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STRUCTURAL MODELS IN THE SUBTERRANEAN WORLD

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ABSTRACT. Graph-theoretic structural models offer one strategy for simplifying complicated real-world systems. Two examples, the New York City subway and the Chicago freight-tunnel systems, are considered in this light. When this conceptual approach is coupled with the evidence of maps, insight into barrier-free subway access emerges, as does insight into a time lag in the 1992 flooding of adjacent locations along the Chicago tunnel system.

FOR many years geographers have employed various indexes based on the ideas and concepts of graph theory to measure connectivity and other facets of network analysis. They have noted the limits of these indexes and have sought other tools, some, such as those drawing on combinatorial topology, intimately related to graph theory but others farther afield (Garrison 1960; Nystuen and Dacey 1961; Tinkler 1988). One difficulty with applying mathematics to real-world settings is that strategies which serve in laboratory sciences often do not function equally well in the real world. Thus indexes that can be used over and over in carefully controlled laboratories may not be well suited to complexities in applications that lack such controls.

Modifications of theoretical tools that yield good results for one project might not be suited when further modified for another project. Thus it is often preferable to return to the original theoretical underpinnings and adjust the mathematics to fit the real-world situation at hand. In such situations tailor-made mathematical suits are superior to those taken from the racks of traditional models. There are geographical examples of this art, and there is room for more (Harary 1969; Arlinghaus 1994; Arlinghaus, Arlinghaus, and Harary forthcoming). In the latter spirit I offer two simple examples that display both the power and the elegance that carefully constructed structural models can bring to geographical analysis.

PROBLEM OF LAYERS

Any system, such as a transportation system, that is forced to have different physical levels for entry and exit becomes a target for policy difficulties of various kinds. Elevated and subterranean trains are a response to providing efficient commuting in a densely populated environment: to prevent collisions these networks generally have a number of different horizontal layers. Clearly it is inconvenient for passengers to move vertically as well as hori-

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zontally during transfers from one route to another. Although the extra dimension removes a collision hazard between trains, it increases the potential for collisions among passengers. Different layers add a host of security problems for security personnel on the lookout for muggers and thieves who prey on a population closely confined underground. Unless elevators or ramps are installed, multiple-level stations exclude transfer possibilities for individuals confined to wheelchairs. In some locales funding is directly tied to the extent of handicap access to be provided in a public project. Different access layers offer a way to overcome congestion and collisions; however, their mere presence can keep an otherwise worthwhile project from being funded, if they are not constructed in a manner that permits barrier-free access. To begin, therefore, I propose a theorem that determines when a set of layers is arranged to achieve the goal of barrier-free access.

ELEVATOR THEOREM

A persistent issue in any logical approach to a problem is knowing whether or not a solution exists. It is counterproductive to search for solutions that are known not to exist. One standard theorem from the realm of continuous mathematics is the intermediate-value theorem. Stated informally, this theorem ensures that a continuous function on $[a, b]$ assumes all values between $f(a)$ and $f(b)$ (Fig. 1). That is, choose a value m on the y axis between $f(a)$ and $f(b)$—there exists a value c in the interval on the x axis between a and b such that $f(c) = m$. In this context m is the problem, and the value c is its solution. Stated more formally, the intermediate-value theorem is often cast in the following manner: suppose that a function f is continuous throughout a closed interval $[a, b]$ and that m is any number between $f(a)$ and $f(b)$; then there is at least one number c in $[a, b]$ such that $f(c) = m$.

Geometrically it is not difficult to imagine that instead of a continuous function f over a continuous closed interval $[a, b]$, there might be a discrete function f with separated values. Because the problem of layers is one of discrete or separated levels, I offer a discrete version of the intermediate-value theorem as a way to tailor the mathematics to suit the real-world issues under examination. One straightforward way is to make the formal statement of the new theorem for discrete functions parallel that of the formal statement of the intermediate-value theorem for continuous functions.

The statement for the elevator theorem is thus as follows: suppose that a function f takes on a discrete set of values throughout a closed interval $[a, b]$ and that m is any number in a subset of discrete values between $f(a)$ and $f(b)$; then there is at least one number c in $[a, b]$ such that $f(c) = m$ (Fig. 2). The validity of this theorem depends on how the subset of discrete values for m is chosen. When the discrete values of f are strictly increasing, one might view $f(a)$ as the entry level of a building and $f(b)$ as its top level. Values of f between the top and the bottom are other levels of the building. The y axis is an elevator shaft, and the discrete subset of possible values for

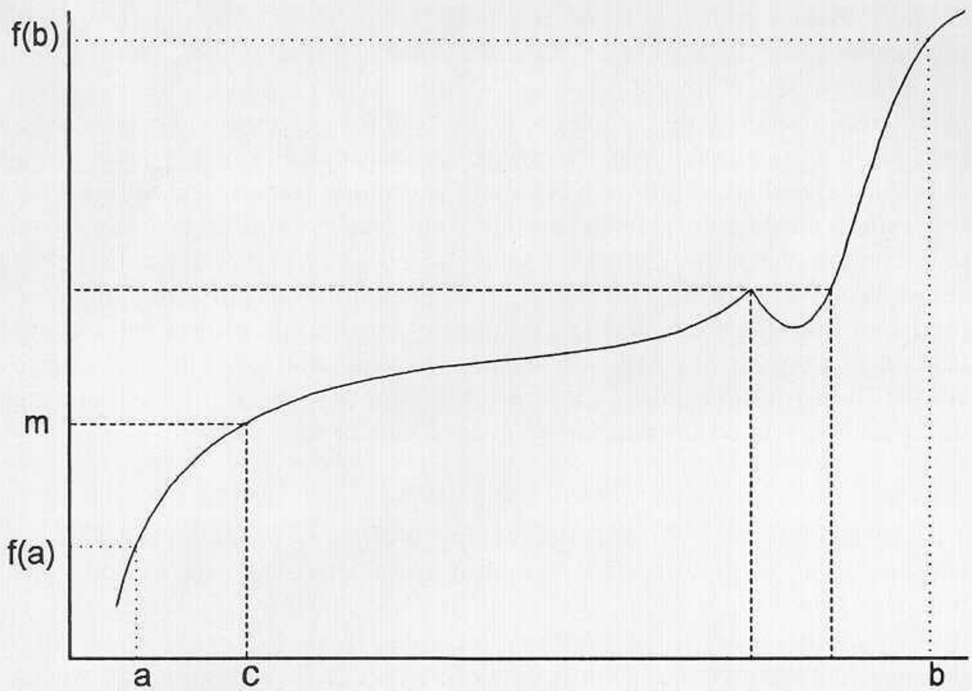

FIG. 1—Intermediate-value theorem for continuous functions. Choose m on the y axis between $f(a)$ and $f(b)$. This theorem ensures that there is at least one, possibly more, value c in $[a, b]$ such that $f(c) = m$.

m is the set of buttons in the elevator. When the set of buttons is exactly the set required to make the elevator theorem hold, that is, one button for each level, there is a strong elevator access throughout the building; any two levels are mutually reachable. It is this sort of strong access that is needed to create a barrier-free environment for wheelchair access. When the elevator theorem holds, the problem of barrier-free access between layers is solved.

In addition, multiple elevators can work together to provide strong access. Many tall buildings partition elevators in banks that go to proper subsets of available floors. This strategy works as long as the sets of the partition intersect; it also moves certain amounts of lateral congestion that might otherwise have remained in the lobby to higher levels, where the sets of the partition intersect. Only when levels are excluded from access, as in the case of locked levels in which there is a button in the elevator with no accessible associated level, does this interpretation of the elevator theorem fail, as it should. The theorem holds when desired and fails when it should, an example of going back to the theoretical underpinnings and modifying them to fit the real-world situation at hand.

New York City Subway System

The New York City subway is a mass-transit system that offers barrier-free wheelchair access at some stations: nine in Manhattan, two in the Bronx, seven in Queens, and two in Brooklyn. The Tauranac map of the systems

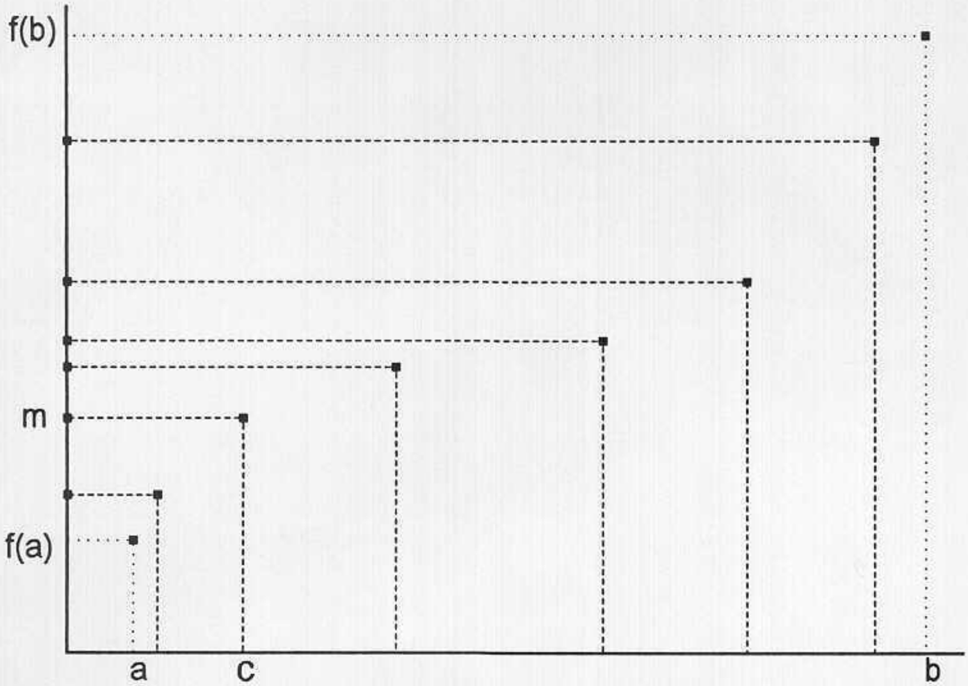

FIG. 2—Elevator theorem. The intermediate-value theorem for a discrete, not a continuous, curve. The value $f(a)$ is the entry level; $f(b)$ the top level; the y axis is an elevator shaft; the values of m are elevator buttons. This theorem ensures that there is least one value c in $[a, b]$ such that $f(c) = m$; the idea of c corresponds with that of a floor in a building.

displays the network as a structural model of edges linking nodes (Tauranac and Eichen 1991). A logical next step is to ask whether the submodel suggested by the barrier-free stations and the existing tracks joining them is itself barrier-free as a whole. Or is it possible that some wheelchair patron could become trapped in the system in a manner reminiscent of the "man who never returned" from the MTA, as portrayed in a song by the Kingston Trio?

To consider this idea, join the twenty nodes representing the barrier-free stations using all the possible edges already present in the entire system. When this linkage pattern is represented abstractly, the structural model has three distinct components (Fig. 3) or, in terms of barrier-free access, three distinct subway subsystems (Table I).

Entrapment in a network can occur in several ways. If the elevator theorem does not hold, vertical entrapment between layers can occur, which does not appear to be a problem in any of the barrier-free subsystems in New York City. There is no exit fee, and one can learn in advance what facilities are available for vertical movement simply by calling a dedicated telephone line. It remains to consider whether or not one can become entrapped by the connectivity (Fadiman 1958).

In subsystem 1 stations are mutually reachable but not always in the most direct manner. A wheelchair patron wishing to travel from the World Trade Center (4) to the Howard Beach–JFK station (16) must go to the end

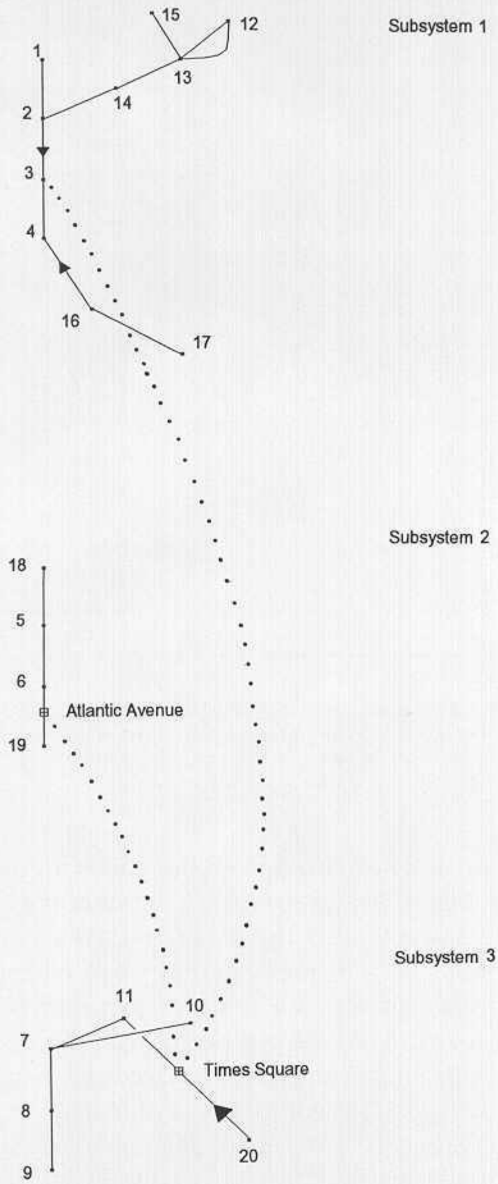

FIG. 3—Graph of three barrier-free subsystems of the New York City subway system. Dotted lines show linkages to improve connectivity. Node numbers correspond to station names, as in Table I.

of the line and return to the accessible northbound side. This maneuver is possible because the Rockaway Beach station is wheelchair accessible. A similar situation exists at the 50th Street–8th Avenue station (2). Wheelchair access is available only on the southbound side, so that a northbound wheelchair rider wishing to exit at that station must go to the 175th Street station (1) and transfer to a southbound train. The access in this subsystem is barrier-free but not optimal. In subsystem 2 access is both barrier-free and optimal; all stations are mutually reachable in the most direct fashion.

In subsystem 3 wheelchair access is available only on the northbound side of the Atlantic Avenue station (20). From the evidence of the subsystem, it appears that one might not be able to return to that station. There is no wheelchair-accessible station beyond this stop; a wheelchair rider cannot go past this station on a southbound train and use another station to leave the train, cross the tracks, and return on a northbound train. Unless the train turns around at the end of the line beyond Atlantic Avenue at either the Flatbush Avenue–Brooklyn College station or the New Lots Avenue station, neither of which is wheelchair accessible, and brings the rider back, the person cannot return to the station of departure. Indeed, planners need to consider access limits at end points of systems, lest one be trapped in the connectivity or its absence.

TABLE I—WHEELCHAIR ACCESS TO NEW YORK CITY SUBWAYS

BOROUGH	BARRIER-FREE STOPS BY STRUCTURAL SUBSYSTEM
Manhattan	Subsystem 1
175th St. (1)	175th St. (1)
50th St.–8th Ave. (2)	50th St.–8th Ave. (2)
42d St.–8th Ave. (3)	42d St.–8th Ave. (3)
World Trade Center (4)	World Trade Center (4)
Roosevelt Island (5)	Jamaica Center (12)
Lexington Ave.–63d St. (6)	Sutphin Blvd.–Archer Ave. (13)
125th St.–Lex. Ave. (7)	Jamaica–Van Wyck (14)
51st St.–Lex. Ave. (8)	Metropolitan Ave. (15)
42d St.–Grand Central (9)	Howard Beach–JFK (16)
Bronx	Rockaway Beach–116th St. (17)
Pelham Bay Park (10)	Subsystem 2
Simpson St. (11)	Roosevelt Island (5)
Queens	Lexington Ave.–63d St. (6)
Jamaica Center (12)	21st St.–Queensbridge (18)
Sutphin Blvd.–Archer Ave. (13)	Stillwell Ave.–Coney Is. (19)
Jamaica–Van Wyck (14)	Subsystem 3
Metropolitan Ave. (15)	125th St.–Lex. Ave. (7)
Howard Beach–JFK (16)	51st St.–Lex. Ave. (8)
Rockaway Beach–116th St. (17)	42d St.–Grand Central (9)
21st St.–Queensbridge (18)	Pelham Bay Park (10)
Brooklyn	Simpson St. (11)
Stillwell Ave.–Coney Is. (19)	Atlantic Ave. (20)
Atlantic Ave. (20)	

The situation at the Atlantic Avenue station (20) could be changed by making the southbound side wheelchair accessible or by installing such facilities at some station beyond. That adjustment would make all stops in subsystem 3 mutually reachable, though not in an optimal fashion. Far greater barrier-free access might be provided by linking the three subsystems. If wheelchair access were installed at the Times Square station on the route for subsystem 3 with suitable regard for the problem of layers, it would provide a node of access to subsystem 1. In addition, if a wheelchair-accessible stop were added at Atlantic Avenue on the route of subsystem 2 with due regard for the elevator theorem, subsystems 2 and 3 would join. Such a juncture could offer a way for a wheelchair rider to leave a southbound train there in subsystem 3.

The subterranean world of the New York City subway wheelchair rider is restricted to three separate parts of the large system that is accessible to walking patrons. Because surface congestion is extreme, subterranean access patterns mold the sets of surface contacts that are readily available. Thus one set of wheelchair New Yorkers might find their sphere of contacts ranging the full length of Manhattan west of Central Park and extending across lower Manhattan through Brooklyn and south to Rockaway Beach, with a midtown spur crossing past Queens to Jamaica. An entirely different wheelchair environment might center on the subway routes of subsystem 2, linking Queens to Coney Island via east-side midtown Manhattan. A third distinct enclave of wheelchair patrons might situate themselves near subsystem 3, linking the Bronx to the east side of Manhattan and to Brooklyn.

When planned sequences of structural models are carefully made to fit the needs of a population, efficient systems result. Over time, however, even systems that are well suited to fit the needs of society eventually fall out of political favor or into disuse because new ideas, which may or may not be better, replace them. Subterranean systems that are shut down must also be cared for. If not, the result can be devastating, as exemplified by the "Great Chicago Flood" of 1992.

Chicago Freight Tunnels

On 13 April 1992 a hole was accidently made in a tunnel under the Chicago River. The river swirled through the Kinzie Street opening as valiant efforts were made to stop the huge mass of water from filling the abandoned freight-tunnel system, which extended more than fifty miles forty feet below the streets of the Loop (Fig. 4). In the days when coal was used for central heating in Loop buildings, barges delivered coal to designated docks along the river (Moffat 1982). The loads were put into trains of small cars that then dove below the streets, delivering coal to subbasements of one building after another. The cars were seldom empty: when coal was delivered, ashes from the previous load were picked up, often by gravity, and trainloads of ashes returned to a river dock to be dumped into empty barges for shipment downstream. In the early twentieth century, when this system functioned well, plans were laid for the eventual introduction of a passenger subway; forty feet was thought to be sufficiently deep to leave space for an underground mass-transit system that would run between the freight-tunnel layer and the surface but not interfere with the former (Moffat 1982).

As coal-powered heating systems were replaced with gas-powered ones, the gas was piped directly to furnaces, and the system to transport coal and ash was no longer needed. The trains were called into service in conjunction

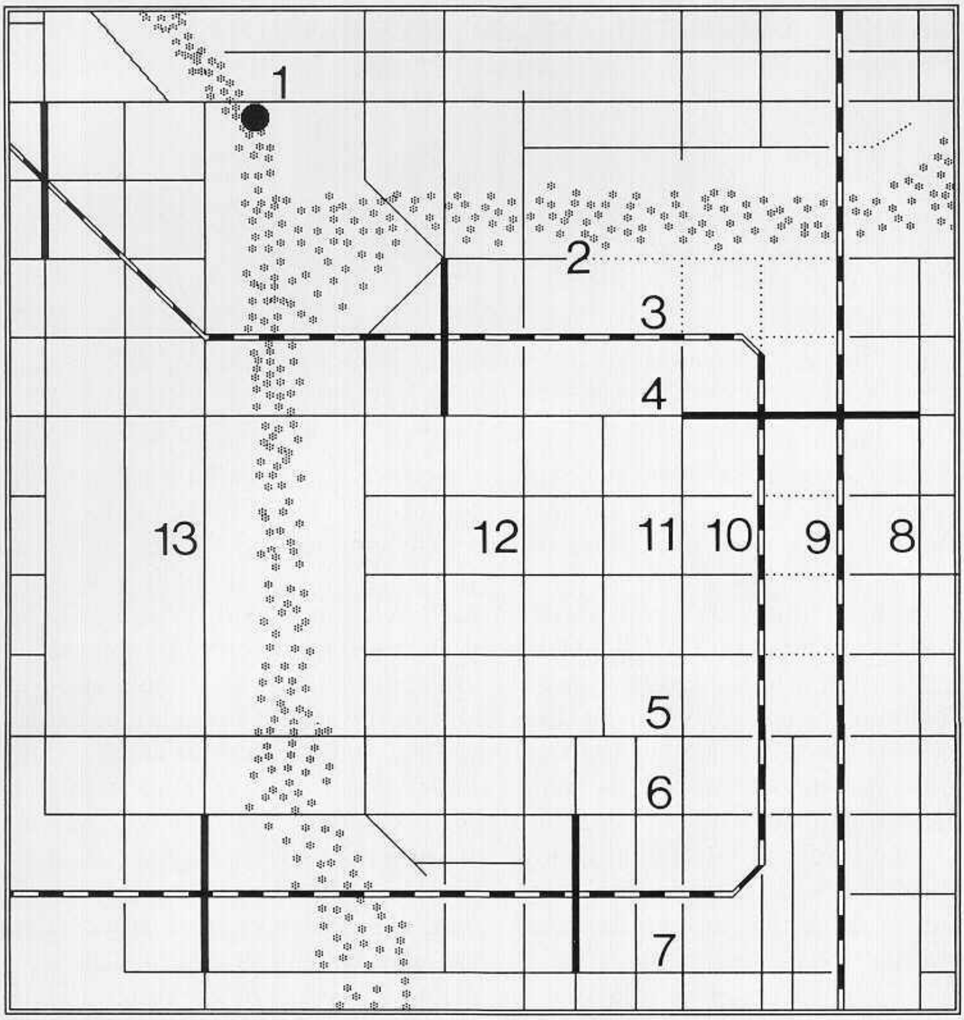

FIG. 4—Chicago freight-tunnel system. Dotted lines indicate isolated tunnels. Heavy black segments indicate tunnel bypasses. Striped double line represents Chicago subway system. Circle indicates location of 1992 hole in Kinzie Street freight tunnel. North is top; stream of bubbles suggests Chicago River. Key to streets—east-west: 1 Kinzie, 2 Wacker, 3 Lake, 4 Randolph, 5 Jackson, 6 Van Buren, 7 Harrison; north-south: 8 Wabash, 9 State, 10 Dearborn, 11 Clark, 12 Wells, 13 Canal. *Source:* Adapted from maps in Moffat 1982.

with the postal service, often hauling bags of mail as a supplement to surface or pneumatic movement. Over the years the trains and tunnels were used in various imaginative ways. For example, before central air-conditioning was common in cinemas, the ornate theaters along State Street circulated the constantly cool air from the freight tunnel through their furnaces during hot, muggy weather.

What was crucial in causing this apparently forward-looking urban system to fall into disuse was an alteration of its structure. The structural model

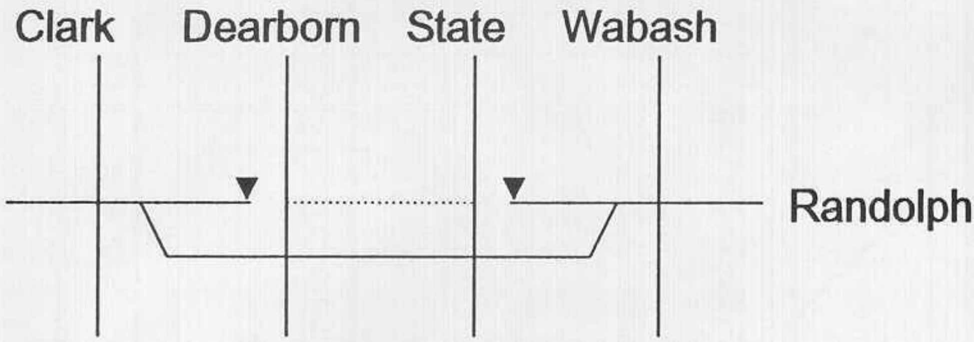

FIG. 5—Randolph Street bypass in the Chicago freight-tunnel system (see Fig. 4). Cul-de-sacs created by bypass are displayed and noted with arrows. *Source:* Adapted from maps in Moffat 1982.

of the system in 1932 had two components: when the mass-transit subway tunnels were built in 1942, the little freight-tunnel trains helped to haul out the dirt to excavate the larger tunnels that were to lead to their eventual demise. The space left above the freight-tunnel system, thought to be adequate by engineering standards of the early twentieth century, proved to be inadequate by 1942. The Lake-Dearborn-Harrison route forced tunnel reconstruction at selected crossings with north-south streets. That route and the State Street route forced major disruptions in the freight tunnels along east-west streets between State and Dearborn (Fig. 4). The effect was to sever the transmission capacity of the freight-tunnel system and to alter it from a structural model with two separate graphical components to one with eleven.

The implications of that change in connectedness became apparent in the 1992 flood. Records of the Chicago Fire Department and Commonwealth Edison show that one site reported a water-related emergency at 6:22 A.M. but that the site across the street did not report one until almost 10:00 A.M. What might account for this time lag, given that people in the buildings had ample opportunity to notice the internal flooding? When one notes the structure of the bypass introduced by the subway at State and Randolph streets, the seemingly confusing situation is easily explained (Fig. 5).

The bypass joining Randolph between Clark and Dearborn to Randolph between State and Wabash did not attach to the ends of the severed tunnels. Cul-de-sacs were created in constructing the bypass. Consequently, the full surge of the eastbound water could flow downstream through the bypass south of the original, now truncated tunnels. Only later did water back up into the cul-de-sac across the street on the north side of Randolph between State and Wabash. The structural model explains, when other sources do not, why there could be a significant time lag in reporting an emergency at seemingly adjacent sites north and south of Randolph.

Closing Comments

Structural models offer an easily used strategy to disentangle crucial components from complicated real-world systems. Because much of the complexity often arises from having different layers in a system, it is important not only to look at spatial structure and patterns of connectivity in each layer but also to have systematic tools such as the elevator theorem for observing such structure among layers. When these are coupled with historical evidence and the evidence of maps, explanations that are unavailable from other sources can emerge.

Barrier-free subway stations in New York City, together with their associated linkages, create three distinct subnetworks in a subterranean layer of that city. A planner needing to consider the requirements of a wheelchair-mobile population at the surface layer would be well advised to note these subterranean patterns, to locate surface elements, and to check that vertical access from the subterranean to surface layers is ensured.

A disaster at the surface layer wreaked havoc in a subterranean layer in Chicago; use of a carefully constructed structural model coupled with historical evidence of the construction of the subway system led to a structural reason that might have caused a delay in reporting the emergency. An understanding of structure at one layer can explain activities in another; armed with such knowledge, planners and municipal authorities can be better prepared to understand the likely structure of future interaction between surface and subterranean layers.

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