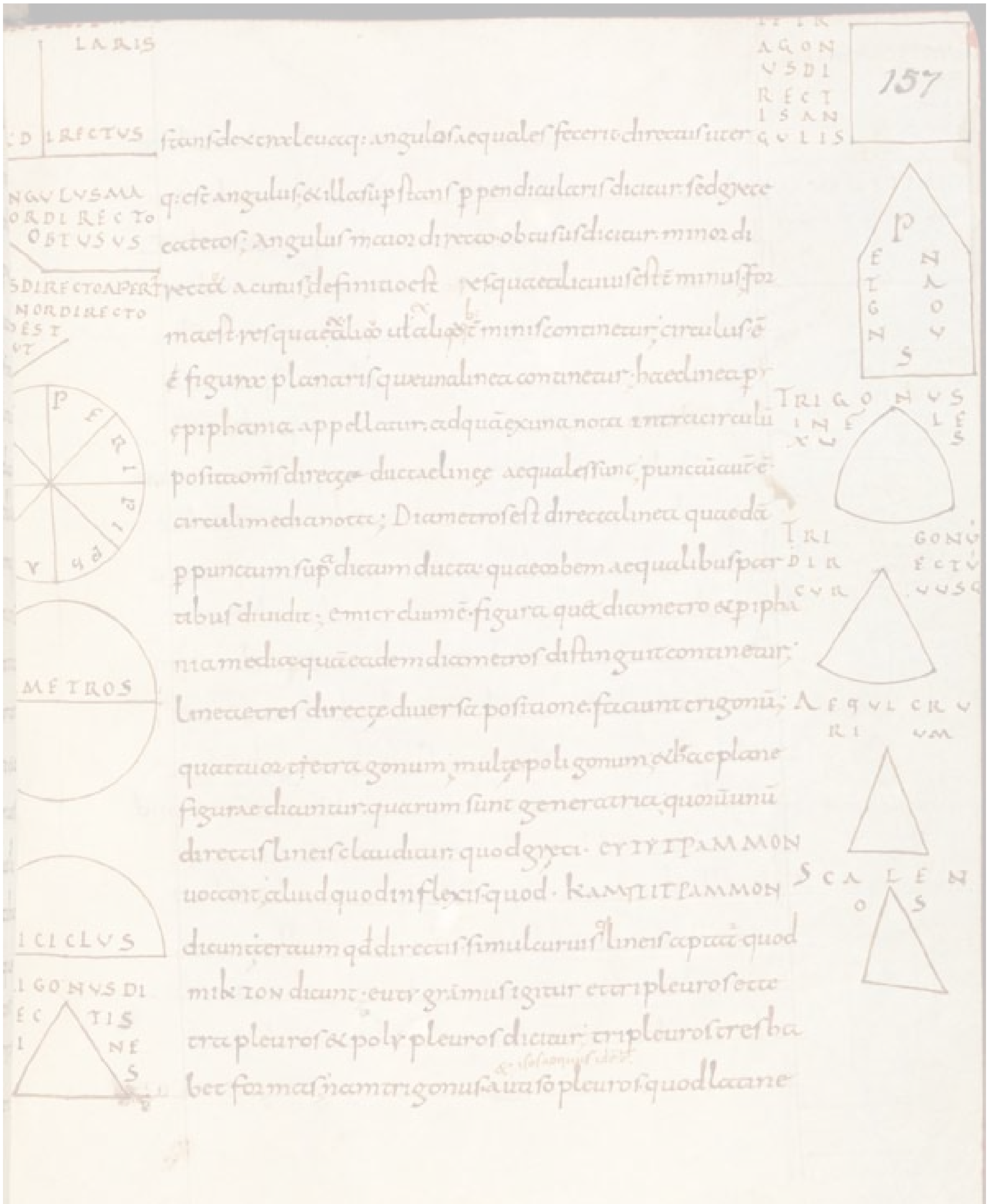


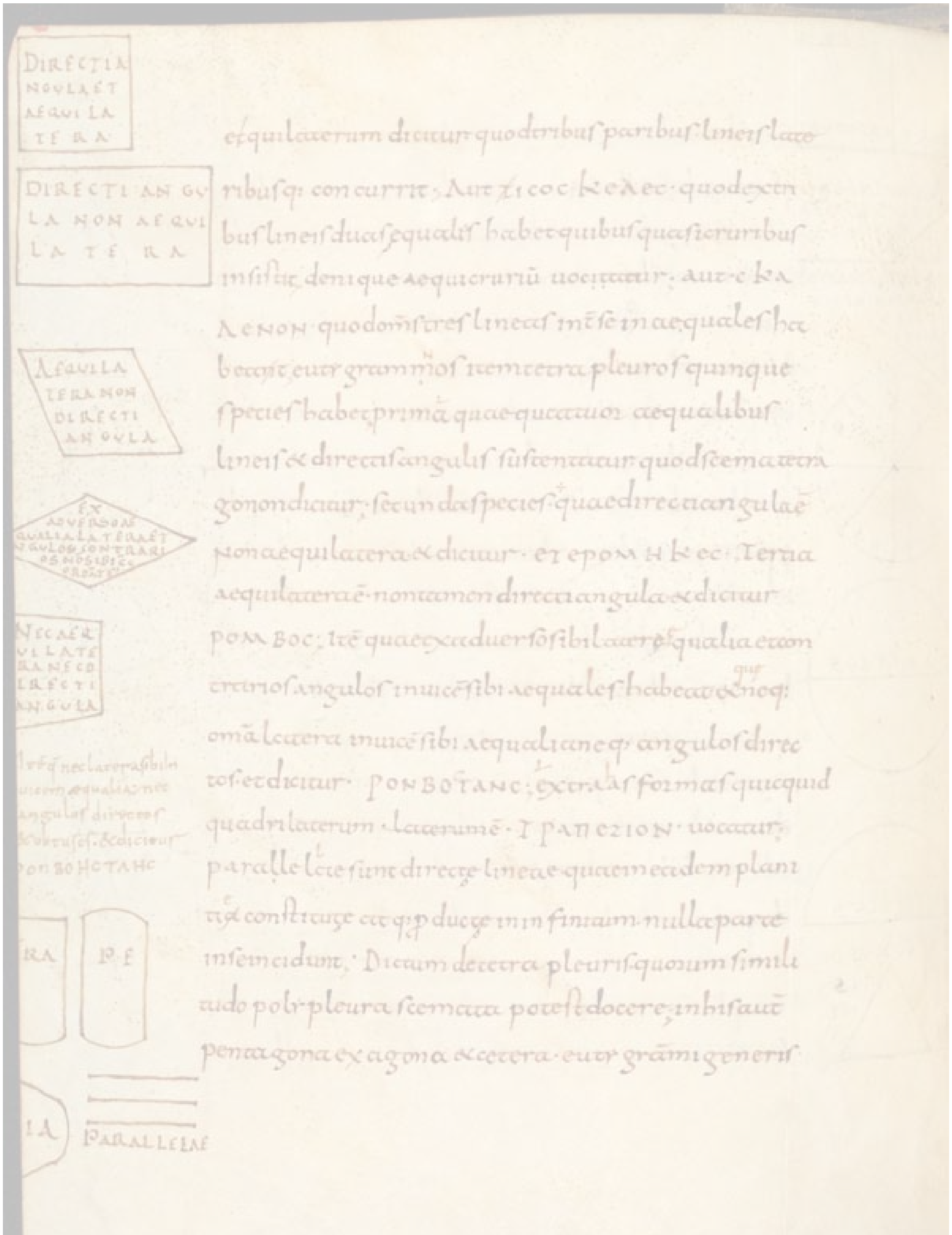
1.51 Bamberg, Staasbibliothek MS Msc.Class.55, fol. 7v.


planaris· quam epipedon graece soleo memorare alia
solida quam stereon dicimus; et prioris principium est
simion· quod punctum uel signum latinae appellatur,
consequens est superficies· quae epiphania dicitur, puncti
uero est cui pars nihil est; quae si duo fuerint lineam iacente
iunguntur; linea uero est· quam gramen uocamus sine la
titudine longitudo linearum aliae directae sunt quas
euthias dico· aliae in gyrum reflexae quas cyclicas·
nonnullas etiam elicoides· alias campylas pro obliqui
tate discriminis. quae tamen lineae punctis utrimque
secus includuntur· sicut ipse quoque superficiem circum
cingunt, superficies est· quae longitudinem et latitu
dinem tantum habet· profunditate deseritur ut est
color in corpore, hanc epiphaniam graeci dixere· et ut
dixi eius termini lineae sunt siue planae sit siue sinuosae;
Planus autem fit angulus in planitiae duabus li
neis se inuicem tangentibus et non in una facientibus
ad alterutrum inclinationem; quando autem aequae
intra se tenent angulum lineae et directae fuerint
directa lineus dicitur angulus ut graece eythygram
mon; quando autem directa super directam iacentem

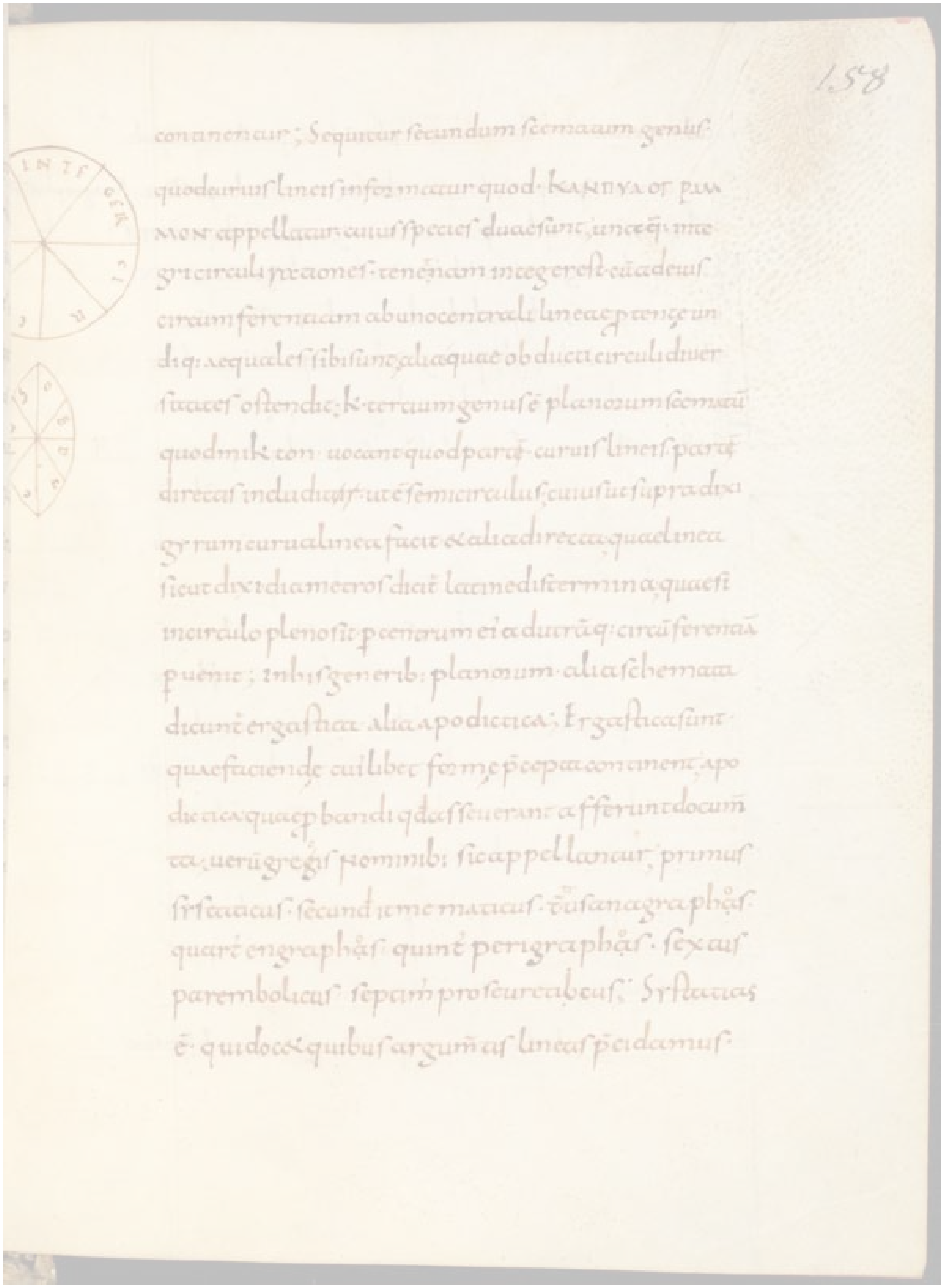
1.52 Bamberg, Staasbibliothek MS Msc.Class.39, fol. 156v.



1.53 Bamberg, Staasbibliothek MS Msc.Class.39, fol. 157r.

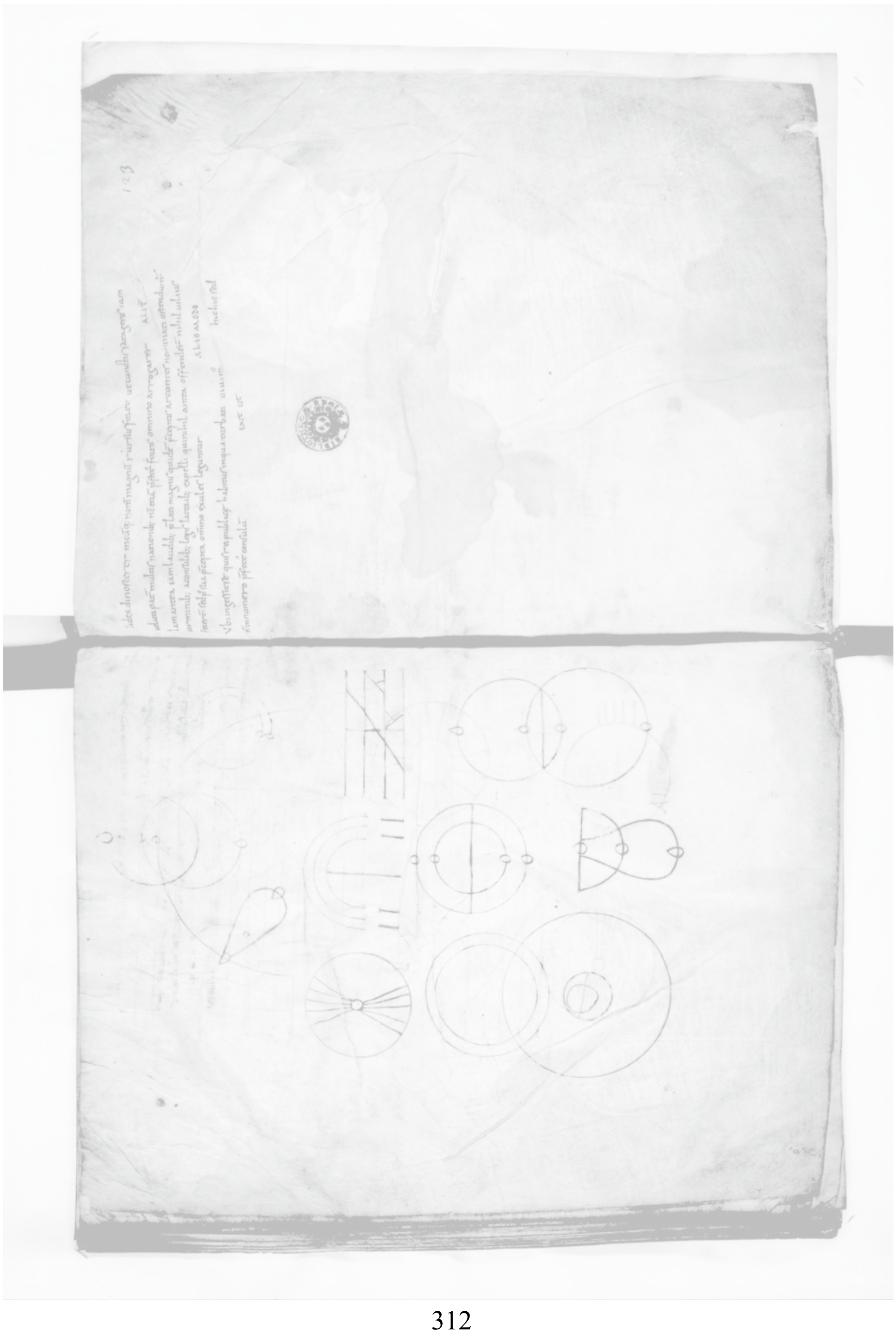


1.54 Bamberg, Staasbibliothek MS Msc.Class.39, fol. 157v.

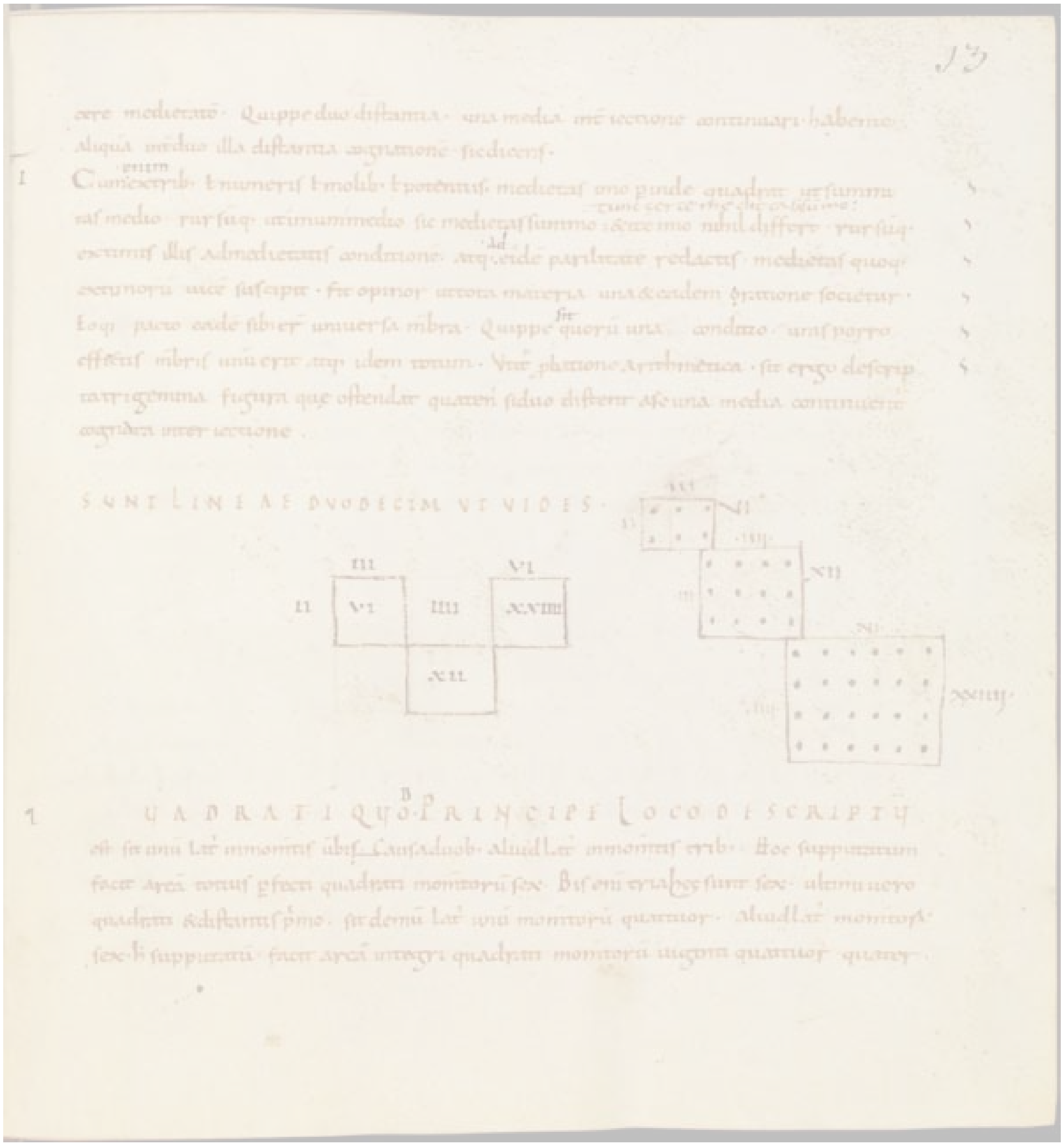


1.55 Bamberg, Staasbibliothek MS Msc.Class.39, fol. 158r.

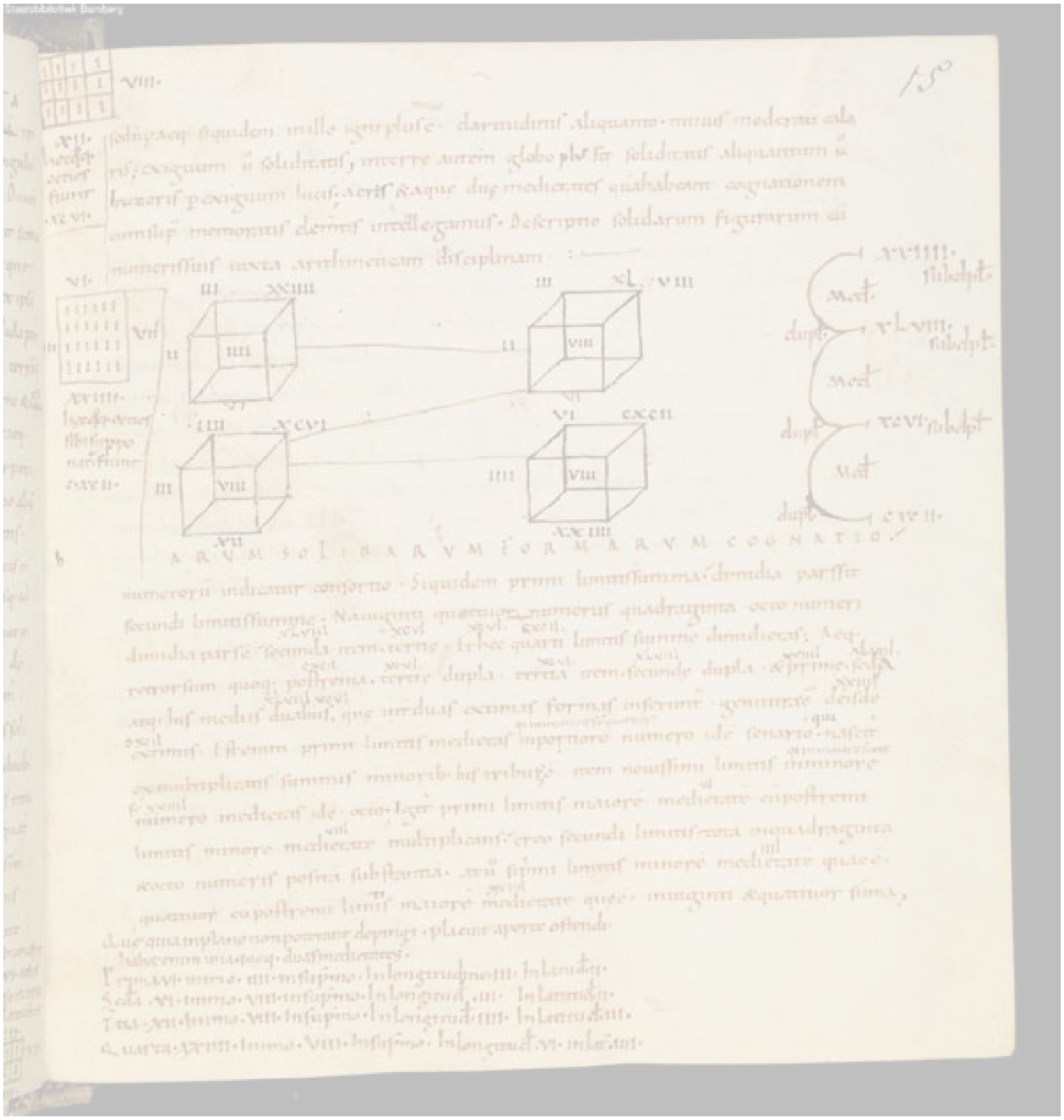
1.56 Paris, Bibliotheèque nationale de France lat. MS 8669, fols. 121v–122r. © Bibliothèque nationale de France



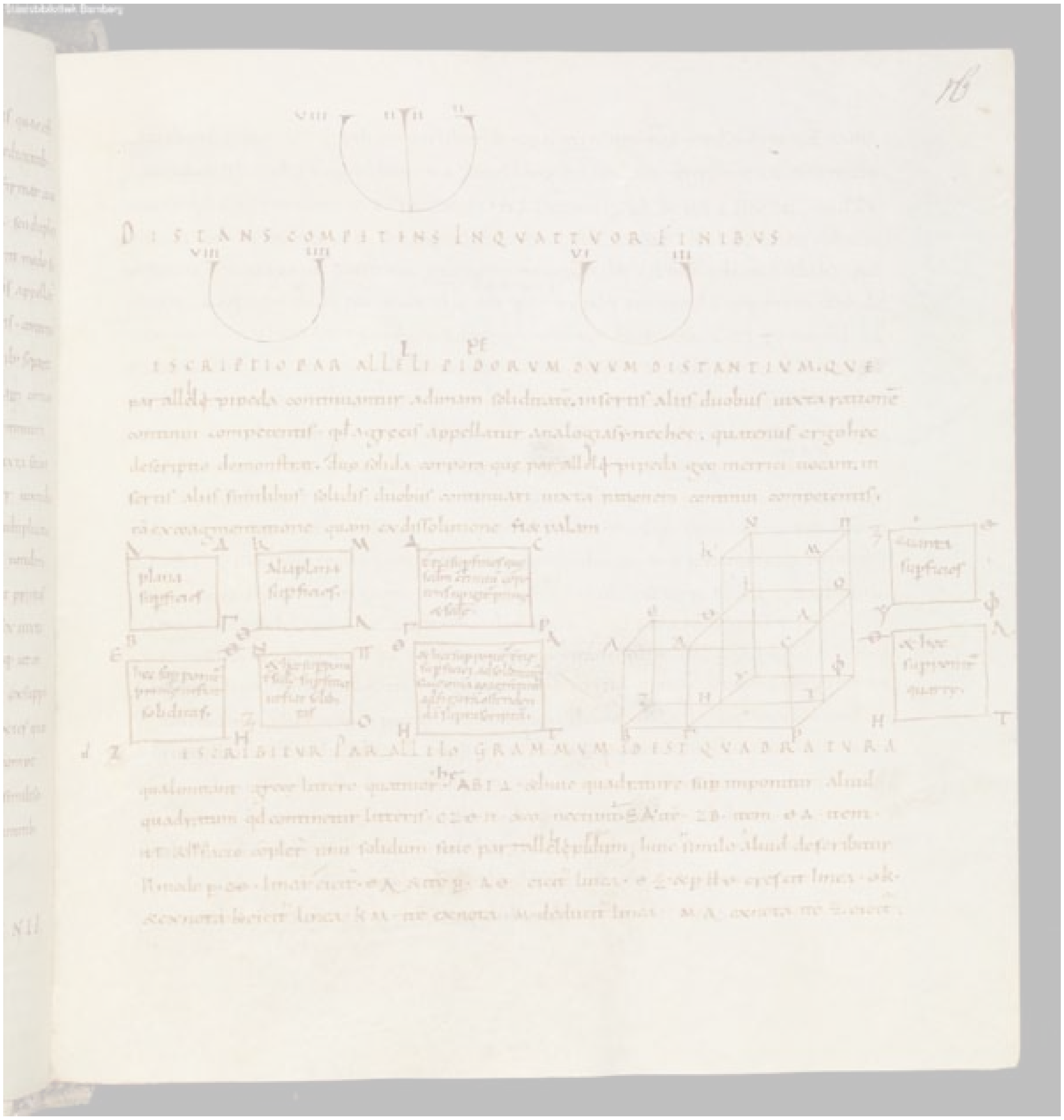
1.57 Paris, Bibliotheèque nationale de France lat. MS 8669, fols. 122v–123r. © Bibliothèque nationale de France



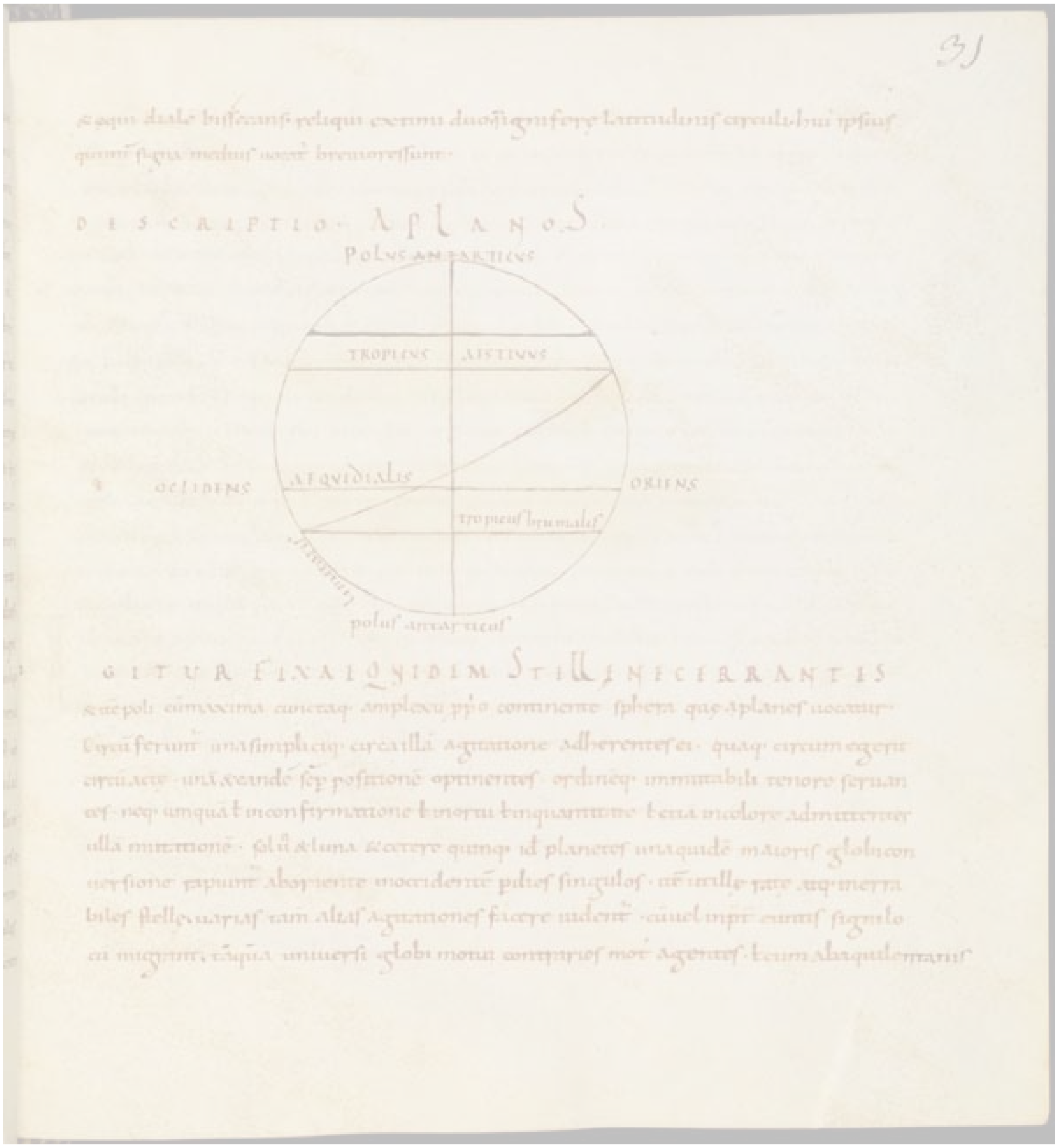
1.58 Bamberg, Staasbibliothek MS Msc.Class.18, fol. 13r. © Staatsbibliothek Bamberg



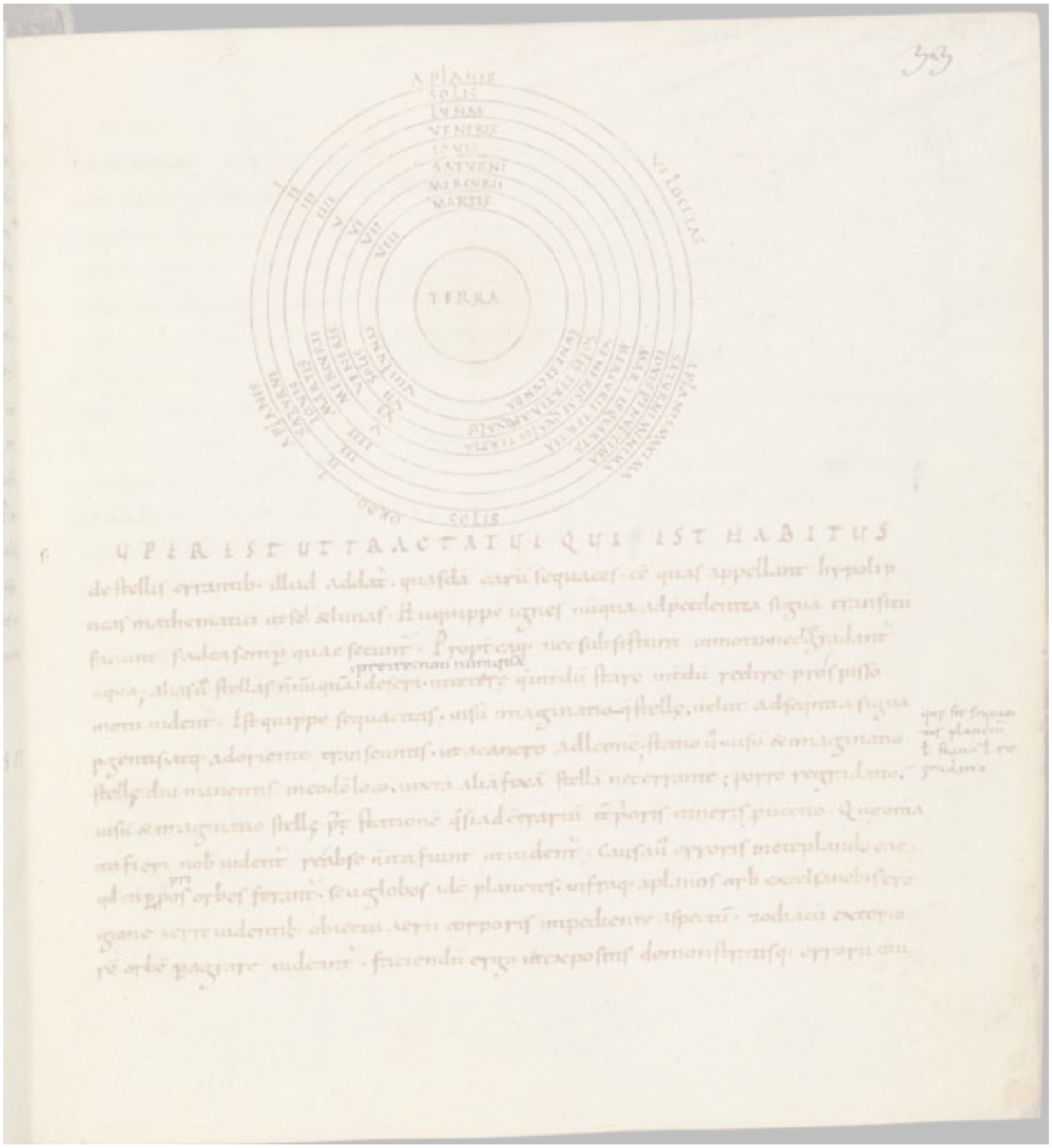
1.59 Bamberg, Staasbibliothek MS Msc.Class.18, fol. 15r. © Staatsbibliothek Bamberg



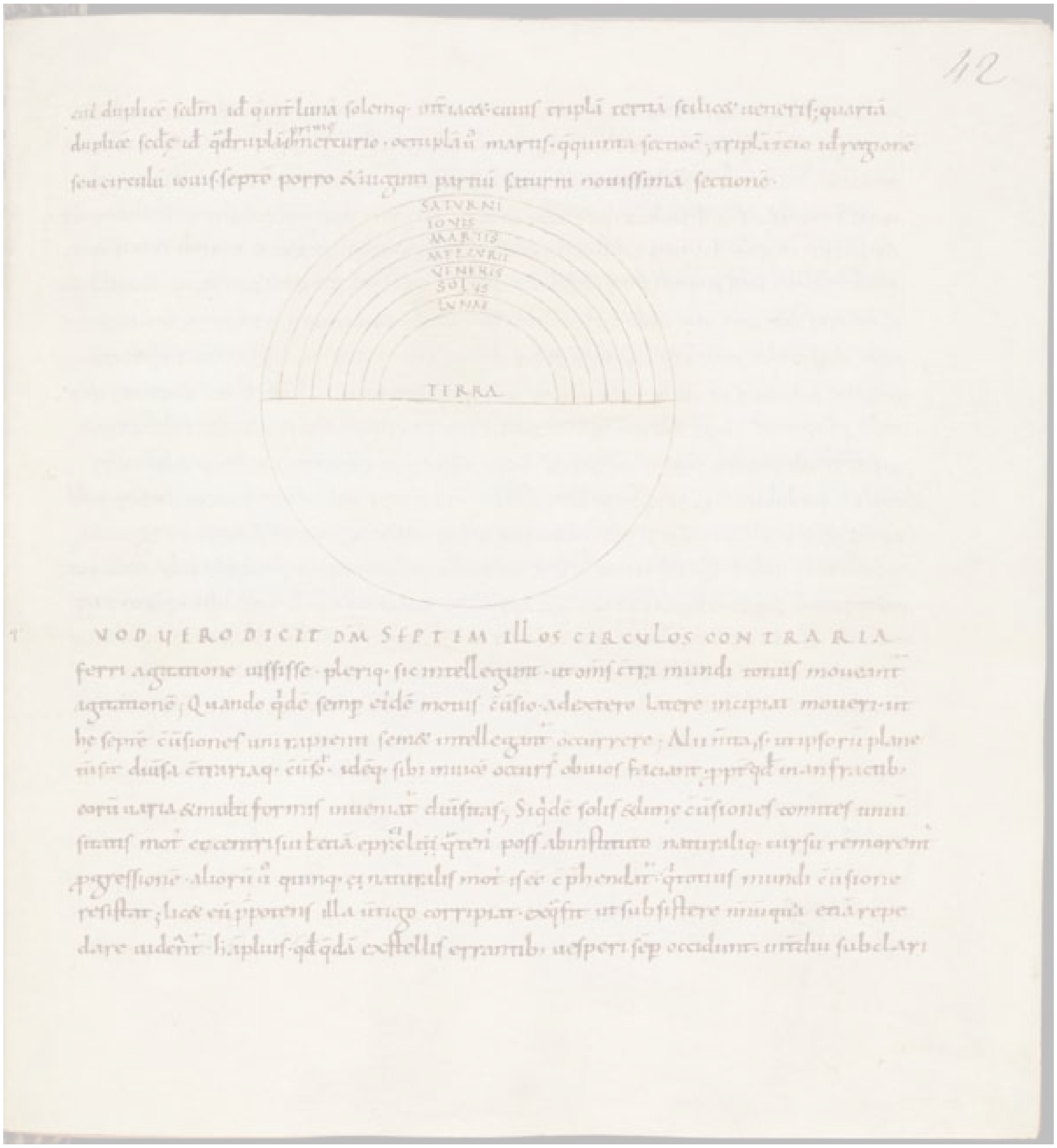
1.60 Bamberg, Staasbibliothek MS Msc.Class.18, fol. 16r. © Staatsbibliothek Bamberg



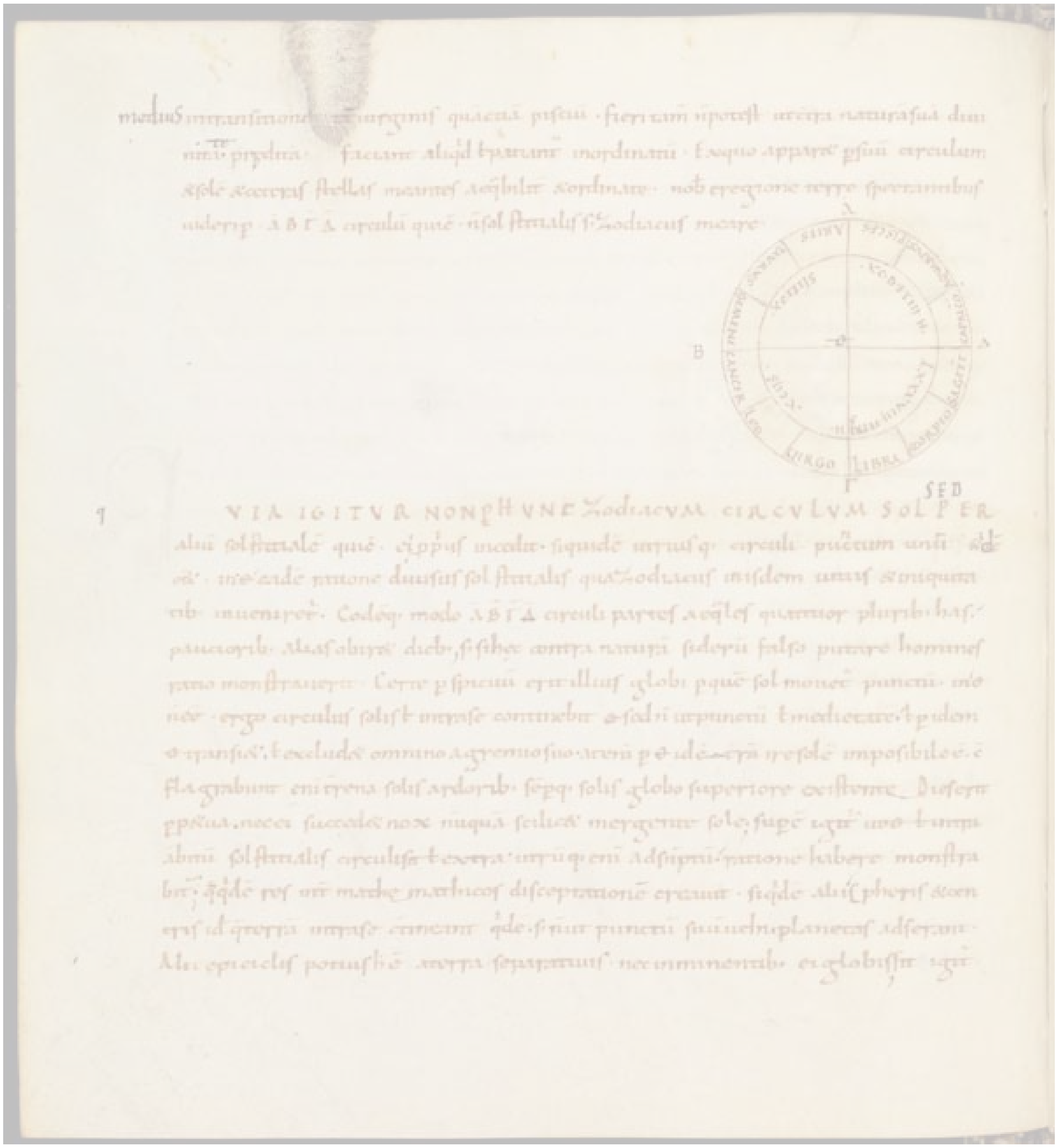
1.61 Bamberg, Staasbibliothek MS Msc.Class.18, fol. 31r. © Staatsbibliothek Bamberg



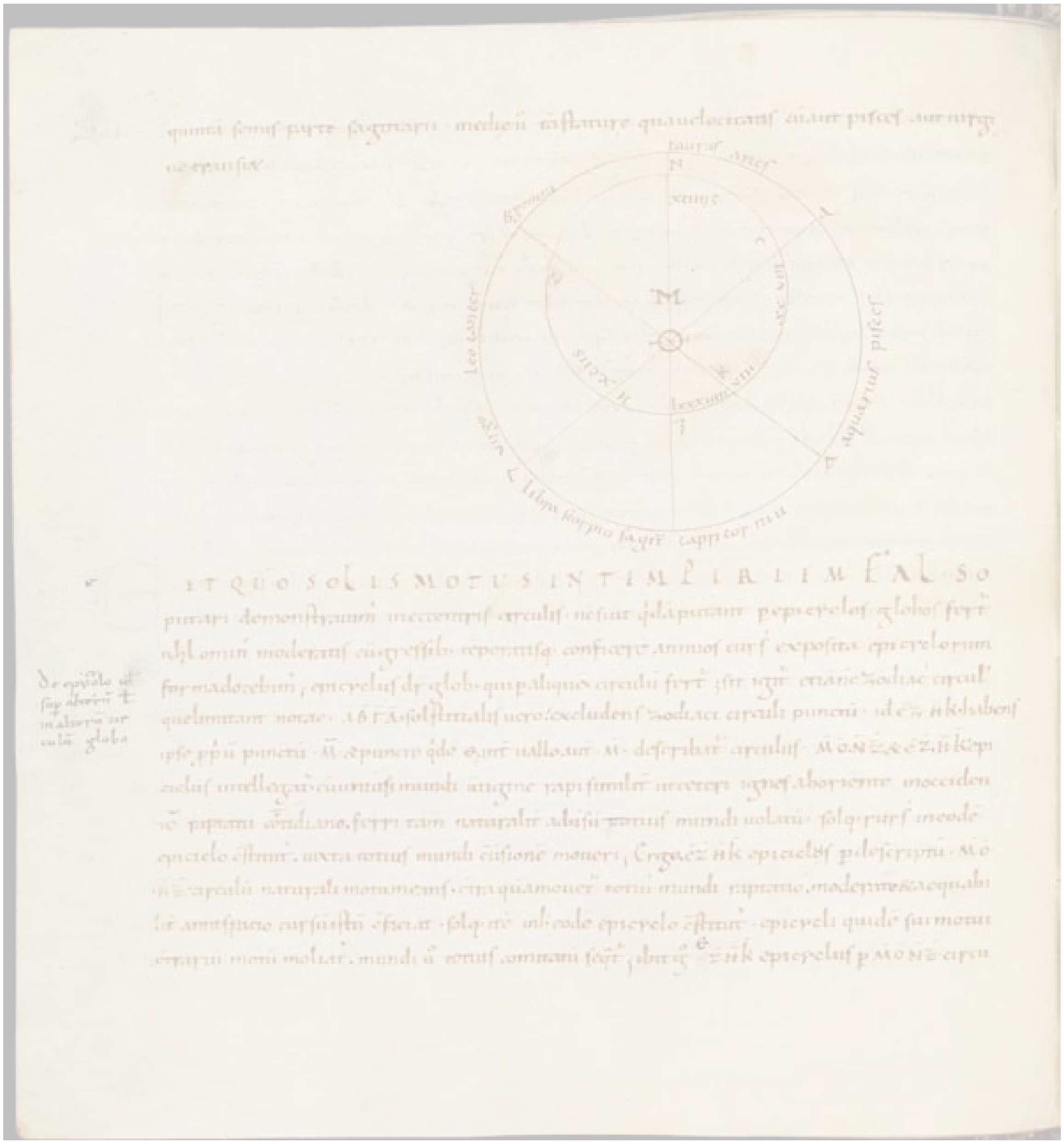
1.62 Bamberg, Staasbibliothek MS Msc.Class.18, fol. 33r. © Staatsbibliothek Bamberg



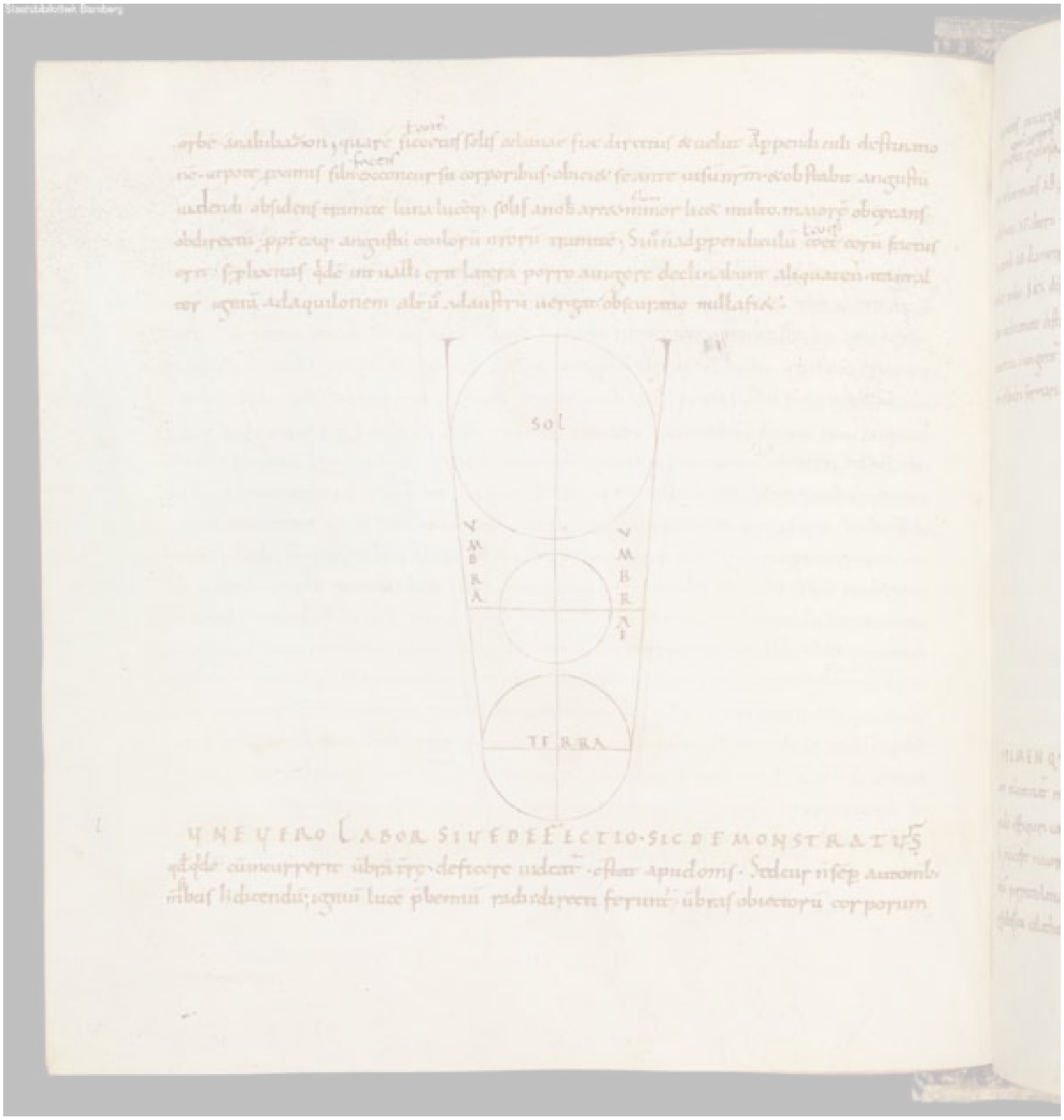
1.63 Bamberg, Staasbibliothek MS Msc.Class.18, fol. 42r. © Staatsbibliothek Bamberg



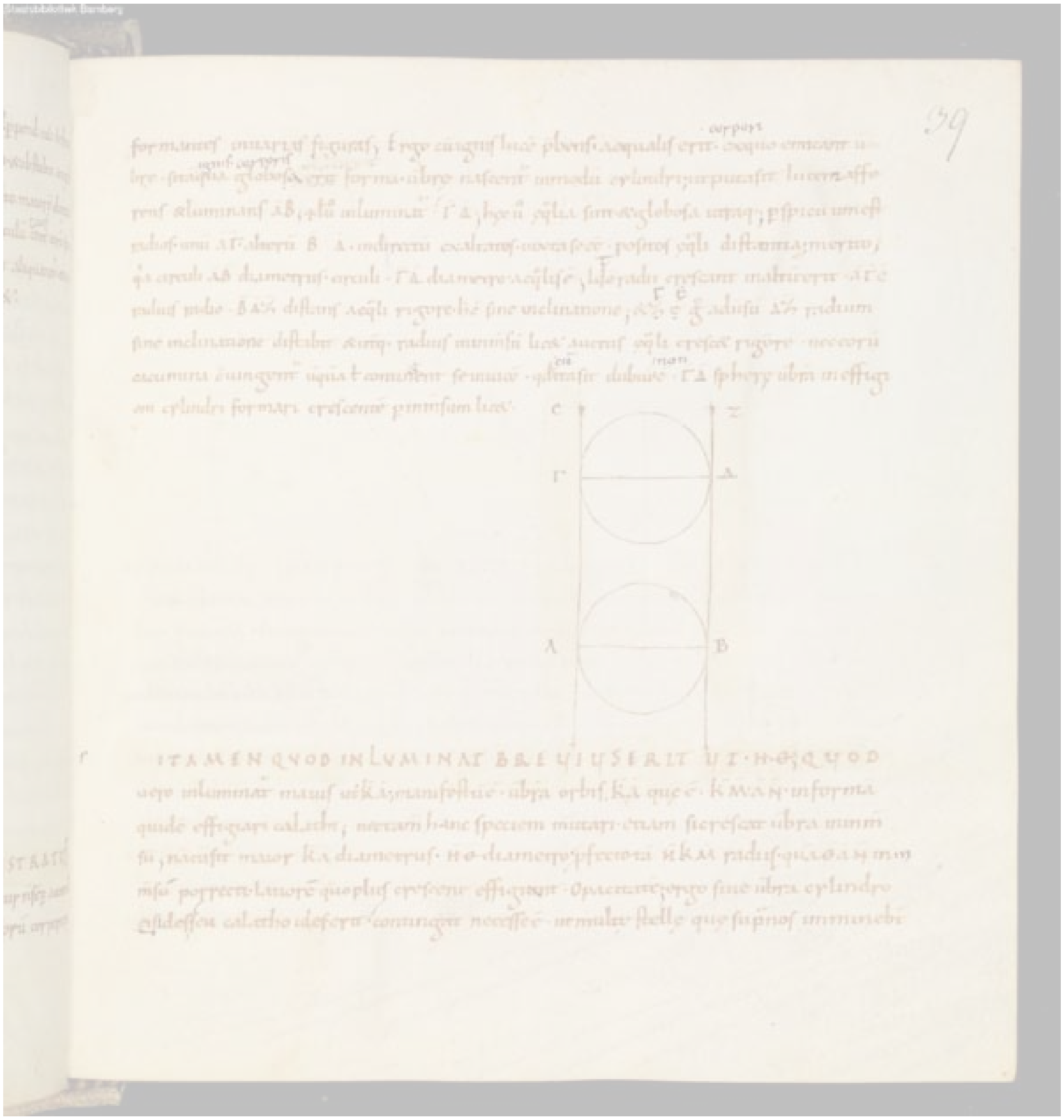
1.64 Bamberg, Staasbibliothek MS Msc.Class.18, fol. 34v.



1.65 Bamberg, Staasbibliothek MS Msc.Class.18, fol. 35v.



1.66 Bamberg, Staasbibliothek MS Msc.Class.18, fol. 38v.



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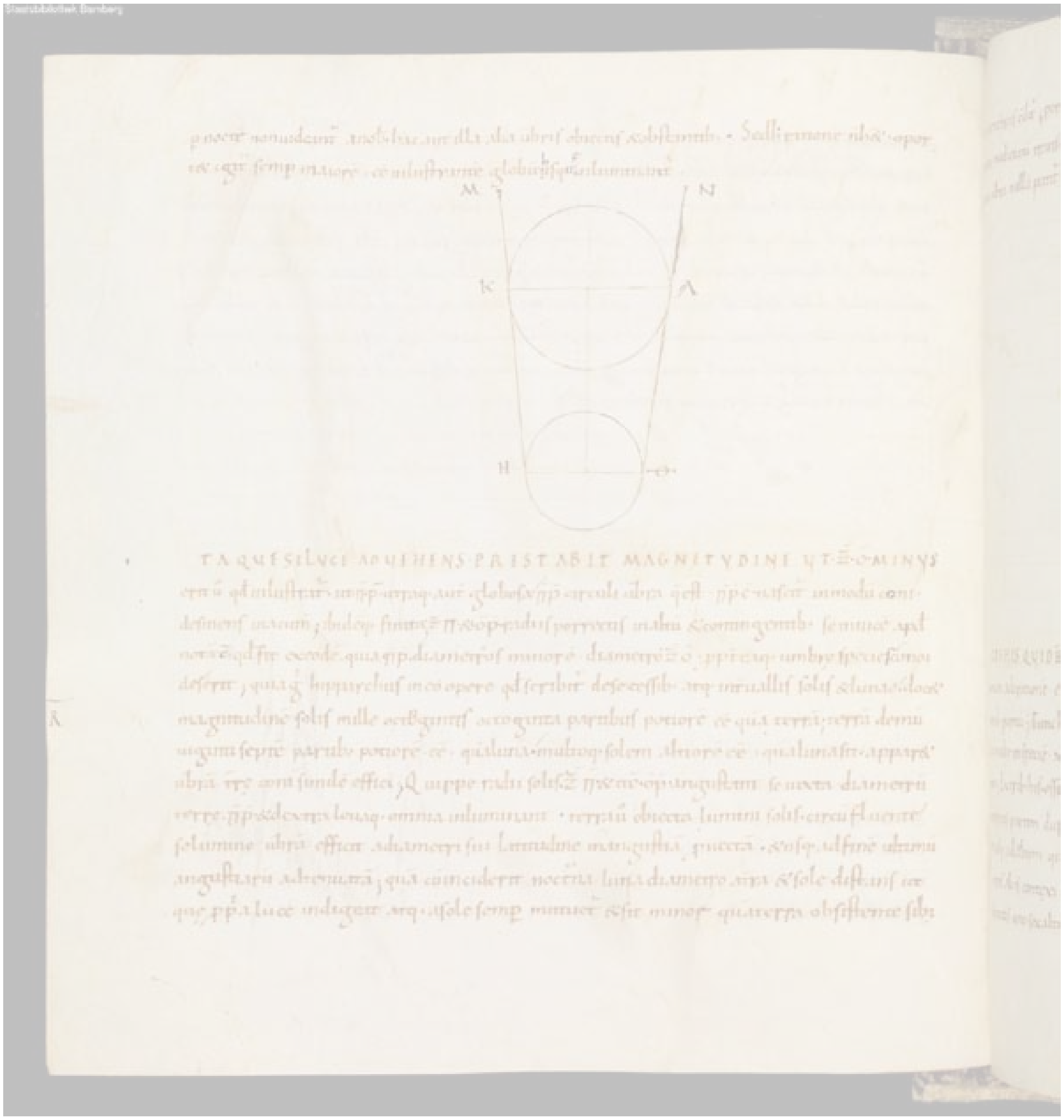
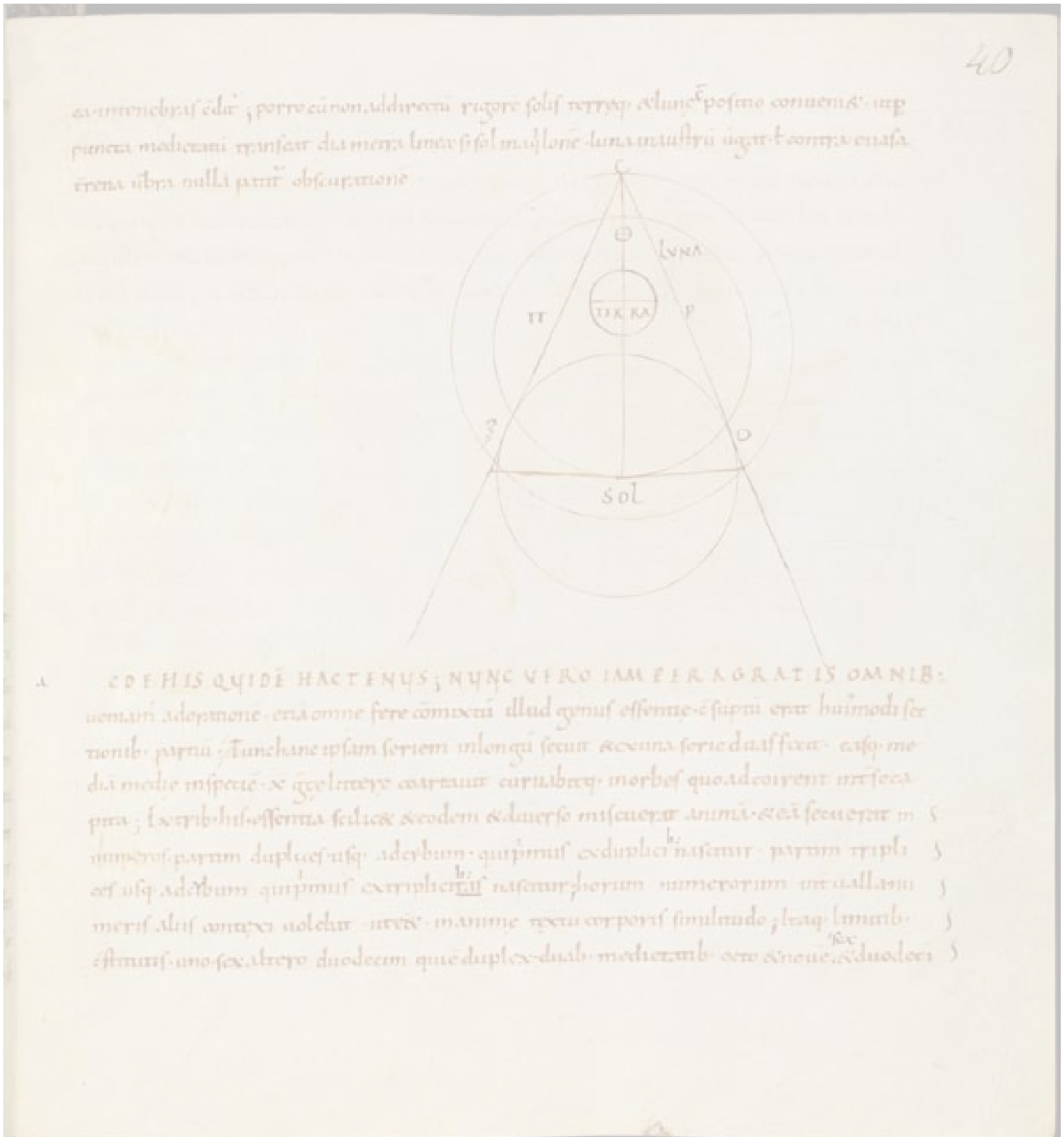
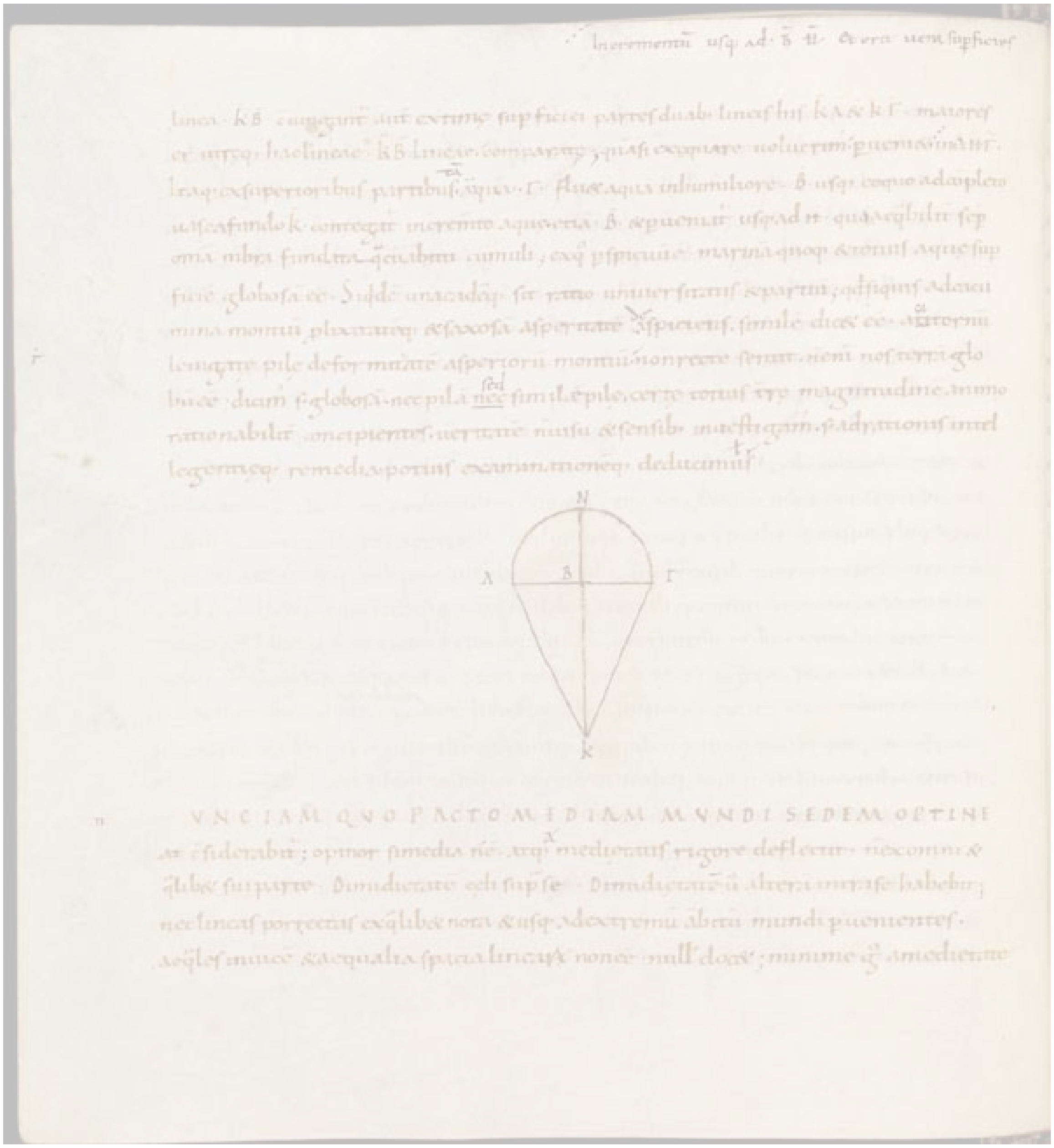


1.68 Bamberg, Staasbibliothek MS Msc.Class.18, fol. 39v.
© Staatsbibliothek Bamberg



1.69 Bamberg, Staasbibliothek MS Msc.Class.18, fol. 40r.



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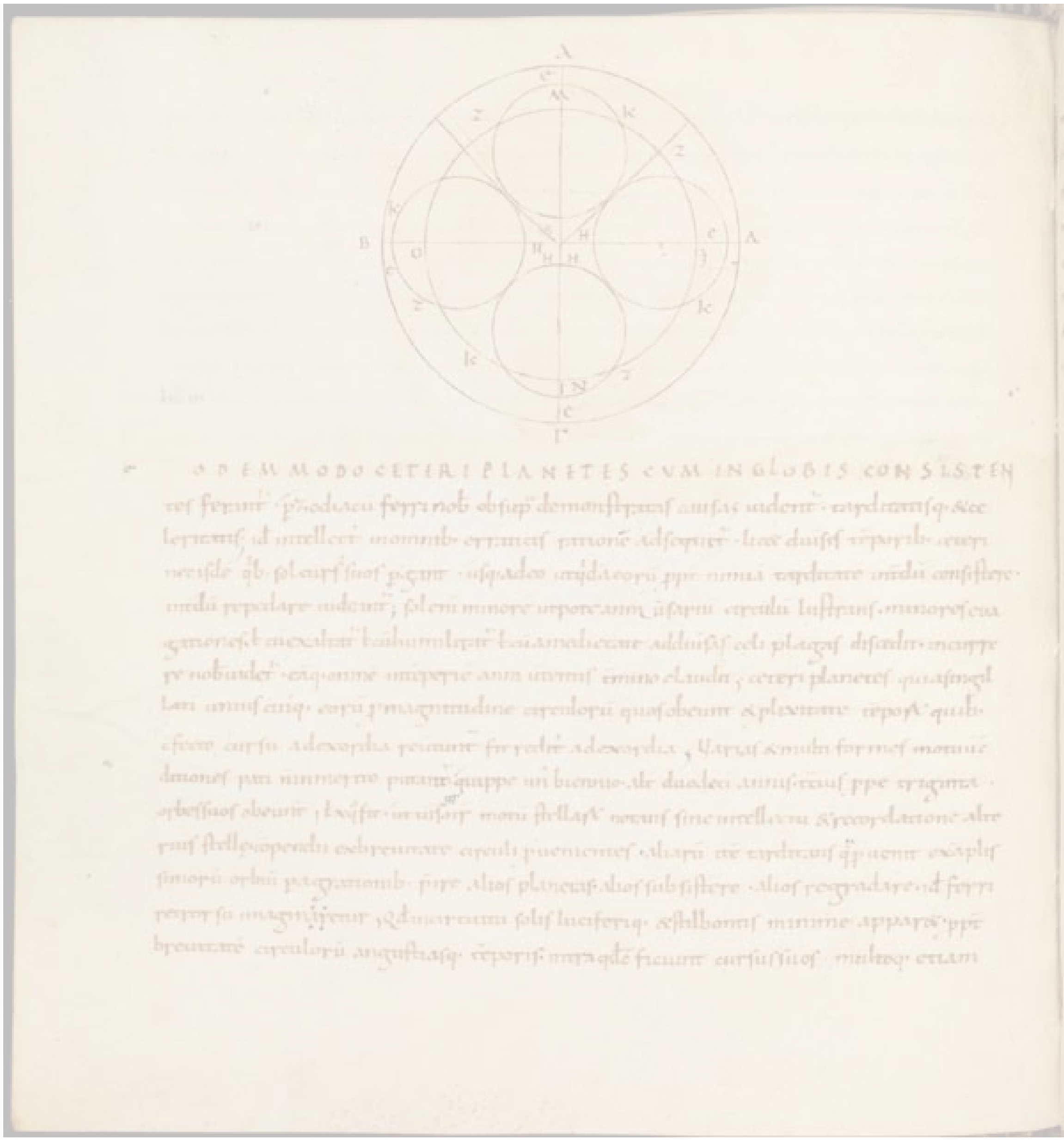
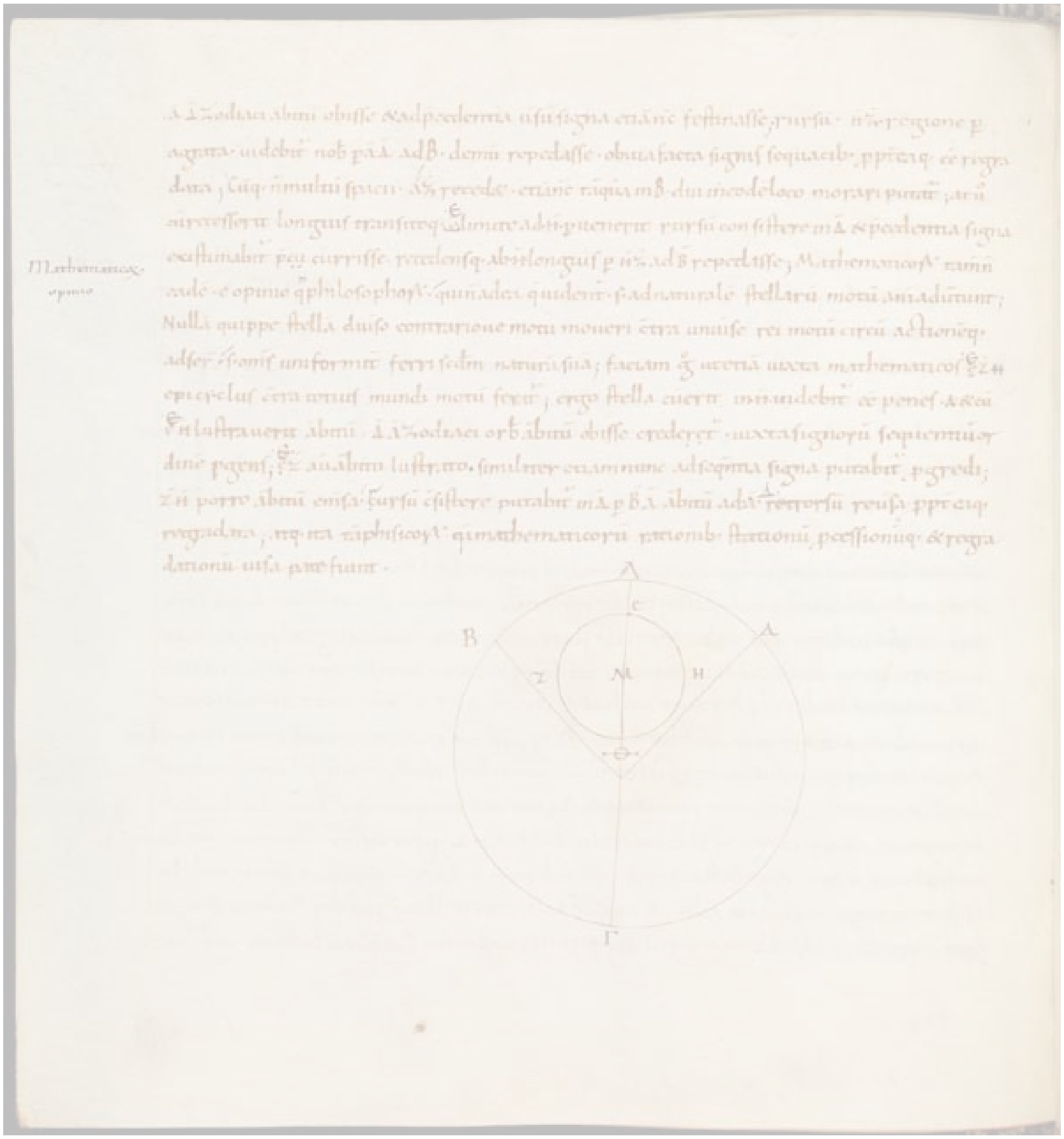
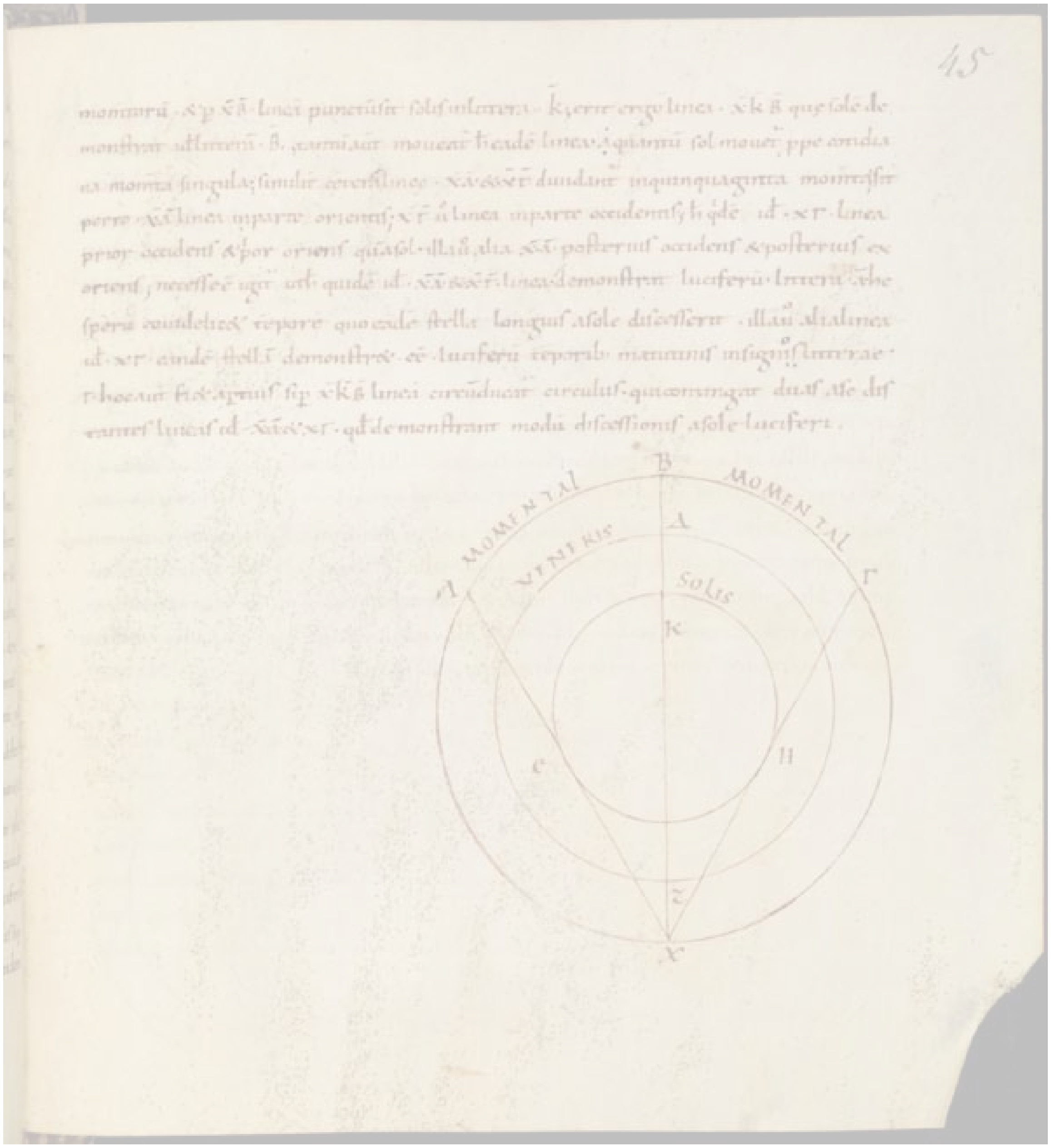


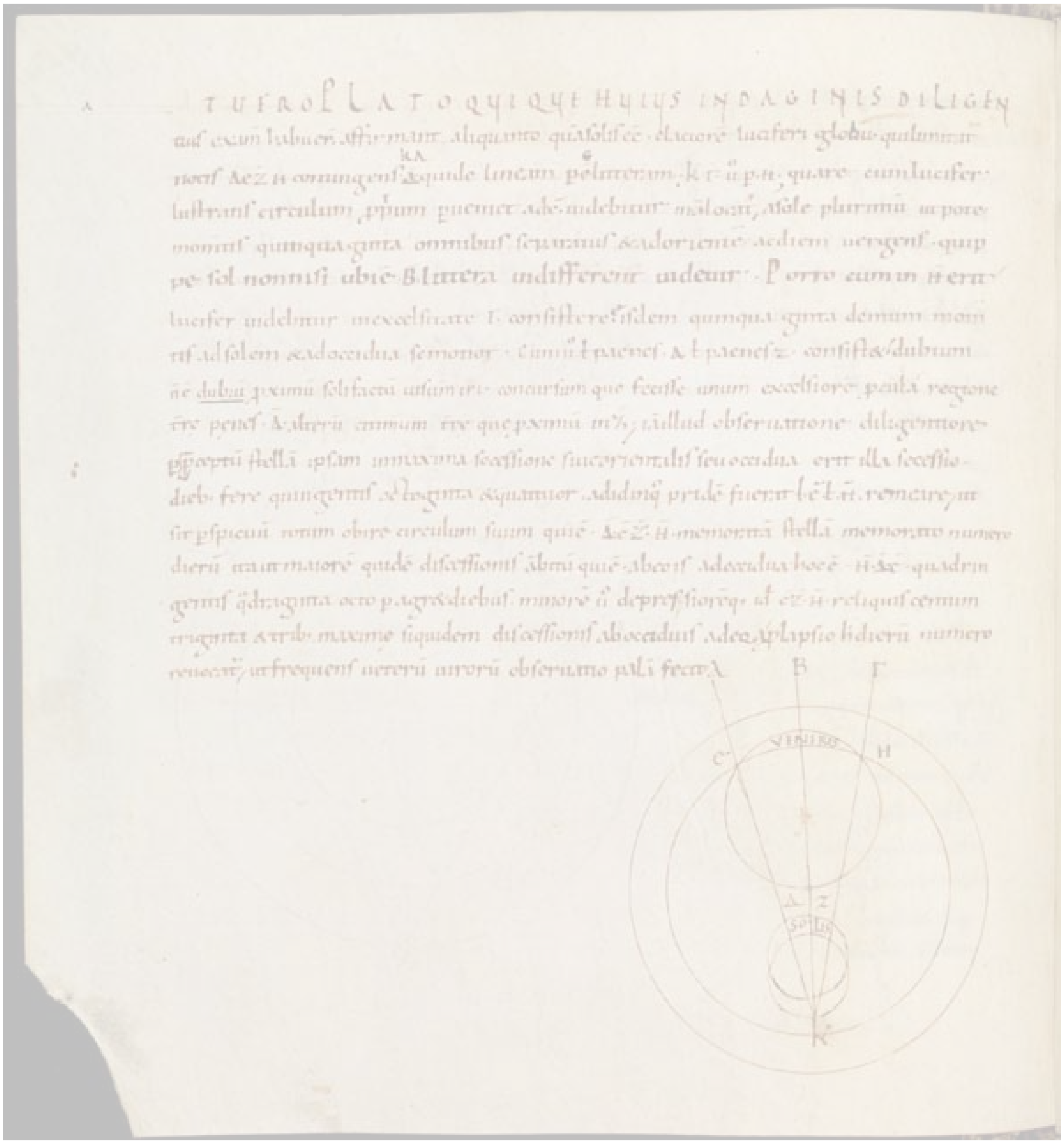
1.71 Bamberg, Staasbibliothek MS Msc.Class.18, fol. 36v.



1.72 Bamberg, Staasbibliothek MS Msc.Class.18, fol. 37v.



1.73 Bamberg, Staasbibliothek MS Msc.Class.18, fol. 45r.



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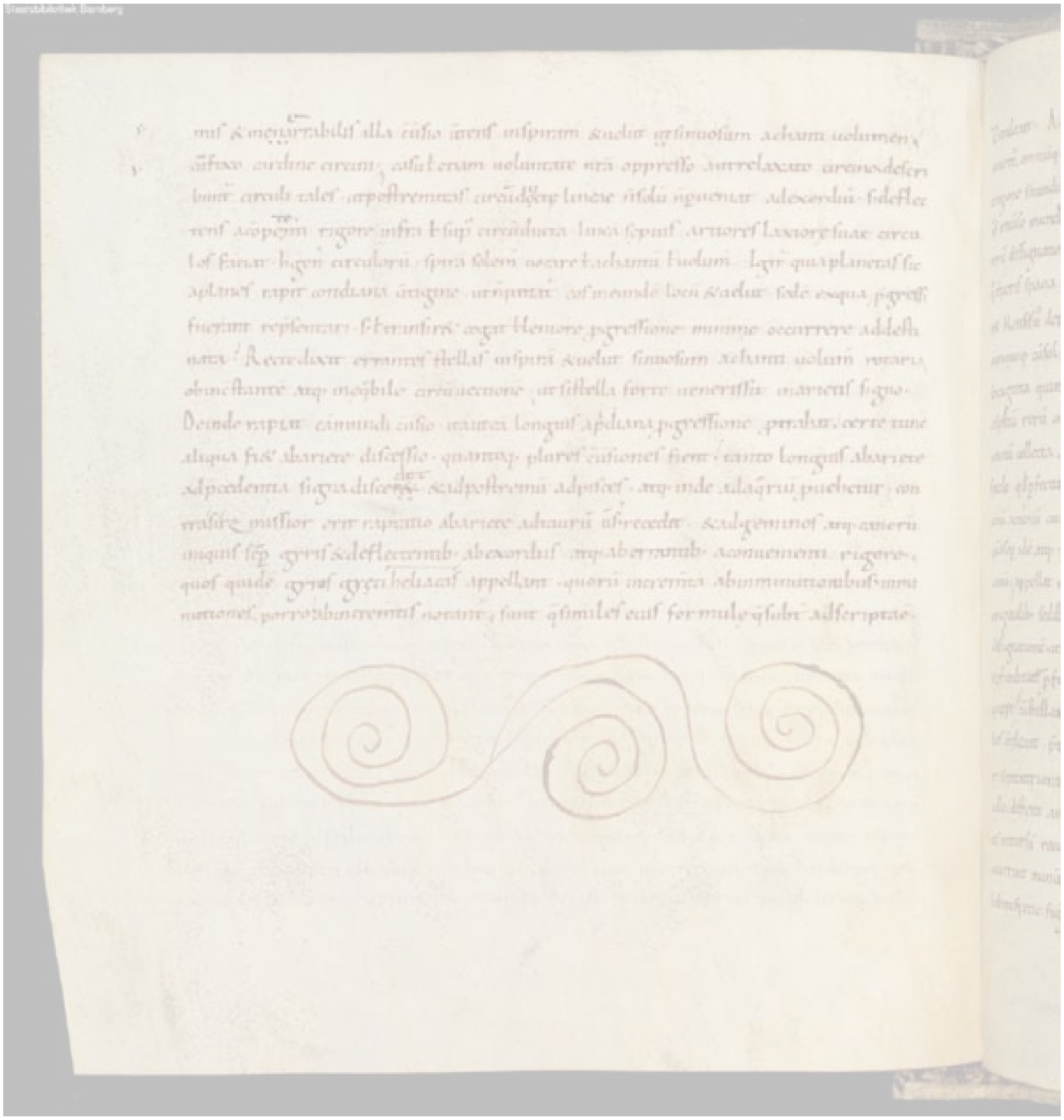

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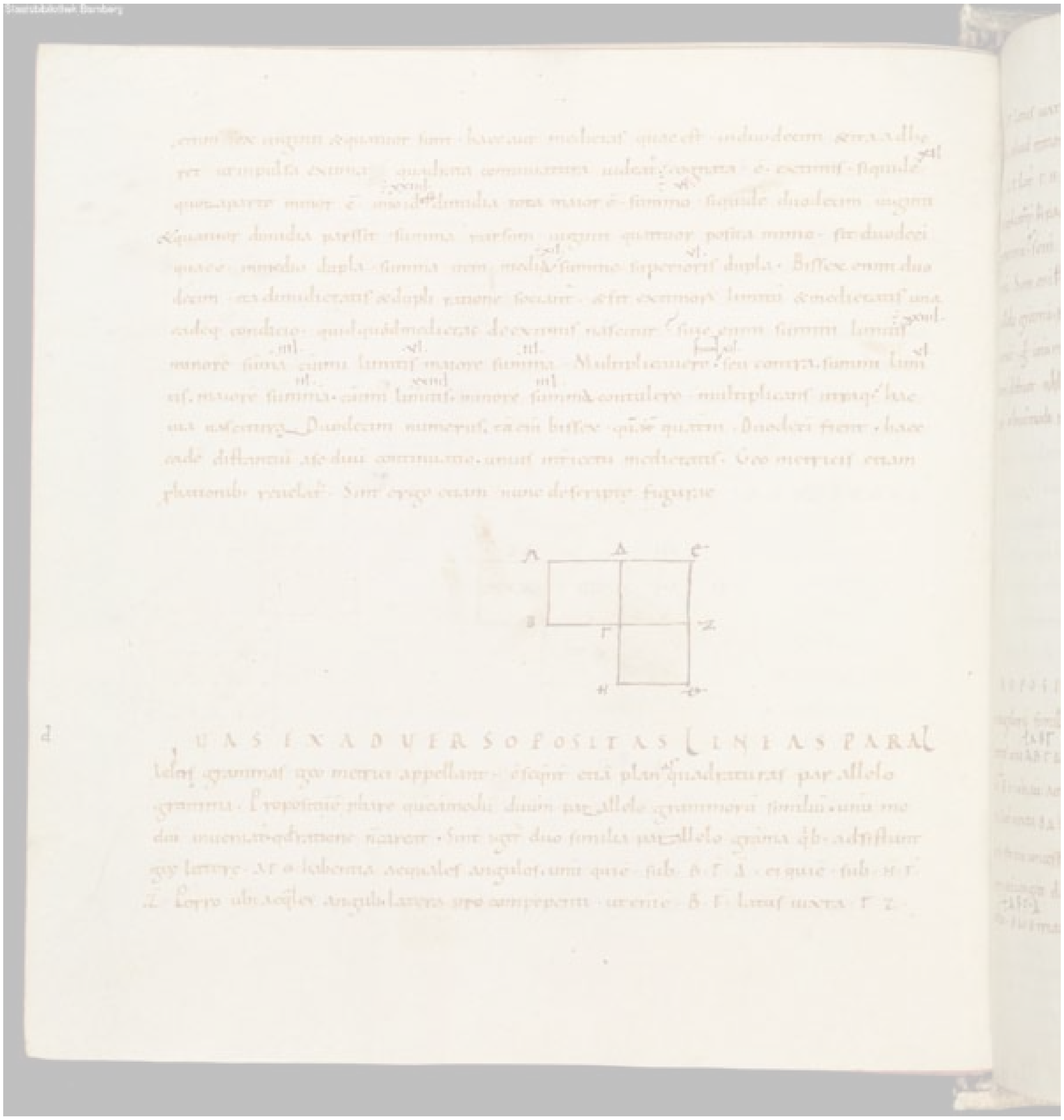

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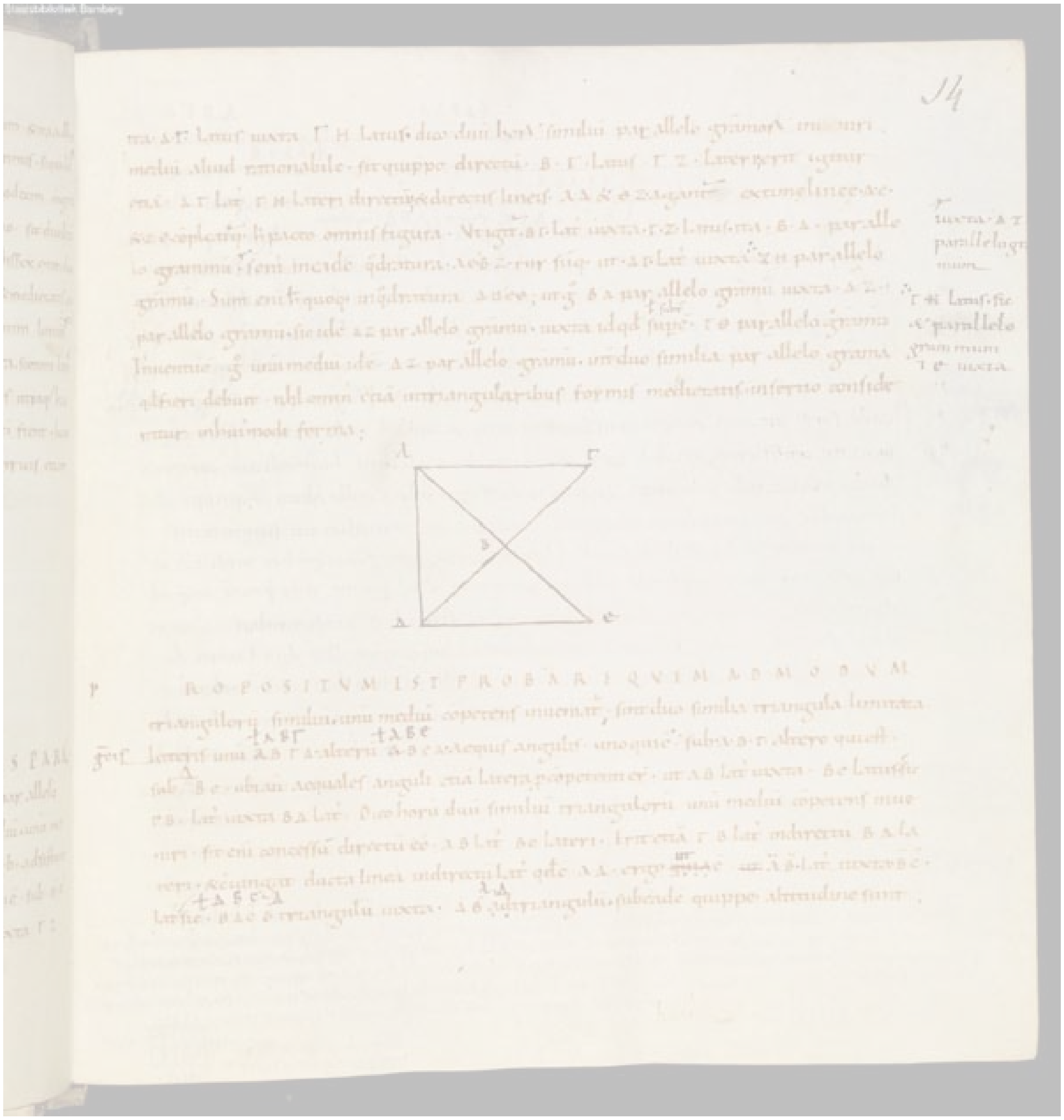
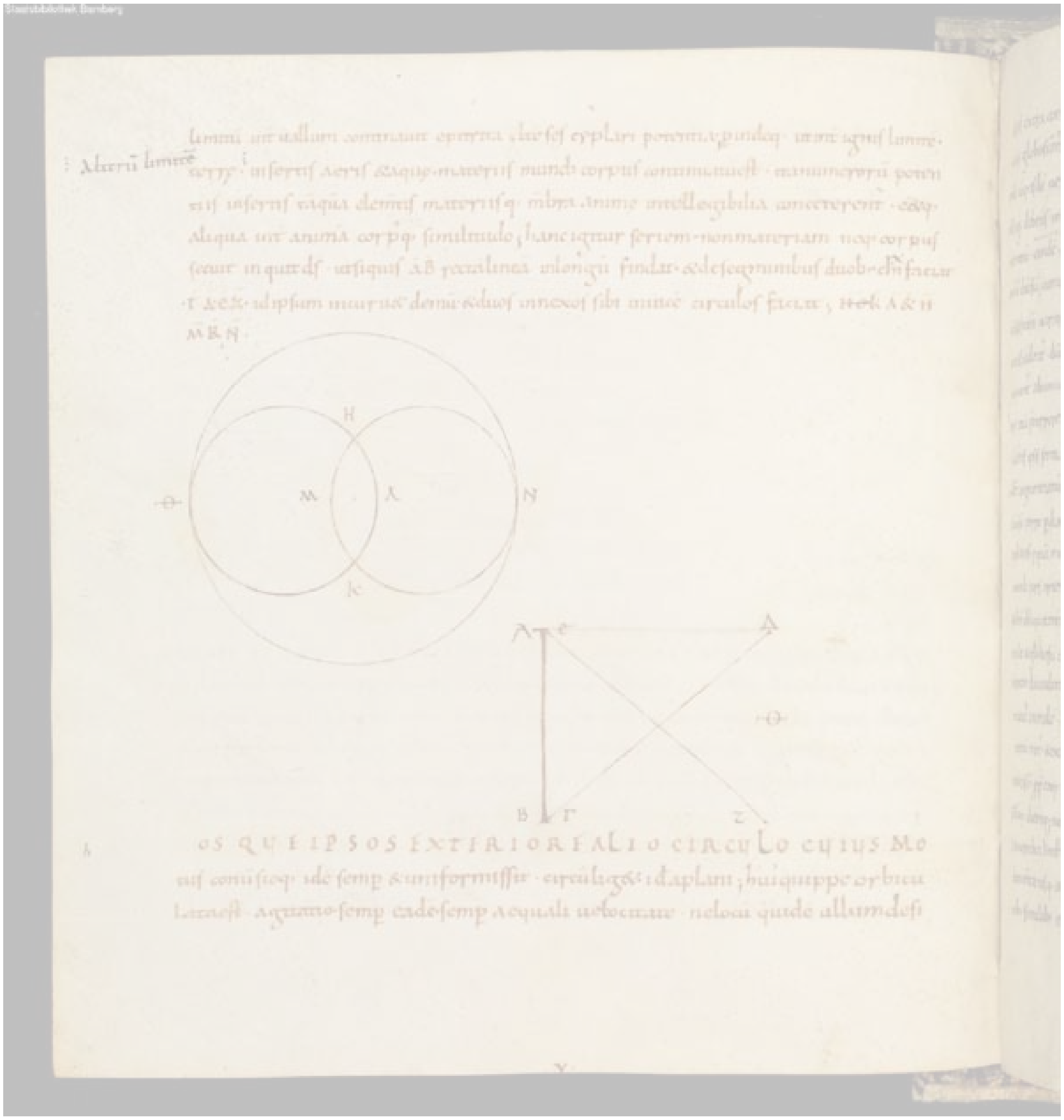


1.77 Bamberg, Staasbibliothek MS Msc.Class.18, fol. 14r.



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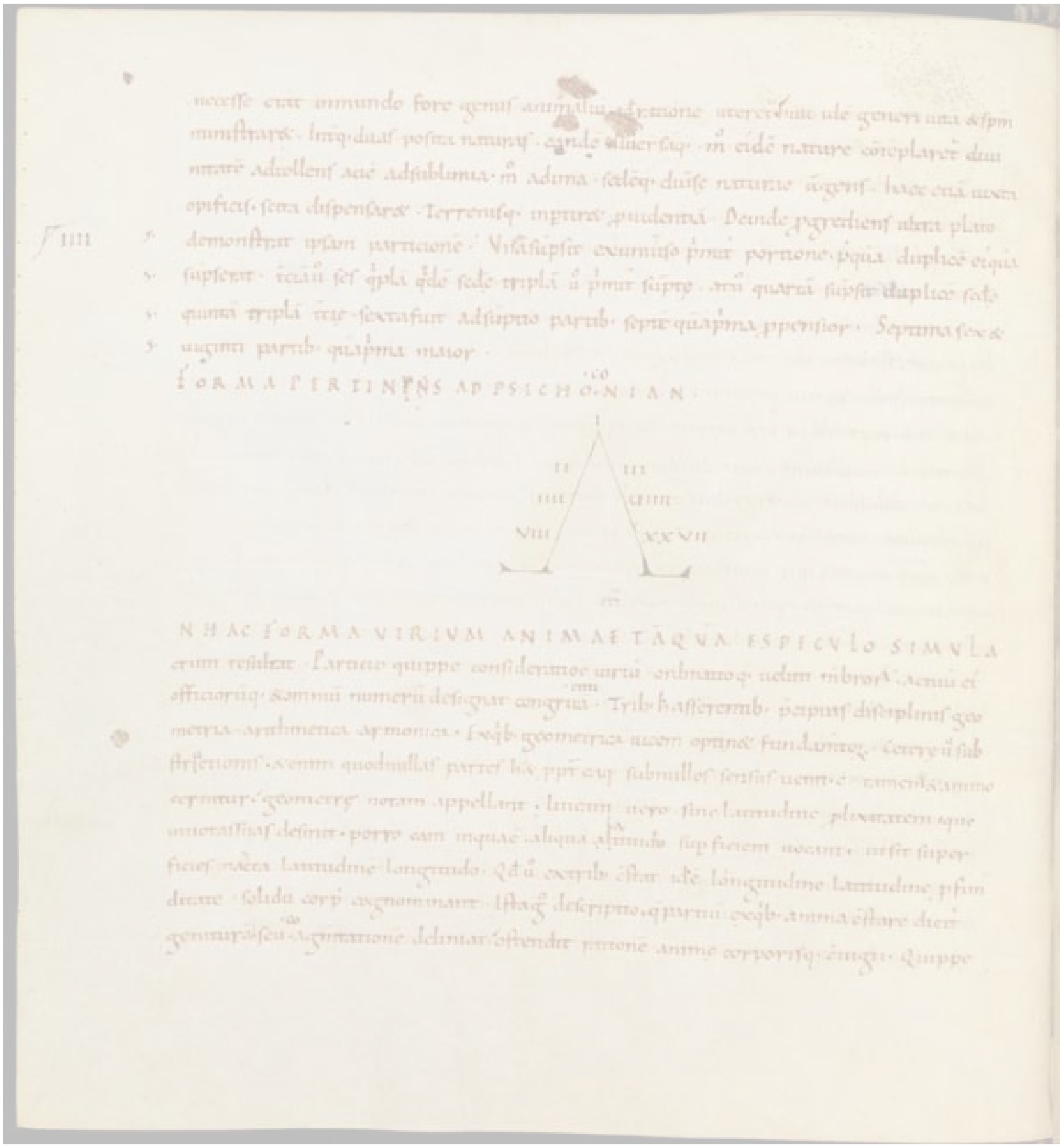

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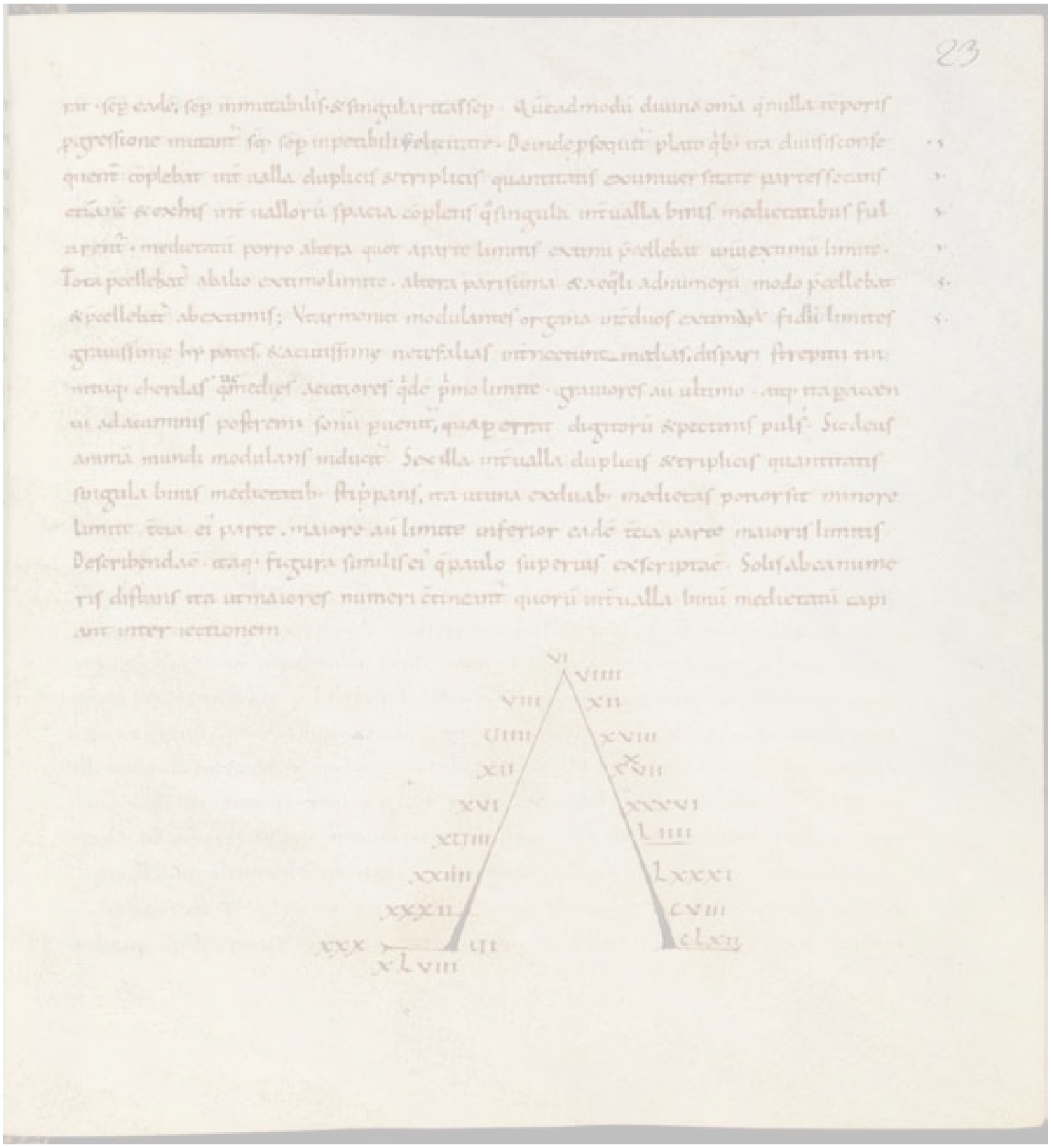

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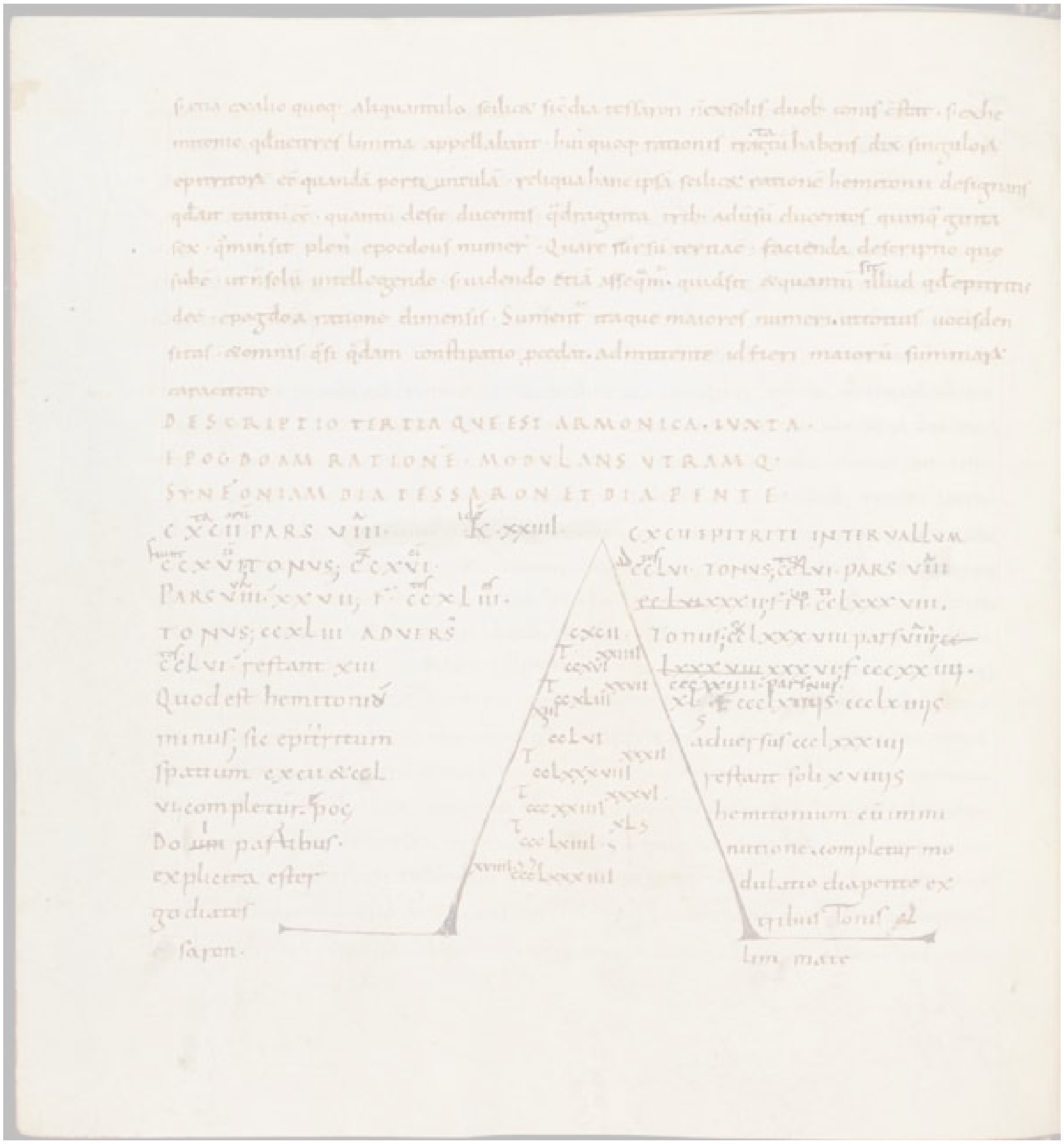
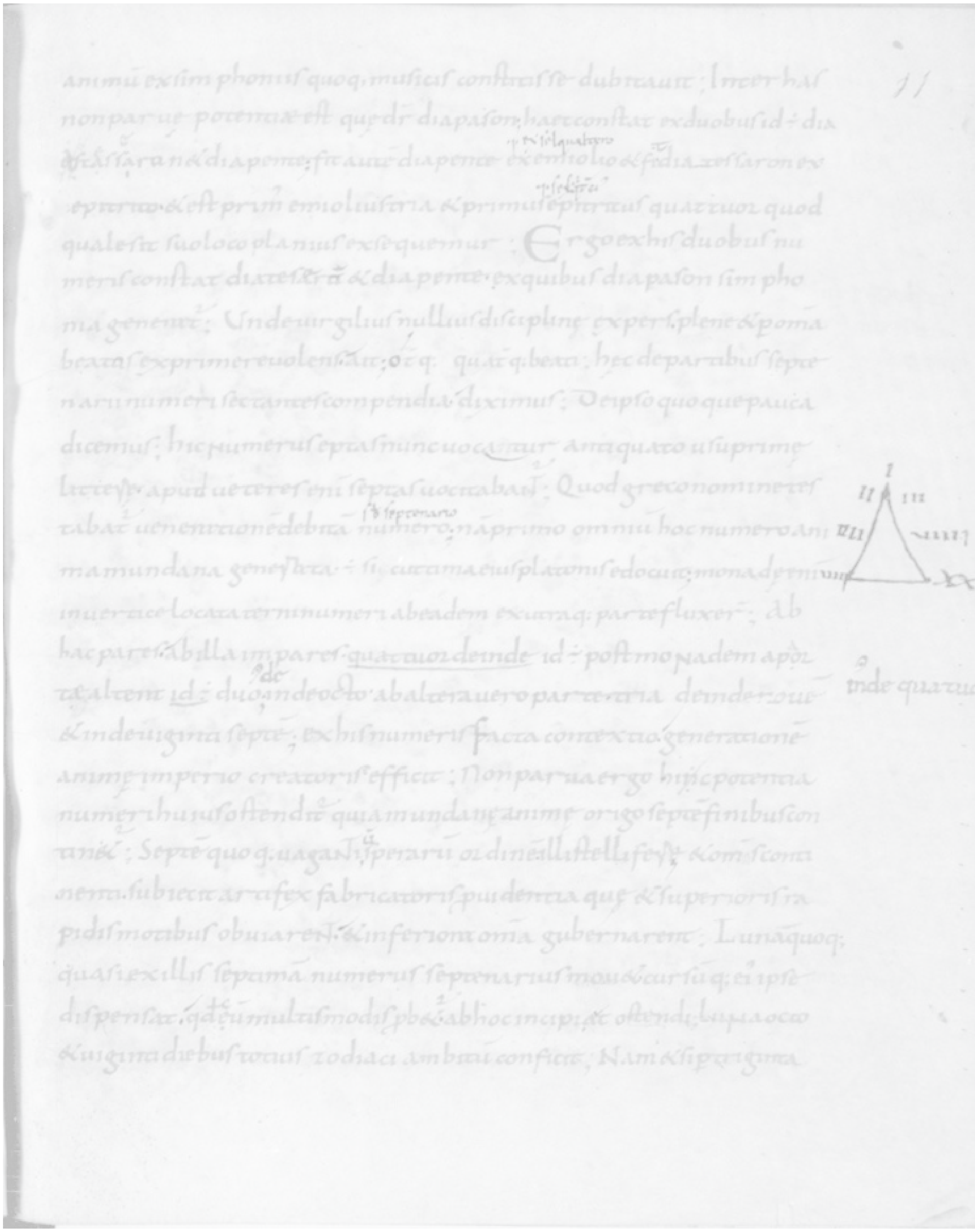
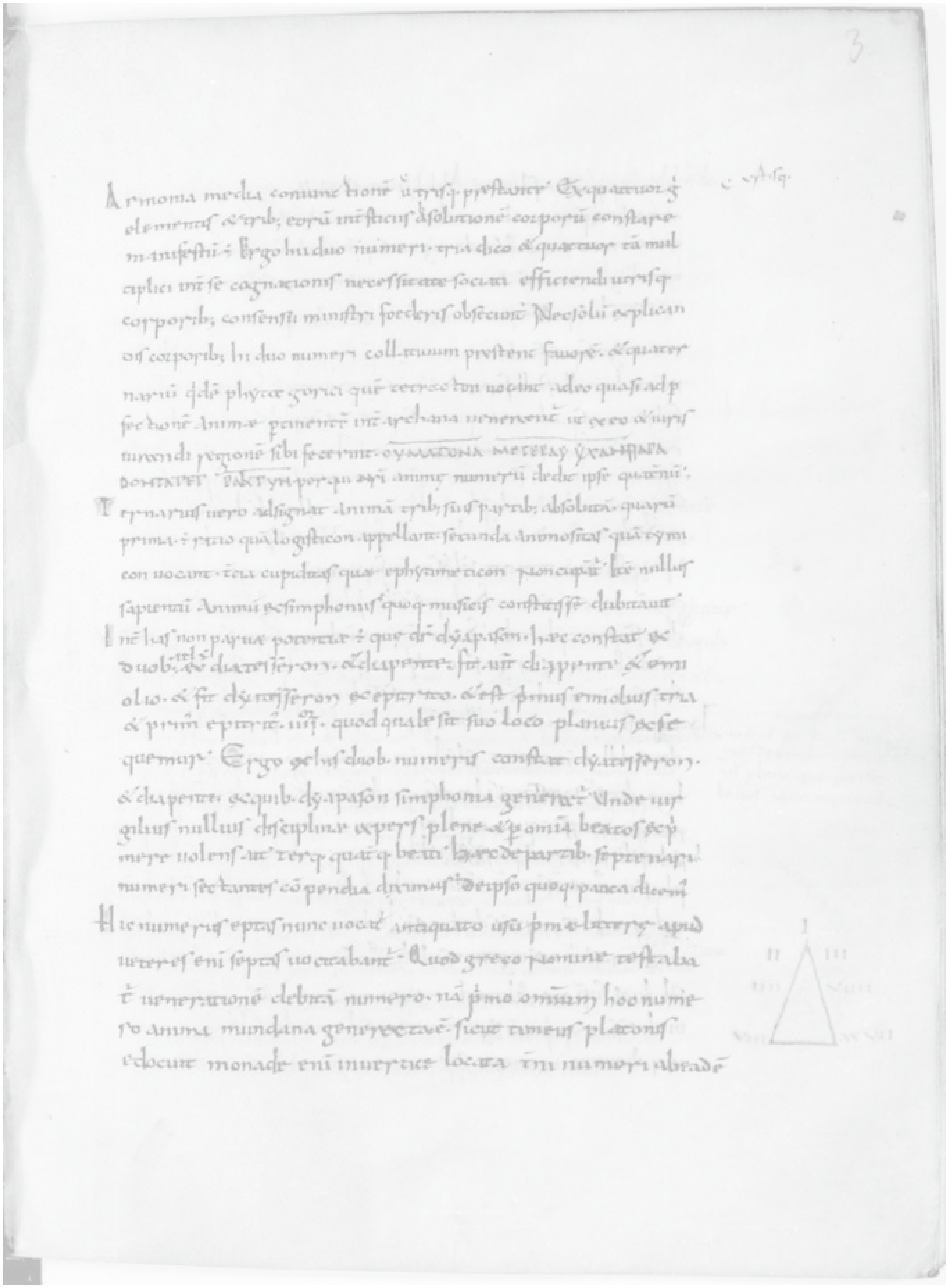


DESCRIPTIO TERTIA QUE EST ARMONICA IUXTA EPOGDOAM RATIONE MODULANS UTRAMQ SYNFONIAM DIATESSARON ET DIAPENTE

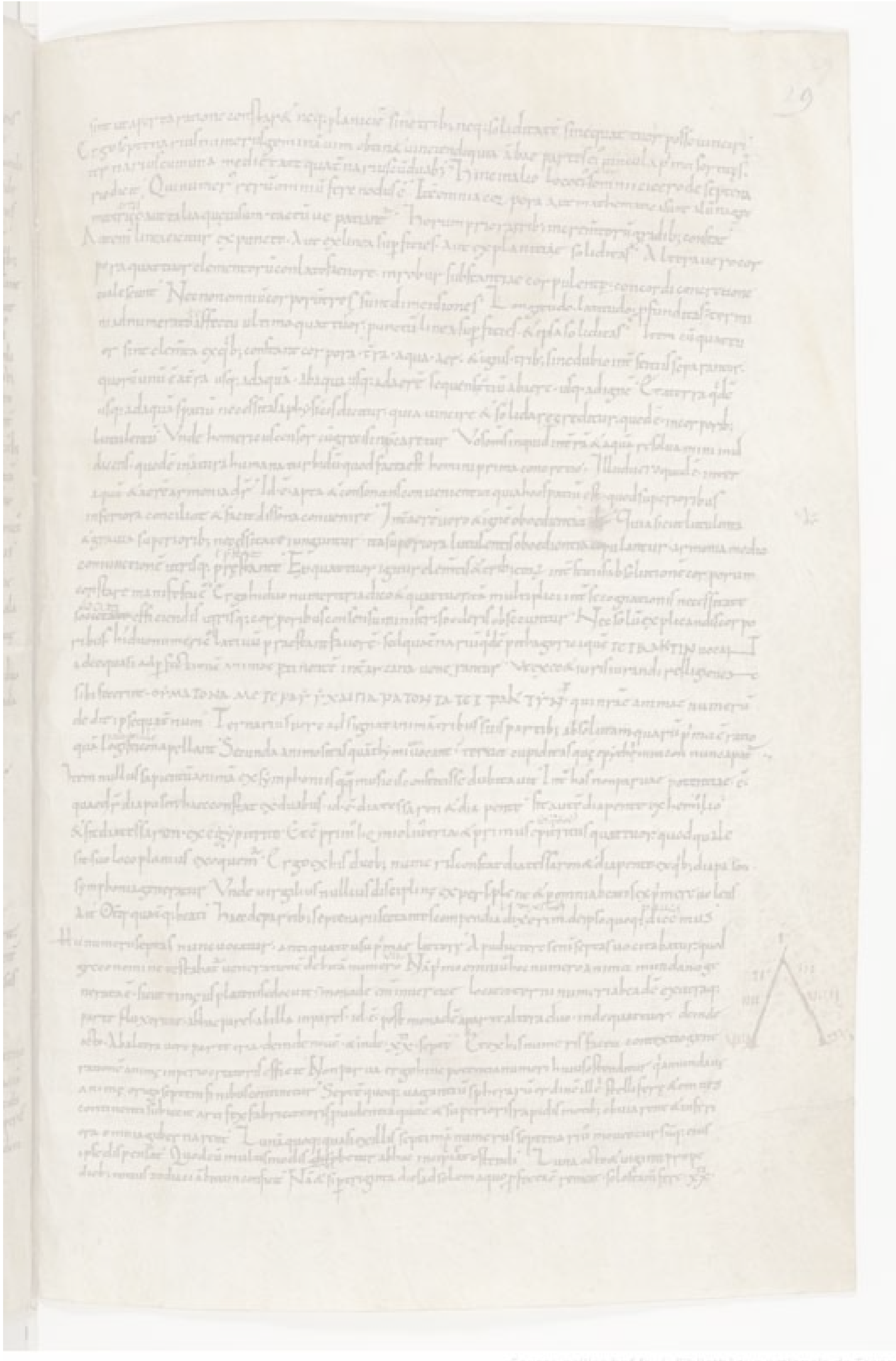
1.81 Bamberg, Staasbibliothek MS Msc.Class.18, fol. 25v.



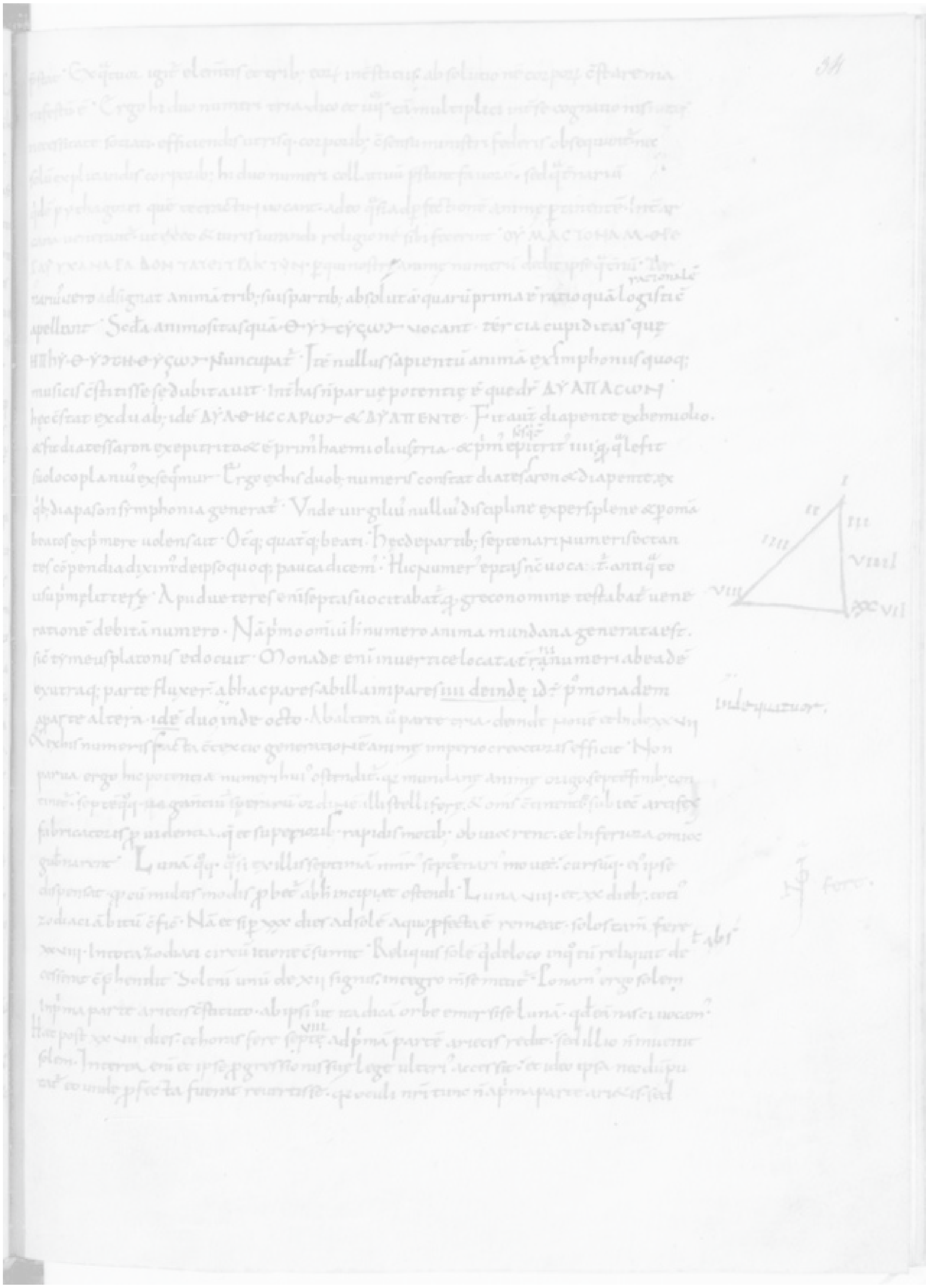
1.82 Paris, Bibliothèque nationale de France,MS lat. 16677, fol. 11r.



1.83 Paris, Bibliothèque nationale de France lat. MS 6620, fol. 3r.



1.84 Paris, Bibliothèque nationale de France lat. MS 8663, fol. 29r.



1.85 Paris, Bibliothèque nationale de France lat. MS 7299, fol. 34r.
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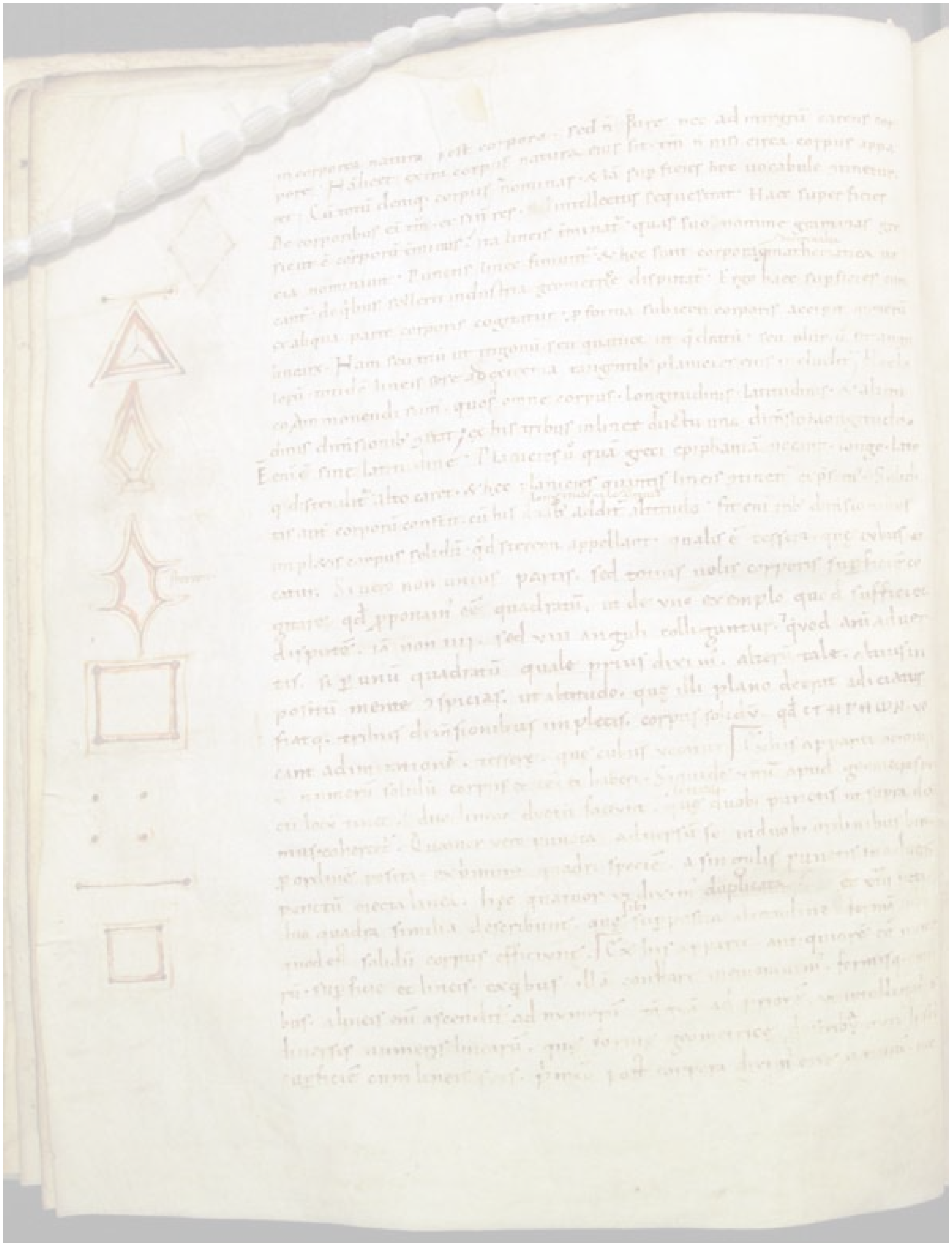
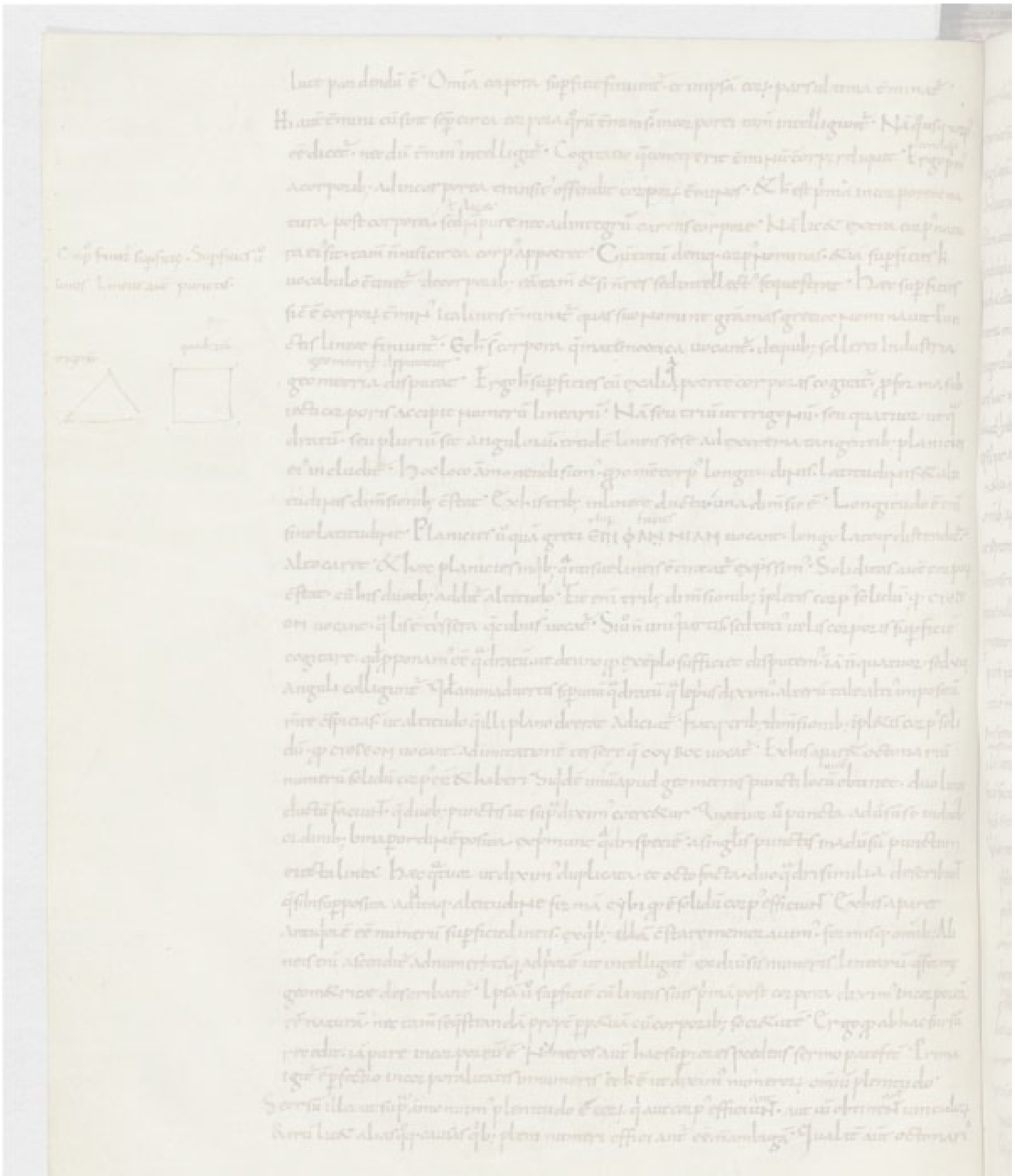
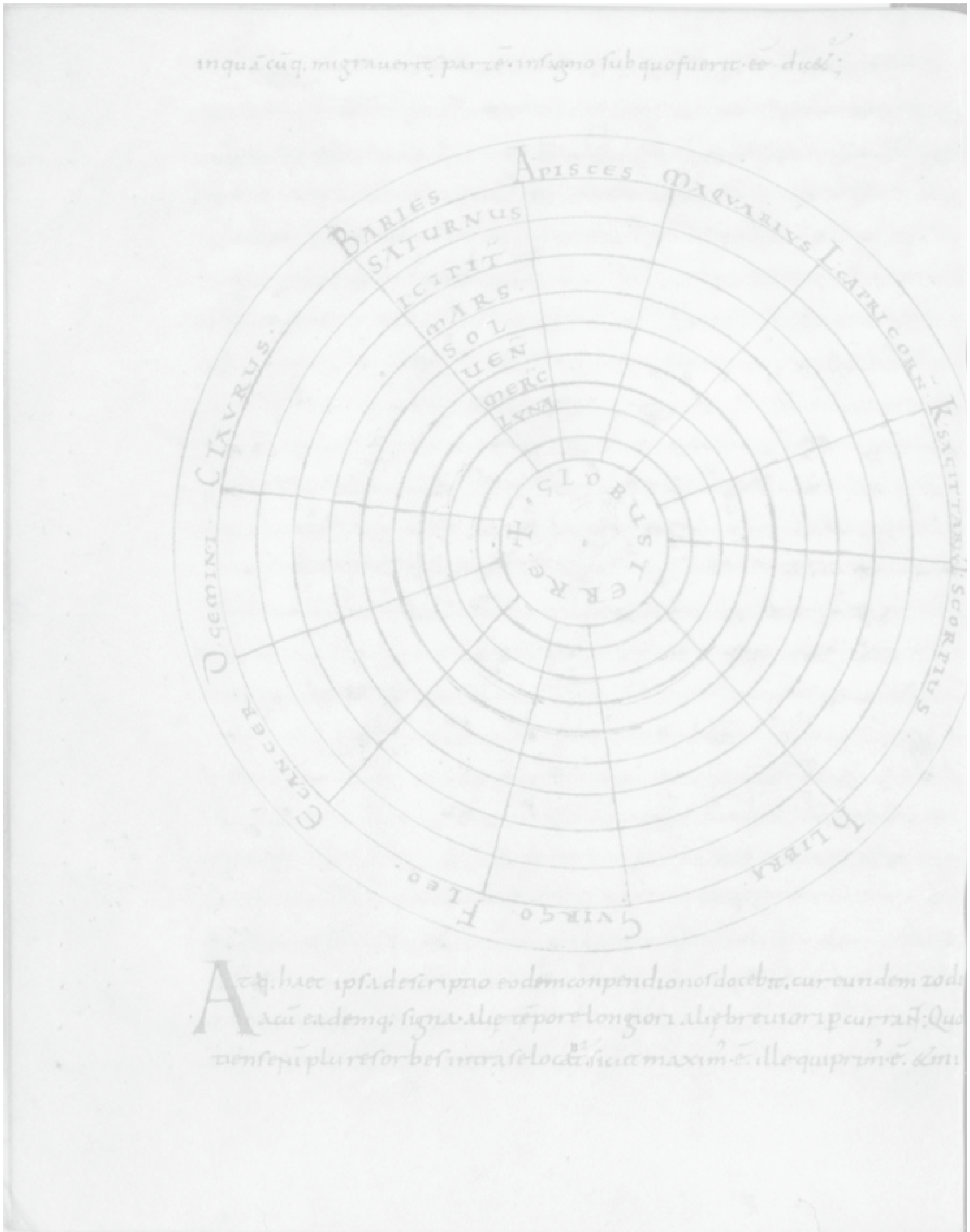


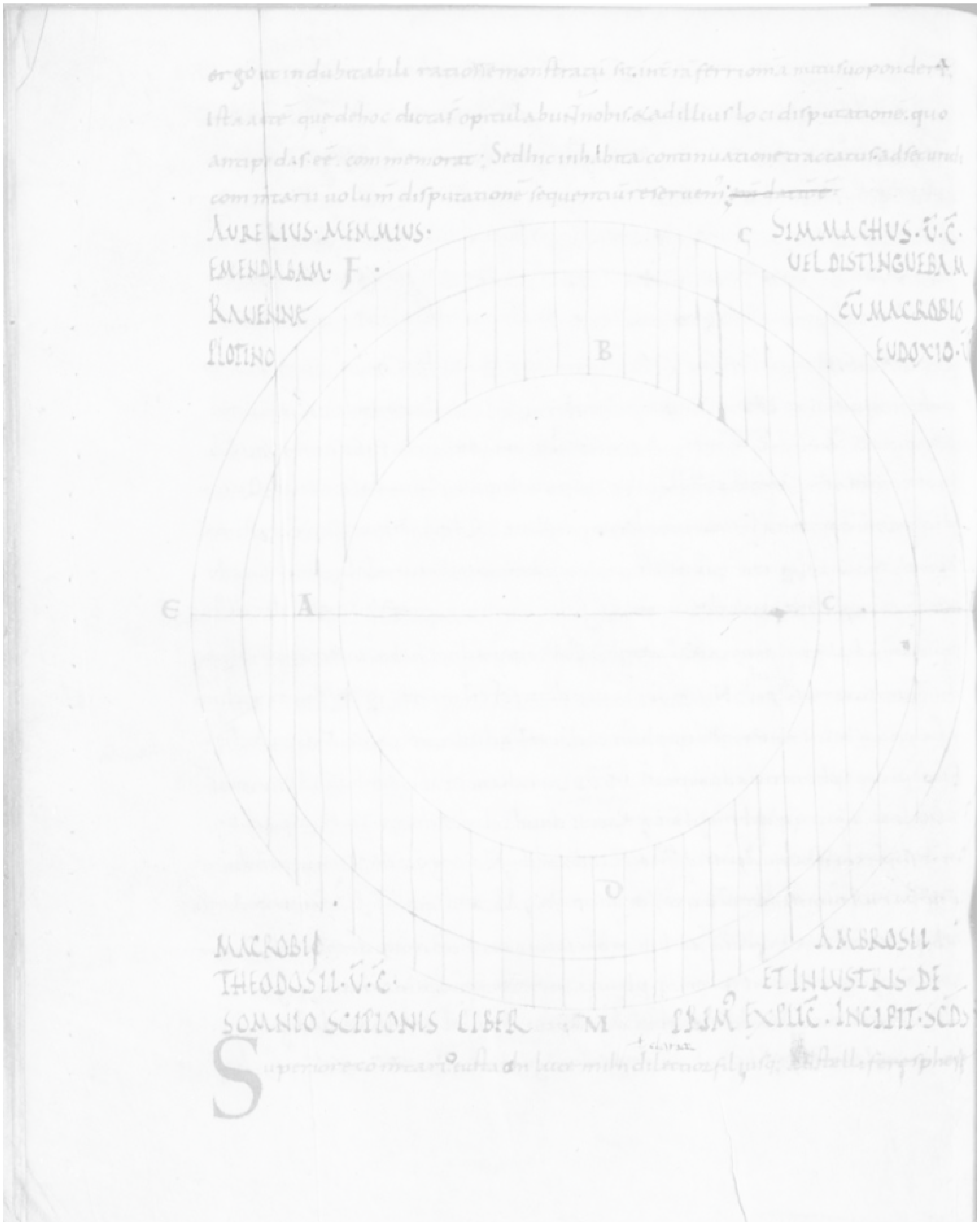
1.86 Oxford, Bodleian Library MS Auct. T. 2.27, fol. 14v.
Photo taken with permission of the Bodleian Library, Oxford University



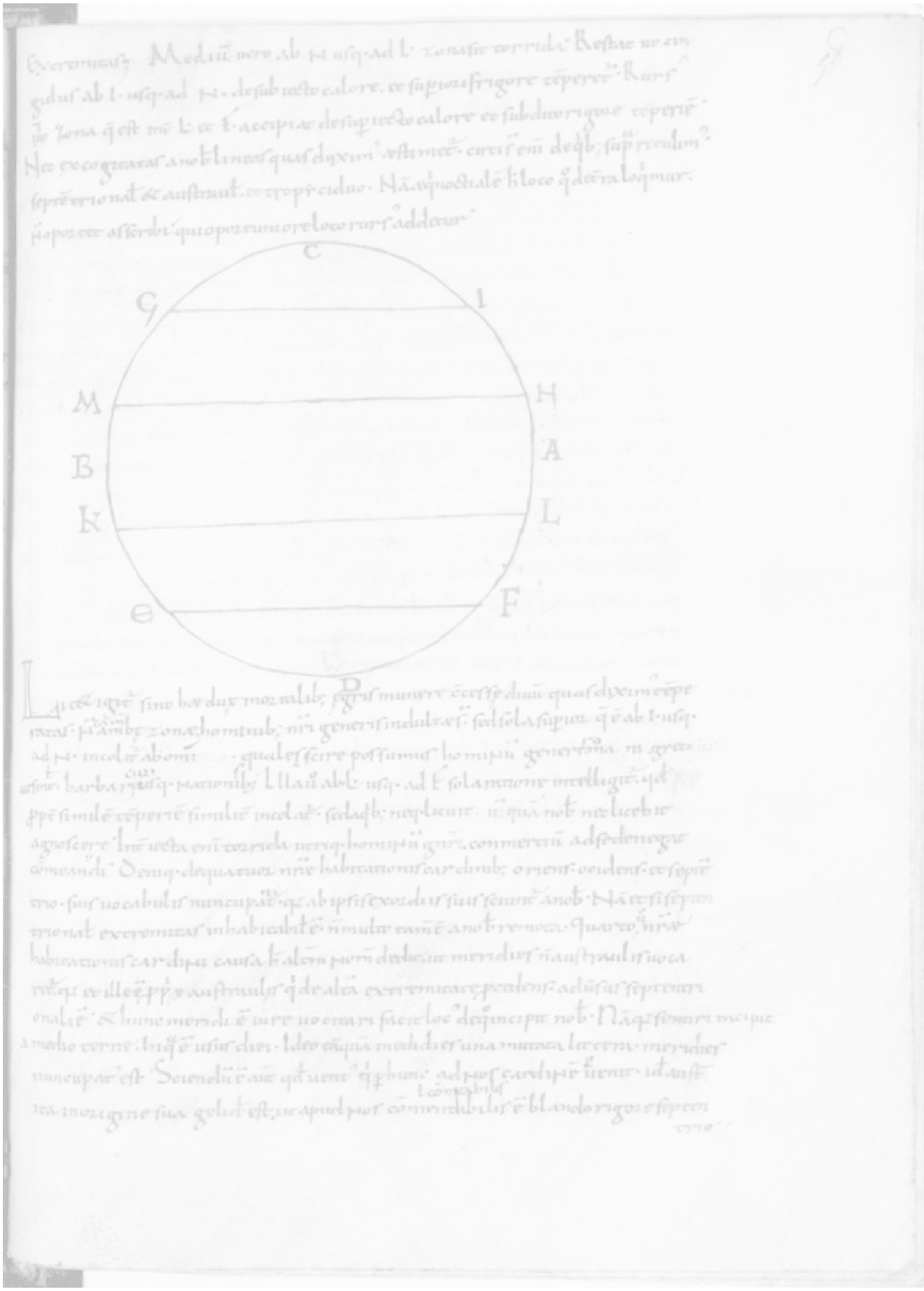
1.87 Paris, Bibliothèque nationale de France lat. MS 7299, fol. 33v.



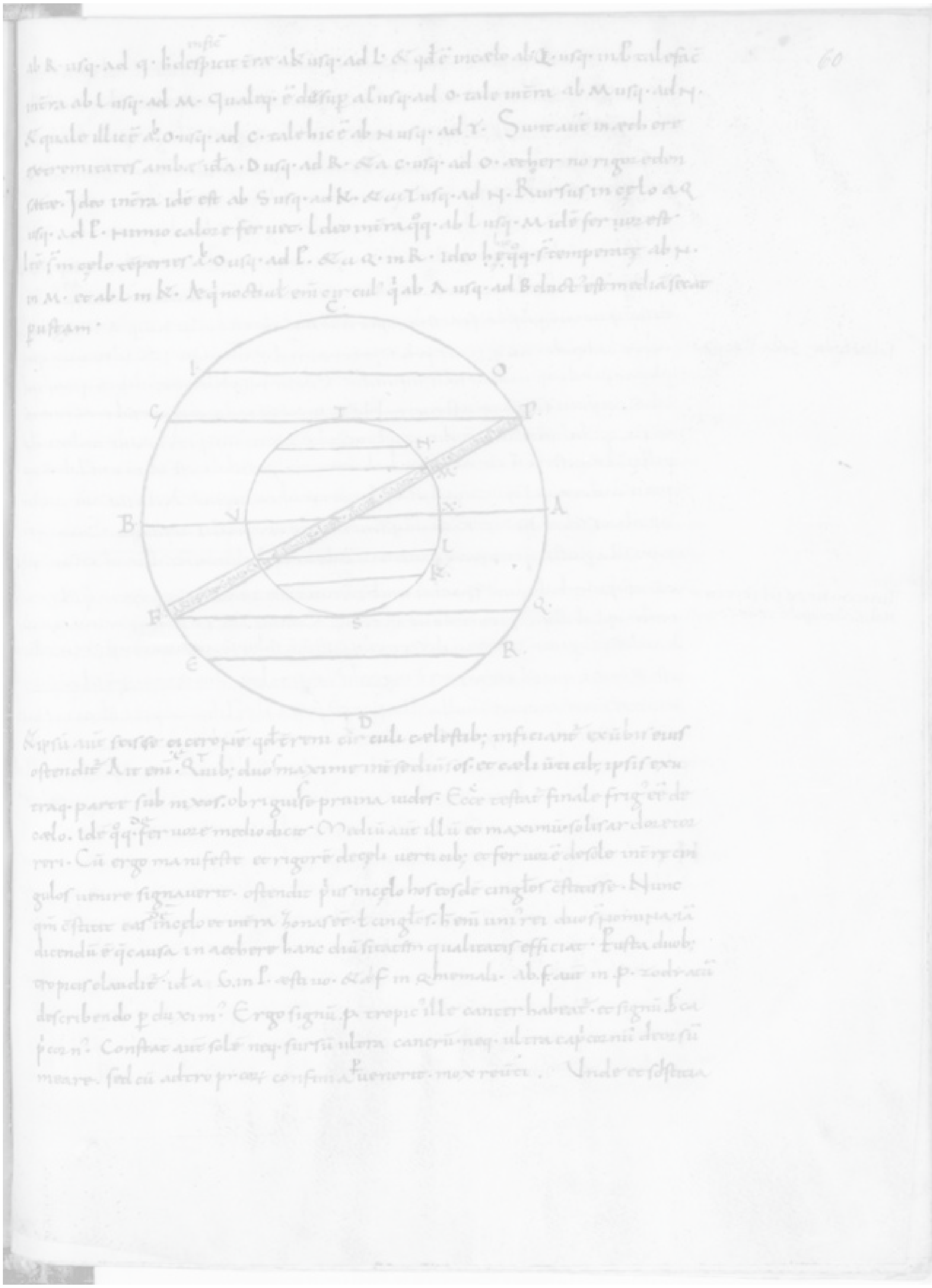
1.88 Paris, Bibliothèque nationale de France lat. MS 16677, fol. 37v.



1.89 Paris, Bibliothèque nationale de France lat. MS 16677, fol. 41v.



1.90 Paris, Bibliothèque nationale de France lat. MS 7299, fol. 58r.



1.91 Paris, Bibliothèque nationale de France, MS lat. 7299, fol. 60r.
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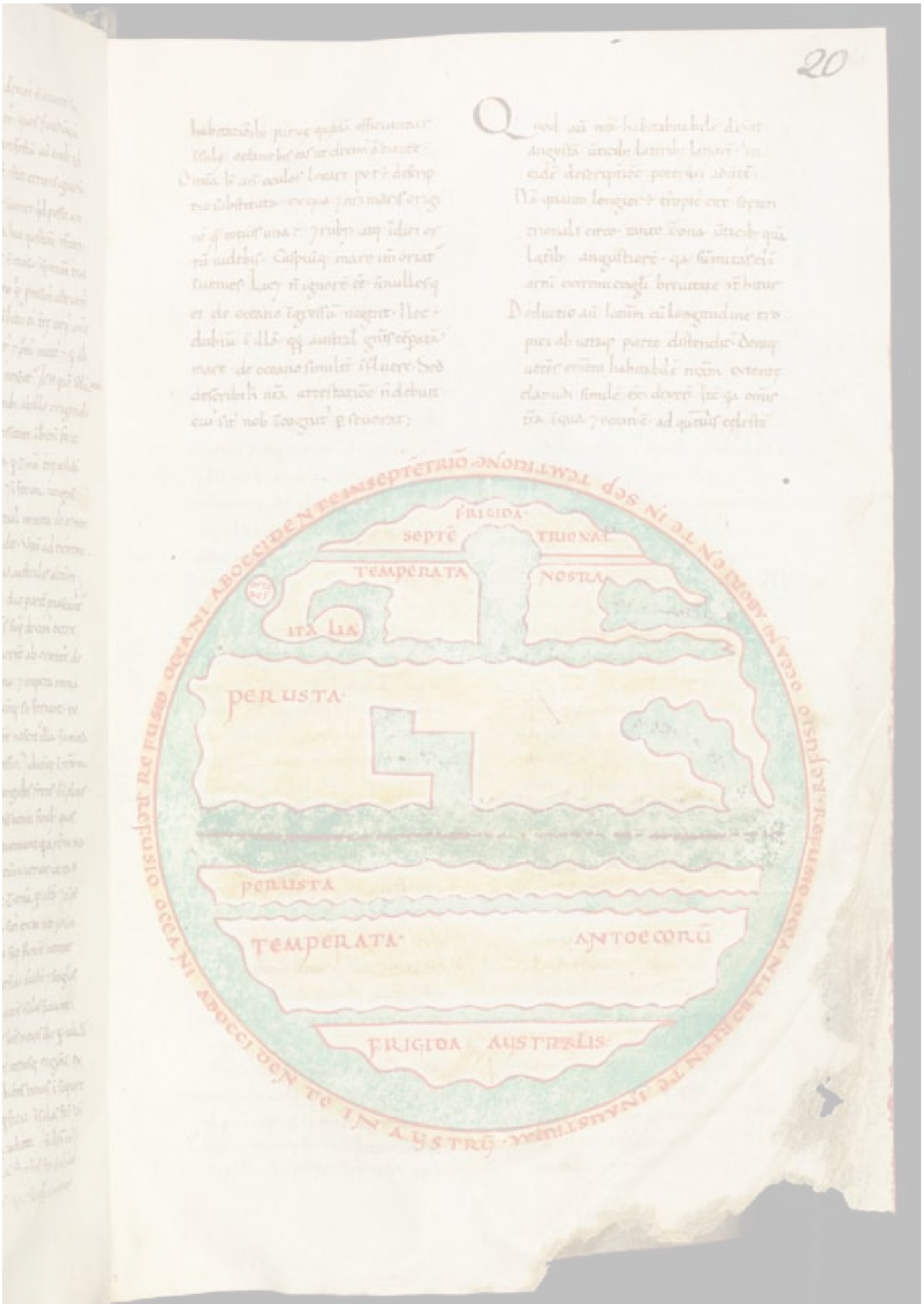

1.92 Bamberg, Staasbibliothek MS Msc.Class.38, fol. 20r.
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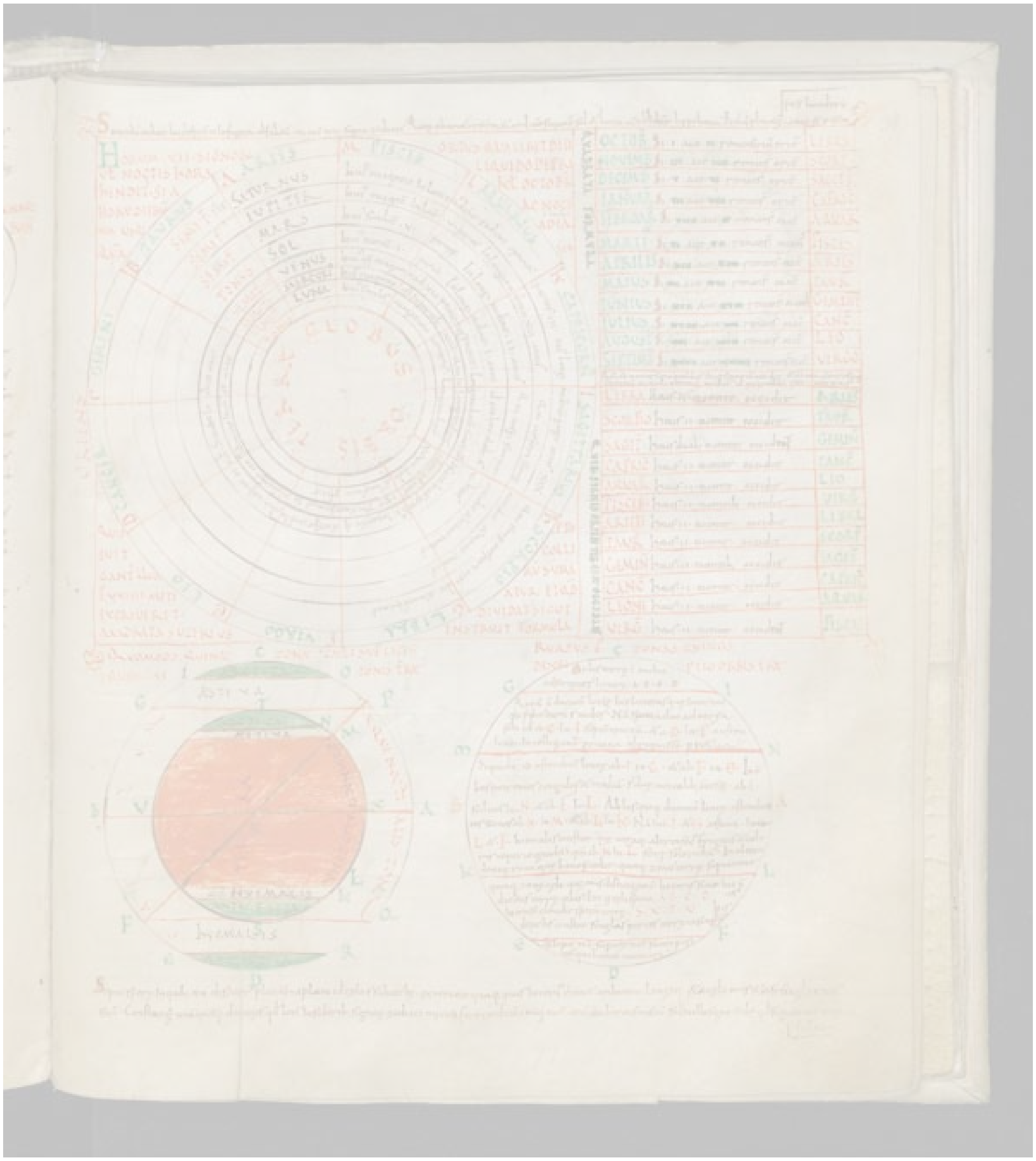
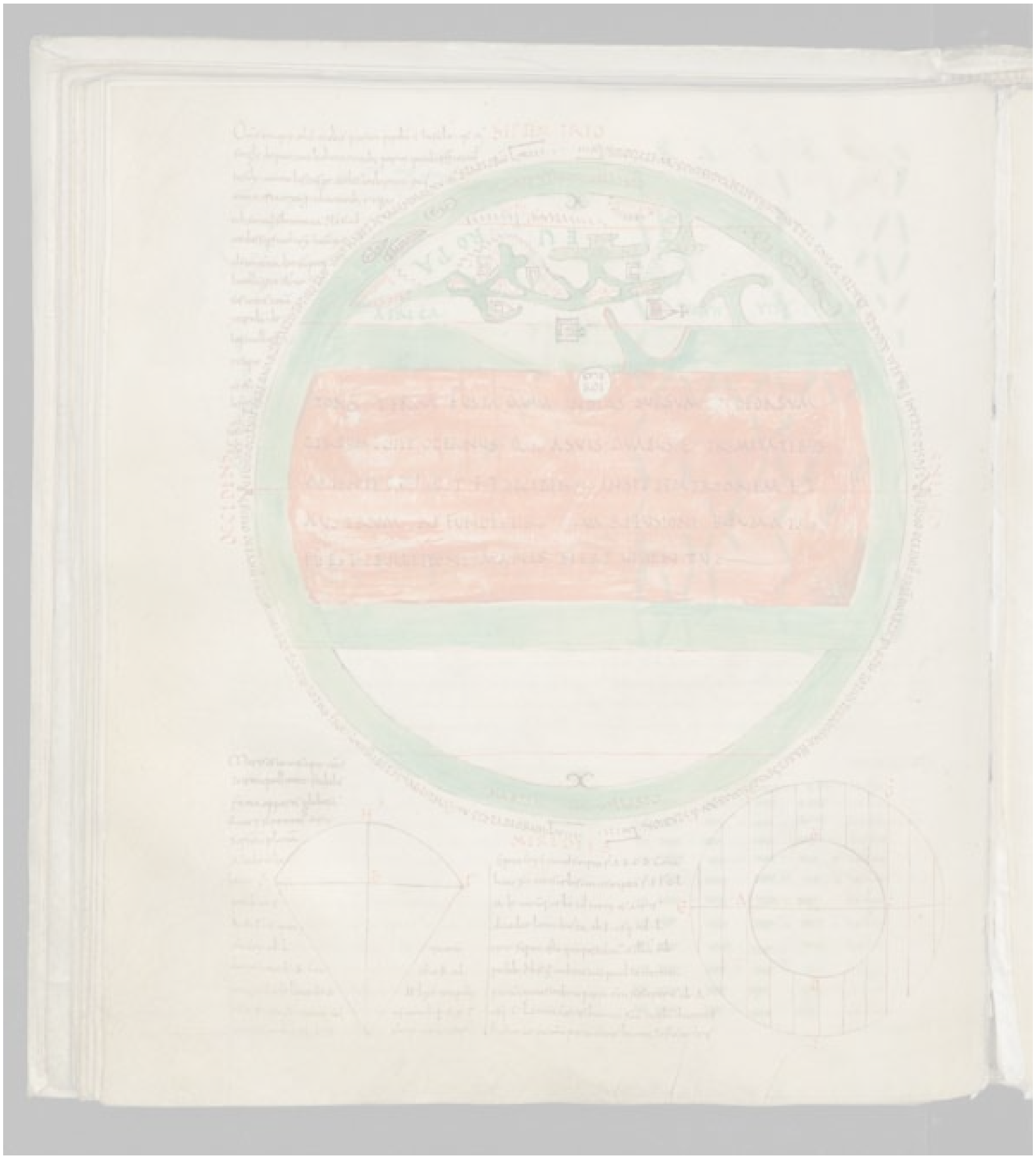
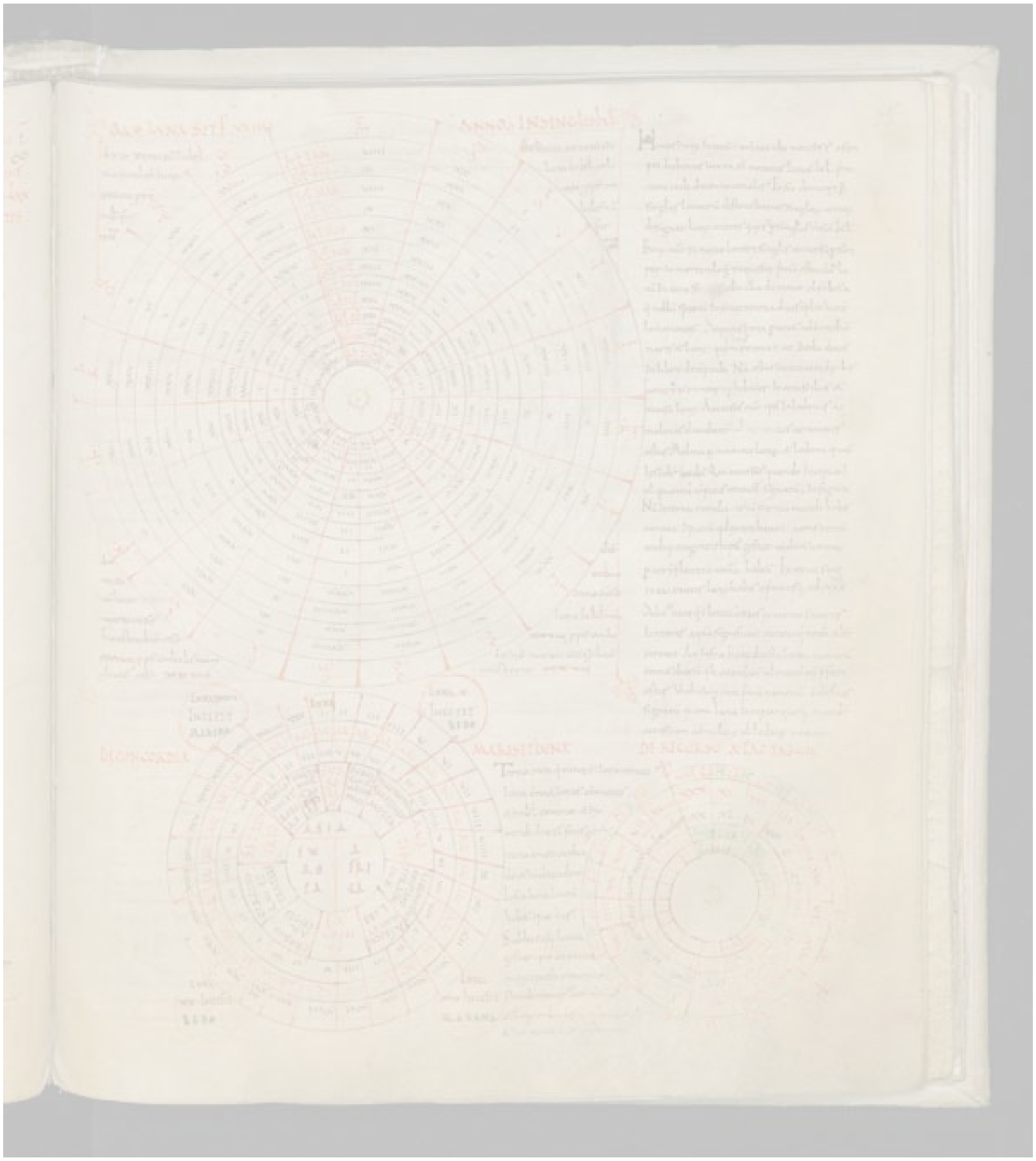


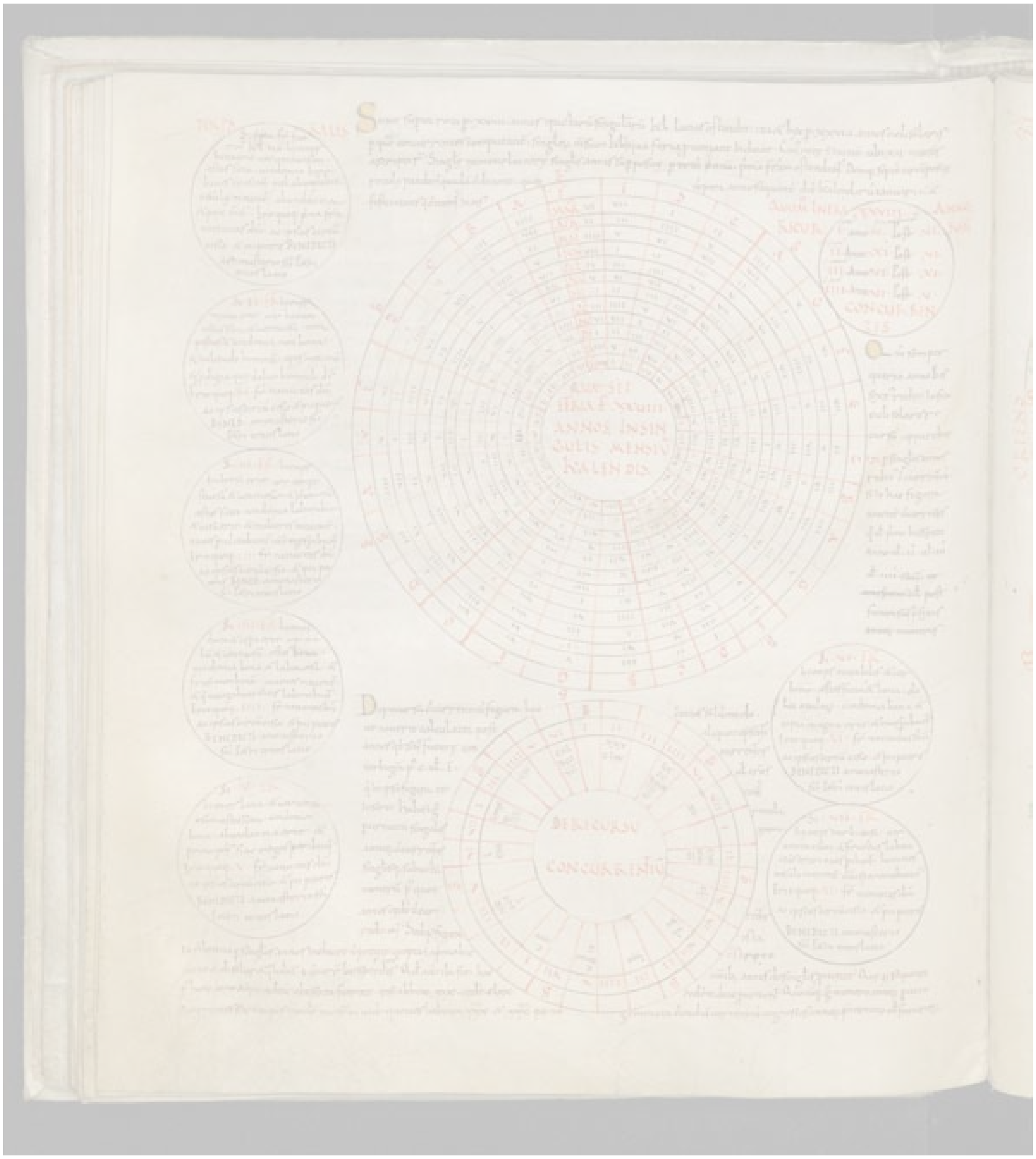
1.93 Berlin, Staatsbibliothek Cod. 138, fol. 36r.



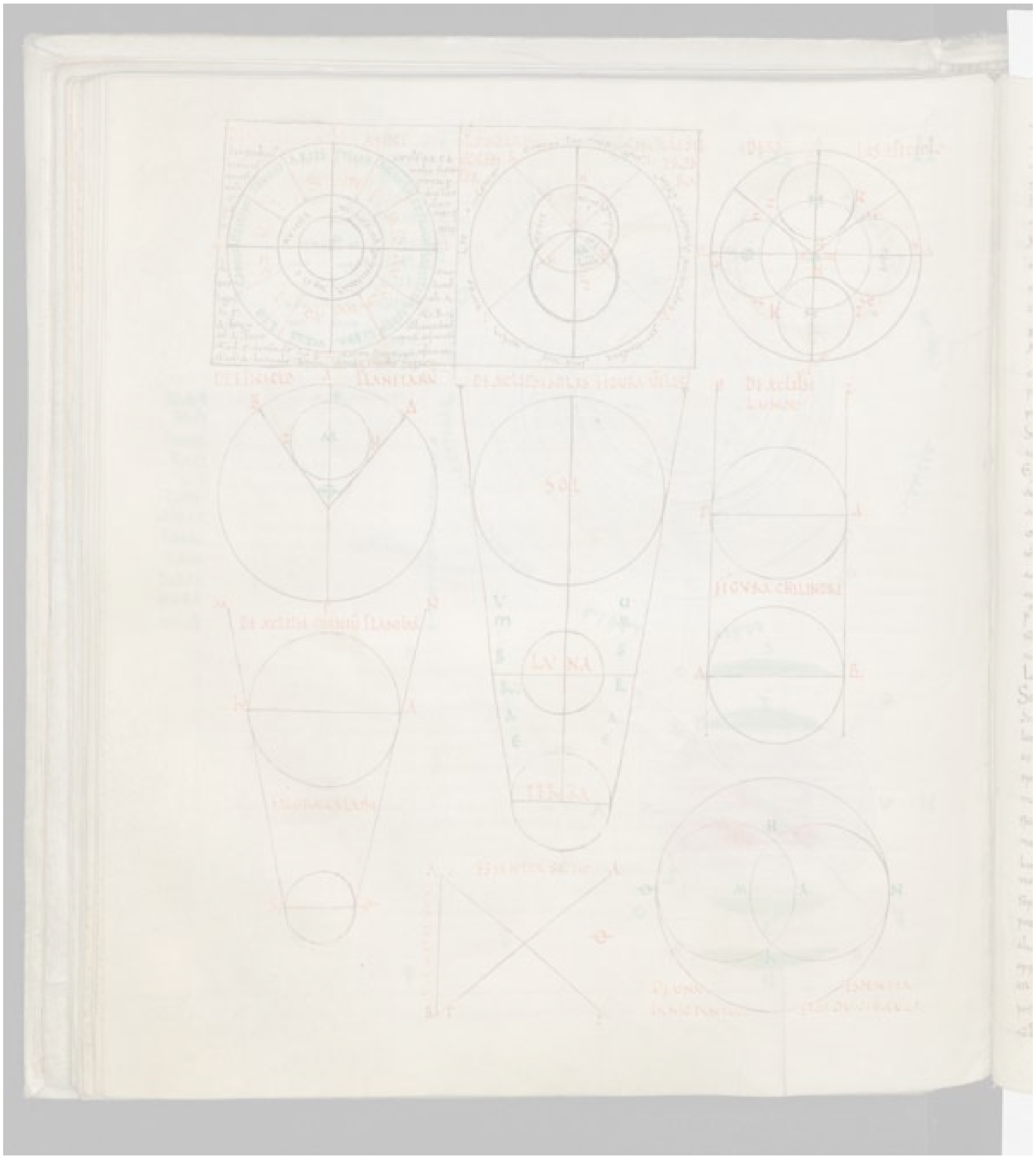
1.94 Berlin, Staatsbibliothek Cod. 138, fol. 99v.



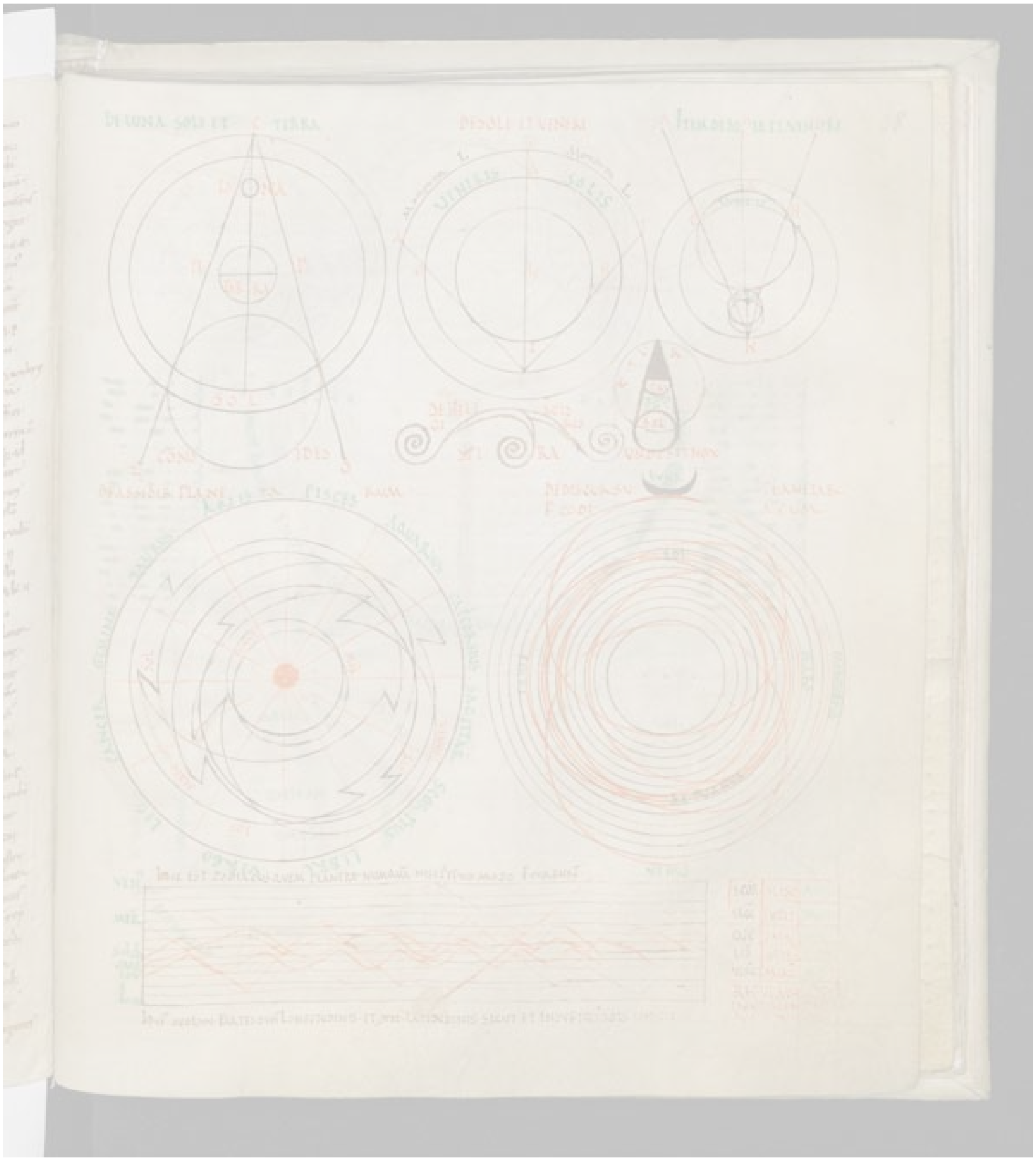
1.95 Berlin, Staatsbibliothek Cod. 138, fol. 35r.



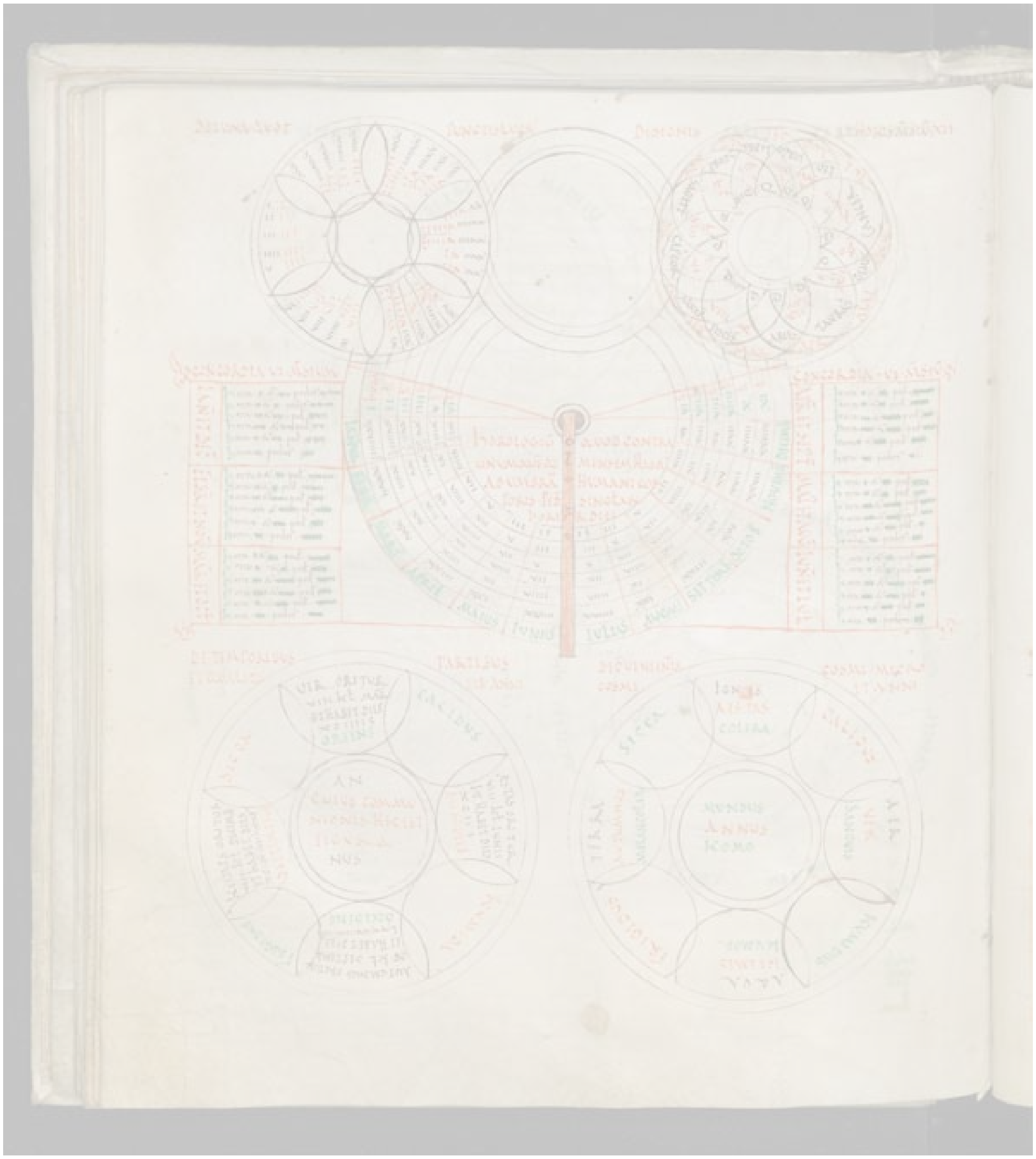
1.96 Berlin, Staatsbibliothek Cod. 138, fol. 35v.



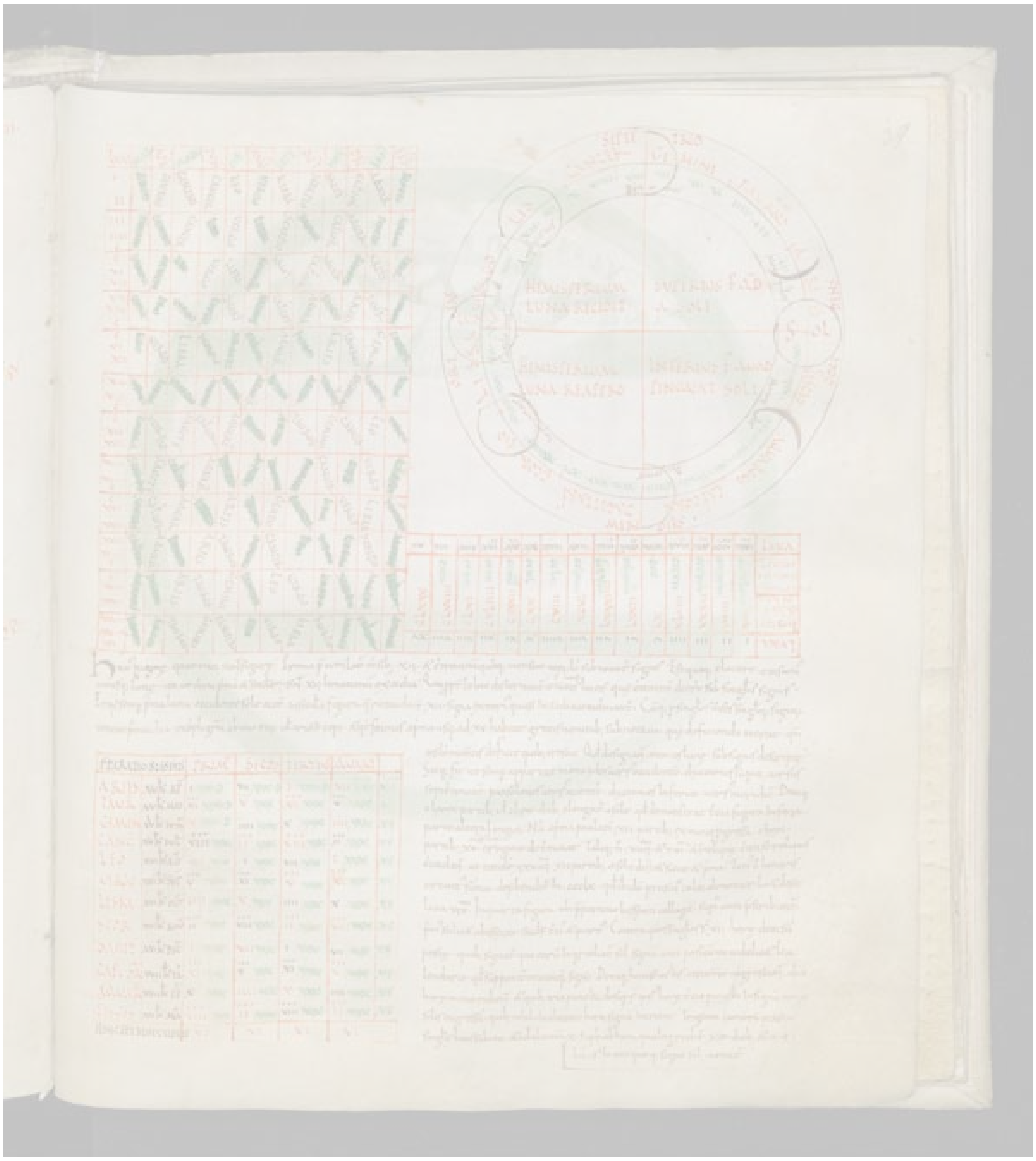
1.97 Berlin, Staatsbibliothek Cod. 138, fol. 36v.



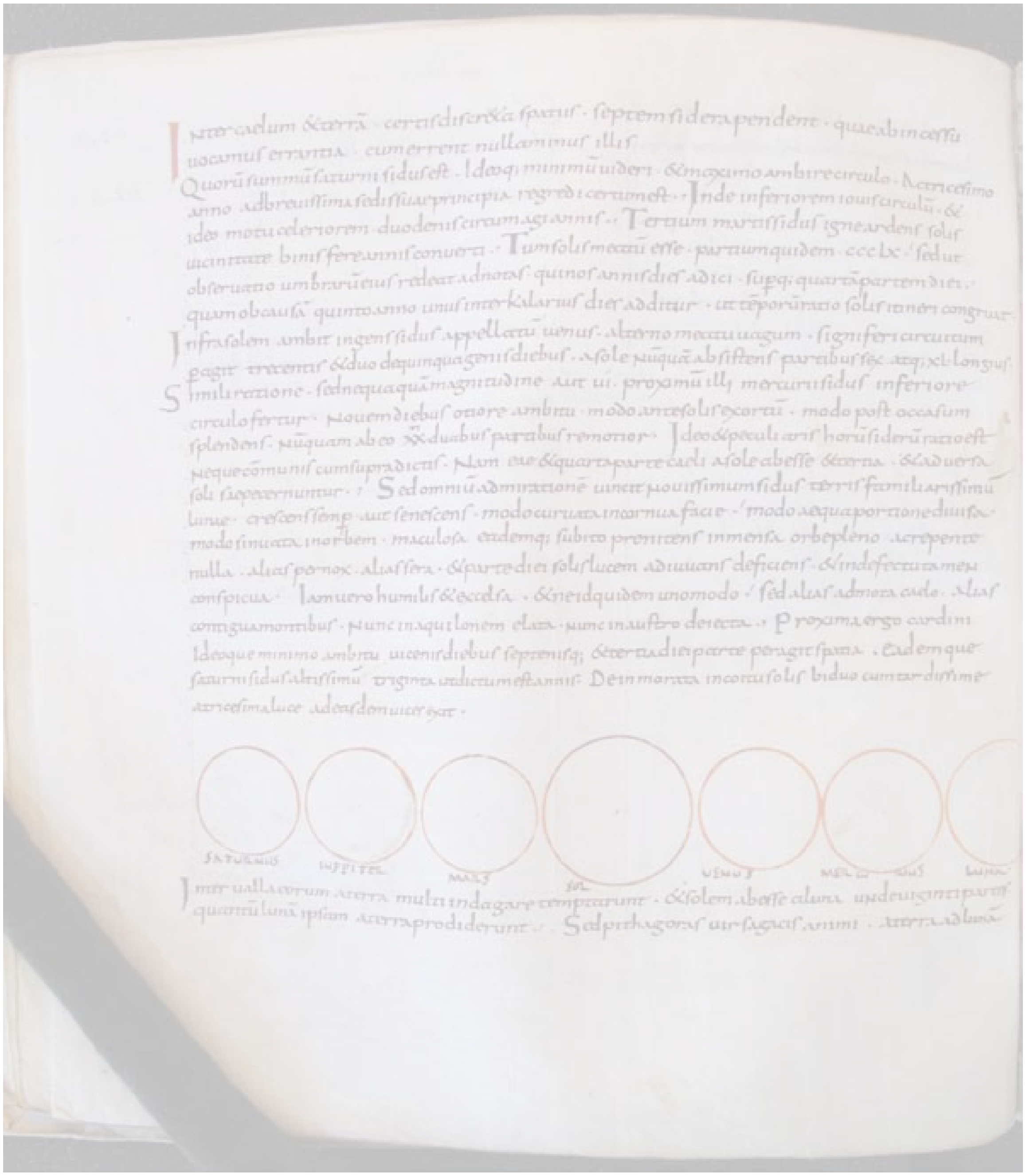
1.98 Berlin, Staatsbibliothek Cod. 138, fol. 38r.



1.99 Berlin, Staatsbibliothek Cod. 138, fol. 38v.



1.100 Berlin, Staatsbibliothek Cod. 138, fol. 39r.



1.101 Bern, Burgerbibliothek Cod. 347, fol. 22v.
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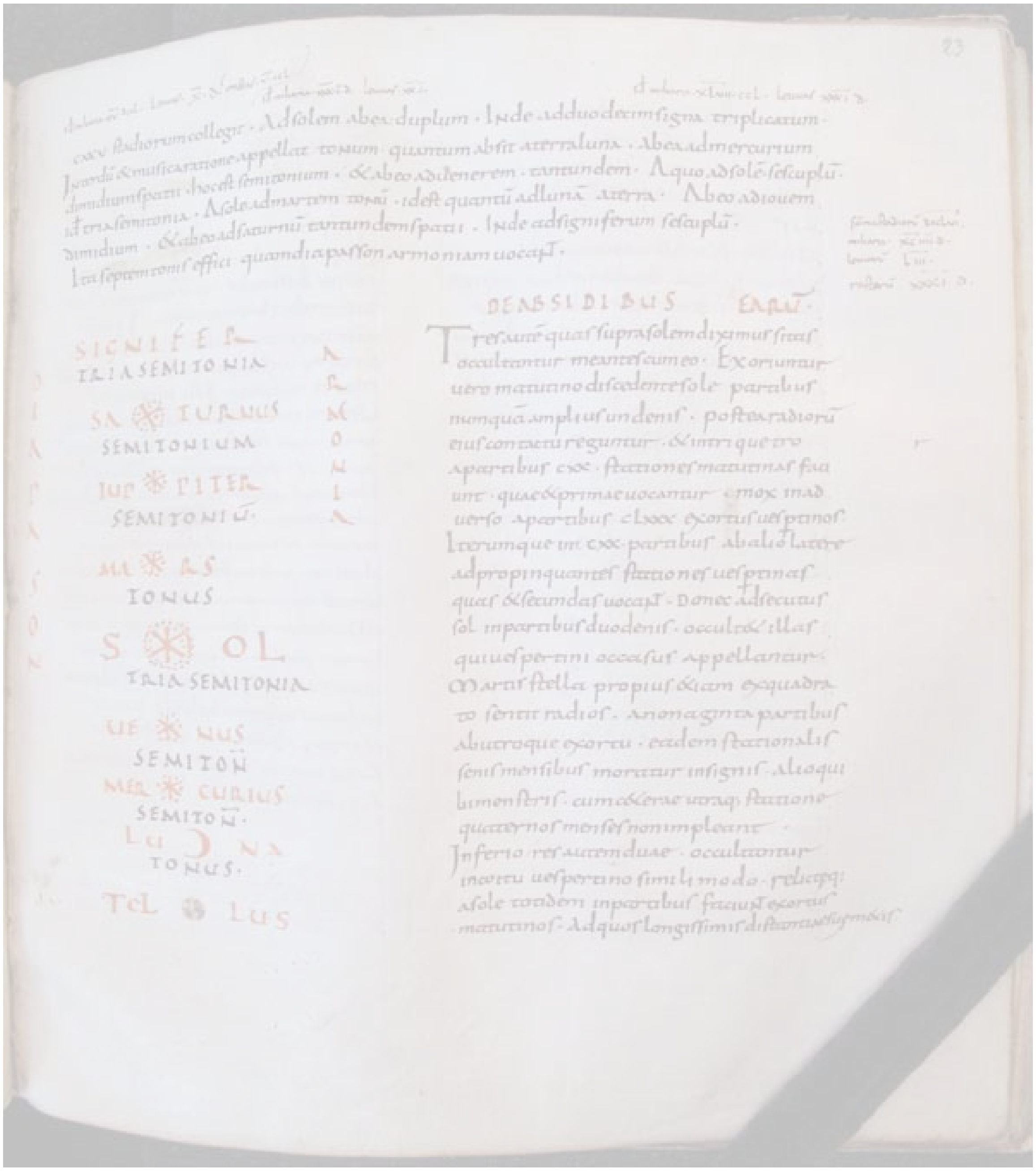
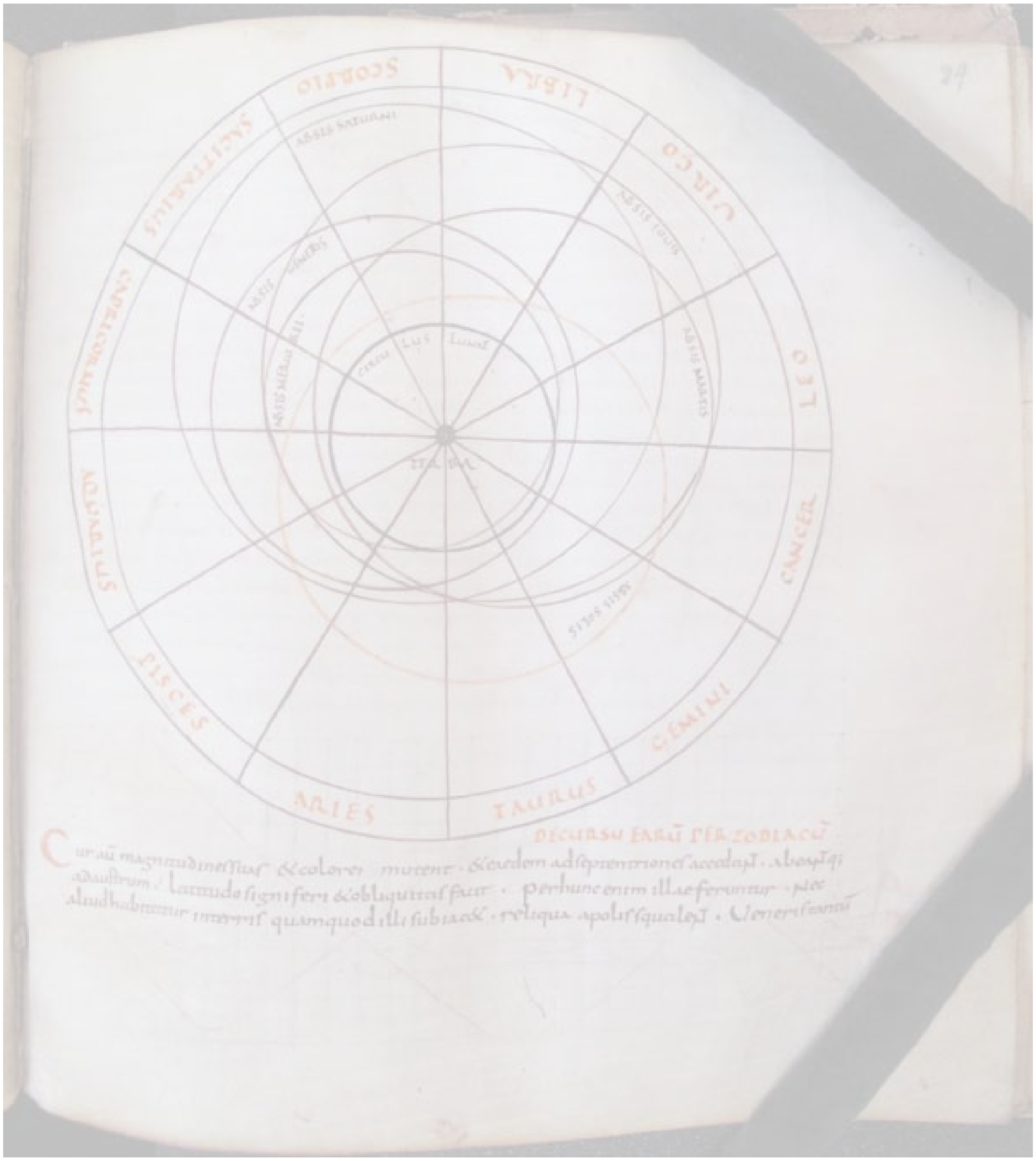
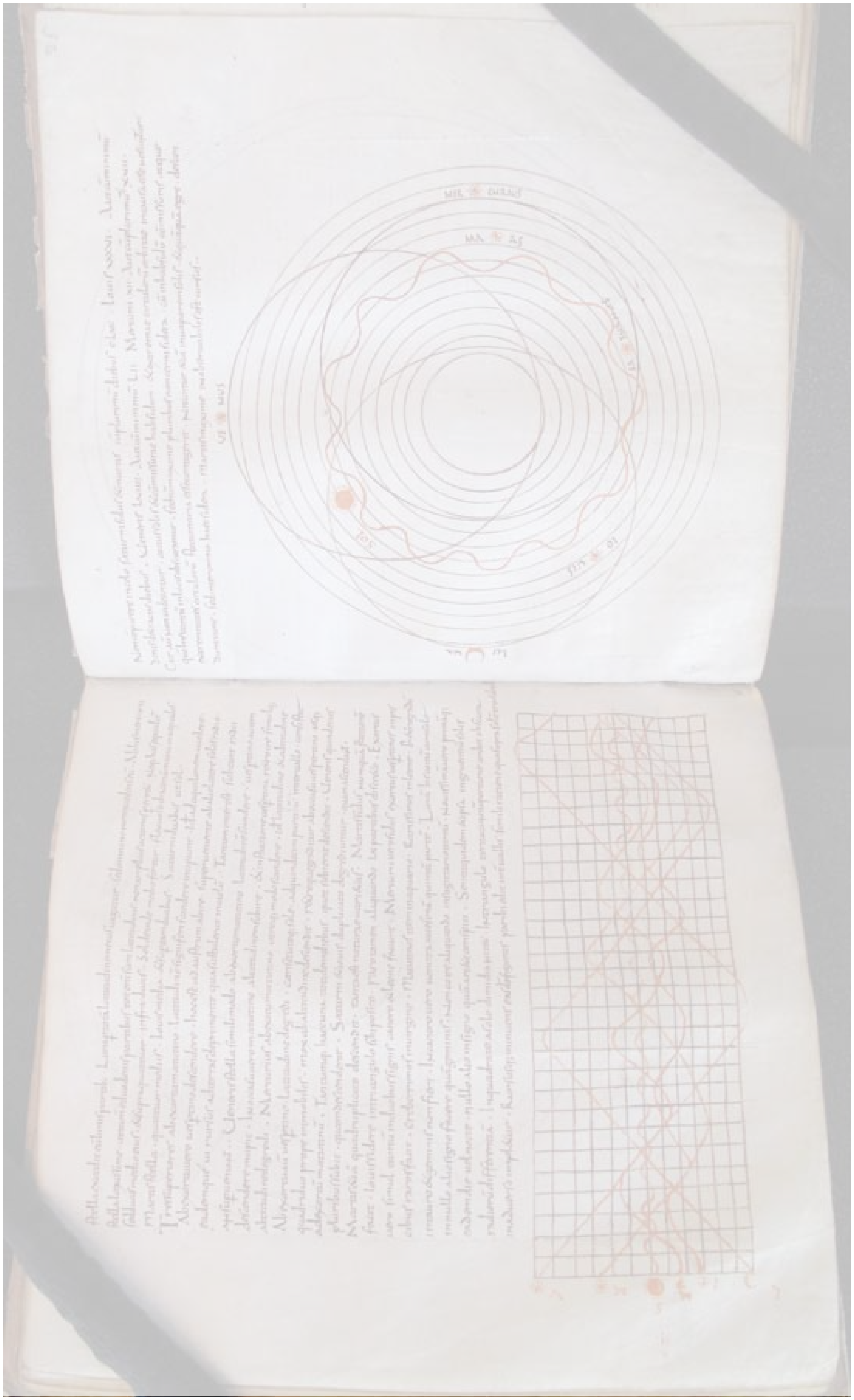


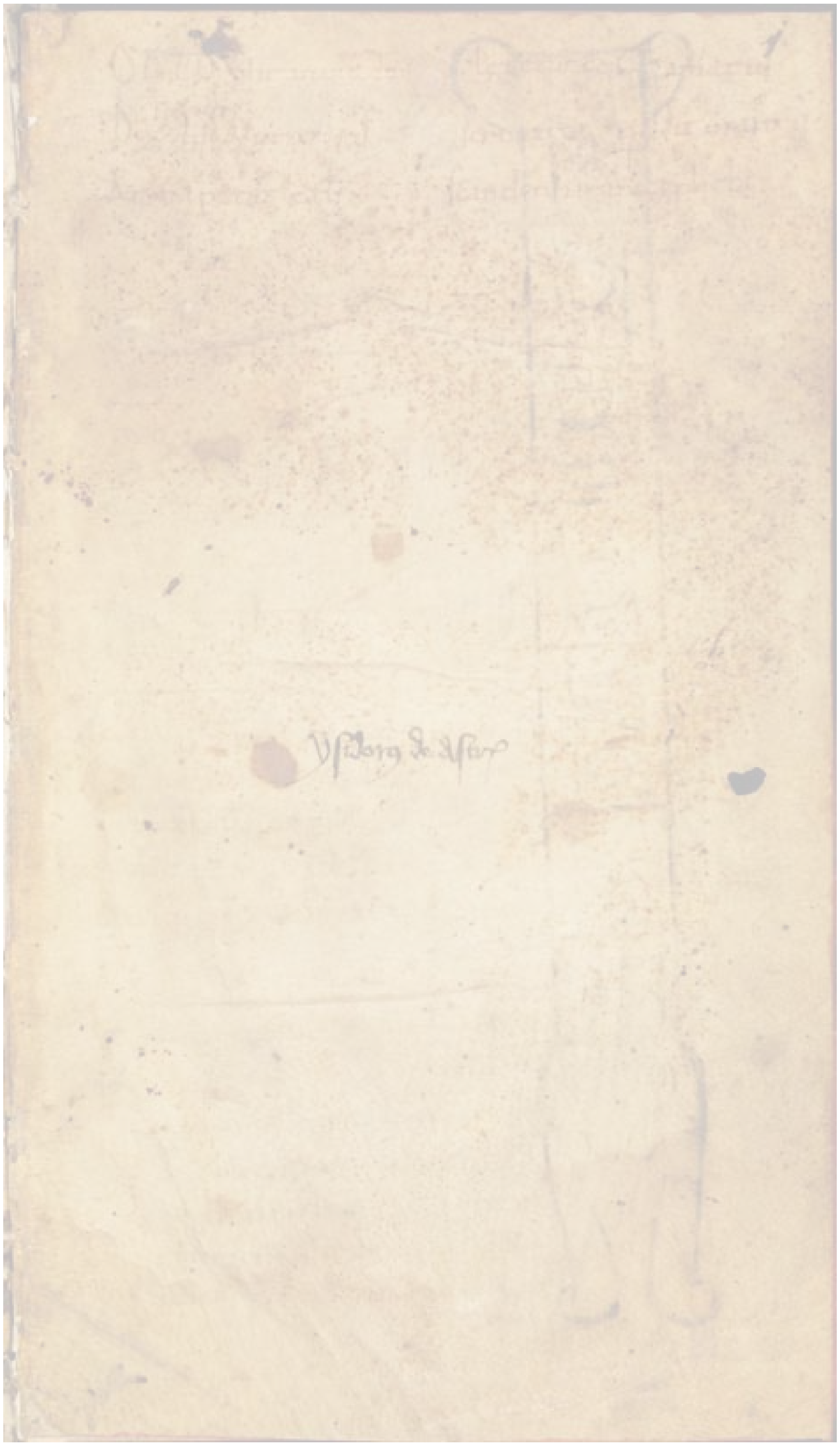
1.102 Bern, Burgerbibliothek Cod. 347, fol. 23r.
Photo taken with permission of the Burgerbibliothek Bern



1.103 Bern, Burgerbibliothek Cod. 347, fol. 24r.
Photo taken with permission of the Burgerbibliothek Bern

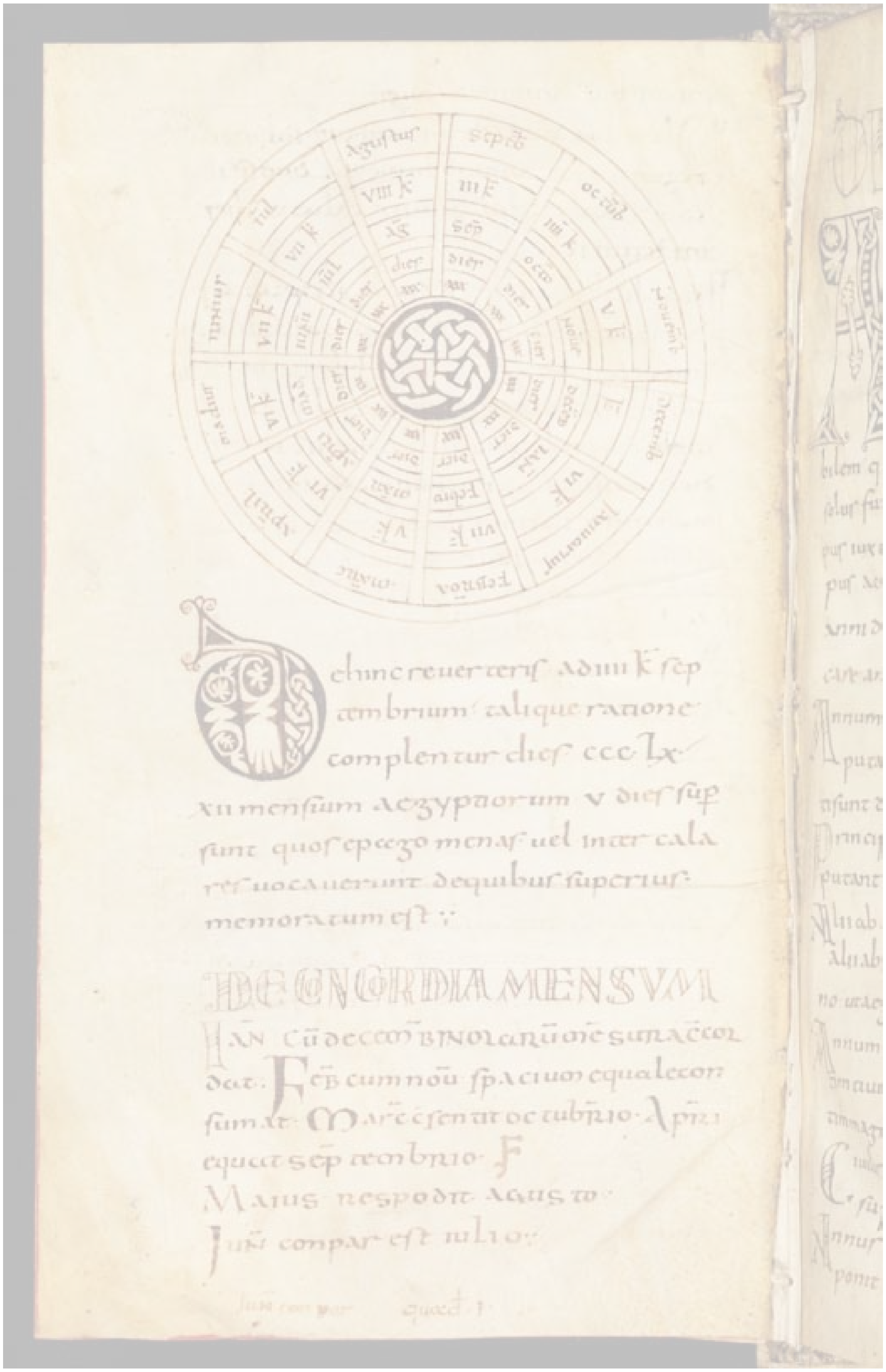


1.104 Bern, Burgerbibliothek Cod. 347, fol. 24v–25r. Photo taken with permission of the Burgerbibliothek Bern

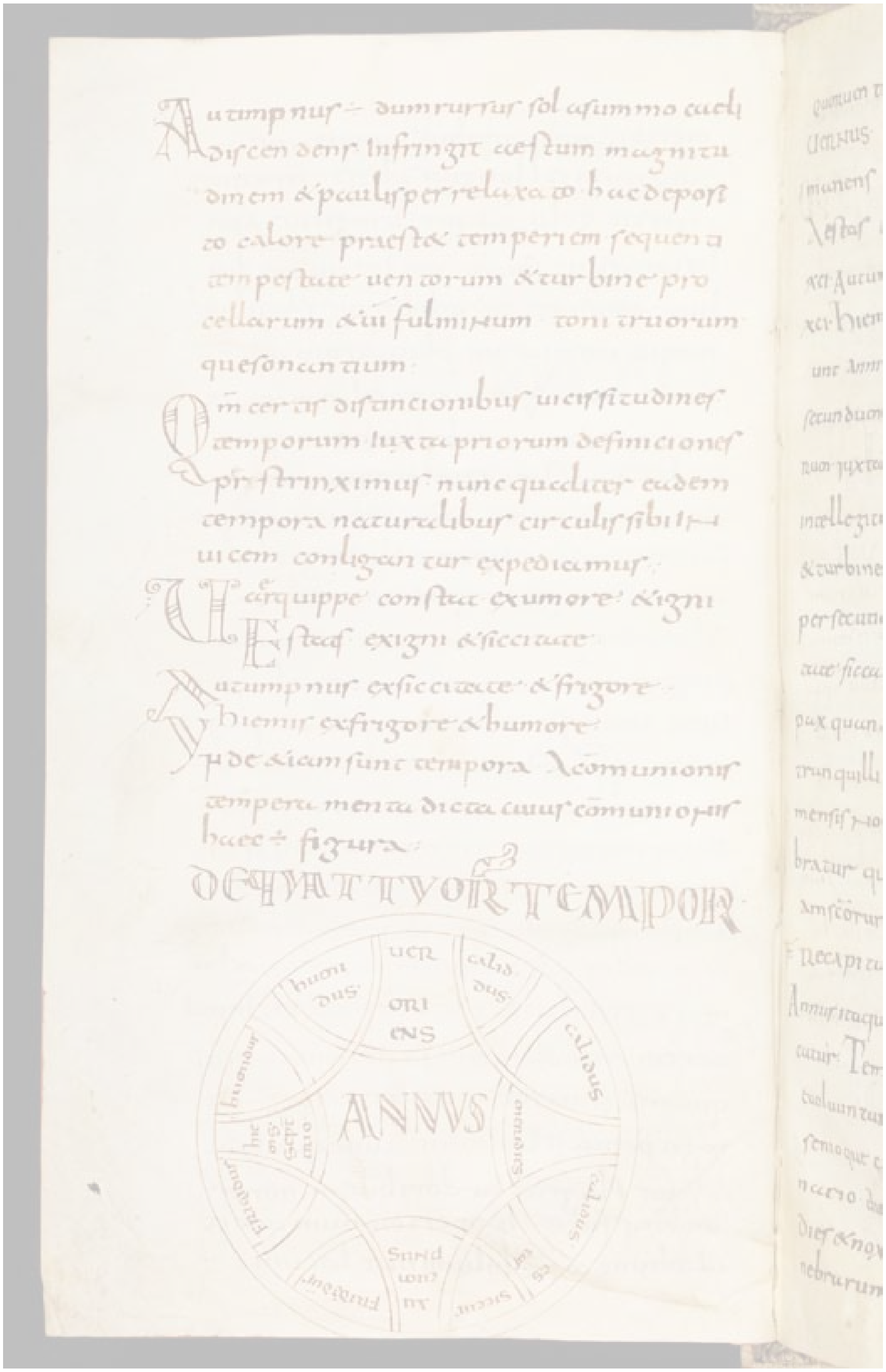


1.105 Bamberg, Staatsbibliothek MS Msc.Nat.1, fol. 1r.
© Staatsbibliothek Bamberg

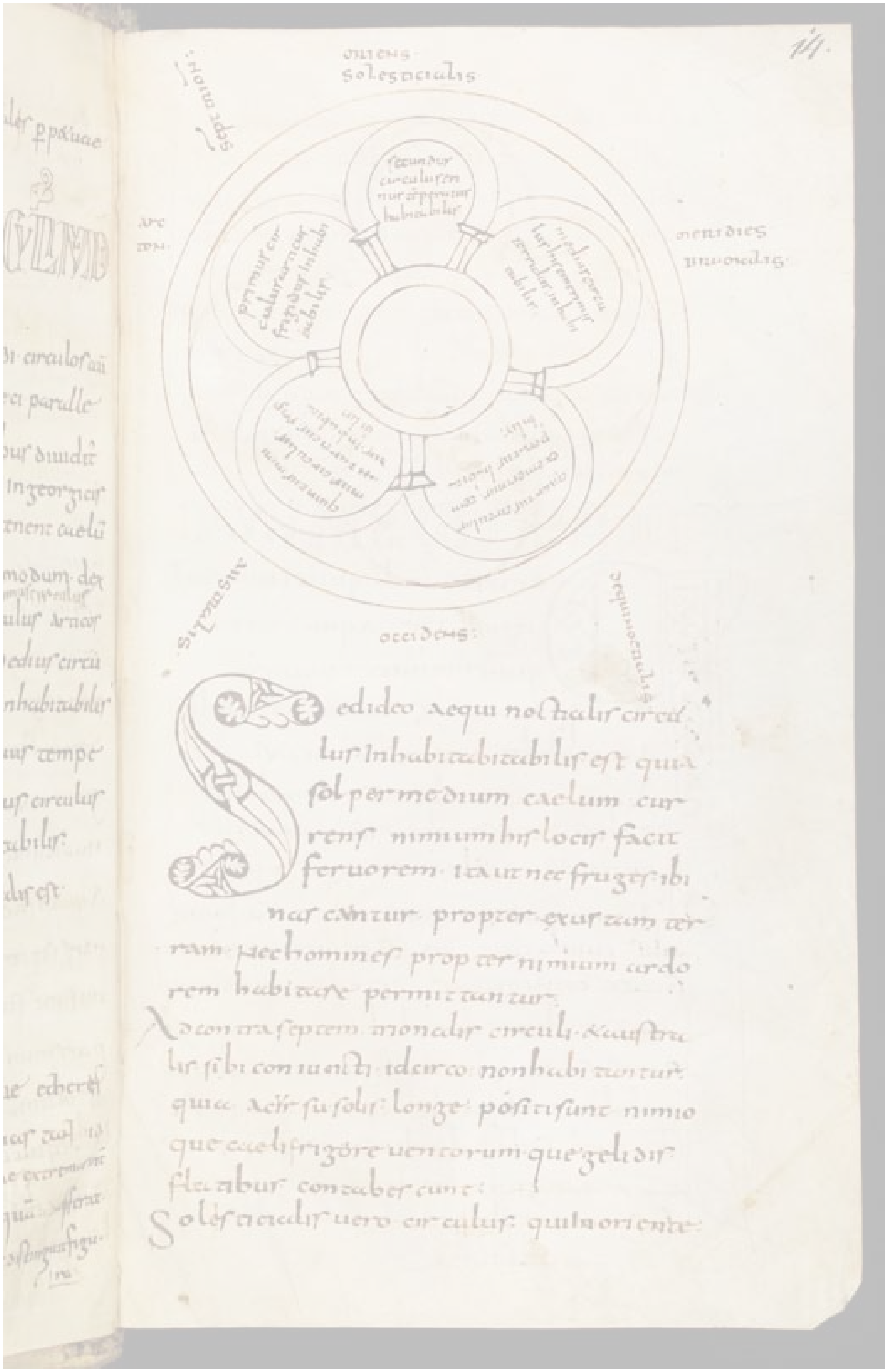
1.106 Bamberg, Staatsbibliothek MS Msc.Nat.1, fol. 1r, detail. © Staatsbibliothek Bamberg



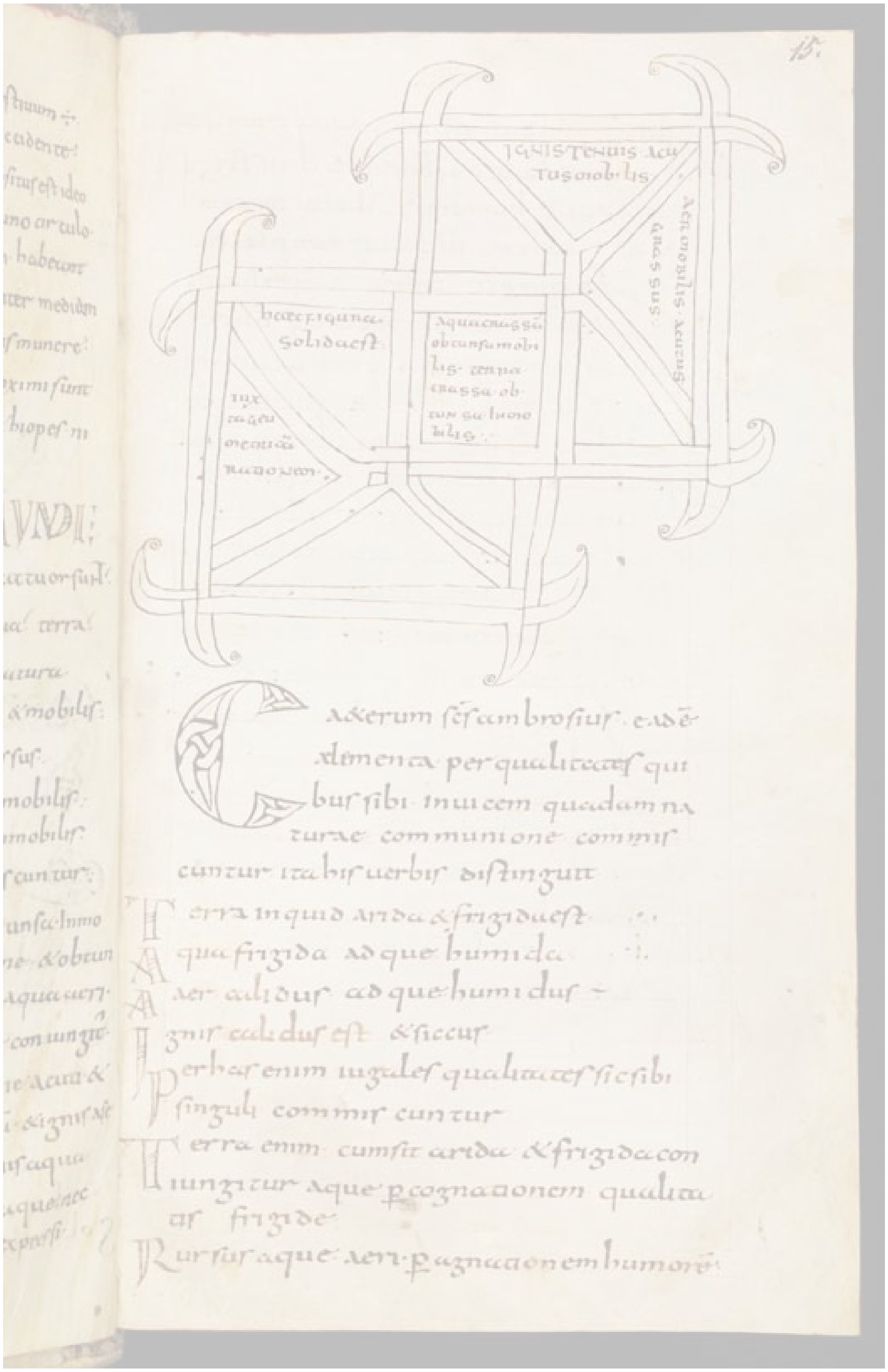
1.107 Bamberg, Staatsbibliothek MS Msc.Nat.1, fol. 8v.



1.108 Bamberg, Staatsbibliothek MS Msc.Nat.1, fol. 11v.

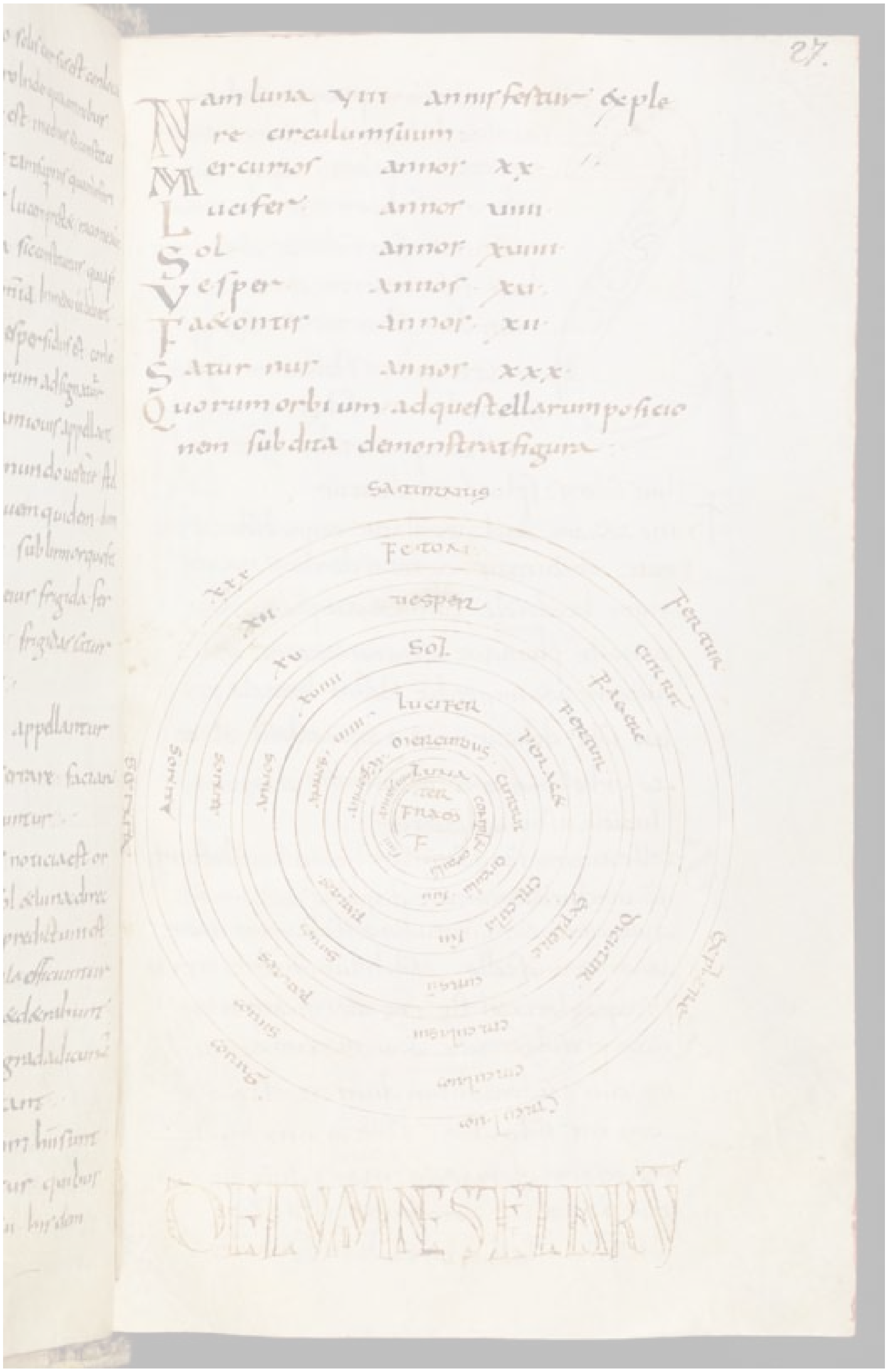


1.109 Bamberg, Staatsbibliothek MS Msc.Nat.1, fol. 14r.



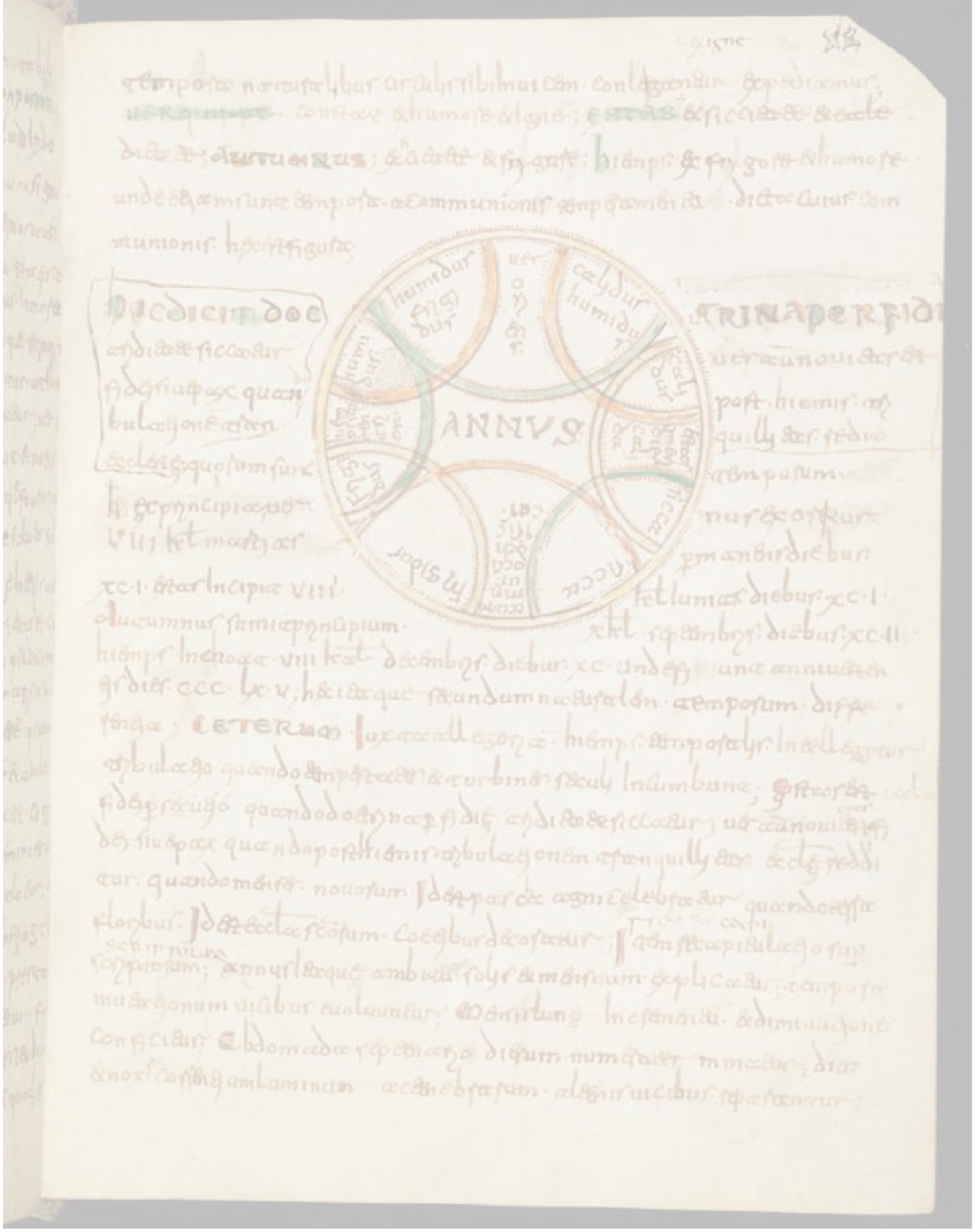
1.110 Bamberg, Staatsbibliothek MS Msc.Nat.1, fol.15r.

1.111 Bamberg, Staatsbibliothek MS Msc.Nat.1, fol. 16r.

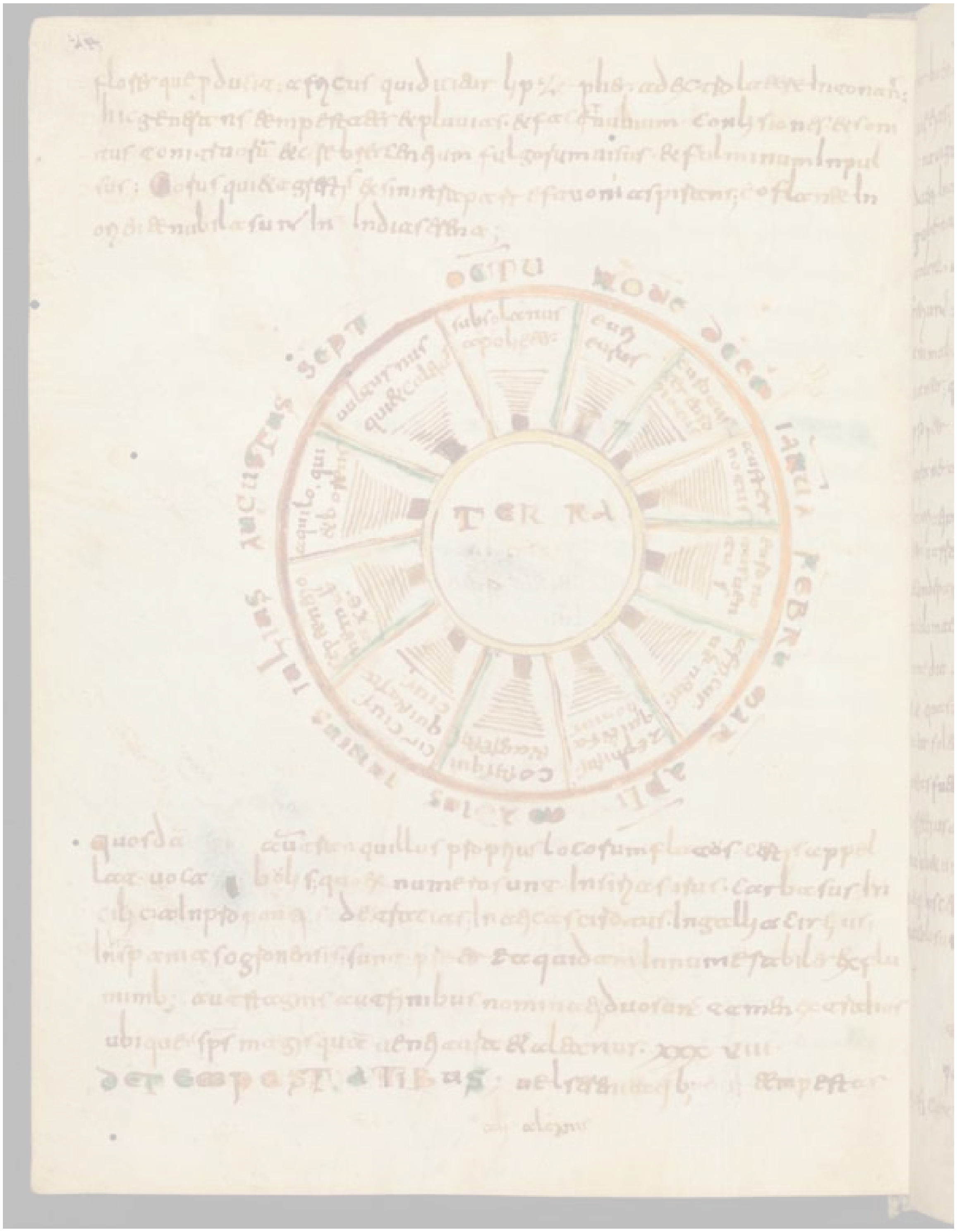


1.112 Bamberg, Staatsbibliothek MS Msc.Nat.1, fol. 27r.

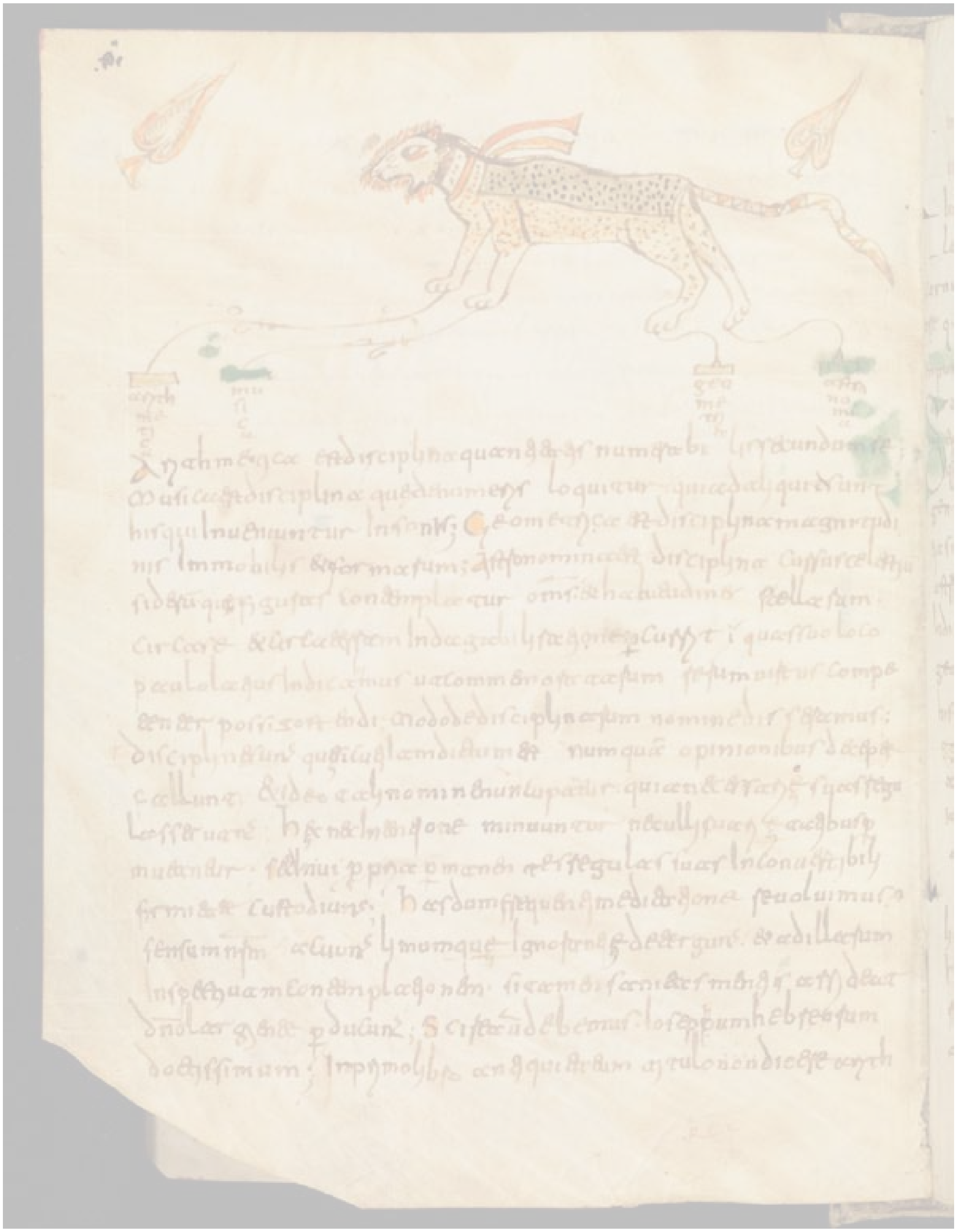
1.113 Bamberg, Staatsbibliothek MS Msc.Nat.1, fol. 35v.



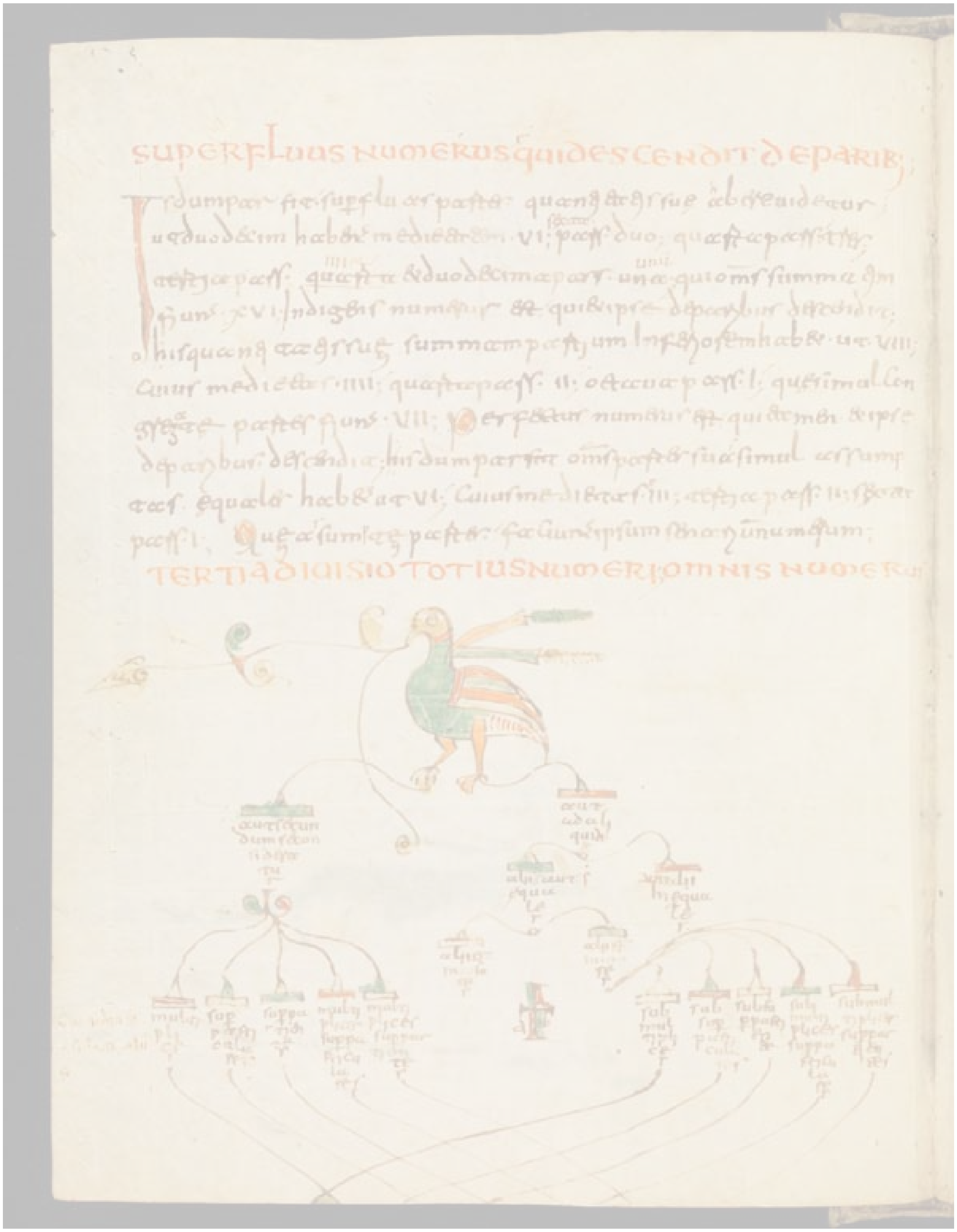
1.114 Bamberg, Staatsbibliothek MS Msc.Patr.61, fol. 87r.



1.115 Bamberg, Staatsbibliothek MS Msc.Patr.61, fol. 99r.



1.116 Bamberg, Staatsbibliothek MS Msc.Patr.61, fol. 53v.



1.117 Bamberg, Staatsbibliothek MS Msc.Patr.61, fol. 55v.
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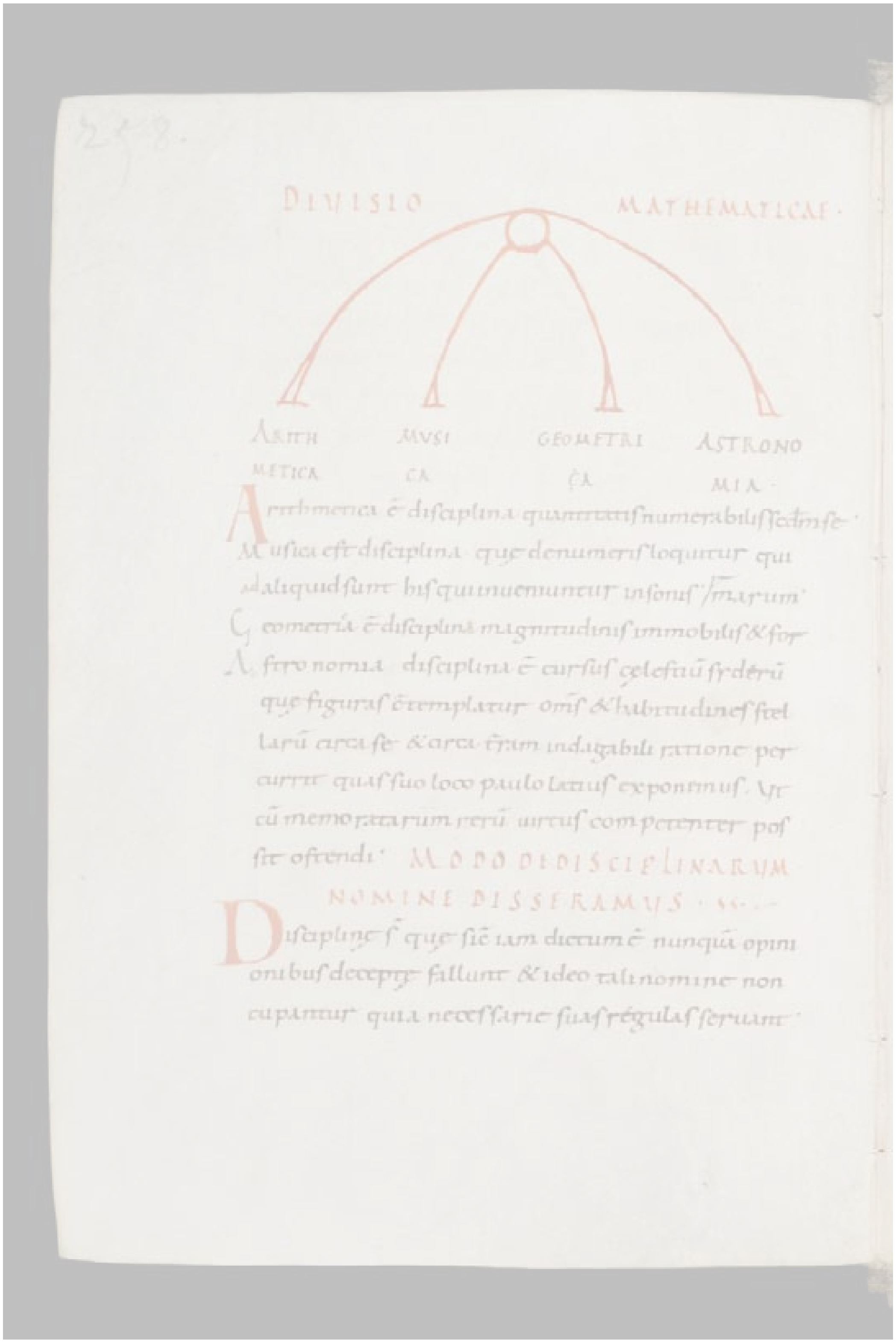
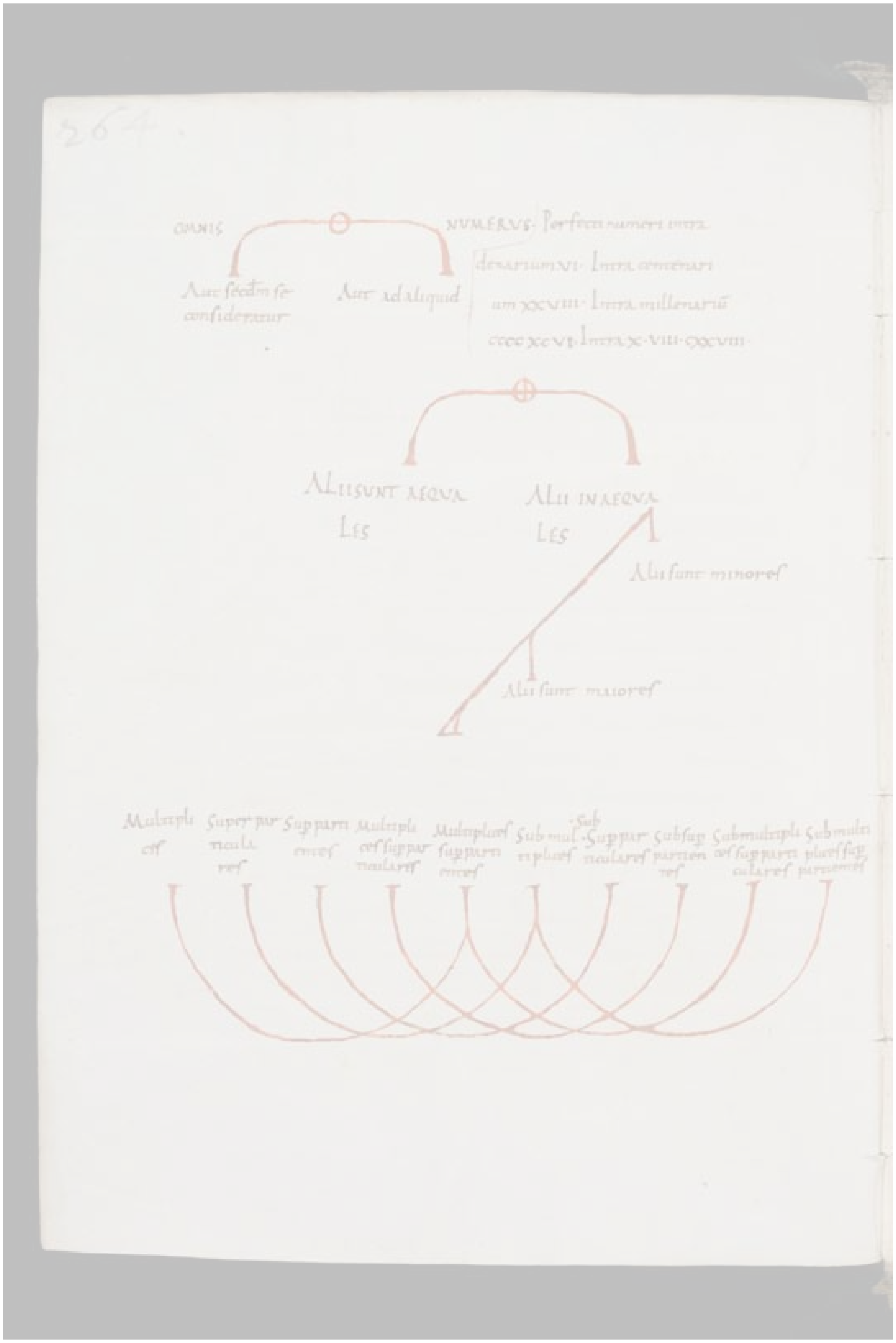
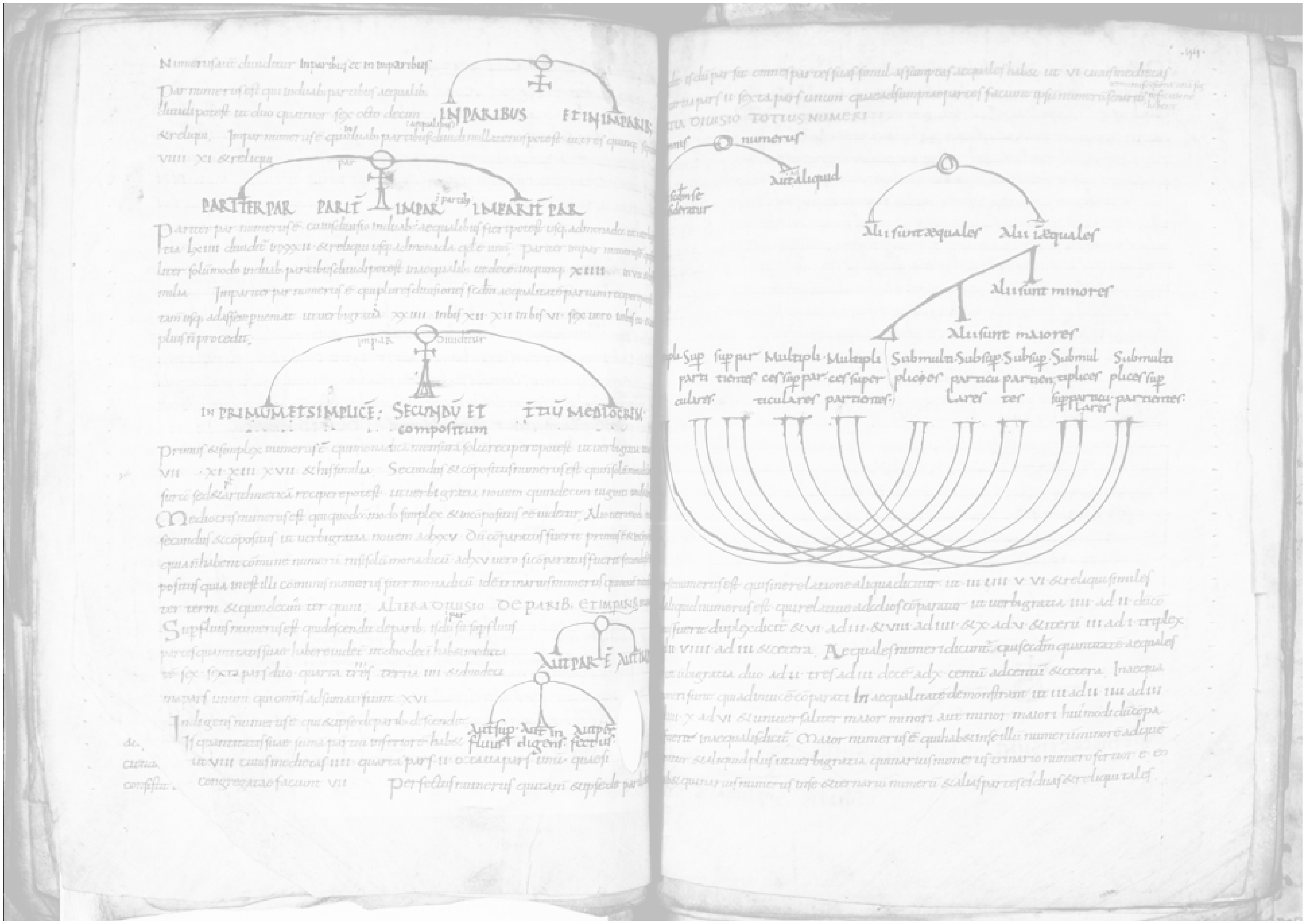


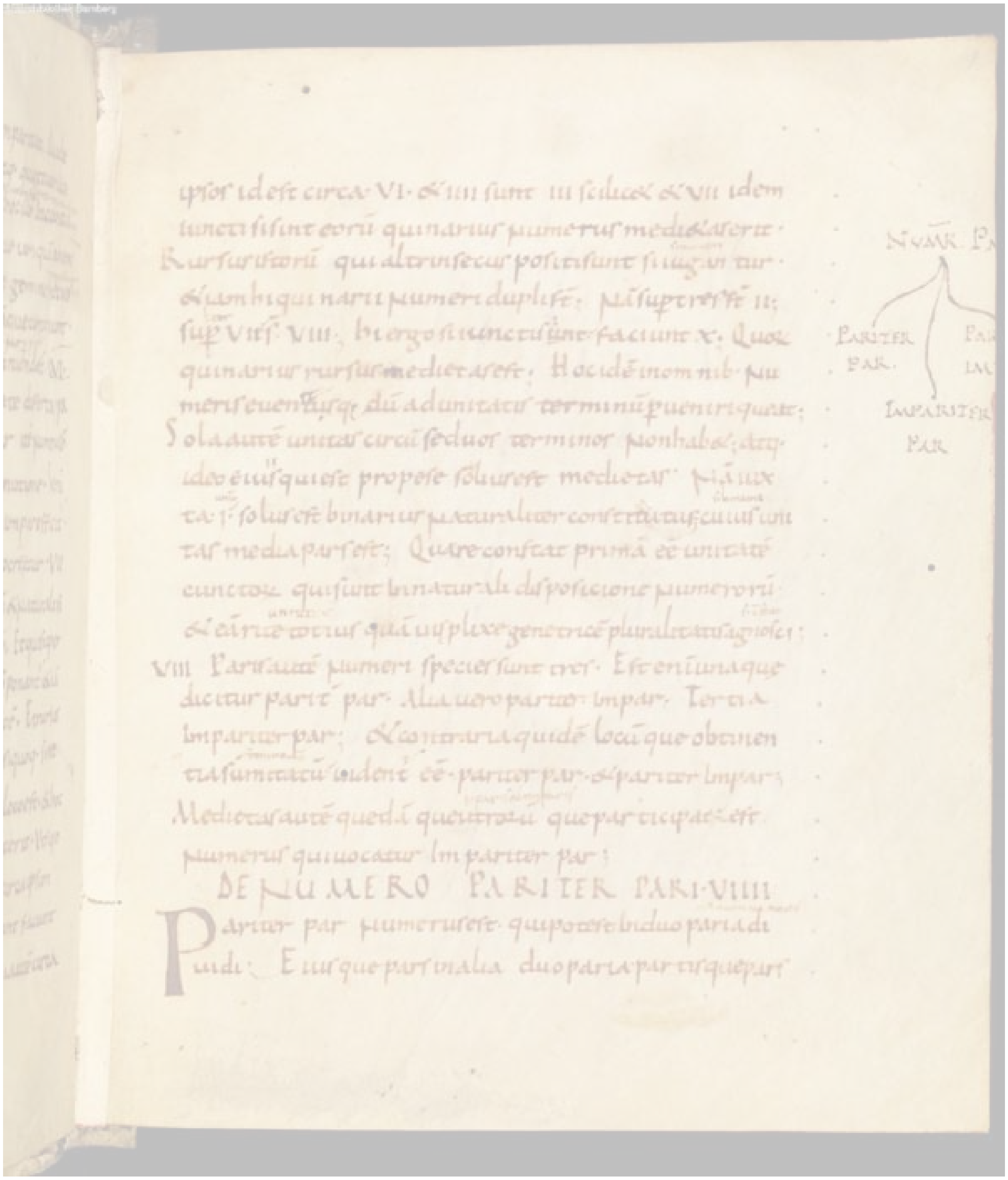
1.118 St. Gall, Stiftsbibliothek Cod. 199, p. 258.



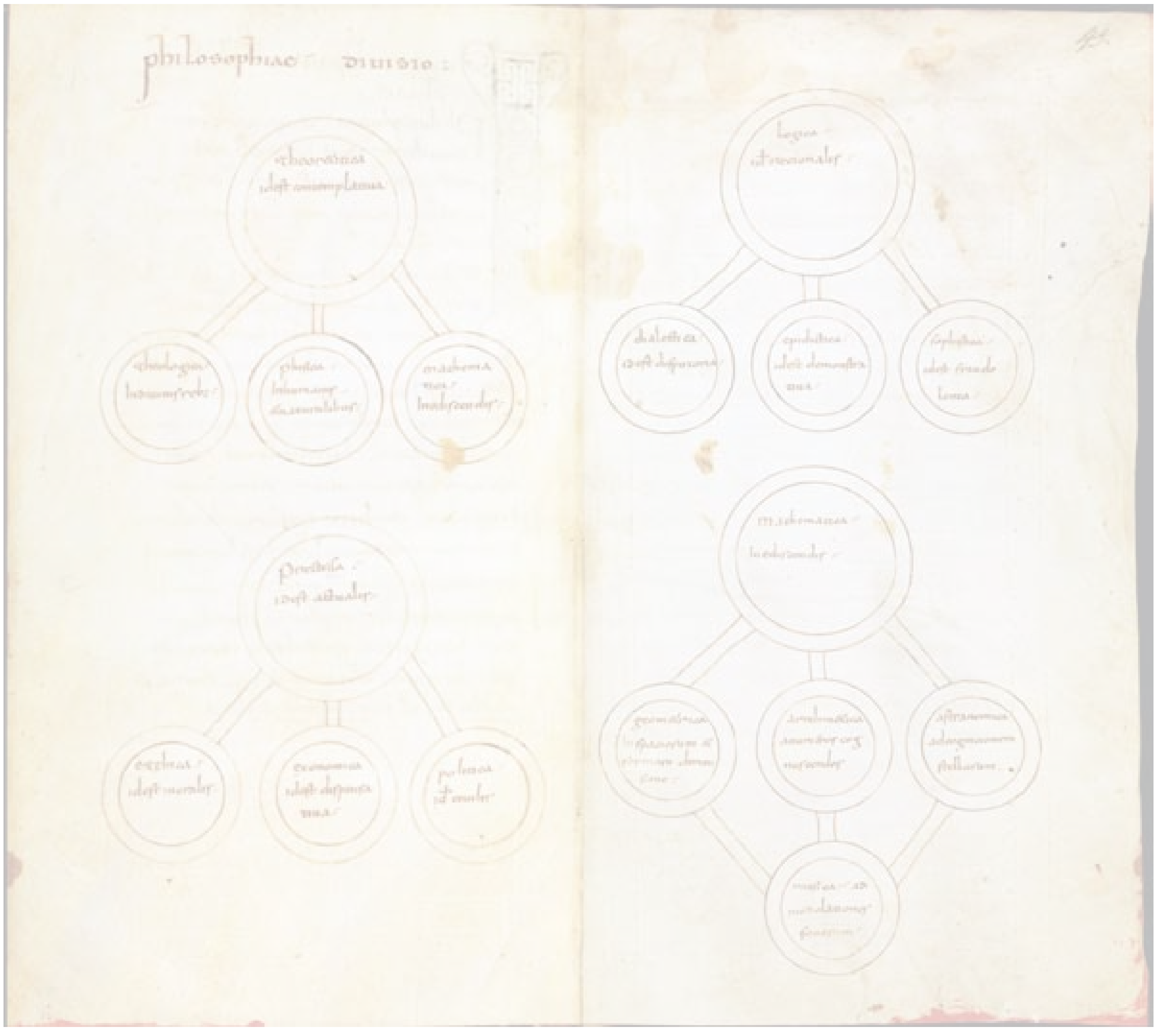
1.119 St. Gall, Stiftsbibliothek Cod. 199, p. 264.



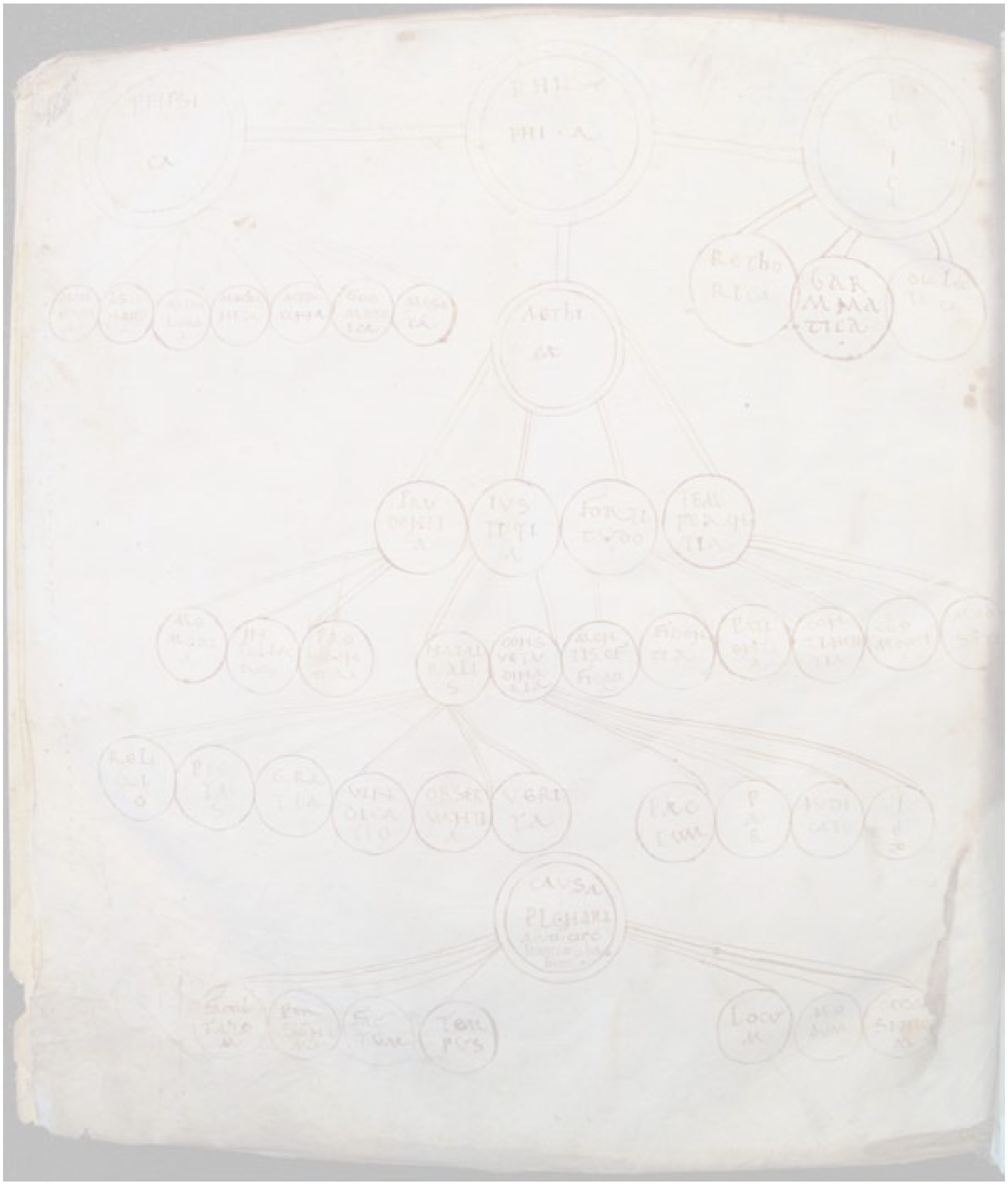
1.120 Reims, Bibliothèque municipale MS 975, fols. 54v–55r. © Bibliothèque municipale Reims



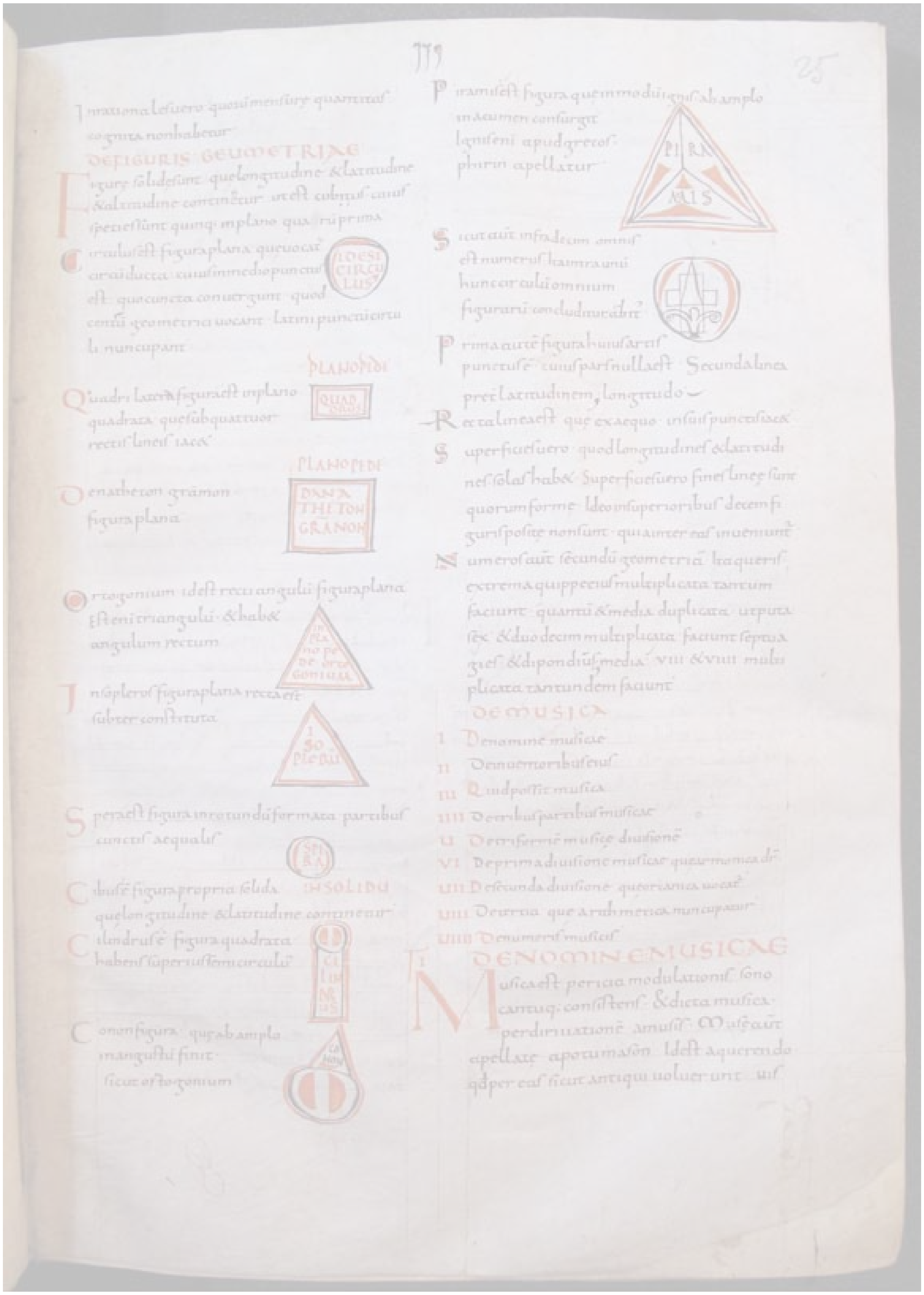
1.121 Bamberg, Staatsbibliothek MS Msc.Class.8, fol. 9r.



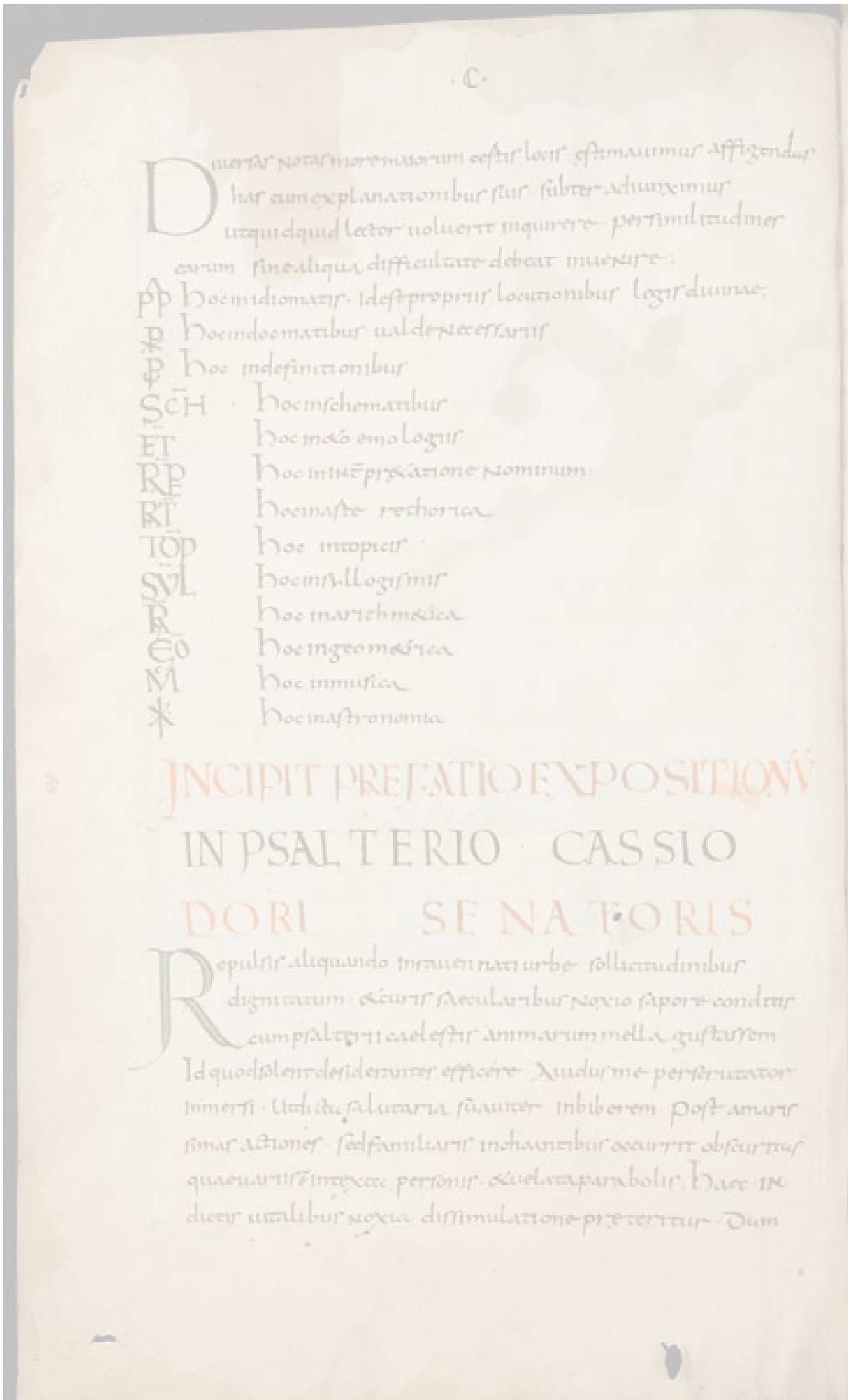
1.122 Bamberg, Staatsbibliothek MS Msc.Nat.1, fols. 44v–45r.



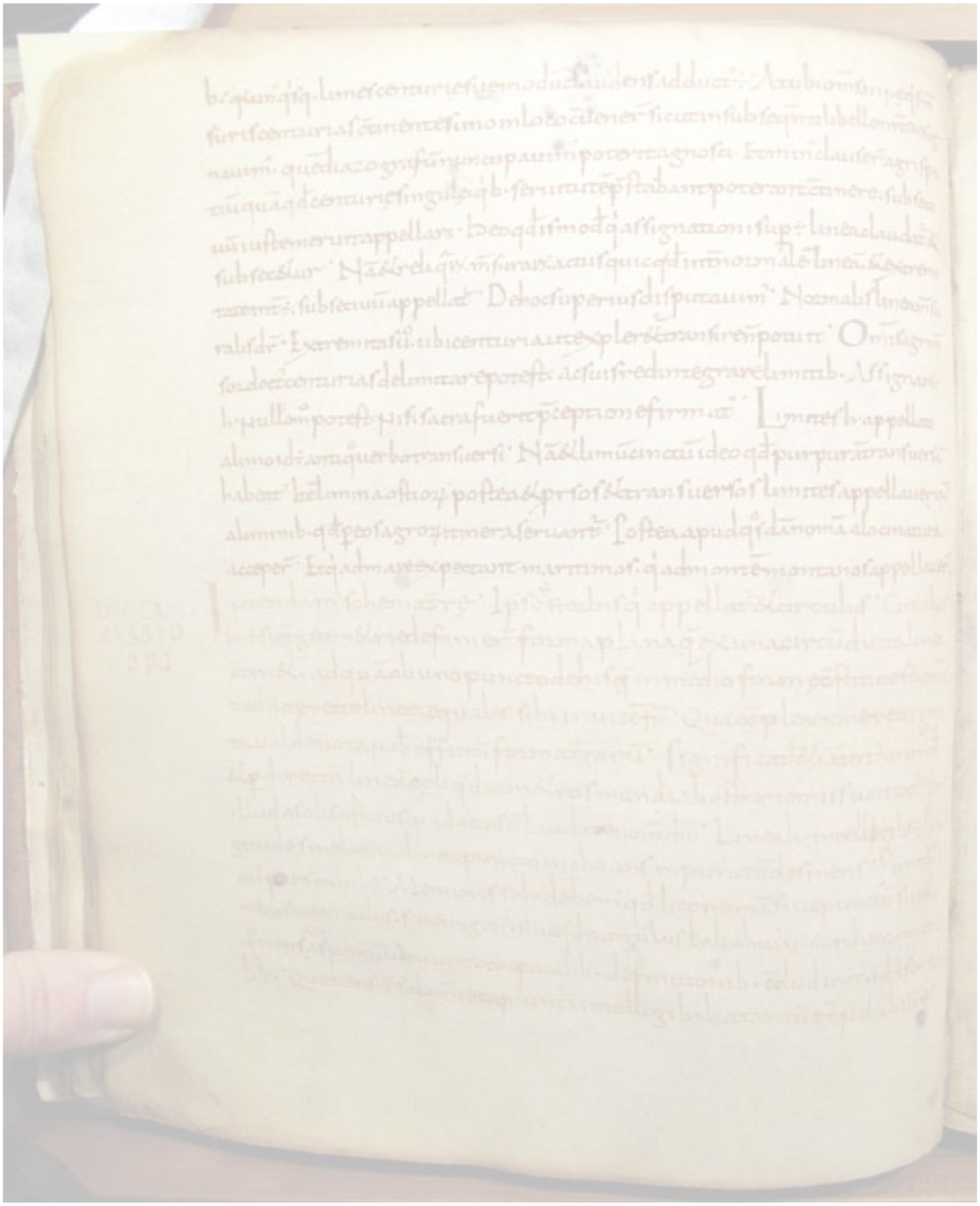
1.123 Bern, Burgerbibliothek Cod. B 56, fol. 183v.
Photo taken with permission of the Burgerbibliothek Bern



1.124 Bern, Burgerbibliothek Cod. 36, fol. 25r.
Photo taken with permission of the Burgerbibliothek Bern

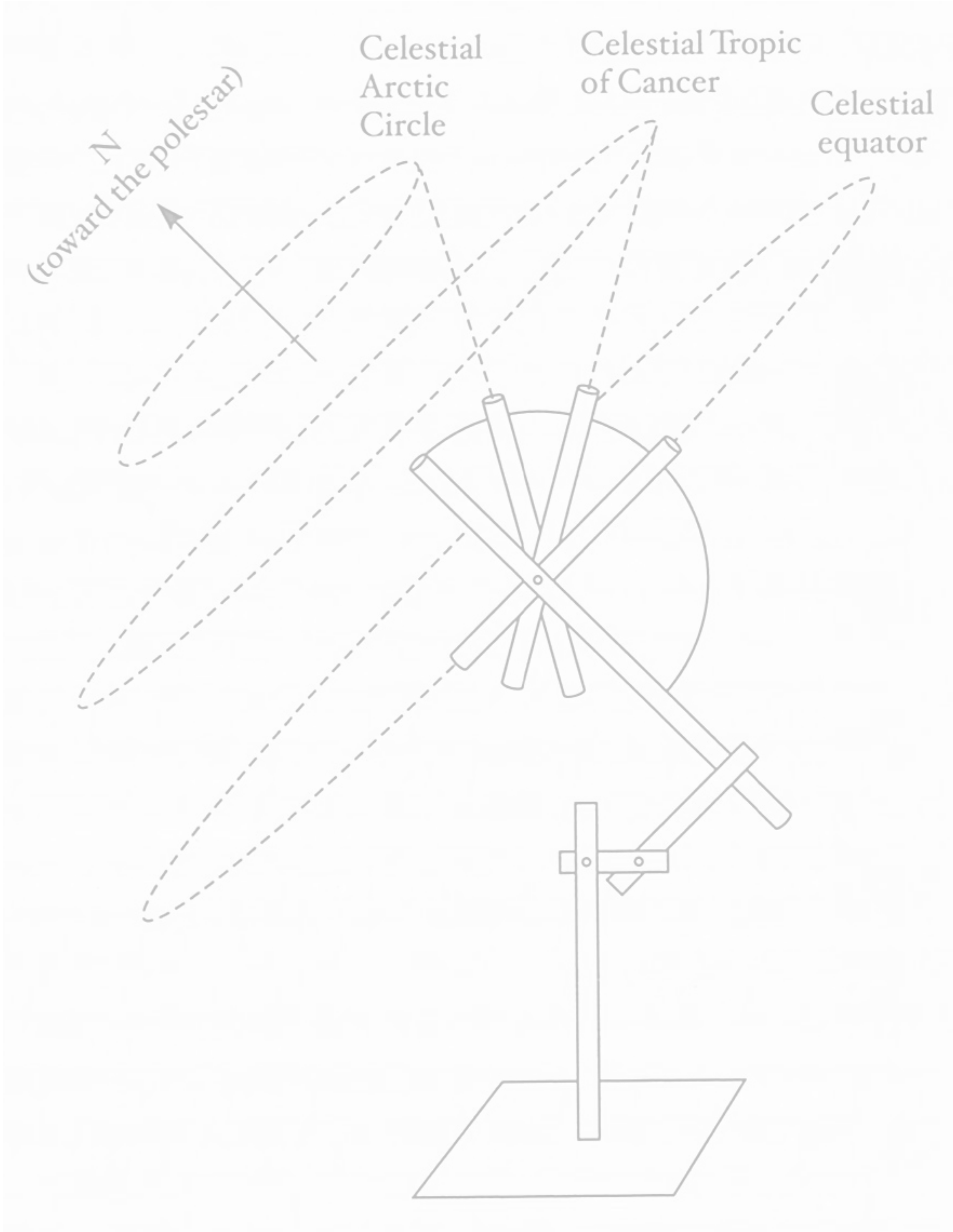


1.125 Wolfenbüttel, Herzog August Bibliothek Cod. Guelf. 4 Weiss., fol. 1v.



1.126 Naples, Biblioteca Nazionale Vittorio Emanuele III MS V.A.13, fol. 27v.
Photo taken with permission of the Biblioteca Nazionale Vittorio Emanuele III

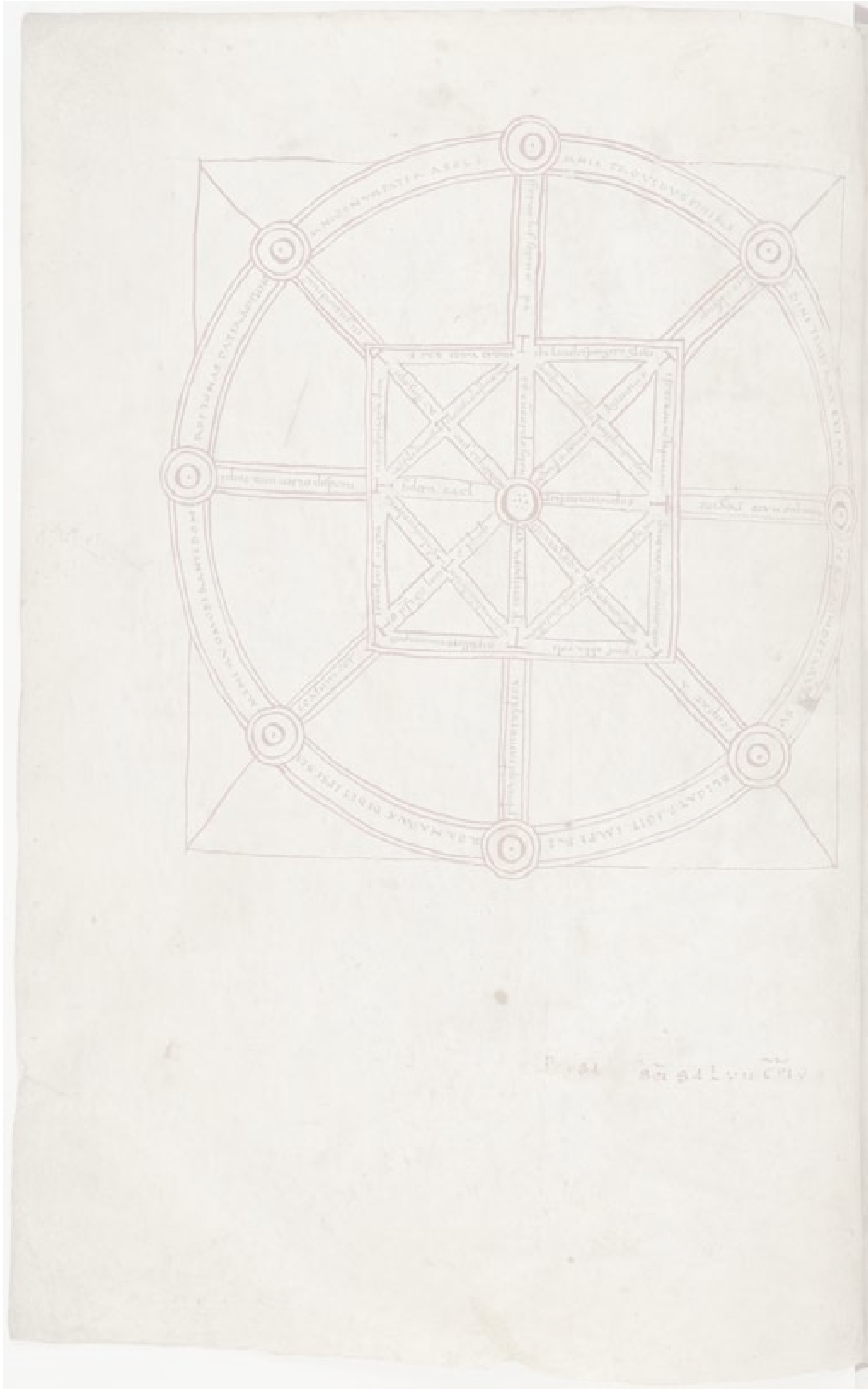
2.1 Paris, Musée du Louvre, Islamicate celestial sphere, Bagdad, c. 1145
Engraved brass with silver inlay
Photo © Musée du Louvre



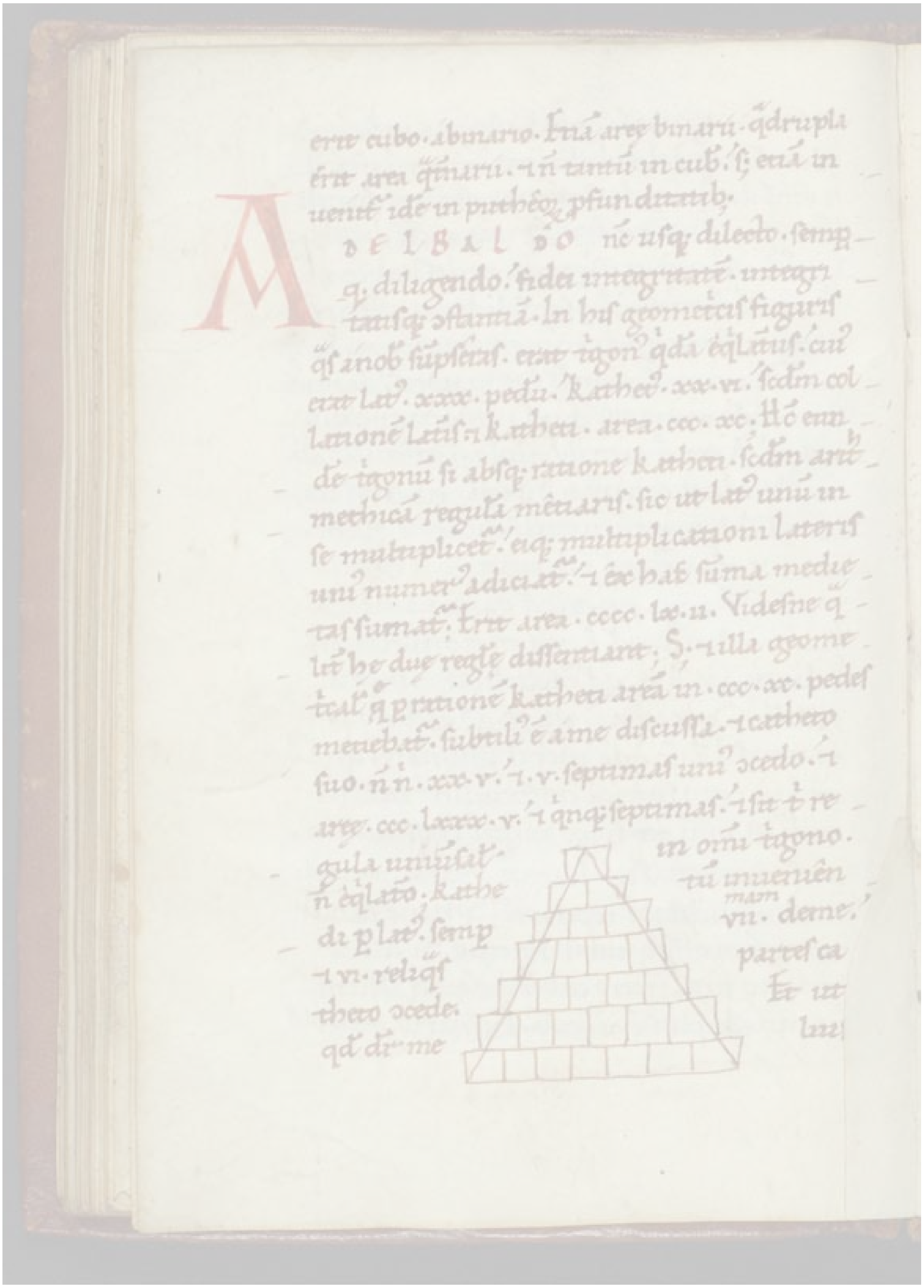
2.2 Compound sighting tube or *dioptra*.
Image from Justin Lake, in Richer, *Historiae*, 460, n. 59.

OTTOVALENSCAESARNOSTROTVCEDECOTVRNO
TOTFELIXATAVISQVOTCAELOSIDERALVCENT
TEDOMINVMSIBISAXOTVLITTEROMANOTAVIT
ORBISETIPSECVPITSOLOCONTENTVSALVMNO
VIRTVTVMTITVLISETVIRCOGNOSCERISACTV
ACDOMITORPATRIAEPACISSECTATORINAVLA
LVMENVBIQ:MICANSSOLVSLVCENDOVELVTSOL
ERGODEISOLITOREDDENTVRSANCTABENIGNE
NECDEERITVIRTVSOMNISQVAGRATIACVLMEN
SCANDITETOCCVLTISSECEDITNENIACAVSIS
CERTENOSOMNESTIBICAESARDESERISISTIC
AVSTRASIOSQVATERRAMANETFERALISOPIMA
ETFECVNDASITVPOLLETSATISVBEREGLEBAE
SVMMISCARAVIRISACSAEVISPLENACOLONIS
APATRISIMPERIONONABSTITITISMAHELITA
REXITEVMSOLLERSETREGNANSINDVPERATOR
NVNCAVGVSTETVVMPONAMVENERABILENOMEN
OTTOVALENSCAESARNOSTROTVCEDECOTVRNO
SOLVSENIMREGNANSABSENSOCESARISHERES
TOTVSAVOSIMILISSITENOVAVITARESIGNAT
REXFVITILLEPOTENSROMANAELEGISAMATOR
OMNEDECVSPATRIAESOLIOPROGNATVSAVITO
TEMPORAPACISERANTTALIDVMIVREVIGERET
VIRTANTVSQVEMSICDVXIDESCRIBEREVERSV
CVRERGONATALETVVMCVRCONTRAHISETNVNC
EXVLISINBELLISDEFERSPIADEBITAPOMPAE
DVMVATESBONVSOPTODARIMIRABILISISTVD
EXPANDESOPVSIPSEMEVMTRACTABILISINDE
CAESARVTINVICTVSSCVTOMVNITVSETEXHOC
OMNIBVSVTILIQRMIRODATVSANTETRIVMPHO
TERRIBILISCLEMENSTVTODIADEMATERISIT
VVLTVSAVIPATRISQ:TVIPRAECLARVSAMICTV
RVRSVSVTERQ:FVITDIROSVBTEMPOREVICTOR
NVNCVNVMVIVENSDIGNVMCVMPATREVOCAMEN
OTTOVALENSCAESARNOSTROTVCEDECOTVRNO

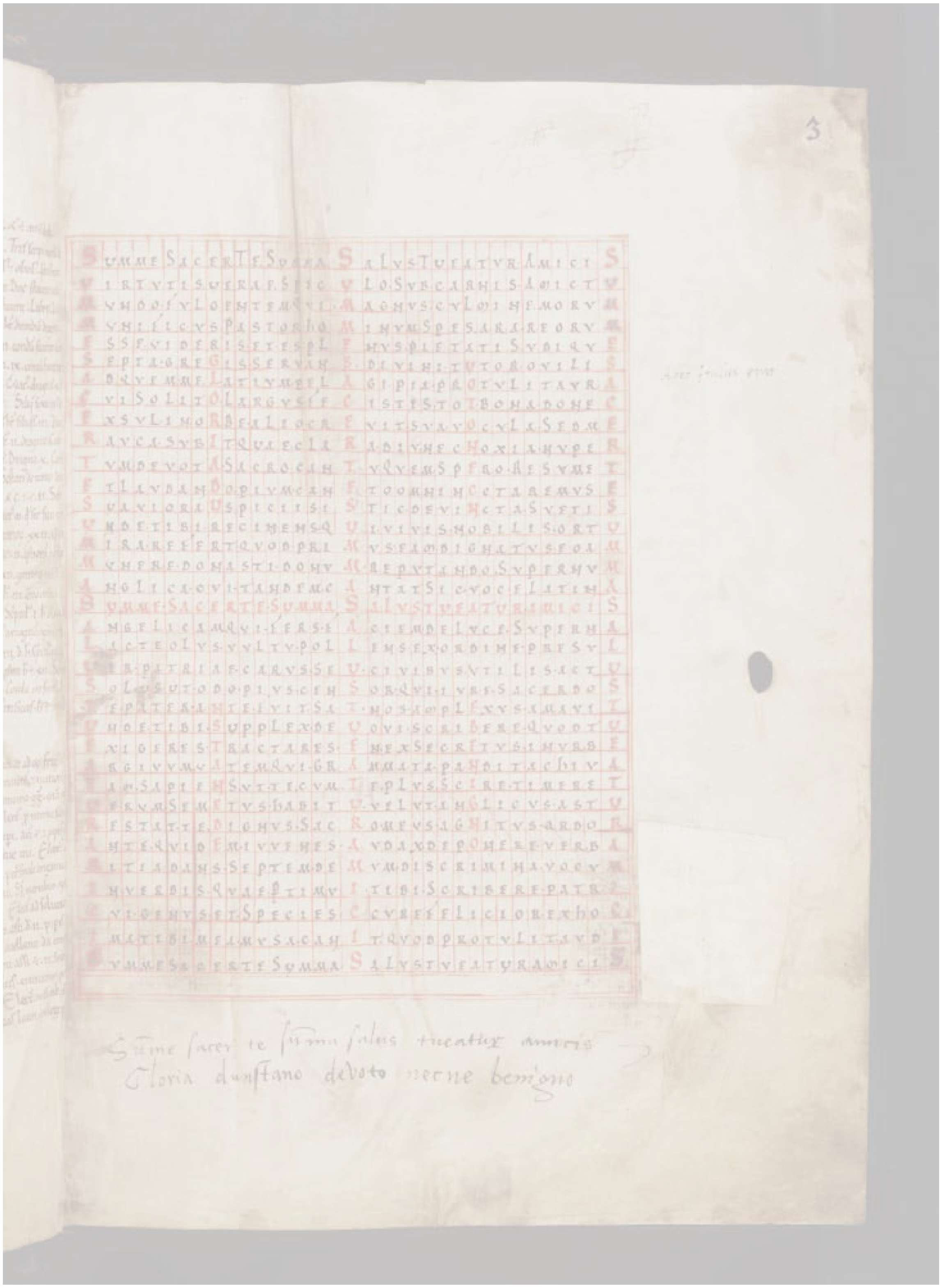
2.3 Reconstruction of Abbo's acrostic for Otto III, *Otto valens*
Image from Scott Gwara, "Three Acrostic Poems by Abbo of Fleury," J*ournal of Medieval Latin* 2 (1992): 227.



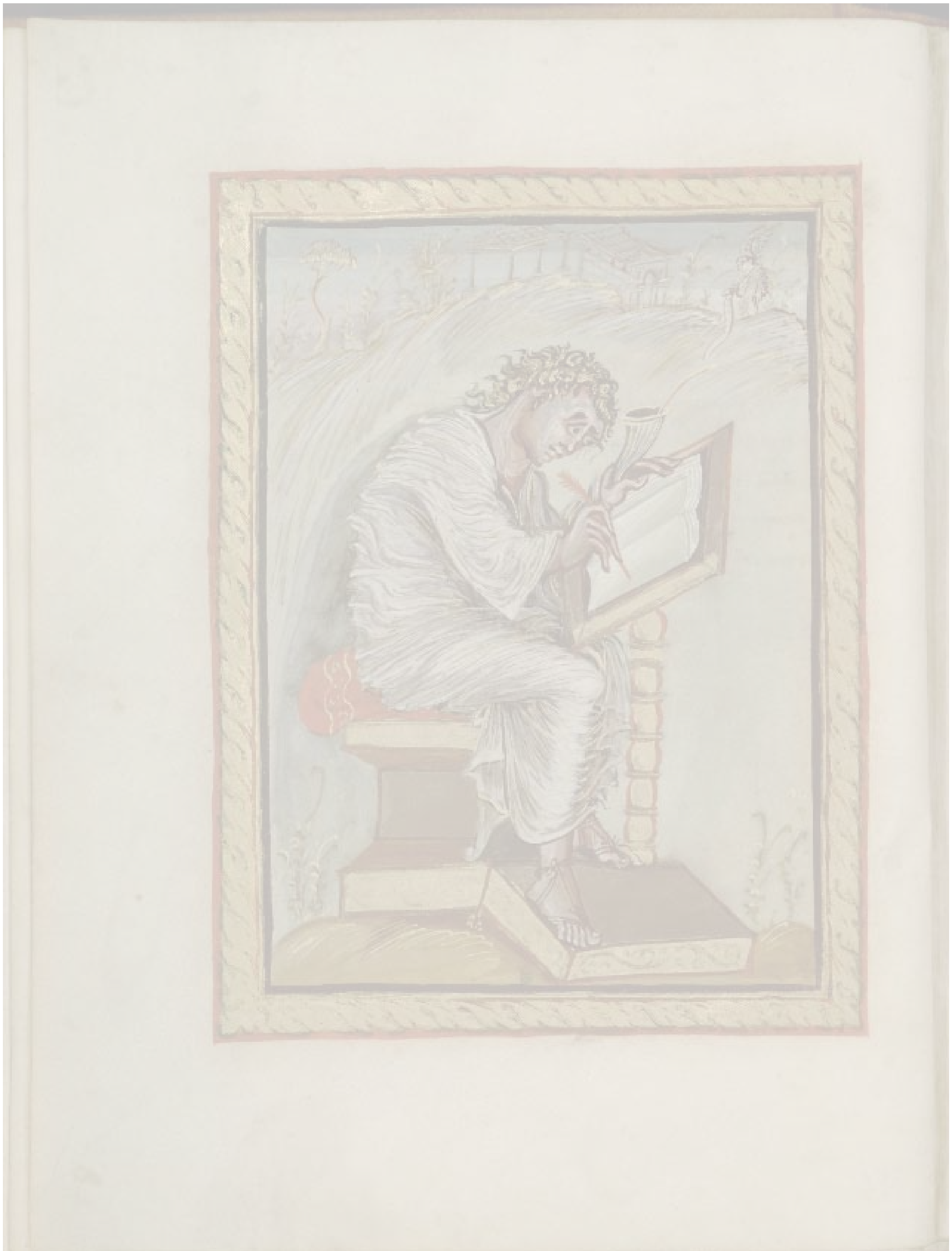
2.4 Paris, Bibliothèque nationale de France lat. MS 776, fol. 1v.
© Bibliothèque nationale de France



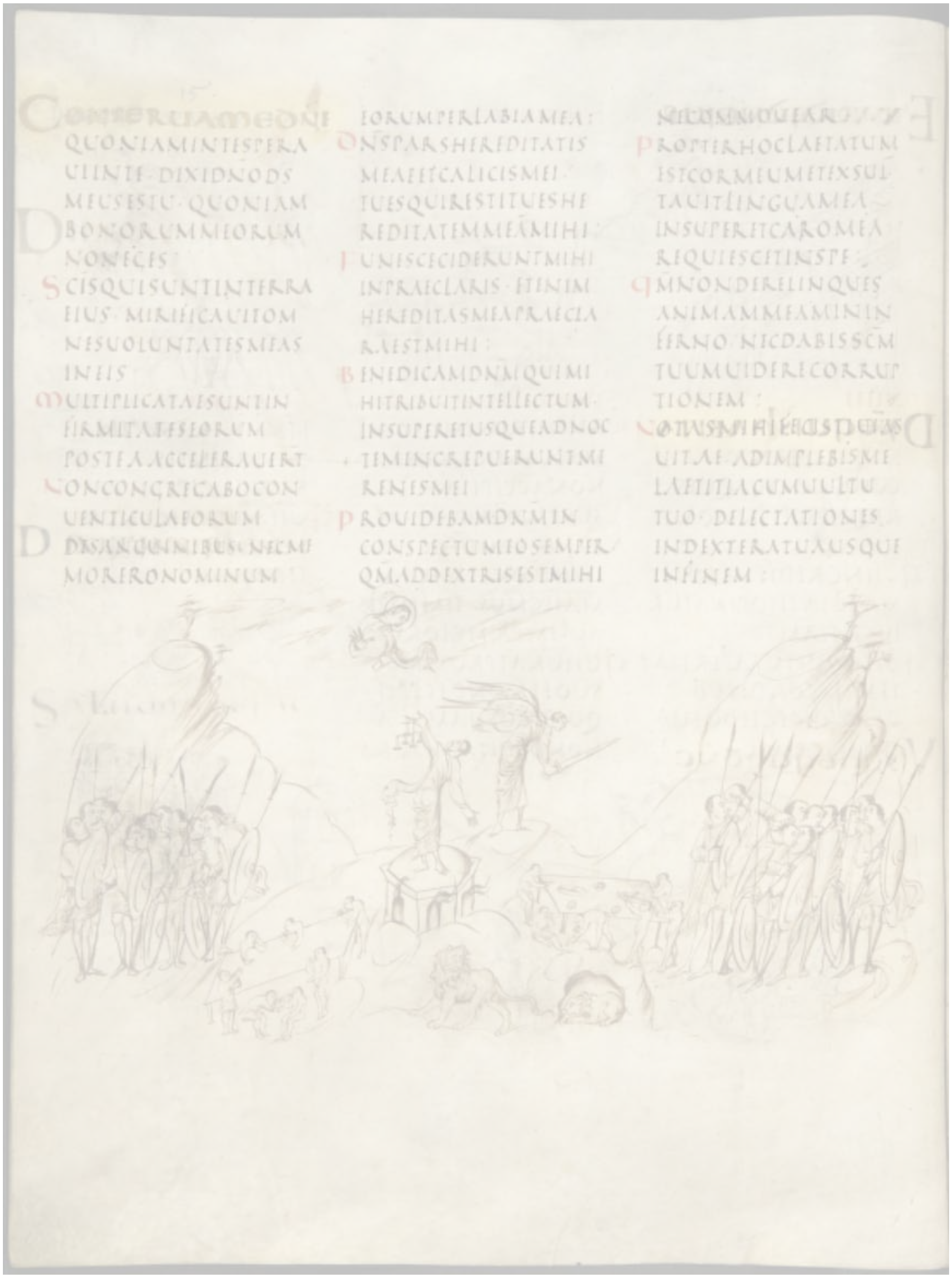
2.5 Philadelphia, University of Pennsylvania, Schoenberg Center MS LJS 194, fol. 51v.
© University of Pennsylvania



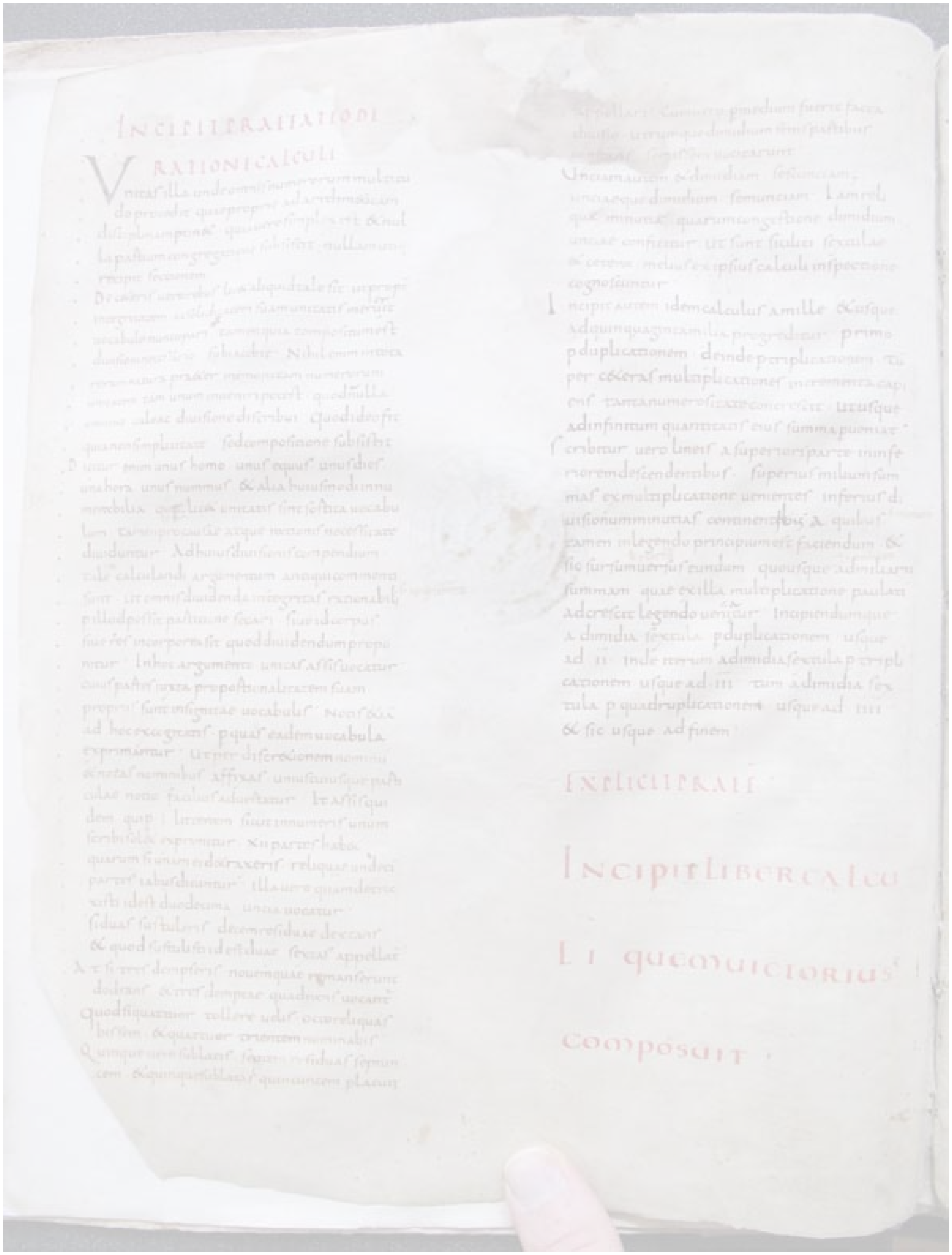
2.6 Oxford, St. John’s College MS 17, fol. 3r.
© St. John’s College, Oxford University



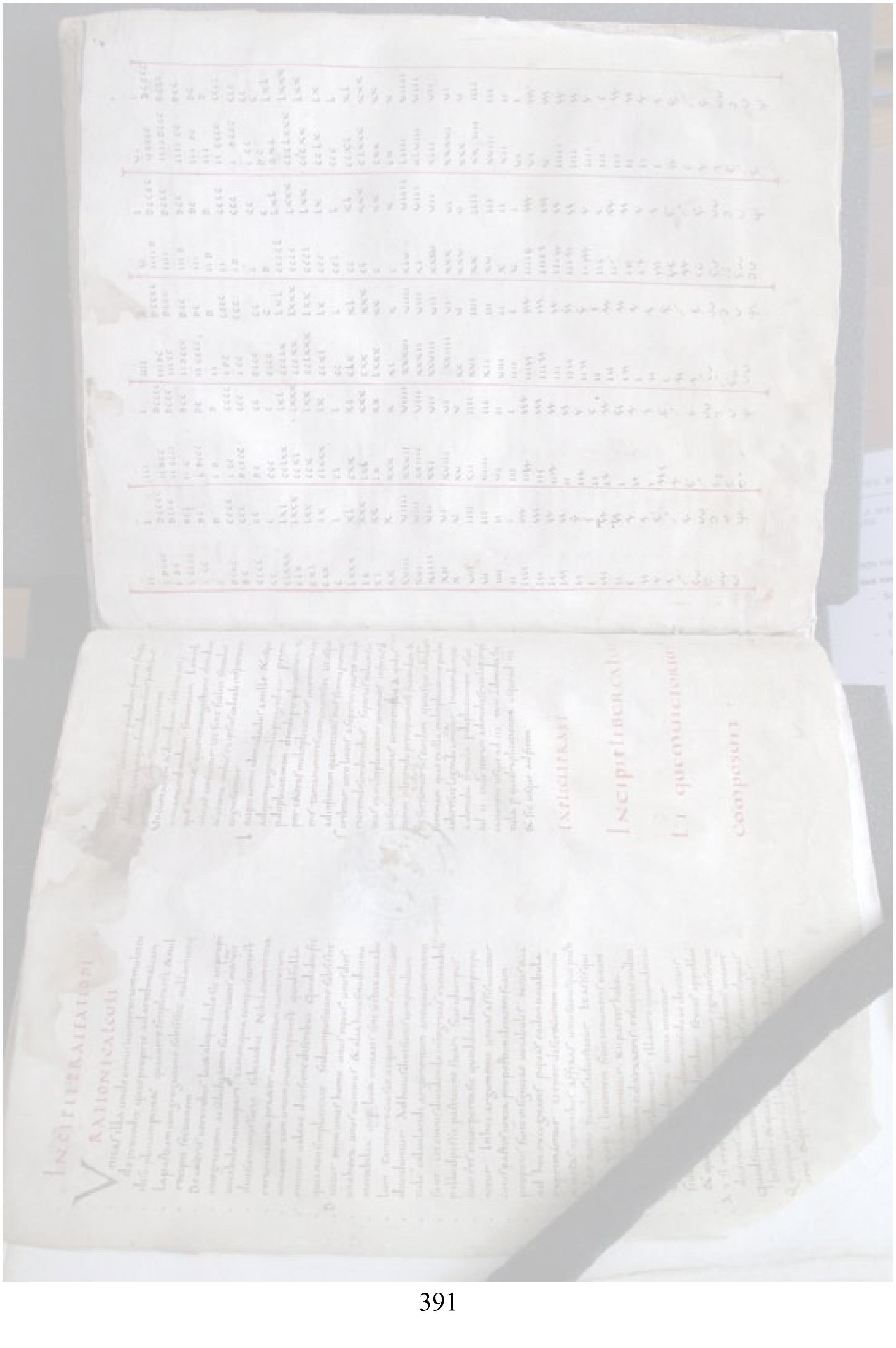
2.7 Épernay, Bibliothèque municipale MS 1 (“Ebo Gospels”), fol. 18v.
© Bibibliothèque municipale de Épernay



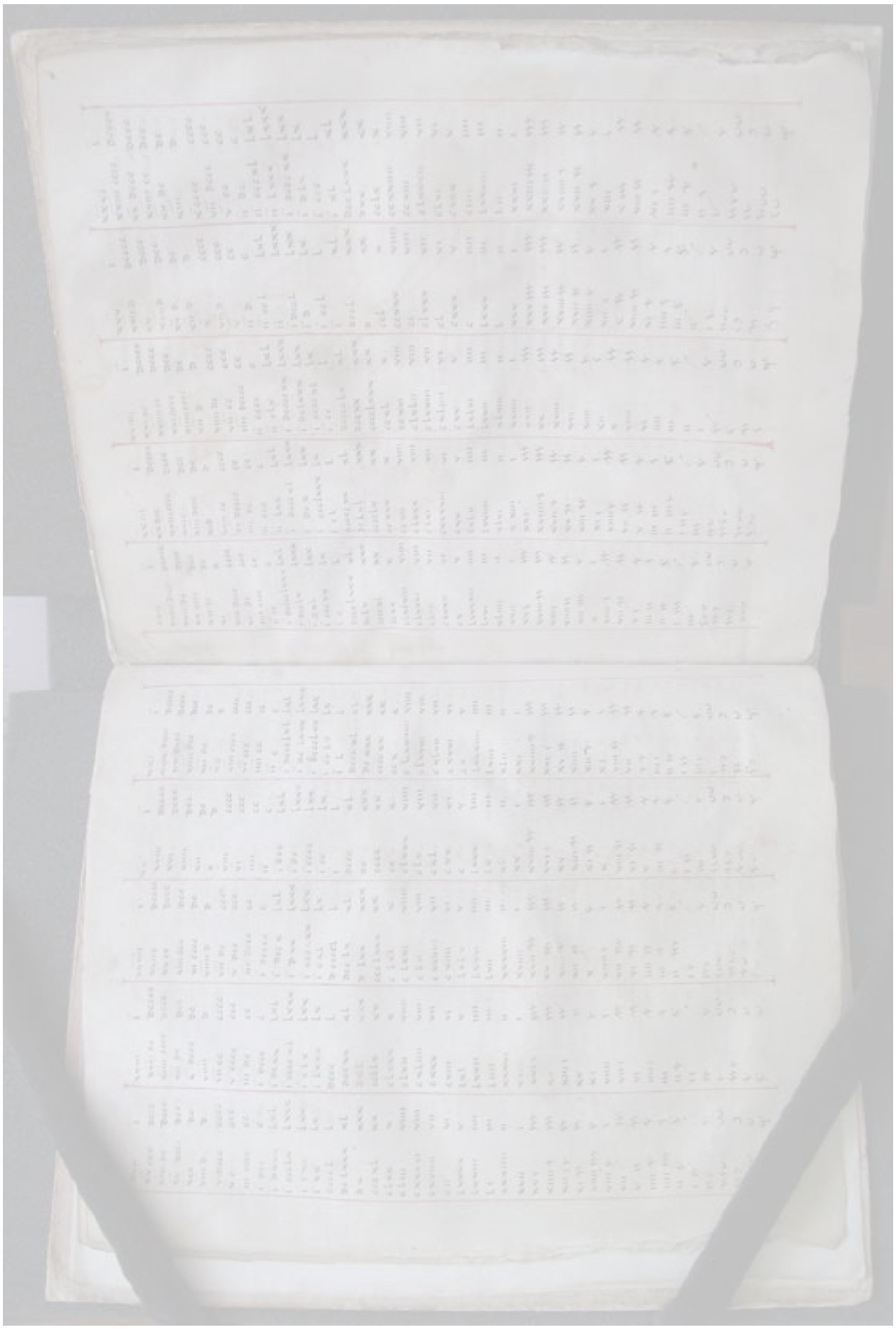
2.8 Utrecht, Universiteitsbibliotheek Hs. 32 (“Utrecht Psalter”), p. 23.



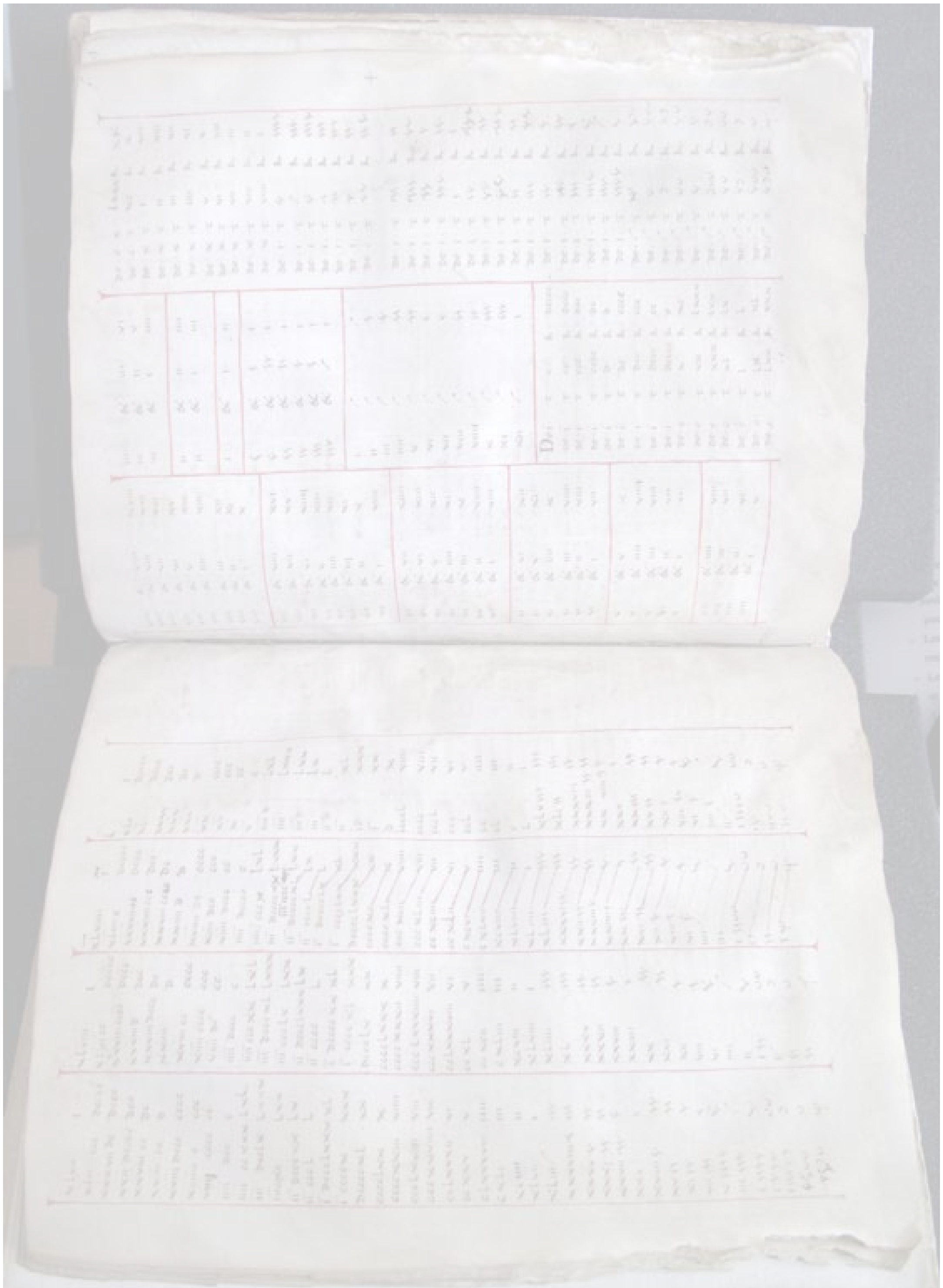
3.1 Bern, Burgerbibliothek Cod. 250, fol. 1v.
Photo taken with permission of the Burgerbibliothek Bern



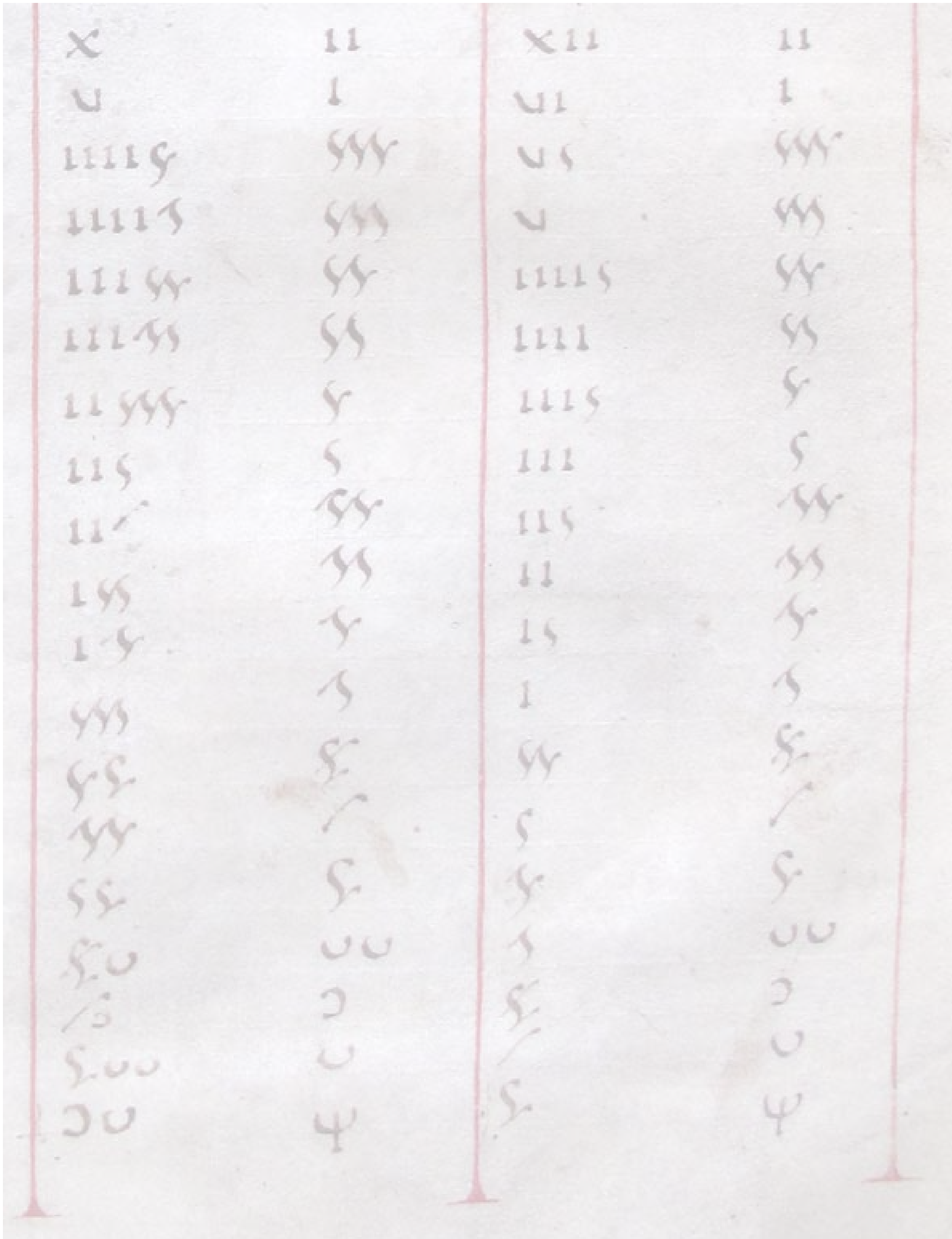
3.2 Bern, Burgerbibliothek Cod. 250, fols. 1v and 2r. Photo taken with permission of the Burgerbibliothek Bern



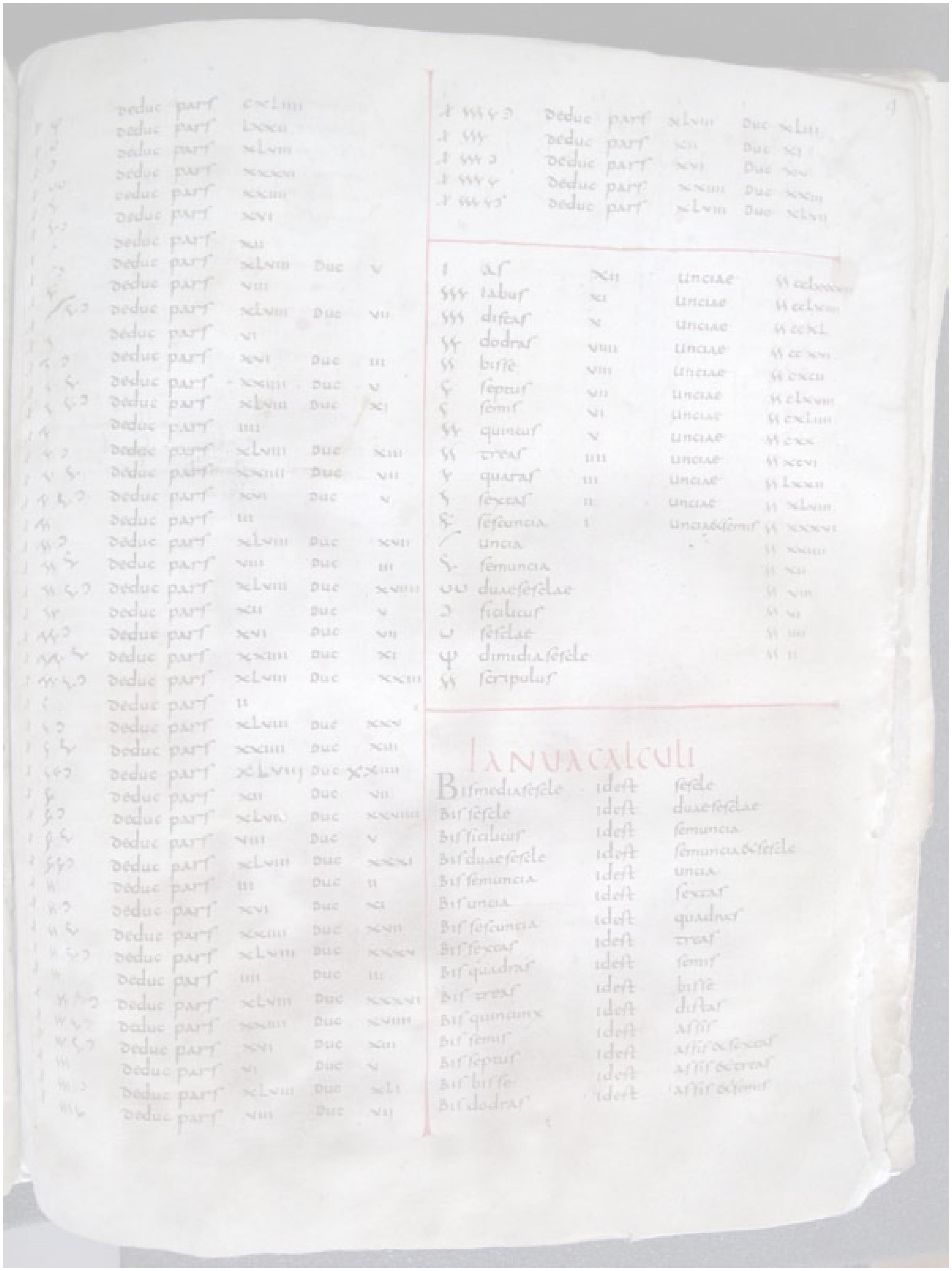
3.3 Bern, Burgerbibliothek Cod. 250, fols. 3v and 4r. Photo taken with permission of the Burgerbibliothek Bern



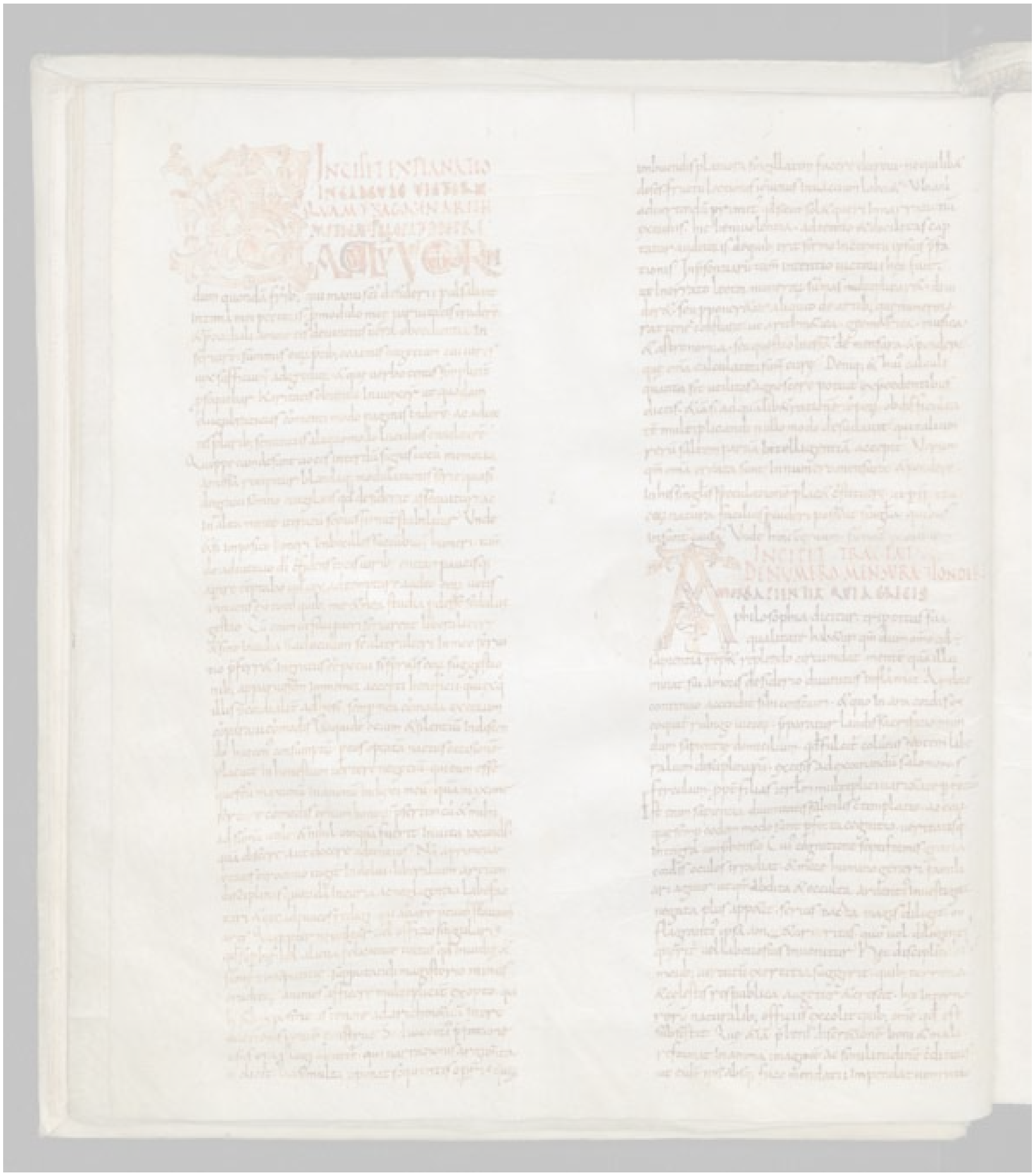
3.4 Bern, Burgerbibliothek Cod. 250, fols. 6v and 7r. Photo taken with permission of the Burgerbibliothek Bern



3.5 Bern, Burgerbibliothek Cod. 250, fol. 2r, detail.
Photo taken with permission of the Burgerbibliothek Bern



3.6 Bern, Burgerbibliothek Cod. 250, fol. 9r.
Photo taken with permission of the Burgerbibliothek Bern

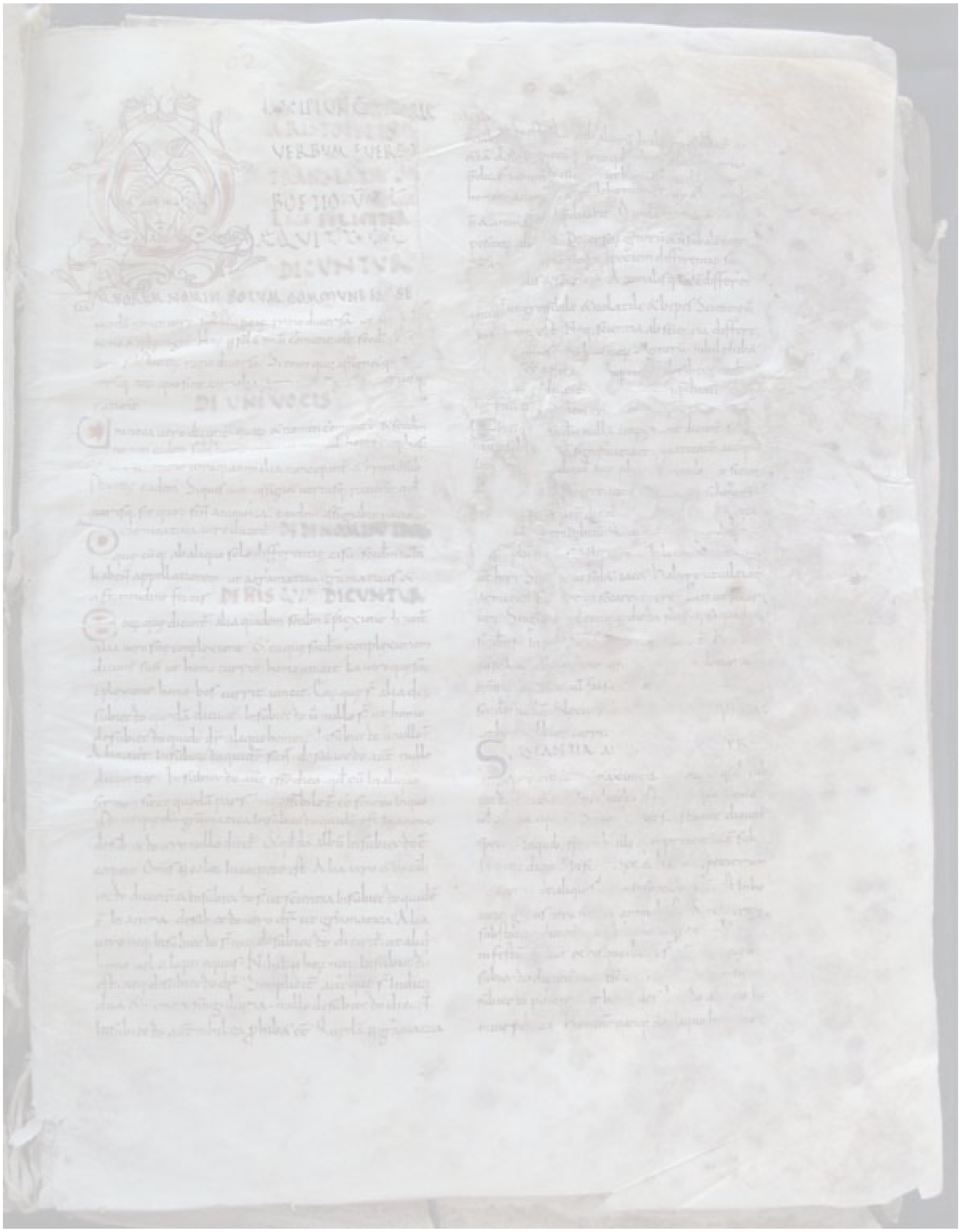


3.7 Berlin, Staatsbibliothek Cod. 138, fol. 7v.

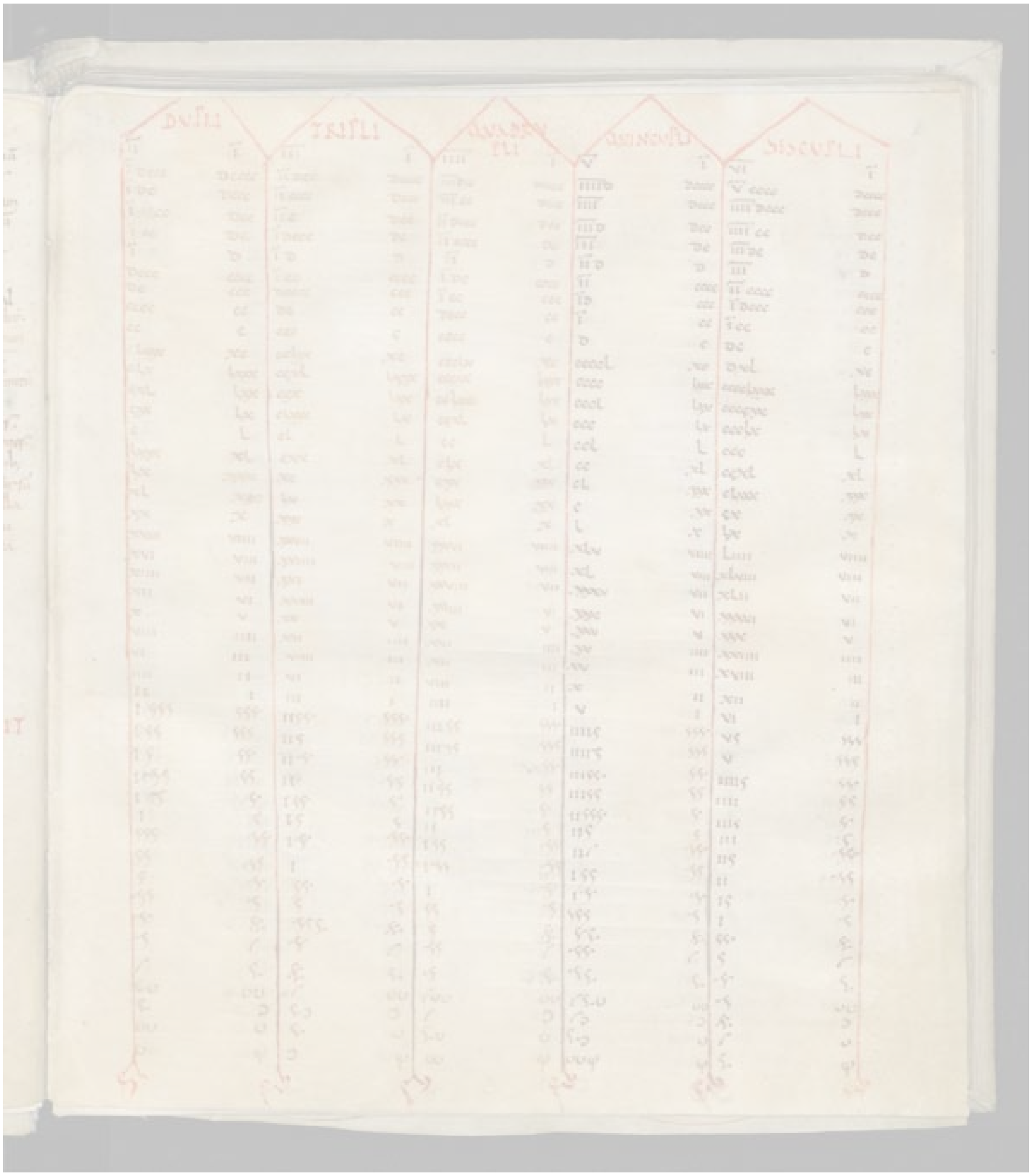
3.8 Berlin, Staatsbibliothek Cod. 138, fol. 7v, detail. © Staatsbibliothek Berlin

3.9 Bern, Burgerbibliothek Cod. 207, fol. 6r, detail. Latin paradigms laid out in an almost tabular form rather than sentence form. Image from Vivian Law, Grammar and Grammarians in the Early Middle Ages (London: Longman, 1997), 255, fig. 12.3.

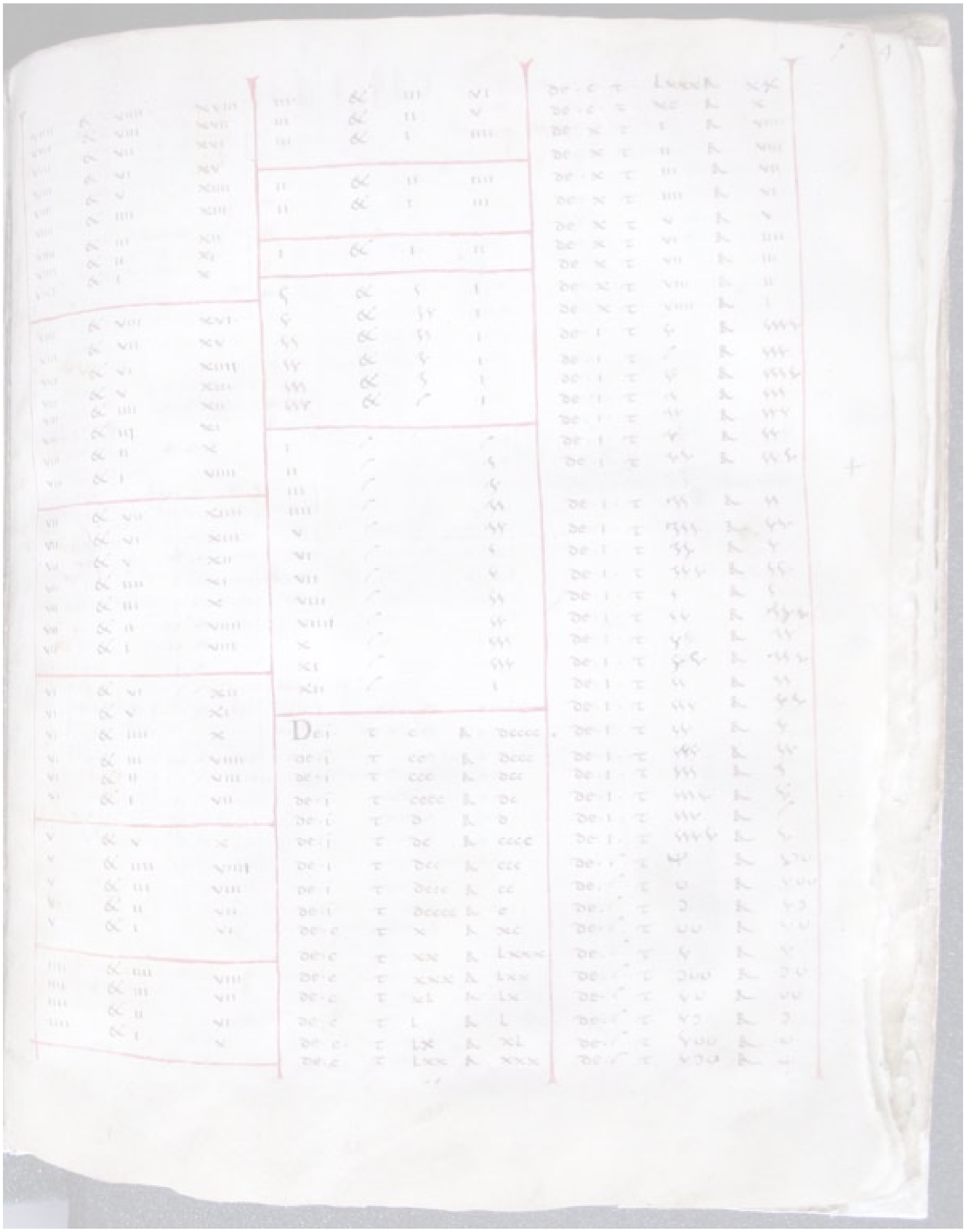
3.10 Milan, Biblioteca Ambrosiana MS C. 128, fol. 1v.
Image from Paul Saenger, *Space between Words: The Origins of Silent Reading* (Stanford: Stanford University Press, 1997), 145, fig. 25.



3.11 Orléans, Bibliothèque municipale MS 277 (233), p. 62.
Photo taken with permission of the Bibliothèque municipale d’Orléans



3.12 Berlin, Staatsbibliothek Cod. 138, fol. 2r.

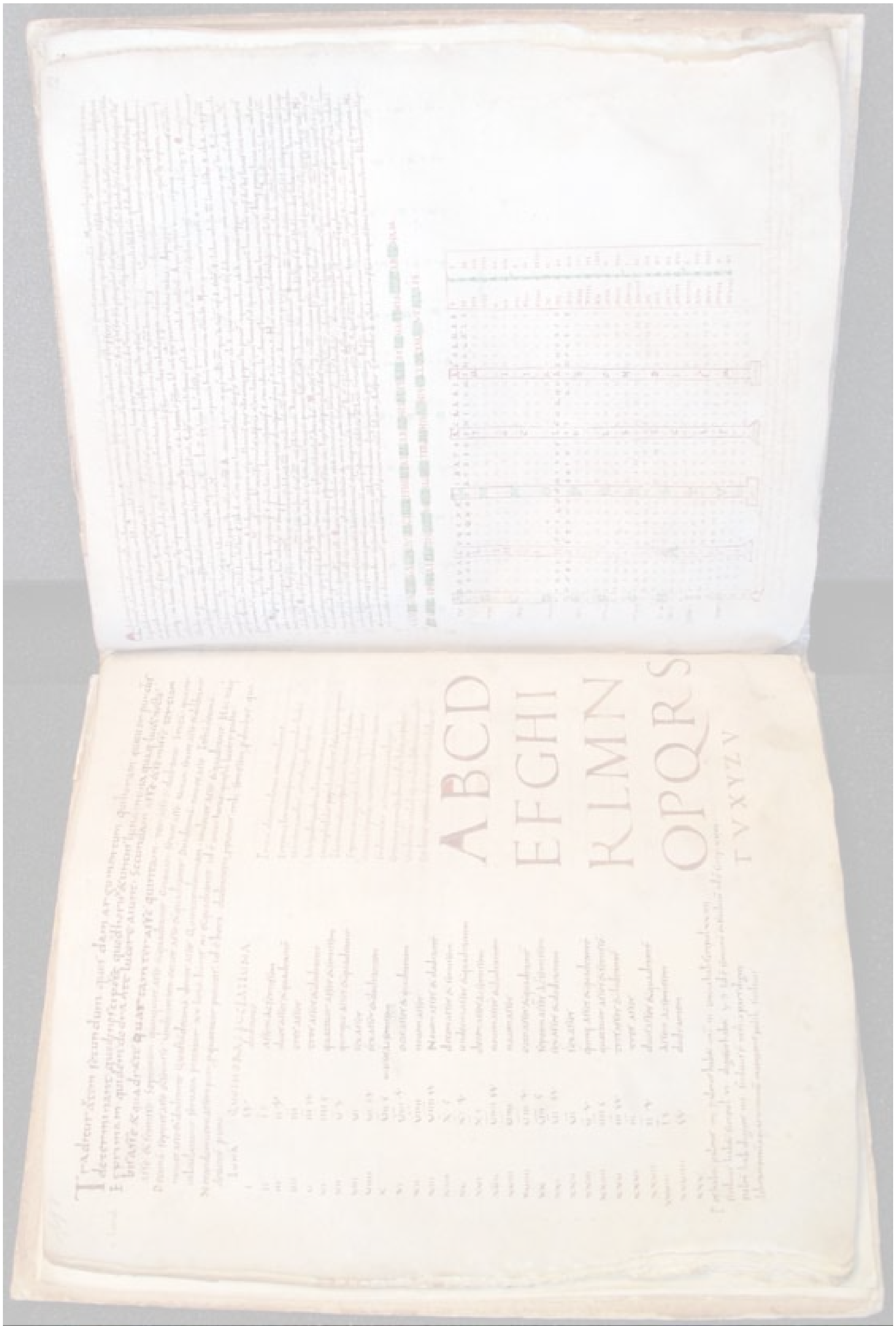


3.13 Bern, Burgerbibliothek Cod. 250, fol. 7r.
Photo taken with permission of the Burgerbibliothek Bern

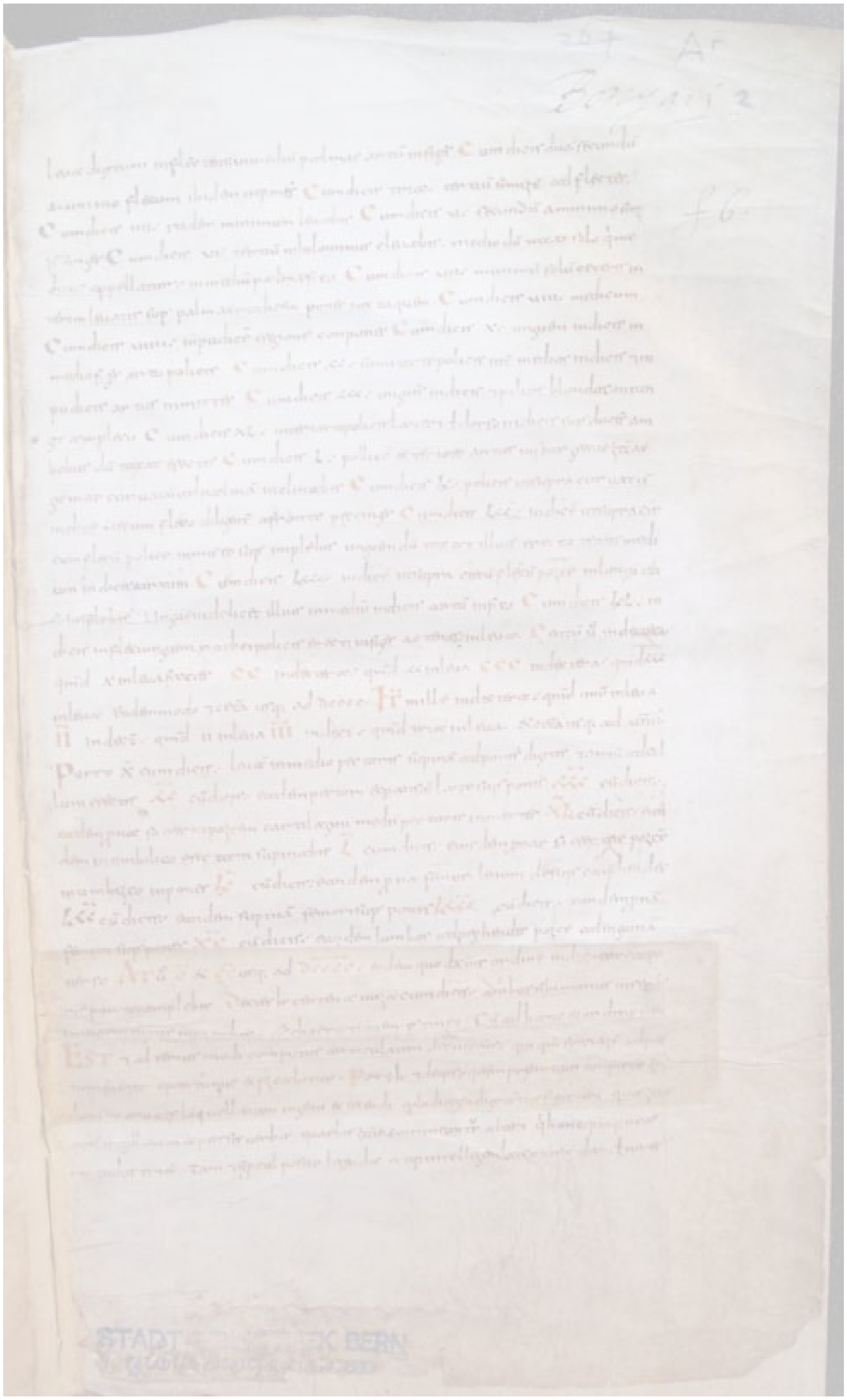
3.14 Berlin, Staatsbibliothek Cod. 138, fol. 15v, detail. © Staatsbibliothek Berlin

siue res incorporea sit quod diuidendum propo
nitur. In hoc argumento unitas assis uocatur.
cuius partes iuxta proportionalitatem suam
propriis sunt insignitae uocabulis. Notis etiam
ad hoc excogitatis. p quas eadem uocabula
exprimantur. Ut per discretionem nominu
et notas nominibus affixas. uniuscuiusque parti
culae notio facilius aduertatur. Ita assis qui
dem quip. I litteram sicut innumeris unum
scribi solet exprimitur. XII partes habet
quarum si unam ei detraxeris. reliquae undeci
partes iabus dicuntur. illa uero quam detrac

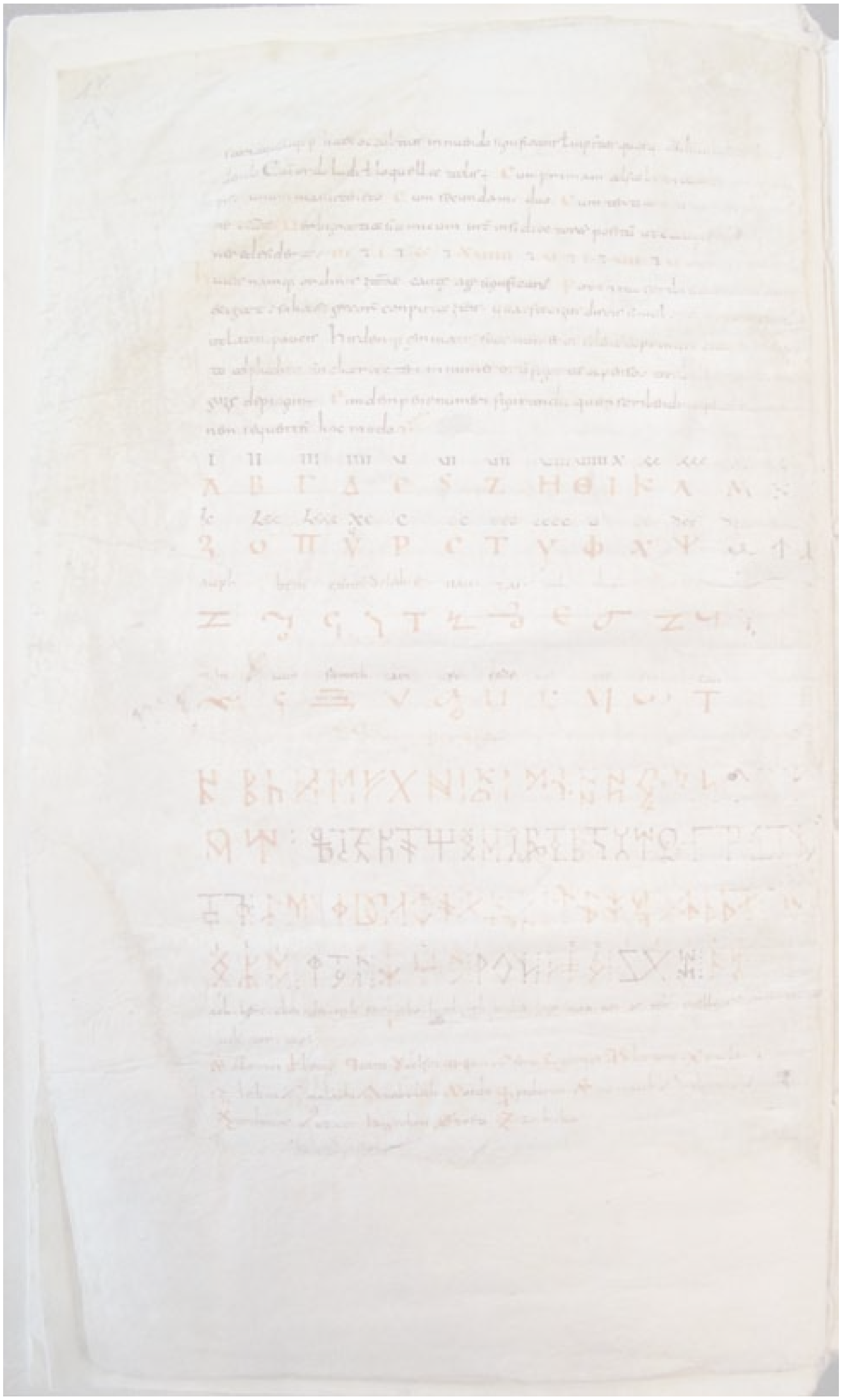
3.15 Bern, Burgerbibliothek Cod. 250, fol. 1v, detail. Photo taken with permission of the Burgerbibliothek Bern

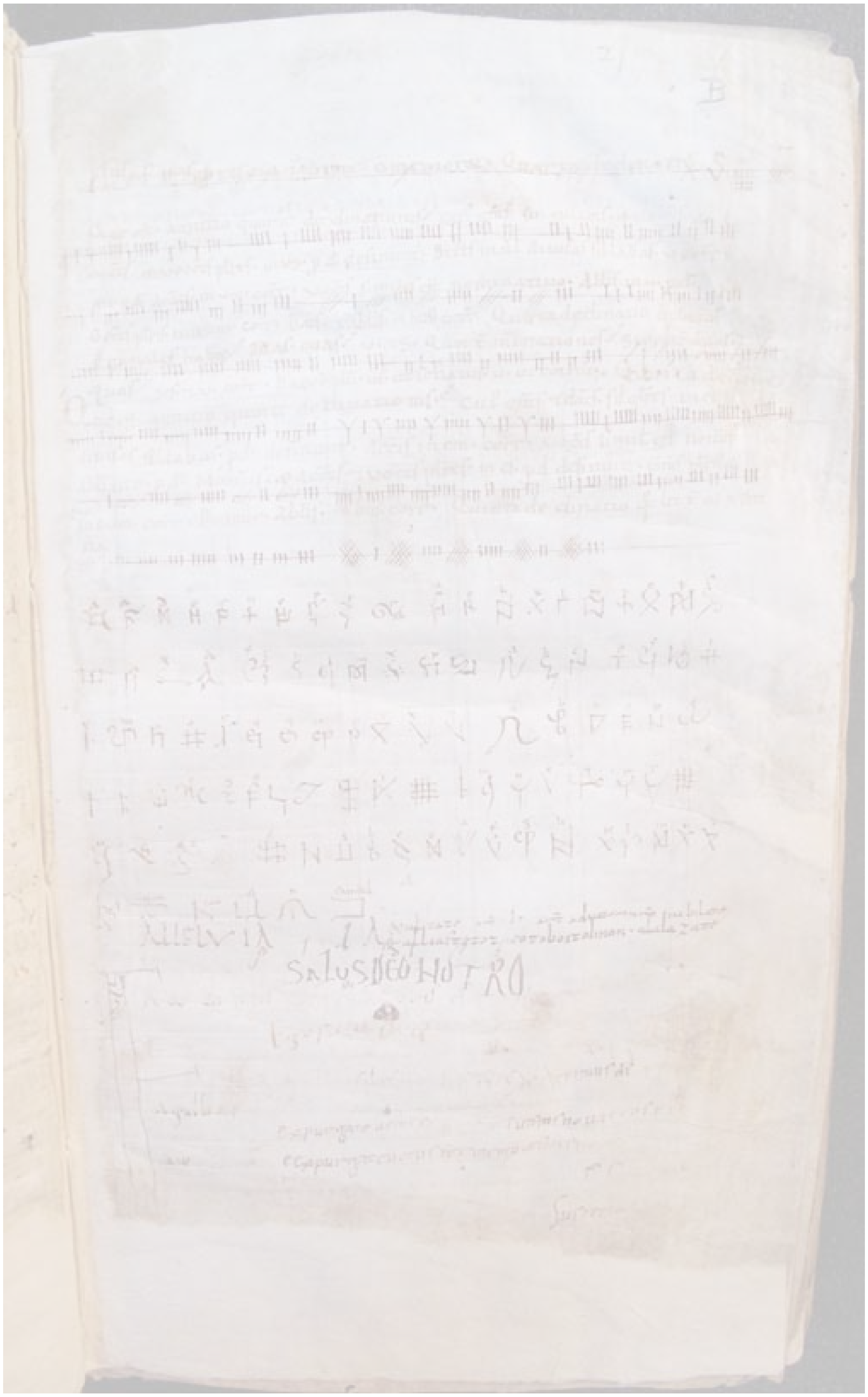


3.16 Bern, Burgerbibliothek Cod. 250, fols. 11v and 12r. Photo taken with permission of the Burgerbibliothek Bern



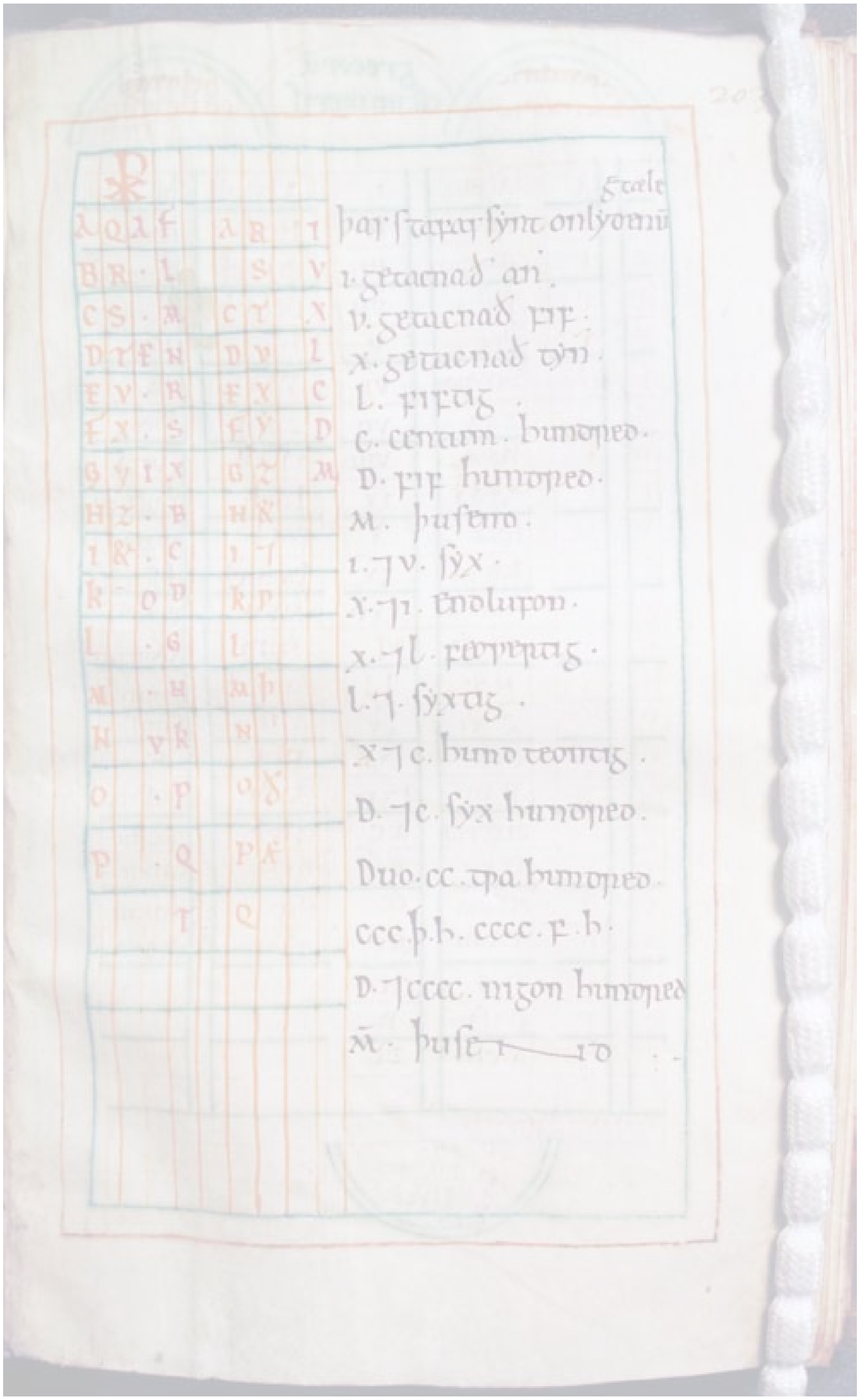
3.17 Bern, Burgerbibliothek Cod. 207, fol. 2r.
Photo taken with permission of the Burgerbibliothek Bern


3.18 Bern, Burgerbibliothek Cod. 207, fol. 2v.
Photo taken with permission of the Burgerbibliothek Bern

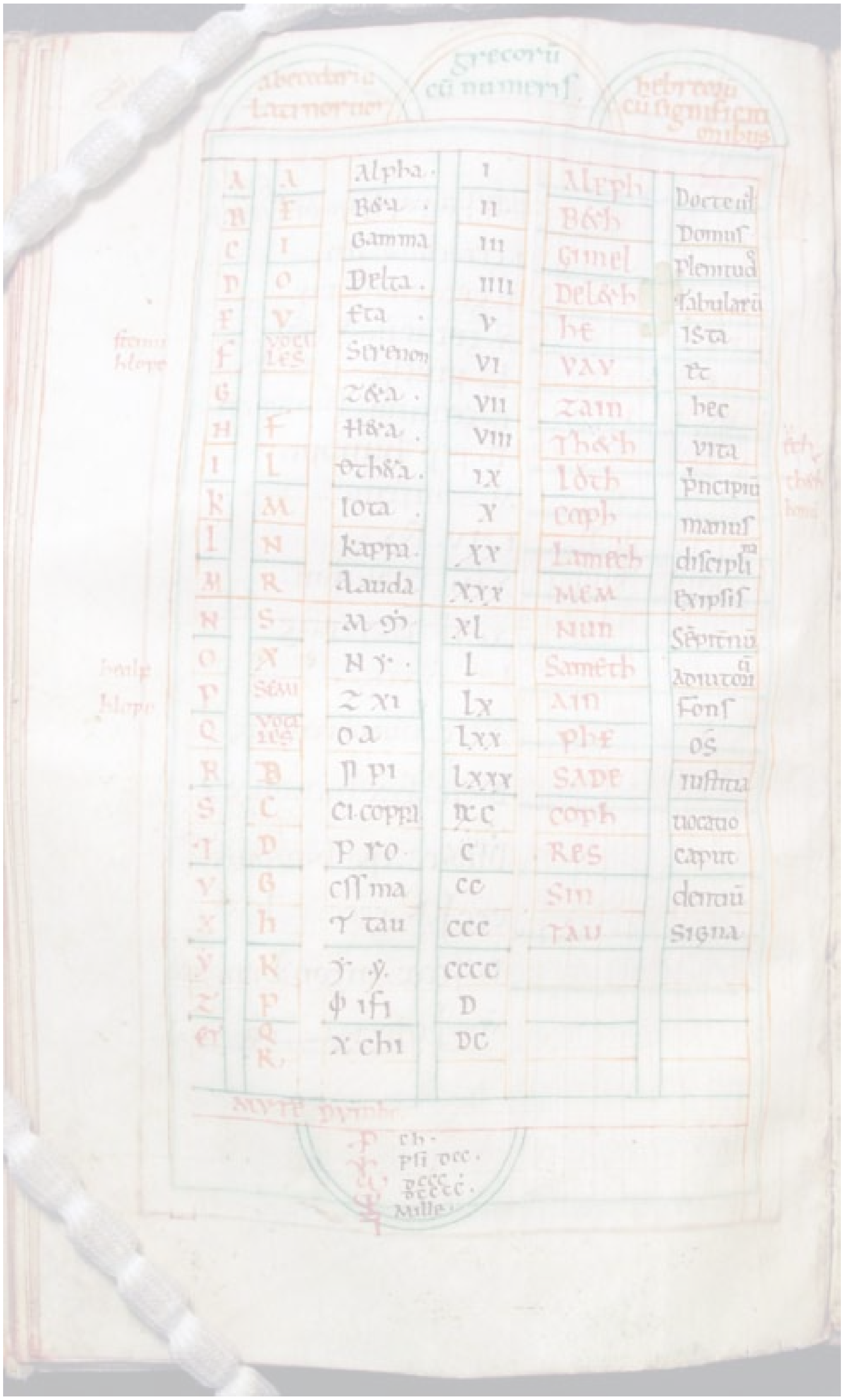


3.19 Bern, Burgerbibliothek Cod. 207, fol. 2b.
Photo taken with permission of the Burgerbibliothek Bern

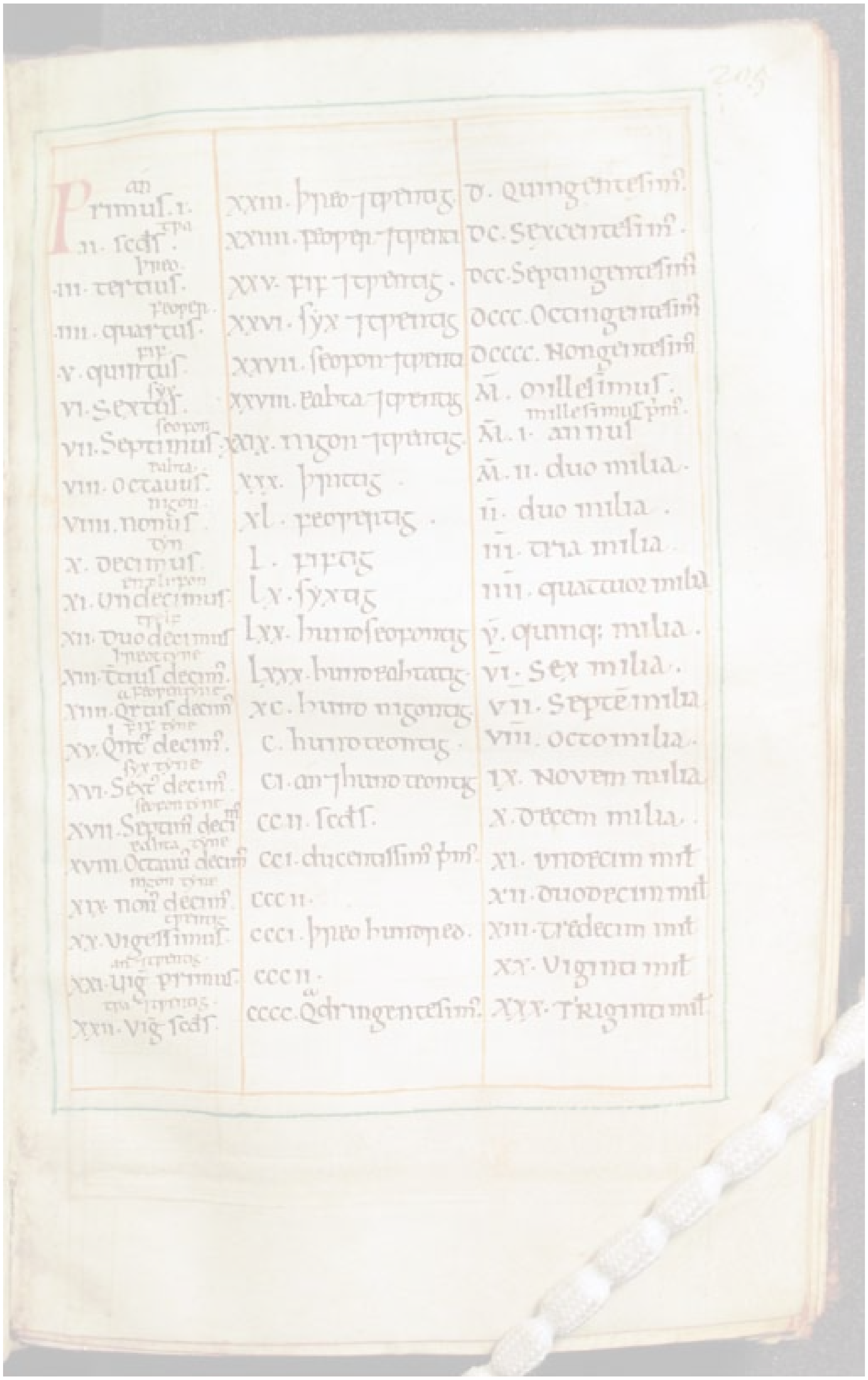
3.20 Vactican City, Biblioteca Apostolica Pal. lat. MS 1449, fol. 118v



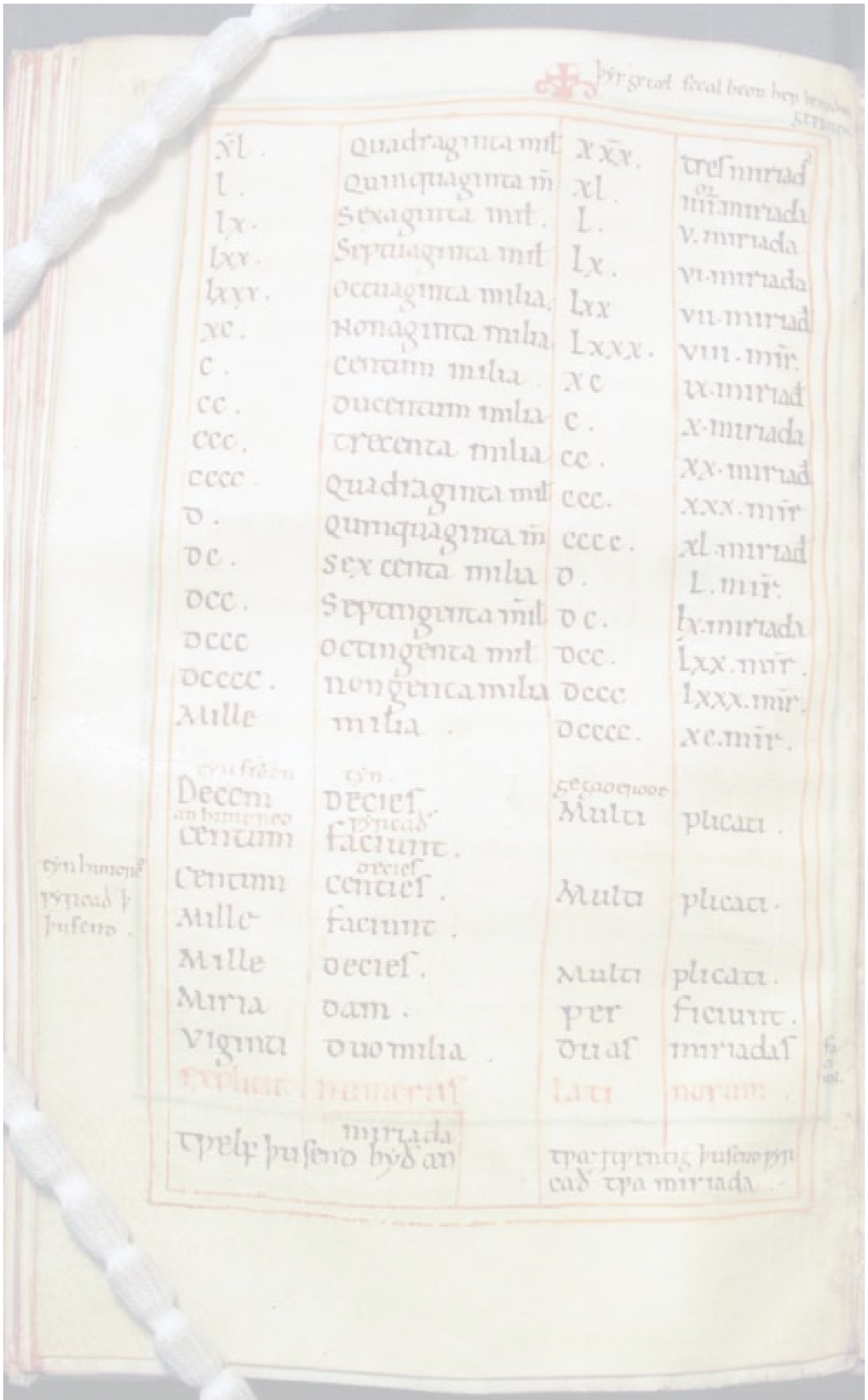
3.21 Oxford, Bodleian Library Ashmole MS 328, p. 203.
Photo taken with permission of the Bodleian Library, Oxford University



3.22 Oxford, Bodleian Library Ashmole MS 328, p. 204.
Photo taken with permission of the Bodleian Library, Oxford University

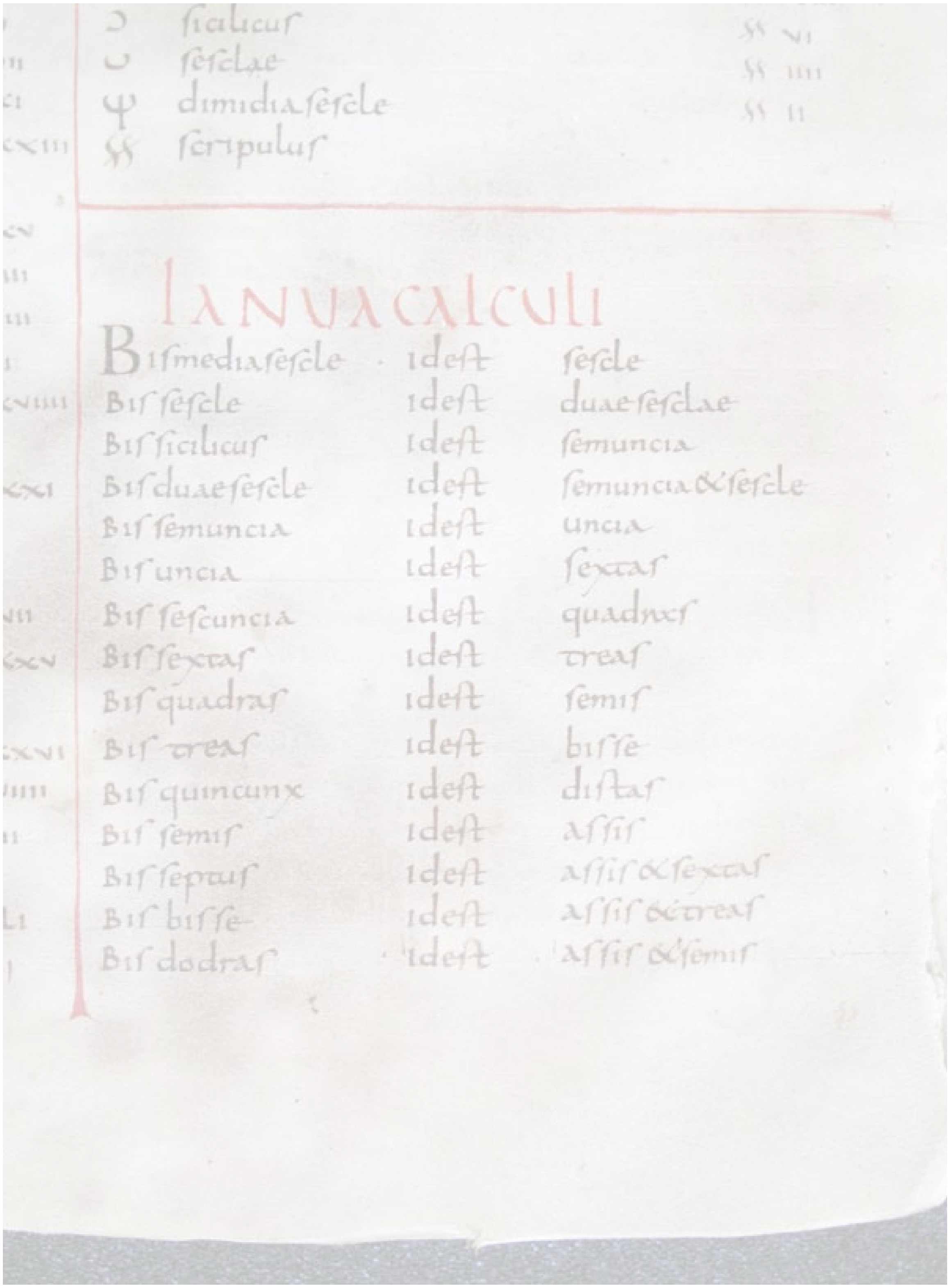


3.23 Oxford, Bodleian Library Ashmole MS 328, p. 205.
Photo taken with permission of the Bodleian Library, Oxford University

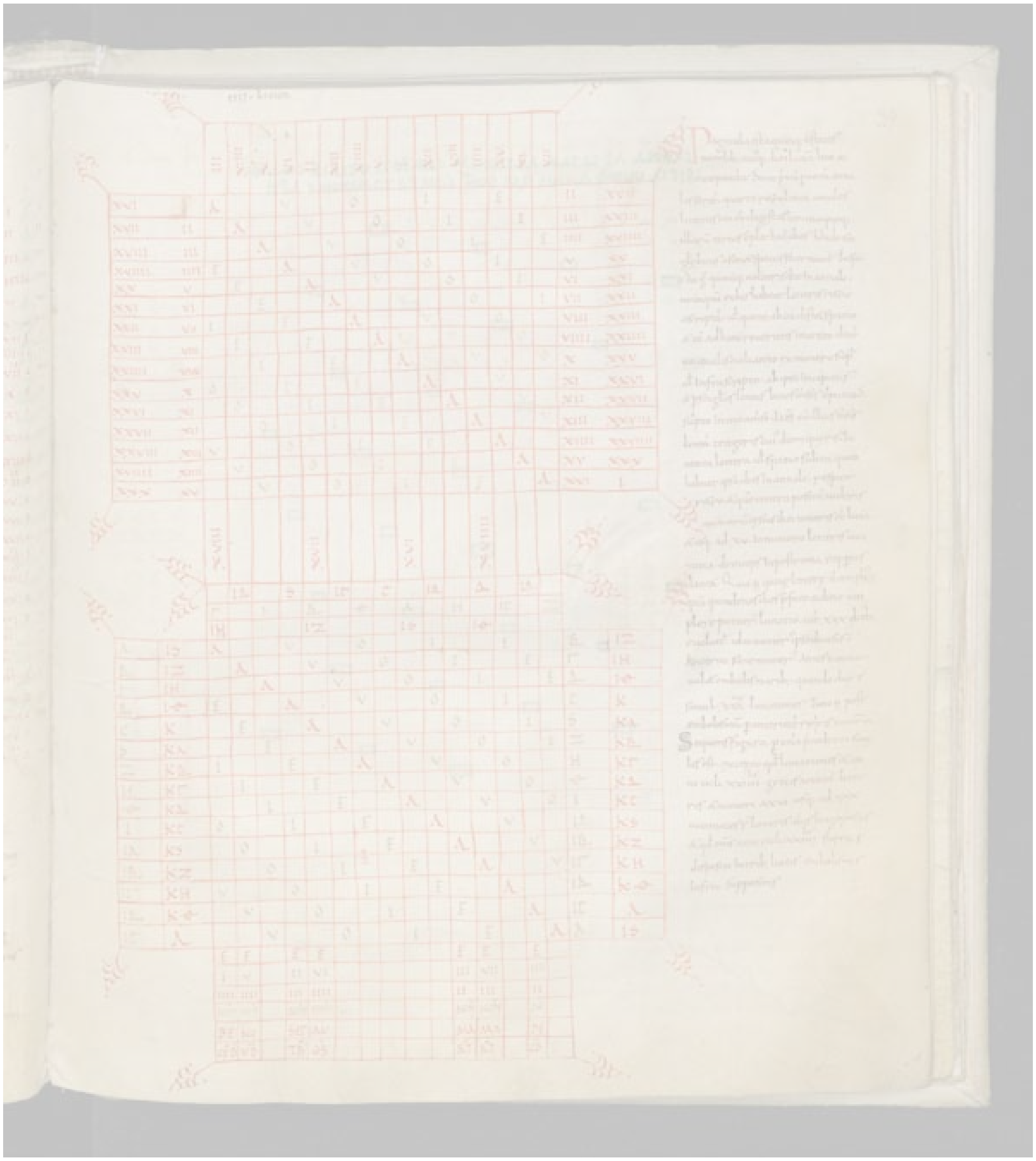


xl.	Quadraginta mil	xxx.	tres miriad
l.	Quinquaginta m	xl.	iiii miriada
lx.	Sexaginta mil.	l.	v. miriada
lxx.	Septuaginta mil	lx.	vi miriada
lxxx.	Octuaginta milia	lxx	vii miriad
xc.	Nonaginta milia	lxxx.	viii. mir.
c.	Centum milia.	xc	ix miriad
cc.	Ducentum milia	c.	x miriada
ccc.	Trecenta milia	cc.	xx miriad
cccc	Quadraginta mil	ccc.	xxx. mir
d.	Quinquaginta m	cccc.	xl miriad
dc.	Sex centa milia	d.	l. mir.
dcc.	Septingenta mil	dc.	lx miriada
dccc	Octingenta mil	dcc.	lxx. mir.
dcccc.	Nongenta milia	dccc	lxxx. mir.
Mille	milia.	dcccc.	xc. mir.
Decem	decies	Multi	plicati.
Centum	faciunt.		
Centum	centies.	Multi	plicati.
Mille	faciunt.		
Mille	decies.	Multi	plicati.
Miria	dam.	per	ficiunt.
Viginti	duo milia	duas	miriadas
Explicit	numerus	Lati	norum.

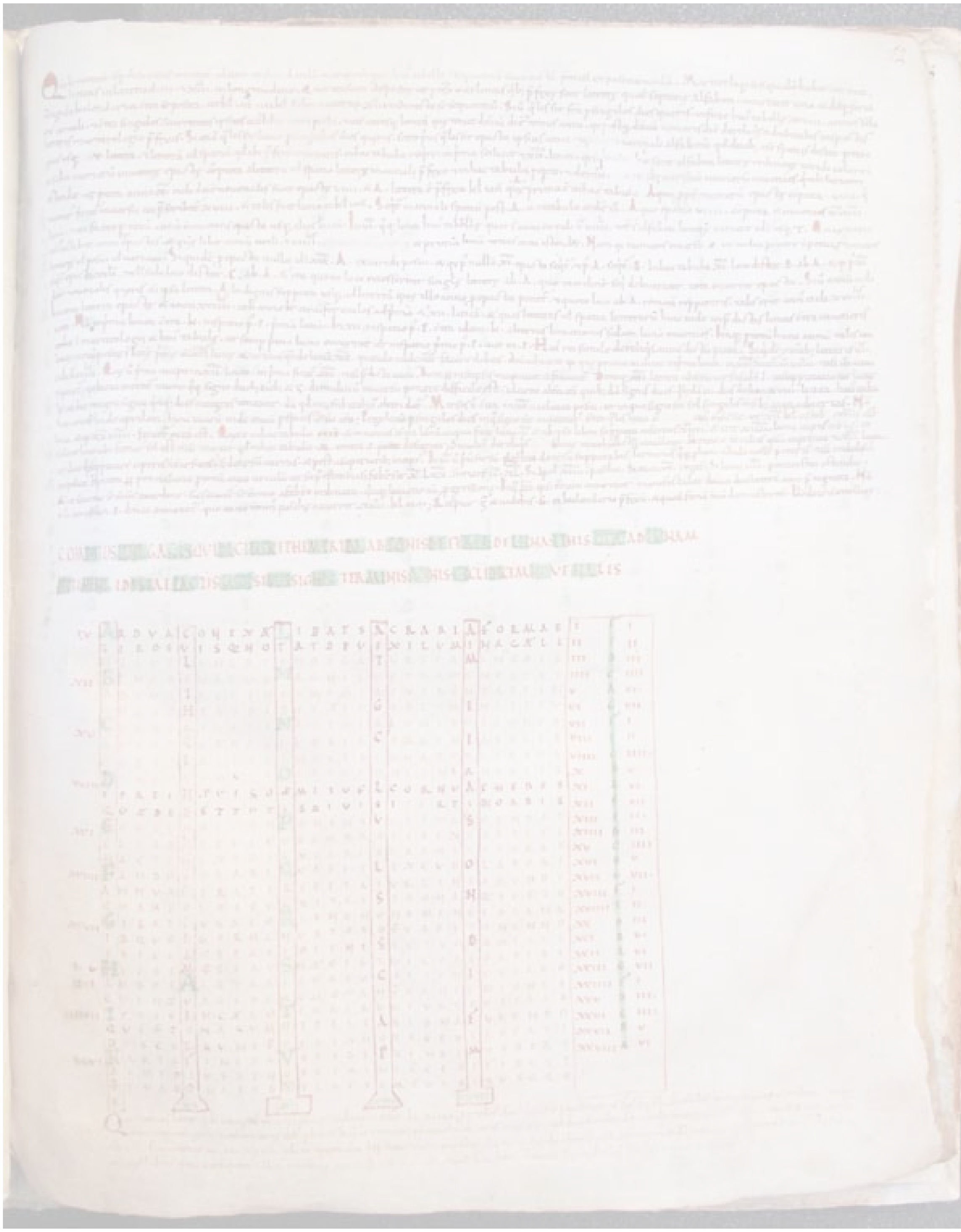
3.24 Oxford, Bodleian Library Ashmole MS 328, p. 206.
Photo taken with permission of the Bodleian Library, Oxford University



3.25 Bern, Burgerbibliothek Cod. 250, fol. 9r, detail.
Photo taken with permission of the Burgerbibliothek Bern

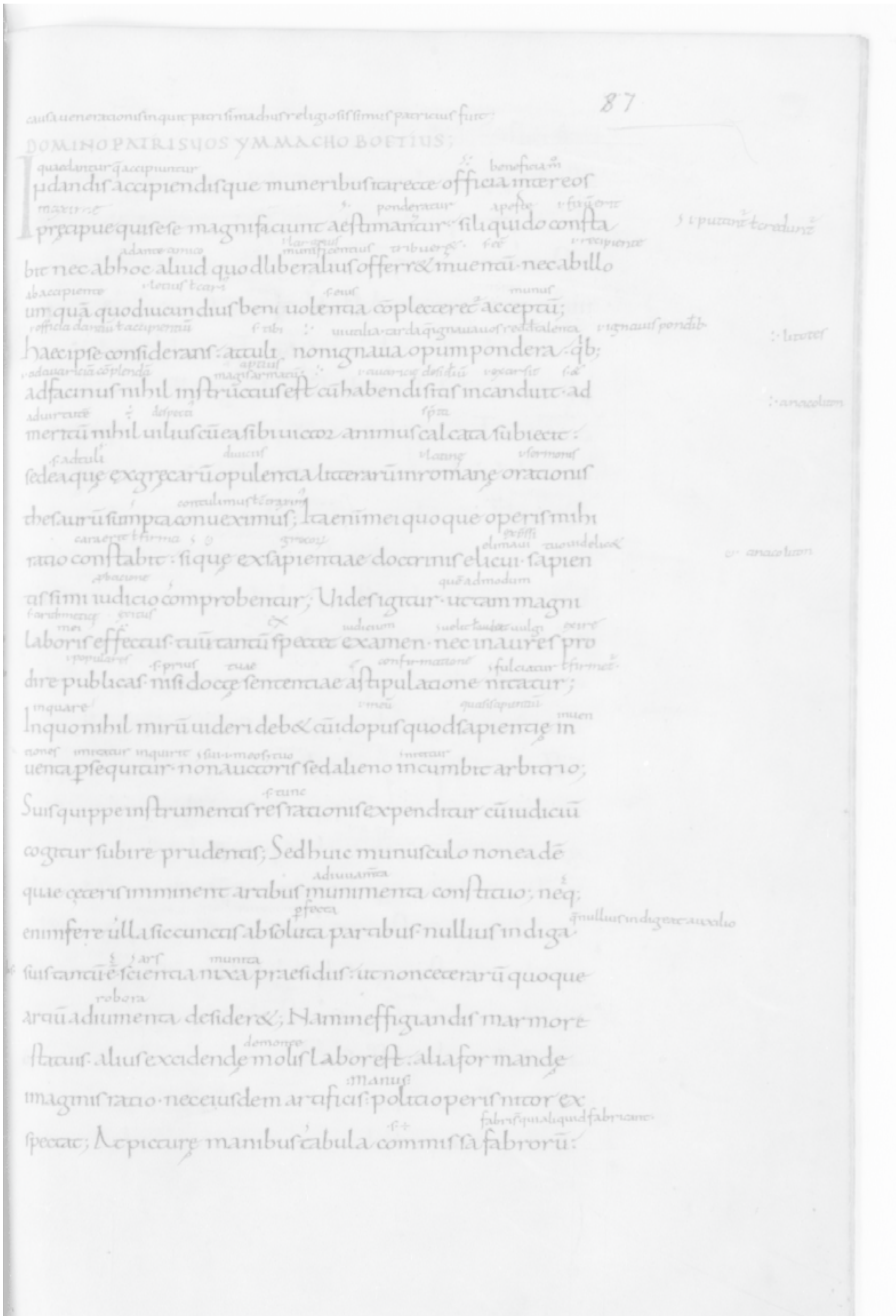


3.26 Berlin, Staatsbibliothek Cod. 138, fol. 34r.

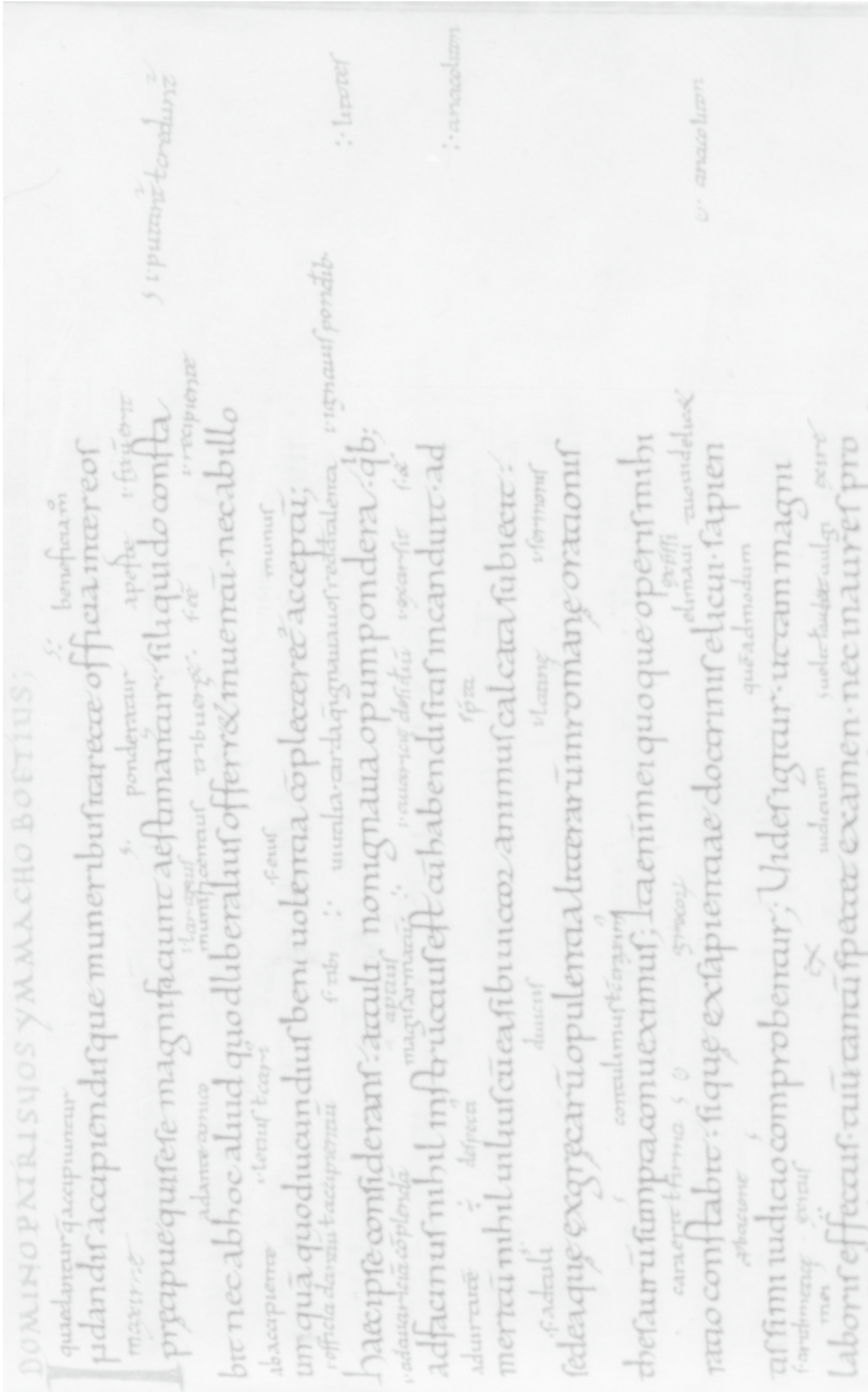


3.27 Bern, Burgerbibliothek Cod. 250, fol. 12r.

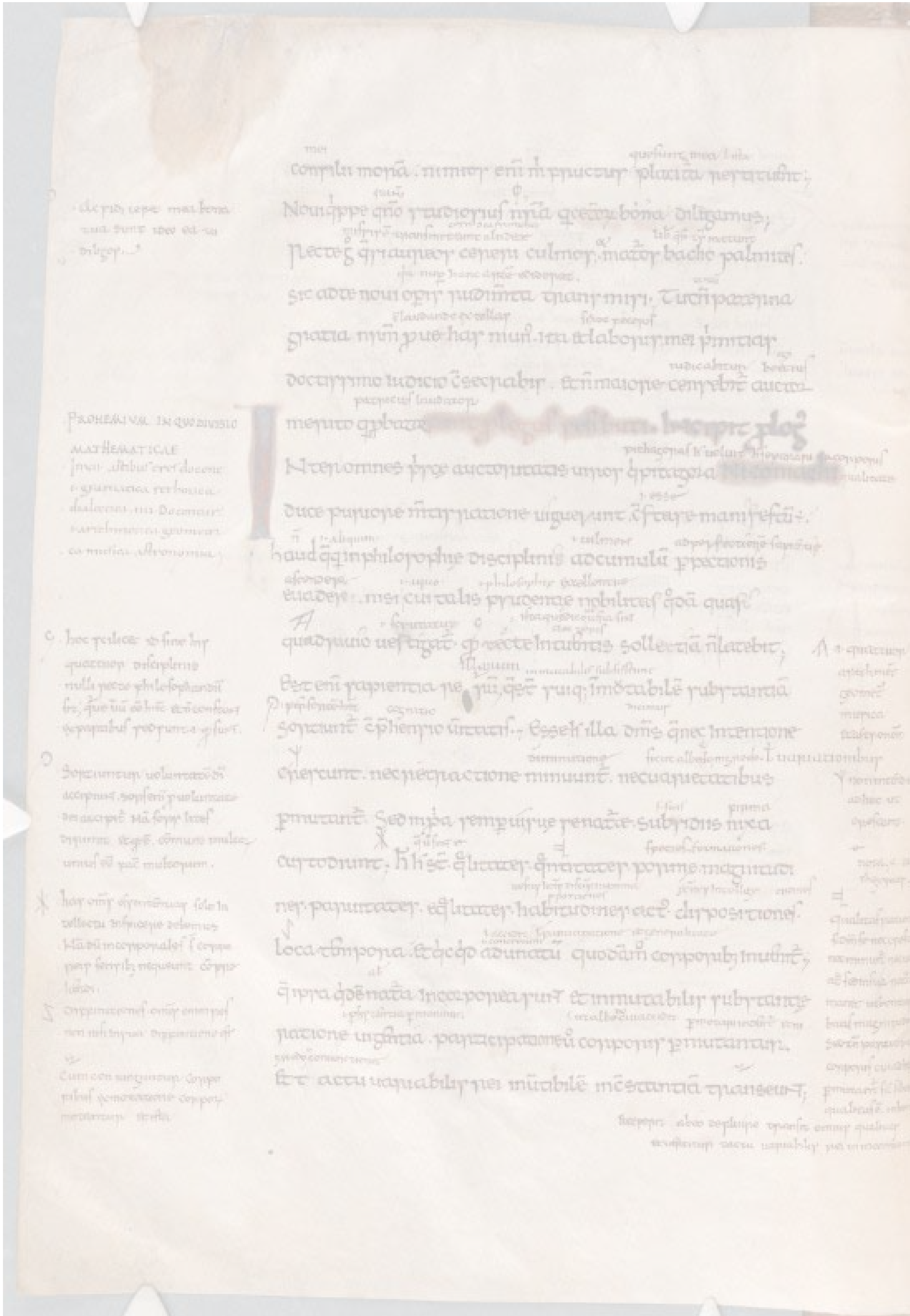
3.28 Bern, Burgerbibliothek Cod. 250, fol. 12r, detail.


DOMINO PATRI SUO SYMMACHO BOETIUS;
In dandis accipiendisque muneribus ita recte officia inter eos
praecipue qui sese magnificiunt aestimantur si liquido consta
bit nec ab hoc aliud quod liberalius offerret inueniri nec ab illo
umquam quod iucundius beneuolentia complecteretur acceptum;
Haec ipse considerans attuli non ignaua opum pondera quibus
ad facinus nihil instructius est cum habendi sitis incanduit ad
mercium nihil uilius cum ea sibi uictor animus calcata subiecit
sed ea quae ex graecarum opulentia litterarum in romanae orationis
thesaurum sumpta conuexi mus; Ita enim mei quoque operis mihi
ratio constabit si quae ex sapientiae doctrinis elicui sapien
tissimi iudicio comprobentur; Vides igitur ut tam magni
laboris effectus tuum tantum spectet examen nec in aures pro
dire publicas nisi docte sententiae adstipulatione nitatur;
In quo nihil mirum uideri debet cum id opus quod sapientiae in
uenta persequitur non auctoris sed alieno incumbit arbitrio;
Suis quippe instrumentis res rationis expenditur cum iudicium
cogitur subire prudentis; Sed huic munusculo non ea deest
quae ceteris imminent artibus munimenta constitutio; neque
enim fere ulla secum ceteris absoluta partibus nullius indiget
sui tantum est scientia nixa praesidiis ut non ceterarum quoque
artium adiumenta desideret; Nam in effigiandis marmore
statuis alius excidendae molis labor est alia formandae
imaginis ratio nec eiusdem artificis polita operis nitor ex
spectat; At picturae manibus tabula commissa fabrorum

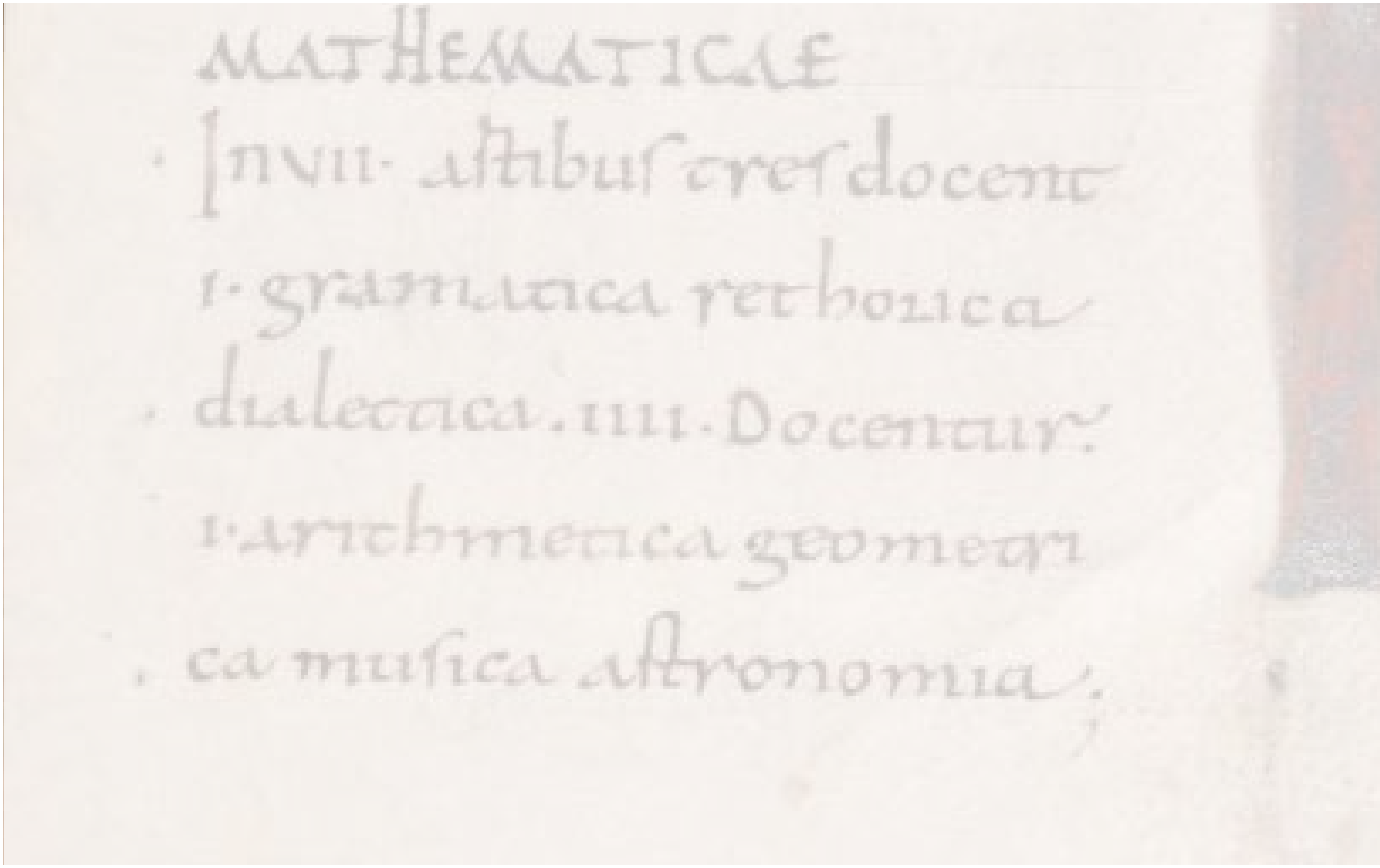
3.29 Paris, Bibliothèque nationale de France lat. MS 6401, fol. 87r.



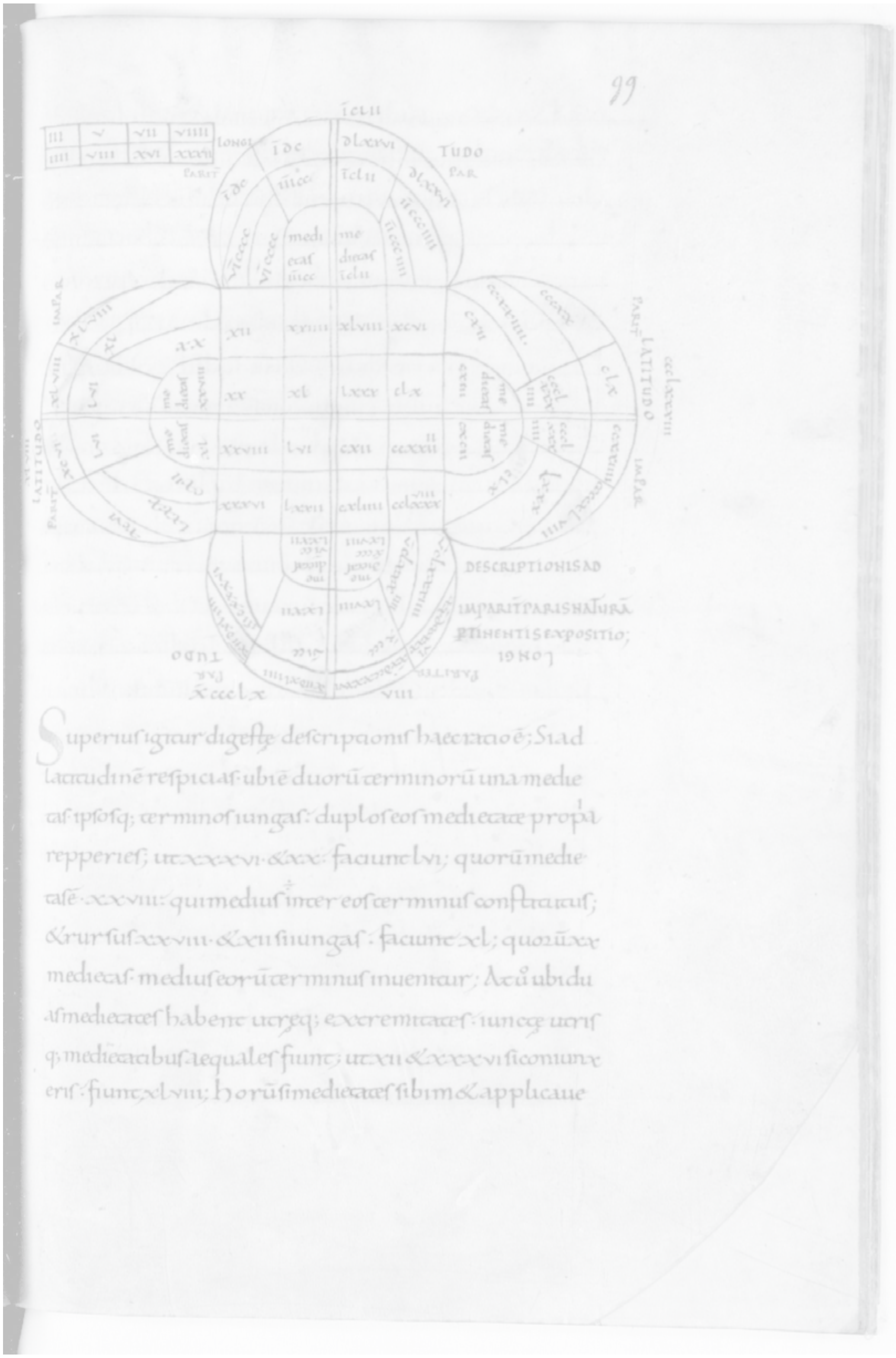
3.20 Paris, Bibliothèque nationale de France lat. MS 6401, fol. 87r, detail. © Bibliothèque nationale de France



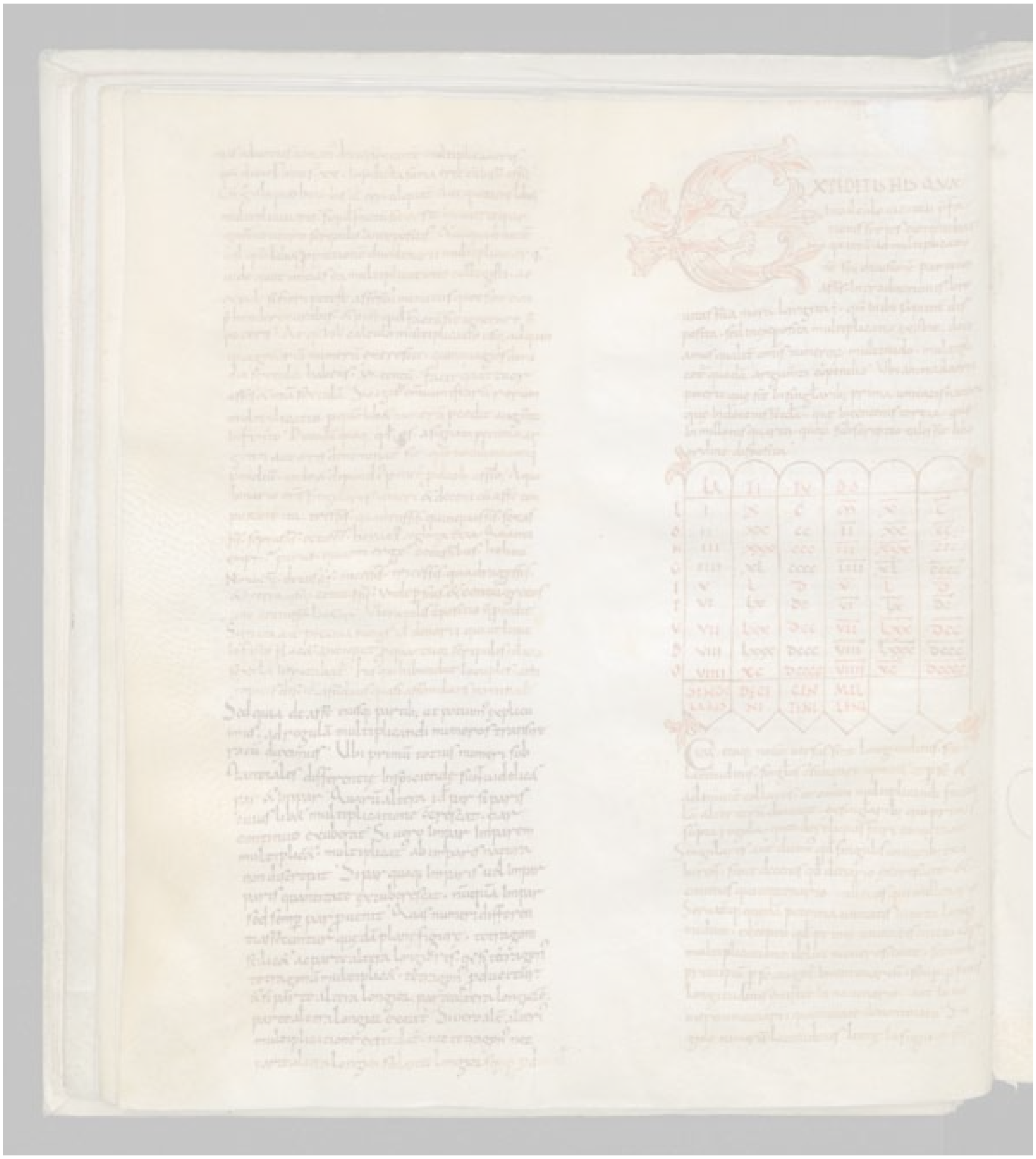
3.31 Cambridge, Corpus Christi College MS 352, fol. 3v.



3.32 Cambridge, Corpus Christi College MS 352, fol. 3v, detail. © Corpus Christi College, Cambridge University

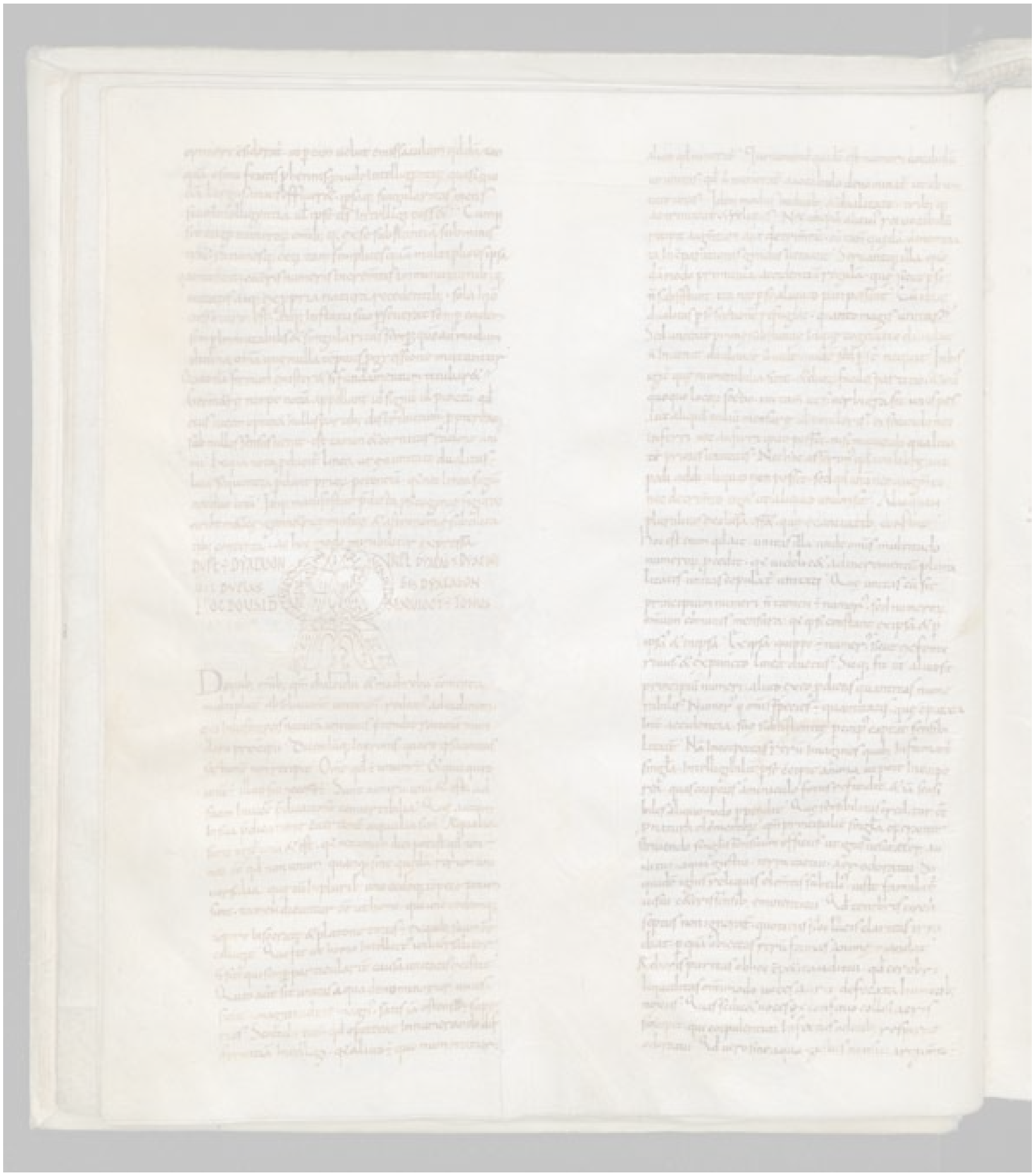


3.33 Paris, Bibliothèque nationale de France lat. MS 6401, fol. 99r.

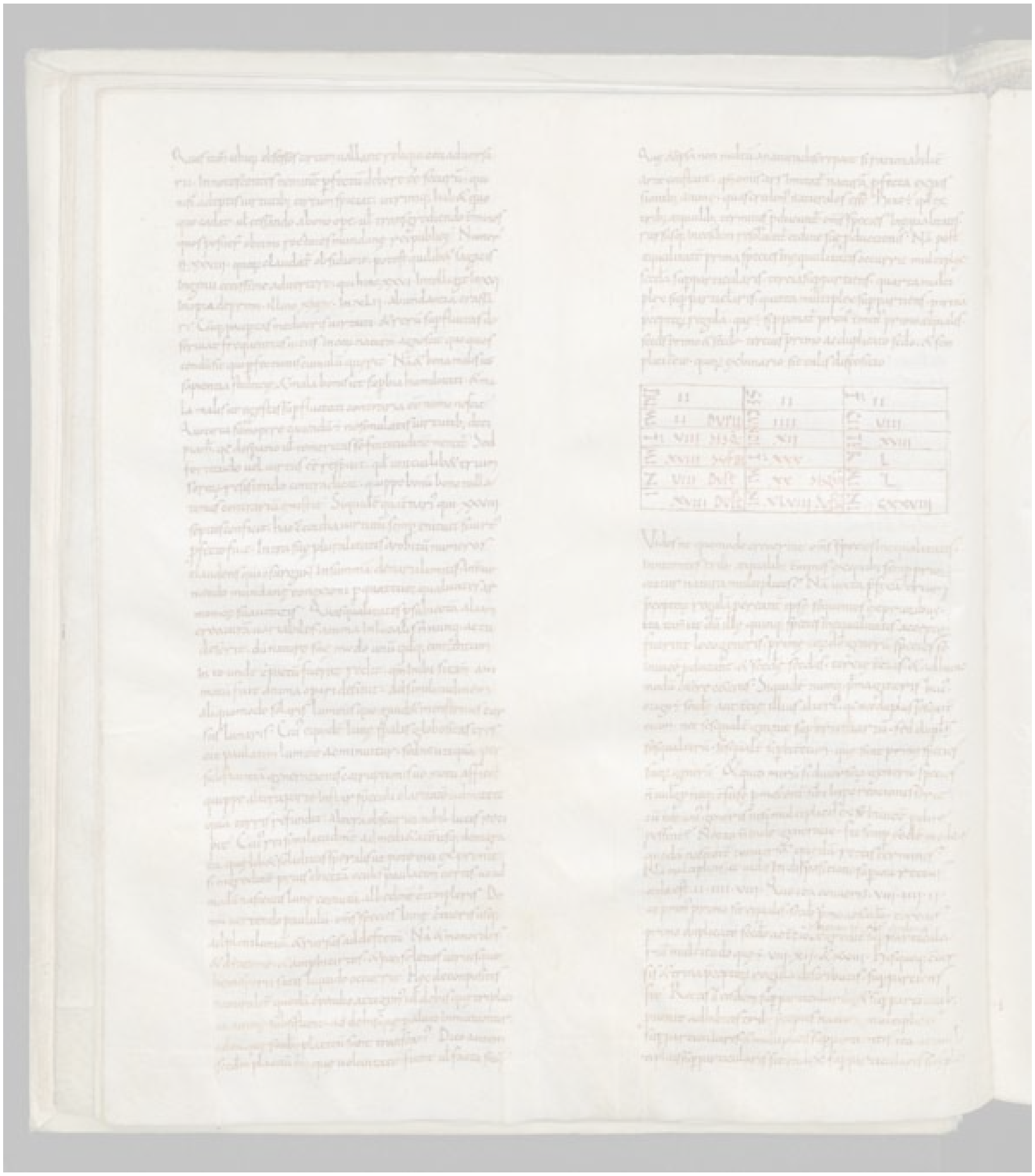


3.34 Berlin, Staatsbibliothek Cod. 138, fol. 16v.

3.35 Berlin, Staatsbibliothek Cod. 138, fol. 16v, detail.



3.36 Berlin, Staatsbibliothek Cod. 138, fol. 9v.



3.37 Berlin, Staatsbibliothek Cod. 138, fol. 11v.
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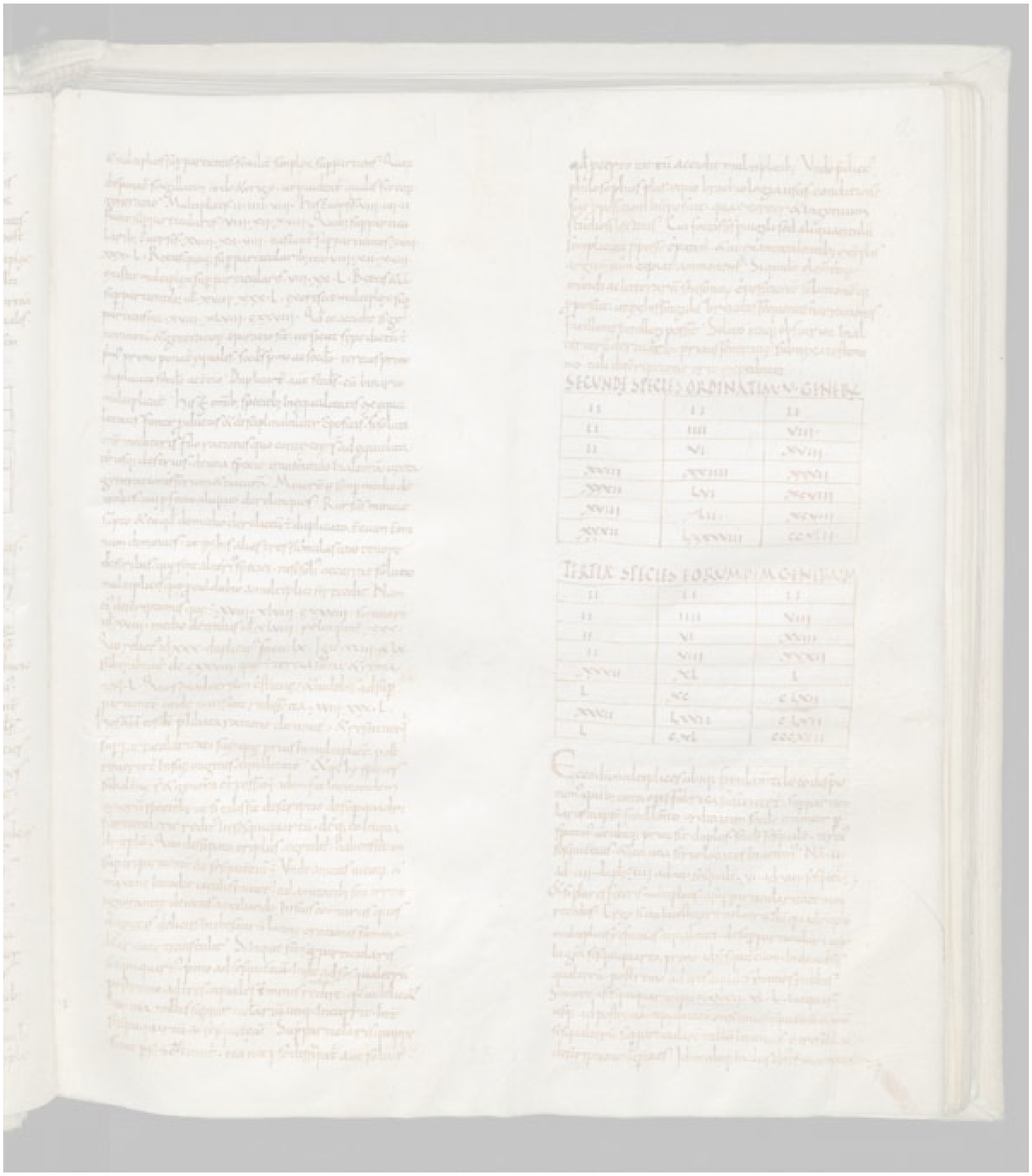
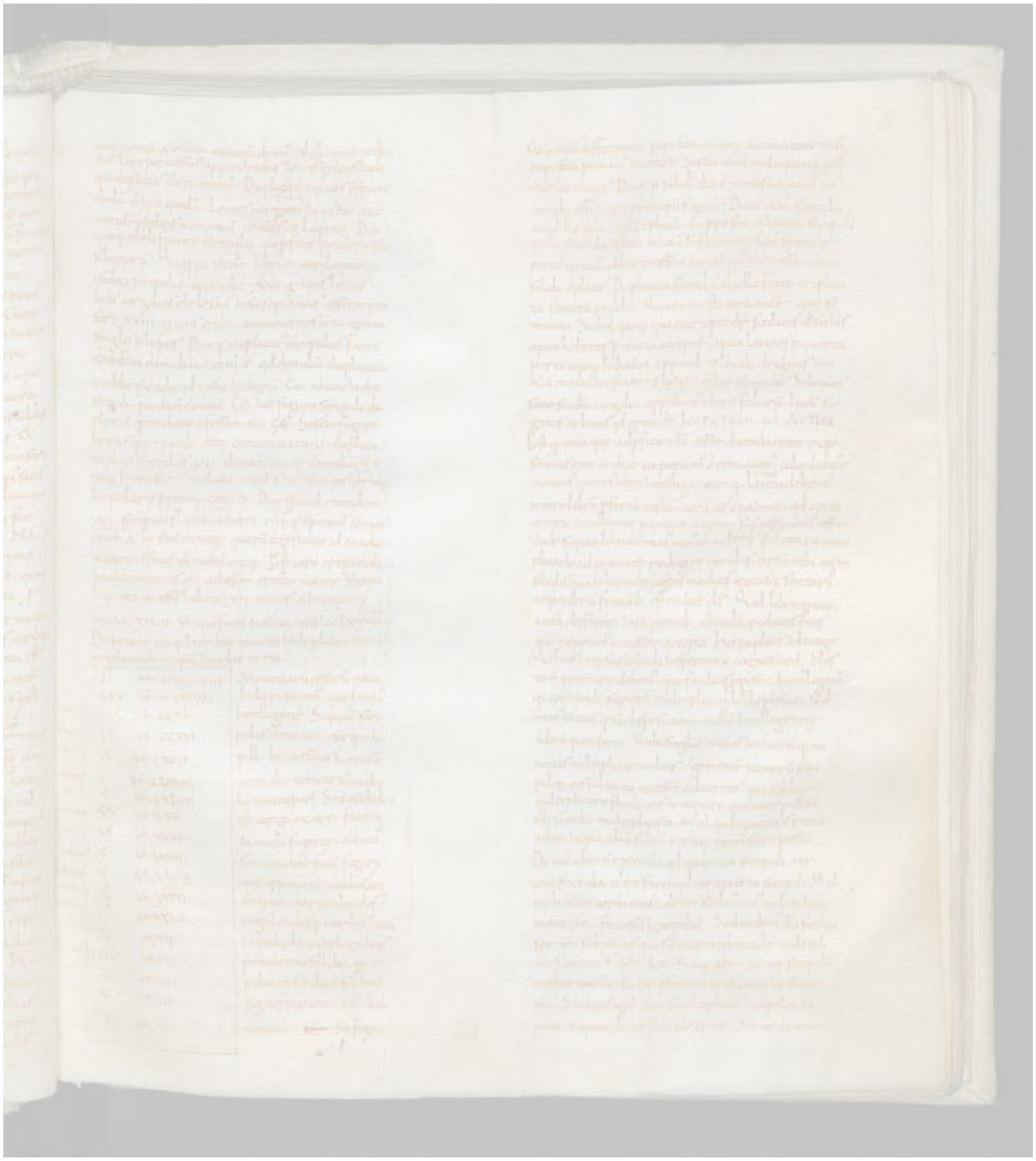
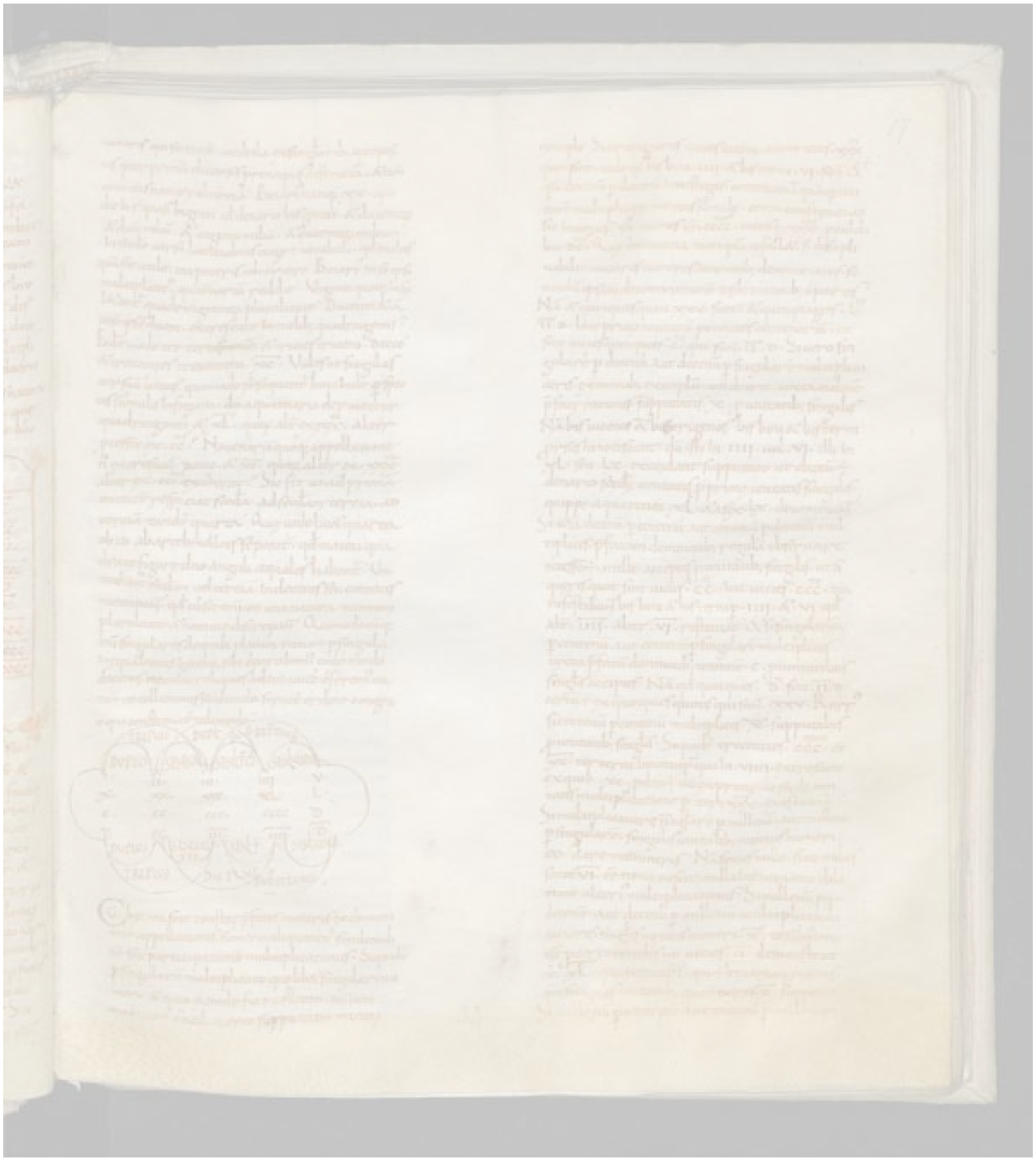


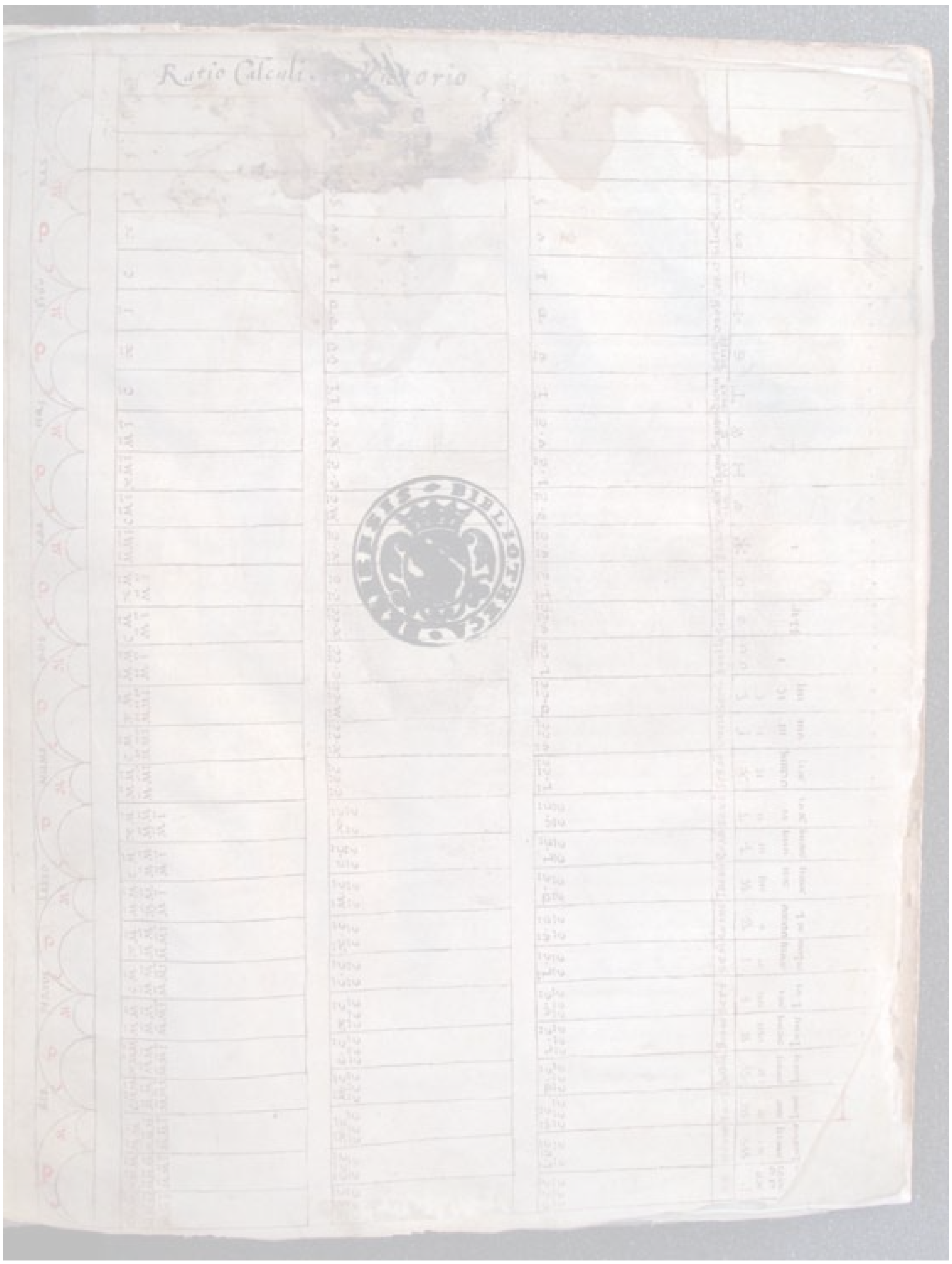
3.38 Berlin, Staatsbibliothek Cod. 138, fol. 12r.



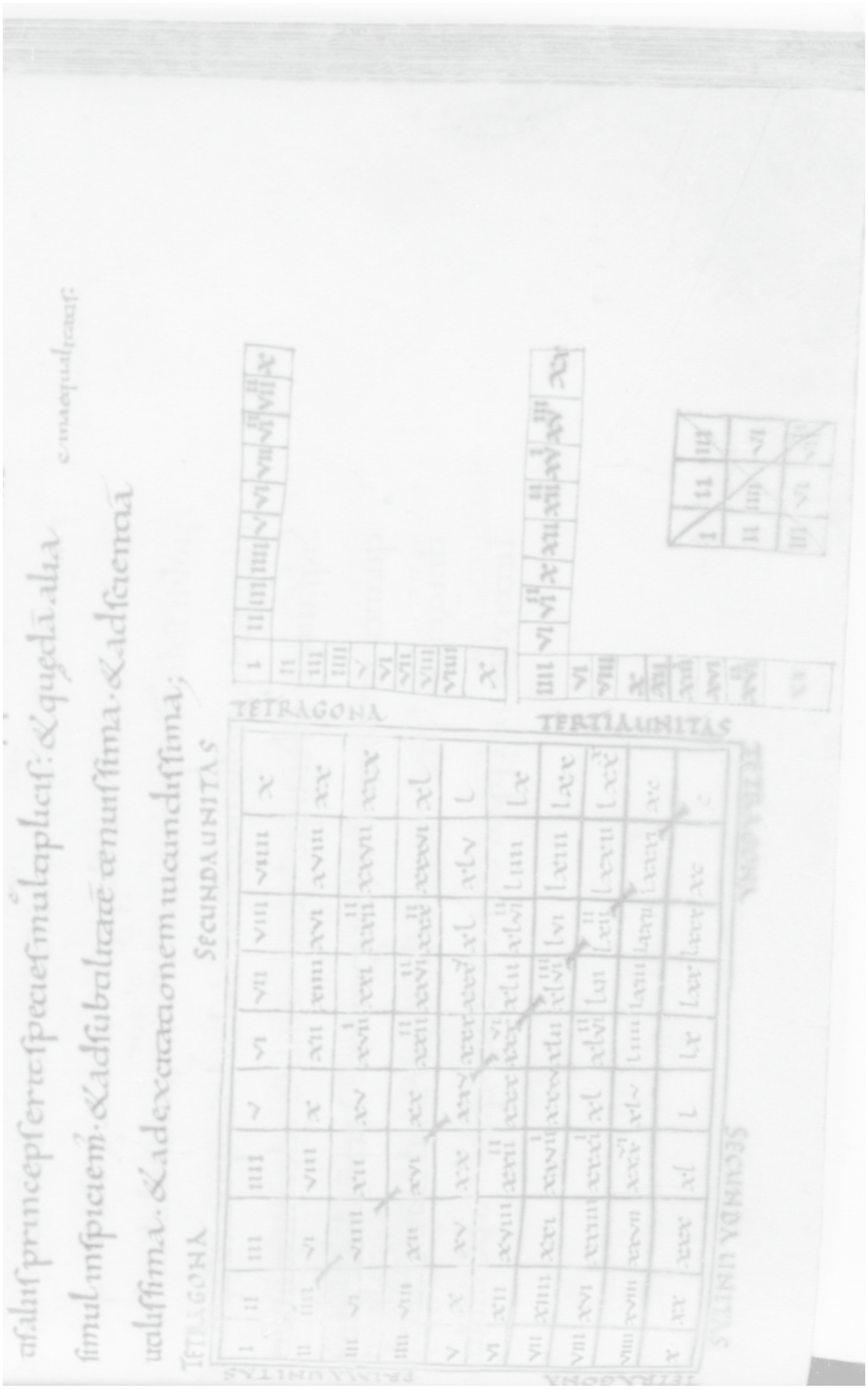
3.39 Berlin, Staatsbibliothek Cod. 138, fol. 16r.



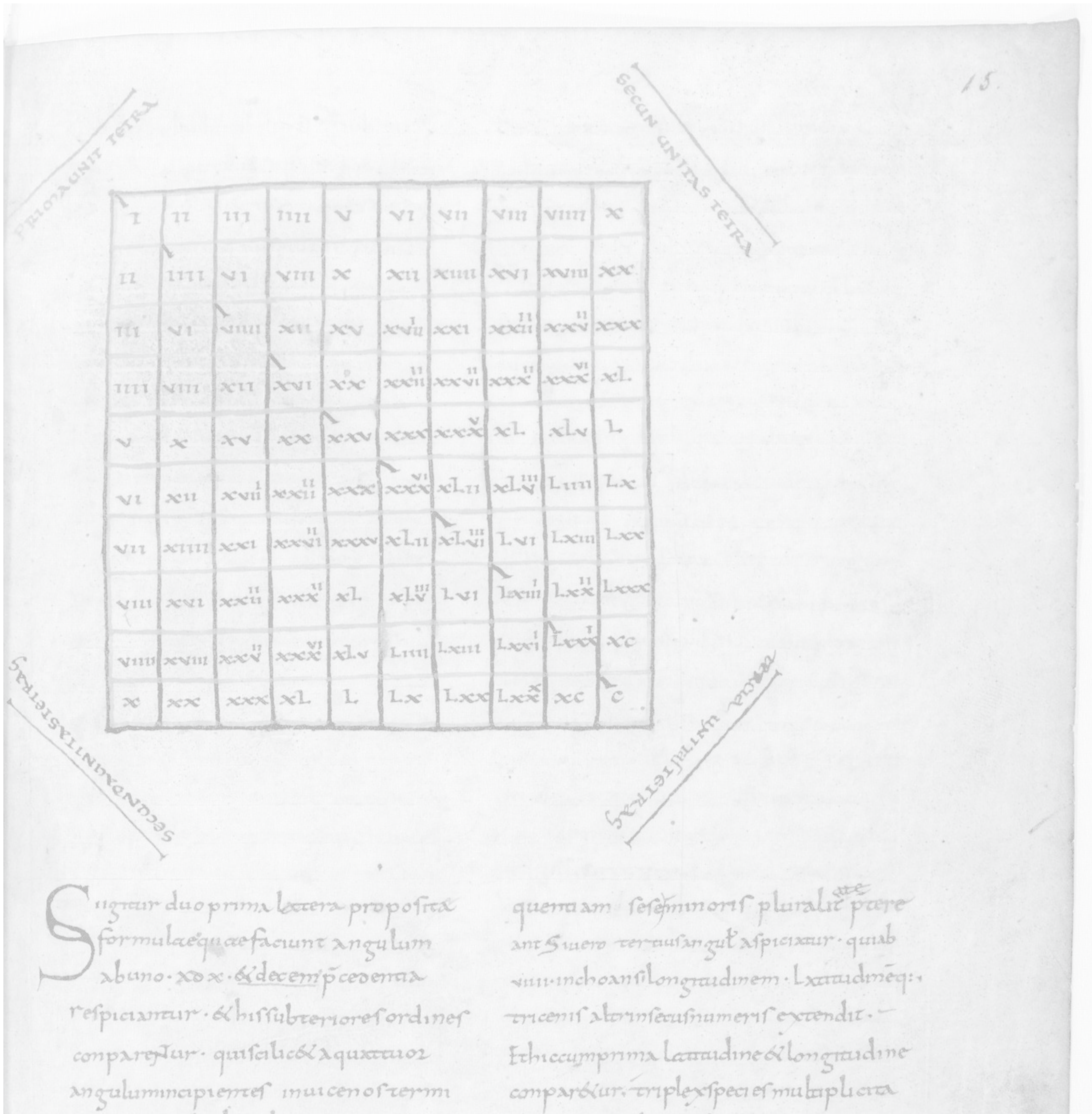
3.40 Berlin, Staatsbibliothek Cod. 138, fol. 17r.



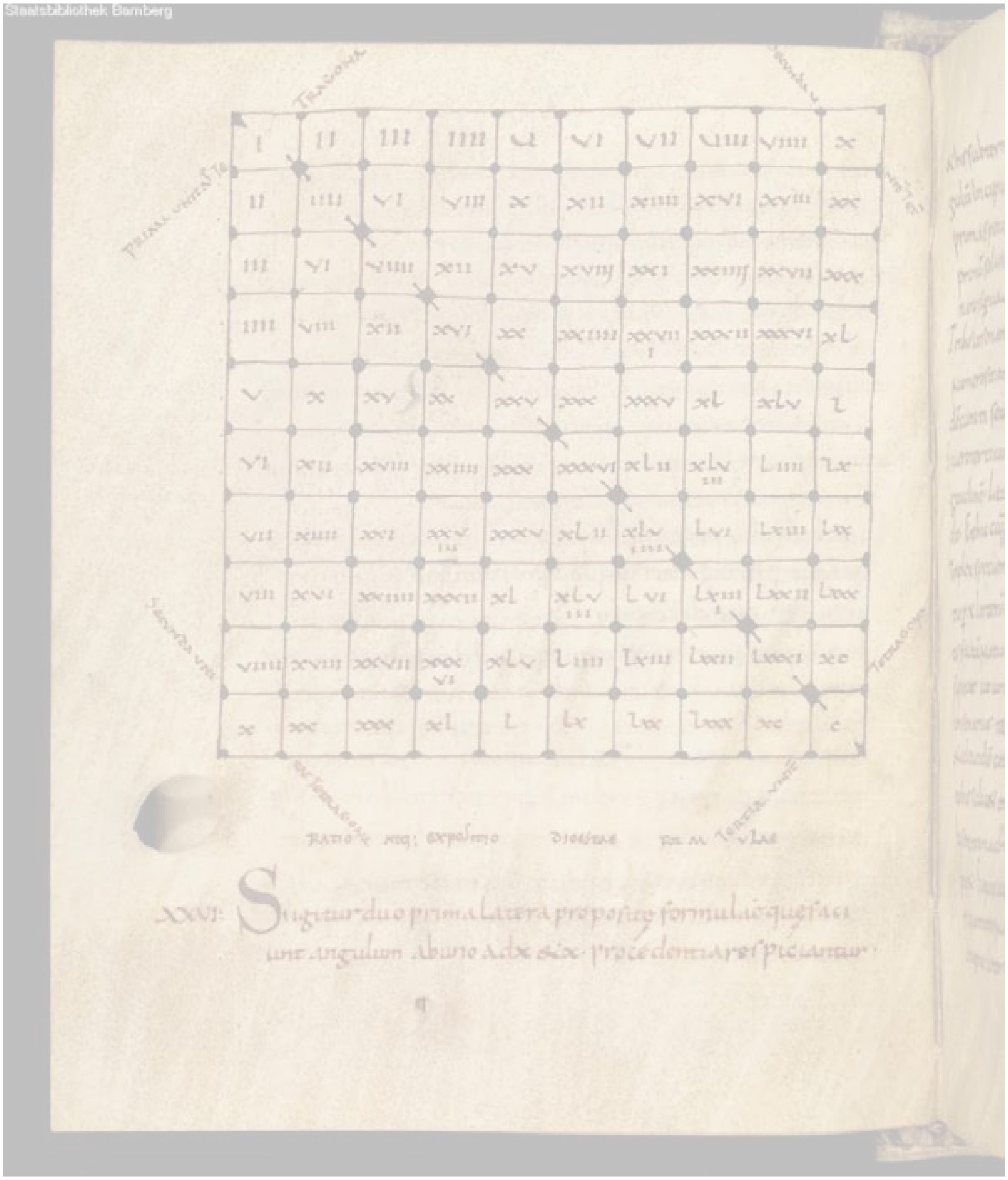
3.41 Bern, Burgerbibliothek Cod. 250, fol. 1r. Photo taken with permission of the Burgerbibliothek Bern



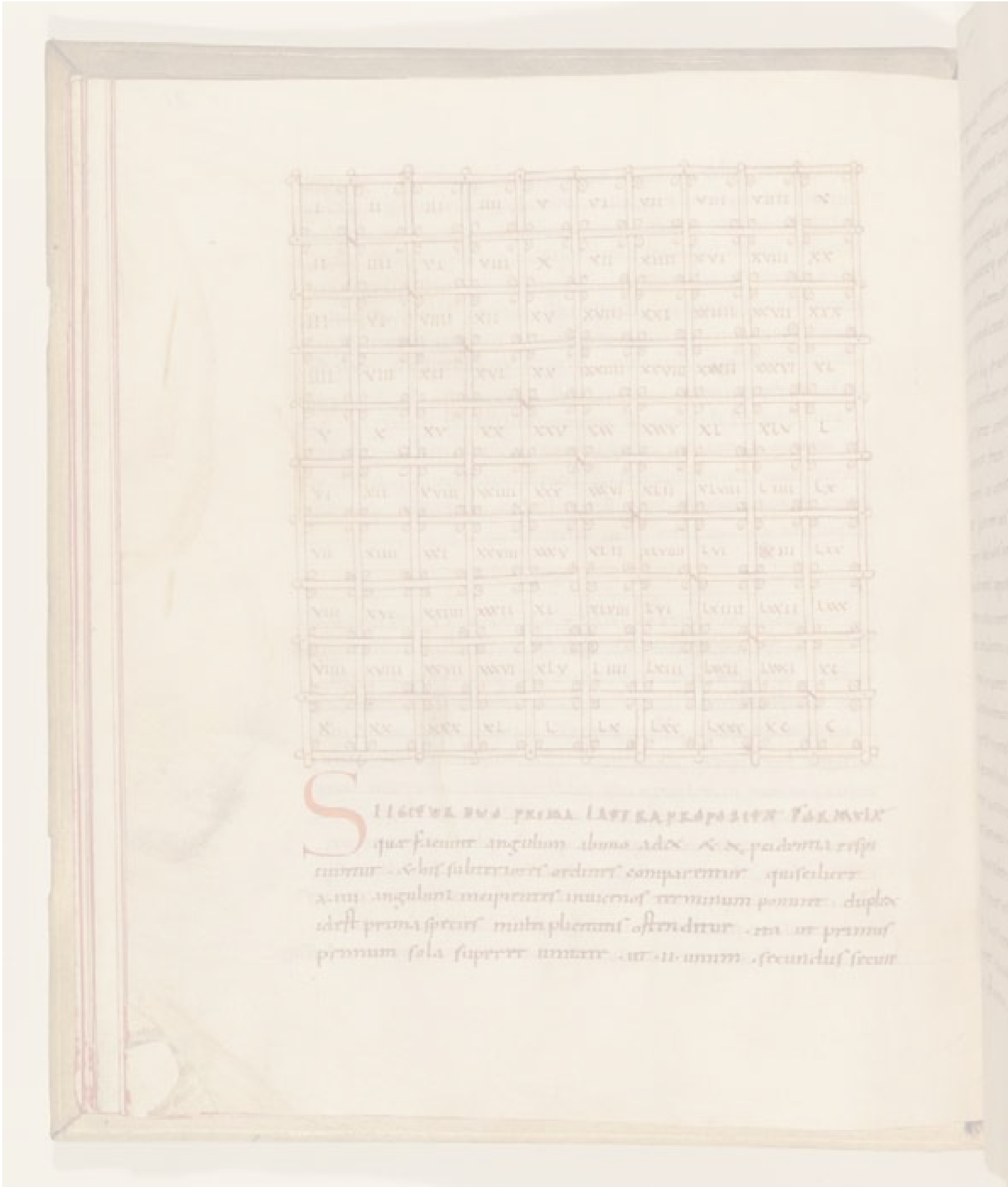
3.42 Paris, Bibliothèque nationale de France lat. MS 6401, fol. 110r. © Bibliothèque nationale de France



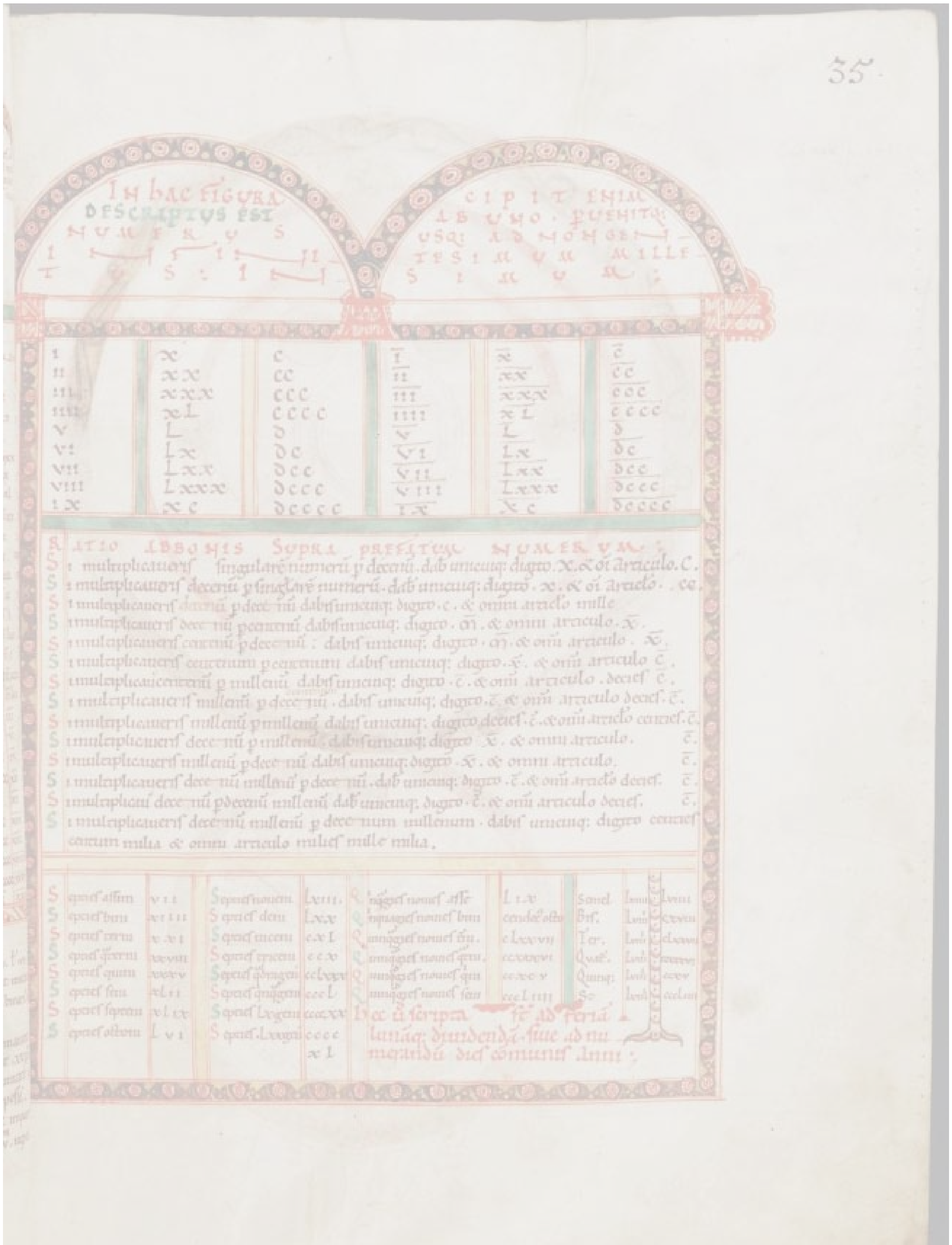
3.43 Paris, Bibliothèque nationale de France lat. MS 7185, fol. 15r.



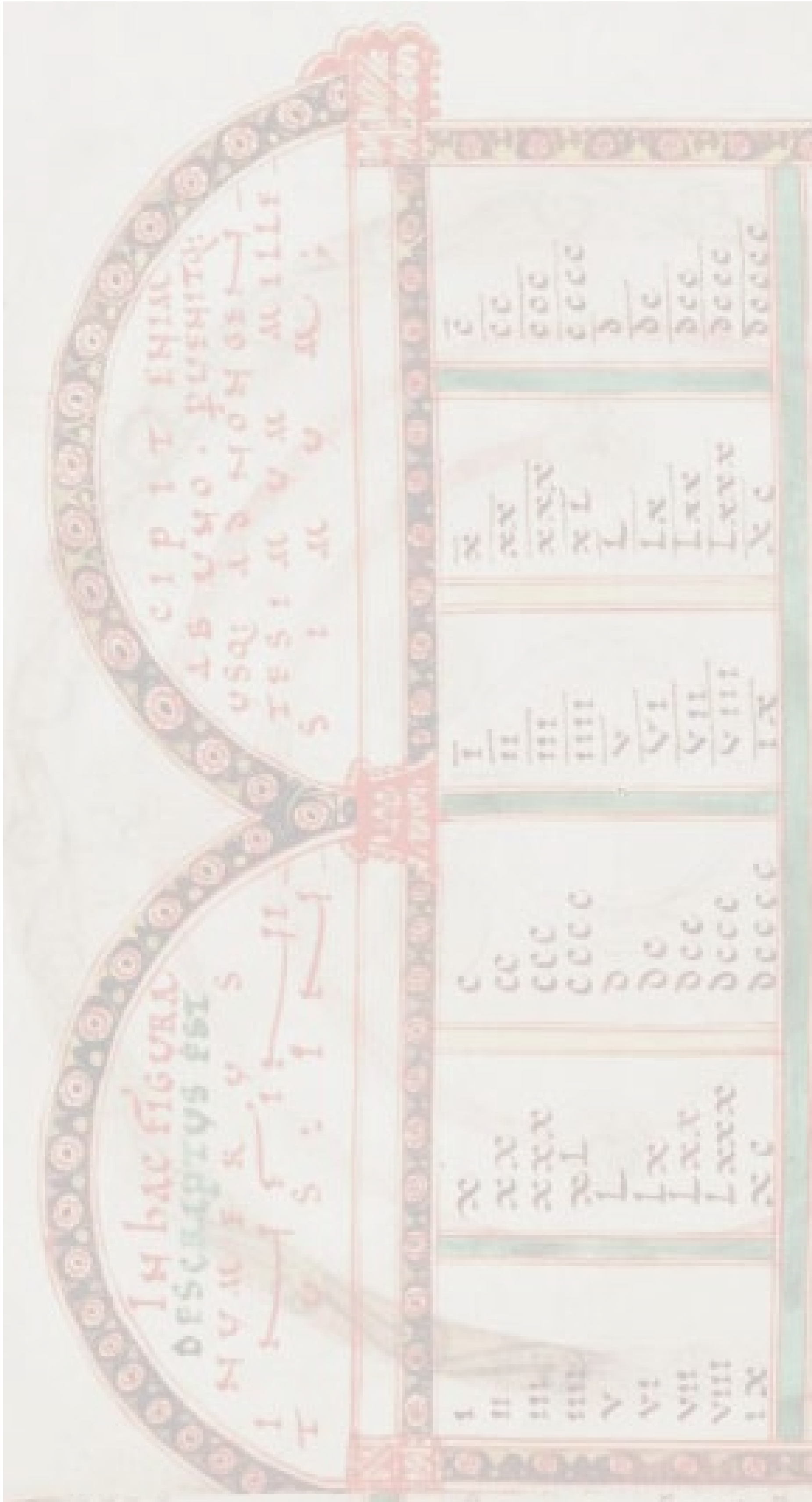
3.44 Bamberg, Staatsbibliothek MS Msc.Class.8, fol. 28v.



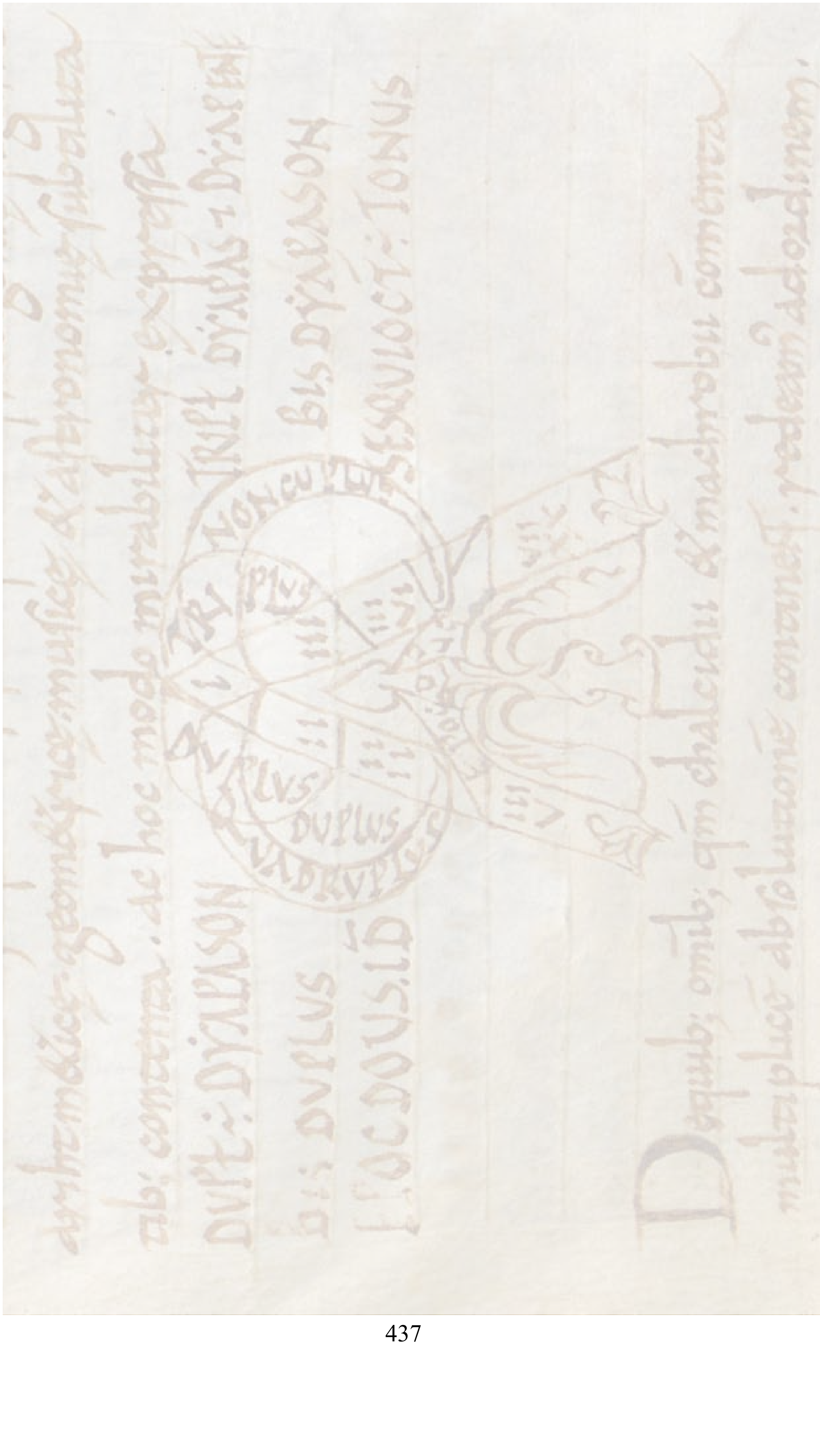
3.45 Bamberg, Staatsbibliothek MS Msc.Class.7, fol. 21v.
© Staatsbibliothek Bamberg



3.46 Oxford, St. Johns College MS 17, fol. 35r.

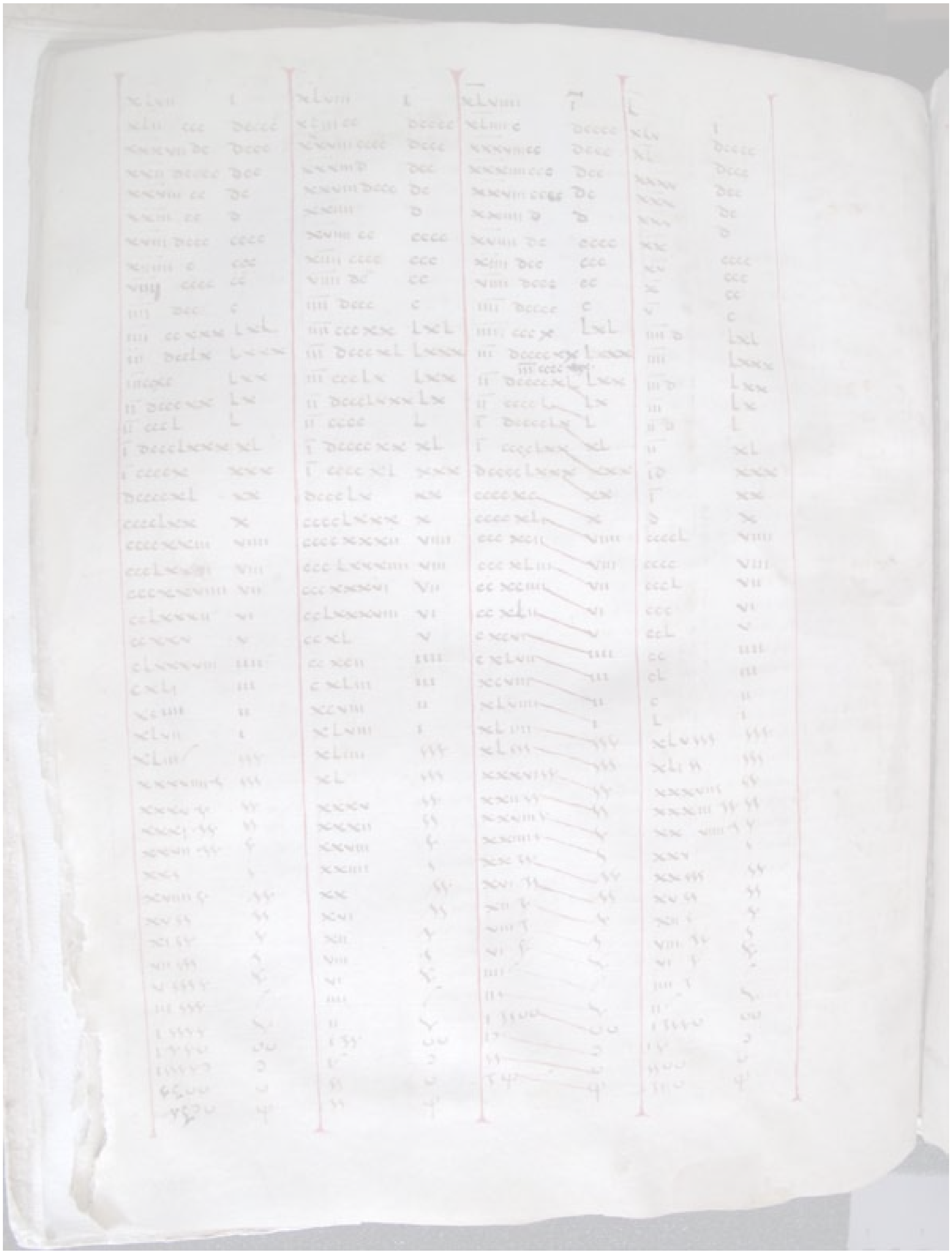


3.47 Oxford, St. Johns College MS 17, fol. 35r, detail. © St. John's College, Oxford University



3.48 Berlin, Staatsbibliothek Cod. 138, fol. 9v, detail. © Staatsbibliothek Berlin

3.49 Bern, Burgerbibliothek Cod. 250, fol. 7r, detail.
Photo taken with permission of the Burgerbibliothek Bern

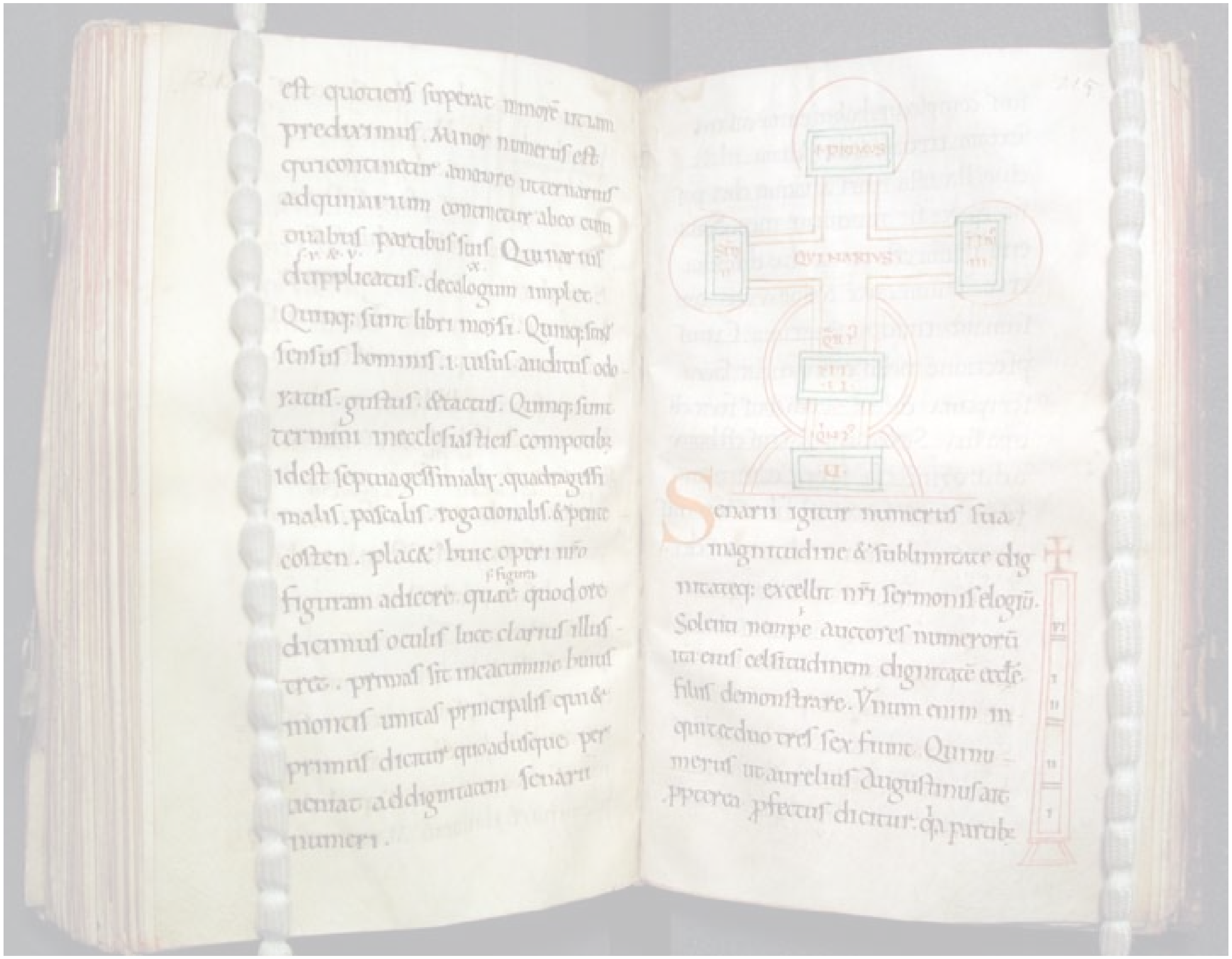


3.50 Bern, Burgerbibliothek Cod. 250, fol. 6v.
Photo taken with permission of the Burgerbibliothek Bern

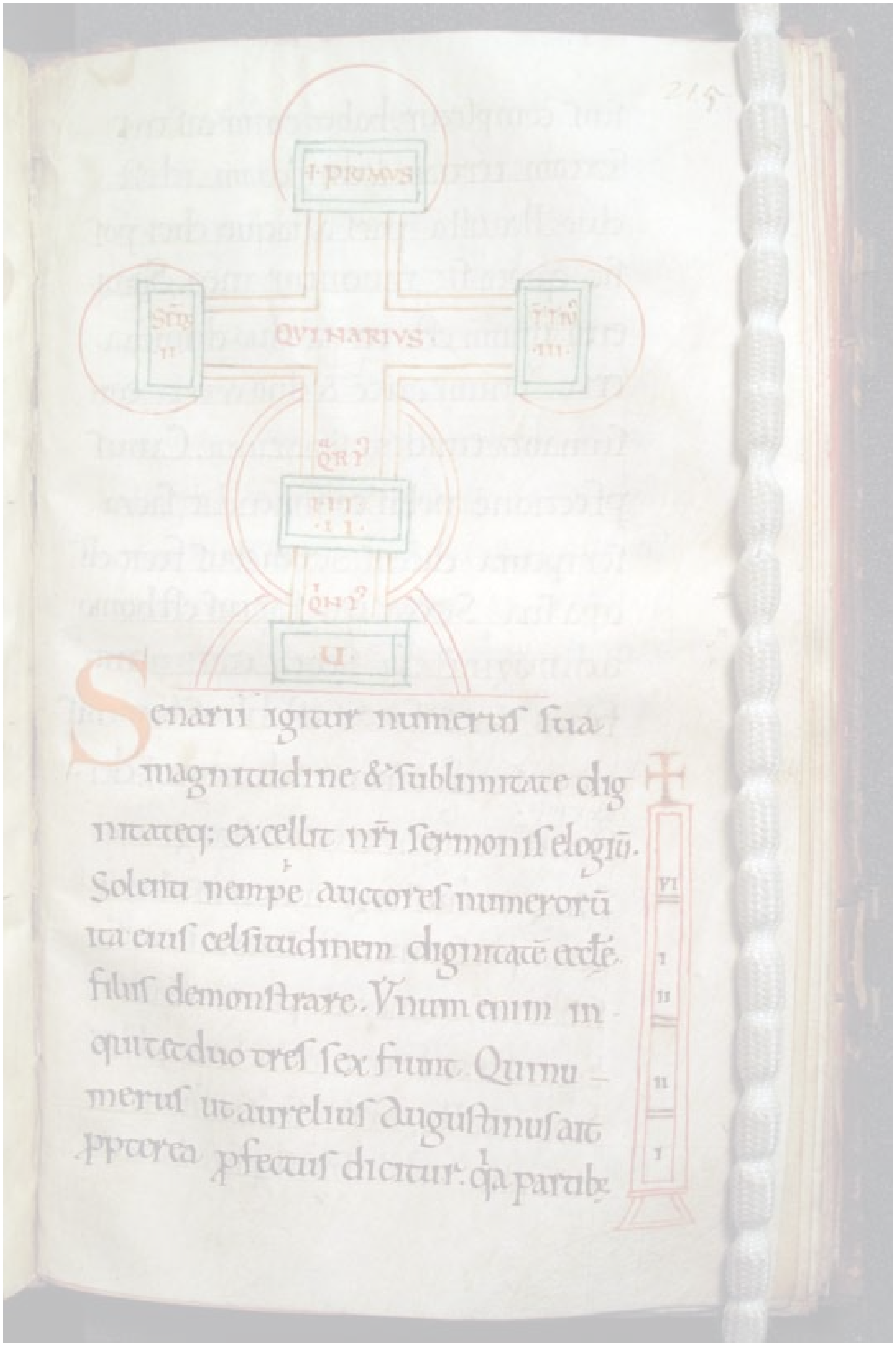
3.51 Bern, Burgerbibliothek Cod. 250, fol. 6v, detail. Photo taken with permission of the Burgerbibliothek Bern

3.52 Berlin, Staatsbibliothek Cod. 138 fol. 11v, detail. © Staatsbibliothek Berlin

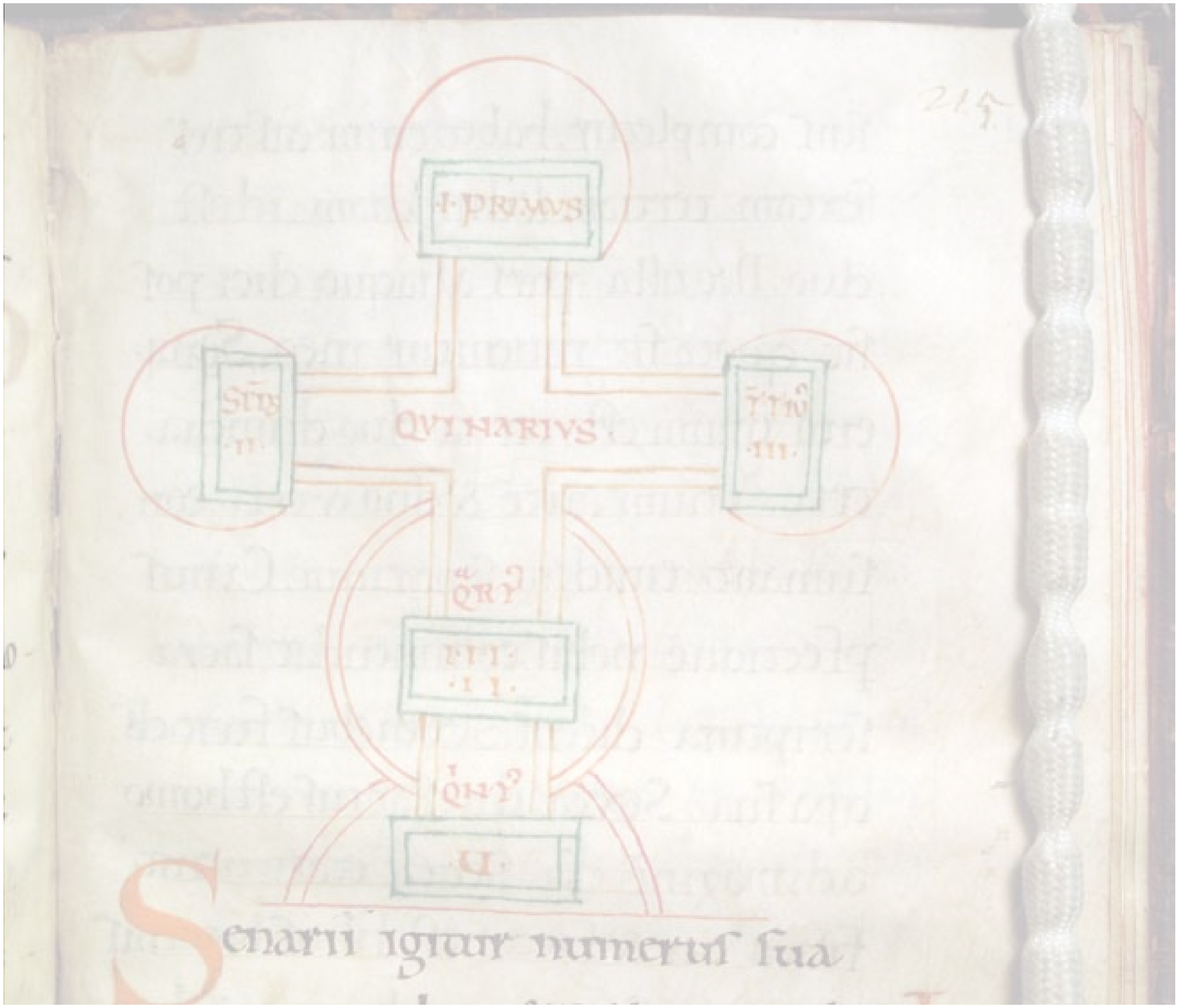
3.53 Berlin, Staatsbibliothek Cod. 138 fol. 16r, detail.



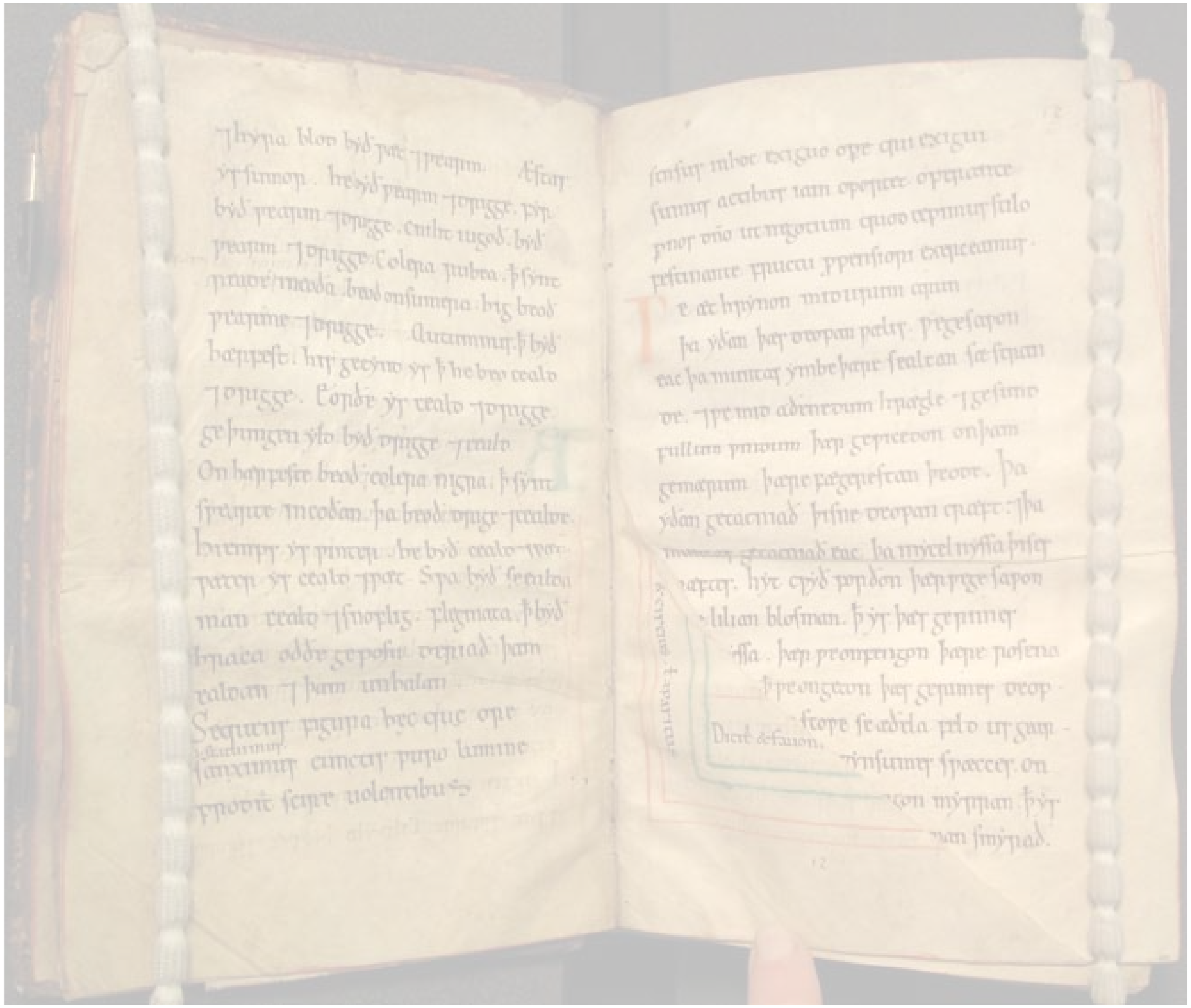
4.1 Oxford, Bodleian Library Ashmole MS 328, p. 214–15. Photo taken with permission of the Bodleian Library, Oxford University



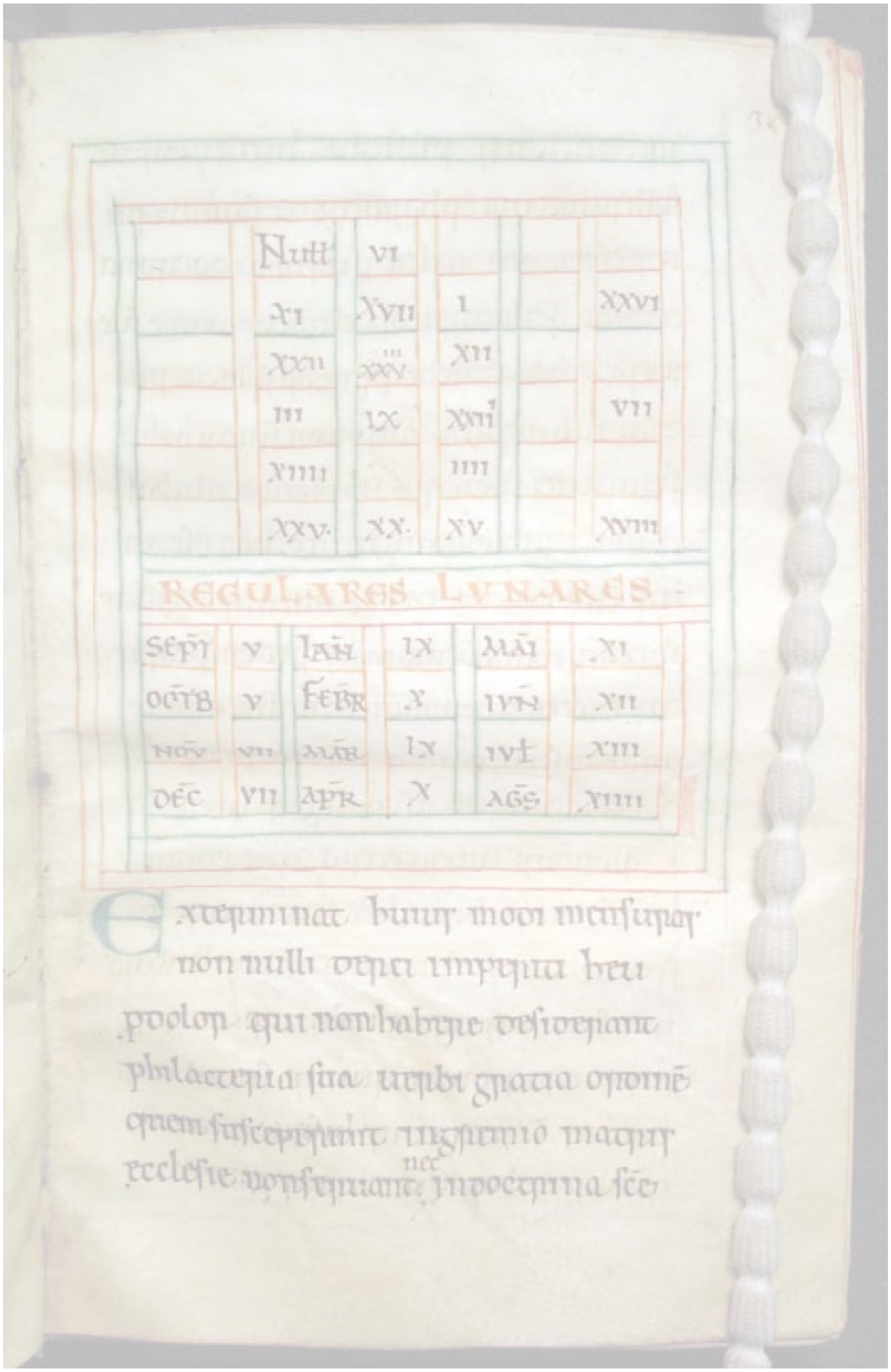
4.2 Oxford, Bodleian Library Ashmole MS 328, p. 215.
Photo taken with permission of the Bodleian Library, Oxford University



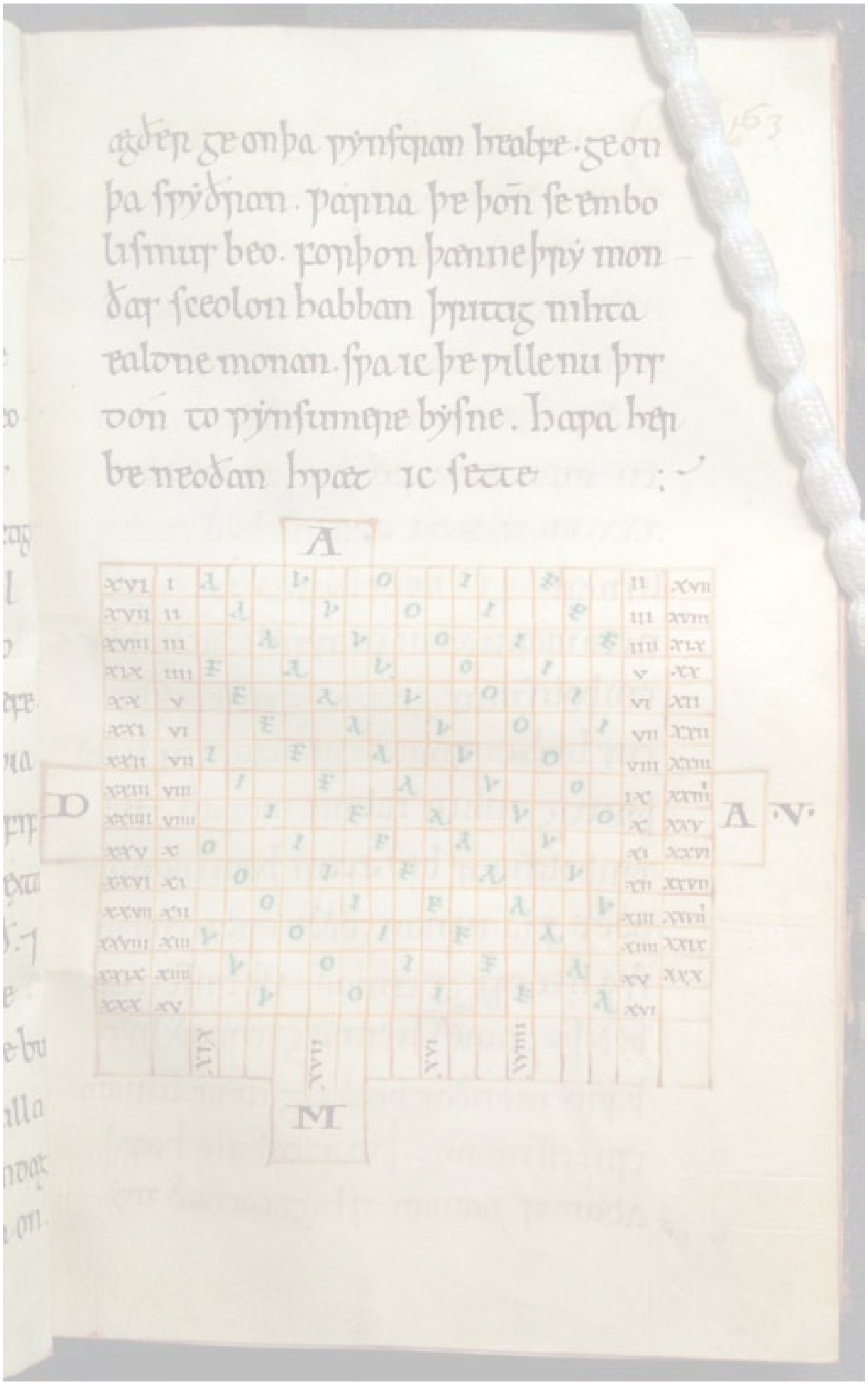
4.3 Oxford, Bodleian Library Ashmole MS 328, p. 215, detail. Photo taken with permission of the Bodleian Library, Oxford University



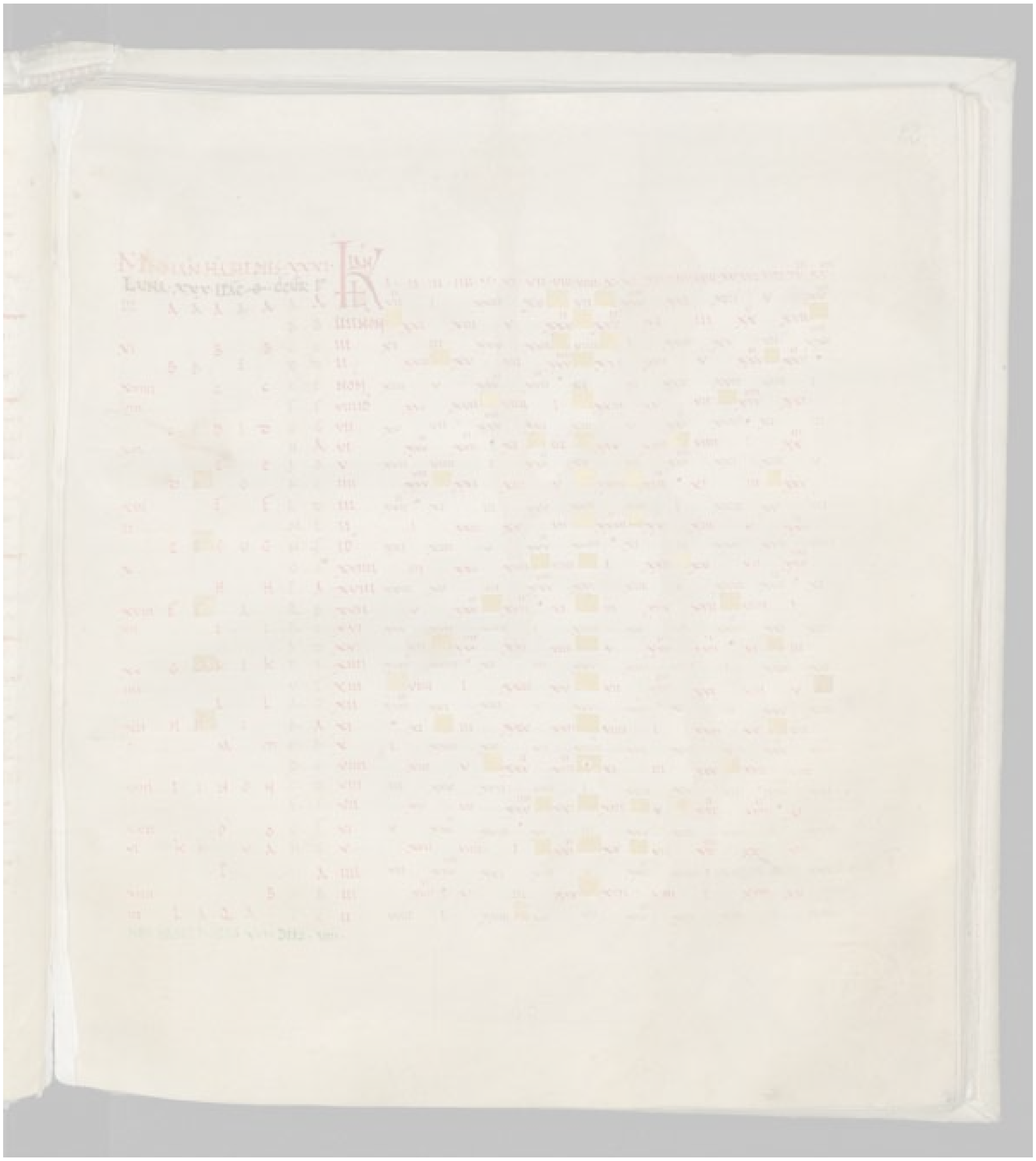
4.4 Oxford, Bodleian Library Ashmole MS 328, p. 12–13. Photo taken with permission of the Bodleian Library, Oxford University



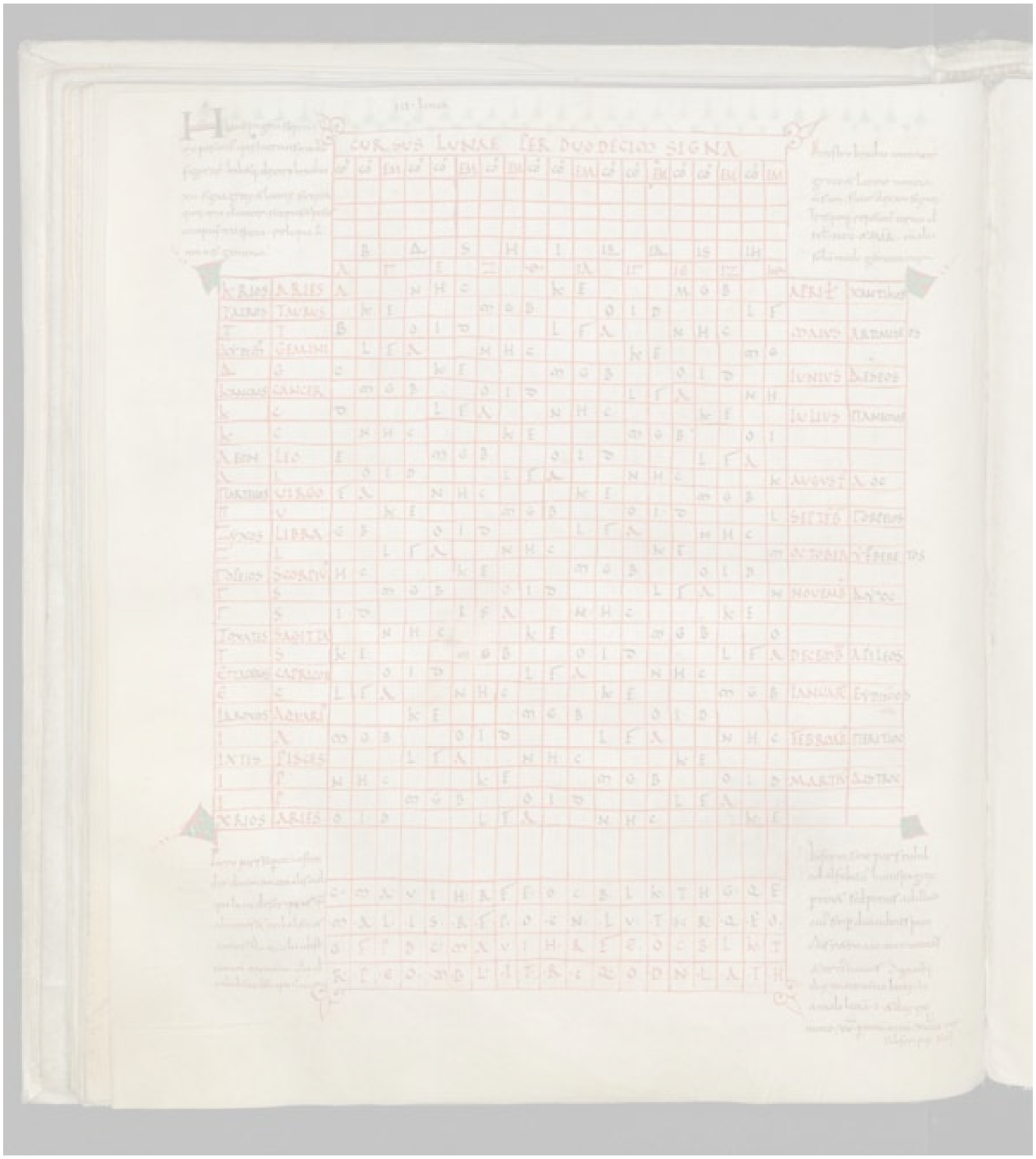
4.5 Oxford, Bodleian Library Ashmole MS 328, p. 35.
Photo taken with permission of the Bodleian Library, Oxford University



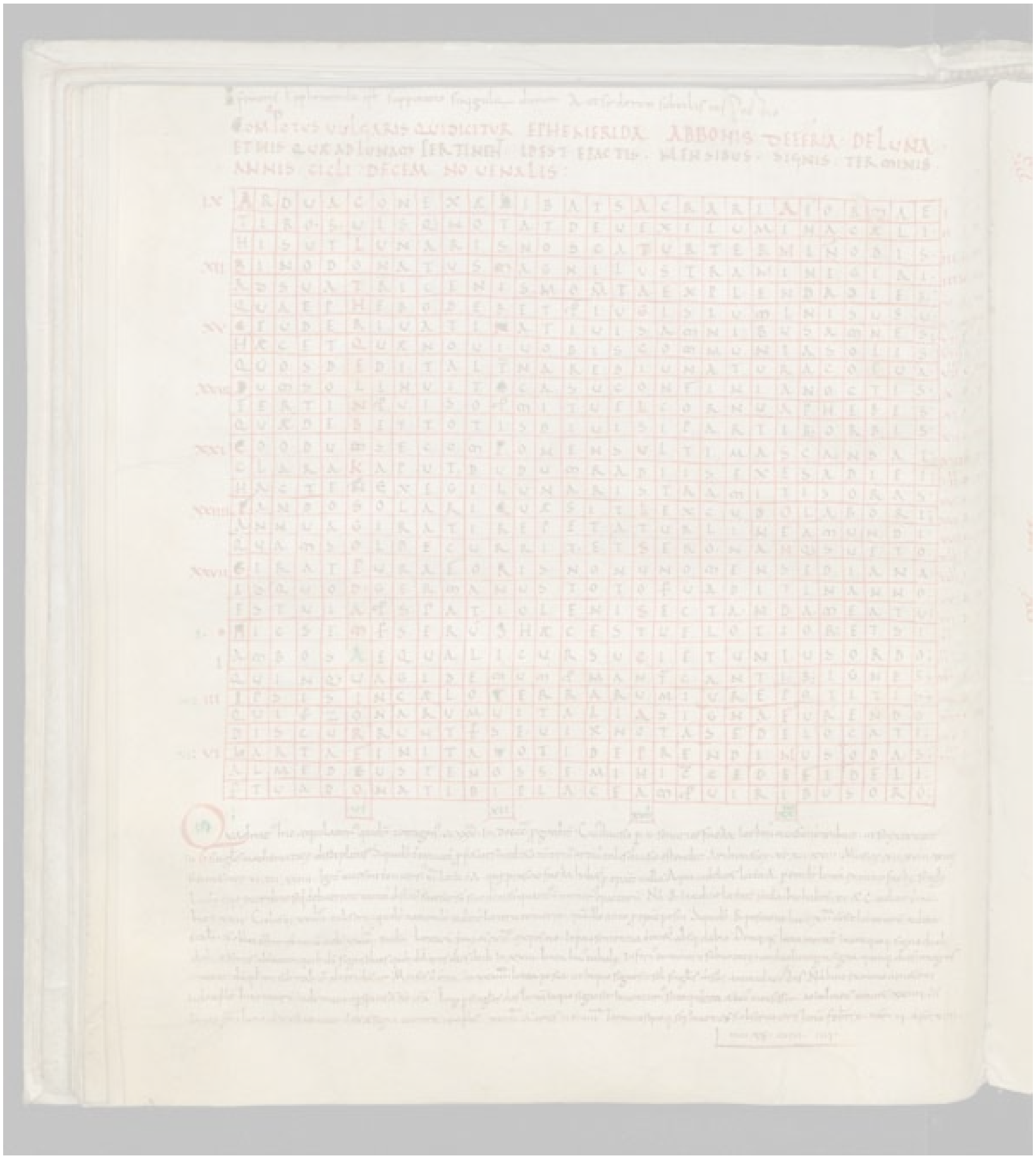
4.6 Oxford, Bodleian Library Ashmole MS 328, p. 163.
Photo taken with permission of the Bodleian Library, Oxford University



4.7 Berlin, Staatsbibliothek Cod. 138 fol. 23r.



4.8 Berlin, Staatsbibliothek Cod. 138 fol. 31v.



4.9. Berlin, Staatsbibliothek Cod. 138 fol. 33v.
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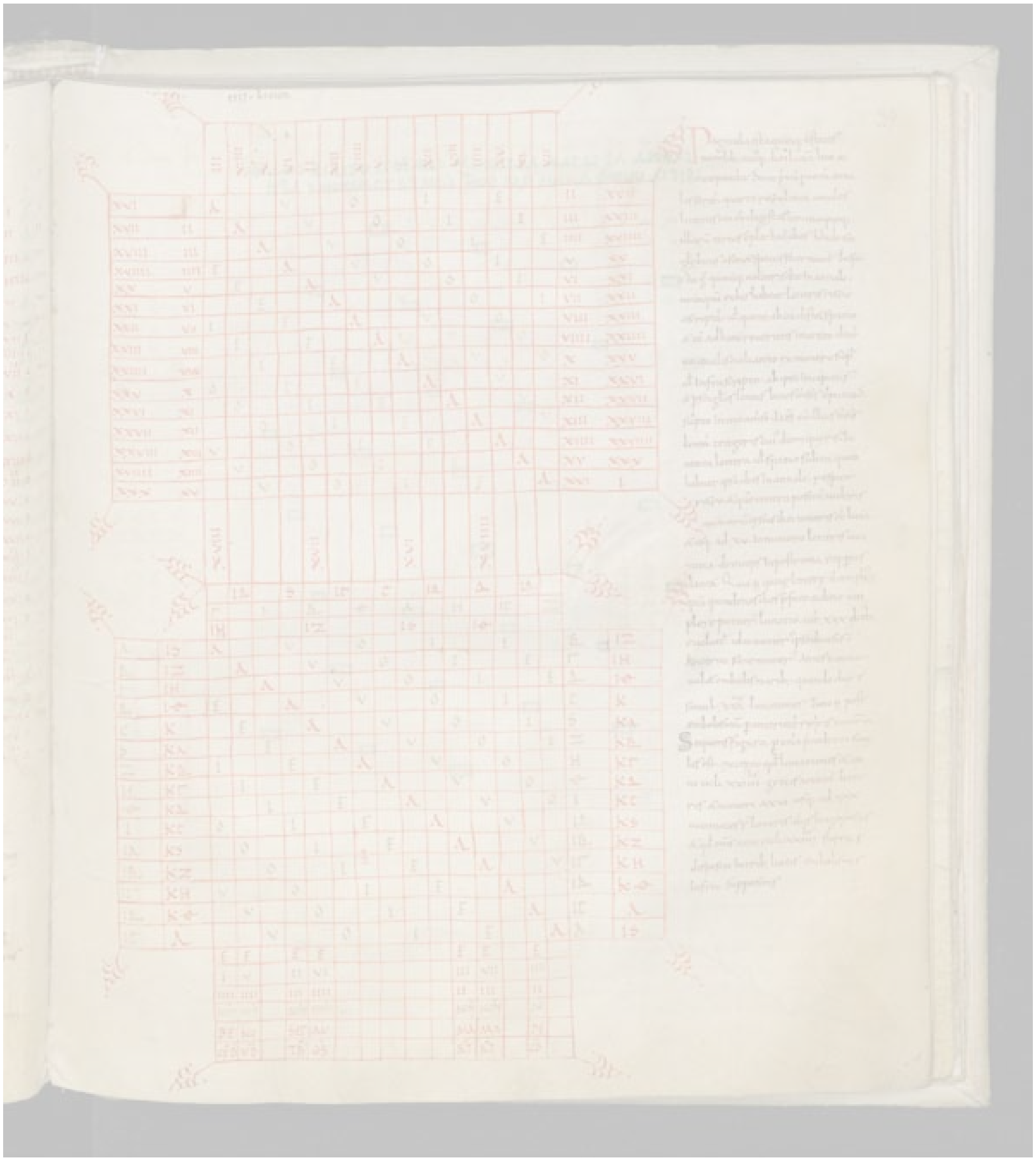
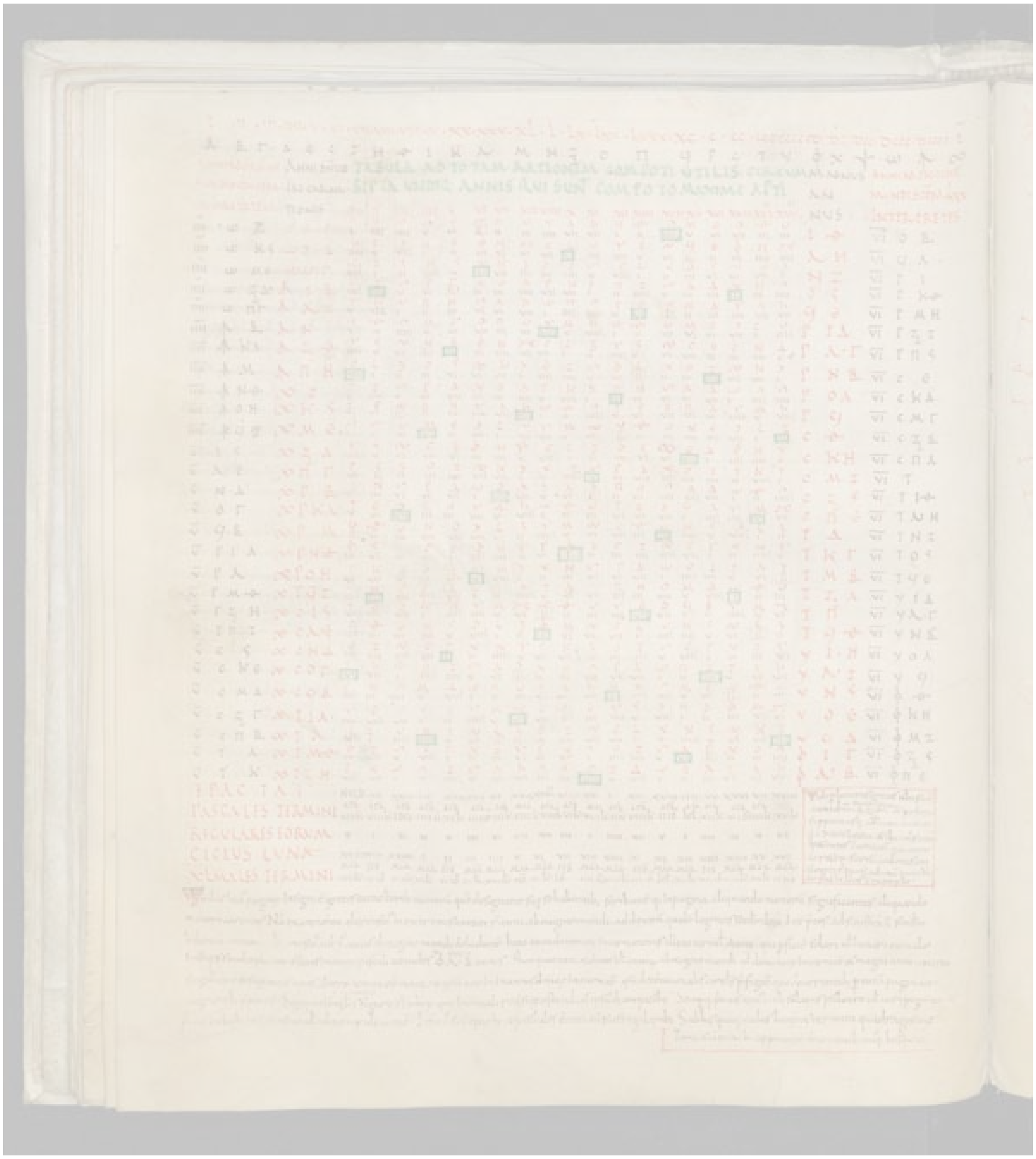
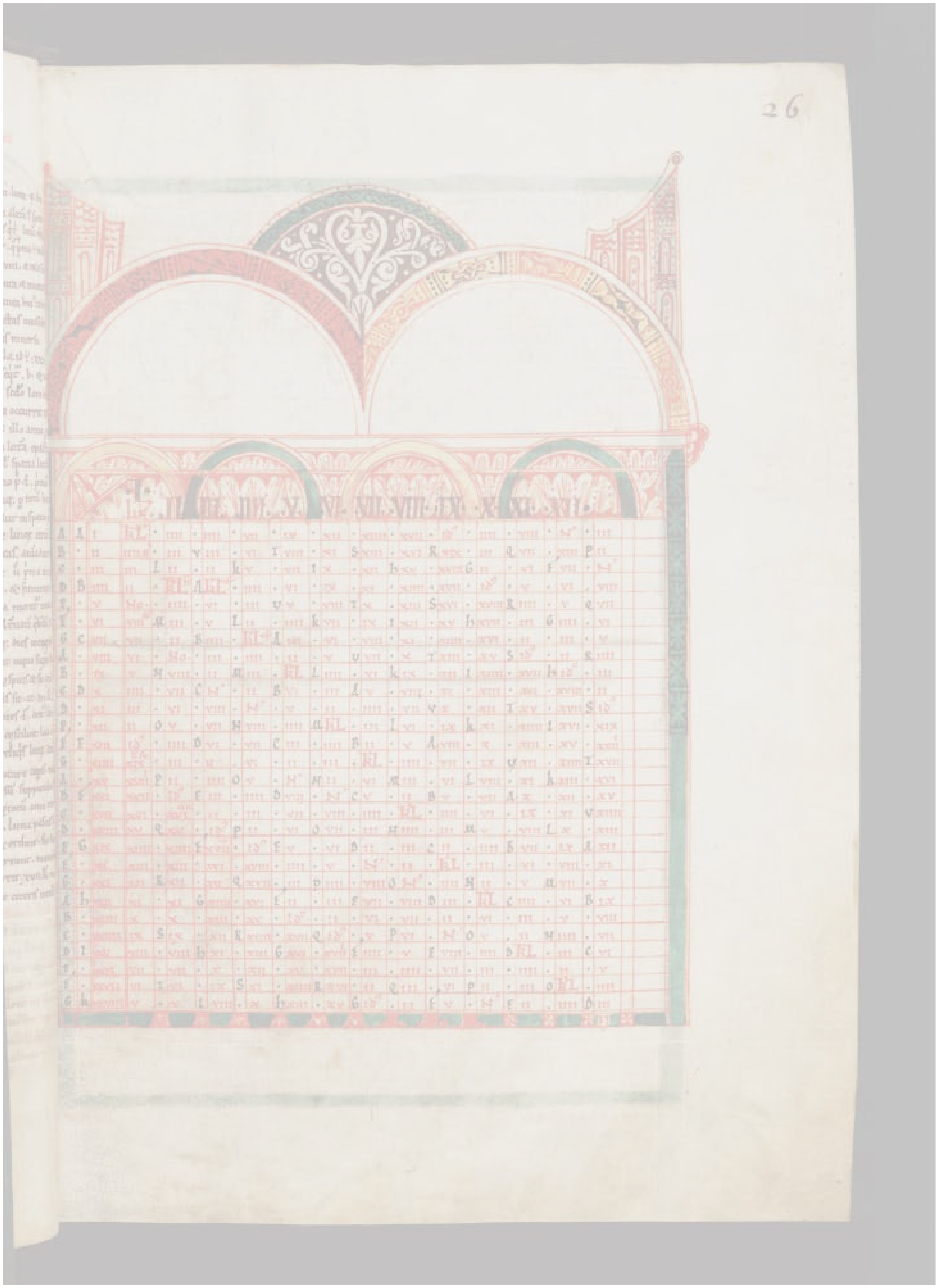


4.10 Berlin, Staatsbibliothek Cod. 138 fol. 34r.

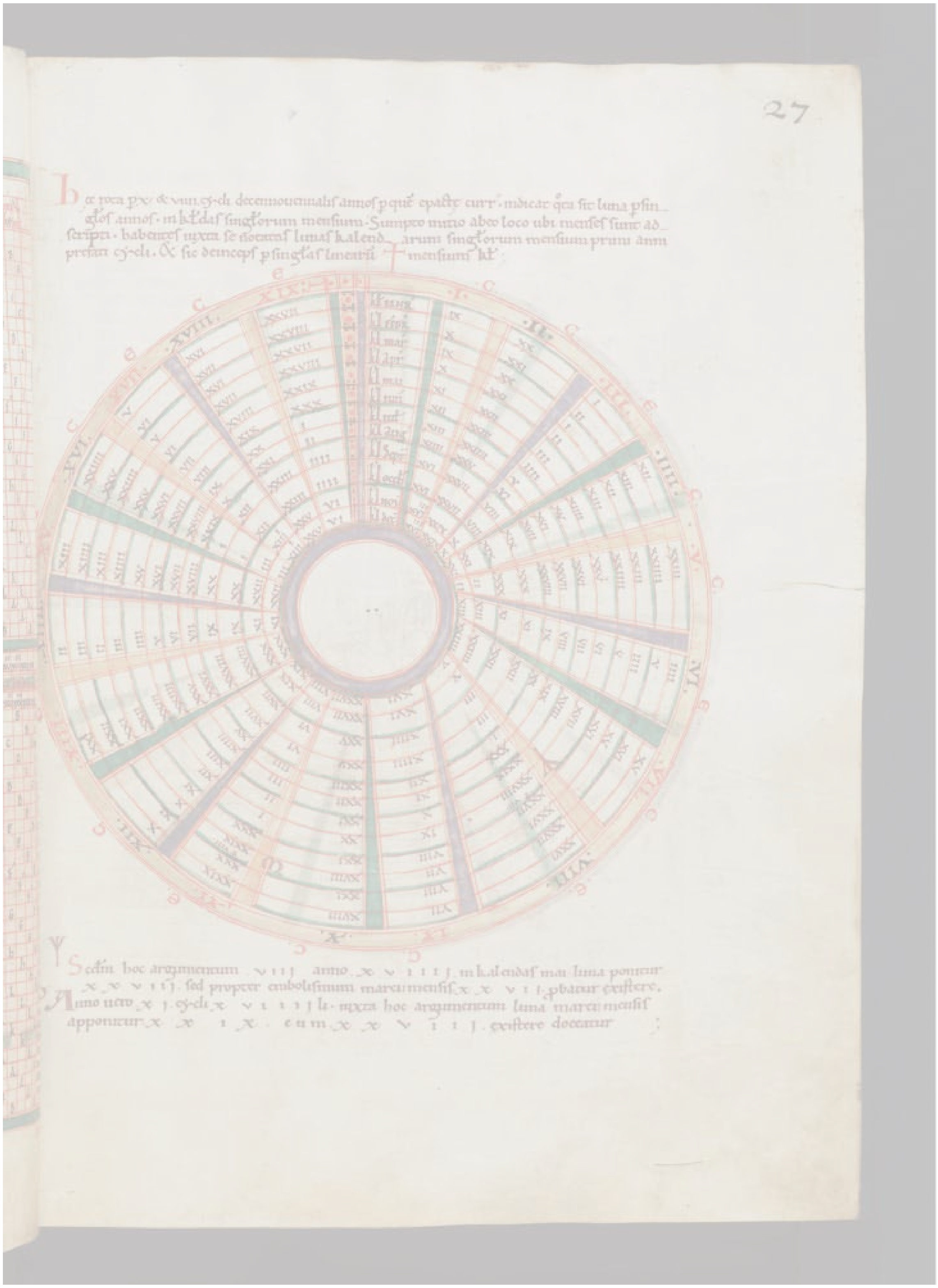


4.11 Berlin, Staatsbibliothek Cod. 138 fol. 34v.

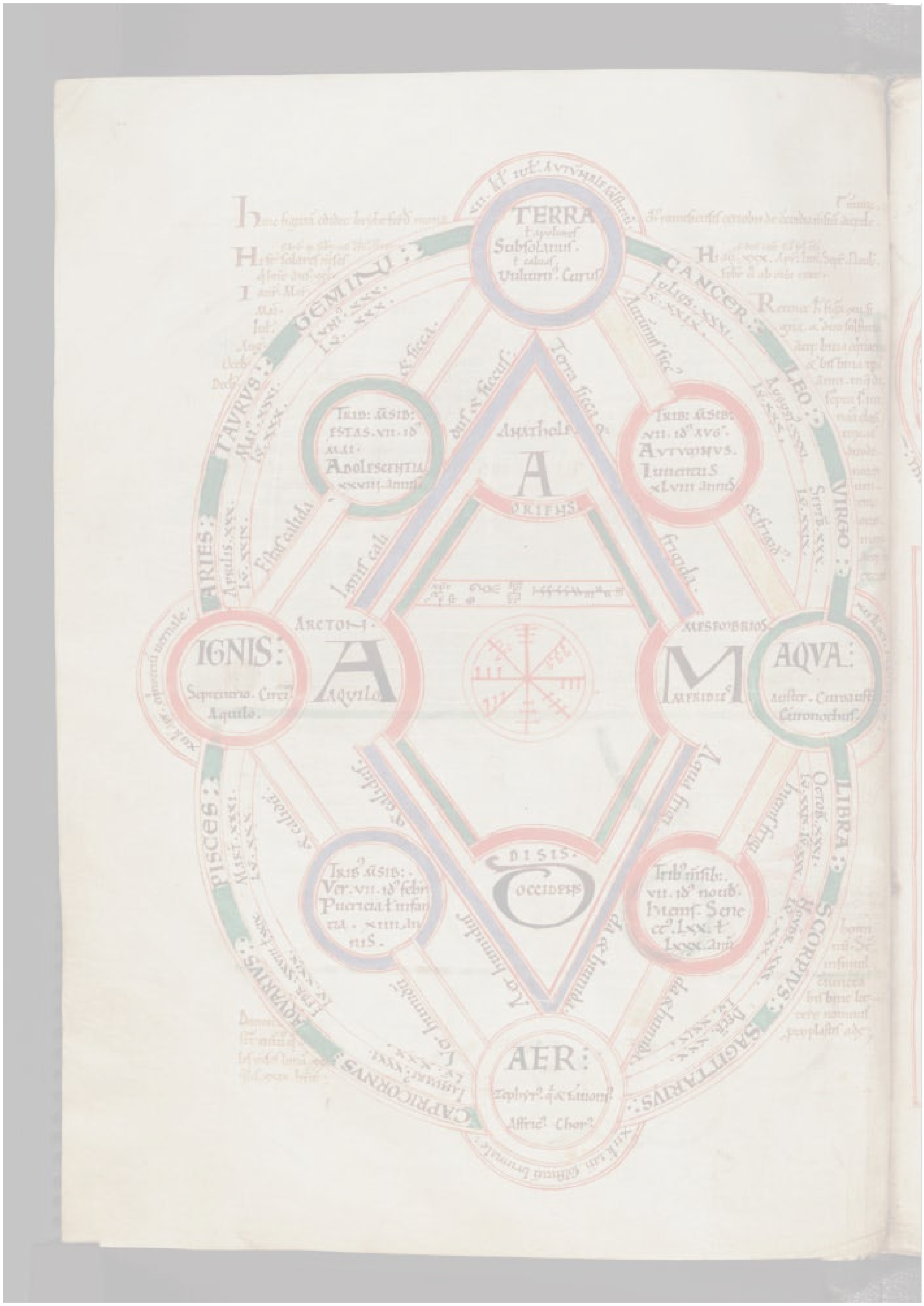


4.12 Oxford, St. John's College MS 17, fol. 26r.

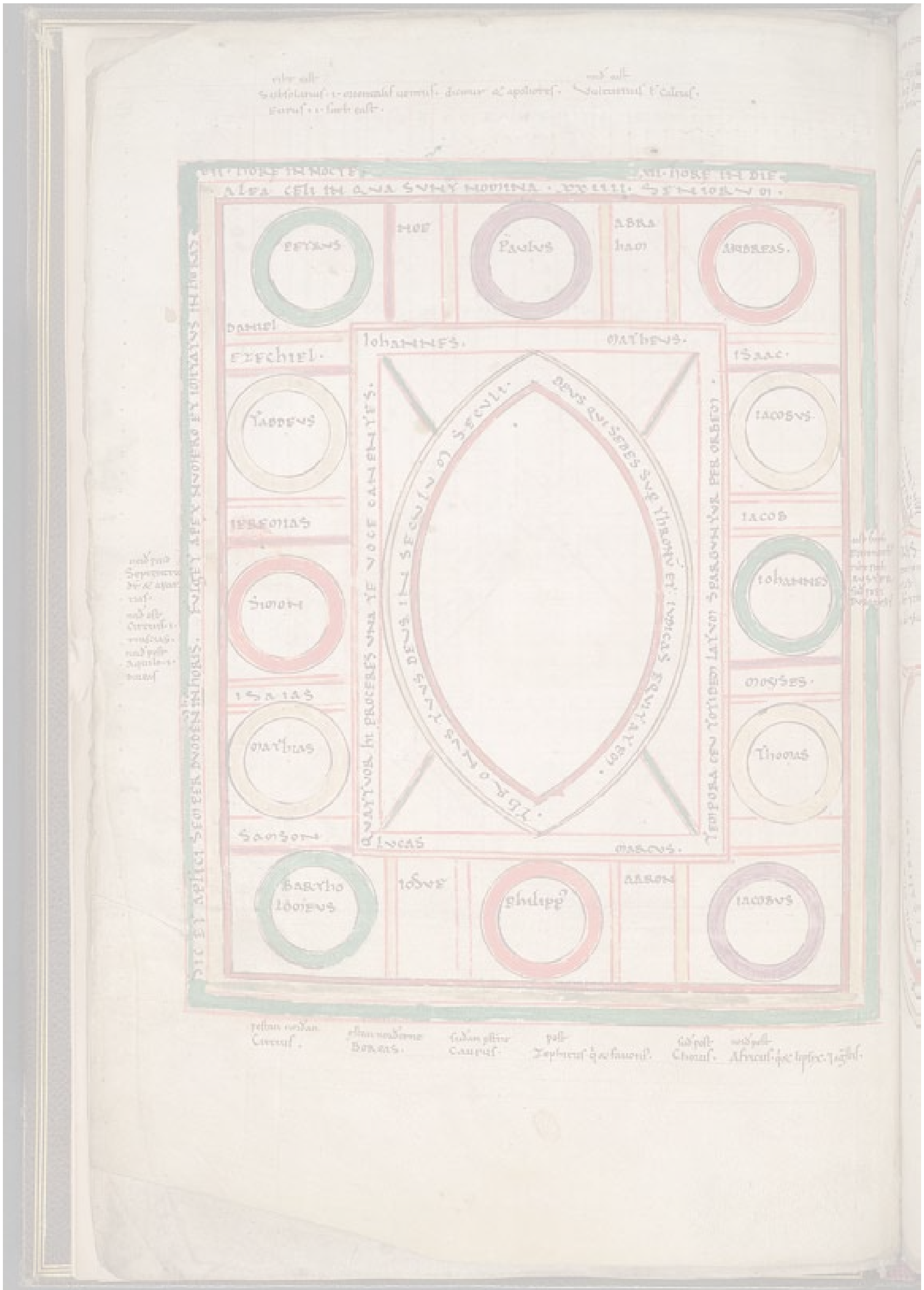
4.13 Oxford, St. John's College MS 17, fol. 26v.



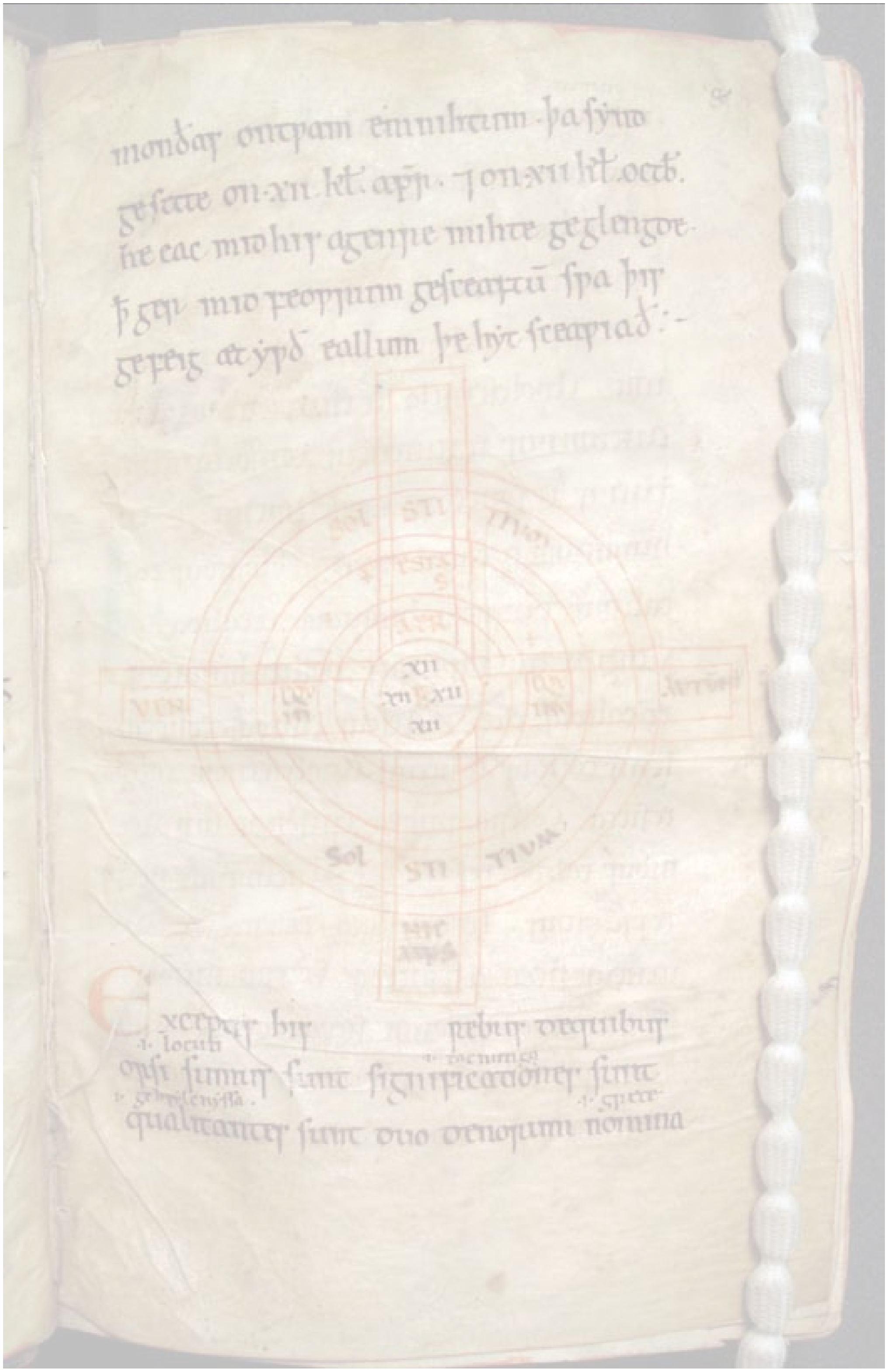
4.14 Oxford, St. John’s College MS 17, fol. 27r.



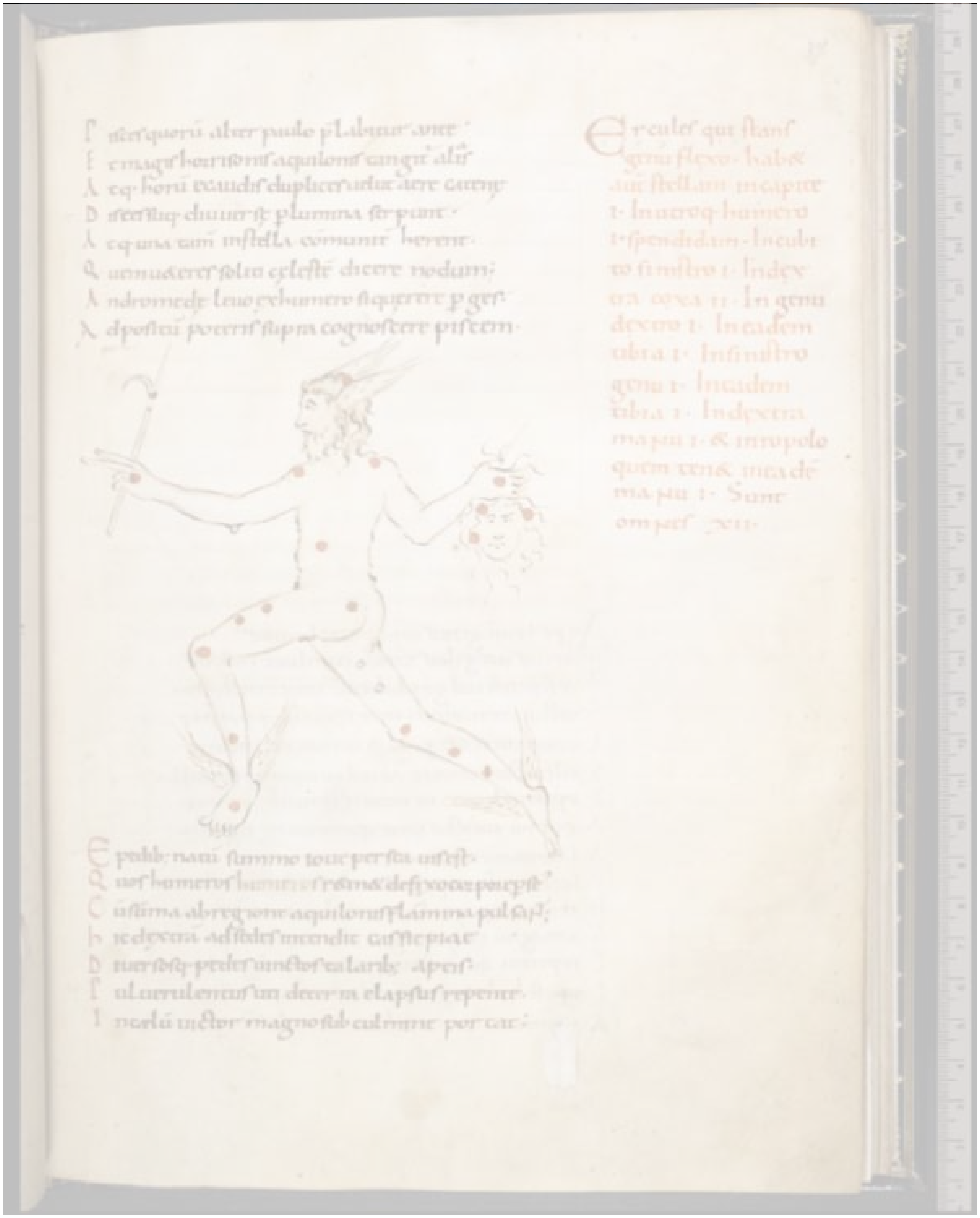
4.15 Oxford, St. John's College MS 17, fol. 7v.



4.16 London, British Library Harley MS 3667, 7v.



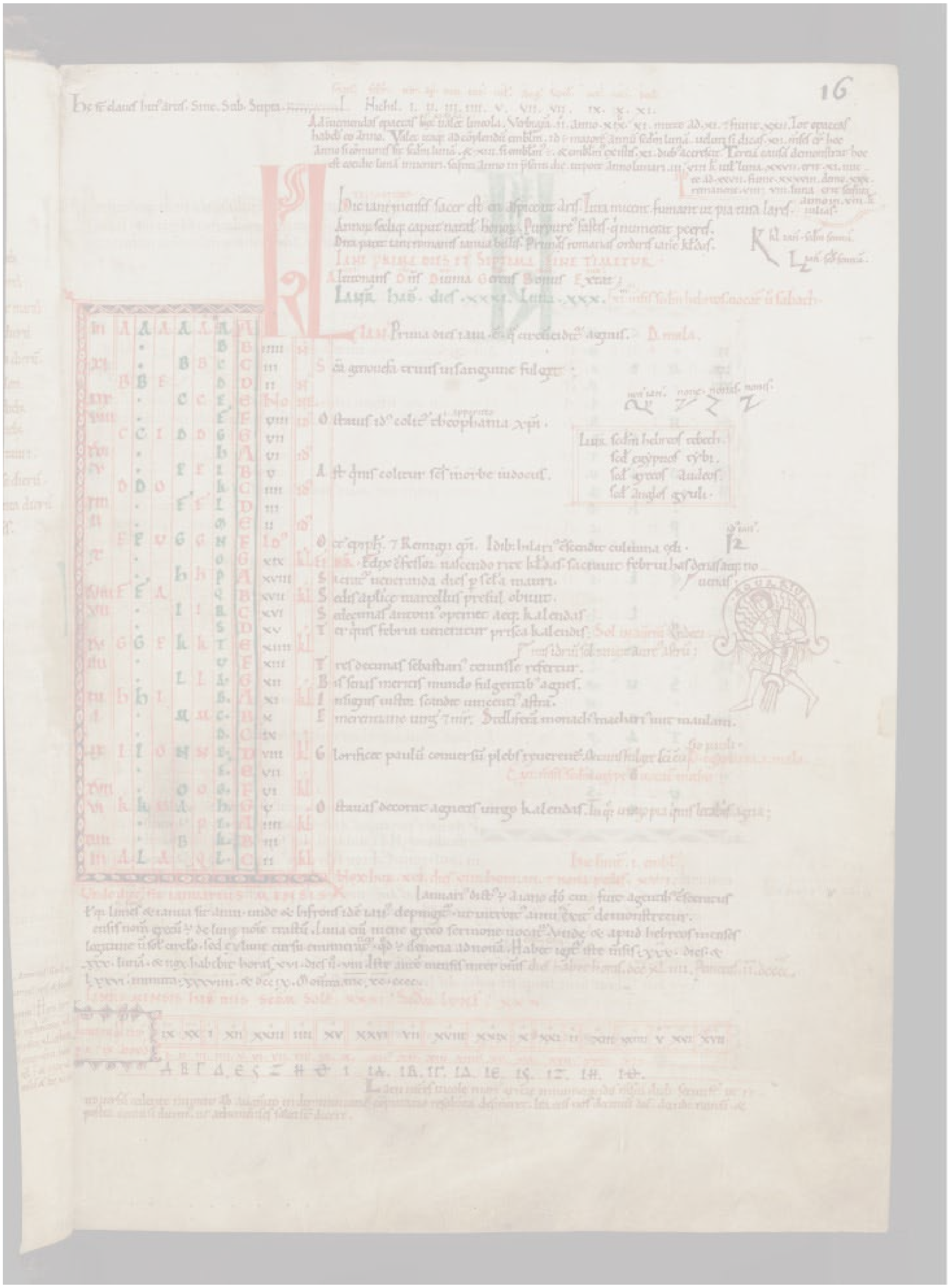
4.17 Oxford, Bodleian Library Ashmole MS 328, p. 8.



4.18 London, British Library Harley MS 2506, fol. 37r.
© The British Library

4.19 Paris, Bibliothèque nationale de France n. a. l. MS 1615, fol. 4r.

4.20 Paris, Bibliothèque nationale de France n. a. l. MS 1615, fol. 3r.

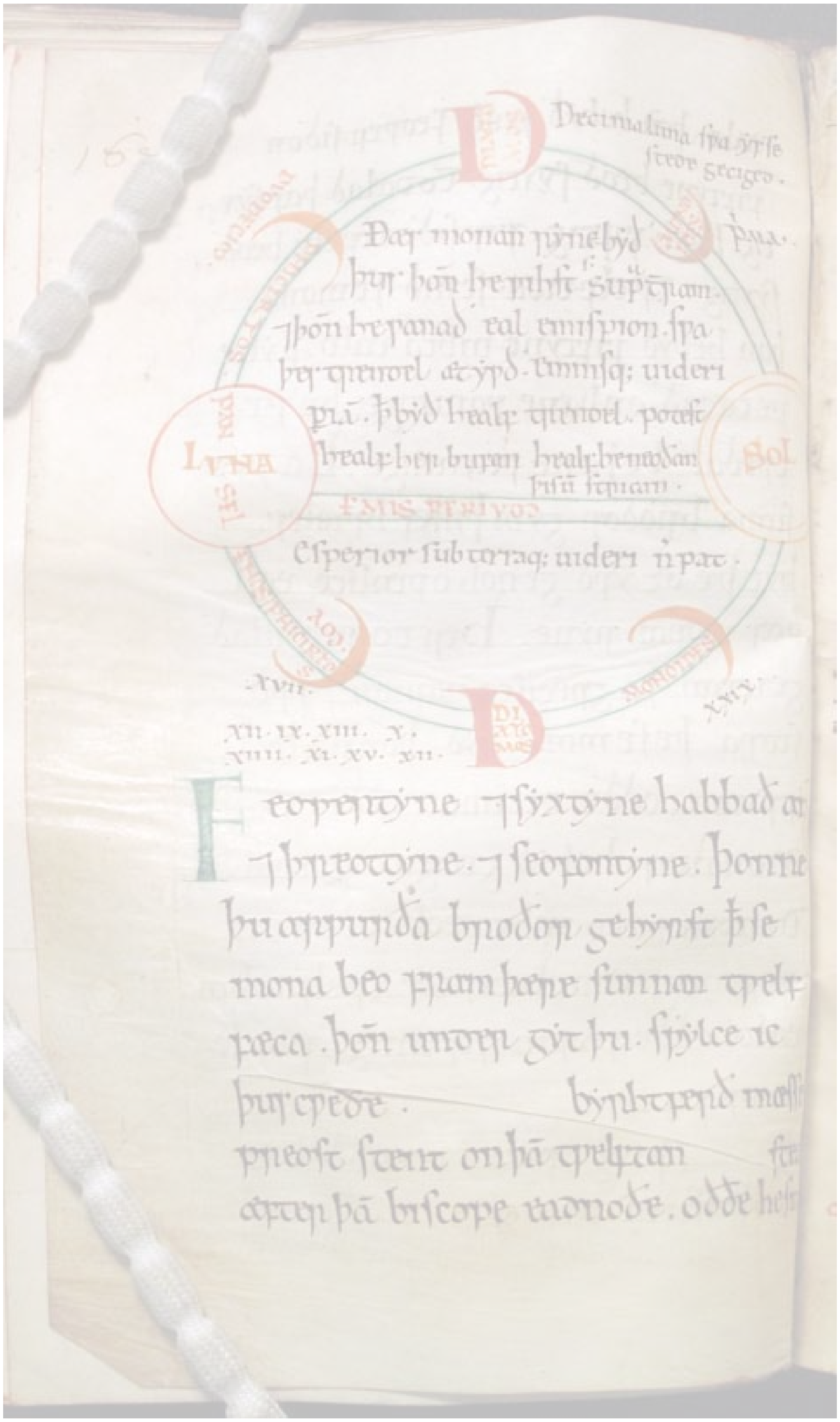


4.21 Oxford, St. John's College MS 17, fol. 16r.

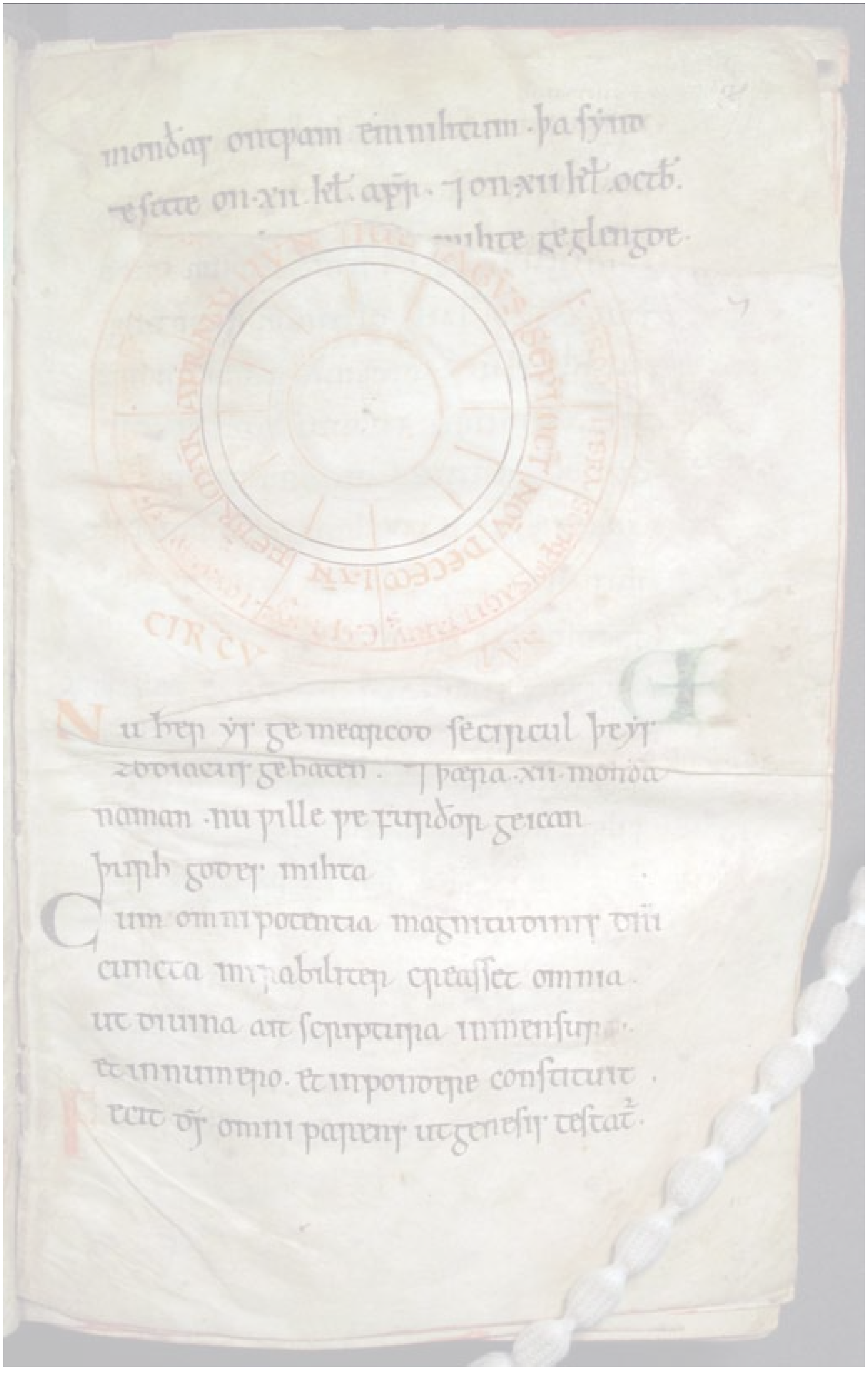
4.22 Berlin, Staatsbibliothek Cod. 138 fol. 34r, detail, upper AEIOU table.

4.23 Berlin, Staatsbibliothek Cod. 138 fol. 34r, detail, lower AEIOU table.
© Staatsbibliothek Berlin

4.24 Oxford, Bodleian Library Ashmole MS 328, p. 118.
Photo taken with permission of the Bodleian Library, Oxford University



4.25 Oxford, Bodleian Library Ashmole MS 328, p. 168.
Photo taken with permission of the Bodleian Library, Oxford University



4.26 Oxford, Bodleian Library Ashmole MS 328, p. 7.
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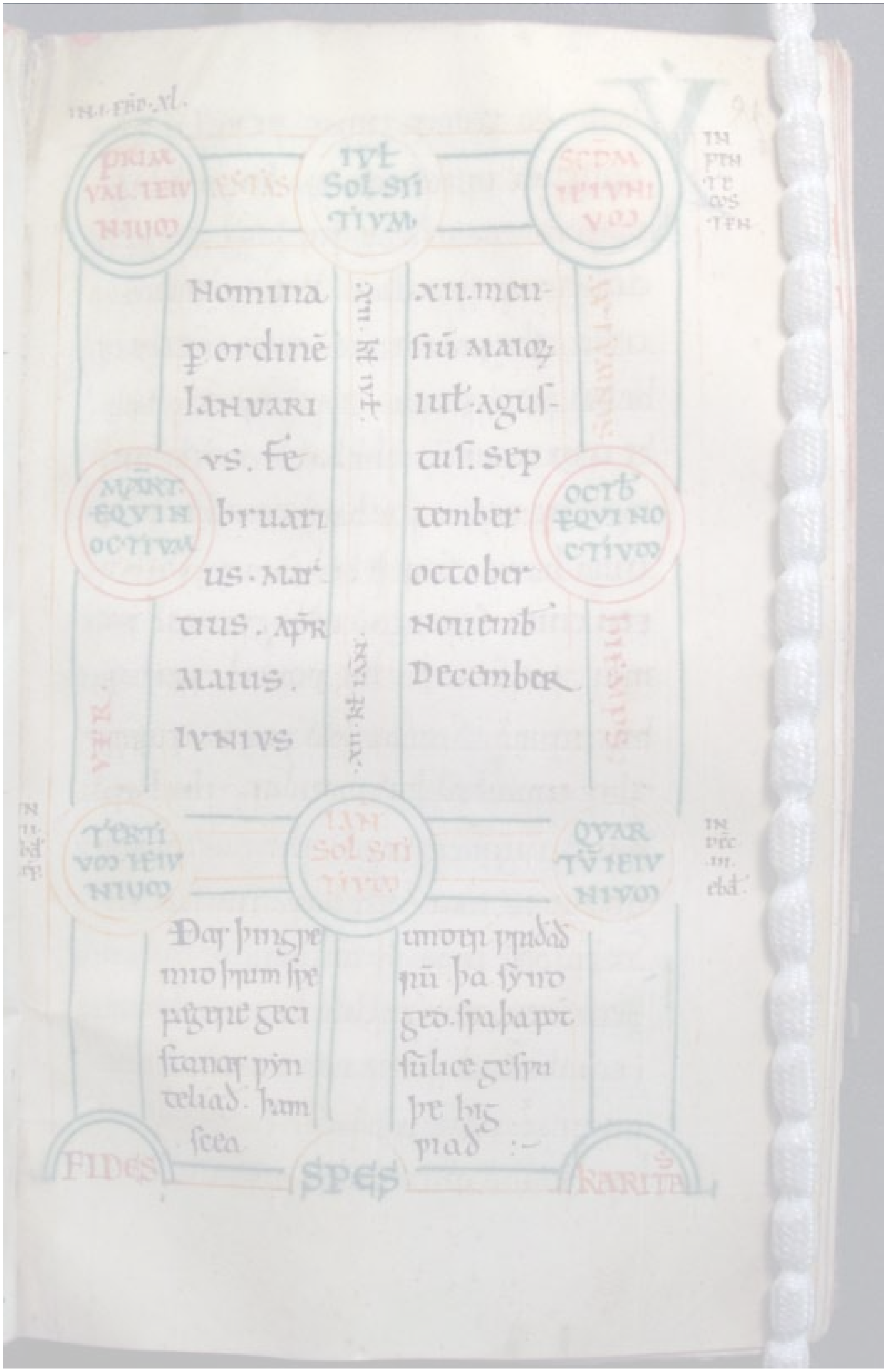

4.27 Oxford, Bodleian Library Ashmole MS 328, p. 91.
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4.28 Oxford, Bodleian Library Ashmole MS 328, p. 94.
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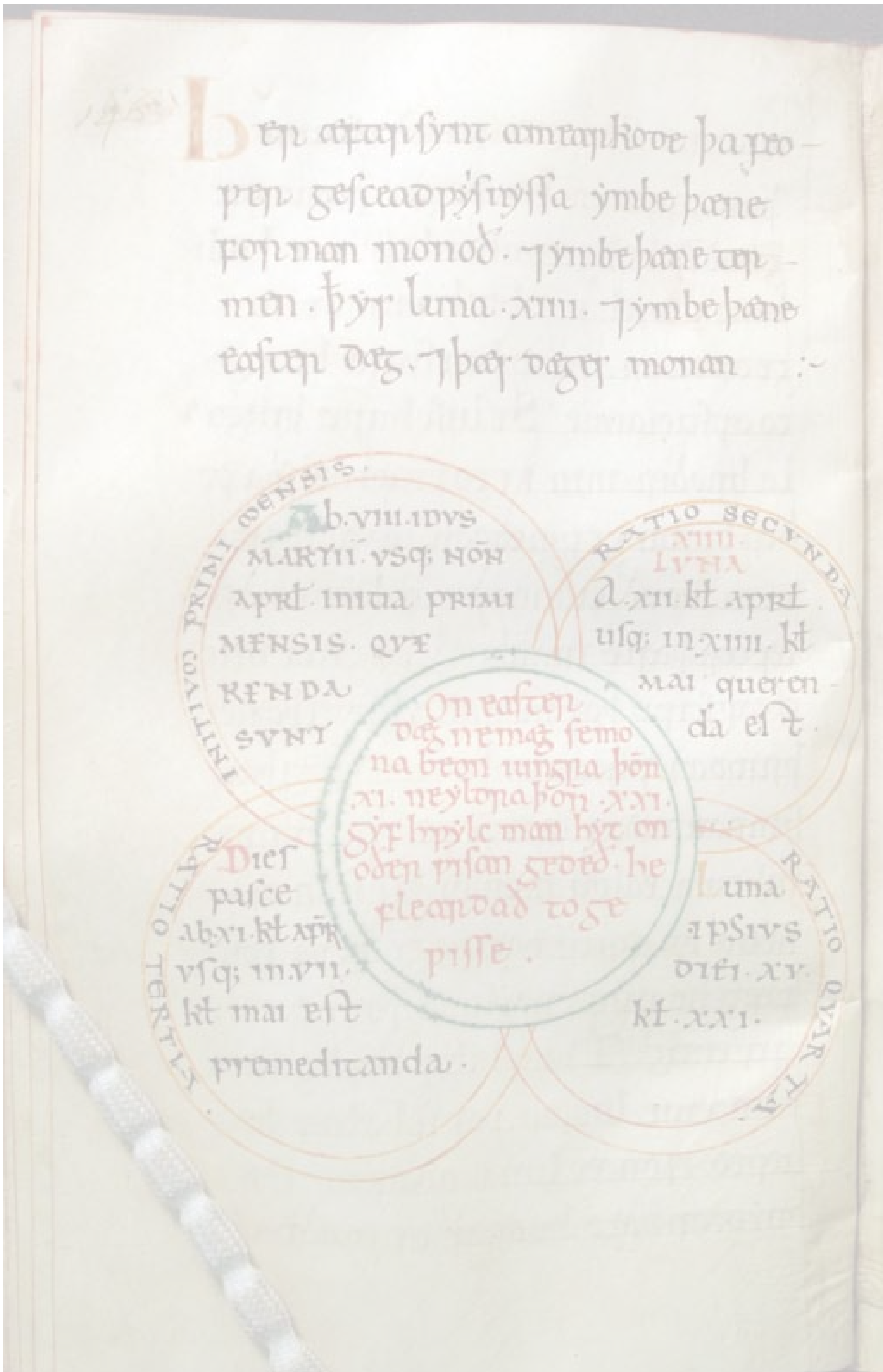


4.29 Oxford, Bodleian Library Ashmole MS 328, p. 147.
Photo taken with permission of the Bodleian Library, Oxford University

4.30 Oxford, Bodleian Library Ashmole MS 328, p. 85.
Photo taken with permission of the Bodleian Library, Oxford University

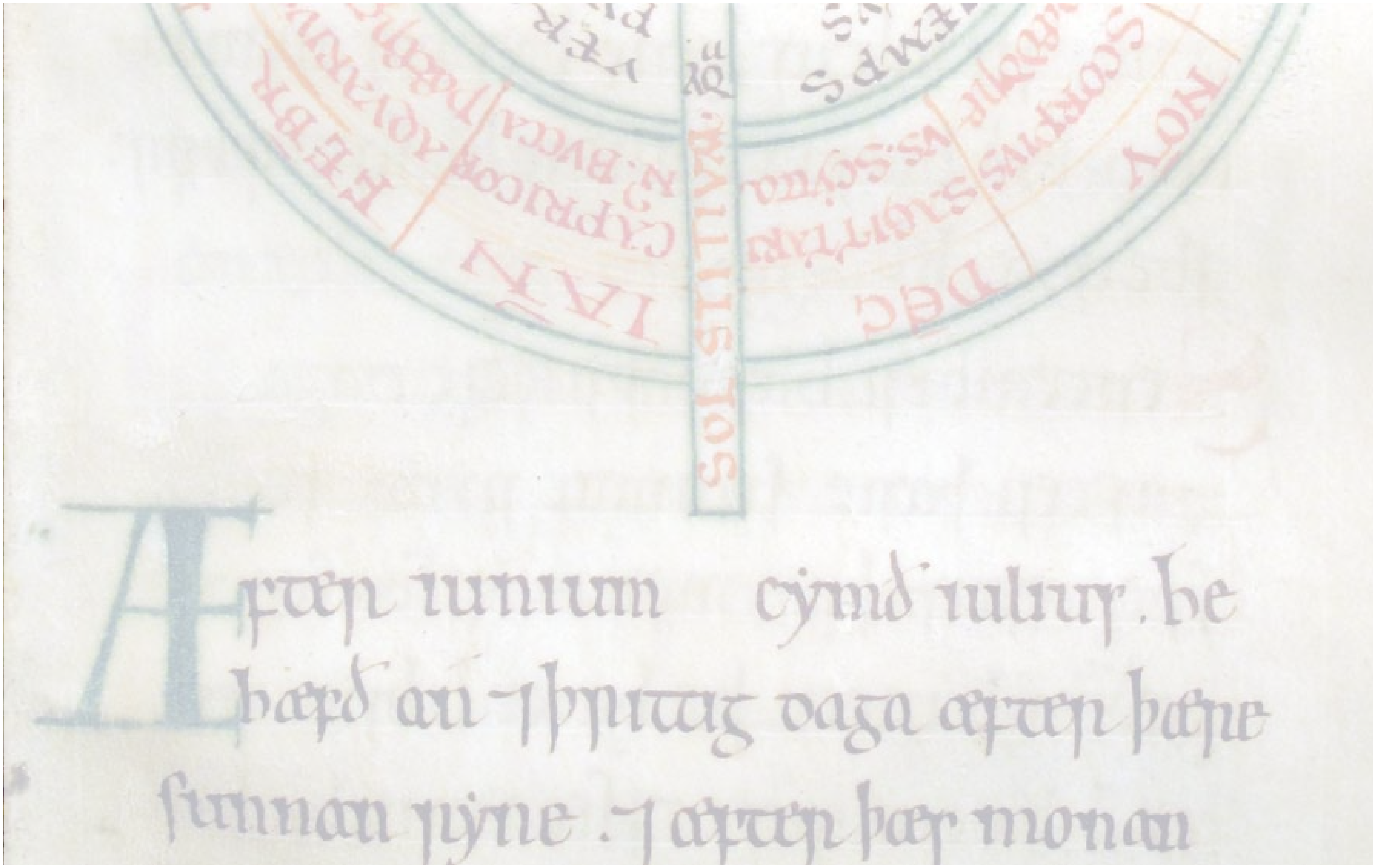
4.31 Berlin, Staatsbibliothek Cod. 138 fol. 36v, detail.

4.32 Berlin, Staatsbibliothek Cod. 138 fol. 38v, detail. © Staatsbibliothek Berlin

4.33 Oxford, St. John’s College MS 17, fol. 39v.

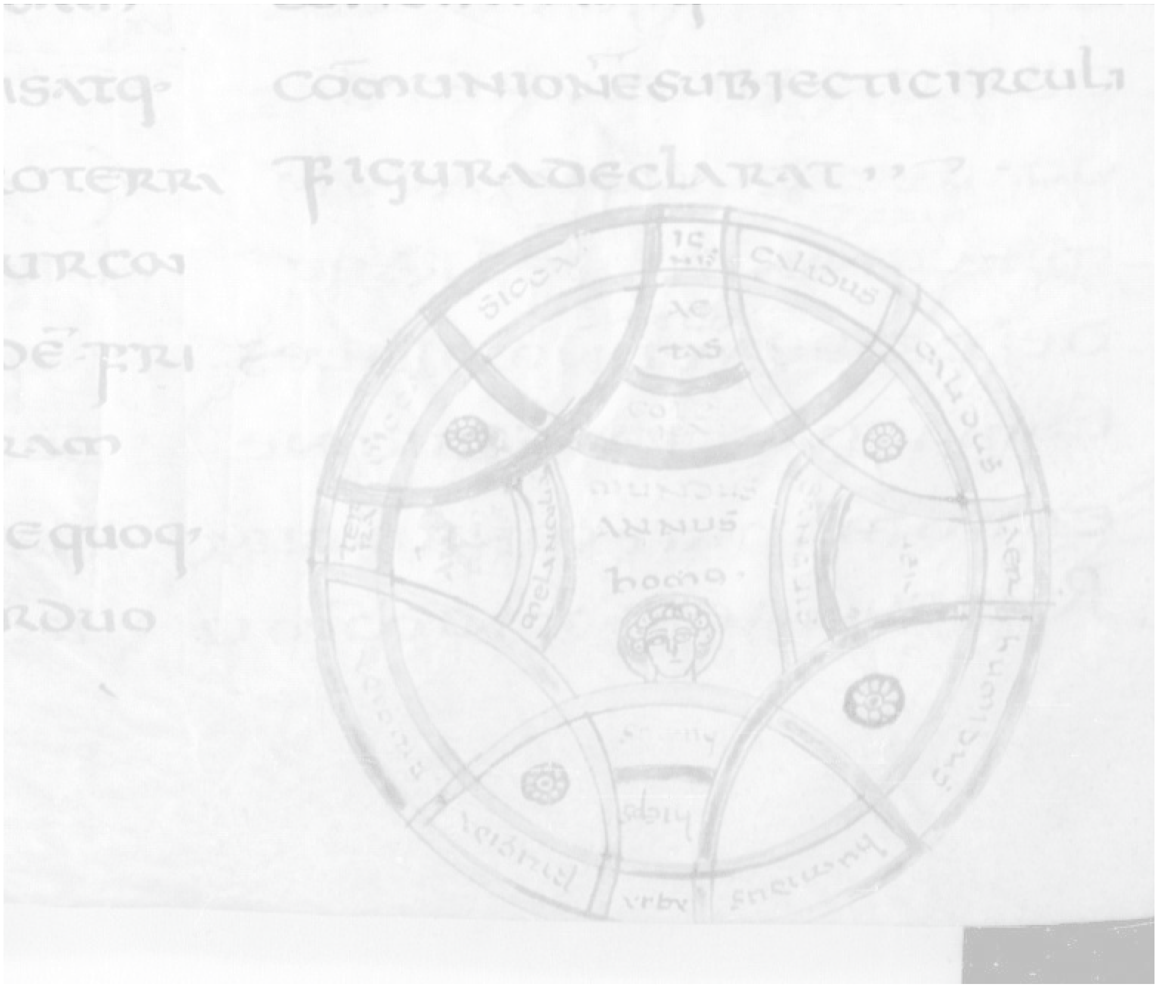
4.34 Oxford, Bodleian Library Ashmole MS 328, p. 85, detail. Photo taken with permission of the Bodleian Library, Oxford University

4.35 Oxford, Bodleian Library Ashmole MS 328, p. 85, detail.
Photo taken with permission of the Bodleian Library, Oxford University

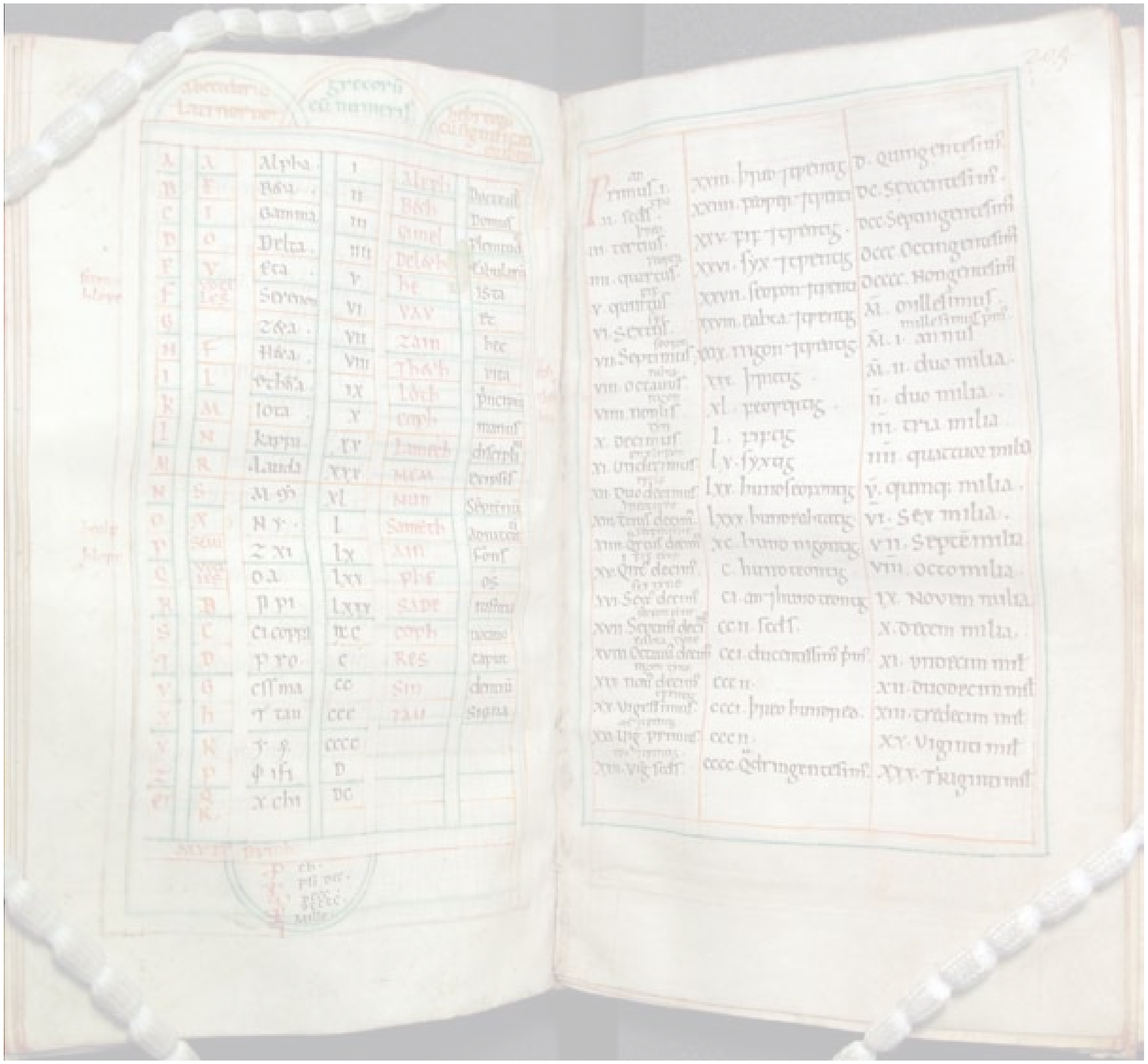


4.36 Oxford, Bodleian Library Ashmole MS 328, p. 85, detail. Photo taken with permission of the Bodleian Library, Oxford University

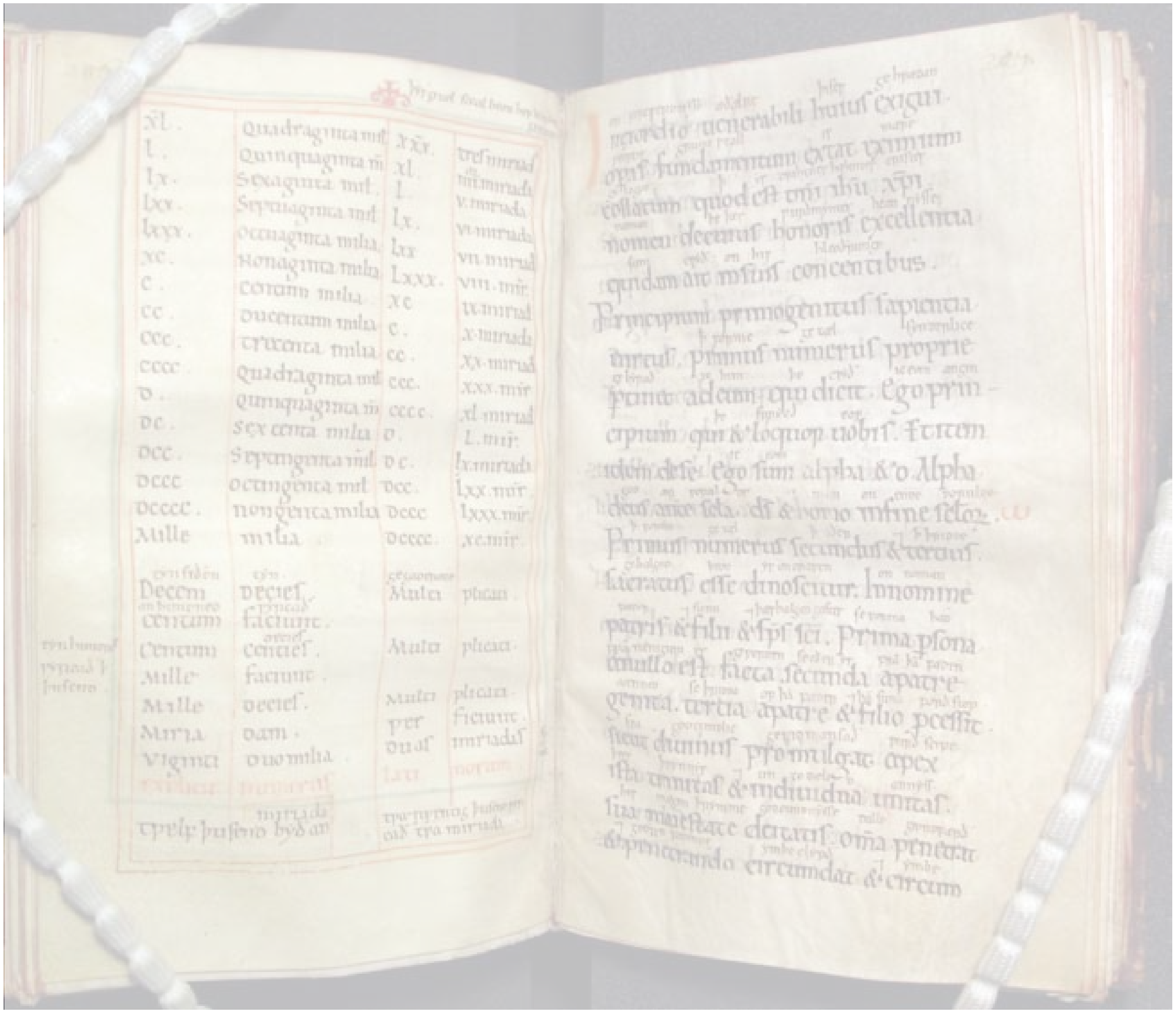
4.37 Paris, Bibliothèque nationale de France lat. MS 6400G, fol. 120v.



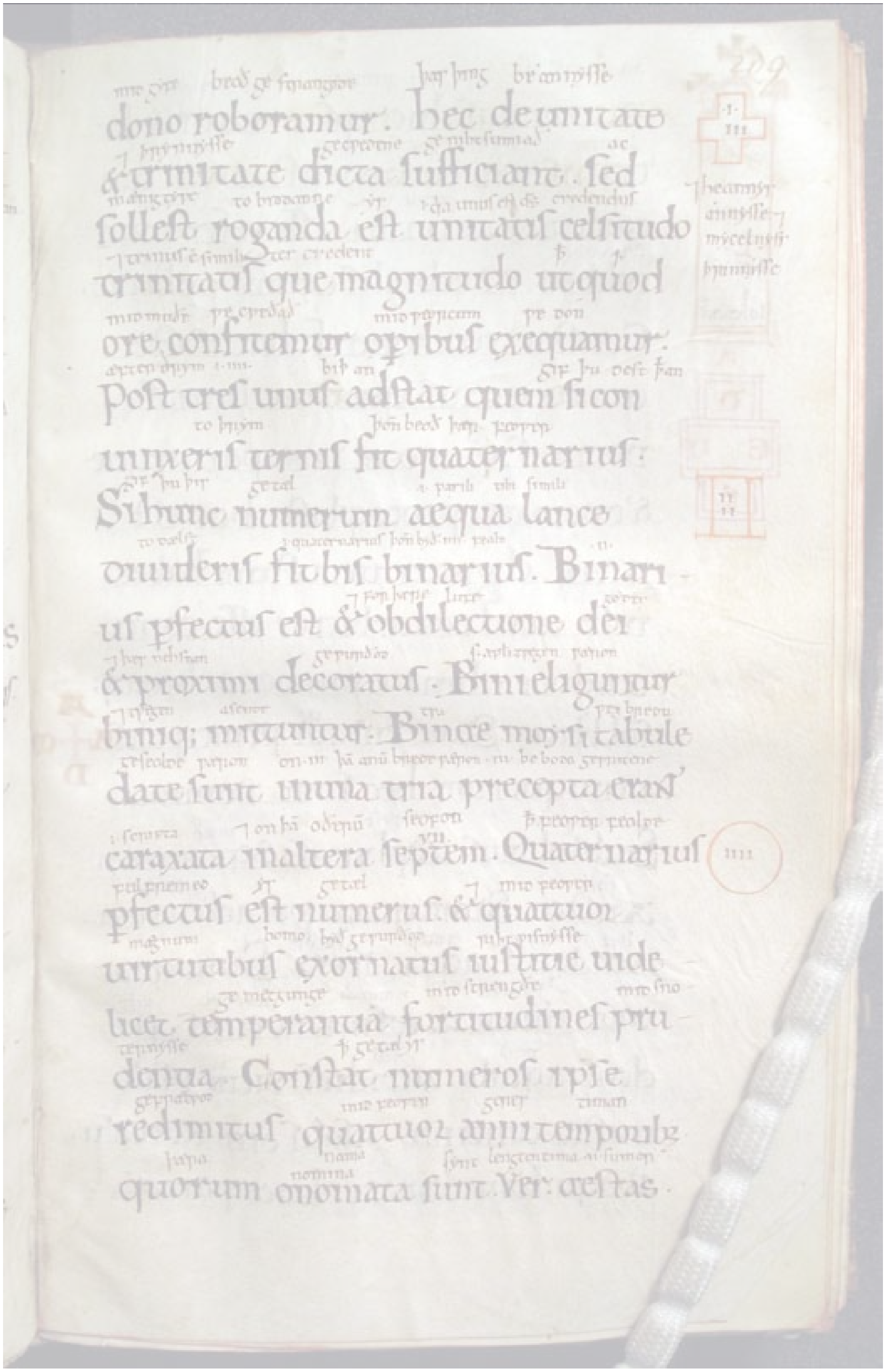
4.38 Paris, Bibliothèque nationale de France lat. MS 6400G, fol. 120v.



4.39 Oxford, Bodleian Library Ashmole MS 328, p. 204–205. Photo taken with permission of the Bodleian Library, Oxford University



4.40 Oxford, Bodleian Library Ashmole MS 328, p. 206–207. Photo taken with permission of the Bodleian Library, Oxford University



4.41 Oxford, Bodleian Library Ashmole MS 328, p. 209.
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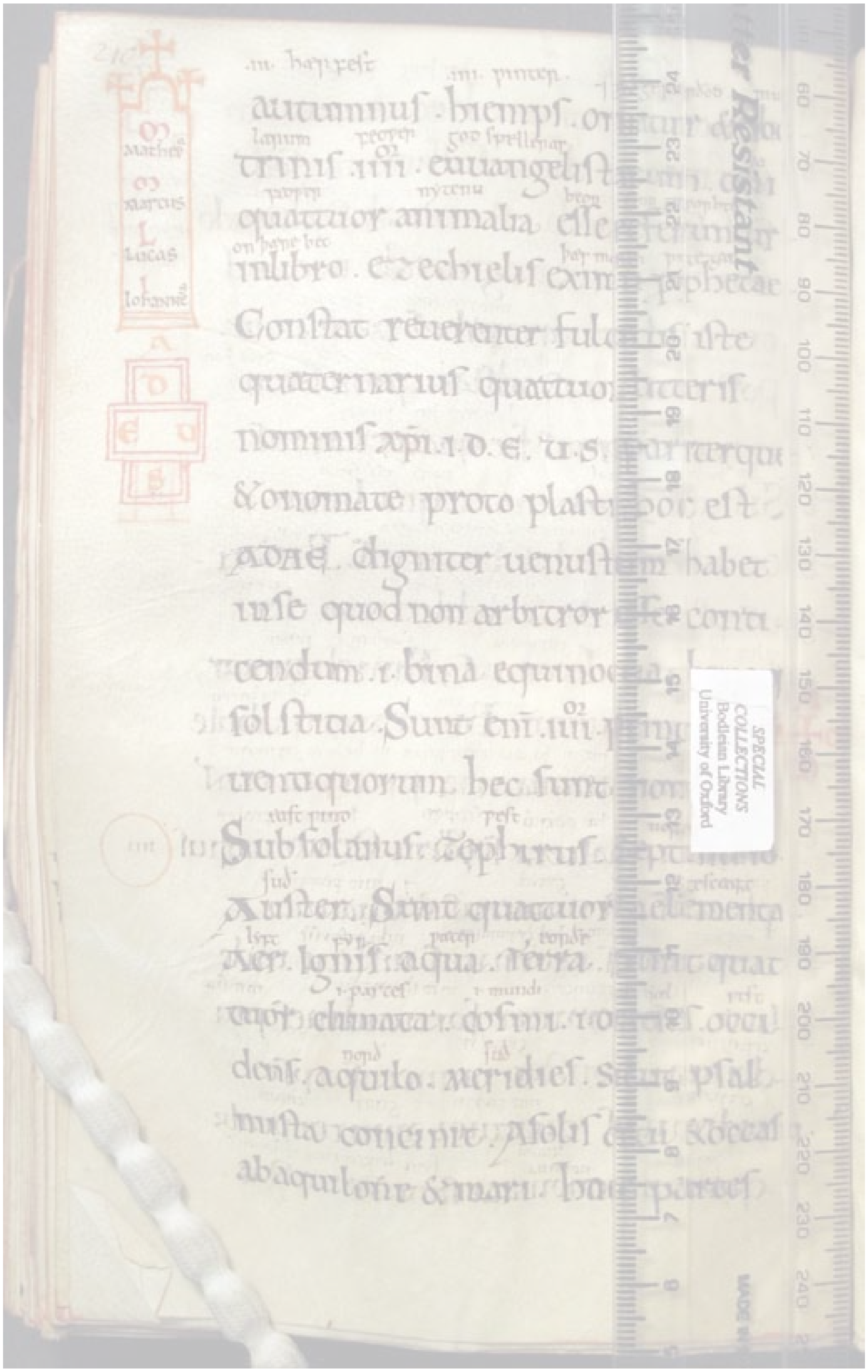
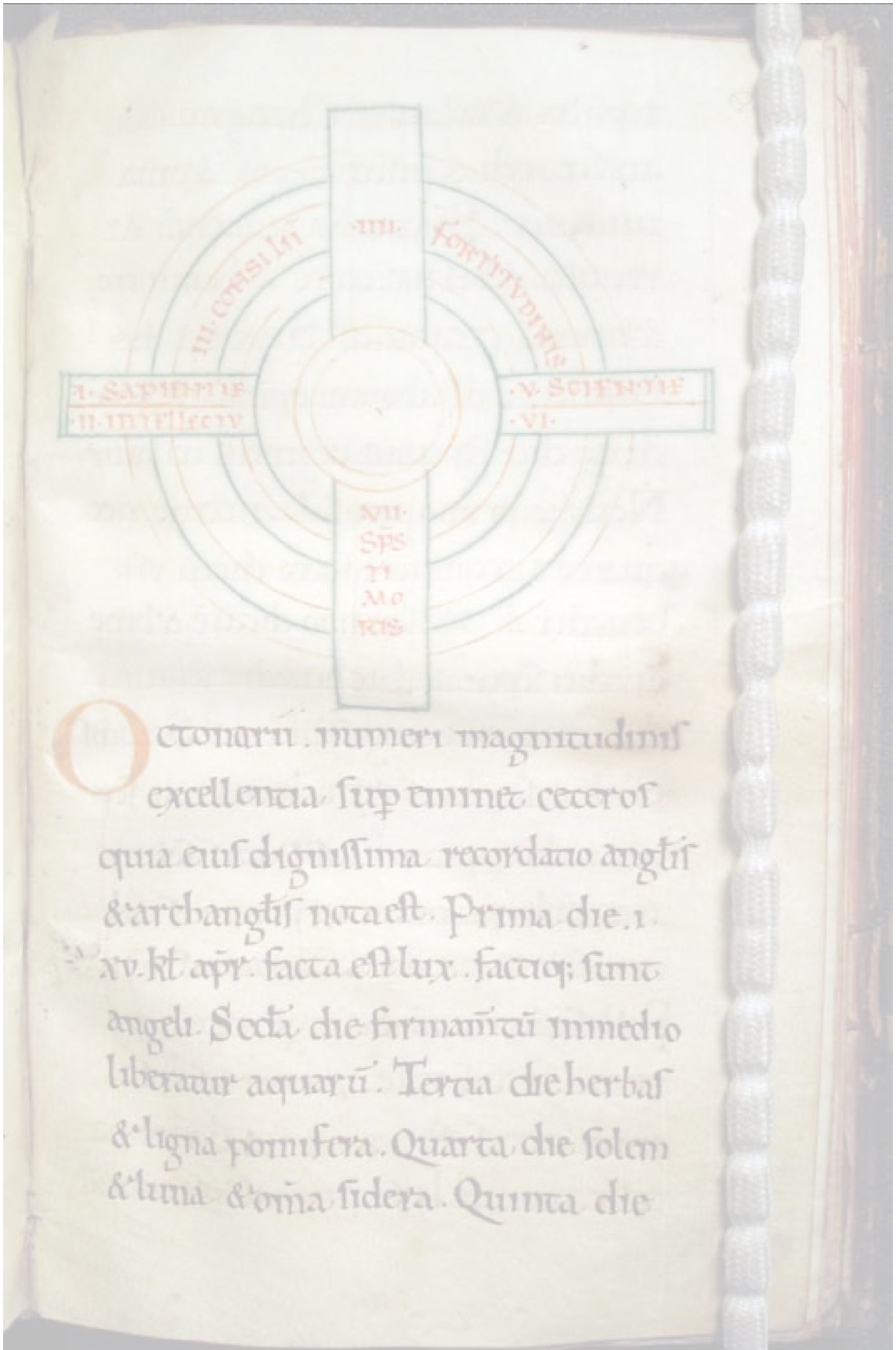
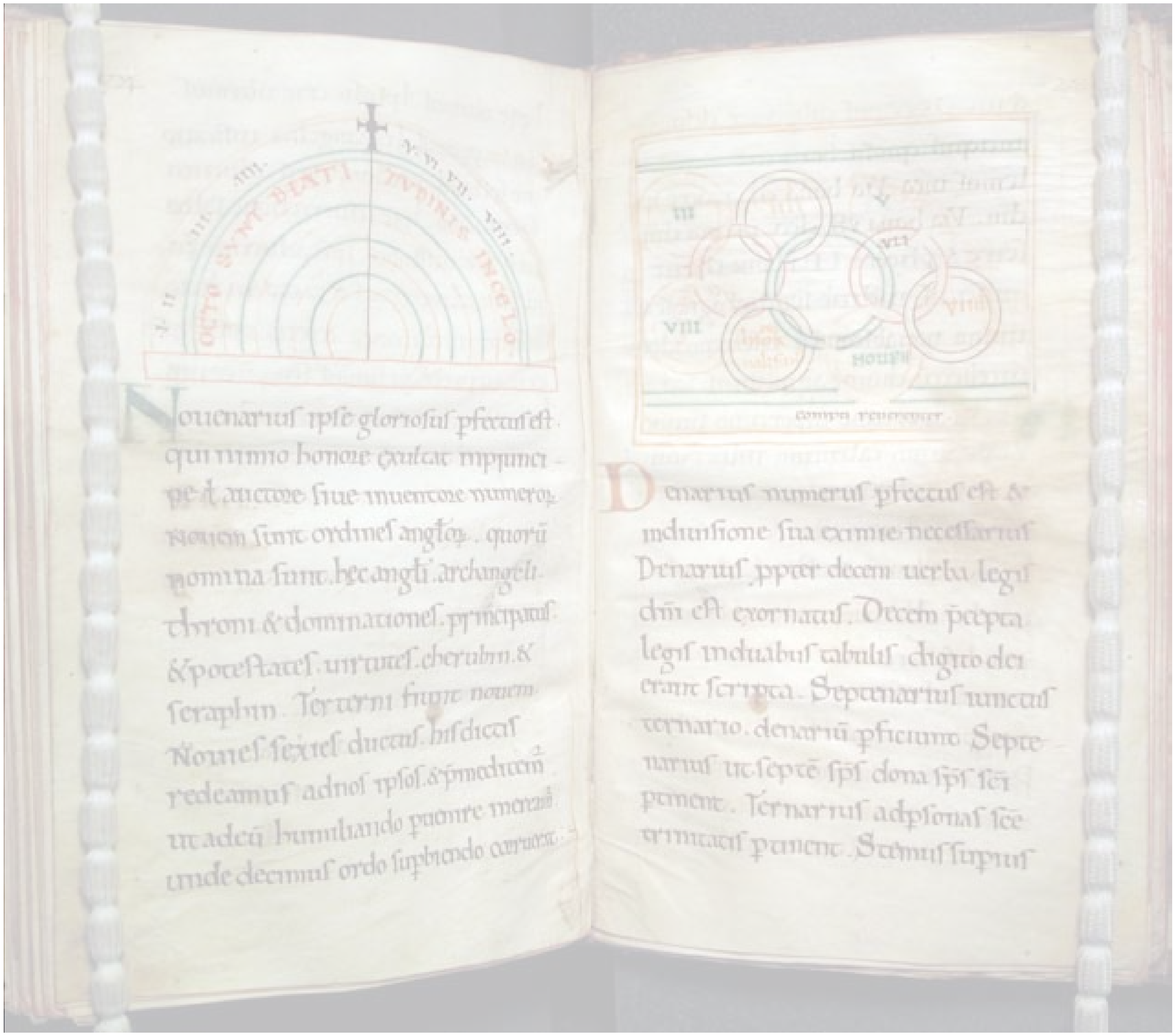


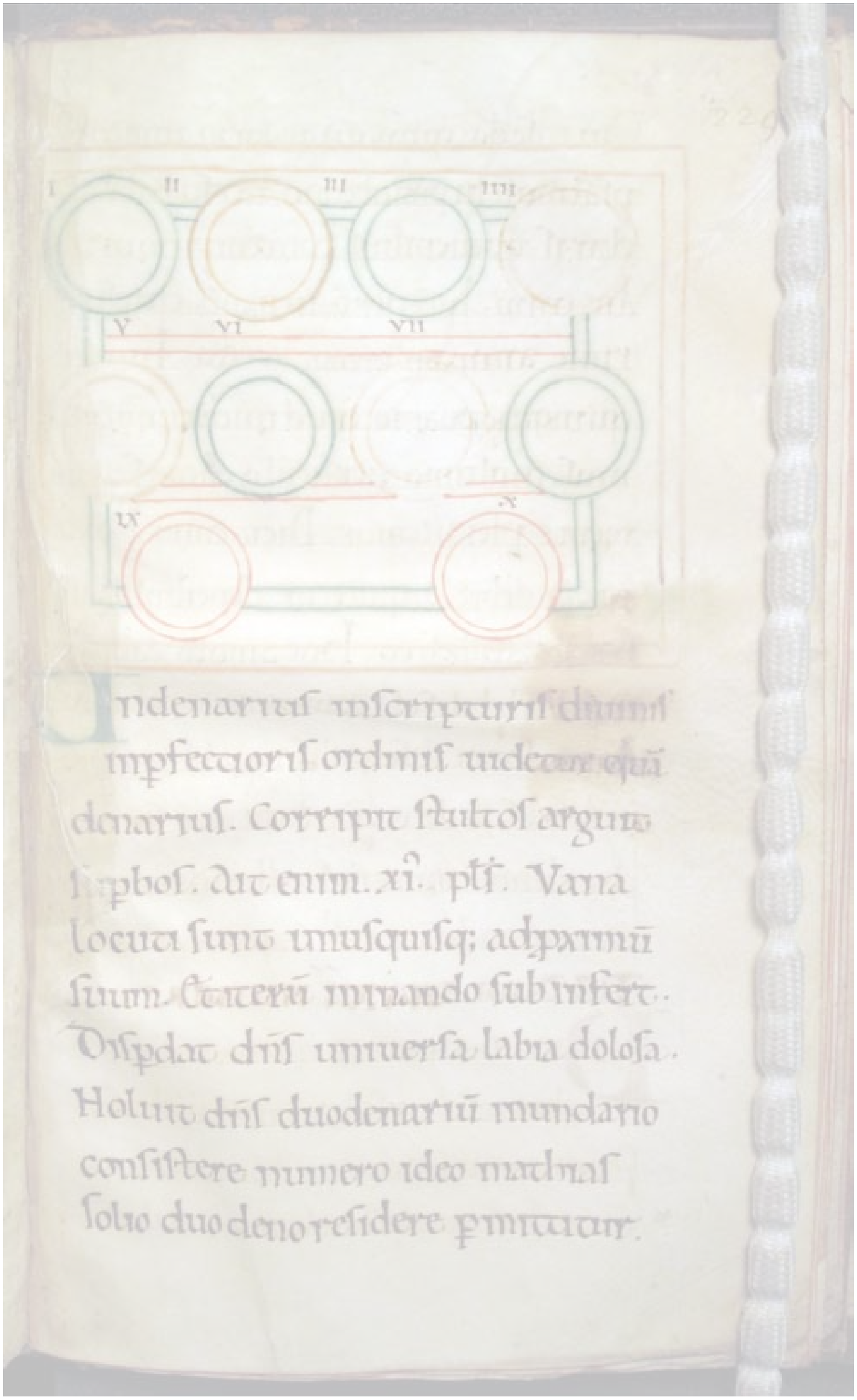
4.42 Oxford, Bodleian Library Ashmole MS 328, p. 210.
Photo taken with permission of the Bodleian Library, Oxford University



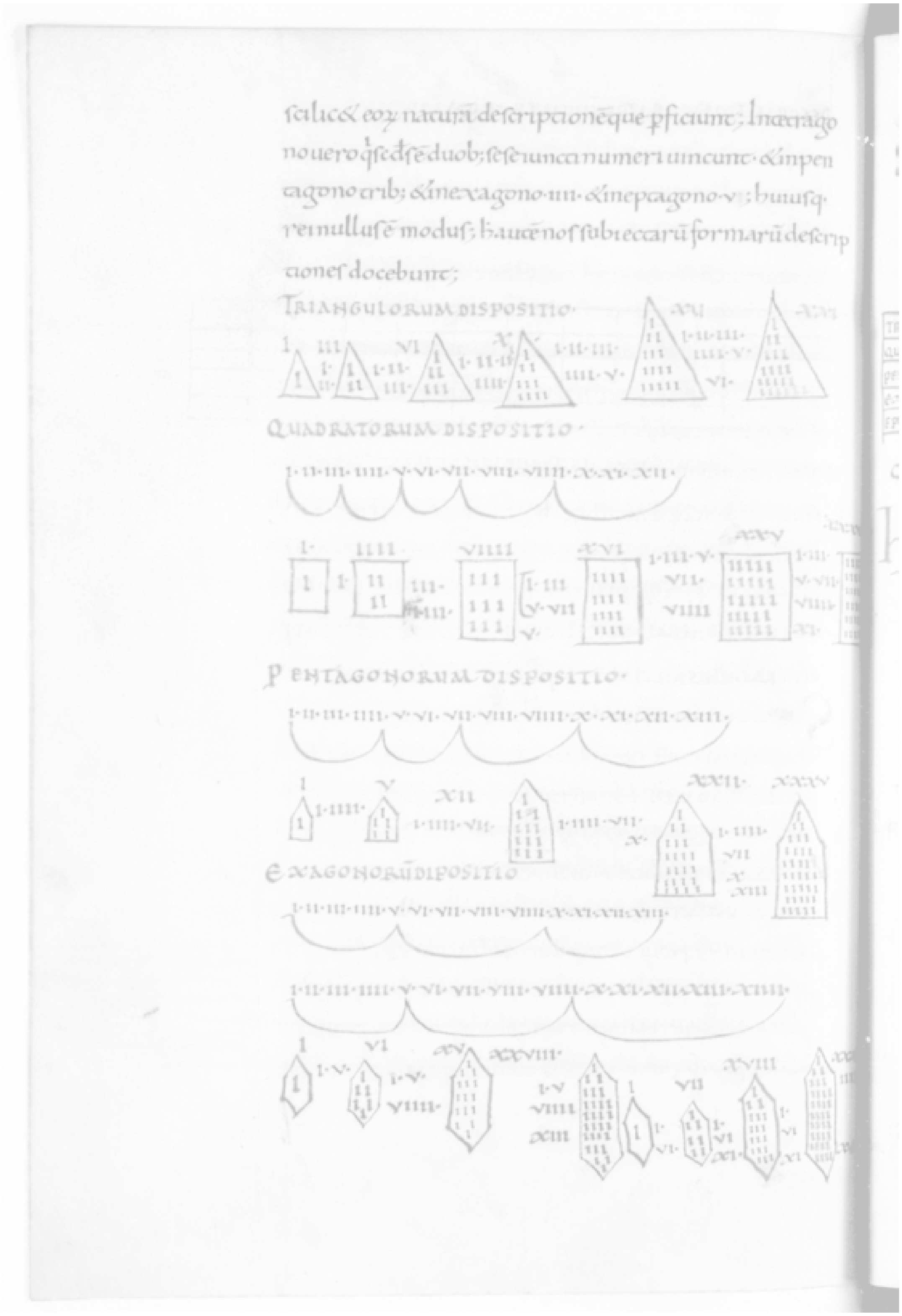
4.43 Oxford, Bodleian Library Ashmole MS 328, p. 221.
Photo taken with permission of the Bodleian Library, Oxford University



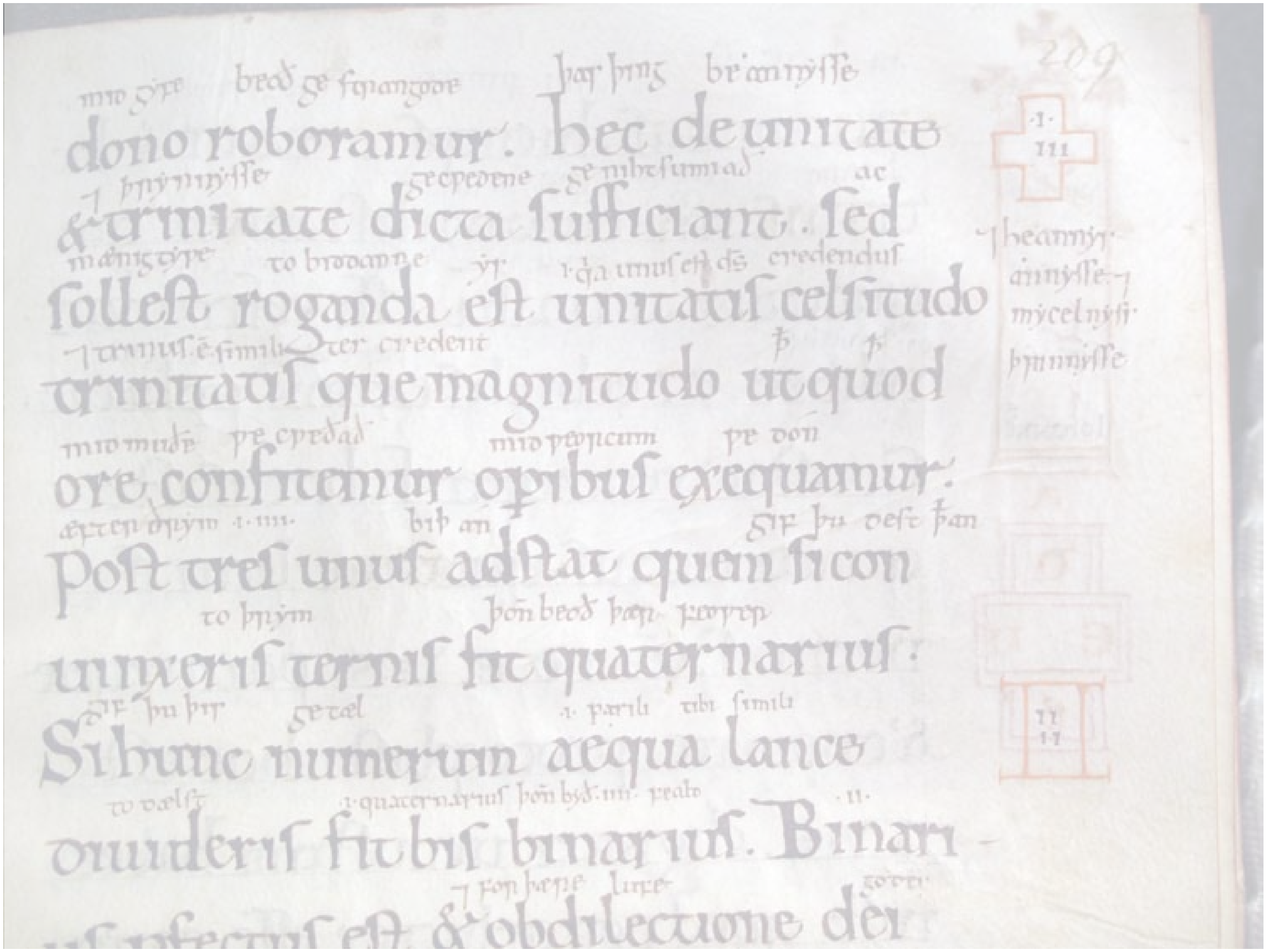
4.44 Oxford, Bodleian Library Ashmole MS 328, p. 223–224. Photo taken with permission of the Bodleian Library, Oxford University



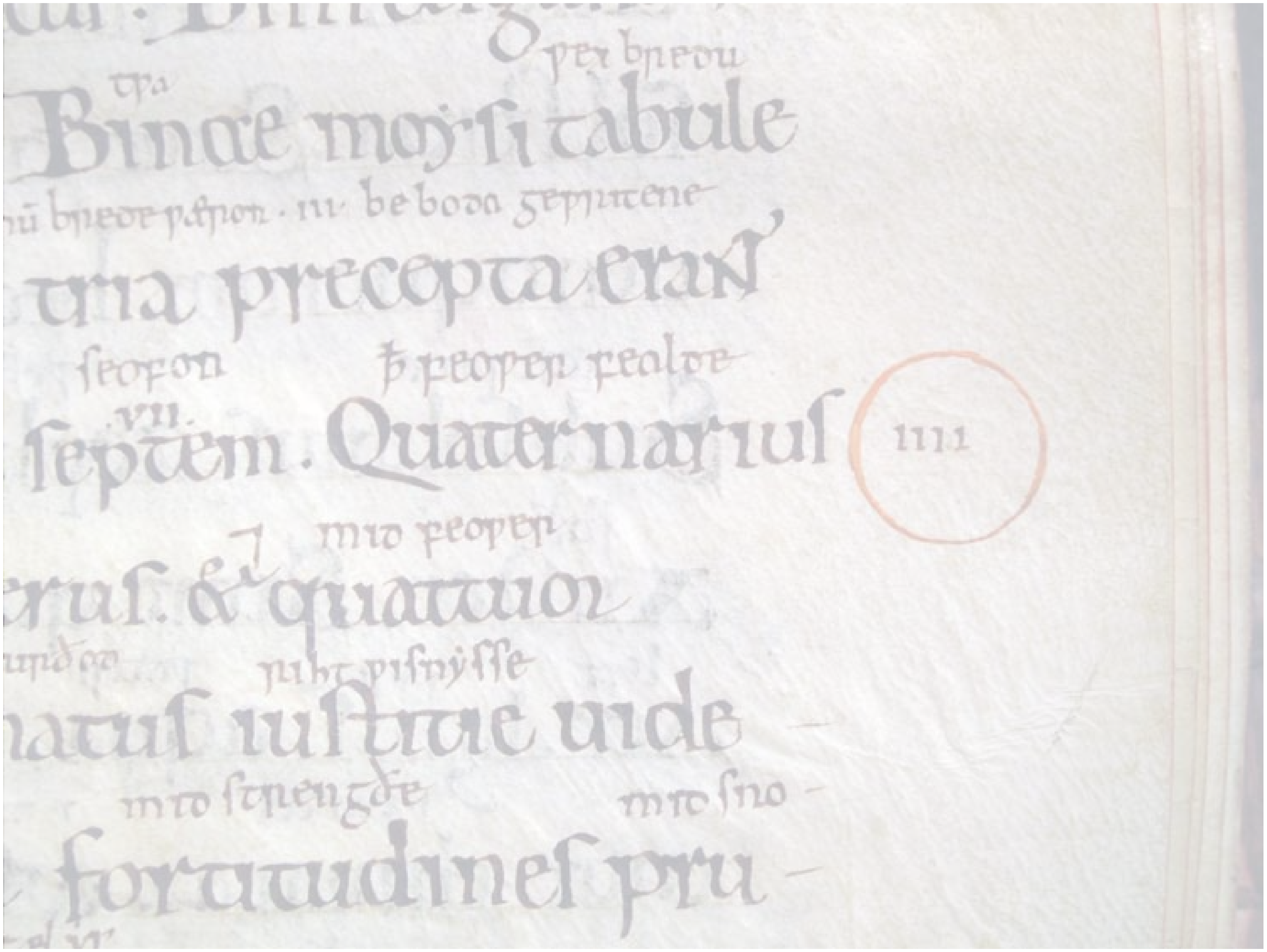
4.45 Oxford, Bodleian Library Ashmole MS 328, p. 229.
Photo taken with permission of the Bodleian Library, Oxford University



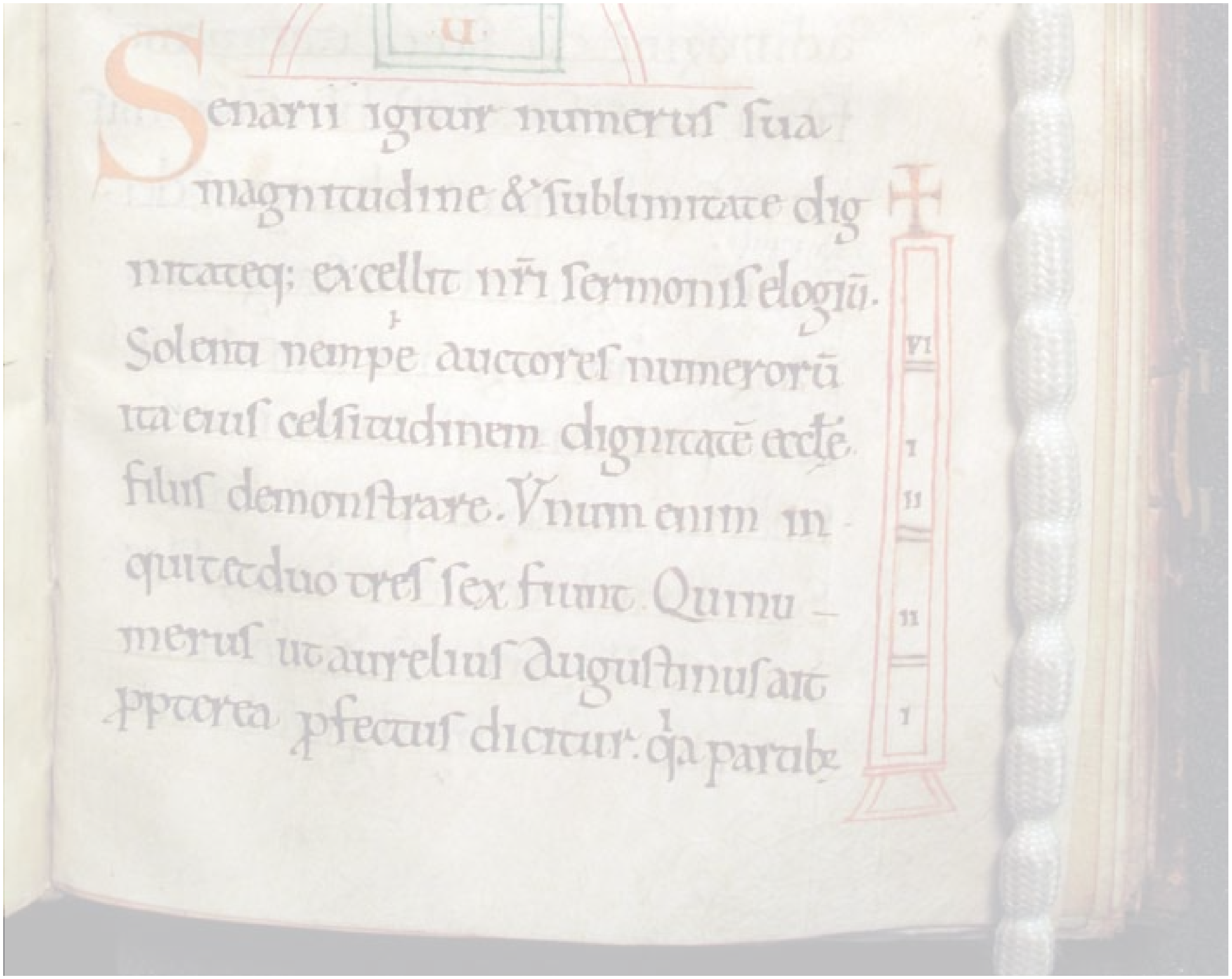
4.46 Paris, Bibliothèque nationale de France lat. MS 6401, fol. 128v.



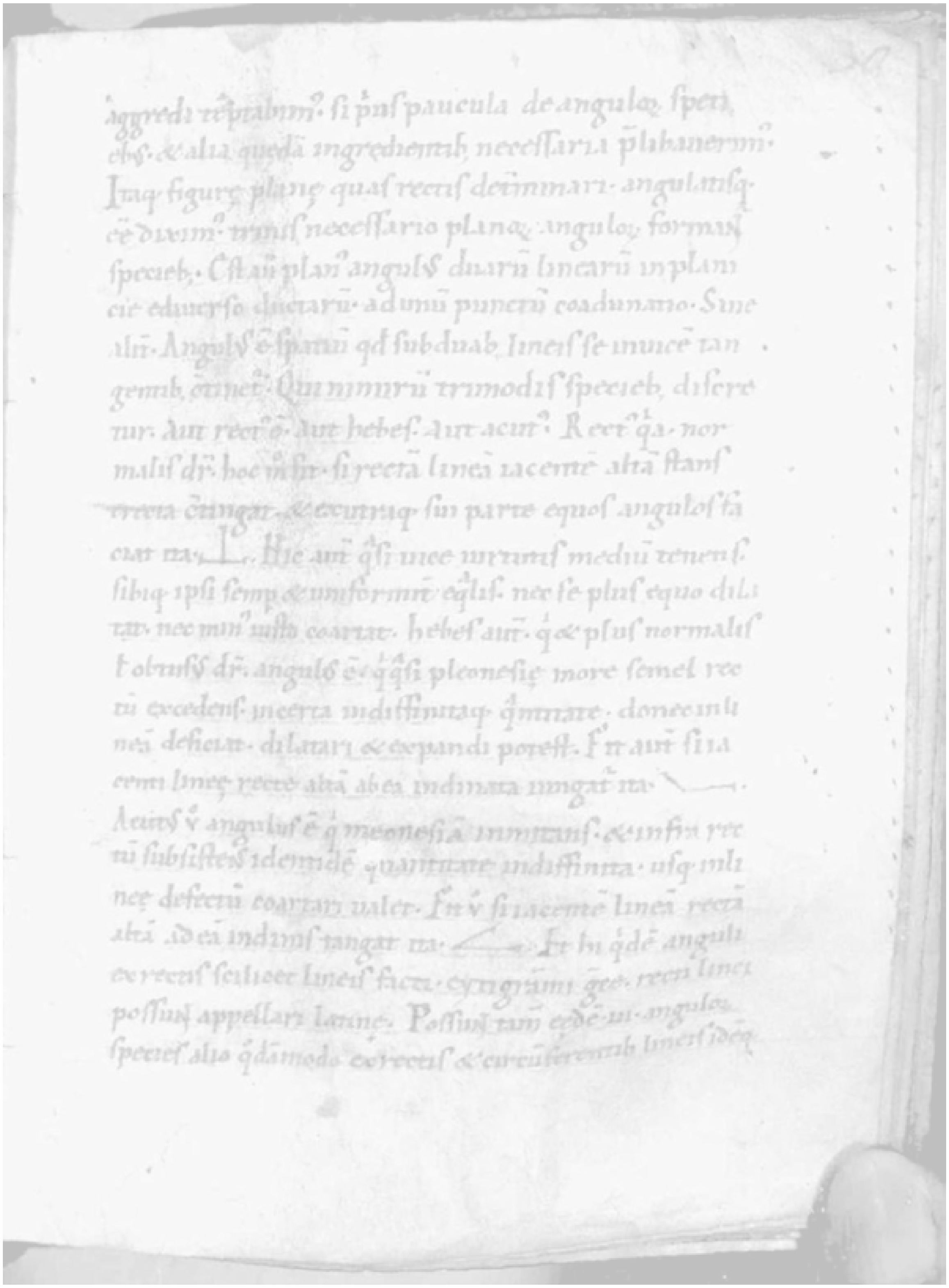
4.47 Oxford, Bodleian Library Ashmole MS 328, p. 209, detail. Photo taken with permission of the Bodleian Library, Oxford University



4.48 Oxford, Bodleian Library Ashmole MS 328, p. 209, detail. Photo taken with permission of the Bodleian Library, Oxford University



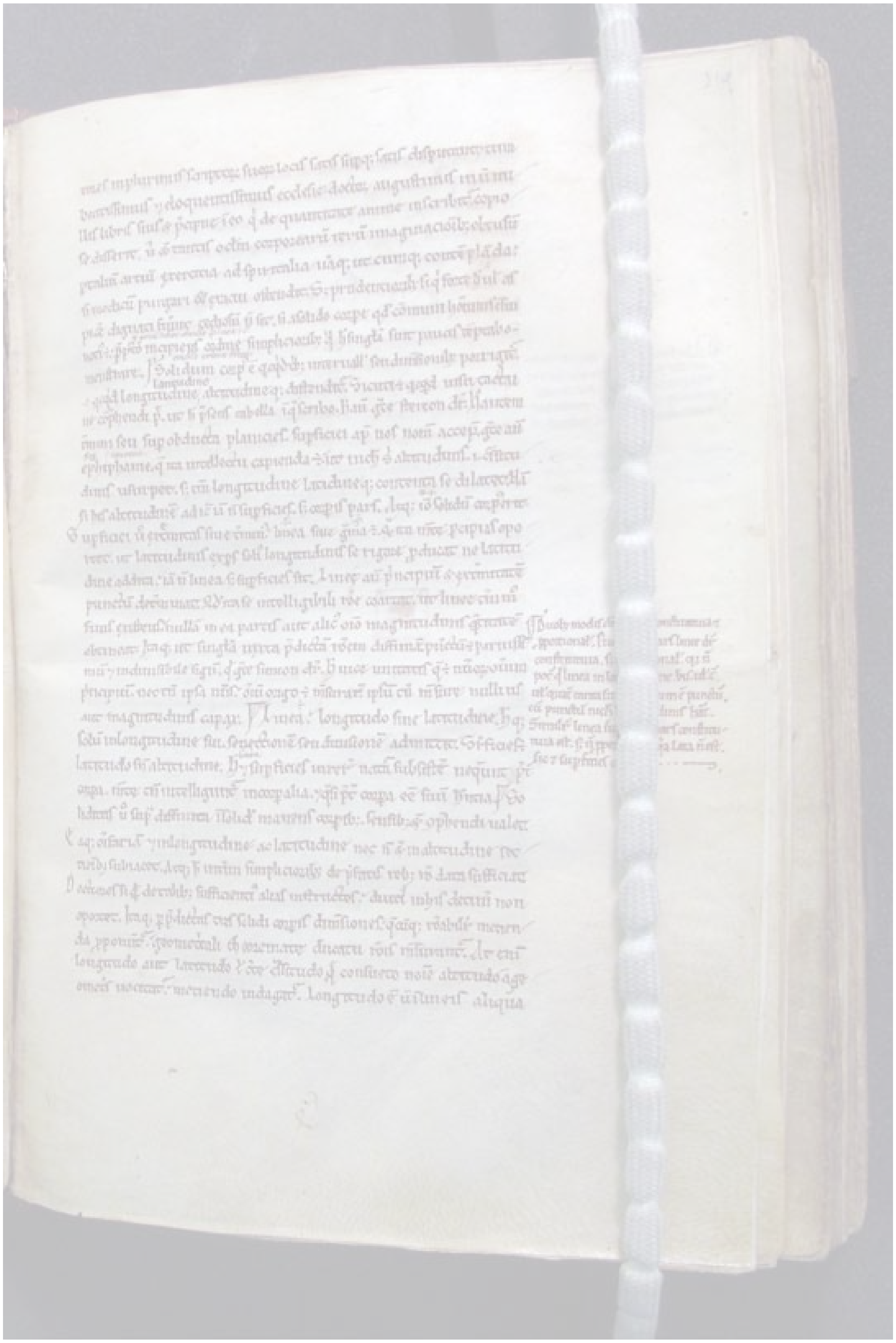
4.49 Oxford, Bodleian Library Ashmole MS 328, p. 215, detail. Photo taken with permission of the Bodleian Library, Oxford University



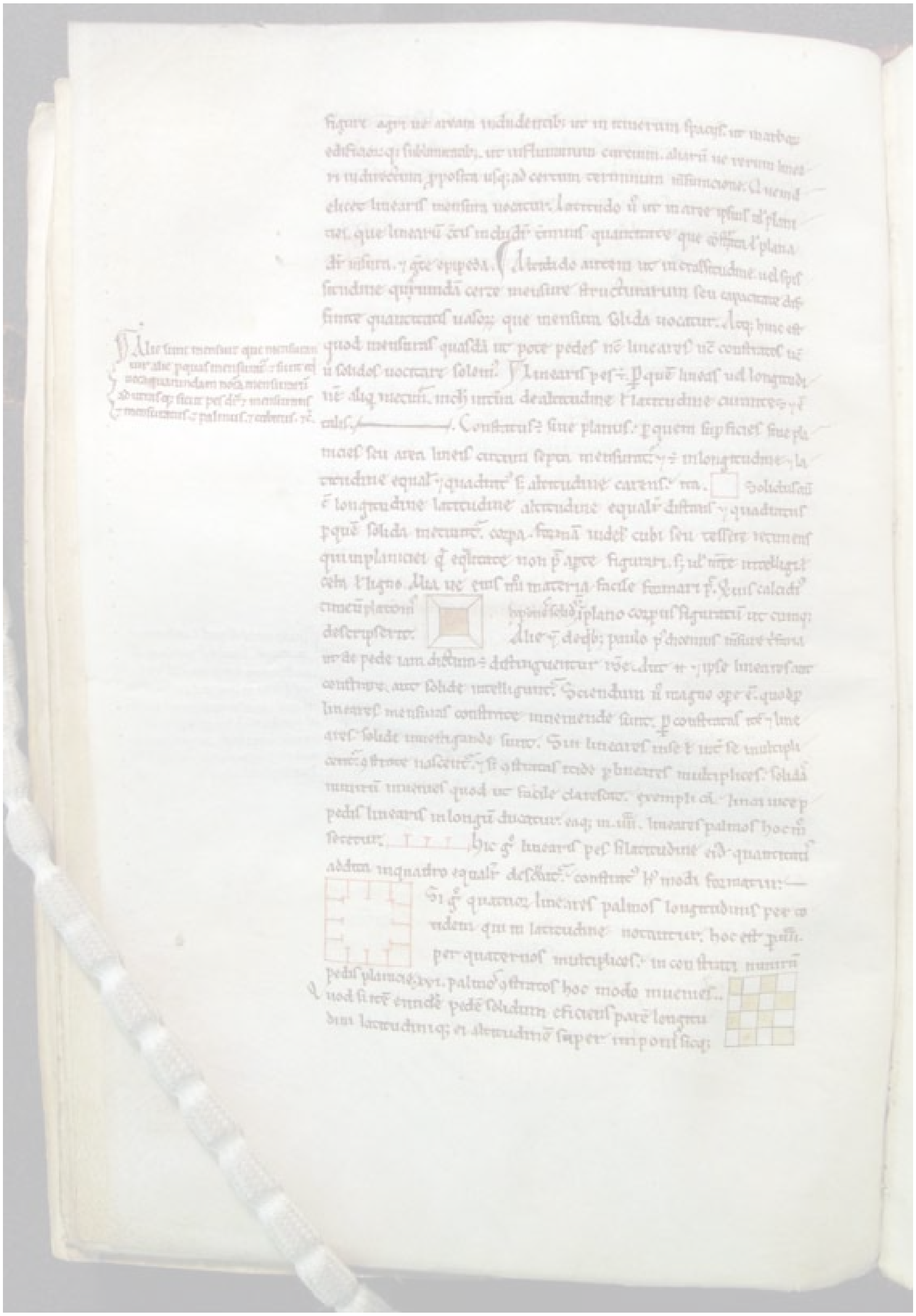
aggredi temptabimus si prius paucula de angulorum speci-
ebus et alia quedam ingredientibus necessaria prelibauerimus.
Itaque figure plane quas rectis determinari angulisque
esse diximus tamen necessario planorum angulorum formis
speciebus. Est autem planus angulus duarum linearum in plani-
cie e diuerso ductarum ad unum punctum coadunatio. Siue
aliter. Angulus est spatium quod sub duabus lineis se inuicem tan-
gentibus continetur. Qui nimirum trimodis speciebus discerni-
tur. aut rectus est aut hebes aut acutus. Rectus qui et nor-
malis dicitur hoc modo fit. si recta linea iacentem aliam stans
recta contingat et ex utraque sui parte equos angulos fa-
ciat ita. Hic autem quasi inter utrumque medium tenens.
sibique ipsi semper et uniformiter equalis. nec se plus equo dila-
tat. nec minus iusto coartat. hebes autem qui et plus normalis
uel obtusus dicitur angulus est qui quasi pleonesie more semel rec-
ti excedens incerta indiffinitaque quantitate donec in li-
nea deficiat dilatari et expandi potest. Fit autem si ia-
centi linee recte aliam ab ea inclinata tangat ita.
Acutus uero angulus est qui meonesia imminutus et infra rec-
ti subsistens idemque quantitate indiffinita usque in li-
nee defectum coartari ualet. Fit uero si iacenti linee recta
alia ab ea inclinis tangat ita. Et hi quidem anguli
ex rectis scilicet lineis facti rectigrammi grece recti linei
possunt appellari latine. Possunt tamen eedem iii angulorum
species alio quodammodo ex rectis et circumferentibus lineis ideoque

5.1 Munich, Bayerische Staatsbibliothek Cod. Clm 14836, fol. 30r.

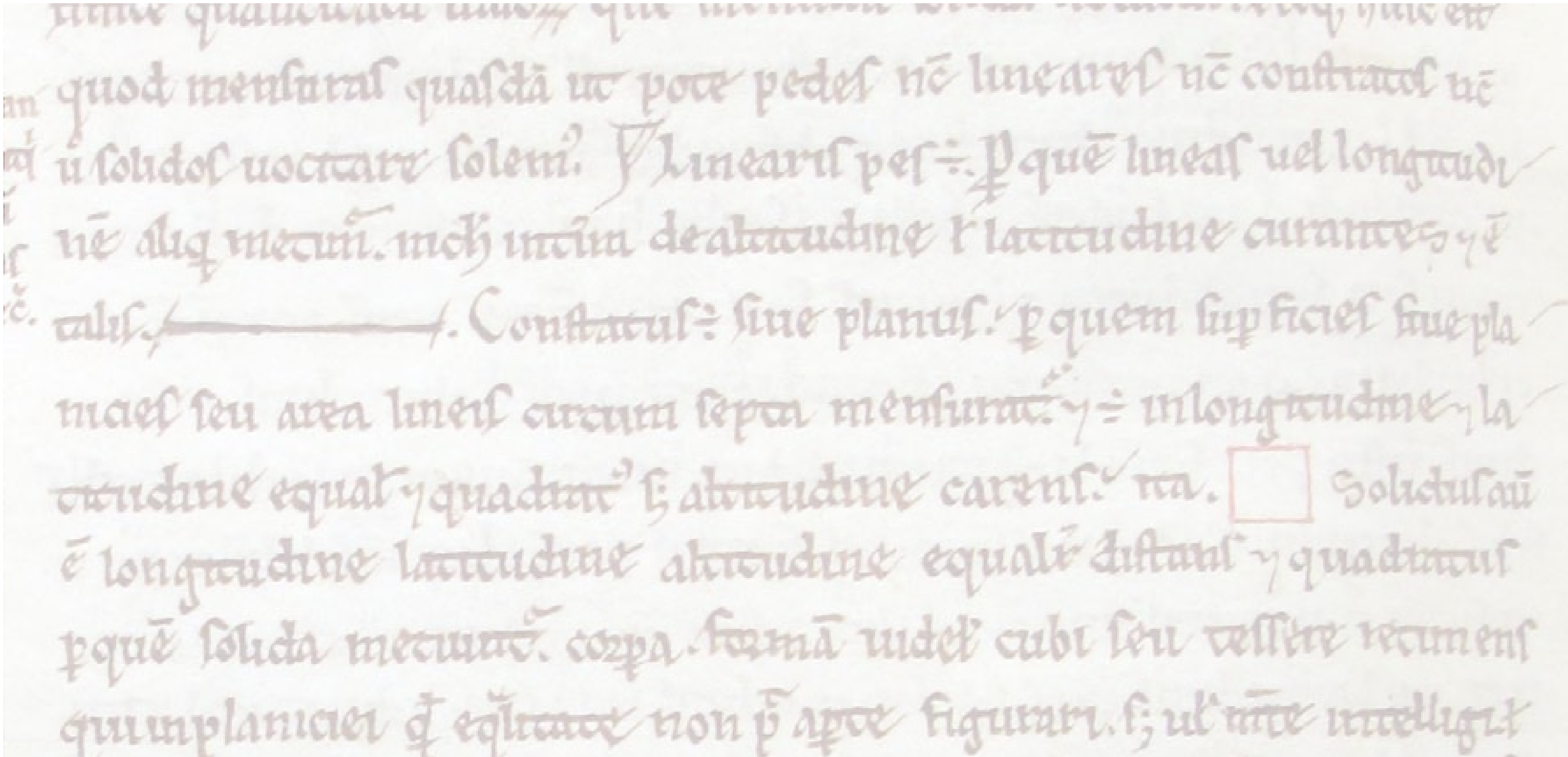
5.2 Brussels, Bibliothèque royale de Belgique MS 4499–4503, fol. 15v. Photo taken with permission of the Bibliothèque royale de Belgique



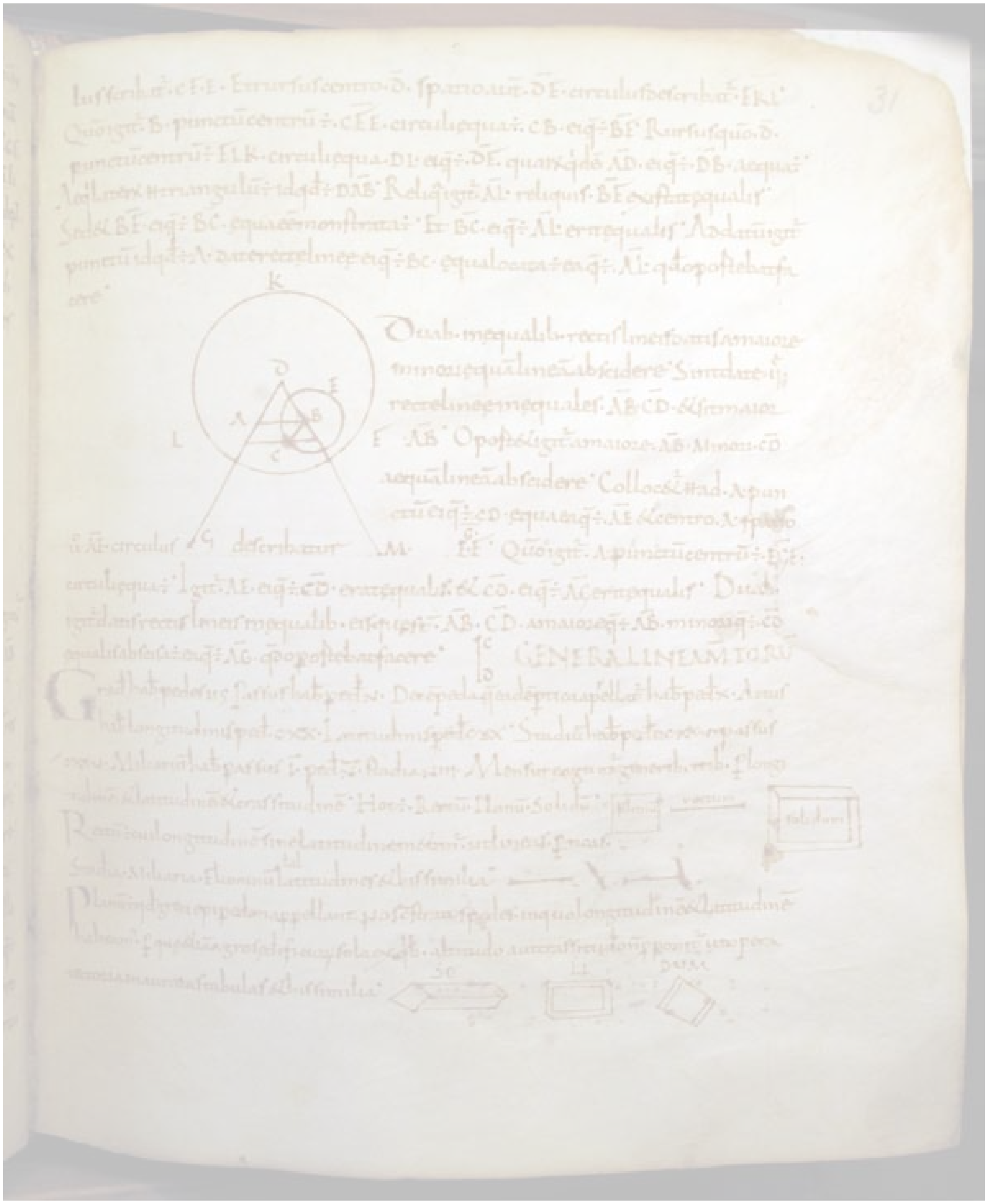
5.3 Oxford, Bodleian Library Selden Supra MS 25, fol. 119r.
Photo taken with permission of the Bodleian Library, Oxford University



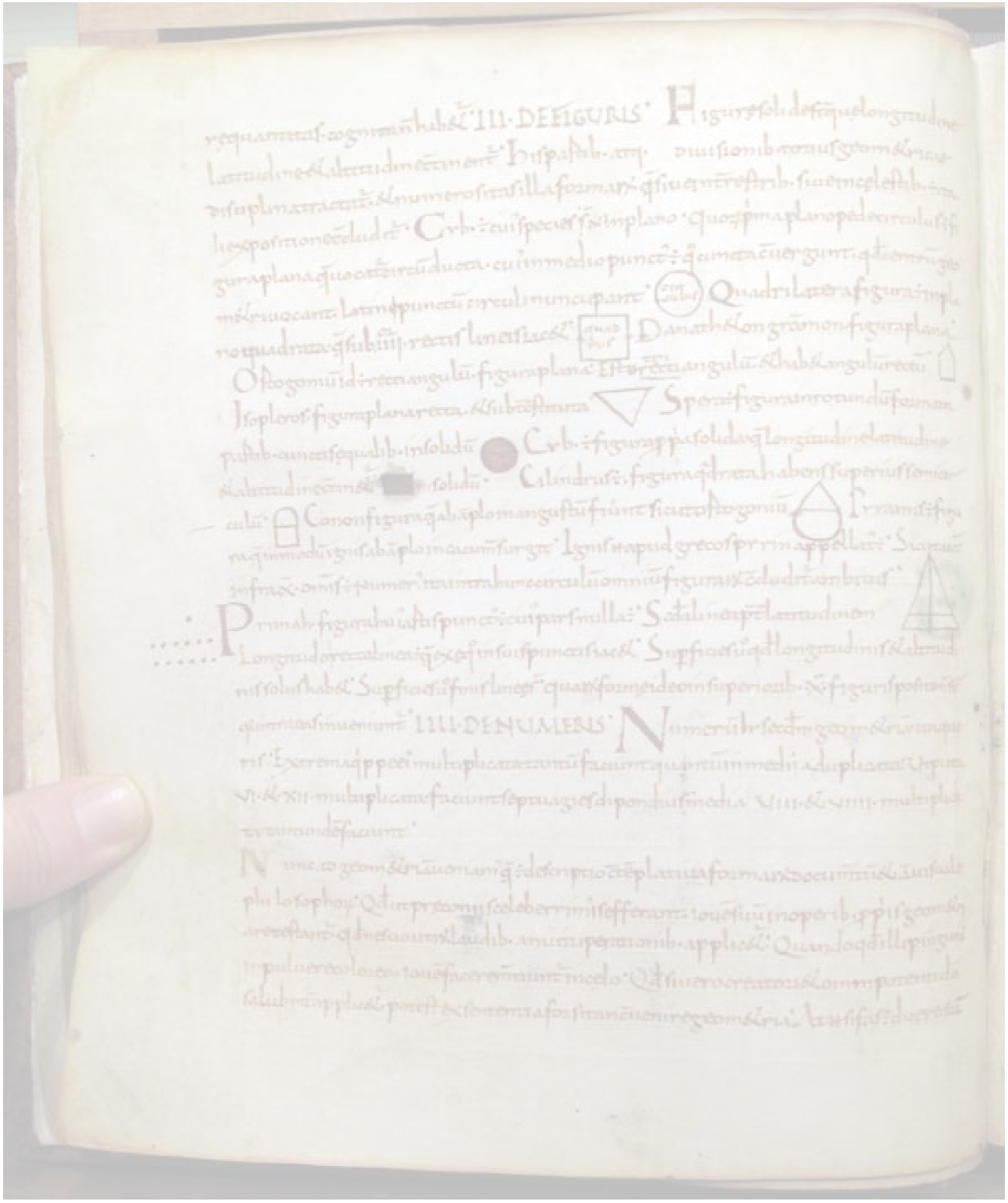
5.4 Oxford, Bodleian Library Selden Supra MS 25, fol. 119v.
Photo taken with permission of the Bodleian Library, Oxford University



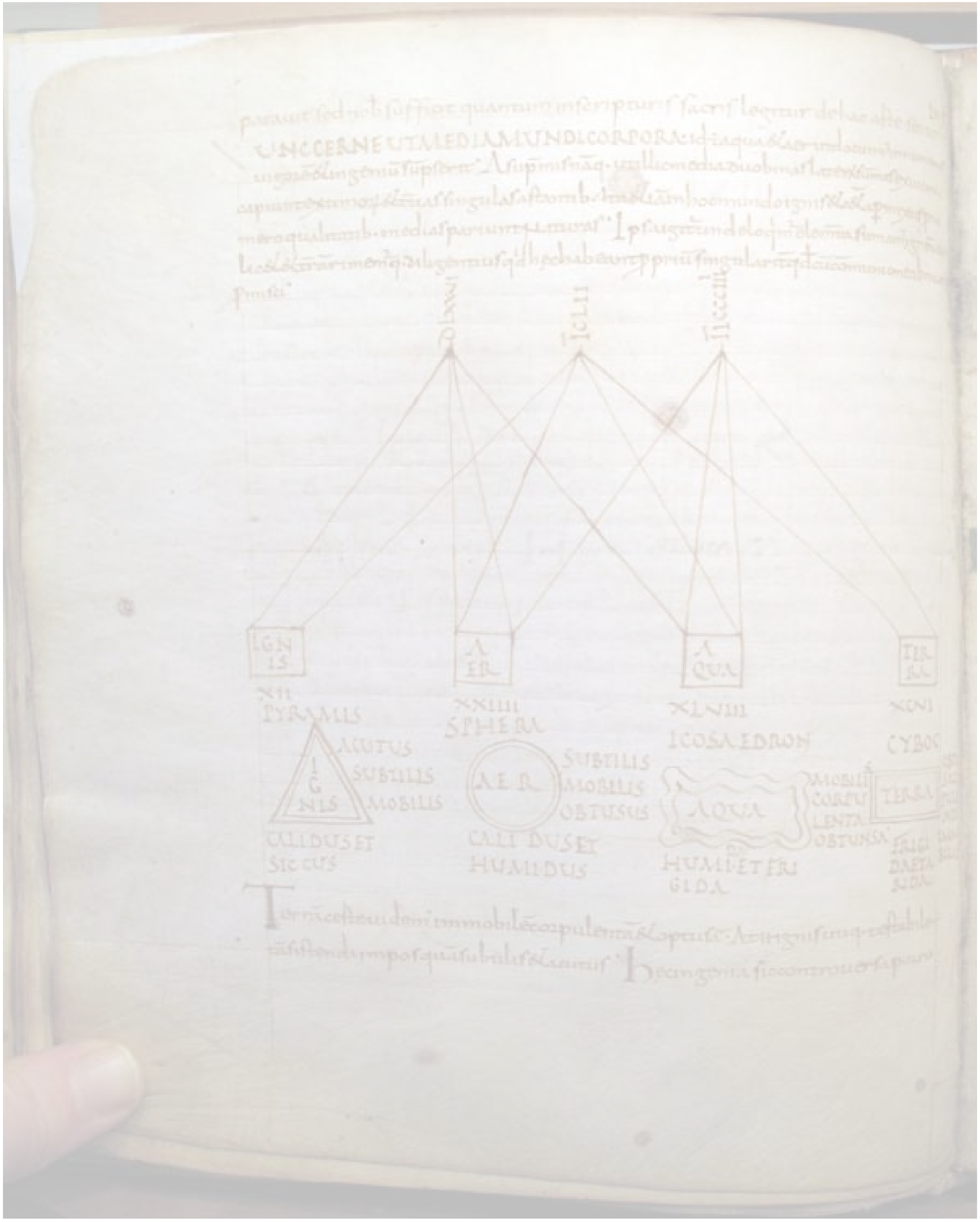
5.5 Oxford, Bodleian Library Selden Supra MS 25, fol. 119v, detail. Photo taken with permission of the Bodleian Library, Oxford University



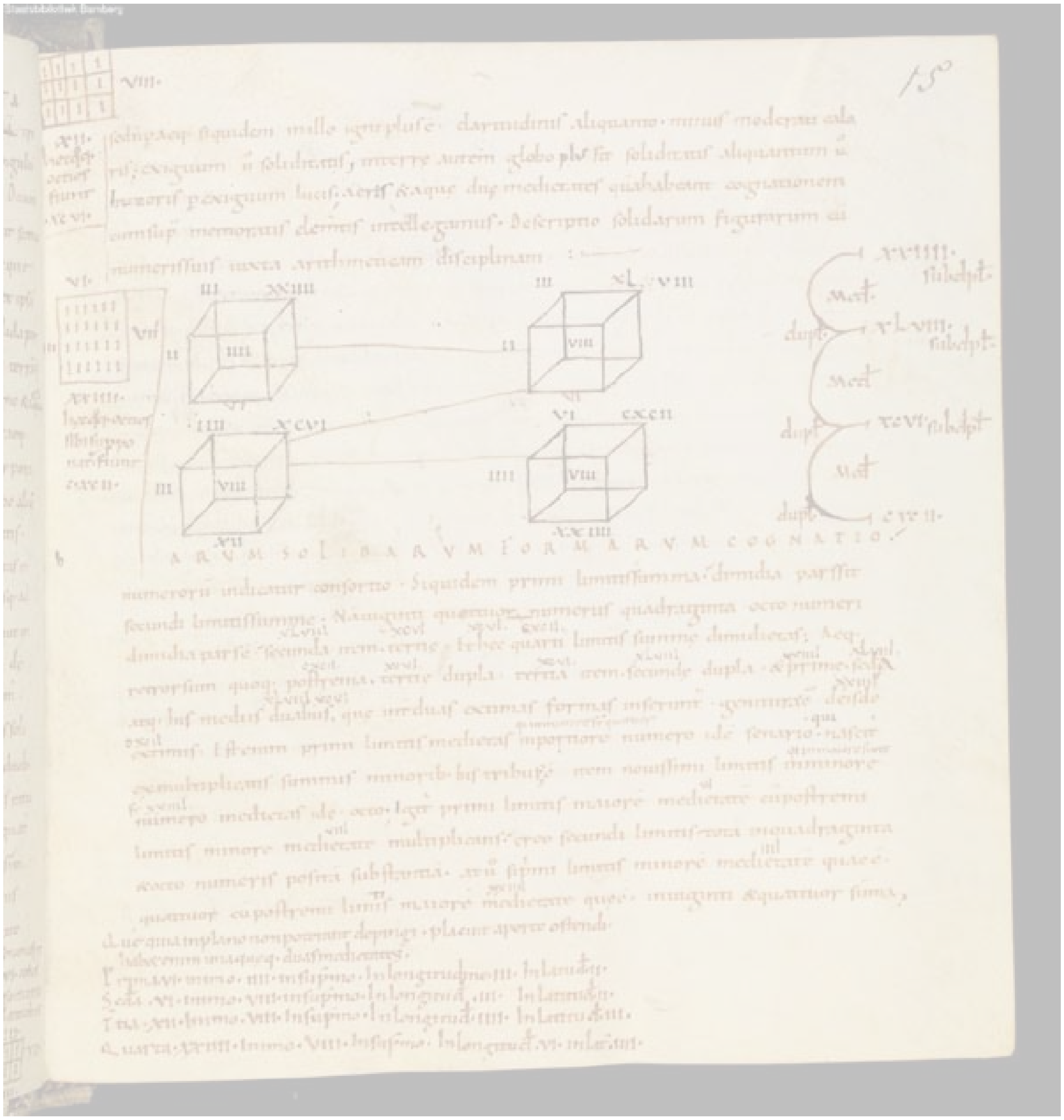
5.6 Naples, Biblioteca Nazionale Vittorio Emanuele III MS V.A.13, fol. 31r. Photo taken with permission of the Biblioteca Nazionale Vittorio Emanuele III



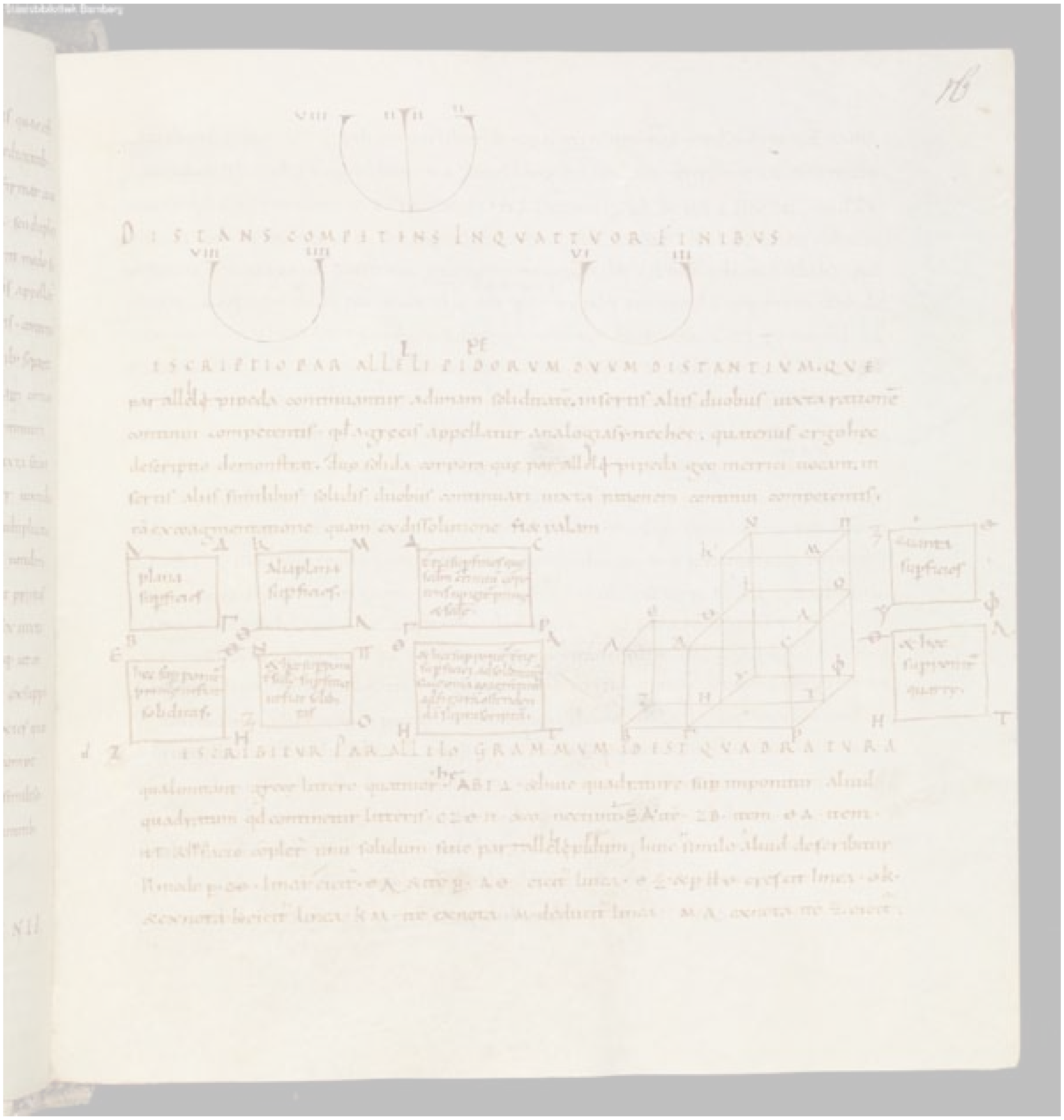
5.7 Naples, Biblioteca Nazionale Vittorio Emanuele III MS V.A.13, fol. 15v.
Photo taken with permission of the Biblioteca Nazionale Vittorio Emanuele III



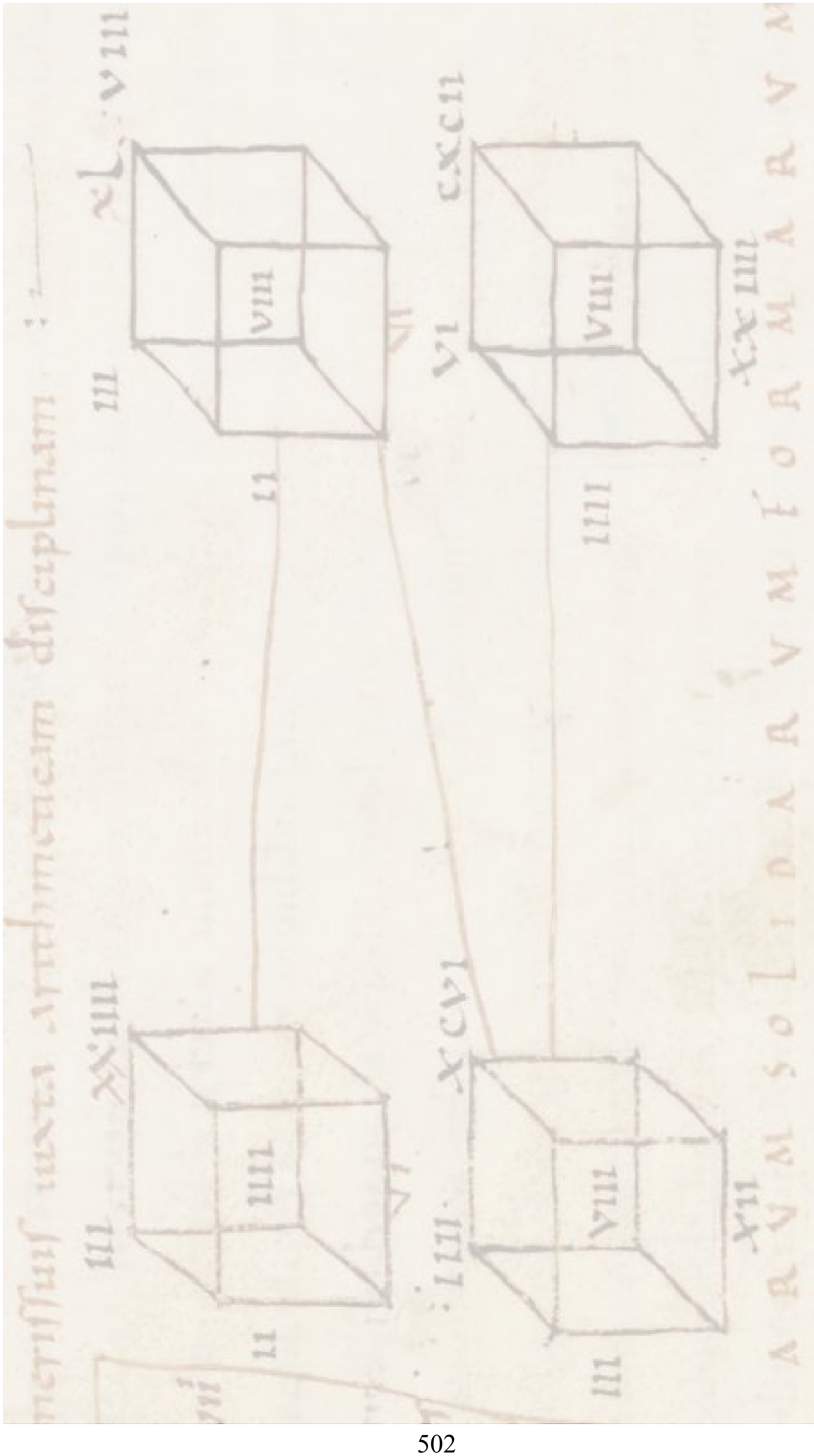
5.8 Naples, Biblioteca Nazionale Vittorio Emanuele III MS V.A.13, fol. 38v.
Photo taken with permission of the Biblioteca Nazionale Vittorio Emanuele III



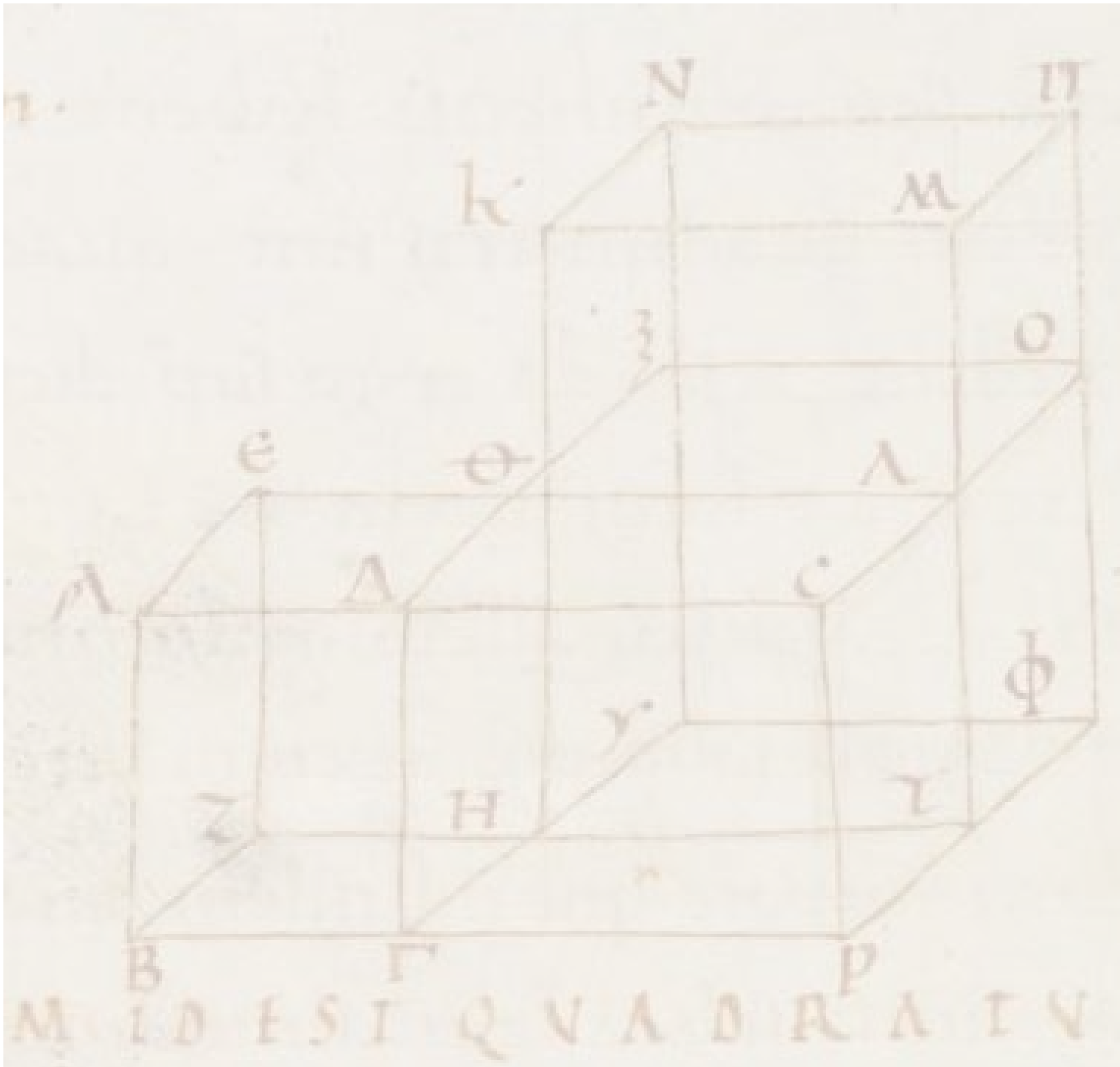
5.9 Bamberg, Staatsbibliothek Cod. Msc.Class.18, fol. 15r.



5.10 Bamberg, Staatsbibliothek Cod. Msc.Class.18, fol. 16r.

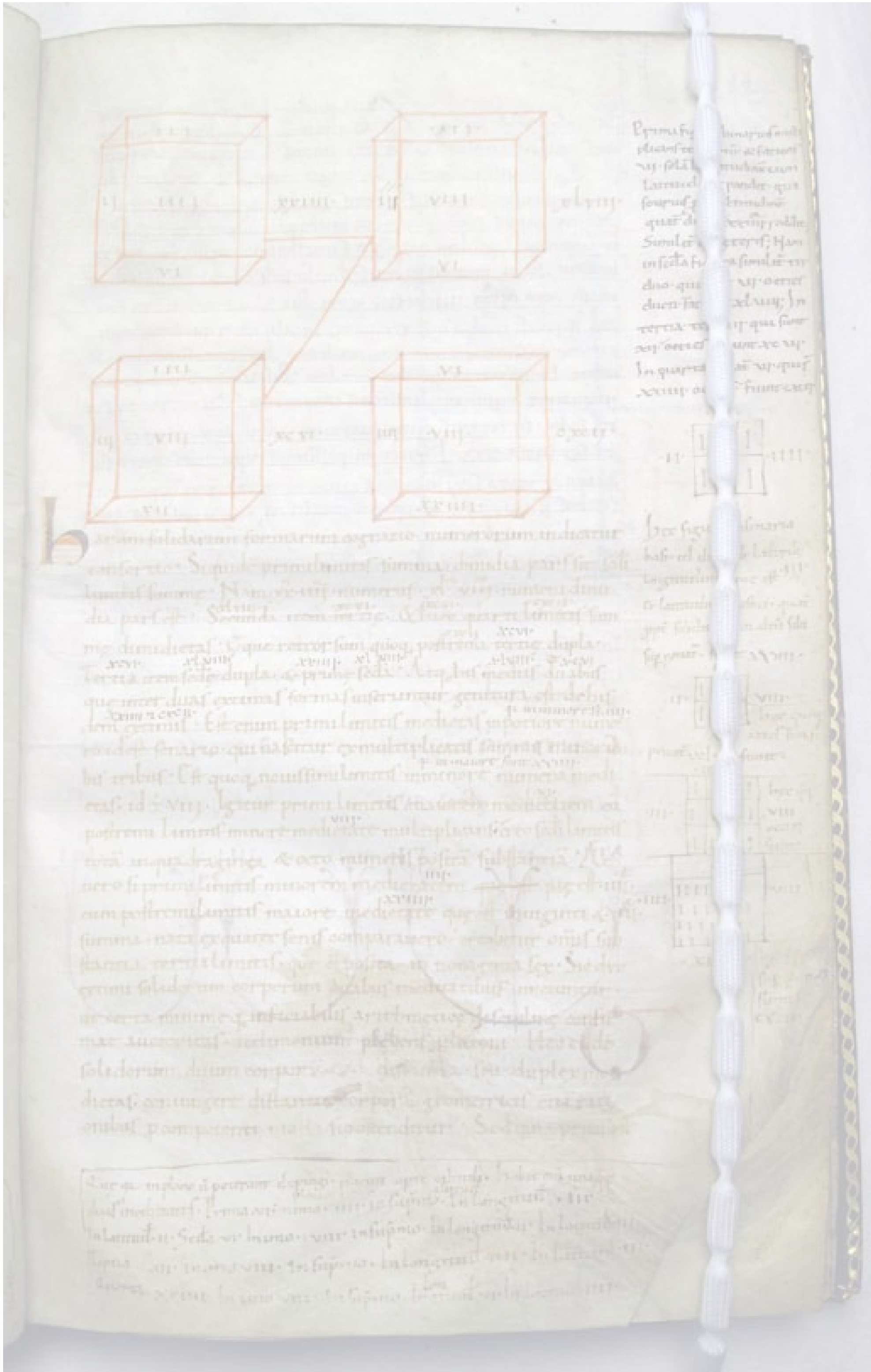


5.11 Bamberg, Staatsbibliothek Cod. Msc.Class.18, fol. 15r, detail. © Staatsbibliothek Bamberg

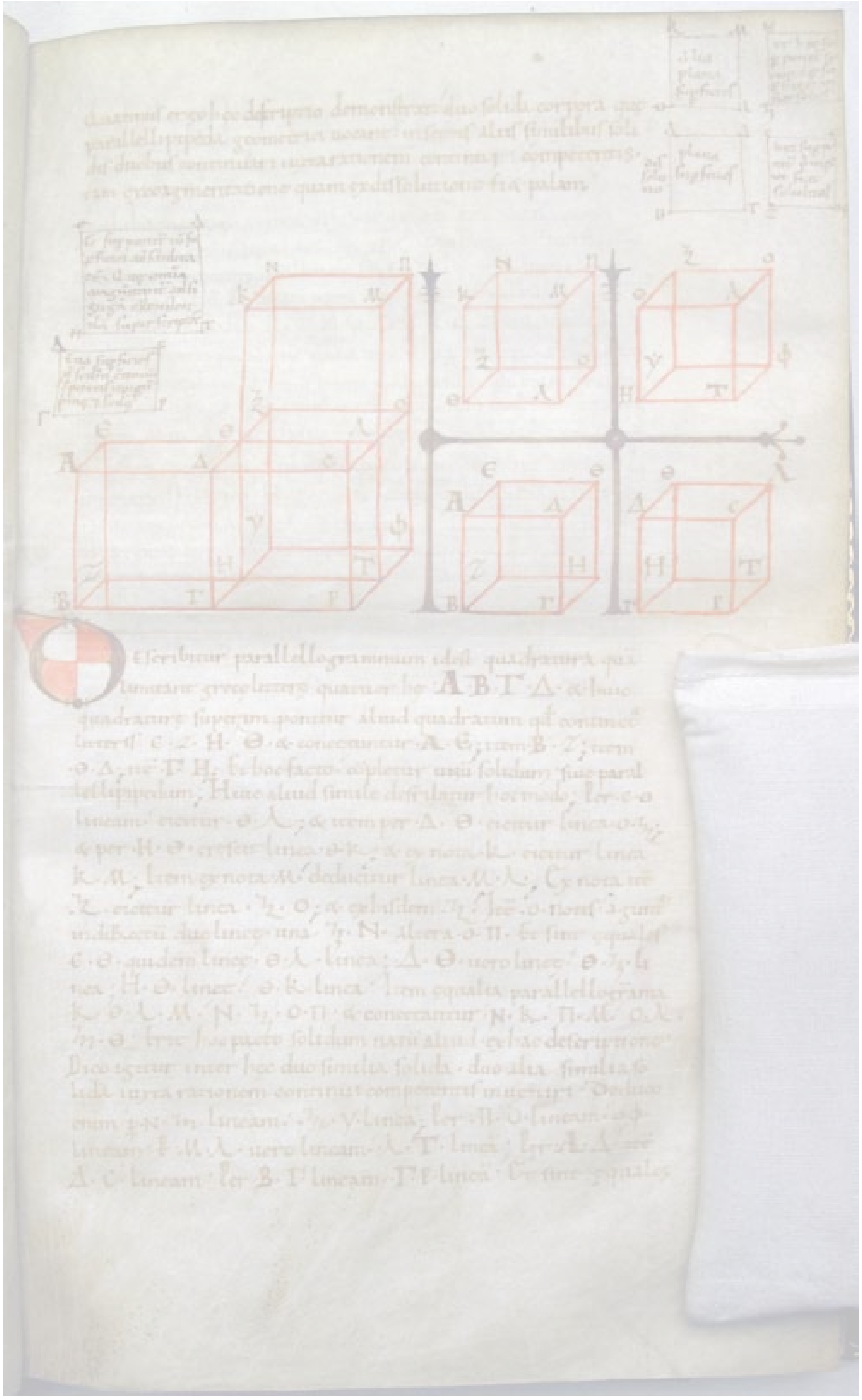


5.12 Bamberg, Staatsbibliothek Cod. Msc.Class.18, fol. 16r, detail.

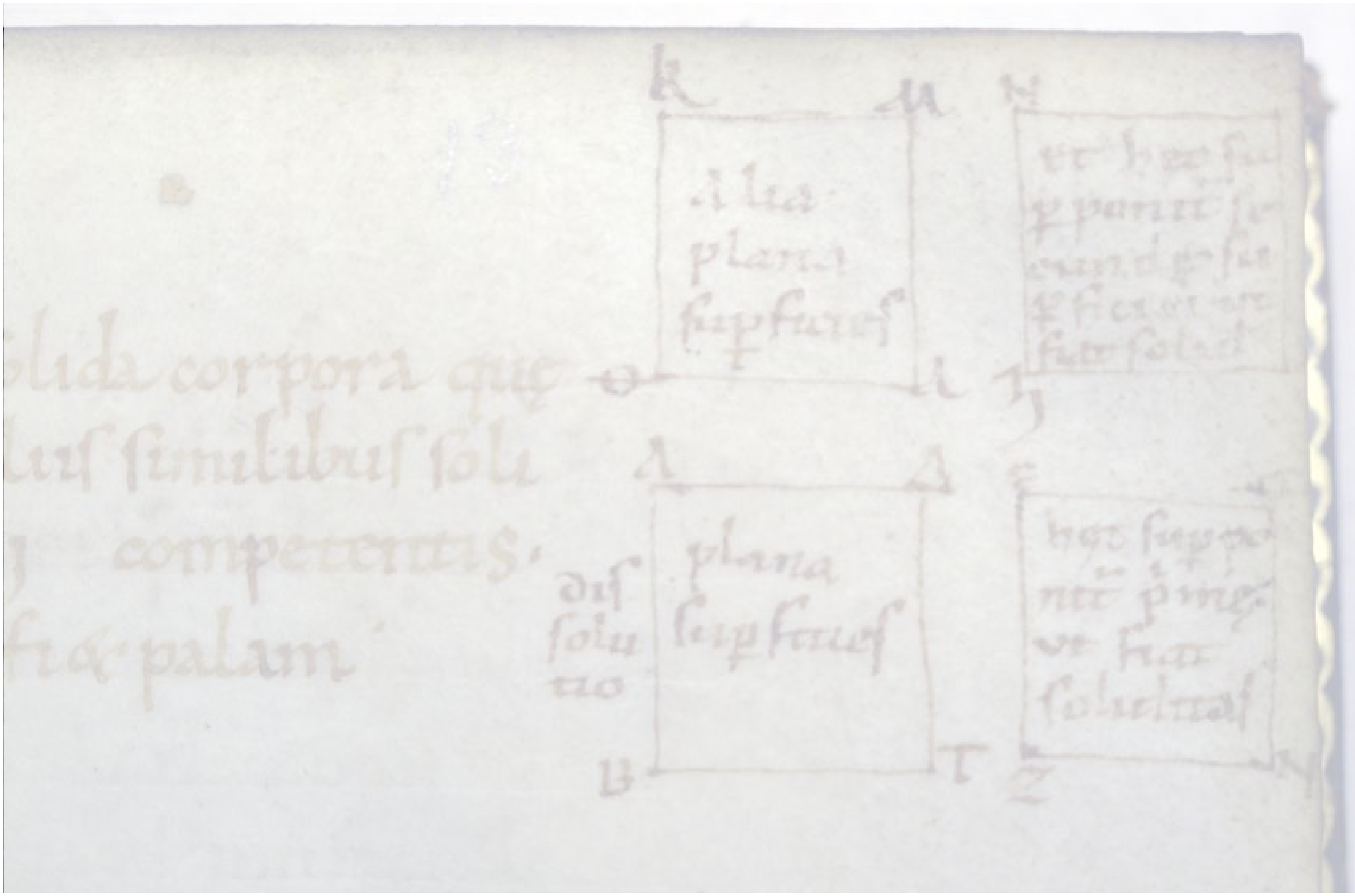
5.13 Paris, Bibliothèque nationale de France lat. MS 2164, fol. 28v, detail.



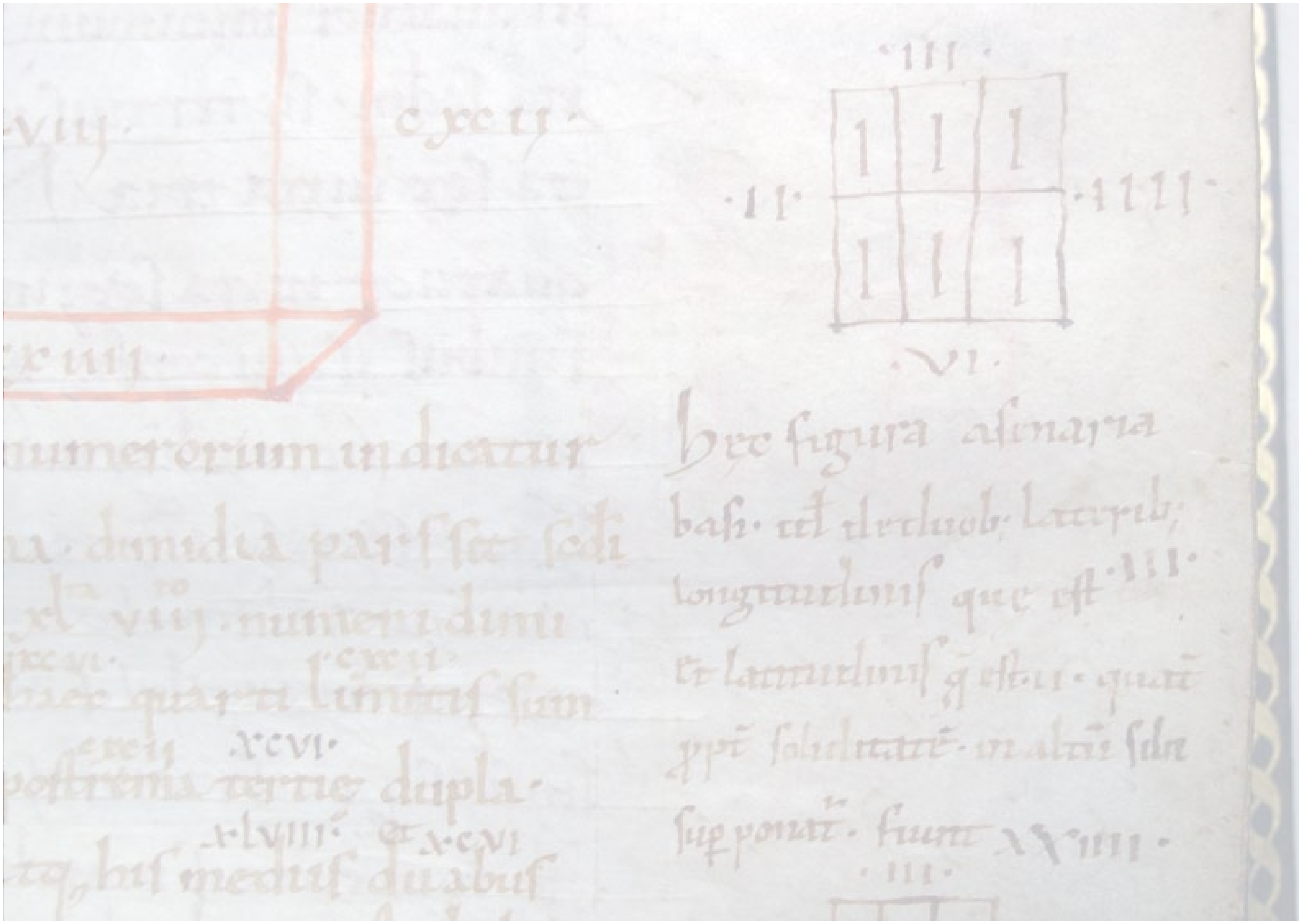
5.14 Brussels, Bibliothèque royale de Belgique MS 9625-26, fol. 12r.
Photo taken with permission of the Bibliothèque royale de Belgique



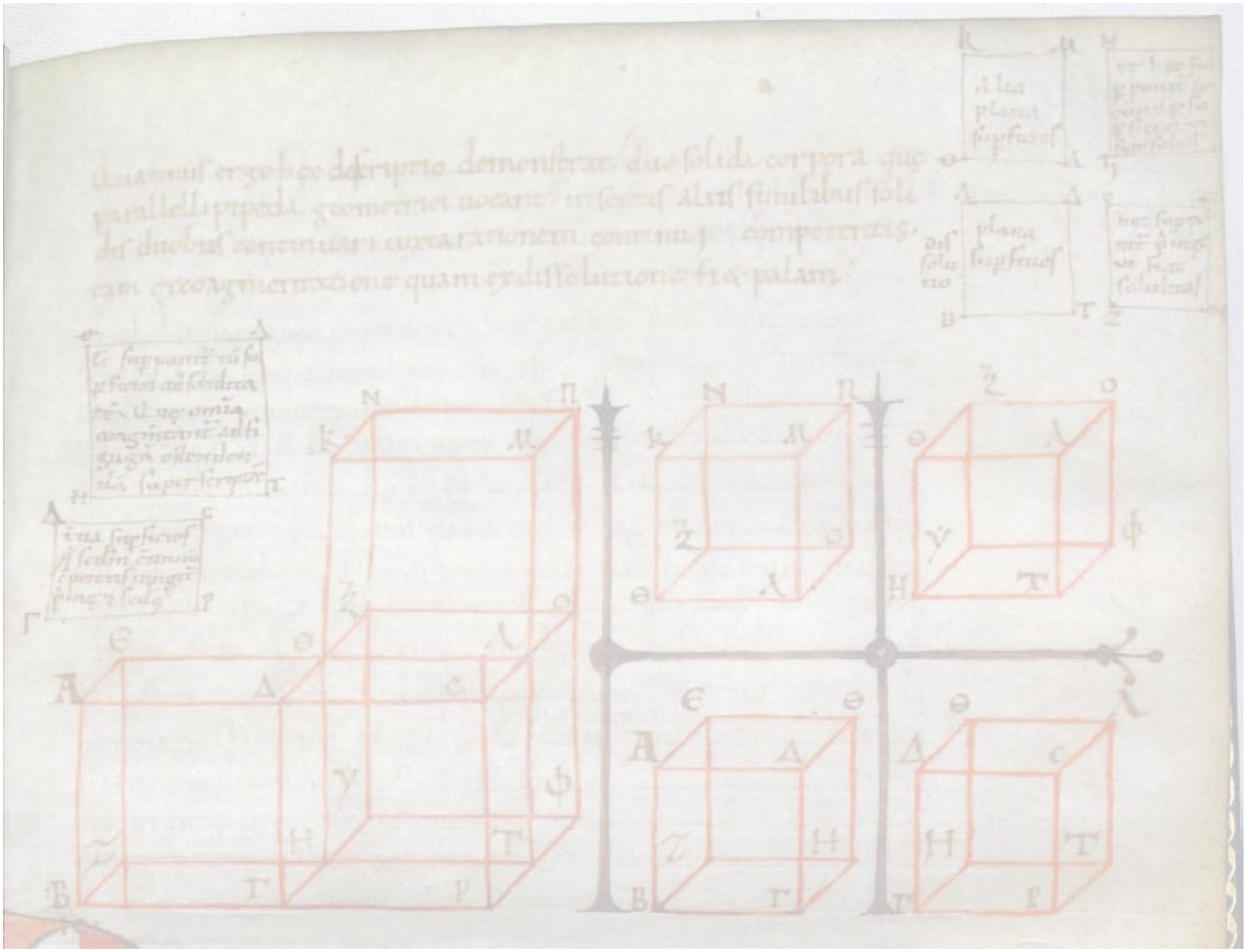
5.15 Brussels, Bibliothèque royale de Belgique MS 9625-26, fol. 19r.
Photo taken with permission of the Bibliothèque royale de Belgique



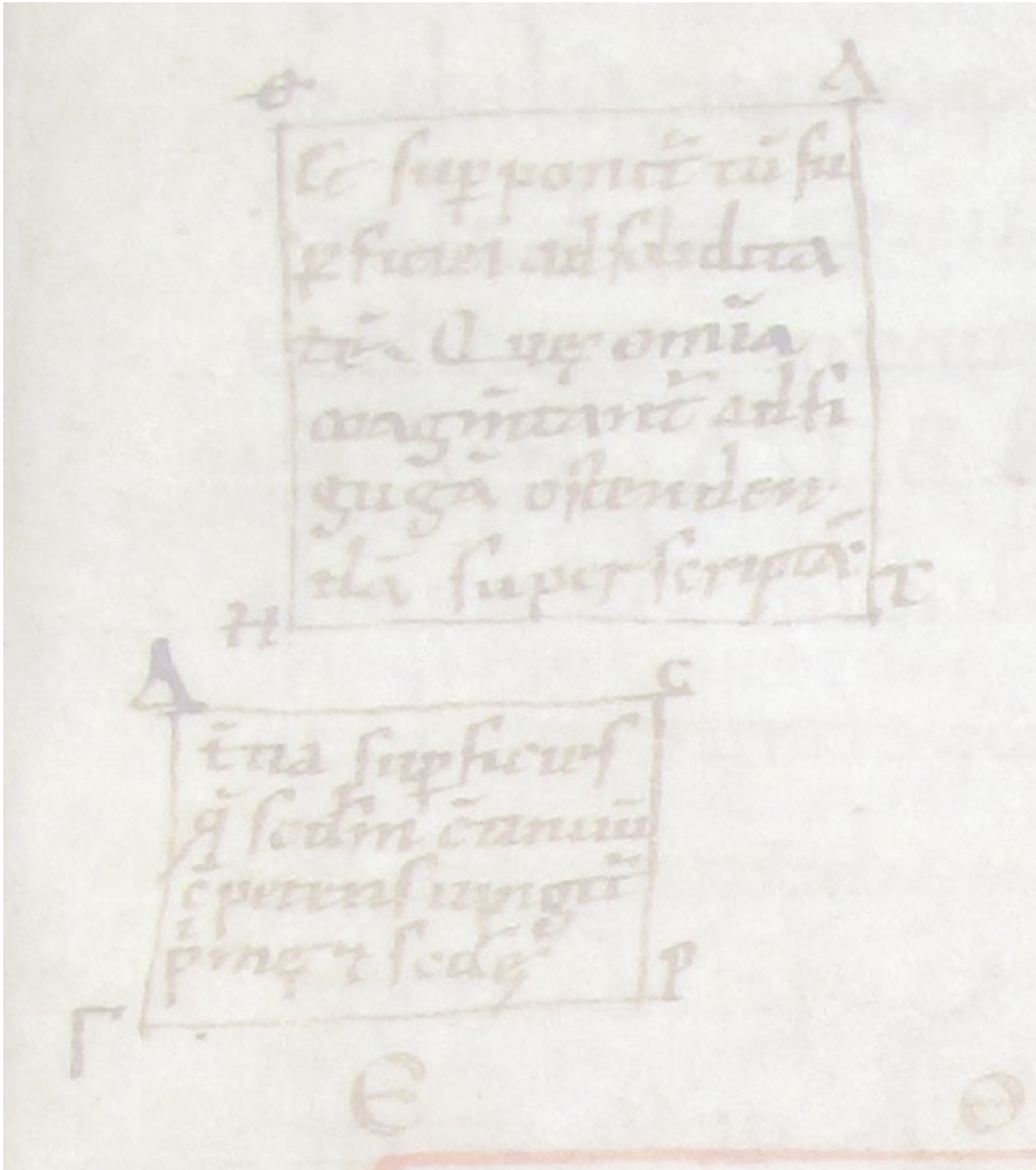
5.16 Brussels, Bibliothèque royale de BelgiqueMS 9625-26, fol. 19r, detail. Photo taken with permission of the Bibliothèque royale de Belgique



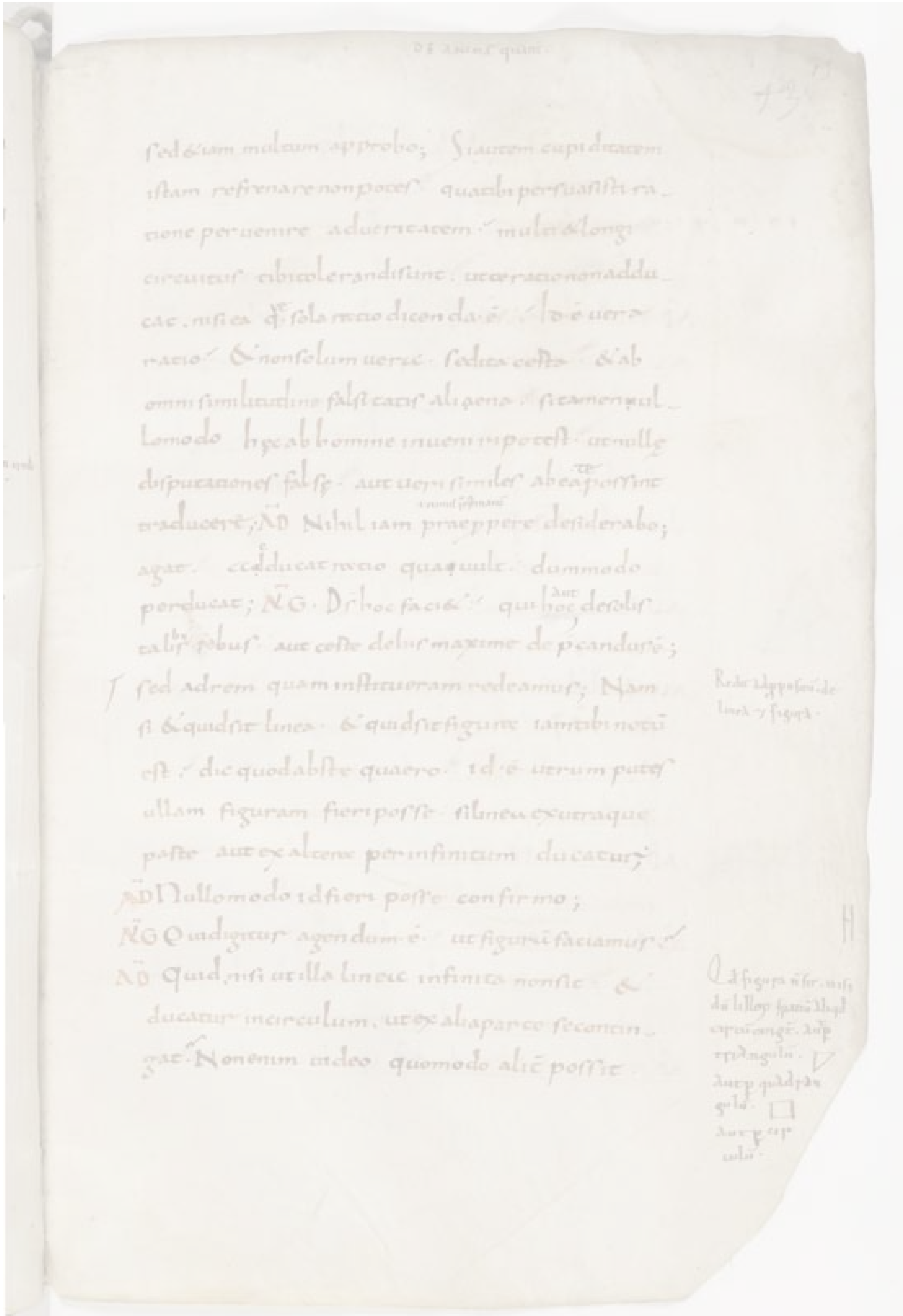
5.17 Brussels, Bibliothèque royale de Belgique MS 9625-26, fol. 12r, detail. Photo taken with permission of the Bibliothèque royale de Belgique



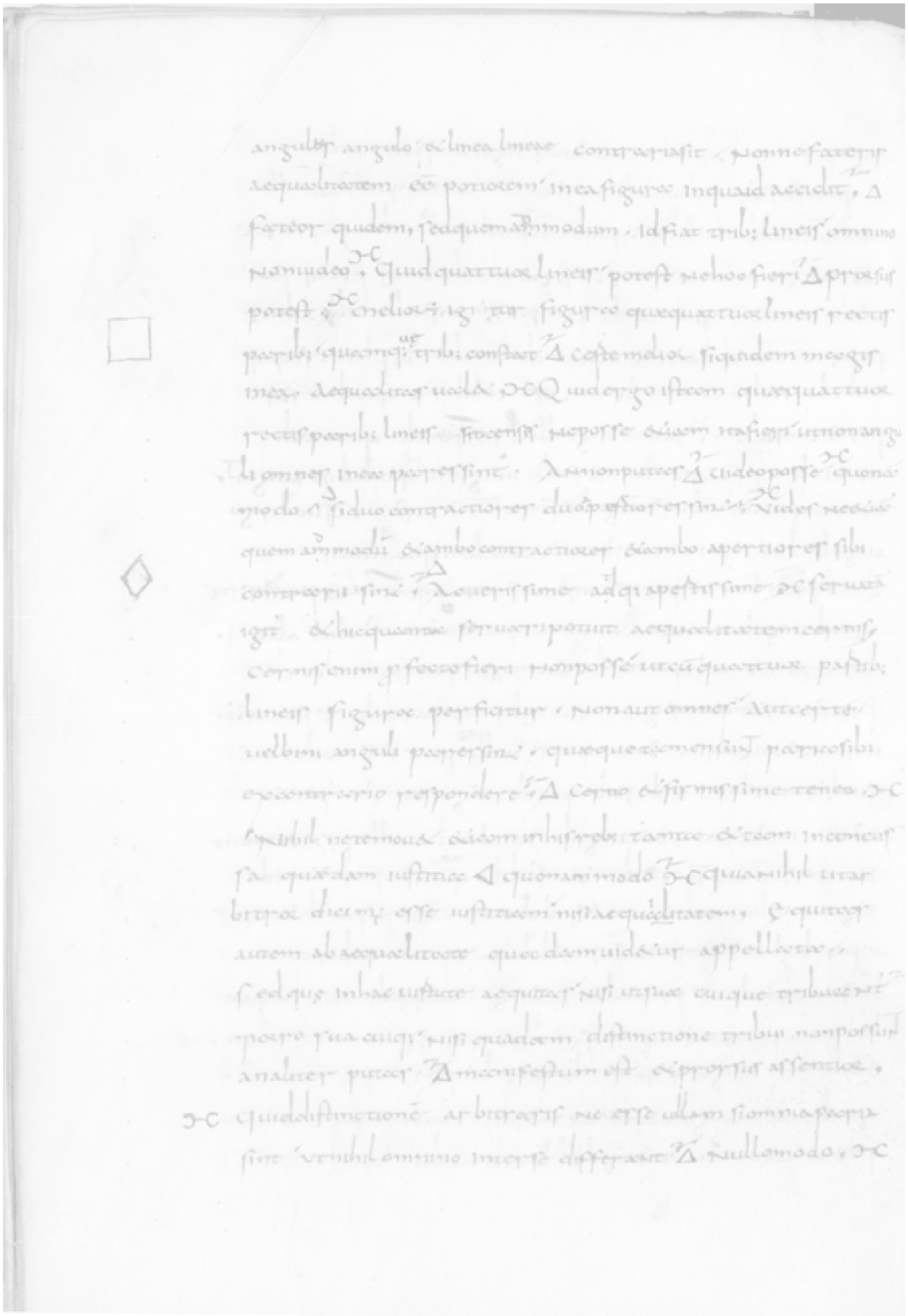
5.18 Brussels, Bibliothèque royale de Belgique MS 9625-26, fol. 19r, detail. Photo taken with permission of the Bibliothèque royale de Belgique



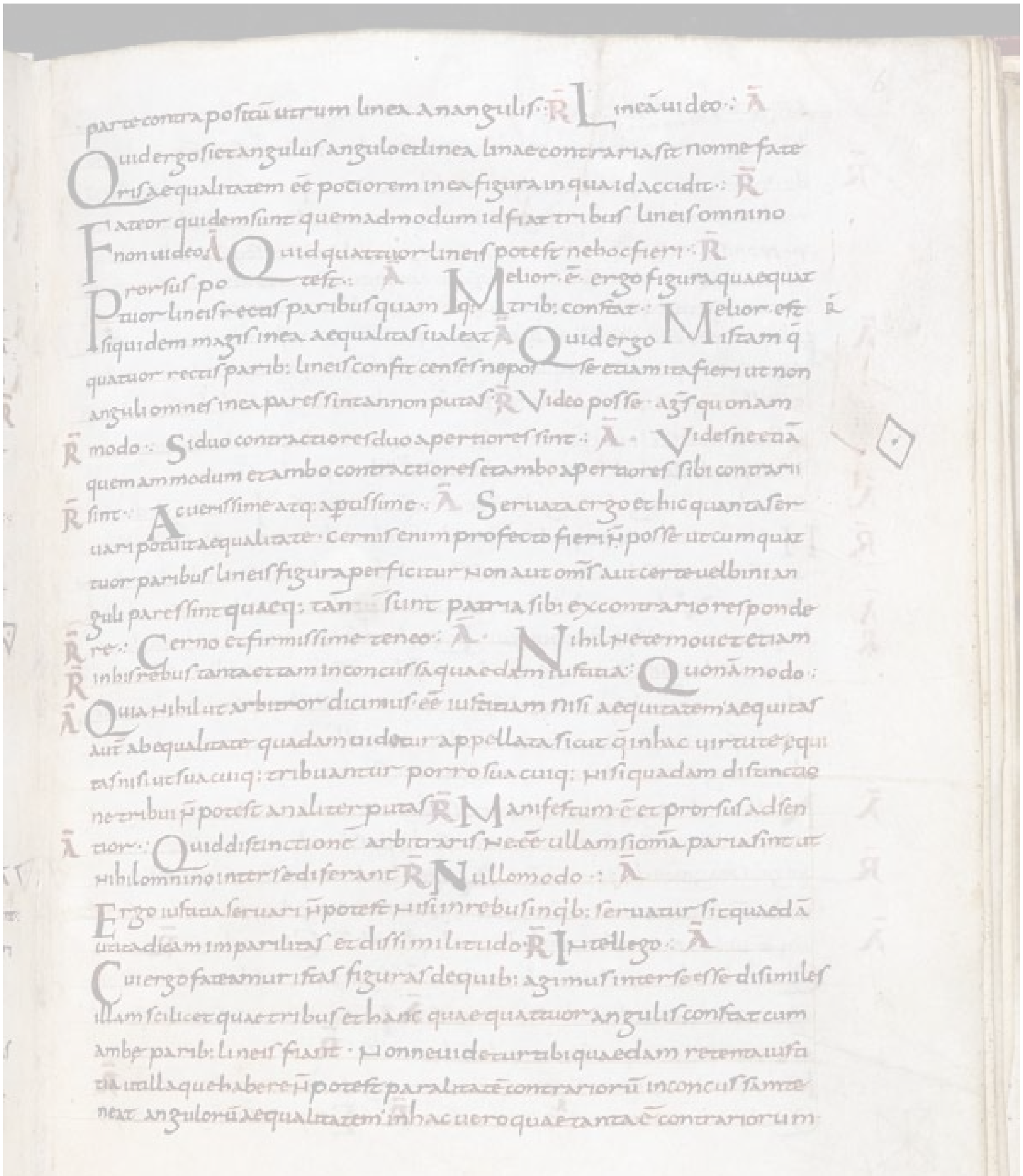
5.19 Brussels, Bibliothèque royale de Belgique MS 9625-26, fol. 19r, detail.
Photo taken with permission of the Bibliothèque royale de Belgique



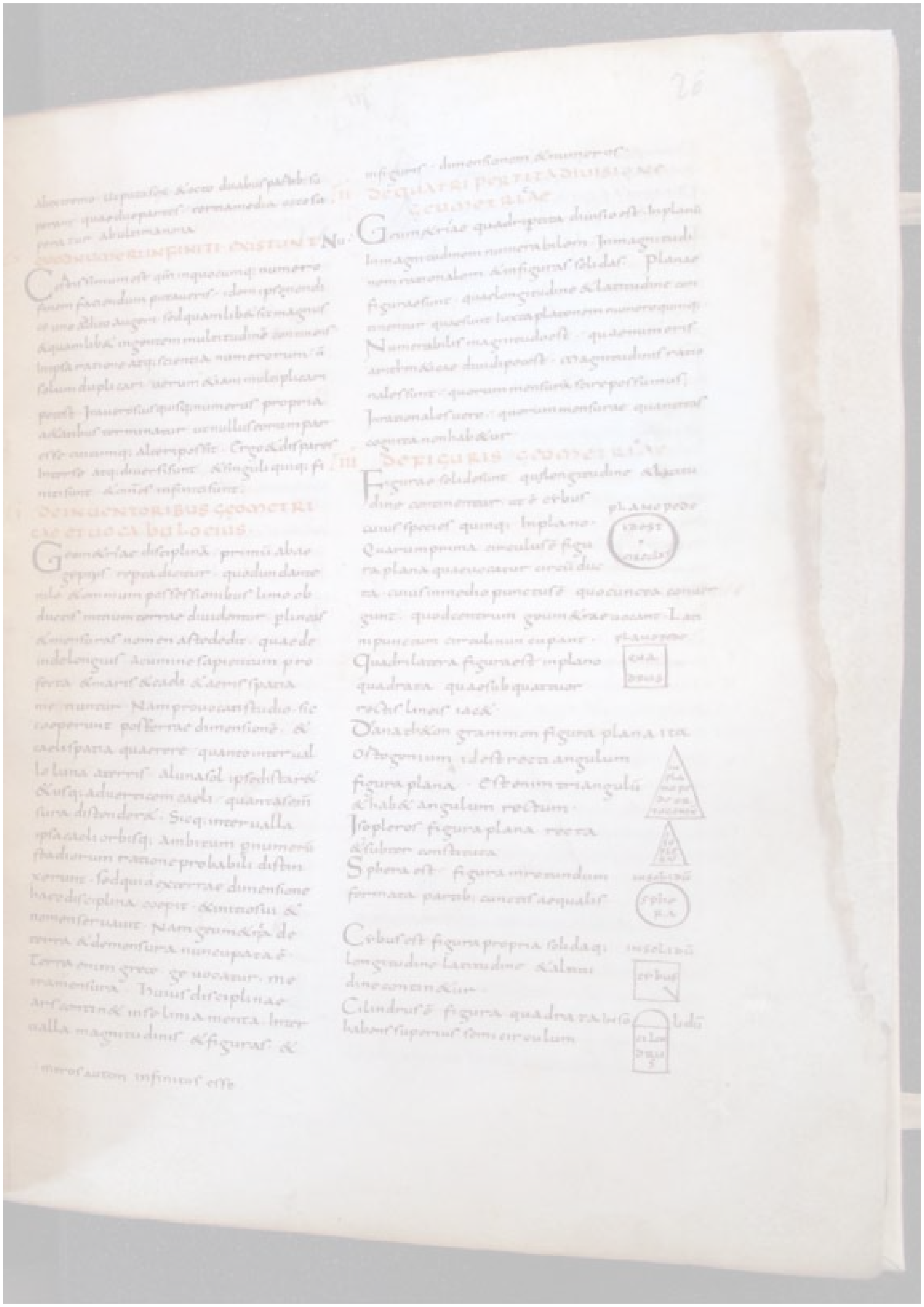
5.20 Valenciennes, Bibliothèque municipale MS 163 (155), fol. 13r.



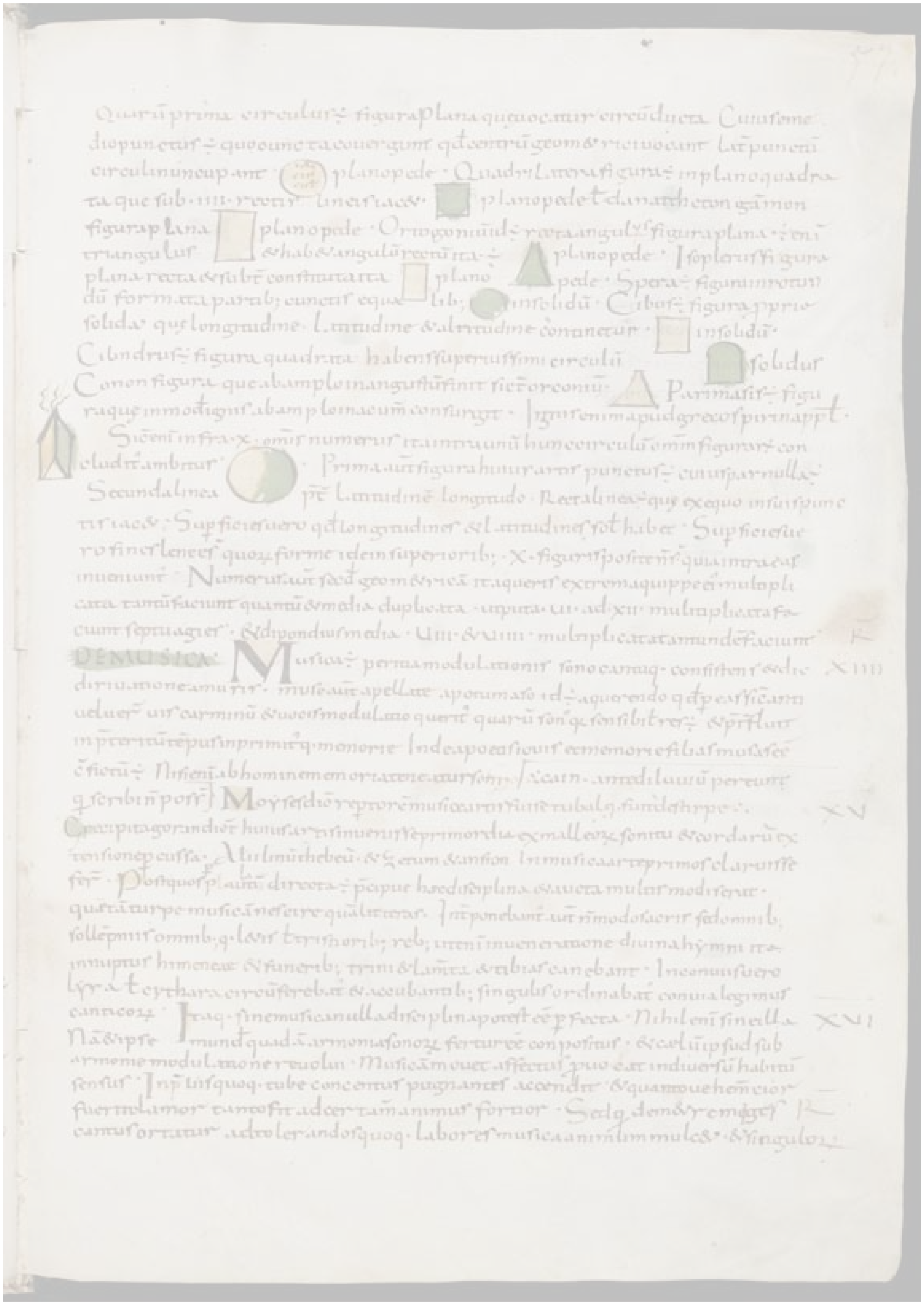
5.21 Paris, Bibliothèque nationale de France lat. MS 13369, fol. 151v.



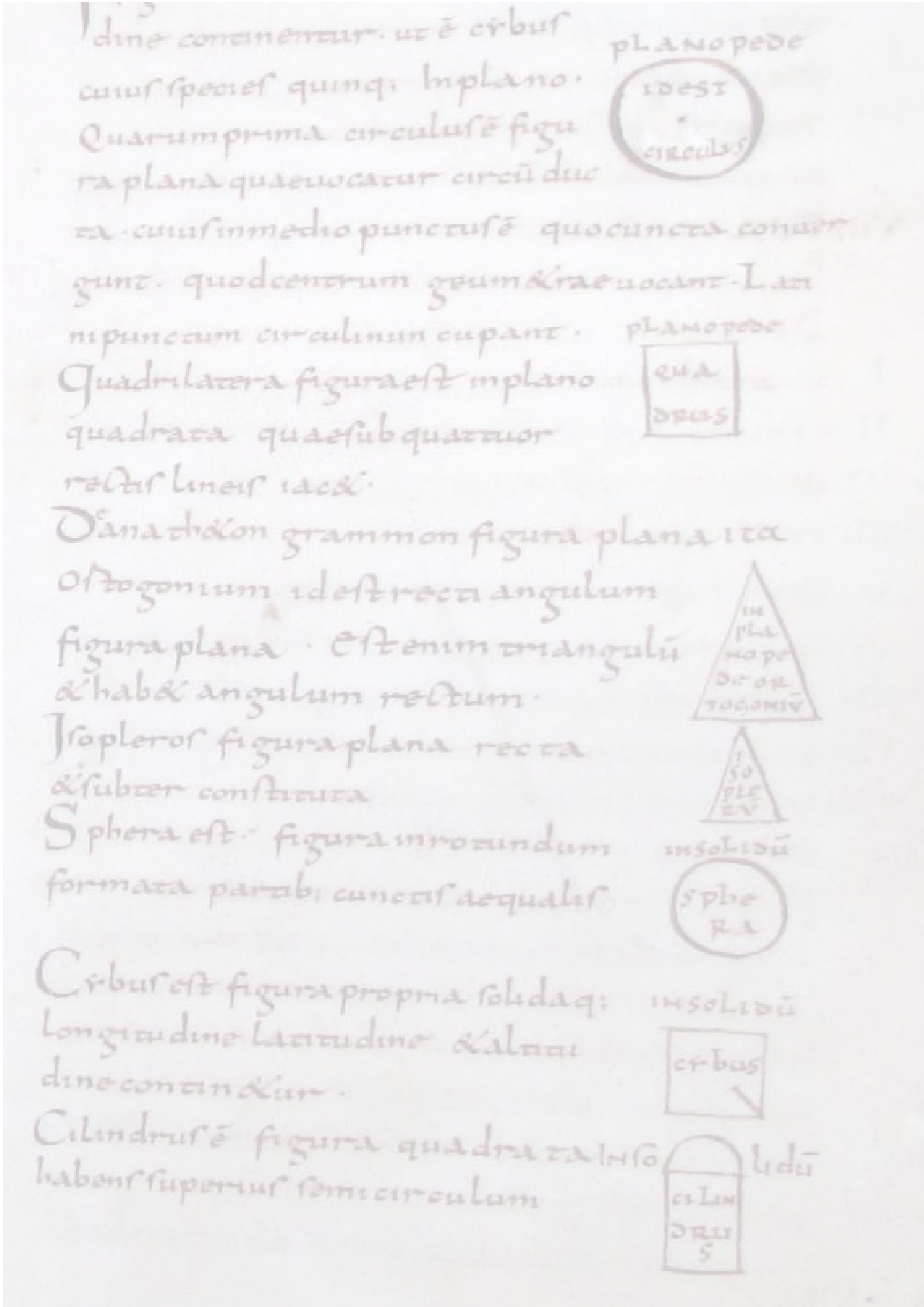
5.22 London, British Library Harley MS 3012, fol. 6r.



5.23 Bern, Burgerbibliothek Cod. 101, fol. 26r.
Photo taken with permission of the Burgerbibliothek Bern



5.24 St. Gall, Stiftsbibliothek Cod. Sang. 237r.



5.25 Bern, Burgerbibliothek Cod. 101, fol. 26r, detail.
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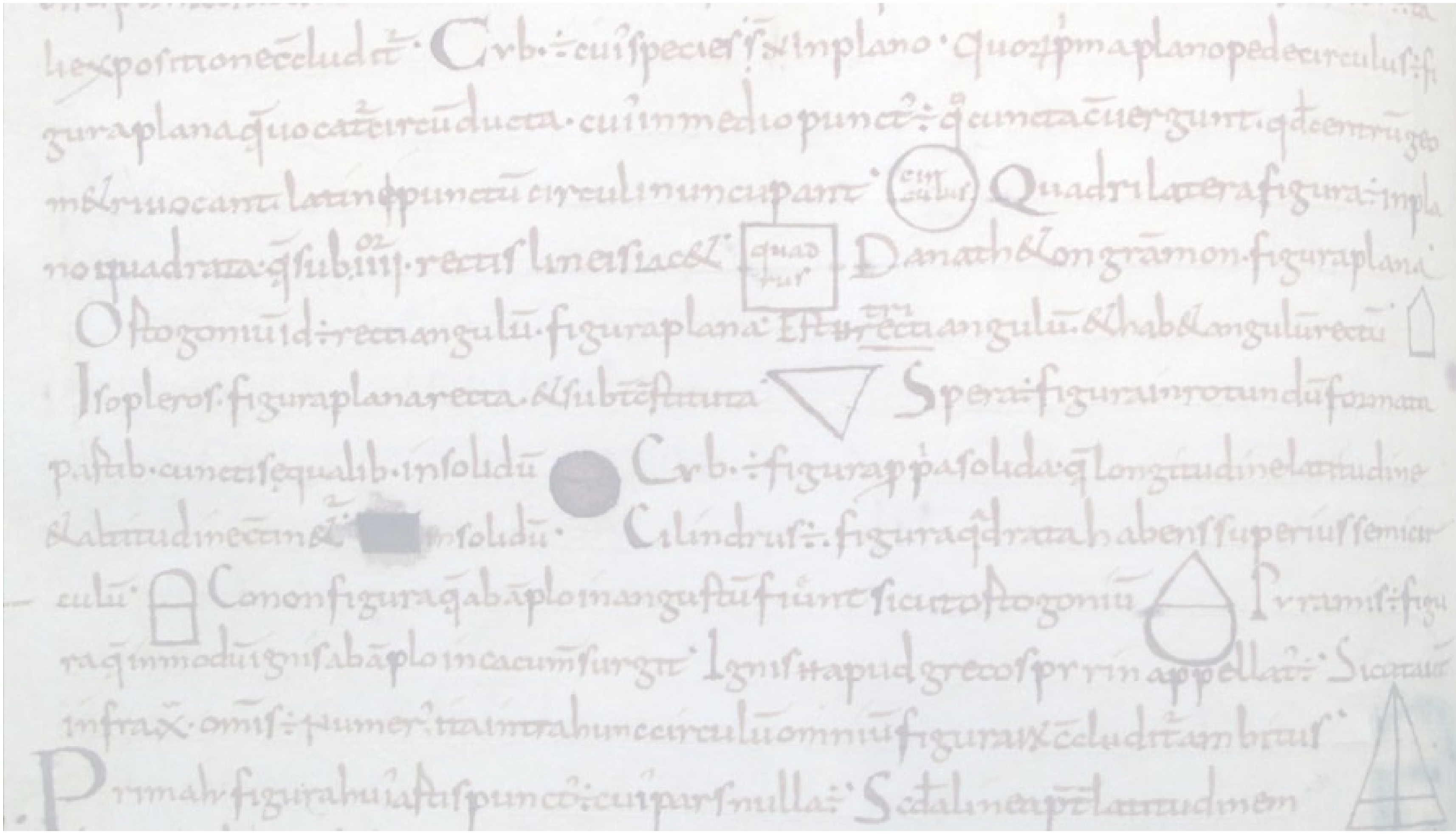
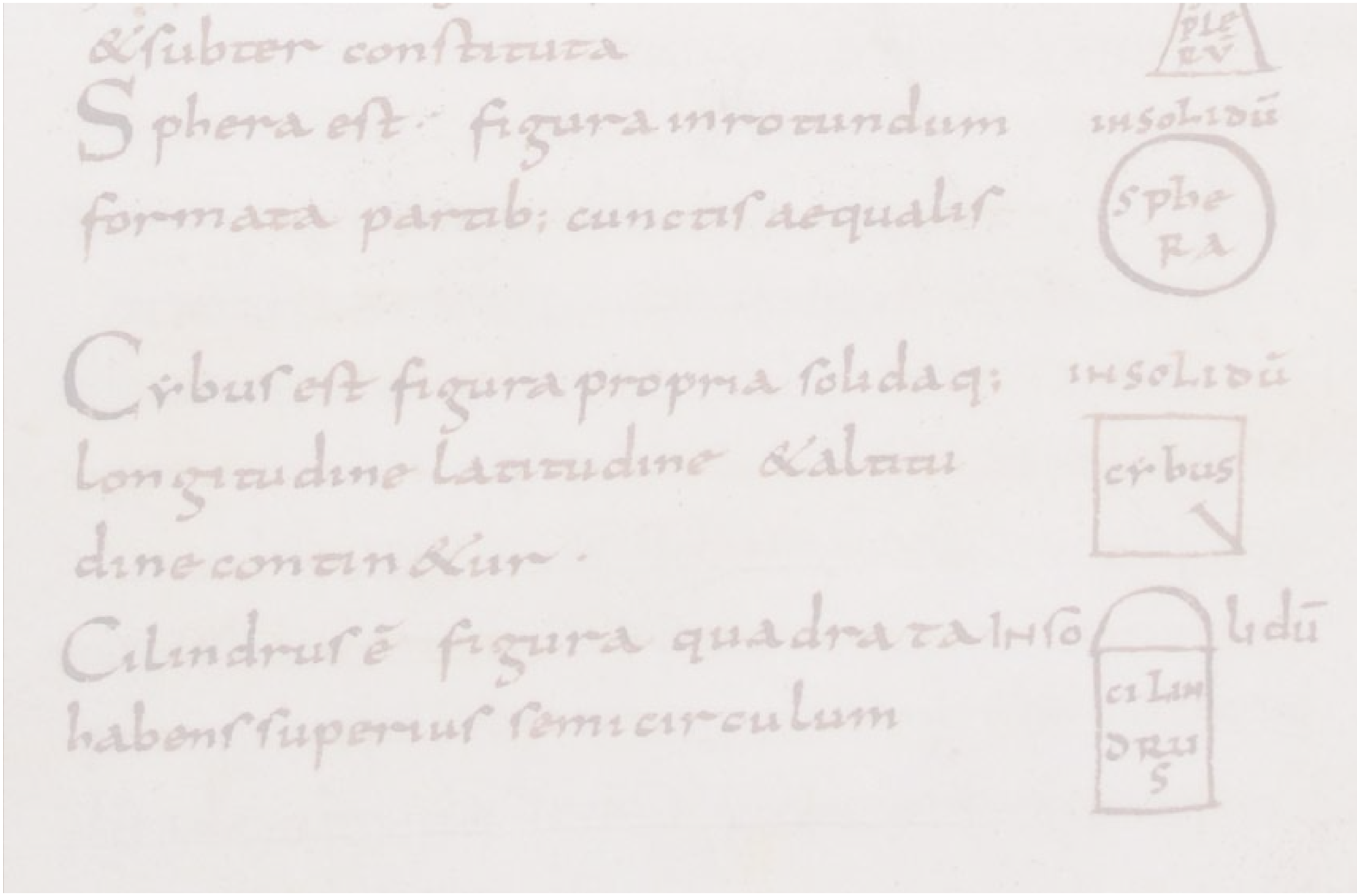
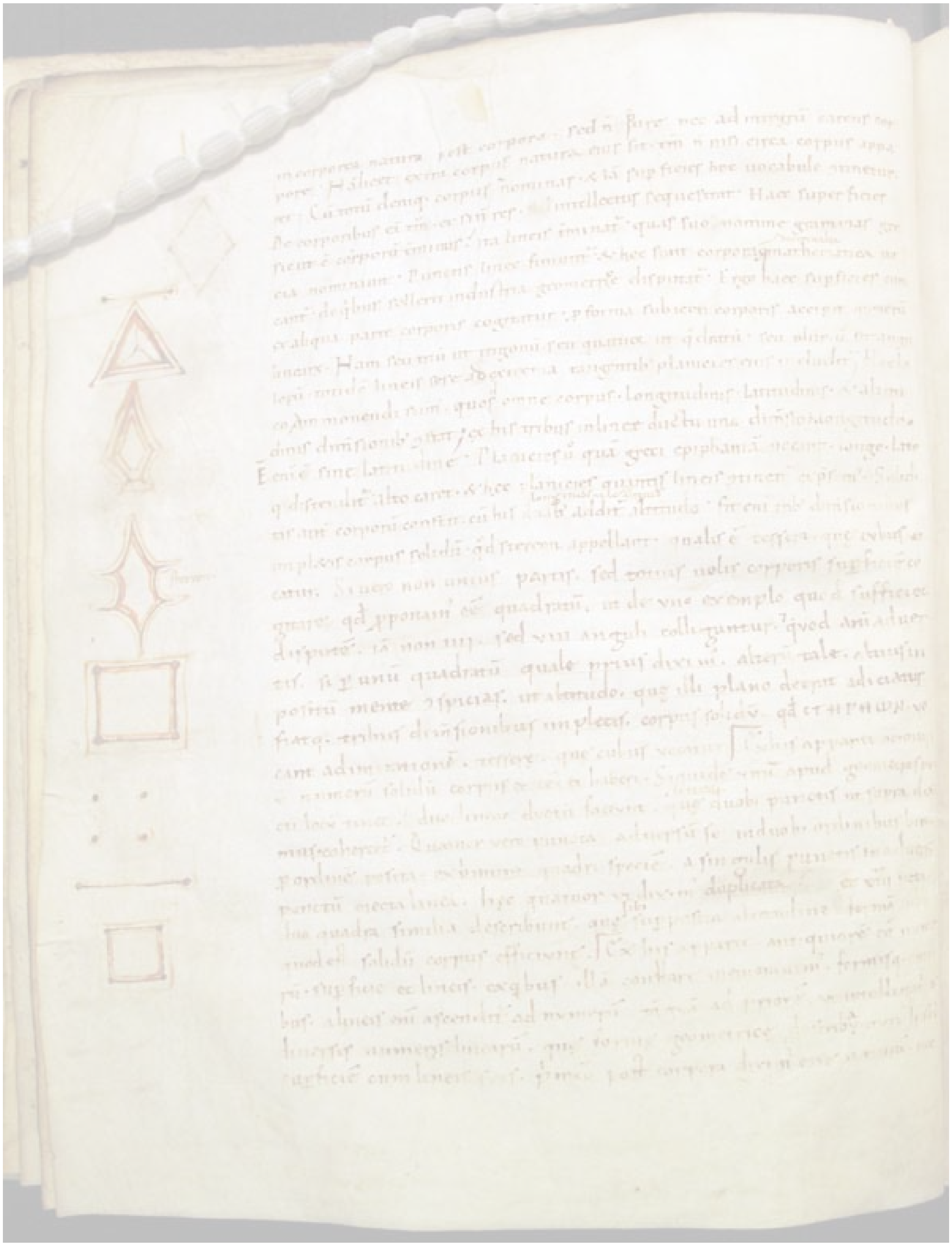


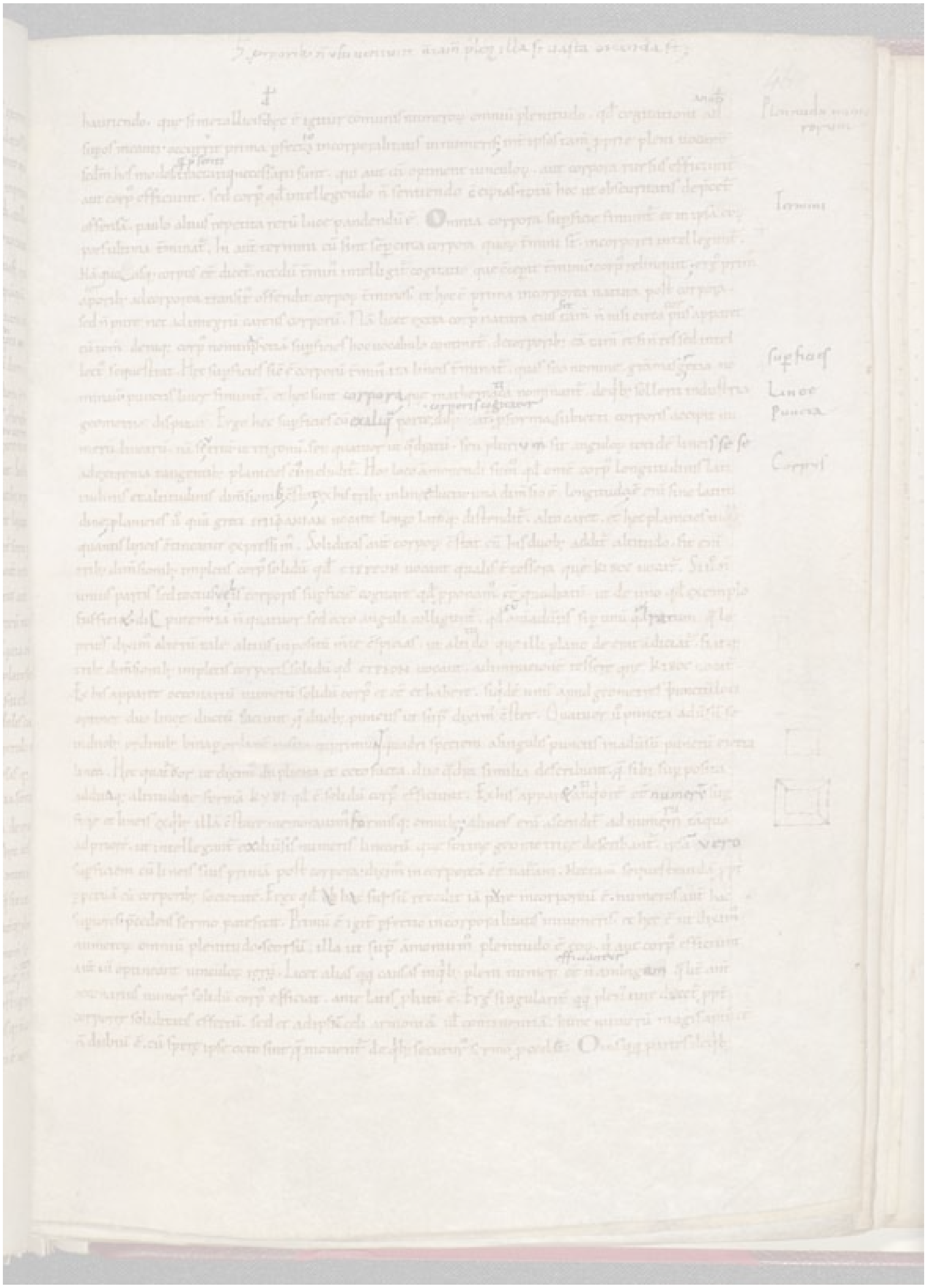
5.26 Naples, Biblioteca Nazionale Vittorio Emanuele III MS V.A.13, fol. 15v, detail. Photo taken with permission of the Biblioteca Nazionale Vittorio Emanuele III


&subter constituta
Sphera est· figura inrotundum
formata partib; cunctis aequalis
insolidū
sphe
RA
Cybus est figura propria solidaq;
longitudine latitudine &altitu
dine contin&ur·
insolidū
cybus
Cilindrus ē figura quadrata inso lidū
habens superius semicirculum
ci lin
dru
s

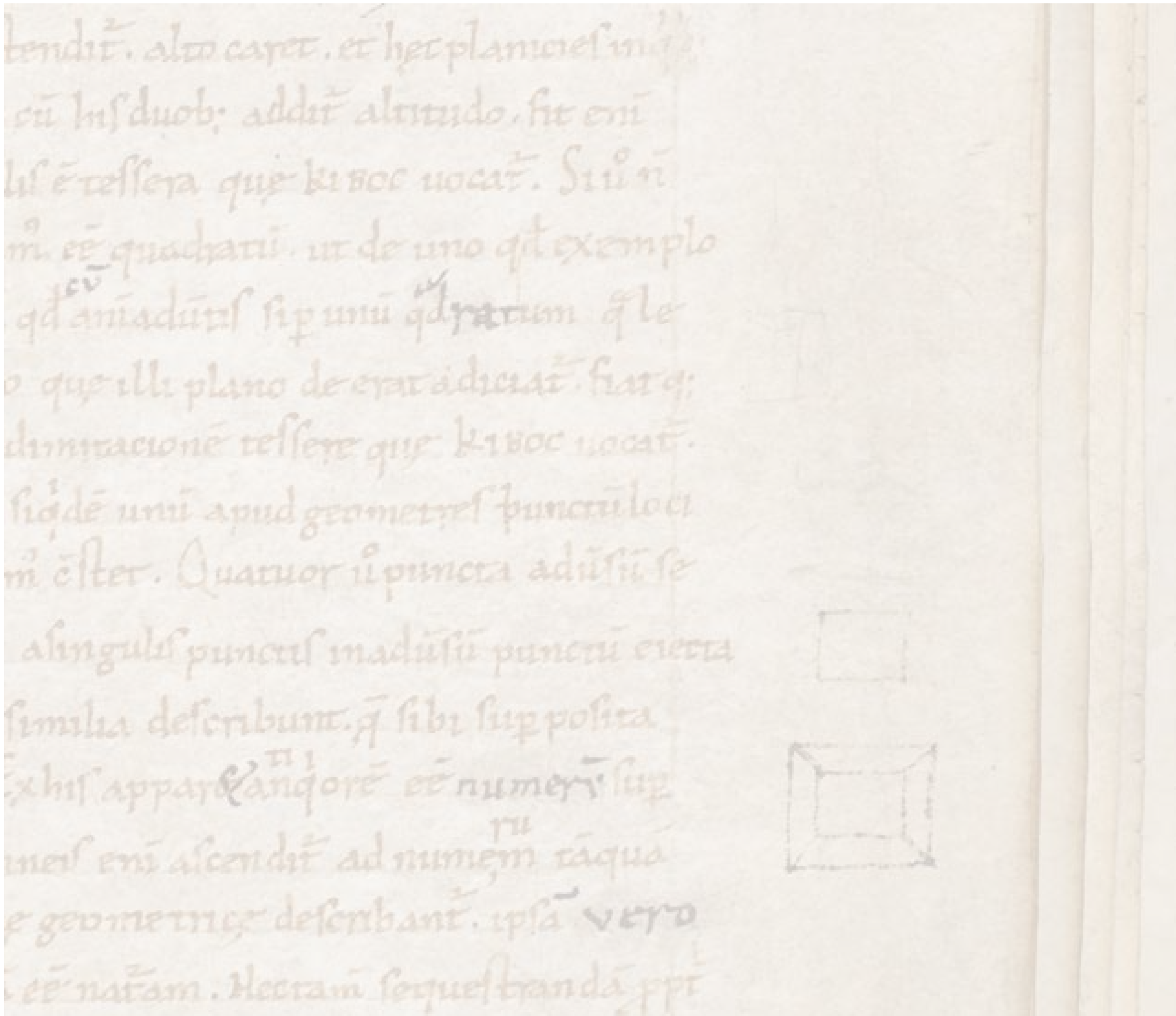
5.27 Bern, Burgerbibliothek Cod. 101, fol. 26r, detail. Photo taken with permission of the Burgerbibliothek Bern



5.28 Oxford, Bodleian Library Auct. MS T.2.27, fol. 14v.
Photo taken with permission of the Bodleian Library, Oxford University

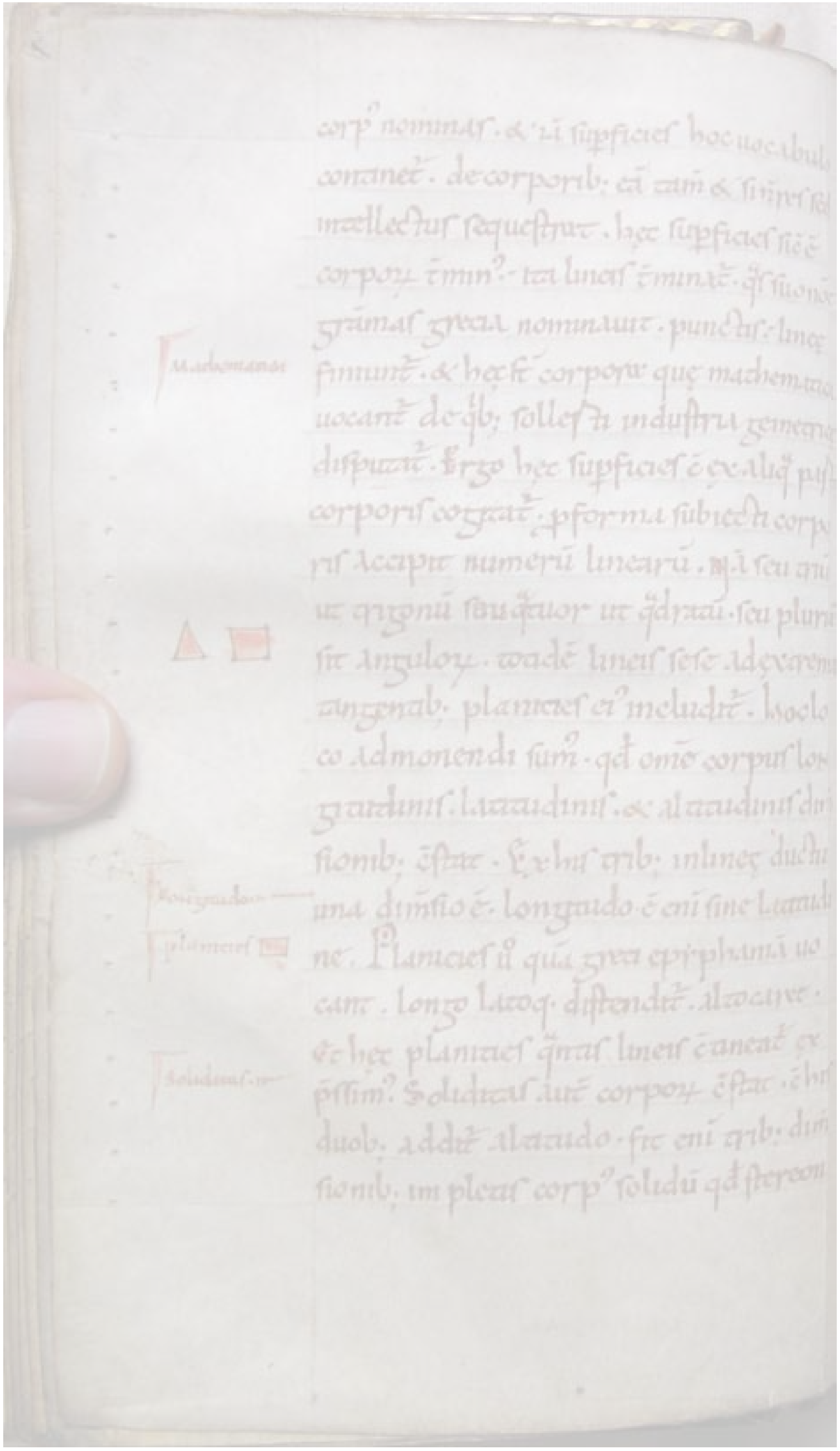


5.29 London, British Library Harley MS 2772, fol. 46r.

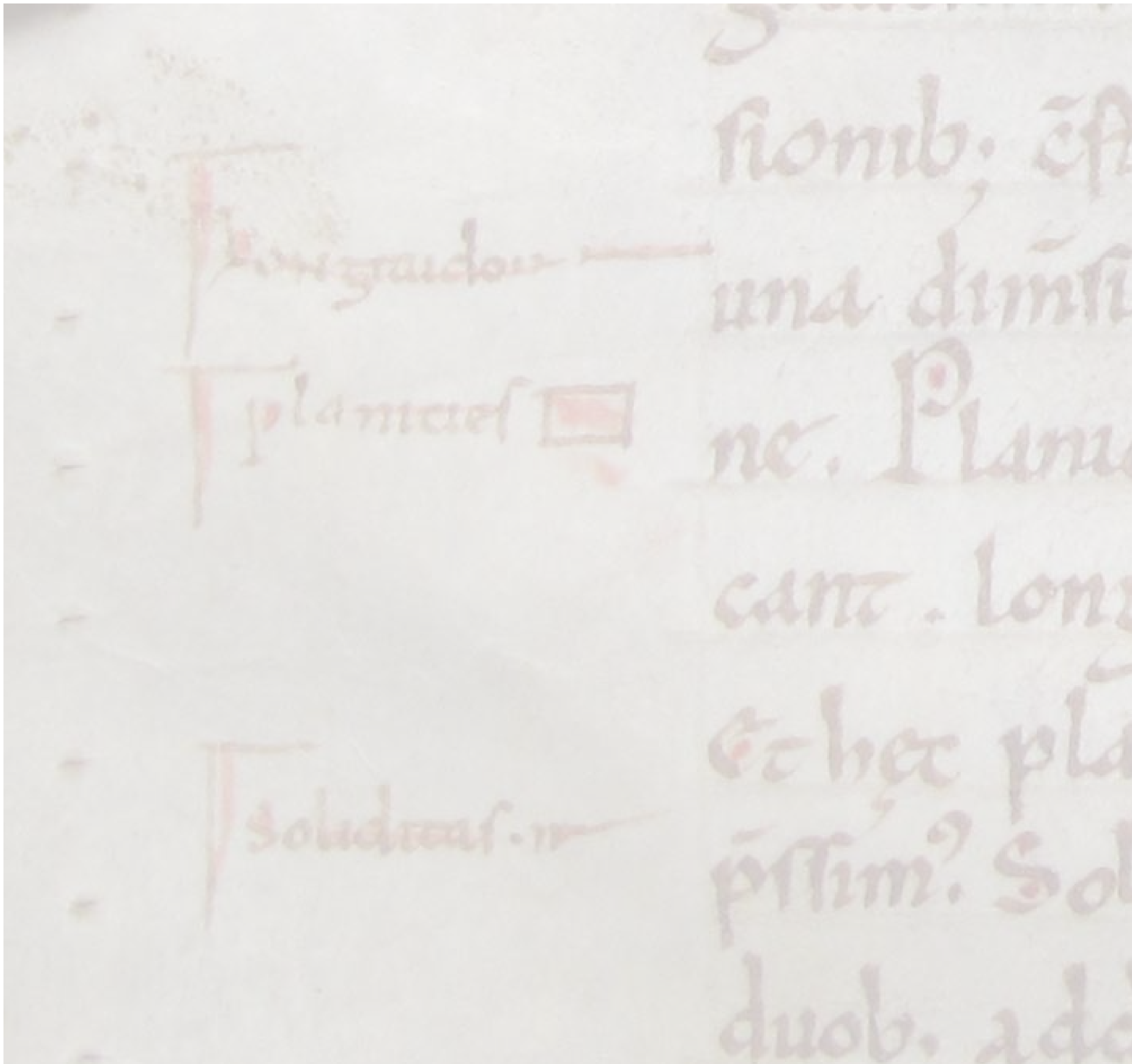


5.30 London, British Library Harley MS 2772, fol. 46r, detail. © The British Library

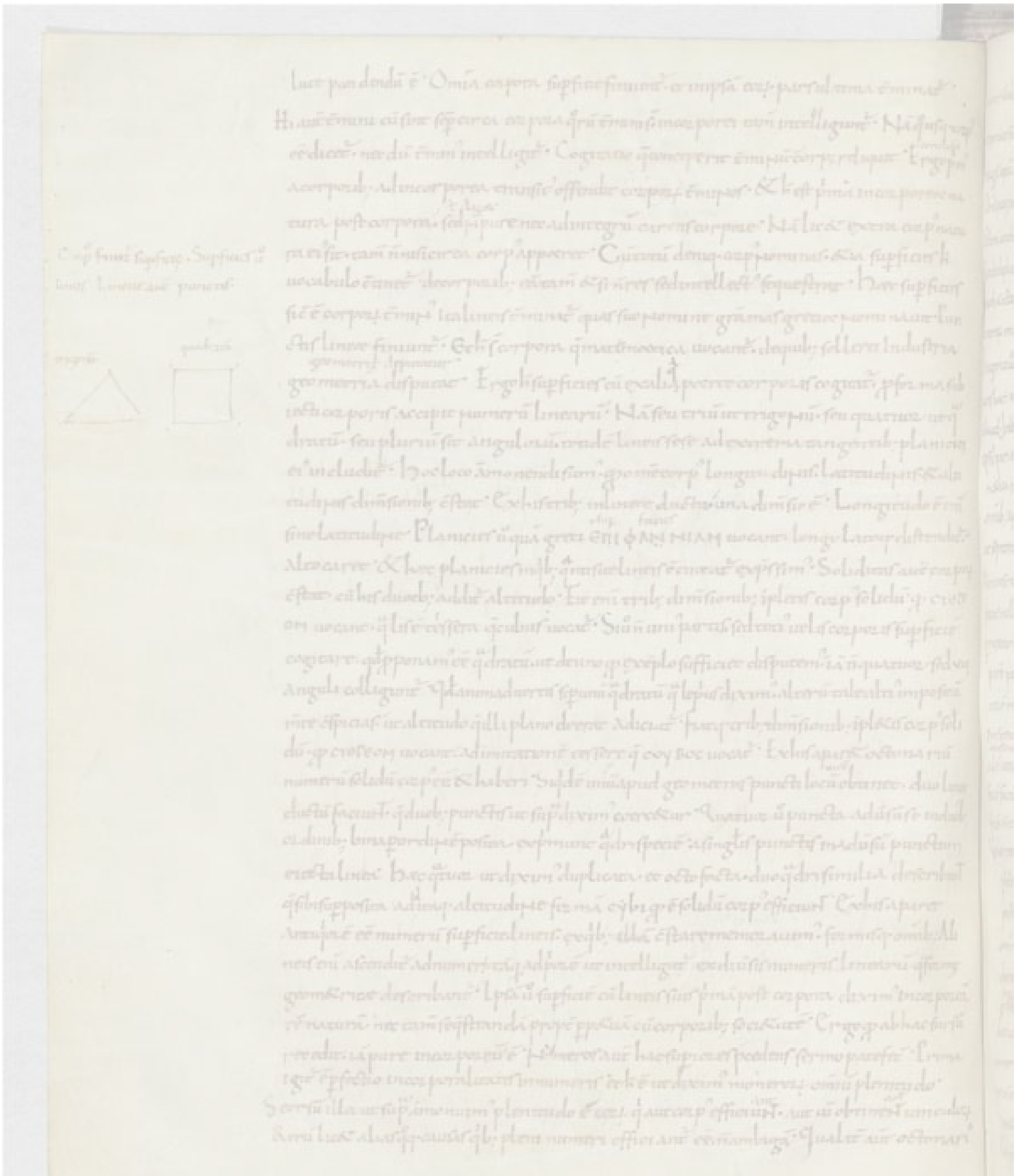
5.31 London, British Library, Harley MS 2772, fol. 46r, detail.



5.32 Brussels, Bibliothèque royale de Belgique MS 10146, fol. 19v. Photo taken with permission of the Bibliothèque royale de Belgique



5.33 Brussels, Bibliothèque royale de Belgique MS 10146, fol. 19v, detail.
Photo taken with permission of the Bibliothèque royale de Belgique



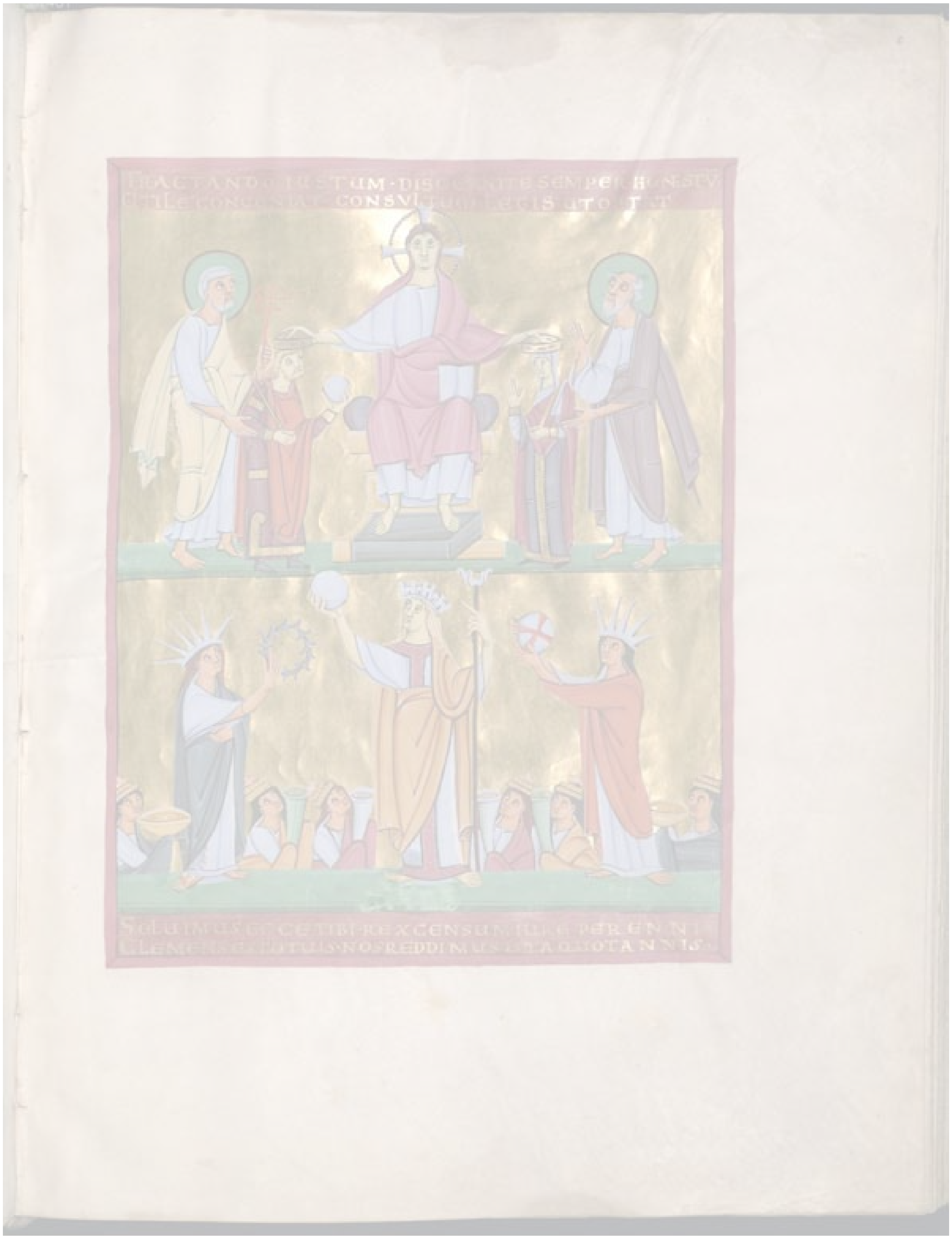
5.34 Paris, Bibliothèque nationale de France lat. MS 7299, fol. 31v.

5.35 Paris, Bibliothèque nationale de France lat. MS 7299, fol. 31v, detail.

6.1 Orléans, Bibliothèque municipale d'Orléans MS 145, p. 11–12. Photo taken with permission of the Bibliothèque municipale d'Orléans

6.2 Vienna, Kunsthistorisches Museum, Weltliche Schatzkammer, Reichskrone, Inv.-Nr. SK_WS_XIII_1.

6.3 Munich, Bayerische Staatsbibliothek Clm. 4452, front cover.



6.4 Munich, Bayerische Staatsbibliothek Clm. 4452, fol. 2r.

6.5 Munich, Bayerische Staatsbibliothek Clm. 4452, back cover.

6.6 Munich, Bayerische Staatsbibliothek Clm. 4452, front cover, detail.
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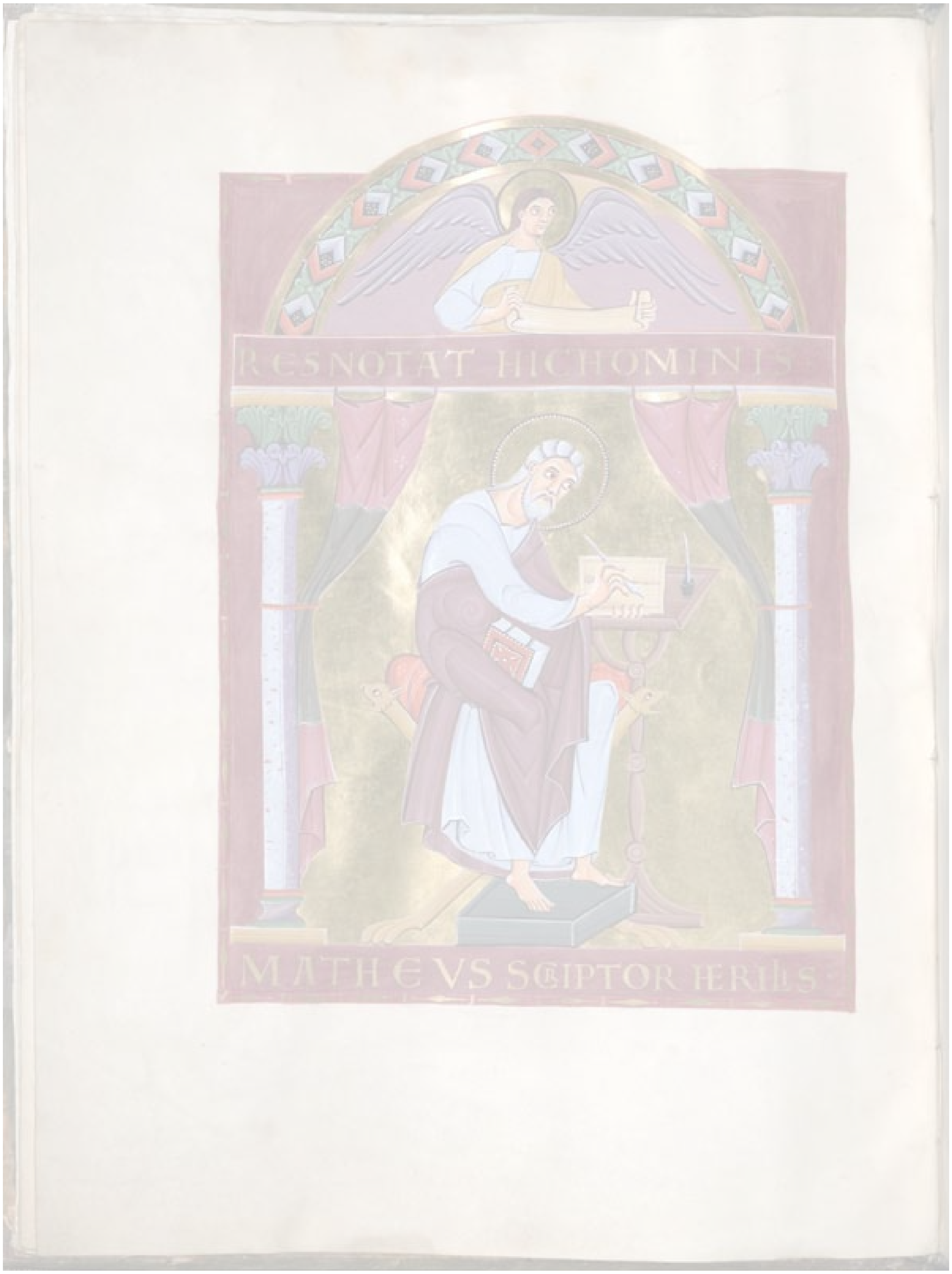
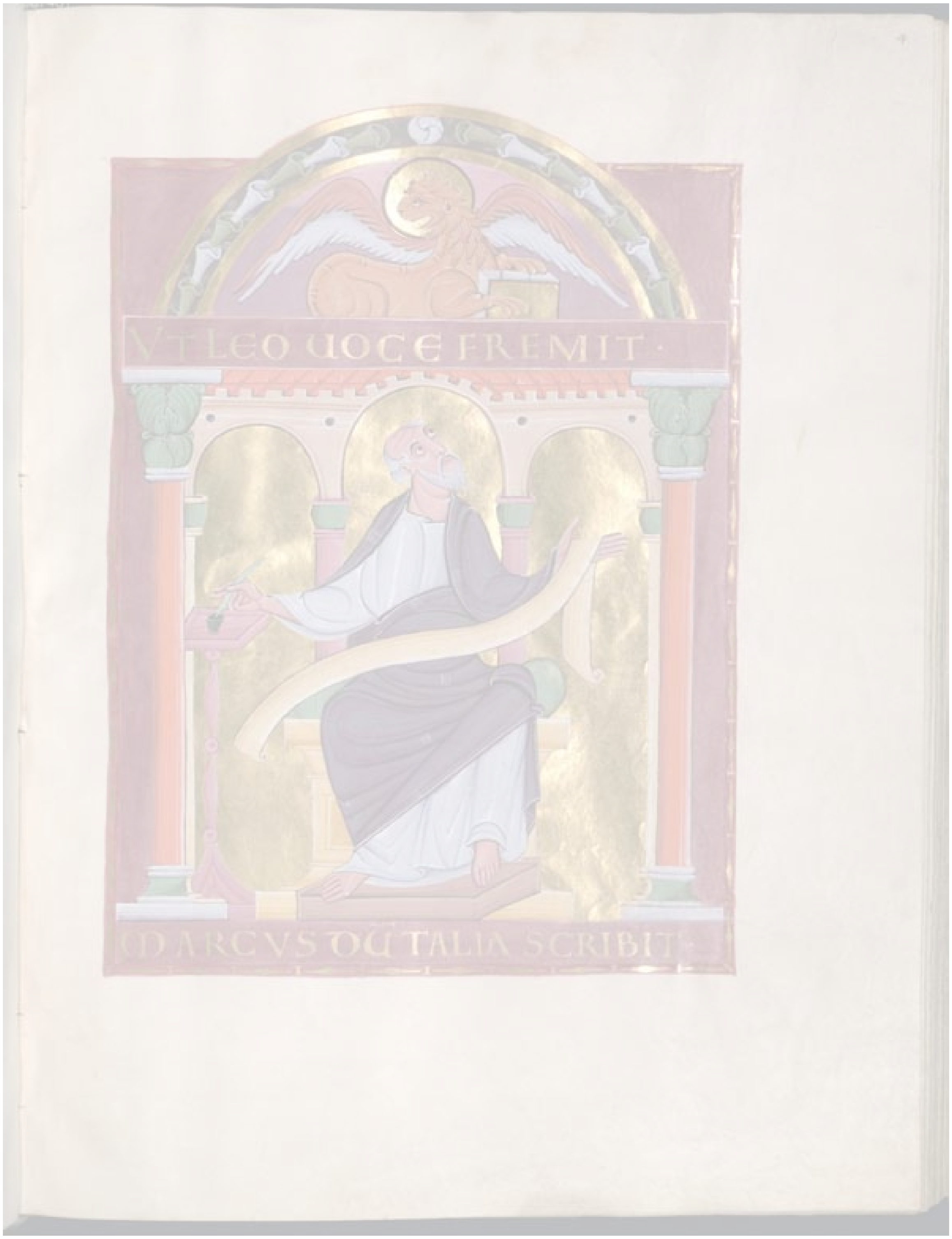
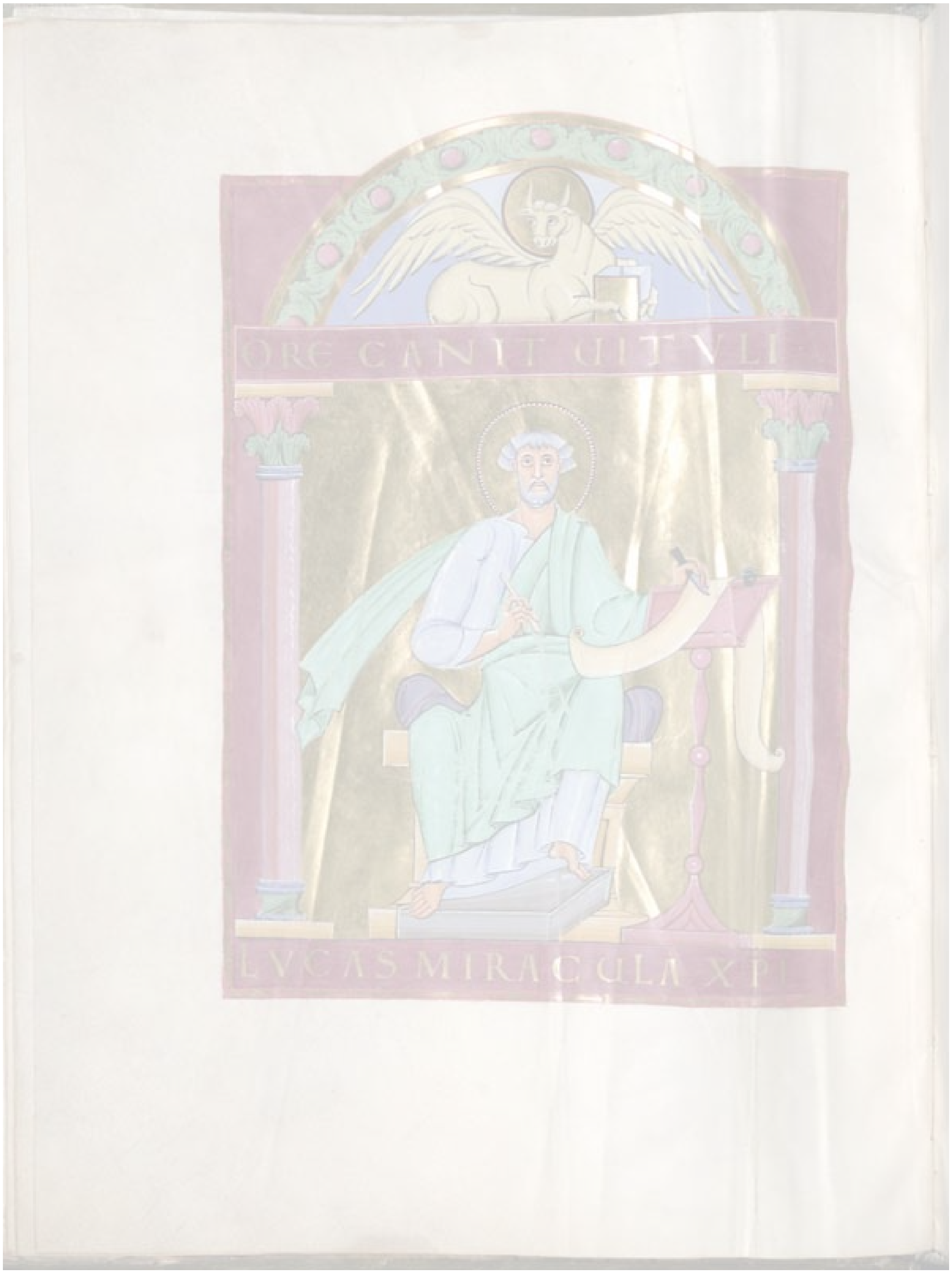


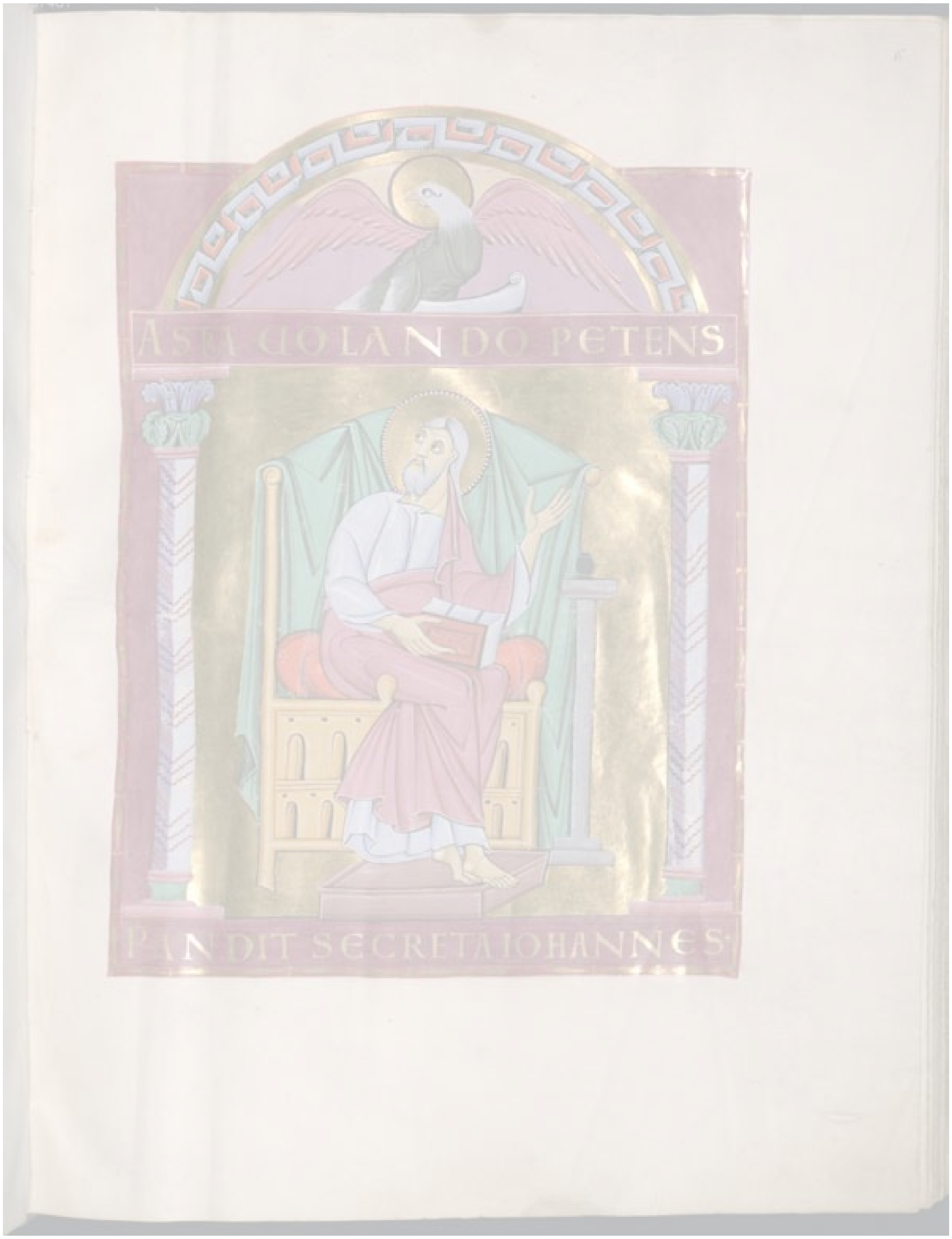
6.7 Munich, Bayerische Staatsbibliothek Clm. 4452, fol. 3v.



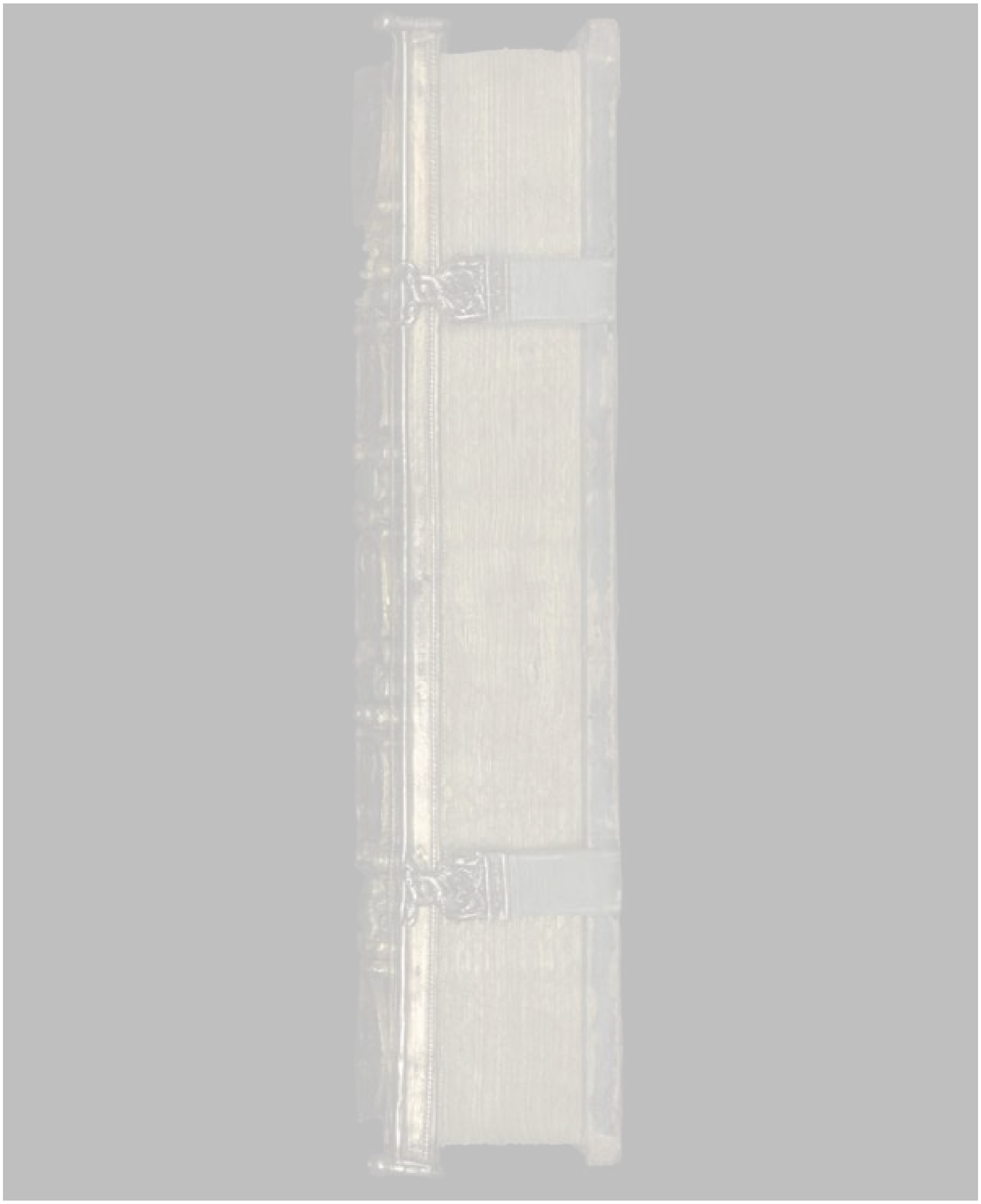
6.8 Munich, Bayerische Staatsbibliothek Clm. 4452, fol. 4r.



6.9 Munich, Bayerische Staatsbibliothek Clm. 4452, fol. 5v.



6.10 Munich, Bayerische Staatsbibliothek Clm. 4452, fol. 6r.



6.11 Munich, Bayerische Staatsbibliothek, Clm. 4452, side.
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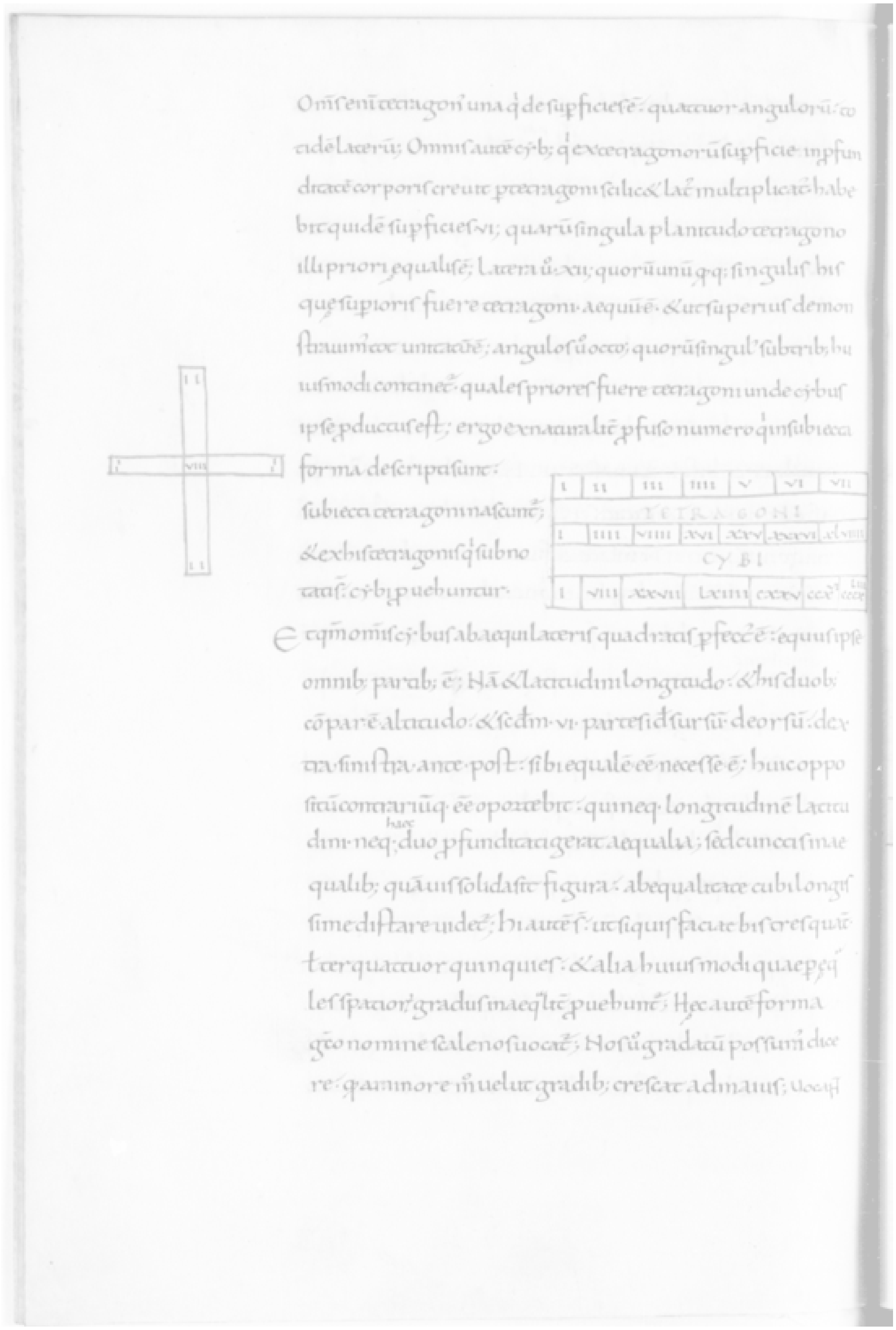
6.12 Munich, Bayerische Staatsbibliothek Clm. 13601, fol. 3v.

6.13 Munich, Bayerische Staatsbibliothek Clm. 13601, fol. 3v, detail.



6.14 Paris, Bibliothèque nationale de France lat. MS 6401, fol. 133v.

6.15 Paris, Bibliothèque nationale de France lat. MS 6401, fol. 133v, detail. © Bibliothèque nationale de France

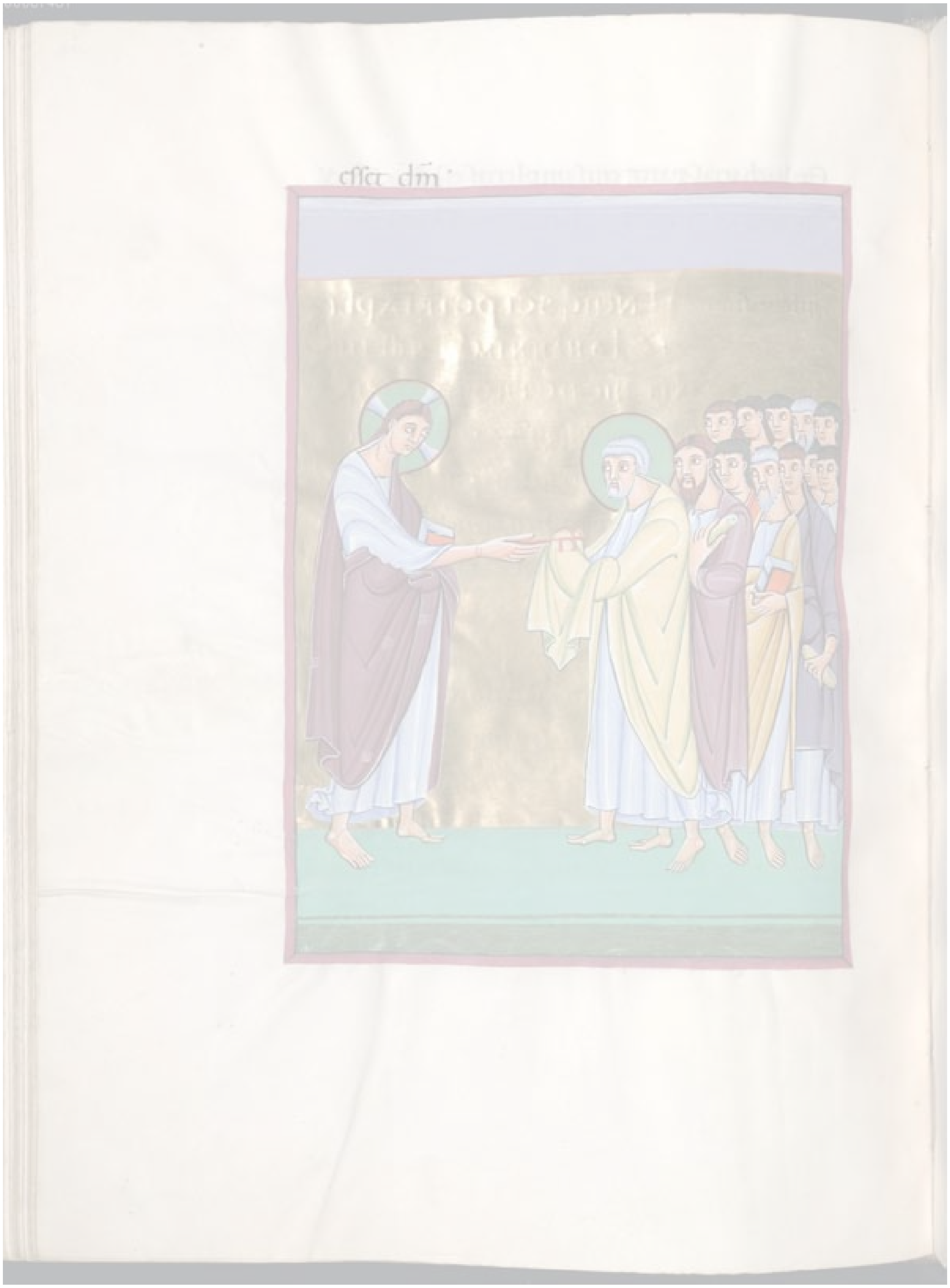
6.16 Munich, Bayerische Staatsbibliothek Clm. 13601, fol. 5v.

6.17 Munich, Bayerische Staatsbibliothek Clm. 13601, fol. 5v, detail. © Bayerische Staatsbibliothek

6.18 Munich, Bayerische Staatsbibliothek Clm. 13601, fol. 5v, detail. © Bayerische Staatsbibliothek

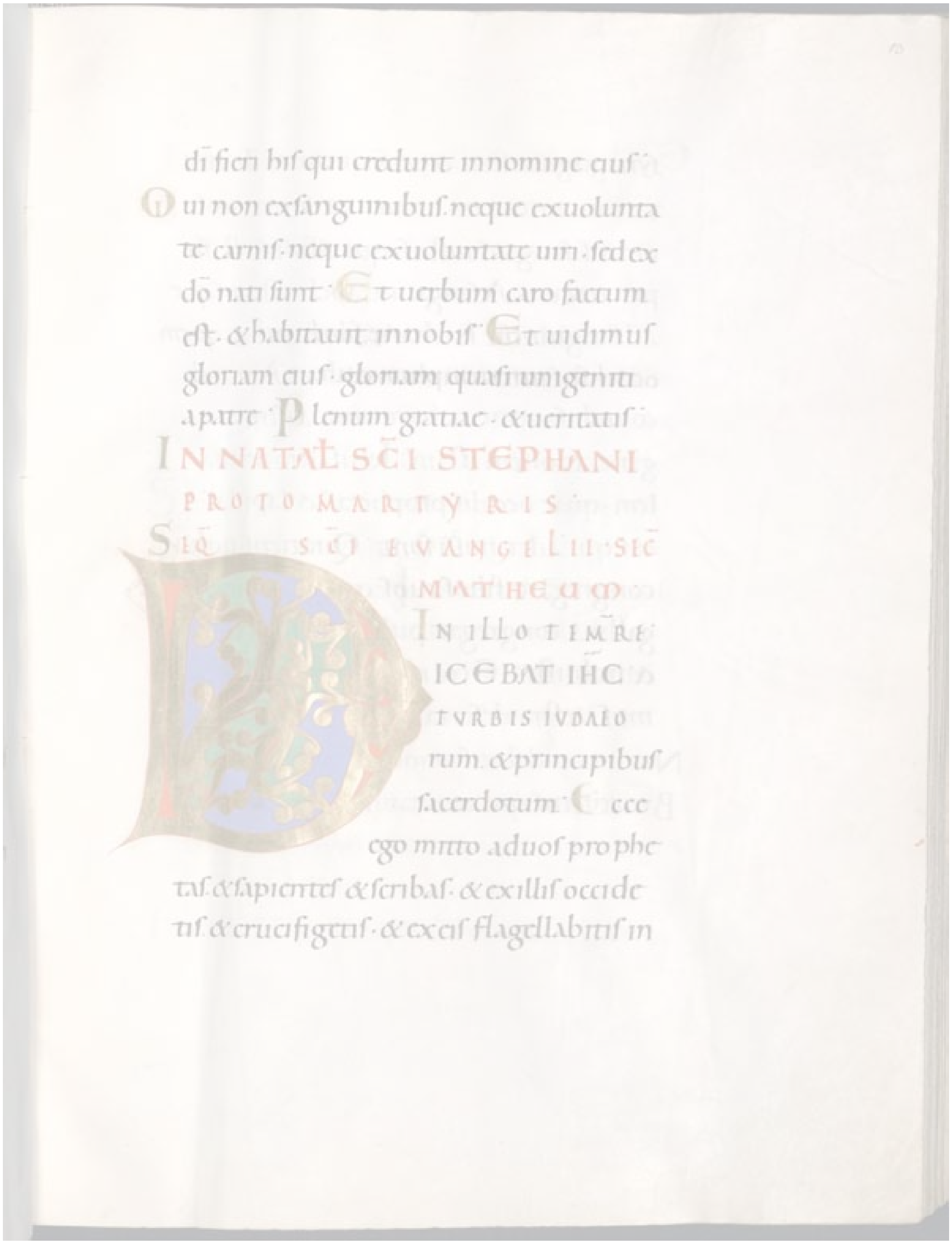
6.19 Munich, Bayerische Staatsbibliothek Clm. 13601, fol. 5v, detail. © Bayerische Staatsbibliothek

6.20 Munich, Bayerische Staatsbibliothek Clm. 13601, fol. 5v, detail. © Bayerische Staatsbibliothek

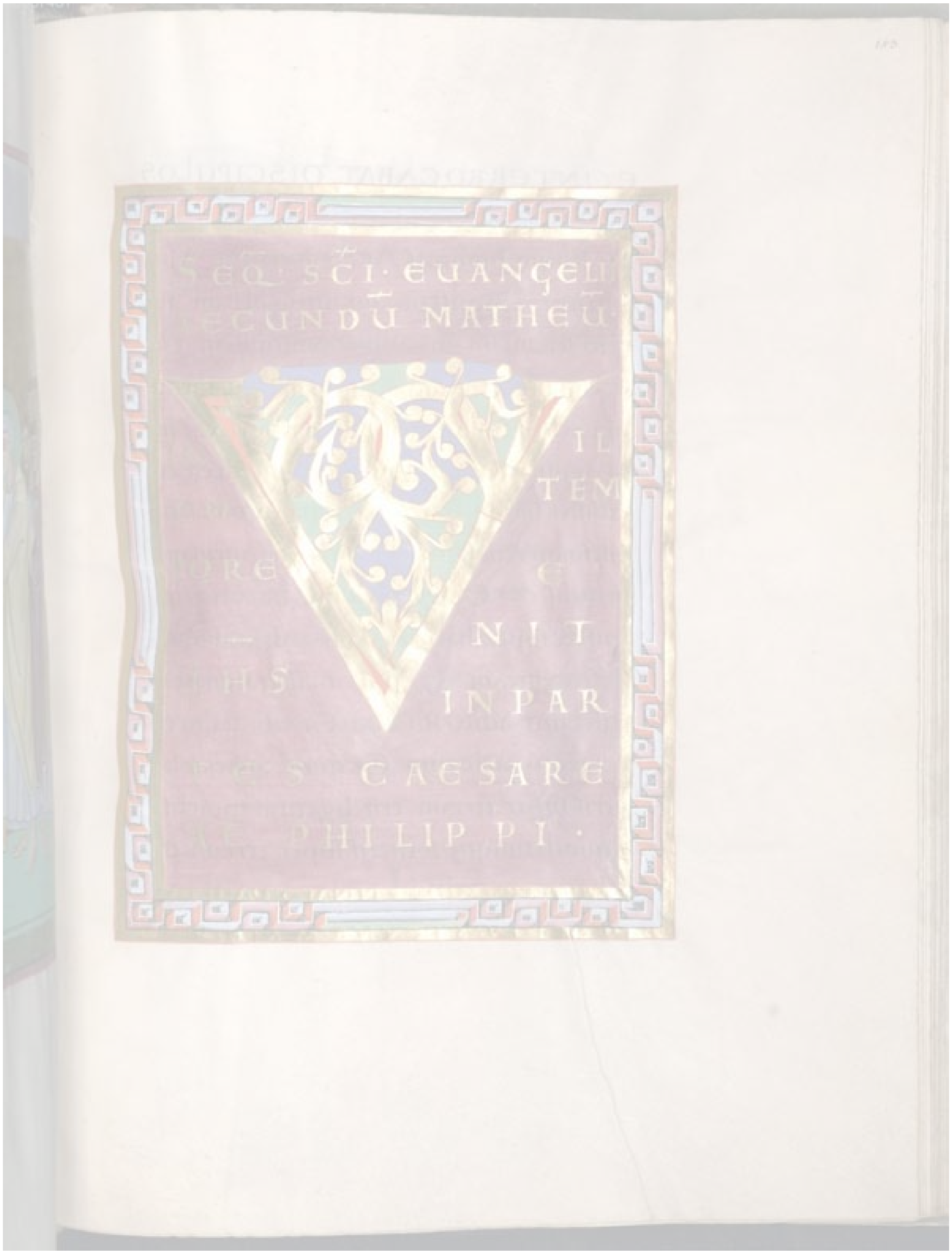


6.21 Munich, Bayerische Staatsbibliothek Clm. 4452, fol. 152v.

6.22 Munich, Bayerische Staatsbibliothek Clm. 4452, fols. 17v–18r. © Bayerische Staatsbibliothek



6.23 Munich, Bayerische Staatsbibliothek Clm. 4452, fol. 13r.



6.24 Munich, Bayerische Staatsbibliothek Clm. 4452, fol. 20r.

6.25 Vienna, Kunsthistorisches Museum, Weltliche Schatzkammer, Inv. Nr. SK_WS_XIII_26, Stephansbursa. © Kunsthistorisches Museum Wien

6.26 Aachen, Domschatzkammer, Lothar Cross, jeweled side.

6.27 Aachen, Domschatzkammer, Lothar Cross, engraved side.

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