EYE-CONTACT GRAPHS

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The structure of nonverbal communication expressed as eye-contact between two human beings is analyzed using graph-theoretic tools involving a theorem of König on bipartite graphs and various results concerning directed graphs (as in Harary). A taxonomy for possible eye-contact configurations is constructed; then a theory, formed from a sequence of theorems proved about classes of eye-contact graphs derived from the taxonomy, is interpreted to analyze possible levels of communication. This theory can apply to any living system but it is interpreted here with respect to human subsystems composed of (1) individuals with normal vision, and (2) individuals with vision disorders which lead to crossed eyes.

KEY WORDS: living systems, human subsystem, nonverbal communication, normal vision, directed graph, eye-contact graph, cross-eyed.

Nor do they trust their tongue alone,
But speak a language of their own;
Can read a nod, a shrug, a look,
Far better than a printed book;
Convey a libel in a frown
And wink a reputation down.

(Jonathan Swift, 1729)

FREQUENTLY, nonverbal communication is transmitted between two individuals via eye contact. Many factors influence the nature of that exchange in communication, but independent of these, there is always present an underlying structure that describes the pattern of that interaction from one eye to the other. The material below presents a structural taxonomy, based in graph theory, for all possible eye-contact configurations between two individuals. This classificatory structure will be used to generate theorems whose proofs can be interpreted to analyze the extent of communication exchanged between individuals with normal vision as well as between those with crossed eyes.

CLASSIFICATORY MATERIAL

Throughout this material, X will be an individual with left eye x and right eye x', and Y will be an individual with left eye y and right eye y'. The sequence of lemmas in this section, that follows directly from definitional material, leads to a taxonomy which will be used to generate results that are not as straightforward.

Definition 1

An eye-contact graph is a directed graph with vertex set $\{x, x', y, y'\}$ and edge set composed of directed edges (expressed as ordered pairs) chosen from the set of possible linkages among x, x', y, and y'. Two vertices are adjacent if and only if one of the eyes they represent is looking at the other; the direction assigned to the edge linking these vertices represents the direction of gaze from one eye to the other.

Thus in the pair (x, y), the vertex x is adjacent to the vertex y, and these vertices are linked by an edge representing the direction of gaze from eye x to eye y.

Definition 2

An adjacency matrix that represents an eye-contact graph is a matrix of 0's and 1's; a 1 (0) in the ith row and jth column indicates the presence (absence) of an edge from vertex i to vertex j in the graph (Harary, 1969, p. 202).

Definition 3

A bipartite directed graph G is a directed graph whose vertex set V may be partitioned into two subsets, V_1 and V_2, such that every edge in V links a vertex of V_1 to a vertex of V_2 (or of V_2 to V_1) (Harary, 1969, p. 17).

Lemma 1

An eye-contact graph, G, is a bipartite graph.

Proof:

The vertex set V, of G, is $V = \{x, x', y, y'\}$. Since each vertex represents an eye, the two heads of X and Y, that contain pairs of eyes, provide a natural partitioning of V into $V_1 = \{x, x'\}$, $V_2 = \{y, y'\}$. All edges in G must be from V_1 to V_2 (or from V_2 to V_1), for otherwise individual X (or Y) would be required to have his left eye look at his own right eye, or his own right eye look at his own right eye, or vice-versa. This is physically impossible assuming that X (or Y) does not use a mirror.

Corollary 1

In an eye-contact graph G with $V = \{x, x', y, y'\}$, the edges (x, x), (x', x'), (y, y), (y', y'), and (x, x'), (x', x), (y, y'), (y', y) may never appear.

Proof:

This is a direct consequence of Lemma 1.

Definition 4

The indegree of a vertex is a nonnegative integer representing the number of distinct directed edges coming to that vertex from other vertices; the outdegree of a vertex is a nonnegative integer representing the number of distinct directed edges leading from that vertex to other vertices (Harary, 1969, p. 198).

Lemma 2

In an eye-contact graph, the outdegree of any vertex is either 0 or 1.

Proof:

Without loss of generality, analyze the size of the outdegree of vertex x. By Definition 4, (outdegree x) $\geqq 0$.

a) If (outdegree x) $= 0$, then eye x is not looking at either eye of Y.
b) If (outdegree x) $= 1$, then eye x is looking at exactly one of Y's eyes.
c) If (outdegree x) > 1, then eye x is looking, simultaneously, at more than one of Y's eyes, which is physically impossible.

Corollary 2

In an adjacency matrix representing an eye-contact graph, the sum of the entries in any row is less than or equal to 1.

Proof:

This is a direct consequence of Lemma 2 and Definition 2.

Lemma 3

In an eye-contact graph the indegree of any vertex is at most 2.

Proof:

Without loss of generality, analyze the size of the indegree of vertex x. By Definition 4, (indegree x) $\geqq 0$.

a) If (indegree x) $= 0$, then neither eye of Y is looking at x.
b) If (indegree x) $= 1$, then one eye of Y is looking at x.
c) If (indegree x) $= 2$, then both eyes of Y are focused on x.
d) If (indegree x) > 2, then more than two eyes of Y are focused on x, which is not possible.

Corollary 3

In an adjacency matrix representing an eye-contact graph, the sum of the entries in any column is less than or equal to 2.

Proof:

This is a direct consequence of Lemma 3 and Definition 2. (If equality holds, both eyes of one individual are focused on a single eye in the other).

Lemma 4

An eye-contact graph has, at most, four directed edges.

Proof:

An eye-contact graph has four vertices (Definition 1) and each of these has outdegree, at most, 1 (by Lemma 2). Consequently, there can be no more than four distinct edges in this graph; these edges are directed in the manner indicated in Definition 1.

Corollary 4

An adjacency matrix representing an eye-contact graph has at most four nonzero rows.

Proof:

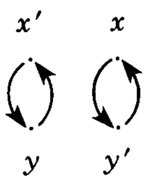
This is a direct consequence of Lemma 4.

These definitions, lemmas, and corollaries lead to the following taxonomy for all possible eye-contact configurations, represented both as graphs and adjacency matrices, between individuals *X* and Y.

Class I

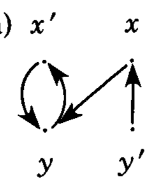
Assume *X* has normal vision (or corrected to normal). Also, assume that both eyes in *X* are engaged in eye-contact. One could exchange *Y* for *X*, or *y* for *x*, throughout to obtain dual configurations; these will not be shown (Harary, 1957, p. 259).

A) Assume *Y* has normal vision and that both eyes are engaged in eye-contact. By Lemma 4, the graphs representing eye-contact will have four edges.

i)


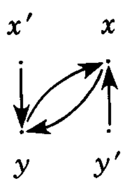
$$\begin{matrix} & x & x' & y & y' \\ x & 0 & 0 & 0 & 1 \\ x' & 0 & 0 & 1 & 0 \\ y & 0 & 1 & 0 & 0 \\ y' & 1 & 0 & 0 & 0 \end{matrix}$$

ii) *X* focuses on *y*.

a)


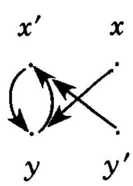
$$\begin{matrix} & x & x' & y & y' \\ x & 0 & 0 & 1 & 0 \\ x' & 0 & 0 & 1 & 0 \\ y & 0 & 1 & 0 & 0 \\ y' & 1 & 0 & 0 & 0 \end{matrix}$$

b) *Y* focuses on *x*.



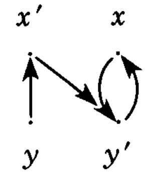
$$\begin{matrix} & x & x' & y & y' \\ x & 0 & 0 & 1 & 0 \\ x' & 0 & 0 & 1 & 0 \\ y & 1 & 0 & 0 & 0 \\ y' & 1 & 0 & 0 & 0 \end{matrix}$$

c) *Y* focuses on *x′*.



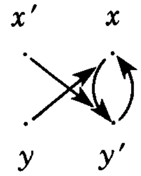
$$\begin{matrix} & x & x' & y & y' \\ x & 0 & 0 & 1 & 0 \\ x' & 0 & 0 & 1 & 0 \\ y & 0 & 1 & 0 & 0 \\ y' & 0 & 1 & 0 & 0 \end{matrix}$$

iii) *X* focuses on *y′*.

a)


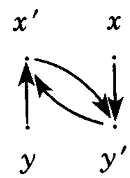
$$\begin{matrix} & x & x' & y & y' \\ x & 0 & 0 & 0 & 1 \\ x' & 0 & 0 & 0 & 1 \\ y & 0 & 1 & 0 & 0 \\ y' & 1 & 0 & 0 & 0 \end{matrix}$$

b) *Y* focuses on *x*.



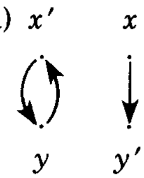
$$\begin{matrix} & x & x' & y & y' \\ x & 0 & 0 & 0 & 1 \\ x' & 0 & 0 & 0 & 1 \\ y & 1 & 0 & 0 & 0 \\ y' & 1 & 0 & 0 & 0 \end{matrix}$$

c) *Y* focuses on *x′*.

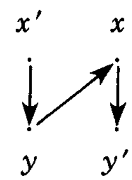


$$\begin{matrix} & x & x' & y & y' \\ x & 0 & 0 & 0 & 1 \\ x' & 0 & 0 & 0 & 0 \\ y & 0 & 1 & 0 & 0 \\ y' & 0 & 1 & 0 & 0 \end{matrix}$$

B) Suppose that *Y* has only one eye engaged in eye-contact. The graphs representing eye-contact configurations will have exactly three edges.

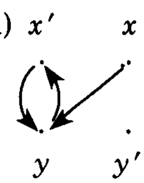
i) a)


$$\begin{matrix} & x & x' & y & y' \\ x & 0 & 0 & 1 & 0 \\ x' & 0 & 0 & 0 & 1 \\ y & 0 & 1 & 0 & 0 \\ y' & 0 & 0 & 0 & 0 \end{matrix}$$

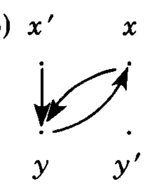
b)


$$\begin{matrix} & x & x' & y & y' \\ x & 0 & 0 & 0 & 1 \\ x' & 0 & 0 & 1 & 0 \\ y & 1 & 0 & 0 & 0 \\ y' & 0 & 0 & 0 & 0 \end{matrix}$$

ii) *X* focuses on *y*.

a)


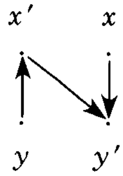
$$\begin{matrix} & x & x' & y & y' \\ x & 0 & 0 & 1 & 0 \\ x' & 0 & 0 & 1 & 0 \\ y & 0 & 1 & 0 & 0 \\ y' & 0 & 0 & 0 & 0 \end{matrix}$$

b)


$$\begin{matrix} & x & x' & y & y' \\ x & 0 & 0 & 1 & 0 \\ x' & 0 & 0 & 1 & 0 \\ y & 1 & 0 & 0 & 0 \\ y' & 0 & 0 & 0 & 0 \end{matrix}$$

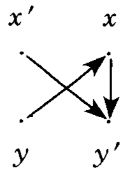
iii) X focuses on y'.

a)



$$\begin{matrix} & x & x' & y & y' \\ x & 0 & 0 & 0 & 1 \\ x' & 0 & 0 & 0 & 1 \\ y & 0 & 1 & 0 & 0 \\ y' & 0 & 0 & 0 & 0 \end{matrix}$$

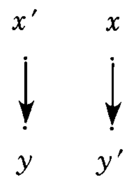
b)



$$\begin{matrix} & x & x' & y & y' \\ x & 0 & 0 & 0 & 1 \\ x' & 0 & 0 & 0 & 1 \\ y & 1 & 0 & 0 & 0 \\ y' & 0 & 0 & 0 & 0 \end{matrix}$$

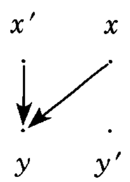
C) Suppose that Y has no eyes engaged in eye-contact. The graphs representing eye-contact configurations will have exactly two edges.

i)



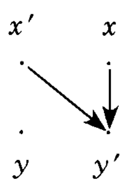
$$\begin{matrix} & x & x' & y & y' \\ x & 0 & 0 & 0 & 1 \\ x' & 0 & 0 & 1 & 0 \\ y & 0 & 0 & 0 & 0 \\ y' & 0 & 0 & 0 & 0 \end{matrix}$$

ii) X focuses on y.



$$\begin{matrix} & x & x' & y & y' \\ x & 0 & 0 & 1 & 0 \\ x' & 0 & 0 & 1 & 0 \\ y & 0 & 0 & 0 & 0 \\ y' & 0 & 0 & 0 & 0 \end{matrix}$$

iii) X focuses on y'



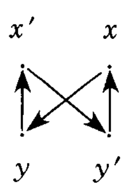
$$\begin{matrix} & x & x' & y & y' \\ x & 0 & 0 & 0 & 1 \\ x' & 0 & 0 & 0 & 1 \\ y & 0 & 0 & 0 & 0 \\ y' & 0 & 0 & 0 & 0 \end{matrix}$$

Class II

Assume X is cross-eyed and that both eyes in X are engaged in eye-contact. (Duals will not be shown.)

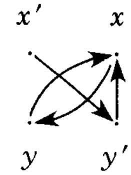
A) Assume Y has normal vision and that both eyes are engaged in eye-contact. The graphs representing eye-contact configurations will have four edges.

i)



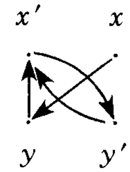
$$\begin{matrix} & x & x' & y & y' \\ x & 0 & 0 & 1 & 0 \\ x' & 0 & 0 & 0 & 1 \\ y & 0 & 1 & 0 & 0 \\ y' & 1 & 0 & 0 & 0 \end{matrix}$$

ii)



$$\begin{matrix} & x & x' & y & y' \\ x & 0 & 0 & 1 & 0 \\ x' & 0 & 0 & 0 & 1 \\ y & 1 & 0 & 0 & 0 \\ y' & 1 & 0 & 0 & 0 \end{matrix}$$

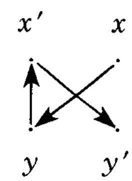
iii)



$$\begin{matrix} & x & x' & y & y' \\ x & 0 & 0 & 1 & 0 \\ x' & 0 & 0 & 0 & 1 \\ y & 0 & 1 & 0 & 0 \\ y' & 0 & 1 & 0 & 0 \end{matrix}$$

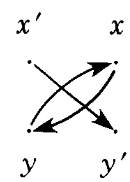
B) Suppose that Y has only one eye engaged in eye-contact. The graphs representing eye-contact configurations will have exactly three edges.

i)



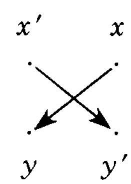
$$\begin{matrix} & x & x' & y & y' \\ x & 0 & 0 & 1 & 0 \\ x' & 0 & 0 & 0 & 1 \\ y & 0 & 1 & 0 & 0 \\ y' & 0 & 0 & 0 & 0 \end{matrix}$$

ii)



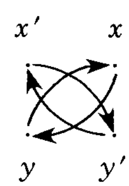
$$\begin{matrix} & x & x' & y & y' \\ x & 0 & 0 & 1 & 0 \\ x' & 0 & 0 & 0 & 1 \\ y & 1 & 0 & 0 & 0 \\ y' & 0 & 0 & 0 & 0 \end{matrix}$$

C) Suppose that Y has no eyes engaged in eye-contact; the graphs representing eye-contact configurations will have exactly two edges.



$$\begin{matrix} & x & x' & y & y' \\ x & 0 & 0 & 1 & 0 \\ x' & 0 & 0 & 0 & 1 \\ y & 0 & 0 & 0 & 0 \\ y' & 0 & 0 & 0 & 0 \end{matrix}$$

D) Suppose that Y is also cross-eyed.



$$\begin{matrix} & x & x' & y & y' \\ x & 0 & 0 & 1 & 0 \\ x' & 0 & 0 & 0 & 1 \\ y & 1 & 0 & 0 & 0 \\ y' & 0 & 1 & 0 & 0 \end{matrix}$$

Class III

This includes all trivial cases, such as no eye-contact on the part of either X or Y, no contact from Y and contact from one eye of X only, and contact from one eye for each of X and Y.

THEOREMS ABOUT EYE-CONTACT GRAPHS

Definition 5

An n-cycle ($n \geq 2$) in a directed graph is an alternating sequence of n vertices and n-edges, v_0, (v_0, v_1), v_1, (v_1, v_2), $\cdots$, v_{n-1}, (v_{n-1}, v_n), v_n, in which all vertices are distinct except the first and last (Harary, 1969, p. 198).

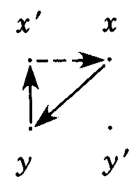
Theorem 1

A cycle in an eye-contact graph has either two or four edges.

Proof:

This is a direct consequence of Lemma 1 and of a theorem of König that a graph is bipartite if and only if all of its cycles are even (Harary, 1969, p. 17). A constructive proof, expressed in terms of eye-contact graphs, is exhibited below.

By Definition 5, any cycle has at least two edges. Since there are at most four edges in an eye-contact graph (Lemma 4), it follows that a cycle could have 2, 3, or 4 edges (a cycle with 2 edges is exhibited in I.A.i and one with four edges in II.A.i). So it remains to determine if a cycle of three edges can exist in an eye-contact graph. Suppose, without loss of generality, that x is adjacent to y. The sequence in the 3-cycle under construction begins x, (x, y), $y \cdots$



and must continue by linking y to x' (a 2-cycle is formed if y is linked to x, and Lemma 1 is violated if y is linked to y'). Thus, the sequence in the 3-cycle reads x, (x, y), y, (y, x'), x', $\cdots$ and to force the sequence to represent a cycle with three edges, it must be completed, to satisfy Definition 5, as x, (x, y), y, (y, x'), x', (x', x), x. But this contradicts Corollary 1 and so no 3-cycle may exist in an eye-contact graph. Q.E.D.

Theorem 2

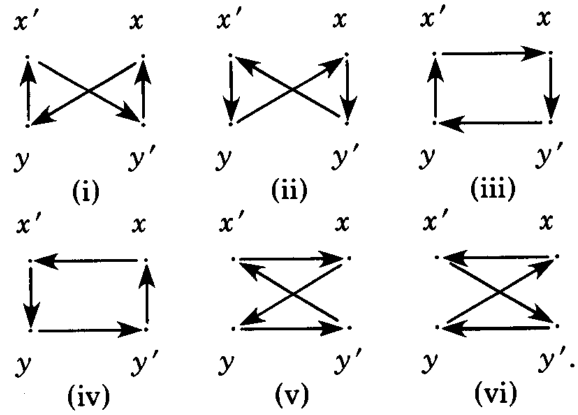
In an eye-contact graph,

a) If a 4-cycle is present, then exactly one of X and Y is cross-eyed.
b) If 4 edges are present and neither X nor Y has crossed eyes, then the only cycles that may exist are 2-cycles.
c) If both X and Y have crossed eyes, only 2-cycles exist.

Proof:

This is a direct consequence of the taxonomy and the proof of Theorem 1; enumeration of possibilities will yield the results.

a) All possible 4-cycles are enumerated below:



In (i), X is cross-eyed and in (ii), Y is cross-eyed. Cases (iii), (iv), (v), and (vi) violate Lemma 1 and so are not eye-contact graphs. Thus (a) follows.

b) Since neither X nor Y has crossed-eyes, the only 4 cycles that can be created are those in (iii), (iv), (v), and (vi); but these are not eye-contact graphs. So there can be no 4-cycle in this case. Theorem 1 insures that there can be no 3-cycle in this case. Thus the only cycles that may exist are 2-cycles, and the result follows.
c) In this case, there is only one possible eye-contact graph, (Class II.D) and it has two 2-cycles. Q.E.D.

The following assumption will permit further interpretation of eye-contact graphs, and is based on the idea that an eye is both a transmitter and a receiver in any eye-contact exchange.

Basic Assumption

From a structural viewpoint, the size of the indegree (outdegree) of a vertex represents directly the extent of nonverbal communication coming to (from) the eye which that vertex represents.

The condition set forth in Definition 6 is suggested by the basic assumption, and is consistent with the "concept of equal exchange" set forth by Berger and Snell (1957, p. 114).

Definition 6

The interchange in communication, resulting from eye-contact between X and Y, is maximal (structurally) if for every vertex v in the eye-contact graph representing the exchange,

$$\text{indegree } v = \text{outdegree } v.$$

Definition 7

The row rank of an adjacency matrix representing an eye-contact graph is the number of distinct, nonzero, rows in that matrix.

Theorem 3

In an eye-contact graph with four edges, there is maximal interchange of communication between X and Y if the row rank of the adjacency matrix is 4.

Proof:

The row rank of the adjacency matrix is 4, by hypothesis. By Corollary 2 and the hypothesis, each row contains exactly one entry of 1. Thus the sum of the entries in each row is 1 and so the outdegree of each vertex is 1 (Lemma 2). Since the four rows are distinct, by Definition 7, each column contains exactly one entry of 1. Thus the sum of the entries in each column is 1 and so the indegree of each vertex is 1 (Lemma 3). Therefore, indegree v = outdegree v for every vertex v, and so, by Definition 6, there is maximal interchange of communication between X and Y. Q.E.D.

Theorem 4

In an eye-contact graph with four edges, there is nonmaximal interchange of communication between X and Y if the row rank of the adjacency matrix is less than 4.

Proof:

a) Suppose the row rank is 3.

By Corollary 2 and the hypothesis, each row contains exactly one entry of 1. Thus the sum of the entries in each row is 1 and so the outdegree of each vertex is 1 (Lemma 2). There are three distinct rows, by Definition 7. By hypothesis all four rows are nonzero, so two rows must be identical. Thus one column contains two entries of 1, another contains only entries of 0, and the remaining two each contain a single entry of 1. Thus the sums of the entries in the columns are 2, 0, and 1, respectively. Therefore the indegrees of the vertices are 2, 0, and 1 (Lemma 3). Thus indegree $v \neq$ outdegree v for two vertices v and there is nonmaximal interchange of communication between X and Y (Definition 6).

b) Suppose the row rank is 2.

As above, the outdegree of each vertex is 1. There are two distinct rows, by Definition 7. By hypothesis, all four rows are nonzero, so there must be two pairs of identical rows or three rows the same (the latter is impossible for then one column would contain three 1's in violation of Corollary 3). So, there are two pairs of identical rows, generating two columns each with two entries of 1, and two columns each with only 0 entries. The indegree of two vertices is 2, and of the remaining two, is 0. Thus degree $v \neq$ outdegree v, for all v. By Definition 6, there is nonmaximal exchange between X and Y.

c) The row rank cannot be 1.

If it were, all rows would be the same, generating a column with four entries of 1 in violation of Corollary 3. Q.E.D.

Combining Theorems 3, 4, and Corollary 4 produces:

Theorem 5

In an eye-contact graph with four edges, there is maximal interchange of communication between X and Y if and only if the row rank of the adjacency matrix is 4.

Definition 8

Given an eye-contact graph, and its adjacency matrix, that represent an exchange of communication between X and Y; the interchange will be called

a) unbalanced, if the row rank is 3
b) balanced, if the row rank is 2 or 4.

This is distinct from, but not unrelated to, Harary, Norman, and Cartwright's use of balance in signed graphs; in the case of eye-contact graphs, all cycles of the signed digraph associated with the underlying digraph would be positive so that strict use of their notion of balance would not distinguish situations in which exchange of eye-contact is uneven (Harary, Normal, & Cartwright, 1965, p. 341).

Corollary 5

In an eye-contact graph with four edges,

a) indegree v − outdegree v = 0 for all vertices v if and only if there is maximal (balanced) exchange of communication.
b) $|\text{indegree } v - \text{outdegree } v|$

$$= \begin{cases} 0, & \text{for two vertices} \\ 1, & \text{for the other two vertices} \end{cases}$$

if and only if there is an unbalanced (nonmaximal) interchange of communication.
c) $|\text{indegree } v - \text{outdegree } v| = 1$ for all vertices if and only if there is balanced nonmaximal interchange of communication.

Proof:

The proof of (a) is a consequence of Theorem 5. The proofs of (b) and (c) are consequences of the proof of Theorem 4 and of Definition 8.

Examples of each type of interchange are represented, in graph and matrix form, in the taxonomy:

a) Graphs exhibiting maximal (balanced) exchange: I.A.i, II.A.i, II.D.
b) Graphs exhibiting (nonmaximal) unbalanced exchange: I.A.ii.a, I.A.iii.a, II.A.ii, II.A.iii.
c) graphs exhibiting nonmaximal balanced exchange: I.A.ii.b, I.A.ii.c, I.A.iii.b, I.A.iii.c.

This exhausts all cases with four edges.

Definition 9

Using the Basic Assumption, a cycle in an eye-contact graph will be considered to represent direct feedback in nonverbal communication between X and Y.

Definition 10

A sink (source) of eye-contact is a vertex in an eye-contact graph which has only edges leading to (from) it (Harary, 1969, p. 201).

Theorem 6

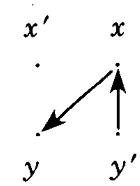
In an unbalanced (nonmaximal) interchange, delete all direct feedback (cycles). Then the individual who receives the larger share of the remaining communication has the eye which is a sink.

Proof:

That an eye which is a sink receives a larger share of the remaining communication than does an eye which is not, is a consequence of the basic assumption. It remains to show that there is exactly one such eye in any graph that satisfies the hypothesis. Only I.A.ii.a, I.A.iii.a, II.A.ii, and II.A.iii satisfy the hypothesis.

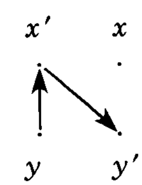
Analyze each graph which satisfies the hypothesis.

a) Delete the cycle in graph I.A.ii.a.



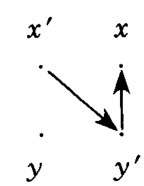
Then y is the only sink by Definition 10. So Y receives the larger share of the remaining communication.

b) Delete the cycle in graph I.A.iii.a.



Then y' is the only sink by Definition 10. So Y receives the larger share of the remaining communication.

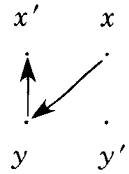
c) Delete the cycle in graph II.A.ii.



Then x is the only sink by Definition 10. So

X receives the larger share of the remaining communication.

d) Delete the cycle in graph II.A.iii.



Then x' is the only sink by Definition 10. So X receives the larger share of the remaining communication. Q.E.D.

Corollary 6

The sink in an eye-contact graph satisfying the hypotheses of Theorem 6 is the only vertex v in the graph such that indegree v − outdegree $v = 1$.

Proof:

From the proof of Theorem 6,

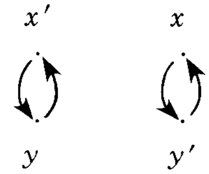
indegree of sink − outdegree of sink = 1 in all cases

indegree of source − outdegree of source = −1 in all cases

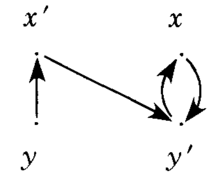
indegree of another vertex − outdegree of another vertex = 0 in all cases.

INTERPRETATIONS

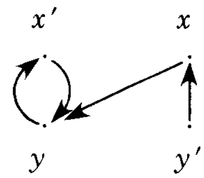
The following examples represent the types of inference, concerning the structural characteristics of eye-contact, that may be made as a consequence of the taxonomy and of the sequence of theorems based on it. If both individuals have normal vision (class I.A in the taxonomy), directness in feedback, fullness of contact, or openness between the individuals is maximized through creating two cycles (Definitions 6 and 9) by focusing x' on y, y on x' and x on y', y' on x (as in I.A.i). Since this is the only configuration in Class I.A whose row rank is four, a shift of the eyes, on the part of X, forces the situation from one of maximal interchange to one of nonmaximal interchange between X and Y (by Theorem 5). Reasons for X to shift away from the maximal position could include discomfort in open contact with someone to whom he is lying, or from whom he is hiding something. For whatever reason, suppose that Y maintains the steady gaze of I.A.i (with y on x' and y' on x) while X shifts his eyes to focus them both on only one of Y's eyes (as in I.A.ii.a or I.A.iii.a).



I.A.i

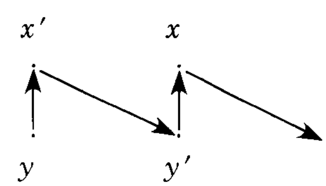


I.A.ii.a



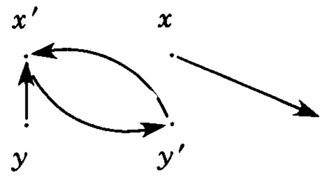
I.A.iii.a

If X wishes to retain complete eye-contact with Y (i.e., with four edges present), in order to keep the appearance of willingness to engage openly with Y in eye-contact, then these are the only two configurations that can result. Both represent unbalanced nonmaximal interchange of communication (Corollary 5), but the larger share of the communication outside the cycle structure belongs to Y since indegree y' − outdegree $y' = 1$ (in I.A.ii.a) and indegree y − outdegree $y = 1$ (in I.A.iii.a) by Theorem 6 and Corollary 6. So, as long as Y maintains his steady gaze of I.A.i, X may not, by shifting his eyes, both retain complete eye-contact and maneuver the configuration to a position in which he receives the larger share of communication. If X is willing to sacrifice complete eye-contact with Y, then he may tip the balance to receive the larger share in communication by focusing x' on y' and x on Y's right ear (or elsewhere) as in I.B.i.b.



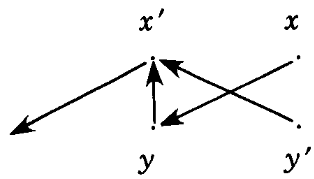
I.B.i.b

In this case, x is a sink since indegree x − outdegree $x = 1$ (Corollary 6), and so X receives the larger share in communication. If Y senses this strategy on the part of X (which seems likely unless X has close-set, deep-set, small, bespectacled eyes) he may shift his eyes to create a cycle by focusing on x' (I.B.iii.a).



I.B.iii.a

For X to restore his position, he must break this newly formed cycle that provides Y with some feedback. This requires a shift so that x looks at y, and x' looks at Y's left ear (I.B.iii.b).

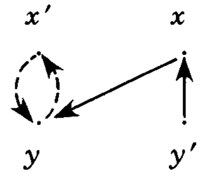


I.B.iii.b

Presumably Y would notice a shift of X's eyes as wide as this, and would respond by once again creating a cycle; a sequence of shifts of this sort could be carried on indefinitely. But by now, consistent with what we have learned about "shifty" eyes, Y should suspect that X has something to hide (Harary, 1982).

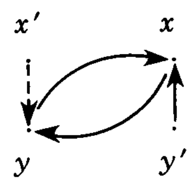
Arguments of this sort, involving shiftiness of gaze on the part of one individual may all be analyzed using Theorems 5 and 6, their corollaries, and the taxonomy, independent of the original configuration representing initial eye-contact.

Another strategy that could be employed by X in modifying the structure of an initial eye-contact configuration is to wink (a wink is a deliberate action, distinct from a blink which is involuntary). A wink may be friendly, or it may suggest something mysterious. From an initial position of eye-contact such as I.A.ii.a, a wink of x' temporarily breaks complete eye-contact; in this case Y has an advantage for an instant since indegree y − outdegree $y = 1$ (Theorem 6 and Corollary 6).



I.A.ii.a

By winking, X has deliberately increased his vulnerability in terms of eye-contact exchange, and so this configuration could be viewed as one that represents trust and friendliness. (Vulnerability, as used here, cannot be aligned with Harary's use of the word, since no eye-contact graph is a complete graph [Harary, Norman, and Cartwright, 1965, p. 194]). On the other hand, from a position such as I.A.ii.b, a wink of x' creates a temporary advantage for X, since indegree x − outdegree $x = 1$, and this strategy could be used by X to dodge contact during a small part of a conversation as well as to receive more communication (from both y and y') than is given (from x).



I.A.ii.b

Still a third method of forcing change in eye-contact structure is by changing the distance between X and Y. Since vision is binocular (in Class I.A), moving Y very close to X would force a break in the linkages of I.A.i (for example). If X and Y are sufficiently close, each would be forced to focus on a single eye in the other and so only positions I.A.ii.b, I.A.ii.c, I.A.iii.b, and I.A.iii.c would be possible.

Finally, if one individual has crossed eyes and the other does not, maximal interchange occurs in configuration II.A.i via the 4-cycle present in that configuration (Definition 5). Since this configuration is impossible between two individuals with normal eyesight, by Theorem 2, this provides a structural reason for the sense of discomfort felt by individuals with normal vision when engaging in eye-contact with a cross-eyed individual. From this position, the person who is not cross-eyed may restore comfort in eye-contact by shifting his gaze to a single eye in the cross-eyed person (II.A.ii or II.A.iii). He does so, however, at the expense of retaining balance in the exchange. For, in II.A.ii, indegree x—outdegree $x = 1$ and so X receives the larger share of communication in this unbalanced exchange (Theorem 6, Corollary 6). Alternately, the person who is not cross-eyed may keep a balanced configuration by crossing his own eyes, restoring the eye-contact to II.D, but probably the discomfort is not worth it.

DIRECTIONS FOR FURTHER RESEARCH

Extensions in the application of these theorems to eye-contact systems with more than two individuals could suggest strategy for dealing with crowds, both in terms of controlling contagious effects such as panic transmitted through a sequence of individuals (Rashevsky, 1951, p. 81), as well as in dispersing academic or social material from a point-source to a region of points, as a professor to a class, singer to an audience, or a clergyman to a congregation.

Further, a theory of eye-contact for species that do not possess binocular vision could be constructed parallel to the one for humans. Coupling of these might yield insight into the role eye-contact plays in determining the shapes of territorial boundaries associated with various species (Ardrey, 1966, pp. 131, 154; Neutra, 1969, p. 314).

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(Manuscript received October 30, 1983)