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GEOGRAPHY OF CITY TERRAIN BASED ON BUS ROUTES

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ABSTRACT. *A geography of city terrain, based on bus routes, supplies information about steepness that is vital to maintenance of equipment. This article presents a technique to determine vertical profiles of bus routes to evaluate topography and service demands in terrain stress on vehicles.*

AN arbitrary abstract bus-route network, superimposed on the undulating surface of a city, would logically follow lines of lowest topographic gradient when minimal terrain-imposed stress on equipment was a factor. The resultant routing strategy would fail to provide effective service to the population, because bus-route networks should also follow service gradients. The key issue centers on how to thread routes through an urban area so that they touch a set of high-demand locations like places of employment and then connect with areally spread residential zones. Another way to pose the question is how can the linear form of a bus route get as close as possible to an area, or how can a one-dimensional linear form be routed along a winding path to fill a portion of a two-dimensional market area?

The latter question appears from general context to be one that is likely to respond to an abstract approach based on fractal geometry, and we conducted an empirical study of one set of bus routes. Bus-route vertical profiles were viewed as wiggly lines, attempting to fill some sector of the market area. To understand how these routes might fill space, we developed a procedure to measure the displacement of a bus-route vertical profile from a topographic baselevel. This base was established as a slope between adjacent points like river crossings on the profile. These points force a change in elevation of a route as the bus travels from origin to terminus. Routes along paths that would maintain baselevel are analogous to the arbitrary net dropped on a city. Displacement of all others from this minimal routing response is a function of the need to serve distinct points with high demands for service as well as areally spread residential markets with different demands for service.

With this broad idea expressed in a terrain context, the test case begins by displaying briefly a general procedure to classify city terrain at a 1:250,000 scale; the results of applying this procedure to Ann Arbor, Michigan, as well

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as San Francisco, Washington, D.C., and Detroit illustrate extreme positions in the taxonomy.[1] Although this classification is useful to make broad terrain comparisons among cities, it does not permit identification of variations in elevation that result from residential-service needs to be made at the city level. Therefore the report proceeds with a comprehensive analysis of the terrain in Ann Arbor at a scale of 1:24,000, based on vertical profiles of local bus routes. These profiles are partitioned by a discrete set of critical values that force variation in elevation into continuous intervals over which route-elevation displacement from a baselevel is measured. The study concludes by mapping the results of the displacement and discussing the implications of a geography of terrain based on bus routes for transit managers.

TERRAIN CLASSIFICATION

Terrain with a gradient in excess of 8 percent causes problems for virtually any type of vehicle, and most railroad tracks follow a grade of less than 2 percent.[2] Thus a city with a large percentage of 8 percent grade might be classified as steep, one with terrain largely less than 2 percent as flat, and all others as intermediate. To determine these percentages for a specific city, we employed the following technique. The city boundary was represented as a circle to facilitate classificatory comparisons among cities; allometry was used to represent the city as a circle with a radius proportional to population. Then evenly spaced lines were used to sample the unevenly spaced contour lines within the allometric circle and to classify the underlying terrain as steep, intermediate, or flat.[3]

The mechanics of analyzing the terrain within a circle required sampling of the spacing between the pattern of contour lines. Generally contour lines are wiggly; however, locally all are topologically equivalent to short straight-line segments. Thus a sequence of parallel short straight-line segments became a contour comb to disentangle contour lines. When the segments were spaced to represent 2 percent and 8 percent grades on a 1:250,000 topographical map with a 50-foot contour interval, they were in a form suitable for use with a topographic map of the same scale (Fig. 1). When the contour combs were applied to the pattern of contour lines representing topography on U.S. Geological Survey maps, the spacing in a set of lines appeared to fall mostly between the 2 percent and 8 percent pattern, but much of it was closer to the 2 percent end (Fig. 2). Ann Arbor was found to have terrain of

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[1] Sandra Arlinghaus and John Nystuen, Terrain Effects on Bus Durability, report prepared for University of Michigan Transportation Research Institute in cooperation with Urban Mass Transportation Administration, U.S. Department of Transportation, National Technical Information Service, Springfield, Va., 1986; Sandra Arlinghaus and John Nystuen, Terrain Effects on Bus Maintenance Performance, *Transportation Research Record,* forthcoming.

[2] Edward H. Hammond, Analysis of Properties in Land Form Geography: An Application to Broad-Scale Land Form Mapping, *Annals of the Association of American Geographers* 54 (1964): 11–19; Edward L. Ullman, The Railroad Pattern of the United States, *Geographical Review* 39 (1949), 242–256.

[3] Waldo R. Tobler, The Spectrum of U.S. 40, *Papers of the Regional Science Association* 23 (1969): 45–52; Hammond, footnote 2 above.

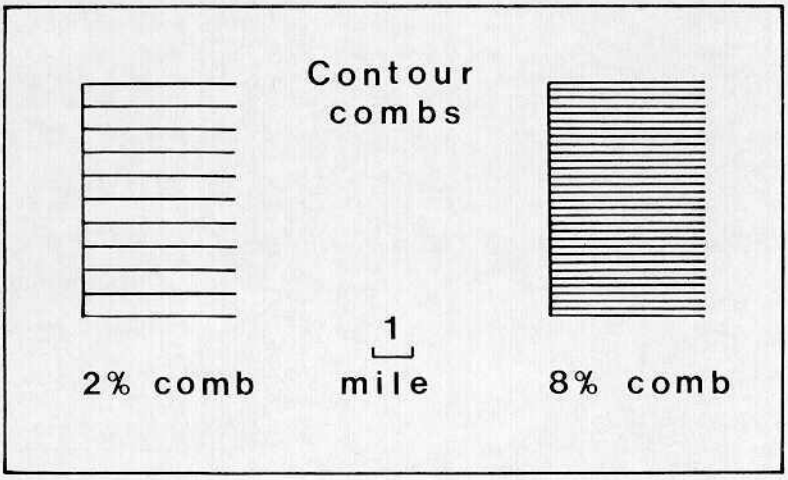

FIG. 1—2 percent and 8 percent contour combs.

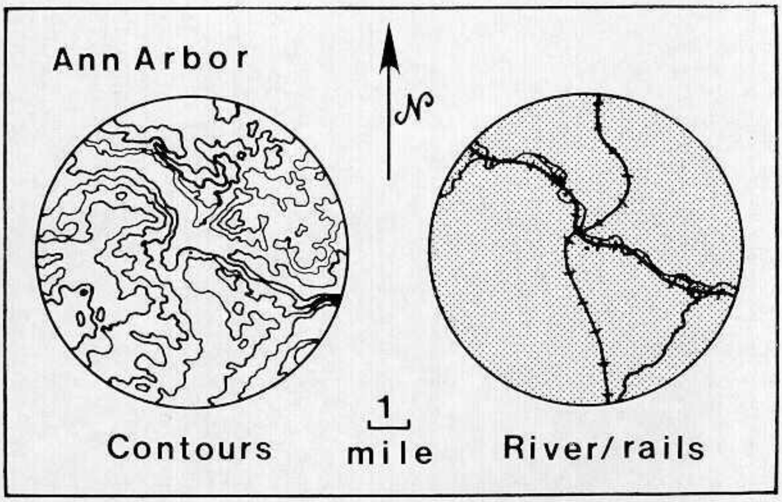

FIG. 2—Ann Arbor: Contour lines within the allometric circle (left); river and railroad lines in intermediate terrain (right). *Source:* U.S. Geological Survey topographic maps 1:250,000, 50-foot contour interval.

intermediate steepness. The river and railroad networks were also suggested by the pattern of contour lines. The same technique showed Detroit to be flat and San Francisco to be steep. These circle cities were shaded, according to coarseness of pattern, to reflect differences in terrain (Fig. 3). Washington, D.C., was generally steep but revealed a mixture of terrain types in zones bounded by rivers and railroad tracks.

BUS-ROUTE TERRAIN IN ANN ARBOR

A general measure of terrain, based on topographic evidence at the city scale, may fail to correspond with the gradient stresses along individual

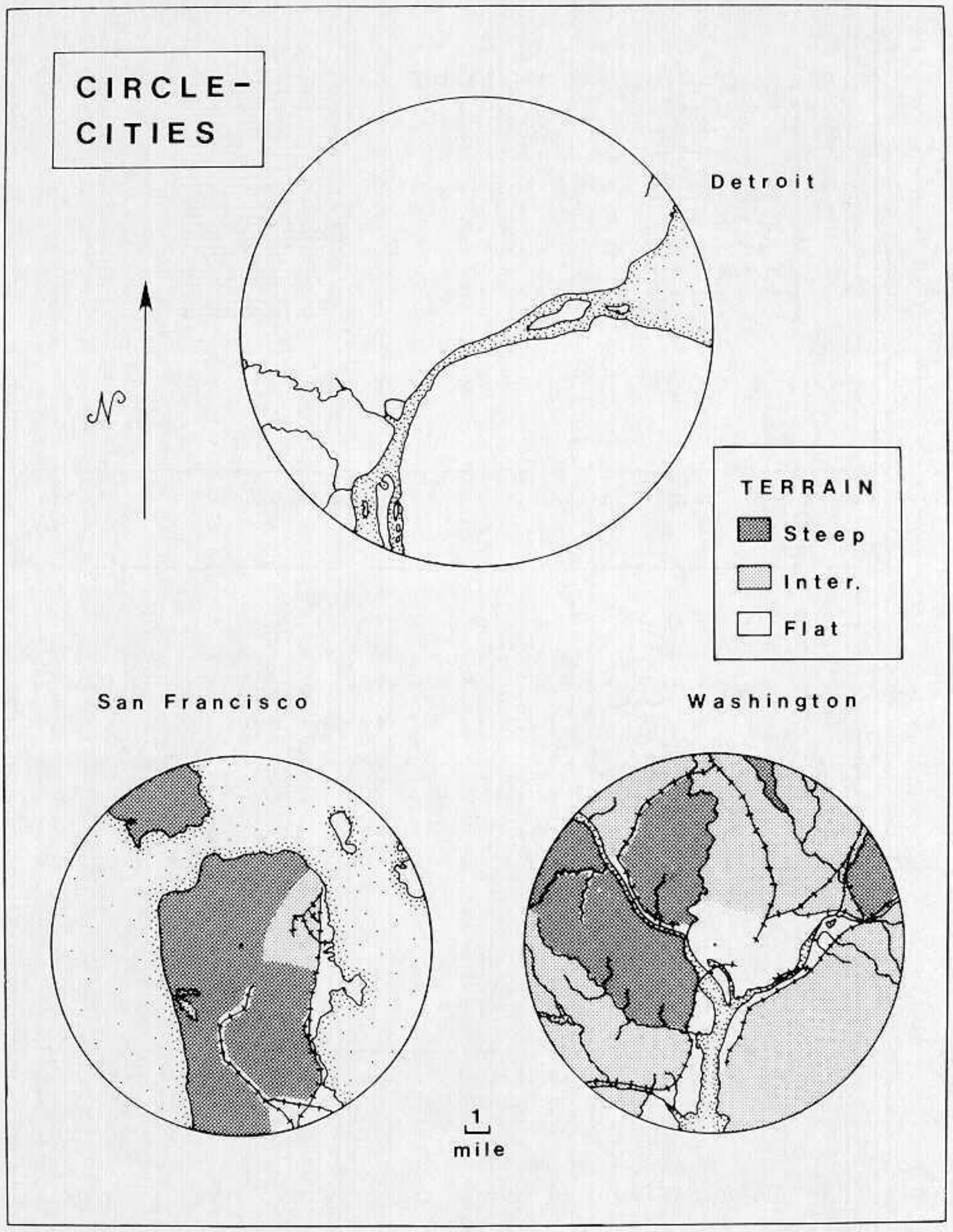

FIG. 3—Circle cities: Detroit, flat terrain (top); San Francisco, steep terrain (bottom left); Washington, D.C., steep terrain (bottom right). *Source:* Calculated by authors.

routes. Traffic engineers may adjust routes along favorable terrain, while demand for service may force buses to routes that have uncharacteristically steep gradients. The application of a taxonomy, based on small-scale topographic evidence, allows placement of an arbitrary city into one of the three general categories of steepness. This taxonomy is not sensitive to local variations in terrain along individual bus routes.

Ann Arbor, the site of a major state university, had a 1980 population of 107,966. The city has eighteen distinct bus routes to serve this population as well as the substantially fewer number of people in nearby Ypsilanti, which is approximately three miles away.[4] The bus garage is a facility located south of the central business district (CBD) in Ann Arbor. A central stop in each downtown serves as the terminus for all routes. Both CBDs are located approximately one-half mile from the Huron River. Except to the southeast, the CBD in Ann Arbor is higher in elevation than the zones immediately around it. Farther from the CBD, glacial features like moraines and hogbacks dominate the landscape and provide a generally rolling surface over which buses travel.[5]

The eighteen bus routes were mapped, and the average percentage of slope was calculated for each of them. The mechanics of this procedure involved determining total relief and length for each route. To obtain resolution of topography finer than that on a 1:250,000 scale, maps at a scale of 1:24,000 were used.[6] Measurement of total length, L, was straightforward. Using the map scale to determine lengths of the linear segments that compose the route, their sum is L. Measurement of total relief for a route, R, was more complicated. We counted the number, N, of contour lines crossed by a route; multiplied N by the value, C, of feet in the contour interval, and added to this total a small amount, E, sufficient to accommodate the ends of a route. Thus the equation $R = (N \times C) + E$. Then the quotient of total route relief divided by total route length gave an average percentage of slope, S, along a route as $S = (R/L) \times 100\%$. The percentage of slope, S, derived from topographic analysis for each of the eighteen routes, ranged from a low of 1 percent along route 10 outbound to a high of 3.4 percent along route 1 outbound (Table I). Not all routes differed in length of inbound and outbound paths.

At the citywide scale, the average of the values for percentage of slope along all routes was 2.039, and the average percentage of slope across the eighteen routes was 1.953 percent. As expected, both proportions are close to the 2 percent figure of the general classification and demonstrate the validity of its results. They suggest by implication the utility of performing additional test cases as empirical support for the general terrain classification.

These results were used to chart a vertical profile for each route at a scale of 1:24,000. Both inbound and outbound profiles along a route were included only if they appeared dissimilar (Fig. 4). The vertical scale of these profiles is 1 inch to 50 feet, and the horizontal scale is 1 inch to 2,000 feet. The

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[4] U.S. Bureau of the Census, County and City Data Book, 1983 (Washington, D.C.: Government Printing Office, 1983); U.S. Geological Survey, topographic maps 1:24,000.

[5] Route maps prepared by the Ann Arbor Transportation Authority.

[6] Maps from Ann Arbor Transportation Authority and the U.S. Geological Survey.

Table I—Topographic Analysis of Ann Arbor Bus Routes

ROUTE	CONTOUR CROSSINGS (N)	LENGTH (L) (in feet)	% SLOPE (S)[a]
1 in	41	14,000	3.0
out	73	21,800	3.4
2 in	84	42,000	2.0
out	82	40,600	2.0
3 in	119	56,400	2.0
out	119	56,400	2.0
4 in	82	44,000	1.8
out	82	44,000	1.8
5 in	77	56,600	1.3
out	77	56,600	1.3
6 in	92	61,000	1.5
out	92	61,000	1.5
7 in	159	64,800	2.5
out	159	64,800	2.5
8 in	32	14,600	2.3
out	28	11,400	2.5
9 in	28	13,800	2.1
out	28	13,800	2.1
10 in	42	19,600	1.1
out	64	31,600	1.0
11 in	34	13,200	1.3
out	57	13,000	2.2
12 in	109	56,200	2.0
out	109	56,200	2.0
13 in	56	20,000	2.9
out	52	18,600	2.9
14 in	59	20,000	3.0
out	50	17,400	2.9
15 in	55	25,400	2.2
out	52	24,800	2.1
16 in	24	17,200	1.5
out	24	17,200	1.5
17 in	35	16,000	2.3
out	35	16,000	2.3
18 in	29	19,000	1.3
out	29	19,000	1.3

[a] $S = (R/L) \times 100\%$.

corresponding vertical exaggeration of the profiles is 40 times that which appears in the landscape. These profiles appear to be quite bumpy; however, the general trend of some is a relatively smooth climb or drop toward the terminus in a CBD. Some of the topographic variation in the profiles is a consequence of landscape features like rivers or dales that cause a drop in elevation along a route. The remainder of the topographic variation arises from demand for bus service and the response of transit engineers to bus-stop placement as well as from various political, economic, and other pressures on the local transit authority.

A critical value along a bus-route vertical profile is a point around which the slope changes sign, that is, from uphill to downhill. To be called a critical value, this point must arise from the rivers or dales that account for shifts of elevation. Streams that appear on a 1:250,000 map produce critical values in the vertical profiles of bus routes crossing them, independent of the steepness of the underlying terrain. In intermediate and steep terrain, railroad lines on a map of the same scale may yield additional critical values in the

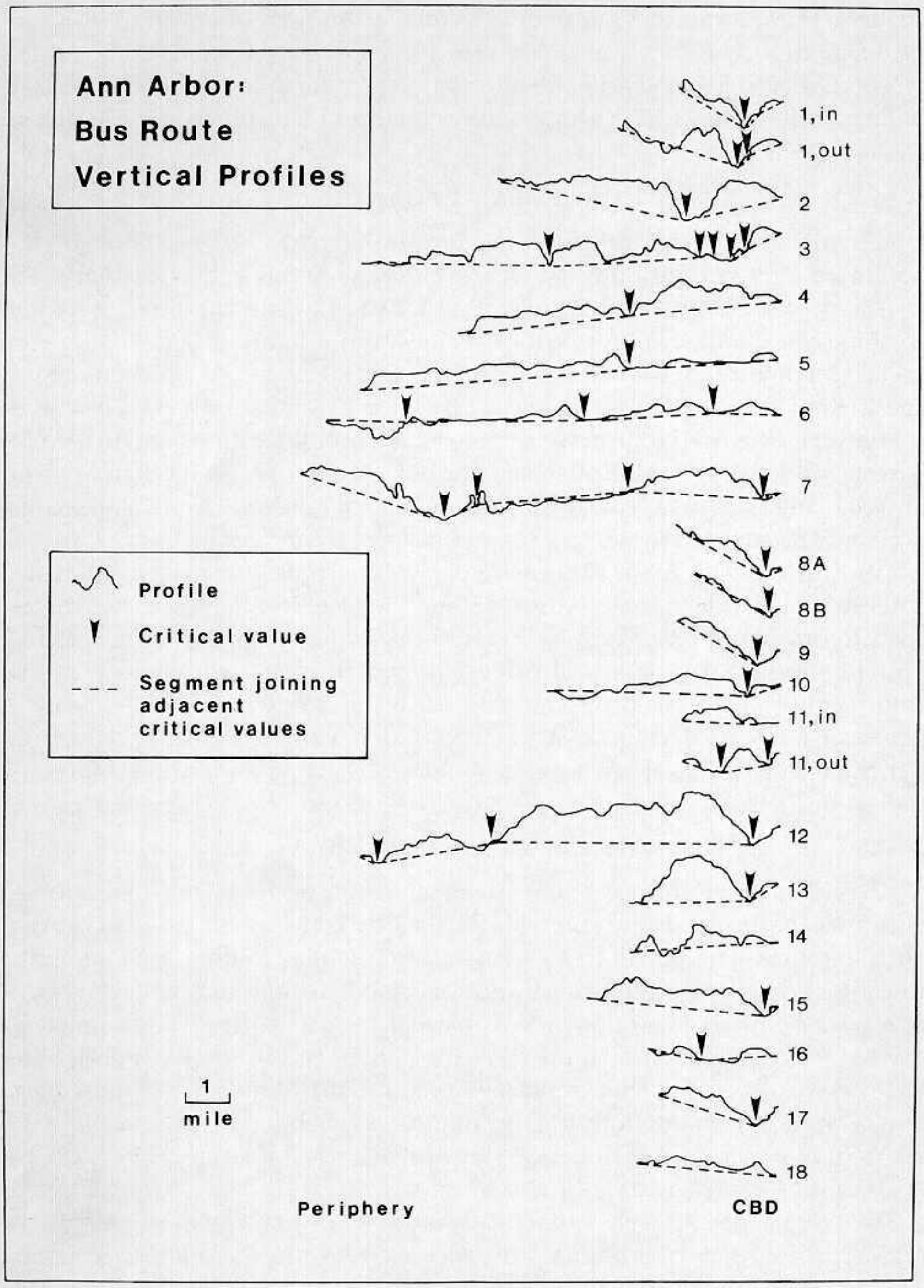

FIG. 4—Vertical profiles for Ann Arbor bus routes. *Source:* Calculated by authors from U.S. Geological Survey topographic maps 1:24,000.

vertical profiles of bus routes that cross them. The railroad lines paralleling rivers contribute little, while ones in valleys with no streams contribute much. We assume that railroad lines are map features that are sensitive to slope. In flat terrain the choice of a route can essentially be a straight line.

In steep terrain, railroad lines curve to follow the path of low grades in the direction of travel; slopes normal to the direction of travel tend to be much steeper. Curved railroad lines are often surrogates for dales that define critical values in the absence of streams wide enough to be displayed on the maps.

TERRAIN PARTITIONING OF INDIVIDUAL ROUTES

Identification of critical values as a means to partition variations in elevation prompted the notion of viewing the continuous intervals between critical values as domains that are well suited to a fine measurement of trends in these variations. Between critical values, route location depends on the social and political needs of various sectors of the population. Decisions about routing in these continuous intervals involve the notion of a service gradient that bridges the need to provide service to important locations and to areally spread residential zones. Reasons for responding to one demand and not to another might be based on historical evidence, gathered from patterns of accidents or congestion, or other issues of local importance.

With the topographic variation along a bus route partitioned into continuous intervals by a set of critical values, the objective here becomes threefold: to develop a procedure to measure bus-route steepness within an interval between adjacent critical values, to apply this procedure to actual bus routes, and to map bus-route intervals according to steepness to delineate a geography of city terrain at a local scale that was not possible with the general classification of city sectors bounded by rivers and railroad lines.

MEASUREMENT BETWEEN CRITICAL VALUES

To characterize the variation in elevation caused by rivers and dales, a route was partitioned into intervals marked by route ends and intervening critical values. The percentage of slope was calculated along each straight-line segment joining these values and was displayed graphically as the slope of a dashed-line segment that linked adjacent critical values. These segments, viewed in route sequence, approximate the shape of the corresponding bus-route vertical profile. They represent the amount of steepness, caused by critical values in the underlying terrain, that a bus must overcome along its route from origin to end, independent of demands for service. This pattern is a minimal representation of city terrain.

In a set of the eighteen actual bus routes, the percentage of slope of each segment linking critical values was calculated within the allometric circle. These slopes ranged in value from 0.0 percent to 2.9 percent, with most values near the intermediate level of 1.5–2 percent, as expected. These values measure absolute steepness and are directly comparable to the percentages used to classify the city as intermediate. The steepest intervals occurred along routes north of the CBD that led toward the river, along routes north of the river, along routes west of the CBD, and along routes in the CBD going west

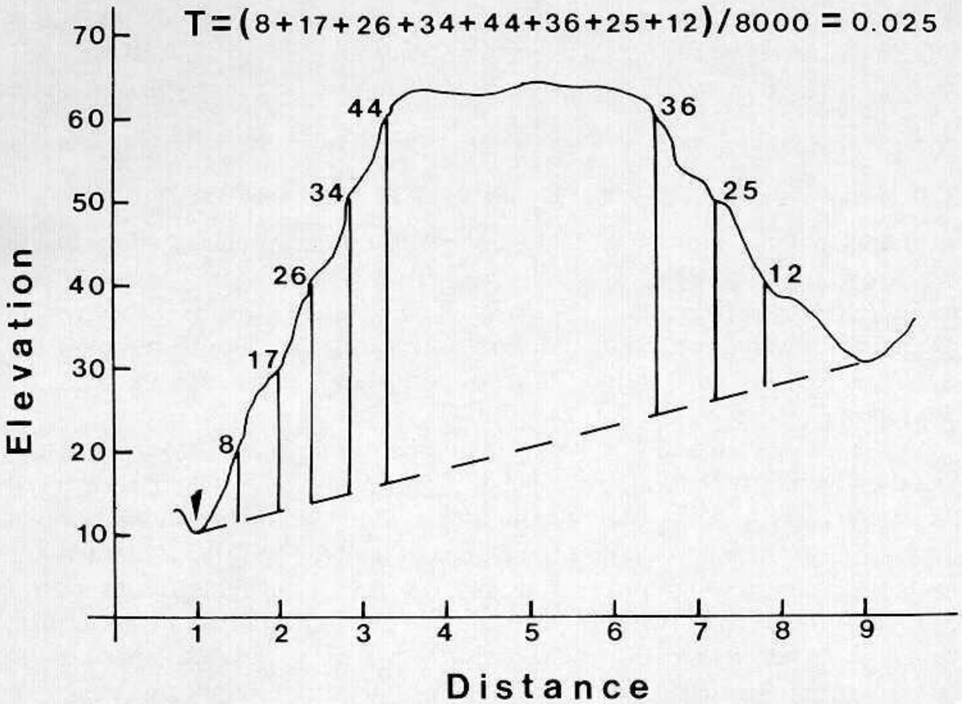

FIG. 5—Sample calculation of topographic variation T between adjacent critical values along a hypothetical bus-route vertical profile. The horizontal axis measures surface distance in thousands of feet between adjacent critical values. The vertical axis measures elevation in feet above some base, spaced to correspond to the contour spacing on suitable U.S. Geological Survey maps. Numerical values along the profile measure the displacement of the bus-route profile from the dashed line.

toward the railroad tracks. Routes to the southeast of the CBD were largely flat.

To characterize the variation in demands for service within bus-route intervals between adjacent critical values, a route was divided into a sequence of n intervals, I1, . . . , In with In in the CBD, marked by route ends and intervening critical values. Within a given interval, between adjacent critical values, D represents the vertical displacement of the actual vertical profile from the dashed line (Fig. 5). This displacement is measured at each point on the profile where a bus route crosses contour lines to produce i values of D, $D(1), \ldots, D(i)$ for each of i contour crossings. The topographic variation T between critical values may then be calculated as a displacement measure:

$$T = [D(1) + \ldots + D(i)] \div \text{(length of segment joining adjacent critical values)}.$$

This variation between critical values represents the topography that a bus must overcome as a result of the transit authority's response to passenger demands and to political pressures; it is one response to finding a service gradient across the urban surface.

Because contour crossings rather than an evenly spaced net of points were used as references for measuring displacement, steepness was reflected directly in T: steeper segments were sampled more frequently than smooth segments.

APPLICATION OF DISPLACEMENT MEASURE

For actual bus routes in Ann Arbor, the topographic variation T was measured along intervals, I, between adjacent critical values. When T was multiplied by 100 to facilitate comparisons, the value of this relative measure ranged from a high of 25.3 for the first interval, I1 of route 13 remote from the CBD, to a low of 0.4 along I2 of route 9. The pattern reflected directly the relative steepness along these route segments (Table II).

When these values of $T \times 100$ were mapped along generalized bus routes, based on actual route locations, a detailed geography of city terrain emerged (Fig. 6). When the rank-ordered values for $T \times 100$ were roughly partitioned into thirds, at the 8 percent and the 2 percent levels, one-third contained bus routes with steep topographic variation between critical values. Another third contained routes with intermediate topographic variation between critical values (between 2 percent [inclusive] and 8 percent [exclusive]), and the remaining one-third comprised bus routes with flat topographic variation between critical values.

The values for $T \times 100$ were mapped along generalized bus routes. Dotted lines represented flat intervals, dashed lines delineated intermediate intervals, and solid lines marked steep intervals. Bus routes were characterized abstractly as radial or circular within the allometric circle for Ann Arbor. Routes 1, 4, 5, 8, 9, 13, 14, 15, 16, and 17 were radial, while routes 2, 3, 6, 7, and 12 were circular. Routes 10, 11, and 18 were outside this circle and not included in the analysis. Along these intervals, relative steepness is a direct consequence of service demands on the local transit authority in routing policy rather than underlying terrain variation due to critical values.

When this relative steepness along intervals is evaluated in the context of the general notion that routes to the north and west have steep intervals and those to the southeast are mainly flat, several implications emerge for routing buses across Ann Arbor city terrain. Firstly, radial route intervals at the western edge of the CBD have minor displacement from the topographic baselevel linking critical values. These intervals have low values for $T \times 100$. However, the underlying terrain in this region is fairly steep. Variation in a profile cutting across it is forced largely by topographic features rather than by service-demand responses. This aspect suggested that the routing of buses across this terrain is effective in making the service gradient coincide with the terrain one. In other words, the routing is "good."

Secondly, route 7 has relatively little variation in elevation caused by critical values, but it displays high values for $T \times 100$ that result from response to service demands. Routes with this pattern experience more terrain stress

TABLE II—TOPOGRAPHIC VARIATION BETWEEN CRITICAL VALUES FROM TRANSIT-AUTHORITY DEMANDS

RANK ORDERING		SEQUENTIAL ORDERING	
$T \times 100$	R & I[a]	$T \times 100$	R & I[a]
25.3	I1 of R3	6.8	I1 of R1 in
21.8	I3 of R12	2.0	I2 of R1 in
15.1	I4 of R7	2.4	I3 of R1 in
14.0	I1 of R1 out	14.0	I1 of R1 out
12.8	I1 of R2	2.5	I2 of R1 out
11.5	I2 of R4	4.0	I3 of R1 out
11.3	I1 of R15	12.8	I1 of R2
10.3	I2 of R3	7.5	I2 of R2
9.6	I1 of R17	10.3	I2 of R3
9.4	I1 of R7	1.5	I3 of R3
8.2	I6 of R3	0.6	I4 of R3
8.0	I2 of R7	4.7	I5 of R3
7.7	I1 of R14	8.2	I6 of R3
7.5	I2 of R2	11.5	I2 of R4
6.8	I1 of R1 in	0.9	I2 of R5
5.3	I1 of R9	2.0	I2 of R6
4.9	I2 of R12	4.0	I3 of R6
4.7	I5 of R3	2.5	I4 of R6
4.0	I3 of R1 out	9.4	I1 of R7
4.0	I3 of R6	8.0	I2 of R7
3.8	I1 of R8A	3.3	I3 of R7
3.3	I3 of R7	15.1	I4 of R7
2.5	I2 of R1 out	0.8	I5 of R7
2.5	I4 of R6	3.8	I1 of R8A
2.4	I3 of R1 in	0.9	I2 of R8A
2.4	I1 of R16	1.7	I1 of R8B
2.0	I2 of R1 in	0.6	I2 of R8B
2.0	I2 of R6	5.3	I1 of R9
1.7	I1 of R8B	0.4	I2 of R9
1.5	I3 of R3	0.8	I1 of R12
1.4	I2 of R13	4.9	I2 of R12
1.2	I2 of R16	21.8	I3 of R12
1.0	I2 of R15	0.6	I4 of R12
0.9	I2 of R5	25.3	I1 of R13
0.9	I2 of R8A	1.4	I2 of R13
0.9	I2 of R17	7.7	I1 of R14
0.8	I5 of R7	11.3	I1 of R15
0.8	I1 of R12	1.0	I2 of R15
0.6	I4 of R3	2.4	I1 of R16
0.6	I2 of R8B	1.2	I2 of R16
0.6	I4 of R12	9.6	I1 of R17
0.4	I2 of R9	0.9	I2 of R17

[a] R denotes route number; I indicates interval number.

than required by critical values. Review of service demand, relative importance of locations that are served, and bus-stop positions in zones of dispersed housing might be in order. Issues of this sort do not appear to be a widespread concern of transit managers, except in cities like Seattle or San Francisco, where response has been to run trolley buses or cable cars on steep slopes. Current cuts in funding for the Urban Mass Transportation Administration might suggest that all transit managers should consider the long-term consequences for maintenance of sometimes subtle environmental factors that affect bus durability.

Route 1 experiences steep variations caused by both critical values and service demands. This consistency suggests that the quality of steepness

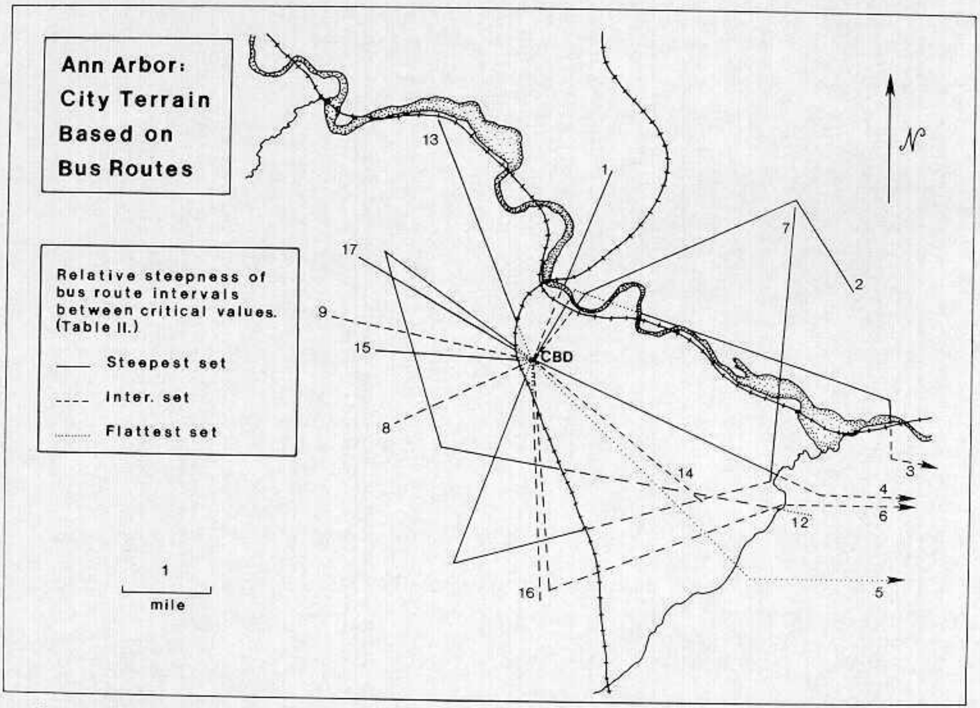

FIG. 6—City terrain based on Ann Arbor bus routes. *Source:* Values from Table II for classification of data as steep, intermediate, or flat.

might be invariant with a shift of scales. The abstract tools of self-similarity and of fractal geometry might be used to investigate steepness along such routes. Insofar as roadbed and water-table levels are a function of steepness, they might be applied locally to predict routes likely to become riddled with potholes.[7]

When values that measure the relative steepness of bus-route intervals between adjacent critical values are coupled with the general trend in terrain dictated by those values, a geography of city terrain is created to suggest which segments of the routes are effectively designed in terms of gradients and which are not. By combining these data with local economic and political issues, transit managers could produce policies that reflect spatial as well as other values.

CLOSING COMMENTS

The principal direct contribution of this study is a demonstration of how to extend a small-scale terrain classification (steep, intermediate, or flat) to a

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[7] B. Mandelbrot, The Fractal Geometry of Nature (San Francisco: W. H. Freeman, 1983); H.-O. Peitgen and P. H. Richter, The Beauty of Fractals: Images of Complex Dynamical Systems (Berlin: Springer-Verlag, 1985); Sandra Arlinghaus, Fractals Take a Central Place, *Geografiska Annaler* 67B (1985): 83–88; Michael Goodchild, The Fractional Brownian Process as a Terrain Simulation Model, *Modeling and Simulation* 13 (1982): 1133–1137.

large scale that is sensitive enough to include terrain variation along bus routes. Because a shift in scale is involved, use of the notion of self-similarity to represent the scale changes along bus routes outward from a CBD or to characterize scale changes with movement from the physiographic province inward to the city might result in terrain fractals.

The geography of city terrain based on Ann Arbor bus routes was crafted to provide insight about general problems in measuring variation of steepness that results from terrain and service demands. At the pragmatic level, implications for transit-authority budgets and local transit policy stem from maintenance associated with steepness of routes. At an abstract level, the ideas in this study might be applied to a broad group of cities. Use of an automatic counter to tabulate contour crossings and to create a data set to produce computer-generated maps could present a static portrait of city terrain based on bus routes, which in turn might lead to a study of the dynamics of bus-route changes.